

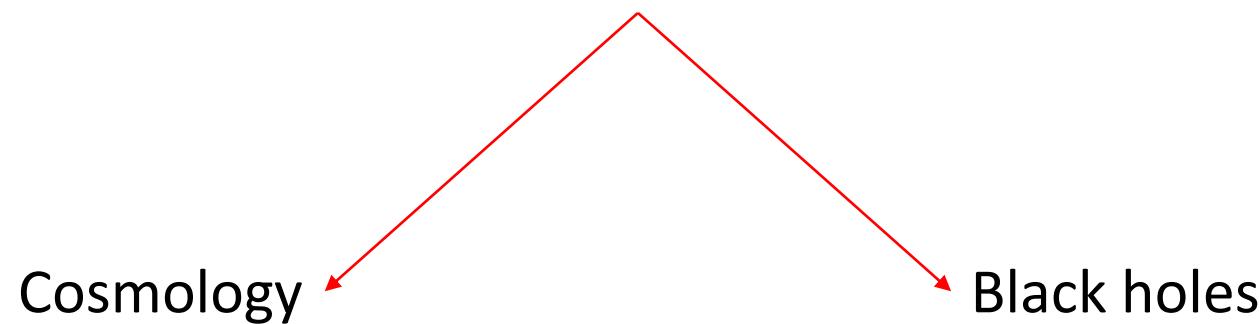
# Does the Weyl invariant based proposal provide an accurate description of gravitational entropy?

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# Question

How is the gravitational entropy of spacetime determined?



Conditions:

Cosmology

$$S_{BH} = \frac{A[B]}{4G\hbar}$$

1. Vanish only in conformally flat spacetimes,
2. Measure local anisotropy,
3. Reduce to Bekenstein-Hawking entropy for black holes

(Penrose 1980)

$g_{\mu\nu} = \Omega^2 \eta_{\mu\nu}$   
Petrov O type

# Gravitational entropy $\rightarrow$ Weyl curvature

$C_{abcd}$  vanishes in conformally flat spacetimes

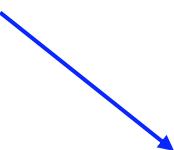


Petrov type O spacetime:  
all NP-Weyl scalars are zero

Measures local anisotropies

We can define estimator for gravitational entropy:  $S_{grav}^{Weyl} = \int_V C_{abcd} C^{abcd} dV$

(arXiv:1510.09027 [gr-qc])



Black hole entropy:  
 $S_{BH} \sim A[B]$

How appropriate is this?

de Sitter gravitational entropy  $\rightarrow$  Horizon entropy  $\neq S_{grav}^{Weyl}$

$D = 4$  won't work due to dimensional problems

Modified gravity = more problems...



Main conditions satisfied

# Gravitational entropy $\rightarrow$ Weyl curvature

Clifton, Ellis and Tavakol: non-zero NP-Weyl scalars  $\Psi_2$  and  $\Psi_4$

“Super energy-density” function

$$W = T_{abcd} u^a u^b u^c u^d$$

“square-root” of the Bel-Robinson tensor

$$t_{ab} = \epsilon |\Psi_4| k_a k_b$$

$$\tau_{ab} = 3|\Psi_2| \epsilon (l_{(a} n_{b)} + m_{(a} \bar{m}_{b)})$$

$$8\pi\rho_{\text{grav}} = \beta \left( \frac{\epsilon}{2} |\Psi_4| - \lambda_2 \right)$$

