# Spin Effects and Semi-Inclusive Deep-Inelastic Scattering

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# Longitudinally Polarized DIS

**spin sum rule**

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}$$

- $\Delta\Sigma = \sum_q \Delta q \equiv \sum_q \int_0^1 dx\, [q_\uparrow(x) - q_\downarrow(x)]$ = fraction of the nucleon spin due to quark spins

**EMC collaboration (1987), SLAC, SMC, HERMES,..**

only small fraction of the proton spin due to quark spins

$$\Delta\Sigma \sim 30\%$$

where is rest?

**$\Delta G$**

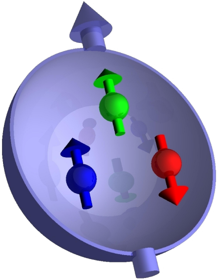
- $\Delta G \equiv \int_0^1 dx \Delta g(x)$ = contribution from gluon spin to nucleon spin
- recent RHIC data $\Rightarrow \int_{0.05}^{0.2} dx \Delta g(x) \approx 0.1$

$\hookrightarrow$ suggests $\Delta G = \mathcal{O}(0.2)$

**$\mathcal{L}$**

- $\mathcal{L}$ = quark & gluon orbital angular momentum
- by subtraction, $\mathcal{L} = \mathcal{O}(0.2)$

$\hookrightarrow$ expect many interesting effects associated with orbital angular momentum



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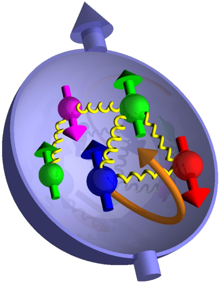


# Outline



- Deeply virtual Compton scattering (DVCS)

$\hookrightarrow$ Generalized parton distributions (GPDs)

$\hookrightarrow$ 'transverse imaging'

- Probabilistic interpretation of GPDs as Fourier transforms of impact parameter dependent PDFs
  - $H(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow q(x, \mathbf{b}_\perp)$
  - $E(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs when the target is $\perp$ polarized

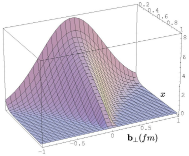
  $\hookrightarrow$ Ji relation

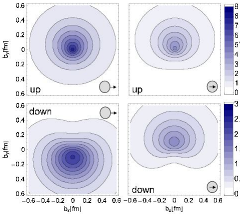
- Chromodynamik lensing and $\perp$ single-spin asymmetries (SSA)

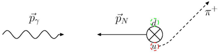
$$\left.\begin{array}{c}\text{transverse distortion of PDFs}\\ +\text{ final state interactions}\end{array}\right\} \Rightarrow$$

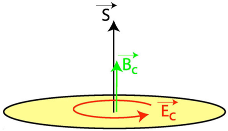
$\hookrightarrow$ SSA in $\gamma N \longrightarrow \pi + X$

