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Optimal control of a radiation pressure limited opto-mechanical resonator

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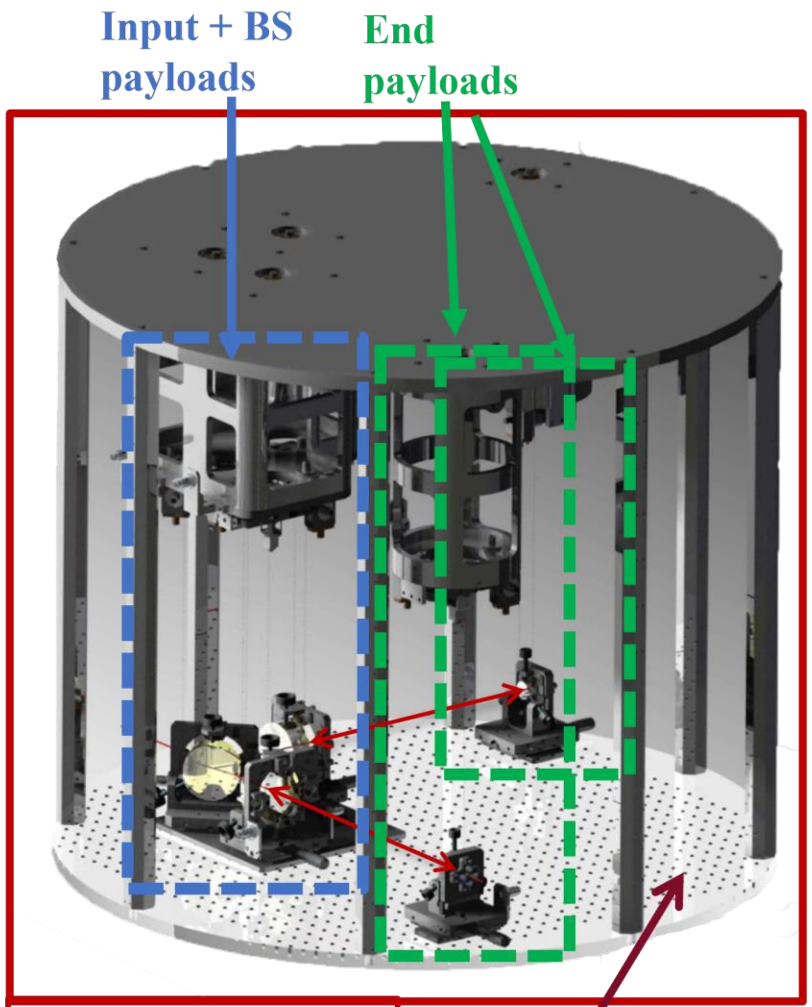
11th International Conference on New Frontiers in Physics (ICNFP 2022)

Crete, *9 September 2022*

Opto-mechanical resonator mechanical design: SIPS experiment



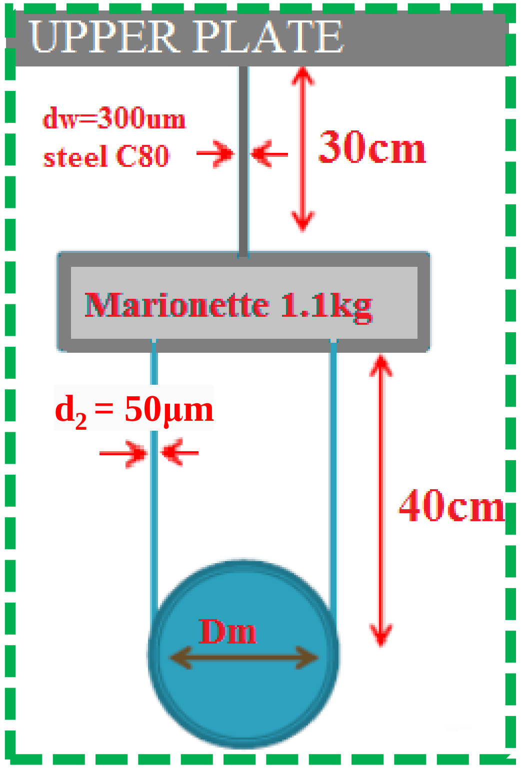
Height:	800 mm
Diameter:	960 mm
Weight:	~ 150 kg

