QUANTUM DISCORD
IN
MACROSCOPIC SYSTEMS

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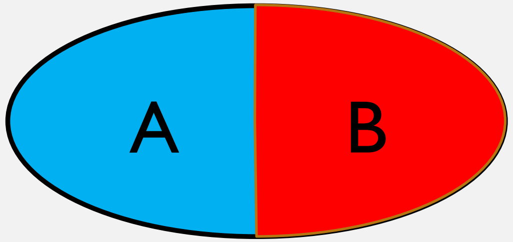
QUANTUM DISCORD
IN
MACROSCOPIC SYSTEMS

How to detect quantumness in solid-state experiments?

- Mixed state
- Many-body state
- Entanglement or something else?

PURE STATES



Pure state: $\rho = |\psi\rangle \langle\psi|$

Quantum or classical correlations between partitions

Product states $|\psi\rangle = |a\rangle \otimes |b\rangle$

Reduced (marginal)
state (density matrix)

$$\rho_A = Tr_B \, \rho = |a\rangle \langle a|$$

Entanglement (by def)
= marginal entropy

$$E(\rho) = S(\rho_A) = S(\rho_B) = 0$$

No correlations at all

PURE STATES
AND MEASURES OF THEIR QUANTUMNESS

Non-product states $$|\psi\rangle = \sum_i \sqrt{p_i}\,|a_i\rangle \otimes |b_i\rangle$$

Entanglement (def) = marginal entropy

$$E(\rho) = S(\rho_A) = S(\rho_B) = -\sum_i p_i \ln p_i \geq 0$$

Product:
no correlations

Non-product:
= correlations
= quantumness
= entanglement

Observation of any type of correlation – not an issue

MIXED STATES

Entanglement cannot be created by Local Operations and Classical Communications

Separable states can be created by LOCC (= non-entangled)

$$\rho = \sum_i p_i \rho_i^A \otimes \rho_i^B$$

… but they are correlated states. ➔ Entanglement is not everything

Question: Correlations must be classical since no entanglement?

⇩

Quantum discord

QUANTUM DISCORD

Mutual information

$$I(\rho) = S(\rho_B) + S(\rho_A) - S(\rho)$$

Projective measurements:

$$\rho \rightarrow \rho' = \sum_i \Pi_i^A \, \rho \, \Pi_i^A$$

Mutual information (post-measurement):

$$I(\rho') = S(\rho'_B) + S(\rho'_A) - S(\rho')$$

$$\boxed{D = \min_{\Pi} \left[I(\rho) - I(\rho') \right]}$$

Discord: minimal loss of correlations after measurement

DISCORD OF PURE STATE

Pre-measurement (pure)

$$|\psi\rangle = \sum_i \lambda_i \; |a_i\rangle \otimes |b_i\rangle$$

Post-measurement (mixed)

$$\rho' = \sum_a p_a \, |a\rangle \langle a| \otimes |b_a\rangle \langle b_a|$$

Discord

$$D = \min_{\{b_a\}} \sum_a p_a S(\cancelto{0}{|b_a\rangle \langle b_a|}) - \cancelto{0}{S(\rho)} + S(\rho_B)$$

$$= S(\rho_B)$$ = Entanglement entropy

Discord = entanglement (pure state)

DISCORD: SEPARABLE STATE

$$\rho = \sum_i p_i \rho_i^A \otimes \rho_i^B$$

$$D(\rho) \neq 0$$

Generically
(quantum correlations)

$$D(\rho) = 0$$

$$\rho_i^A = \Pi_i \qquad \Pi_i \, \Pi_j = \delta_{ij} \, \Pi_i$$

Classical correlations only

Examples:

$$\rho_{non-dis} = \boxed{|\uparrow\rangle \langle\uparrow|} \otimes |\rightarrow\rangle \langle\rightarrow| + \boxed{|\downarrow\rangle \langle\downarrow|} \otimes |\leftarrow\rangle \langle\leftarrow|$$

$$\rho_{dis} = \boxed{|\uparrow\rangle \langle\uparrow|} \otimes |\rightarrow\rangle \langle\rightarrow| + \boxed{|\rightarrow\rangle \langle\rightarrow|} \otimes |\leftarrow\rangle \langle\leftarrow|$$

CAN WE MEASURE DISCORD?

