Fundação para a Ciência e a Tecnologia

LABORATÓRIO DE INSTRUMENTAÇÃO E FÍSICA EXPERIMENTAL DE PARTÍCULAS

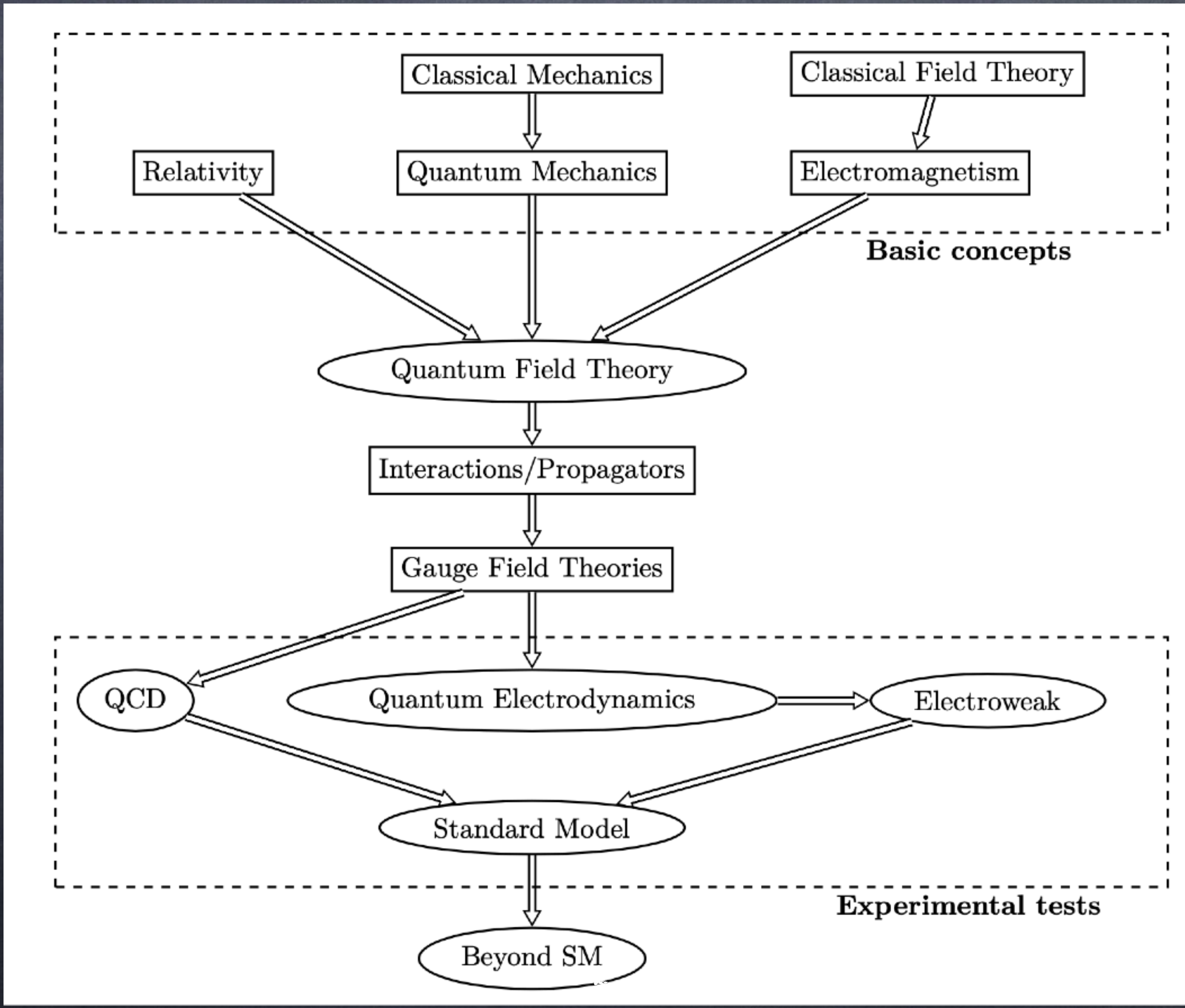
# BFKL and low-x physics
# Lecture 1

## (25-06-2024)

Grigorios Chachamis, LIP Lisbon

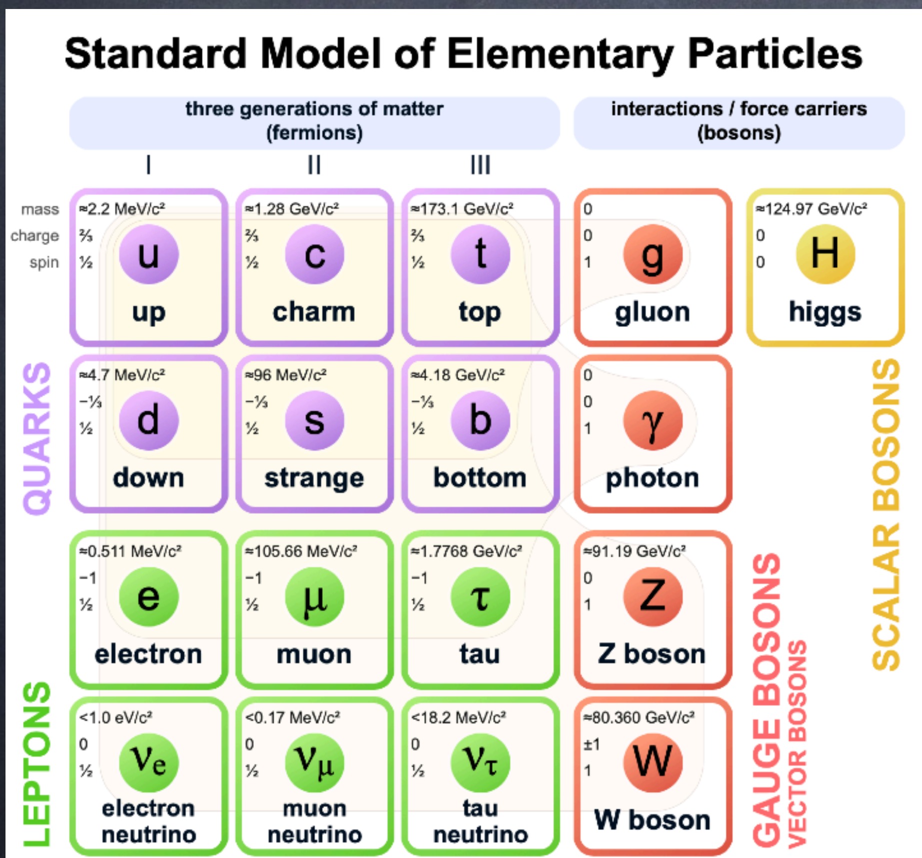
Midsummer School in QCD 2024
Saariselkä, 24 June to 6 July 2024, Finland

# Cornerstones in modern particle physics




A. Rubbia, "Phenomenology of Particle Physics"

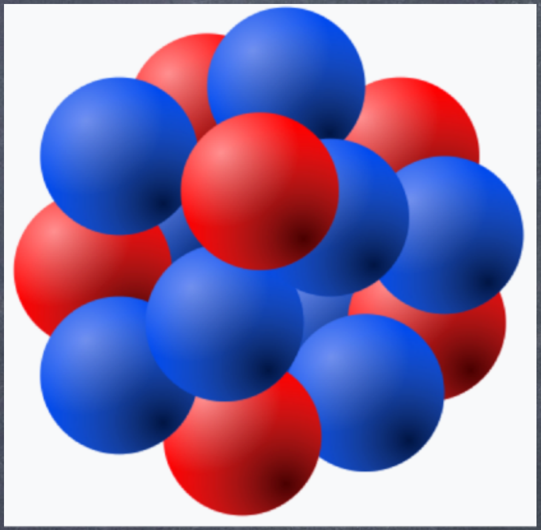
# The Standard Model (SM)




$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
& + i \bar{\psi} \not{D} \psi + h.c. \\
& + \psi_i y_{ij} \psi_j \phi + h.c. \\
& + |D_\mu \phi|^2 - V(\phi)
\end{aligned}
$$

# The strong interaction

- The Strong interaction binds together protons and neutrons in the atomic nuclei
- Back to 1930s: what holds the nucleus together? After all, the positively charged protons should repel one another violently. There must be some other force, more powerful than the force of electrical repulsion, that binds the protons (and neutrons) together. The new force was labeled the strong force



Yukawa: a first attempt at a theory of the strong force, with a new quanta, the meson. The "screened Coulomb potential":

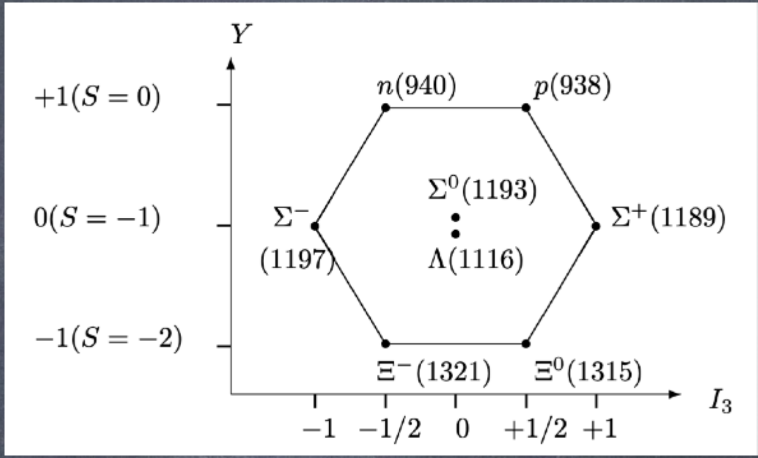
$$U(r) \sim g^2 \frac{e^{-\lambda r}}{r}$$

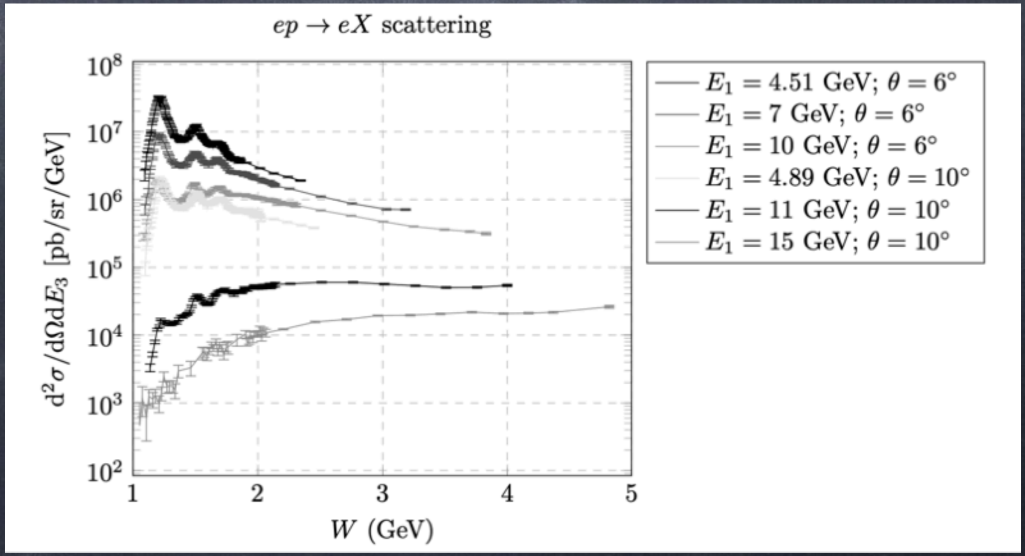
# The strong interaction – early attempts

- In the 1950s and 1960s, lots of new hadrons were discovered, that is, particles that feel the strong interaction.
- Various theoretical attempts to describe the interactions of hadrons:
- S-matrix theory: A framework that focused on the scattering amplitudes of particles, without specifying the underlying dynamics or fields. Unitarity, analyticity, and symmetry.
- Regge theory: A method that used complex angular momentum to analyze the high-energy behavior of scattering processes. It predicted the existence of families of particles, called Regge trajectories, that have linear relationships between their spin and mass squared.
- Bootstrap theory: This was a hypothesis that all hadrons are composed of each other, and that there are no elementary constituents. All particles are resonances of each other.

# The strong interaction – towards QCD

- In the 1950s and 1960s, lots of new hadrons were discovered, that is, particles that feel the strong interaction.
- Attempts to classify these into families were very successful (Eightfold Way) and led to the conclusion that hadrons (protons, neutrons, pions, kaons etc) were not fundamental particles.
- Instead, they are composite, their constituents were named quarks (partons). Another indication of their compositeness were results from deep inelastic e-p scattering experiments



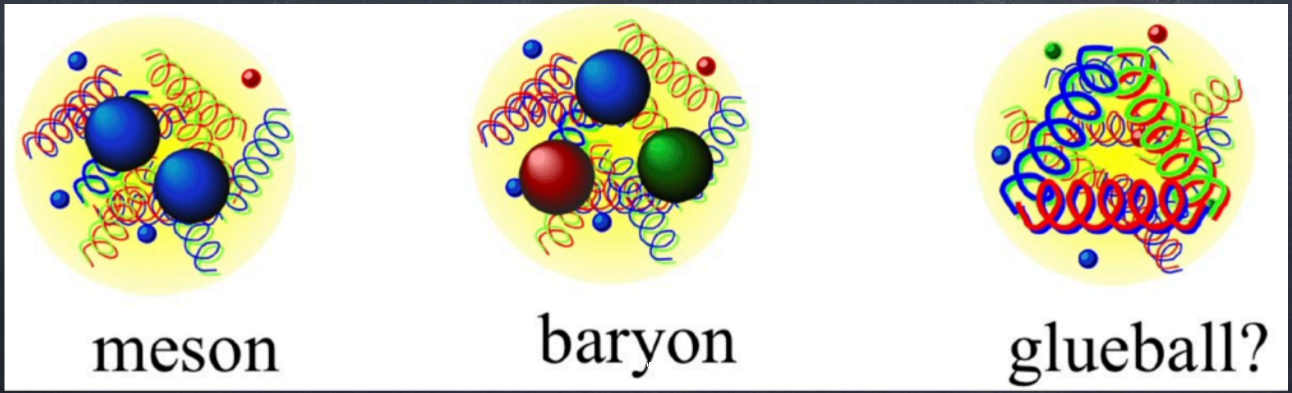



# The strong interaction – the advent of QCD

- In the early 1970s, QCD was formulated as a non-Abelian gauge field theory of quarks that interact by the exchange of spin-1 massless gluons

- QCD is in many aspects similar to Quantum electrodynamics (QED). The QED vertex couples a spin-1 photon with zero rest mass to an electrically charged fermion. In QCD, the strong force is mediated by eight (massless) spin-1 gluons. Quarks carry the "color charge," which comes in three types called R(ed), G(reen) and B(lue).

- Since gluons are colored and therefore carry a strong charge, they interact with other gluons or "self-couple" via QCD vertices involving three or four gluons (non-Abelian).



