Introduction to QCD

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University of Helsinki

Midsummer School in QCD

Saariselkä, 24 June – 6 July 2024

50 Years of Quantum Chromodynamics

F. Gross et al.,
Eur.Phys.J.C 83 (2023) 1125 [2212.11107] (636 p.)

UCLA conference in September 2023
https://indico.cern.ch/event/1276932/overview

The Standard Model

QCD — SU(3) x SU(2)$_L$ x U(1) — Electroweak

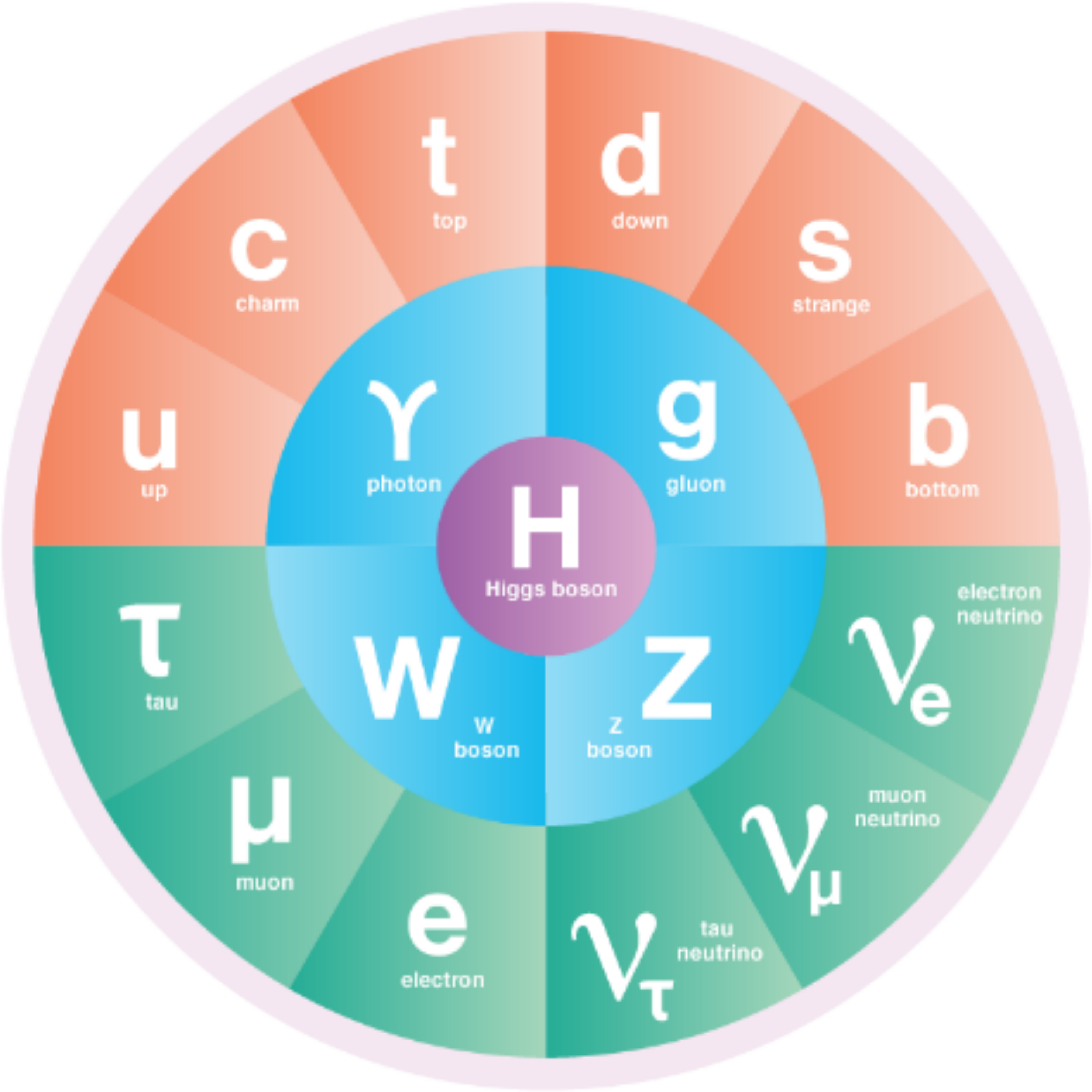
How this Lagrangian?

Confinement

Chiral symmetry breaking

Duality

Abundance of data



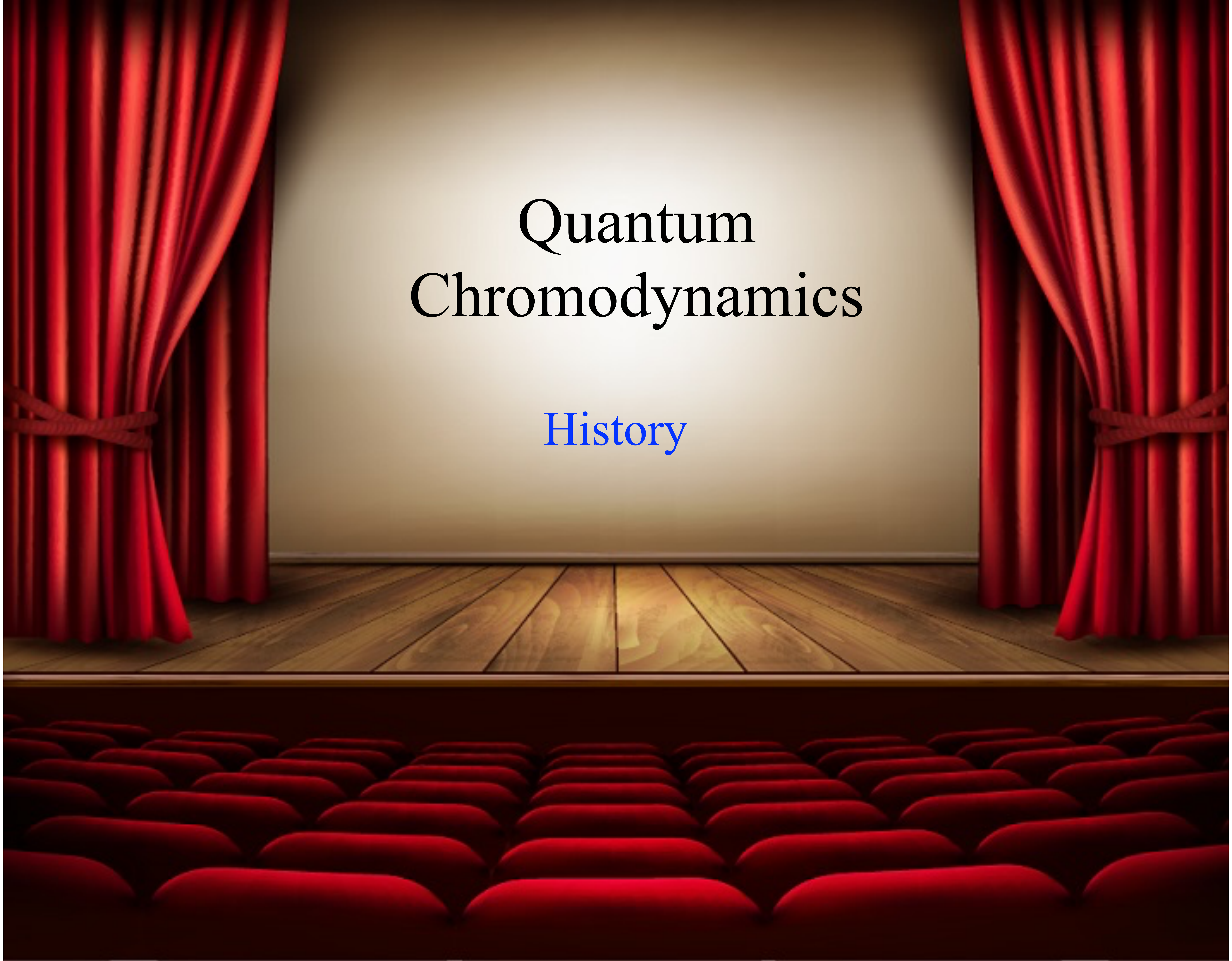
Why this Lagrangian?

Higgs field

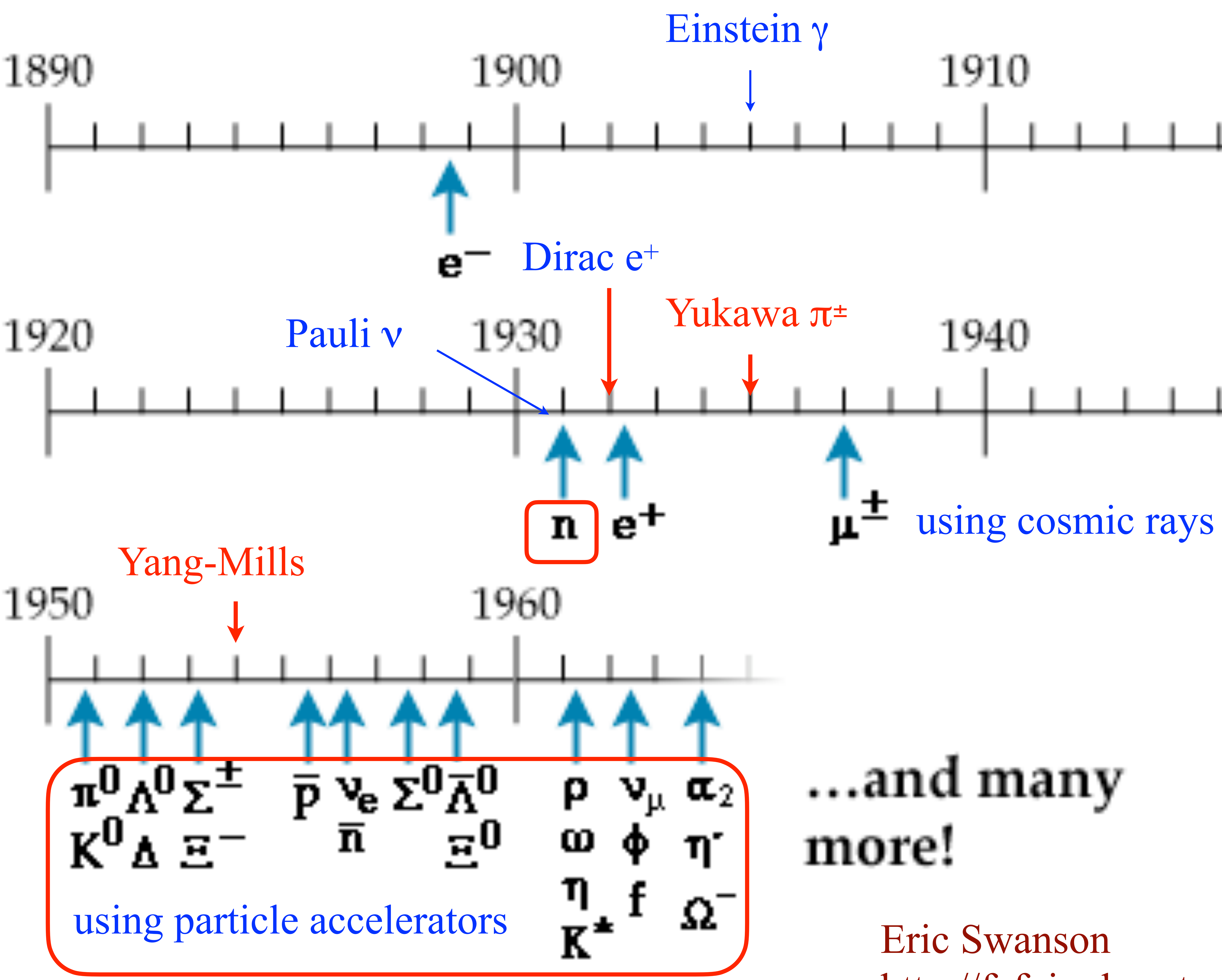
V–A interaction

Masses, Mixings

Few hints of BSM in data

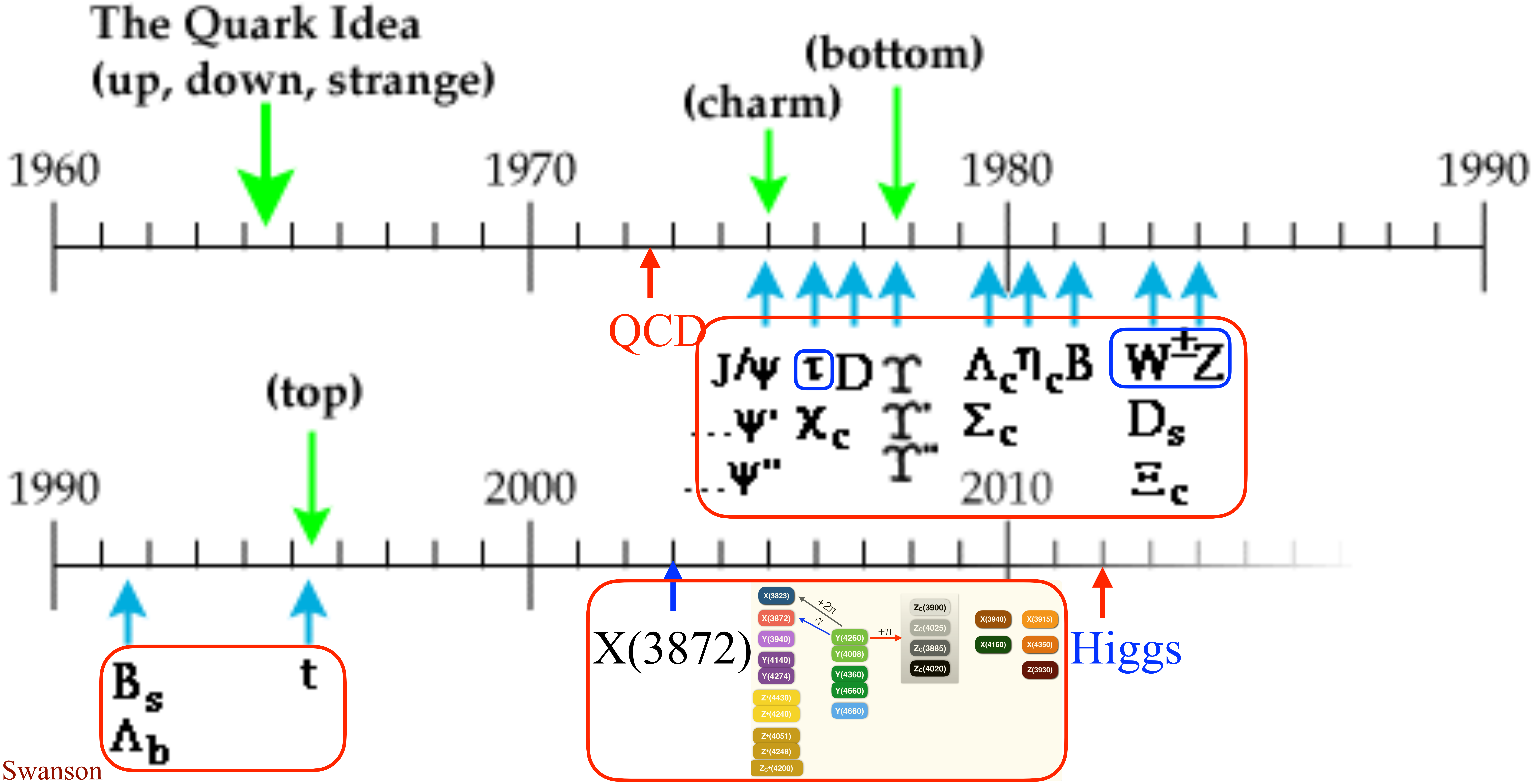

Quantum
Chromodynamics
History

Particle discoveries (< 1964)



Eric Swanson
http://fafnir.phyast.pitt.edu/particles/conuni5.html

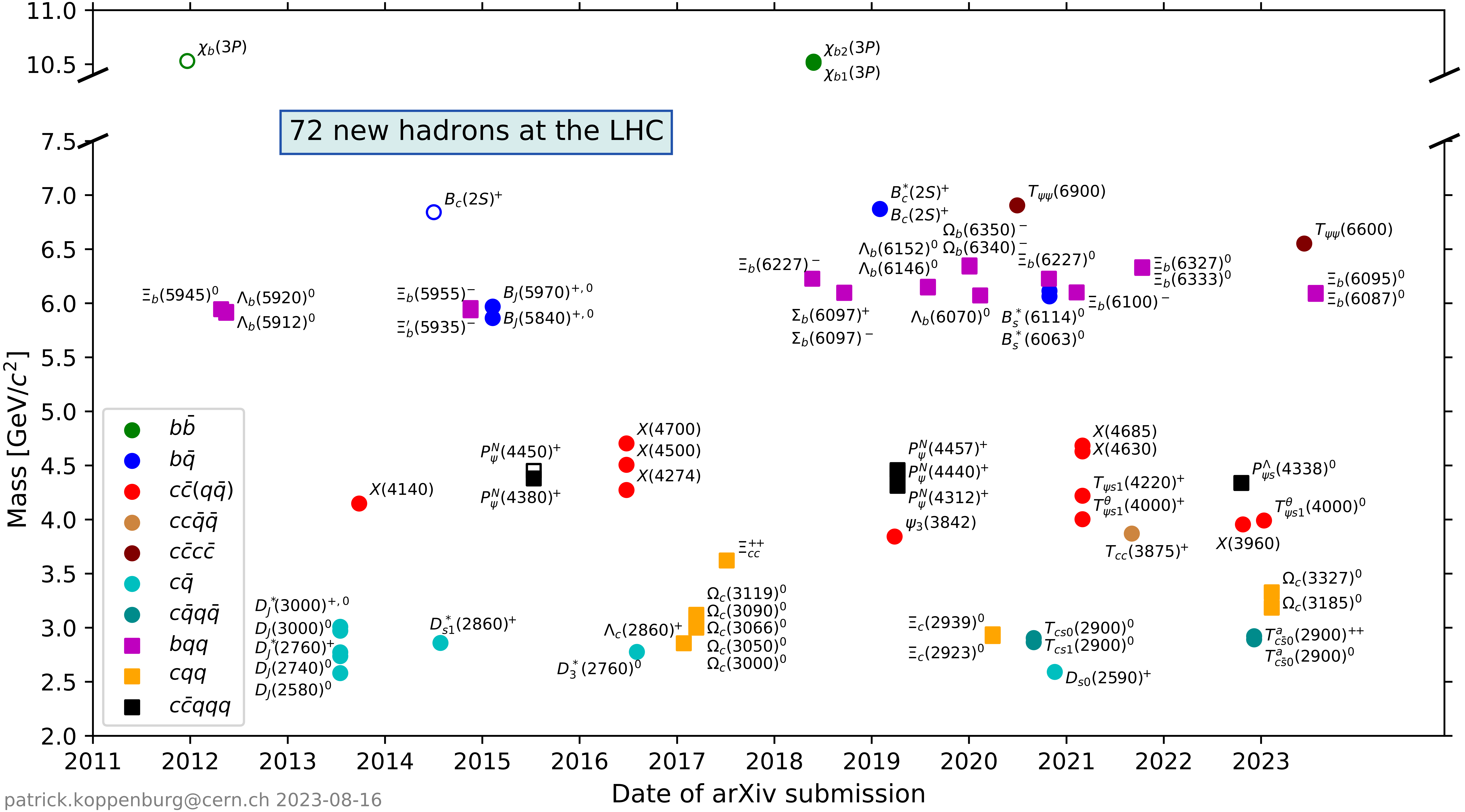
Particle discoveries (1964 - 2012)



Eric Swanson
http://fafnir.phyast.pitt.edu/particles/conuni5.html

H.-X. Chen et al 1601.02092

Hadrons discovered at the LHC (2012 -)



patrick.koppenburg@cern.ch 2023-08-16

The emergence of QCD

Analyticity
Unitarity
S-Matrix Theory
Regge Theory

Era of confusion

Quarks
Confinement
QCD

1950s –1960 → 1972

- Field theory abandoned for strong interactions
- All hadrons equally "fundamental", strictly phenomenological
- No attempt at dynamics (no Hamiltonian)
- Long distances & low energies (and momentum transfer)

- Field theory resurrected!
- All hadrons emergent
- Lagrangian dynamics
- Short distances & high energies

? ← 1972 PH

Quantum Chromodynamics

Basic features

The QCD Fields

$$\mathscr{L}_{QCD}(x) = \sum_f \overline{\psi}_f^B \left(i\partial_\mu \gamma^\mu - g\, A_\mu^a\, t_a^{BC} \gamma^\mu - m_f\right)\psi_f^C - \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a$$

$\psi_f^B(x)$ Quarks have three "colors" $B = 1, 2, 3$ or $B =$ red, blue, green

There are six quark flavors $f = u, d, s, c, b, t$, each with mass m_f

$A_\mu^a(x)$ Gluons have eight colors $a = 1, 2, \ldots, 8$ $\alpha_s = \dfrac{g^2}{4\pi}$ QCD coupling

$2t_a$ 3x3 "Gell-Mann" color matrices (analogous to the 2x2 Pauli matrices)

$$F_{\mu\nu}^a(x) = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\, f_{abc} A_\mu^b A_\nu^c$$

Gluon field strength with self-coupling

f_{abc} are the SU(3) structure constants

Chiral symmetry

$$\mathscr{L}_{QCD}(x) = \sum_f \overline{\psi}_f^B \left(i\partial_\mu\gamma^\mu - g\,A_\mu^a\,t_a^{BC}\gamma^\mu - m_f\right)\psi_f^C - \frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a$$

Invariant under $\psi(x) \to e^{i\gamma_5}\psi(x)$ (global) chiral transformation when $m_f = 0$.

A chiral transformation changes parity and implies parity doubling:
Every hadron should have an identical partner with opposite parity.

Parity degeneracy is not observed in data, even though $m_u, m_d \ll \Lambda_{QCD}$.

Hence chiral symmetry must be spontaneously broken:
The chiral symmetry of $\mathscr{L}_{QCD}(x)$ is not a symmetry of the physical states.

This implies (nearly) massless Goldstone bosons: The $\pi^{\pm,0}$ with $m_\pi \ll m_p$.
Symmetry of $\mathscr{L}_{QCD}(x)$ constrains pion interactions: Chiral perturbation theory.

SU(3) local gauge invariance

A gauge transformation $U(x)$ transforms the quark and gluon fields at each $x = (t, \boldsymbol{x})$:

$$\psi^A(x) \to U^{AB}(x)\psi^B(x) \qquad U(x)U^\dagger(x) = 1\,, \;\; \det U(x) = 1, \qquad 3\times 3$$

$$A_\mu^a(x)\, t_a \to U(x)A_\mu^a(x)\, t_a U^\dagger(x) - \frac{i}{g}U(x)\partial_\mu U^\dagger(x)$$

$$F_{\mu\nu}^a\, t_a \to U(x)F_{\mu\nu}^a\, t_a(x)U^\dagger(x)$$

Color electric and magnetic fields transform

Gauge fixing: $\partial_\mu A^\mu(t, \mathbf{x}) = 0$ (Feynman gauge), $A^0(t, \mathbf{x}) = 0$ (temporal gauge), …

All physical (measurable) quantities have to be gauge invariant

Uniqueness of gauge theory Lagrangians

The form of $\mathscr{L}(x)$ is determined by locality in x and

Gauge symmetry, *ie.*, U(1) for QED and SU(3) for QCD

Relativistic invariance: $x^\mu \to \Lambda^\mu_{\ \nu}\, x^\nu + c^\mu$ (Poincaré = Lorentz + Translations)

Renormalizability: Regularisation of loop integrals without new couplings

Photon and gluon mass terms $\frac{1}{2}m^2 A^\mu A_\mu$ are not gauge invariant, hence $m = 0$:

$m_\gamma < 10^{-18}$ eV $\qquad m_g = 0$ (but confined)

$m_{W,Z} \neq 0$ (gauge invariance spontaneously broken by Higgs)

Universality of QCD coupling

$$\mathscr{L}_{QCD}(x) = \sum_f \bar{\psi}_f^B \left(i\partial_\mu \gamma^\mu - g\, A_\mu^a\, t_a^{BC} \gamma^\mu - m_f\right) \psi_f^C - \tfrac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a$$

$g \neq g_f$ All quarks couple to gluons with the same g

Not so in QED: $g \to e_f\, e$ electric charge $\alpha = \dfrac{e^2}{4\pi} \simeq \dfrac{1}{137}$

$e_u = \frac{2}{3}, \quad e_d = -\frac{1}{3}, \quad e_e = -1$ coupling is not universal for U(1) theories

The proton charge is $e_p = 2e_u + e_d = 2 \cdot \frac{2}{3} - \frac{1}{3} = 1 = -e_e$ Accidentally?

Data: $|e_p + e_e| < 1.0 \cdot 10^{-21}$ Quark and lepton charges seem related!

The symmetry between leptons and quarks hints at physics Beyond the SM: BSM

Strong CP violation?

"Naturality": $\mathscr{L}$ should have all allowed terms, with coefficients of $\mathcal{O}(1)$

$$\mathscr{L}_{QCD} \to \mathscr{L}_{QCD} + \theta \frac{g^2}{64\pi^2} \varepsilon_{\mu\nu\rho\sigma} F_a^{\mu\nu} F_a^{\rho\sigma}$$

gauge and Poincaré invariant

$\theta \neq 0$ breaks CP symmetry. Neutron electric dipole moment: $\theta < 10^{-10}$

Peccei and Quinn (1977): $\theta = 0$ if there are "axion" particles.

Peccei and Quinn, PRL 38, 1440 (1977)

$\Longrightarrow$ Many experimental searches for axions!

But maybe there is no CP violation, even if $\theta \neq 0$?

Ai, Cruz, Garbrecht and Tamarit, Phys. Lett. B 822 (2021), 136616

Issue related to boundary conditions: $A_a^\mu(x \to \infty)$

Confinement?

Physical scales in QED

$$\mathscr{L}_{QED}(x) = \sum_l \bar{\psi}_l\left(i\partial_\mu\gamma^\mu - e_l\, e\, A_\mu\gamma^\mu - m_l\right)\psi_l - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

The lepton masses $m_l = m_e, m_\mu, m_\tau$ determine the scales of QED.

E.g., for the Hydrogen atom $(e^- p)$: Binding energy: $E_B \simeq \frac{1}{2}\alpha^2 m_e$

Radius: $1/r_H \simeq \alpha m_e)$

For Muonium $(\mu^- p)$: $m_e \to m_\mu$

Physical scales in QCD

$$\mathscr{L}_{QCD}(x) = \sum_f \bar{\psi}_f\left(i\partial_\mu\gamma^\mu - g\,A_\mu\gamma^\mu - m_f\right)\psi_f - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

The quark masses $m_f = m_u, m_d, m_s, m_c, m_b, m_t$ provide physical scales.

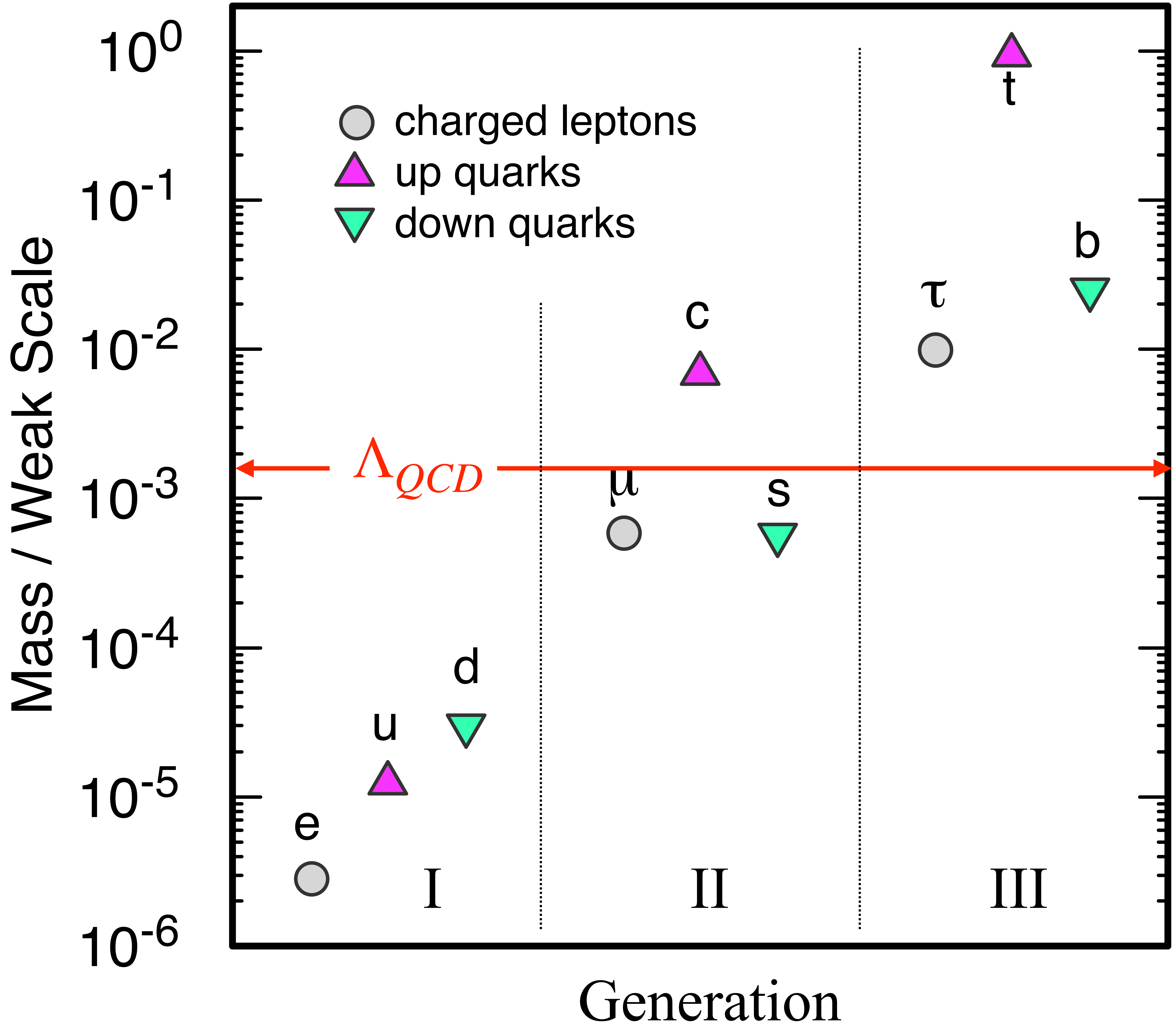
The u and d quarks masses: $m_u \simeq 2.16$ MeV, $m_d \simeq 4.67$ MeV

are small compared to the scale of the proton (uud): $m_p \simeq 938\,\text{MeV}$, $1/r_p \simeq 238\,\text{MeV}$

QCD has a "confinement scale" Λ_{QCD} of $\mathcal{O}(1\,\text{fm}^{-1} \simeq 200\,\text{MeV})$

that is not in $\mathscr{L}_{QCD}$

Physical scales



How can Λ_{QCD} be introduced without changing $\mathscr{L}_{QCD}$?


Quantum
Chromodynamics
Observables

From the QCD Lagrangian to observables

$S_{QCD} = \int d^4x\, \mathscr{L}_{QCD}(x)$ is the gauge and Poincaré invariant Action.

It requires a boundary condition: $A_a^\mu(x \to \infty)$

The expectation value of any functional $\mathcal{O}$ of the gluon and quark fields

[*E.g.*, for a quark propagating from x_1 to x_2 : $\mathcal{O} = \psi(x_2)\,\overline{\psi}(x_1)$] is given by

$$\langle \mathcal{O} \rangle = \int \mathscr{D}(A, \overline{\psi}, \psi)\, \mathcal{O}\, \exp(i\, S_{QCD})$$

Functional integral of QFT

c.f.: Path integral in QM

$\int \mathscr{D}(A, \overline{\psi}, \psi)$ integrates over the values of $A(x), \overline{\psi}(x), \psi(x)$ at all spacetime points x

Thus: An infinite number of integrals!

