High-Energy QCD and Diffraction
Part II

Francesco Giovanni Celiberto, UAH Madrid

Madrid UAH

talento cm Programa de atracción de talento investigador Comunidad de Madrid

ANIVERSARIO PATRIMONIO MUNDIAL

Table of contents

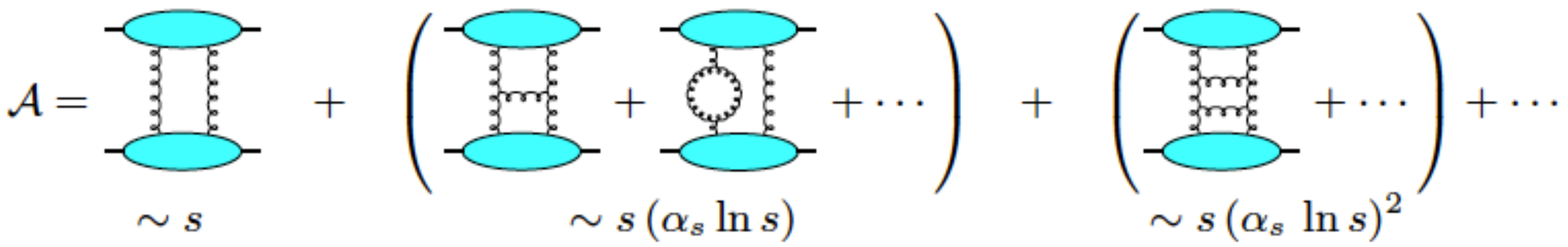
Table of contents

Lecture I Highlights

BFKL resummation: [V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977); Y.Y. Balitskii, L.N. Lipatov (1978)]

$\xrightarrow{\text{based on}}$ **gluon Reggeization**

leading logarithmic approximation (LL): $\alpha_s^n (\ln s)^n$



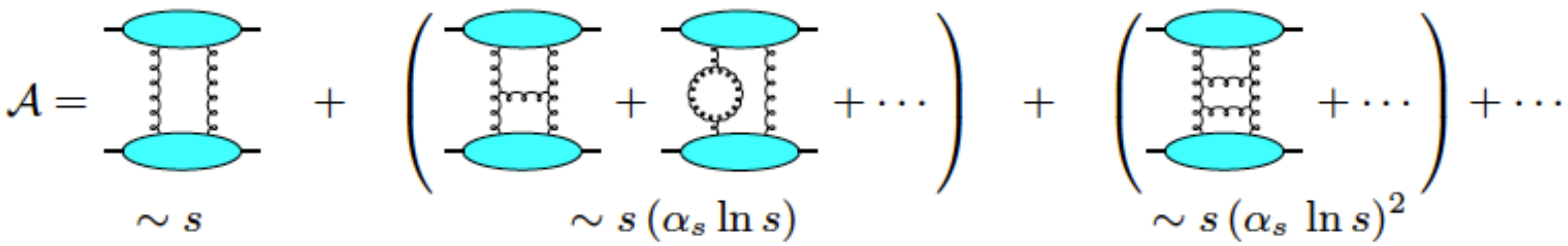
next-to-leading logarithmic approximation (NLL): $\alpha_s^{n+1} (\ln s)^n$

The high-energy resummation

BFKL resummation: [V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977); Y.Y. Balitskii, L.N. Lipatov (1978)]

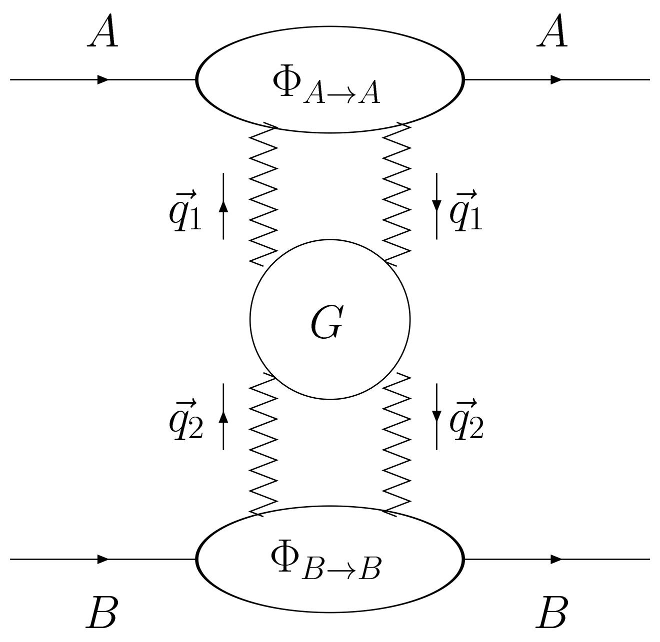
$\xrightarrow{\text{based on}}$ **gluon Reggeization**

leading logarithmic approximation (LL): $\alpha_s^n (\ln s)^n$



next-to-leading logarithmic approximation (NLL): $\alpha_s^{n+1} (\ln s)^n$

Total cross section for $A + B \to X$: $\sigma_{AB}(s) = \frac{\mathcal{I}m_s\{\mathcal{A}_{AB}^{AB}\}}{s}$ $\Leftarrow$ *optical theorem*



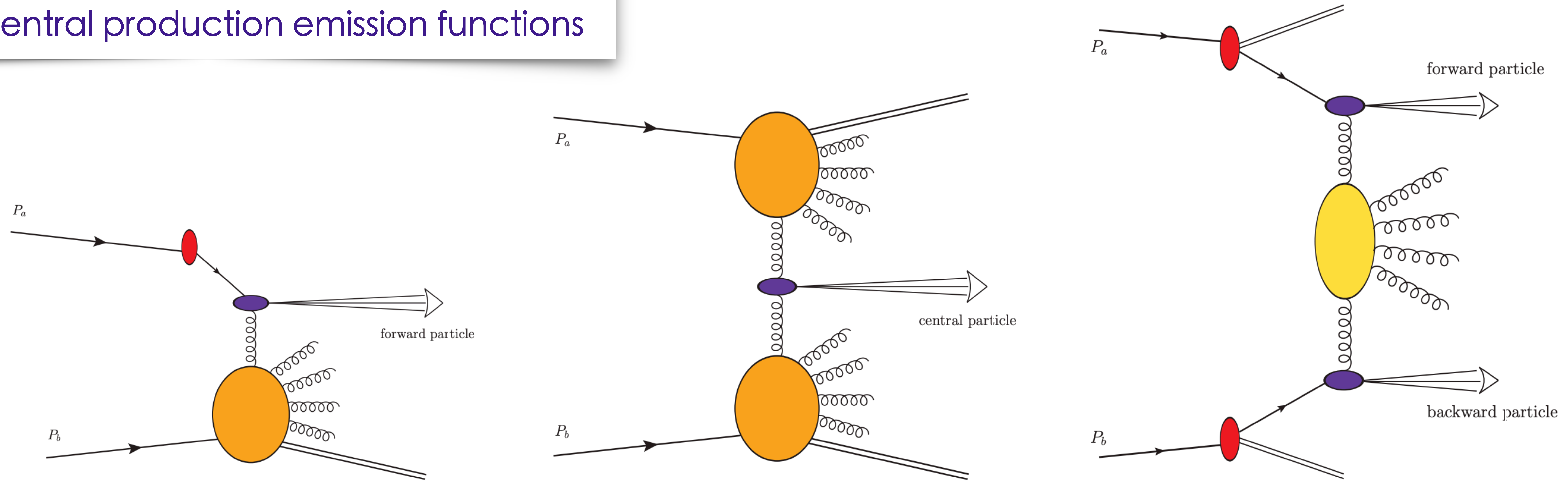
- $\mathcal{I}m_s\left\{\mathcal{A}_{AB}^{AB}\right\}$ factorization:

  convolution of the **Green's function** of two interacting Reggeized gluons with the **impact factors** of the colliding particles

Green's function is **process-independent**, describes energy dependence and obeys BFKL equation; impact factors are known in the **NLL just for few processes**

High-energy factorization at a glance

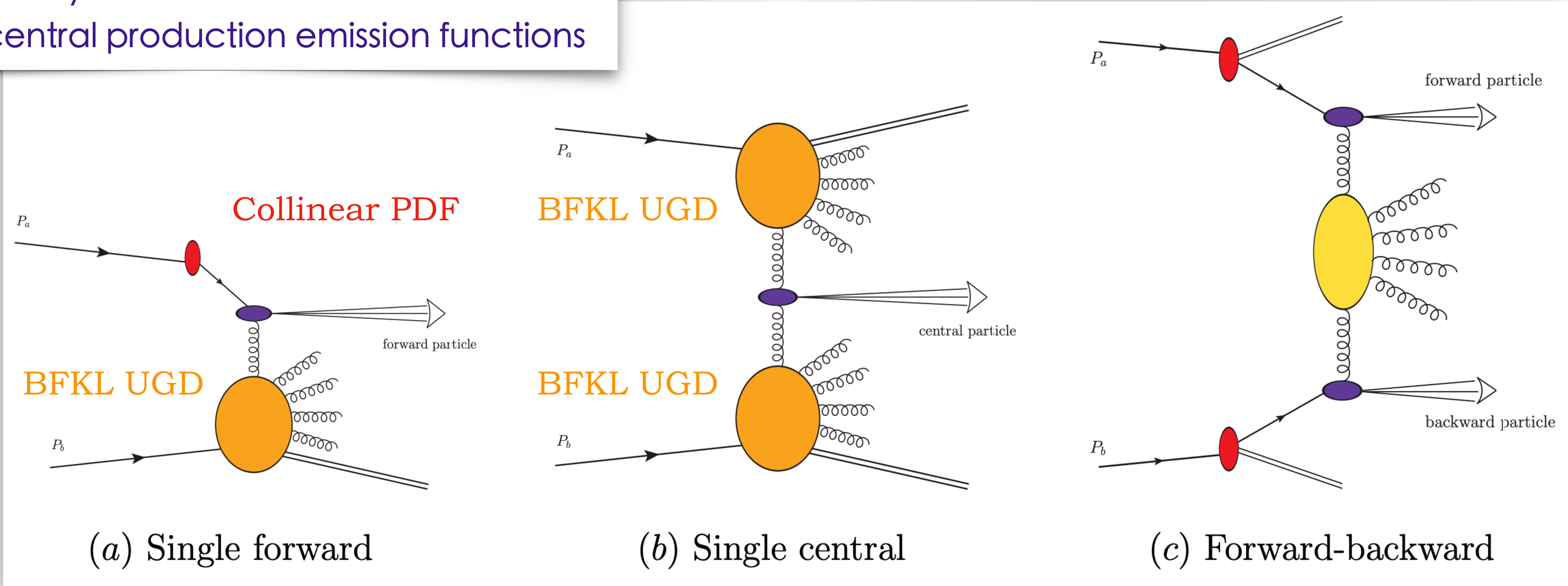
Singly/doubly off-shell coefficient functions
Forward/central production emission functions



(a) Single forward (b) Single central (c) Forward-backward

High-energy factorization at a glance

Singly/doubly off-shell coefficient functions
Forward/central production emission functions



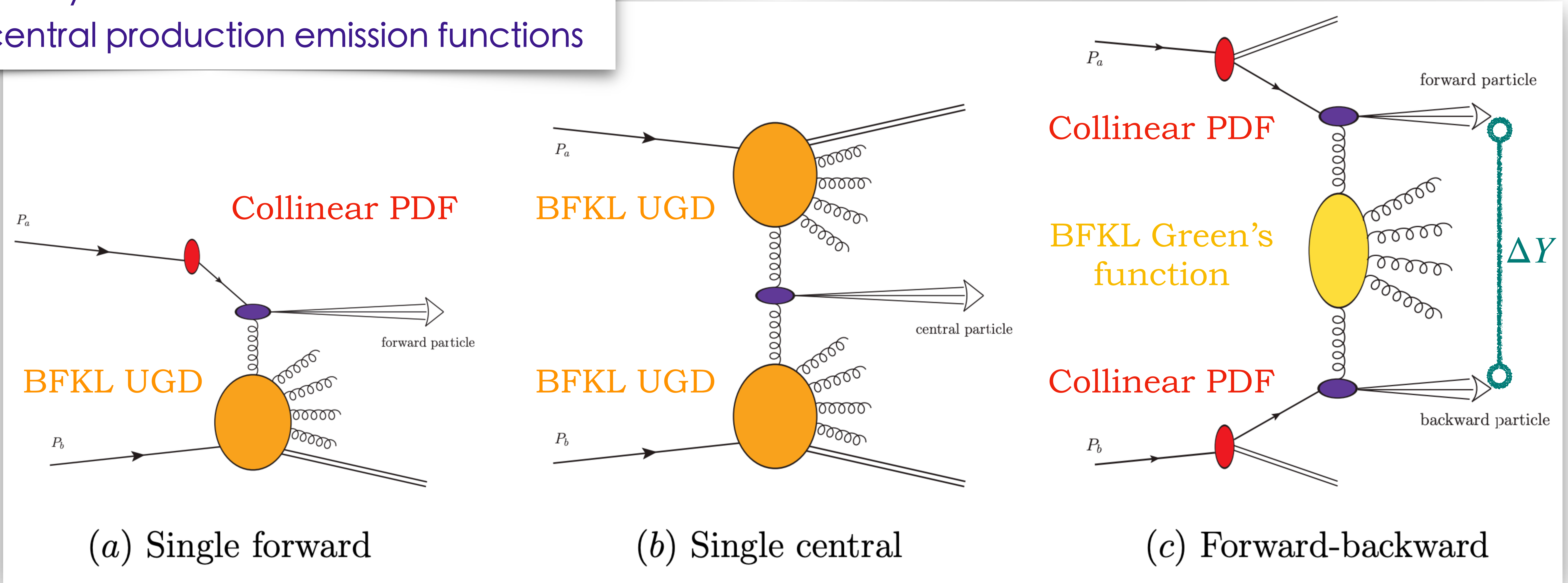
Fast q/g + small-x g	gg induced
Hybrid factorization	High-energy factorization
BFKL + Threshold	BFKL or small-x improved PDFs

[M. Bonvini, S. Marzani (2018)]

Highlights of Lecture I

High-energy factorization at a glance

Singly/doubly off-shell coefficient functions
Forward/central production emission functions



(a) Single forward	(b) Single central	(c) Forward-backward
Fast q/g + small-x g	gg induced	Large rapidity distances, $\Delta Y \gg 1$
Hybrid factorization	High-energy factorization	High energies, moderate x
BFKL + Threshold	BFKL or small-x improved PDFs	PDFs + t-channel BFKL (NLL/NLO HyF)
	[M. Bonvini, S. Marzani (2018)]	Imbalance logs ← back-to-back

