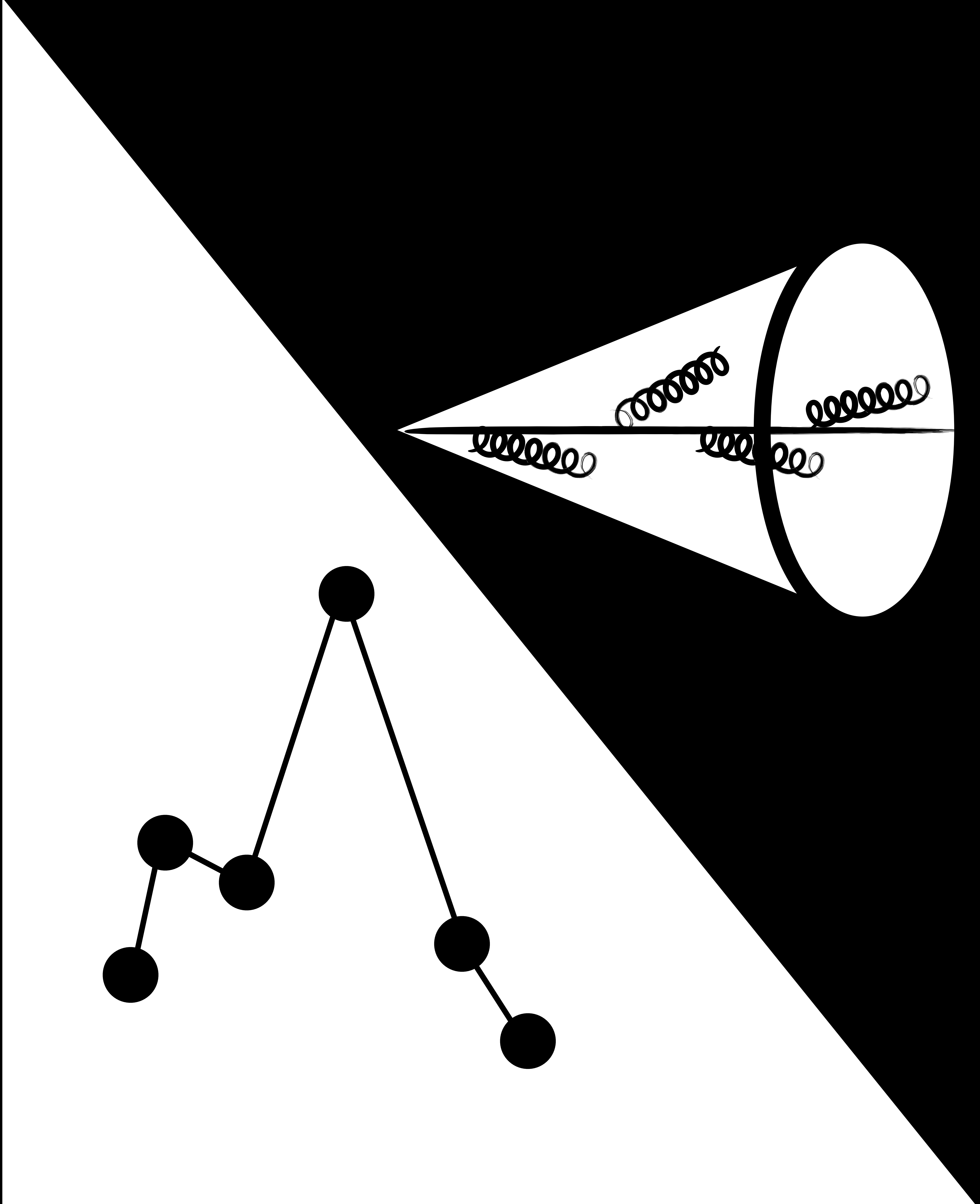
Jet physics in 2024

Alba Soto Ontoso

Midsummer School in QCD

Saariselkä, 25-27th June, 2024



Plan for the course

Lecture 1: big picture

- Why jets?
- $\gamma^* \to q\bar{q}g$: singularity structure
- Resummation and parton showers

Lecture 2: jet algorithms

- Core ideas of jet reconstruction
- Sequential recombination algorithms
- The question of flavour

Lecture 3: jet substructure

- Concepts and tools
- Calculability: groomed jet mass
- Observables at the LHC

A few useful references

- *Towards jetography*, G.P. Salam

- *Looking inside jets: an introduction to jet substructure and boosted-object phenomenology*, S. Marzani, G. Soyez, M. Spannowsky

- *Jet substructure at the LHC: A review of recent advances in theory and machine learning*, A. Larkoski, I. Moult, B. Nachman

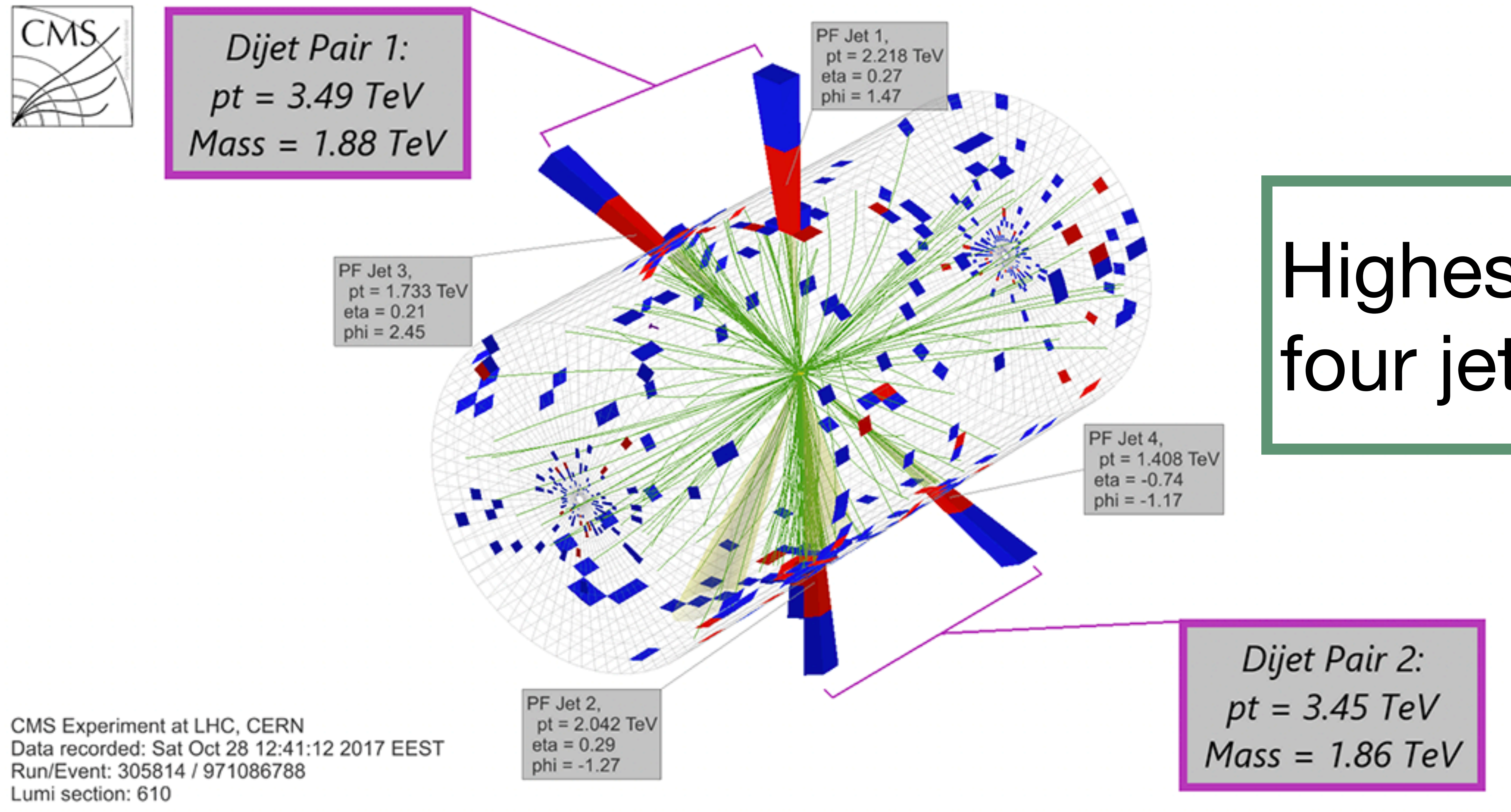
- *Fastjet user manual*, M. Cacciari, G.P. Salam, G. Soyez

Questions? Drop me a line: alba.soto.ontoso at cern.ch

What are jets? Experimental observation

One, of many, definitions: collimated, energetic bunches of hadrons

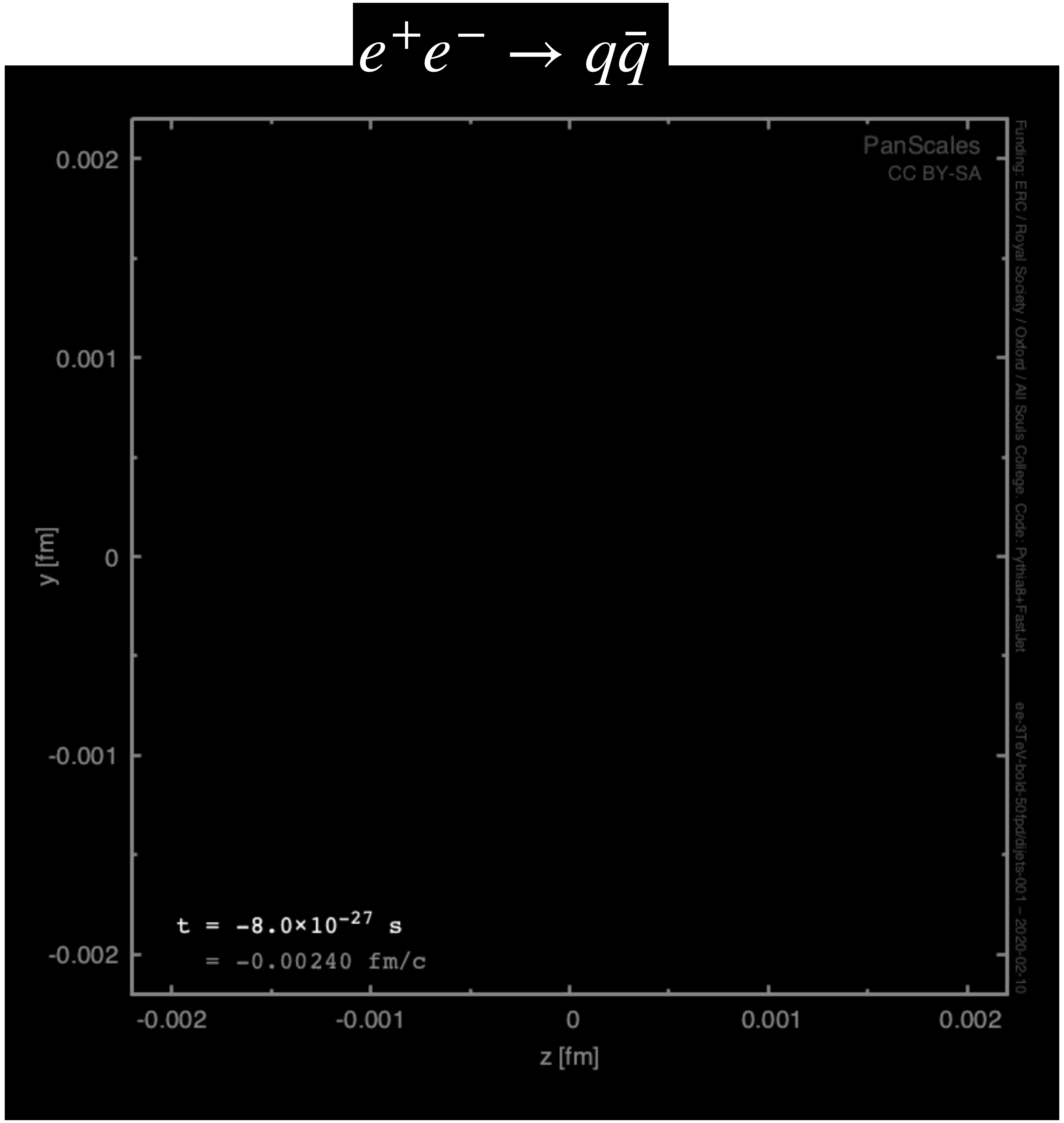
[Source: https://cerncourier.com/a/dijet-excess-intrigues-at-cms/]



Highest mass event: four jet mass of 8 TeV

Interactive view of a dijet event: https://cms3d.web.cern.ch/SMP-20-011/

What are jets? Numerical simulation



[https://gsalam.web.cern.ch/gsalam/panscales/videos.html]

Color coding:

incoming beam particles

intermediate particles (quarks or gluons)

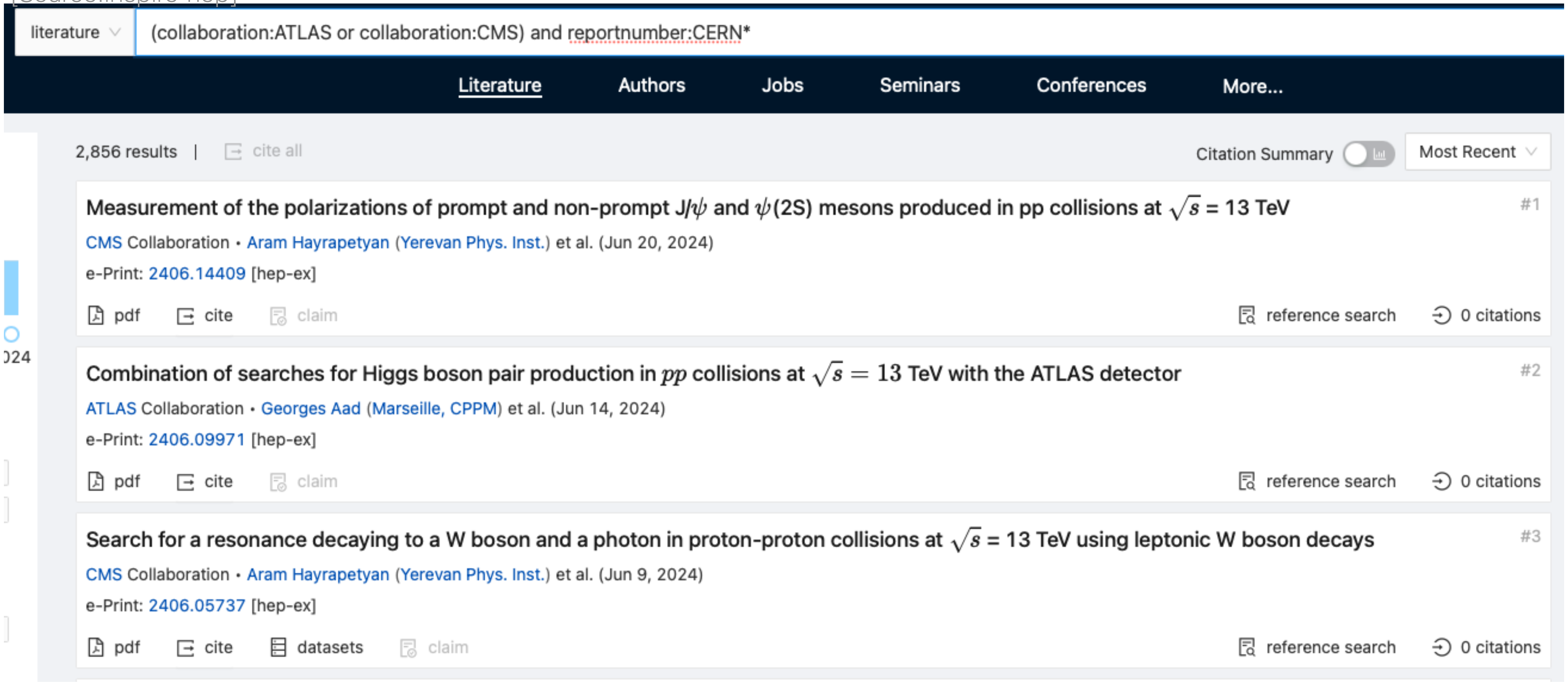
final particle (hadron)

Event evolution spans 7 orders of magnitude in space-time

Jets are very popular at the LHC

Find all papers by ATLAS and CMS

[Source:inspire-hep]

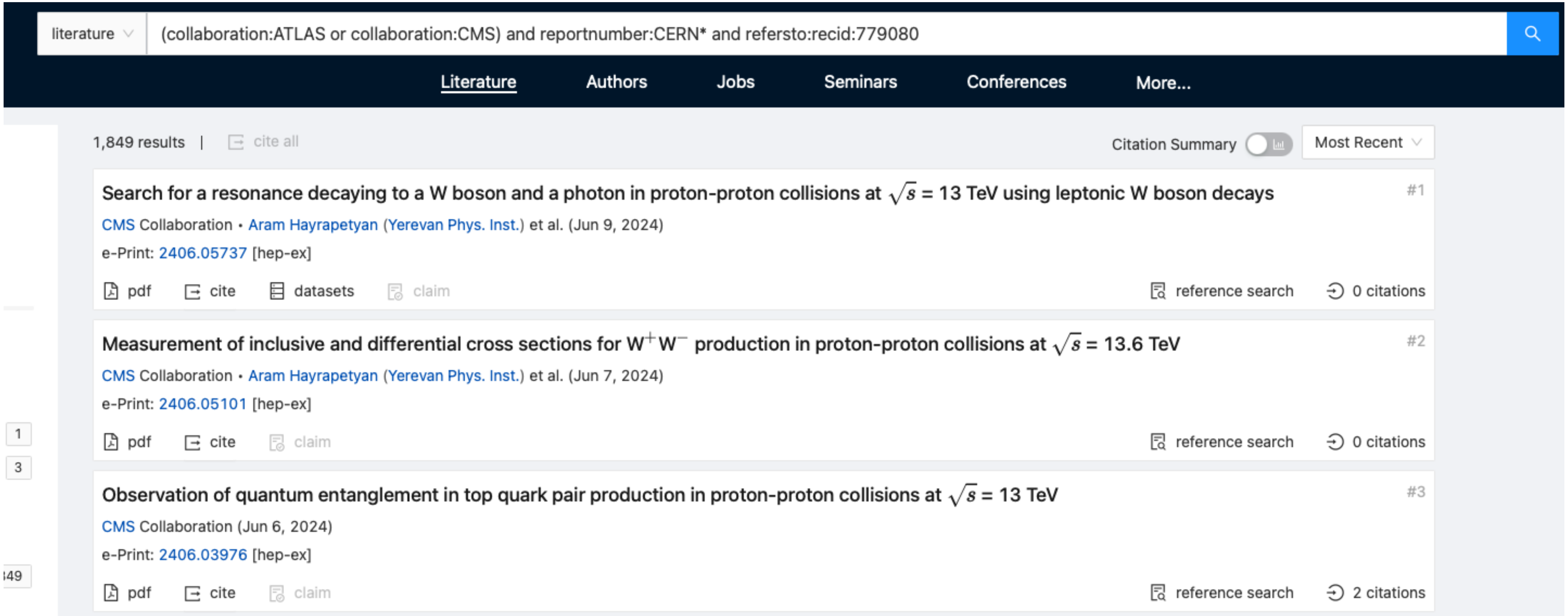


2856 records found

Jets are very popular at the LHC

[Source:inspire-hep]

