Hybrid EDM Ring and Axions

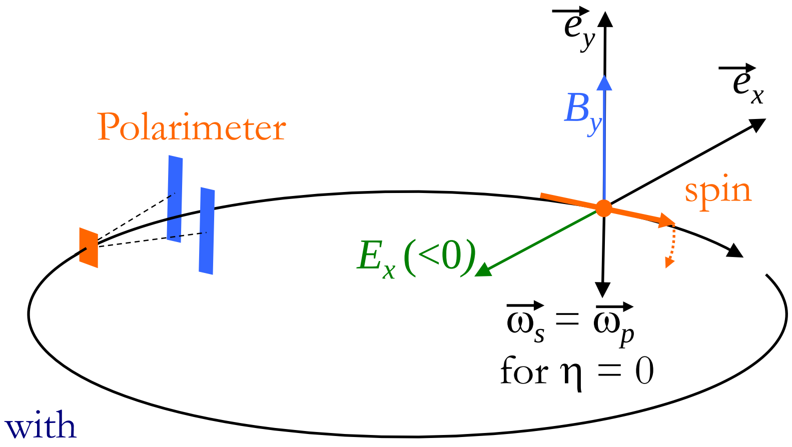
Christian Carli

iFAST BSW22, 29th March to 1st April 2022

- Recap of standard fully electric charged particle Electric Dipole Moment (cpEDM) ring
- Hybrid Ring Concept
- Hybrid Ring with high Periodicity
- Search for oscillating EDMs
- Summary and Conclusions

Recap of standard fully electric charged particle Electric Dipole Moment (cpEDM) ring

- "Frozen spin" concept (for MDM, neglecting EDM)
  - Initial longitudinal polarization is maintained
- "Magic energy" – ideal case
  - "Frozen spin" with radial electric field only choosing rel. factors $g = g_m = \sqrt{1+1/G}$ and $b = b_m = 1/\sqrt{1+G}$ with $G = (g-2)/2 = 1.728...$ describing the proton EDM (average) fields $\bar{E}_x = -\frac{m g_m b_m^2 c^2}{eC/(2p)} = -5.27$ MeV/m and $B_y = 0$ with m and e the proton mass and charge and $C = 500$ m the circumf.



Sketch of a CP-EDM Ring

- Imperfect machine with average radial magnetic field $\bar{B}_x$ ¹ 0 Gravity neglected
  - Vertical electric field $\bar{E}_y = -b_m c \bar{B}_x$ from quadrupoles compensates resulting vertical deflection
- Rotation of spin vector $\vec{S}$ in machine with additional radial magnetic field
  - Described by Thomas-BMT equation (NO subtraction of angular frequency for rotation of direction)

$$\frac{d\vec{S}}{dt} = \vec{\omega}_s \times \vec{S} \qquad \vec{\omega}_s = -\frac{e}{m}\left[\left(G+\frac{1}{\gamma}\right)\vec{B}_\perp + (G+1)\frac{\vec{B}_\parallel}{\gamma} - \left(G+\frac{1}{\gamma+1}\right)\vec{\beta}\times\frac{\vec{E}}{c} + \frac{\eta}{2}\left(\frac{\vec{E}_\perp}{c} + \frac{1}{\gamma}\frac{\vec{E}_\parallel}{c} + \vec{\beta}\times\vec{B}\right)\right]$$

For "magic energy" keep only E_x

  - Gives

$$\bar{\vec{\omega}}_s = \left(-\frac{e}{m}\left(\frac{G+1}{\gamma_m^2}\right)\bar{B}_x + \frac{\eta\gamma_m\beta_m^2 c}{C/\pi}\right)\vec{e}_x + \vec{\omega}_p \quad \text{with} \quad \bar{\vec{\omega}}_p = -\frac{\beta_m c}{C/(2\pi)}$$

describing the rotation of the direction

  - $h = 1.9 \times 10^{-15}$ corresponding to an EDM of $d = 10^{-29}$ e×cm or $\bar{B}_x = -9.3$ aT give both $\bar{w}_x = 1.6$ nrad/s