❖ Discord is a non-linear function of a density matrix
(requires a full quantum state tomography
+optimization over ALL measurements)

❖ Unachievable for a solid-state setup
(too many degrees of freedom)

❖ Only correlation functions are measurable (linear in a density matrix)

Question: can we experimentally measure quantumness of a mixed state?

PROTOCOL: OBSERVATION OF DISCORD MEASURING CORRELATION FUNCTION

0. State preparation: $\hat{\rho}_{AB} = \sum_i p_i \hat{\rho}_i^A \otimes \hat{\rho}_i^B$

1. 'Controlled' unitary evolution:

$$\hat{U}_A = \hat{U}_A(\alpha) \quad \hat{U}_B = \hat{U}_B(\beta) \qquad \hat{\rho}_{AB}^U = (\hat{U}_A \otimes \hat{U}_B)\, \hat{\rho}_{AB}\, (\hat{U}_A^\dagger \otimes \hat{U}_B^\dagger)$$

2. Measure cross-correlation function: ('joint probability distribution') $p(\alpha, \beta) = \mathrm{Tr} \hat{\rho}_{AB}^U \hat{\Pi}_A \otimes \hat{\Pi}_B$

3. Add 'coherence' factor $\hat{U}_A = e^{i\phi\hat{\sigma}_3}\hat{U}(\alpha)$

$$\hat{\rho}_{A|B} = \sum_i w_i(B)\hat{\rho}_i^A$$

Conditional density matrix

$$p(\alpha, \beta) = \mathrm{Tr} e^{i\phi\hat{\sigma}_3}\hat{U}(\alpha)\hat{\rho}_{A|B}\hat{U}^\dagger(\alpha)e^{-i\phi\hat{\sigma}_3}$$

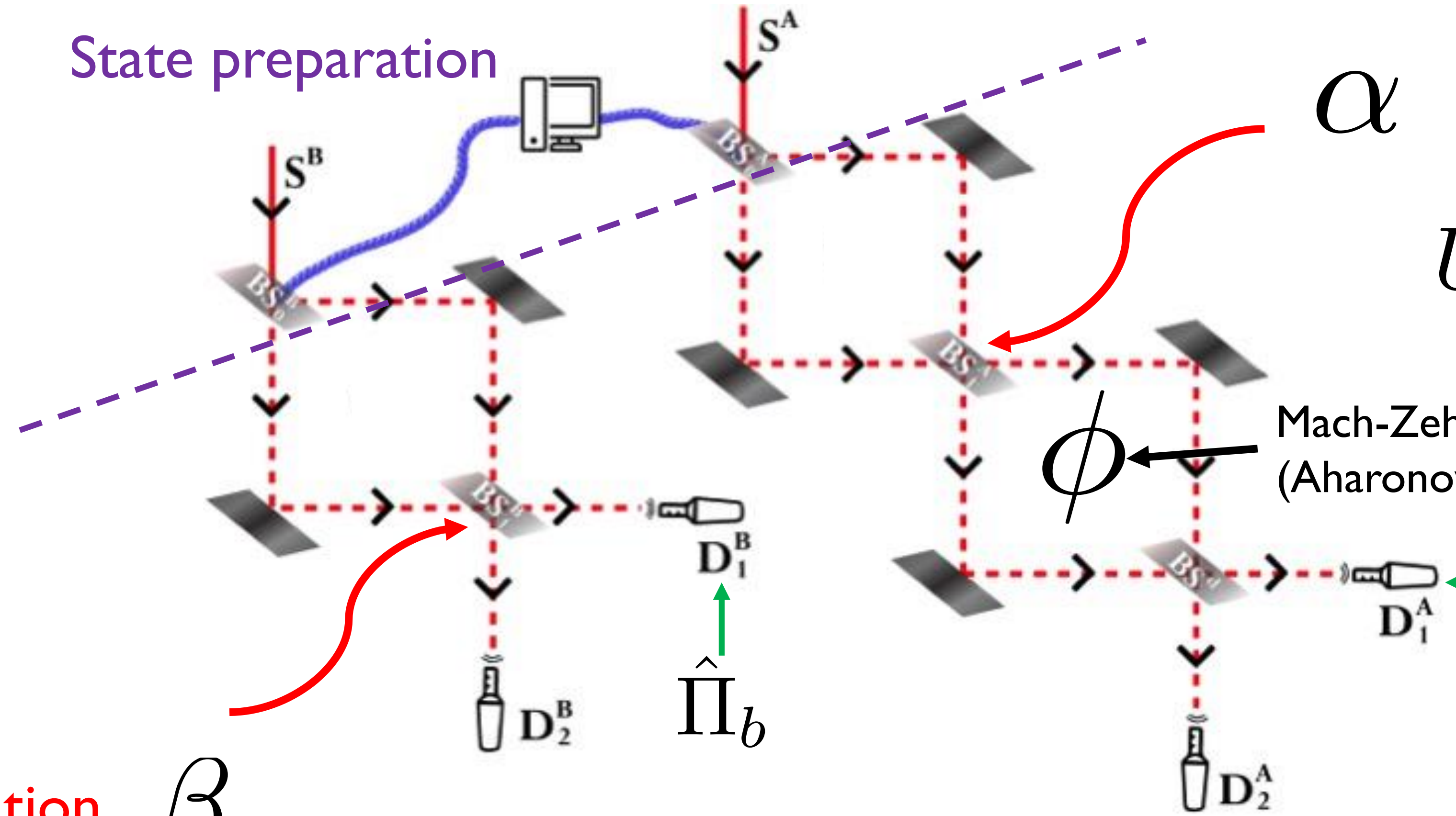
4. Extract amplitude of coherent oscillations $p(\alpha, \beta) = \mathrm{const} + V(\alpha, \beta)\ \cos\phi$