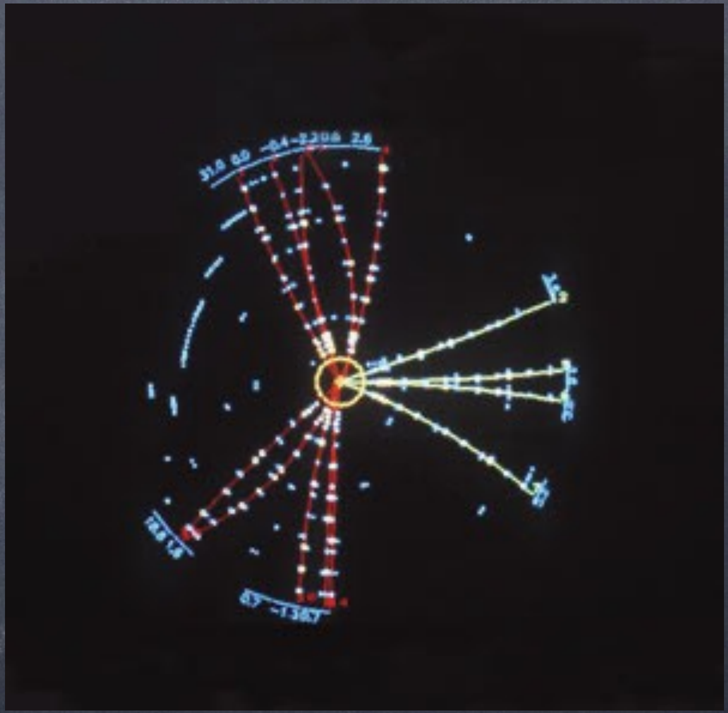
# The advent of QCD, two key theoretical achievements

- 't Hooft & Veltman, showed in 1971 that "non-abelian" gauge theories are renormalizable, a type of quantum field theory that can be made free of infinities by a finite number of redefinitions of the parameters and fields.
- Asymptotic freedom in QCD was discovered in 1973 by David Gross and Frank Wilczek, and independently by David Politzer in the same year. The interactions between quarks and gluons become weaker as the energy scale increases and the distance scale decreases. At low energies, we have confinement: the interactions become stronger and quarks and gluons are confined within hadrons, such as protons and neutrons.

# First evidence of the gluon

- In 1979, experiments at the DESY laboratory in Germany provided the first direct proof of the existence of gluons - the carriers of the strong force that "glue" quarks into protons, neutrons and the other particles known collectively as hadrons. This discovery was a milestone in the history of particle physics, as it helped establish QCD as the theory of the strong interaction.

- "The results followed from an idea that struck theorist John Ellis while walking in CERN's corridors in 1976. As Ellis recounts, he was walking over the bridge from the CERN cafeteria back to his office, turning the corner by the library, when it occurred to him that "the simplest experimental situation to search directly for the gluon would be through production via bremsstrahlung in electron-positron annihilation". In this process, an electron and a positron (the electron's antiparticle) would annihilate and would occasionally produce three "jets" of particles, one of which being generated by a gluon radiated by a quark-antiquark pair." CERN News, 18 JUNE, 2019



TASSO detector at DESY

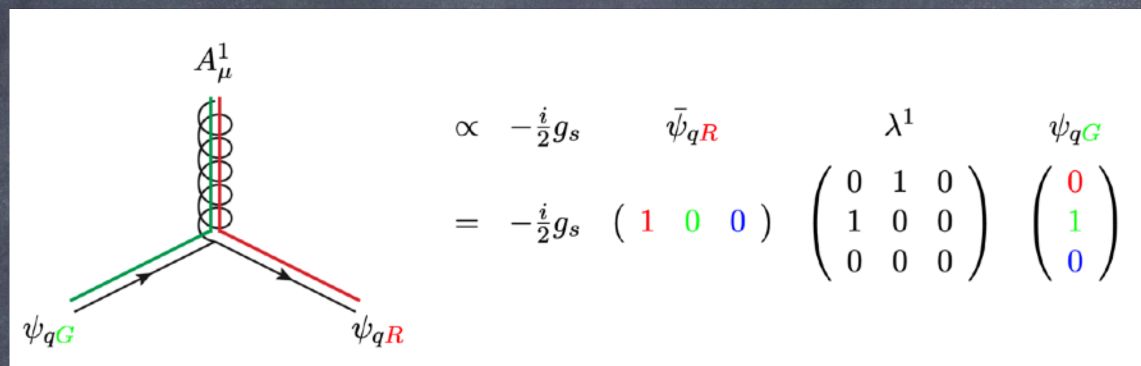
# The QCD Lagrangian

$$\mathcal{L}_{QCD} = \sum_q \left( \bar{\psi}_{qi} i\gamma^\mu \left[ \delta_{ij}\partial_\mu + ig\left(G^\alpha_\mu t_\alpha\right)_{ij} \right] \psi_{qj} - m_q \bar{\psi}_{qi}\psi_{qi} \right) - \frac{1}{4} G^\alpha_{\mu\nu} G^{\mu\nu}_\alpha$$

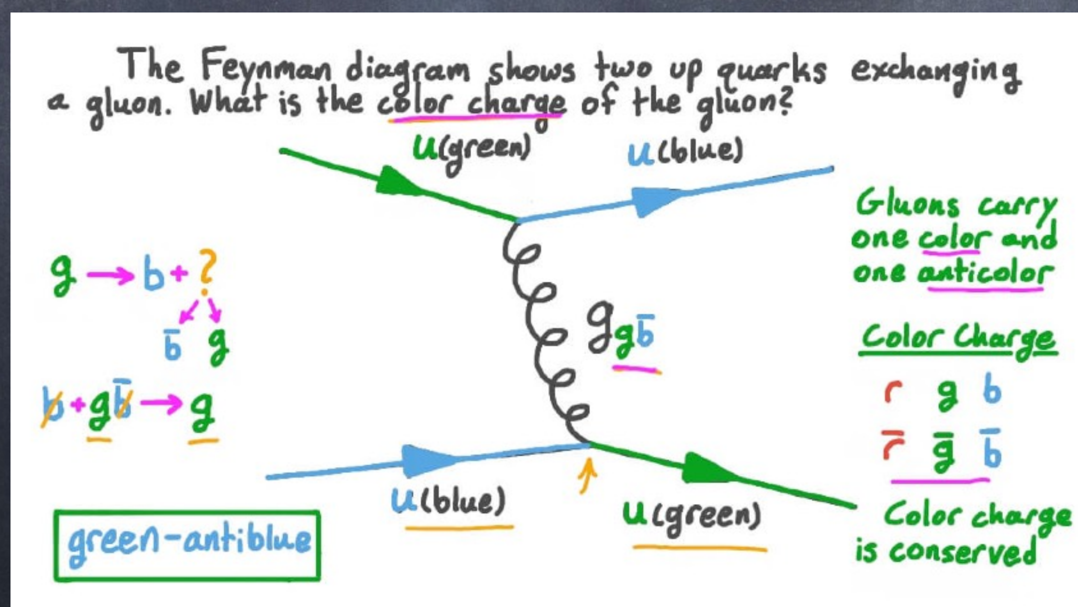
$$\mathcal{L}_{QED} = \bar{\psi}_e i\gamma^\mu \left[ \partial_\mu + ieA_\mu \right] \psi_e - m_e \bar{\psi}_e \psi_e - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- $G^{\mu\nu}_\alpha = \partial^\mu G^\nu_\alpha - \partial^\nu G^\mu_\alpha - g f^{\alpha\beta\gamma} G^\mu_\beta G^\nu_\gamma$ color fields tensor
- $G^\mu_\alpha$ four potential of the gluon fields (α=1,..8)
- $t_\alpha$ 3x3 Gell-Mann matrices; generators of the SU(3) color group
- $f^{\alpha\beta\gamma}$ structure constants of the SU(3) color group
- $\psi_i$ Dirac spinor of the quark field (*i* represents color)
- $g = \sqrt{4\pi\alpha_s}$ $(\hbar = c = 1)$ color charge (strong coupling constant)

# Perturbation theory – Feynman diagrams




P. Skands, arXiv:1207.2389



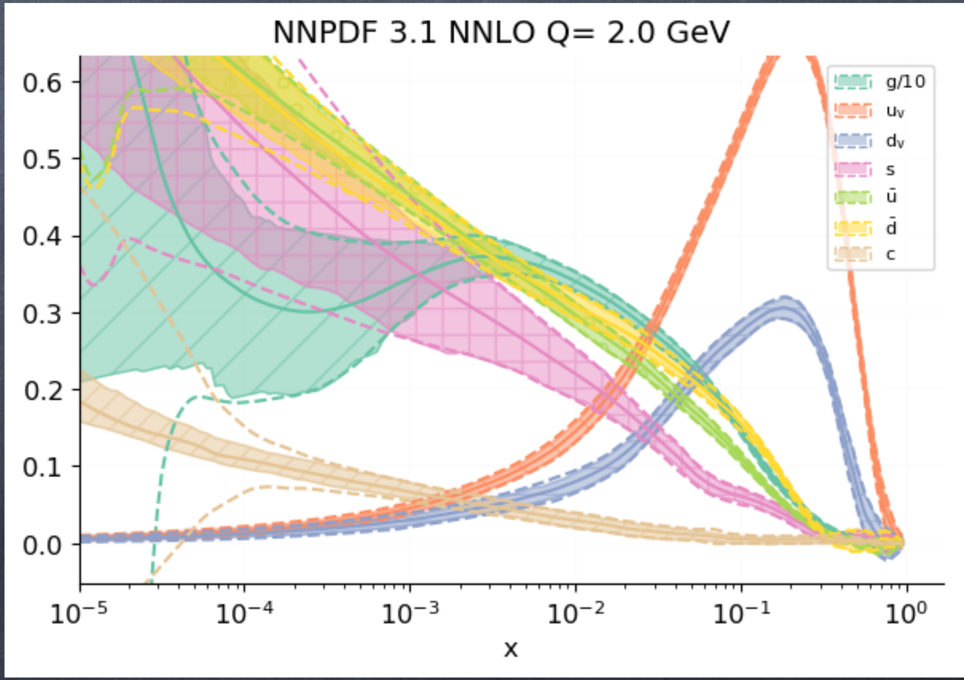
# QCD phenomenology

- Nowadays, QCD is the established theory of the strong interaction. It describes how quarks and gluons interact to form hadrons, such as protons and neutrons and how hadrons interact with each other.
- QCD phenomenology is the study of the observable consequences of QCD, such as the production and decay of hadrons, the structure of nucleons, and the properties of nuclear matter. Most of the work within QCD phenomenology is connected to collider experiments.

# QCD phenomenology

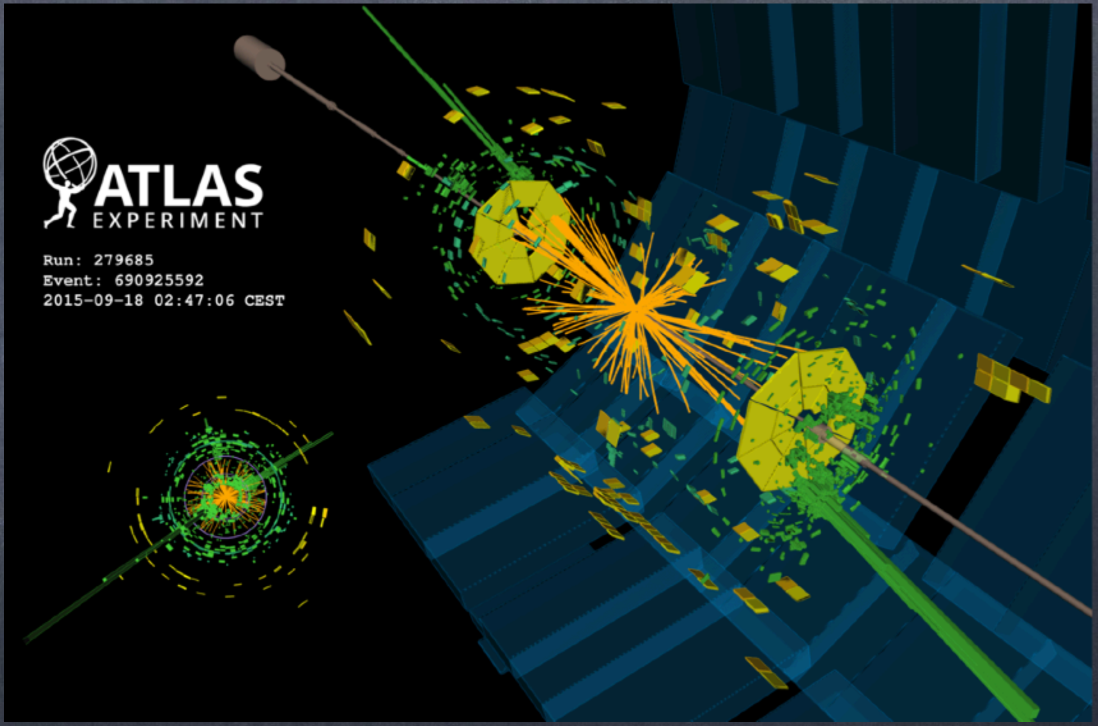
- Some of the main areas of QCD phenomenology are:

- Parton distribution functions (PDFs): These are the probability densities of finding a quark or a gluon with a given fraction of the momentum of a hadron, such as a proton or a neutron. PDFs are essential for predicting the cross sections of hard processes involving hadrons at high energies, such as deep inelastic scattering or hadron collisions.



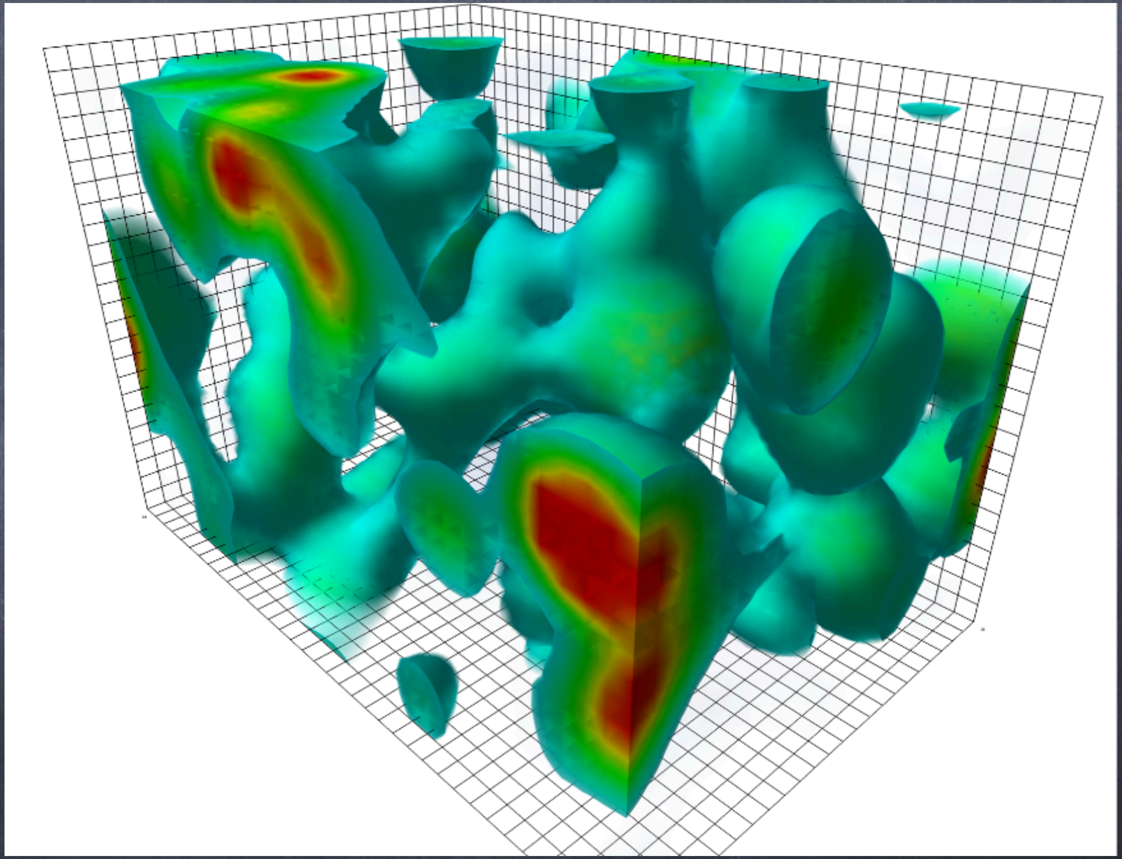
# QCD phenomenology

- Some of the main areas of QCD phenomenology are:
- Jet physics: Jets are collimated sprays of hadrons that originate from the fragmentation and hadronization of high-energy quarks and gluons. Jet physics is important for testing the perturbative aspects of QCD and for probing the structure of the proton and the nucleus.