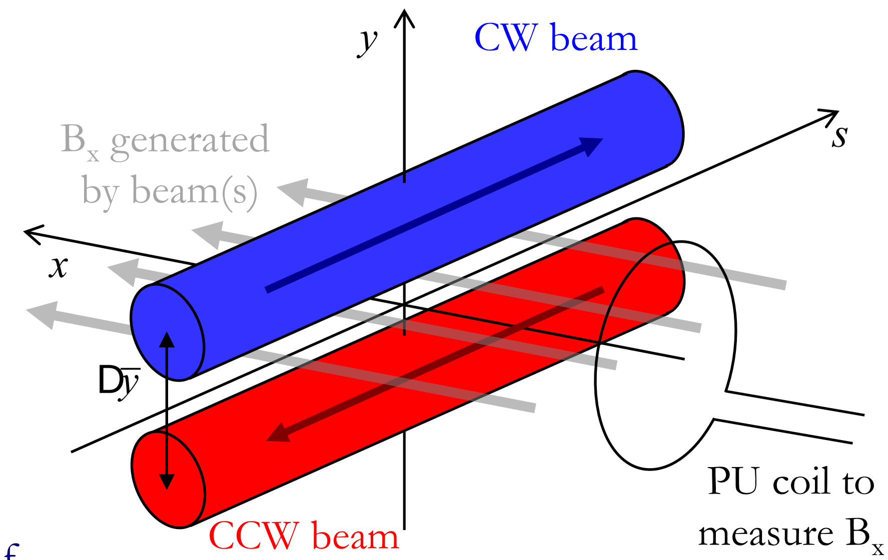
Recap of standard fully electric charged particle Electric Dipole Moment (cpEDM) ring

Orbit difference measurement to assess average radial magnetic field

- With low vertical tune $Q_V = 0.44$, a field of $\bar{B}_x = 9.3$ aT (corresponding to an EDM of -10^{-29} e.cm) and smooth focusing
  - Orbit separation $\Delta \bar{y} = 2\left(\dfrac{C}{2\pi Q_V}\right)^2 \dfrac{e \bar{B}_x}{m \gamma_m \beta_m c} = 0.26$ pm

    (less than 1% of typical radii of atoms!)
  - Measured with SQUID orbit difference Pick-ups
  - In addition tune modulation
    - to alleviate requirements on number and precise spacing of pick-ups with B_x varying around circumf.
    - to measure at a finite frequency with lower background noise
  - On imperfections of orbit difference measurements
    - E.g., 10^{-6} relative difference of beam currents and $y_{co} = 1\ \mu$m common of both beams

    => gives same signal than orbit separation $\Delta y = 1$ pm
    - Ensure that signal sufficiently low with one beam only?

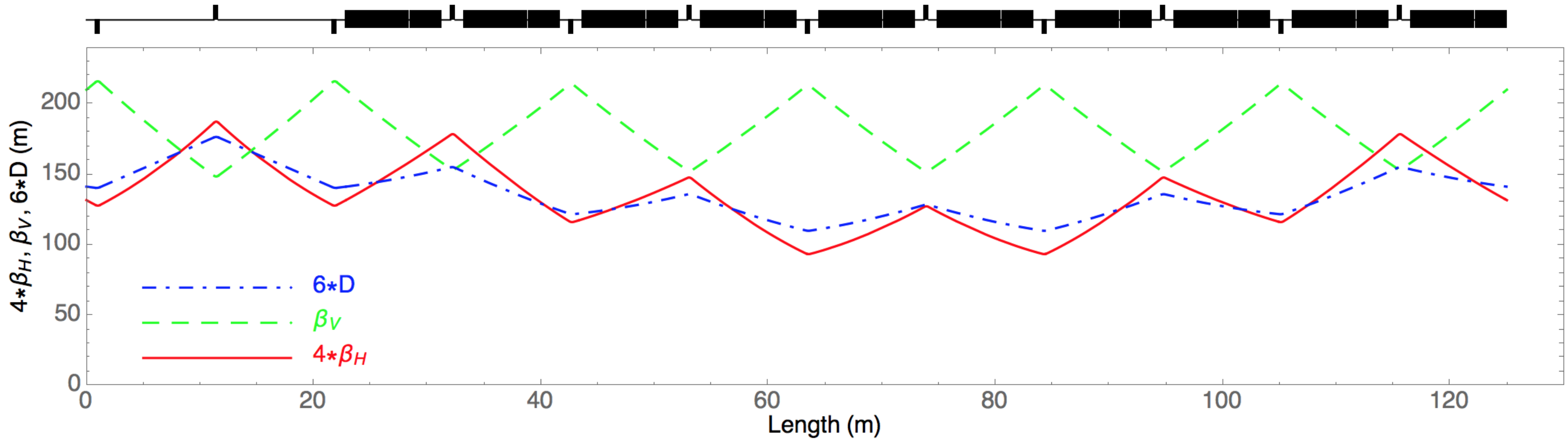


Beam separation and B_x generated

Note:
1) Separation enhances magnetic field
2) With beam parameters and lattice proposed, beams are not round (very large vertical beam size!)