5. Watch zero-visibility lines $V(\alpha, \beta) = 0$

$D = 0 \quad \Leftrightarrow \quad [\hat{\rho}_i^A, \hat{\rho}_j^A] = 0$ Diagonalizing operator is not B-dependent and so 0-visibility condition

$D \neq 0 \quad \Leftrightarrow \quad [\hat{\rho}_i^A, \hat{\rho}_j^A] \neq 0$ Diagonalizing operator is B-dependent and so 0-visibility condition

EXPERIMENTAL SETUP



State preparation

α A-evolution

$$\hat{U}_A = \hat{U}_A(\alpha)$$

ϕ Mach-Zehnder interferometer (Aharonov-Bohm flux)

$\hat{\Pi}_a$

Observables

$\hat{\Pi}_b$

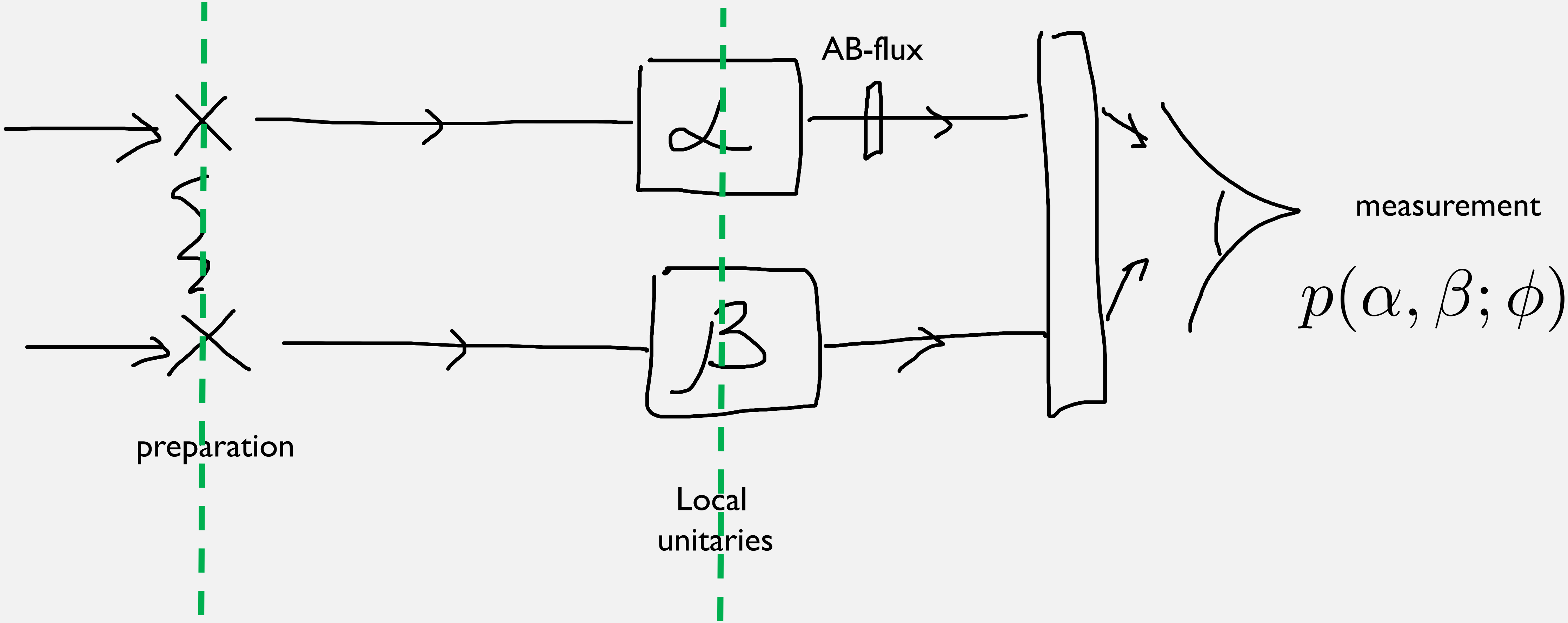
B-evolution β

Current-current cross-correlation function (coincidence rate)