Main optical bench
Michelson ITF with
two **Fabry-Pérot** cavities

Input and BS:
Dm=3'', 300g
End:
Dm=1'', 10 g

They can be suspended with a monolithic Virgo-like technique for thermal noise reduction

Minipayload:
Monolithic suspension system of the main optics
(2 fused silica fibers of 50μm)



Local control of suspended elements with dummy mirrors: status of art

Active control system for alignment and damping of mirror suspension resonance peaks and to recover longitudinal and angular local positions of the mirror.

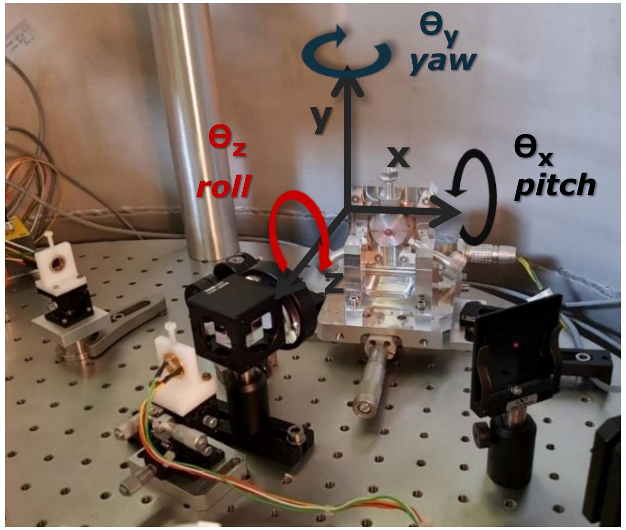
- Actuation system: 4 Coil-magnet actuators for each mirror and marionette.
- Readout system → *Optical Levers* setup of 2 SLED + 3PSDs (ground-based):
  - PSD1→mirror focal plane
  - PSD2→mirror image plane
  - PSD3→marionette focal plane