cf. analysis by V. Lebedev of this scheme and other potential limitation in:
http://collaborations.fz-juelich.de/ikp/jedi/public_files/usual_event/AccPhysLimitationsOnEDMring.pdf)

Recap of standard fully electric charged particle Electric Dipole Moment (cpEDM) ring – limitation of orbit difference measurement



Twiss parameters for one out of four periods of the "strong focusing" pEDM lattice proposal

- Orbit separation at position s in a real machine with Twiss betatron function varying with position $\hat{s}$

$$\Delta y(s) = 2\frac{\sqrt{\beta_v(s)}}{2\sin(\pi Q_V)}\int_0^C d\hat{s}\,\frac{\sqrt{\beta_v(\hat{s})}\,B_x(\hat{s})}{m\gamma_m\beta_m c}\cos\left(\left|\mu(s)-\mu(\hat{s})\right|-\pi Q_V\right)$$ with $\mu(s)$ the betatron phase

- Effect of magnetic field on orbit difference weighted with $\sqrt{\beta_V(s)}$
  - Possible to have finite average separation of beams with vanishing average $\bar{B}_x$
    - Say positive (negative) B_x at positions with large (small) β_V

- Situation similar, somewhat more complicated, in case of tune modulation

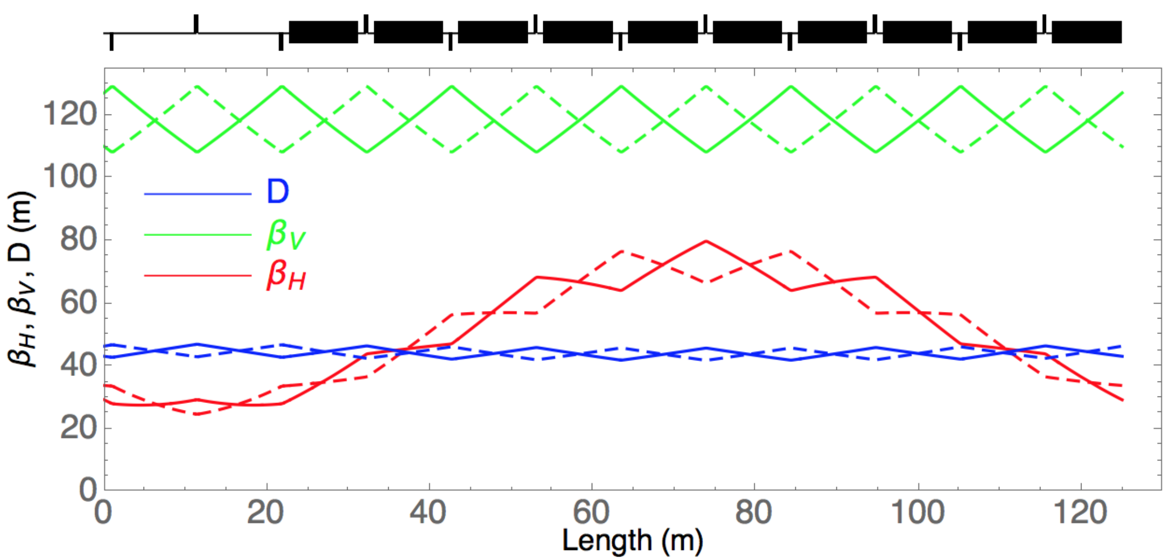
- Horizontal separation of beams due to vertical magnetic field and skew quadrupoles (tilted electrodes of bends) yield similar effects

Hybrid Ring Concept - Proposal

Hybrid ring proposed by S. Haciomeroglu and Y.K. Semertzidis (see PRAB 22, 034001 (2019) and arXiv:1806.09319)

- Ring operated at "magic energy" with electro-static bends without gradient (field index $m=0$)
  - Geometry taken from strong focusing "magic energy" electric ring
  - Quadrupoles magnetic with strength $d B_y / dx = \pm 0.1$ T/m and $k = \pm 0.0428$ m^{-2} (length $l_Q = 0.4$ m)
    - Operation with two counter-rotating beams
    - Thus, quadrupole polarity opposite for CW and CCW beam
  - Working point $Q_H = 1.754$, $Q_V = 0.673$ almost identical for two beams
  - Significant variations of horizontal betatron functions with periodicity four and larger dispersion
    - Impact on IBS to be evaluated
  - Tuning of machine for both beams more delicate
    - Closed orbit
    - Working point
    - Chromaticity and 2nd order dispersion for spin coherence