# QCD phenomenology

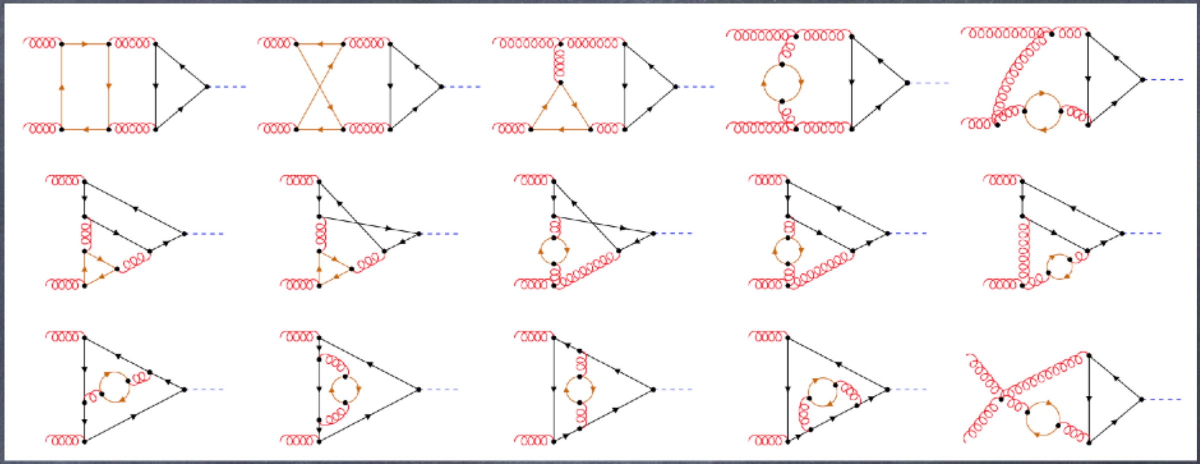
- Some of the main areas of QCD phenomenology are:
- Lattice QCD: Lattice QCD is a numerical method for solving the QCD equations of motion on a discrete space-time grid. Lattice QCD allows for the non-perturbative calculation of various QCD observables, such as hadron masses, decay constants, form factors, and phase transitions.



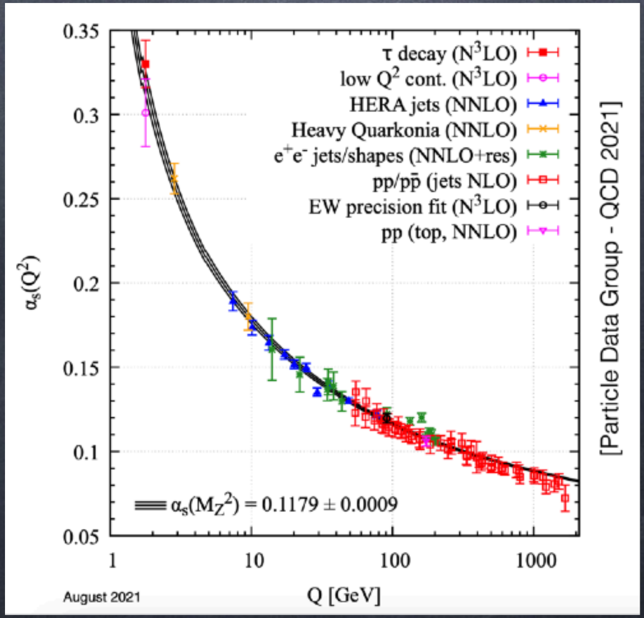
http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/ImprovedOperators/index.html

# QCD phenomenology

- Some of the main areas of QCD phenomenology are:

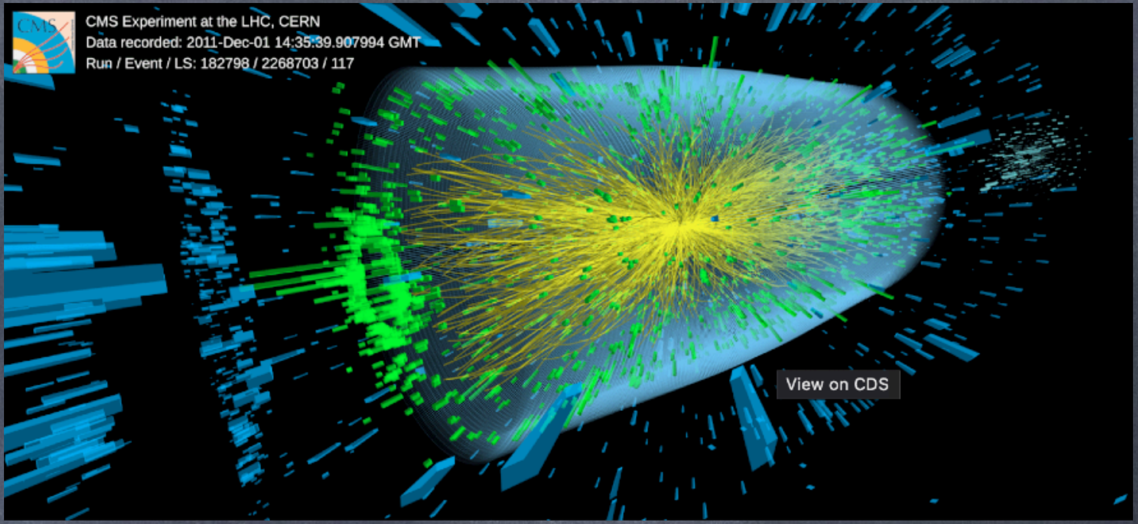


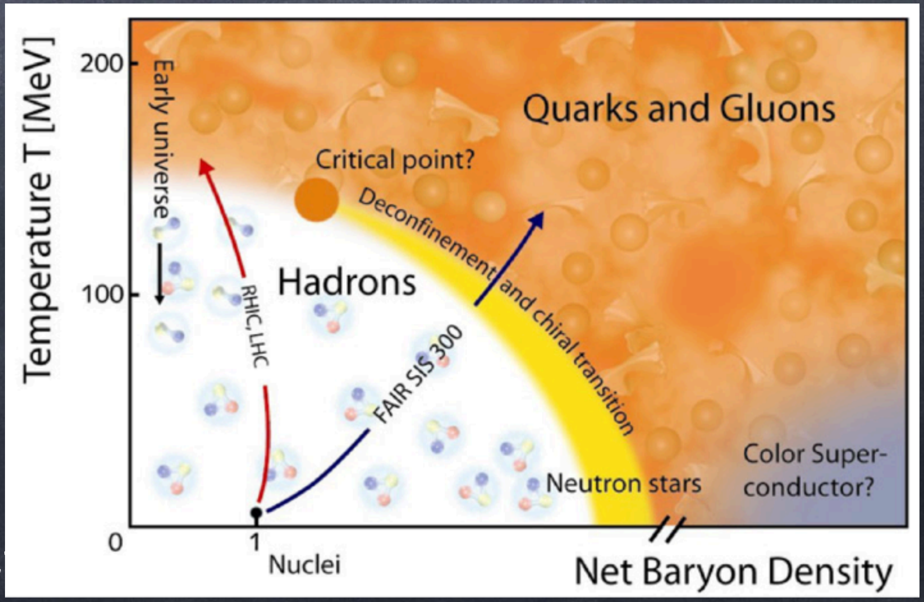
- Precision physics: the study of the effects of quantum corrections and higher-order processes on the observables measured at high-energy particle colliders. Main goal the improvement of the theoretical predictions and experimental measurements of the strong interactions, such as the strong coupling constant, the parton distribution functions, the jet cross sections. This can reduce the uncertainties and enhance the sensitivity of the collider searches and analyses.



# QCD phenomenology

- Some of the main areas of QCD phenomenology are:
- Heavy ion collisions physics: explores the phase diagram of QCD matter and the formation of a new state of matter called quark-gluon plasma, which is believed to exist at extremely high temperatures and densities



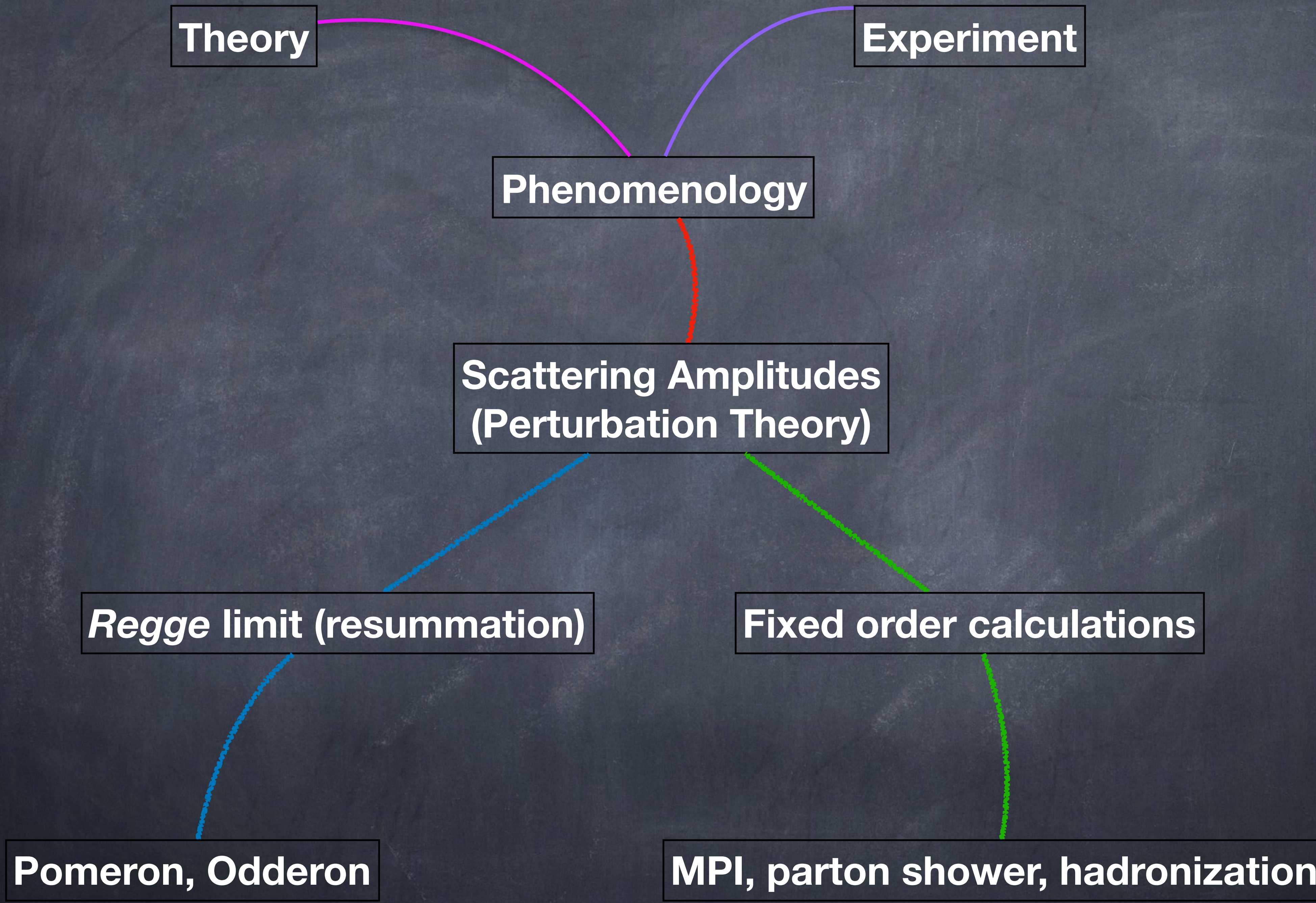



# QCD phenomenology

- Soft and hard diffraction and Forward physics.
- QCD phenomenology is a rich and active field of research that aims to understand the fundamental nature of the strong force and its manifestations in nature.
- It is one of the most exciting and demanding fields of research in particle physics.

# Key concepts

- Phenomenology
- Kinematics vs symmetry vs dynamics
- Scattering amplitudes
- Perturbation theory


Theory
Experiment
Phenomenology
Scattering Amplitudes (Perturbation Theory)
*Regge* limit (resummation)
Fixed order calculations
Pomeron, Odderon
MPI, parton shower, hadronization

# Phenomenology

- Philosophy

**Phenomenology (from Greek: phenomenon = "that which appears" and logos = "study")** is the philosophical study of the structures of subjective experience and consciousness.

- Science in general

Observe "that which appears", a collection of phenomena that share a unifying principle, and try to find patterns to describe it. The patterns might or might not be of fundamental nature or they might be up to a certain degree.

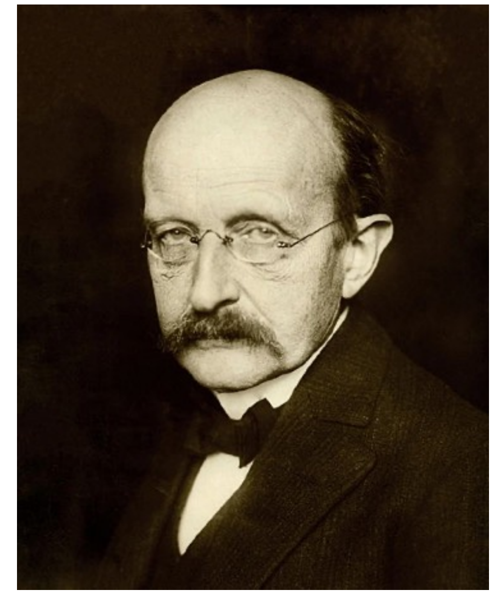
- Particle Physics (our familiar SM phenomenology, for example)

Use assumed fundamental laws to produce theoretical predictions for physical observables and then compare against experimental data to validate or falsify the assumed laws.