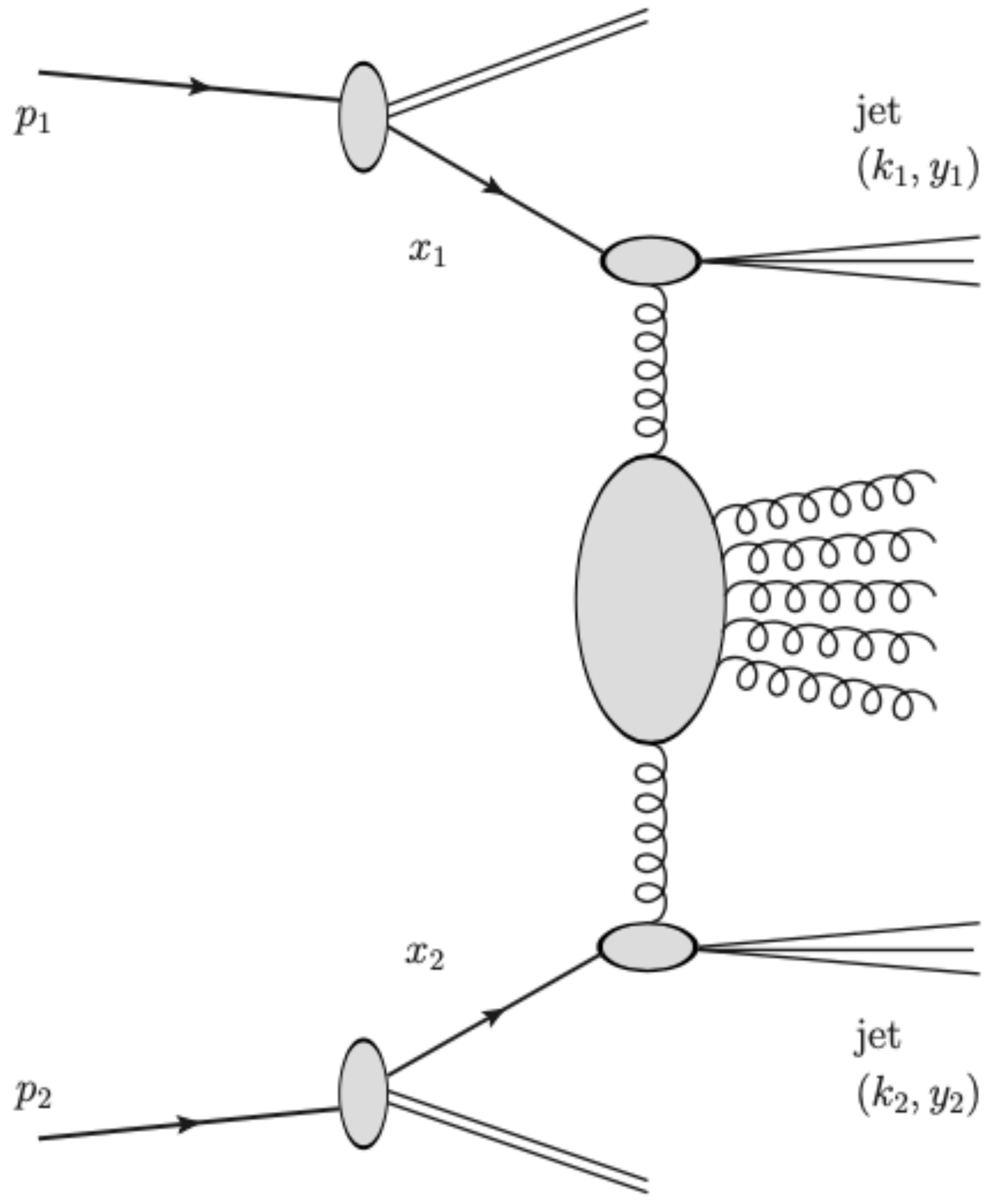
Highlights of Lecture I

1

Mueller-Navelet jets

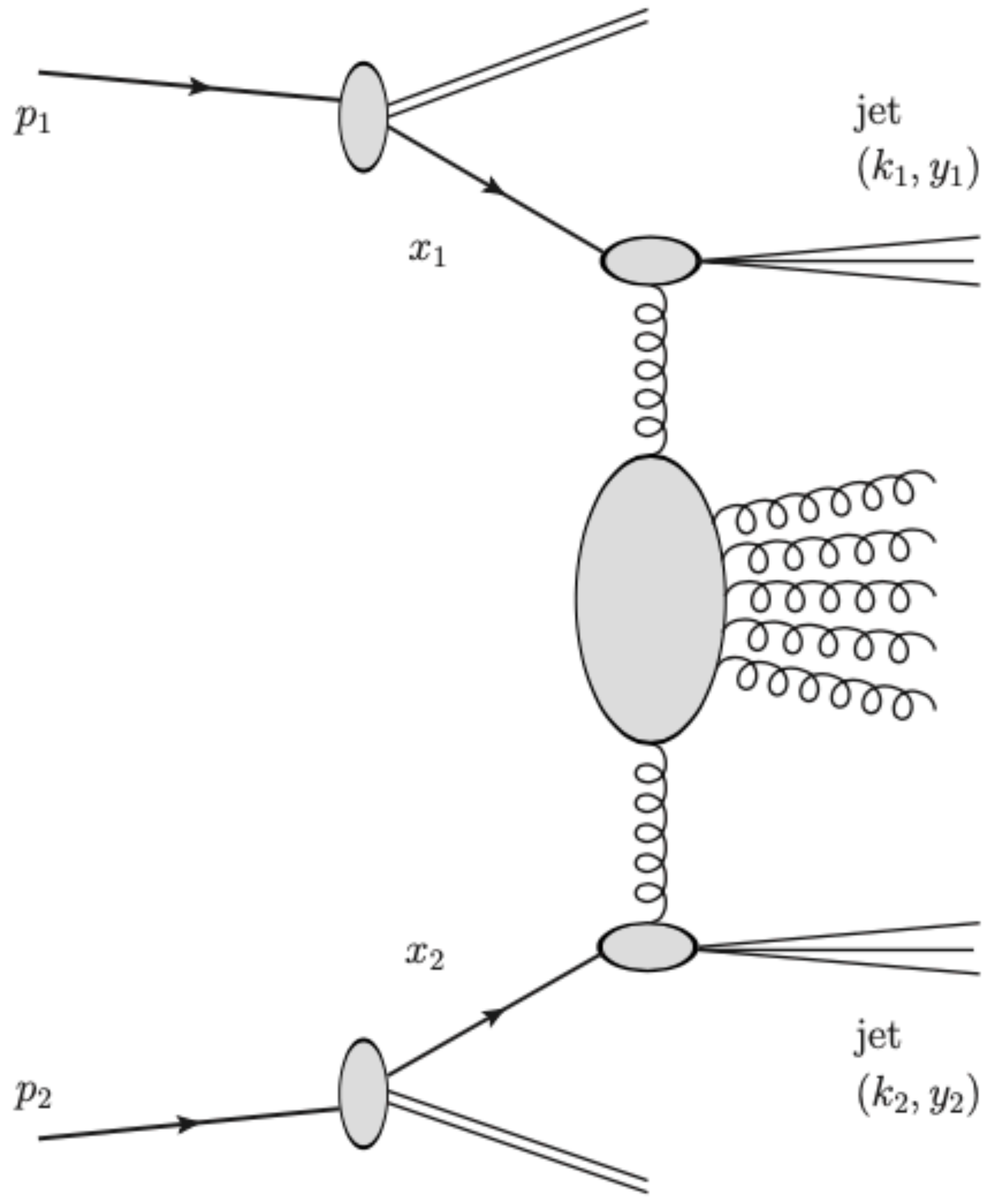
Mueller-Navelet jets at the LHC

$$\text{proton}(p_1) + \text{proton}(p_2) \to \text{jet}_1(k_1, y_1) + X + \text{jet}_2(k_2, y_2)$$



Mueller-Navelet jets at the LHC

$$\text{proton}(p_1) + \text{proton}(p_2) \to \text{jet}_1(k_1, y_1) + X + \text{jet}_2(k_2, y_2)$$

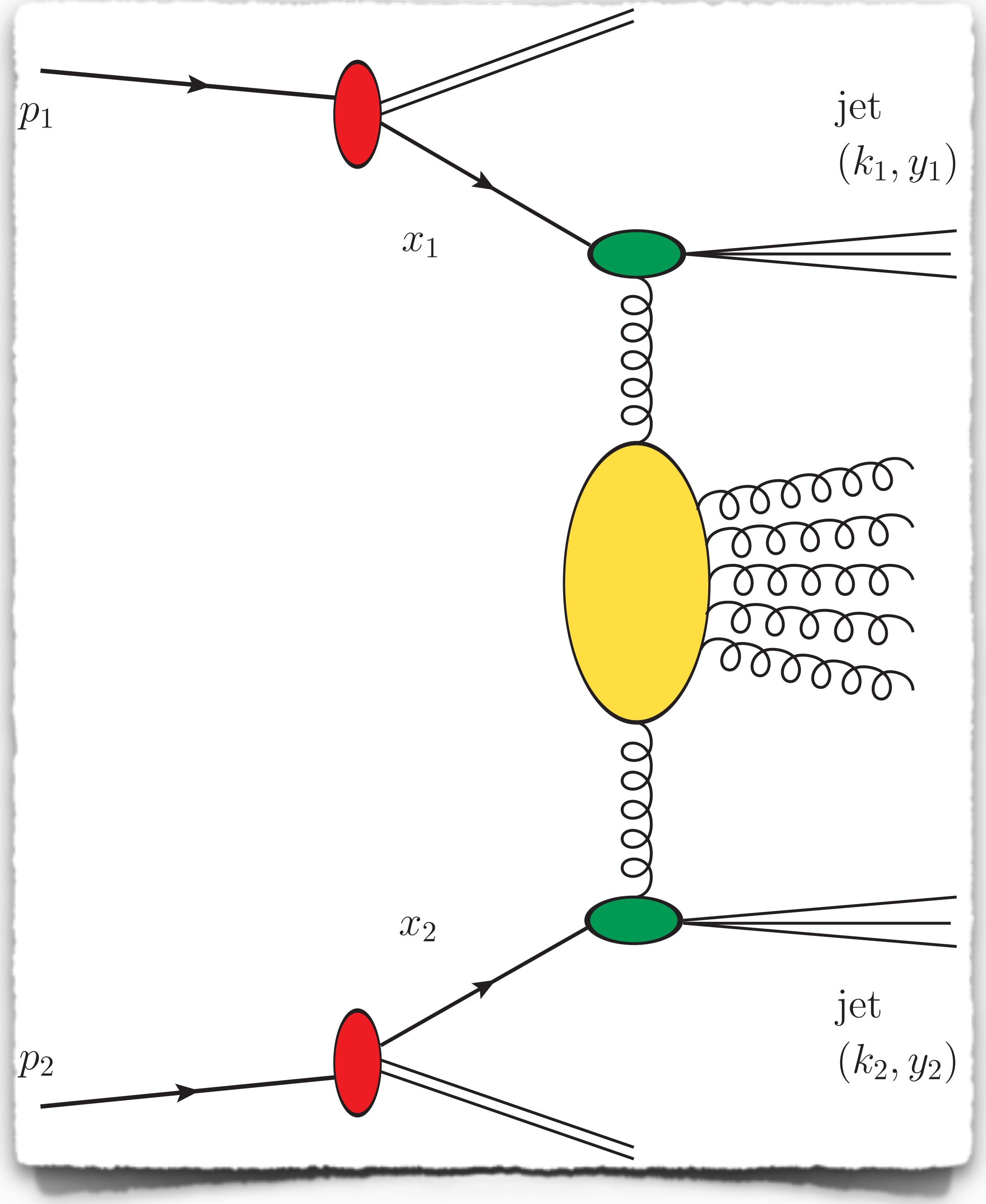


- two hadroproduced jets together with an undetected gluon system, X
- large jet transverse momenta (hard scales): $\vec{k}_1^2 \sim \vec{k}_2^2 \gg \Lambda^2_{\rm QCD}$
- large rapidity distance between jets, $\Delta y \equiv Y = y_1 - y_2$, which leads to large c.o.m energies, $Y \propto \ln s$
- large parton long. fractions (*collinear PDFs*), but non negligible t-channel exchanged momenta (*k_T-factorization*) $\Rightarrow$ **hybrid** approach (next slide)

Mueller-Navelet jet production within HyF

- Inclusive hadroproduction of two jets with high p_T and large rapidity separation, ΔY
- Moderate x (collinear PDFs), but t-channel p_T (BFKL resummation) $\rightarrow$ hybrid factorization

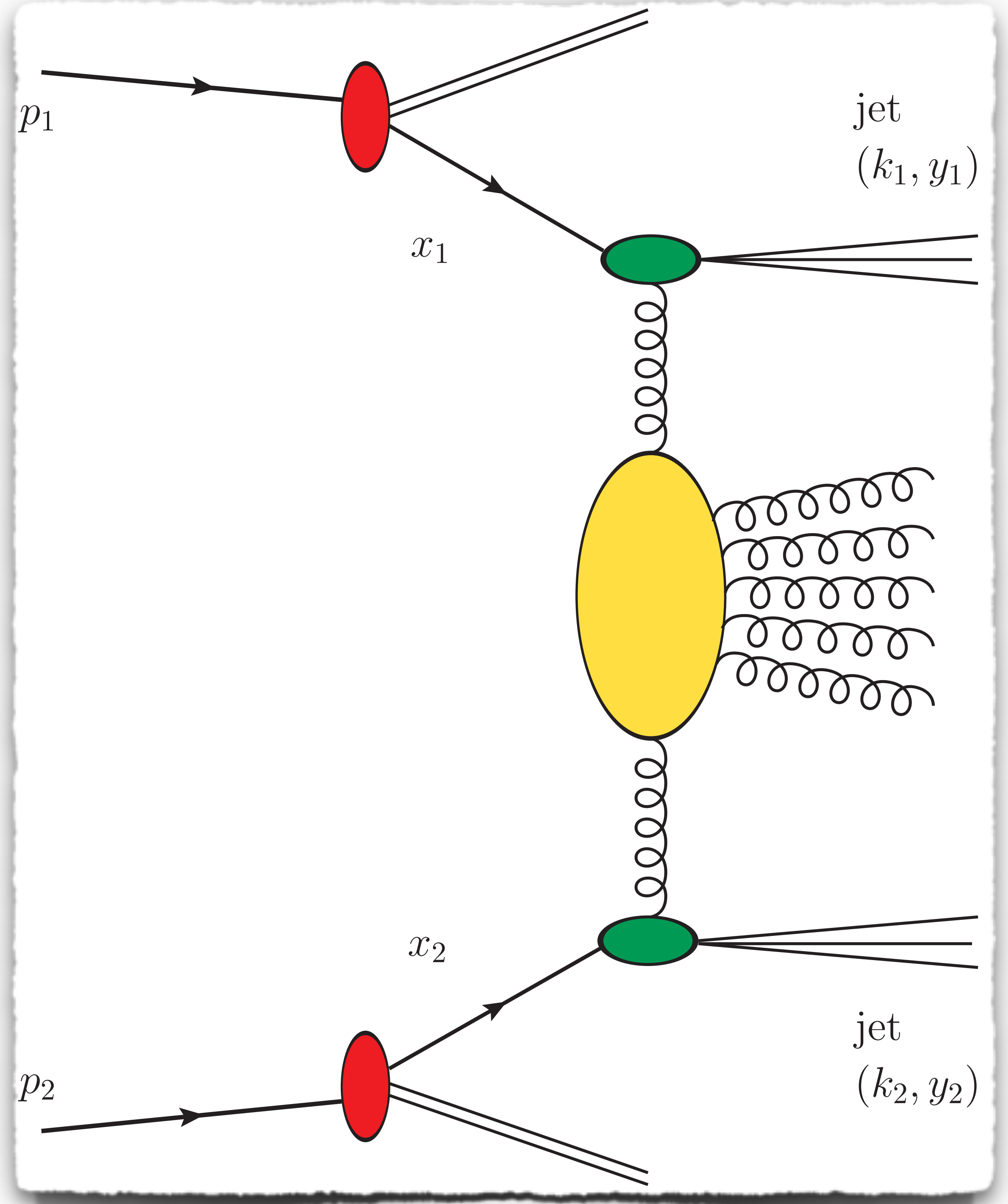
$$\frac{d\sigma}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2\, f_r(x_1, \mu_F)\, f_s(x_2, \mu_F)\, \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2}$$



Mueller-Navelet jet production within HyF

- Inclusive hadroproduction of two jets with high p_T and large rapidity separation, ΔY
- Moderate x (collinear PDFs), but t-channel p_T (BFKL resummation) → hybrid factorization

$$\frac{d\sigma}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2\, f_r(x_1,\mu_F) f_s(x_2,\mu_F) \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2}$$



jet vertices (off-shell amplitudes)

$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2} = \frac{1}{(2\pi)^2}$$

NLO$^{(+)}$ $$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} \mathcal{V}_J^{(r)}(\vec{q}_1, s_0, x_1, \vec{k}_1)$$

NLL $$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2)$$

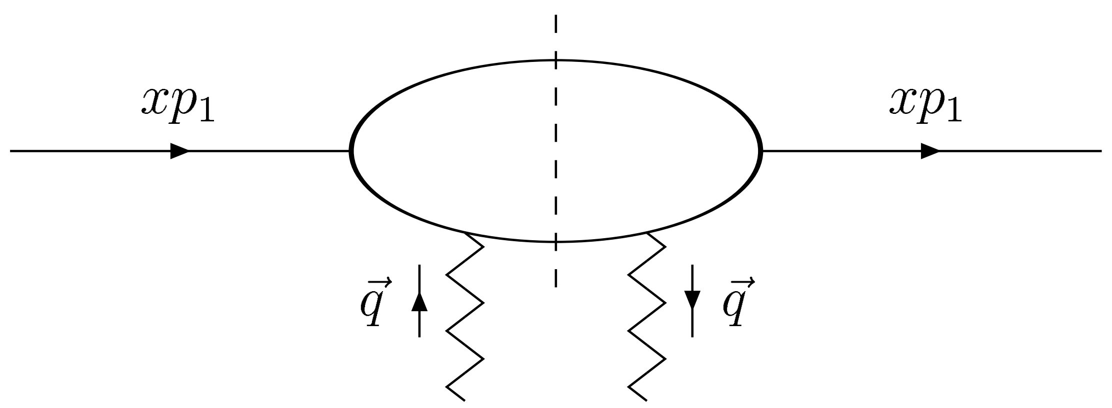
NLO$^{(+)}$ $$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{k}_2)$$