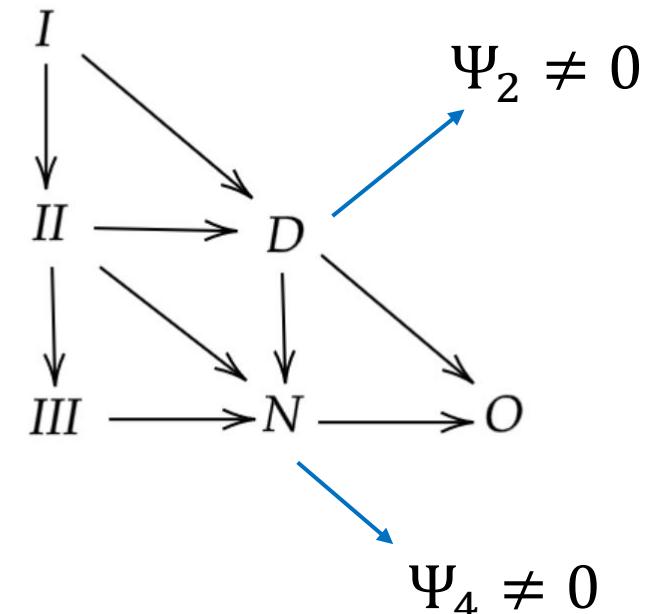
$$8\pi p_{\text{grav}} = \beta \left( \frac{\epsilon}{6} |\Psi_4| + \lambda_2 \right)$$

$$8\pi\pi_{ab}^{\text{grav}} = -\beta \left( \frac{\epsilon}{6} |\Psi_4| (x_a x_b + y_a y_b - 2z_a z_b) \right)$$

$$8\pi q_a^{\text{grav}} = \beta \frac{\epsilon}{2} |\Psi_4| z_a,$$

$$8\pi\rho_{\text{grav}} = \alpha \left( \frac{3}{2} \epsilon |\Psi_2| - f \right),$$

$$8\pi\pi_{ab}^{\text{grav}} = \frac{\alpha}{2} \epsilon |\Psi_2| (x_a x_b + y_a y_b - z_a z_b + u^a u^b),$$



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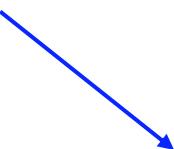


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WAIT.

Black holes have singularities → Divergence of Weyl curvature

$$S_{grav}^{Weyl} = \int_V C_{abcd} C^{abcd} dV \Big|_0^{r_H} \rightarrow \text{div}$$

Define a spherical element of radius  $\epsilon$  around  $r = 0$  and subtract this

$S_{BH}$       (arXiv:1510.09027 [gr-qc])

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What about cosmic censorship?      (arXiv:gr-qc/9705060)

Outgoing geodesics from naked singularity = outgoing geodesics from an initial singularity

Einstein field equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \kappa T_{\mu\nu}$$

Uncharged Vaidya spacetime in asymptotically flat spacetime ( $\Lambda = 0$ )

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right)dv^2 + 2dudv + r^2d\Omega^2$$

Weyl curvature at  $r = 0$  is divergent

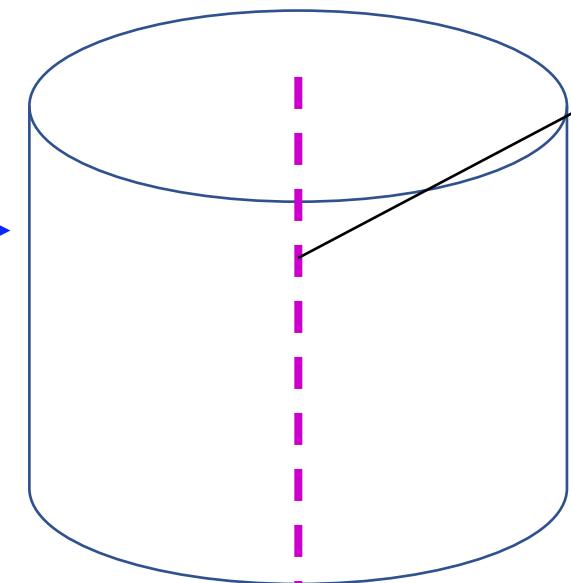
$I^+$

$$\delta\mathcal{P}(x^\mu) < 0$$

$$\mathcal{W} = \frac{12\chi^2 v^2}{r^6}$$

arXiv:2211.11017 [gr-qc]

No horizon



Background conjecture: *Weyl curvature is always increasing along outgoing geodesics from a naked singularity*