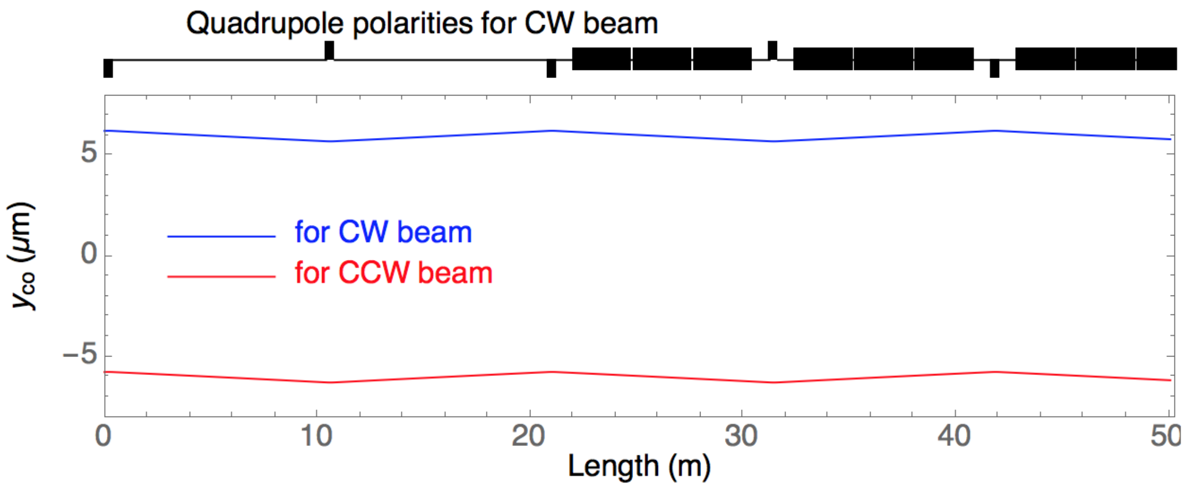
- Note some analogies with, e.g., "doubly magic" proposal (superposition of electric and magnetic fields)



Twiss parameters for one out of four periods for the CW (solid lines) and the CCW (dashed lines) beam

Hybrid Ring Concept - average radial magnetic field

- Average radial magnetic field (in addition to magnetic field from quadrupoles)
  - Again, the vertical acceleration over one turn vanishes (for closed orbit, in average for many turns)



Vertical closed orbits for a constant radial magnetic field B_r = 1 nT

  - With perfect electric bends (field index set to $m_i = 0$), no vertical electric fields
    - Closed orbit such that average radial magnetic field along closed orbit vanishes
    - For particles executing transverse betatron oscillations, radial magnetic field vanishes in average only (as is the case for perfects hybrid ring)

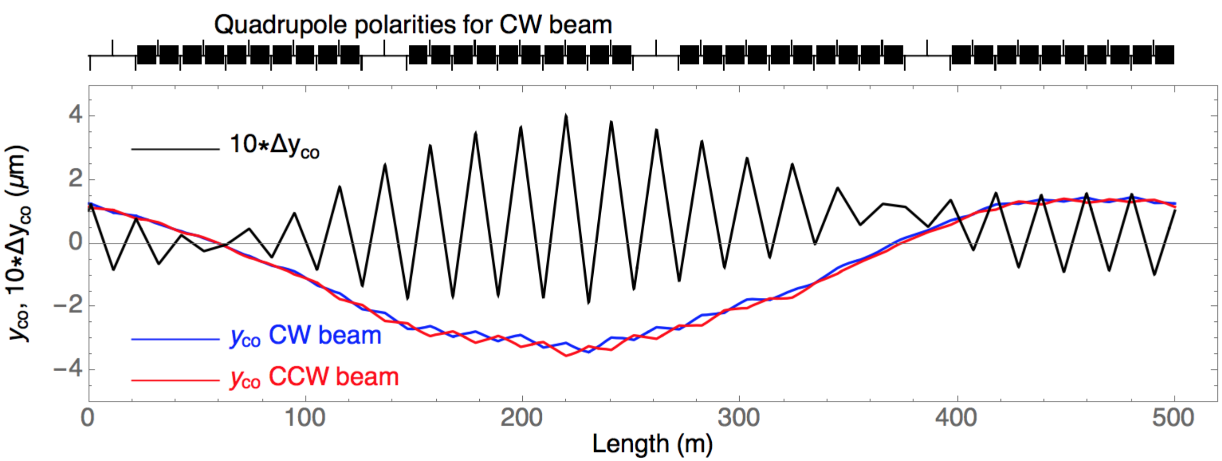
- Average radial magnetic field does not lead to a 1st order effect mimicking EDM (vertical spin build up proportional to the average radial magnetic field)