BFKL Green's function

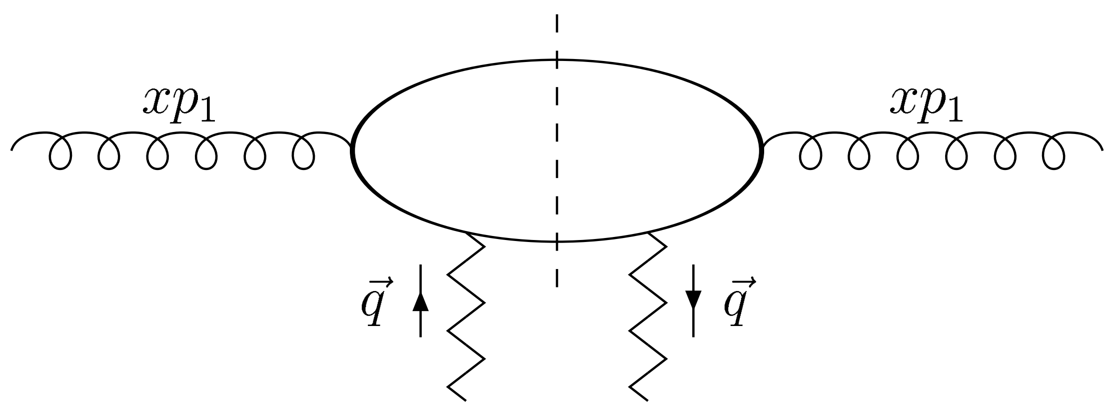
Forward-jet impact factor

- take the impact factors for **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
[M. Ciafaloni and G. Rodrigo (2000)]



quark vertex

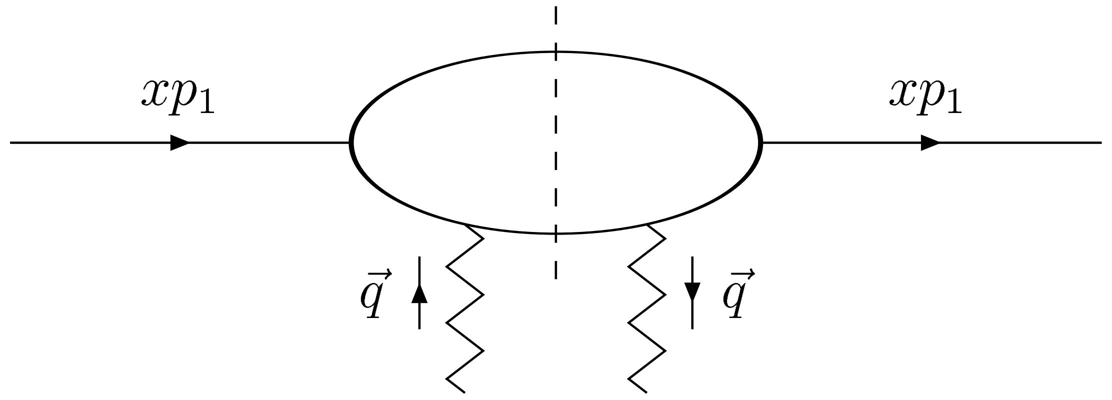


gluon vertex

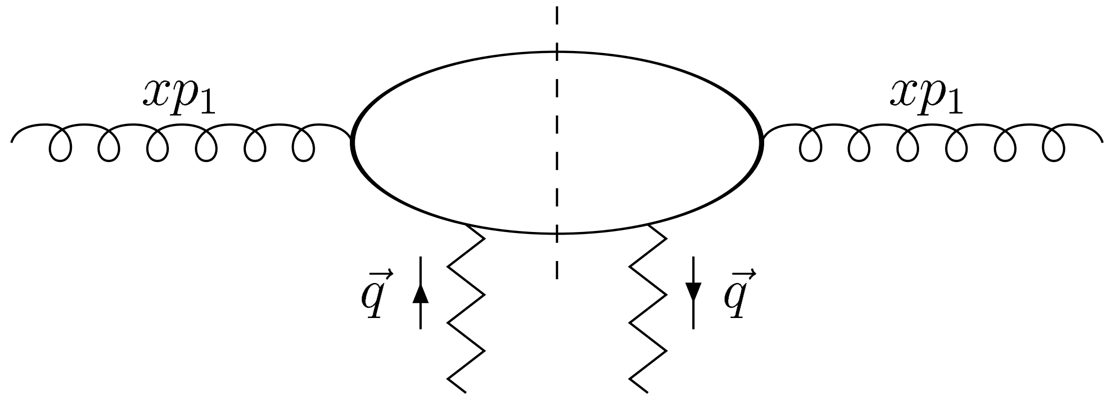
Forward-jet impact factor

- take the impact factors for **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
[M. Ciafaloni and G. Rodrigo (2000)]

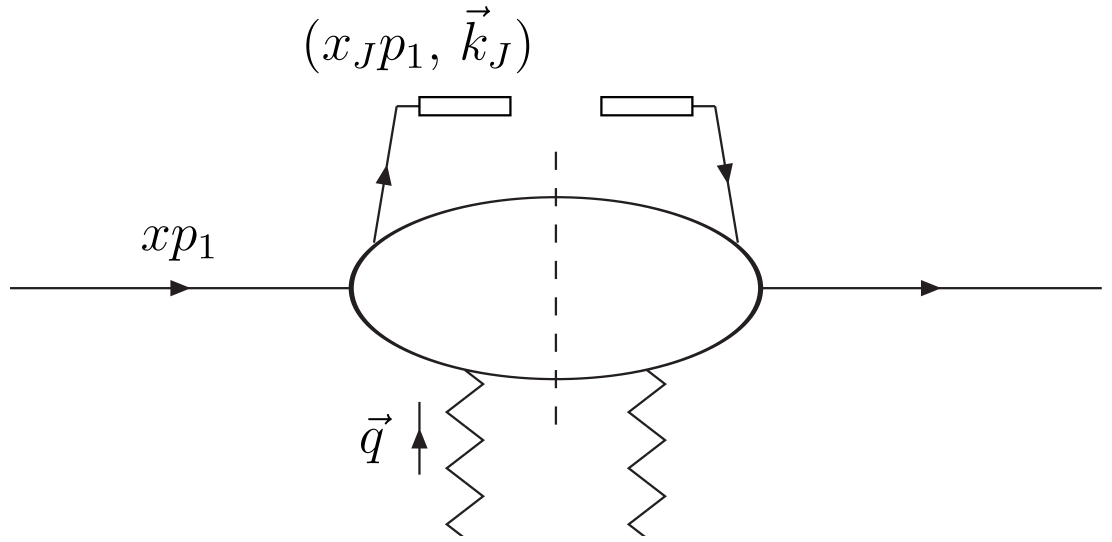


quark vertex

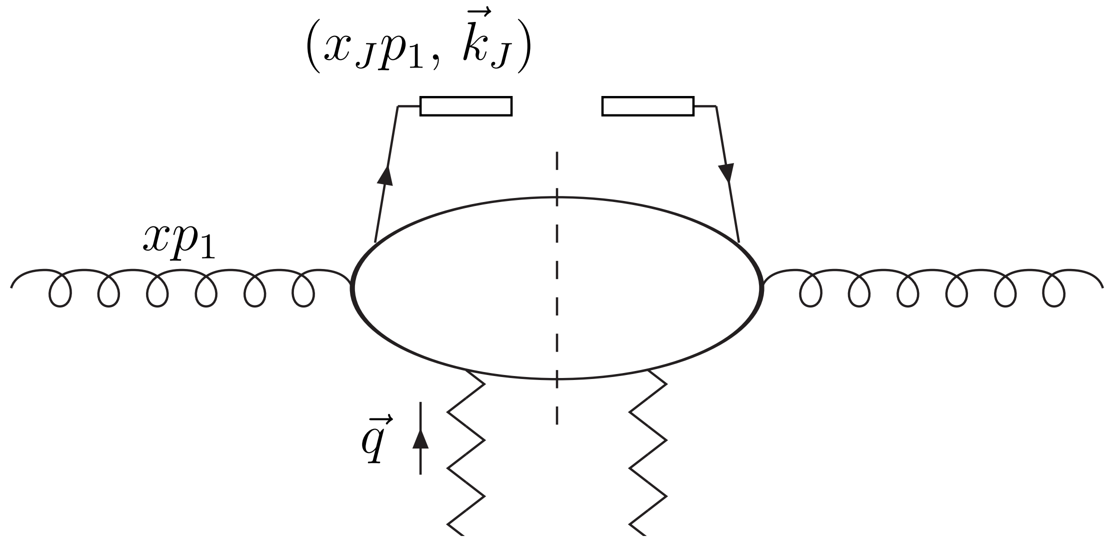


gluon vertex

- "open" one of the integrations over the phase space of the intermediate state to allow one parton to generate the jet



quark jet vertex

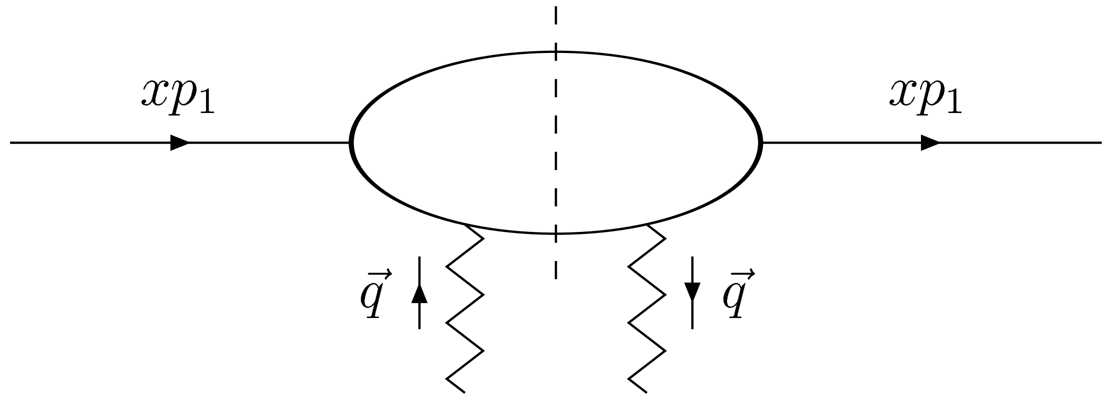


gluon jet vertex

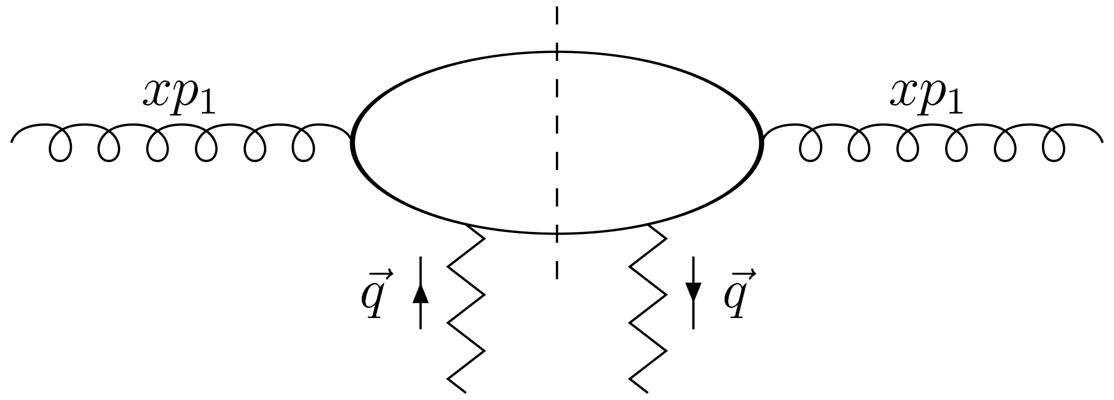
Forward-jet impact factor

- take the impact factors for **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
[M. Ciafaloni and G. Rodrigo (2000)]

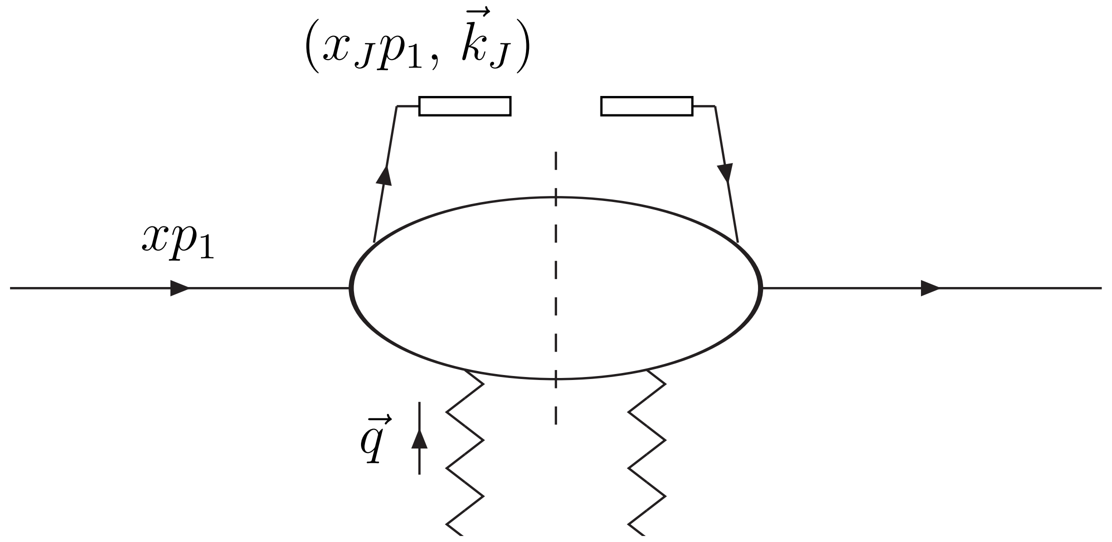


quark vertex

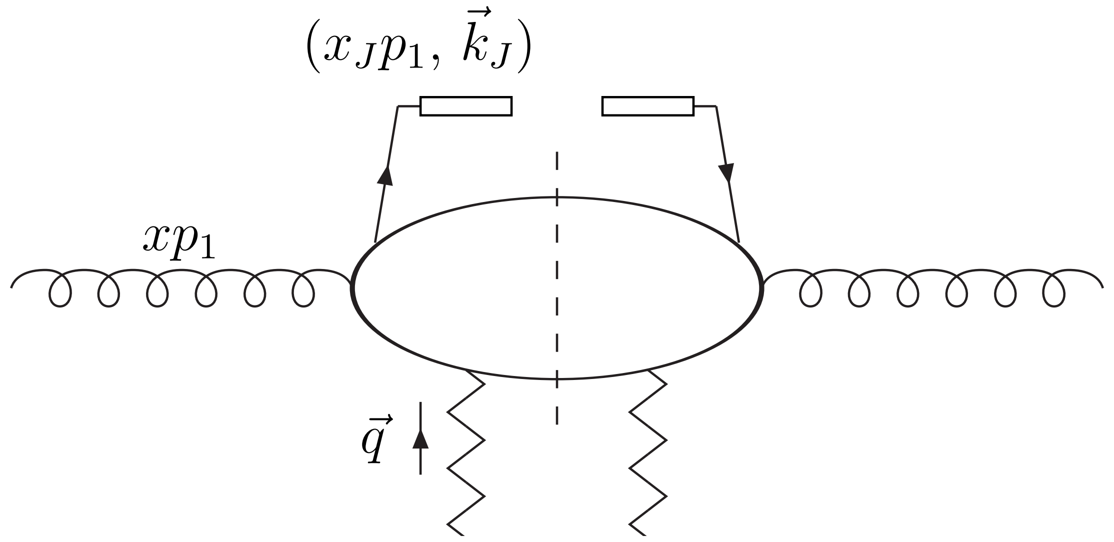


gluon vertex

- "open" one of the integrations over the phase space of the intermediate state to allow one parton to generate the jet



quark jet vertex

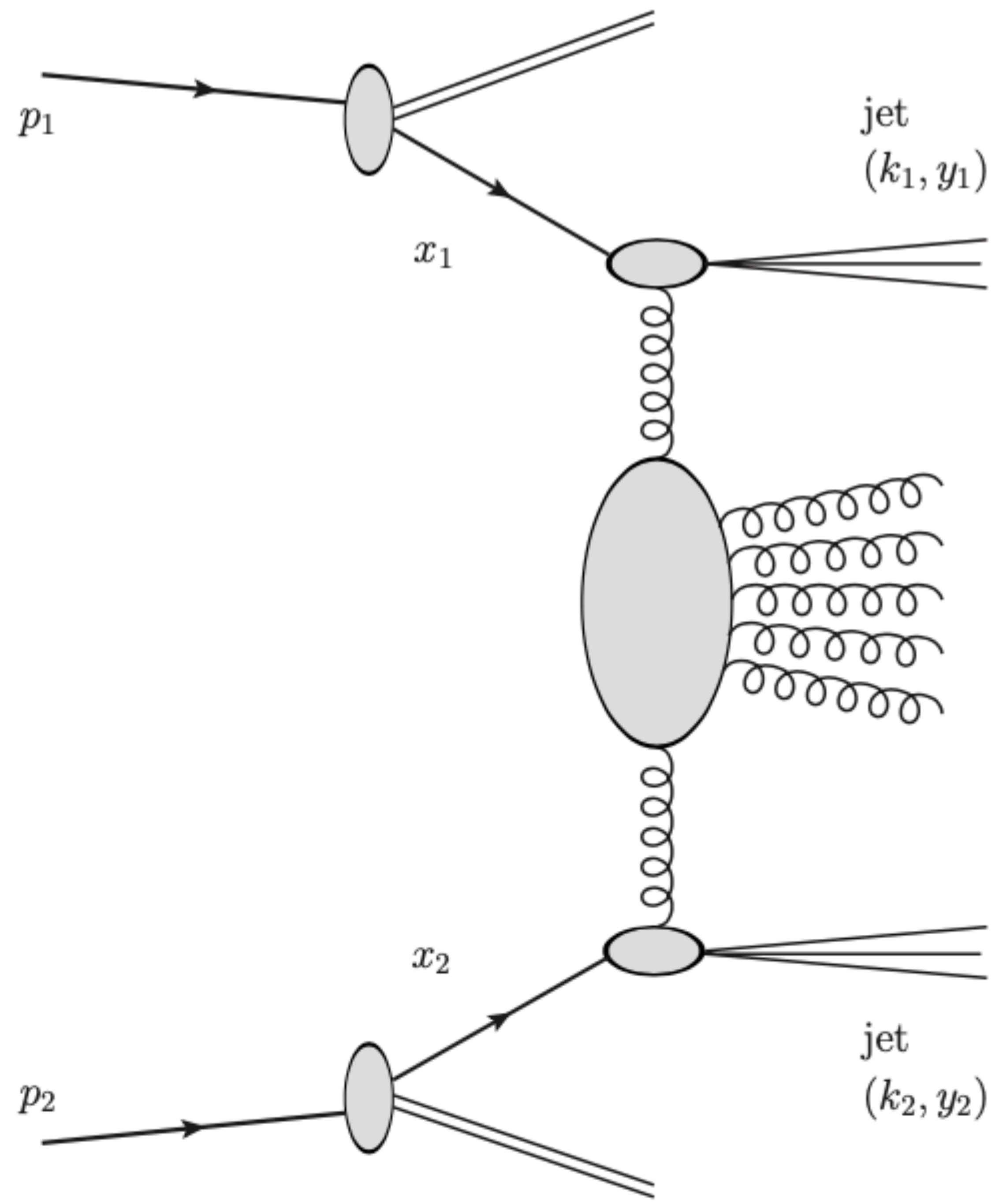


gluon jet vertex

- use QCD collinear factoriz.: $\sum_{s=q,\bar{q}} f_s \otimes$ [quark vertex] $+ f_g \otimes$ [gluon vertex]