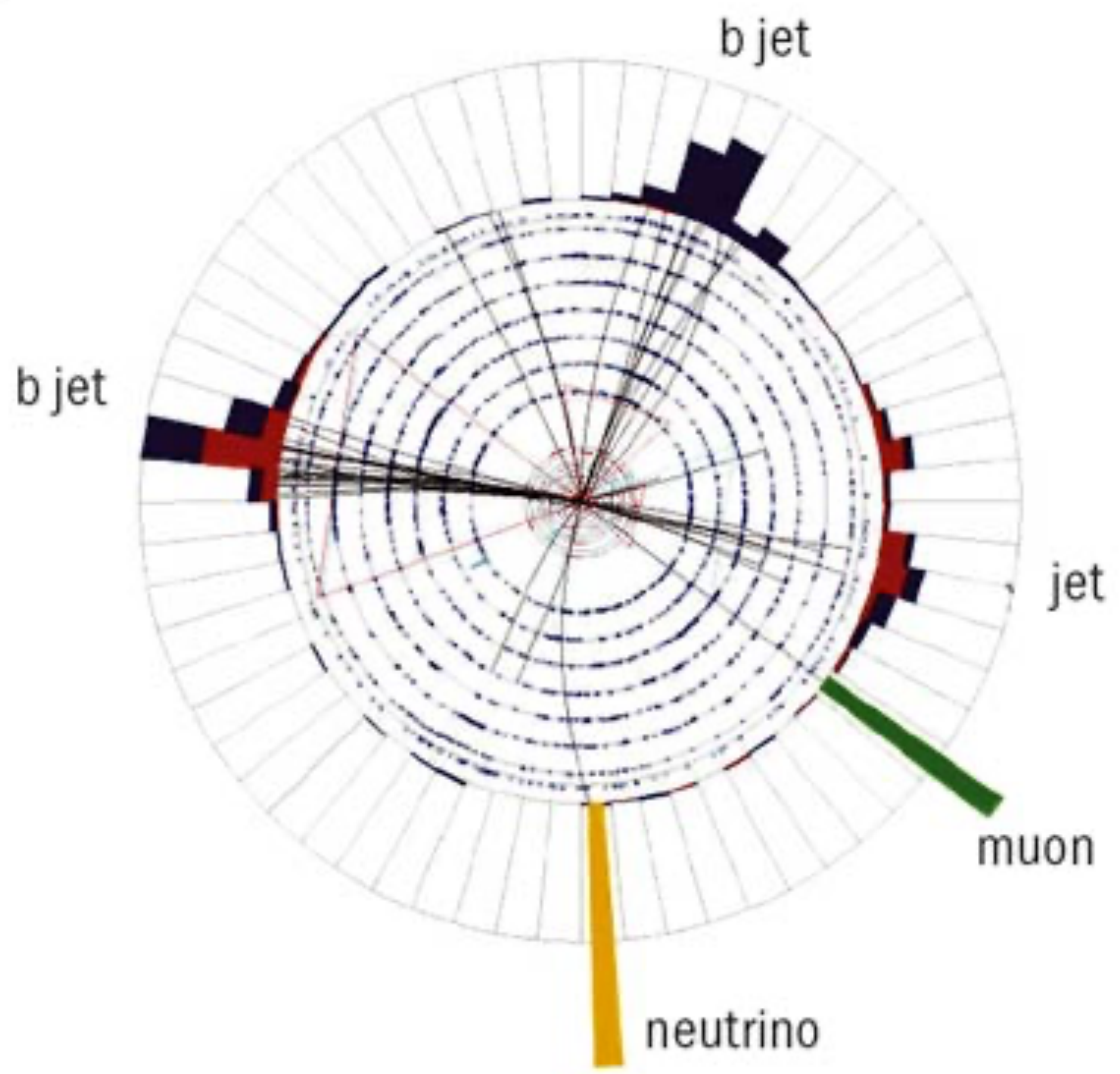
Find all papers by ATLAS and CMS that cite a jet algorithm



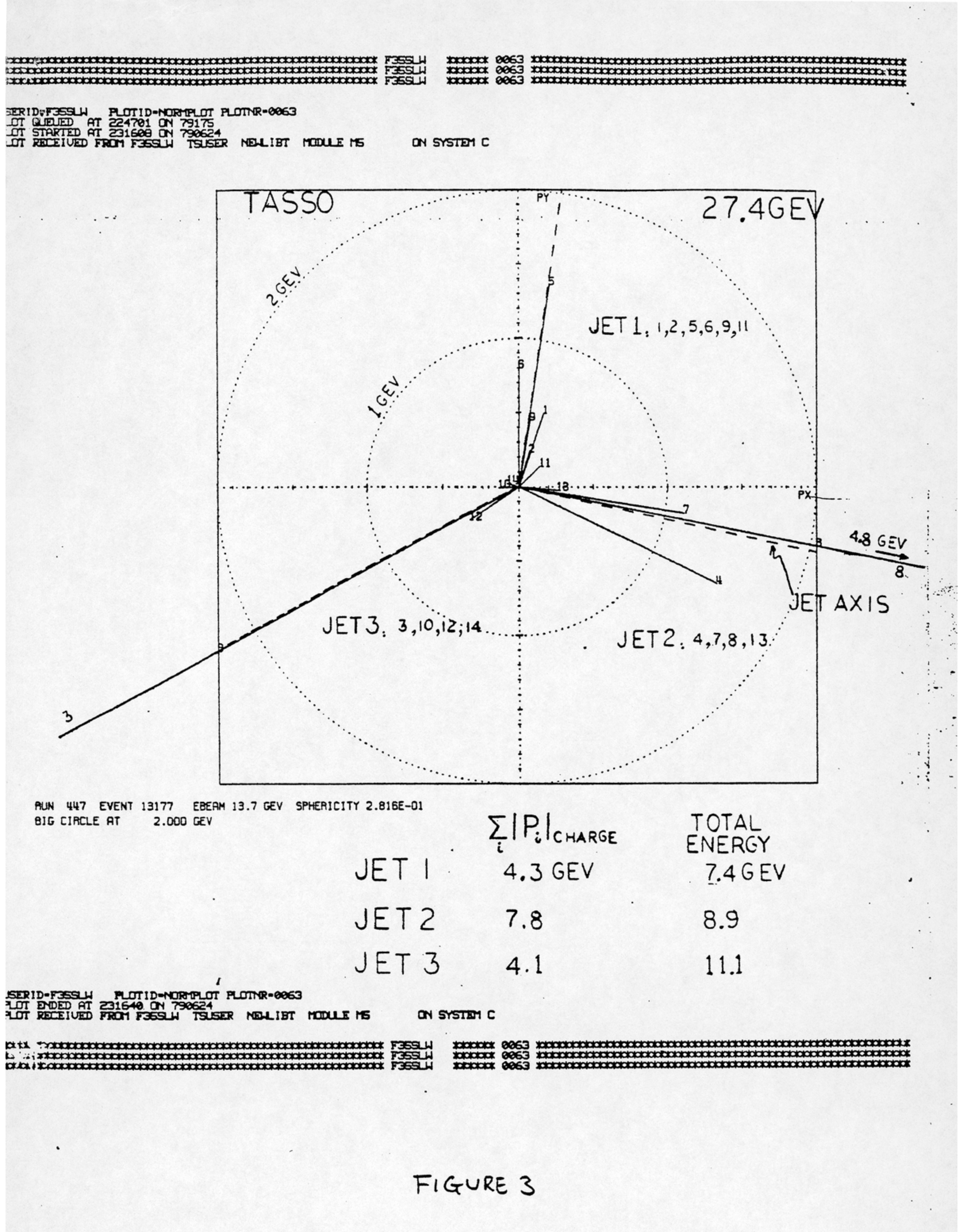
1849 records found: >60% of papers use jets!

Jets have been instrumental for (at least) 2 discoveries

[Source: cern courier]



[Source: symmetry magazine]



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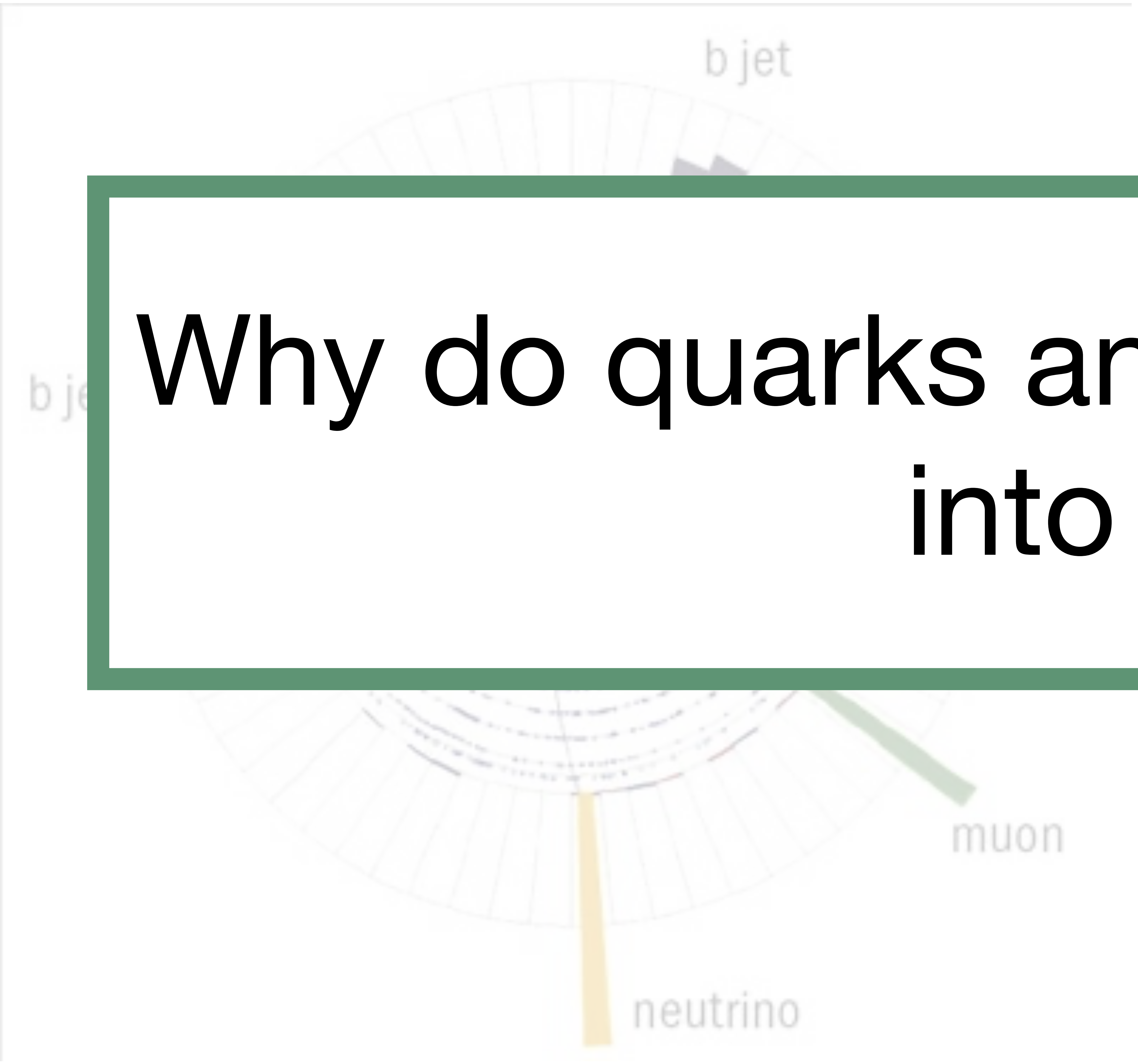
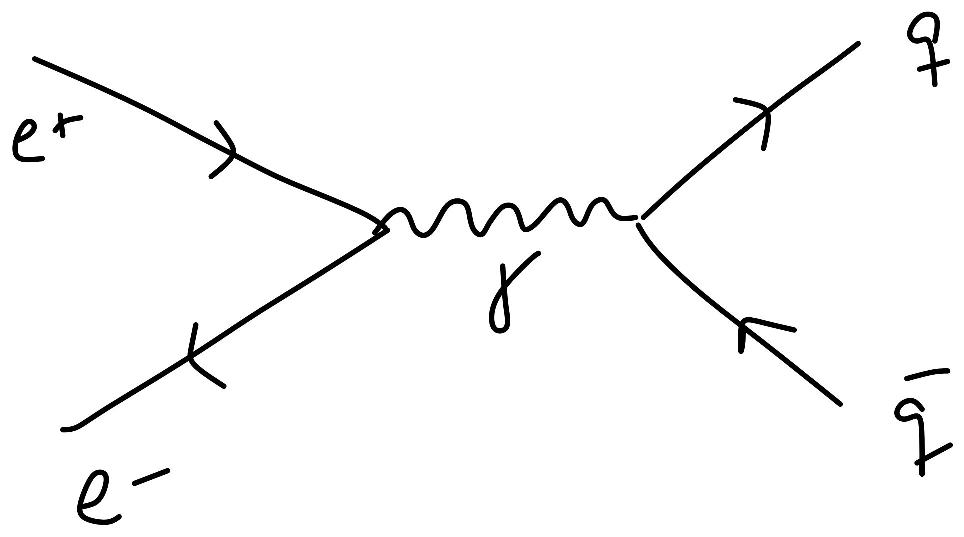


[Source: symmetry magazine]

Why do quarks and gluons fragment into jets?

Leading order calculation: $e^+e^- \to q\bar{q}$ [Adapted from Soyez's lectures]



electron: $p_1 = \left(0, 0, \frac{\sqrt{s}}{2}, \frac{\sqrt{s}}{2}\right)$

positron: $p_2 = \left(0, 0, -\frac{\sqrt{s}}{2}, \frac{\sqrt{s}}{2}\right)$

We neglect masses and use Feynman gauge

Phase-space: $$\int d\Phi_2 = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}(2\pi)^4\delta^{(4)}(p_1+p_2-k_1-k_2)\,(2\pi)\delta(k_1)\,(2\pi)\delta(k_2)$$

$$= \frac{1}{16\pi}\int_{-1}^{1} d\cos\theta \implies$$

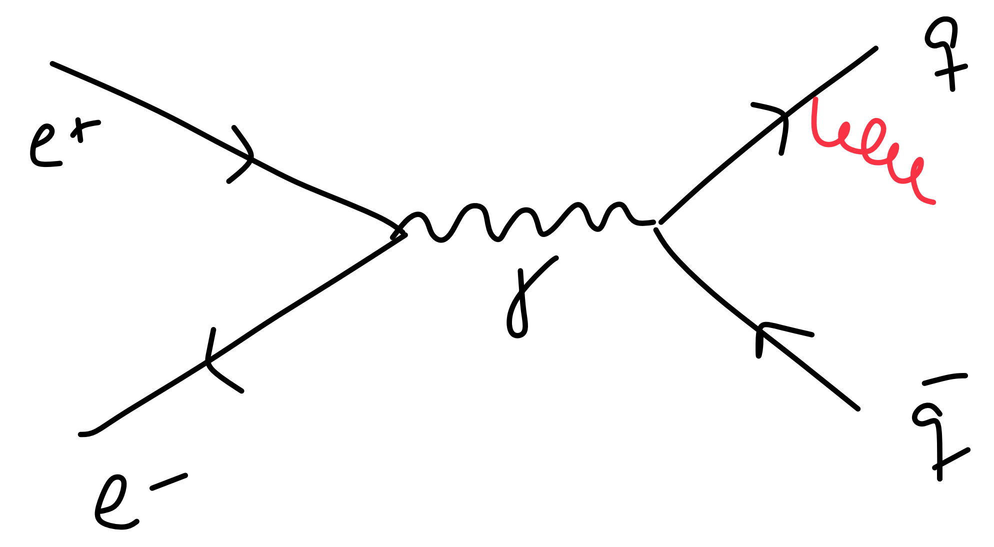
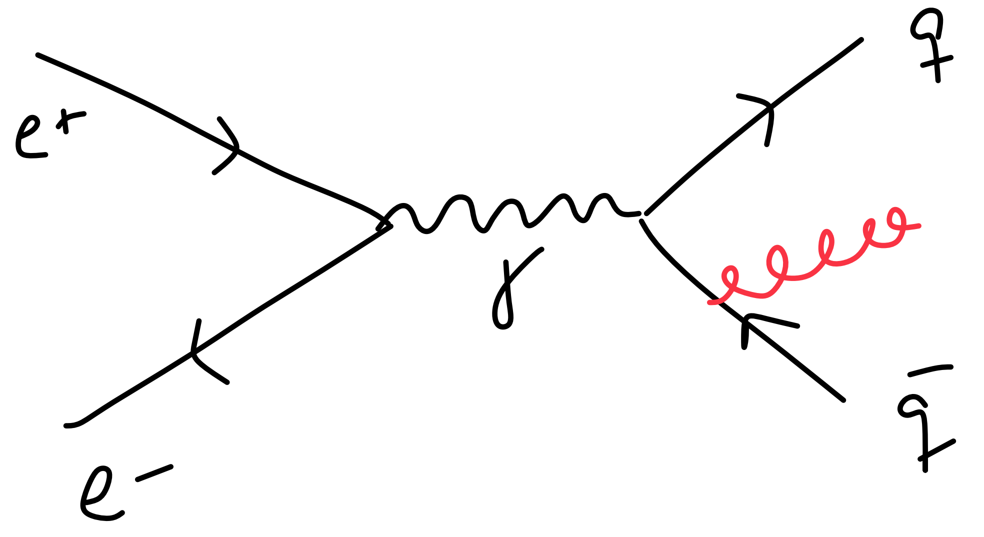
quark: $k_1 = \frac{\sqrt{s}}{2}(\sin\theta, 0, \cos\theta, 1)$