**Extremely important the close collaboration between theorists and experimentalists.**

# A great example: Planck's Law

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{k_\mathrm{B}T}} - 1}$$



Describes the spectral density of electromagnetic radiation emitted by a black body in thermal equilibrium at a given temperature T

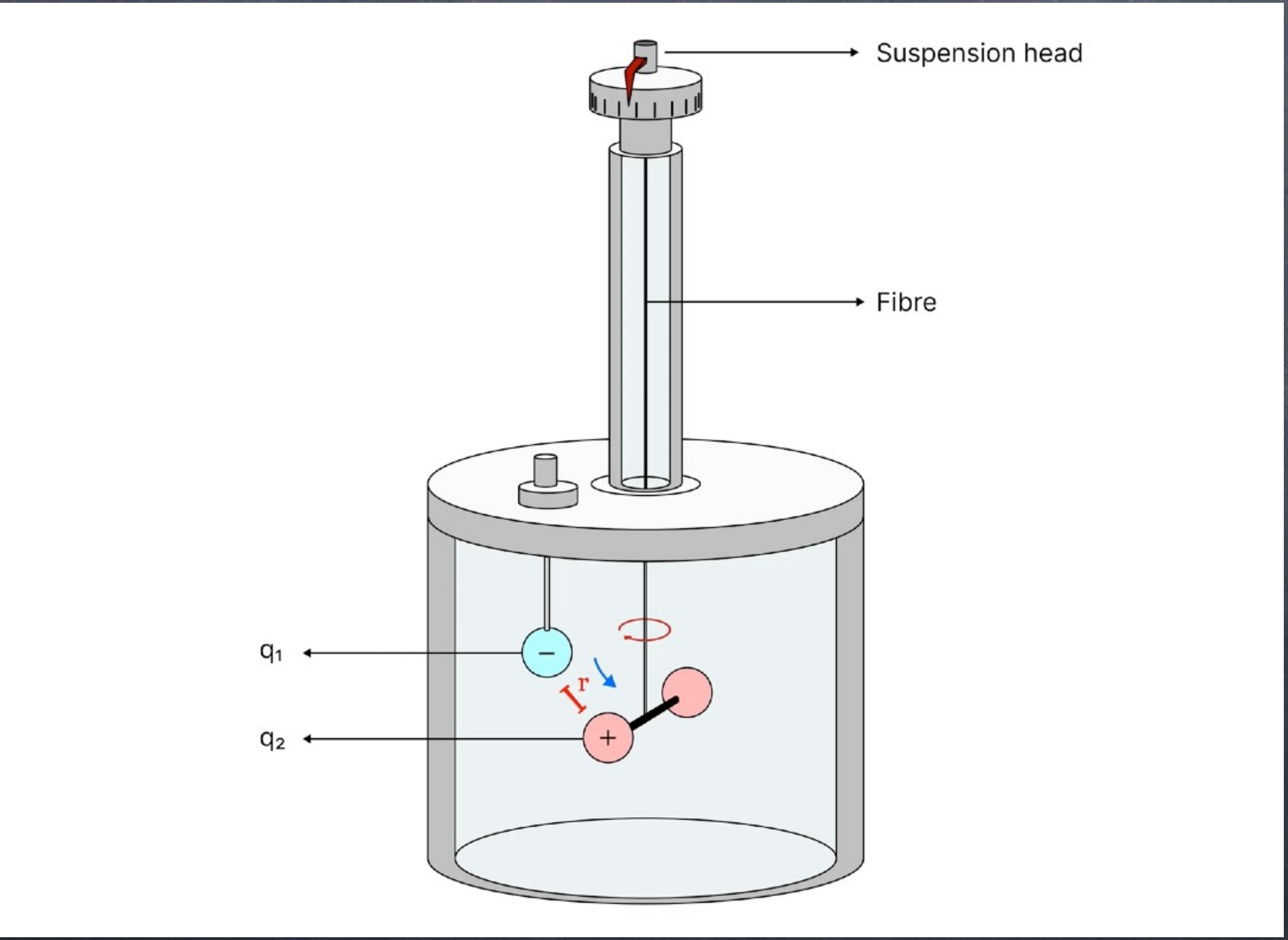
$h$ is the Planck constant… fundamental importance for quantum mechanics

Symmetry

Kinematics

Dynamics

# Coulomb's Law



# Scattering amplitudes

**Experimentally: take particles and smash them against each other with huge velocities**

- It is the main way we have to test our theories against real life (or vice versa)
- The trend is to go to higher colliding energies to see more (probe deeper the internal structure)
- Maybe there is a better way but for now we do not know it

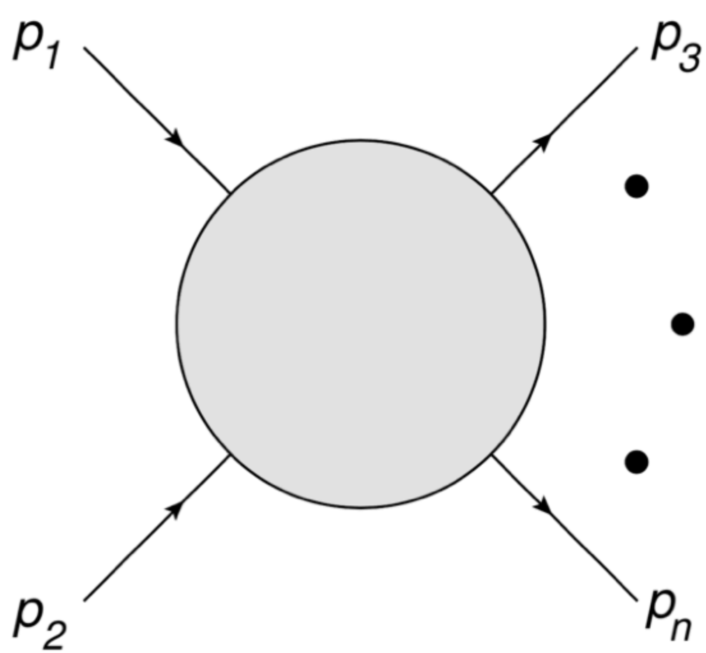
# The central role of scattering amplitudes in modern particle physics

From the excellent recent article by C. White "Aspects of High Energy Scattering" (1909.05177), we quote a few reasons below of the **importance of studying scattering amplitudes**:

- "What physical behaviour occurs in a given theory?" ★
- "What mathematical structures can amplitudes contain?"
- "Can we find common languages, that make e.g. QCD and gravity look the same?"

**Perturbation theory**

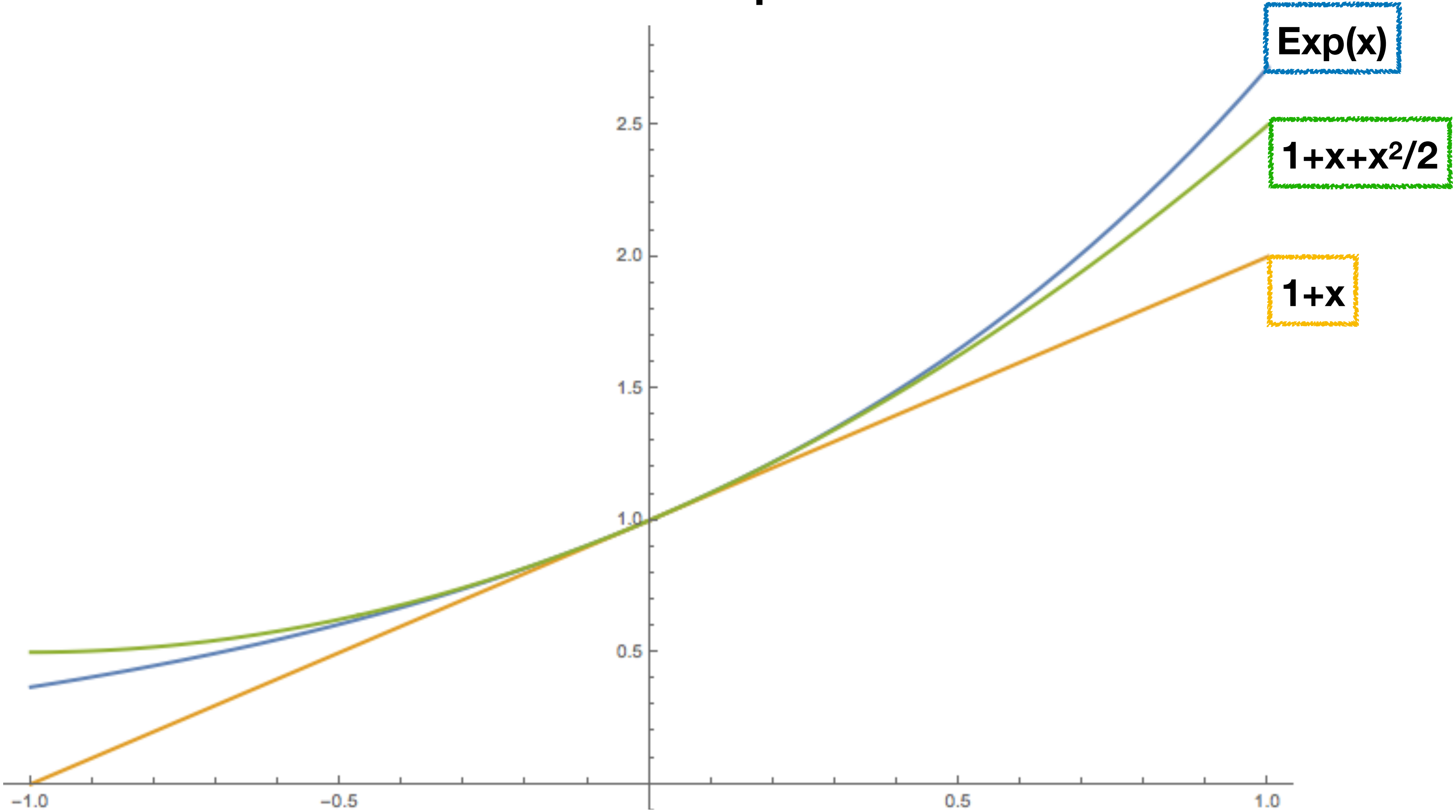
**Feynman Calculus**




★ **Go at a certain limit of the theory**

What does this diagram tell us?

# Fixed order expansion



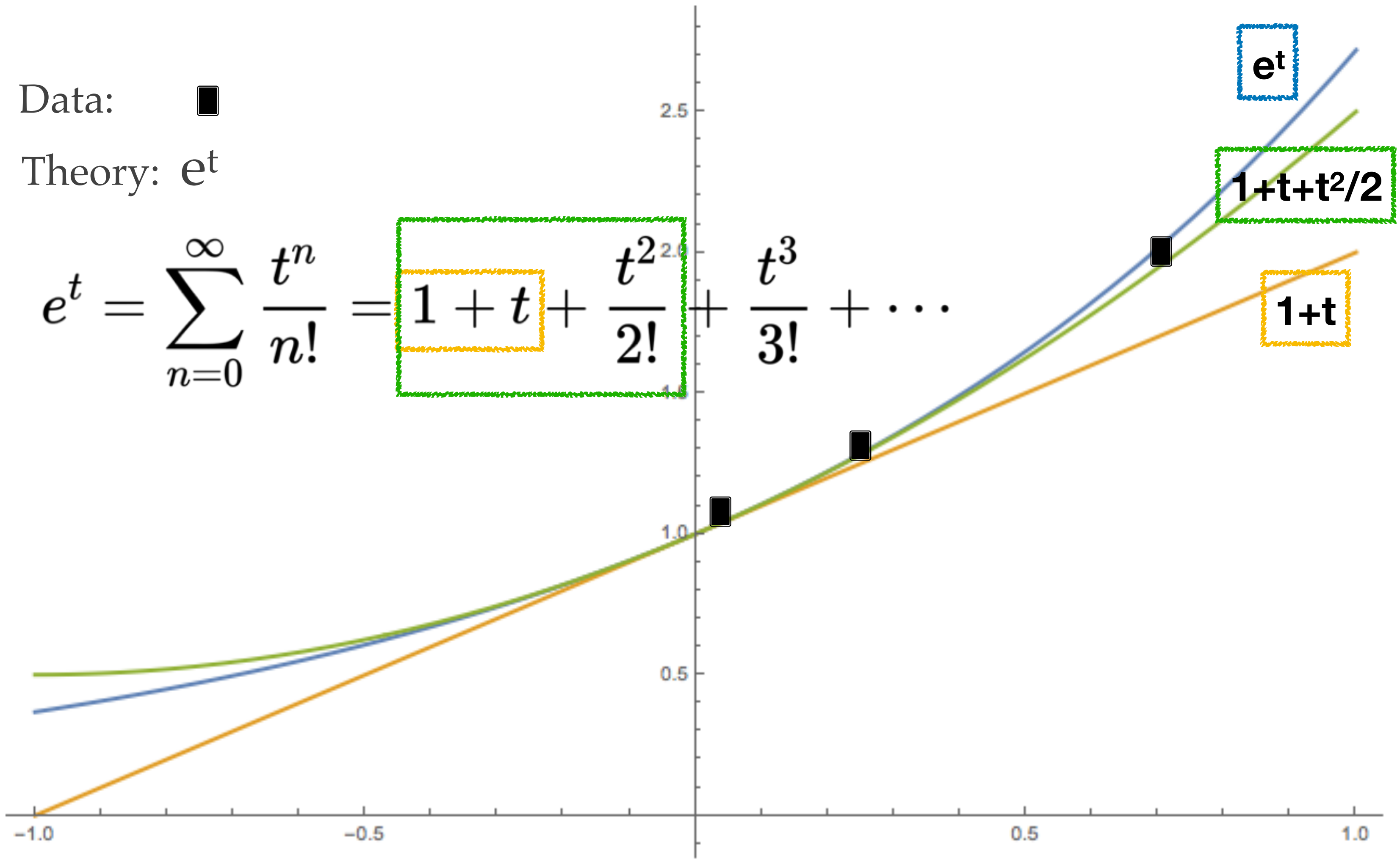

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

# Perturbation theory

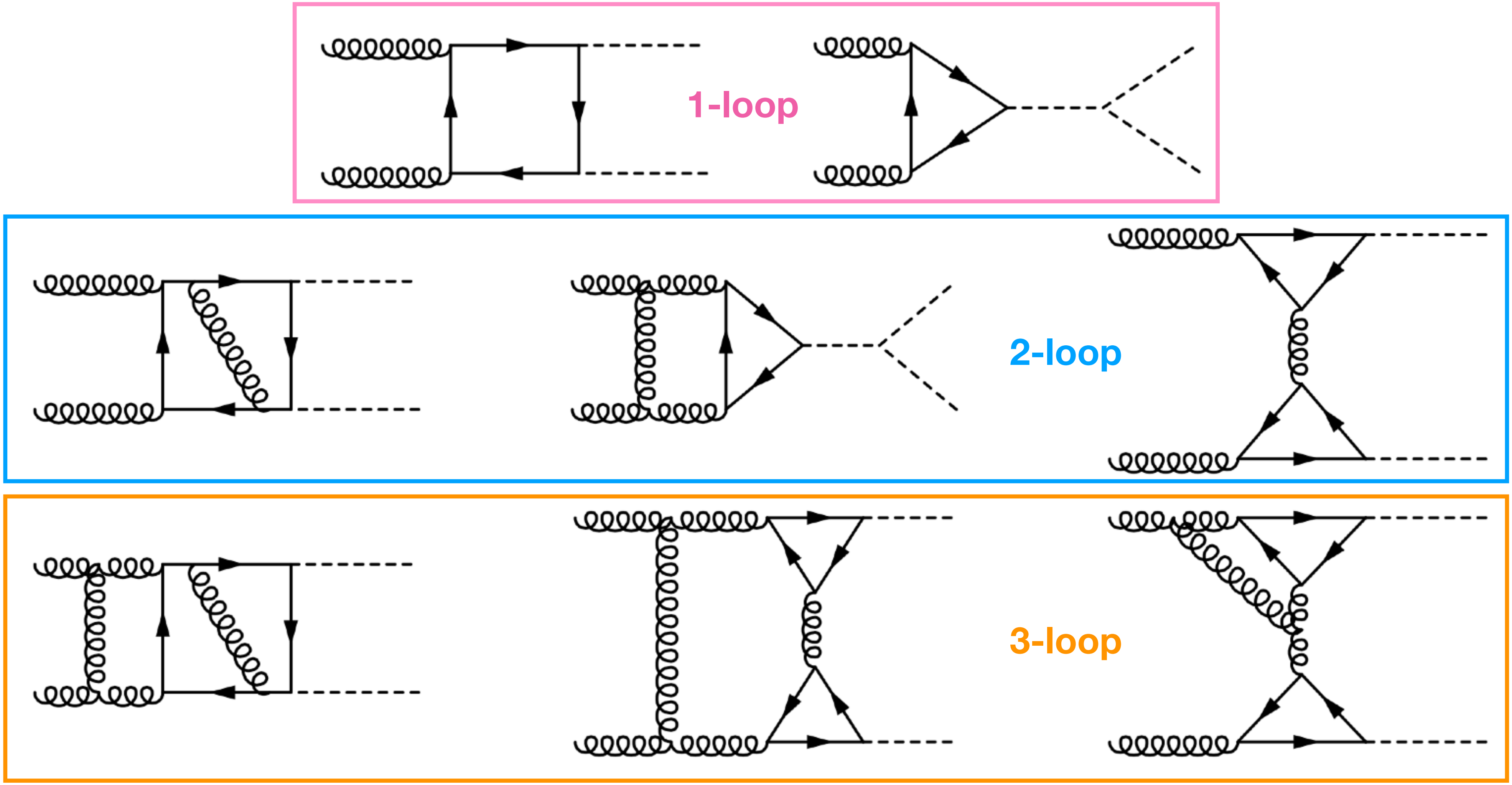
Data:

Theory: $e^t$

$$e^t = \sum_{n=0}^{\infty} \frac{t^n}{n!} = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots$$



# Perturbative expansion (fixed order) in $\alpha_s$

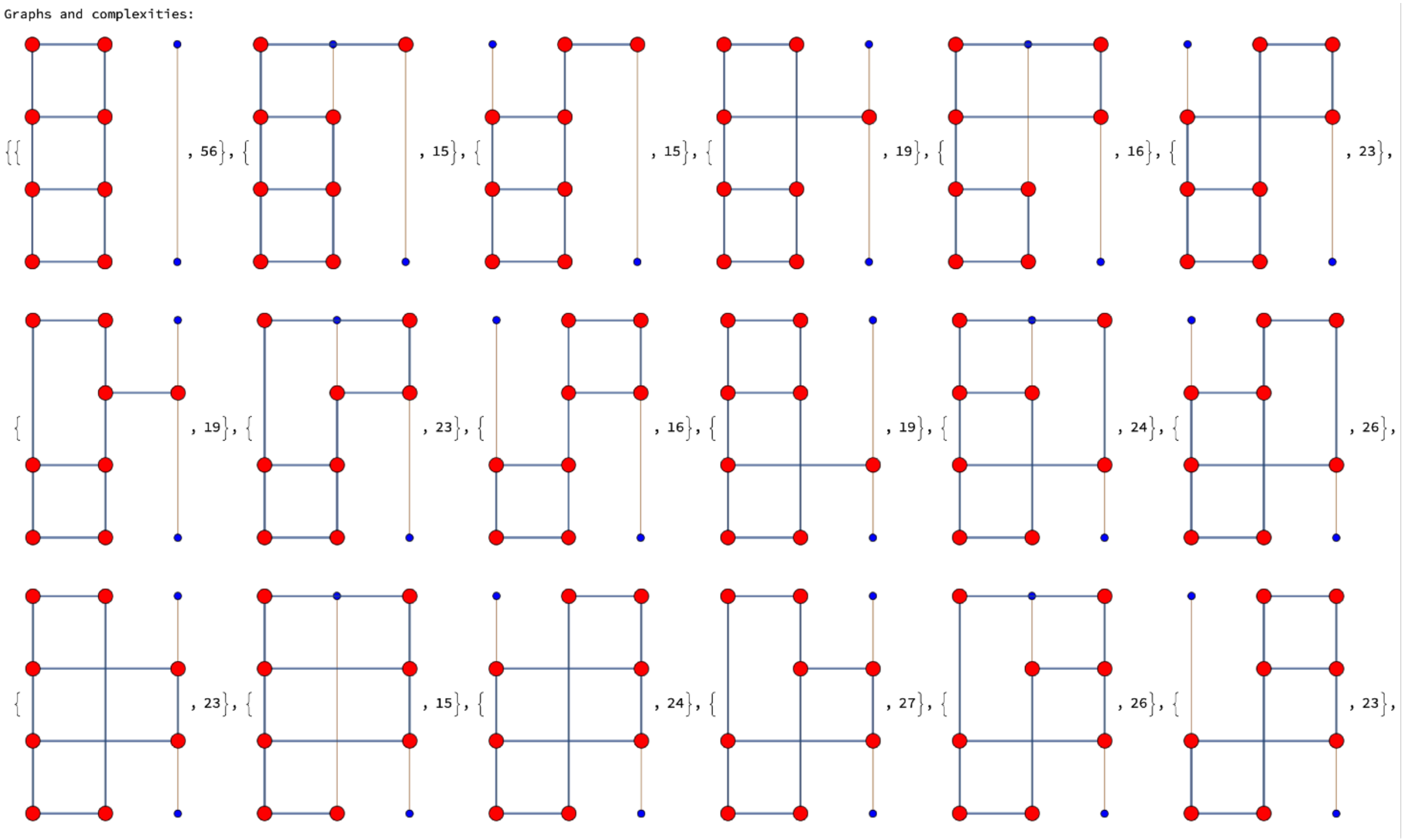



$gg \to HH$

Grigo, Hoff, Steinhauser, 2015

**NOTE: at the end, we study the properties of the expanded amplitudes**

# The need for resummation



# The high energy or *Regge* limit (resummation)

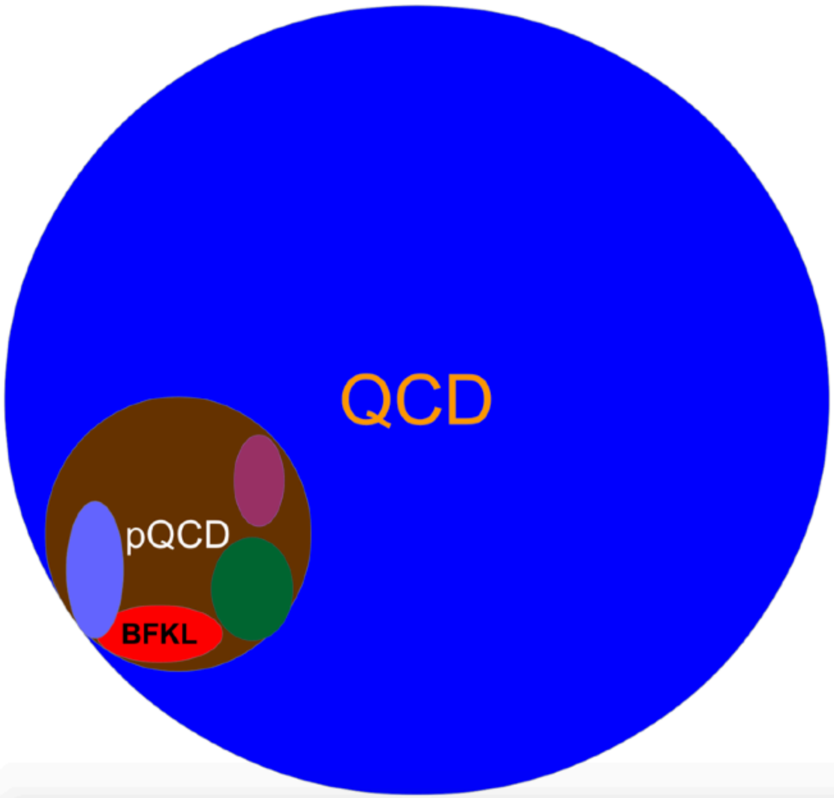
$$s \gg -t \gg m^2$$

There is a plethora or things we access from studying that limit:

- Integrability
- Gravity, black holes
- AdS/CFT
- Bern-Dixon-Smirnov amplitudes
- Factorization
- Separation between transverse and longitudinal d.o.f
- Transition from hard to soft scale physics
- Glueballs
- *Phenomenology*

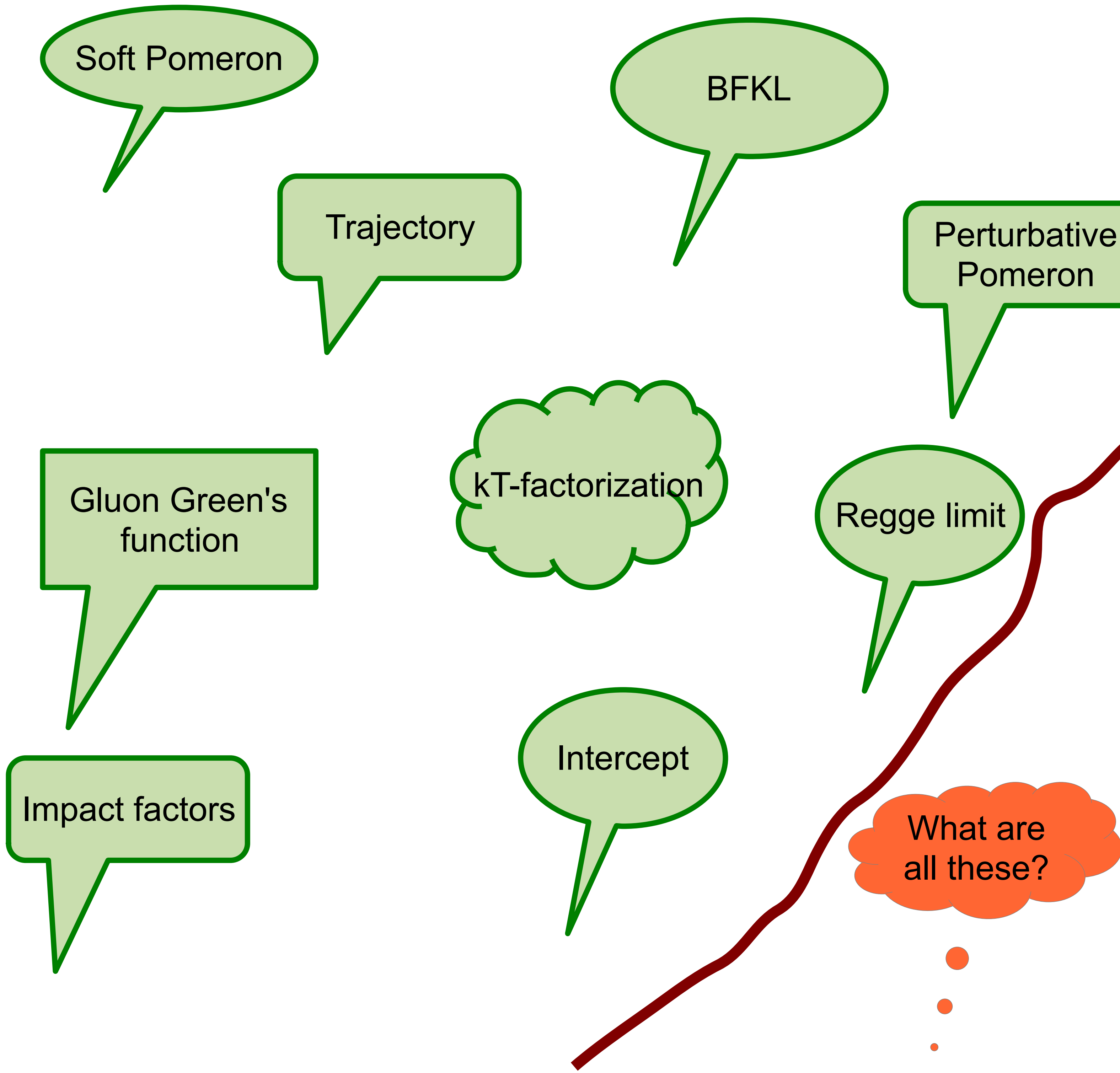
Furthermore, in Mathematics:
number theory, abstract algebra, special functions, …

A crucial tool to study the *Regge* limit is *Balitsky-Fadin-Kuraev-Lipatov* (BFKL) dynamics. In its essence, BFKL resums to all orders diagrams that carry large logarithms in energy. It goes beyond fixed order.



# Terminology

- BFKL
- Pomeron (soft/hard)
- Regge limit
- Regge trajectory
- Intercept
- Gluon Green's function
- Impact factor
- Fixed order calculations VS resummation
- S-Matrix
- Sudakov variables
- Optical theorem
- Unitarity




- The field has lots of internal conventions.
- Particular methods-notation-jargon.
- Different subfields in small-x: BFKL, dipole picture, light cone picture, BK, Color Glass Condensate, saturation...

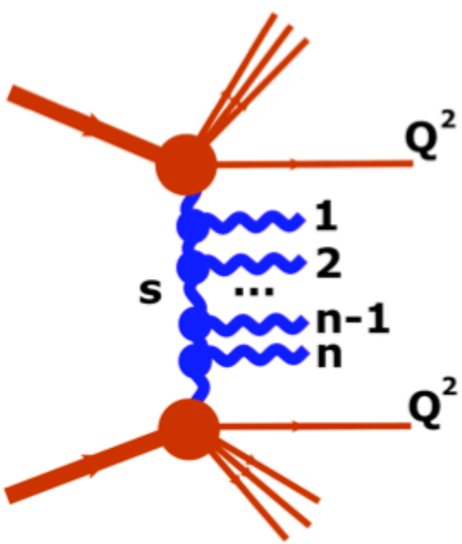
# Why bother?

- Small-x physics studies only a part of the phase space, a certain limit, the limit of scattering at very high energies
- There is a plethora or things though we can learn from studying that limit, to mention but a few:
- - Integrability (The chance to solve QCD exactly)
- - Gravity
- - AdS/CFT
- - BDS amplitudes

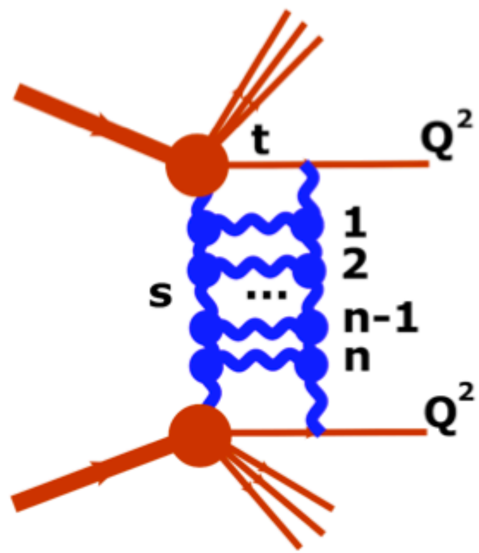
# Why bother?