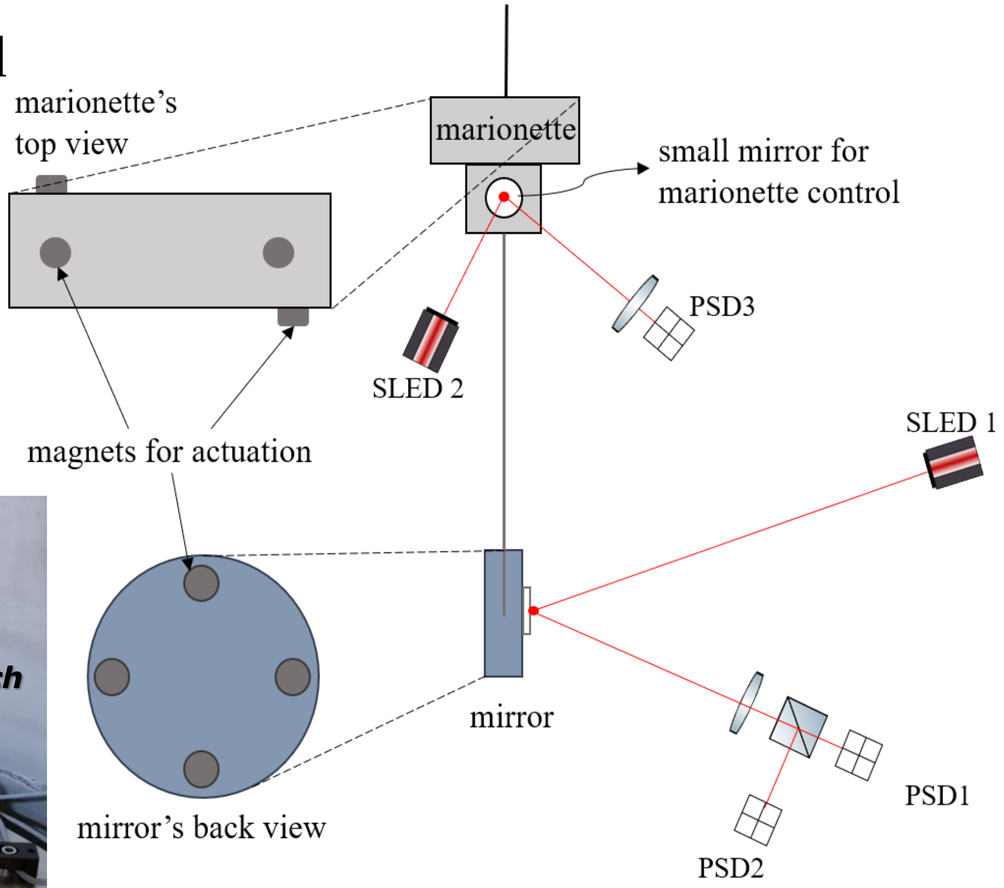
<u>Controlled DOF</u>

Mirror: θ_x, θ_y with PSD1

z with PSD2

Marionette: θ_z, θ_y with PSD3





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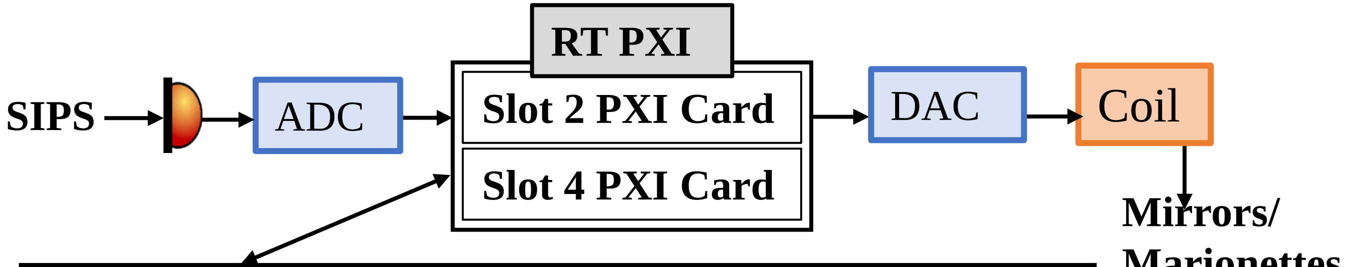
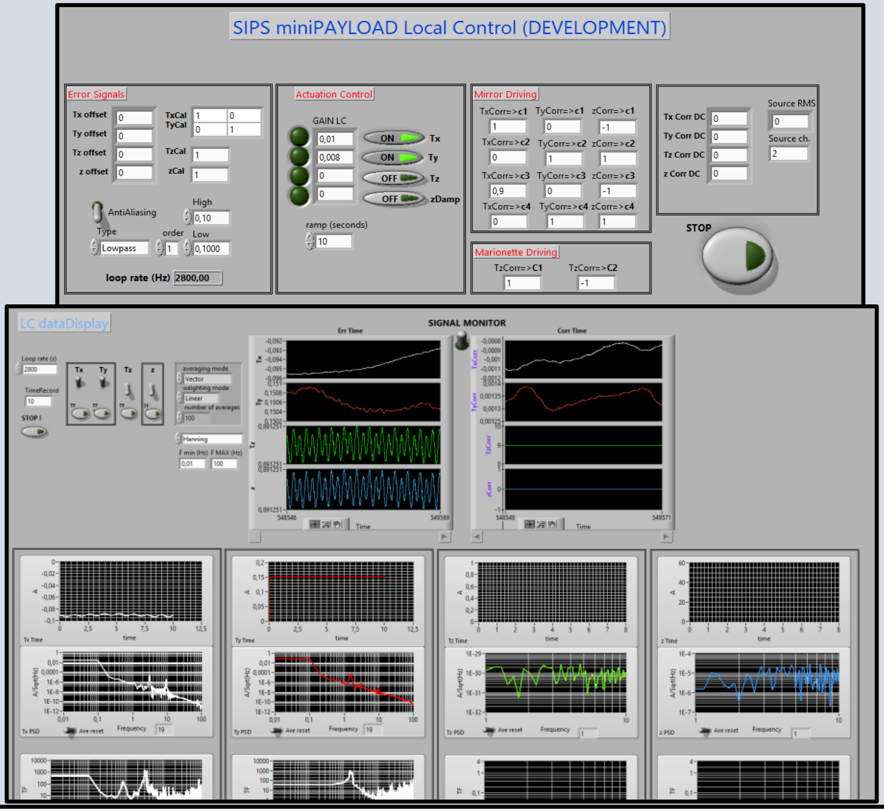
This allows angular and linear control within an accuracy of 10 nrad and 0.1 μm RMS.