antiquark: $k_2 = \frac{\sqrt{s}}{2}(-\sin\theta, 0, -\cos\theta, 1)$

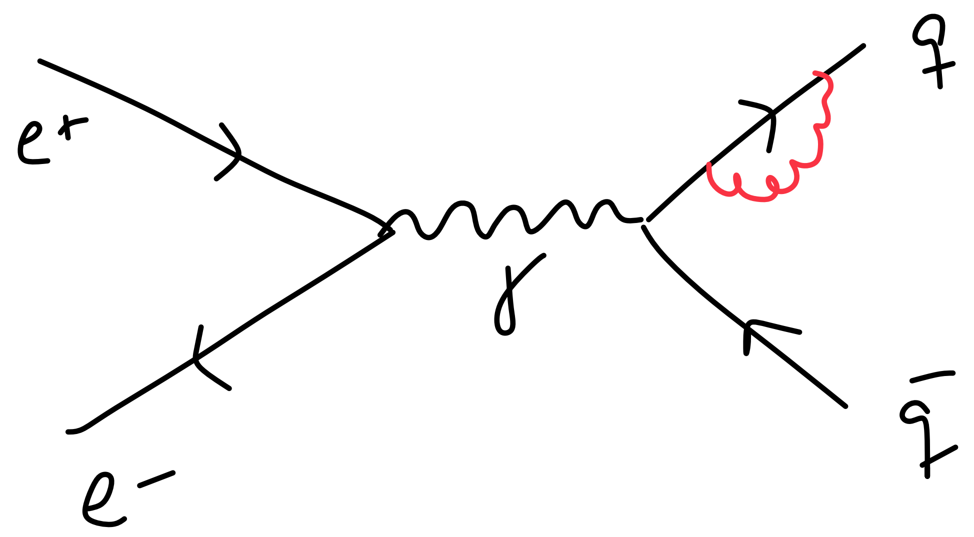
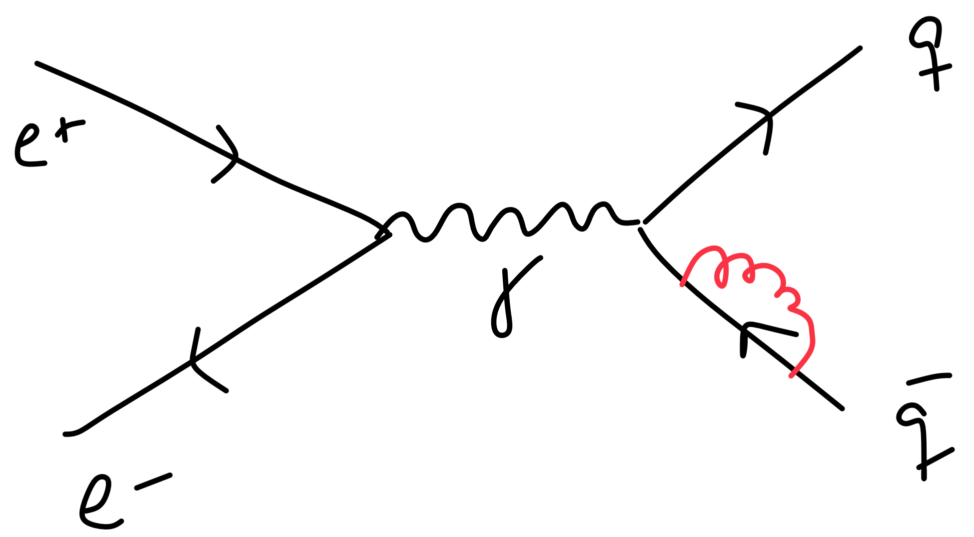
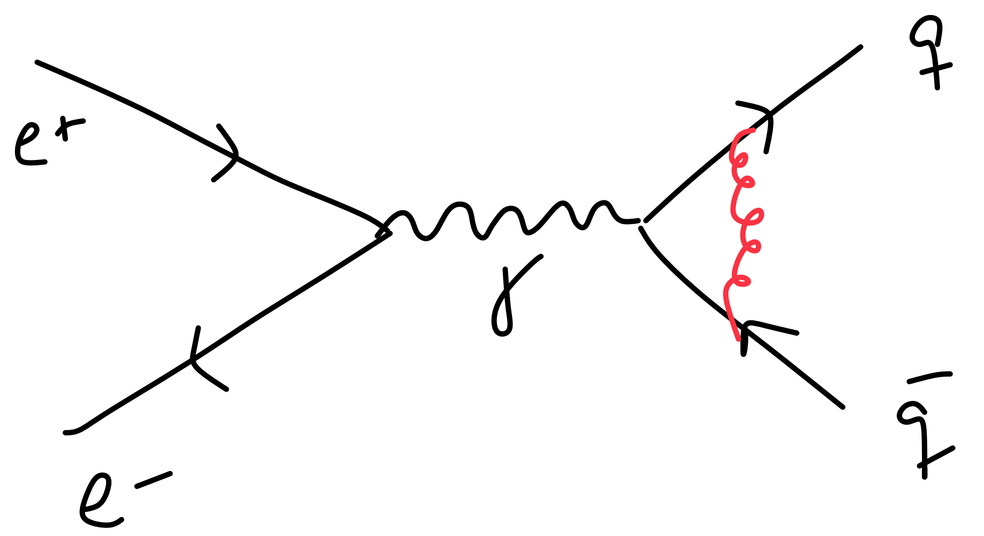
Matrix element: $$\sum |M|^2 \propto \mathrm{Tr}(\not{p}_1\gamma_\mu\not{p}_2\gamma_\rho)\,\frac{g^{\mu\nu}g^{\rho\sigma}}{s^2}\,\underbrace{\mathrm{Tr}(\not{k}_1\gamma_\nu\not{k}_2\gamma_\sigma)}_{k_{1\mu}k_{2\rho}+k_{1\rho}k_{2\mu}-k_1\cdot k_2\, g_{\mu\rho}} \propto 1+\cos^2\theta$$

Cross section: $$d\sigma = \frac{1}{2s}|M|^2 d\Phi_2 \implies \boxed{\frac{d\sigma}{d\cos\theta} = e_q^2 N_c \frac{\pi\alpha_e^2}{2s}(1+\cos^2\theta)}$$

Next-to-leading order calculation: $e^+e^- \to q\bar{q}g$

2 real contributions

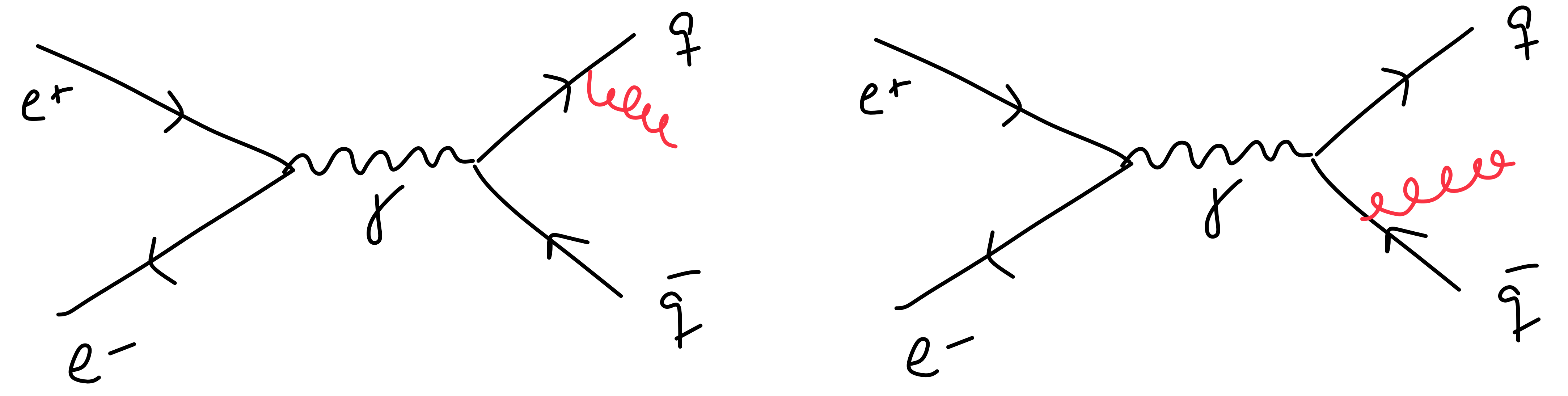
3 virtual contributions

$x_1+x_2+x_3 = 2$

Phase-space: $$\int d\Phi_3 = \prod_{i=1}^{3} \frac{d^3k_i}{(2\pi)^3\, 2E_i}\,(2\pi)^4\,\delta^{(4)}(p_1+p_2-k_1-k_2-k_3)\;; \quad x_i = \frac{2E_i}{\sqrt{s}} \Rightarrow \frac{d^3k_i}{(2\pi)^3\, 2E_i} = \frac{s}{8}\,\frac{1}{(2\pi)^3}\,x_i\,dx_i\,d\Omega_i$$

$$= \frac{s}{32\,(2\pi)^5}\int dx_1\, dx_2\, \underbrace{d\alpha\, d\cos\beta\, d\gamma}_{\text{Euler angles}}$$

Next-to-leading order calculation: $e^+e^- \to q\bar{q}g$



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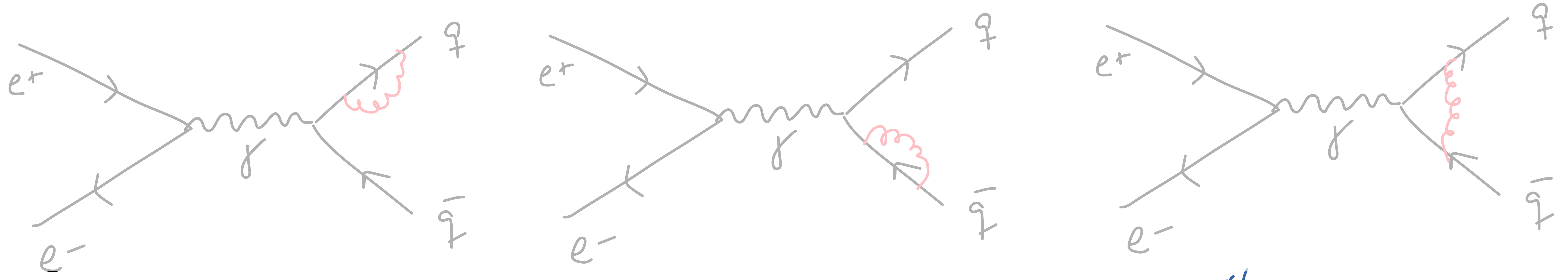
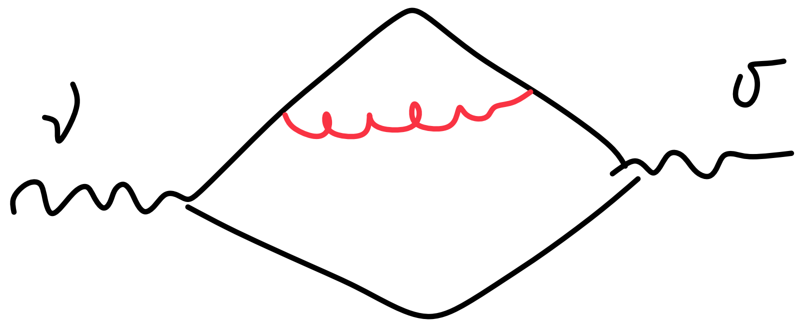


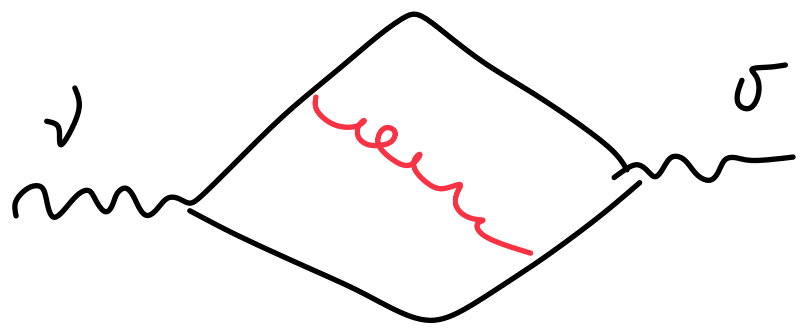
3 virtual contributions

Matrix element:
for real-emission

$$\overline{\sum}|M|^2 \propto \underbrace{\sum \left(t^a_{BA}\, t^a_{AB}\right)}_{C_F N_c} \mathrm{Tr}\left(\not{p}_1 \gamma_\mu \not{p}_2 \gamma_\rho\right) \frac{g^{\mu\nu} g^{\rho\sigma}}{s^2}$$

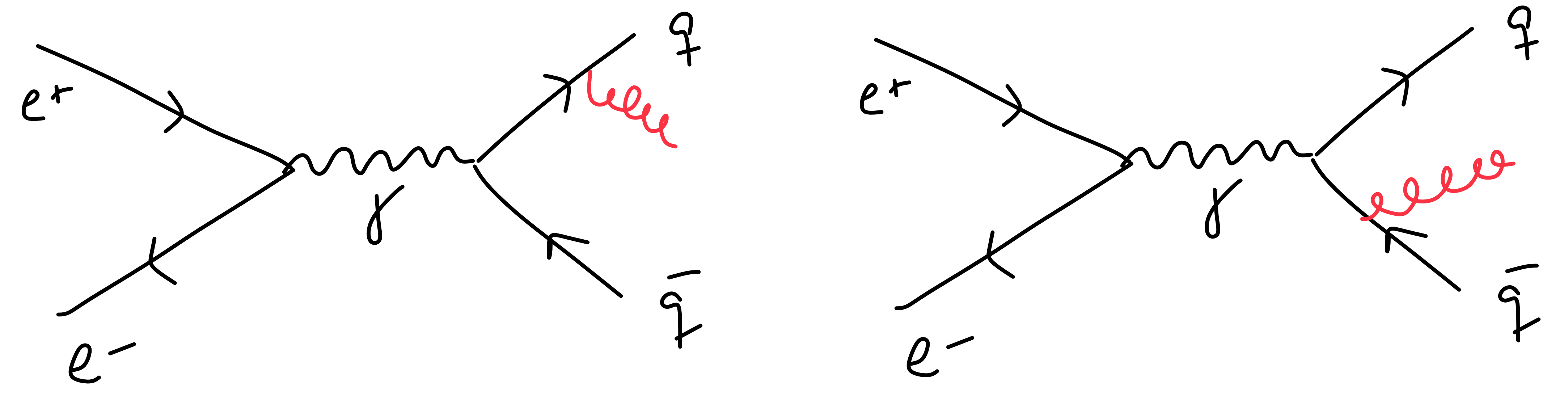
$$\times \Big[\frac{1}{(k_1+k_3)^4} \mathrm{Tr}\left(\not{k}_1 \gamma_\alpha (\not{k}_1+\not{k}_3) \gamma_\nu \not{k}_2 \gamma_\sigma (\not{k}_1+\not{k}_3) \gamma^\alpha\right)$$





$$- \frac{1}{(k_1+k_3)^2 (k_2+k_3)^2} \mathrm{Tr}\left(\not{k}_1 \gamma^\alpha (\not{k}_1+\not{k}_3) \gamma_\nu \not{k}_2 \gamma_\alpha (\not{k}_2+\not{k}_3) \gamma_\sigma\right) + (1 \leftrightarrow 2) \Big]$$

Next-to-leading order calculation: $e^+e^- \to q\bar{q}g$



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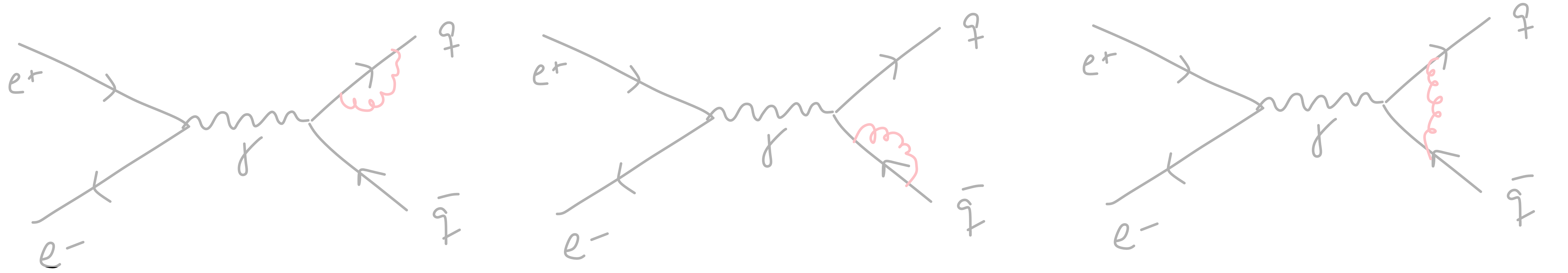


3 virtual contributions

Matrix element: $\overline{\sum}|M|^2 \propto \dfrac{\alpha_e^2\,\alpha_s\,C_F\,N_c}{s}\;\dfrac{(p_1\cdot k_1)^2+(p_1\cdot k_2)^2+(p_2\cdot k_1)^2+(p_2\cdot k_2)^2}{(k_1\cdot k_3)\,(k_2\cdot k_3)}$

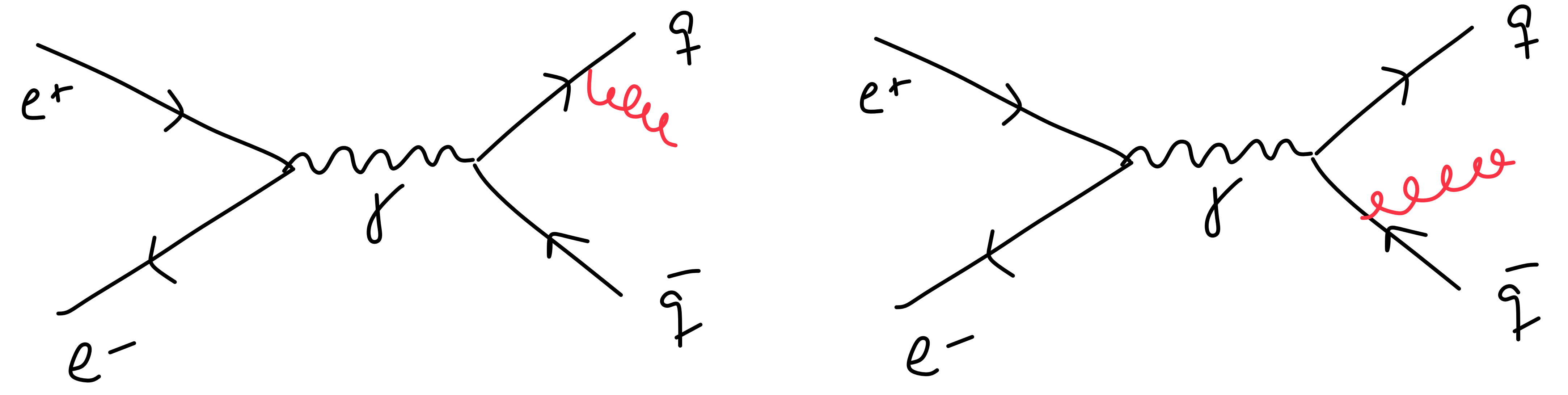
Cross section:
integrated over Euler angles

$$\frac{d^2\sigma}{dx_1\,dx_2} = e_q^2\,N_c\,\sigma_0\,\frac{\alpha_s C_F}{2\pi}\,\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

with $0 \le x_1, x_2 \le 1$

Born-level cross section

Next-to-leading order calculation: $e^+e^- \to q\bar{q}g$



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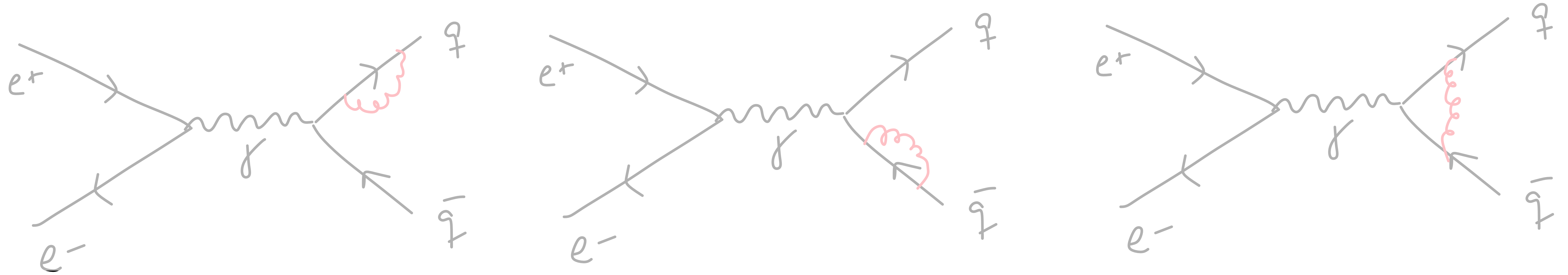