- quark-gluon correlations $\rightarrow \perp$ force on $q$ in DIS
- Summary

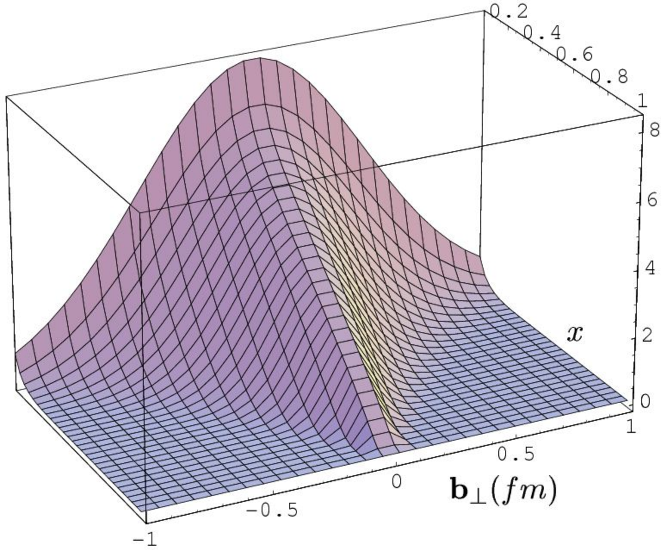






## Impact Parameter Dependent PDFs



# Deeply Virtual Compton Scattering (DVCS)

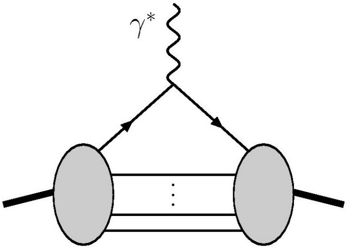


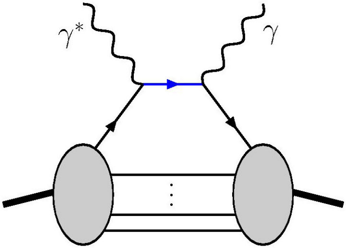
- virtual Compton scattering: $\gamma^* p \longrightarrow \gamma p$ (actually: $e^- p \longrightarrow e^- \gamma p$)
- 'deeply': $-q_\gamma^2 \gg M_p^2, |t| \longrightarrow$ Compton amplitude dominated by (coherent superposition of) Compton scattering off single quarks

$\hookrightarrow$ only difference between form factor (a) and DVCS amplitude (b) is replacement of photon vertex by two photon vertices connected by quark (energy denominator depends on quark momentum fraction $x$)

$\hookrightarrow$ DVCS amplitude provides access to momentum-decomposition of form factor = Generalized Parton Distribution (GPDs).

$$\int dx H_q(x, \xi, t) = F_1^q(t) \qquad \int dx E_q(x, \xi, t) = F_2^q(t)$$



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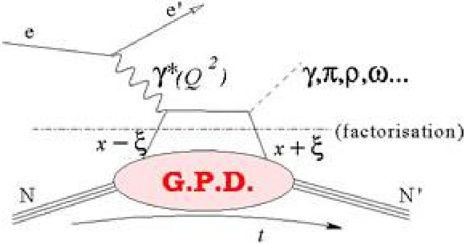



**exploratory studies**

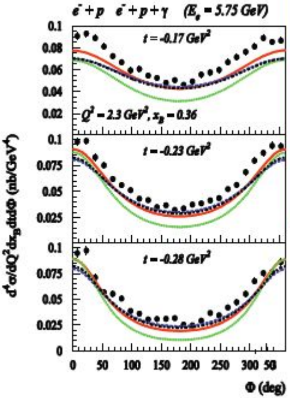
HERA/Hermes, JLab@6GeV, Compass II

**detailed measurements**

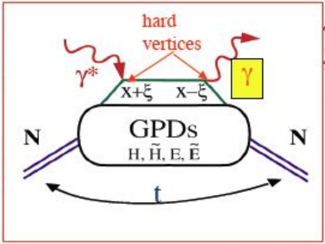
JLab@12GeV, EIC, FAIR/Panda
high luminosity, wide $Q^2$ range

**Unprecedented set of Deeply Virtual Compton Scattering data accumulated in Hall A and with *CLAS in Hall B at JLab***

Hall A




*Phys.Rev.Lett.97:262002,2006*



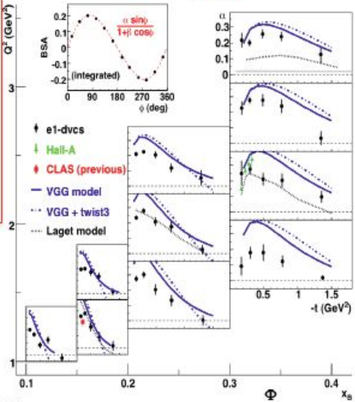

$$A = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} = \frac{\Delta\sigma_{LU}}{2\sigma}$$

Polarized beam, unpolarized target:

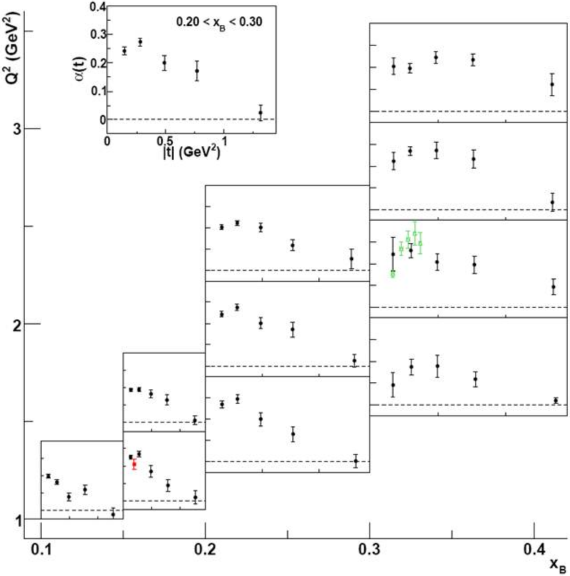
Kinematically suppressed

$$\Delta\sigma_{LU} \sim \sin\phi\{F_1 H + \xi(F_1+F_2)\tilde{H} + kF_2 E\}d\phi$$

CLAS




*Phys.Rev.Lett.100:162002,2008*


$Q^2$ (GeV$^2$)
$x_B$
0.1
0.2
0.3
0.4
1
2
3
$0.20 < x_B < 0.30$
$\alpha(t)$
$|t|$ (GeV$^2$)
0
0.5
1
0.1
0.2
0.3
0.4

- form factors: $\stackrel{FT}{\longleftrightarrow} \rho(\vec{r})$
- $GPDs(x,\vec{\Delta})$: form factor for quarks with momentum fraction $x$

$\hookrightarrow$ suitable FT of $GPDs$ should provide spatial distribution of quarks with momentum fraction $x$

- careful: cannot measure longitudinal momentum ($x$) and longitudinal position simultaneously (Heisenberg)

$\hookrightarrow$ consider purely transverse momentum transfer

**Impact Parameter Dependent Quark Distributions**

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x,\xi=0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

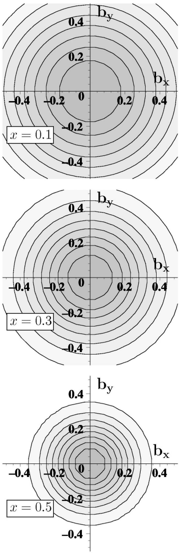
$q(x,\mathbf{b}_\perp)$ = parton distribution as a function of the separation $\mathbf{b}_\perp$ from the transverse center of momentum $\mathbf{R}_\perp \equiv \sum_{i\in q,g} \mathbf{r}_{\perp,i} x_i$
MB, Phys. Rev. D62, 071503 (2000)

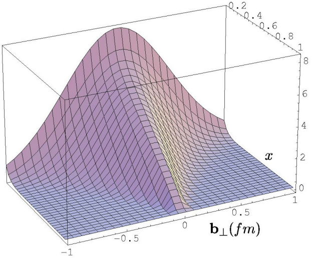
- No relativistic corrections (Galilean subgroup!)

$\hookrightarrow$ corollary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free of relativistic corrections

- probabilistic interpretation

$q(x, \mathbf{b}_\perp)$ for unpol. p

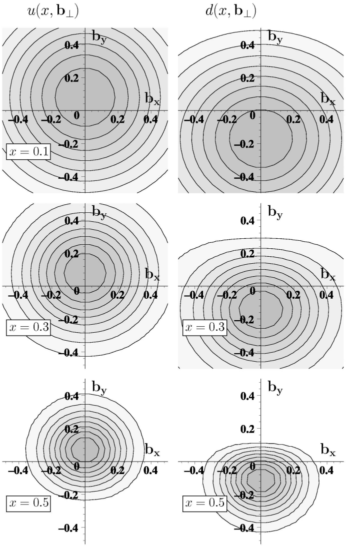


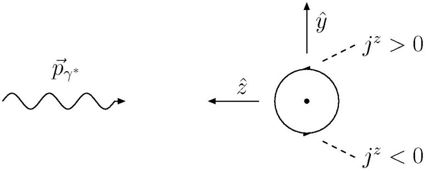




## unpolarized proton

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$
- $x$ = momentum fraction of the quark
- $\vec{b} = \perp$ distance of quark from $\perp$ center of momentum
- small $x$: large 'meson cloud'
- larger $x$: compact 'valence core'
- $x \to 1$: active quark becomes center of momentum

$\hookrightarrow$ $\vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$



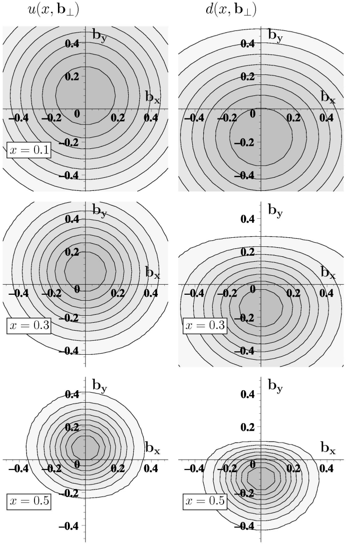

**proton polarized in $+\hat{x}$ direction**

no axial symmetry!