Rich Phenomenology:

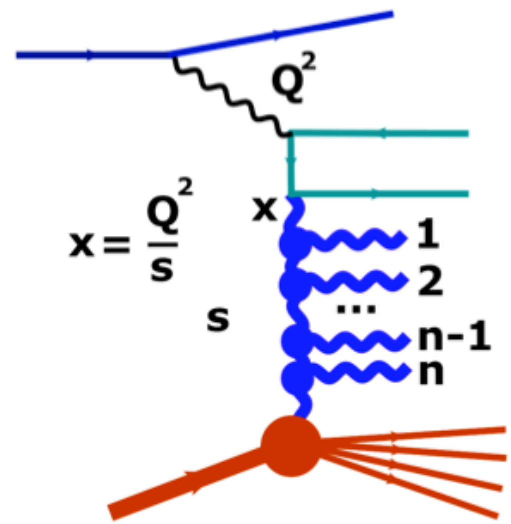
Multijets




Rapidity Gaps



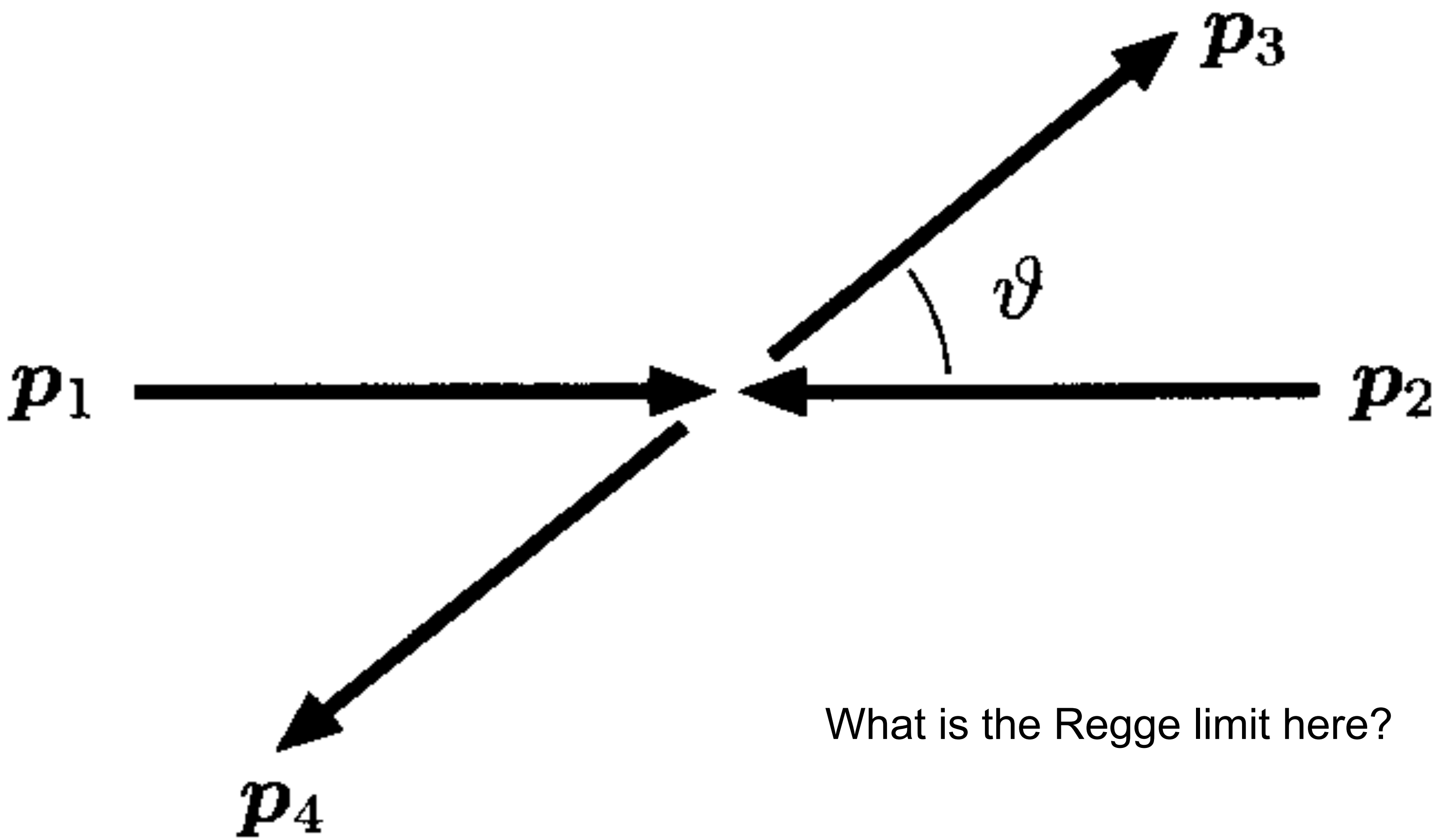

Deep Inelastic Scattering



# List of prerequisites

- S-matrix approach
- Cutkosky rules / Optical theorem (used a lot in the Regge theory but also in BFKL)
- Sudakov parametrization
- Again, which is the kinematical limit we are (which part of the phase space)? Can we picture it? **Regge limit**

# A picture that should be familiar



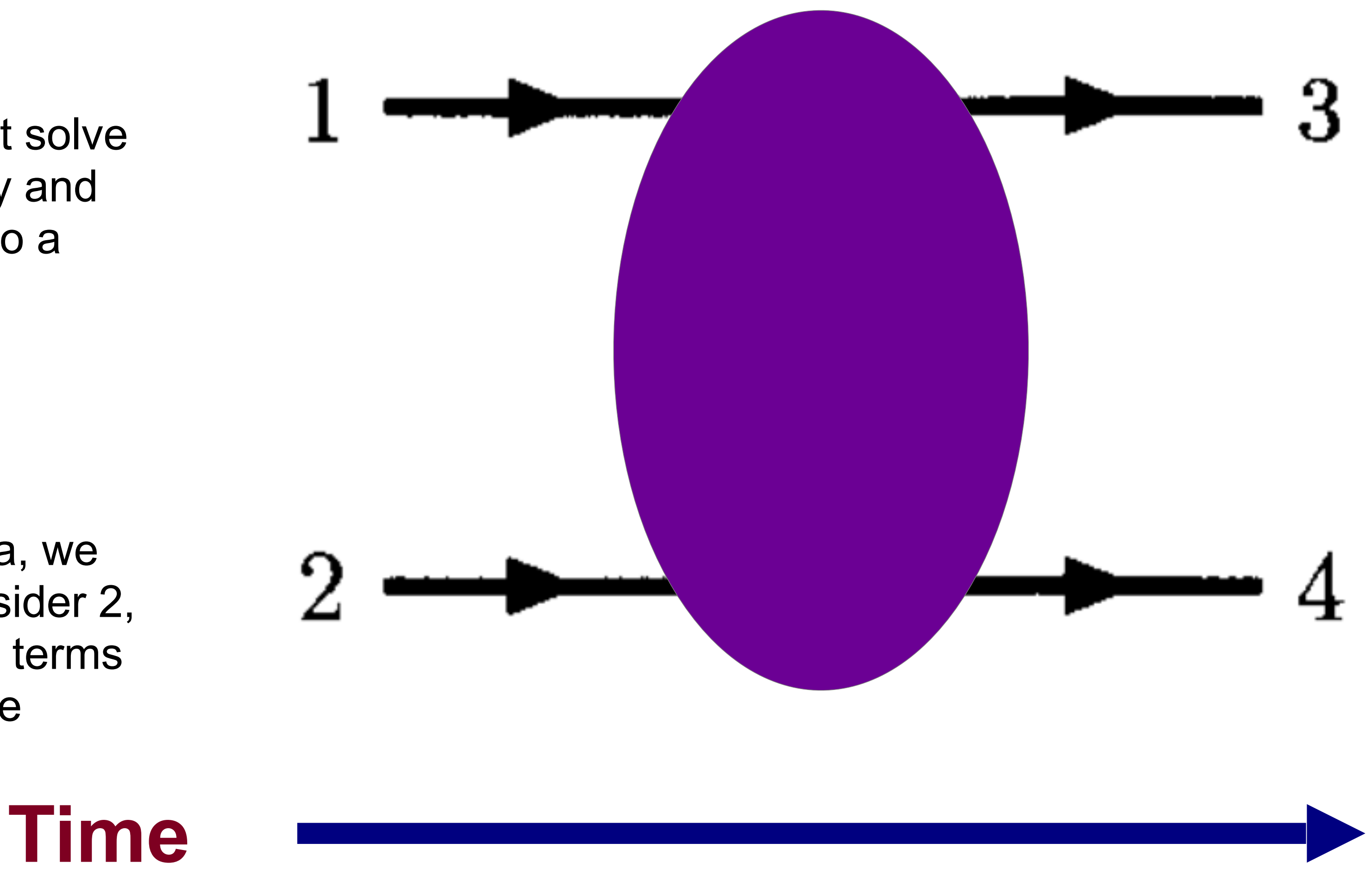

What is the Regge limit here?

# Scattering in our language of Feynman Calculus?

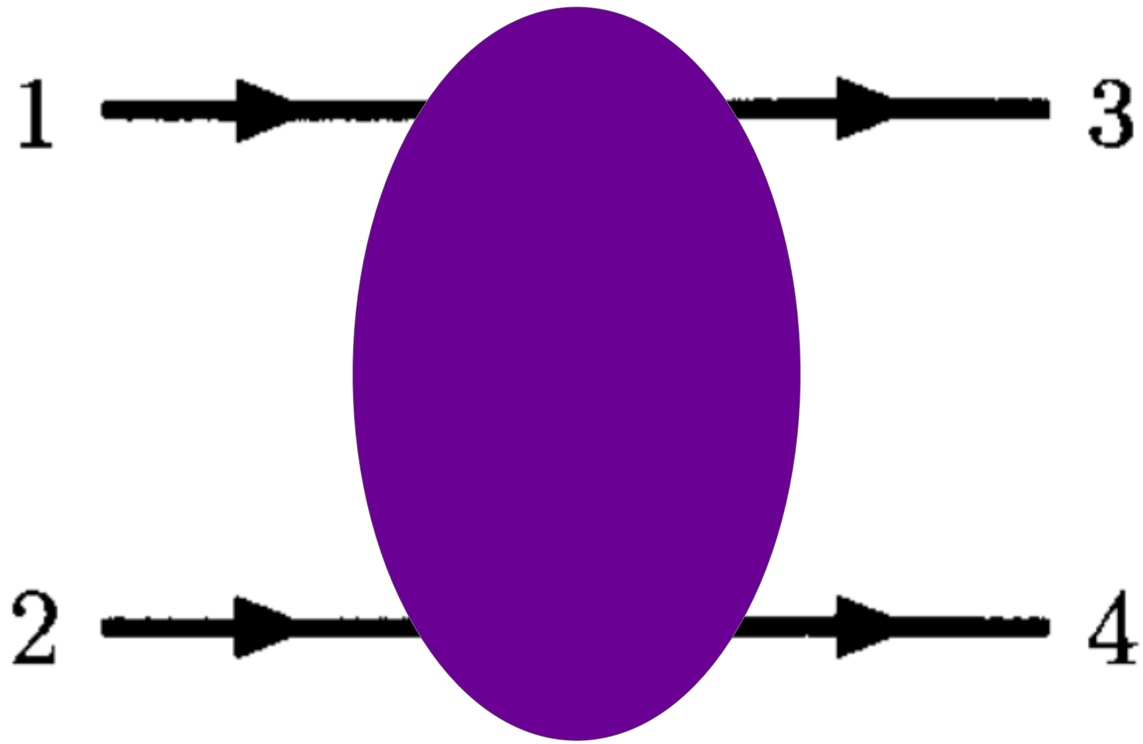
We assume that we have a theory that tells us what is going on inside the blob.

Maybe we cannot solve the theory exactly and then we should do a series expansion around a small parameter.

To compare with experimental data, we may have to consider 2, 3 or maybe more terms in our perturbative expansion



# s, t, u




$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 ,$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2 ,$$
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2 .$$

$$s + t + u = \sum_{i=1}^{4} m_i^2$$

# Sudakov parametrization

$$A^\mu = (A^0, A^1, A^2, A^3) = (A^0, \boldsymbol{A}_\perp, A^3) = (A^0, \boldsymbol{A})$$

$$A^\pm = \frac{1}{\sqrt{2}}(A^0 \pm A^3)$$

$$A^\mu = (A^+, A^-, \boldsymbol{A}_\perp)$$

Light-cone components

$$p^\mu = \frac{1}{\sqrt{2}}(\Lambda, 0, 0, \Lambda), \quad n^\mu = \frac{1}{\sqrt{2}}(\Lambda^{-1}, 0, 0, -\Lambda^{-1})$$

$$p^2 = n^2 = 0, \quad p \cdot n = 1, \quad n^+ = p^- = 0$$

$$A^\mu = \alpha\, p^\mu + \beta\, n^\mu + A_\perp^\mu = (A \cdot n)\, p^\mu + (A \cdot p)\, n^\mu + A_\perp^\mu$$

$$A^2 = 2\,\alpha\beta - \boldsymbol{A}_\perp^2$$

# S-matrix

Before the advent of QCD, the S-matrix approach was what people used to describe hadronic interactions.

The idea was that there is a linear operator called scattering matrix or S-matrix that transforms the initial state of the particles into the final state.

$$\text{initial state } |i\rangle \xrightarrow{\quad S \quad} \text{final state } |f\rangle$$

$$S\,|i\rangle = |f\rangle$$

$$P_{i\to f} = |\langle f|S|i\rangle|^2$$

Clearly, knowing the S-matrix would allow the description of the scattering process.

# S-matrix

Postulate 1: The S-matrix is Lorentz invariant

$$a + b \to c + d$$

$2 \to 2$ scattering process

$$\begin{aligned} s &= (p_a + p_b)^2 \\ t &= (p_a - p_c)^2 \\ u &= (p_a - p_d)^2 \end{aligned}$$

Kinematical invariants

The amplitude will be a function of s and t (and of the masses of the particles)

# S-matrix

Postulate 2: The S-matrix is unitary

$$\sum_k P_{i\to k} = \sum_k |\langle k|S|i\rangle|^2$$
$$= \sum_k \langle i|S^\dagger|k\rangle\langle k|S|i\rangle$$
$$= \langle i|S^\dagger S|i\rangle = 1 \; ,$$

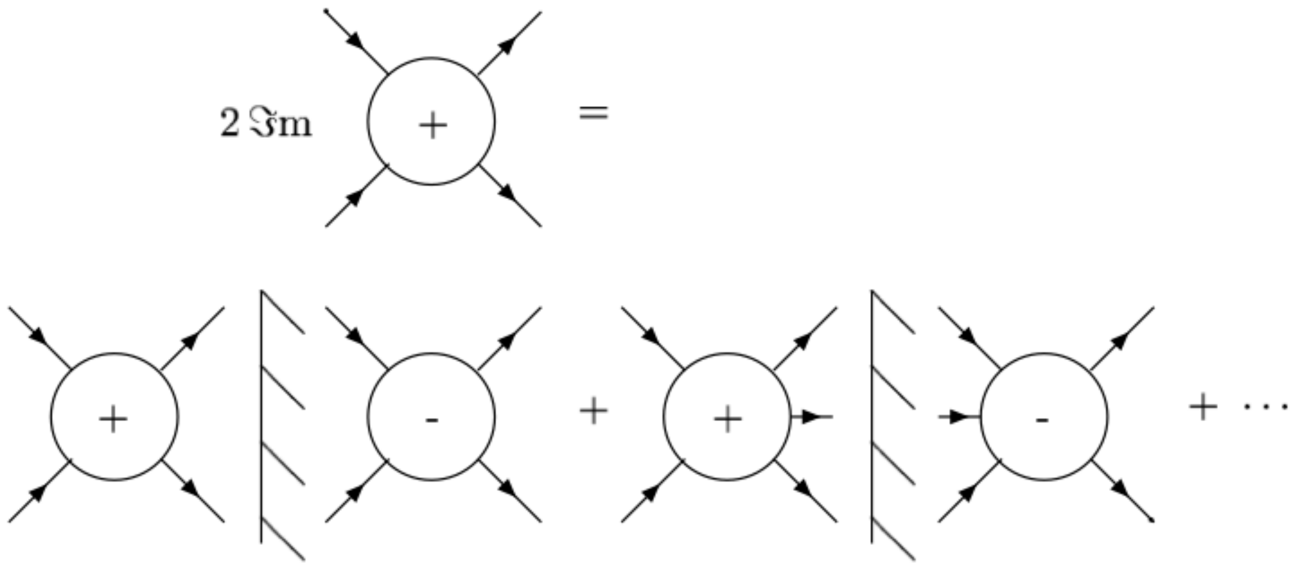
$$SS^\dagger = S^\dagger S = \mathbb{1}$$

A statement of conservation of probability: The probability of an initial state to end up in a particular final state summed over all possible final states has to be unity.