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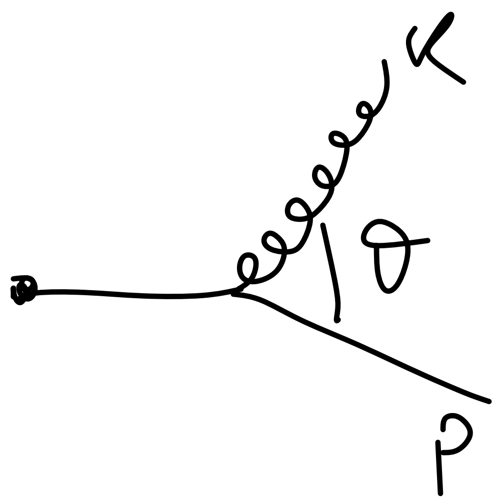
$$1-x_1 = \frac{1}{2}x_2 x_3\left[1-\cos\theta_{23}\right]$$

$$1-x_2 = \frac{1}{2}x_1 x_3\left[1-\cos\theta_{13}\right]$$

Divergences
cancelled by
virtual terms

- Soft: $x_3 \to 0$ (or $E_{gluon} \to 0$)
- Collinear: θ_{13} or $\theta_{23} \to 0$



$\frac{1}{(p+k)^2} \to 0$ if $E_k \to 0$ or $p\cdot k \to 0$

Interlude: IRC safety

For inclusive cross sections, cancellation of divergences can be proven to all orders in the perturbative expansion (KLN theorem)

Beyond inclusive observables, concept of IRC safety emerges

> *For an observable's distribution to be calculable in [fixed-order] perturbation theory, the observable should be infra-red safe, i.e. insensitive to the emission of soft or collinear gluons. In particular if $\vec{p}_i$ is any momentum occurring in its definition, it must be invariant under the branching*
>
> $$\vec{p}_i \to \vec{p}_j + \vec{p}_k$$
>
> *whenever $\vec{p}_j$ and $\vec{p}_k$ are parallel [collinear] or one of them is small [infrared].*
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> [QCD and Collider Physics (Ellis, Stirling & Webber)]

Examples

Multiplicity of gluons

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Examples

Multiplicity of gluons is not IRC safe

[modified by soft/collinear splitting]

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Examples

Multiplicity of gluons is not IRC safe

Energy of hardest particle

[modified by soft/collinear splitting]

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Examples

Multiplicity of gluons is not IRC safe [modified by soft/collinear splitting]

Energy of hardest particle is not IRC safe [modified by collinear splitting]

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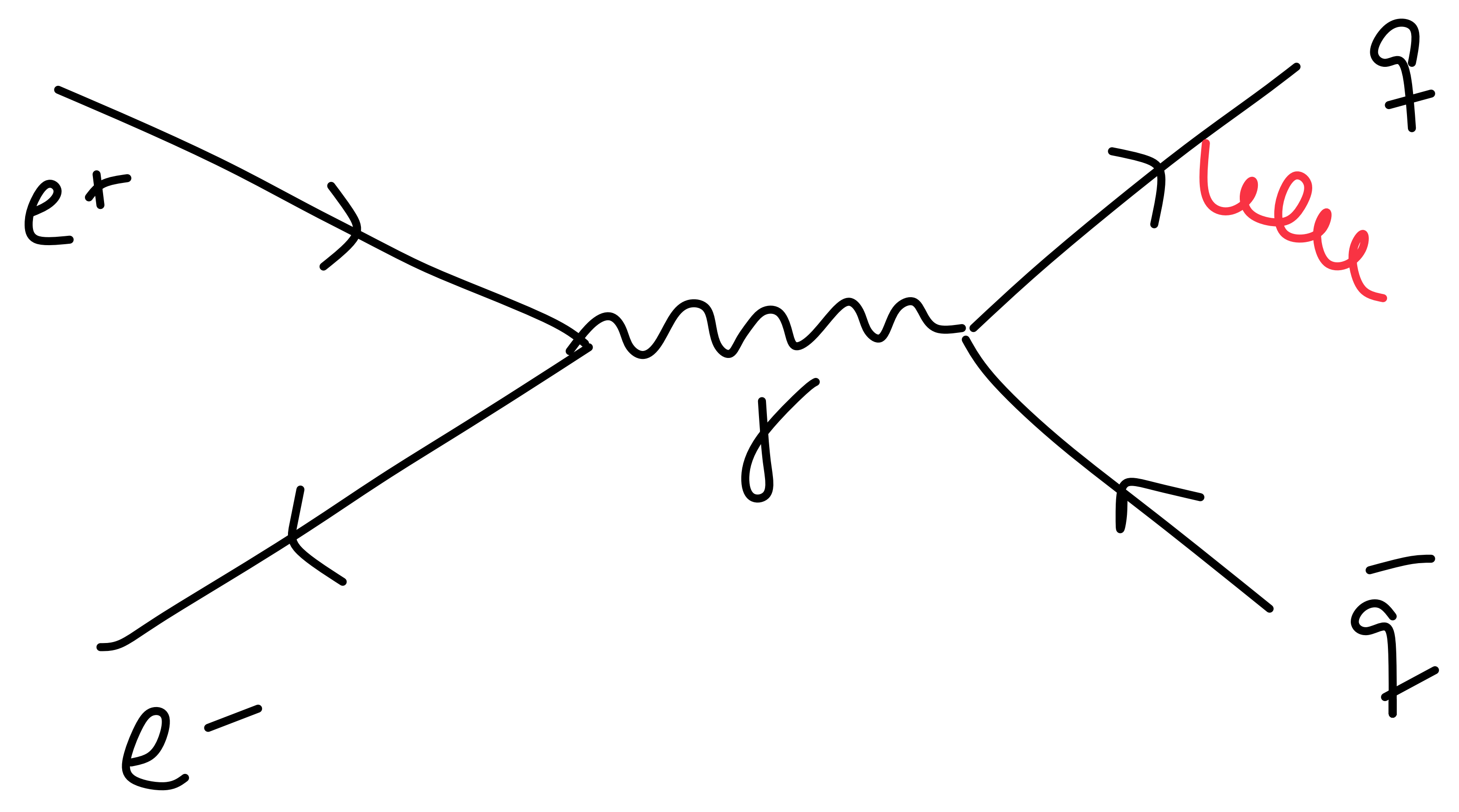
Examples

Multiplicity of gluons is not IRC safe [modified by soft/collinear splitting]

Energy of hardest particle is not IRC safe [modified by collinear splitting]

Energy flow into a cone is IRC safe [soft emissions don't change energy flow, collinear emissions don't change its direction]

Next-to-leading order calculation: $e^+e^- \to q\bar{q}g$



$z = x_g = E_g/E_q$

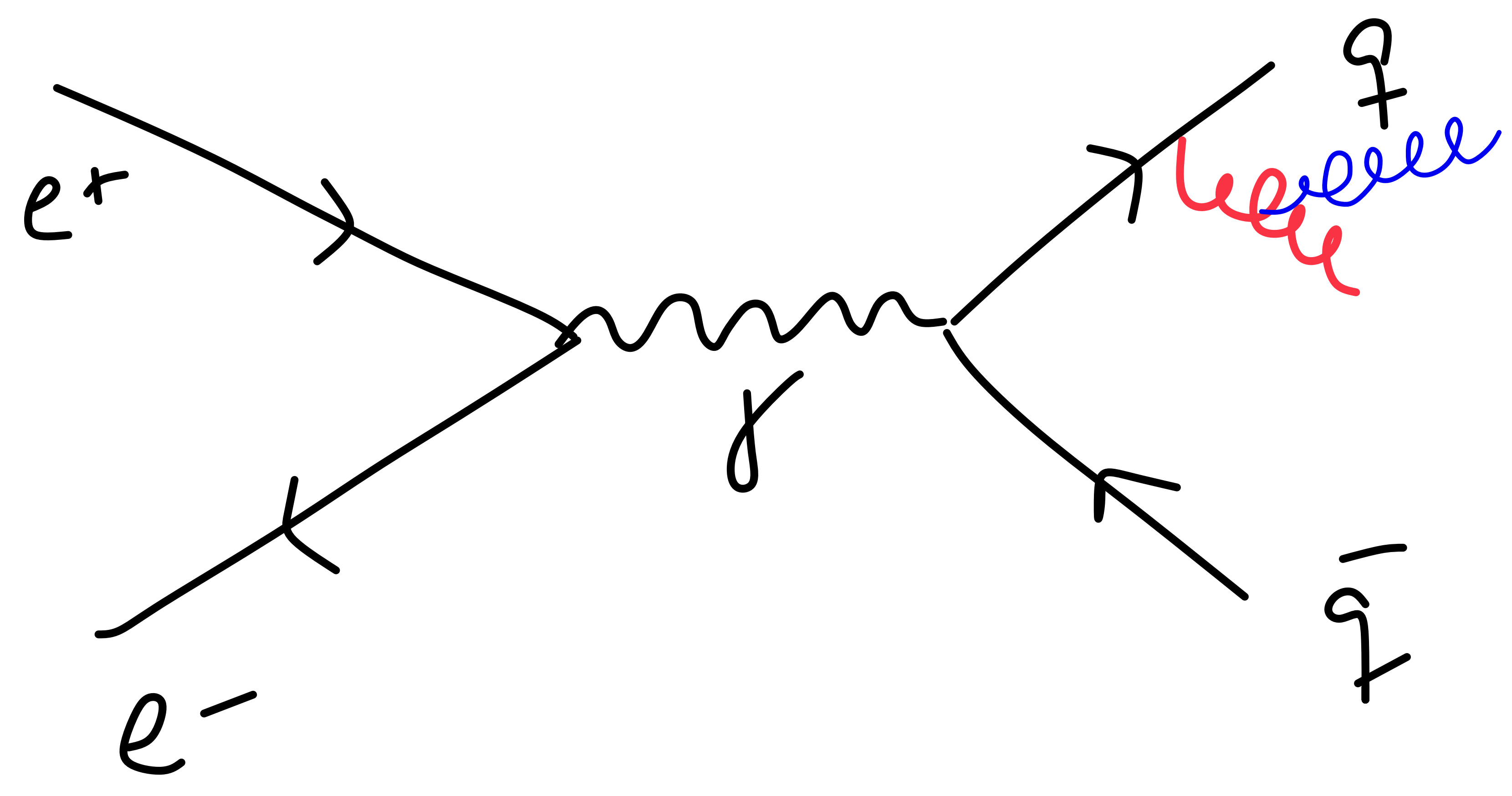
Collinear limit: $d\sigma = (e_q^2 \sigma_0 N_c) \frac{\alpha_s C_F}{2\pi} \underbrace{\frac{1+(1-z)^2}{z}}_{\substack{\text{splitting function} \\ P_{gq}(z)}} dz \underbrace{\frac{d^2\theta}{\theta^2}}_{\text{log divergence}}$

Soft limit: $d\sigma = \underset{\substack{\uparrow \\ \text{factorization}}}{(d\sigma_{q\bar{q}})} \frac{\alpha_s C_F}{2\pi} \underbrace{\frac{k_1 \cdot k_2}{(k_1 \cdot k_3)(k_2 \cdot k_3)}}_{\text{antenna/eikonal factor}} \delta(k_3)\, d^4k_3$

Soft and collinear limit: $d\sigma = (e_q^2 \sigma_0 N_c) \underbrace{\frac{\alpha_s C_F}{2\pi} \frac{dz}{z} \frac{d\theta}{\theta}}_{\alpha_s L^2}$

$\boxed{\text{QCD radiation logarithmically enhanced in soft and collinear limits}}$

Next-to-next-to-leading order calculation: $e^+e^- \to q\bar{q}gg$



Soft and collinear limit:

$$dP_2 = \frac{C_A \alpha_s}{\pi} \frac{dz_2}{z_2} \frac{d\theta_2}{\theta_2} \Theta(\theta_2 < \theta_1) \quad \text{(also factorizes)}$$

C_A ← gluon emission

$$\langle N_{gluon} \rangle \propto \int_{\lambda_{QCD}/E_1}^{\theta_1} \frac{d\theta_2}{\theta_2} \int_{\lambda_{QCD}/\theta_2}^{E_1} \frac{dE_g}{E_g} = \frac{\alpha_s C_A}{\pi} \ln^2 \frac{E_1 \theta_1}{\lambda_{QCD}}$$

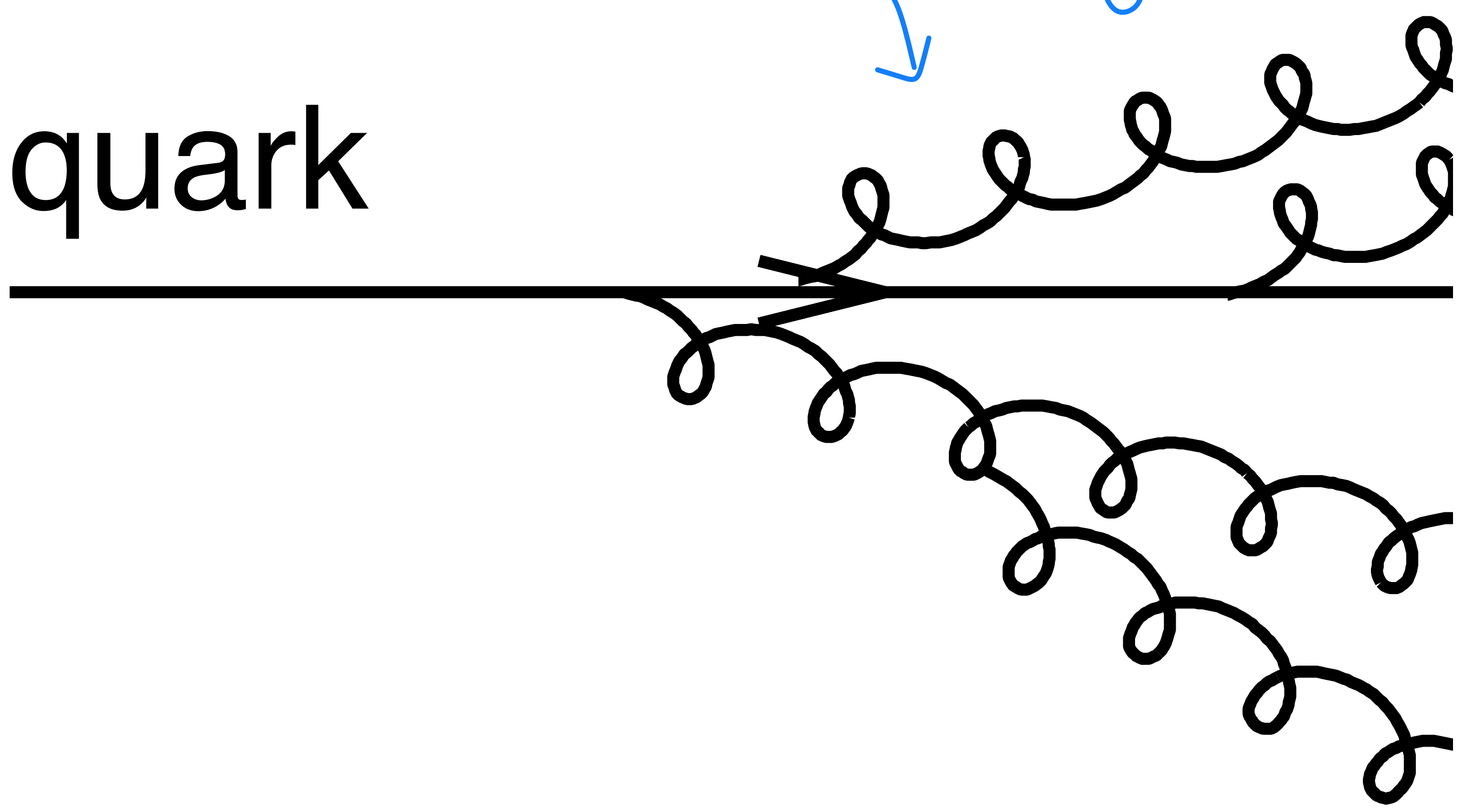
λ_{QCD} ← non-perturbative scale

Additional gluon radiation is angular ordered, i.e. confined within a cone of angle $\theta_2 < \theta_1$. Fundamental property for parton showers.

Why do we see jets? [Adapted from Salam's lectures]

Starting from energetic quark, emit a cascade of many low-energy (soft) and small-angle (collinear) gluons

$$dP = \frac{\alpha_s C_{F/A}}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$$



giving a collimated jet of partons (mostly gluons) that hadronize at $E \sim \Lambda_{QCD}$

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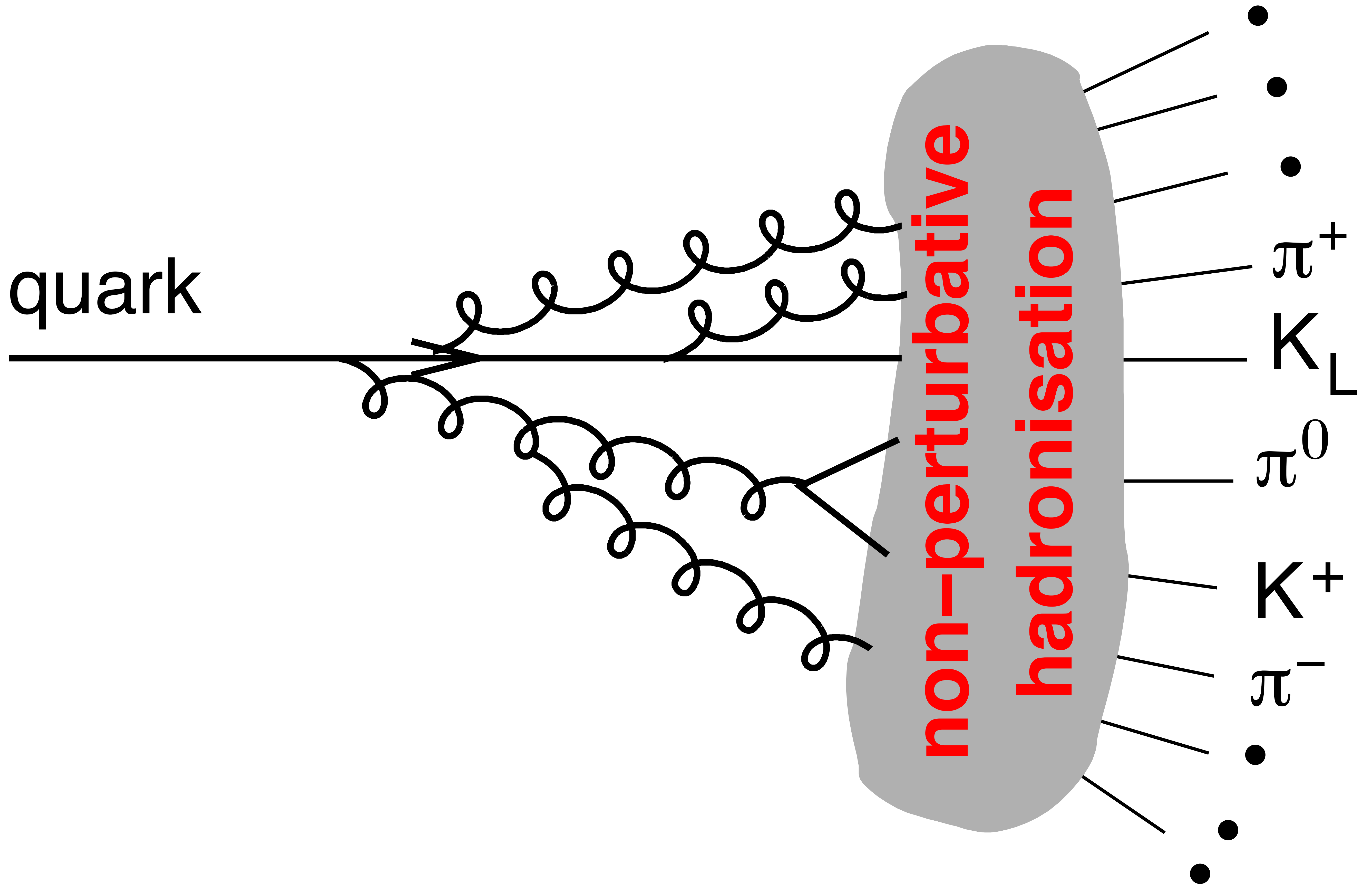

The hadrons go in similar directions to the partons.

