Chiral magnetic effect
in Keldysh technique

M.Zubkov

Ariel University Israel

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collaborators:

C.Banerjee (Ariel University, Israel)
M.Lewkowicz (Ariel University, Israel)

Abstract

We consider the *Chiral Magnetic Effect*
out of equilibrium using the unification of two techniques:
Wigner – Weyl calculus and
Keldysh technique

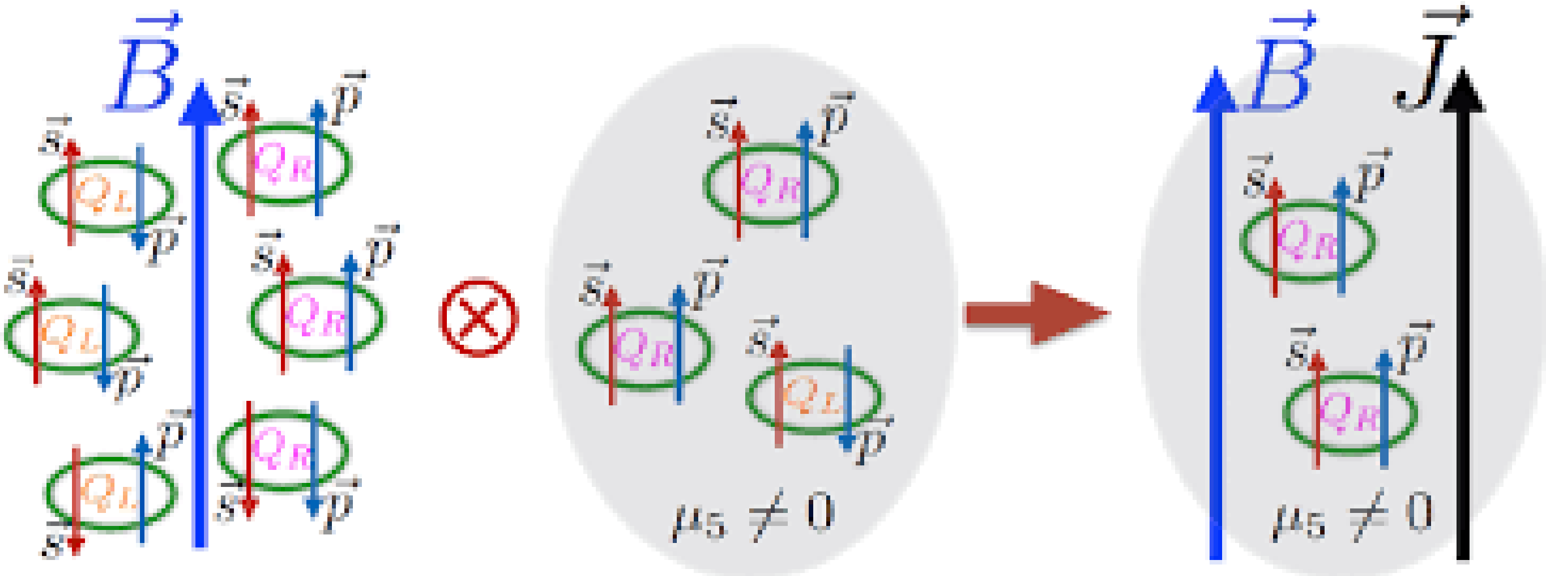
What is CME?

(chiral magnetic effect)

Appearance of electric current along magnetic field in the presence of chiral imbalance

It is one of the non – dissipative transport effects

Chiral Magnetic Effect



D.E. Kharzeev, J. Liao, S.A. Voloshin, G. Wang,
Progress in Particle and Nuclear Physics, Volume 88, 2016, Pages 1-28,

What is non – dissipative transport?

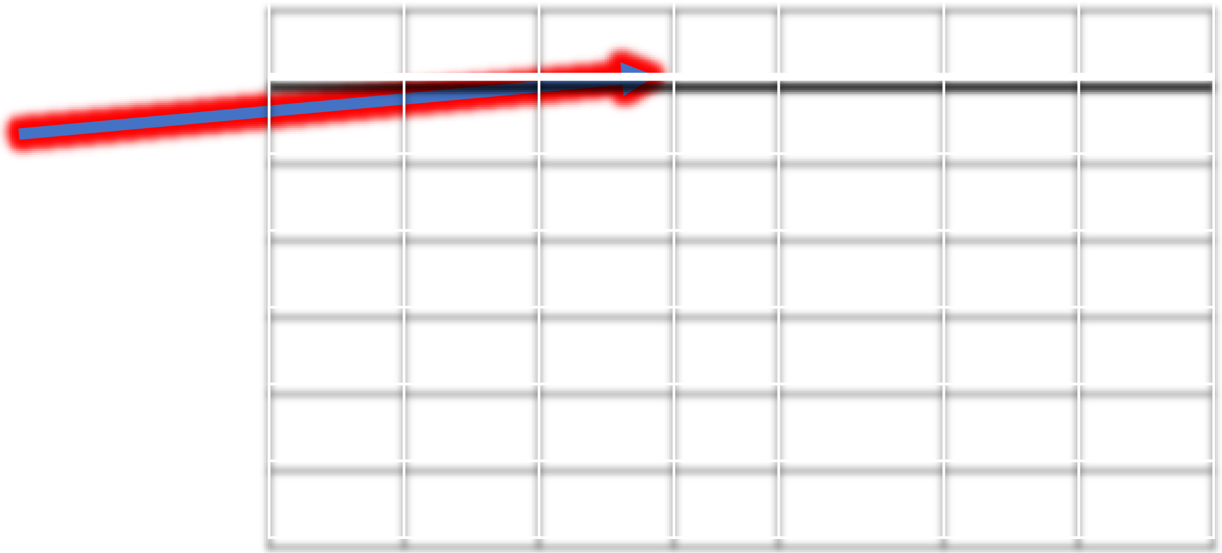
Appearance of a current (electric, axial, etc) that flows without dissipation.