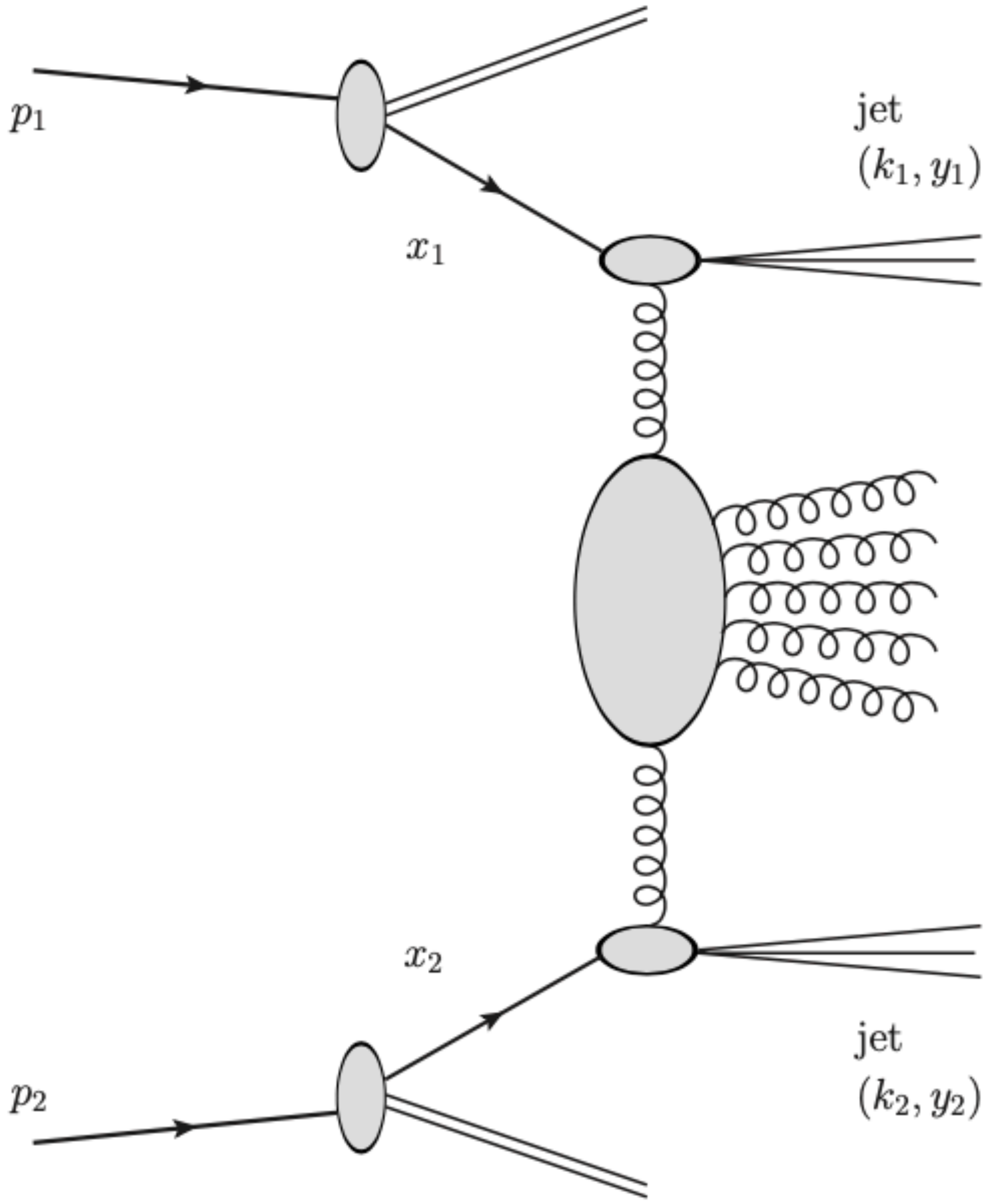
HyF cross section for Mueller-Navelet jets

$$\frac{d\sigma}{dy_1\,dy_2\,d^2\vec{k}_1\,d^2\vec{k}_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2\, f_r(x_1,\mu_F) f_s(x_2,\mu_F) \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s,\mu_F)}{dy_1\,dy_2\,d^2\vec{k}_1\,d^2\vec{k}_2}$$



HyF cross section for Mueller-Navelet jets

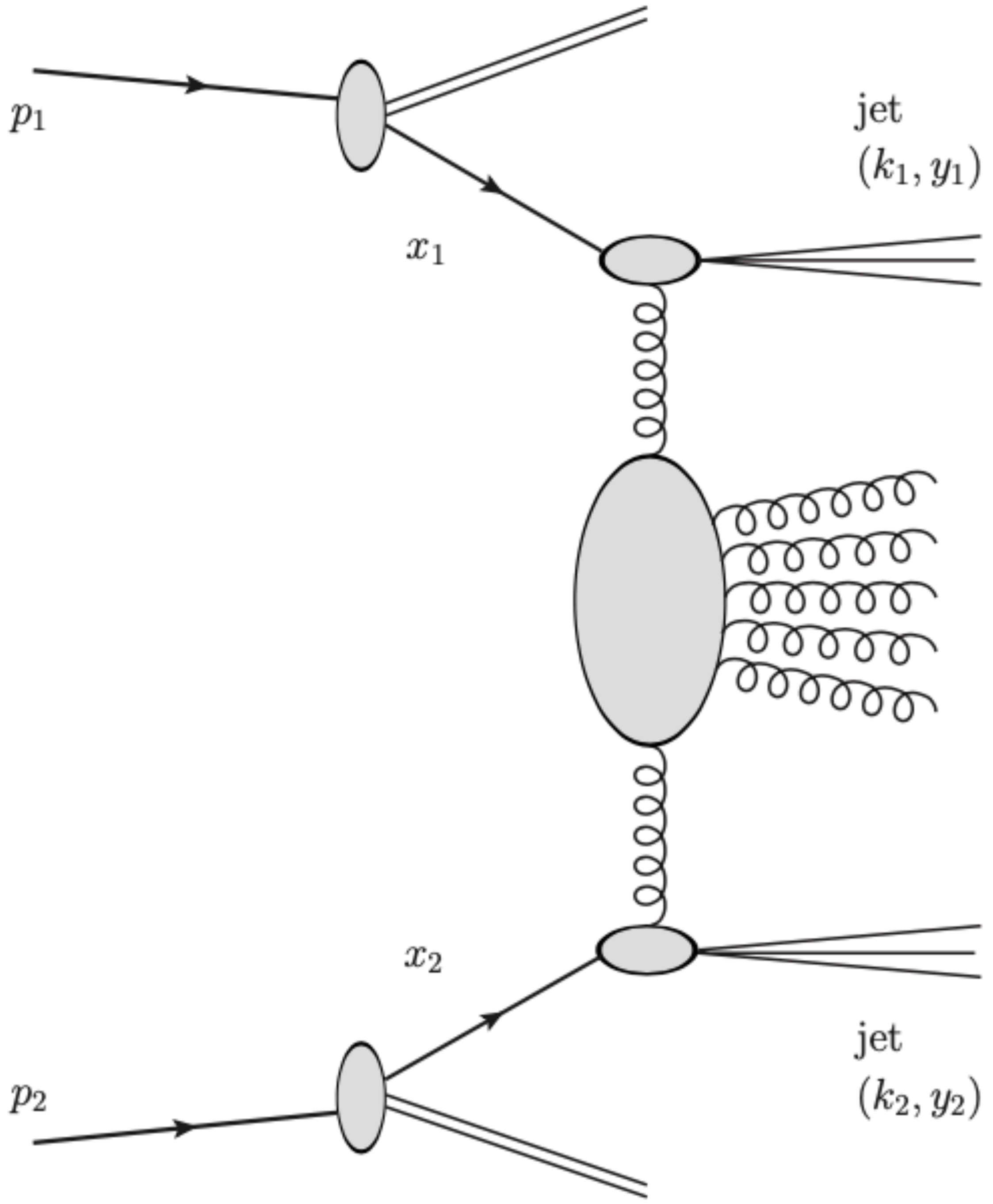
$$\frac{d\sigma}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2\, f_r(x_1, \mu_F) f_s(x_2, \mu_F)\, \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2}$$



- slight change of variable in the final state
- project onto the eigenfunctions of the LO BFKL kernel, i.e. transfer from the reggeized gluon momenta to the (n, ν)-representation
- suitable definition of the **azimuthal coefficients**

HyF cross section for Mueller-Navelet jets

$$\frac{d\sigma}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2\, f_r(x_1,\mu_F) f_s(x_2,\mu_F)\, \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2}$$



- slight change of variable in the final state
- project onto the eigenfunctions of the LO BFKL kernel, i.e. transfer from the reggeized gluon momenta to the (n,ν)-representation
- suitable definition of the **azimuthal coefficients**

$$\frac{d\sigma}{dx_1 dx_2\, d|\vec{k}_1|\, d|\vec{k}_2| d\varphi_1 d\varphi_2} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\varphi)\, \mathcal{C}_n\right]$$

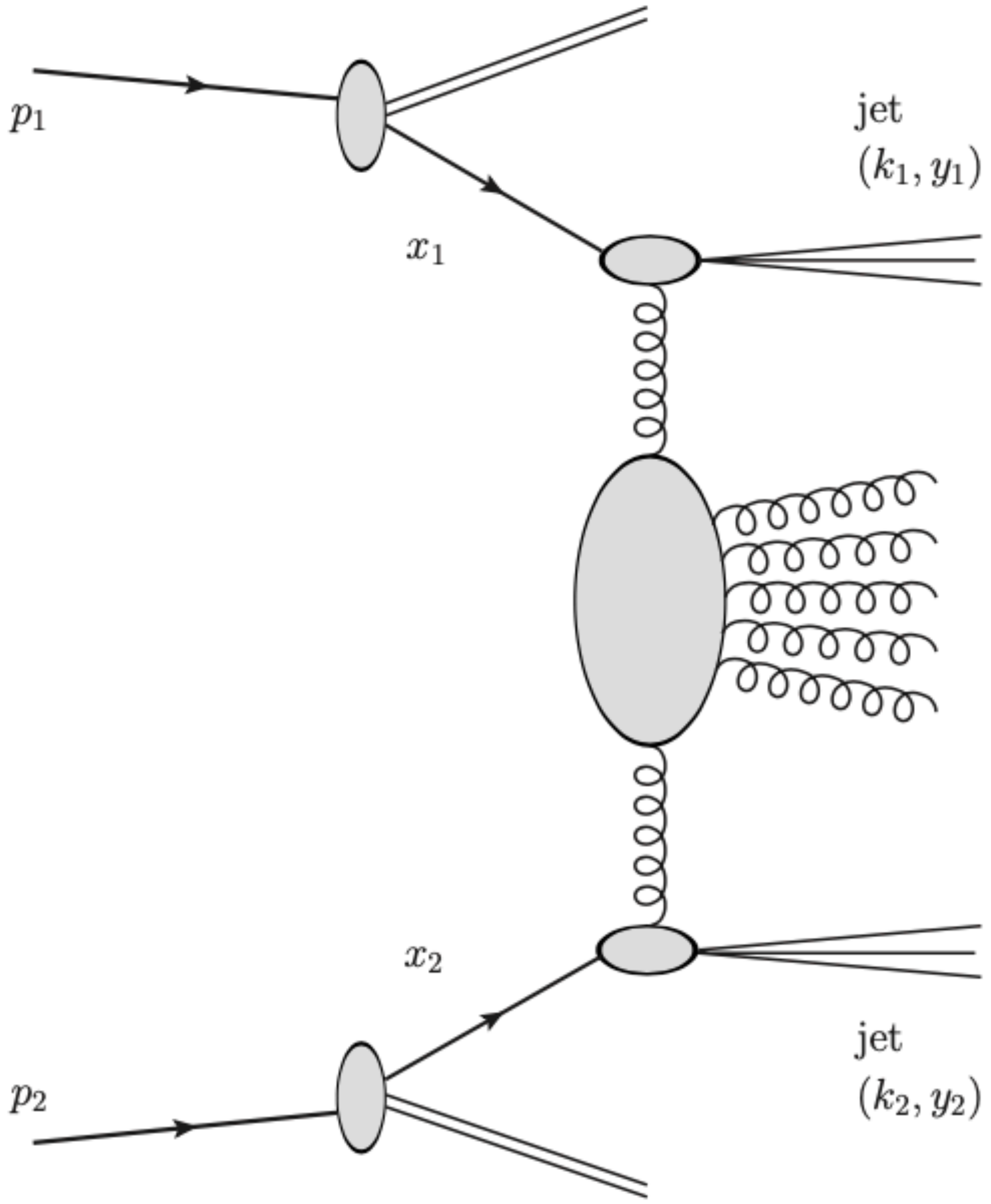
with $\varphi = \varphi_1 - \varphi_2 - \pi$

...useful definitions:

$$Y = \ln \frac{x_1 x_2 s}{|\vec{k}_1||\vec{k}_2|}, \qquad Y_0 = \ln \frac{s_0}{|\vec{k}_1||\vec{k}_2|}$$

HyF cross section for Mueller-Navelet jets

$$\frac{d\sigma}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2\, f_r(x_1, \mu_F) f_s(x_2, \mu_F) \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2}$$



- **Observables:**

ϕ-averaged cross section $\mathcal{C}_0$, $\langle \cos[n(\phi_{J_1} - \phi_{J_2} - \pi)] \rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0}$, with $n = 1, 2, 3$

$$\frac{\langle \cos[2(\phi_1 - \phi_2 - \pi)] \rangle}{\langle \cos(\phi_1 - \phi_2 - \pi) \rangle} \equiv \frac{\mathcal{C}_2}{\mathcal{C}_1} \equiv R_{21}, \quad \frac{\langle \cos[3(\phi_1 - \phi_2 - \pi)] \rangle}{\langle \cos[2(\phi_1 - \phi_2 - \pi)] \rangle} \equiv \frac{\mathcal{C}_3}{\mathcal{C}_2} \equiv R_{32}.$$

◇ *Integrated coefficients:*

$$C_n = \int_{y_{1,\min}}^{y_{1,\max}} dy_1 \int_{y_{2,\min}}^{y_{2,\max}} dy_2 \int_{k_{J_1,\min}}^{\infty} dk_{J_1} \int_{k_{J_2,\min}}^{\infty} dk_{J_2}\, \delta(y_1 - y_2 - Y)\, \mathcal{C}_n(y_{J_1}, y_{J_2}, k_{J_1}, k_{J_2})$$

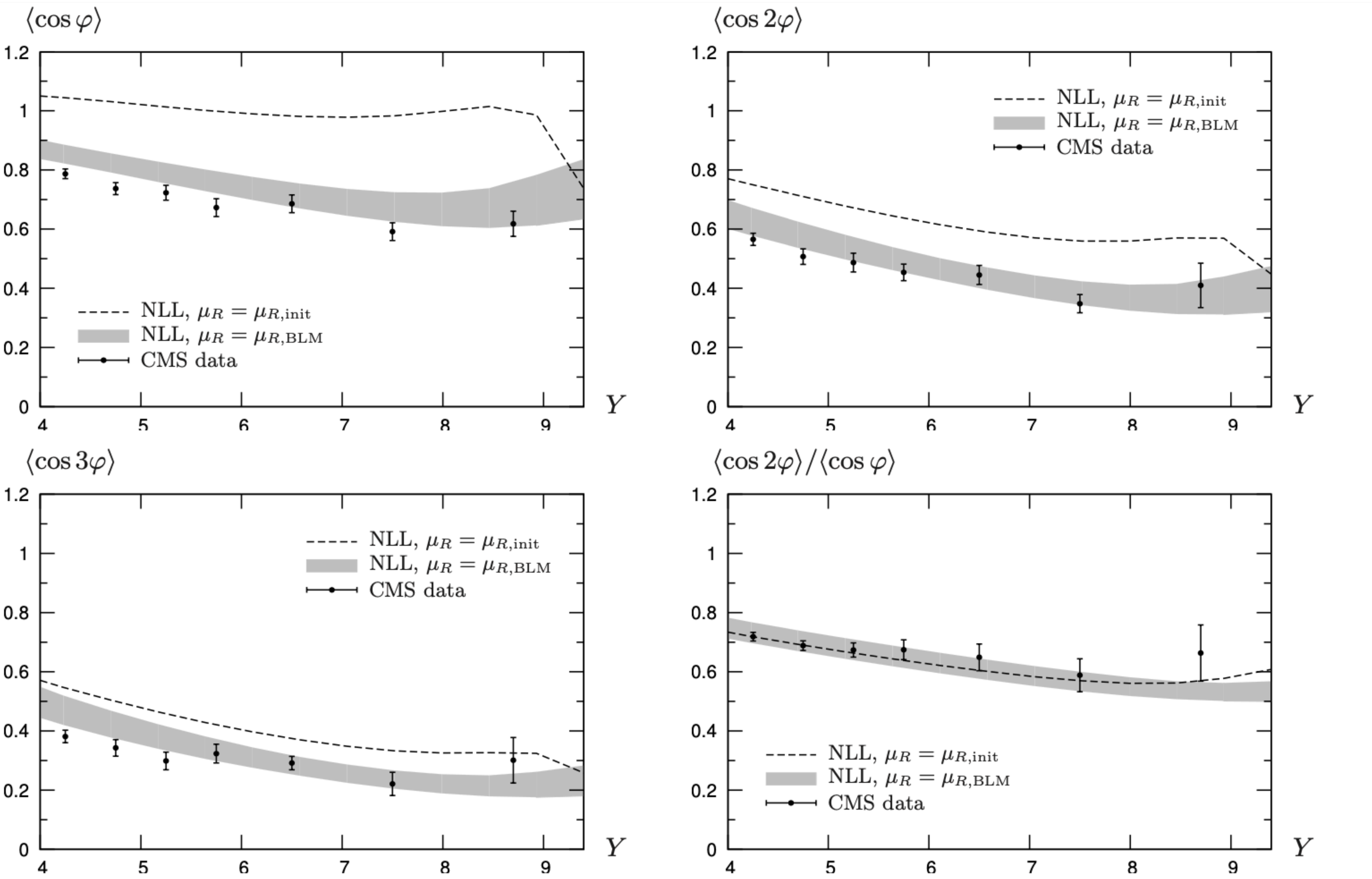
$$\frac{d\sigma}{dx_1 dx_2\, d|\vec{k}_1|\, d|\vec{k}_2| d\varphi_1 d\varphi_2} = \frac{1}{(2\pi)^2} \left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\varphi)\, \mathcal{C}_n \right]$$

with $\varphi = \varphi_1 - \varphi_2 - \pi$

...useful definitions:

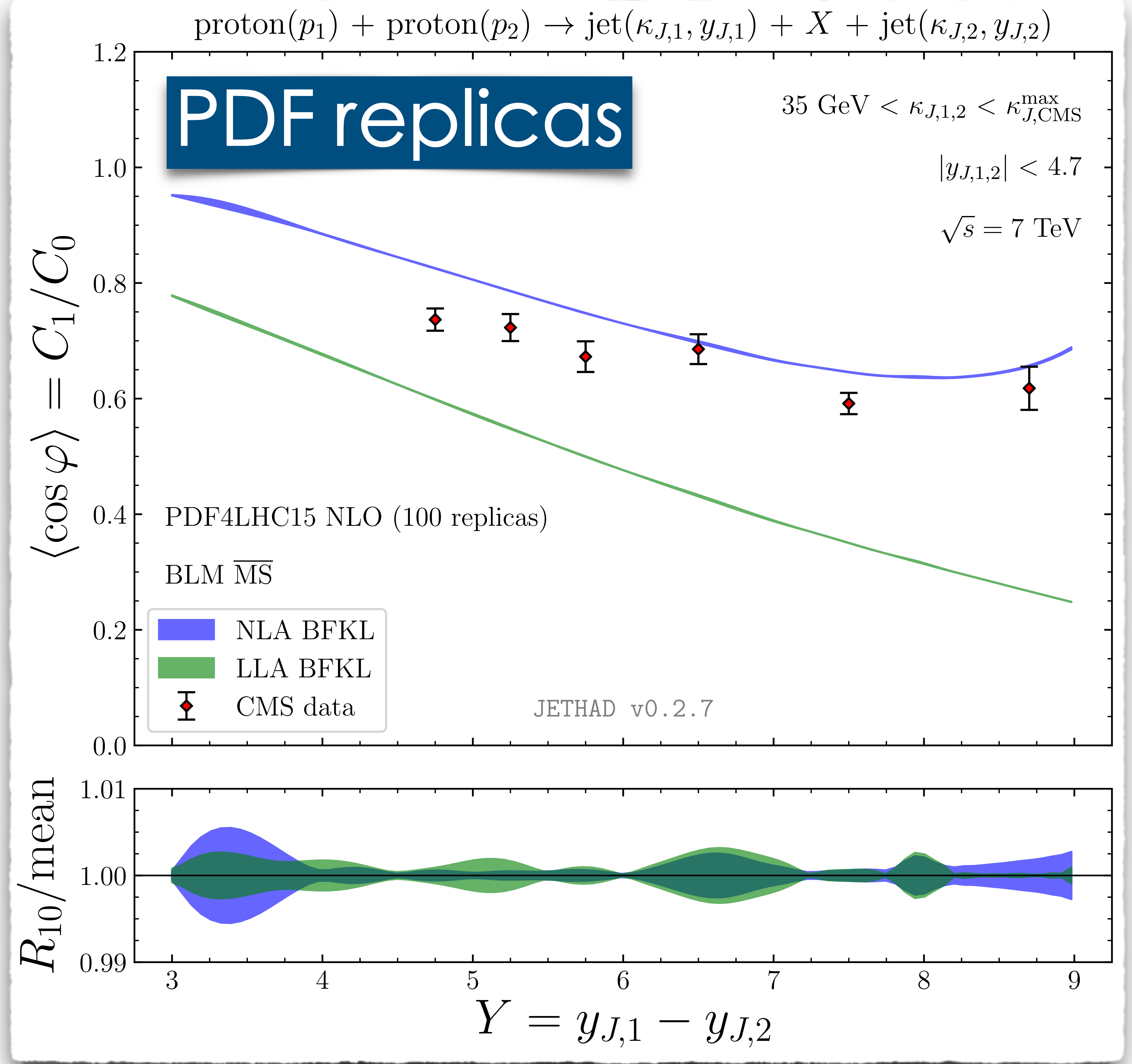
$$Y = \ln \frac{x_1 x_2 s}{|\vec{k}_1||\vec{k}_2|}, \qquad Y_0 = \ln \frac{s_0}{|\vec{k}_1||\vec{k}_2|}$$

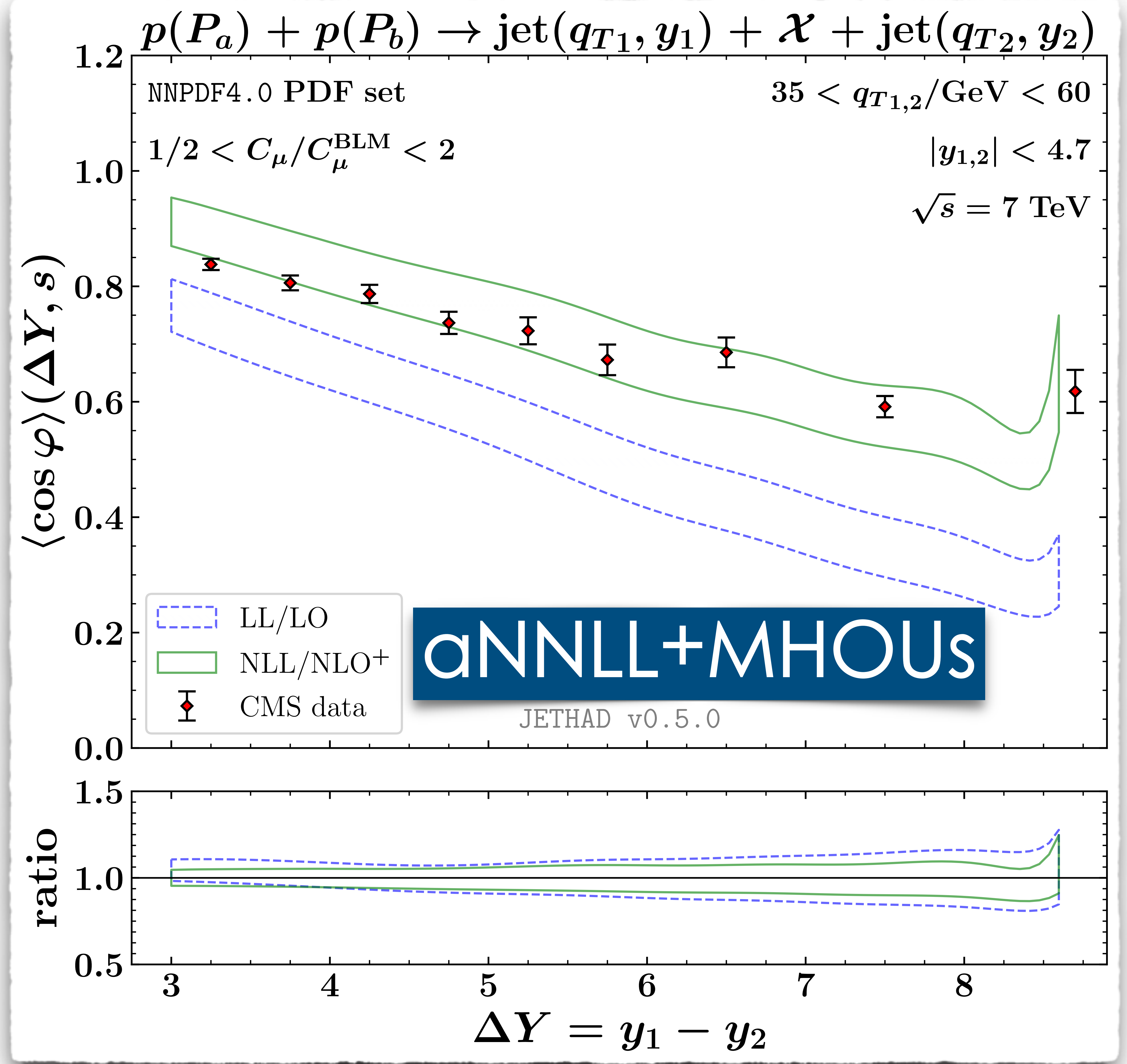
MN jets: Theory vs experiment @7TeV CMS



(figures in this slide; **7 TeV BFKL + sym.**) [B. Ducloué, L. Szymanowski, S. Wallon (2014)]
(similar analysis; **7 TeV BFKL + sym.**) [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

MN jets: Hunting BFKL @7TeV CMS





(left) [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

(right) [F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]

Mueller–Navelet jets

- inclusive hadroproduction of two jets featuring high transverse momenta and well separated in rapidity
- possibility to define *infrared-safe* observables and constrain PDFs
- theory vs experiment (CMS @7 TeV with symmetric p_T-ranges, **only!**)

[B. Ducloué, L. Szymanowski, S. Wallon (2014)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014); F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015, 2016)]

Mueller–Navelet jets

- inclusive hadroproduction of two jets featuring high transverse momenta and well separated in rapidity
- possibility to define *infrared-safe* observables and constrain PDFs
- theory vs experiment (CMS @7 TeV with symmetric p_T-ranges, **only!**)

[B. Ducloué, L. Szymanowski, S. Wallon (2014)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014); F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015, 2016)]

"[...] the reasonable data-theory agreement shown by the NLL BFKL analytical calculations at large Δy, may be considered as indications that the kinematical domain of the present study lies in between the regions described by the DGLAP and BFKL approaches."

[CMS Collaboration (2016)]

Mueller–Navelet jets

- inclusive hadroproduction of two jets featuring high transverse momenta and well separated in rapidity
- possibility to define *infrared-safe* observables and constrain PDFs
- theory vs experiment (CMS @7 TeV with symmetric p_T-ranges, **only!**)

[B. Ducloué, L. Szymanowski, S. Wallon (2014)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014); F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015, 2016)]

"[...] the reasonable data-theory agreement shown by the NLL BFKL analytical calculations at large Δy, may be considered as indications that the kinematical domain of the present study lies in between the regions described by the DGLAP and BFKL approaches."

[CMS Collaboration (2016)]

What's next?

- BFKL vs fixed-order DGLAP adopting **asymmetric** p_T-ranges (next slide)
- need for *more exclusive* final states as well as *more sensitive* observables

Progress in high-energy QCD phenomenology

◇ NLA BFKL expressions for the observables truncated to $\mathcal{O}\left(\alpha_s^3\right)$ **!**

- first analysis: **MN jets** with **partial asymmetric** cuts for jet transv. momenta

(**CMS-jet**, **7 TeV**) [F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]

◇ NLA BFKL expressions for the observables truncated to $\mathcal{O}\left(\alpha_s^3\right)$ **!**

- first analysis: **MN jets** with **partial asymmetric** cuts for jet transv. momenta

(**CMS-jet**, **7 TeV**) [F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]

Why asymmetric cuts?

▶ suppress Born contribution to φ-averaged cross section C_0 (back-to-back)

◇ avoid instabilities observed in NLO fixed-order calculations

[J.R. Andersen, V. Del Duca, S. Frixione, C.R. Schmidt, W.J. Stirling (2001)]
[M. Fontannaz, J.P. Guillet, G. Heinrich (2001)]

◇ **enhance effects of additional hard gluons** $\xrightarrow{\text{emphasize}}$ **BFKL effects**

▶ violation of energy-momentum in NLA strongly suppressed respect to LLA

[B. Ducloué, L. Szymanowski, S. Wallon (2014)]

- **3 NLA BFKL** reactions: **MN jets**, **hadron-jet**, **di-hadron** with **disjoint** k-windows

Looking for new observables

- BFKL feature: factorization between transverse and longitudinal (rapidities) degrees of freedom
- Usual "growth with energy" signal mainly probes the longitudinal degrees of freedom
- Mueller–Navelet correlation momenta mainly probe one of the transverse components, the azimuthal angles

! We would like to study observables for which the p_T (any p_T along the BFKL ladder) enters the game...

- ...to probe not only the general properties of the BFKL ladder, but also "to peek into the interior"...
- ...by studying azimuthal decorrelations where the p_T of extra particles introduces a new dependence...

LHC final states with three or four tagged jets

Looking for new observables

- BFKL feature: factorization between transverse and longitudinal (rapidities) degrees of freedom
- Usual "growth with energy" signal mainly probes the longitudinal degrees of freedom
- Mueller–Navelet correlation momenta mainly probe one of the transverse components, the azimuthal angles

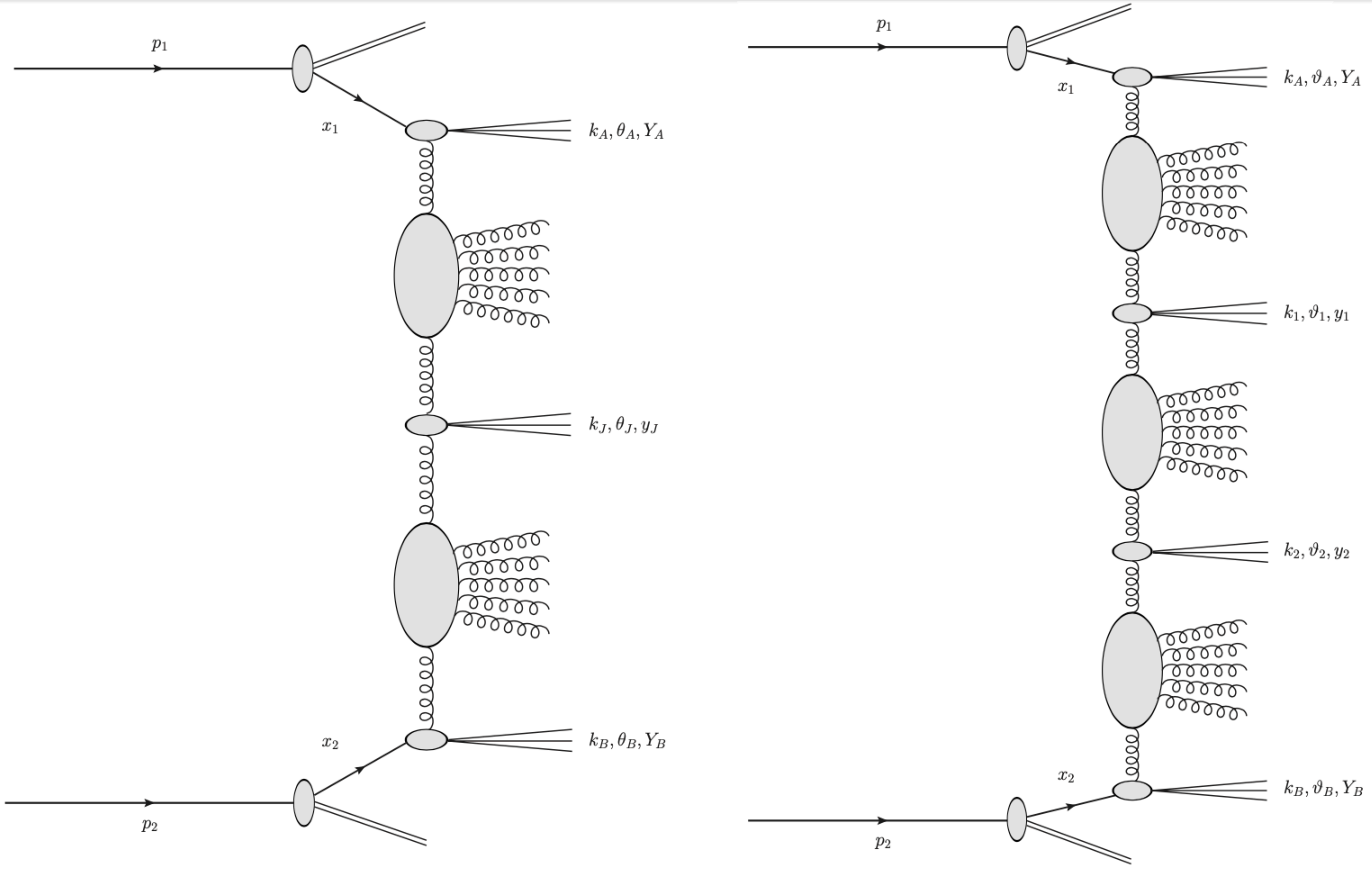
! We would like to study observables for which the p_T (any p_T along the BFKL ladder) enters the game...

- ...to probe not only the general properties of the BFKL ladder, but also "to peek into the interior"...
- ...by studying azimuthal decorrelations where the p_T of extra particles introduces a new dependence...

...multi-jet production!