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

$$-\frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

Physics: relevant density in DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from $j^3$

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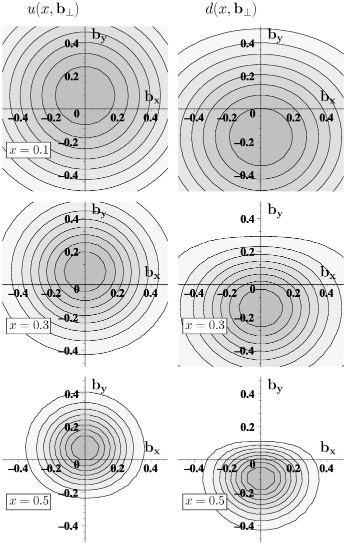
## sign & magnitude of the average shift

model-independently related to p/n anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x,\mathbf{b}_\perp) b_y$$
$$= \frac{1}{2M}\int dx E_q(x,0,0) = \frac{\kappa_q}{2M}$$

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$\kappa^p = 1.913 = \frac{2}{3}\kappa_u^p - \frac{1}{3}\kappa_d^p + ...$

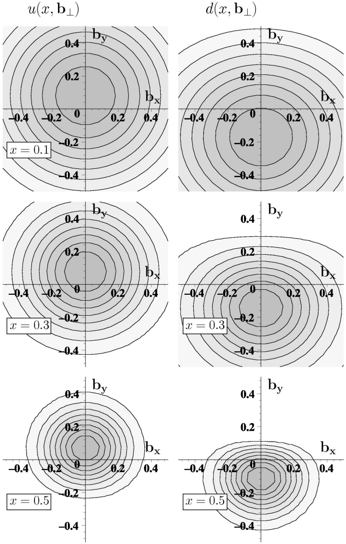
- $u$-quarks: $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$

$\hookrightarrow$ shift in $+\hat{y}$ direction

- $d$-quarks: $\kappa_d^p = 2\kappa_n + \kappa_p = -2.033$

$\hookrightarrow$ shift in $-\hat{y}$ direction

- $\langle b_y^q \rangle = \mathcal{O}(\pm 0.2 fm)$ !!!!

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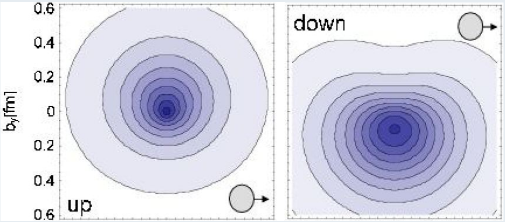
## lattice QCD (QCDSF): lowest moment



**transverse images ↔ Ji relation for quark angular momentum:**

- $J_q^x = m_N \int dx\, x r^y q(x,\mathbf{r}_\perp)$ with $b^y = r^y - \frac{1}{2m_N}$, where $q(x,\mathbf{r}_\perp)$ is distribution relative to CoM of whole nucleon
- recall: $q(x,\mathbf{b}_\perp)$ for nucleon polarized in $+\hat{x}$ direction

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} - \frac{1}{2M_N}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

$$\Rightarrow J_q^x = M_N \int dx\, x r^y q(x,\mathbf{r}_\perp) = \int dx\, x \left(m_N b^y + \frac{1}{2}\right) q(x,\mathbf{r}_\perp)$$
$$= \frac{1}{2}\int dx\, x\left[H(x,0,0) + E(x,0,0)\right]$$