# A small digression: the Cutkosky rules

$$| a \rangle \rightarrow | b \rangle$$

$$S_{ab} = \delta_{ab} + i(2\pi)^4 \delta^4 \left( \sum_a p_a - \sum_b p_b \right) \mathcal{A}_{ab}$$




$$2\Im\mathrm{m}\, \mathcal{A}_{ab} = (2\pi)^4 \delta^4 \left( \sum_a p_a - \sum_b p_b \right) \sum_c \mathcal{A}_{ac} \mathcal{A}^\dagger_{cb}$$

# A small digression: … and the optical theorem

$$2\Im\mathrm{m}\,\mathcal{A}_{ab} = (2\pi)^4\delta^4\left(\sum_a p_a - \sum_b p_b\right)\sum_c \mathcal{A}_{ac}\mathcal{A}^\dagger_{cb}$$

Forward scattering,
in state |α> = out state |b>

$$2\Im\mathrm{m}\mathcal{A}_{aa}(s,0) = (2\pi)^4\sum_n \delta^4\left(\sum_f p_f - \sum_a p_a\right)|\mathcal{A}_{a\to n}|^2 = F\,\sigma_{\mathrm{tot}}$$

# S-matrix

Postulate 3: The analyticity of S-matrix

The S-matrix is an analytic function of Lorentz invariants (these seen as complex variables) with only those singularities as required by unitarity.

This postulate is a consequence of causality.

Another property of the S-matrix, namely the crossing symmetry is a result of the analyticity.

# Timeline

- The problem of scattering amplitudes
- Old Regge theory - soft Pomeron (60's, 70's)
- Advent of pQCD - mid 70's achievements – BFKL and the perturbative Pomeron
- HERA @ DESY: input from the experimental side / boom in the field of small-x physics (90's)
- Collinear / $k_T$-factorization scheme (90's)
- Connection of BFKL dynamics with other fields (last three decades)
- Phenomenology at the LHC @ CERN

# THE OLD REGGE THEORY

# The old Regge theory

- Recall scattering in Quantum mechanics:

- Assume that there is a function $F$ that controls the outcome of a collision between 2 particles, $F$ is actually the scattering amplitude.

- In the simple case, $F = F(E, \theta)$, where $E$ is the energy and $\theta$ is the scattering angle. $F$ can be found by solving the Schroedinger eq.

- Generally, it is easier to study the collision at a fixed value of the angular momentum, $l$, where $l$ is a positive integer.

- This way, one obtains a set of amplitudes $f_l(E), l = 0,1,2,\ldots$ from which one could reconstruct $F = F(E, \theta)$.

# The old Regge theory

- Recall scattering in Quantum mechanics:
- Imagine now that the energy E is a complex number and each $f_l(E)$ is a complex function.
- Imagine further that the two colliding particles can bind together to form a bound state with angular momentum $L$ and energy $E_B$.
- Then the $L$th one from the $f_l(E)$ functions, that is $f_L(E)$ would be found to have a "pole" at $E = E_B$, that is a simple infinity $1/(E - E_B)$
- Then push the formalism further to allow the angular momentum to take non-integer values (this step taken 33 years after the original Regge paper).

# The old Regge theory

- Recall scattering in Quantum mechanics:
- More formally we are talking about the “partial wave expansion”:

$$\mathcal{A}_{ab\to cd}(s,t) = \sum_{l=0}^{\infty}(2l+1)\, a_l(t)\, P_l\,(1+2s/t)$$

- Regge's idea: complexify energy and angular momentum and focus on the function

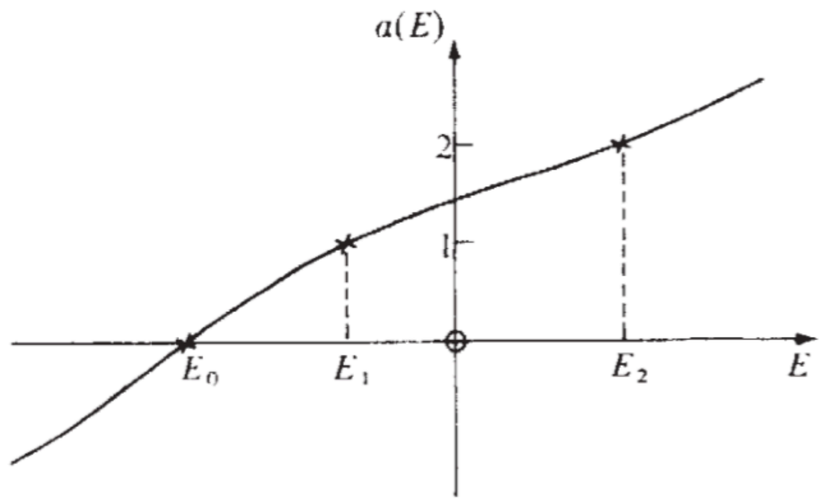
$$\alpha_{(l)}(t)$$

- Gribov, Chew, Frautschi

Note that s, t and u are the Mandelstam variables and cos(θ) = 1 + 2 t/s

# The old Regge theory

- The function $\alpha_l(t)$, the trajectory function, has the property that if an energy $E$ exists such that $\alpha(E)$ is a positive integer $L$, then a bound state would exist at energy $E$ with angular momentum $l = L$



# The old Regge theory

Why the previous result is so important?

- A new way emerged for classifying bound states and resonances into families.

- Instead of focussing on one bound state with a given angular momentum, say $l = 1$ with energy $E_1$ and then on another with $l = 2$ and energy $E_2$, one may focus on a single object, the "trajectory" function $\alpha(E)$ with the property that if a particular value of $E$ causes $\alpha(E)$ to be equal to a positive integer $L$, then a bound state would exist at that energy $E$ and with angular momentum $l = L$.

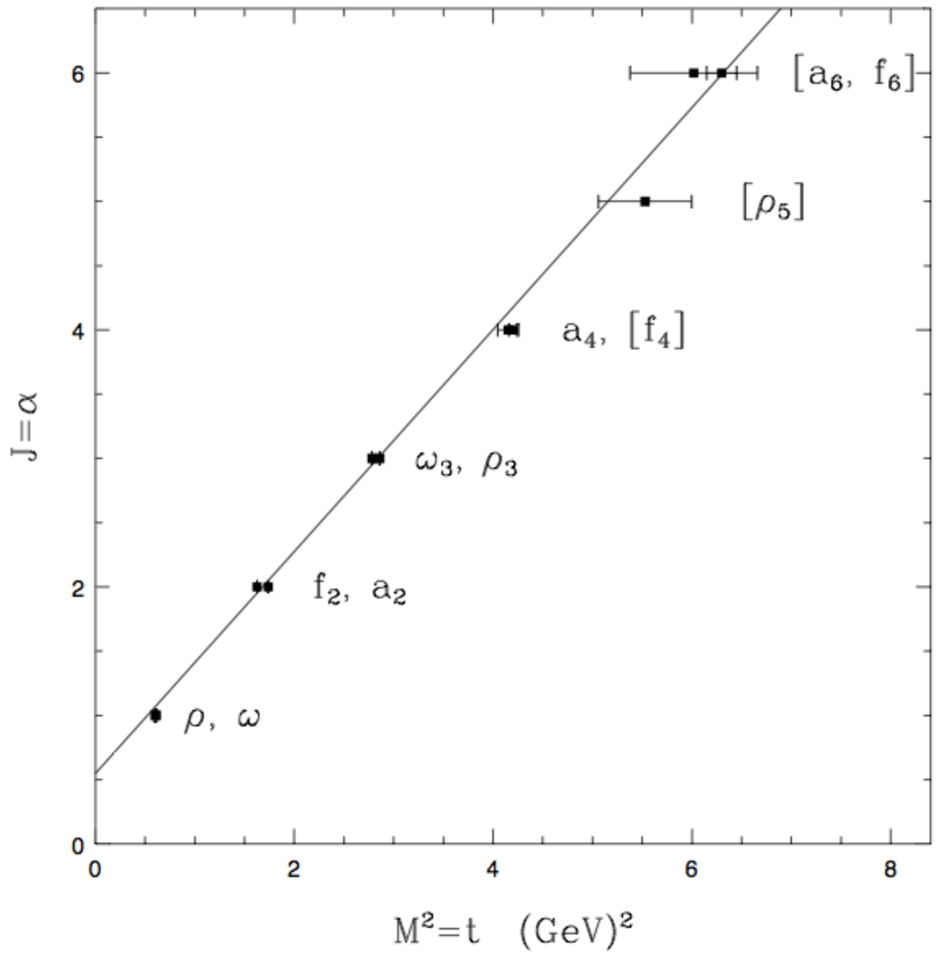
# The old Regge theory

Chew and Frautschi ('61, '62) plotted the spins of low lying mesons against mass squared and noticed that they lie in a straight line.

$$\alpha(t) = \alpha(0) + \alpha' t$$

$$\begin{aligned} \alpha(0) &= 0.55 \\ \alpha' &= 0.86\ \mathrm{GeV}^{-2} \end{aligned}$$

$$\frac{d\sigma}{dt} \propto s^{(2\alpha(0)-2\alpha' t-2)}$$




How to use this fact?

# The old Regge theory

- First, find a hadronic process that can be "characterized" by the particles that lie in the trajectory [same quantum numbers]
- Second, extrapolate the trajectory to negative values of t by using

$$\begin{aligned}\alpha(0) &= 0.55\\ \alpha' &= 0.86 \text{ GeV}^{-2}\end{aligned}$$

into

$$\frac{d\sigma}{dt} \propto s^{(2\alpha(0)-2\alpha' t-2)}$$

- Plot both the extrapolated trajectory and the experimental data

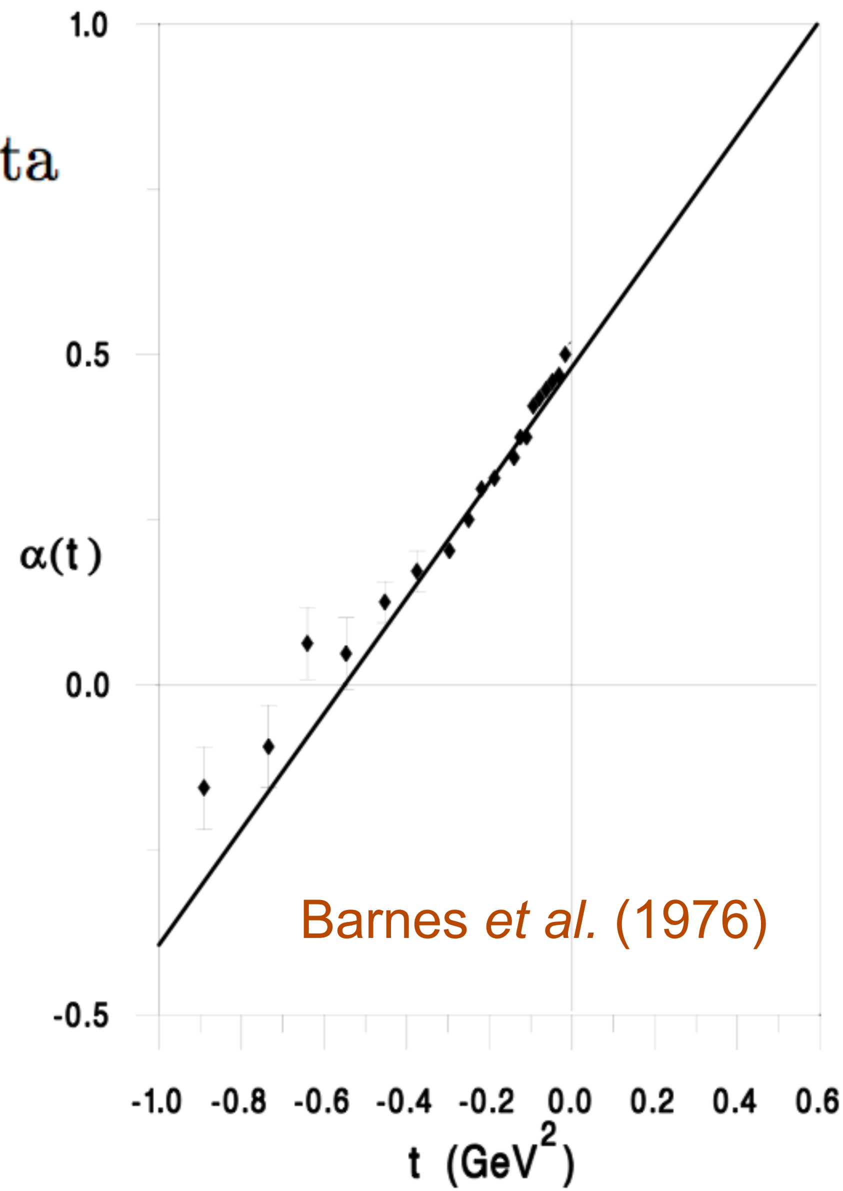
# The old Regge theory

$\alpha(t)$ obtained from $\pi^- p \rightarrow \pi^0 n$ data

Characterized by Isospin=1, even Parity:
ρ mesons

The interaction is mediated by the
“exchange of a trajectory”.