The conductivities of all known non – dissipative transport phenomena are given by topological invariants.

Plan

1. Introduction. CME in equilibrium.
2. Keldysh technique
- *Basics of Keldysh technique*
- *Unification with Wigner – Weyl calculus*
3. CME out of equilibrium
- *Lattice model with Wilson fermions and CME*
- *Chiral chemical potential depending on time*
5. Conclusions.

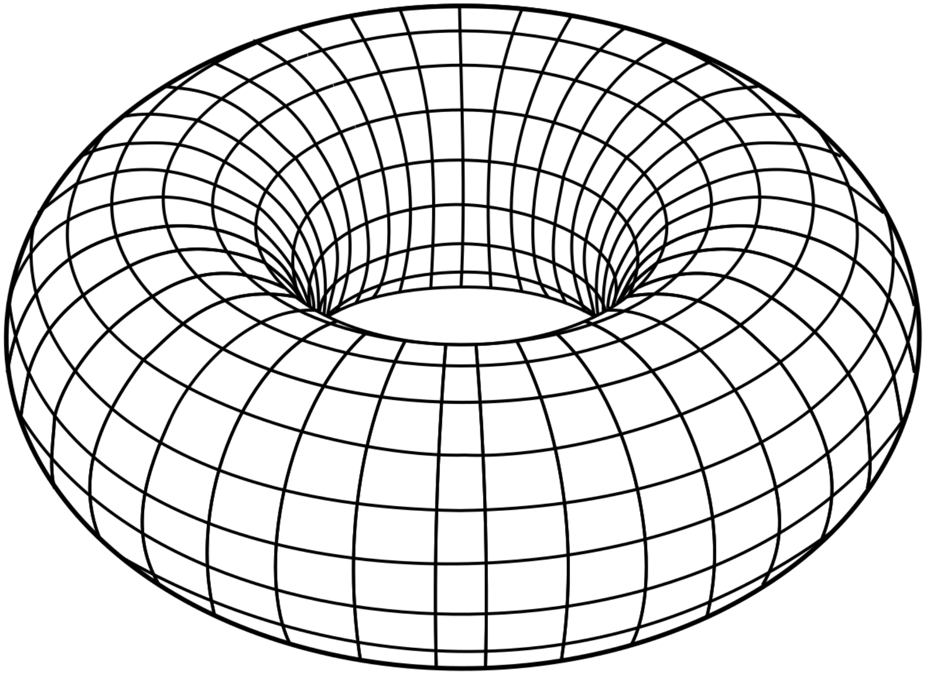
fermions live on the lattice sites



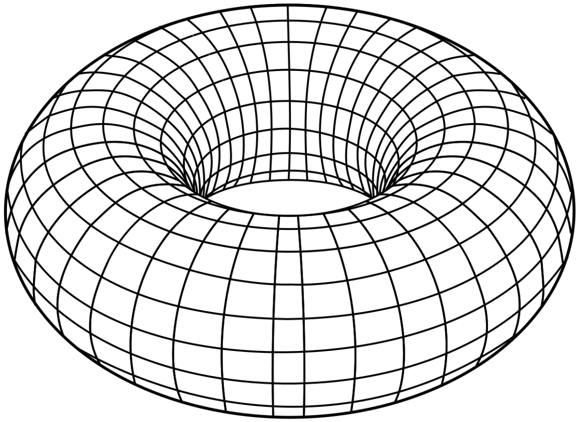
Momentum space

$$\psi(r_n) = \int_{\mathcal{M}} \frac{d^D p}{|\mathcal{M}|} e^{i r_n p} \psi(p)$$

For rectangular lattice Momentum space has the topology of torus



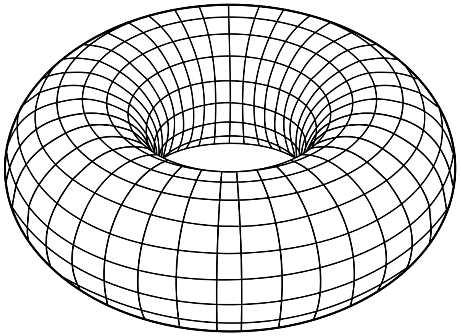
Approximate *Wigner – Weyl calculus for the lattice models*



Weyl symbol of operator (momentum space)

$$[\hat{A}]_W(x_n, p) = \int_{\mathcal{M}} dq e^{iqx_n} \langle p + \frac{q}{2} | \hat{A} | p - \frac{q}{2} \rangle$$

Approximate Wigner – Weyl calculus for the lattice models



Weyl symbol of operator (momentum space)

$$[\hat{A}]_W(x_n,p) = \int_{\mathcal{M}} dq e^{iqx_n} \langle p+\frac{q}{2}|\,\hat{A}\,|p-\frac{q}{2}\rangle$$

Weyl symbol of the product of two operators

$$\boxed{(AB)_W(x_n,p) \equiv A_W(x_n,p) \star B_W(x_n,p)}$$

This identity is approximate. It is valid for the near diagonal operators

This identity is approximate.
It is valid for the near diagonal operators

$$\boxed{(AB)_W(x_n, p) \equiv A_W(x_n, p) \star B_W(x_n, p)}$$

partition function

$$Z = \int D\bar{\psi}D\psi \, e^{S[\psi,\bar{\psi}]}$$

Action

$$S[\psi, \bar{\psi}] = \int_{\mathcal{M}} \frac{d^D p}{|\mathcal{M}|} \bar{\psi}(p)\hat{Q}(i\partial_p, p)\psi(p)$$

Lattice model for the regularization of continuum quantum field theory:

The typical Lattice Dirac operator ***Q*** *is almost diagonal when we approach continuum limit of the lattice model.*

$$\boxed{(\hat{Q}\hat{G})_W = Q_W \star G_W = 1}$$

Applications to Chiral Magnetic Effect

non-homogeneous system, equilibrium, T=0

Average electric current

3 + 1 D:

$$\bar{J}^k = \frac{1}{4\pi^2}\epsilon^{ijkl}\mathcal{M}_l F_{ij}$$

topological invariant:

$$\mathcal{M}_l = \frac{-iT\epsilon_{ijkl}}{3!V\,8\pi^2}\int d^D x \int_{\mathcal{M}} d^D p \,\mathrm{Tr}\left[\mathrm{G}_{\mathrm{W}}^{(0)} \star \partial_{\mathrm{p_i}}\mathrm{Q}_{\mathrm{W}}^{(0)}(\mathrm{p},\mathrm{x}) \star \mathrm{G}_{\mathrm{W}}^{(0)} \star \partial_{\mathrm{p_j}}\mathrm{Q}_{\mathrm{W}}^{(0)}(\mathrm{p},\mathrm{x}) \star \mathrm{G}_{\mathrm{W}}^{(0)} \star \partial_{\mathrm{p_k}}\mathrm{Q}_{\mathrm{W}}^{(0)}\right]$$

external magnetic field:

$$F_{ij} = \epsilon_{ijk} B_k$$

C. Banerjee, M. Lewkowicz, M.A. Zubkov
Physics Letters B, 136457

Applications to Chiral Magnetic Effect
non-homogeneous system, equilibrium, T>0
Average electric current

$$\bar{J}^k = \frac{1}{4\pi^2}\epsilon^{ijk4}\mathcal{M}_4 F_{ij}$$

topological invariant:

$$\mathcal{M}_4 = 2\pi T \sum_{\omega} \mathcal{N}_4(\omega) \qquad \omega = 2\pi T(n+1/2),\ n \in Z,\ 0 \le n < N, \text{ where } N = 1/T$$

$$\mathcal{N}_4(\omega) = \frac{-i\epsilon_{ijk4}}{3!V\,8\pi^2}\int d^{D-1}x \int_{\mathcal{B}} d^{D-1}p \,\mathrm{Tr}\left[\mathrm{G}_{\mathrm{W}}^{(0)} \star \partial_{\mathrm{p}_\mathrm{i}}\mathrm{Q}_{\mathrm{W}}^{(0)}(\mathrm{p},\mathrm{x}) \star \mathrm{G}_{\mathrm{W}}^{(0)} \star \partial_{\mathrm{p}_\mathrm{j}}\mathrm{Q}_{\mathrm{W}}^{(0)}(\mathrm{p},\mathrm{x}) \star \mathrm{G}_{\mathrm{W}}^{(0)} \star \partial_{\mathrm{p}_\mathrm{k}}\mathrm{Q}_{\mathrm{W}}^{(0)}\right]$$

Response of N to chiral chemical potential is zero

No CME at T>0

The absence of CME at T>0 **for homogeneous** systems has been reported earlier in
C.G. Beneventano, M. Nieto, E.M. Santangelo J. Phys. A, 53 (46) (2020), Article 465401,

Keldysh technique
Green functions (lower sign for fermions)

$$\left\{\hat{G}^R\right\}_{(\alpha_1;\alpha_2)}(x_1;x_2) \equiv -i\theta(t_1-t_2)\left\langle \left[\Psi_{\alpha_1}(x_1),\Psi^\dagger_{\alpha_2}(x_2)\right]_{\mp}\right\rangle$$

$$\left\{\hat{G}^A\right\}_{(\alpha_1;\alpha_2)}(x_1;x_2) \equiv \ i\theta(t_2-t_1)\left\langle \left[\Psi_{\alpha_1}(x_1),\Psi^\dagger_{\alpha_2}(x_2)\right]_{\mp}\right\rangle$$

$$\left\{\hat{G}^K\right\}_{(\alpha_1;\alpha_2)}(x_1;x_2) \equiv -i\left\langle \left[\Psi_{\alpha_1}(x_1),\Psi^\dagger_{\alpha_2}(x_2)\right]_{-}\right\rangle,$$

$$\left\{\hat{G}^<\right\}_{(\alpha_1;\alpha_2)}(x_1;x_2) \equiv \mp i\left\langle \Psi^\dagger_{\alpha_2}(x_2)\Psi_{\alpha_1}(x_1)\right\rangle$$

Keldysh Green function

$$\hat{G}(t,x|t',x') = -i\begin{pmatrix} \langle T\Phi(t,x)\Phi^+(t',x')\rangle & -\langle \Phi^+(t',x')\Phi(t,x)\rangle \\ \langle \Phi(t,x)\Phi^+(t',x')\rangle & \langle \tilde{T}\Phi(t,x)\Phi^+(t',x')\rangle \end{pmatrix}$$

$$\begin{pmatrix} G^{--} & G^{-+} \\ G^{+-} & G^{++} \end{pmatrix}$$

$$G^{\mathrm{A}} = G^{--} - G^{+-} = G^{-+} - G^{++}$$

$$G^{\mathrm{R}} = G^{--} - G^{-+} = G^{+-} - G^{++}$$

$$G^< = G^{-+}$$

Keldysh technique and Wigner – Weyl calculus.