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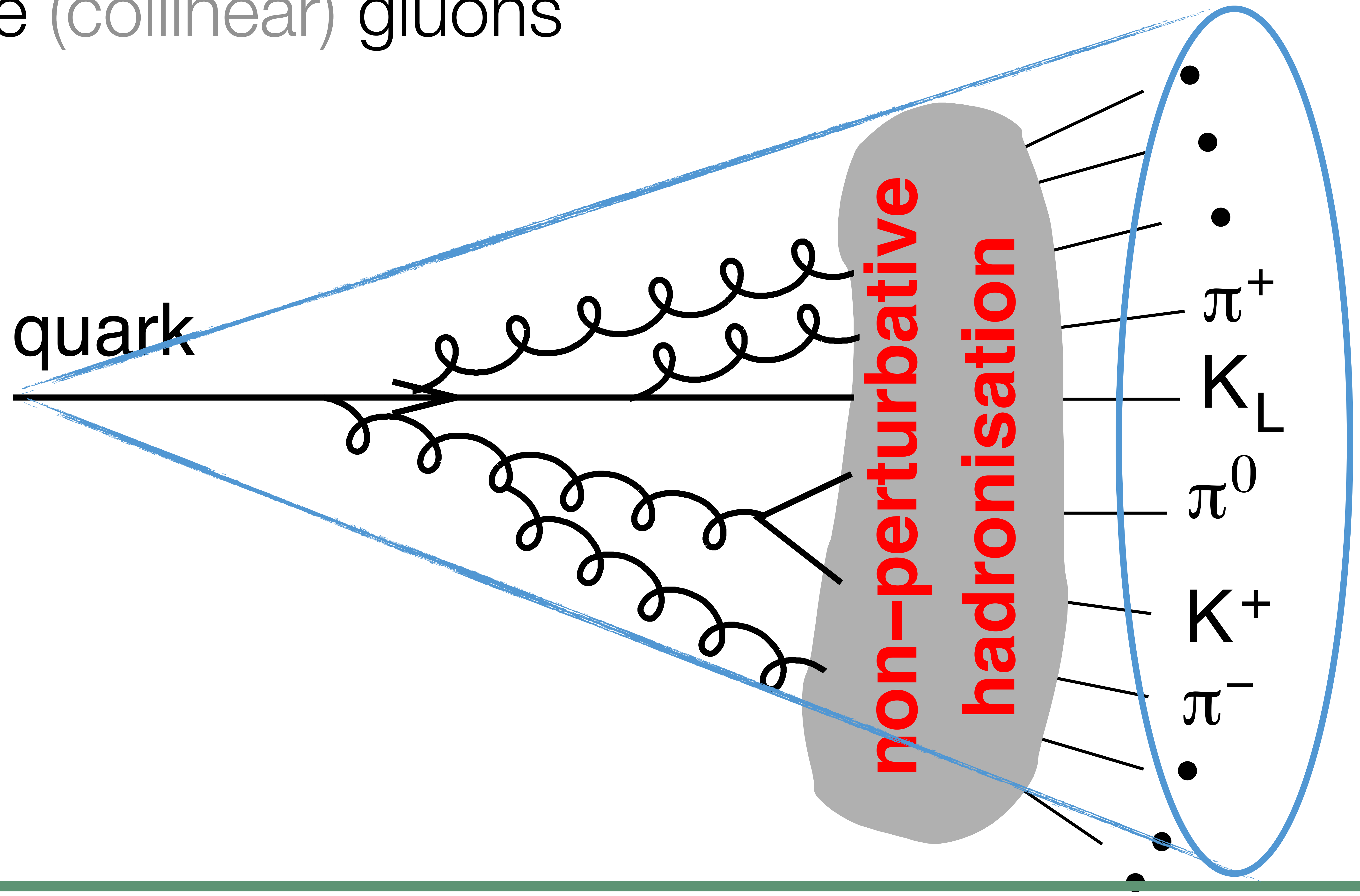

Jets as cones of radius R around QCD radiation

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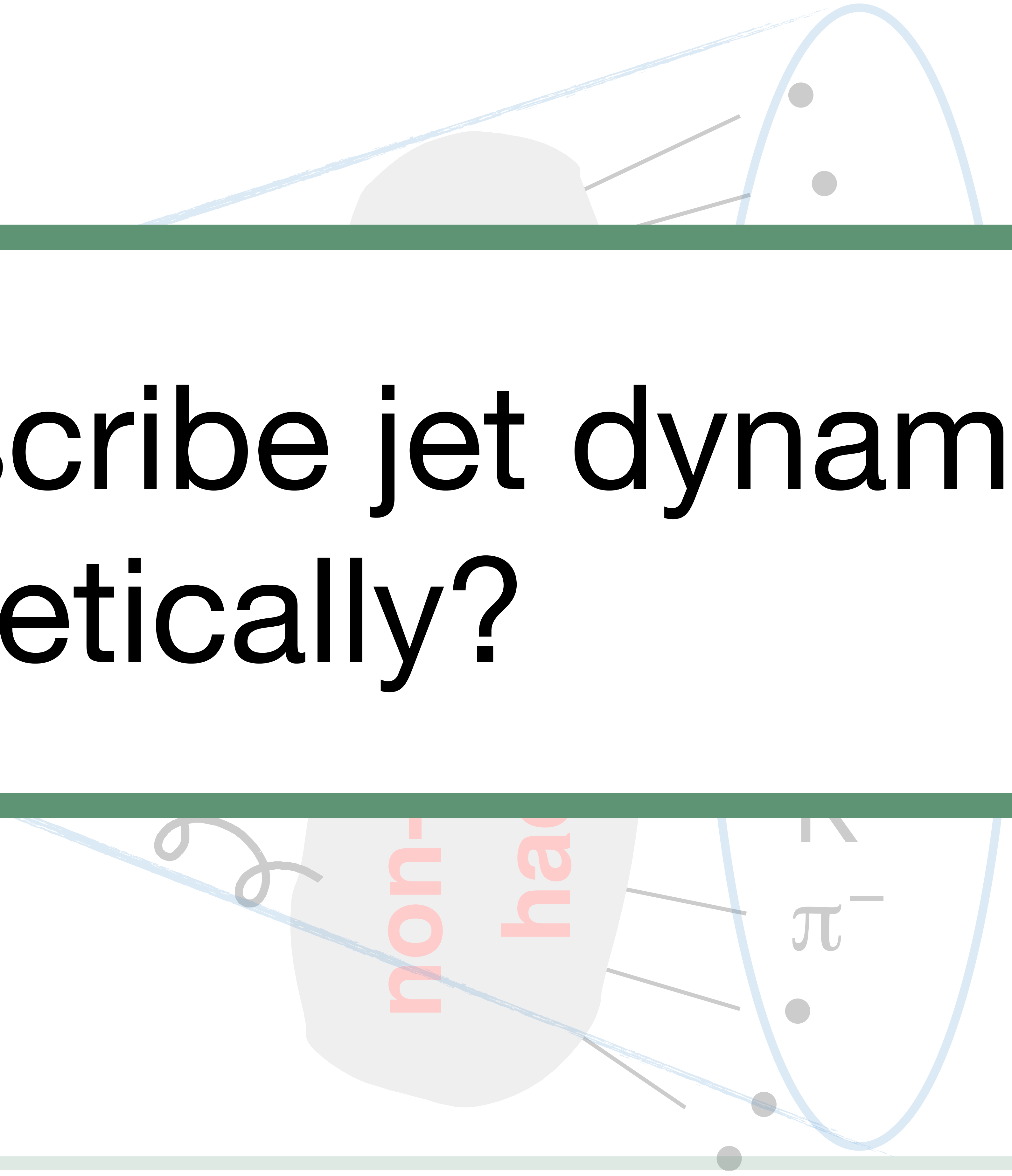
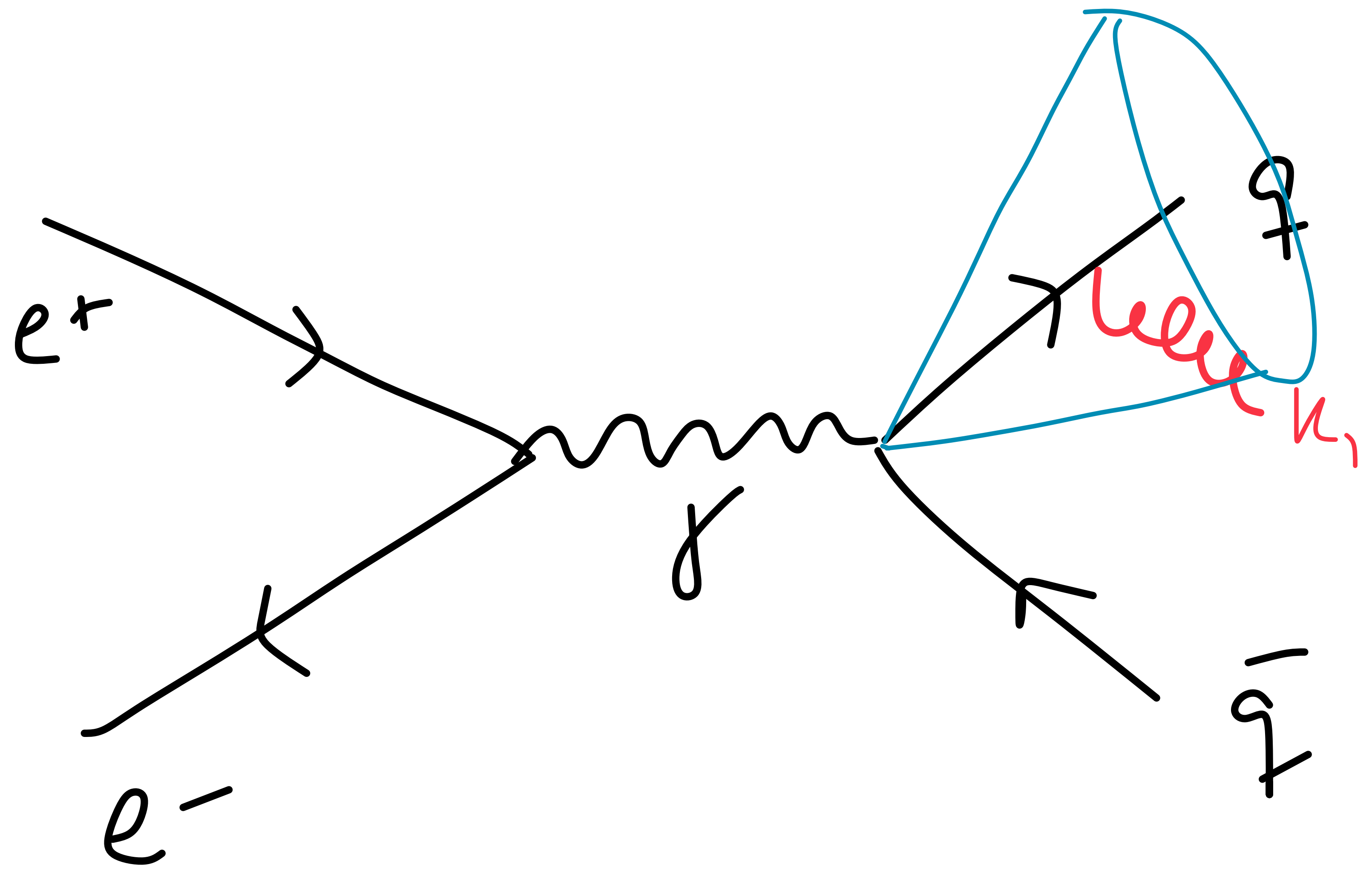


How do we describe jet dynamics theoretically?

Jets as cones of radius R around QCD radiation

Calculating the jet mass at next-to-leading order [Adapted from Marzani et al book]



Definitions: $m^2 = \left(\sum_{i \in jet} k_i\right)^2$; $\Sigma(m^2) = \frac{1}{\sigma}\int_0^{m^2} dm'^2 \frac{d\sigma}{dm'^2} = 1 + \alpha_s \Sigma^{(1)} + \mathcal{O}(\alpha_s^2)$

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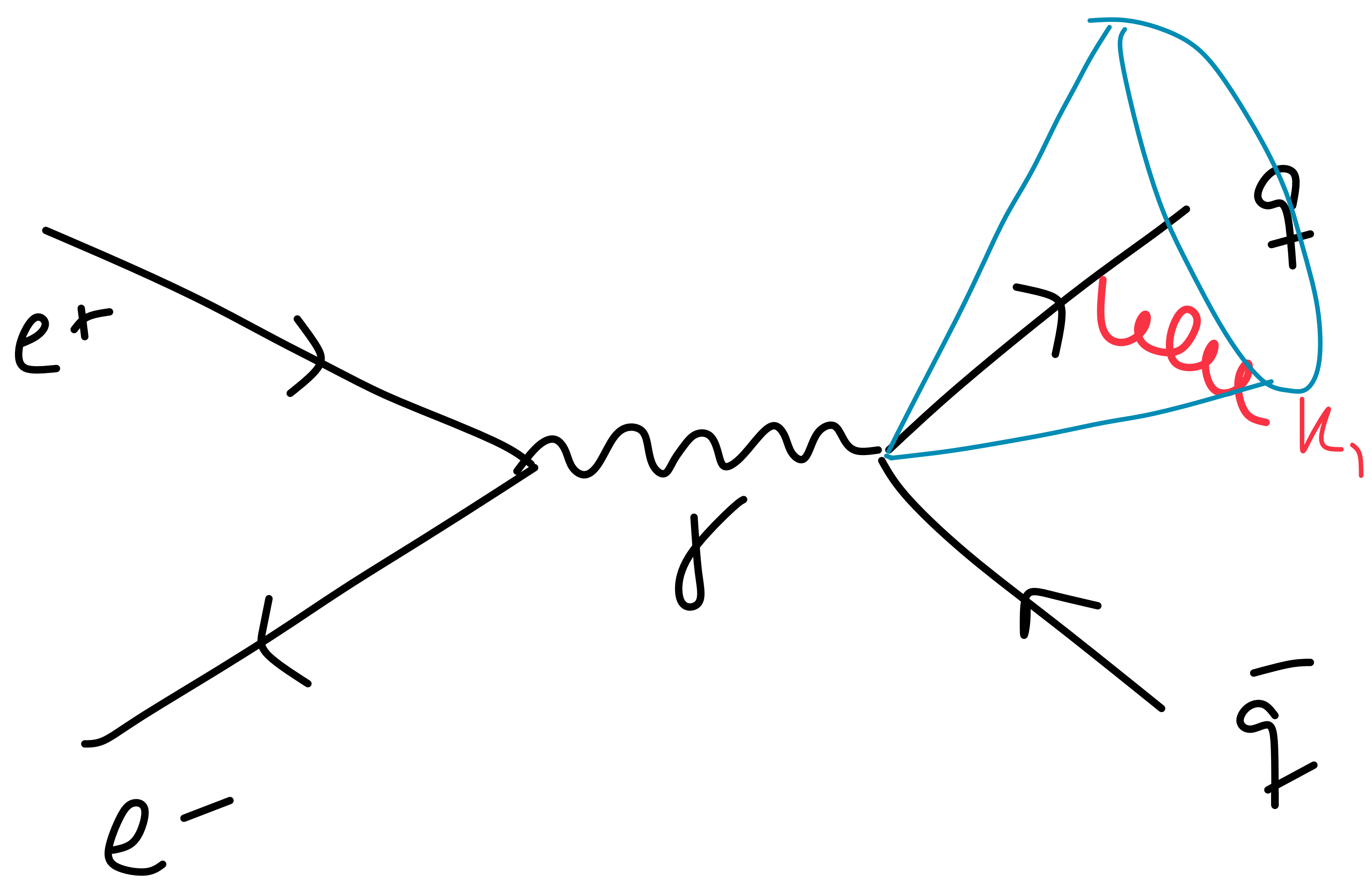

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Soft limit: $|M_R|^2 = \frac{\alpha_s}{2\pi}(2C_F)\frac{k_1\cdot k_2}{(k_1\cdot k_3)(k_2\cdot k_3)}$

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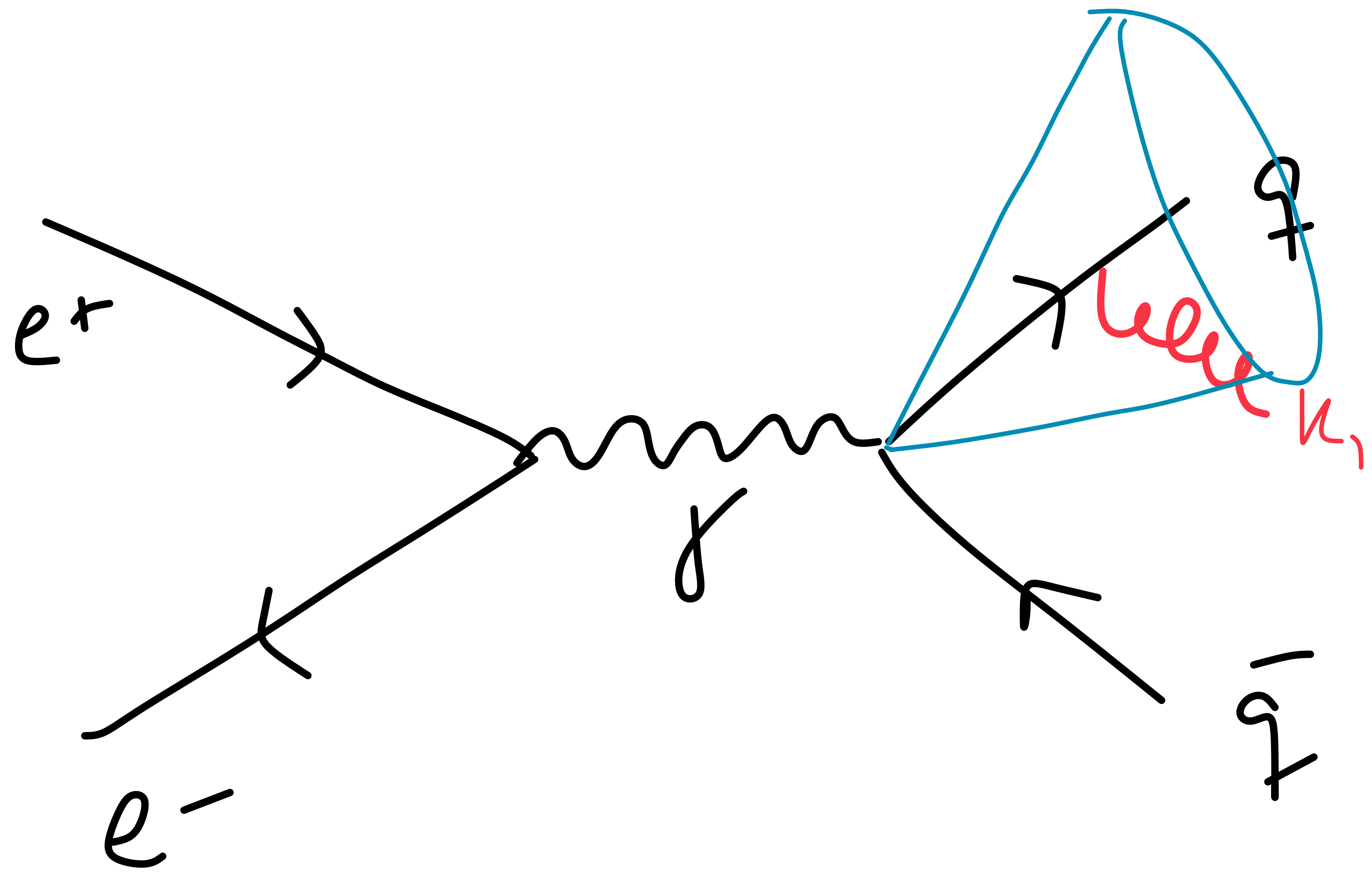