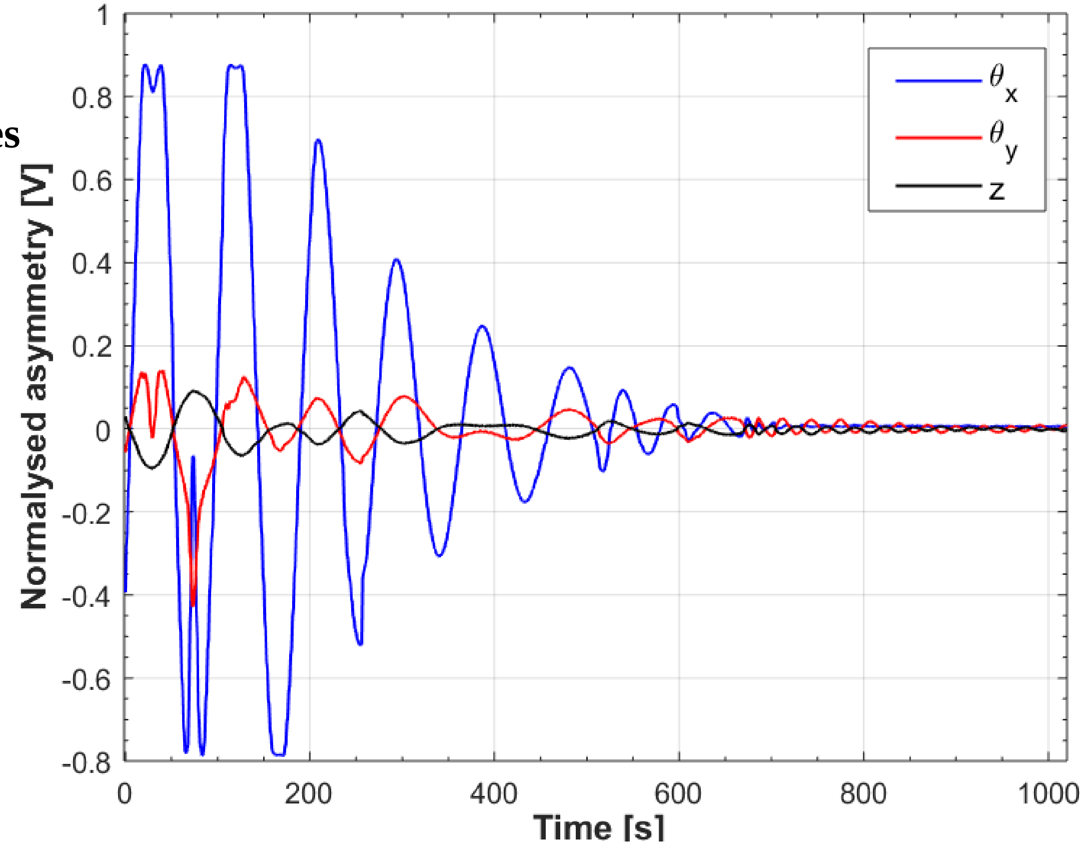
Local control digital system: status of art



Labview software: implement the digital control loops for monitor and real-time feedback cancellation



Pre-alignment recovery of the end mirror



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Attenuation of the amplitude relative to θ_x , θ_y and **z** DOFs.

θ_z control will be done acting on the marionette

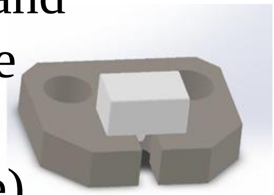
Optimization of SIPS control: status of art

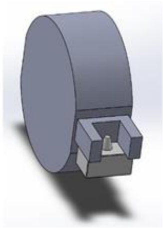
➢ Improvement of mechanical design of the suspension system:

I) *New Monolithic Suspensions*

<u>Old design:</u>
- Two suspension fibers (one for each flat side).