Keldysh Green function

$$\hat{G}(t,x|t',x') = -i \begin{pmatrix} \langle T\Phi(t,x)\Phi^+(t',x')\rangle & -\langle\Phi^+(t',x')\Phi(t,x)\rangle \\ \langle\Phi(t,x)\Phi^+(t',x')\rangle & \langle\tilde{T}\Phi(t,x)\Phi^+(t',x')\rangle \end{pmatrix}$$

$$= \begin{pmatrix} G^{--} & G^{-+} \\ G^{+-} & G^{++} \end{pmatrix}$$

$$G^{\mathrm{A}} = G^{--} - G^{+-} = G^{-+} - G^{++}$$

$$G^{\mathrm{R}} = G^{--} - G^{-+} = G^{+-} - G^{++}$$

$$G^{<} = G^{-+}$$

Wigner transformation

$$\hat{G}(X_1,X_2) = \langle X_1|\hat{\mathbf{G}}|X_2\rangle \qquad A(X_1,X_2) = \langle X_1|\hat{A}|X_2\rangle$$

$$A_W(X|P) = \int d^{D+1}Y\, e^{\mathrm{i}Y^\mu P_\mu} A(X+Y/2, X-Y/2)$$

Moyal product

$$(A\star B)\,(X|P) = A(X|P)\, e^{-\mathrm{i}(\overleftarrow{\partial}_{X^\mu}\overrightarrow{\partial}_{P_\mu} - \overleftarrow{\partial}_{P_\mu}\overrightarrow{\partial}_{X^\mu})/2} B(X|P)$$

Lesser representation

$$\hat{\mathbf{G}}^{(<)} = U\hat{\mathbf{G}}V,$$

$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$ $V = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$

$$\hat{\mathbf{G}}^{(<)} = \begin{pmatrix} G^{\mathrm{R}} & 2G^{<} \\ 0 & G^{\mathrm{A}} \end{pmatrix}$$

$$G^{\mathrm{A}} = G^{--} - G^{+-} = G^{-+} - G^{++}$$

$$G^{\mathrm{R}} = G^{--} - G^{-+} = G^{+-} - G^{++}$$

$$G^{<} = G^{-+}$$

The inverse Q of Green function

$$\hat{\mathbf{Q}}\hat{\mathbf{G}} = 1$$

After Wigner transformation

$$\hat{Q} * \hat{G} = 1$$

In non – interacting systems

$$G^{\mathrm{R}} = (\mathrm{i}\partial_t - \hat{H}e^{+\epsilon\partial_t})^{-1} = (\mathrm{i}\partial_t - \hat{H} + \mathrm{i}\epsilon)^{-1}$$
$$G^{\mathrm{A}} = (\mathrm{i}\partial_t - \hat{H}e^{-\epsilon\partial_t})^{-1} = (\mathrm{i}\partial_t - \hat{H} - \mathrm{i}\epsilon)^{-1}$$
$$G^{<} = (G^{\mathrm{A}} - G^{\mathrm{R}})\frac{\rho}{\rho+1}.$$

distribution function

In general case without interactions electric current

$$J^i(X) = -\frac{\mathrm{i}}{2}\int \frac{d^{D+1}\pi}{(2\pi)^{D+1}}\,\mathrm{tr}\left((\partial_{\pi_i}\hat{Q})\hat{G}\right)^{<} - \frac{\mathrm{i}}{2}\int \frac{d^{D+1}\pi}{(2\pi)^{D+1}}\,\mathrm{tr}\left(\hat{G}(\partial_{\pi_i}\hat{Q})\right)^{<}$$

A.Shitade, J. Phys. Soc. Jpn. 86, 054601 (2017)

product of triangle matrices is triangle matrix

$$\hat{\mathbf{Q}}^{(<)} = \begin{pmatrix} Q^{\mathrm{R}} & 2Q^{<} \\ 0 & Q^{\mathrm{A}} \end{pmatrix}$$

lesser component for any matrix is defined as

Response of electric current to external field strength

$$J^i = -\frac{1}{4}\int \frac{d^{D+1}\pi}{(2\pi)^{D+1}} \,\mathrm{tr}\Big(\hat{G}\star\partial_{\pi^\mu}\hat{Q}\star\hat{G}\star\partial_{\pi^\nu}\hat{Q}\star\hat{G}\partial_{\pi_i}\hat{Q}\Big)^{<}\mathcal{F}^{\mu\nu}$$
$$-\frac{1}{4}\int \frac{d^{D+1}\pi}{(2\pi)^{D+1}} \,\mathrm{tr}\Big(\partial_{\pi_i}\hat{Q}\hat{G}\star\partial_{\pi^\mu}\hat{Q}\star\hat{G}\star\partial_{\pi^\nu}\hat{Q}\star\hat{G}\Big)^{<}\mathcal{F}^{\mu\nu}.$$

Electric conductivity tensor for non – homogeneous systems

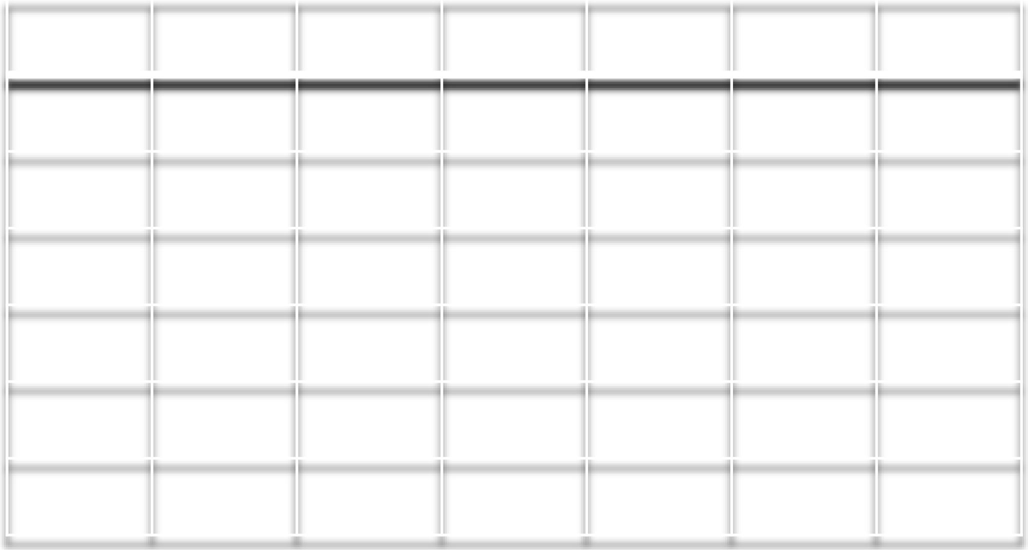
$$J^i = \sigma^{ij}\mathcal{F}_{0j}$$

$$\sigma^{ij} = \frac{1}{4}\int \frac{d^{D+1}\pi}{(2\pi)^{D+1}} \,\mathrm{tr}\left(\partial_{\pi_i}\hat{Q}\left[\hat{G}\star\partial_{\pi_{[0}}\hat{Q}\star\partial_{\pi_{j]}}\hat{G}\right]\right)^{<} + \mathrm{c.c.}$$