There are two main methods to evaluate $\langle \mathcal{O} \rangle$: Lattice QCD and Peturbation Theory

Lattice QCD

K. G. Wilson (1974)

In a finite, discrete space-time (lattice) there is a finite number of integrals: Do them numerically

Wilson discretization preserves exact gauge invariance. Poincaré invariance is restored in the continuum limit

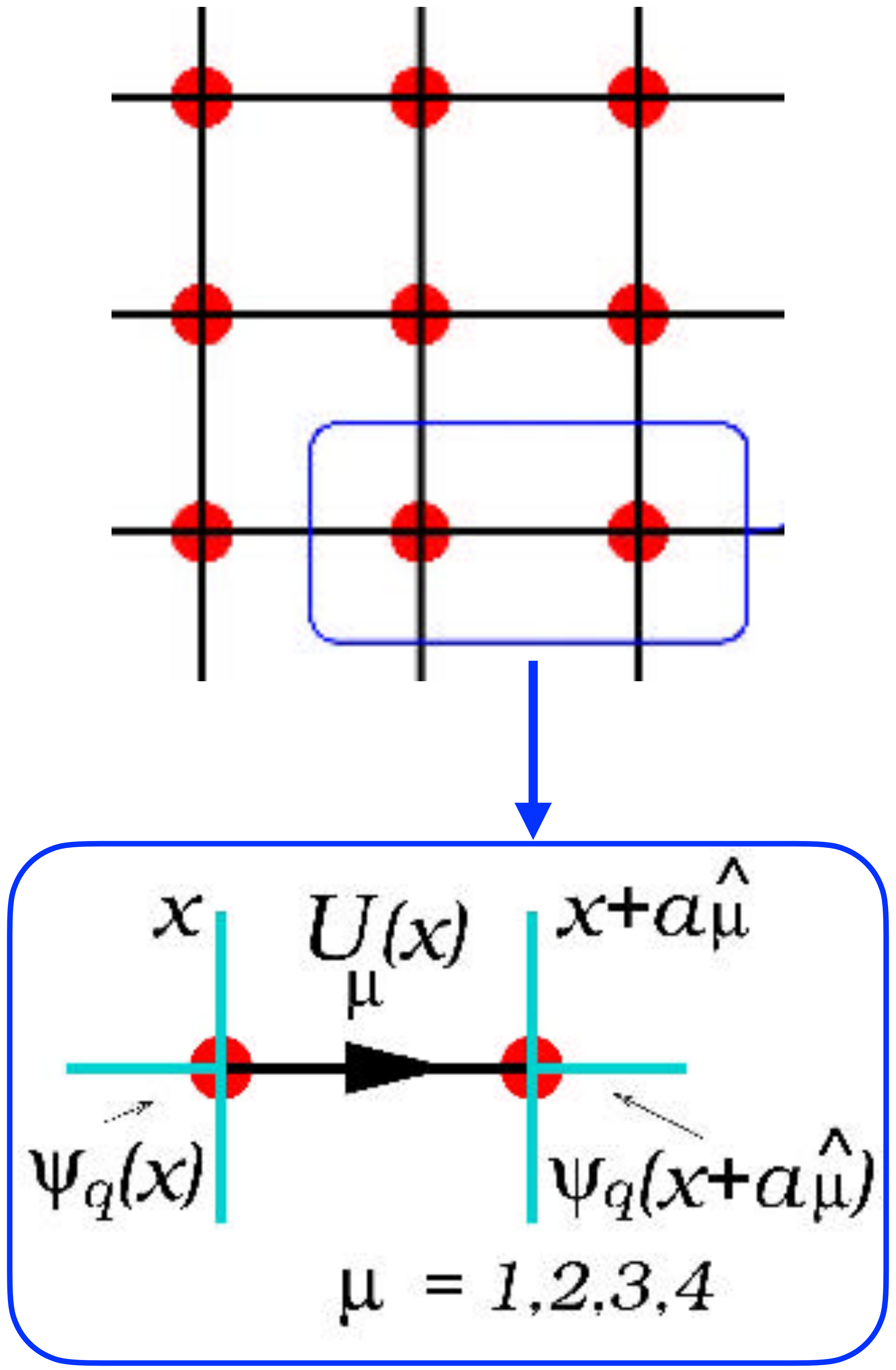
To avoid cancellations in the functional integral, go to

Euclidean space: $t \to i\tau$: $\exp(iS) \to \exp(-S)$

Allows to determine static quantities (masses, form factors)

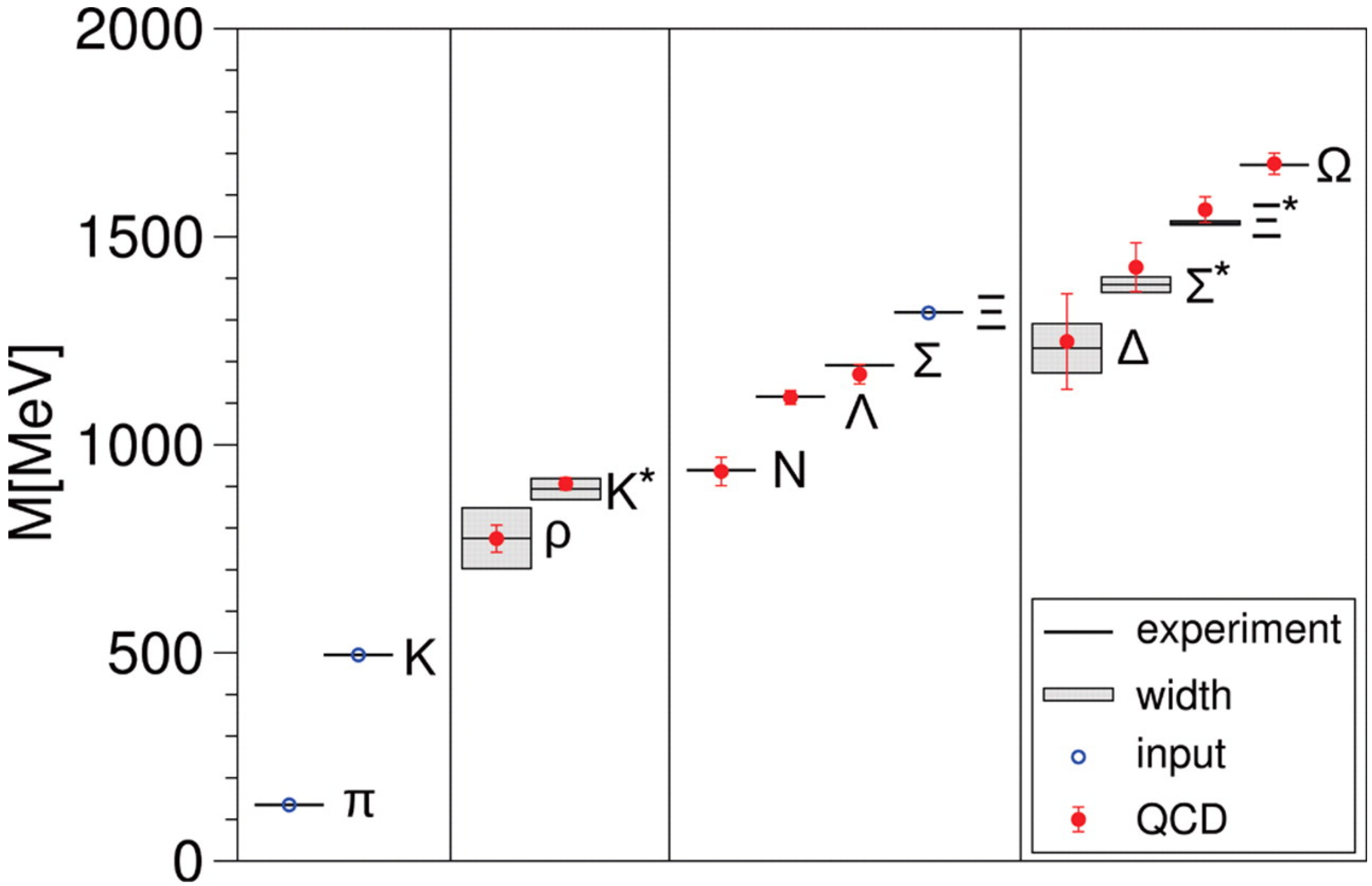
Confirms confinement and breaking of chiral invariance

Scattering and decays challenging: Require Minkowski space (real time t)



R. Soualah (2008)

Hadron masses from Lattice QCD



BMW Collaboration,
Science 322 (2008) 1224 [0906.3599]

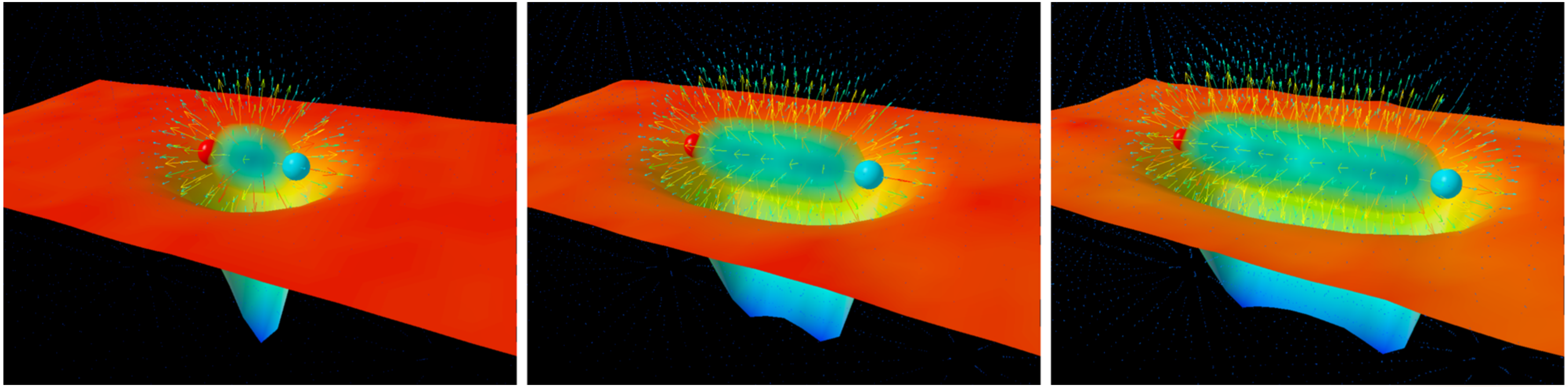
Results have been confirmed by other lattice calculations

A. S. Kronfeld, Annu. Rev. Nucl. Part. Sci. **62** (2012) 265 [1203.1204]

The QCD lattice view of confinement

Numerical simulations show the emergence of a color string between quarks.

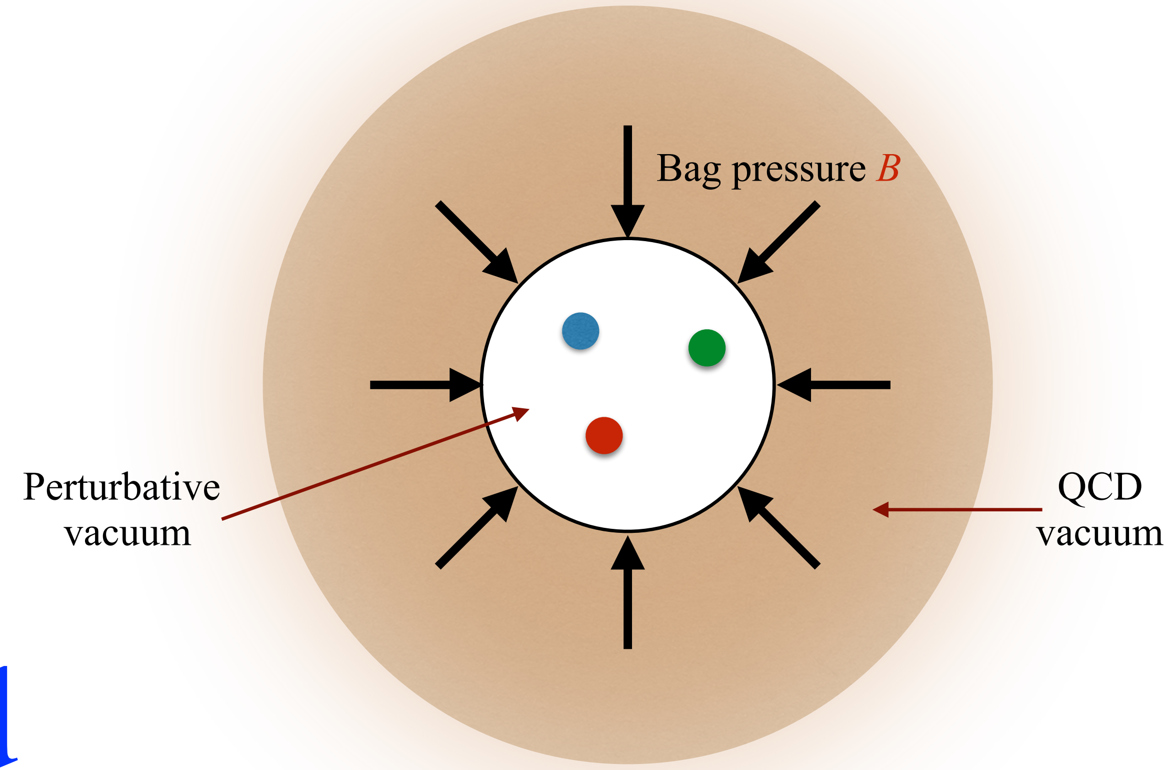
F. Gross, et al., Eur.Phys.J.C 83 (2023) 1125 [2212.11107]



Suggests a linear confinement potential: $V_C(r) = c\, r$

Recalls "Bag model": $\mathscr{L}_{bag} = (\mathscr{L}_{QCD} - B)\, \theta(bag)$

A. Chodos, et al., Phys. Rev. **D9** (1974) 3471



Λ_{QCD} scale can arise from a boundary condition on the gluon field

The unexpected success of the Quark Model

$n^{2s+1}\ell_J$	J^{PC}	I $= 1$ $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$
1^1S_0	0^{-+}	π
1^3S_1	1^{--}	$\rho(770)$
1^3P_0	0^{++}	$a_0(1450)$
1^1P_1	1^{+-}	$b_1(1235)$
1^3P_1	1^{++}	$a_1(1260)$
1^3P_2	2^{++}	$a_2(1320)$
1^3D_1	1^{--}	$\rho(1700)$
1^1D_2	2^{-+}	$\pi_2(1670)$
1^3D_3	3^{--}	$\rho_3(1690)$
1^3F_4	4^{++}	$a_4(1970)$
1^3G_5	5^{--}	$\rho_5(2350)$
2^1S_0	0^{-+}	$\pi(1300)$
2^3S_1	1^{--}	$\rho(1450)$
2^3P_1	1^{++}	$a_1(1640)$
2^3P_2	2^{++}	$a_2(1700)$
2^1D_2	2^{-+}	$\pi_2(1880)$
3^1S_0	0^{-+}	$\pi(1800)$

Particle Data Group

Mesons are $q\bar{q}$, baryons are qqq + nuclei ($\sim$ molecules)

No gluons or sea quarks required by quantum numbers

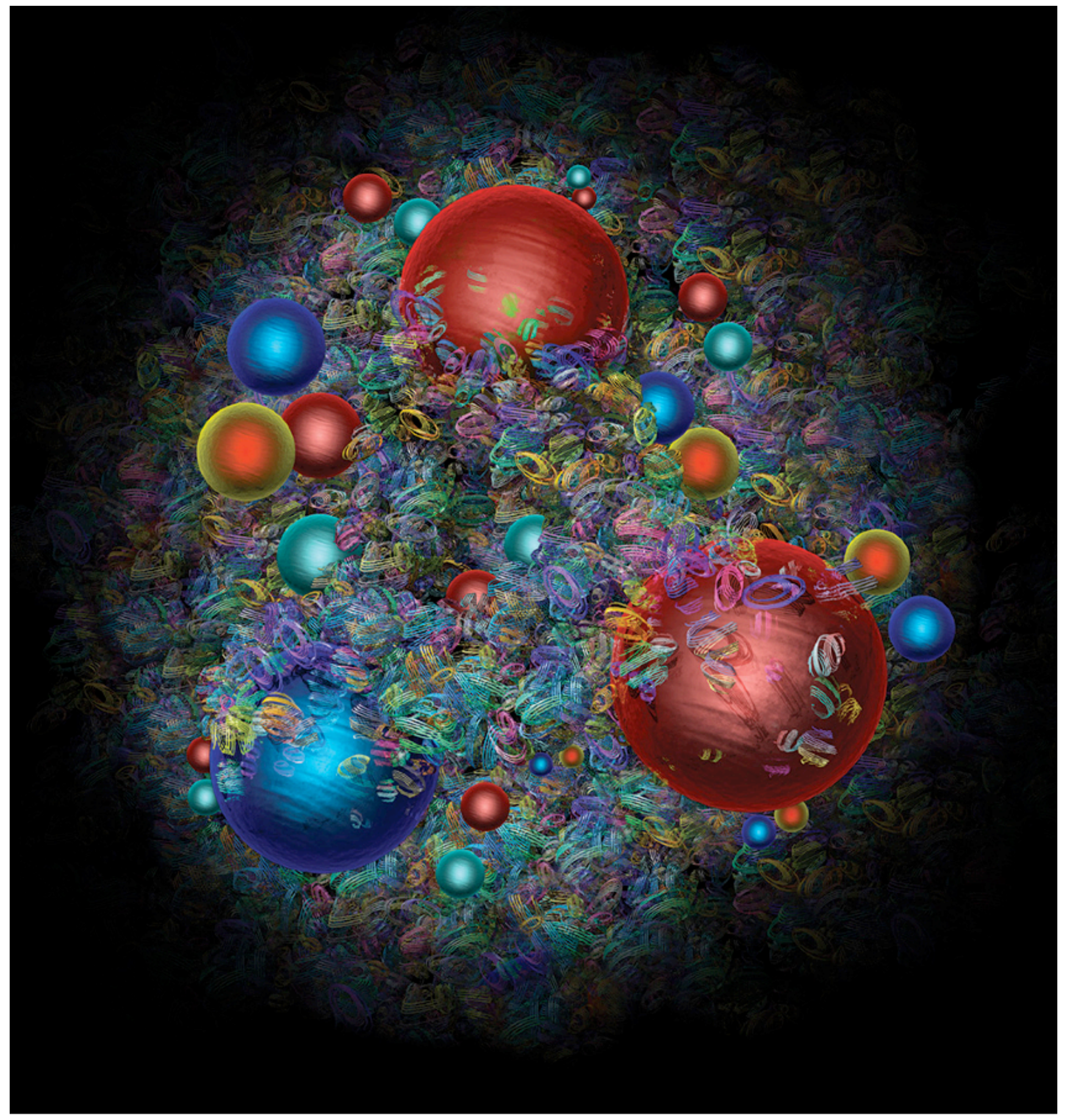
Large excitation energies: Strong binding

Mystery: Why does the strong field not create g, $q\bar{q}$?

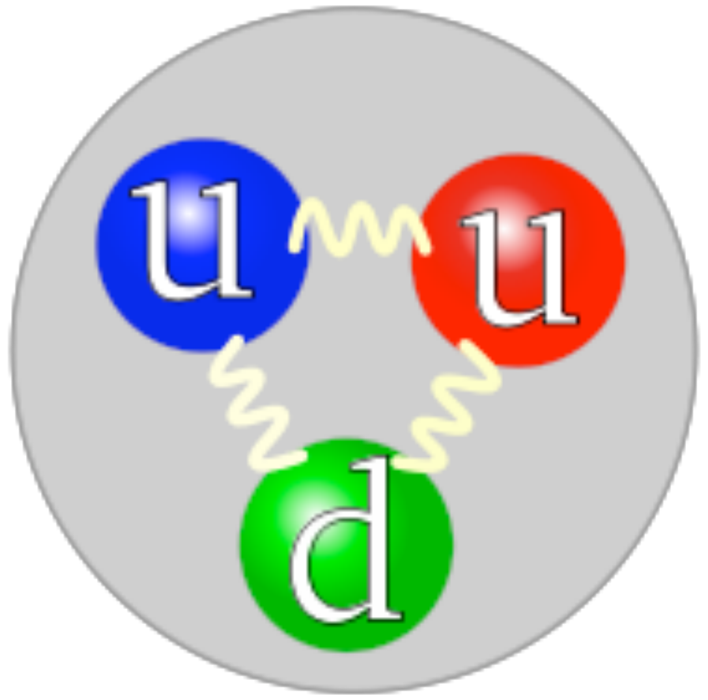
$m_u \simeq 2.16$ MeV

$m_d \simeq 4.67$ MeV

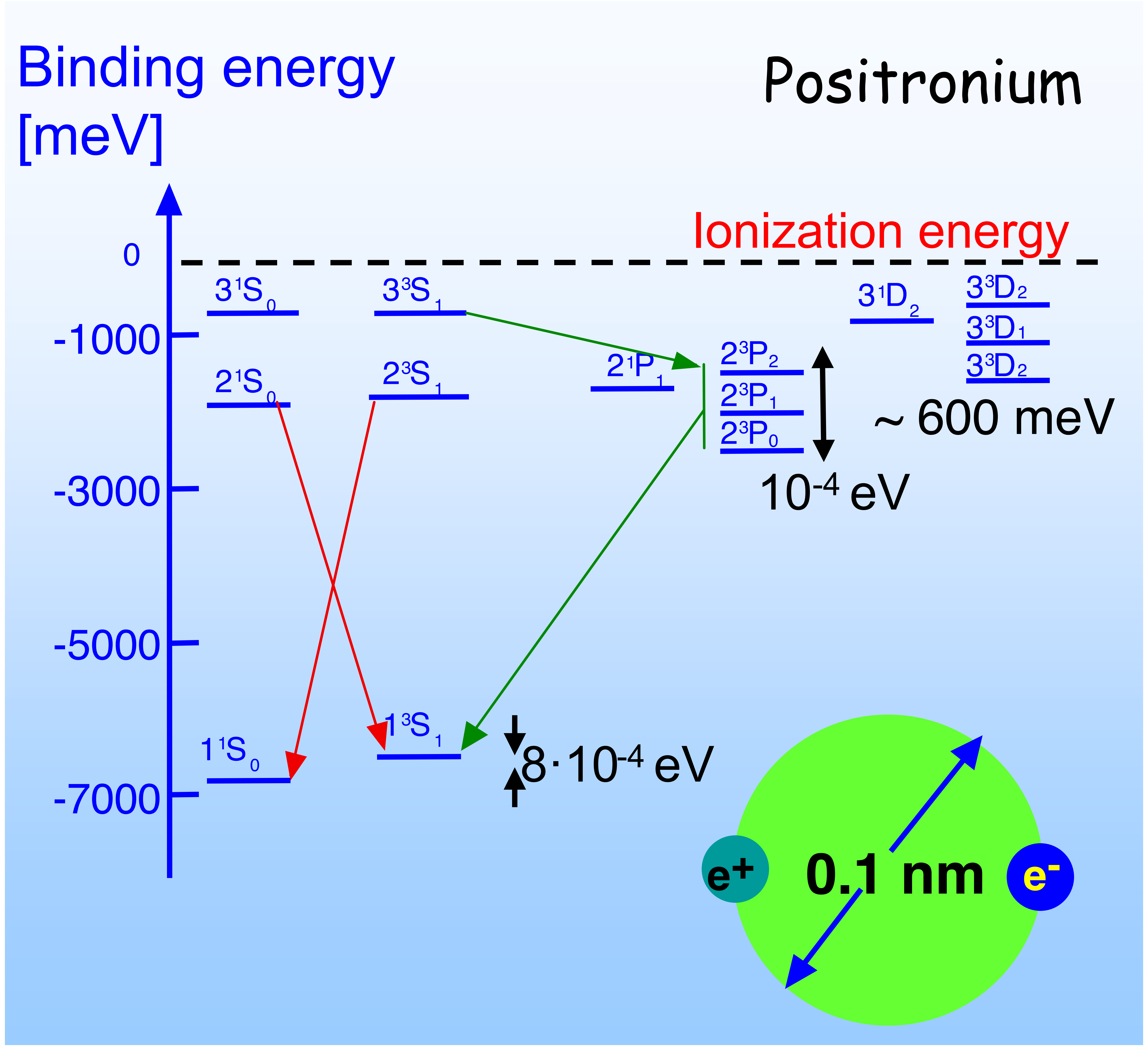
Expected:

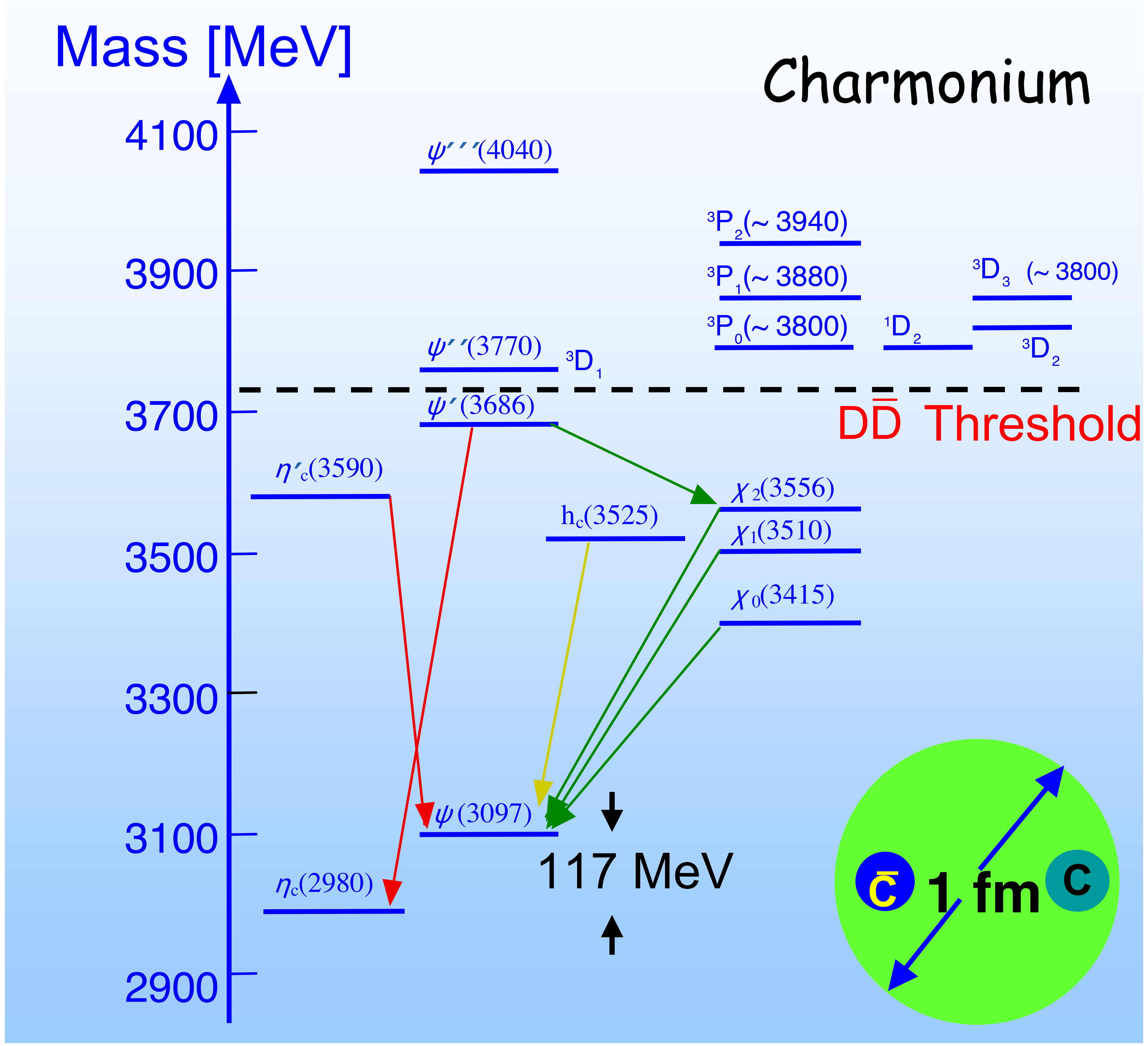


Seen:



Quarkonia are like atoms with confinement





$$V(r) = -\frac{\alpha}{r}$$

$$V(r) = V'r - \frac{4}{3}\frac{\alpha_s}{r} \quad (1980)$$

Charmonia ($c\bar{c}$) and bottomonia ($b\bar{b}$)

$m_c, m_b \gg \Lambda_{QCD}$ Quarkonia have confined, non-relativistic heavy quarks

Successfully described by Schrödinger equation, with phenomenological

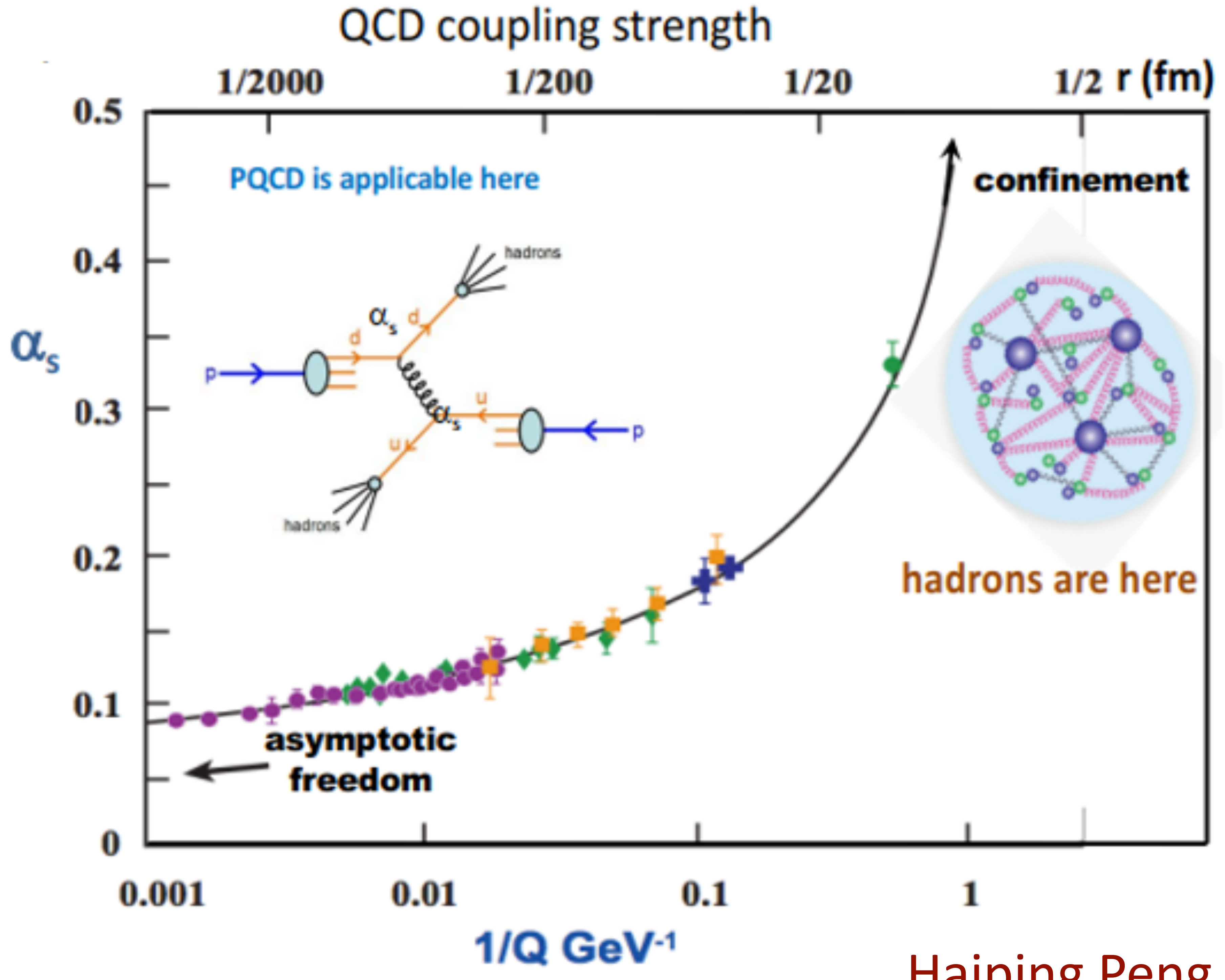
"Cornell potential" $V(r) = V'r - \frac{4}{3}\frac{\alpha_s}{r}$ $V' \simeq 0.18\ \mathrm{GeV}^2,\ \ \alpha_s \simeq 0.39$

Decays calculated perturbatively:

$\Gamma[J/\psi \to ggg] \propto \alpha_s^3$

E. Eichten et al, Phys. Rev. **D21** (1980) 203, Rev. Mod. Phys. **80** (2008) 1161

Confinement does not require large α_s !

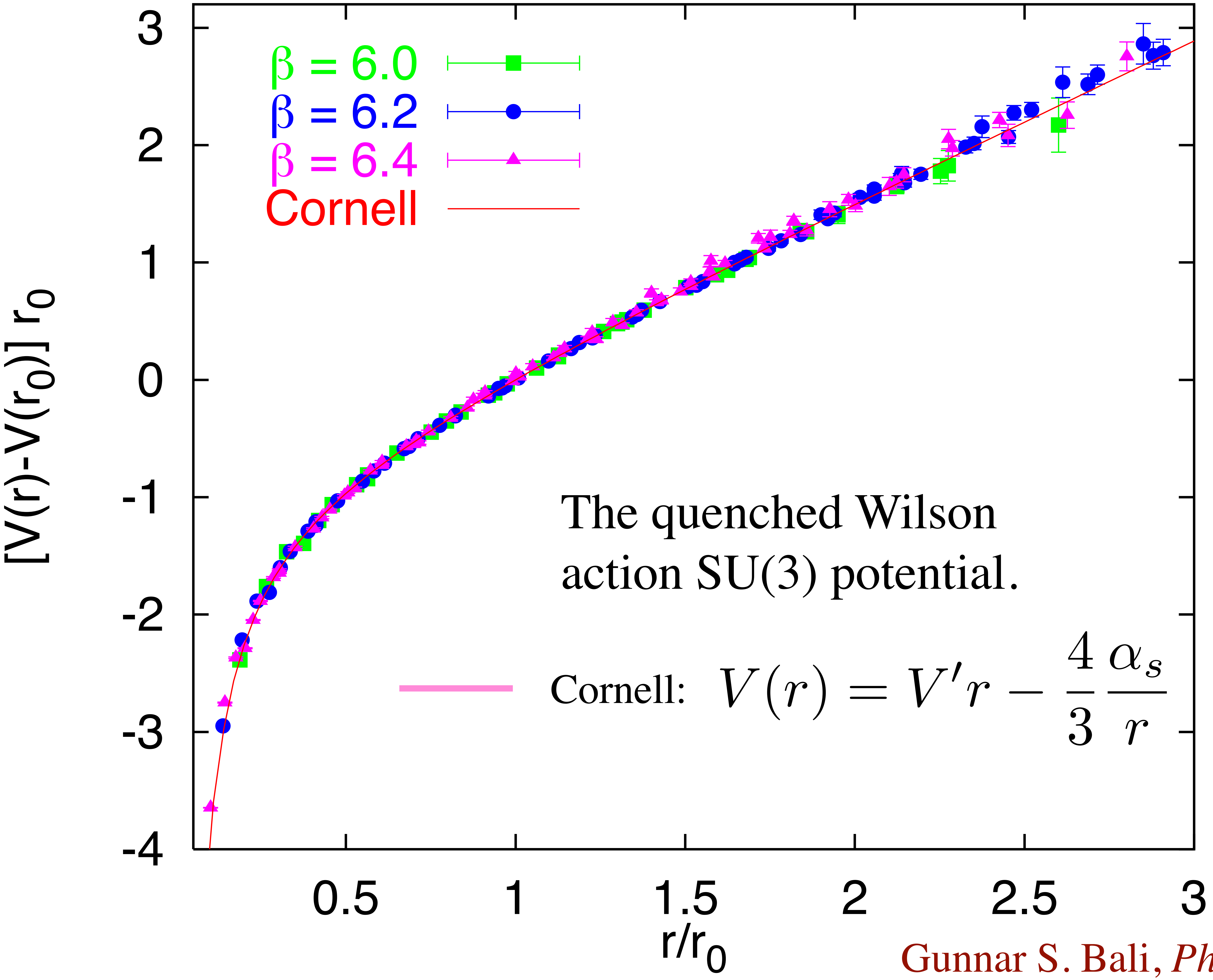


Despite common belief:

Haiping Peng

https://indico.pnp.ustc.edu.cn/event/91/contributions/6657/attachments/1856/3047/STCF-Workshop 20240113.pptx

Lattice QCD agrees with the Cornell potential



Gunnar S. Bali, *Phys.Rept.* **343** (2001) 1

Bound states are omitted in QFT textbooks

A.S. Blum, Stud. Hist. Phil. Sci. B60 (2017) 46 [2011.05908]:

THE STATE IS NOT ABOLISHED, IT WITHERS AWAY:
HOW QUANTUM FIELD THEORY BECAME
A THEORY OF SCATTERING

"Learning quantum field theory (QFT) for the first time, after first learning quantum mechanics (QM), one is (or maybe, rather, I was) struck by the change of emphasis: The notion of the quantum state, which plays such an essential role in QM, from the stationary states of the Bohr atom, over the Schrödinger equation to the interpretation debates over measurement and collapse, seems to fade from view when doing QFT."