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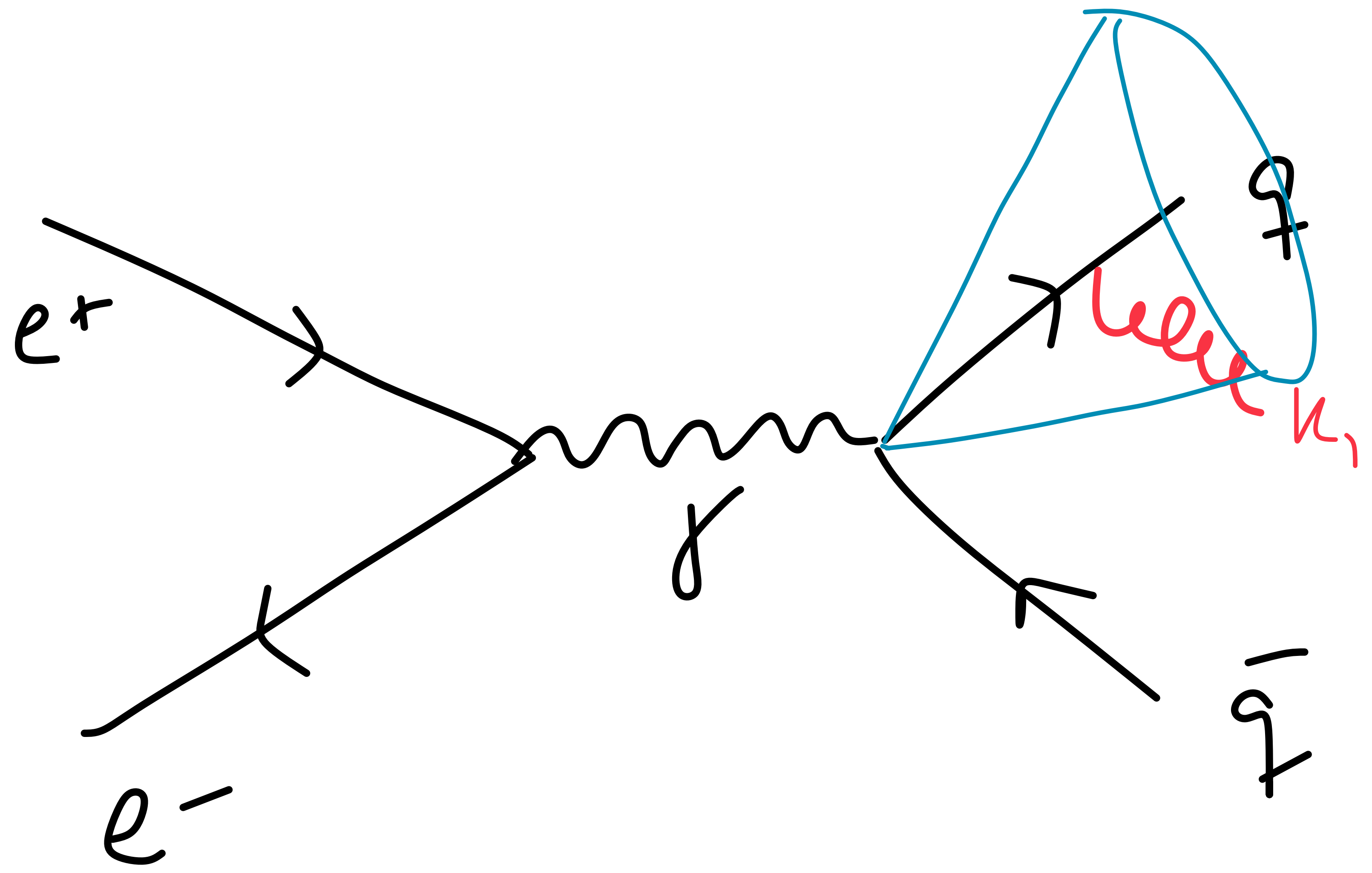
$k_1 = \frac{Q}{2}(1,0,0,1)$

$k_2 = \frac{Q}{2}(1,0,0,-1)$

$k_3 = \omega(1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

Phase-space: $\int d\Phi = \int_0^\infty \omega\, d\omega \int_{-1}^{1} d\cos\theta \int_0^{2\pi} \frac{d\phi}{2\pi}$

Calculating the jet mass at next-to-leading order [Adapted from Marzani et al book]



Definitions: $m^2 = \left(\sum_{i \in jet} k_i\right)^2 ; \quad \Sigma(m^2) = \frac{1}{\sigma}\int_0^{m^2} dm'^2 \frac{d\sigma}{dm'^2} = 1 + \alpha_s \Sigma^{(1)} + \mathcal{O}(\alpha_s^2)$

Soft limit: $|M_R|^2 = \frac{\alpha_s}{2\pi}(2C_F)\frac{k_1 \cdot k_2}{(k_1\cdot k_3)(k_2 \cdot k_3)}$

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Phase-space: $\int d\Phi = \int_0^\infty \omega\, d\omega \int_{-1}^{1} d\cos\theta \int_0^{2\pi}\frac{d\phi}{2\pi}$

$$\alpha_s \Sigma^{(1)}(m^2) = \int_{-1}^{1} d\cos\theta \int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^{Q/2}\omega\, d\omega\, \frac{2C_F\alpha_s}{\pi}\,\frac{1}{\omega^2(1-\cos\theta)(1+\cos\theta)}$$

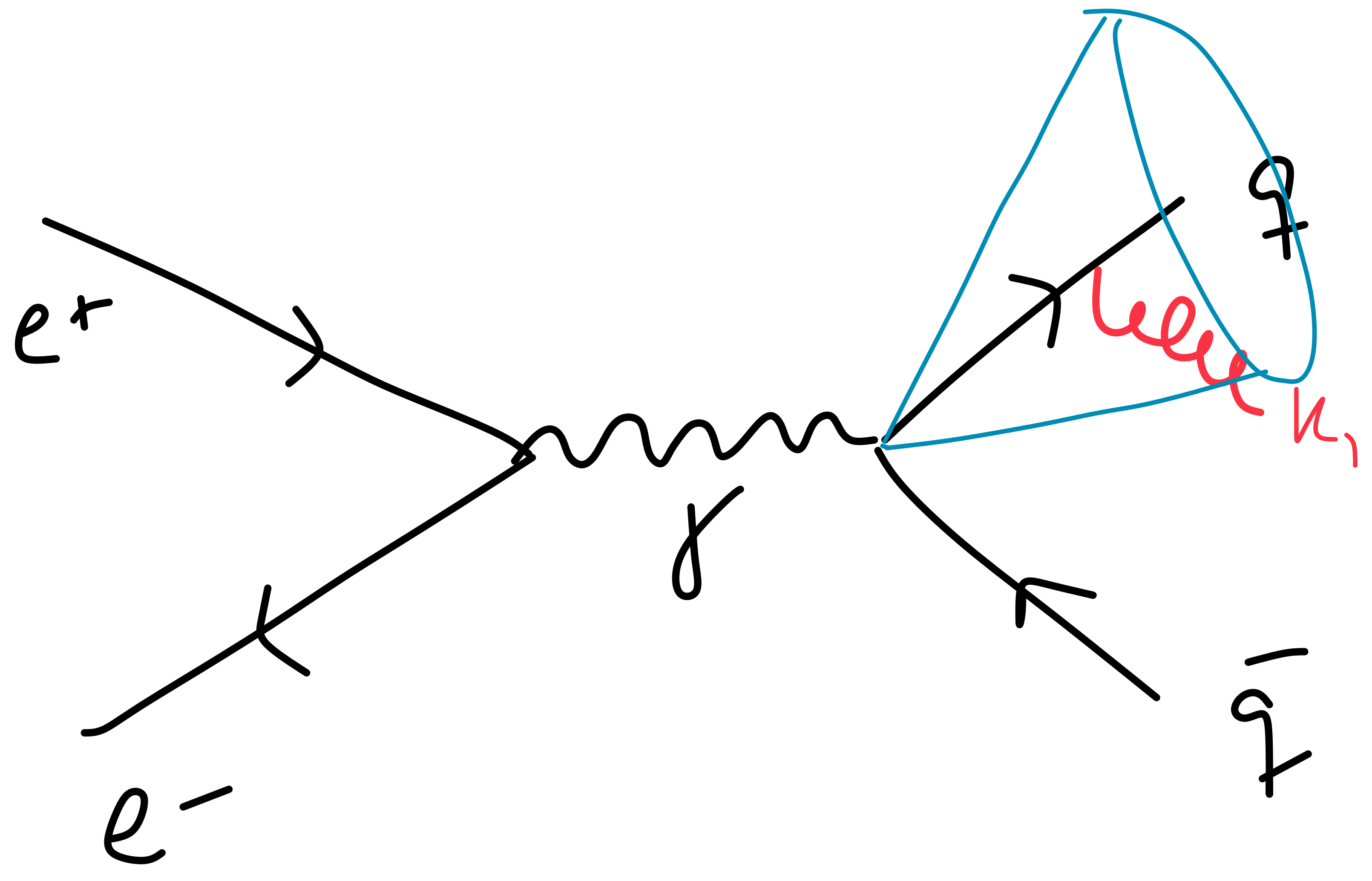
$$\times \left[\underset{\uparrow}{\Theta_{in\text{-}jet}}\,\Theta\left(2\frac{Q\omega}{2}(1-\cos\theta) < m^2\right) + \underset{\uparrow}{\Theta_{out\text{-}jet}} - \underset{\uparrow}{1}\right]$$

$\Theta(1-\cos\theta < 1-\cos R)$

$1-\Theta_{in\text{-}jet}$

Virtual

Calculating the jet mass at next-to-leading order [Adapted from Marzani et al book]



Definitions: $m^2 = \left(\sum_{i \in \text{jet}} k_i\right)^2$; $\Sigma(m^2) = \frac{1}{\sigma}\int_0^{m^2} dm'^2 \frac{d\sigma}{dm'^2} = 1 + \alpha_s \Sigma^{(1)} + \mathcal{O}(\alpha_s^2)$

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Phase-space: $\int d\Phi = \int_0^\infty \omega\, d\omega \int_{-1}^{1} d\cos\theta \int_0^{2\pi} \frac{d\phi}{2\pi}$

$$\alpha_s \Sigma^{(1)}(m^2) = -\frac{\alpha_s C_F}{2\pi} \underbrace{\ln^2\left(\frac{Q^2R^2}{4m^2}\right)}_{\text{double-log enhancement}} + \mathcal{O}(R^2) + \mathcal{O}\left(\frac{m^2}{Q^2}\right) \equiv -\frac{\alpha_s C_F}{2\pi}\ln^2\left(\frac{1}{\rho}\right) \qquad \rho = \frac{4m^2}{Q^2R^2}$$

Adding collinear limit:

$$\boxed{\alpha_s \Sigma^{(1)}(\rho) = \underbrace{-\frac{\alpha_s C_F}{\pi}\left[\frac{1}{2}\ln^2\left(\frac{1}{\rho}\right)\right.}_{\alpha_s^n L^{2n}\,!!} \left. + \overbrace{B_q \ln\left(\frac{1}{\rho}\right)}^{\text{single log correction}}\right]}$$

$$B_q = \int_0^1 dz \left[\frac{P_q(z)}{2C_F} - \frac{1}{z}\right] = -\frac{3}{4}$$

Calculating the jet mass at next-to-leading order [Adapted from Marzani et al book]

Definitions: $m^2 = \left(\sum_{i\in \text{jet}} k_i\right)^2$; $\Sigma(m^2) = \frac{1}{\sigma}\int_0^{m^2} dm'^2 \frac{d\sigma}{dm'^2} = 1 + \alpha_s \Sigma^{(1)} + \mathcal{O}(\alpha_s^2)$

This simple exercise reveals 2 regimes:

$m \sim Q$: perturbative expansion valid

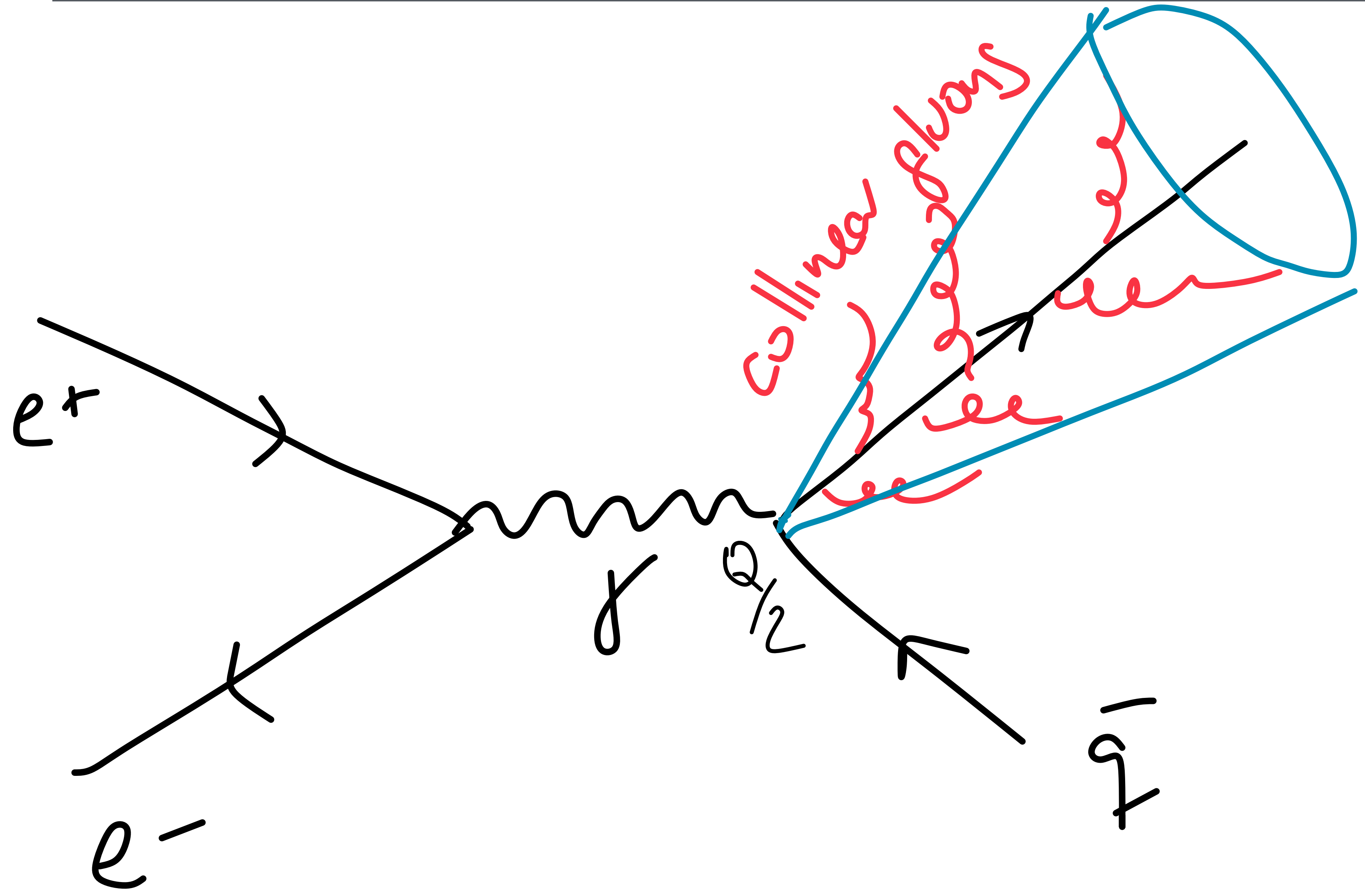
$m \ll Q$: potentially-large logarithms, need to resum them!