C Banerjee, IV Fialkovsky, M Lewkowicz, CX Zhang, MA Zubkov
arXiv preprint arXiv:2009.10704

Lattice model with Wilson fermions

Out of equilibrium



Thermal equilibrium (in Euclidean space - time)

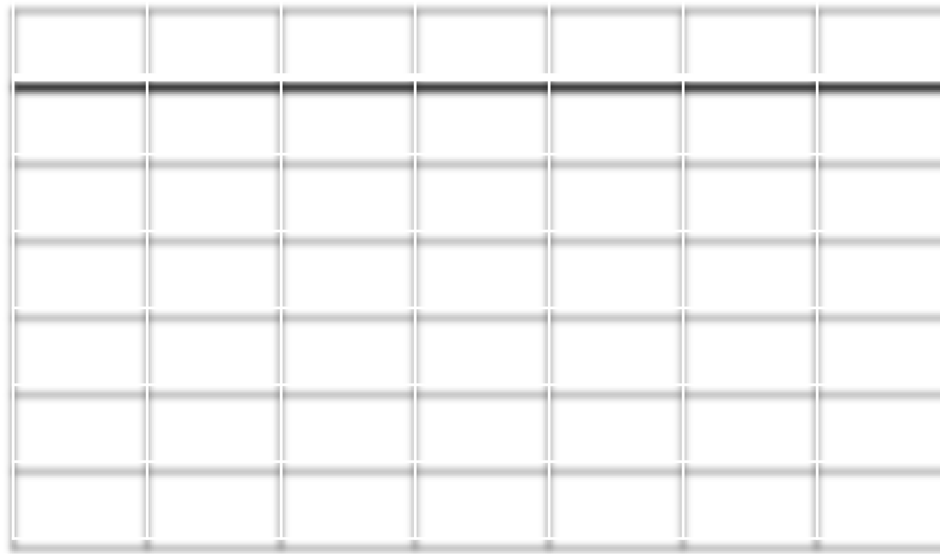
$$Q_W^M(\pi) = \sum_{\mu=1}^{3} \gamma^\mu g_\mu(\pi) - im(\pi) + \gamma^4 g_4(\pi_4)$$

$$g_i = \sin(\pi_i)$$

$$m(\pi) = m^{(0)} + \sum_{i=1}^{4} (1 - \cos(\pi_i))$$

Lattice model with Wilson fermions

Out of equilibrium



Real time dynamics (in Minkowski space - time)

$$Q_W^M(\pi)|_{\pi_4=-i\pi_0} = \sum_{\mu=1}^{3} \gamma^\mu g_\mu(\pi) \qquad ($$

$$-i\left(\sum_{i=1}^{3}(1-\cos(\pi_i)) + (1-\mathrm{ch}(\pi_0))\right) - i\gamma^4 \mathrm{sh}(\pi_0)$$

Lattice model with Wilson fermions

Out of equilibrium

Keldysh Green function

$$\hat{Q} = \left(\begin{array}{cc} Q_{--} & Q_{-+} \\ Q_{+-} & Q_{++} \end{array} \right)$$

$$Q_{++} = -\mathcal{Q}(\pi_0, \vec{\pi}) + \mathrm{i}\epsilon\partial_{\pi_0}\mathcal{Q}(\pi_0, \vec{\pi})\frac{1-\rho(\pi_0)}{1+\rho(\pi_0)}$$

$$Q_{--} = \mathcal{Q}(\pi_0, \vec{\pi}) + +\mathrm{i}\epsilon\partial_{\pi_0}\mathcal{Q}(\pi_0, \vec{\pi})\frac{1-\rho(\pi_0)}{1+\rho(\pi_0)}$$

$$Q_{+-} = -2\mathrm{i}\epsilon\partial_{\pi_0}\mathcal{Q}(\pi_0, \vec{\pi})\frac{1}{1+\rho(\pi_0)},$$

$$Q_{-+} = 2\mathrm{i}\epsilon\partial_{\pi_0}\mathcal{Q}(\pi_0, \vec{\pi})\frac{\rho(\pi_0)}{1+\rho(\pi_0)}.$$

$$\pi = P - A(X)$$

initial one – particle distribution

$$f(\pi_0) = \rho(\pi_0)(1 + \rho(\pi_0))^{-1}$$

$$\hat{Q} = \begin{pmatrix} Q_{--} & Q_{-+} \\ Q_{+-} & Q_{++} \end{pmatrix}$$

$$Q_{++} = -\Big(\sum_{\mu=1}^{3} \gamma^\mu g_\mu(\pi) - im(\vec{\pi}, -i\pi_0 - i\mu_5(t)\gamma^5)$$
$$+\gamma^4 g_4(-i\pi_0 - i\mu_5(t)\gamma^5) - \gamma^4 \epsilon\, e^{-\pi_0\gamma^4} \frac{1-\rho(\pi_0)}{1+\rho(\pi_0)}\Big),$$
$$Q_{--} = \sum_{\mu=1}^{3} \gamma^\mu g_\mu(\pi) - im(\vec{\pi}, -i\pi_0 - i\mu_5(t)\gamma^5)$$
$$+\gamma^4 g_4(-i\pi_0 - i\mu_5(t)\gamma^5) + \gamma^4 \epsilon\, e^{-\pi_0\gamma^4} \frac{1-\rho(\pi_0)}{1+\rho(\pi_0)},$$
$$Q_{+-} = -2\gamma^4 \epsilon\, e^{-\pi_0\gamma^4} \frac{1}{1+\rho(\pi_0)},$$
$$Q_{-+} = 2\gamma^4 \epsilon\, e^{-\pi_0\gamma^4} \frac{\rho(\pi_0)}{1+\rho(\pi_0)}. \qquad (32)$$