Quantum
Chromodynamics
Perturbative methods

The Scattering matrix

Determine the generator of time translations (Hamiltonian): $\mathscr{L}_{QCD} \to H_{QCD}$

$H = H_0 + gH_{int}$ where H_0 is the free part, of $\mathcal{O}(g^0)$

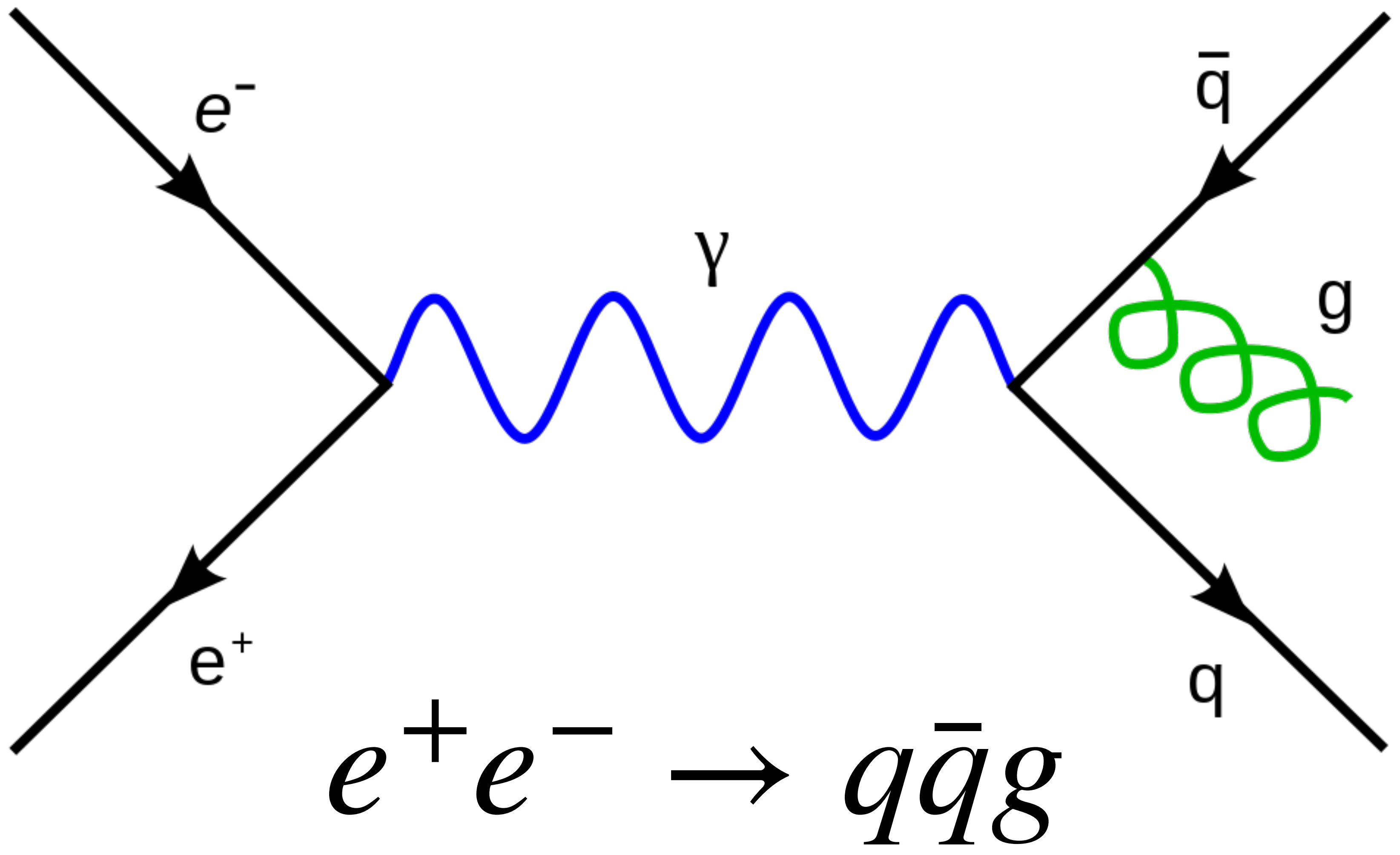
Use the free state basis (Interaction Picture): $H_0 \,|\, \psi, \overline{\psi}, A; t\rangle_0 = E_0 \,|\, \psi, \overline{\psi}, A; t\rangle_0$

$$S_{fi} = {}_0\langle f, t \to \infty \,|\, \left\{ \mathrm{T} \exp\left[-i \int_{-\infty}^{\infty} dt\, gH_{int}(t) \right] \right\} |\, i, t \to -\infty \rangle_0$$

The initial and final states i, f at $t = \pm\infty$ are free, as required for scattering

S_{fi} can, at each order in g, be pictured in terms of Feynman diagrams

Feynman diagrams



Each order in g is Poincaré invariant

Gauge invariance is not explicit: The propagation $A_a^\mu(x_1) \to A_b^\nu(x_2)$ depends on the gauge at x_1, x_2

S_{fi} must not depend on the choice of gauge, at any order of g.

S_{fi} has no bound state poles (at finite order in g): Expansion in free propagators

Lattice QCD and the perturbative S-matrix are complementary

The perturbative expansion diverges

QED predictions are based on expansions in powers of α.
Yet the series diverges for any α (zero radius of convergence)

Physics for $\alpha = e^2/4\pi < 0$ is very different:
The S-matrix is not unitary (e is imaginary).
Unlike charges repel: $V = -\alpha/r > 0$

F. Dyson,
Phys. Rev. 85, 631 (1952)

The perturbative expansion is believed to be an asymptotic series, which starts to diverge after some finite number ($\sim 1/\alpha$?) of terms.

Asymptotic series:
Y. Meurice, hep-th/0608097

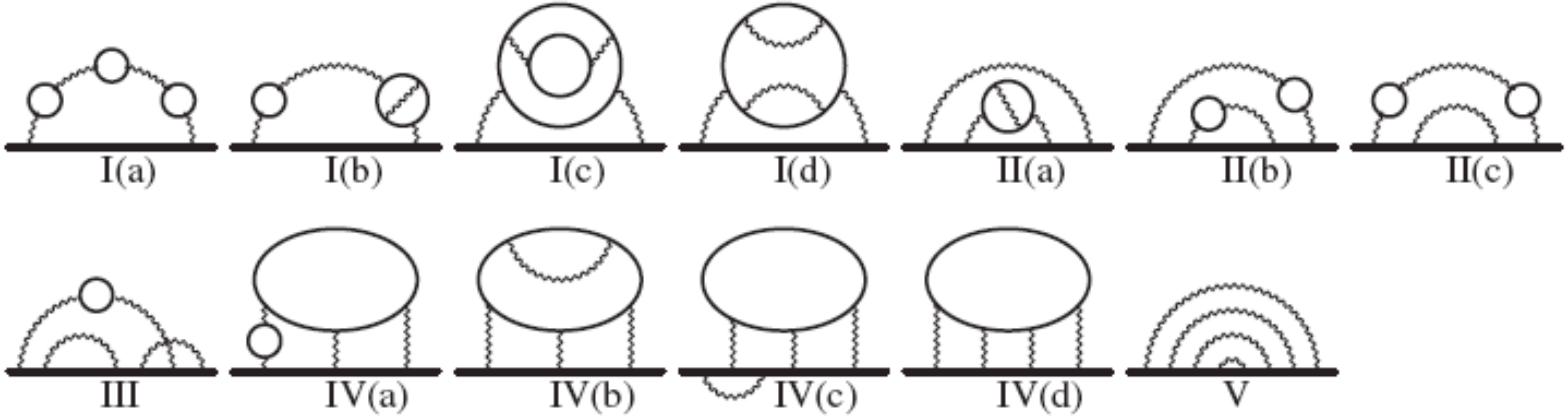
QCD perturbation theory is similar to that of QED, but $\alpha_s > \alpha$.

The electron magnetic moment g_e in the SM

The SM prediction: $$\frac{g_e}{2} = 1 + \sum_{n=1}^{\infty} C_n \left(\frac{\alpha}{\pi}\right)^n + a_{\mu\tau} + a_{QCD} + a_{EW}$$

The QED coefficients $C_1 \ldots C_5$ are known. Contributions from μ, τ and QCD, W, Z are added. *E.g.*, C_4 includes 891 QED diagrams of the following type:

$\mathcal{O}(\alpha^4)$:



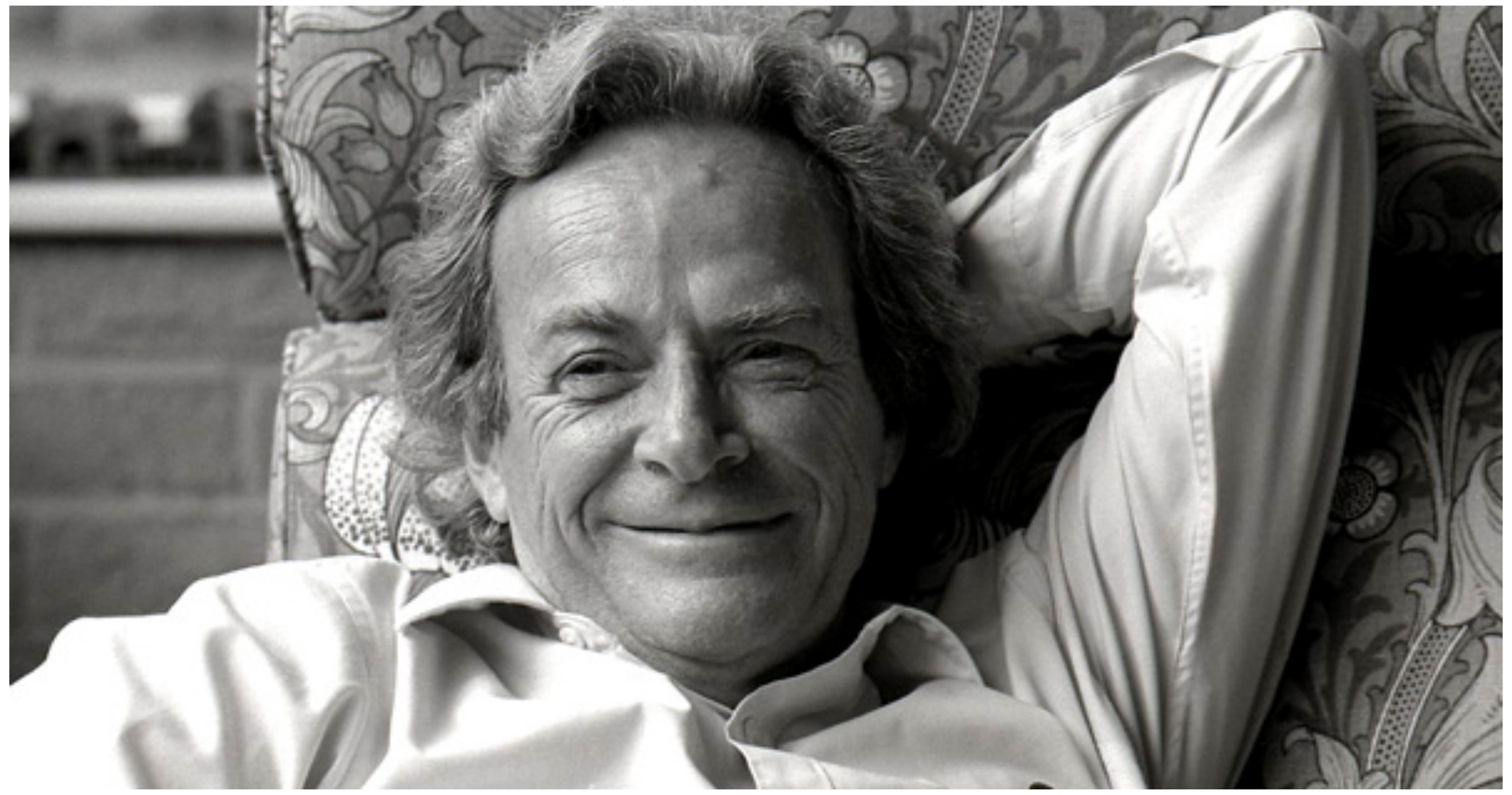
A precision measurement of the electron magnetic moment gives:

$\alpha^{-1} = 137.035999166(15)$

Fan et al., Phys. Rev. Lett. 130 (2023) 071801

S. D. Drell and H. R. Pagels, Phys. Rev. 140 (1965) B397

Feynman's challenge

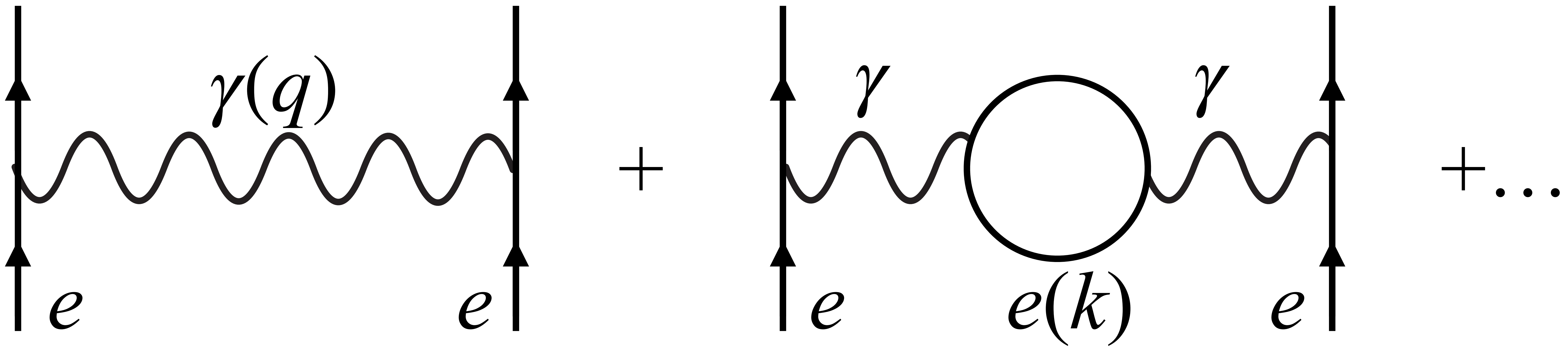


In his report to the 12th Solvay Congress (1961) on "The Present Status of Quantum Electrodynamics" (QED), Feynman called for more insight and physical intuition in QED calculations. To quote from a particularly relevant passage: *"It seems that very little physical intuition has yet been developed in this subject. In nearly every case we are reduced to computing exactly the coefficient of some specific term. We have no way to get a general idea of the result to be expected. To make my view clearer, consider, for example, the anomalous electron moment,* $(g-2)/2 = \alpha/2\pi - 0.328\,\alpha^2/\pi^2$*. We have no physical picture by which we can easily see that the correction is roughly* $\alpha/2\pi$*, in fact, we do not even know why the sign is positive (other than by computing it). In another field we would not be content with the calculation of the second-order term to three significant figures without enough understanding to get a rational estimate of the order of magnitude of the third. We have been computing terms like a blind man exploring a new room, but soon we must develop some concept of this room as a whole, and to have some general idea of what is contained in it. As a specific challenge, is there any method of computing the anomalous moment of the electron which, on first rough approximation, gives a fair approximation to the* α *term and a crude one to* α^2*; and when improved, increases the accuracy of the* α^2 *term, yielding a rough estimate to* α^3 *and beyond?"*

Pesking and Schroeder:
An Introduction to Quantum Field Theory

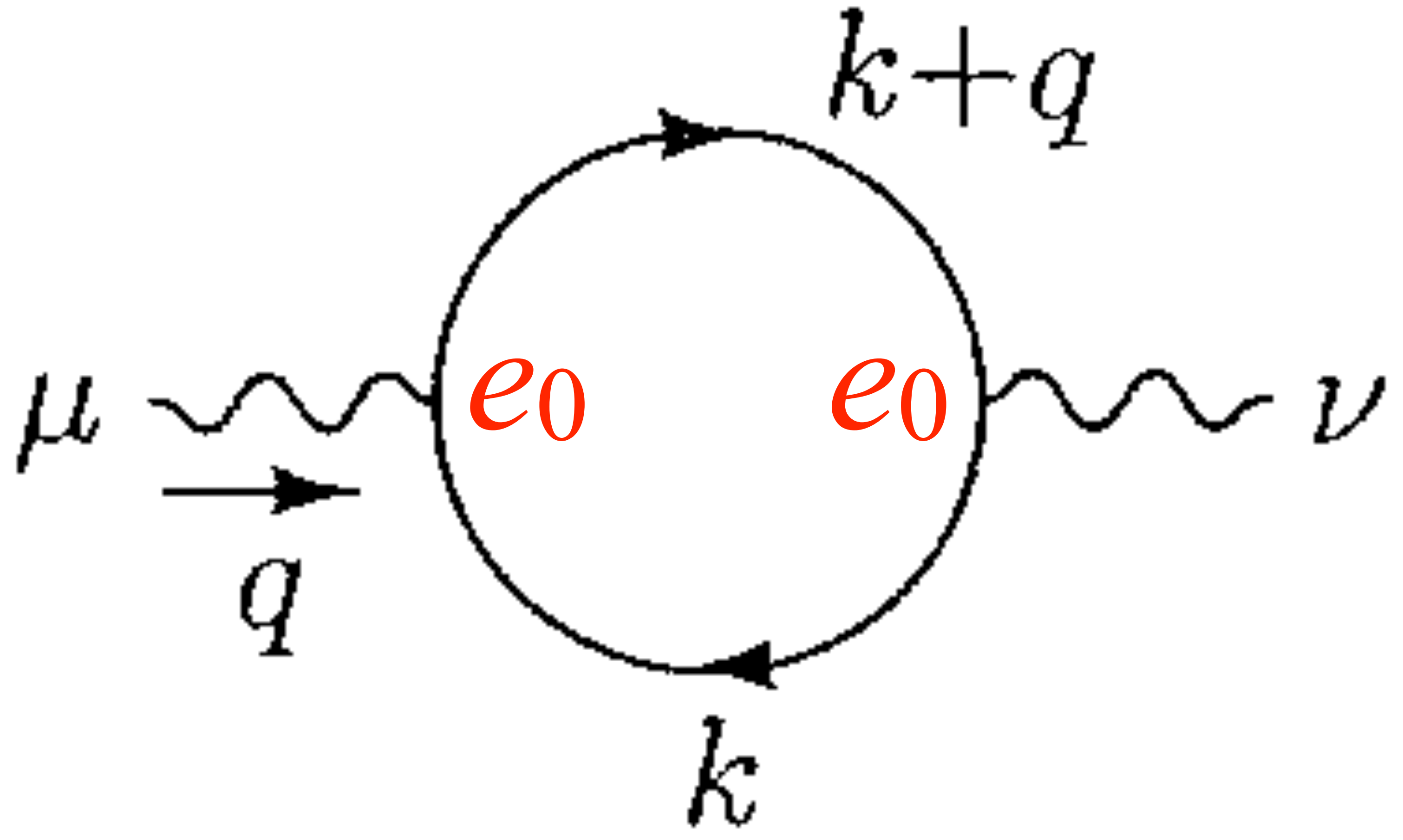
Running of the QED coupling α (I)

The electric charge is defined by *ee* scattering:



Loop integral diverges as $k^\mu \to \infty$: Becomes a pointlike interaction

$$i\Pi_2^{\mu\nu}(q) \equiv \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4}$$



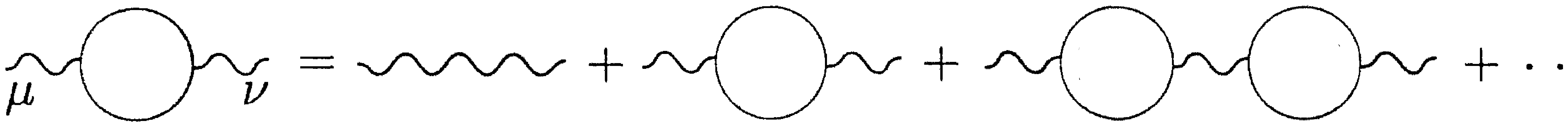
$$\equiv i(g^{\mu\nu}q^2 - q^\mu q^\nu)\Pi_2(q^2)$$

Regularize by subtraction:

$\Pi_2(q^2) - \Pi_2(0)$

Running of the QED coupling α (II)

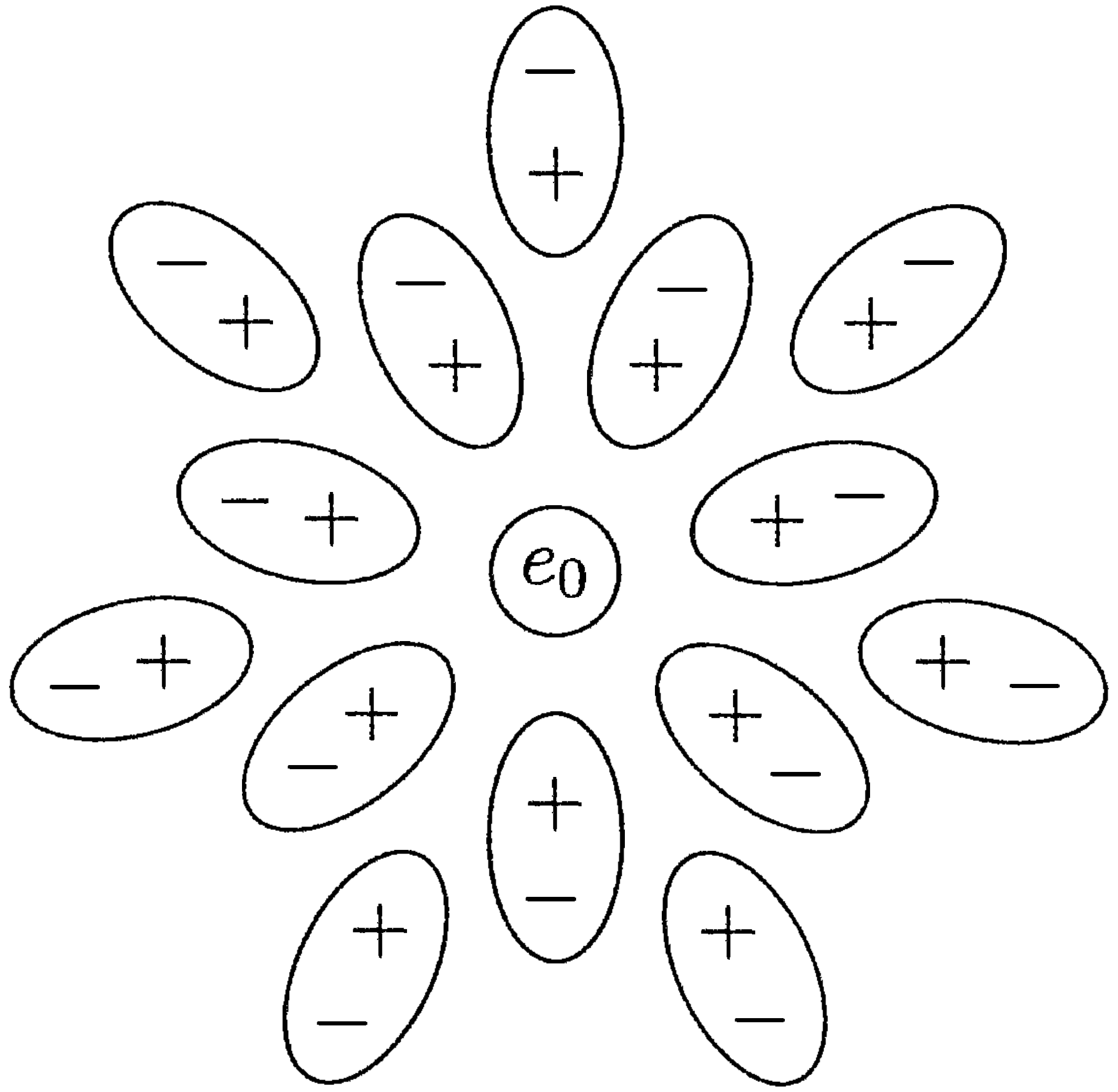
Summing the geometric series



makes the coupling “run”:

$$\alpha_0 \to \alpha_{eff}(q^2) = \frac{\alpha}{1 - \left[\Pi_2(q^2) - \Pi_2(0)\right]} + \mathrm{O}(\alpha^2)$$

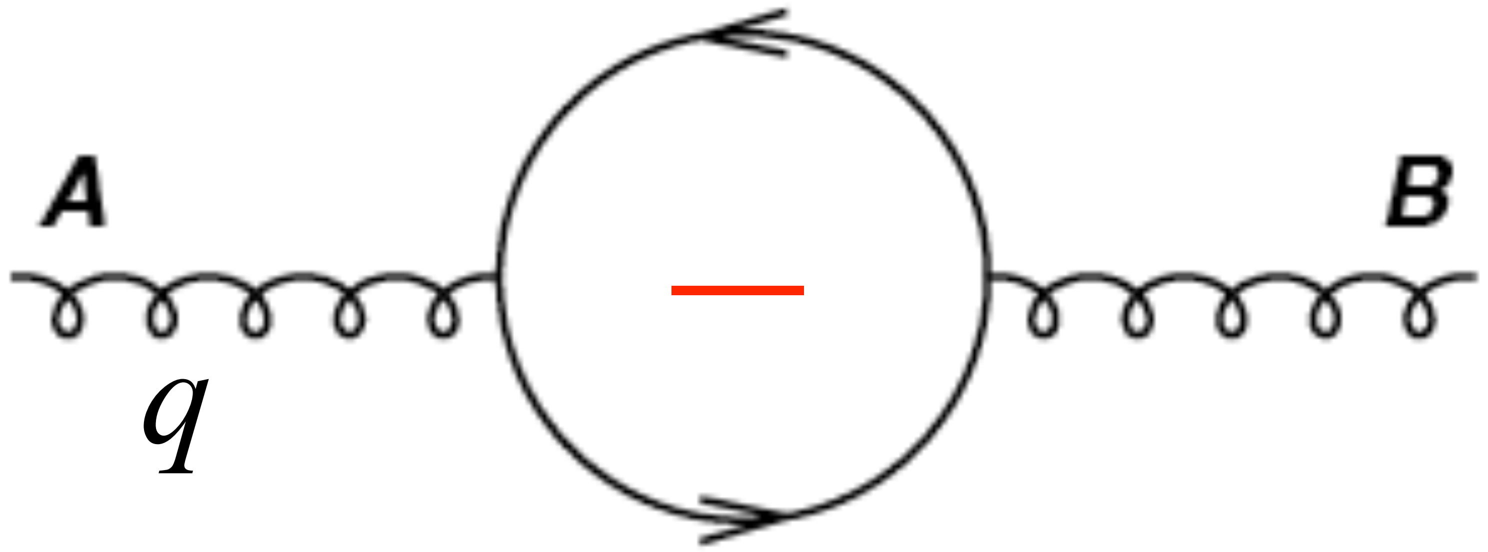
$$= \frac{\alpha}{1 - \frac{\alpha}{3\pi}\log(-q^2 e^{-5/3}/m_e^2)} \quad \text{for } -q^2 \gg m_e^2$$

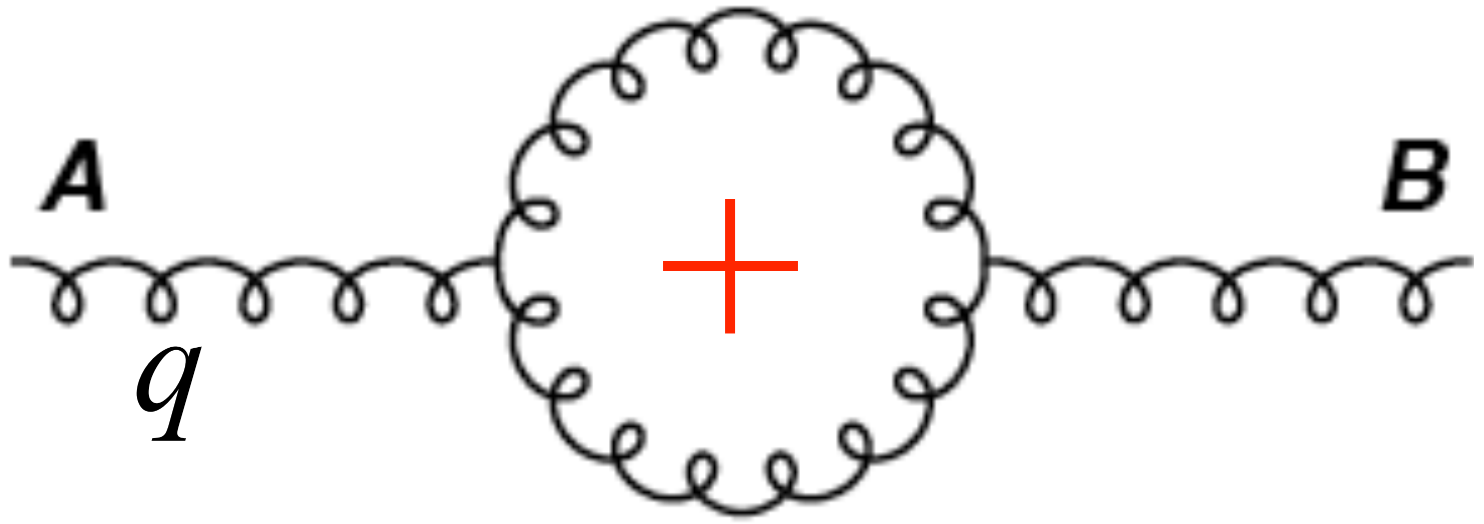


$\alpha_{eff}(q^2)$ increases with $-q^2$ in QED, as one probes shorter distances, closer to the infinite bare charge e_0.

The running of α_s in QCD

Surprise: In QCD the effective coupling decreases with $-q^2 \equiv Q^2$





The gluon loop diagram contributes with opposite sign compared to the fermion loop.

Gross, Politzer, Wilczek (1973); Nobel 2004

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\log(Q^2/\Lambda^2)}$$

"Asymptotic freedom"

The Q^2-dependence of α_s has been verified experimentally, with

$$\Lambda \simeq 200\,\text{MeV} \simeq 1\,\text{fm}^{-1}$$

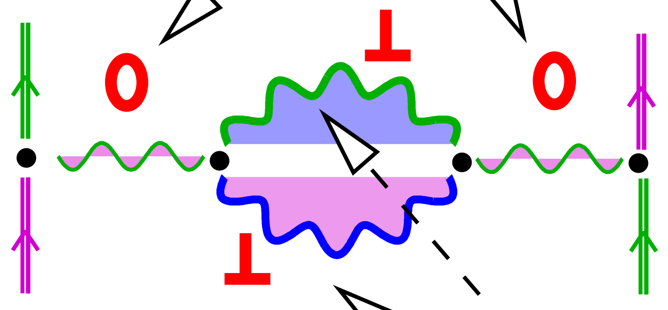
The origin of anti-screening

Due to the coupling of the instantaneous Coulomb gluons to transverse gluons in the vacuum

Yu. Dokshitzer, hep-ph/0306287

Note: Gauge theories have instantaneous interactions, arising from the gauge-dependent A^0 and A_L fields!