LHC final states with three or four tagged jets

Three- and four-jet production



(Three-jets) [F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]
(Three-jets) [F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016, 2017)]
(Four-jets) [F. Caporale, F.G. C., G. Chachamis, A. Sabio Vera (2016)]
(Four-jets) [F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2017)]

LHC final states with three or four tagged jets

Three-jets: generalized azimuthal correlations

Prescription: integrate over all angles after using the projections on the two azimuthal angle differences indicated below...to define:

$$\int_0^{2\pi} d\theta_A \int_0^{2\pi} d\theta_B \int_0^{2\pi} d\theta_J \cos\left(M\left(\theta_A - \theta_J - \pi\right)\right) \cos\left(N\left(\theta_J - \theta_B - \pi\right)\right) \frac{d^3\hat{\sigma}^{3-\text{jet}}}{dk_J d\theta_J dy_J}$$

$$= \bar{\alpha}_s \sum_{L=0}^{N} \binom{N}{L} \left(k_J^2\right)^{\frac{L-1}{2}} \int_0^{\infty} dp^2 \left(p^2\right)^{\frac{N-L}{2}} \int_0^{2\pi} d\theta \frac{(-1)^{M+N} \cos\left(M\theta\right) \cos\left((N-L)\theta\right)}{\sqrt{\left(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta\right)^N}}$$

$$\times\ \phi_M\left(k_A^2, p^2, Y_A - y_J\right) \phi_N\left(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta, k_B^2, y_J - Y_B\right)$$

Main observables: **generalized azimuthal correlation momenta**

$$\mathcal{R}_{PQ}^{MN} = \frac{\mathcal{C}_{MN}}{\mathcal{C}_{PR}} = \frac{\langle \cos(M(\theta_A - \theta_J - \pi)) \cos(N(\theta_J - \theta_B - \pi)) \rangle}{\langle \cos(P(\theta_A - \theta_J - \pi)) \cos(Q(\theta_J - \theta_B - \pi)) \rangle}$$

- Remove the contribution from the zero conformal spin

  $\xrightarrow{\text{to}}$ drastically reduce the dependence on collinear configurations

  study $\mathcal{R}_{PQ}^{MN}$ with integer $M, N, P, Q > 0$

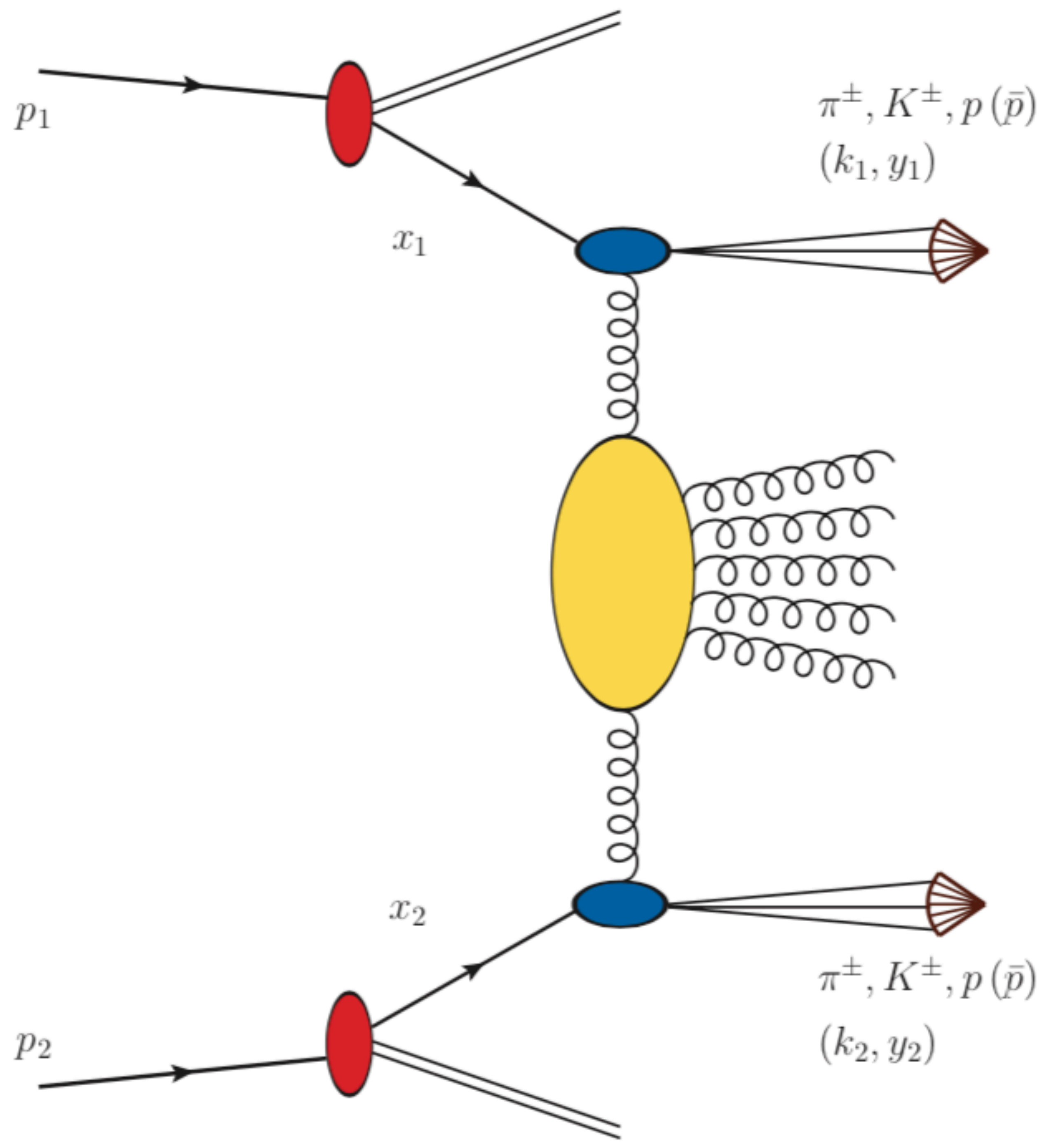
2

Light-flavored hadrons

Di-hadron and hadron-jet correlations

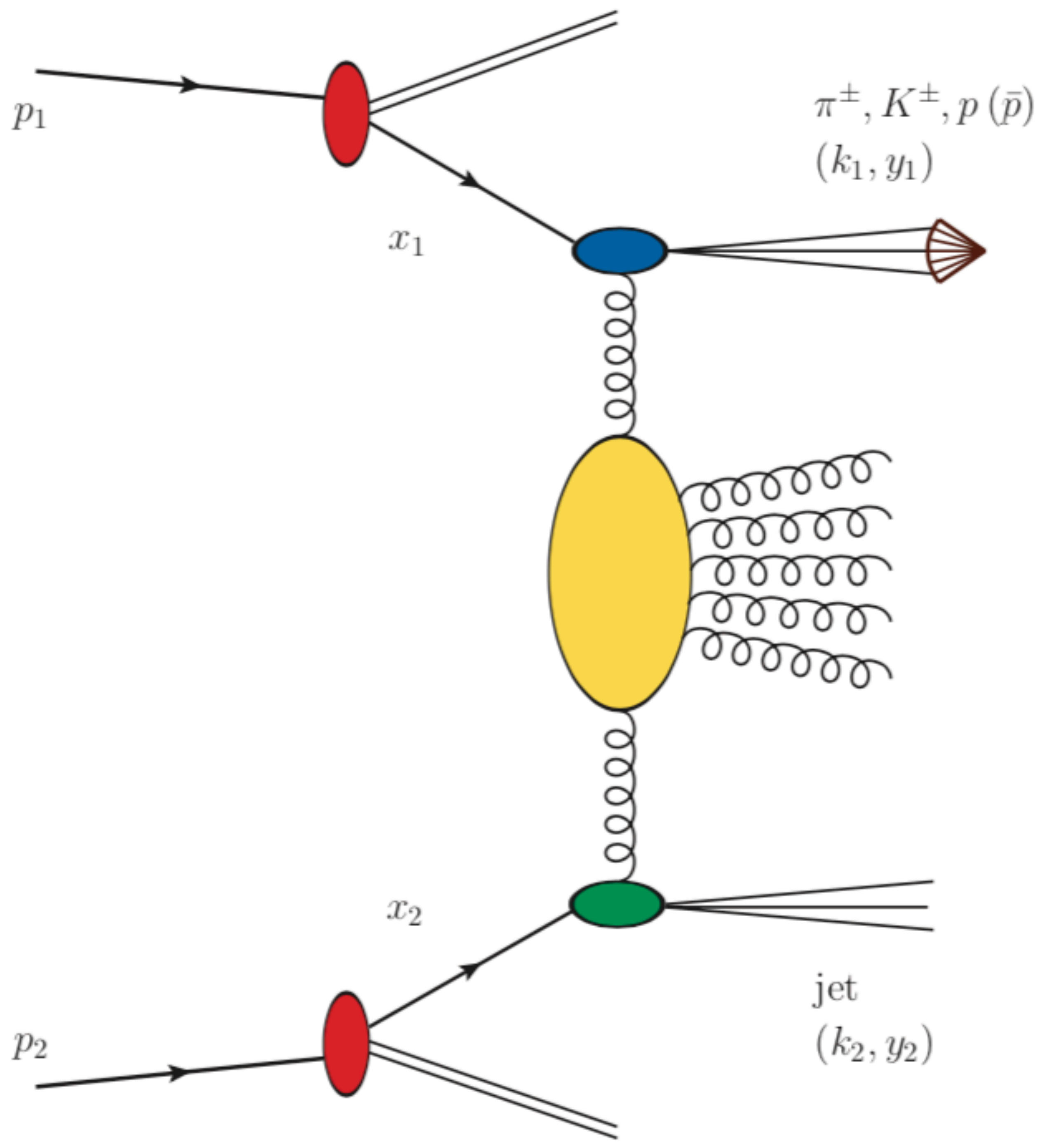
Inclusive di-hadron production

[D.Yu. Ivanov, A. Papa (2012)] (NLO forward-hadron impact factor)
[F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]



Inclusive hadron-jet production

[A.D. Bolognino, F.G.C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]
[F.G.C. (in preparation)]



From jets to hadrons

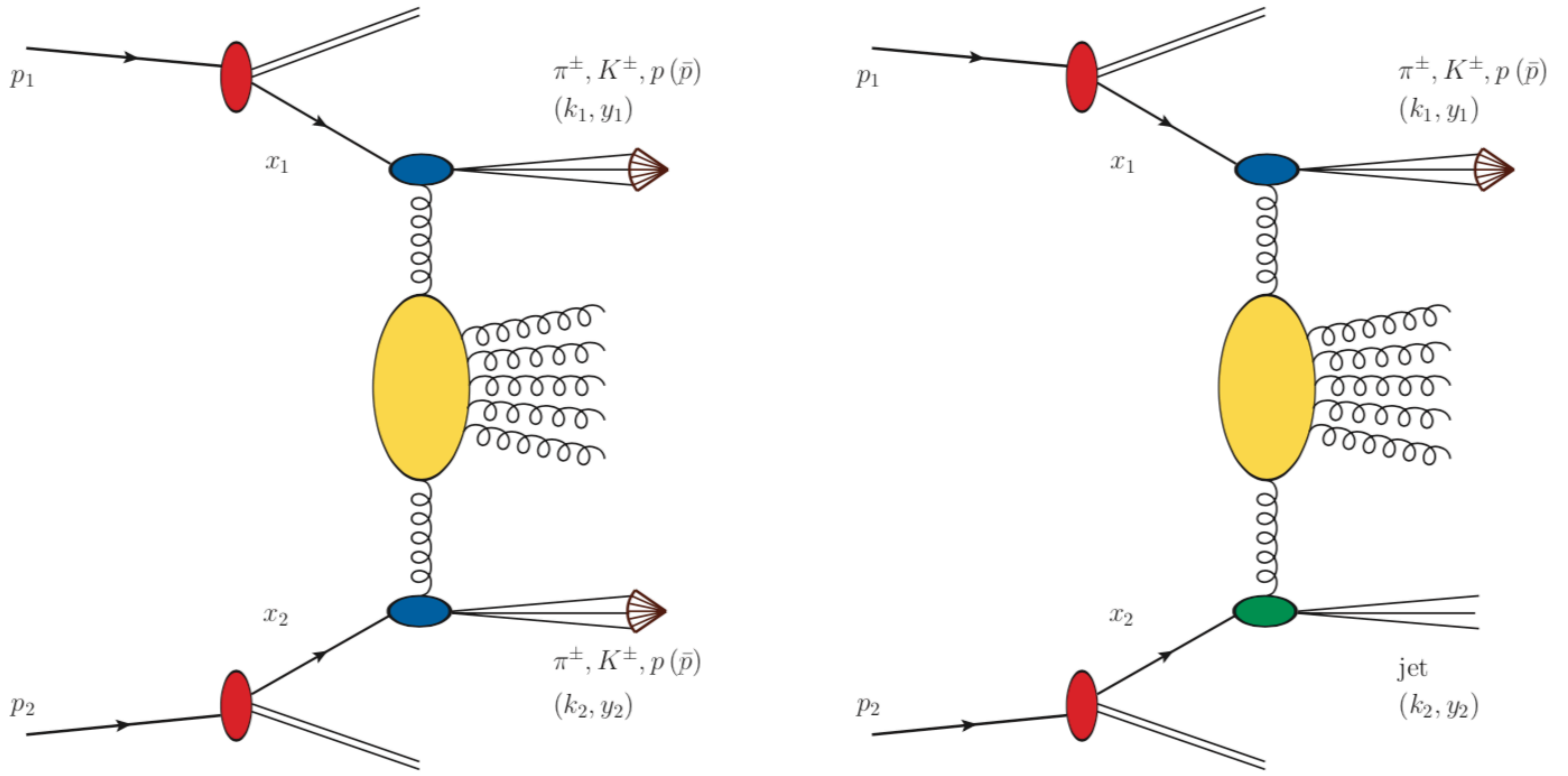
Di-hadron and hadron-jet correlations

Inclusive di-hadron production

[D.Yu. Ivanov, A. Papa (2012)] (NLO forward-hadron impact factor)
[F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]

Inclusive hadron-jet production

[A.D. Bolognino, F.G.C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]
[F.G.C. (in preparation)]

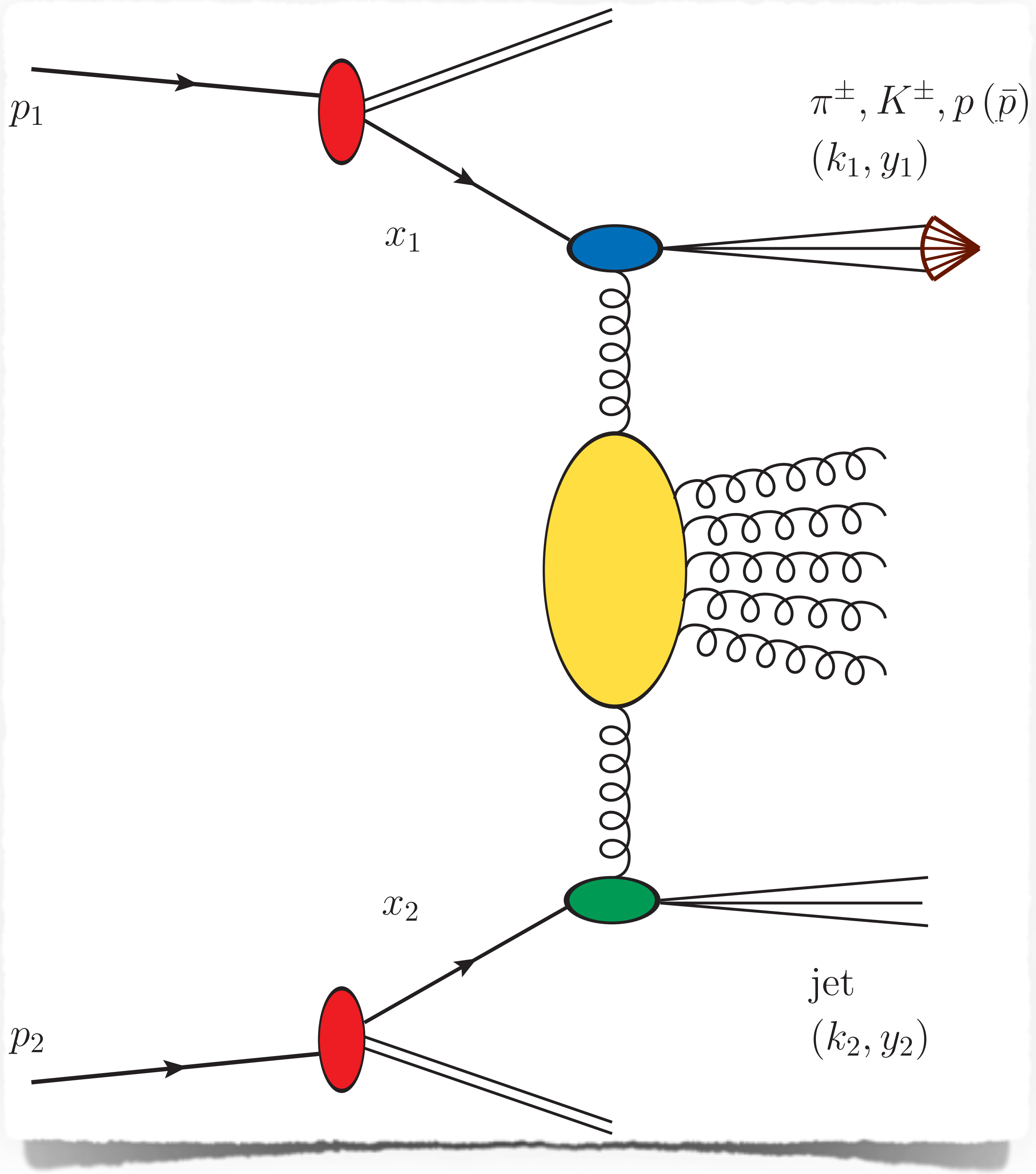


- ◇ NLO impact factors known $\Rightarrow$ full NLA BFKL analysis feasible
- ◇ PDFs + FFs at work (both), hadrons at smaller rapidities than jets (di-hadron)
- ◇ genuine *asymmetric* cuts in transverse momenta (hadron-jet)