$$p(\alpha,\beta) = \text{const} + V(\alpha,\beta)\,\cos\phi$$

Aharonov-Bohm oscillations

SCHEMATICAL REPRESENTATION

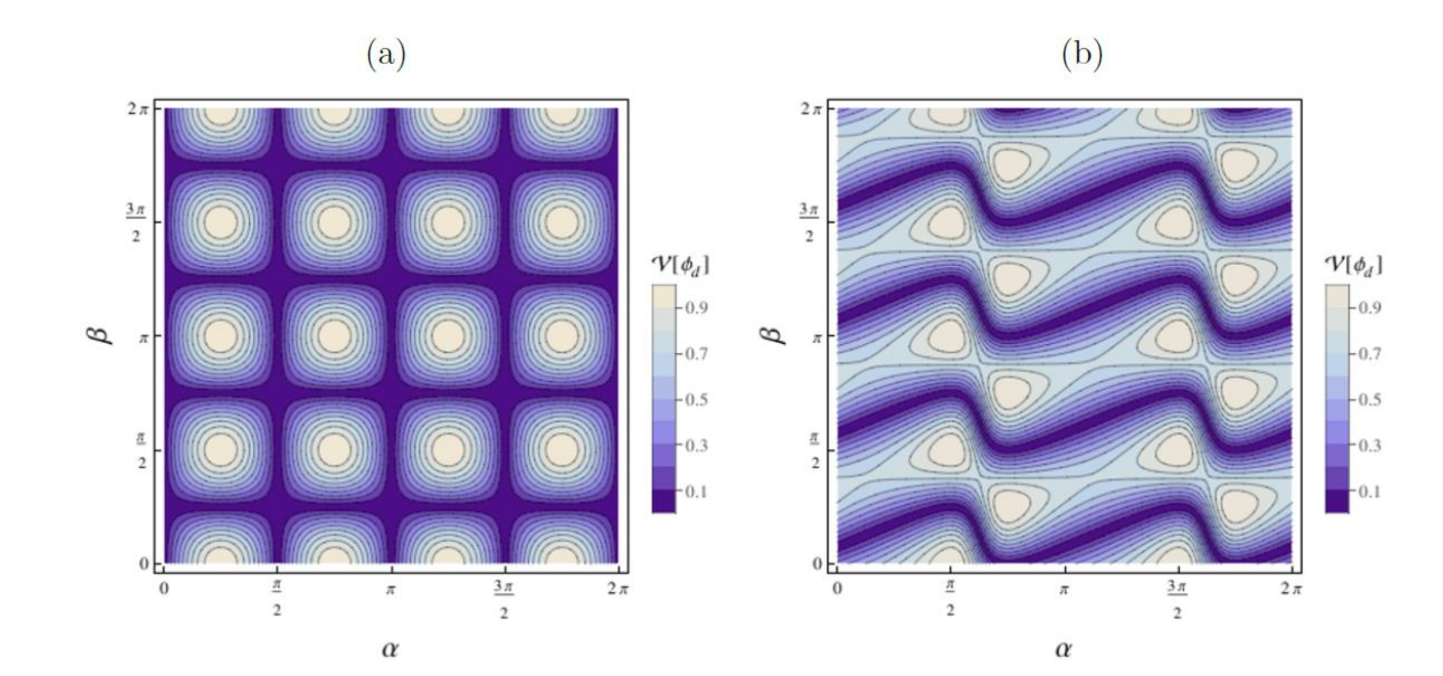


AHARONOV-BOHM OSCILLATIONS

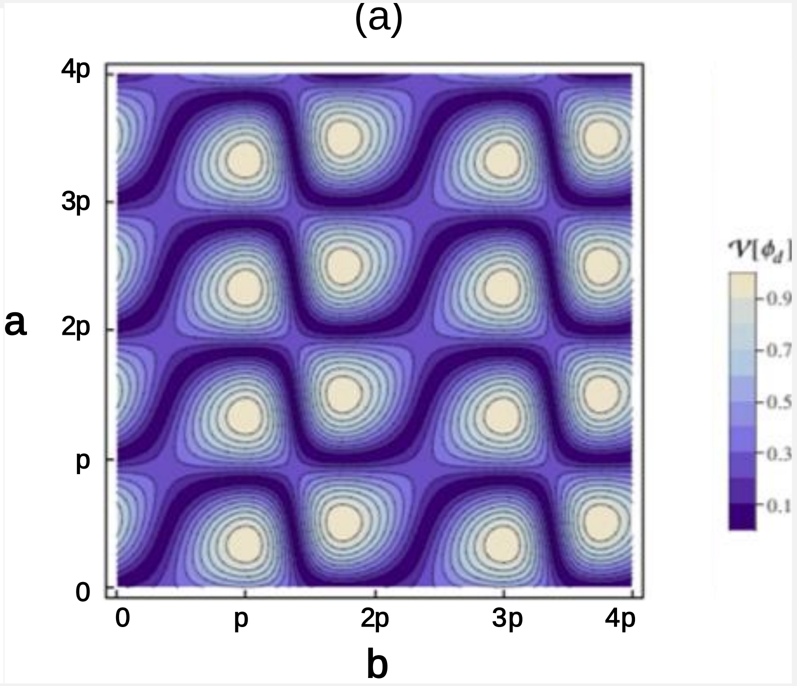
$$p(\alpha, \beta) = \mathrm{const} + V(\alpha, \beta)\ \cos\phi$$

Visibility *V* of AB-oscillations for

zero-discord state:



discordant states

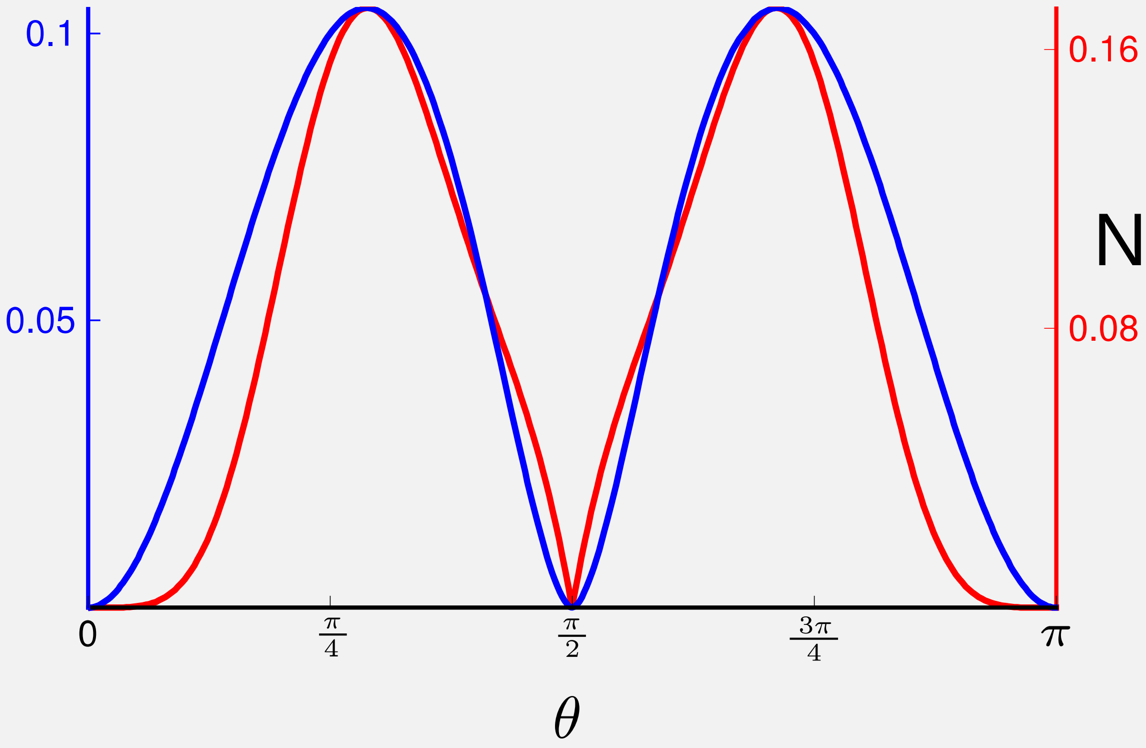


NEW QUANTIFIER VS DISCORD

$$\hat{\rho}(\theta) = \frac{1}{2}\left[\left|\uparrow\right\rangle\left\langle\uparrow\right| \otimes \left|\uparrow\right\rangle\left\langle\uparrow\right| + \left|\theta\right\rangle\left\langle\theta\right| \otimes \left|\theta\right\rangle\left\langle\theta\right|\right]$$

D

Quantum discord



Δ

New quantifier

$$\Delta = \int d\beta \left|\alpha'(\beta)\right|^{1/2}$$

DISCORD IN METROLOGY

Random variable

$$p(x|\theta)$$

Measurements