time depending chiral chemical potential

$$\delta\mu_5(t) = \delta\mu_5^{(0)} \cos\omega_0 t$$

response to external field strength

$$J^i = \Sigma^{ijk} \mathcal{F}_{jk}$$

$$\Sigma^{ijk} = -\frac{1}{4} \int \frac{d^{D+1}\pi}{(2\pi)^{D+1}} \, \mathrm{tr} \left(\partial_{\pi_i} \hat{Q} \left[\hat{G} \star \partial_{\pi_{[j}} \hat{Q} \star \hat{G} \star \partial_{\pi_{k]}} \hat{Q} \star \hat{G} \right] \right)^{<}$$

$$- \frac{1}{4} \int \frac{d^{D+1}\pi}{(2\pi)^{D+1}} \, \mathrm{tr} \left(\left[\hat{G} \star \partial_{\pi_{[j}} \hat{Q} \star \hat{G} \star \partial_{\pi_{k]}} \hat{Q} \star \hat{G} \right] \partial_{\pi_i} \hat{Q} \right)^{<} .$$

response to external magnetic field

$$J^i = \Sigma_{CME} B^i$$

$$\Sigma_{CME} = \frac{\epsilon_{ijk}}{3!2} \int \frac{d^{D+1}\pi}{(2\pi)^{D+1}} \, \mathrm{tr} \left(\partial_{\pi_i} \hat{Q} \left[\hat{G} \star \partial_{\pi_j} \hat{Q} \star \partial_{\pi_k} \hat{G} \right] \right)^{<} + \mathrm{c.c.}$$

Response of electric current both to magnetic field and to chiral chemical potential

$$J^i = \Sigma_{CME} B^i$$

response to chiral chemical potential $\delta\mu_5(t) = \delta\mu_5^{(0)} \cos\omega_0 t$

$$\Delta\Sigma_{CME} = \frac{1}{4\pi^2}\sigma_{CME}(\omega_0)\delta\mu_5^{(0)} e^{\mathrm{i}\omega_0 t} + (c.c.)$$

two parts of conductivity $\sigma_{CME}(\omega_0) = \sigma_{CME}^{(I)}(\omega_0) + \sigma_{CME}^{(II)}(\omega_0)$

$$\tilde{\sigma}_{CME}^{(I)} = -\frac{2\pi T\epsilon^{ijk}}{48\pi^2} \sum_{\pi_4=2\pi T(n+1/2)} \int d^3\pi \, \mathrm{tr}\Big(\gamma^5 \hat{G}_0^{[0]}$$
$$\partial_{\pi_j}\hat{Q}_0^{[0]}\hat{G}_0^{[0]}\partial_{\pi_k}\hat{Q}_0^{[0]}\hat{G}_0^{[0]}\partial_{\pi_i}\hat{Q}_0^{[-]}\hat{G}_0^{[--]}\frac{\partial Q^{[-]}}{\partial\pi_4}\Big)^M$$
$$-\frac{2\pi T\epsilon^{ijk}}{48\pi^2} \sum_{\pi_4=2\pi T(n+1/2)} \int d^3\pi \, \mathrm{tr}\Big(\gamma^5 \hat{G}_0^{[++]}\partial_{\pi_i}\hat{Q}_0^{[+]}\hat{G}_0^{[0]}$$
$$\partial_{\pi_j}\hat{Q}_0^{[0]}\hat{G}_0^{[0]}\partial_{\pi_k}\hat{Q}_0^{[0]}\hat{G}_0^{[0]}\frac{\partial Q^{[+]}}{\partial\pi_4}\Big)^M \qquad \text{(A1)}$$

Notations:

$$K^{[\pm]} \equiv K(\omega \pm \omega_0/2)$$

$$K^{[0]} \equiv K(\omega)$$

two parts of conductivity

$$\sigma_{CME}(\omega_0) = \sigma^{(I)}_{CME}(\omega_0) + \sigma^{(II)}_{CME}(\omega_0)$$

$$\tilde{\sigma}^{(I)}_{CME} = -\frac{2\pi T \epsilon^{ijk}}{48\pi^2} \sum_{\pi_4 = 2\pi T(n+1/2)} \int d^3\pi \,\mathrm{tr}\,\Big(\gamma^5 \hat{G}^{[0]}_0$$

$$\partial_{\pi_j} \hat{Q}^{[0]}_0 \hat{G}^{[0]}_0 \partial_{\pi_k} \hat{Q}^{[0]}_0 \hat{G}^{[0]}_0 \partial_{\pi_i} \hat{Q}^{[-]}_0 \hat{G}^{[--]}_0 \frac{\partial Q^{[-]}}{\partial \pi_4}\Big)^M$$

$$-\frac{2\pi T \epsilon^{ijk}}{48\pi^2} \sum_{\pi_4 = 2\pi T(n+1/2)} \int d^3\pi \,\mathrm{tr}\,\Big(\gamma^5 \hat{G}^{[++]}_0 \partial_{\pi_i} \hat{Q}^{[+]}_0 \hat{G}^{[0]}_0$$

$$\partial_{\pi_j} \hat{Q}^{[0]}_0 \hat{G}^{[0]}_0 \partial_{\pi_k} \hat{Q}^{[0]}_0 \hat{G}^{[0]}_0 \frac{\partial Q^{[+]}}{\partial \pi_4}\Big)^M \qquad \text{(A1)}$$