- Ear-anchor system and ear-flat mirror surface silica bonding (Advanced Virgo like)

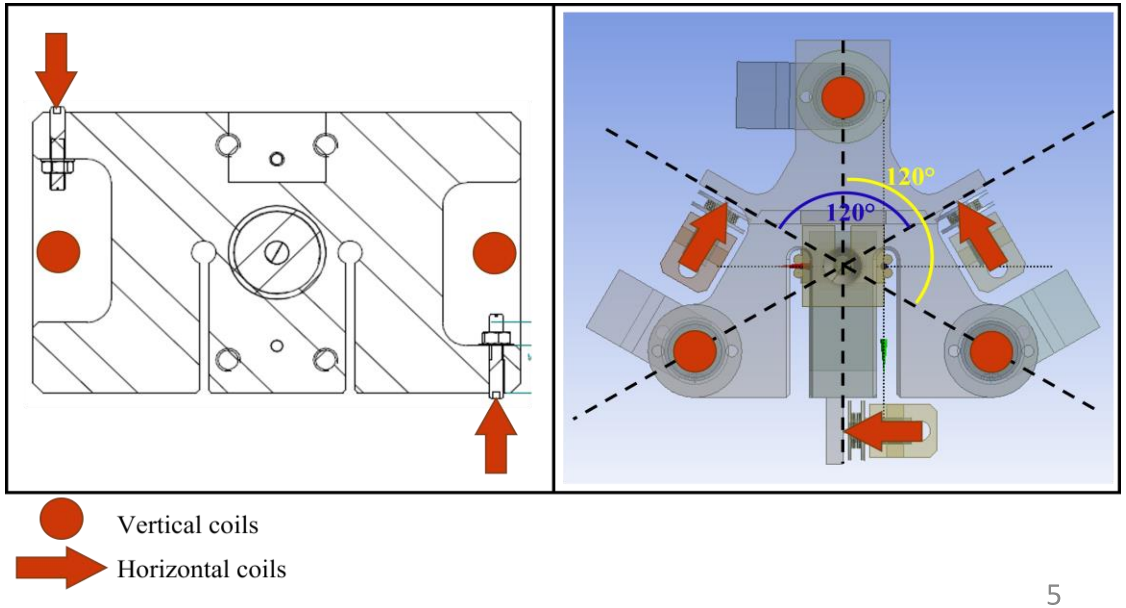




<u>New design:</u>
- Four suspension fibers, bonded to flat part of the mirrors
- New geometry for the bonding between mirror and fibers (in progress).

II) New Marionette design

- implementation of two more coil-magnet systems for the actuation to improve the control of θ_x and θ_y degrees of freedom.
- presence of a SiO_2 block, as a component of the new monolithic suspension system, with two SiO_2 fibers attached on each side of it.



Optimization of SIPS control system

➢ Engineering control problem approach:

- ❑ Problem: control of **coupled dynamical systems** → Develop a model for the interaction between mirrors and wave field of the laser;
- ❑ Mathematical approach, **Pontryagin control equations** → complete control of all the optical elements, simultaneously, to optimize the performance of the system;

Optimization of SIPS control: analytical model development

- We first develop a **nonlinear theory** of the scattering phenomenon describing the expression for the **radiation pressure** acting on a body moving in the wave field by using **Hamilton's variational principle**.

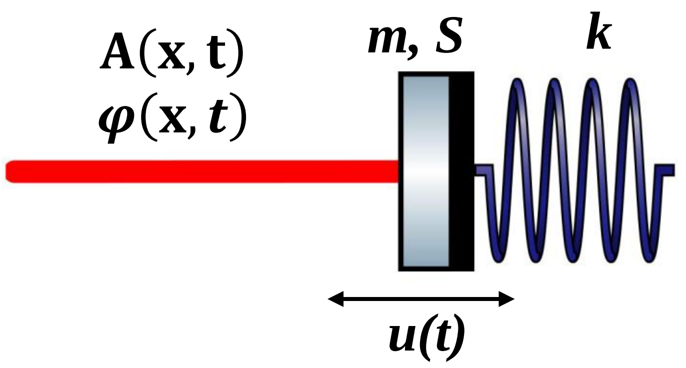
- The model presented uses the **equations of the wave field coupled with the resonator** (end mirror) **by the boundary conditions**

- The final expressions of the wave field and the resonator motion $\boldsymbol{u(t)}$ are determined.

