Inflating and Reheating the Universe with an Independent Affine Connection

Alberto Salvio

TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA
Dipartimento di Fisica

September 5, 2022

ICNFP2022

Mainly based on
A.S. 2207.08830

Introduction

- Einstein's general relativity (GR) explains gravity in geometrical terms:
  - distances are measured through the metric $g_{\mu\nu}$
  - the gravitational force is determined by the (affine) connection $\mathcal{A}_{\mu\ \sigma}^{\ \rho}$

This beautiful construction accounts for all gravitational observations performed so far, including today's nearly-exponential accelerated expansion of the universe if the cosmological constant is present.

Another, but much more rapid, nearly-exponential expansion occurred during the early stages of the universe (inflation).

- It is driven by a spin-0 field, the inflaton
- The inflaton has an appropriate potential, which guarantees that such an expansion not only occurred, but also eventually came to an end: a reheating must take place after inflation in order to generate all particles we observe

Introduction

- Einstein's general relativity (GR) explains gravity in geometrical terms:
  - distances are measured through the metric $g_{\mu\nu}$
  - the gravitational force is determined by the (affine) connection $\mathcal{A}_{\mu}{}^{\rho}{}_{\sigma}$

This beautiful construction accounts for all gravitational observations performed so far, including today's nearly-exponential accelerated expansion of the universe if the cosmological constant is present.

Another, but much more rapid, nearly-exponential expansion occurred during the early stages of the universe (inflation).

- It is driven by a spin-0 field, the inflaton
- The inflaton has an appropriate potential, which guarantees that such an expansion not only occurred, but also eventually came to an end: a reheating must take place after inflation in order to generate all particles we observe

From the purely geometrical point of view $g_{\mu\nu}$ and $\mathcal{A}_{\mu}{}^{\rho}{}_{\sigma}$, unlike in GR, can be **completely independent objects** and, moreover, **can contain extra degrees of freedom** besides the spin-2 graviton. This generalized scenario is known as metric-affine gravity.

Introduction

→ Can the inflaton be identified with an extra dynamical component of the connection?

The main motivation for answering this question is to understand whether the inflaton can have a geometrical origin too

Introduction

→ Can the inflaton be identified with an extra dynamical component of the connection?

The main motivation for answering this question is to understand whether the inflaton can have a geometrical origin too

Who is the inflaton? Are the observational bounds satisfied?

To do list

- Find whether the connection contains a spin-0 field with an appropriate potential
- Identify the region of parameter space with viable values of the scalar spectral index n_s, the tensor-to-scalar ratio r and the curvature power spectrum P_R [Planck collaboration (2018); BICEP/Keck collaboration (2021) (BK18)]
- Understand whether an efficient production of known particles, such as electrons, quarks and Higgs bosons (reheating) can take place after inflation

The key idea

When ${\mathcal{A}_\mu}^\rho{}_\sigma$ and $g_{\mu\nu}$ are independent there are 2 rather than 1 invariant that are linear in

$$ {\mathcal{R}_{\mu\nu}}^\rho{}_\sigma \equiv \partial_\mu {\mathcal{A}_\nu}^\rho{}_\sigma + {\mathcal{A}_\mu}^\rho{}_\lambda {\mathcal{A}_\nu}^\lambda{}_\sigma - (\mu \leftrightarrow \nu) $$

1. The usual Ricci-like scalar $\mathcal{R} \equiv {\mathcal{R}_{\mu\nu}}^{\mu\nu}$

2. The parity-odd Holst invariant $\mathcal{R}' \equiv \epsilon^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}/\sqrt{-g}$
   [Hojman, Mukku, Sayed (1980); Nelson (1980); Holst (1995)]

In the GR case, where ${\mathcal{A}_\mu}^\rho{}_\sigma$ equals the Levi-Civita connection, $\mathcal{R}$ coincides with the Ricci scalar, R, but $\mathcal{R}'$ vanishes. For this reason in metric-affine gravity $\mathcal{R}'$ can be understood as a component of the connection

The key idea

When $\mathcal{A}_\mu{}^\rho{}_\sigma$ and $g_{\mu\nu}$ are independent there are 2 rather than 1 invariant that are linear in

$$\mathcal{R}_{\mu\nu}{}^\rho{}_\sigma \equiv \partial_\mu \mathcal{A}_\nu{}^\rho{}_\sigma + \mathcal{A}_\mu{}^\rho{}_\lambda \mathcal{A}_\nu{}^\lambda{}_\sigma - (\mu \leftrightarrow \nu)$$