Hybrid Ring Concept -
average vertical electric field

- Vertical electric generated typically by tilted (rotated around longitudinal axis) bendings
  - E.g. one out of $N_B = 120$ bends tilted by $\alpha = 1$ mrad in $C = 500$ m ring generates deflection $\Delta y' = \alpha\, 2\pi / N_B = 0.052$ mrad and average electric field $\bar{E}_y = -2\pi \frac{\beta_m^2 \gamma_m m c^2}{e} \frac{\alpha}{N_B} = -43.9$ V/m
  - Again compensation by average radial magnetic field $\bar{B}_x = -\frac{1}{\beta_m c} \bar{E}_y = 2\pi \frac{\beta_m \gamma_m m c}{e} \frac{\alpha}{N_B} = 0.245\ \mu T$
  - Resulting spin rotation around radial axis $\bar{\omega}_x = -\frac{e}{m}\left[\left(G + \frac{1}{\gamma_m}\right)\bar{B}_x + \left(G + \frac{1}{\gamma_m - 1}\right)\frac{\beta_m \bar{E}_y}{c}\right] = -\frac{\alpha\, 2\pi\, \beta_m c}{N_B C \gamma_m}(G - 1) = -11.9\ \mathrm{s}^{-1}$
  - Vertical spin build-up proportional to vertical electric field – 1st order effect!
  - Can be disentangled from EDM observing CW and CCW beams =>does not mimic EDM
- Vertical closed orbit due to random tilts (rms 0.1 mrad) of bendings

Sign and direction of B_x for CCW beam?!

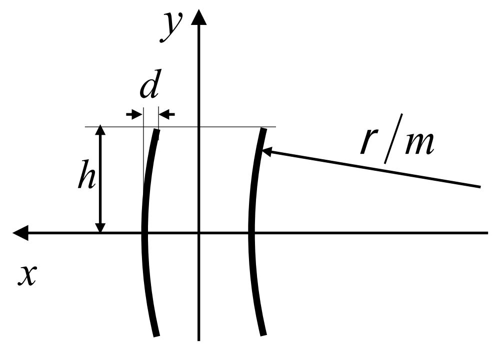
Fast spin rotations not mimicking EDM a feature of proposals combining electric and magnetic fields for large G



Separation of CW and CCW orbits (by a few 100 um under the assumptions made)

Hybrid Ring Concept - Splitting of counter-rotating beams and electric gradients

- Small field indices say $m_i = 0.01$ difficult to control
  - For $h = 20$ cm and $r = 52$ m, $d = 3.8$ μm
  - Corresponding gradient $\dfrac{dE_y}{dy} = -m_i \dfrac{\gamma_m \beta_m^2 m c^2}{e r^2} = -1530\ \text{V/m}^2$



Cross section of deflectors with finite field index m

- One single bending (length L_B) with gradient and orbit separation Δy_{co}
  - Difference of electric field seen by CW and CCW beam $\Delta \bar{E}_y = \dfrac{L_B}{C} \dfrac{dE_y}{dy} \Delta y_{co}$
  - Compensated by difference of magnetic field seen by CW and CCW beam $\Delta \bar{B}_x = -\dfrac{1}{\beta_m c} \Delta \bar{E} = -\dfrac{L_B}{\beta_m c C} \dfrac{dE_y}{dy} \Delta y_{co}$
  - Resulting spin rotation cannot be disentangled from EDM

$$\Delta \bar{\omega}_x = -\frac{e}{m}\left[\left(G + \frac{1}{\gamma_m}\right)\bar{B}_x + \left(G + \frac{1}{\gamma_m - 1}\right)\frac{\beta_m \bar{E}_y}{c}\right] = \frac{e}{m}\frac{L_B}{\gamma_m^2 \beta_m c C}\frac{d\bar{E}_y}{dy}(G-1)\Delta y_{co}$$

Effects mimics EDM!