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Uncharged Vaidya spacetime in asymptotically flat spacetime ( $\Lambda = 0$ )

$$ds^2 = - \left( 1 - \frac{2m(v)}{r} + \frac{q^2(v)}{r^2} - \frac{Ar^2}{3} \right) dv^2 + 2dudv + r^2 d\Omega^2$$

Slope condition:

$$\frac{dr}{dv} = \frac{1}{2} \left( 1 - 2\lambda X + \theta^2 X^2 - \frac{Ar^2}{3} \right)$$

$$X_0 = \lim_{r,v \rightarrow 0} \frac{v}{r}$$

$$\theta^2 X_0^3 - 2\lambda X_0^2 + X_0 - 2 = 0$$

arXiv:2211.11017 [gr-qc]

Which has a positive root, implying a locally naked singularity

$$\mathcal{W}(X, r) = \frac{48}{r^4} (\lambda^2 X^2 - 2\lambda \theta^2 X^3 + \theta^4 X^4)$$

Divergent at the singularity

$\rightarrow \delta\mathcal{P}(x^\mu) < 0$

# Black hole entropy

At  $r = 0$   $C_{abcd}$  diverges  $\longrightarrow$  Remove a small area element around it

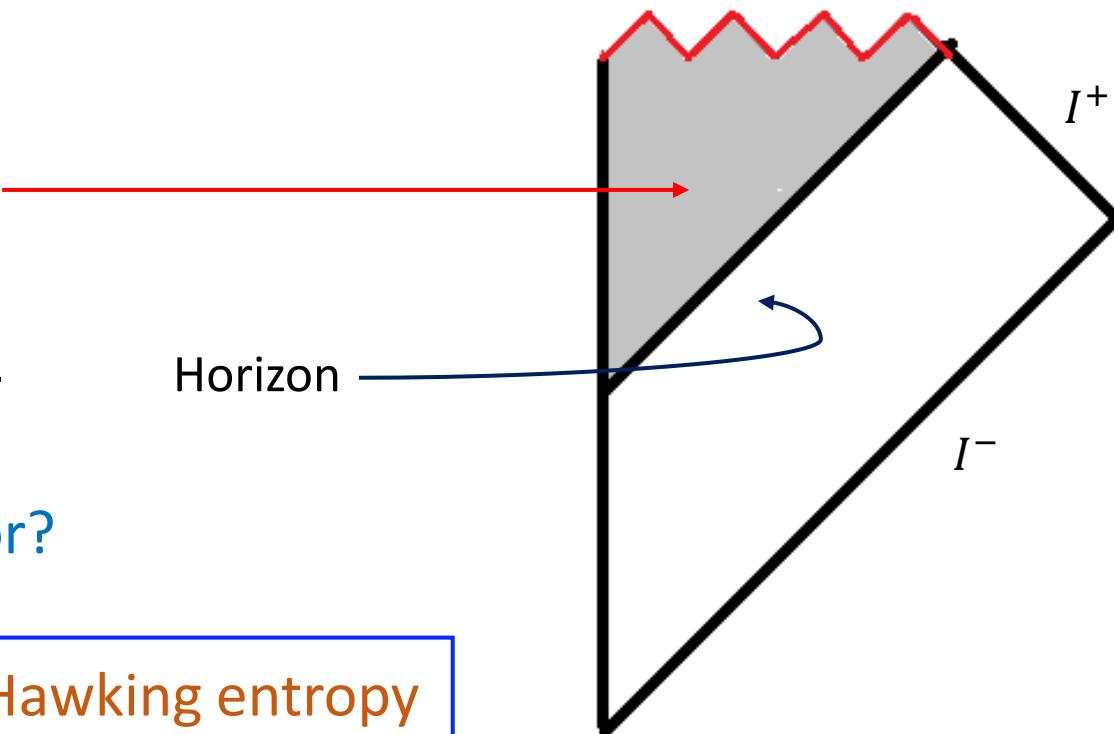
Interior does not affect exterior thermodynamically – ongoing work

Causally disconnected region

$$\text{Bekenstein-Hawking entropy: } S_{BH} = \frac{A[B]}{4G\hbar}$$

When are you allowed to consider interior?

