Nonperturbative corrections and hypothesis of vacuum dominance

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IUPAP

XI International Conference New Frontiers in Physics 2022

Abstract

We check the range of values of four-quark condensate, relying on the processing of experimental data on e^+e^--annihilation into even number of pions (BaBar, CMD-2, OLYA) and extracting from them the values of the nonperturbative corrections. According to the hypothesis of vacuum dominance, the vacuum average of the four-quark condensate is expressed in terms of quark condensate while only vacuum intermediate states contribution is taken into account. However, assuming a possible non-zero effect of non-vacuum intermediate states, we check its influence on the four-quark condensate, or coefficient C_6 in operator product expansion. Moreover, we explore how other nonperturbative corrections, C_2 (operator with dimension 2), and C_4 (which includes gluon condensate), are changed when assuming this effect. It is shown that values of four-quark condensate, or coefficient C_6, can vary not more than 15-20%. Other parameters (C_2 and C_4) take values close to the currently available ones.

Motivation

Studies of quark and gluon condensates in the framework of operator product expansion (OPE) were initiated by M.A. Shifman, A.I. Vainstein and V.I. Zakharov and reflected in the seminal review [1]. Nonperturbative effects are related to non-vanishing vacuum average values for the local operators in OPE. These topics are discussed for example in work [2]. In our previous works [3–5] the operator of dimension two, C_2, related to short string, mostly is explored, while the 4-quark condensate or C_6 was taken as a constant value. Is shown that C_2 values are strongly (anti)correlated with C_4 and compared with zero.

Short string is characterized by potential kr (string potential), in well-known Cornell potential, $V(r) \approx -4\,\alpha_s(r)/(3r) + kr$, and it is connected to the phenomenon of confinement. At short distances is leads to the correction $\sim k/Q^2$. The concept of short strings was suggested in the pioneering paper [6].

In this work, we do not fix the four-quark condensate and we estimate the range in which the value of C_6 can vary, comparing gluon condensate to the available estimates of SVZ [1], Geshkenbein, Ioffe, Zyablyuk [7, 8].

As a reference point we use the vacuum dominance approach, when C_6 is related to the square of the quark condensate estimated of using the Gell-Mann-Oakes-Renner (GMOR) formula [9] for the two lightest flavors:

$$(m_u + m_d)\langle 0|\bar{u}u + \bar{d}d|0\rangle = -m_\pi^2 f_\pi^2,$$

where m_π and f_π are pion mass and decay constant. The data on quark and pion masses and decay constant are taken from PDG [10]. We estimate quark condensate and calculate the coefficient C_6 through 4-quark operatorand C_4 through gluon condensate:

$$C_6 = -\frac{448\pi^3}{27}\alpha_s\langle 0|\bar{q}q|0\rangle^2, \quad C_4 = \frac{2\pi^2}{3}\left\langle 0\left|\frac{\alpha_s\,GG}{\pi}\right|0\right\rangle.$$

The purpose of the work is to check how vacuum dominance works: the possibility of other intermediate states, connected with non-diagonal elements of the scattering matrix that make a non-zero contribution to four-quark condensate. How other nonperturbative corrections (C_2 and C_4 or gluon condensate) change when considering additional contribution of intermediate states?

In our analysis we consider contemporary experimental data: $e^+e^- \to \pi^+\pi^-$, CMD-2, OLYA [11], $e^+e^- \to 2\pi^+2\pi^-$ and $e^+e^- \to \pi^+\pi^-2\pi^0$, BaBar [12, 13], $e^+e^- \to 3\pi^+3\pi^-$ and $e^+e^- \to 2\pi^+2\pi^-2\pi^0$ BaBar [14].