  - For $\Delta y_{co} = 10$ μm (due to 1 nT radial magnetic field): $\Delta \bar{\omega}_x = -2.0\ \text{s}^{-1}$
  - For $\Delta y_{co} = 0.3$ mm (0.1 mrad rms tilts of bends): $\Delta \bar{\omega}_x = -61\ \text{s}^{-1}$

=> Control of closed orbit and unwanted electric gradients!

Subtlety with the meaning of ΔB_x and $\Delta \omega_x$ (radial axis pointing inwards for CCW beam)

- Similar effect from horizontal orbit separation and skew quadrupolar components (inner and outer plate of bend not perfectly parallel)

Hybrid Ring Concept –
Quadrupole polarity swaps & geometric phase effects

- Runs with opposite (magnetic) quadrupole polarities
  - Important ingredient of concept!
  - Analogy with inversion of all magnetic fields for "frozen spin" rings with electric and magnetic fields
    - Time reversal argument applies to CW beam with one polarity and CCW beam with other polarity …. but in addition operation with counter-rotating beams
    - Imperfection of magnetic field inversion limit to sensitivity
  - Quadrupolar fields generated with currents only (no yoke) inside state-of-the-art shielding?
    - General reproducibility (between fills and longer term) of machine
    - Achievable average magnetic field of the two polarities
    - Orbit difference (both transverse planes) measurements?
- Geometric phase effects
  - Magnetic field due to quadrupole with strength $dB_y/dx = \pm 0.1$ T/m, length $l_Q = 0.4$ m and offset $\Delta y_Q = 10\ \mu$m averaged over half-cell length $L_{HC} = 10.42$ m:

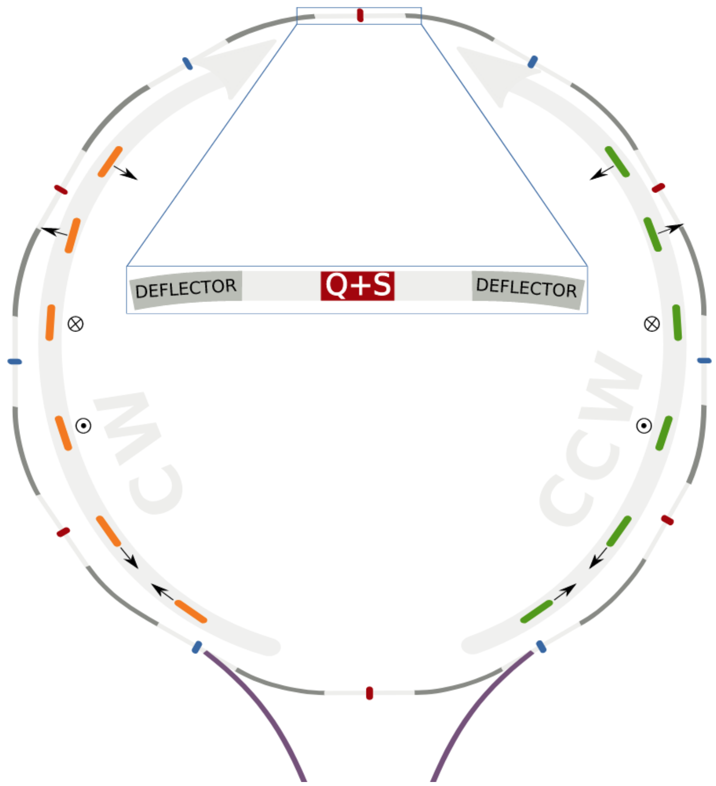
$$\Delta B_x = \frac{dB_y}{dx} \Delta y_Q \frac{L_Q}{L_{HC}} = 38.4 \text{ nT}$$

    - Expect enhanced geometric phase effects – beam based alignment to mitigate?
  - Again cancellations combining results with different quadrupole polarities and with counter-rotating beams

Hybrid Ring Concept – Recent proposal for high periodicity lattice

- "Symmetric-Hybrid" ring
  - Each quadrupole is symmetry point
  - "Vertical velocity effect" due to vertical quadrupole misalignments disappear
    - Vertical velocity effect is transfer of radial spin component into vertical direction
    - Proportional to average slope inside bending elements
    - (Kind of) rotation around longitudinal axis
    - Misaligned quadrupole at symmetry point gives vanishing average slope and effect
  - Beam based methods for mitigation of systematic effects
    - Intentional quad movement to
  - Many other possible effects not (yet) studied (see list below with some possible effects)