Gravitational entropy  $\rightarrow$  Bekenstein-Hawking entropy



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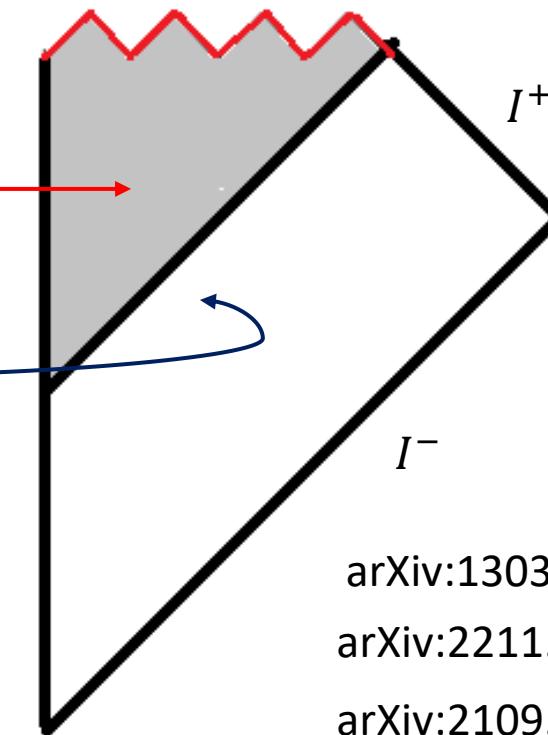
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Horizon

When are you allowed to consider interior?

Gravitational entropy  $\rightarrow$  Bekenstein-Hawking entropy



arXiv:1303.5612 [gr-qc]

arXiv:2211.11017 [gr-qc]

arXiv:2109.11968 [gr-qc]

# Horizon appropriate measure

Difference between Bekenstein-Hawking and Weyl?

- Thermodynamic entropy -- Gravitational entropy
- No effect from interior -- Interior affects total measure
- Does not work for arbitrary  $r$  -- Defines BH-like entropy