Theoretical framework: R-ratio and D-function

R-ratio: $$R(s) = \frac{\sigma_{e^+e^-\to\text{hadrons}}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)}, \quad \sigma_{e^+e^-\to\mu^+\mu^-}(s) = \frac{4\pi\alpha_{\text{em}}^2}{3s}.$$

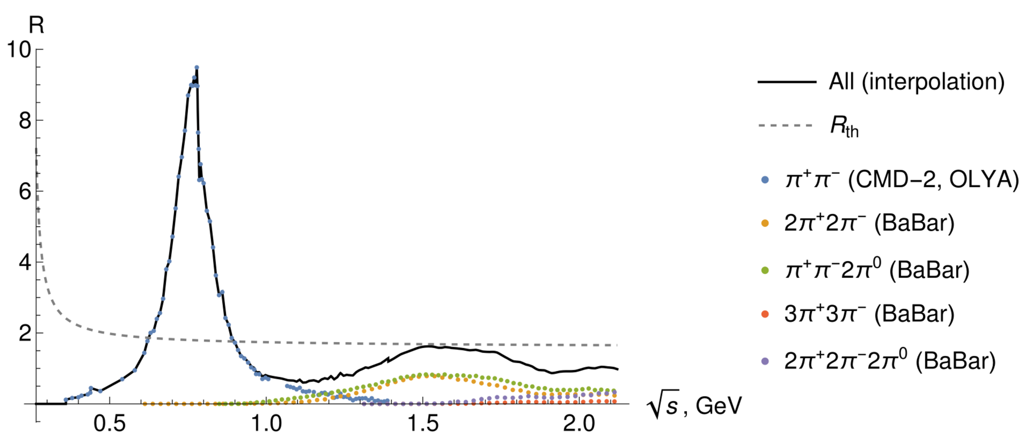


Fig. 1: The full R-ratio in dependence on $\sqrt{s}$ *at* $\sqrt{s} \leq 3$ GeV (black), the experimental data (blue, green, orange, violet and red dots), the theoretical representation R_{th} in the PT (red, dashed) and in the APT (blue, dashed). The continuum threshold is $s_0 \approx 1.52^2$ GeV^2.

Theoretical form: $$R_{\text{th}}^{\text{PT}}(s) = N_c \sum_q e_q^2\left(1 + \frac{\alpha_s^{\text{PT}}(s)}{\pi}\right).$$

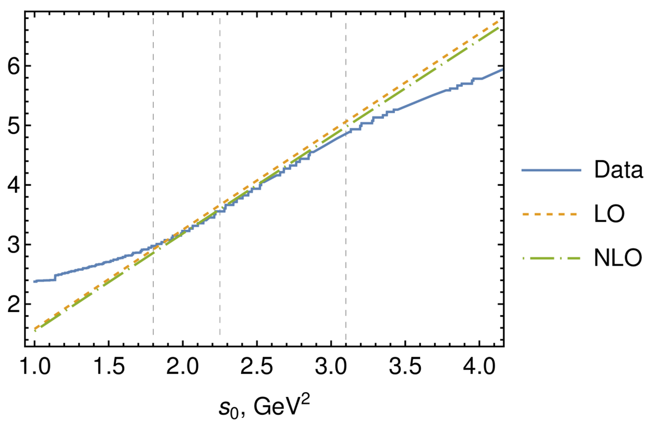
where $$\alpha_s(Q^2) = \frac{4\pi}{\beta_0}\frac{1}{\ln(Q^2/\Lambda^2)}.$$

Dispersional D-function: $$D_{\exp}(Q^2) = Q^2\int_{4m_\pi^2}^{\infty}\frac{R_{\text{exp-th}}(s)ds}{(s+Q^2)^2}.$$

D-function in OPE framework with using perturbation theory approach:

$$D_{\text{PT+OPE}}(Q^2) = \frac{3}{2}\left(1 + \frac{\alpha_s(Q^2)}{\pi} + \sum_{n\geq 1}\Gamma(n)\frac{C_{2n}}{Q^{2n}}\right),$$

Fig. 2: The quark-hadron duality integral $I_{dual} = \int_{4m_\pi^2}^{s_0} ds\, R_{exp}(s) = \int_{4m_\pi^2}^{s_0} ds\, R_{th}(s)$ versus the upper integration limit, s_0. The integral for the experimental data corresponds to the blue solid line (Data). The theoretical curves are the dashed orange (LO) and dot-dashed green (NLO) lines. $s_0 \approx 1.52^2$ GeV^2.