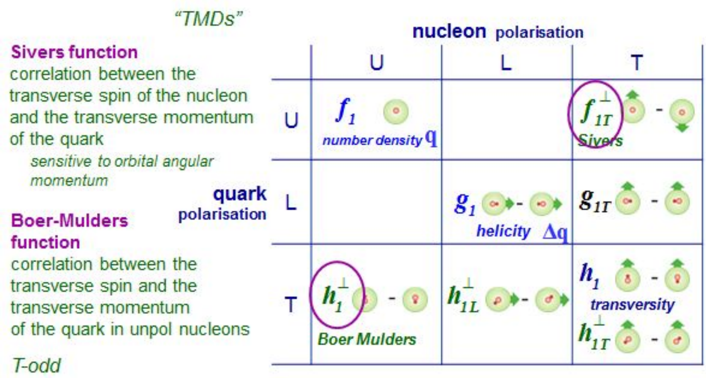
- X.Ji(1996): rotational invariance ⇒ apply to all components of $\vec{J}_q$
- partonic interpretation exists only for ⊥ components!

## TMDs

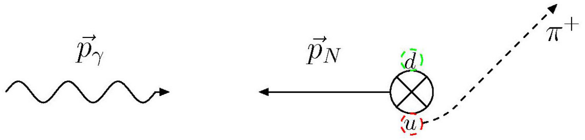
- Transverse Momentum Dependent Parton Distributions
- 8 structures possible at leading twist (only 3 for PDFs)
- $f_{1T}^{\perp}$ and $h_1^{\perp}$ require both orbital angular momentum and final state interaction
- can be measured in SIDIS and DY

### facilities

JLab@6GeV & 12GeV, HERMES, COMPASS I & II, RHIC, FAIR/PANDA, EIC



example: semi-inclusive deep-inelastic scattering (SIDIS) $\gamma p \to \pi X$




- $u, d$ distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign 'determined' by $\kappa_u$ & $\kappa_d$
- attractive FSI deflects active quark towards the CoM

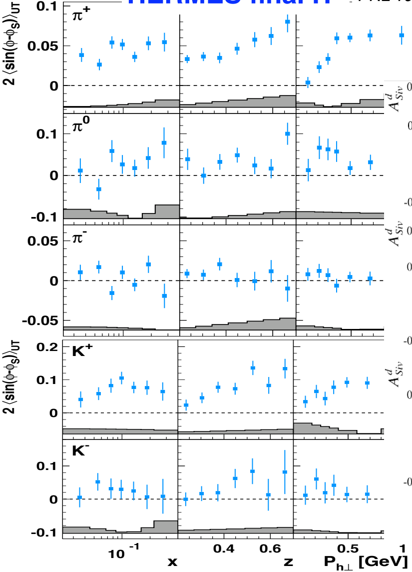
$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction $\to$ 'chromodynamic lensing'

$\Rightarrow$ $\kappa_p \, , \kappa_n \quad \longleftrightarrow \quad$ sign of SSA!!!!!!!!

- confirmed by HERMES (and recent COMPASS) $p$ data; consistent with vanishing isoscalar Sivers (COMPASS)