Response of electric current both to magnetic field and to chiral chemical potential

$$J^i = \Sigma_{CME} B^i$$

response to chiral chemical potential $\delta\mu_5(t) = \delta\mu_5^{(0)} \cos\omega_0 t$

$$\Delta\Sigma_{CME} = \frac{1}{4\pi^2}\sigma_{CME}(\omega_0)\delta\mu_5^{(0)} e^{\mathrm{i}\omega_0 t} + (c.c.)$$

*two parts of **conductivity*** $\sigma_{CME}(\omega_0) = \sigma_{CME}^{(I)}(\omega_0) + \sigma_{CME}^{(II)}(\omega_0)$

$$\sigma_{CME}^{(II)}(\omega_0) = I_1 + I_2$$

$$\begin{aligned}
\mathrm{I}_1 = -\frac{\epsilon^{ijk}}{3!2}\int d\pi_0 \Big(f(\pi_0 - \omega_0/2) - f(\pi_0 + \omega_0/2)\Big)\int \frac{d^D\vec{\pi}}{(2\pi)^{D-1}} \\
\mathrm{tr}\Big(\partial_{\pi_i}\hat{Q}_0^{[0]R}\Big[(\hat{G}_0^{[-]A} - \hat{G}_0^{[-]R})\partial_{\pi_0}Q^{[0]A}\gamma^5\hat{G}_0^{[+]A} \\
\partial_{\pi_j}\hat{Q}_0^{[+]A}\hat{G}_0^{[+]A}\partial_{\pi_k}\hat{Q}_0^{[+]A}\hat{G}_0^{[+]A}\Big]\Big) \qquad \text{(B1)}
\end{aligned}$$

Response of electric current both to magnetic field and to chiral chemical potential

$$J^i = \Sigma_{CME} B^i$$

response to chiral chemical potential $\delta\mu_5(t) = \delta\mu_5^{(0)} \cos\omega_0 t$

$$\Delta\Sigma_{CME} = \frac{1}{4\pi^2}\sigma_{CME}(\omega_0)\delta\mu_5^{(0)} e^{\mathrm{i}\omega_0 t} + (c.c.)$$

two parts of conductivity $\sigma_{CME}(\omega_0) = \sigma_{CME}^{(I)}(\omega_0) + \sigma_{CME}^{(II)}(\omega_0)$

$$\sigma_{CME}^{(II)}(\omega_0) = I_1 + I_2$$

$$\mathrm{I}_2 = +\frac{\epsilon^{ijk}}{3!2}\int d\pi_0 \Big(f(\pi_0 - \omega_0/2) - f(\pi_0 + \omega_0/2) \Big)$$
$$\int \frac{d^D\vec{\pi}}{(2\pi)^{D-1}} \,\mathrm{tr}\,\Big(\partial_{\pi_i}\hat{Q}_0^{[0]R}\Big[\hat{G}_0^{[-]R}\partial_{\pi_j}\hat{Q}_0^{[-]R}\hat{G}_0^{[-]R}\partial_{\pi_k}$$
$$\hat{Q}_0^{[-]R}\hat{G}_0^{[-]R}\partial_{\pi_0}Q^{[0]R}\gamma^5(\hat{G}_0^{[+]A} - \hat{G}_0^{[+]R})\Big]\Big) \qquad \text{(B2)}$$

Response of electric current both to magnetic field and to chiral chemical potential

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response to chiral chemical potential $\delta\mu_5(t) = \delta\mu_5^{(0)} \cos\omega_0 t$

$$\Delta\Sigma_{CME} = \frac{1}{4\pi^2}\sigma_{CME}(\omega_0)\delta\mu_5^{(0)} e^{\mathrm{i}\omega_0 t} + (c.c.)$$

two parts of conductivity $\sigma_{CME}(\omega_0) = \sigma^{(I)}_{CME}(\omega_0) + \sigma^{(II)}_{CME}(\omega_0)$

$$\sigma^{(II)}_{CME}(\omega_0) = I_1 + I_2$$

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$$\int \frac{d^D\vec{\pi}}{(2\pi)^{D-1}}\ \mathrm{tr}\,\Big(\partial_{\pi_i}\hat{Q}_0^{[0]R}\Big[\hat{G}_0^{[-]R}\partial_{\pi_j}\hat{Q}_0^{[-]R}\hat{G}_0^{[-]R}\partial_{\pi_k}$$
$$\hat{Q}_0^{[-]R}\hat{G}_0^{[-]R}\partial_{\pi_0}Q^{[0]R}\gamma^5(\hat{G}_0^{[+]A} - \hat{G}_0^{[+]R})\Big]\Big) \qquad ($$

Response of electric current both to magnetic field and to chiral chemical potential

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response to chiral chemical potential $\delta\mu_5(t) = \delta\mu_5^{(0)} \cos\omega_0 t$

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two parts of conductivity $\sigma_{CME}(\omega_0) = \sigma_{CME}^{(I)}(\omega_0) + \sigma_{CME}^{(II)}(\omega_0)$

$$\sigma_{CME}^{(II)}(\omega_0) = I_1 + I_2$$

$$\mathrm{I}_1 = \mathrm{I}_2 = \frac{2}{3}\int_{-\infty}^{+\infty} \mathrm{p}^2 \mathrm{dp} \frac{(4\mathrm{p} - 3\omega_0)}{\omega_0^2(-2\mathrm{p} + \omega_0 - \mathrm{i}0)^2}$$

$$\left(\frac{1}{e^{-p\beta} + 1} - \frac{1}{e^{(-p+\omega_0)\beta} + 1}\right)$$