Optimization of SIPS control: analytical model development

- We examine the **one dimensional** problem of a light beam impinging on a dielectric mirror with surface ***S***, and mass ***m***, connected to a spring of stiffness ***k***.

- introducing the scalar φ and vector potential A for the electric and magnetic field, E and B respectively: $\boldsymbol{E} = -\nabla\varphi - \boldsymbol{A}_t$ $\boldsymbol{B} = \nabla \times \boldsymbol{A}$



- ***Variational principle approach:***

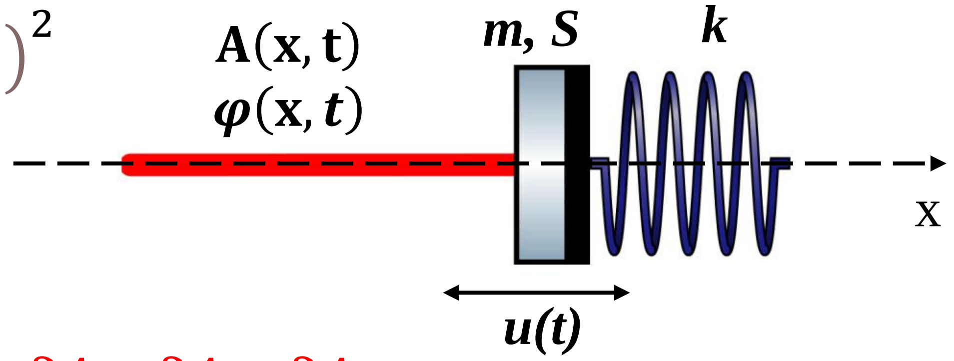
  - $L_{mech} = \frac{1}{2}m\dot{u}^2 - \frac{1}{2}ku^2$
  - $\mathcal{L}_{em} = \frac{1}{\mu_0}|\boldsymbol{B}|^2 - \varepsilon_0|\boldsymbol{E}|^2 = \frac{1}{\mu_0}|\nabla \times \boldsymbol{A}|^2 - \varepsilon_0|\nabla\varphi + \boldsymbol{A}_t|^2$
  - Total Action of system:

$$\mathcal{A}_{tot} = \mathcal{A}_{mech}(u, \dot{u}) + \mathcal{A}_{em}(\varphi_x, \boldsymbol{A}_t, \boldsymbol{A}_x)$$
$$= S\int_{t_1}^{t_2}\left\{\frac{1}{2}m\,\dot{u}^2(t) - \frac{1}{2}k\,u^2(t) + \left(\int_u^l \mathcal{L}_{em}\,dx\right)\right\}dt$$

Optimization of SIPS control: analytical model development

- Assuming the only non-zero derivatives are those with respect to the x propagation axis, the 1-D case simplifies as: $|\boldsymbol{E}|^2 = (\varphi_x + A_{x,t})^2 + (A_{y,t})^2 + (A_{z,t})^2$

$$|\boldsymbol{B}|^2 = (A_{y,x})^2 + (A_{z,x})^2$$



- According to variational principle:

$\boxed{\delta\mathcal{A}_{tot} = 0}$

➢ we compute the variations δu, $\delta\varphi$, δA_x, δA_y, δA_z

$$= -\int_{t_1}^{t_2}\{m\,\ddot{u}(t) + k\,u(t) + S[\mathcal{L}_{em}]_u\}\delta u\,dt$$

$$+ S\int_{t_1}^{t_2}\int_u^l\left[2\varepsilon_0(\varphi_{xx} + A_{x,xt})\delta\varphi + 2\varepsilon_0(\varphi_{xt} + A_{x,tt})\delta A_x + \left(2\varepsilon_0 A_{y,tt} - \frac{2}{\mu_0}A_{y,xx}\right)\delta A_y\right.$$

$$\left. + \left(2\varepsilon_0 A_{z,tt} - \frac{2}{\mu_0}A_{z,xx}\right)\delta A_z\right]dx\,dt$$

$$+ \boxed{S\int_{t_1}^{t_2}\left\{\frac{2}{\mu_0}[A_{y,x}\delta A_y]_u^l + \frac{2}{\mu_0}[A_{z,x}\delta A_z]_u^l - 2\varepsilon_0[(\varphi_x + A_{x,t})\delta\varphi]_u^l\right\}dt} \rightarrow \textbf{Boundary terms}$$