Borel transform (BT) and D-function

BT of D-function: $$\hat{B}_{Q^2\to M^2}\left[D_{\exp}(Q^2)\right] = \Phi_{\exp}(M^2) = \int_{4m_\pi^2}^{\infty} R_{\text{exp-th}}(s)\left(1 - \frac{s}{M^2}\right)e^{-s/M^2}\frac{ds}{M^2},$$

in **PT**: $$\hat{B}_{Q^2\to M^2}\left[D_{\text{PT+OPE}}(Q^2)\right] = \Phi_{\text{PT+OPE}}(M^2) = \frac{3}{2}\left(\frac{\hat{B}_{Q^2\to M^2}\left[\alpha_s(Q^2)\right]}{\pi} + \frac{C_2}{M^2} + \frac{C_4}{M^4} + \frac{C_6}{M^6}\right),$$

where $$\hat{B}_{Q^2\to M^2}\left[\alpha_s(Q^2)\right] = \frac{4\pi}{b_0}\left[\frac{1}{M^2}\int_0^{\infty}\frac{e^{-s/M^2}ds}{\ln^2(s/\Lambda^2) + \pi^2} + \frac{\Lambda^2}{M^2}e^{\Lambda^2/M^2}\right]$$

Equating both forms gives sum rule: $$\Phi_{\exp}(M^2) = \Phi_{\text{PT+OPE}}(M^2).$$

Extracting of the condensates

Firstly, the coefficient $C_6 = -\frac{448\pi^3}{27}\alpha_s\langle 0|\bar{q}q|0\rangle^2 \approx -0.102$ GeV^2 is fixed, see panel a).

Then we find C_2 and C_4 by equating both forms of Adler function using sum rule. The chosen range on squared Borel mass is $M^2 \in [0.75; 3.50]$GeV^2.