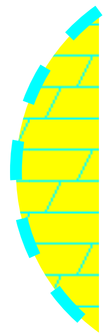
Instantaneous Coulomb interaction



$$= -N_c * \frac{1}{3} \quad -n_f * \frac{2}{3}$$

Transverse gluons **(and quarks)**

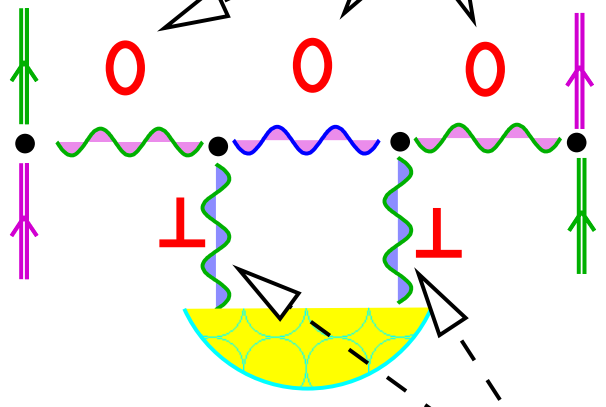
ANTI screening



Khriplovich (1969)

Gribov (1976)

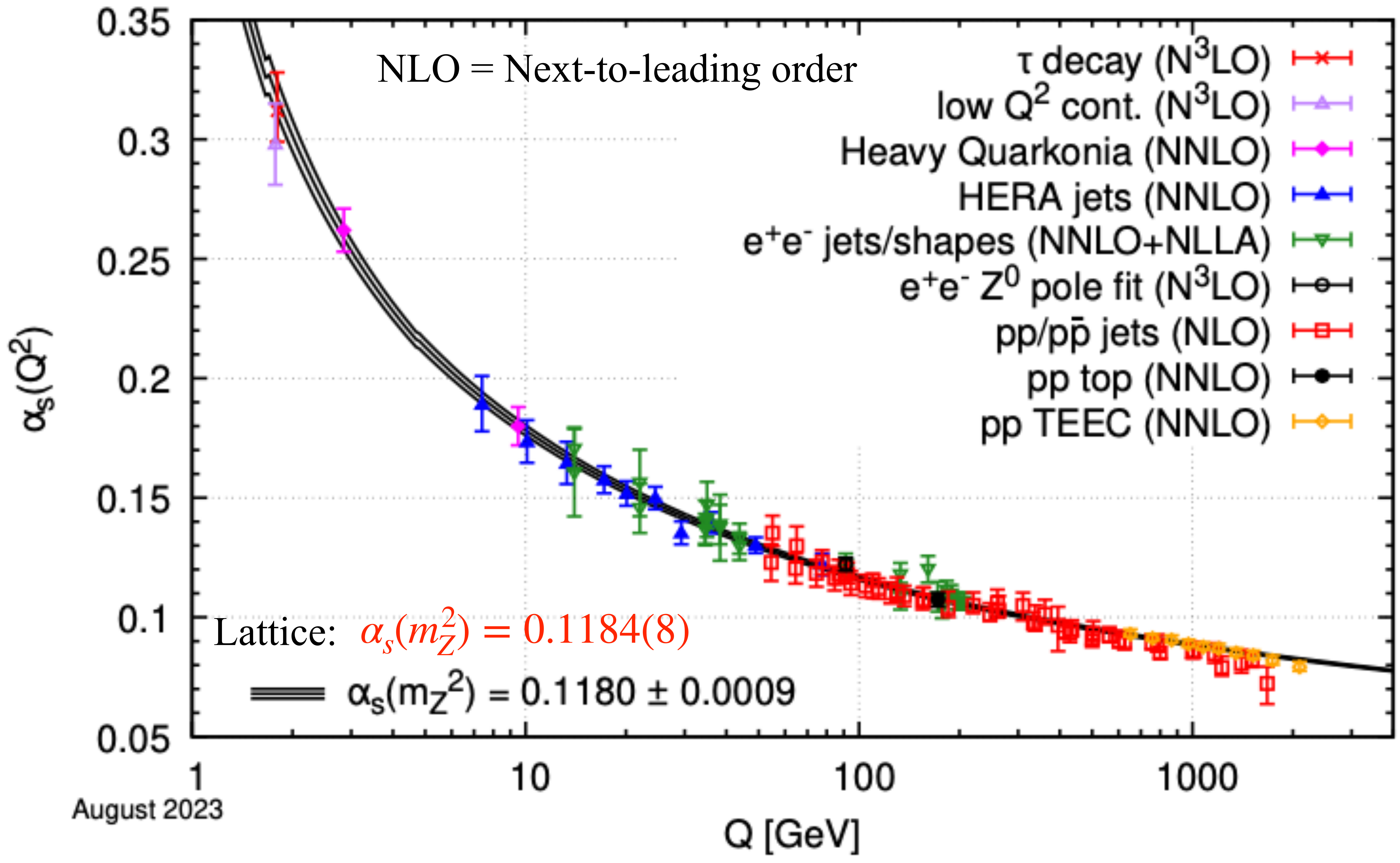
Instantaneous Coulomb interaction



$$= +N_c * 4$$

Vacuum fluctuations of transverse fields

The running of $\alpha_s(Q^2)$

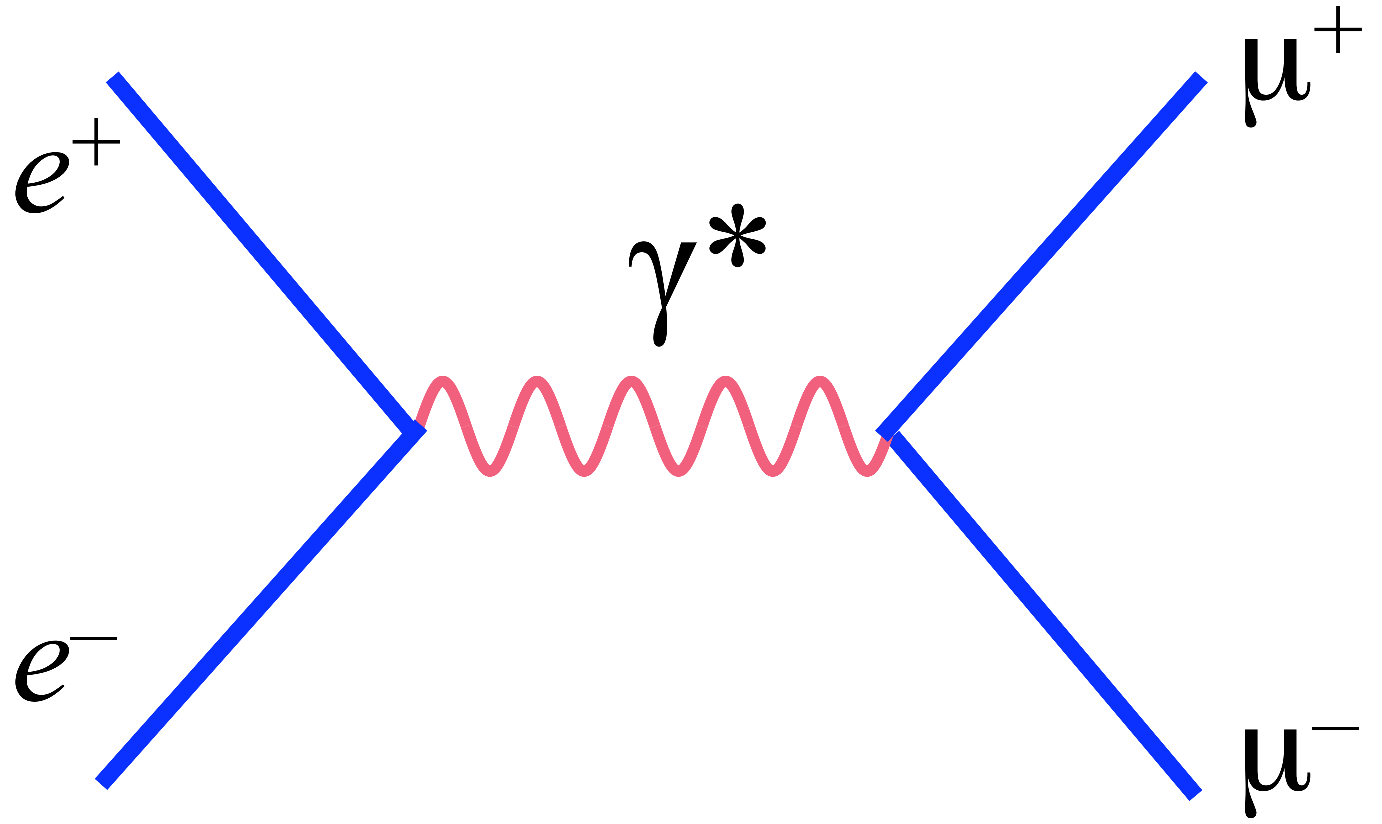


Measurements of α_s in various processes, and in Lattice QCD

Particle Data Group, 2023

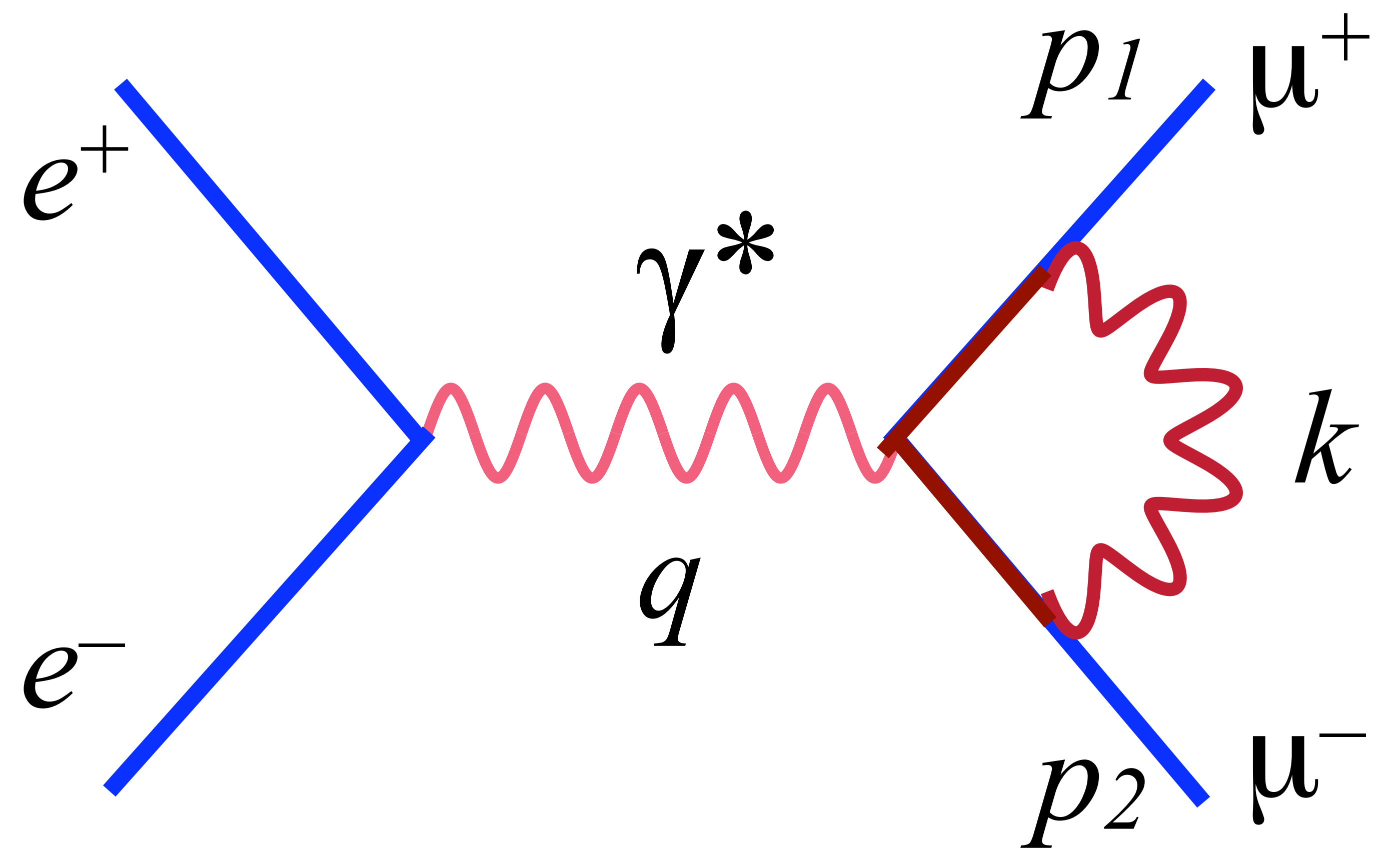
Infrared singularities in QED

The O(α) Born term for $e^+e^- \to \mu^+\mu^-$ is regular, and given by the Feynman rules:



At order α^2 there is an infrared singularity in the loop integral for $k \to 0$:

$$\int_0 \frac{d^4k}{k^4}$$



The two fermion denominators $\propto k$:

$$(p_1 - k)^2 - m_\mu^2 = -2p_1 \cdot k + k^2 \propto k$$

The photon denominator $\propto k^2$, giving a log singularity at $k = 0$

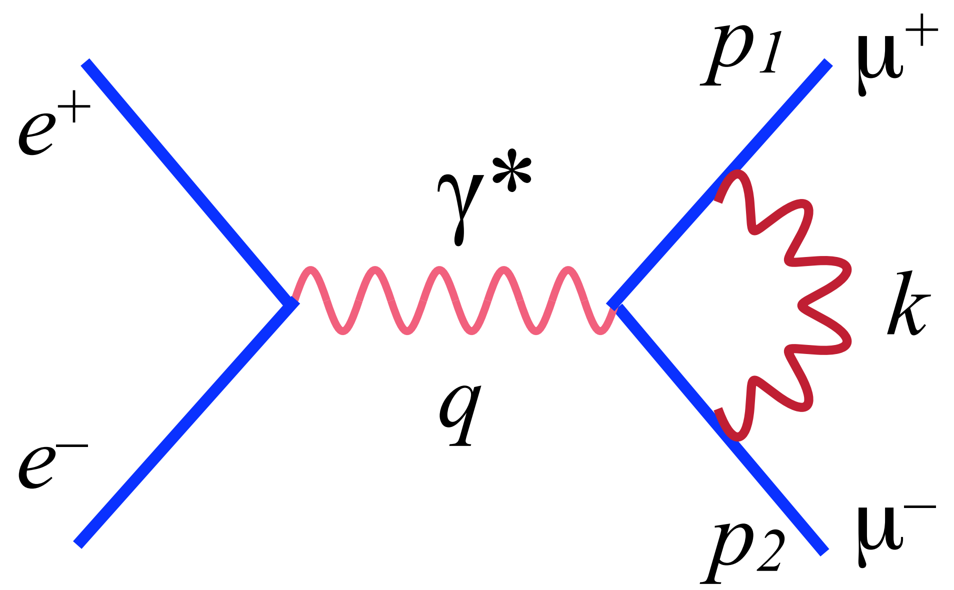
$\Rightarrow$ The exclusive process $e^+e^- \longrightarrow \mu^+\mu^-$ is ill defined.

There are no exclusive amplitudes for charged particles

Gauge invariance dictates that amplitudes with external charged particles vanish:

$$A(e^+e^- \to \mu^+\mu^-) = 0$$

This is because the amplitude must be invariant under local $U(1)$ gauge transformations. Multiplying one of the external fermions by $U = e^{i\pi} = -1$ we get $A \to -A$.



Two charged particles at different positions *x*, *y* must be connected by a gauge field exponential to be gauge invariant:

$$\bar{\psi}(y)\exp\left(ie\int_x^y dz_\nu A^\nu(z)\right)\psi(x)$$

The photon (gauge) field serves as a connection, which "informs" about the choice of gauge at each point in space.

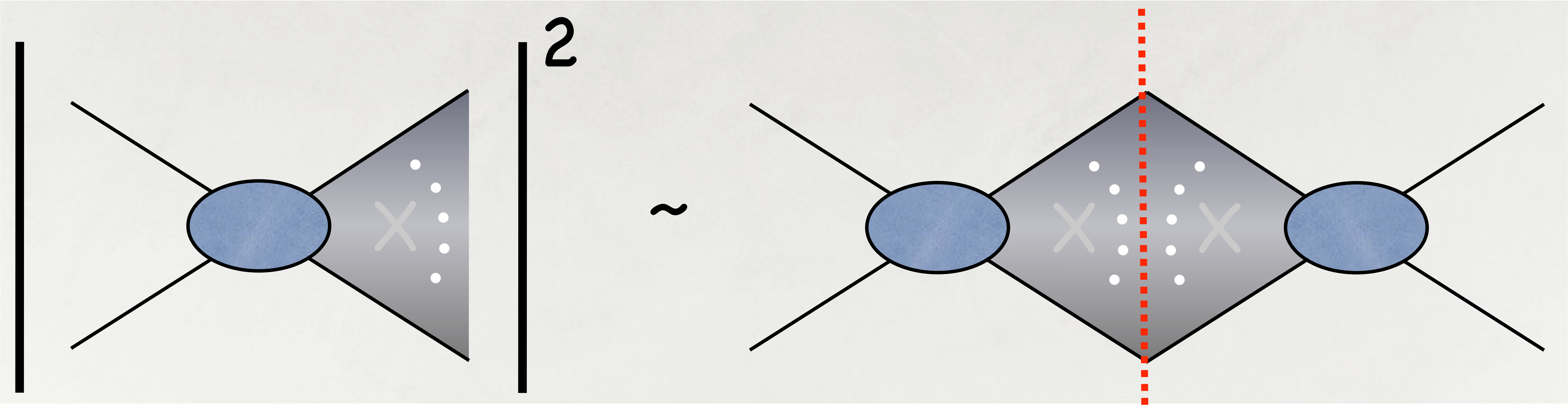
Optical Theorem

As a consequence of the unitarity of the scattering matrix: the total cross section may be expressed in terms of the imaginary part of the forward elastic amplitude:

$$S\,S^\dagger = 1$$

$$\sigma_{tot}(s) = \sum_X \int d\Phi_X\, |M_X|^2 = \frac{8\pi}{\sqrt{s}}\, \mathrm{Im}\left[M_{el}(\theta = 0)\right]$$

Nonlinear in M !



Completeness sum on the rhs.

QED and QCD satisfy unitarity at each order of α (non-trivial!)

Unitarity holds also for the physical hadron states

Optical Theorem for $\sigma_{tot}(e^+e^-)$ in QED

$\mathcal{O}(\alpha)$ $\mathcal{O}(\alpha^2)$

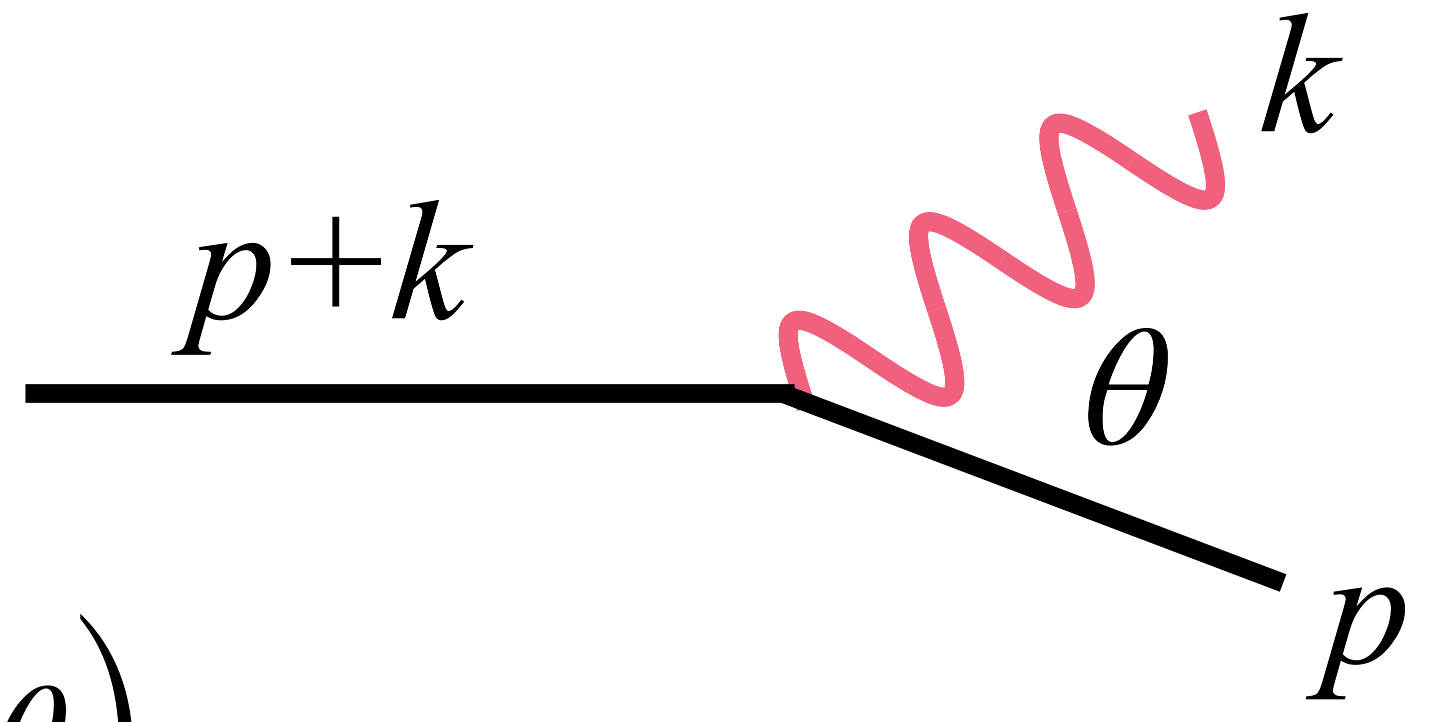
$\gamma^* \to \mu^+\mu^-$ $\gamma^* \to \mu^+\mu^-\gamma$ $\gamma^* \to \mu^+\mu^-$

At $\mathcal{O}(\alpha^2)$ the IR singular contributions to the imaginary part cancel.
The cancellations are between different final states!

The $\gamma_T^* \to \gamma_T^*$ amplitude is regular because it is gauge invariant.

Finite cross sections include (arbitrarily soft, $k \to 0$) photons.
There are no free, “bare” charged particles.

Collinear singularity in QED



The cross section for collinearly emitted, high energy photons is also enhanced

$$(p+k)^2 - m^2 = 2p\cdot k = 2\,|\mathbf{k}|\left(\sqrt{\mathbf{p}^2+m^2}) - |\mathbf{p}|\cos\theta\right)$$

$$\propto 1 - \cos\theta + \mathcal{O}(m^2/\mathbf{p}^2) \qquad\qquad (|\mathbf{p}| \gg m)$$

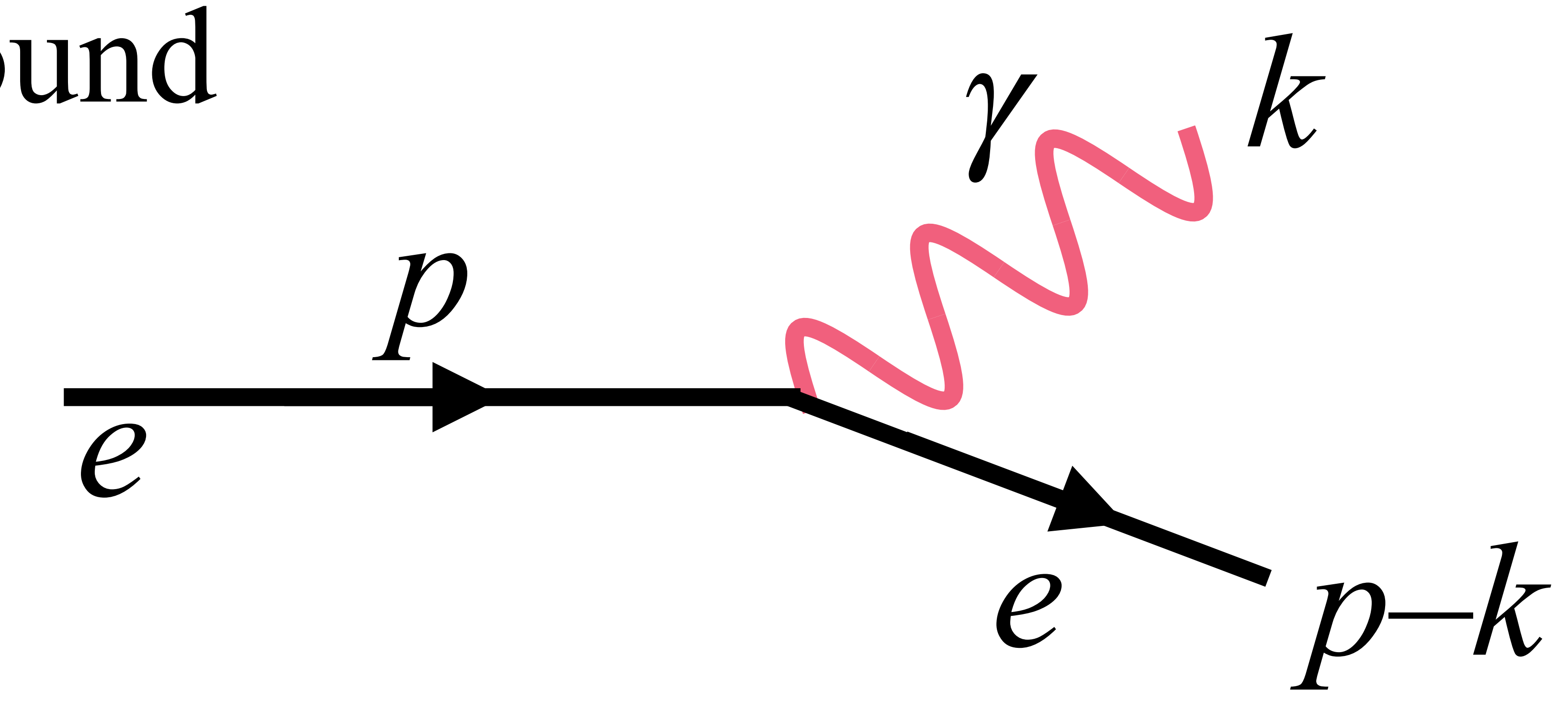
$$\sigma \sim \alpha \int^1 d\cos\theta \, \frac{1}{1-\cos\theta + m^2/2\mathbf{p}^2} \propto \alpha \log\left(\frac{\mathbf{p}^2}{m^2}\right)$$

Large radiative corrections

Also this collinear logarithm is cancelled by the virtual correction in σ_{tot}

Sudakov form factor

We need not sum over all final states as in σ_{tot} , only around the singular regions with soft and collinear photons



If the detector is insensitive to photons with $k < k_{det}$, any measurement will include soft photons:

$$\sigma_{meas} = \sigma\big[e(p) \to e(p)\big] + \sigma\big[e(p) \to e(p-k) + \gamma(k)\big]_{k<k_{det}}$$

Keeping the initial electron off-shell, $p^2 - m_e^2 = q^2$, regularizes the singularities:

$$\sigma_{meas} = \sigma_0\left[1 - \frac{\alpha}{\pi}\log\left(\frac{q^2}{m_e^2}\right)\log\left(\frac{q^2}{k_{det}^2}\right)\right] + \mathcal{O}(\alpha^2)$$

Summing to all orders:

$$= \sigma_0 \exp\left[-\frac{\alpha}{2\pi}\log\left(\frac{q^2}{m_e^2}\right)\log\left(\frac{q^2}{k_{det}^2}\right)\right]$$

Sudakov form factor vanishes faster than any power ($q^2 \gg m_e^2$)


Quantum
Chromodynamics
Hard scattering

Infrared Safe observables

QCD perturbation theory is reliable only at large virtualities, $|q^2| \gg \Lambda_{QCD}^2$, which excludes the IR and collinear singularities: The calculation is "IR Safe".

An observable is infrared safe if it is **insensitive** to

SOFT radiation:

Adding any number of infinitely soft particles should not change the value of the observable

COLLINEAR radiation:

Splitting an existing particle up into two comoving particles each with half the original momentum should not change the value of the observable

P. Skands

QCD result for $\sigma_{tot}(e^+e^- \to q, \bar{q}, g)$

$$\frac{\sigma(e^+e^- \to q, \bar{q}, g)}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\Big(\sum_q e_q^2\Big)\left[1 + \frac{\alpha_s(Q^2)}{\pi} + \sum_{n=2}^{\infty} c_n \Big(\frac{\alpha_s(Q^2)}{\pi}\Big)^n\right] + \mathcal{O}\Big(\frac{\Lambda^4}{Q^4}\Big)$$

$$c_2 = 1.9857 - 0.1152\, n_f$$

$$c_3 = -\,6.63694 - 1.20013\, n_f - 0.00518\, n_f^2 - 1.240\, \eta$$

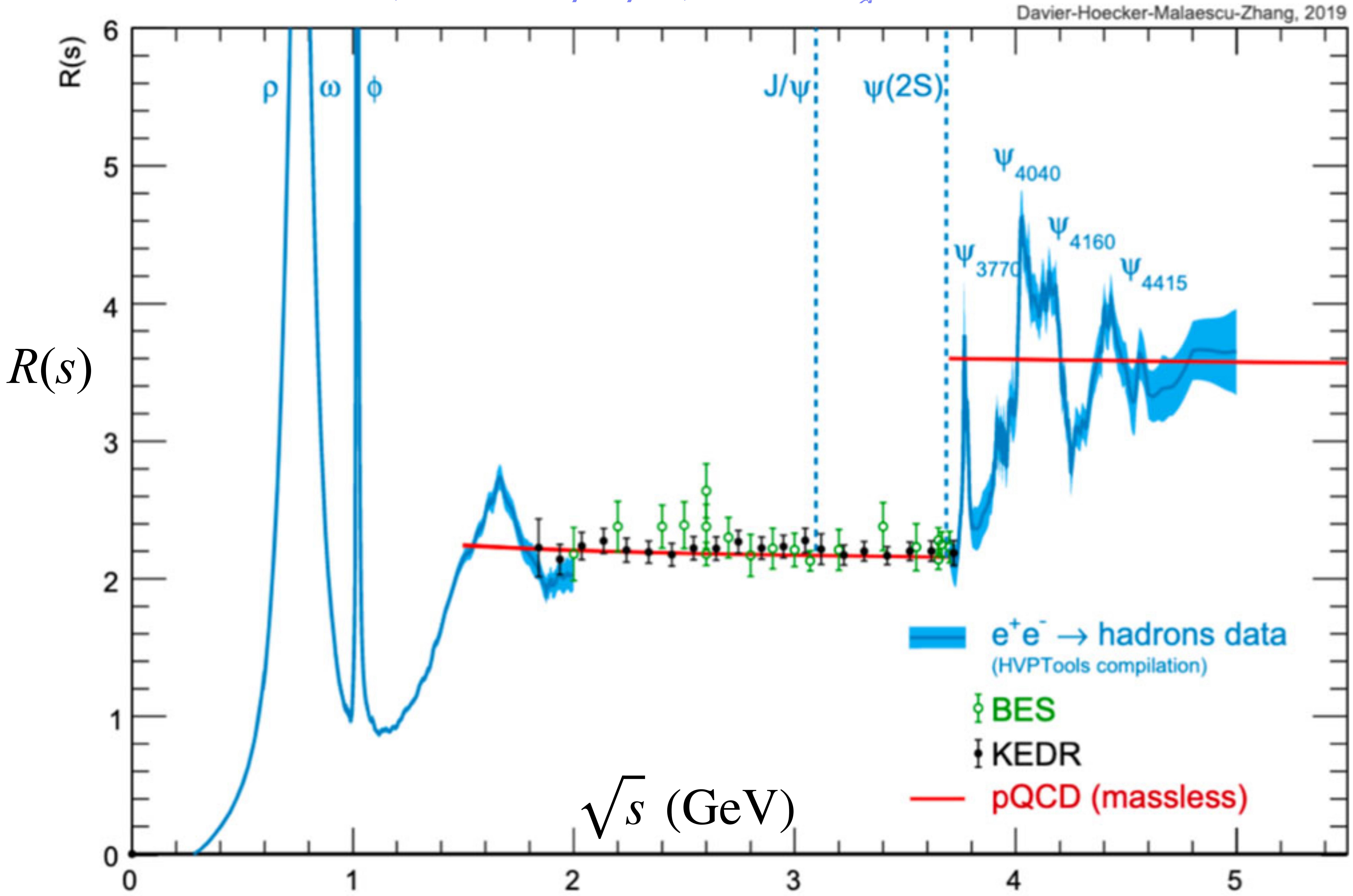
$$c_4 = -\,156.61 + 18.775\, n_f - 0.7974\, n_f^2 + 0.0215\, n_f^3 - (17.828 - 0.575\, n_f)\eta$$

$$\eta = \Big(\sum_q e_q\Big)^2 \Big/ \Big(3 \sum_q e_q^2\Big)$$

pdg (2023)

The perturbative expression for $\sigma_{tot}(e^+e^- \to q, \bar{q}, g)$ is infrared safe and may thus be compared with data on $\sigma_{tot}(e^+e^- \to \text{hadrons})$

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q e_q^2\left(1 + \frac{\alpha_s}{\pi} + \ldots\right)$$



PQCD curve averages resonance contributions (duality)