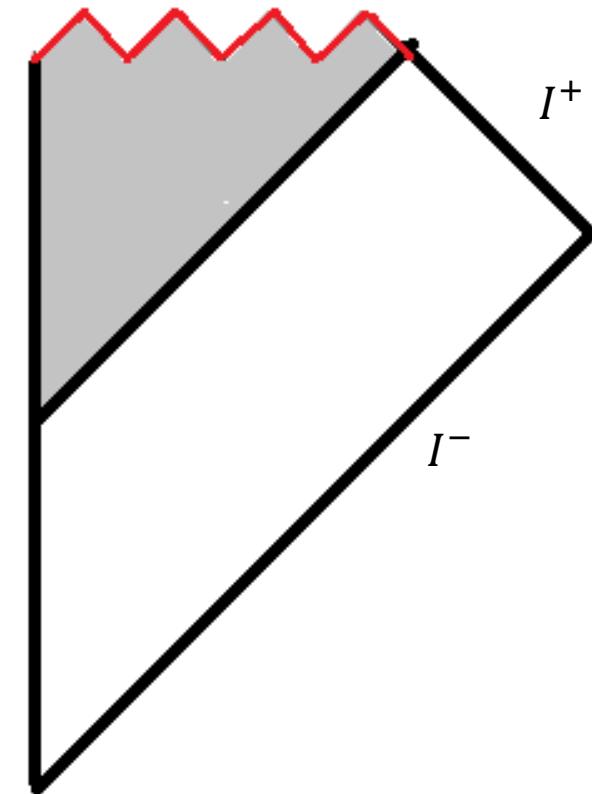
**NO HORIZON = NO BOUNDARY**

Bekenstein-Hawking entropy

Gravitational entropy

Thermodynamics

$$S \neq \frac{A[B]}{4G\hbar}$$



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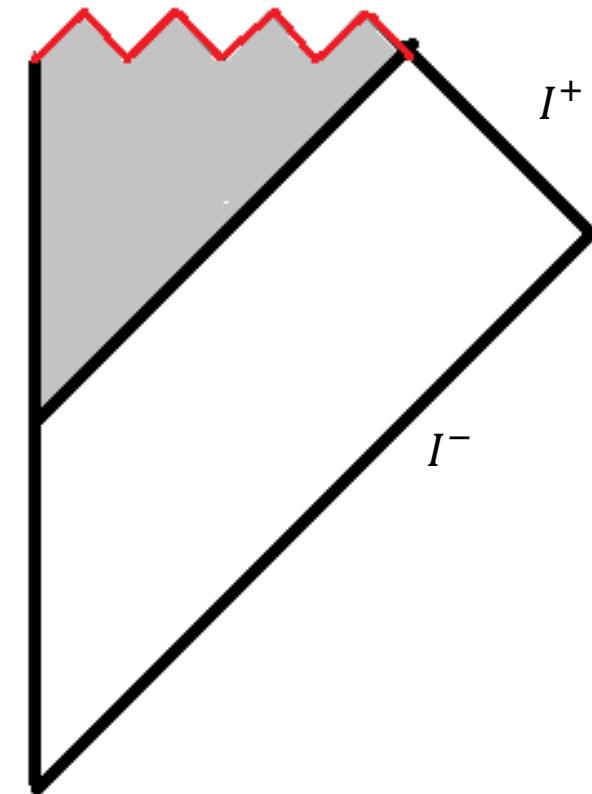
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Bekenstein-Hawking entropy      Gravitational entropy      Thermodynamics

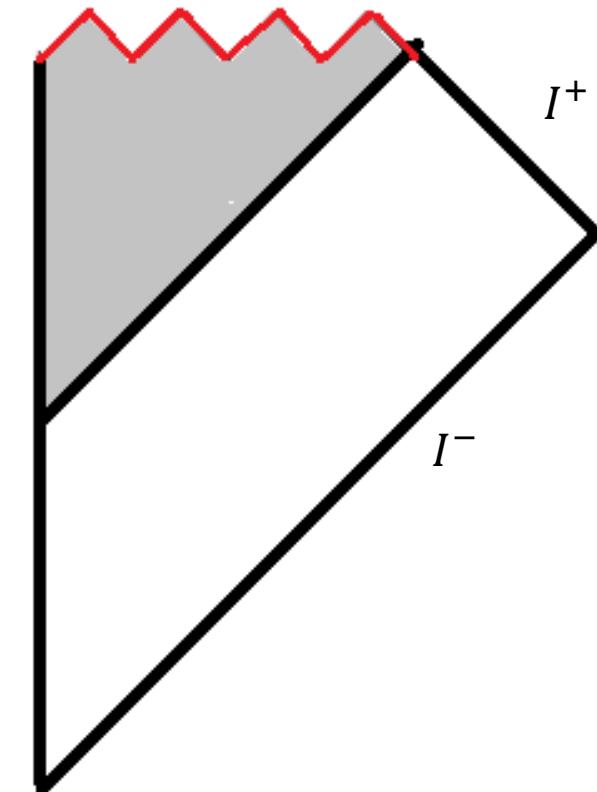
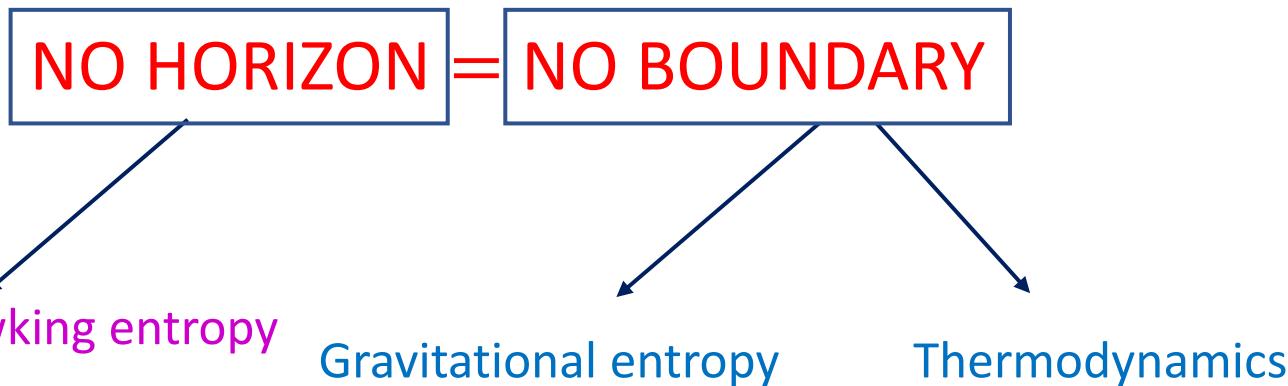
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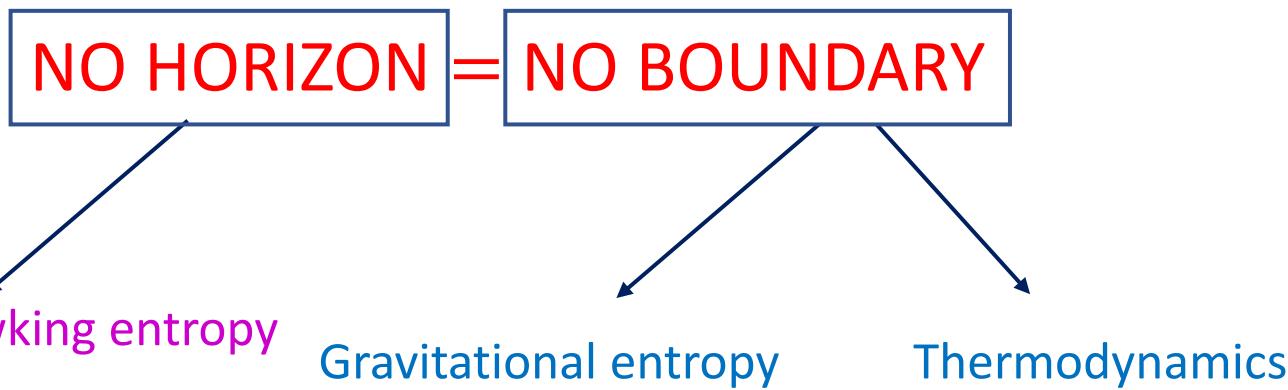