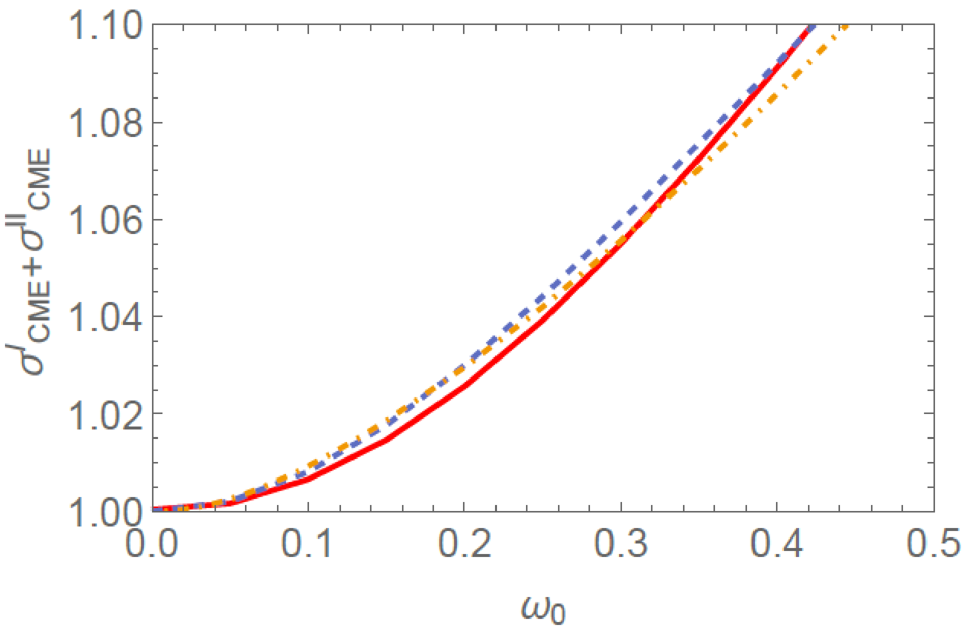
Response of electric current both to magnetic field and to chiral chemical potential

$$J^i = \Sigma_{CME} B^i$$

response to chiral chemical potential $\delta\mu_5(t) = \delta\mu_5^{(0)} \cos\omega_0 t$

$$\Delta\Sigma_{CME} = \frac{1}{4\pi^2}\sigma_{CME}(\omega_0)\delta\mu_5^{(0)} e^{i\omega_0 t} + (c.c.)$$

two parts of conductivity $\sigma_{CME}(\omega_0) = \sigma_{CME}^{(I)}(\omega_0) + \sigma_{CME}^{(II)}(\omega_0)$



$T = \frac{1}{10a}$ (solid line), $\frac{1}{20a}$ (dashed line), $\frac{1}{50a}$ (dashed - dotted line)

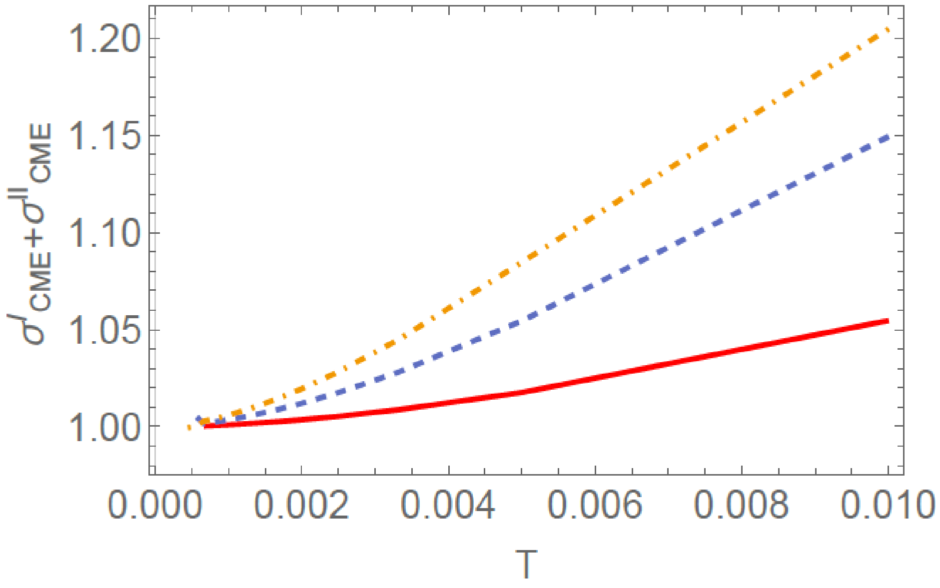
Response of electric current both to magnetic field and to chiral chemical potential

$$J^i = \Sigma_{CME} B^i$$

response to chiral chemical potential $\delta\mu_5(t) = \delta\mu_5^{(0)} \cos\omega_0 t$

$$\Delta\Sigma_{CME} = \frac{1}{4\pi^2}\sigma_{CME}(\omega_0)\delta\mu_5^{(0)} e^{\mathrm{i}\omega_0 t} + (c.c.)$$

two parts of conductivity $\sigma_{CME}(\omega_0) = \sigma_{CME}^{(I)}(\omega_0) + \sigma_{CME}^{(II)}(\omega_0)$



$x = \omega_0/T = 30$ (solid line), $x = 60$ (dashed line), $x = 80$ (dashed dotted line)

Conclusions

- Wigner – Weyl calculus allows to represent in compact form the conductivities of non – dissipative transport phenomena **in non – uniform systems**.
- Equilibrium systems at finite temperatures: CME response of electric current to magnetic field is the topological invariant in phase space. As a result the equilibrium CME does not exist.

Conclusions

- Non – equilibrium systems, Keldysh technique and Wigner – Weyl calculus allow to express in compact form electric current.
- Out of equilibrium, when chiral chemical potential is time dependent, the CME conductivity depends on frequency w. In the continuum limit the conventional value of CME conductivity is reproduced for any ratio w/T.