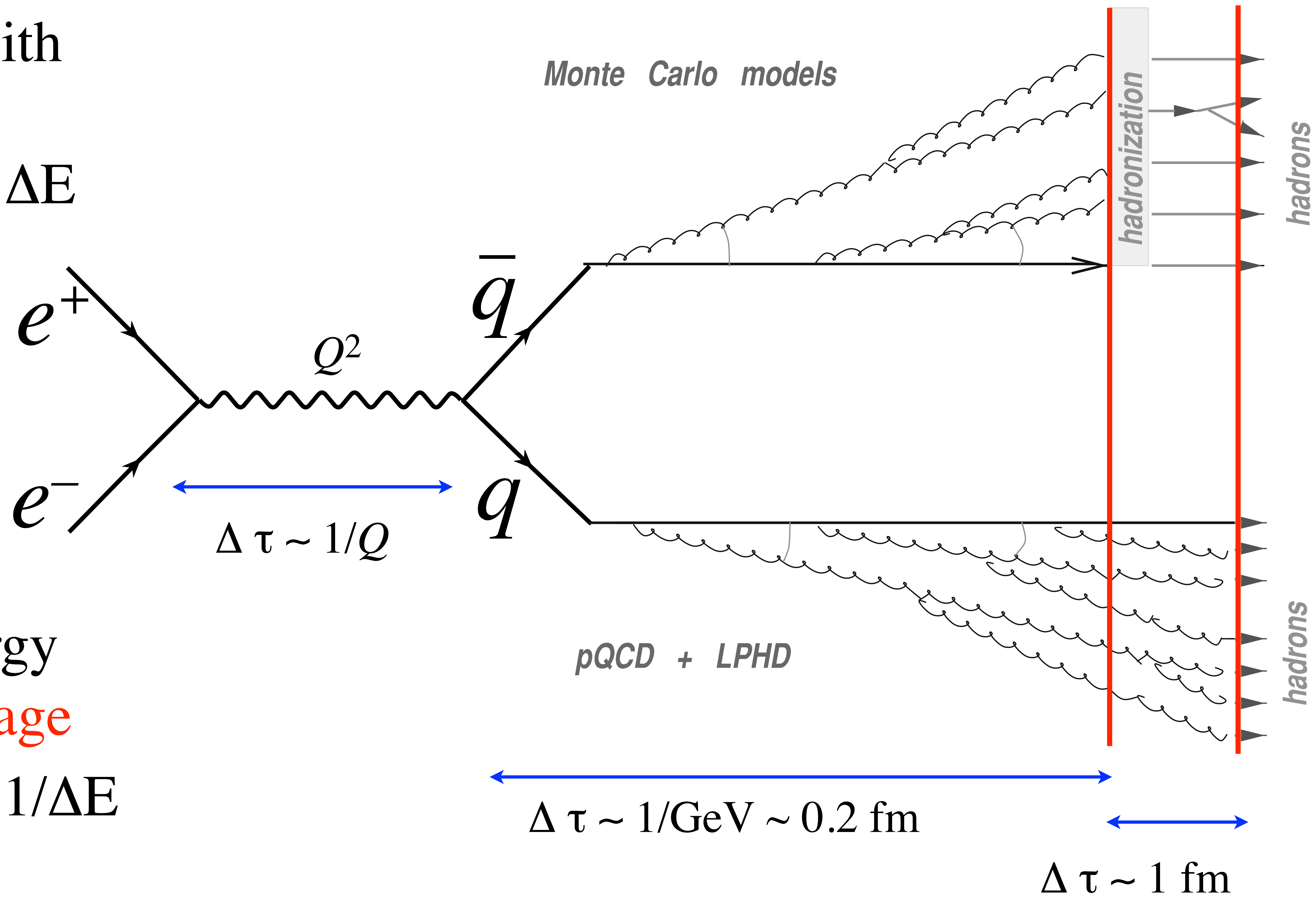
F. Gross et al., Eur.Phys.J.C 83 (2023) 1125 [2212.11107]

Time evolution in e+e- → of hadrons

Final state evolves in time τ with decreasing virtuality and thus decreasing energy uncertainty ΔE

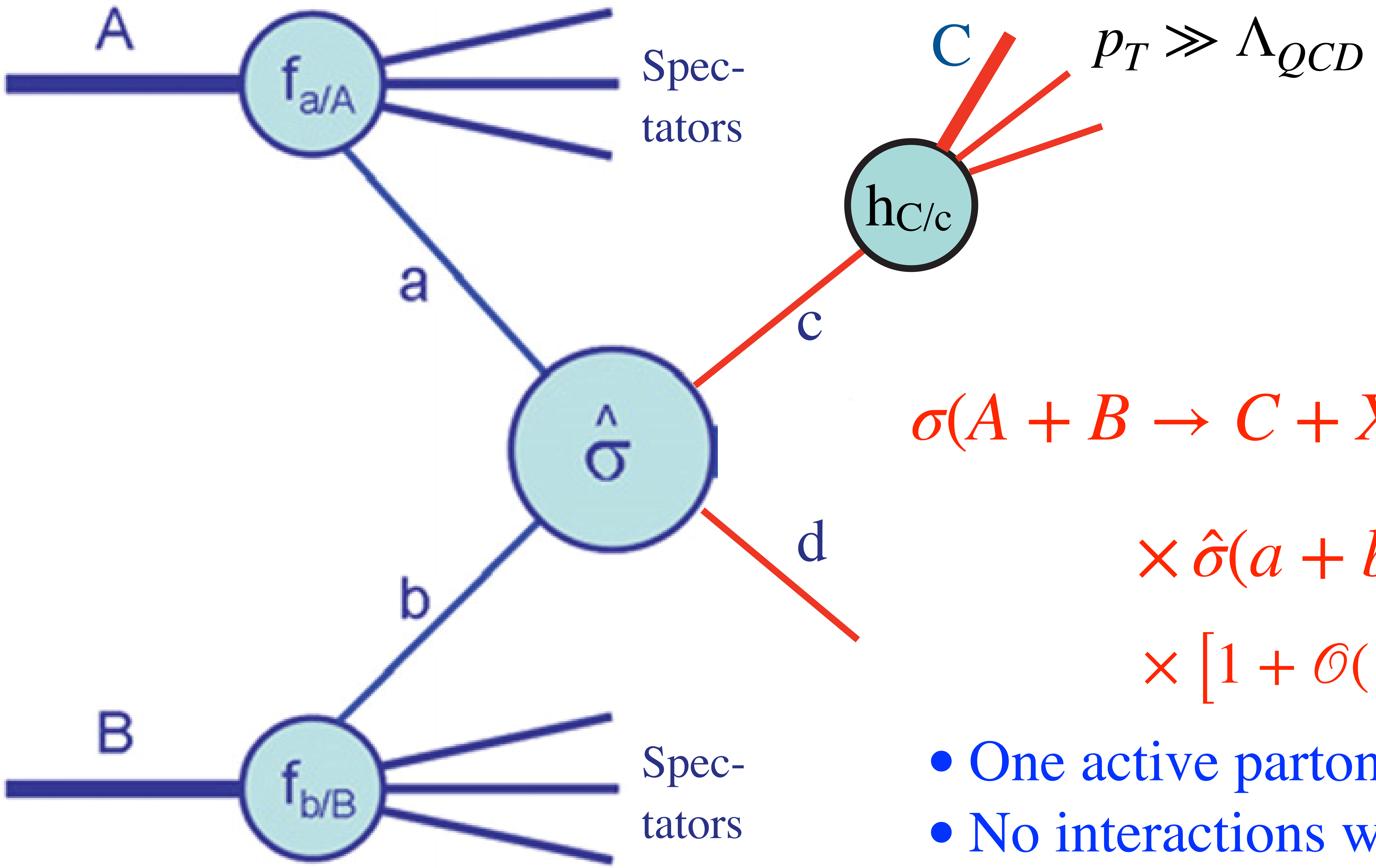
$$\Delta\tau \, \Delta E \gtrsim \hbar$$

Evolution is unitary:
Measured cross section in energy interval $E_{CM} \pm \Delta E$ must average to (parton) cross section at $\tau \sim 1/\Delta E$



The perturbative evolution is imprinted on the hadrons (duality)

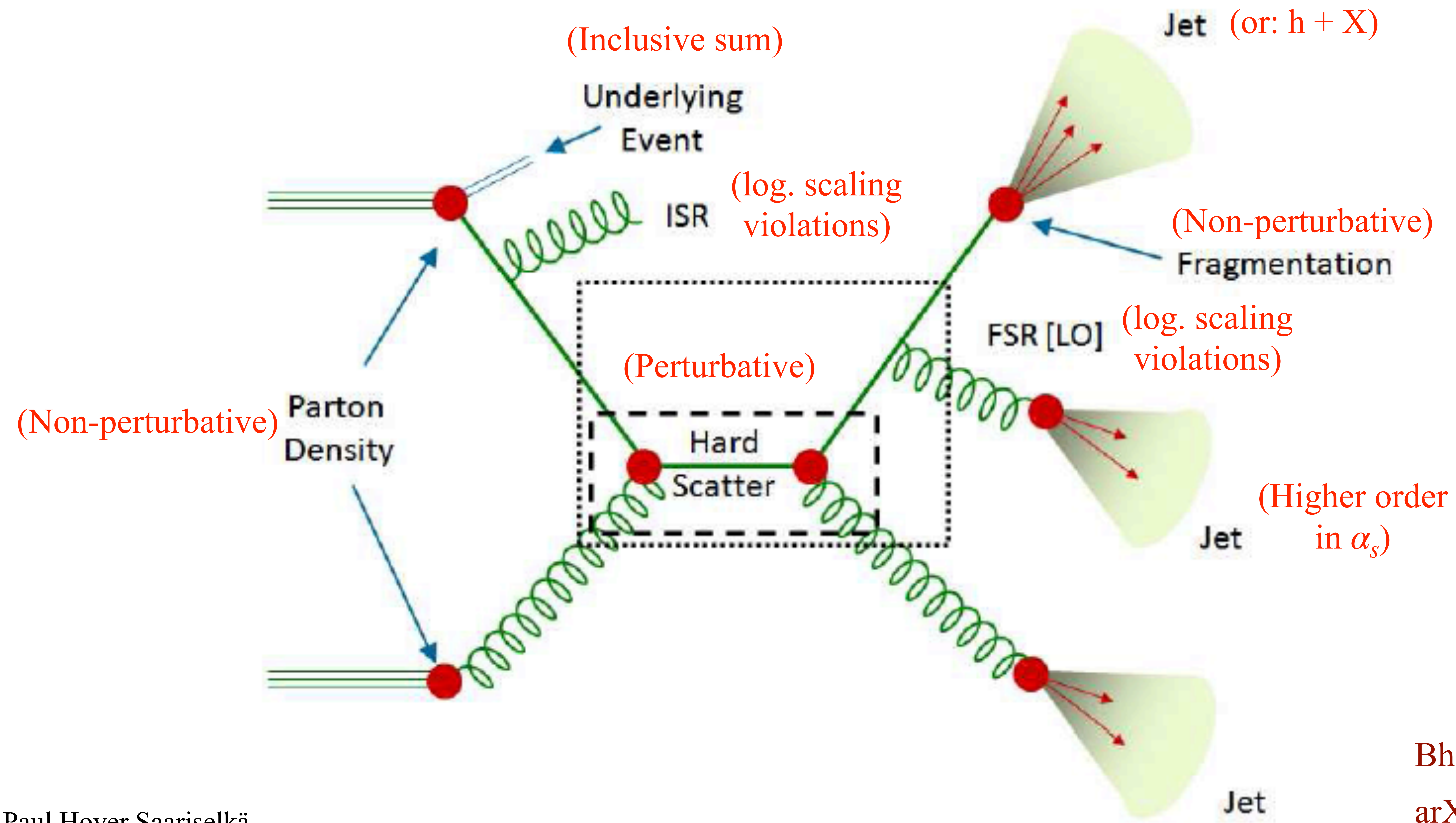
QCD Factorization in Hard Inclusive Processes



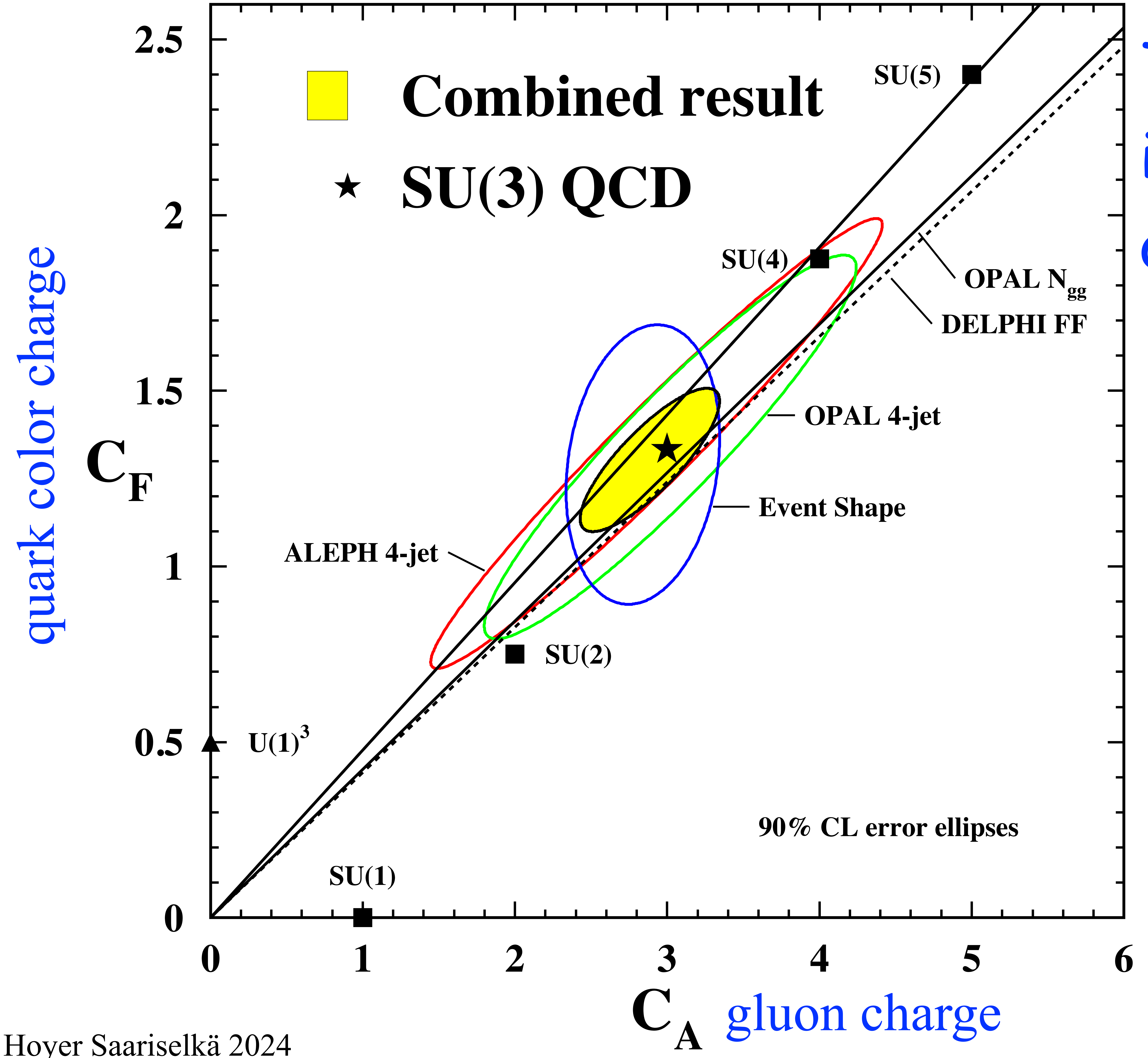
$$\sigma(A+B\to C+X) = f_{a/A}(x_A)\, f_{b/B}(x_B) \times \hat{\sigma}(a+b\to c+d)\, h_{C/c}(z_C) \times \left[1+\mathcal{O}(1/p_T^2)\right]$$

- One active parton in each hadron
- No interactions with spectators
- Hard subprocess $\hat{\sigma}$ is perturbative

Jet production in hadron collisions



Bhatti et al,
arXiv:1002.1708



The QCD Lagrangian is verified by data on hard scattering

Measurement of the quark and gluon color charges

S. Kluth, hep-ex/0603011

Gluon vs. Quark jets

P. Abreu et al. / Physics Letters B 449 (1999) 383–400

The PQCD splittings of gluon and quark jets gives a multiplicity ratio

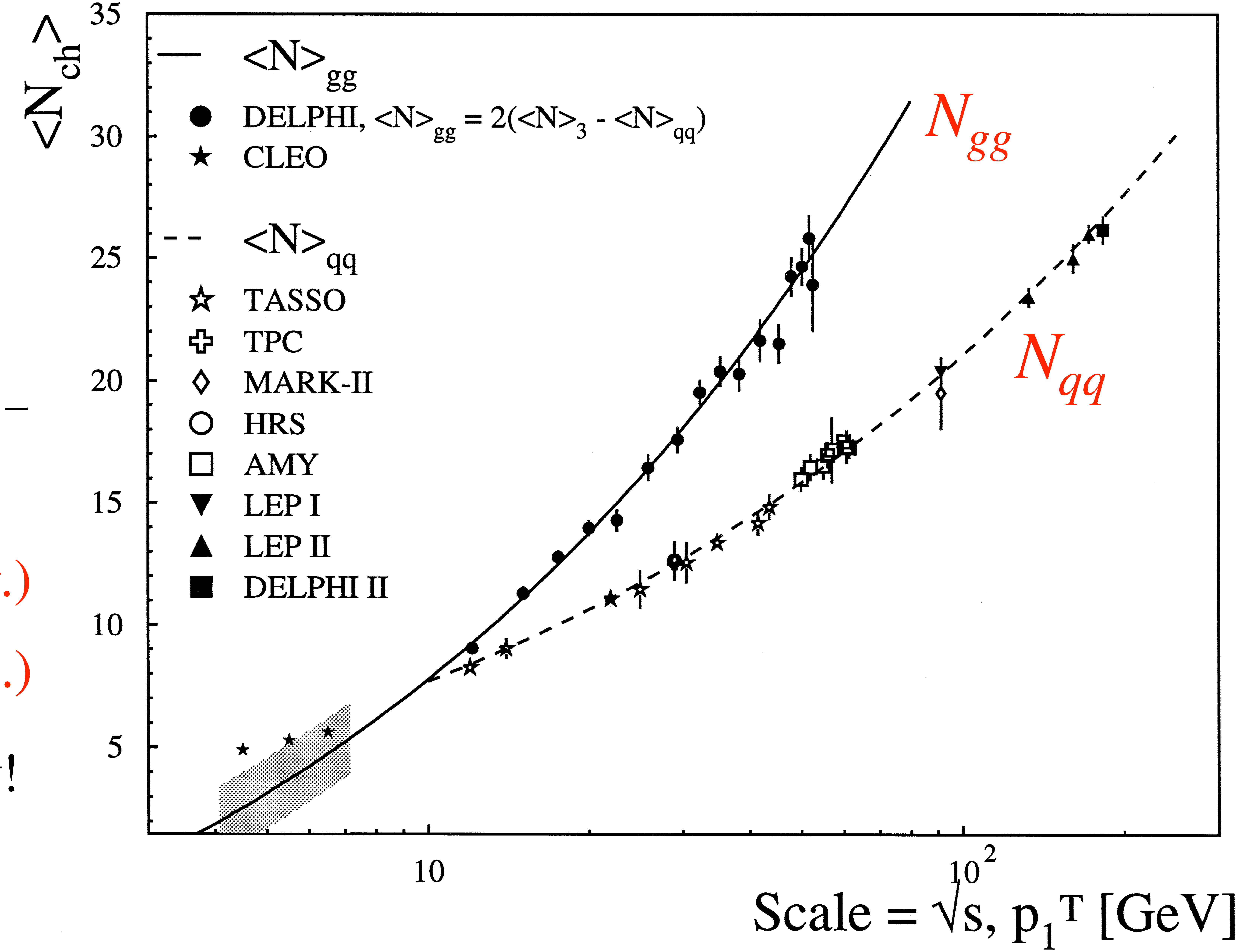
$$\frac{C_A}{C_F} = \frac{9}{4} = 2.25$$

Hadron multiplicities in e^+e^- data gave

$$C_A/C_F = 2.246 \pm 0.062\,(stat.)$$
$$\pm 0.080\,(syst.)\ \pm 0.095\,(theo.)$$

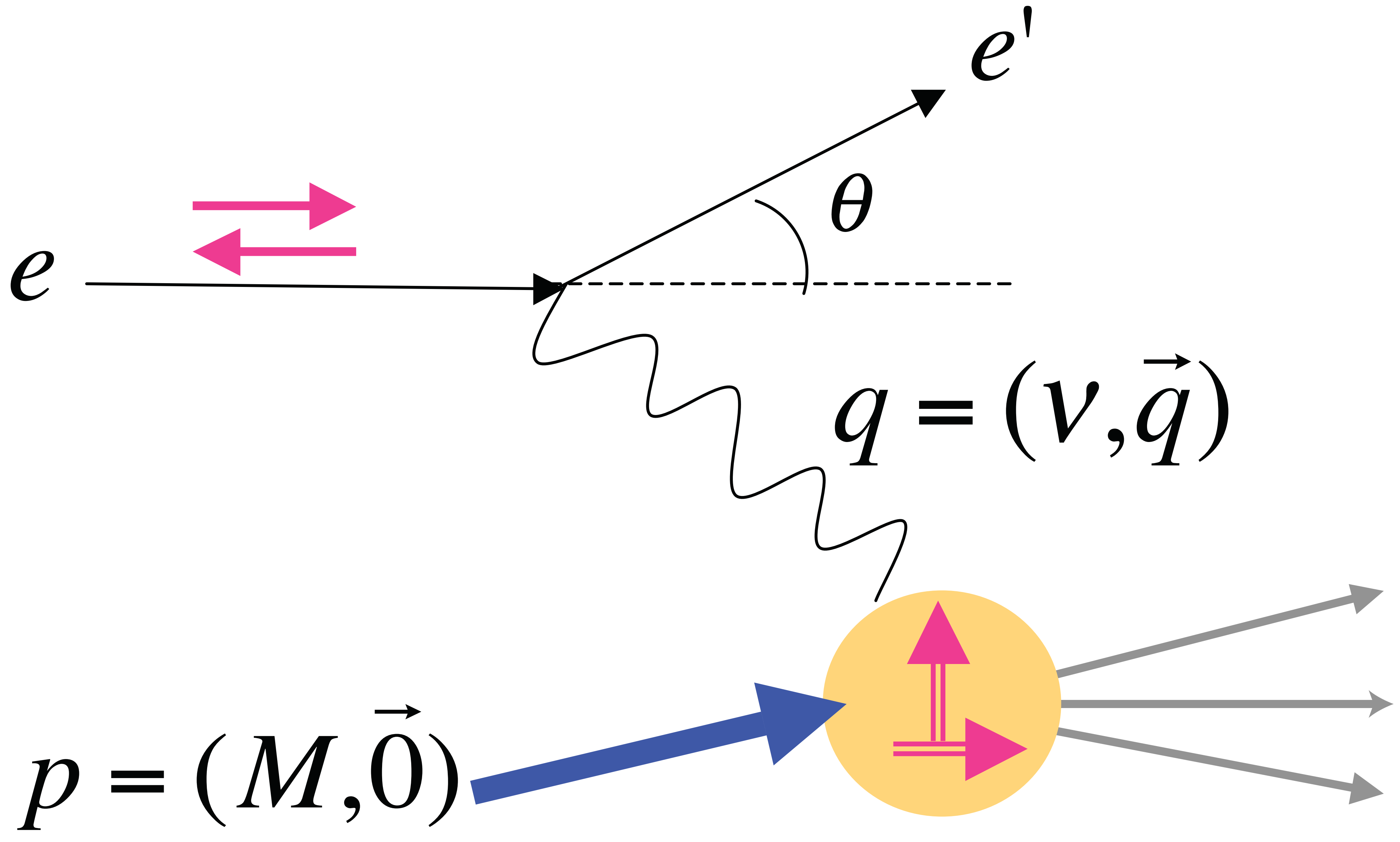
Local Parton-Hadron duality!

Yu. Dokshitzer, hep-ph/0306287

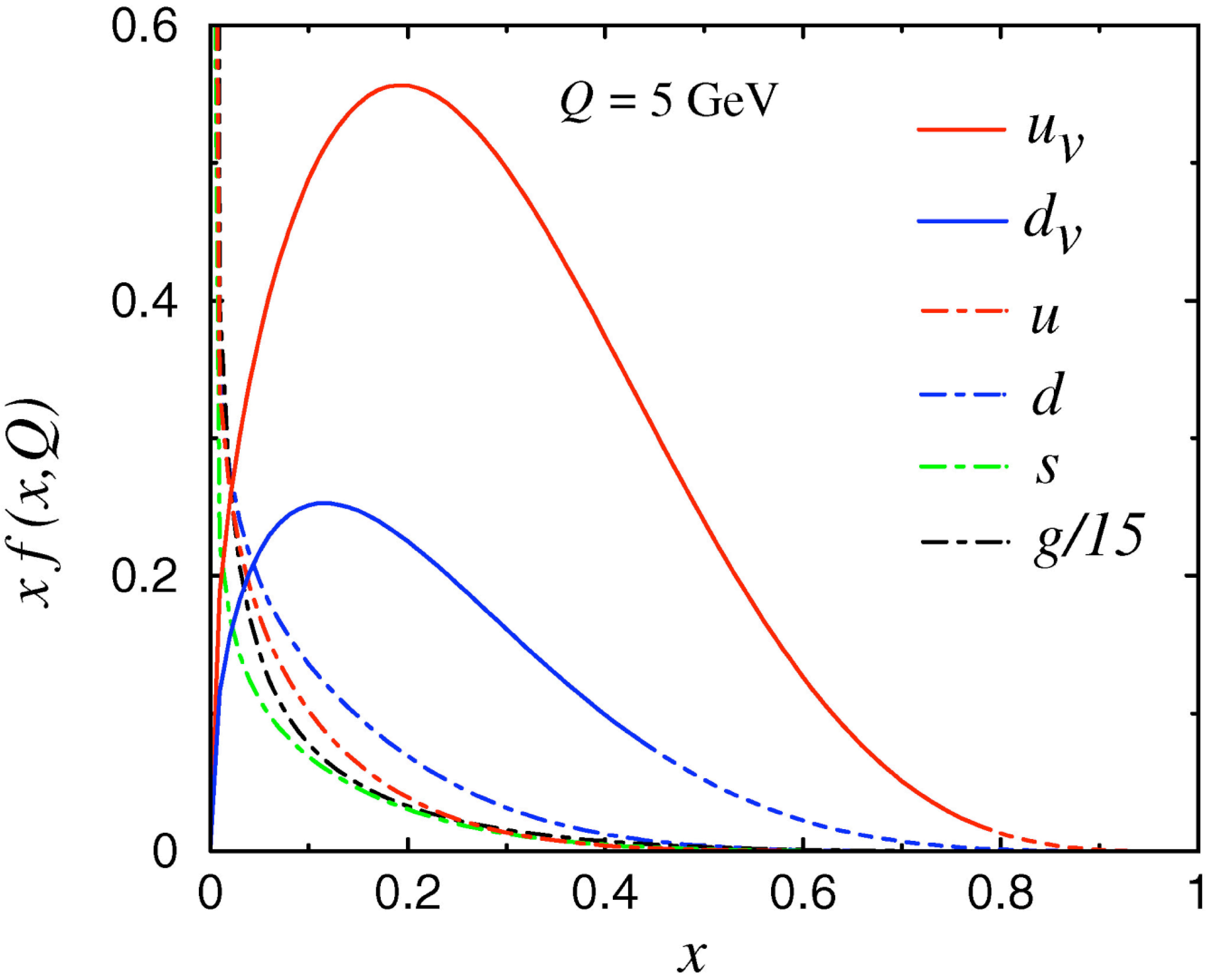


Dynamics of DIS: e + p → e + X

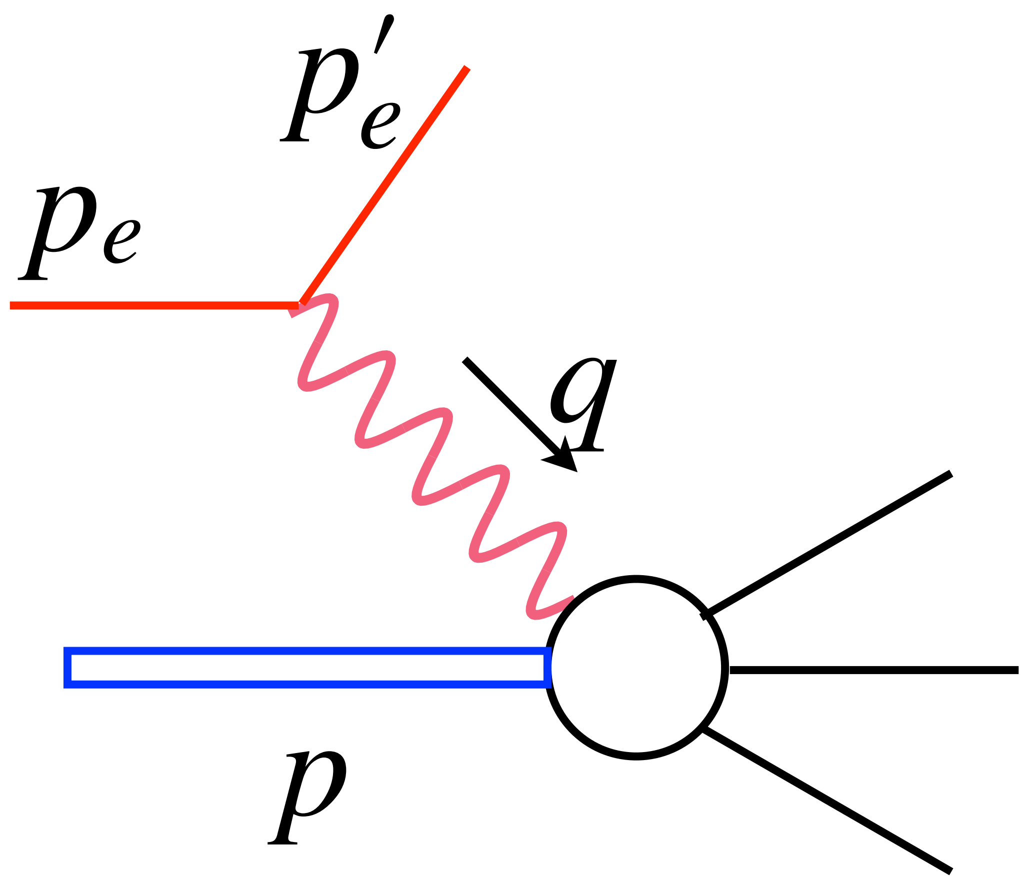
Deep Inelastic Scattering (DIS) was the key to discovering quarks as physical, pointlike constituents of the proton (SLAC, 1969)



Parton distributions in the proton



Transverse resolution in DIS



Target rest frame: $p = (m, \mathbf{0}), \quad p_e = (E_e, 0, 0, -E_e)$

For $Q^2 = -q^2$ and $q^0 = \nu$ both large and

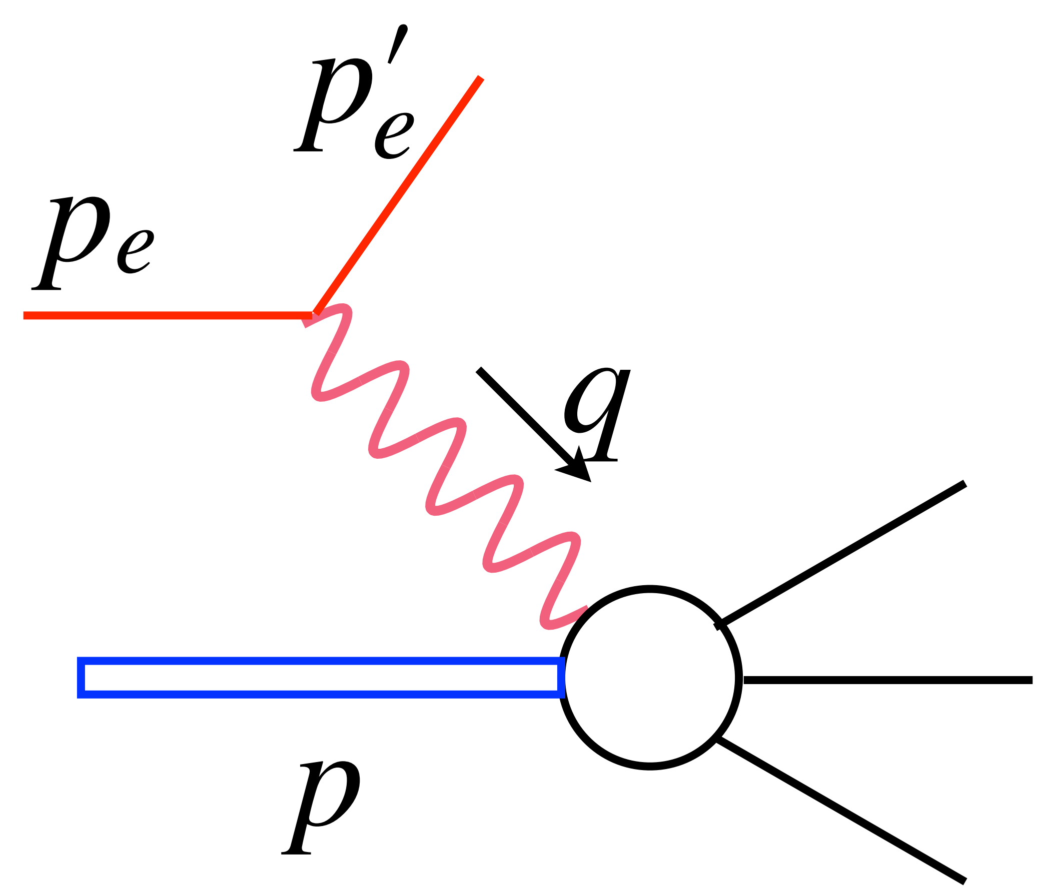
$x_B = \dfrac{Q^2}{2p \cdot q} = \dfrac{Q^2}{2m\nu}$ fixed (Bjorken limit) Note: $\nu \propto Q^2$

the transverse resolution is $r_\perp \sim 1/q_\perp \sim 1/Q$ *e.g.*, $r_\perp \sim 0.1$ fm, $Q^2 = 4$ GeV^2

The probability to hit a single parton is $\sim \Lambda^2_{QCD}/Q^2$
hence $\sigma_{DIS} \sim 1/Q^2$ (dimensional scaling)

Probability to hit two partons is $\sigma_{HT} \sim \Lambda^4_{QCD}/Q^4$ (higher twist contribution)

Longitudinal resolution in DIS



Through a small rotation $\theta \sim 1/Q$ align $\boldsymbol{q}$ along the negative z -axis $q = (q^0, q^x, q^y, q^z) = (\nu, 0, 0, -\sqrt{\nu^2 + Q^2})$

Define: $q^\pm \equiv q^0 \pm q^z$ Then $q = (q^+, q^-, \mathbf{q}_\perp) \simeq \left(-\frac{Q^2}{2\nu}, 2\nu, \mathbf{0}\right)$

The Fourier transform $\propto \exp(ir \cdot q) = \exp\left[i(r^+q^- + r^-q^+)/2 - \mathbf{r}_\perp \cdot \mathbf{q}_\perp\right]$ implies

$r^+ \sim 1/q^- \sim 1/\nu \to 0$ The photon probes the proton at an instant of Light-Front (LF) time, $r^+ = t + z \approx 0$

$r^- \sim 1/q^+ \sim 2\nu/Q^2 = 1/mx_B$ "Ioffe length"

The resolution is finite in $r^- = t - z$

Note: The photon moves in the negative z-direction!