Adding collinear limit:

$$\alpha_s \Sigma^{(1)}(\rho) = -\frac{\alpha_s C_F}{\pi}\left[\frac{1}{2}\ln^2\left(\frac{1}{\rho}\right) + B_q \ln\left(\frac{1}{\rho}\right)\right]$$

$$B_q = \int_0^1 dz\left[\frac{P_{gq}(z)}{2C_F} - \frac{1}{z}\right] = -\frac{3}{4}$$

Calculating the jet mass at leading-log accuracy [Adapted from Marzani et al book]



Collinear limit:

$$m^2 = 2\sum_{(i<j)\in\text{jet}} k_i\cdot k_j = \sum_{(i<j)\in\text{jet}} \omega_i\omega_j\theta_{ij}^2 + \mathcal{O}(\theta_{ij}^4) = \frac{Q}{2}\sum_i \omega_i\theta_i^2$$

All-orders expression:

$$\Sigma(\rho) = \sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}\int\frac{d\theta_i^2}{\theta_i^2}\int dz_i\, P_q(z_i)\,\frac{\alpha_s\left(z_i\theta_i\frac{Q}{2}\right)}{2\pi}\,\Theta_{i\in\text{jet}}\,\Theta\left(\sum_{i=1}^{n} z_i\frac{\theta_i^2}{R^2}<\rho\right)$$

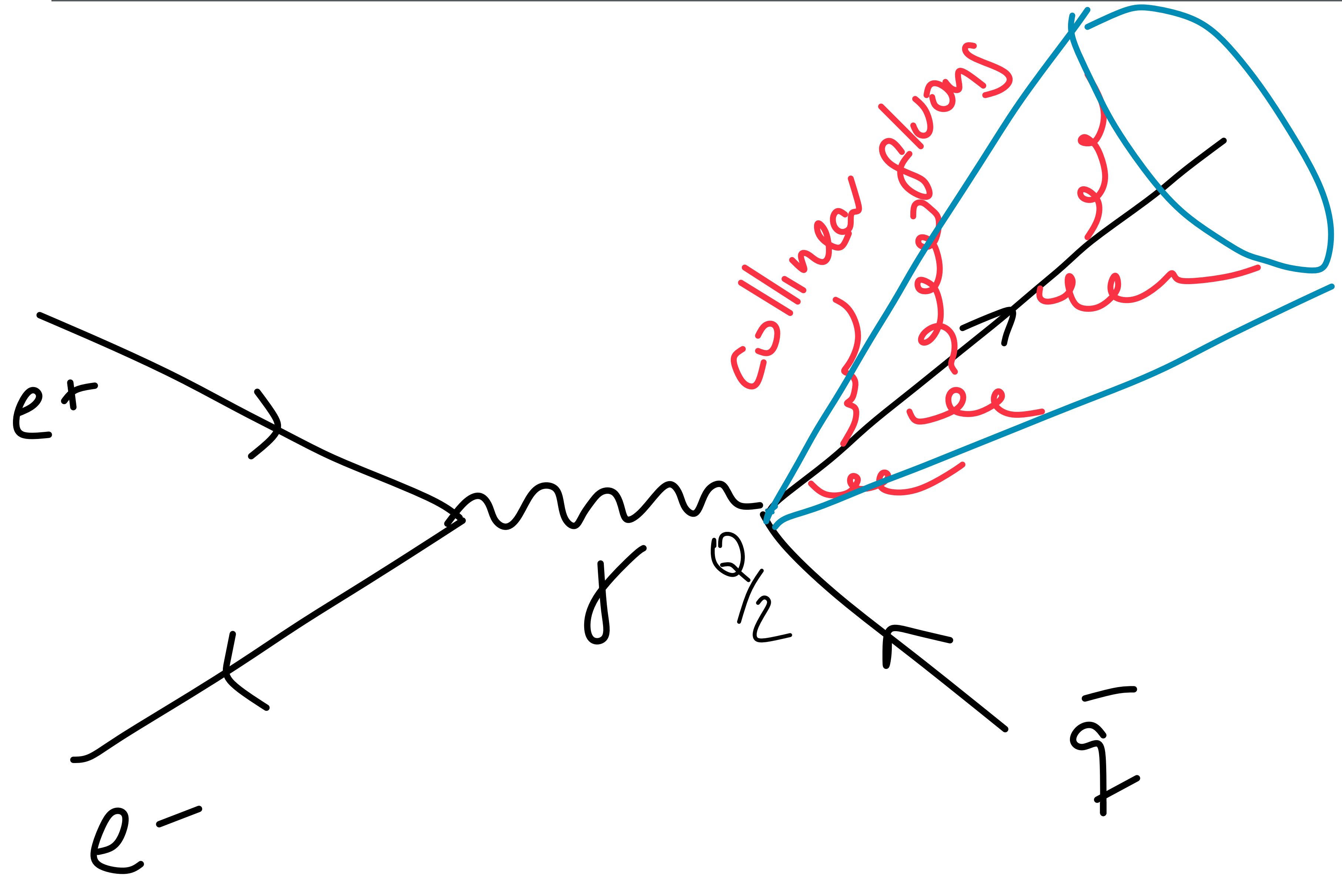
$\Theta(\theta_i<R)$

$$+\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}\int\frac{d\theta_i^2}{\theta_i^2}\int dz_i\, P_q(z_i)\,\frac{\alpha_s\left(z_i\theta_i Q/2\right)}{2\pi}\left[\Theta_{i\notin\text{jet}}-1\right]$$

virtual

angular ordering

Calculating the jet mass at leading-log accuracy [Adapted from Marzani et al book]



Leading-log accuracy = strong ordering

$$E_1 \gg E_2 \gg \ldots \gg E_n$$
$$\theta_1 \gg \theta_2 \gg \ldots \gg \theta_n$$

$\Big\}$ $z_i \theta_i^2$ also ordered

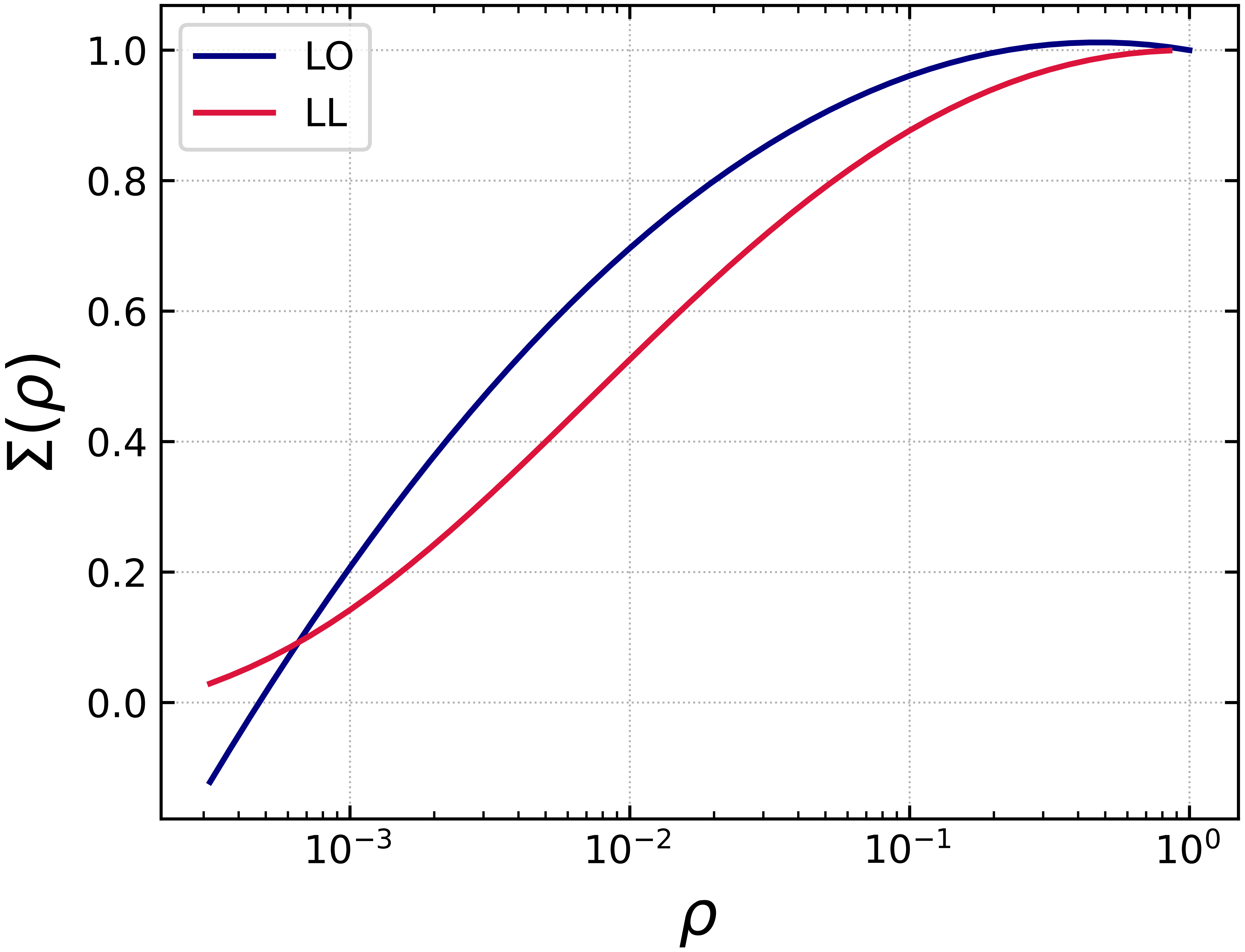
$$\Theta\left(\sum_{i=1}^{n} \rho_i < \rho\right) \simeq \underbrace{\Theta\left(\max_i \rho_i < \rho\right)}_{\text{a single emission dominates the result}} = \prod_{i=1}^{n} \Theta(\rho_i < \rho)$$

The cumulative distribution at leading-log reads

$$\Sigma^{LL}(\rho) = -\sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int \frac{d\rho_i}{\rho_i} \int dz_i\, P_q(z_i)\, \frac{\alpha_s\left(\sqrt{z_i \rho_i}\, \frac{QR}{2}\right)}{2\pi}\, \Theta(\theta_i < R)\, \Theta(\rho_i > \rho)$$

$$= \exp\left[-\int_{\rho}^{1} \frac{d\rho'}{\rho'} \int dz\, P_q(z)\, \frac{\alpha_s\left(\sqrt{\rho' z}\, \frac{QR}{2}\right)}{2\pi}\right]$$ = Sudakov exponent

Fixed-order vs resummation at lowest order

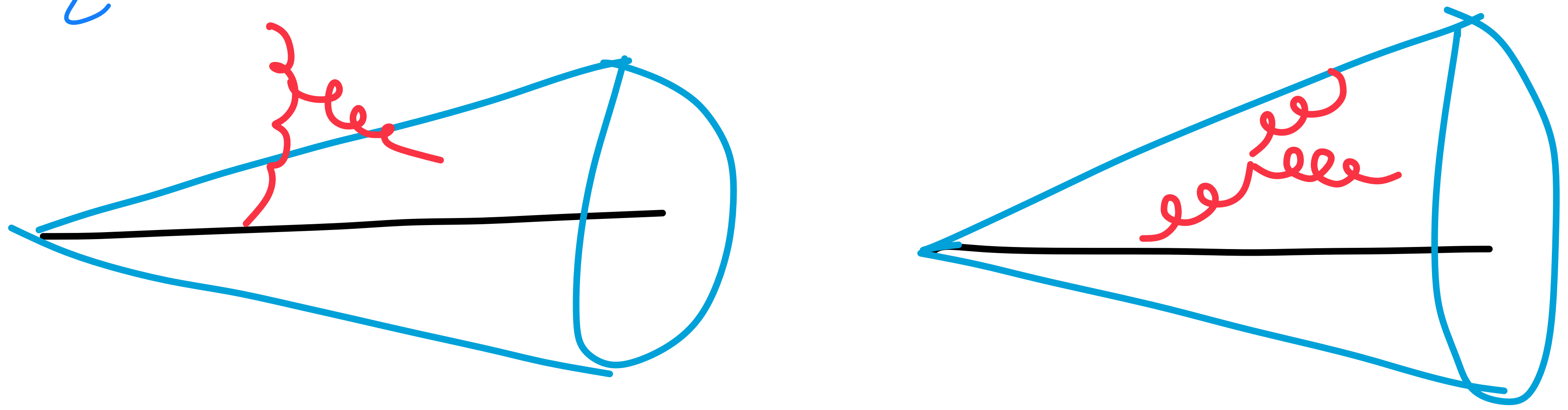


Resummation cures the divergence when $\rho \ll 1$

Dynamics beyond leading-log accuracy for the jet mass

So far, we have considered emissions to be soft and collinear. Corrections

- Collinear but not soft emissions $\frac{1}{z} \to P(z)$
- Soft but not collinear emissions



- Correction to observable definition $\Theta\left(\sum_{i=1}^{n} \rho_i < \rho\right) \neq \Theta\left(\max_i \rho_i < \rho\right)$
- Running coupling at two loops β_0, β_1
- Much more beyond NLL!

Dynamics beyond leading-log accuracy for the jet mass

So far, we have considered emissions to be soft and collinear.

- Collinear but not soft emissions
- S
- C
- Running coupling at two loops
- Much more beyond NLL!

Is there a way of automating logarithmic resummation? YES!

Parton shower basics: example of radioactive decay

[Adapted from Gavin Salam]

Consider decay rate μ per unit time, total time t_{max}. Find distribution of emissions

$$\frac{dP_n}{dt} = -\mu P_n(t) \qquad n \to n+1$$

How to solve this with Monte Carlo methods?

(a) start with $n = 0,\ t_0 = 0$

(b) choose random number $r (0 < r < 1)$ and find t_{n+1} that satisfies

$$r = e^{-\mu(t_{n+1} - t_n)}$$

(c) if $t_{n+1} < t_{\text{max}}$, increment n go to step (b)

Parton shower basics: example of radioactive decay

E.g. for decay rate $\mu = 1$, $t_{\max} = 2$

- start with $n = 0,\ t_0 = 0$
- $r = 0.6 \to t_1 = t_0 + \ln(1/r) = 0.51$ [emission 1]
- $r = 0.3 \to t_2 = t_1 + \ln(1/r) = 1.71$ [emission 2]
- $r = 0.4 \to t_3 = t_2 + \ln(1/r) = 2.63$ [$> t_{\max}$, stop]

How to solve this with Monte Carlo methods?

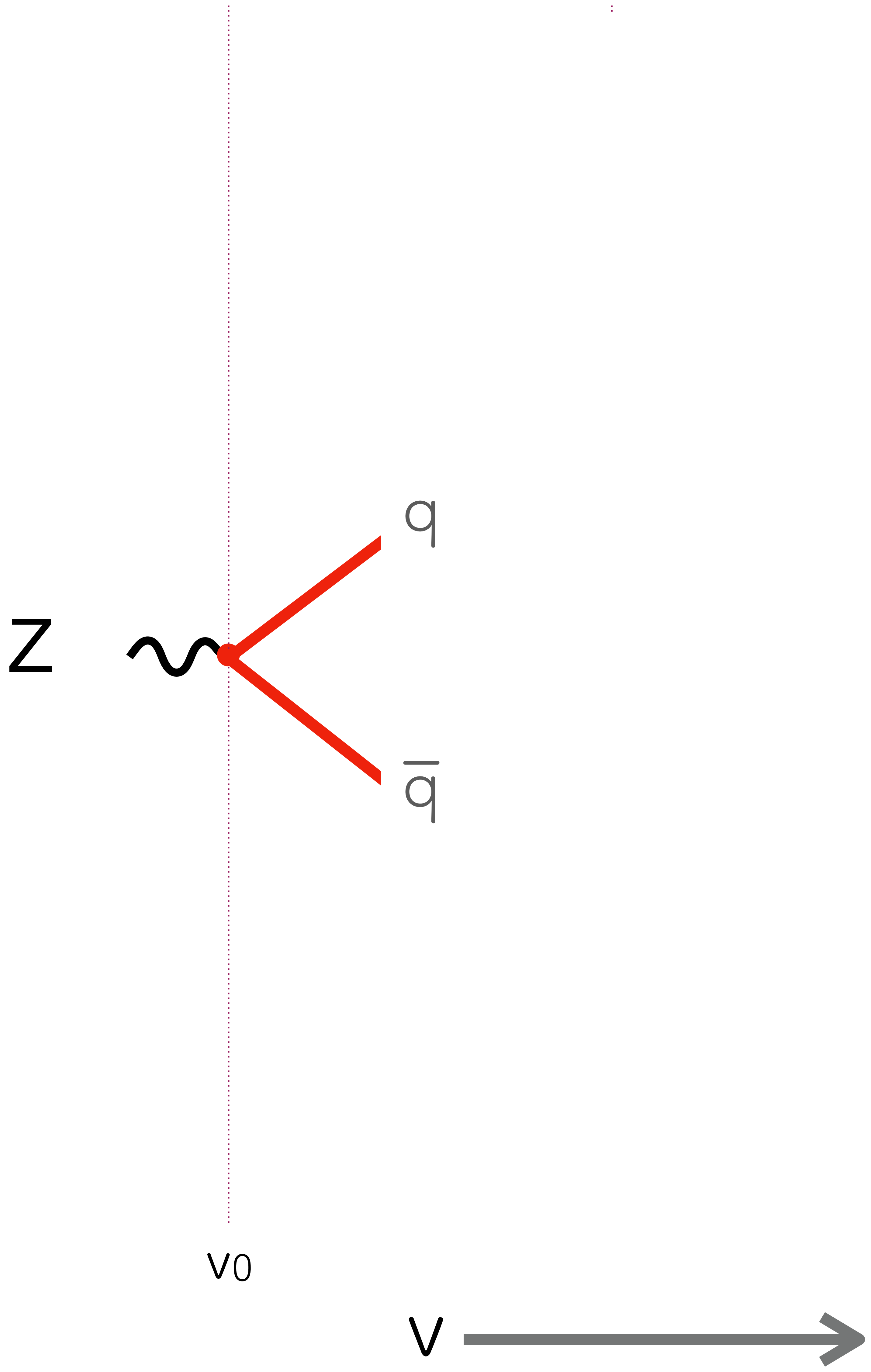
(a) start with $n = 0,\ t_0 = 0$

(b) choose random number $r (0 < r < 1)$ and find t_{n+1} that satisfies

$$r = e^{-\mu(t_{n+1} - t_n)}$$

(c) if $t_{n+1} < t_{\max}$, increment n go to step (b)

QCD shower: an evolution equation



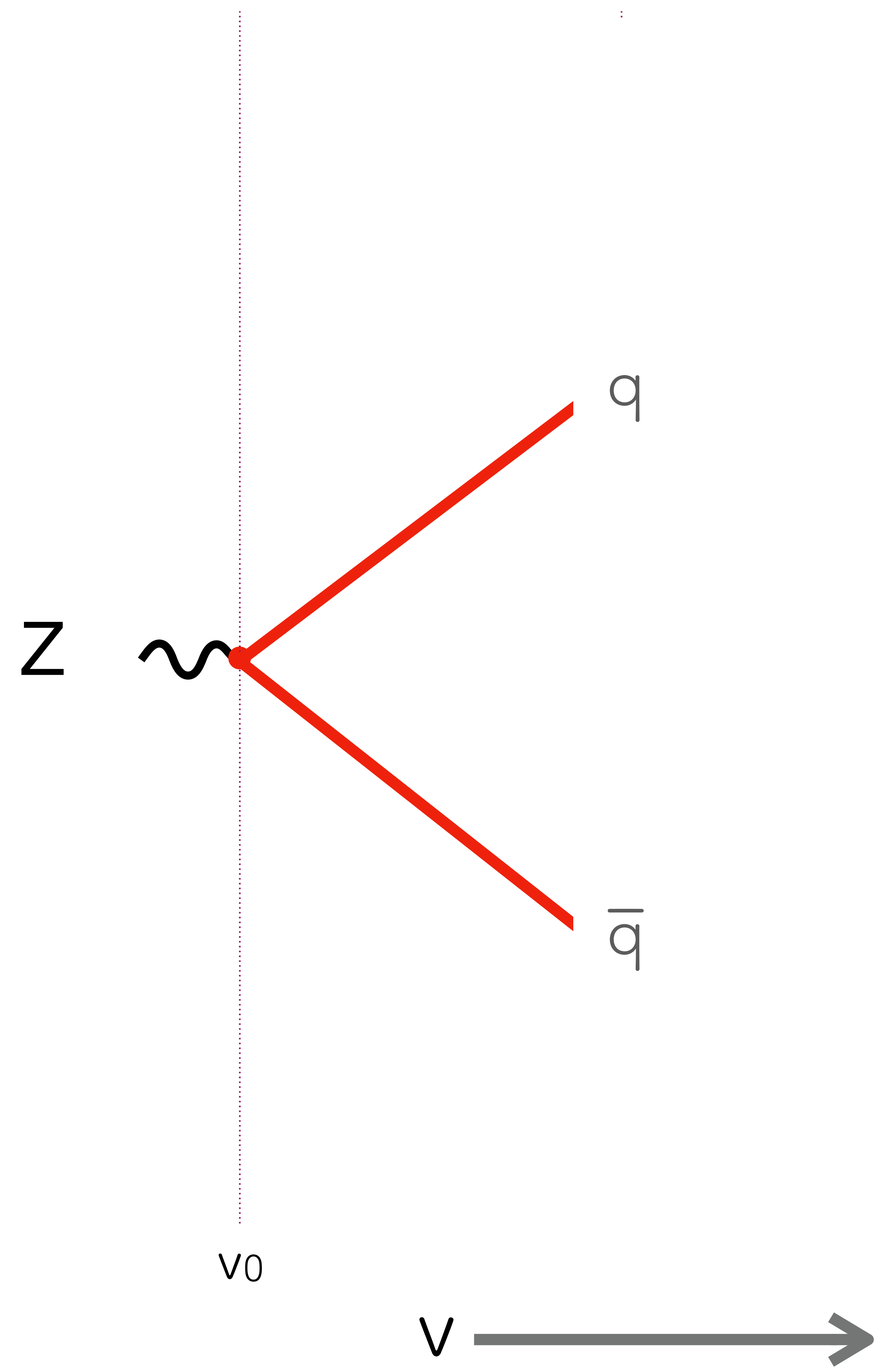
Start with $q\bar{q}$ state.