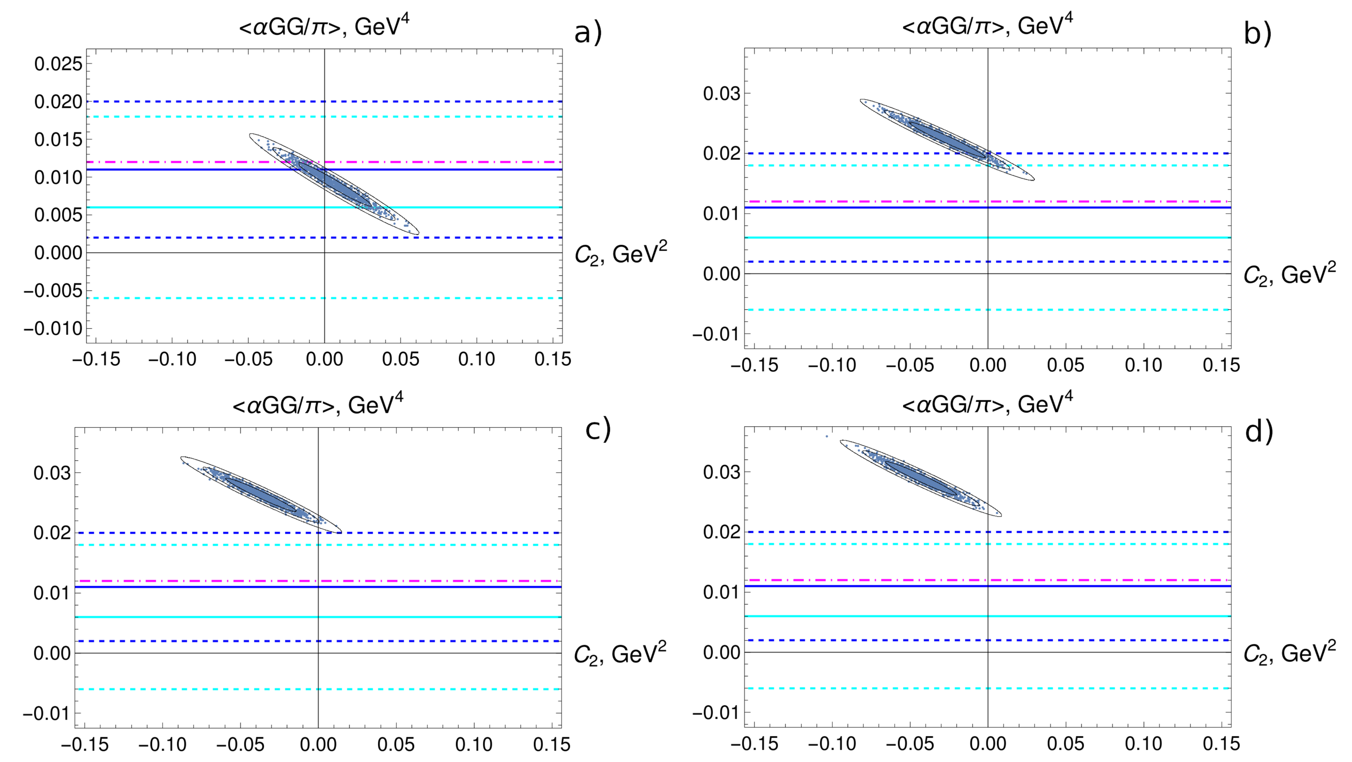


Fig. 3: The allowed regions of C_2 and gluon condensate with various multiplication factors k for $C_6 = k \cdot C_6^{\text{GMOR}}$ within 1, 2 and 3 standard deviations at the panels: $\mathrm{k} = 1$; b) $\mathrm{k} = 1.5$; c) $\mathrm{k} = 1.65$; d) $\mathrm{k} = 1.75$; e) $\mathrm{k} = 1.8$; f) $\mathrm{k} = 2$. The existing estimates for gluon condensate from SVZ [1] are marked by dot-dashed violet line, those from works of Geshkenbein, Ioffe, Zyabluyk [7], [8] are marked by solid blue and light-blue lines with their limits marked by dashed lines with the same colors.

C_6	C_2, GeV^2	C_4, GeV^4	$\langle\frac{\alpha_s GG}{\pi}\rangle$, GeV^4	χ^2	$\langle\alpha_s GG/\pi\rangle(C_2)$, GeV^4
C_6^{GMOR}	0.006 ± 0.016	0.060 ± 0.013	0.009 ± 0.002	0.968	$-0.120C_2 + 0.010$
$C_6^{\text{GMOR}} \times 1.5$	-0.026 ∓ 0.016	0.147 ± 0.013	0.022 ± 0.002	0.440	$-0.121C_2 + 0.019$
$C_6^{\text{GMOR}} \times 1.65$	-0.037 ∓ 0.015	0.173 ± 0.012	0.026 ± 0.002	0.411	$-0.123C_2 + 0.022$
$C_6^{\text{GMOR}} \times 1.75$	-0.043 ∓ 0.015	0.190 ± 0.012	0.029 ± 0.002	0.428	$-0.123C_2 + 0.024$
$C_6^{\text{GMOR}} \times 1.8$	-0.046 ∓ 0.015	0.199 ± 0.012	0.030 ± 0.002	0.453	$-0.122C_2 + 0.025$
$C_6^{\text{GMOR}} \times 2.0$	-0.059 ∓ 0.015	0.234 ± 0.012	0.035 ± 0.002	0.603	$-0.120C_2 + 0.028$

Table 1: The extracted values for the interval $M^2 \in [0.75; 3.50]$GeV^2 in PT. The obtained values of condensates with errors, $\Lambda = 0.25$ GeV, $s_0 = 1.5^2$ GeV^2 and using $C_6^{\text{GMOR}} = -0.102$ GeV^6 fixed. In the last column the estimate of anticorrelation between gluon condensate and C_2 is given.