Note: Since $t \approx -z$, the resolution in z is $1/2mx_B$

$x_B = 0.1 \Rightarrow \Delta z = 1$ fm

The Handbag

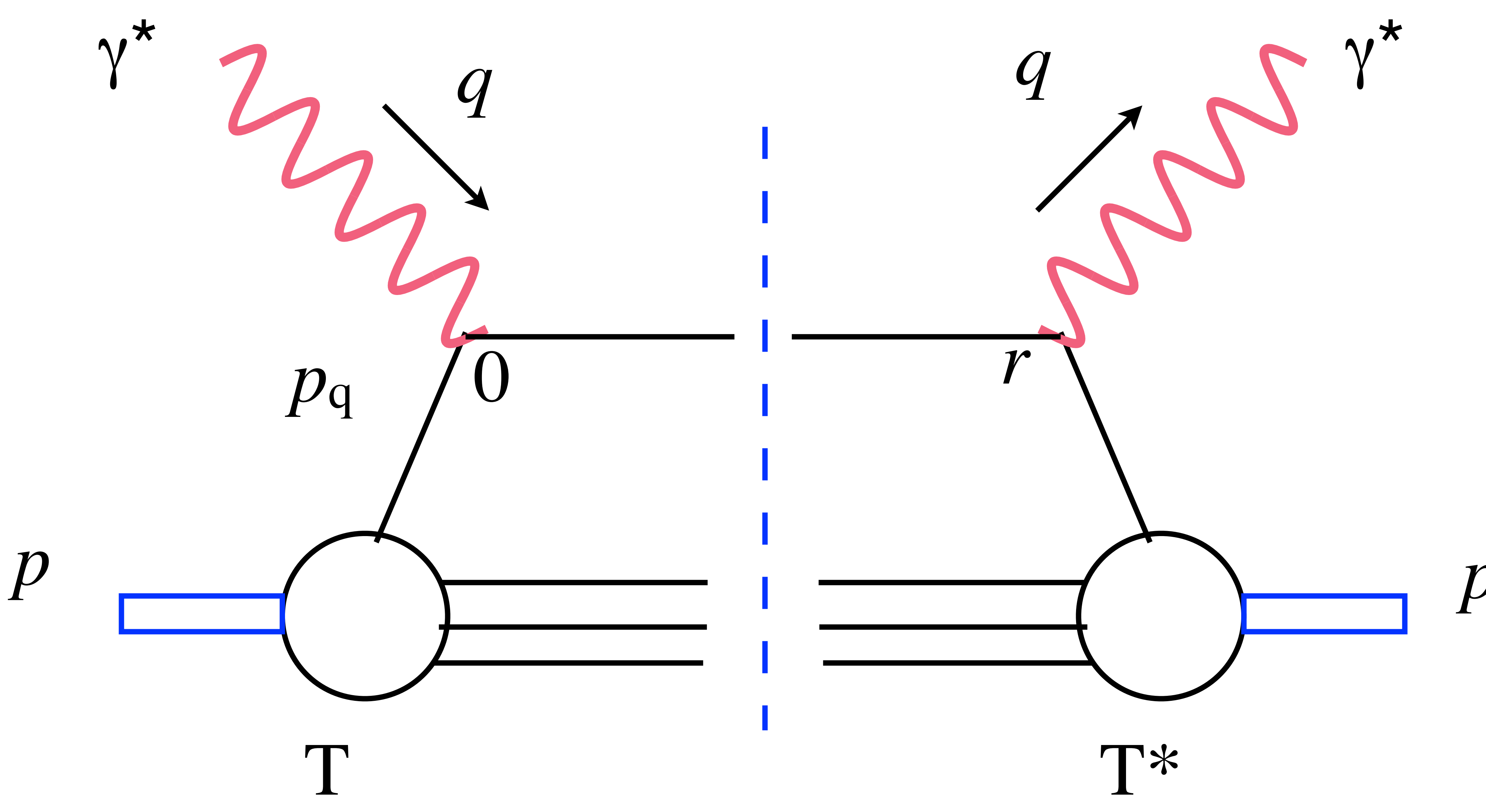
According to the optical theorem, the inclusive cross section is given by the discontinuity (imaginary part) of the handbag (forward) amplitude:

$$\sum_X |T(\gamma^* + p \to X)|^2 = \mathrm{Disc}\, T(\gamma^* + p \to \gamma^* + p)$$

The scaling (leading twist) contribution to σ_{DIS} arises when the same quark is hit in the amplitude and (amplitude)*.

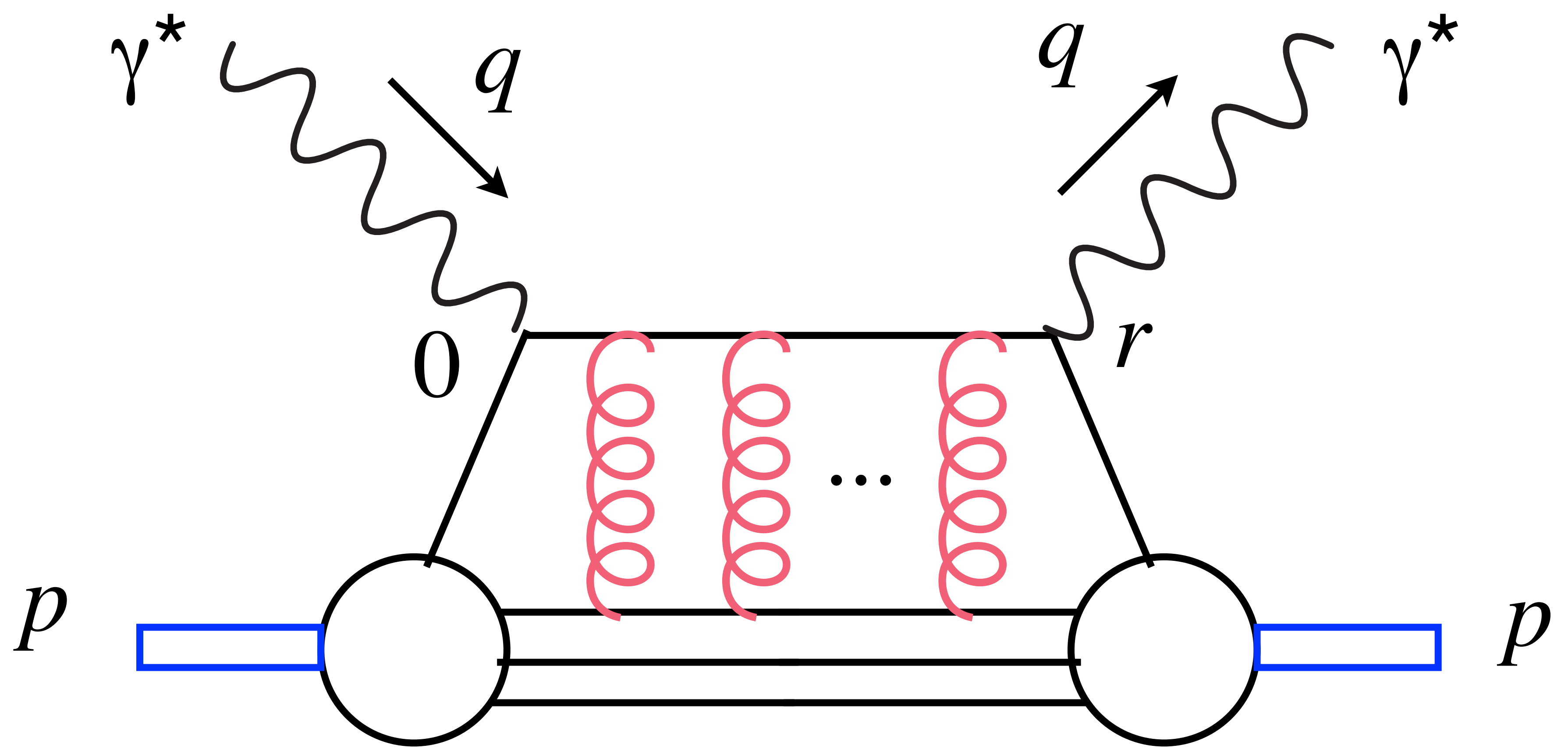
The photon vertices are separated by the finite resolution distance $r^- \sim 1/mx_B$

$r^+ \sim 1/\nu \sim 1/Q^2$ and $r_\perp \sim 1/Q$



Parton distribution with rescattering

Soft rescattering of the struck parton on the color field of the spectators gives rise to the "gauge link" in the matrix element that defines the gauge invariant parton distribution



$$f_{q/N}(x_B, Q^2) = \frac{1}{8\pi}\int dr^- e^{-imx_B x^-/2}\langle N(p)|\bar{q}(r^-)\gamma^+ W[r^-,0] q(0)|N(p)\rangle\Big|_{\substack{r^+=0\\ r_\perp\sim 1/Q}}$$

where the gauge link $W[r^-,0] \equiv \mathrm{P}\exp\left[\frac{ig}{2}\int_0^{r^-} dx^- A^+(x^-)\right]$

arises from rescattering of the struck quark on the color field of target spectators

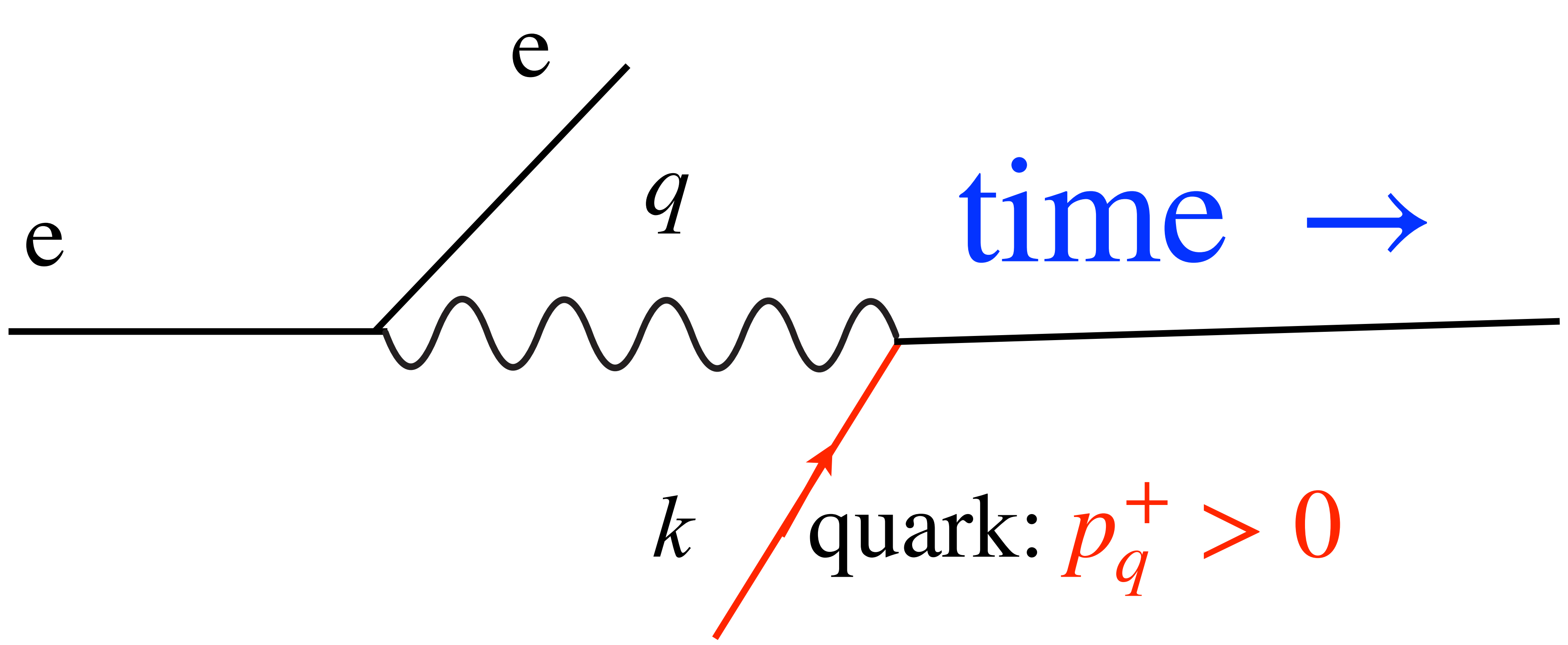
– Only instantaneous Coulomb exchange $A^+ = A^0 + A^z$ (specific to gauge theory)

– The gauge link ensures gauge invariance of the matrix element

The two views of DIS

The LF time (x^+) development in DIS depends on the electron beam direction:

$p_e^z < 0$: $q^+ = -mx_B$

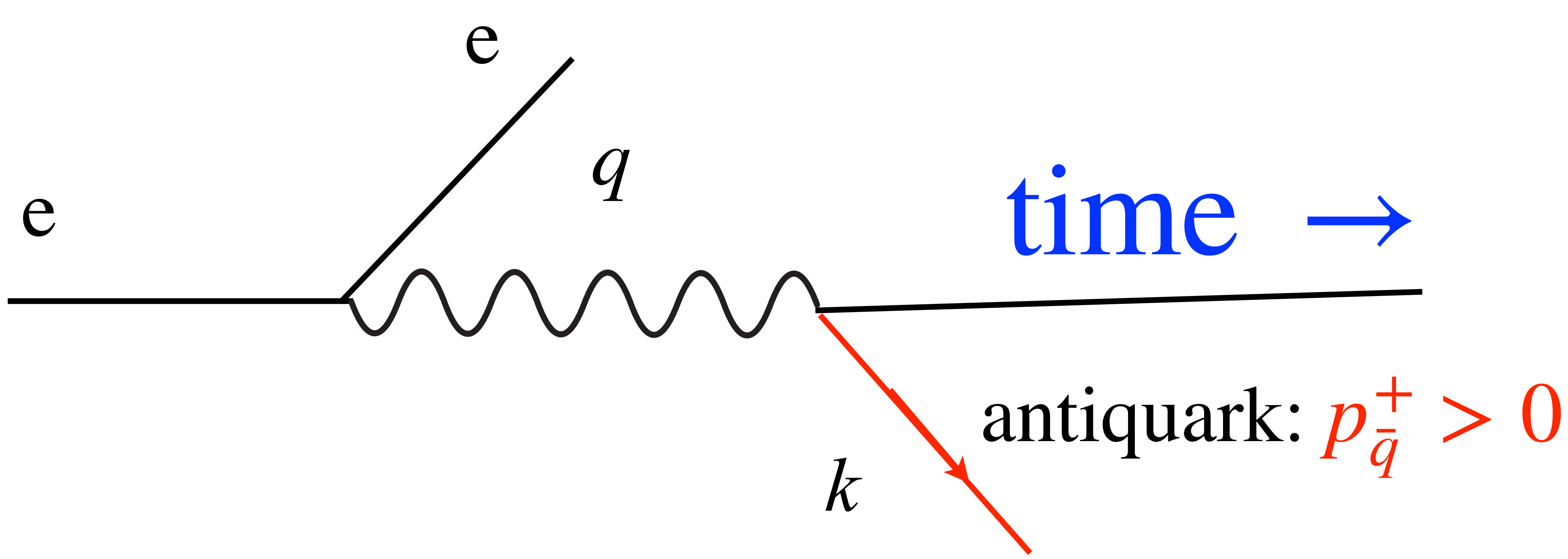


Virtual photon scatters on a target quark

σ_{DIS} ~ quark probability in the target

"Infinite momentum frame"

$p_e^z > 0$: $q^+ = 2\nu$



Virtual photon splits into a $q\bar{q}$ pair.

σ_{DIS} ~ $\sigma(q\bar{q})$ in the target

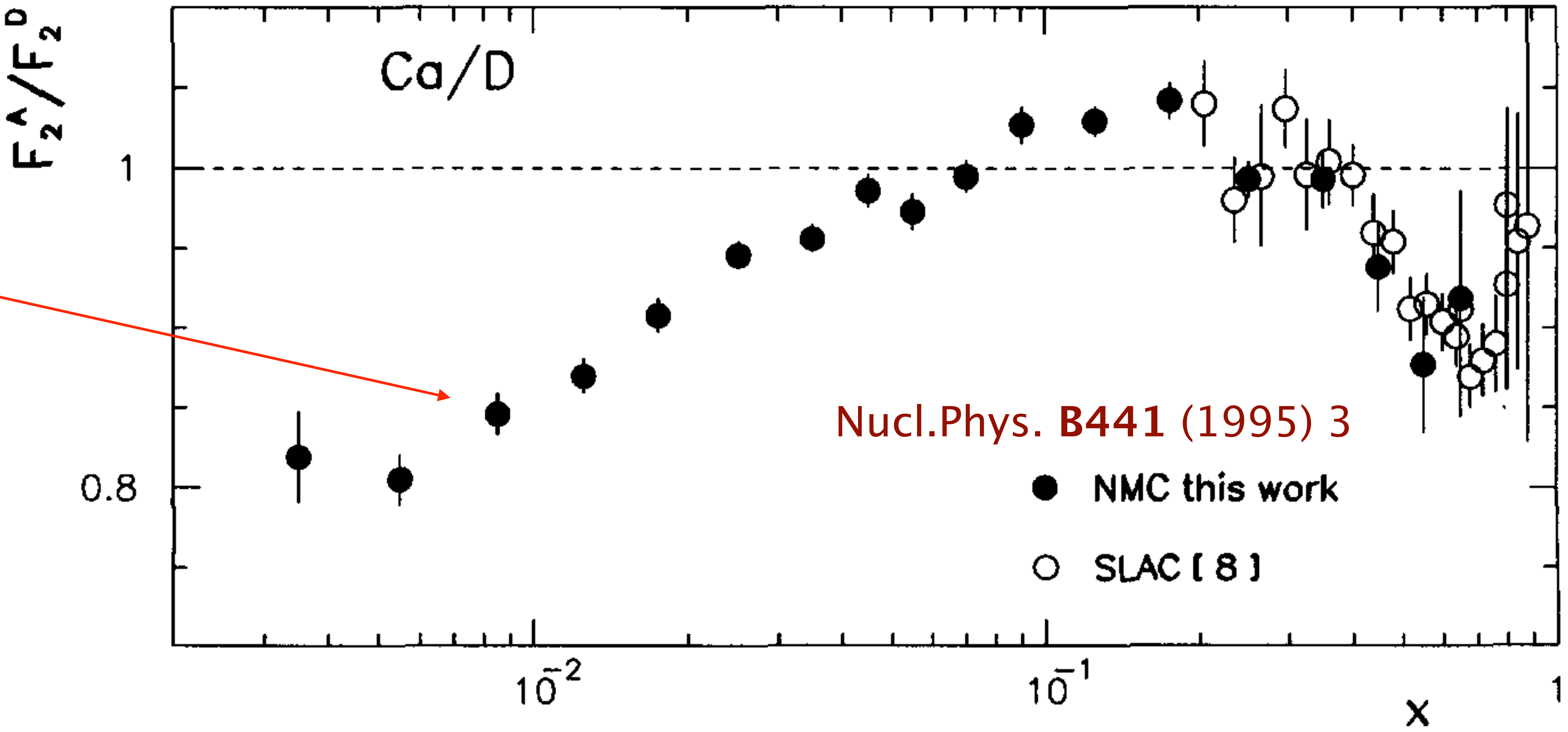
"Target rest frame"

The two views are related by a rotation of 180°, but rotations are not kinematic (explicit) symmetries on the Light Front.

Shadowing in DIS for nuclear targets

$$\frac{F_2(e+A \to e+X)}{A\,F_2(e+N \to e+X)}$$

is suppressed a small x_B



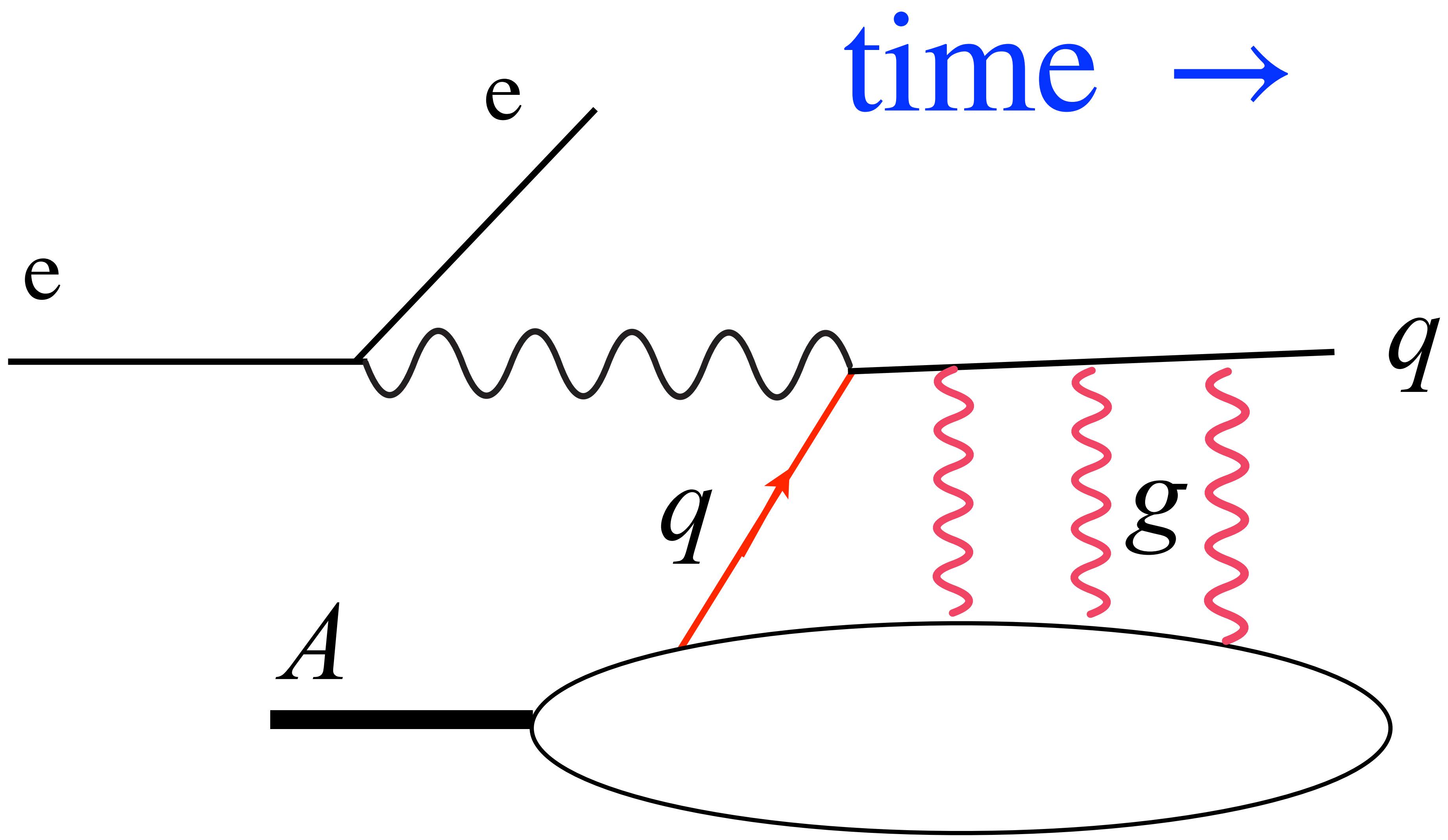
Intuitive explanation:
Nucleons in front "shadow" those behind them.

Requires DIS to be coherent on more than one nucleon in nucleus A

Longitudinal resolution of γ^*: $\dfrac{1}{2m_N x_B} \geq 2\,\text{fm}$ implies: $x_B \lesssim 0.05$

Shadowing: the two views of DIS

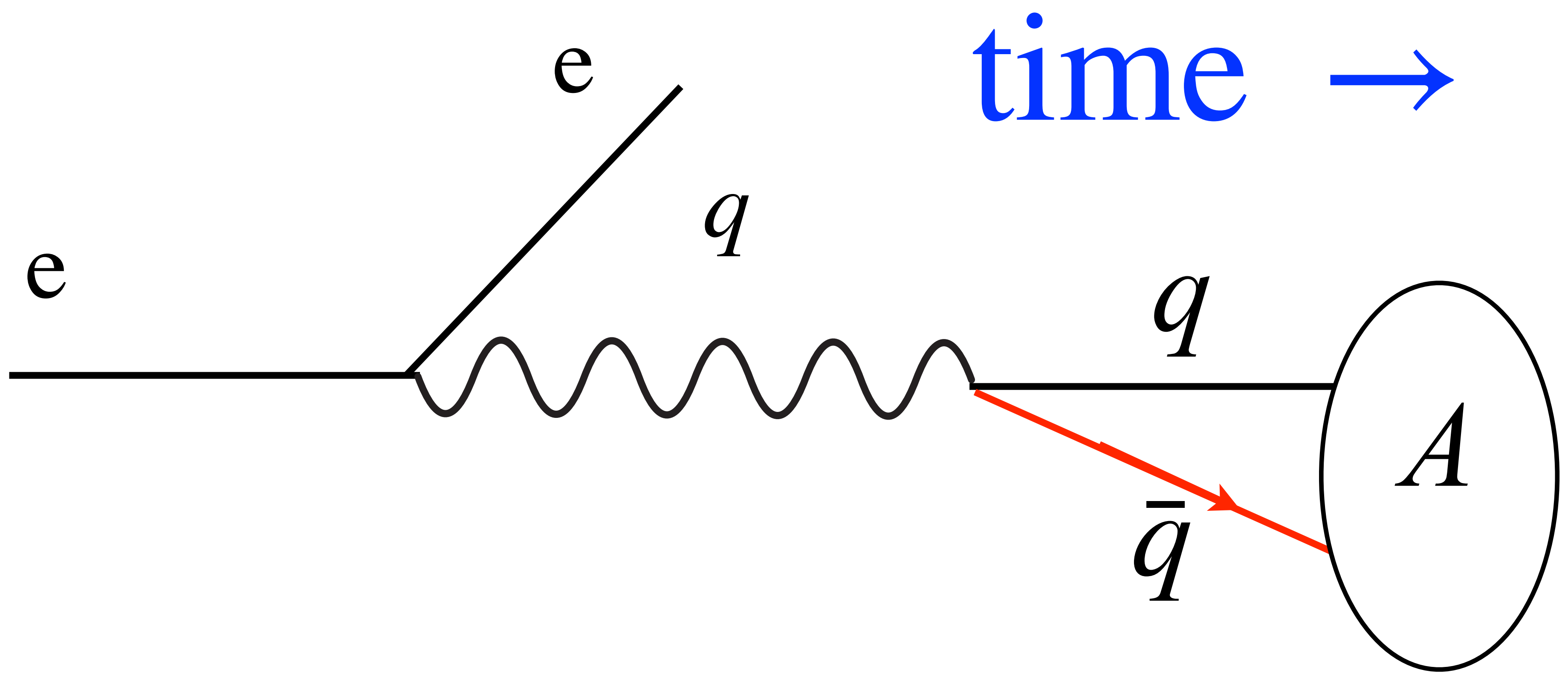
$p_e^z < 0$



Rescattering on several nucleons

"Infinite momentum frame"

$p_e^z > 0$



$q\bar{q}$ absorbed on front surface of A

"Target rest frame"

The two views are related by a rotation of 180°, but rotations are not kinematic (explicit) symmetries on the Light Front.


Quantum
Chromodynamics
Soft scattering

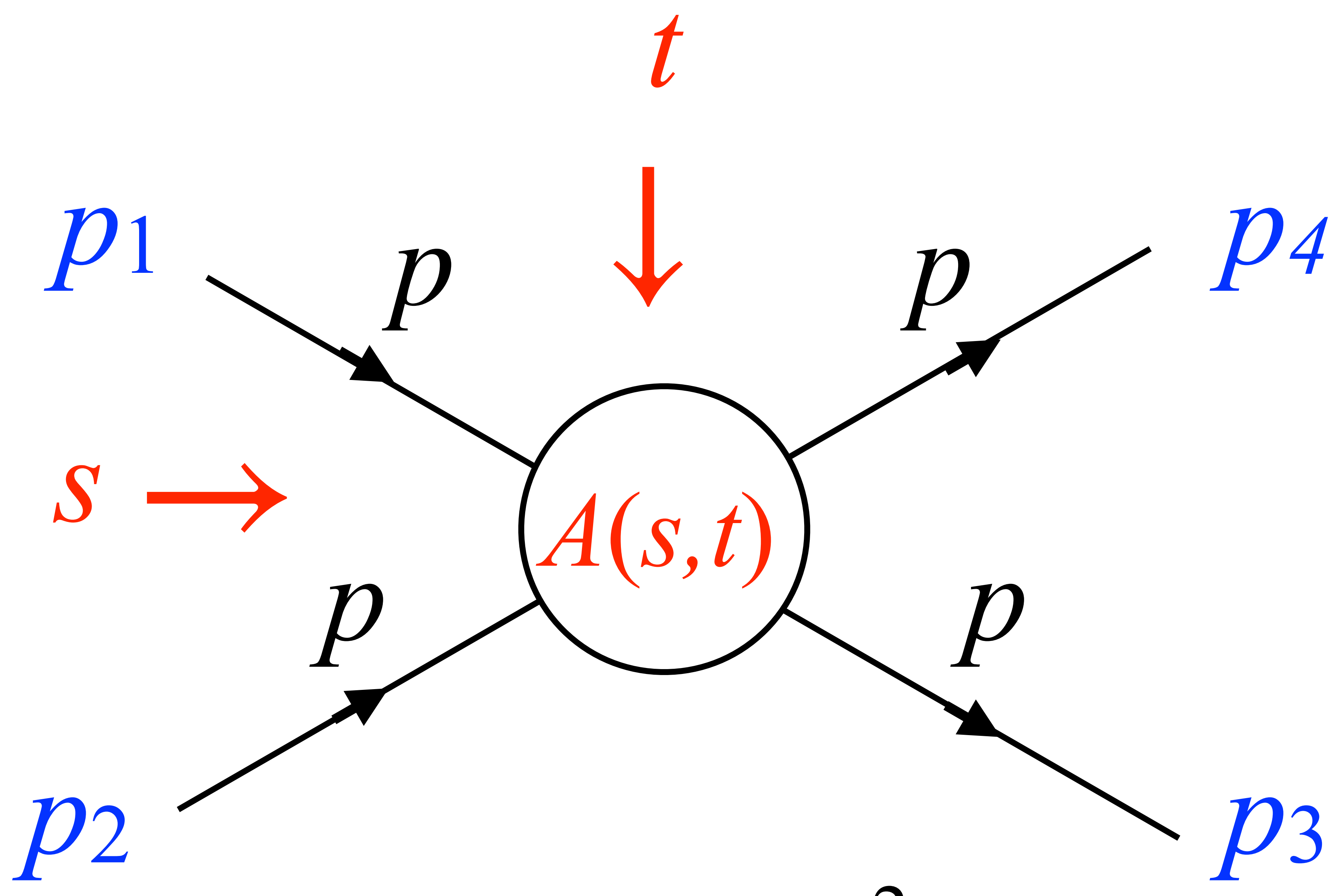
Soft hadron scattering: $pp \to pp$

Three processes related by crossing symmetry:

s-channel: $pp \to pp$ $\quad s \geq 4m_p^2\,;\; t, u \leq 0$

t-channel: $p\bar{p} \to p\bar{p}$ $\quad t \geq 4m_p^2\,;\; s, u \leq 0$

u-channel: $p\bar{p} \to p\bar{p}$ $\quad u \geq 4m_p^2\,;\; s, t \leq 0$



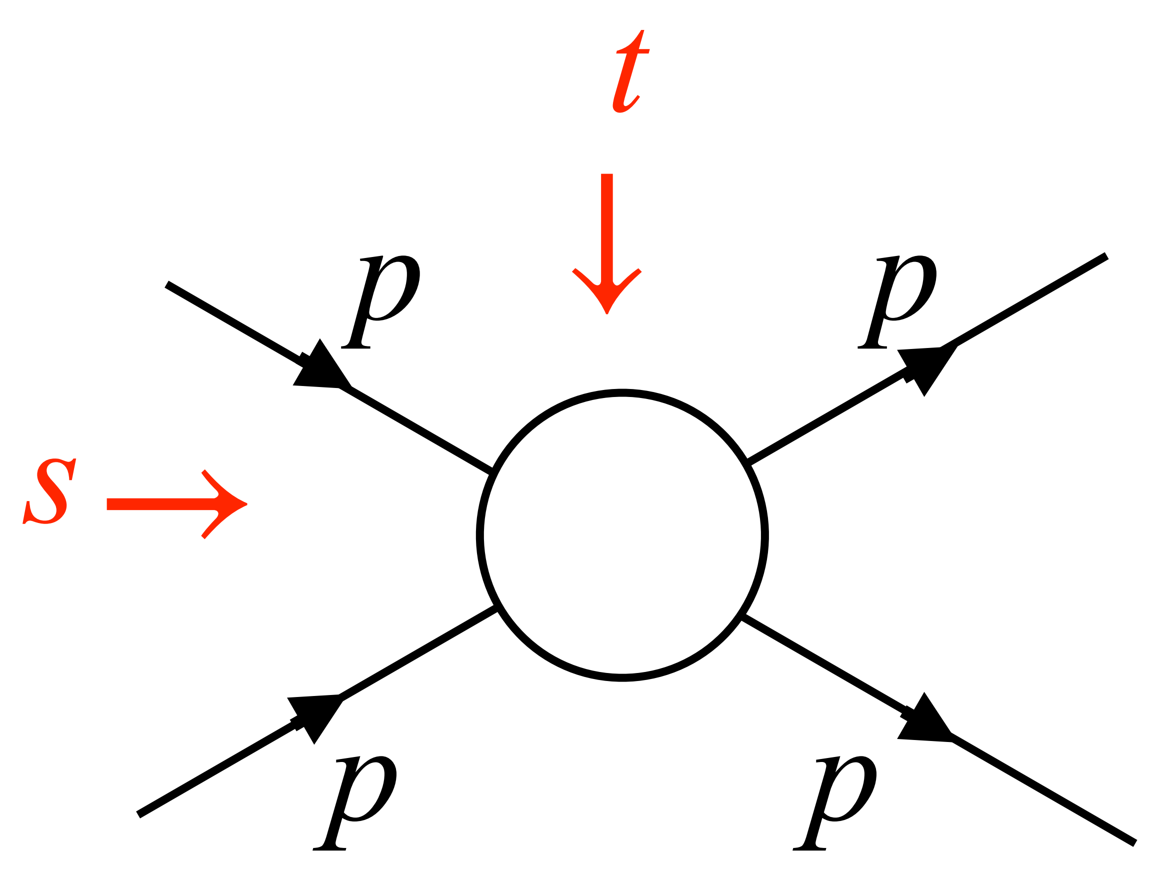
$$s = (p_1 + p_2)^2$$
$$t = (p_1 - p_4)^2$$
$$u = (p_1 - p_3)^2$$
$$s + t + u = 4m_p^2$$

Lorentz invariant Mandelstam variables s, t and u have distinct values for the three scattering processes.