1. The usual Ricci-like scalar $\mathcal{R} \equiv \mathcal{R}_{\mu\nu}{}^{\mu\nu}$
2. The parity-odd Holst invariant $\mathcal{R}' \equiv \epsilon^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}/\sqrt{-g}$
[Hojman, Mukku, Sayed (1980); Nelson (1980); Holst (1995)]

In the GR case, where $\mathcal{A}_\mu{}^\rho{}_\sigma$ equals the Levi-Civita connection, $\mathcal{R}$ coincides with the Ricci scalar, R, but $\mathcal{R}'$ vanishes. For this reason in metric-affine gravity $\mathcal{R}'$ can be understood as a component of the connection

The key idea here is to identify the inflaton with $\mathcal{R}'$.
To do so $\mathcal{R}'$ has to be a dynamical field, which is independent of the metric

The minimal model

The simplest inflationary action that realizes this is

$$S_I = \int d^4x\sqrt{-g}\left(\alpha\mathcal{R} + \beta\mathcal{R}' + c\mathcal{R}'^2\right)$$

Indeed,

- for $c = 0$ one can easily show, by solving the connection equations, that S_I is equivalent to the Einstein-Hilbert action, having identified $\alpha = M_P^2/2$
- for $c \neq 0$, standard auxiliary field methods show that an extra spin-0 parity odd dynamical field ζ' (the "pseudoscalaron") is present and equals $\mathcal{R}'$ on shell [Hecht, Nester, Zhytnikov (1996); Beltrán Jiménez, Maldonado Torralba (2019); Pradisi, Salvio (2022)]
- the $\beta\mathcal{R}'$ term, a.k.a the Holst term, is also necessary to obtain a suitable inflaton potential, as we will see;
  the quantity $M_P^2/(4\beta)$ is called the Barbero-Immirzi parameter [Immirzi (1996)]

S_I can be recast in the following metric form (where the connection equals the Levi-Civita one)

$$S_I = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{(\partial\omega)^2}{2} - U(\zeta'(\omega))\right]$$

where $U(\zeta') = c\zeta'^2$ ($c \geq 0$ for stability reasons) and

$$\zeta'(\omega) = \frac{1}{2c}\left(\frac{M_P^2\tanh X(\omega)}{4\sqrt{1-\tanh^2 X(\omega)}} - \beta\right), \quad X(\omega) \equiv \sqrt{\frac{2}{3}}\frac{\omega}{M_P} + \tanh^{-1}\left(\frac{4\beta}{\sqrt{16\beta^2 + M_P^4}}\right)$$

Inflation

The slow-roll approximation can be used when

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{1}{U}\frac{dU}{d\omega}\right)^2 \ll 1, \quad \eta \equiv \frac{M_P^2}{U}\frac{d^2U}{d\omega^2} \ll 1$$

and in this case the number of e-folds N_e as a function of the field ω is given by

$$N_e(\omega) = N(\omega) - N(\omega_{\text{end}}), \qquad N(\omega) = \frac{1}{M_P^2}\int^{\omega} d\omega' \, U\left(\frac{dU}{d\omega'}\right)^{-1}$$

and ω_{end} satisfies $\epsilon(\omega_{\text{end}}) = 1$. Then n_s, r and P_R (at horizon exit) are

$$n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon, \quad P_R = \frac{U/\epsilon}{24\pi^2 M_P^4}$$

Inflation

The slow-roll approximation can be used when

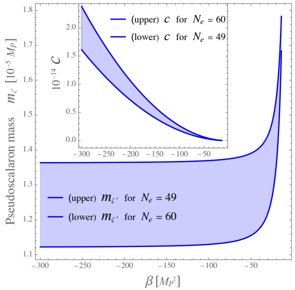
$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{1}{U}\frac{dU}{d\omega}\right)^2 \ll 1, \quad \eta \equiv \frac{M_P^2}{U}\frac{d^2U}{d\omega^2} \ll 1$$