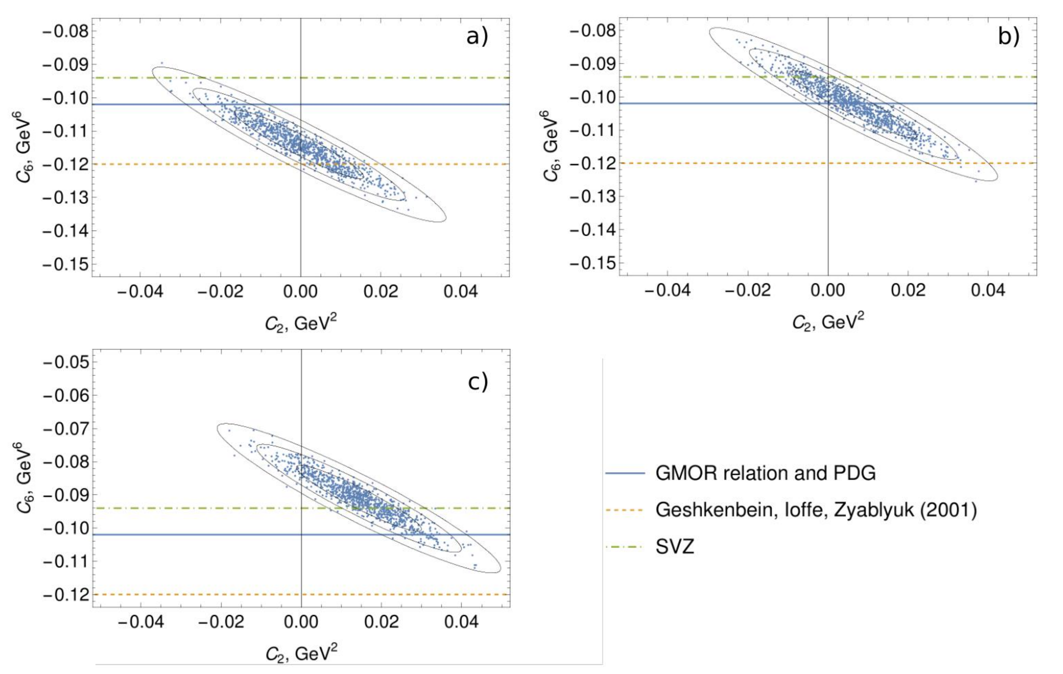


Fig. 4: The regions of existence of condensates C_2 and C_6 for different fixed gluon condensates (or C_4) at the panel a) $\langle\alpha_s\,\mathrm{GG}/\pi\rangle = 0.012$ GeV^4 (SVZ); b) $\langle\alpha_s\,\mathrm{GG}/\pi\rangle = 0.009$ GeV^4 (average of SVZ and value of Ioffe, Zuablyuk); c) $\langle\alpha_s\,\mathrm{GG}/\pi\rangle = 0.006$ GeV^4 (Ioffe, Zyablyuk).

The known values of quark condensate recalculated into C_6 are shown by horizontal lines.

C_4	$\langle\frac{\alpha_s GG}{\pi}\rangle$, GeV^4	C_2, GeV^2	C_6, GeV^6	χ^2	$C_6(C_2)$, GeV^6
C_4^{SVZ}	0.012	-0.0004 ∓ 0.011	-0.114 ∓ 0.007	0.786	$-0.637C_2 - 0.114$
C_4^{mid}	0.009	0.006 ± 0.010	-0.102 ∓ 0.007	0.969	$-0.642C_2 - 0.098$
C_4^{IZ}	0.006	0.014 ± 0.010	-0.091 ∓ 0.007	1.181	$-0.636C_2 - 0.082$

Table 2: The extracted values for the interval of $M^2 \in [0.75; 3.50]$GeV^2 in PT. The obtained values of condensates with errors, $\Lambda = 0.25$ GeV , $s_0 = 1.5^2$ GeV^2 and different C_4 fixed. In the last column the estimate of anticorrelation between C_6 and C_2 is given.