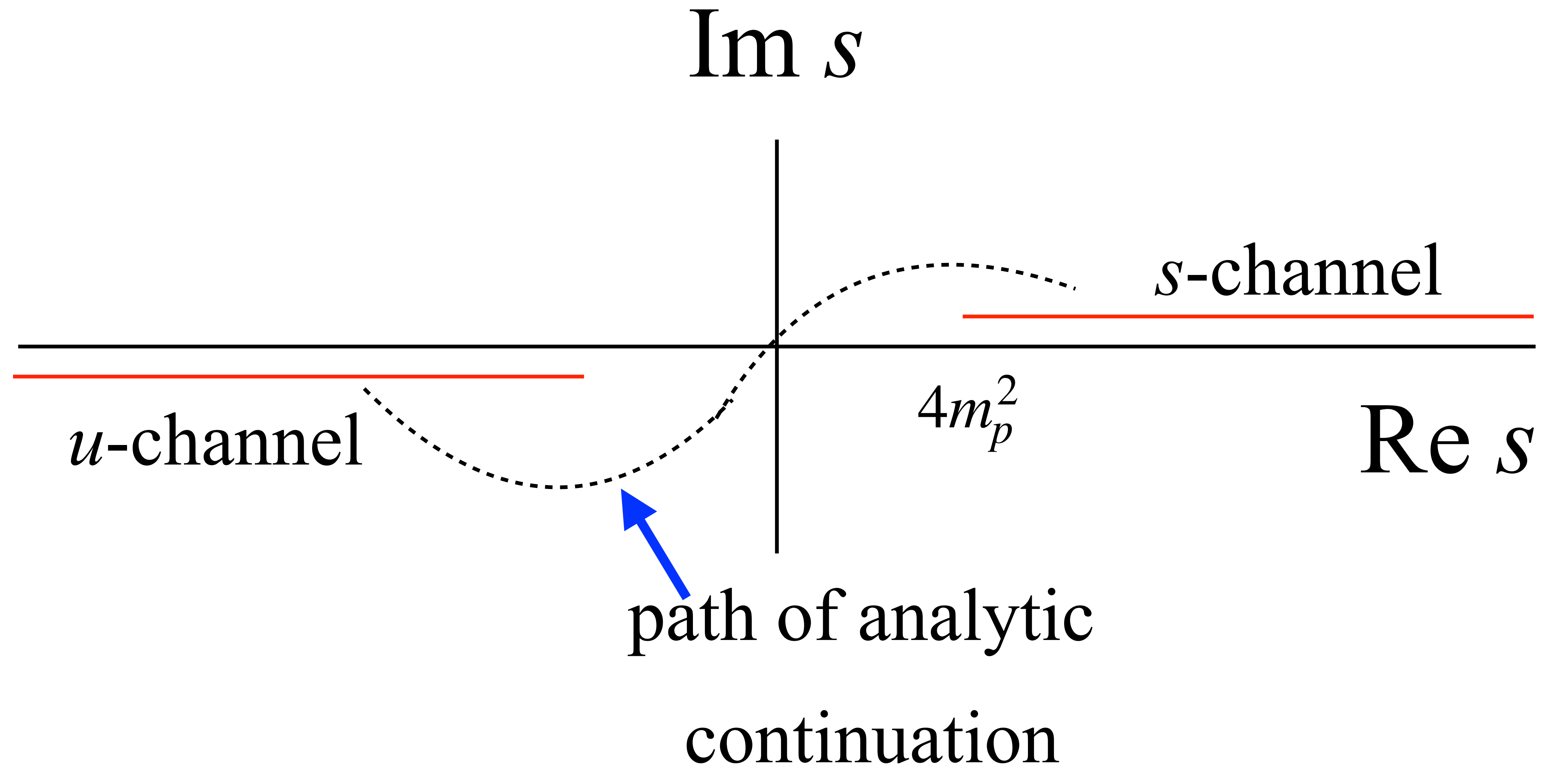
The same scattering amplitude $A(s,t)$ describes all three processes (crossing symmetry)

Analytic continuation of $pp \to pp$

Keeping $t < 0$ fixed, we may analytically continue the amplitude $A(s,t)$ from the $pp \to pp$ region, where $\mathrm{Im}(s) = +i\varepsilon$ to the $p\bar{p} \to p\bar{p}$ region, where $\mathrm{Im}(s) = -i\varepsilon$



This requires an exact knowledge of $A(s,t)$ for a finite range of s.



Crossing symmetry is an exact property of QFT's.

Particle poles in $p\bar{p} \to p\bar{p}$

For $s \to \infty$ with t fixed, a spin J resonance pole in the t-channel is $\propto s^J$

$$\cos\theta_t = \frac{2s + t - 4M^2}{t - 4M^2}$$

$$A(s,t) \propto \frac{P_J(\cos\theta_t)}{t - M^2 + iM\Gamma} \propto s^J$$

Unitarity: $\sigma_{tot}(pp) = \dfrac{\mathrm{ImA(s, t = 0)}}{s} < \dfrac{\pi}{m_\pi^2} \log^2 s$

Violated by resonances with spin ≥ 2 ?

Yes, but $t = M^2 - iM\Gamma$ is not in the physical region of the s-channel.
Pole contribution is finite, and can be canceled by other terms.

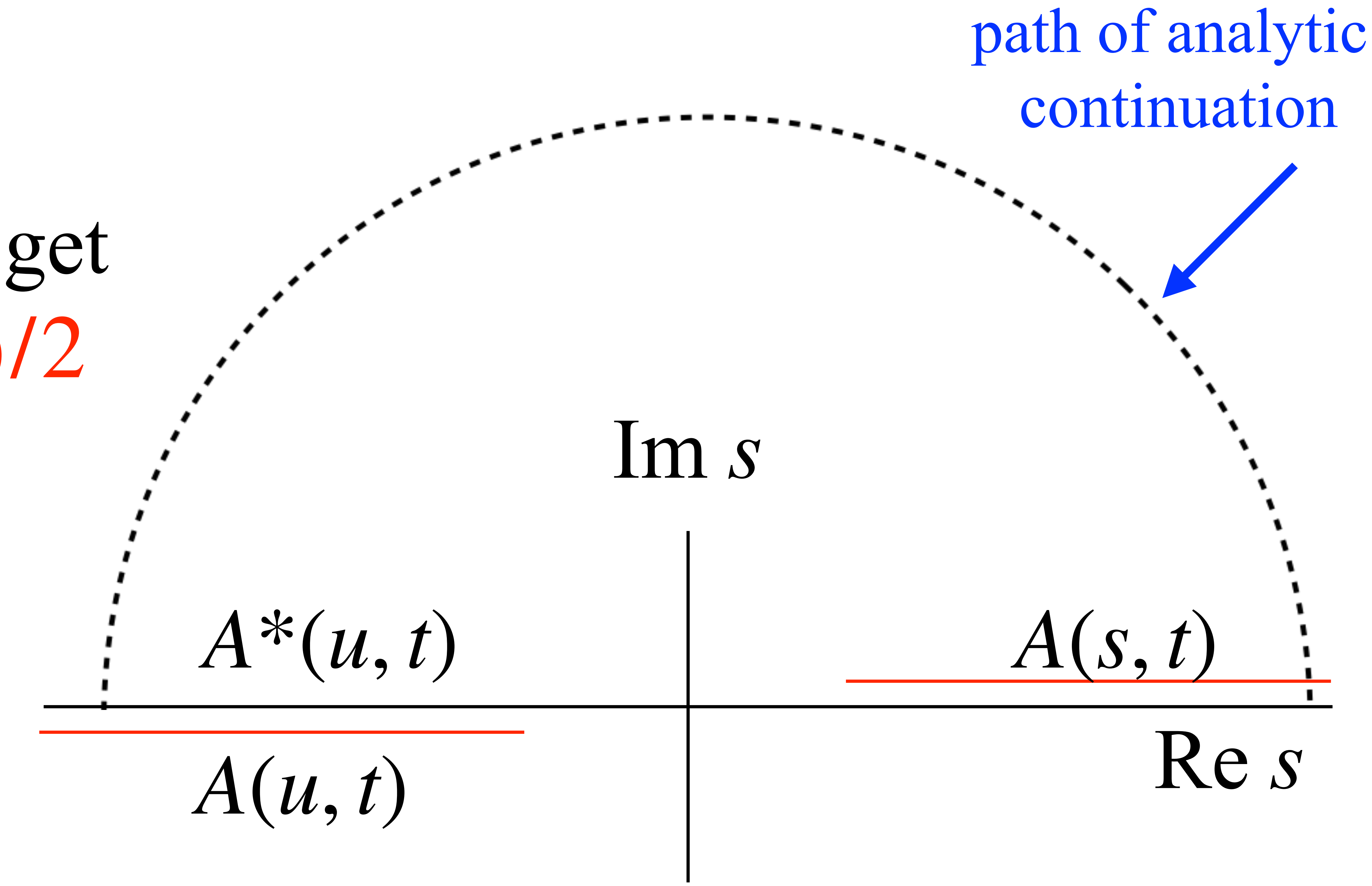
High energy behavior of $A(pp \to pp)$

Assuming $A(s \to +\infty, t) = \beta(t)\, e^{i\phi}\, s^{\alpha}$ we may analytically continue

to $A(s \to -\infty + i\varepsilon, t) = \beta(t)\, e^{i\phi}\, e^{i\pi\alpha}(-s)^{\alpha}$ along the large semicircle.

Take complex conjugate to cross the cut: $A(u \to \infty + i\varepsilon, t) = A^*(s \to -\infty + i\varepsilon, t)$

For combinations that are (anti)symmetric under $s \to u$, i.e., $A(pp \to pp) \pm A(p\bar{p} \to p\bar{p})$ get the "Regge" phases: $\phi = -\pi\alpha/2$ or $\pi(1-\alpha)/2$



LHC data: Up to log's:

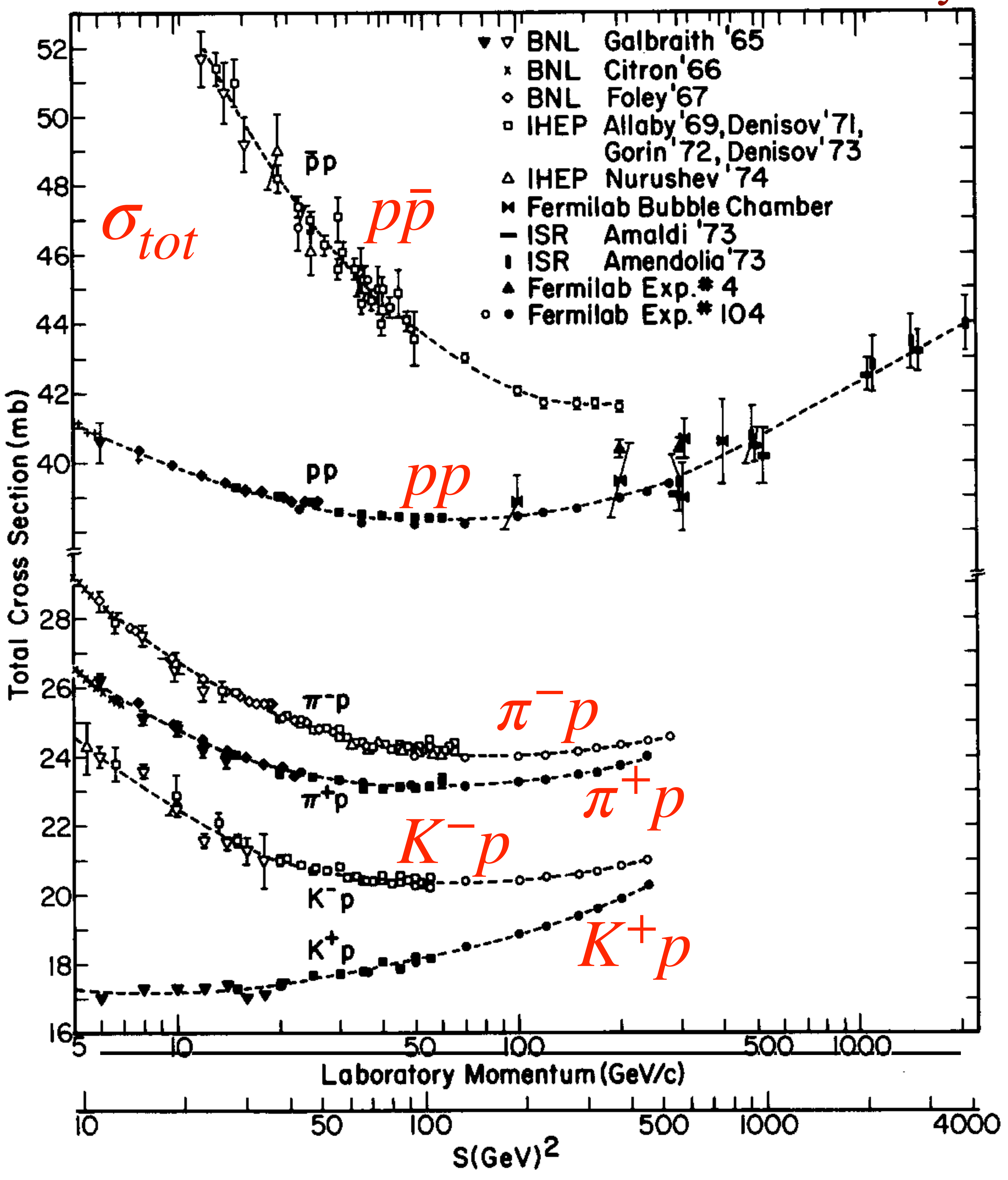
$\sigma_{tot}(pp) \simeq \sigma_{tot}(p\bar{p}) \propto s^0$

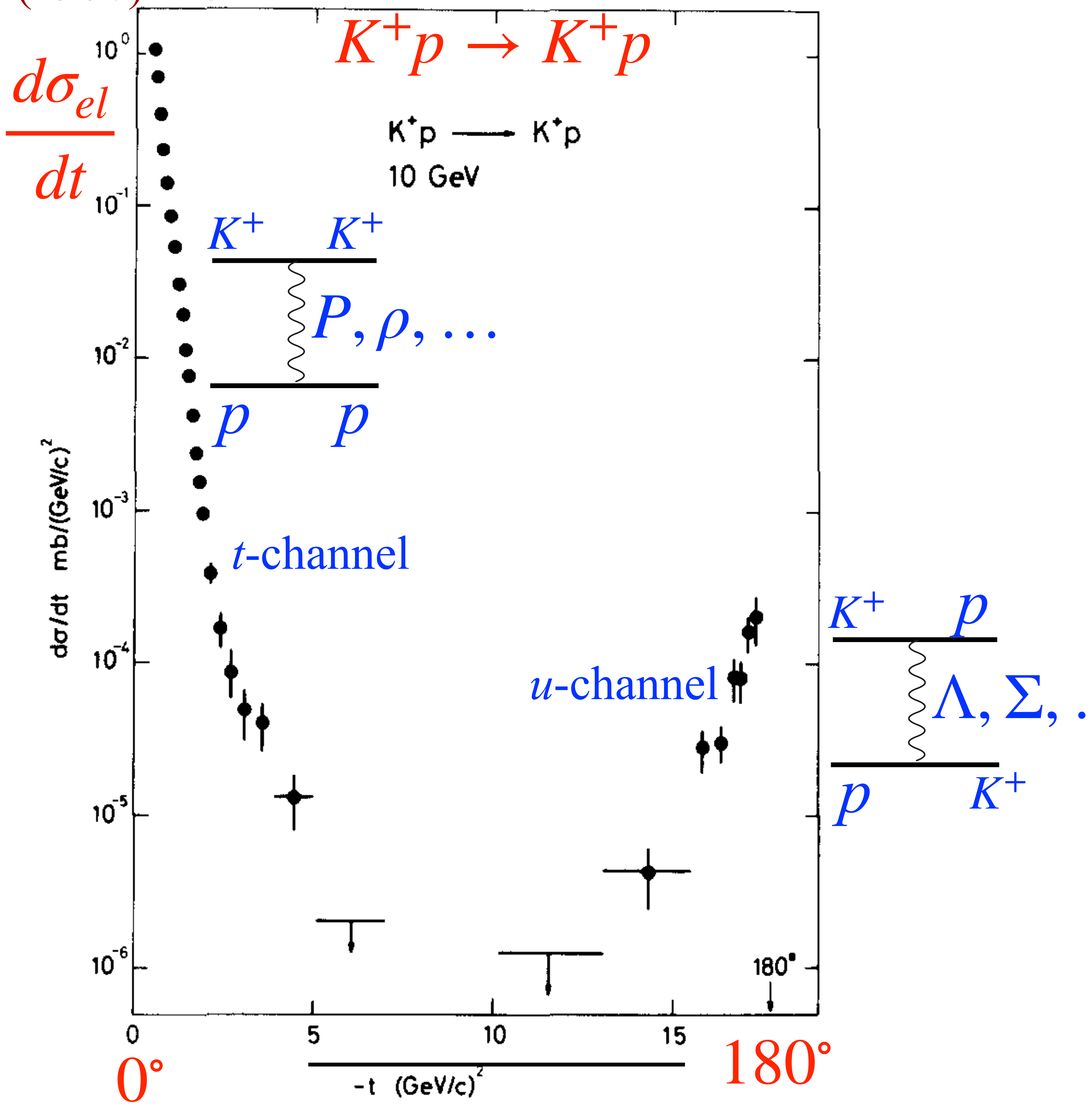
Hence $\alpha_P(t=0) \simeq 1$

The "Pomeron" exchange amplitude ($C = +1$) should be dominantly imaginary

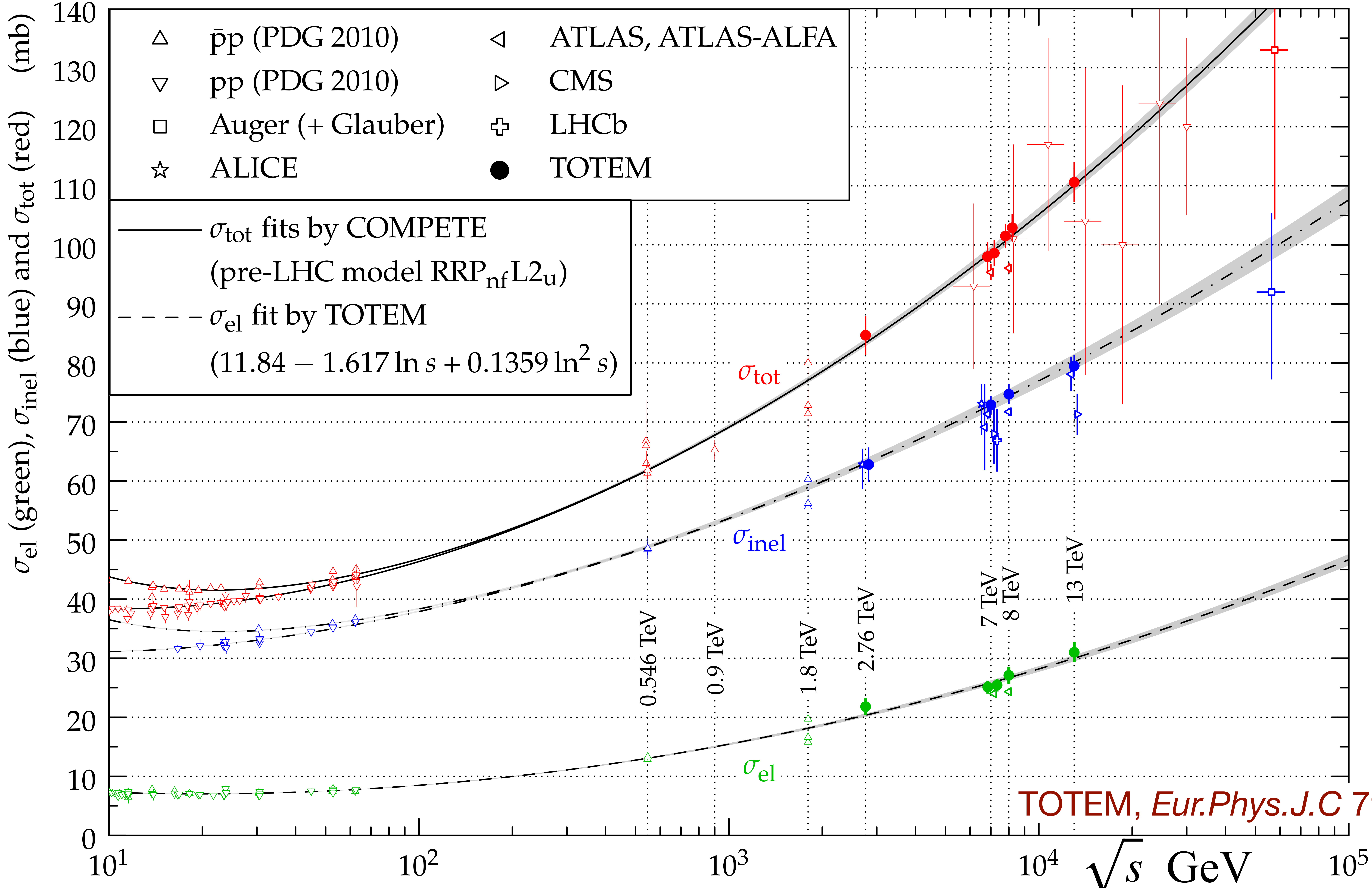
G. Giacomelli, Total cross sections and elastic scattering at high energies

Phys. Rept. **23** (1976) 123



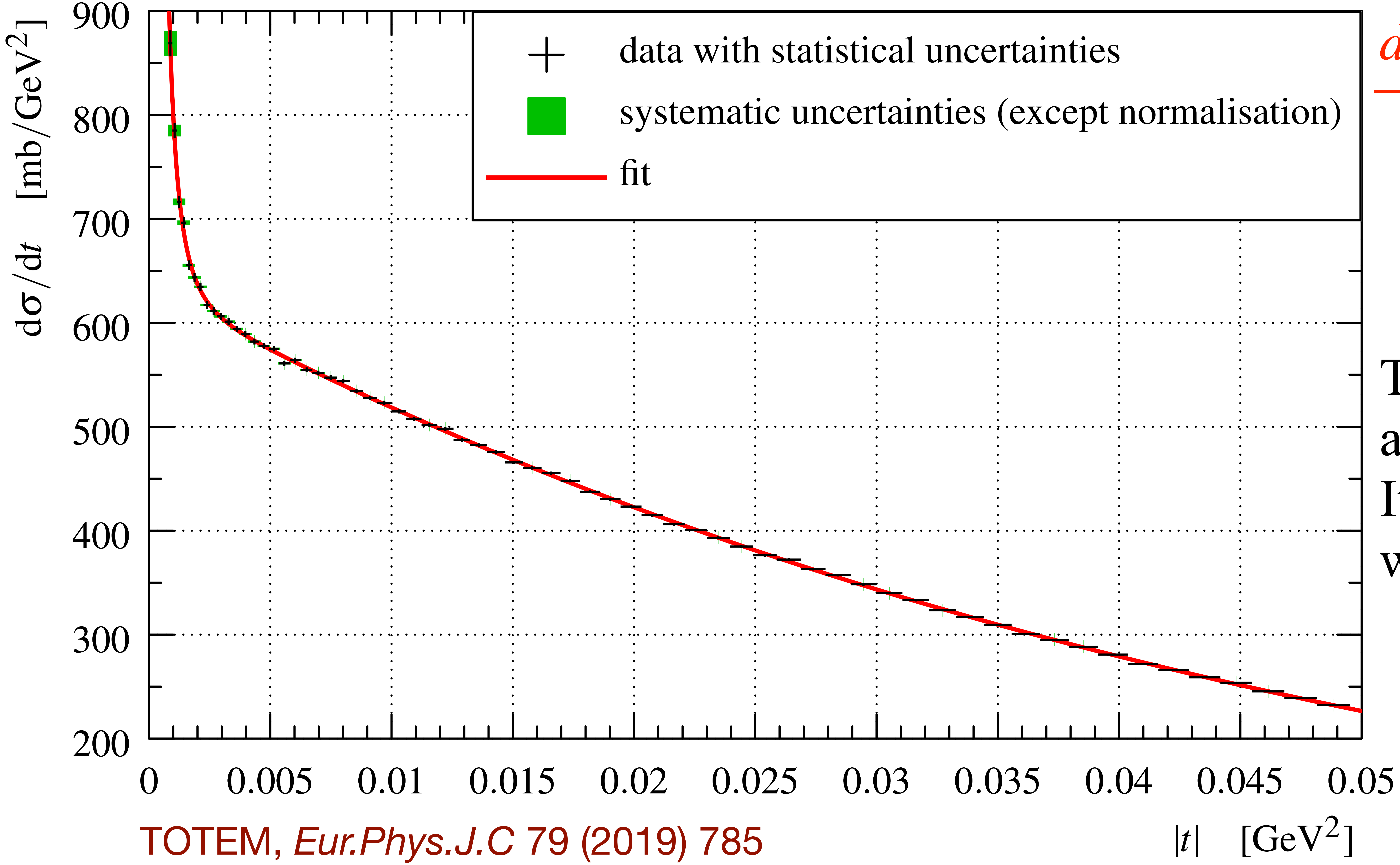


σ_{tot}, σ_{inel} and σ_{el} at high $\sqrt{s}$



TOTEM, *Eur.Phys.J.C* 79 (2019) 785

Photon exchange dominates at small |t| in $pp \to pp$



TOTEM, *Eur.Phys.J.C* 79 (2019) 785

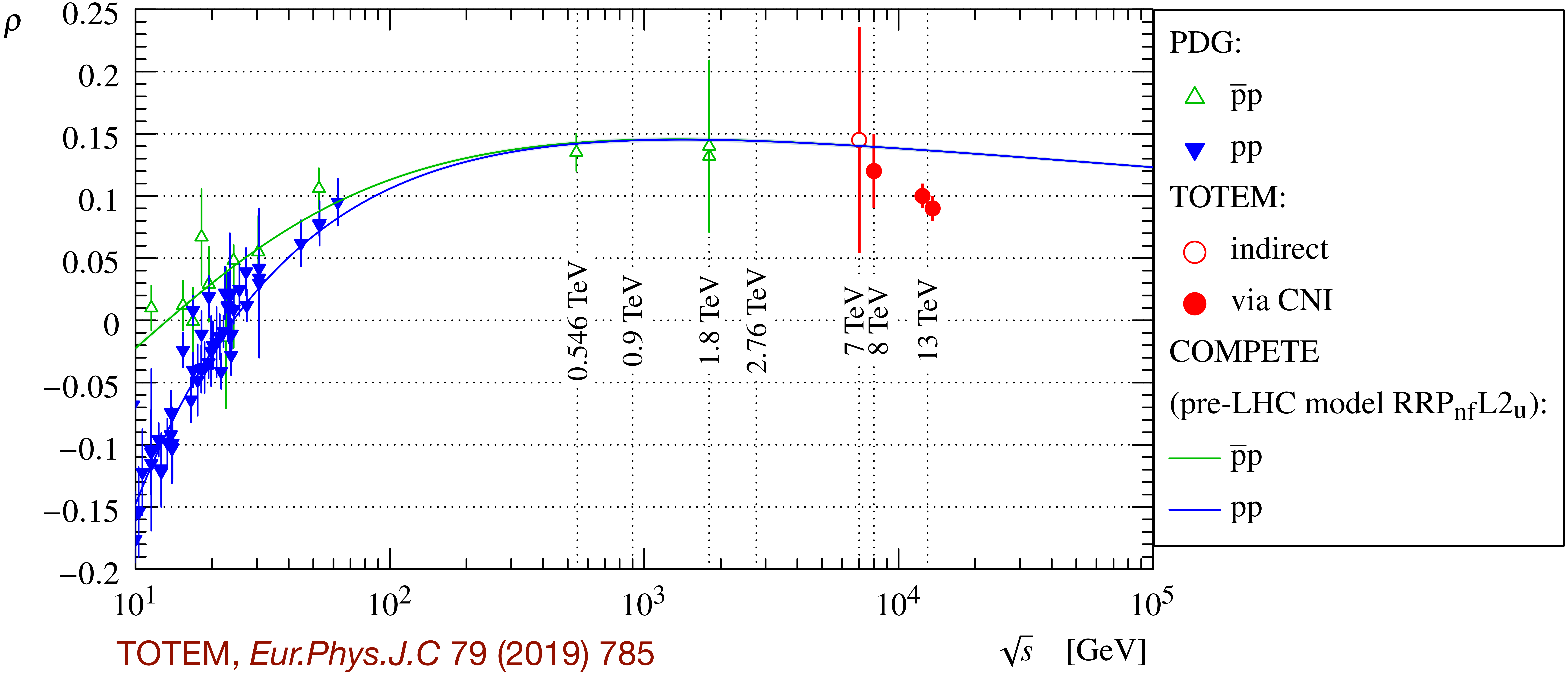
$$\frac{d\sigma_{QED}}{dt} = \frac{4\pi\alpha^2}{t^2}\mathcal{F}^4(t)$$

The photon exchange amplitude is real. Its interference with QCD gives

$$\rho = \frac{\mathrm{Re}A_{QCD}}{\mathrm{Im}A_{QCD}}$$

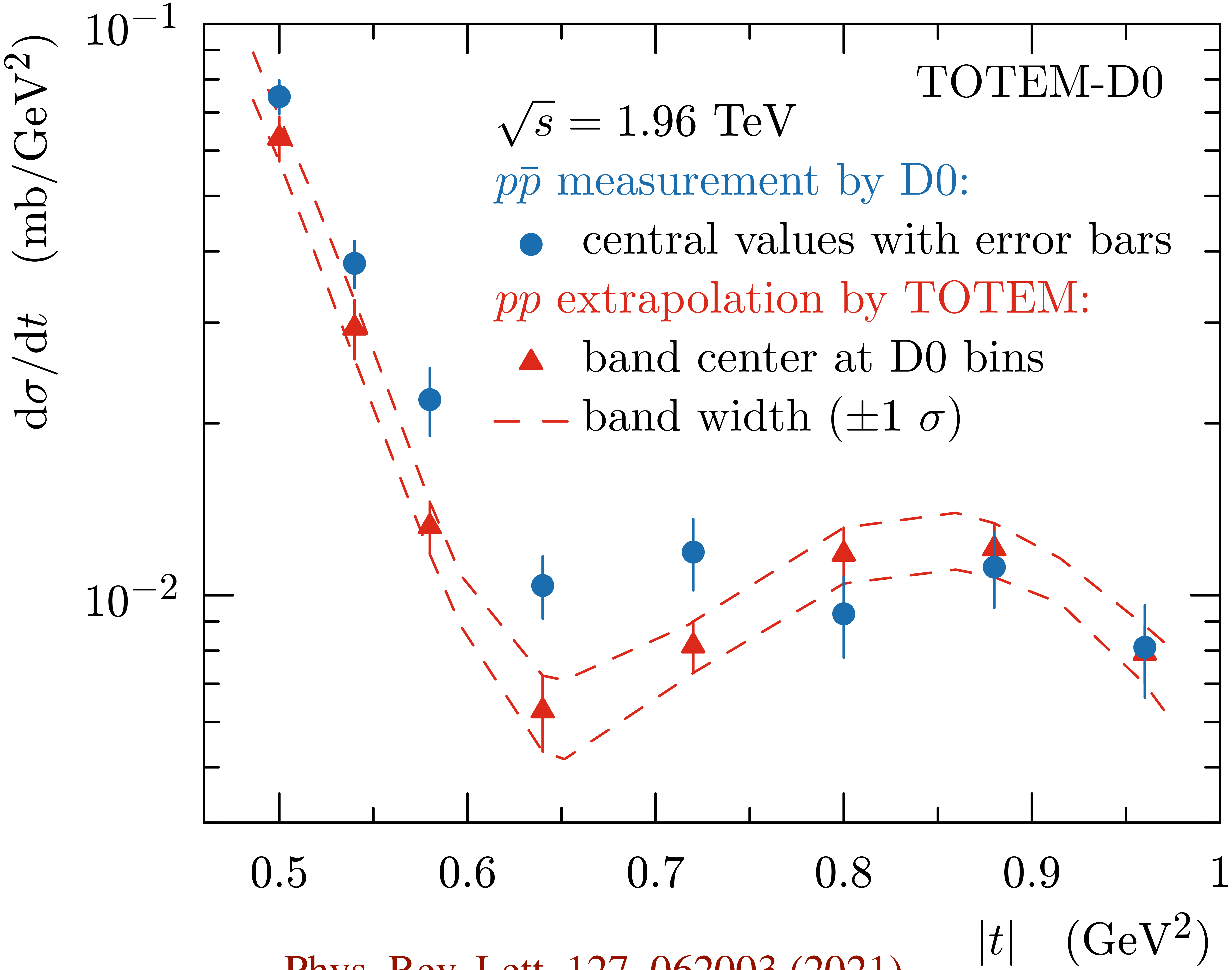
Real part of $A(pp \to pp, t=0)$ is small

$$\rho = \frac{\mathrm{Re}A_{QCD}}{\mathrm{Im}A_{QCD}}$$



TOTEM, *Eur.Phys.J.C* 79 (2019) 785

Odderon: $\sigma(pp \to pp) - \sigma(p\bar{p} \to p\bar{p})$



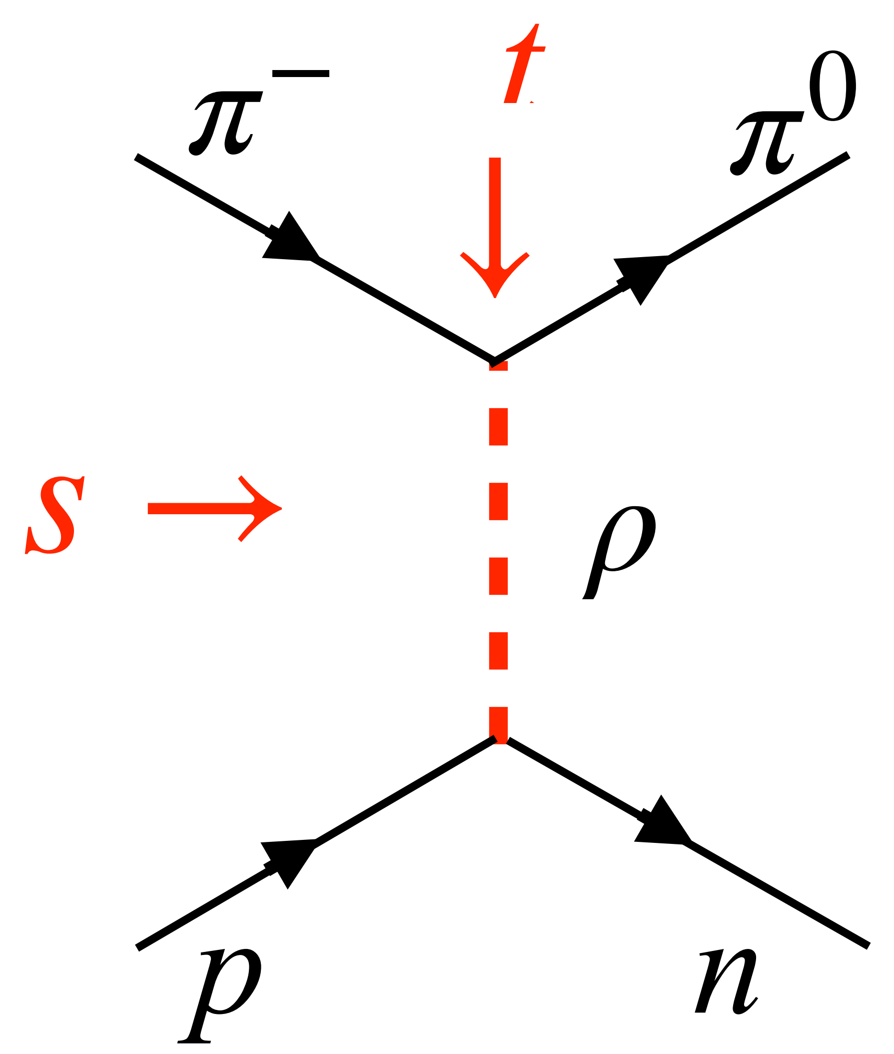
Phys. Rev. Lett. 127, 062003 (2021)

Search for an exchange with $\alpha \simeq 1$ and odd charge conjugation: The Odderon

Odderon exchange implies $\sigma(pp \to pp) - \sigma(p\bar{p} \to p\bar{p}) \neq 0$ at LHC energies.