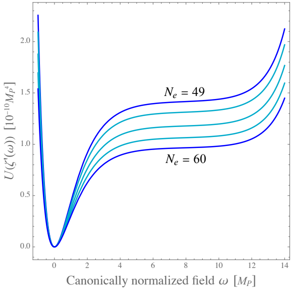
and in this case the number of e-folds N_e as a function of the field ω is given by

$$N_e(\omega) = N(\omega) - N(\omega_{\rm end}), \qquad N(\omega) = \frac{1}{M_P^2}\int^{\omega} d\omega' \, U\left(\frac{dU}{d\omega'}\right)^{-1}$$

and $\omega_{\rm end}$ satisfies $\epsilon(\omega_{\rm end}) = 1$. Then n_s, r and P_R (at horizon exit) are

$$n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon, \quad P_R = \frac{U/\epsilon}{24\pi^2 M_P^4}$$





Left plot: *the pseudoscalaron mass $m_{\zeta'}$ and the corresponding value of c that gives the observed P_R at N_e e-folds before the end of inflation*
Right plot: *the corresponding pseudoscalaron potential for $\beta = -80M_P^2$*

Inflation

The slow-roll approximation can be used when

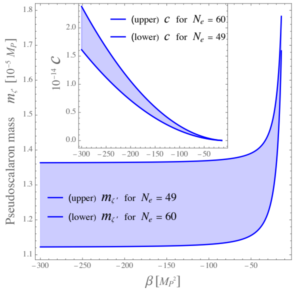
$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{1}{U}\frac{dU}{d\omega}\right)^2 \ll 1, \quad \eta \equiv \frac{M_P^2}{U}\frac{d^2U}{d\omega^2} \ll 1$$

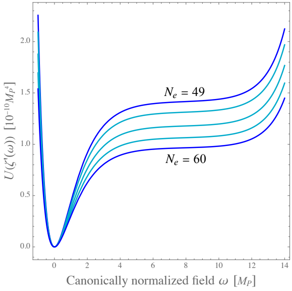
and in this case the number of e-folds N_e as a function of the field ω is given by

$$N_e(\omega) = N(\omega) - N(\omega_{\rm end}), \qquad N(\omega) = \frac{1}{M_P^2}\int^{\omega} d\omega' \, U\left(\frac{dU}{d\omega'}\right)^{-1}$$

and $\omega_{\rm end}$ satisfies $\epsilon(\omega_{\rm end}) = 1$. Then n_s, r and P_R (at horizon exit) are

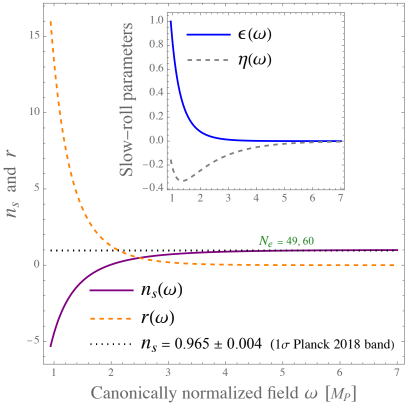
$$n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon, \quad P_R = \frac{U/\epsilon}{24\pi^2 M_P^4}$$





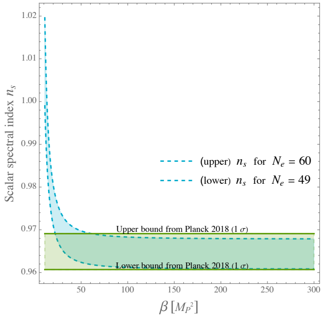
In the ω potential there is a plateau, which is larger the larger $|\beta|$ is and disappears when $\beta = 0$. This is the reason why the $\beta\mathcal{R}'$ term in S_I is necessary

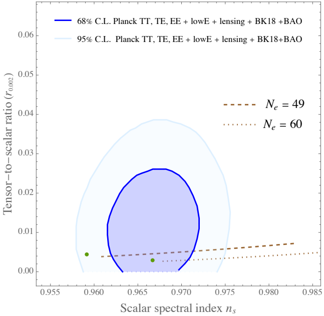
Predictions for n_s and r

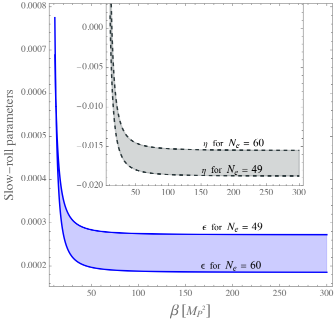


n_s and r as functions of the canonically normalized pseudoscalaron ω. In the inset the slow-roll parameters are given. This plot shows that viable slow-roll inflation with an appropriate N_e occurs for ω slightly above the Planck scale. As an example we have set $\beta = -300M_P^2$.