## Sivers Moments for π and K from H↑ & D↑

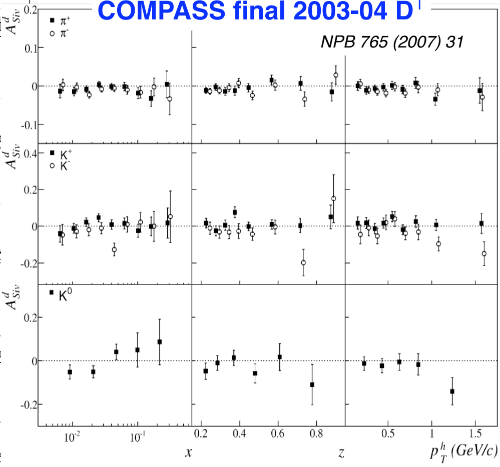
**HERMES final H↑** *PRL 103 (2009)*

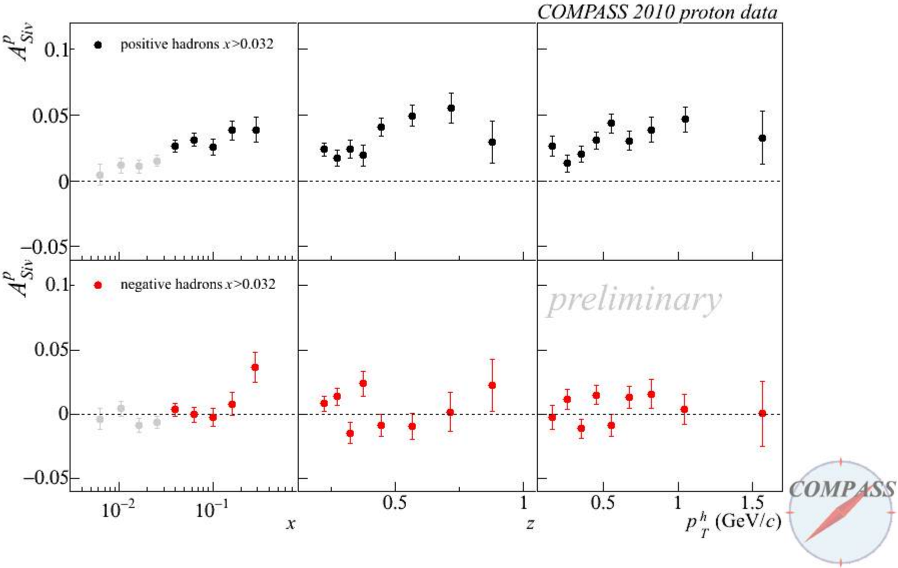



$$f_{1T}^{\perp}(x, k_T) \otimes D_1^{\perp}(z)$$

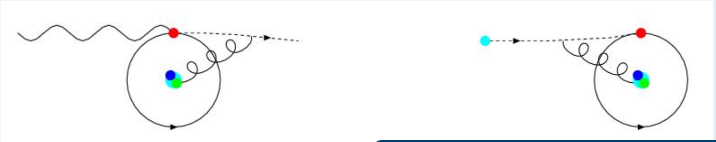
**COMPASS final 2003-04 D↑**

*NPB 765 (2007) 31*




COMPASS 2010 proton data
$A^p_{Siv}$
positive hadrons $x$>0.032
negative hadrons $x$>0.032
preliminary
0.1
0.05
0
−0.05
$10^{-2}$
$10^{-1}$
$x$
0.5
1
$z$
1.5
$p^h_T$ (GeV/$c$)
COMPASS

compare FSI for 'red' $q$ that is being knocked out of nucleon with ISI for 'anti-red' $\bar{q}$ that is about to annihilate with a 'red' target $q$



**FSI in SIDIS**

- knocked-out $q$ 'red'
- ↪ spectators 'anti-red'
- ↪ interaction between knocked-out quark and spectators attractive

**ISI in DY**

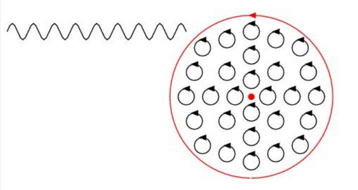
- incoming $\bar{q}$ 'anti-red'
- ↪ struck target $q$ 'red'
- ↪ spectators also 'anti-red'
- ↪ interaction between incoming $\bar{q}$ and spectators repulsive

test of $f_{1T}^{\perp}(x,\mathbf{k}_\perp)_{DY} = -f_{1T}^{\perp}(x,\mathbf{k}_\perp)_{SIDIS}$ critical test of TMD factorization approach

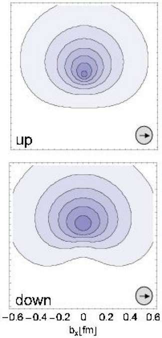
**facilities**

COMPASS II, RHIC, J-PARC, Fermilab/SeaQuest, FAIR/PANDA

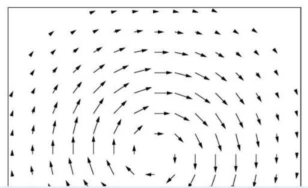
## $q$ with polarization $\odot$



## lattice calculation (QCDSF)




## unpolarized target



- transversity distribution in unpol. target described by chirally odd GPD $\bar{E}_T$
- $\bar{E}_T > 0$ for $u$ & $d$ (QCDSF)
- connection $h_1^\perp(x, \mathbf{k}_\perp) \leftrightarrow \bar{E}_T$ similar to $f_{1T}^\perp(x, \mathbf{k}_\perp) \leftrightarrow E$.