Linear Regge trajectories $\alpha(t)$

For $s \to \infty$ the $\pi^- p \to \pi^0 n$ amplitude is dominated by ρ Regge exchange in the t-channel:



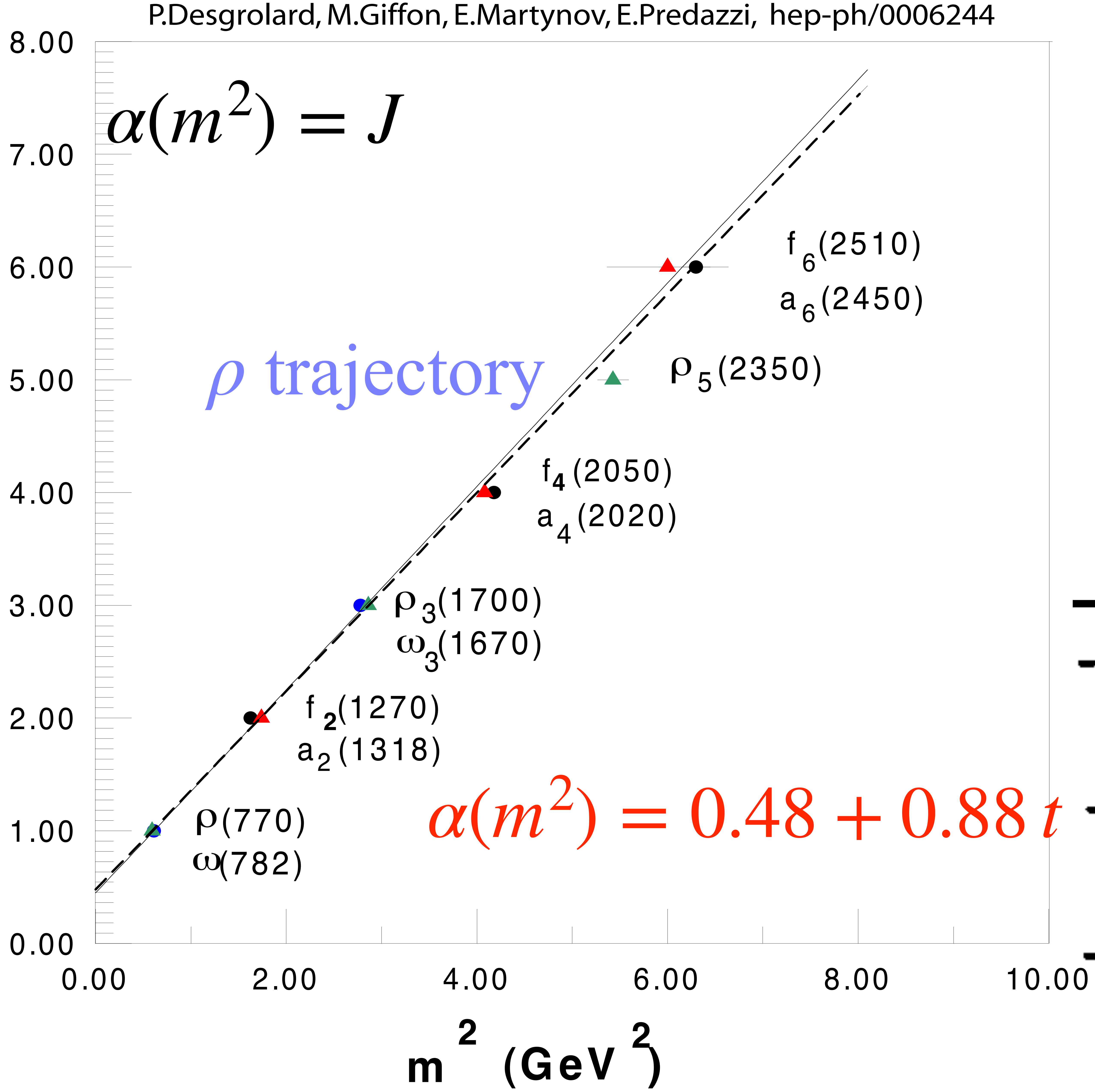
$$A(\pi^- p \to \pi^0 n) = \beta(t)\, i\, e^{-i\pi\alpha_\rho(t)/2}\, s^{\alpha_\rho(t)}$$

At particle poles $t = m^2 > 0$ the s-dependence is determined by the pole residues to be $\propto s^J$, where J is the spin of the resonance. E.g., $\alpha_\rho(m_\rho^2) = 1$.

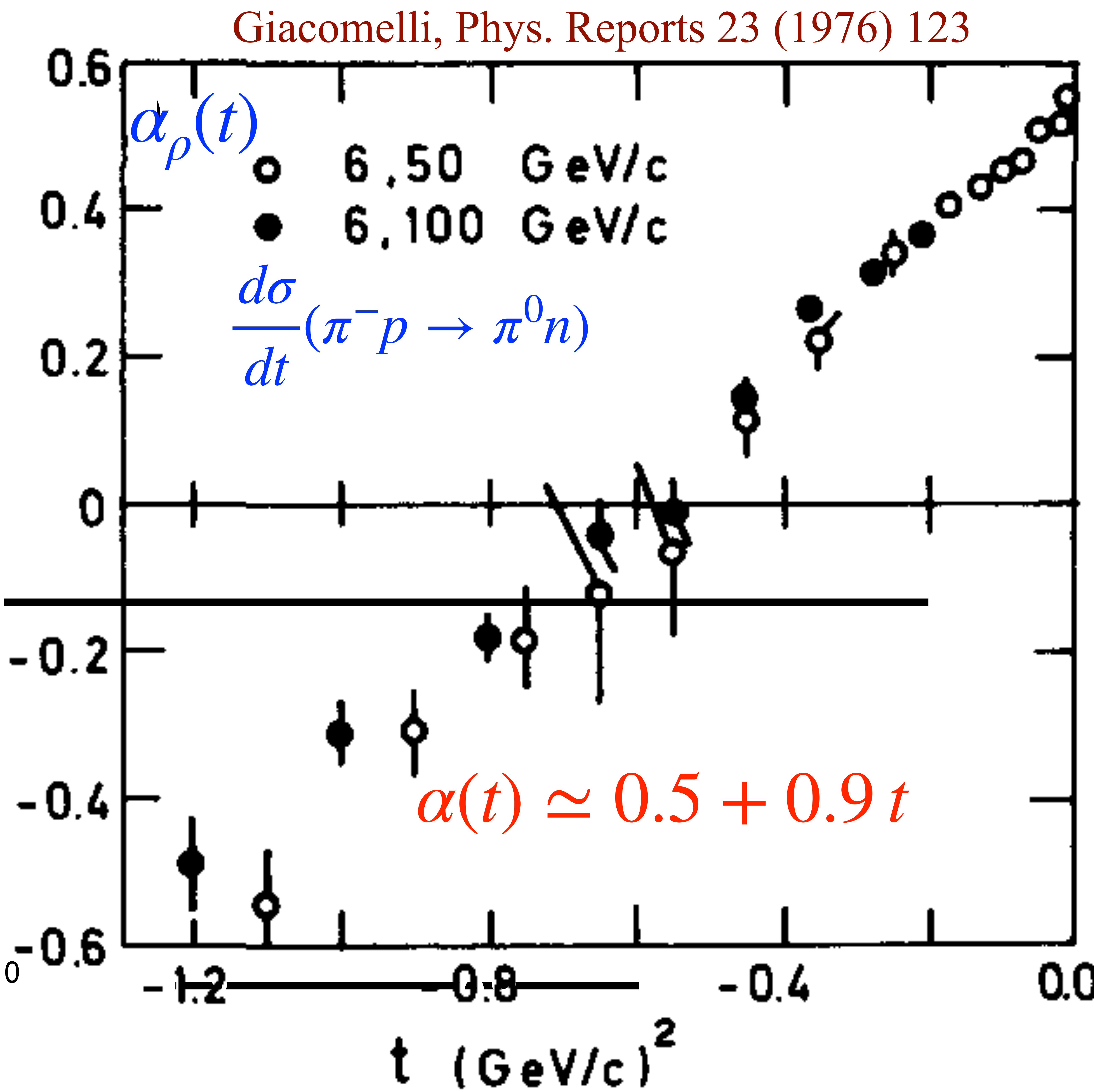
In the physical scattering region ($t \leq 0$), $\alpha(t)$ can be determined from the s-dependence of the cross section.

The data on the resonances and the scattering agree on $\alpha_\rho(t) \simeq 0.5 + 0.9\, t$

The ρ Regge trajectory $\alpha_\rho(t)$



$$\alpha(m^2) = 0.48 + 0.88\, t$$



$$\alpha(t) \simeq 0.5 + 0.9\, t$$

Duality in hadron scattering

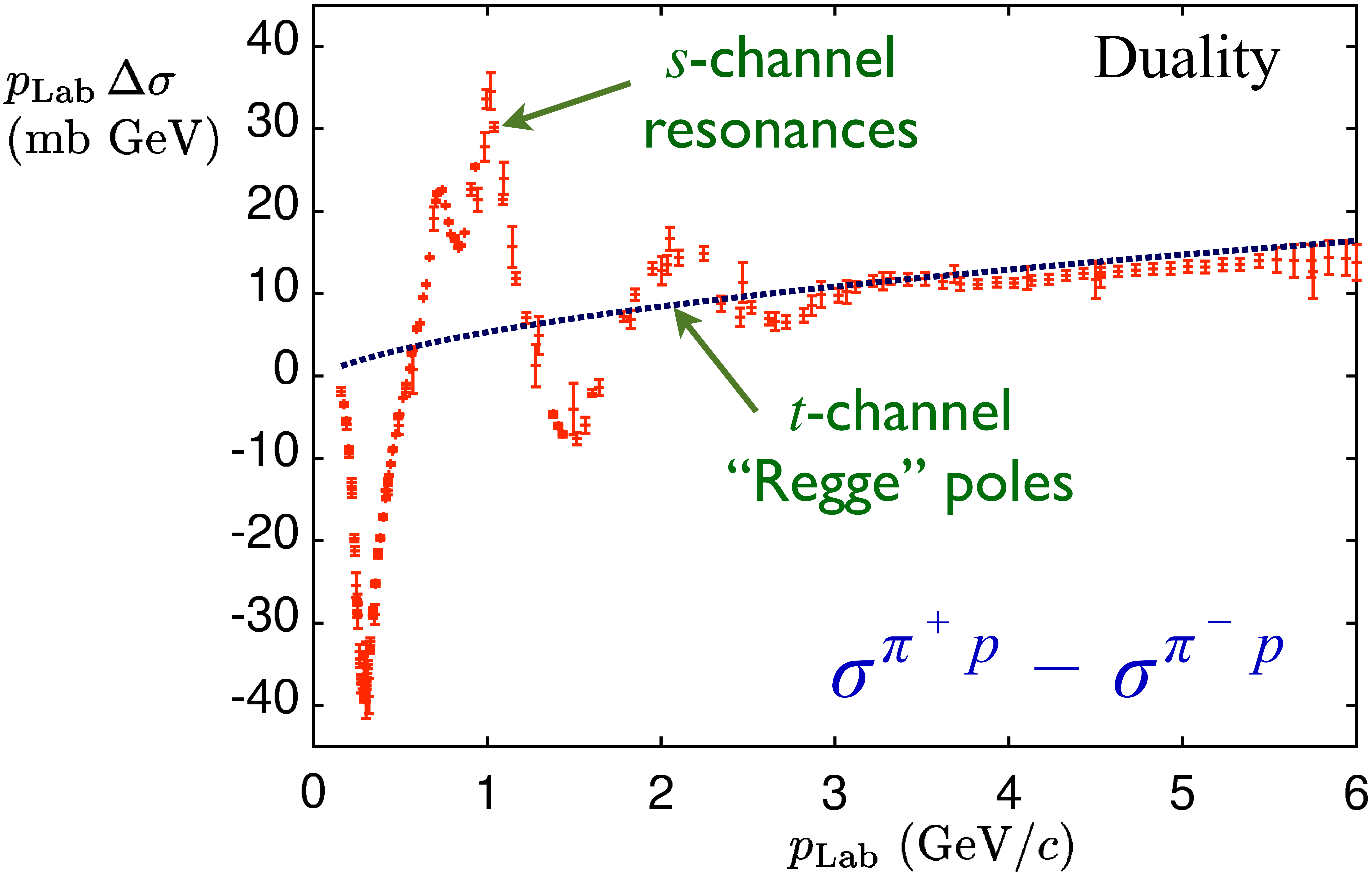
"finite energy sum rules"

Igi (1962), Dolen, Horn, Schmidt (1968)

Resonances in s-channel or Regge exchange in t-channel build Im $A(s,t)$

Analogous duality phenomena seen in $e^+e^- \to$ hadrons and in DIS, $eN \to eX$

W. Melnitchouk (2010)
https://www.jlab.org/conferences/HiX2010/program.html



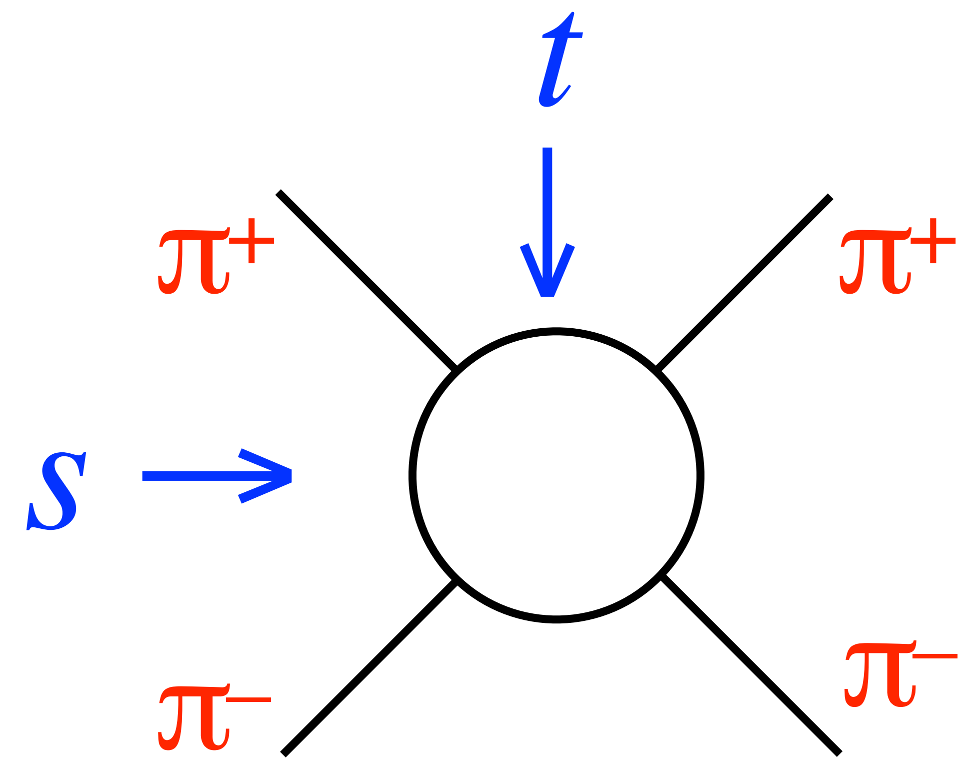
Duality
s-channel
resonances
t-channel
"Regge" poles
$\sigma^{\pi^+ p} - \sigma^{\pi^- p}$
$p_{\mathrm{Lab}}\,\Delta\sigma$ (mb GeV)
p_{Lab} (GeV/c)
40
30
20
10
0
-10
-20
-30
-40
0
1
2
3
4
5
6

Analytic example: Dual amplitudes

In 1968, Veneziano found a simple analytic function with many of the properties required for scattering amplitudes, including duality.
Lovelace applied this idea to the $\pi^+\pi^- \to \pi^+\pi^-$ scattering amplitude

$$A(\pi^+\pi^- \to \pi^+\pi^-) = \frac{\Gamma(1-\alpha_s)\,\Gamma(1-\alpha_t)}{\Gamma(1-\alpha_s-\alpha_t)}$$

$$\alpha_s \equiv \alpha(s) = \tfrac{1}{2} + s \qquad (\alpha' \equiv 1)$$



The amplitude has poles at $\alpha = 1, 2, \ldots$: the $\rho, \omega, f, \ldots$ resonances.
The residues are polynomials of degree $\alpha = n$ in $\cos\Theta = 1+2t/s$

Thus the pole at $\alpha_s = n$ is a superposition of bound states with $J = 1, \ldots, n$

$$\lim_{s\to\infty} A(s,t) = \Gamma(1-\alpha_t)e^{-i\pi\alpha_t}s^{\alpha_t}$$

Regge behavior

G. Veneziano, Nuovo Cim. **57A** (1968) 190
C. Lovelace, Phys. Lett. **28B** (1968) 264

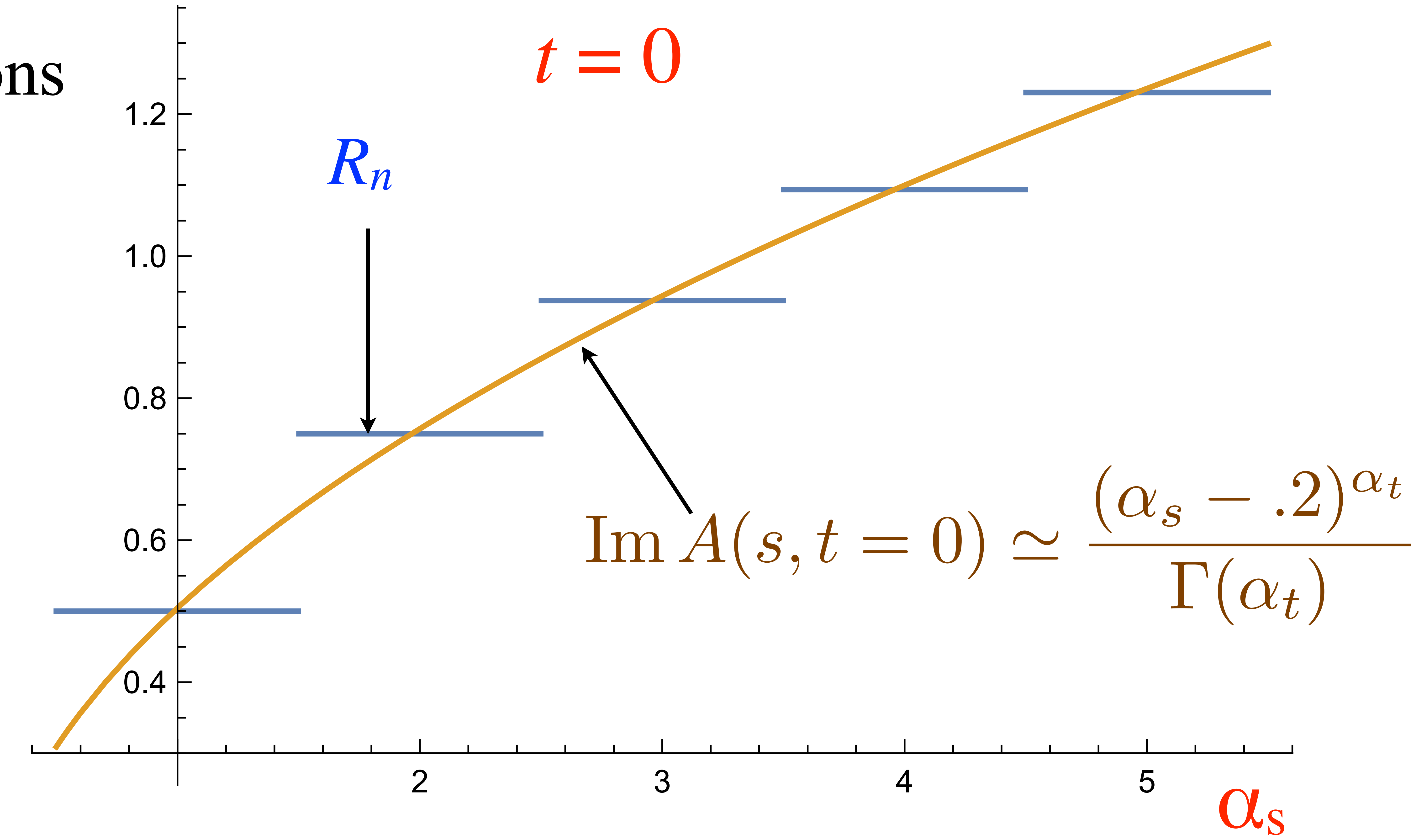
The π+π- → π+π- dual amplitude A(s,t)

$$A(s,t) = \frac{R_n(\alpha_t)}{\alpha_s - n} + \ldots$$

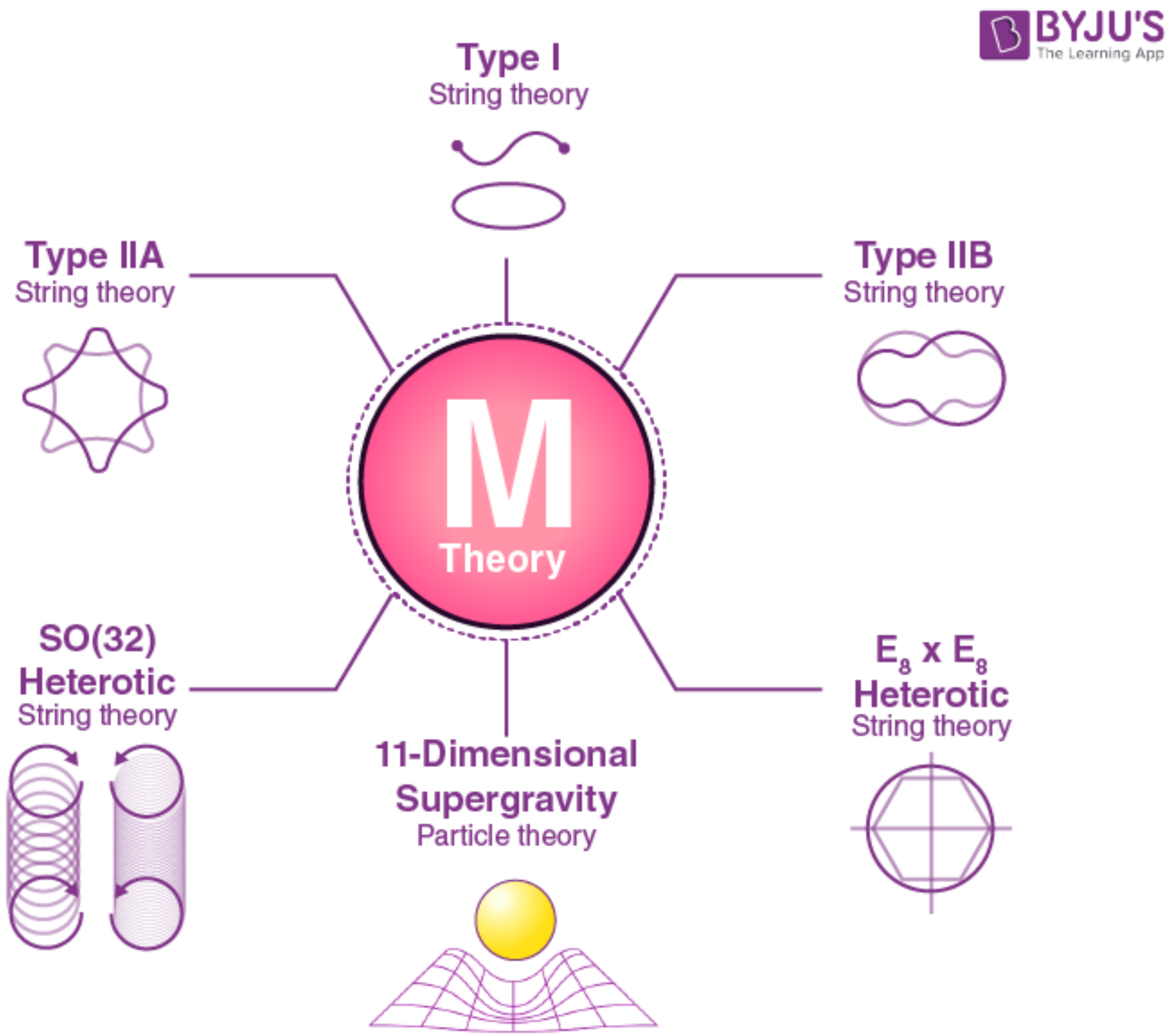
Resonances vs Regge
in forward scattering

Resonance contributions smeared over $\alpha_s \pm 0.5$

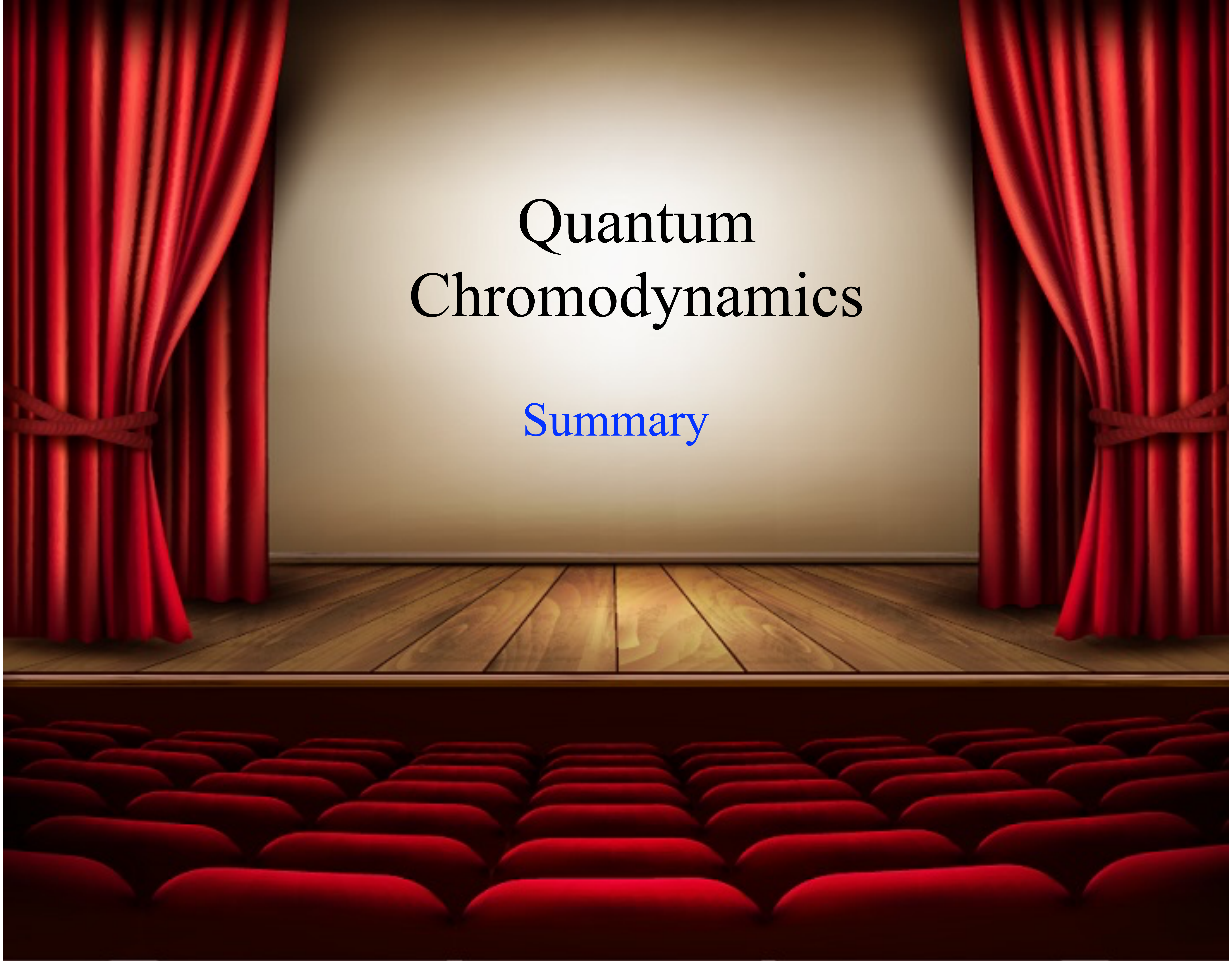
(m_π=0)



The Veneziano model morphed into String Theory...



while duality in hadron physics is waiting for a QCD explanation



Quantum Chromodynamics

Summary

Take-home messages

QCD is the theory of the strong interactions

Theoretically self-consistent
Lagrangian verified by hard scattering data
Soft features verified using lattice methods

Perturbative methods

(Generalized) parton and hadronization distributions
Cross sections ($gg \rightarrow$ Higgs, BSM physics)
Nuclear targets (shadowing, saturation)
High temperature (quark-gluon plasma)

Lattice methods

Confinement and chiral symmetry breaking
Hadron masses, form factors, …
Strong coupling α_s

New methods

Experimental facilities
Theoretical developments

QCD is a remarkable theory, and much remains to be explored!

Perspective: The divisibility of matter

One has wondered since ancient times whether matter can be divided into smaller parts *ad infinitum*, or whether there is a smallest constituent.

Democritus, ~ 400 BC; Vaisheshika school

Common sense suggest that these are the only possibilities, but

Nature has provided other alternatives.

Quantum mechanics shows that atoms (or molecules) are the identical smallest constituents of a given substance,

– yet they can be taken apart into electrons, protons and neutrons.

Hadron physics gives a new twist to this age-old puzzle: Quarks can be removed from the proton, but cannot be isolated. Relativity – the creation of matter from energy – is the new feature which makes this possible.

We are fortunate to be here to study – and hopefully develop an understanding of – this essentially novel phenomenon!