Predictions for n_s and r







Upper plots: n_s and r compared with the observational data. The green dots are the predictions of Starobinsky inflation.
Down plot: slow-roll parameters.

These plots show that slow-roll inflation not only occurs, but is also remarkably compatible with the most recent CMB observations provided by Planck and BK18 for large $|\beta|$ (i.e. small values of the Barbero-Immirzi parameter) and for an appropriate number of e-folds N_e

Reheating: generalities

If ω decays into some SM particles with width Γ_ω the reheating temperature $T_{\rm RH}$ is

$$T_{\rm RH} \gtrsim \min\left(\left(\frac{45\Gamma_\omega^2 M_P^2}{4\pi^3 g_*}\right)^{1/4}, \left(\frac{30\rho_{\rm vac}}{\pi^2 g_*}\right)^{1/4}\right),$$

where g_* is the effective number of relativistic species in thermal equilibrium at temperature $T_{\rm RH}$ and $\rho_{\rm vac}$ is the vacuum energy density due to ω

Reheating: $\omega \to$ fermion pair

Let us first consider a fermion f represented by a Dirac spinor Ψ with action

$$S_f = \int \sqrt{-g}\,\frac{1}{2}\overline{\Psi}(i\not{D} - m_f)\Psi + \text{h.c.}\,,$$

By using the connection equations with the formalism of [Pradisi, Salvio (2022)], one finds the following effective pseudoscalaron-fermion-fermion interaction

$$\mathscr{L}_{\omega ff} = \frac{c_{\omega ff}}{M_P}\,\partial_\mu\omega\,\overline{\Psi}\gamma_5\gamma^\mu\Psi,$$

where

$$c_{\omega ff} = \left[\frac{3M_P}{1+16B^2}\frac{dB}{d\omega}\right]_{\omega=0} = \sqrt{\frac{3M_P^4}{8(M_P^4+16\beta^2)}}, \quad B = (\beta + 2c\zeta'(\omega))/M_P^2$$

This effective interaction leads to the decay $\omega \to ff$ with width

$$\Gamma_{\omega\to ff} = |c_{\omega ff}|^2\,\frac{m_\omega m_f^2}{2\pi M_P^2}\sqrt{1-\frac{4m_f^2}{m_\omega^2}}$$

This can efficiently reheat the universe up to a temperature above the electroweak scale if m_f is very large compared to that scale.

Such a fermion is not present in the SM, but it is possible to engineer a model where there is a very heavy fermion with sizable couplings to SM particles

Reheating: $\omega \to$ scalar pair (e.g. a pair of Higgs bosons)

In order to keep our analysis as model independent as possible, we consider another channel: the decay of ω into two identical real scalar particles, **e.g. two Higgs bosons**.

This is possible when there is a non-minimal coupling between the real (canonically normalized) scalar field ϕ in question and $\mathcal{R}$ in the action:

$$S_{\rm nm} = \int \sqrt{-g}\frac{\xi\phi^2}{2}\mathcal{R}$$

$S_{\rm nm}$ is known to be generated by quantum corrections (it is more natural to include it)

Solving the connection equations with the results of [Pradisi, Salvio (2022)], one finds

$$\mathscr{L}_{\omega\phi\phi} = \frac{c_{\omega\phi\phi}}{M_P}\,\partial_\mu\omega\,\phi\partial^\mu\phi \quad \text{where} \quad c_{\omega\phi\phi} = \left[\frac{48\xi M_P B}{1+16B^2}\,\frac{dB}{d\omega}\right]_{\omega=0} = \frac{4\sqrt{6}\beta\xi}{\sqrt{M_P^4+16\beta^2}}$$

$\mathscr{L}_{\omega\phi\phi}$ only arises through the Holst term because $c_{\omega\phi\phi} \to 0$ as $\beta \to 0$ and gives

$$\Gamma_{\omega\to\phi\phi} = |c_{\omega\phi\phi}|^2\,\frac{m_\omega^3}{16\pi M_P^2}\sqrt{1-\frac{4m_\phi^2}{m_\omega^2}}$$

where m_ϕ is the mass of ϕ. The channel $\omega \to \phi\phi$ can efficiently and naturally reheat the universe up to a temperature much above the electroweak scale, even if one identifies ϕ with the Higgs, so *per se* it does not require any beyond-the-SM physics.