Hadron + jet production within HyF

- Inclusive hadron + jet hadroproduction: high p_T and large rapidity separation, ΔY
- Moderate x (collinear PDFs), but t-channel p_T (BFKL resummation) → hybrid factorization

$$\frac{d\sigma}{dy_H\, dy_J\, d^2\vec{k}_H\, d^2\vec{k}_J} = \sum_{r,s=q,g} \int_0^1 \int_0^1 dx_1\, dx_2\, f_r(x_1, \mu_F)\, f_s(x_2, \mu_F)\, \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_H\, dy_J\, d^2\vec{k}_J\, d^2\vec{k}_J}$$

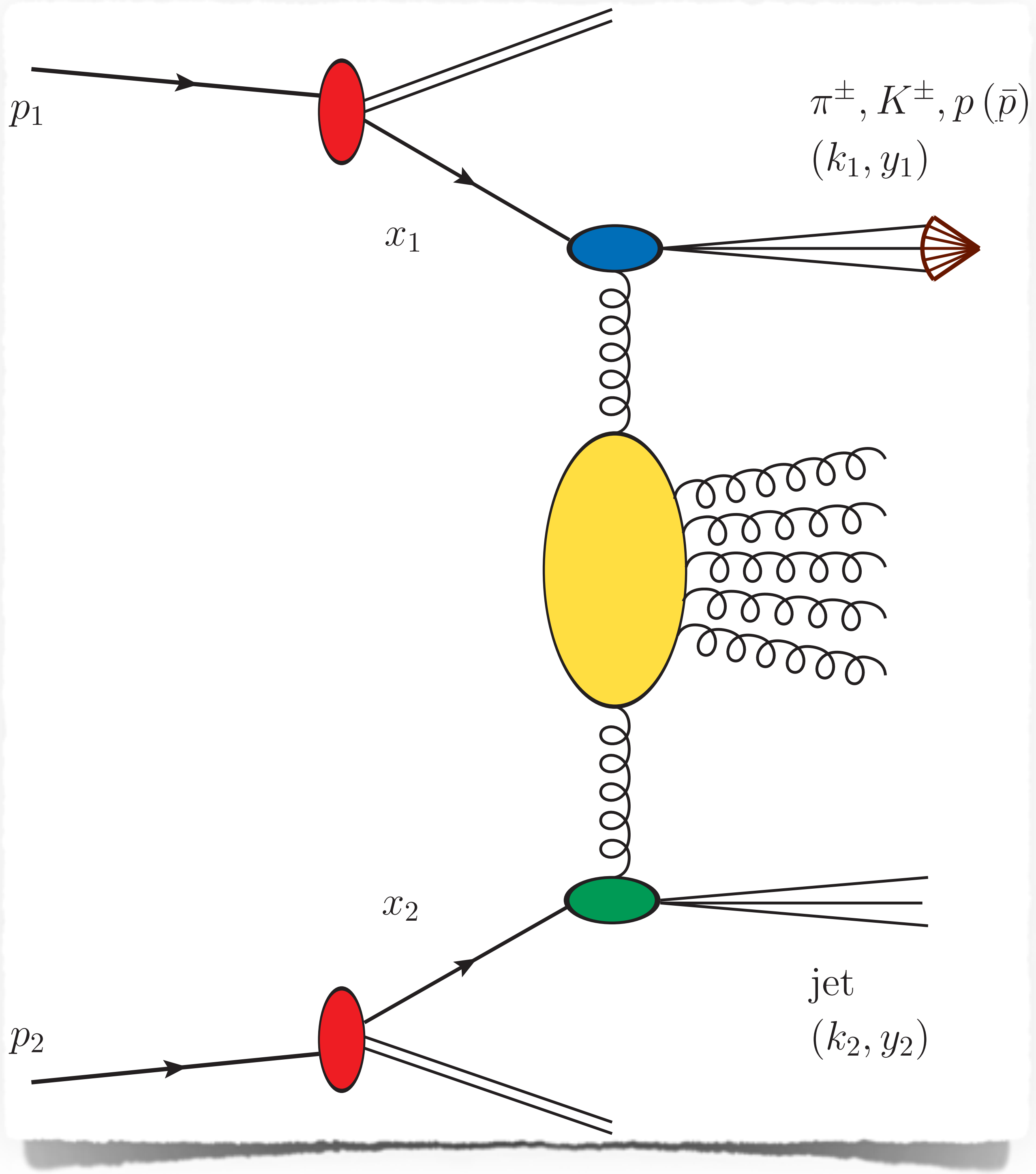


II.2 Light hadrons

Hadron + jet production within HyF

- Inclusive hadron + jet hadroproduction: high p_T and large rapidity separation, ΔY
- Moderate x (collinear PDFs), but t-channel p_T (BFKL resummation) → hybrid factorization

$$\frac{d\sigma}{dy_H\, dy_J\, d^2\vec{k}_H\, d^2\vec{k}_J} = \sum_{r,s=q,g} \int_0^1 \int_0^1 dx_1\, dx_2\, f_r(x_1, \mu_F)\, f_s(x_2, \mu_F)\, \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_H\, dy_J\, d^2\vec{k}_J\, d^2\vec{k}_J}$$



$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_H\, dy_J\, d^2\vec{k}_H\, d^2\vec{k}_J} = \frac{1}{(2\pi)^2}$$

NLO$^{(+)}$ $$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\, \mathcal{V}_H^{(r)}(\vec{q}_1, s_0, x_1; \vec{k}_H, x_H)\, D_r^H\left(\frac{x_H}{x_1}, \mu_F\right)$$

NLL $$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2)$$

NLO$^{(+)}$ $$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\, \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2; \vec{k}_J, x_J)$$

hadron vertex (off-shell amplitude)

hadron FF

BFKL Green's function

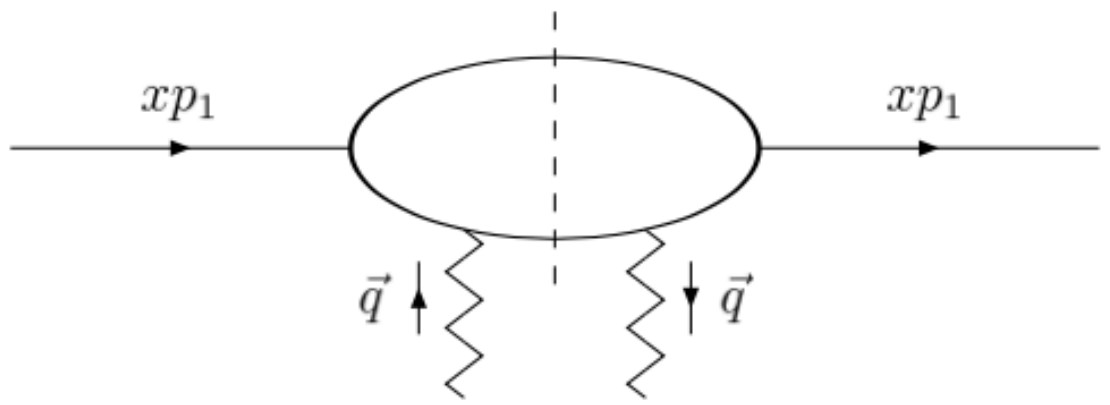
jet vertex (off-shell amplitude)

II.2 Light hadrons

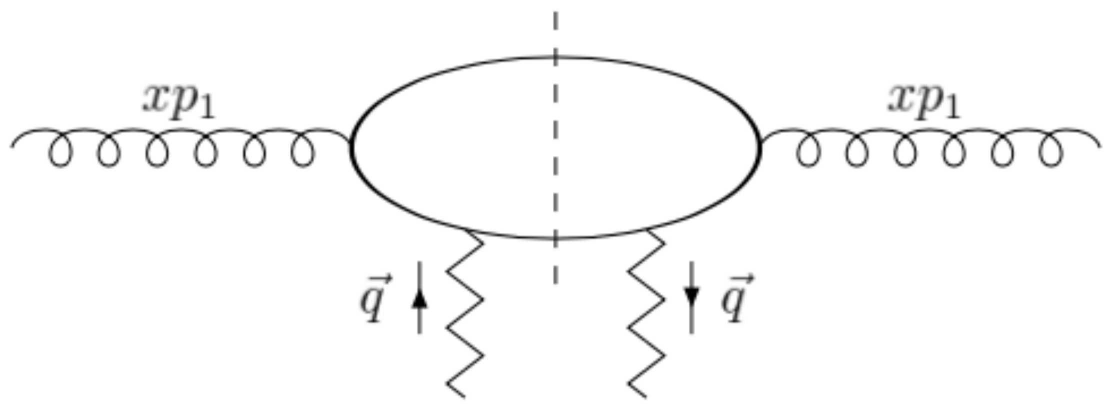
Forward-hadron impact factor

- take the impact factors for **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000); M. Ciafaloni and G. Rodrigo (2000)]



quark vertex

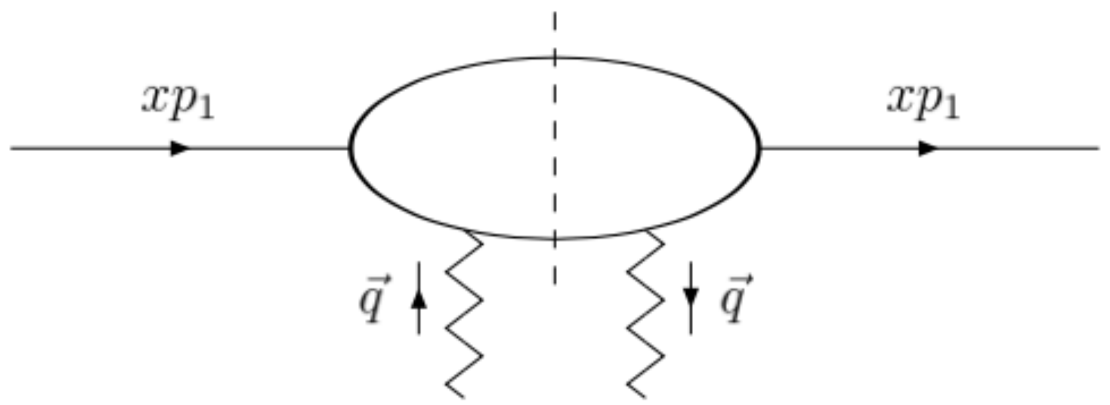


gluon vertex

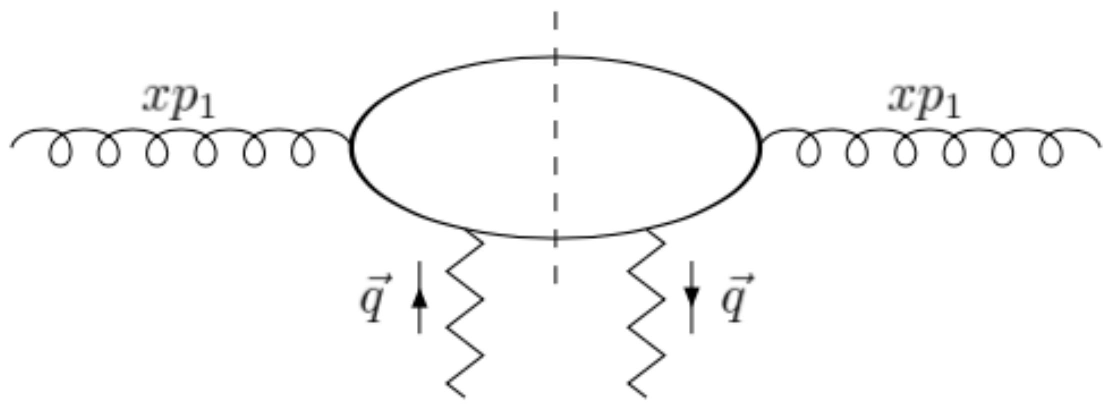
Forward-hadron impact factor

- take the impact factors for **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000); M. Ciafaloni and G. Rodrigo (2000)]

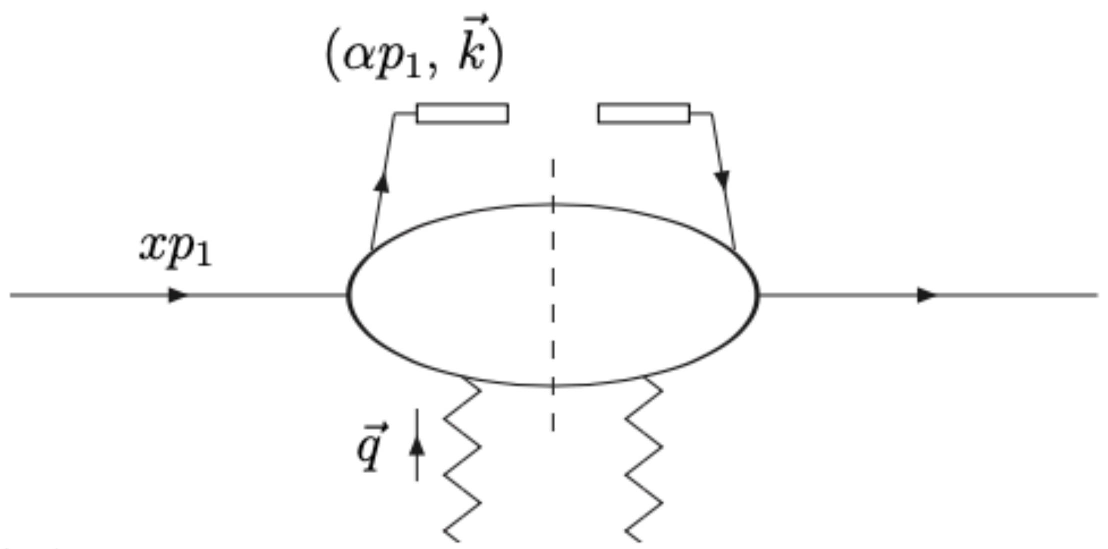


quark vertex

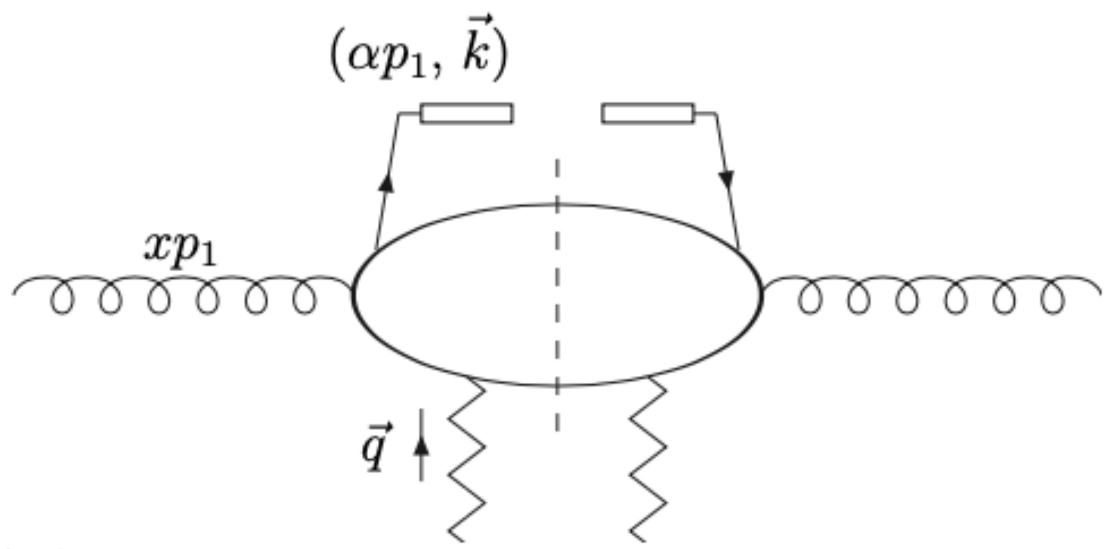


gluon vertex

- “open” one of the integrations over the phase space of the intermediate state to allow one parton to generate the jet



$\mathcal{V}^{(q)}_{H/J}$ (quark hadron/jet vertex)

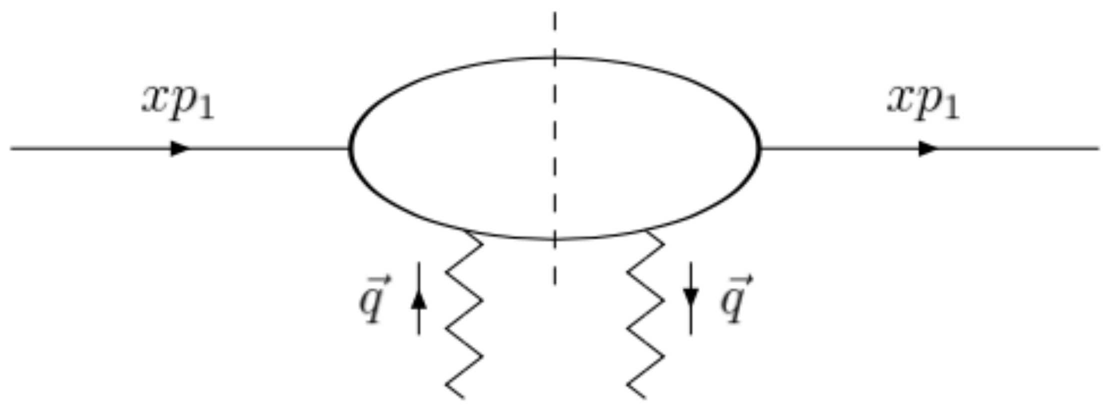


$\mathcal{V}^{(g)}_{H/J}$ (gluon hadron/jet vertex)