$\hookrightarrow$ $h_1^\perp(x, \mathbf{k}_\perp) < 0$ for $u/p$, $d/p$, $u/\pi$, $\bar{d}/\pi$

- $h_1^\perp{}_{SIDIS} = -h_1^\perp{}_{DY}$

## facilities

SIDIS (no pol. needed): JLab@6 & 12 GeV, Compass , EIC
DY (one beam $\perp$ pol.): RHIC, FAIR/Panda

## higher twist in polarized DIS

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $g_1 = \frac{1}{2}\sum_q e_q^2 g_1^q$ with $g_1^q = q^\uparrow(x) + \bar{q}^\uparrow(x) - q^\downarrow(x) - \bar{q}^\downarrow(x)$
- $g_2$ involves quark-gluon correlations

$\hookrightarrow$ no parton interpret. as difference between number densities for $g_2$

- for $\perp$ pol. target, $g_1$ & $g_2$ contribute equally

$$\sigma_{LT} \propto g_T \equiv g_1 + g_2$$

$\hookrightarrow$ 'clean' separation between $g_2$ and $\frac{1}{Q^2}$ corrections to $g_1$

## What can we learn from $g_2$?

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3\int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x}\left\langle P,S\left|\bar{q}(0)gG^{+y}(0)\gamma^+ q(0)\right|P,S\right\rangle$$

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$$\sqrt{2} G^{+y} = G^{0y} + G^{zy} = -E^y + B^x$$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

**color Lorentz force**

$$\sqrt{2} G^{+y} = G^{0y} + G^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

$$d_2 \equiv 3\int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x}\left\langle P,S\left|\bar{q}(0)gG^{+y}(0)\gamma^+q(0)\right|P,S\right\rangle$$

## color Lorentz force

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↪ $d_2 \leftrightarrow$ average **color Lorentz force** acting on quark moving with $v = c$ in $-\hat{z}$ direction in the instant after being struck by $\gamma^*$

$$\langle F^y\rangle = -2M^2 d_2 = -\frac{M}{P^{+2}S^x}\left\langle P,S\left|\bar{q}(0)gG^{+y}(0)\gamma^+q(0)\right|P,S\right\rangle$$

## sign of $d_2 \leftrightarrow \perp$ imaging

- $\kappa_q/p \longrightarrow$ sign of deformation

↪ direction of average force

↪ $d_2^u > 0,\ d_2^d < 0$

- *cf.* $f_{1T}^{\perp u} < 0,\ f_{1T}^{\perp u} < 0$

## lattice (Göckeler et al., 2005)

$d_2^u \approx 0.010,\ d_2^d \approx -0.0056$

## magnitude of $d_2$

- $\langle F^y\rangle = -2M^2 d_2 = -10\frac{GeV}{fm} d_2$
- expect partial cancellation of forces in SSA

↪ $|\langle F^y\rangle| \ll \sigma \approx 1\frac{GeV}{fm}$

↪ $d_2 = \mathcal{O}(0.01)$

## facilities

SLAC, JLab@6GeV, JLab@12GeV

- Deeply Virtual Compton Scattering (DVCS) $\longrightarrow$ GPDs

$\hookrightarrow$ impact parameter dependent PDFs $q(x, \mathbf{b}_\perp)$

- $E^q(x, 0, -\boldsymbol{\Delta}_\perp^2) \leftrightarrow \kappa_{q/p}$ (contribution from quark flavor $q$ to anomalous magnetic moment)
- $E^q(x, 0, -\boldsymbol{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target
- $\perp$ deformation $\leftrightarrow$ (sign of) SSA (Sivers; Boer-Mulders)
- parton interpretation for Ji-relation
- higher-twist $\left(\int dx\, x^2 \bar{g}_2(x), \int dx\, x^2 \bar{e}(x)\right) \leftrightarrow \perp$ force in DIS
- $\perp$ deformation $\leftrightarrow$ (sign of) quark-gluon correlations $\left(\int dx\, x^2 \bar{g}_2(x), \int dx\, x^2 \bar{e}(x)\right)$

combine complementary information from deeply-virtual Compton scattering, semi-inclusive DIS & Drell-Yan to study orbital angular momentum and map uncover the 3-d structure of hadrons