Z. Omarov et al., *Comprehensive symmetric-hybrid ring design for a proton EDM experiment at below 10^{-29} e.cm*, Phys. Rev. D 105, 032001 (2022)



Schematic of the Symmetric-Hybrid ring

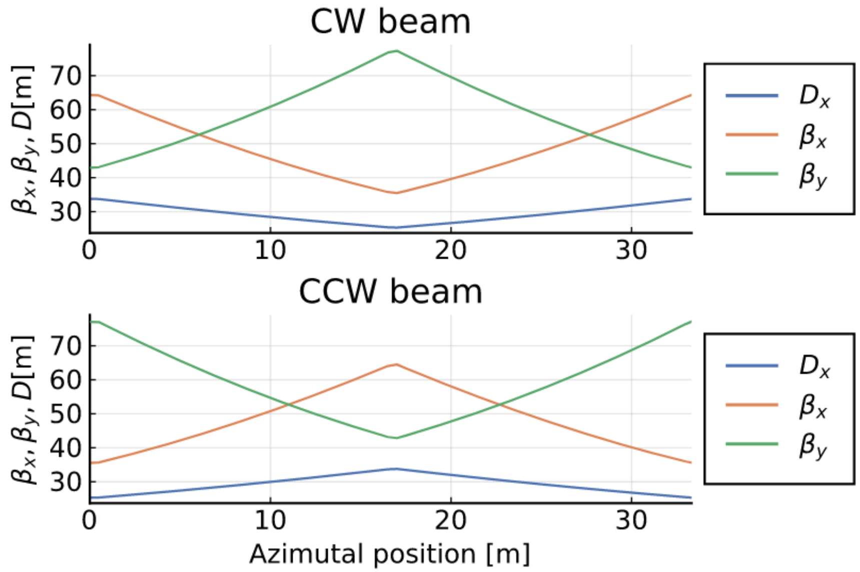
Hybrid Ring Concept –
Recent proposal for high periodicity lattice

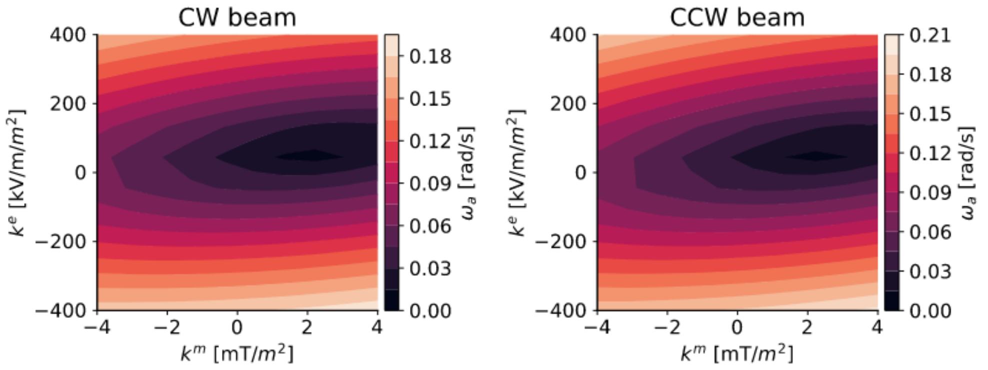
- Spin coherence optimization with combined magnetic and electric sextupoles
  - Polarity of quadrupoles swapped between CW and CCW beam
  - Sextupoles with combined electric and magnetic components

  => Electric part average strength between CW and CCW beam

  => Magnetic part generating a difference

  - Correction of transverse oscillations to spin de-coherence only?