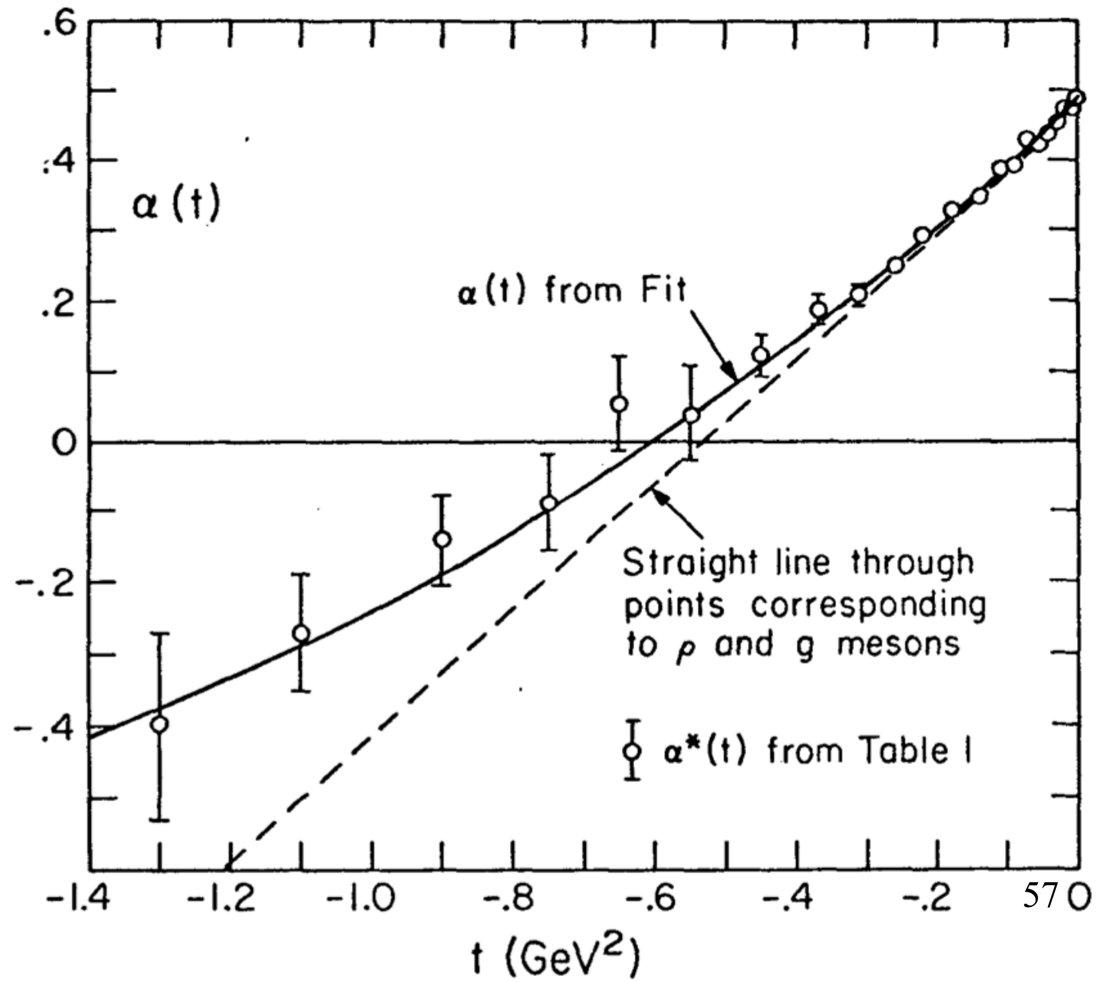
$$A(s,t) \sim \beta(t) s^{\alpha(t)}$$



# The old Regge theory

$\alpha(t)$ obtained from $\pi^- p \to \pi^0 n$ data

Barnes *et al.* (1976)




Actually, in the paper, they offer an effective trajectory, plotted with the continuous line whereas the dashed curve is the continuation of the ρ meson trajectory.

# Toward the (soft) Pomeron

$$\sigma_{\text{tot}} \propto s^{(\alpha(0)-1)}$$

- The asymptotic behavior of the total cross section for a process can be obtained by the intercept of the Regge trajectory that dominates that process.
- Pomeranchuk and Okun &Pomeranchuk (1956) proved that if in a process there is charge exchange, then the cross section vanishes asymptotically.
- Pomeranchuk theorem:

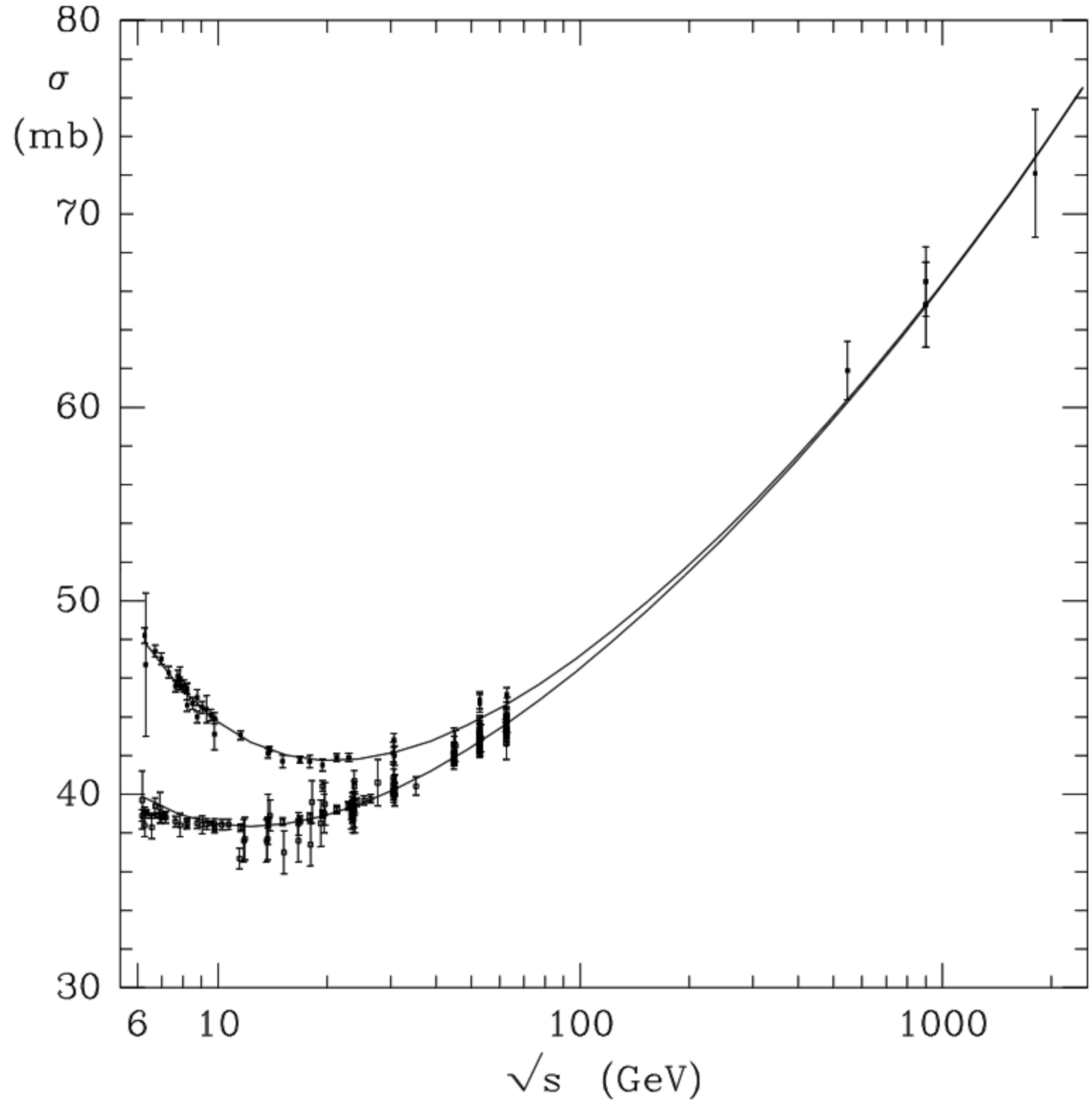
$$\sigma_{\text{tot}}(ab) \underset{s\to\infty}{\simeq} \sigma_{\text{tot}}(a\bar{b})$$

- Froissart-Martin bound: $\sigma_{\text{tot}} \leq C \ln^2 s \,, \quad \text{as } s \to \infty$

# Toward the (soft) Pomeron

$$\sigma_{\text{tot}} \propto s^{(\alpha(0)-1)}$$

But the total cross sections do rise with energy! This is not compatible with an exponent smaller than 1.



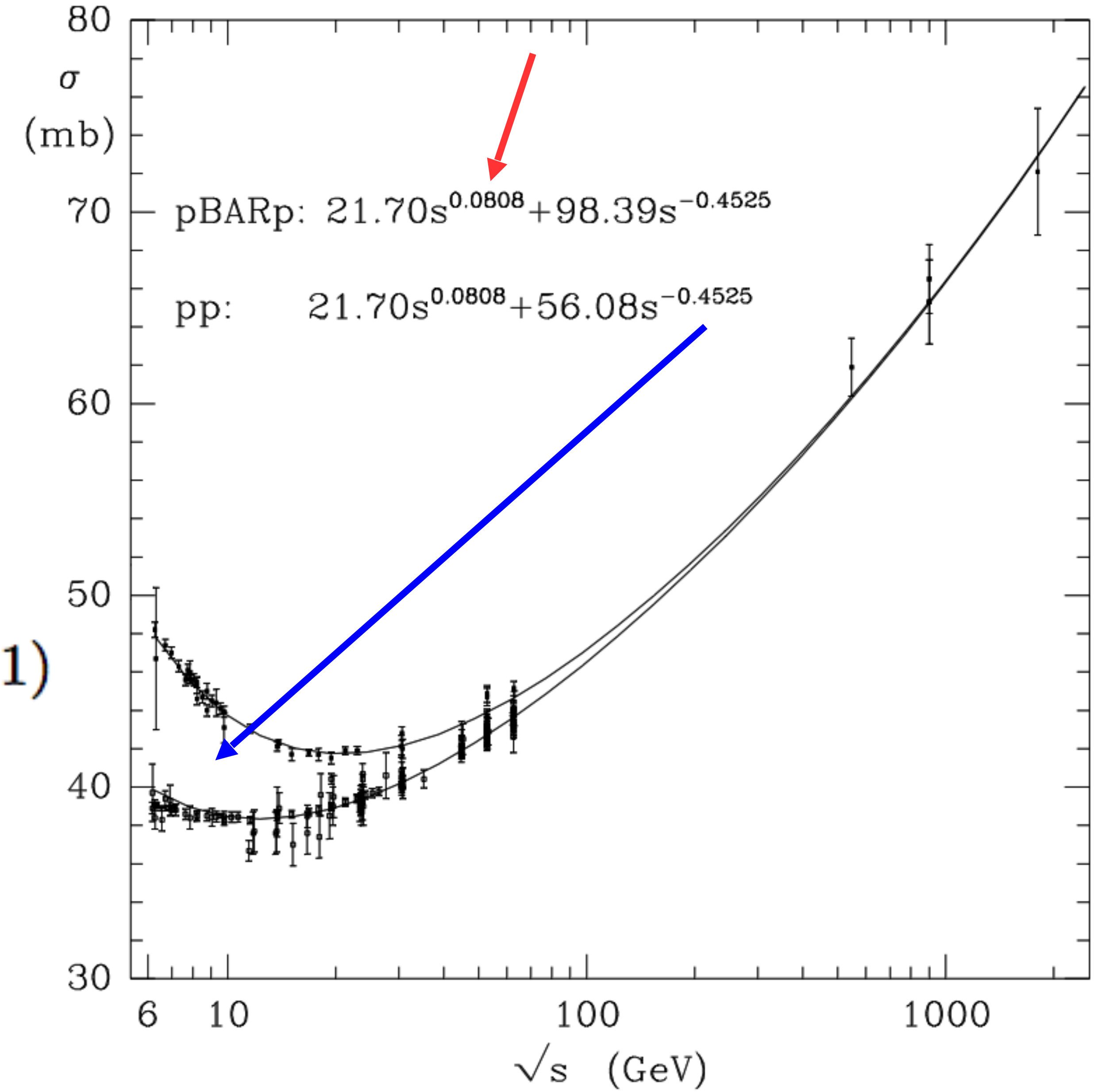
# The (soft) Pomeron

- Gribov introduced (1961) a Regge trajectory with intercept 1: the Pomeron (named after Pomeranchuk)
- It does not correspond to any known particle (glueballs?)
- It carries the quantum numbers of the vacuum, C-even, P-even, Charge 0, Isospin 0.
- Intercept consistent with fits (~1.08)

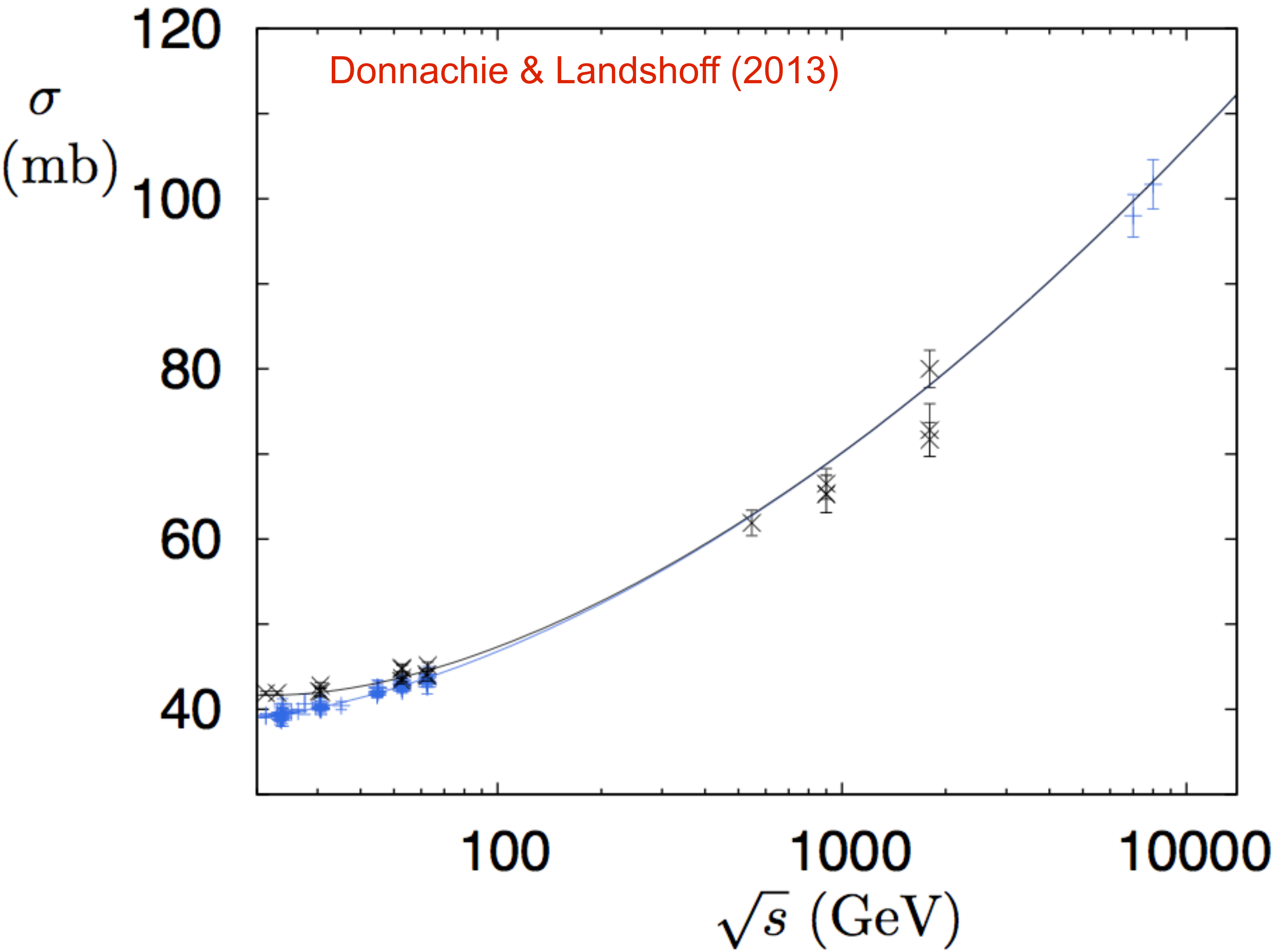
# Toward the (soft) Pomeron

Fitting proton-proton & proton-antiproton scattering data.
NOTE: the parameters of the fit were determined before the measurements at Tevatron by using data below 100 c.o.m energy

$$\sigma_{tot} \propto s^{(\alpha(0)-1)}$$




Donnachie & Landshoff (1992)


Donnachie & Landshoff (2013)
σ
(mb)
120
100
80
60
40
100
1000
10000
√s (GeV)

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See also a nice talk by Poghosyan: ***An introduction to Regge Field Theory***