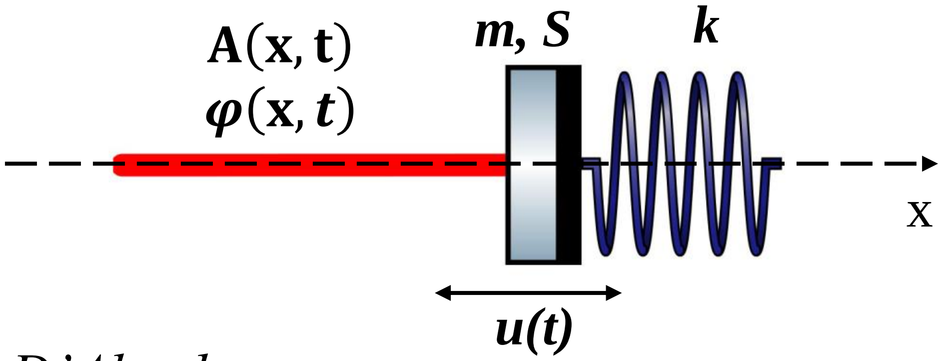
Optimization of SIPS control: analytical model development

- Hamilton's principle produces:

1. the potential field equations:

$$\begin{cases} \varphi_{xx} + A_{x,xt} = 0 \\ \varphi_{xt} + A_{x,tt} = 0 \end{cases} \rightarrow \begin{cases} \frac{1}{c_0^2}\varphi_{tt} - \varphi_{xx} = 0 \\ \frac{1}{c_0^2}A_{y,tt} - A_{y,xx} = 0 \\ \frac{1}{c_0^2}A_{z,tt} - A_{z,xx} = 0 \end{cases}$$

D'Alembert wave equations



2. boundary conditions

$$\begin{cases} \left(\varphi_x + A_{x,t}\right)\delta\varphi\Big|_u = 0 \\ A_{y,x}\delta A_y\Big|_u = 0 \\ A_{z,x}\delta A_z\Big|_u = 0 \end{cases}$$

3. ***mirror equation of motion***

$$m\ddot{u} + ku = \mathcal{L}\Big|_u$$
$$= S\left\{\frac{1}{\mu_0}\left[\left(A_{y,x}\right)^2 + \left(A_{z,x}\right)^2\right]\right.$$
$$\left.- \varepsilon_0\left[\left(\varphi_x + A_{x,t}\right)^2 + \left(A_{y,t}\right)^2 + \left(A_{z,t}\right)^2\right]\right\}\Big|_u$$

Optimization of SIPS control: analytical model development

- we can **find the wave solution** in the general propagation D'Alembert form, for the ***local non-resonant coupling model***:

$$\varphi(x,t) = \boxed{\varphi_i(x+ct)} + \boxed{\varphi_r(x-ct)}$$
$$A_y(x,t) = \boxed{A_{yi}(x+ct)} + \boxed{A_{yr}(x-ct)}$$
$$A_z(x,t) = \boxed{A_{zi}(x+ct)} + \boxed{A_{zr}(x-ct)}$$

unknown

known

- and we can **solve *reflection problem***, calculating in terms of the four unknowns $\varphi_r, A_{yr}, A_{zr}, u$, by solving the system:

$$\begin{cases} m\ddot{u} + ku = S\left\{\frac{1}{\mu_0}\left[\left(A_{y,x}\right)^2 + \left(A_{z,x}\right)^2\right] - \varepsilon_0\left[\left(\varphi_x + A_{x,t}\right)^2 + \left(A_{y,t}\right)^2 + \left(A_{z,t}\right)^2\right]\right\}\Big|_u \\ \left(\varphi_x + A_{x,t}\right)\delta\varphi\Big|_u = 0 \\ A_{y,x}\delta A_y\Big|_u = 0 \\ A_{z,x}\delta A_z\Big|_u = 0 \end{cases}$$

- this can be manipulated to both permits the theoretical analysis of the opto-mechanical coupling and to be useful for ***numerical simulations*** and to ***tune an optimal control technique*** → ongoing work.