Throw a random number to determine down to what scale, v, state persists unchanged

$$\frac{\mathrm{d}P_2(v)}{\mathrm{d}v} = -f^{q\bar{q}}_{2\to 3}(v)\,P_2(v)$$

- Evolution variable: $v = k_t, \theta, t_f$

QCD shower: an evolution equation



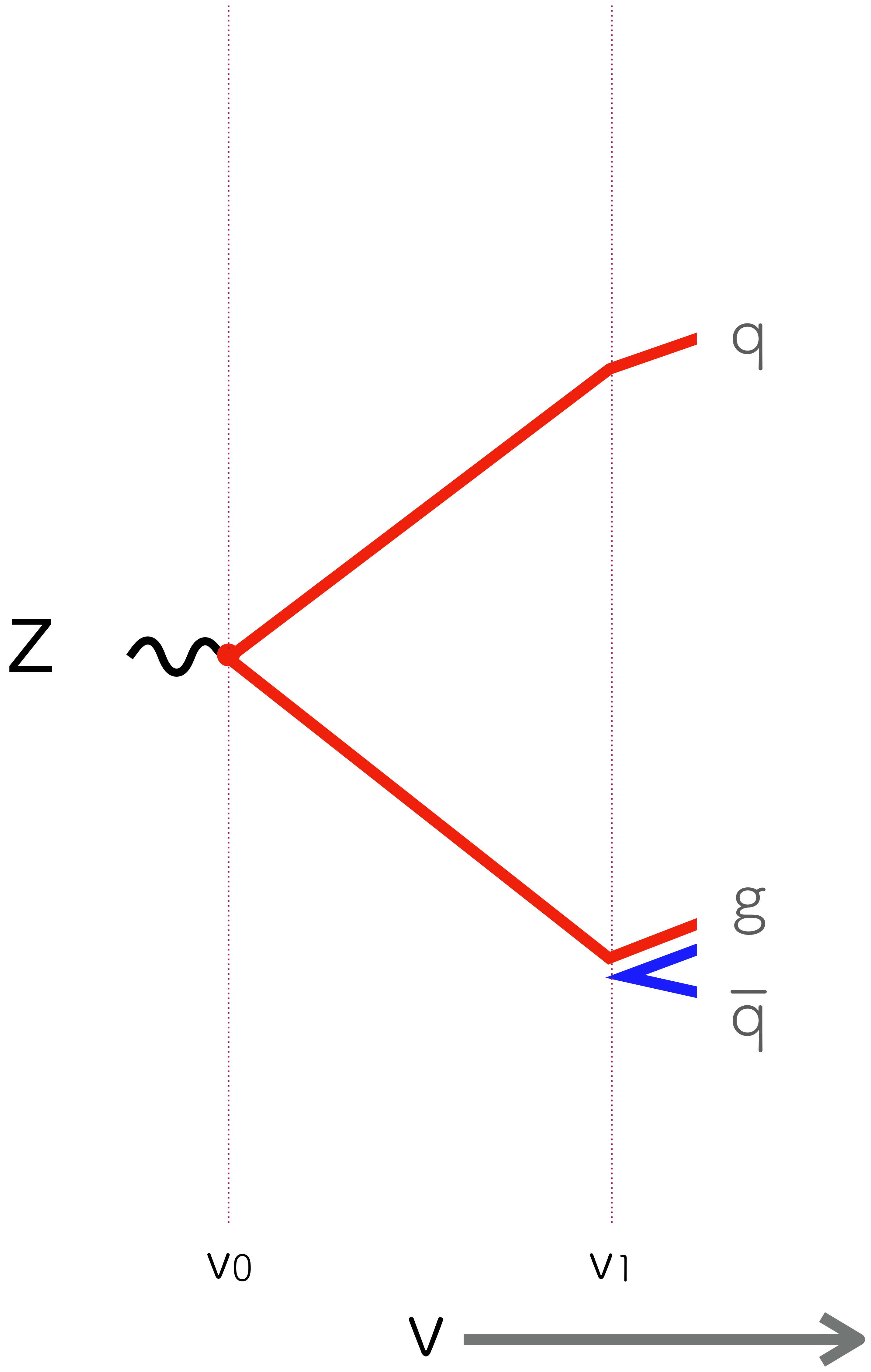
Start with $q\bar{q}$ state.

Throw a random number to determine down to what scale, v, state persists unchanged

$$\frac{\mathrm{d}P_2(v)}{\mathrm{d}v} = -f_{2\to 3}^{q\bar{q}}(v)\,P_2(v)$$

- Evolution variable: $v = k_t, \theta, t_f$

QCD shower: an evolution equation

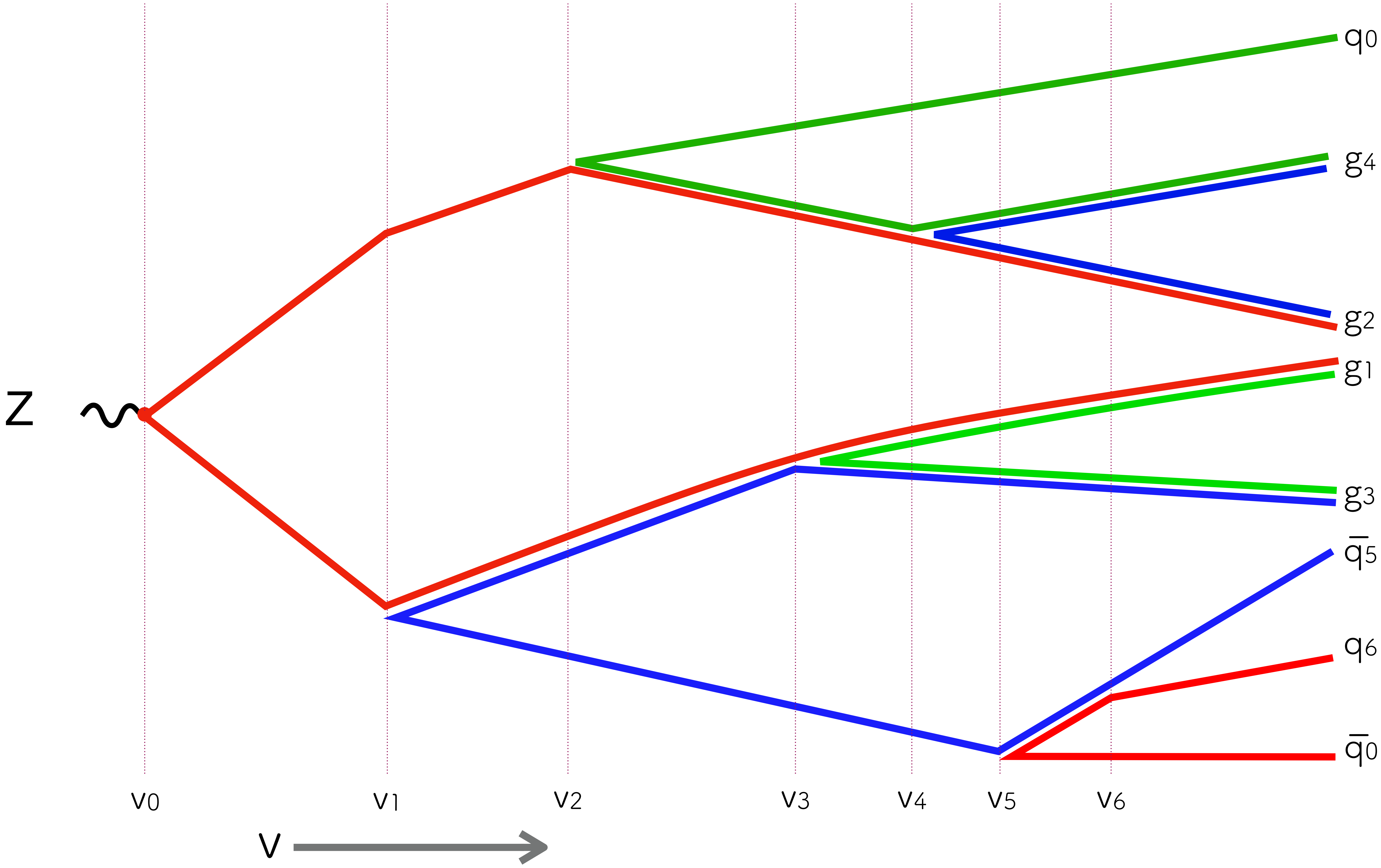


At some point, state splits (2→3, i.e. emits gluon). Evolution equation changes

$$\frac{\mathrm{d}P_3(v)}{\mathrm{d}v} = -\left[f_{2\to 3}^{qg}(v) + f_{2\to 3}^{g\bar{q}}(v)\right] P_3(v)$$

- Recoil scheme: $\tilde{p}_{q,\bar{q}} \to p_{q,\bar{q},g}$

QCD shower: an evolution equation

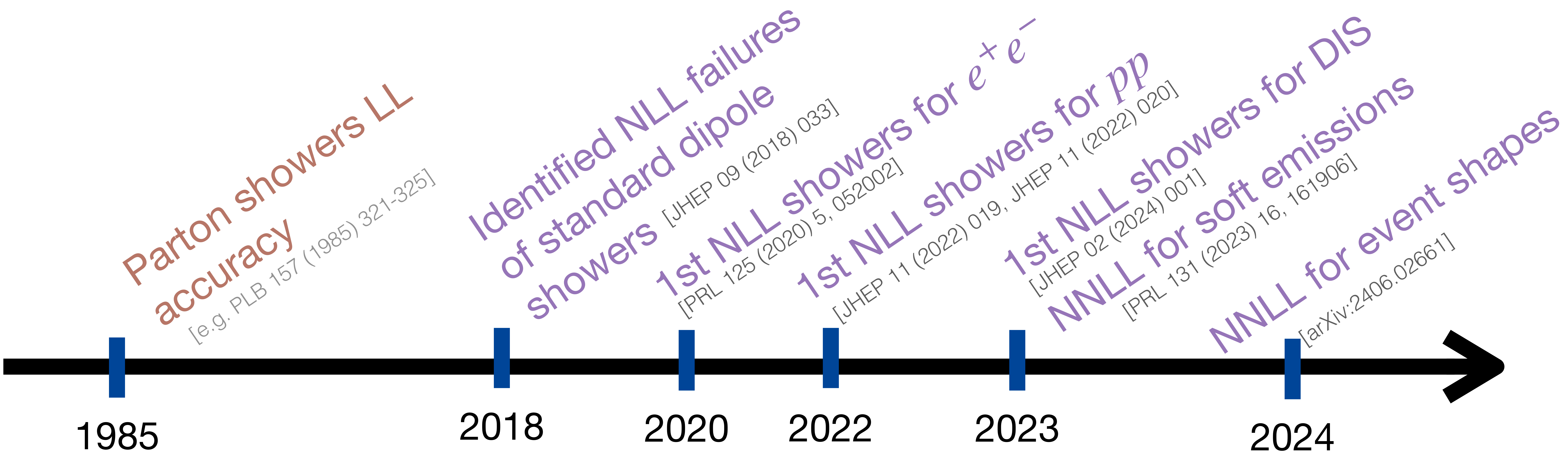


$(E^1, p_x^1, p_y^1, p_z^1)$

$(E^n, p_x^n, p_y^n, p_z^n)$

self-similar evolution continues until it reaches a non-perturbative scale

Current status of parton shower development



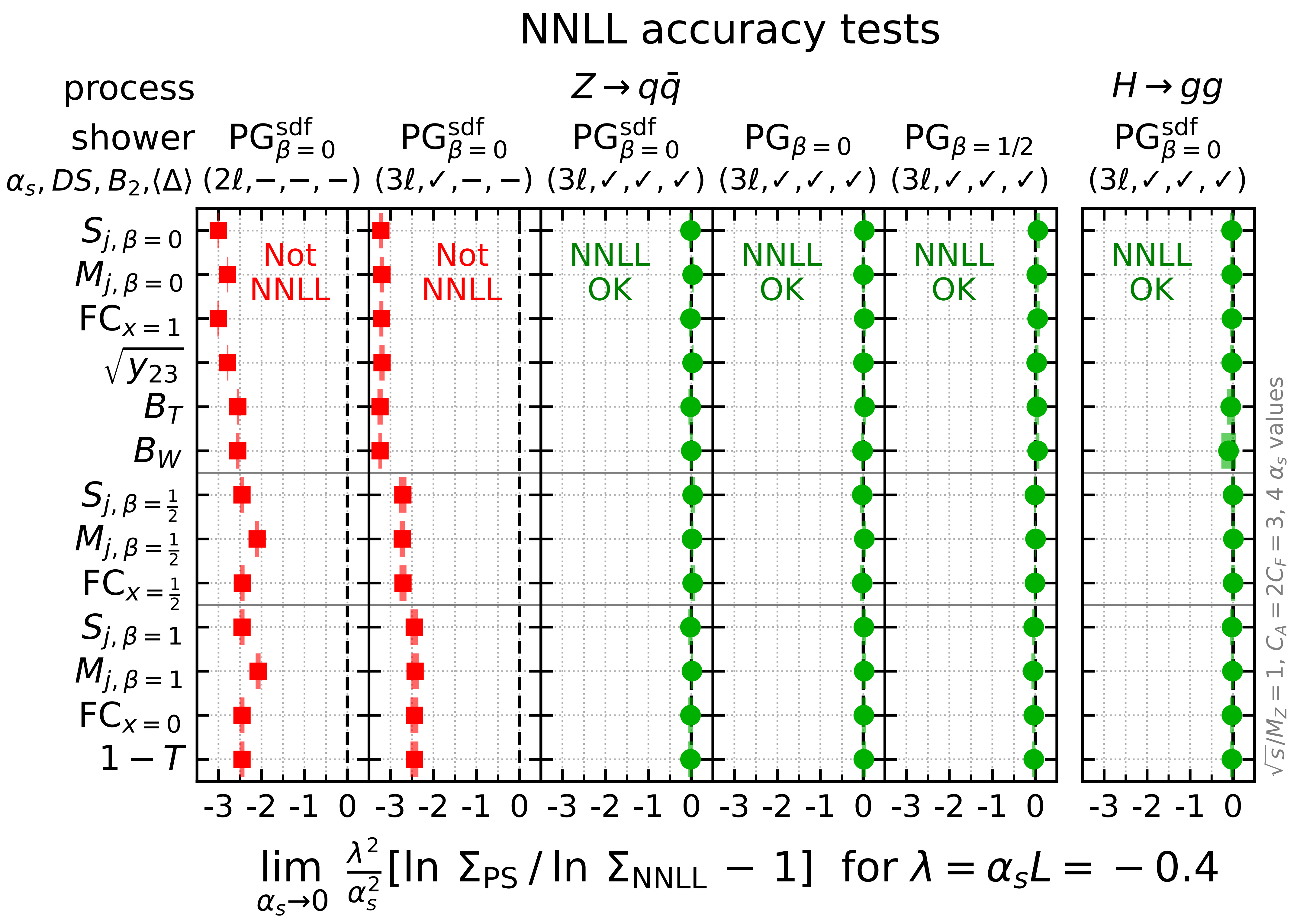
[PanScales code]

Note work by other groups: JHEP 09 (2020) 014, PRD 104 (2021) 5, 054049, JHEP 10 (2023) 091

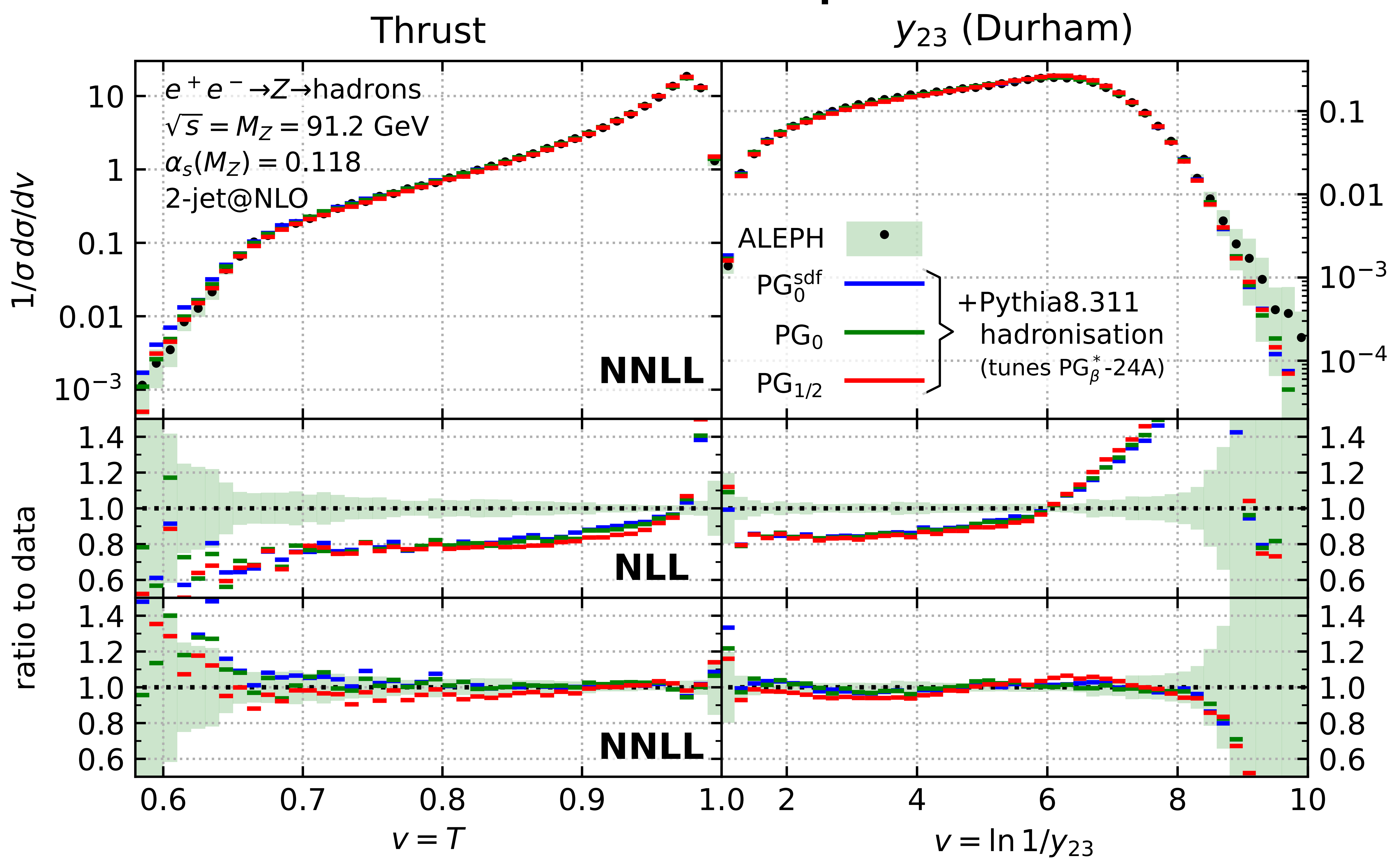
Understanding and improving the accuracy of parton showers is a very active field of research

A single tool to resum them all!

[PanScales collaboration, arXiv:2406.02661]



Pheno impact



NNLL parton showers for e^+e^- collisions is the new standard

Some conclusions

- Jets are a consequence of the soft & collinear enhancements of gluon emission (even at small coupling), followed by hadronisation

- Theoretical tools to understand jet dynamics

Simulation: Parton showers ⟷ **Analytic approach: Logarithmic resummation**

- Tomorrow: jets are not just rigid cones!