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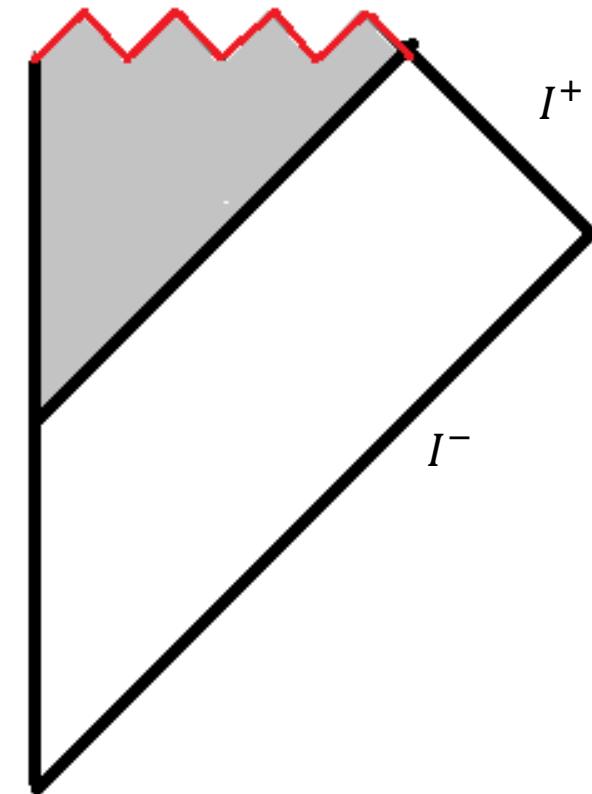
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Thank you for your  
attention!!!!!!