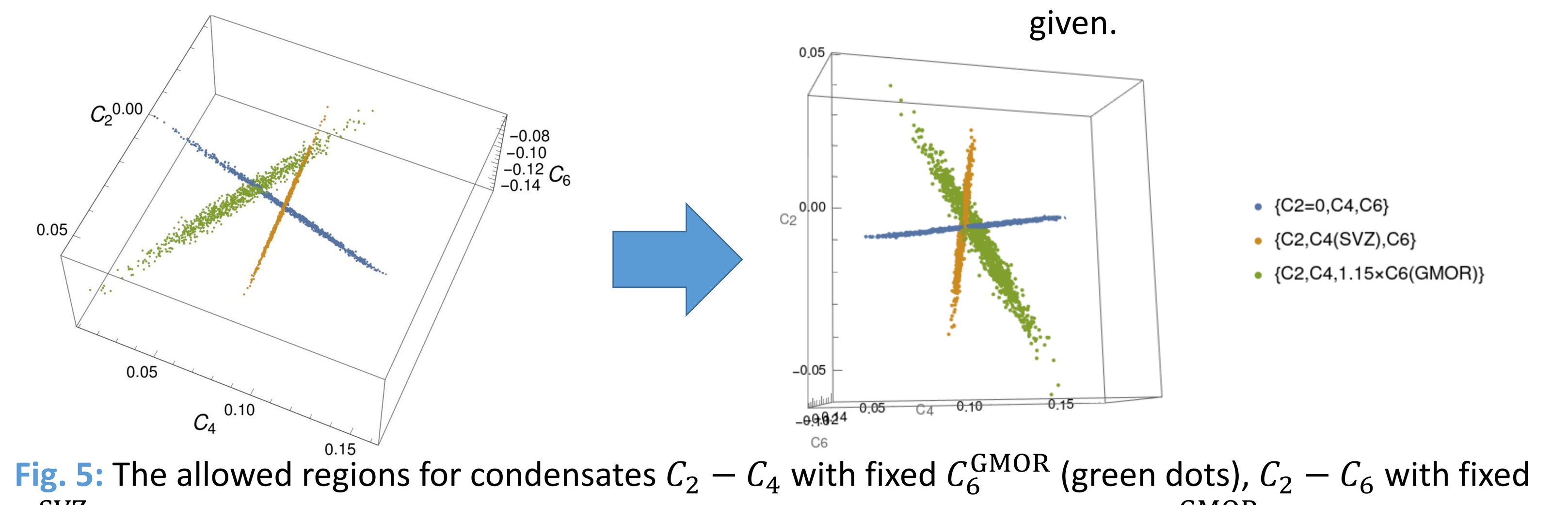


Fig. 5: The allowed regions for condensates $C_2 - C_4$ with fixed C_6^{GMOR} (green dots), $C_2 - C_6$ with fixed C_4^{SVZ} (orange dots) and $C_4 - C_6$ with fixed $C_2 = 0$ (blue dots). The $C_6 = k \cdot C_6^{\text{GMOR}}$ with scaled vacuum dominance contributions are shown on the panels a) k = 1, there regions do not intersect in one point; b) k = 1.15, there all the regions intersect in one point.

Summary

We have presented the check of hypothesis of vacuum dominance based on processing of modern data on e^+e^- -annihilation into an even number of pions.

- The method: construction of the Adler, the Borel transform, sum rule construction, extraction the nonperturbative corrections in OPE is developed later [3–5] and implemented in the present work. To improve the quality of calculations in this work the Monte-Carlo simulation of events with taking into account errors of the data is applied.
- The anticorrelation between OPE corrections, previously found [3–5] and confirmed at present work, remains strong.
- The possible deviation from vacuum dominance and its interplay with short strings contribution. We checked that it is possible to increase the coefficient C_6 in OPE (and the 4-quark condensate) not more than by 20%, but better by 15% — that may reflect the possible contribution of non-vacuum intermediate states. In this case the value of C_2 is compatible with zero, and C_4 is within reasonable values.

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