$$\{x_1, x_2, \cdots, x_N\}$$

Estimator (trial f-n): $\theta_{est}(x_1, \cdots, x_N)$

$$\langle \theta_{est}(x_1, \cdots, x_N) \rangle = \theta$$

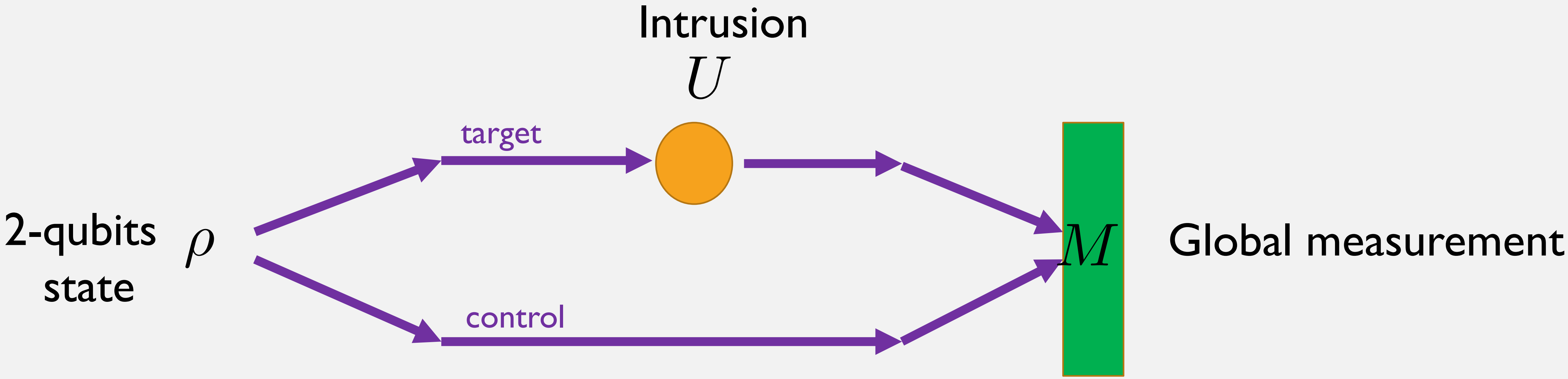
Cramer-Rao bound

$$\delta\theta \geq \frac{1}{\sqrt{N\,F}}$$

Fisher information

$$F = \int \frac{dx}{p(x|\theta)} \left[\frac{\partial p(x|\theta)}{\partial \theta} \right]^2$$

DISCORD RULES



$$F(\rho; U, M) \leq c\, D(\rho)$$

Intrusion can be hidden from observer for zero-discord states $D(\rho) = 0$

Cramer-Rao bound $\delta\theta = \infty$ (no detection)

THANK YOU