Optimization of SIPS control: Pontryagin's approach for control technique

- From analytical model → ***nonlinear* optomechanical coupling** → spurious frequencies in the reflected light spectrum (treated analytically in another work, compared with acoustic case)

↓

Pontryagin's maximum principle

allows finding efficient control equations for complex systems like SIPS case, based on wave models close to boundaries (mirrors)

→ more complicated approach with respect to those techniques that make direct use of the transfer function, but it gives more accurate results from the point of view of the system response to control actuation.

Optimization of SIPS control: Pontryagin's approach for control technique

The prototype of the local control problem is represented by the equations

$$\begin{cases} m\ddot{u} + ku = S\left\{\frac{1}{\mu_0}\left[\left(A_{y,x}\right)^2 + \left(A_{z,x}\right)^2\right] - \varepsilon_0\left[\left(\varphi_x + A_{x,t}\right)^2 + \left(A_{y,t}\right)^2 + \left(A_{z,t}\right)^2\right]\right\}\Big|_u \\ \left(\varphi_x + A_{x,t}\right)\delta\varphi\Big|_u = 0 \\ A_{y,x}\delta A_y\Big|_u = 0 \\ A_{z,x}\delta A_z\Big|_u = 0 \end{cases}$$

+

Control vector term $\underline{f_c}$ → in terms of $u(t)$, φ_r, A_{yr}, A_{zr}

(i.e. control force applied to the mirror)

- transform a partial derivative system into an ordinal differential equation system:

$$\underline{\dot{x}} = A\underline{x} + B\,\underline{f_c}(\underline{s})$$

- define a criterion for the choice of the function $\underline{f_c}(\underline{s})$, where $\underline{s} = G\underline{x}$ represents vector of signal from the sensors

Optimization of SIPS control: Pontryagin's approach for control technique

- build a suitable ***objective function***: $\boldsymbol{J} = \int_0^T Qu^2 + Rf_c^2 \, dt$

control force applied to the mirror, operated by coil-magnet pairs

- introducing a *modified functional* $\tilde{J}$, adding a constrain:

$$\tilde{J}(\underline{x}, \underline{f_c}, \underline{\lambda}) = \int_0^T \left[\mathcal{L} + \underline{\lambda}^T \left(\underline{\dot{x}} - A\underline{x} - B\underline{f_c}\right)\right] dt, \qquad \text{with } \underline{\lambda} = \underline{\lambda}(\mathrm{t})$$

- $\underline{f_c}$ must satisfy $\delta J \rightarrow 0$, using the method of ***Lagrange multipliers*** → find the <u>*independent variables*</u> $\underline{x}(\mathrm{t}), \underline{f_c}, \underline{\lambda}(\mathrm{t})$ (easly obtained analytically) → simultaneously obtain all the components of $\underline{f_c}$

→ Obtain a **simultaneous integrated feedback control**

→ Ensure the system **stability**

Conclusions and next developments

1. Finalize theoretical analysis of the opto-mechanical coupling → ***numerical simulations***
2. Analyze signals from PSDs → Empirical Mode Decomposition technique: allows to identify the signal components, in order to find and classify possible non-linearities of the system, based on analytical model developed
3. Start from the described <u>mathematical approach</u>, defining the requirements for the ***Objective function J***, which better represents the desired system performance
4. Apply Pontryagin's approach, together with an extended Kalman's filter, and using the new FLOP technique, to be able to produce the desired feedback→ ***optimize the performance of SIPS***
5. SIPS represents a suitable test bench for the new proposed optimal control technique, with also the outlook on a possible application in GW interferometer.

THANKS FOR YOUR ATTENTION