E.g. taking $m_\phi \ll m_\omega$, $g_* \sim 10^2$ and $\beta \gtrsim M_P^2$ one finds $T_{\rm RH} \gtrsim 10^9|\xi|$ GeV.

Conclusions

- It has been found that a pseudoscalar component of a dynamical connection, which is independent of the metric, can drive inflation in agreement with current data.
- This pseudoscalaron is identified with the parity odd Holst invariant and inflationary predictions in excellent agreement with data have been found for small values of the Barbero-Immirzi parameter, where the inflaton potential develops a plateau.
- The predictions approach, but do not quite reach, those of Starobinsky inflation as the Barbero-Immirzi parameter goes to zero; for finite values, on the other hand, the predictions significantly differ.
- Pseudoscalaron inflation can be tested by future CMB observations, such as those of LiteBIRD.
- Moreover, the decays of the pseudoscalaron into Higgs particles can efficiently reheat the universe after inflation up to a high enough temperature. This temperature could be further increased by other channels, such as decays into very massive fermions

Conclusions

- It has been found that a pseudoscalar component of a dynamical connection, which is independent of the metric, can drive inflation in agreement with current data.
- This pseudoscalaron is identified with the parity odd Holst invariant and inflationary predictions in excellent agreement with data have been found for small values of the Barbero-Immirzi parameter, where the inflaton potential develops a plateau.
- The predictions approach, but do not quite reach, those of Starobinsky inflation as the Barbero-Immirzi parameter goes to zero; for finite values, on the other hand, the predictions significantly differ.
- Pseudoscalaron inflation can be tested by future CMB observations, such as those of LiteBIRD.
- Moreover, the decays of the pseudoscalaron into Higgs particles can efficiently reheat the universe after inflation up to a high enough temperature. This temperature could be further increased by other channels, such as decays into very massive fermions

Thank you very much for your attention!

Extra slides

Analytical formulæ for inflationary observables

$$\epsilon(\omega) = \frac{4M_P^4 \cosh^2 X(\omega)}{3\left(M_P^2 \sinh X(\omega) - 4\beta\right)^2},$$

$$\eta(\omega) = \frac{4M_P^2\left(M_P^2 \cosh\left(2X(\omega)\right) - 4\beta \sinh X(\omega)\right)}{3\left(M_P^2 \sinh X(\omega) - 4\beta\right)^2},$$

$$N(\omega) = \frac{3}{4}\log\left(\cosh X(\omega)\right) - \frac{3\beta \arctan\left(\sinh X(\omega)\right)}{M_P^2},$$

$$n_s(\omega) = 1 - \frac{8M_P^4 \cosh^2 X(\omega)}{\left(M_P^2 \sinh X(\omega) - 4\beta\right)^2} + \frac{8M_P^2\left(M_P^2 \cosh\left(2X(\omega)\right) - 4\beta \sinh X(\omega)\right)}{3\left(M_P^2 \sinh X(\omega) - 4\beta\right)^2},$$

$$r(\omega) = \frac{64M_P^4 \cosh^2 X(\omega)}{3\left(M_P^2 \sinh X(\omega) - 4\beta\right)^2},$$

$$P_R(\omega) = \frac{\left(\beta - \frac{M_P^2 \sinh X(\omega)}{4}\right)^2 \left(M_P^2 \sinh X(\omega) - 4\beta\right)^2 \mathrm{sech}^2 X(\omega)}{128\pi^2 c M_P^8}.$$

Moreover, the analytic expressions of $\omega_\pm$ (the two solutions of $\epsilon(\omega_{\text{end}}) = 1$) are

$$\omega_\pm = \sqrt{\frac{3}{2}} M_P \left(\sinh^{-1}\left(\pm\sqrt{\frac{192\beta^2}{M_P^4} - 4} - \frac{12\beta}{M_P^2}\right)\right.$$