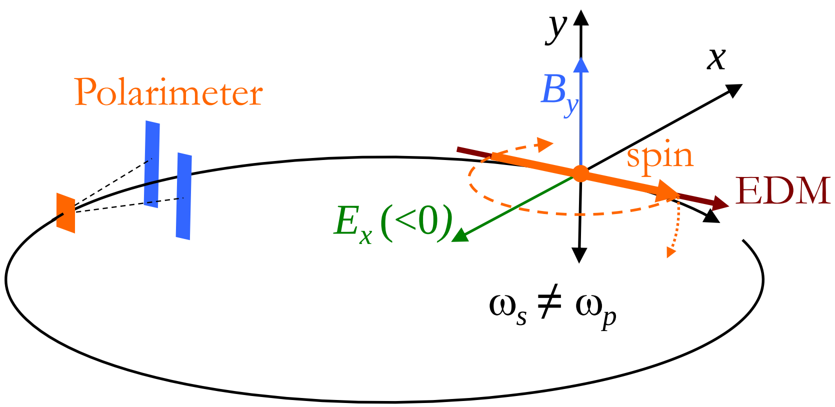




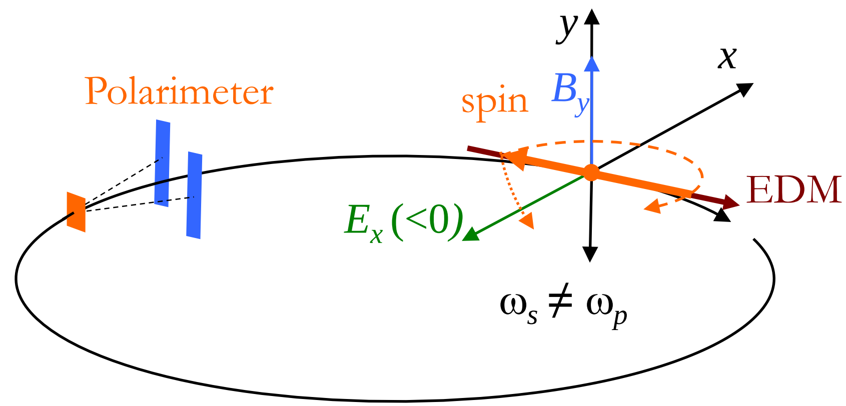
Hybrid Ring Concept –
Recent proposal for high periodicity lattice

- Potential systematic effects – some ideas, not an exhaustive list!
  - Bending elements with (simultaneous) horizontal offset and tilt (around longitudinal axis)
    - Offset shifts energy from magic one, vertical field from tilt generates spin rotation with respect to tangential
  - "Vertical velocity effect" from radial magnetic fields not caused by quadrupole offsets (residual fields even with state-of-the-art shielding)
  - Geometric phase effects due to bending elements with horizontal offset and longitudinal magnetic fields
    - Offset shifts energy from magic one and generates radial spin component
    - Rotation around longitudinal axis from magnetic field generates vertical spin component
  - Offset of sextupole and beam separation from magnetic fields
    - Sextupole offset generates quadrupolar and skew quadrupolar components (proportional to offset whereas deflection is proportional to square of offset)
    - With beam separation leads to systematic effects (mimicking EDM)
  - Spin coherence time optimisation with two sextupole families
    - Oscillations in three phase spaces (and emittances), each contributing to spin de-coherence
    - Expect to need three sextupole families for optimisation

Search for oscillating EDMs –
Signature of coupling from axions



Initial situation
Spin and EDM parallel



Half an oscillation period later
Spin and EDM antiparallel

- Spin rotation w.r.t. particle direction with frequency equal to EDM oscillation
  - Oscillating EDM means that ratio between EDM and spin oscillates $\eta = \eta_0 + \hat{\eta}\sin\left(\omega_{axion} t + \varphi_0\right)$
    - Frequency range and stability?
    - Resonance condition $\left|\omega_s - \omega_p\right| = \omega_{axion}$!
    - Long-term build up of vertical spin component
  - Many systematic effects strongly mitigated!
  - Severely limited by statistics (need for runs with different possible spin oscillation frequencies)?
    - Say frequencies fixed over one 1000 s store
    - Build-up of vertical polarization over full duration for below 1 mHz frequency range!

Summary, Conclusion and Outlook

- "Hybrid ring" proposal triggered by limitations of scheme to asses average radial field with standard fully electric cpEDM ring
  - Focusing using magnetic quadrupoles
    - Different optics for counter-rotating beams
    - Tuning simultaneously for both beams delicate
  - Average radial magnetic field not leading in first order to systematic effect mimicking EDM
  - Average vertical electrical fields (tilted bends) generate fast spin rotations around radial axis
    - Can be disentangled from EDM combining observations with CW and CCW beam
  - Recent proposal for high periodicity and symmetry lattice
  - Still many systematic effects to be understood and evaluated
- Proposal to search for oscillating EDM from axion coupling
  - Systematic effects strongly mitigated
  - Expect statistical limit orders of magnitudes larger than for static EDMs
- Conclusion and Outlook
  - Careful assessment of different proposals to be done to come realistic sensitivity estimates and choose best one