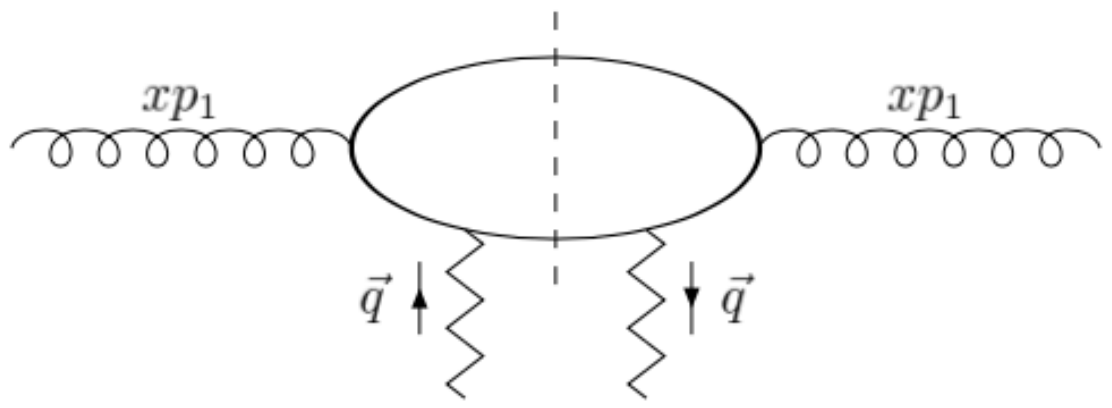
Forward-hadron impact factor

- take the impact factors for **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000); M. Ciafaloni and G. Rodrigo (2000)]

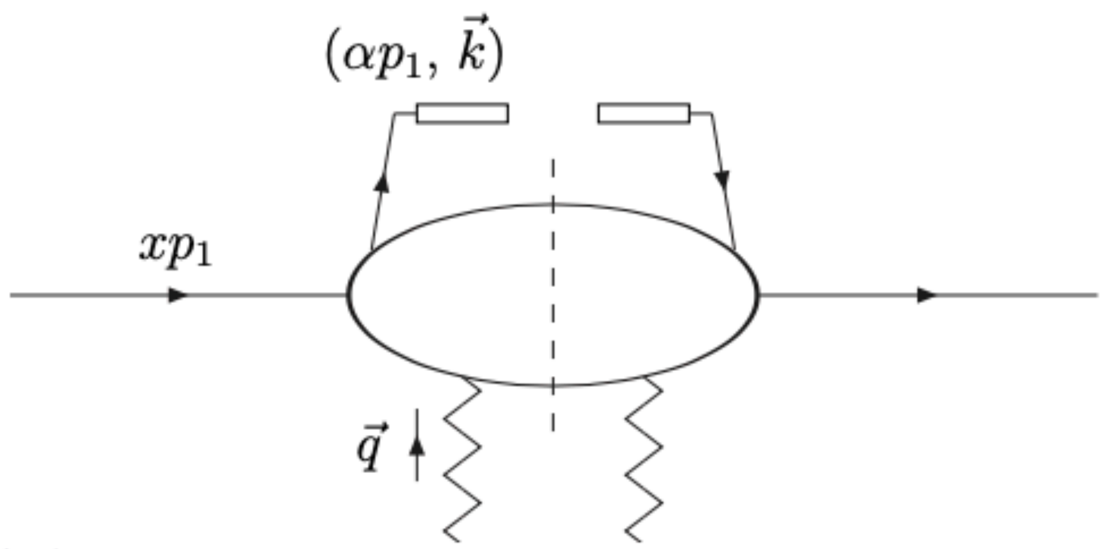


quark vertex

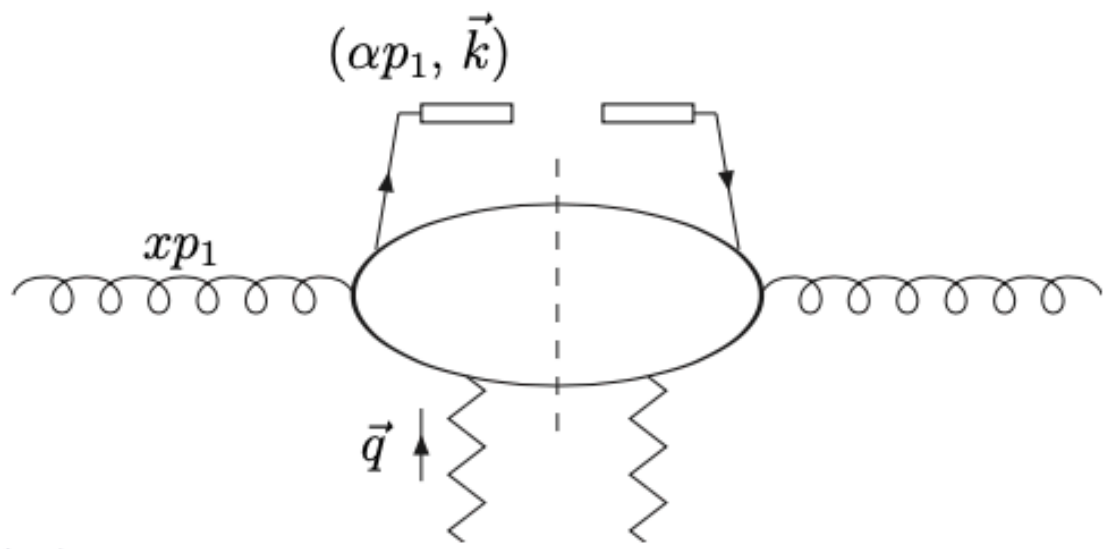


gluon vertex

- "open" one of the integrations over the phase space of the intermediate state to allow one parton to generate the jet



$\mathcal{V}^{(q)}_{H/J}$ (quark hadron/jet vertex)



$\mathcal{V}^{(g)}_{H/J}$ (gluon hadron/jet vertex)

- use QCD collinear factorization

$$\text{hadron} \;\to\; \sum_{r=q,\bar{q}} f_r \otimes \mathcal{V}^{(r)}_H \otimes D^H_r + f_g \otimes \mathcal{V}^{(g)}_H \otimes D^H_g$$

$$\text{jet} \;\to\; \sum_{s=q,\bar{q}} f_s \otimes \mathcal{V}^{(s)}_J + f_g \otimes \mathcal{V}^{(g)}_J$$

Hadron + jet: Observables & kinematics

- **Observables:**

  φ-averaged cross section, $\mathcal{C}_0$; $\langle \cos(n\varphi) \rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0} \equiv R_{n0}$, with $n = 1, 2, 3$;

  $\langle \cos(2\varphi) \rangle / \langle \cos(\varphi) \rangle \equiv \mathcal{C}_2/\mathcal{C}_1 \equiv R_{21}$, $\langle \cos(3\varphi) \rangle / \langle \cos(2\varphi) \rangle \equiv \mathcal{C}_3/\mathcal{C}_2 \equiv R_{32}$;

  Azimuthal distribution

$$\frac{1}{\sigma}\frac{d\sigma}{d\varphi} = \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty}\cos(n\varphi)\langle\cos(n\varphi)\rangle\right\} \equiv \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty}\cos(n\varphi)R_{n0}\right\}$$

Hadron + jet: Observables & kinematics

- **Observables:**

  φ-averaged cross section, $\mathcal{C}_0$; $\langle \cos(n\varphi) \rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0} \equiv R_{n0}$, with $n = 1, 2, 3$;

  $\langle \cos(2\varphi) \rangle / \langle \cos(\varphi) \rangle \equiv \mathcal{C}_2/\mathcal{C}_1 \equiv R_{21}$, $\langle \cos(3\varphi) \rangle / \langle \cos(2\varphi) \rangle \equiv \mathcal{C}_3/\mathcal{C}_2 \equiv R_{32}$;

  Azimuthal distribution

$$\frac{1}{\sigma}\frac{d\sigma}{d\varphi} = \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty}\cos(n\varphi)\langle\cos(n\varphi)\rangle\right\} \equiv \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty}\cos(n\varphi)R_{n0}\right\}$$

- **Kinematic settings (CMS- and CASTOR-jet detection):**
  - **3 NLA BFKL** reactions at $\sqrt{s} = 13$ TeV: MN jets, hadron-jet, di-hadron
  - $|y_H| \leqslant 2.4$; $|y_J^{(\mathrm{CMS})}| \leqslant 4.7$; $-6.6 \leqslant y_J^{(\mathrm{CST})} \leqslant -5.2$
  - $k_H \geqslant 5, 10$ GeV; $k_J^{(\mathrm{CMS})} \geqslant 20, 35, 45$ GeV; $k_J^{(\mathrm{CST})} \geqslant 10$ GeV
  - **5** final-state configurations with **disjoint** k-windows

Hadron + jet: Observables & kinematics

- **Observables:**

  φ-averaged cross section, $\mathcal{C}_0$; $\langle \cos(n\varphi) \rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0} \equiv R_{n0}$, with $n = 1, 2, 3$;

  $\langle \cos(2\varphi) \rangle / \langle \cos(\varphi) \rangle \equiv \mathcal{C}_2/\mathcal{C}_1 \equiv R_{21}$, $\langle \cos(3\varphi) \rangle / \langle \cos(2\varphi) \rangle \equiv \mathcal{C}_3/\mathcal{C}_2 \equiv R_{32}$;

  Azimuthal distribution

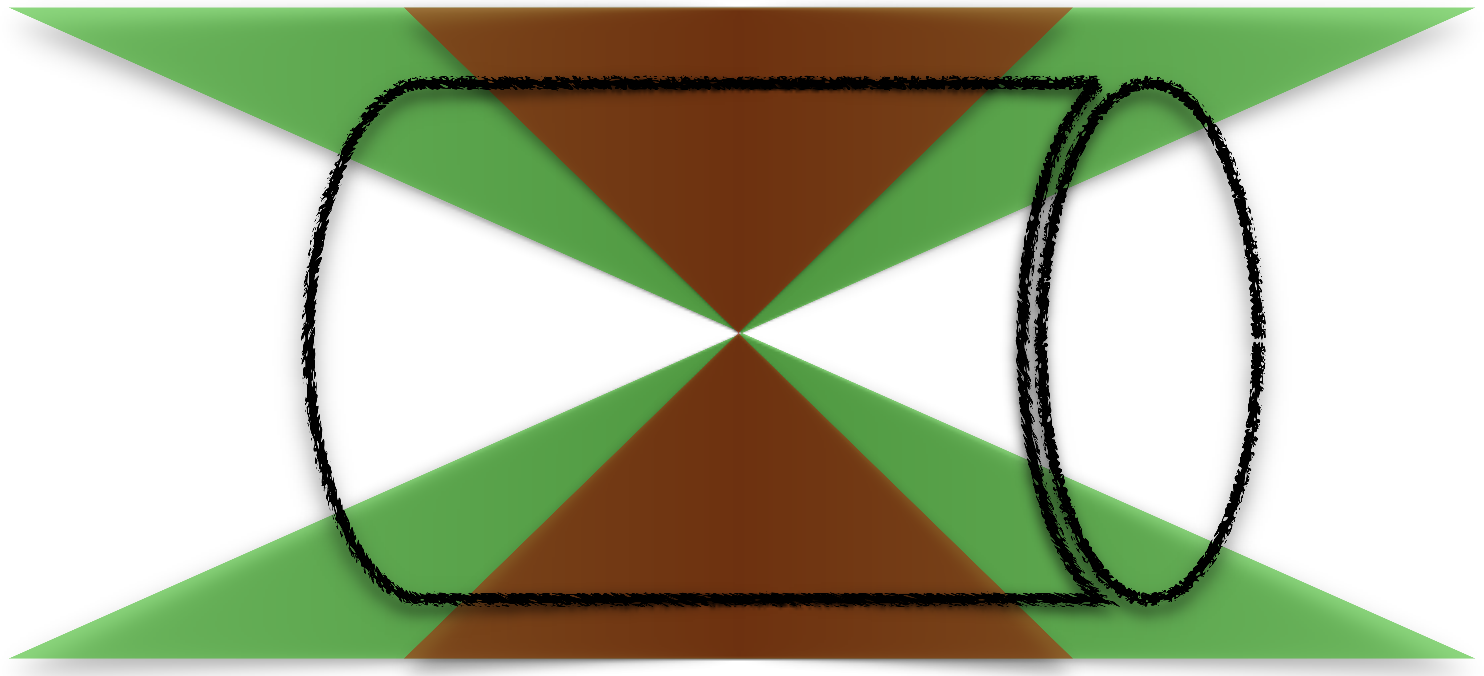
$$\frac{1}{\sigma}\frac{d\sigma}{d\varphi} = \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty} \cos(n\varphi)\langle \cos(n\varphi) \rangle\right\} \equiv \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty} \cos(n\varphi) R_{n0}\right\}$$

- **Kinematic settings (CMS- and CASTOR-jet detection):**
  - ◇ **3 NLA BFKL** reactions at $\sqrt{s} = 13$ TeV: MN jets, hadron-jet, di-hadron
  - ◇ $|y_H| \leqslant 2.4$; $|y_J^{(\mathrm{CMS})}| \leqslant 4.7$; $-6.6 \leqslant y_J^{(\mathrm{CST})} \leqslant -5.2$
  - ◇ $k_H \geqslant 5, 10$ GeV; $k_J^{(\mathrm{CMS})} \geqslant 20, 35, 45$ GeV; $k_J^{(\mathrm{CST})} \geqslant 10$ GeV
  - ◇ **5** final-state configurations with **disjoint** k-windows

- **Numerical specifics:**
  - ◇ **JETHAD** (**HEP@WORK**, FORTRAN08/PYTHON3)
  - ◇ PDF4LHC15 ⊛ (AKK08, DSS07, HKNS07, NNFF1.0)

Jet and hadrons @CMS+CASTOR

Forward + backward CMS detections



$|y_{\rm jet}| < 4.7$

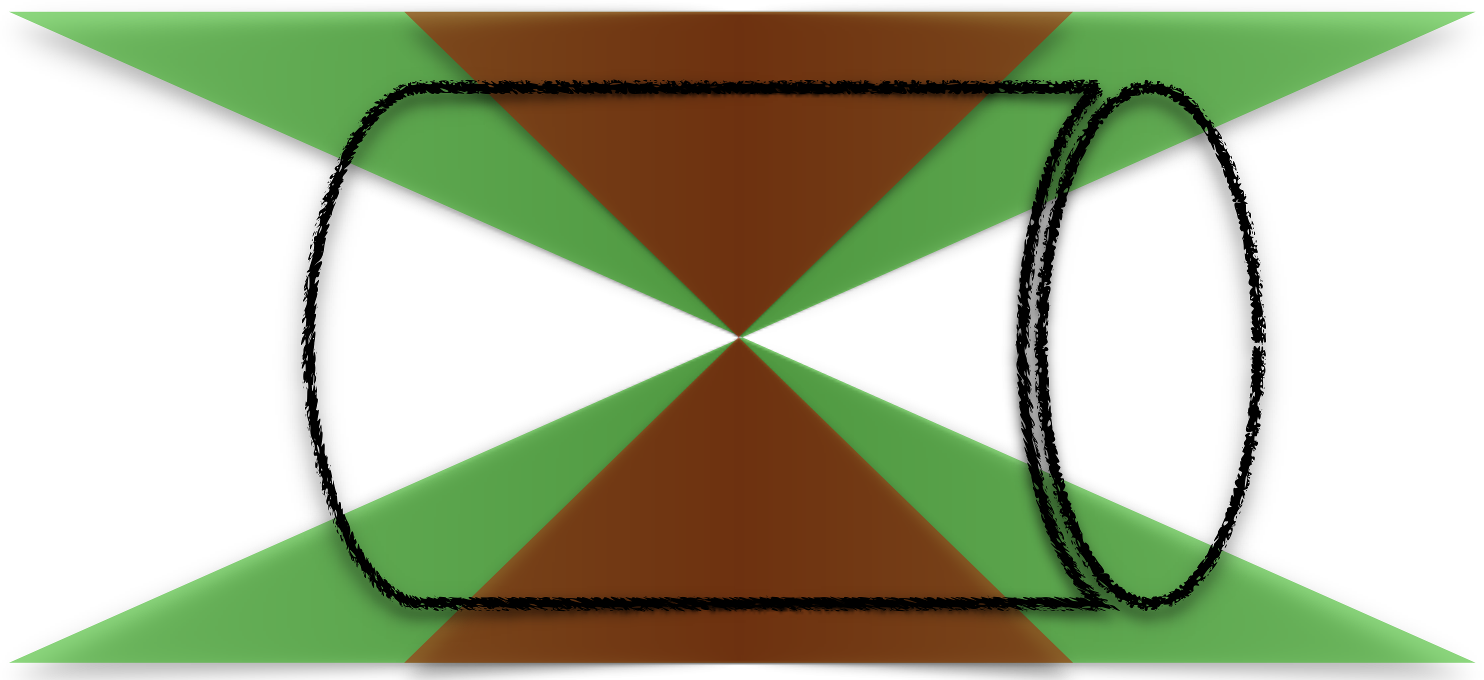
CMS barrel + endcaps

$|y_{\rm hadron}| < 2.4$

CMS barrel

Jet and hadrons @CMS+CASTOR

Forward + backward CMS detections



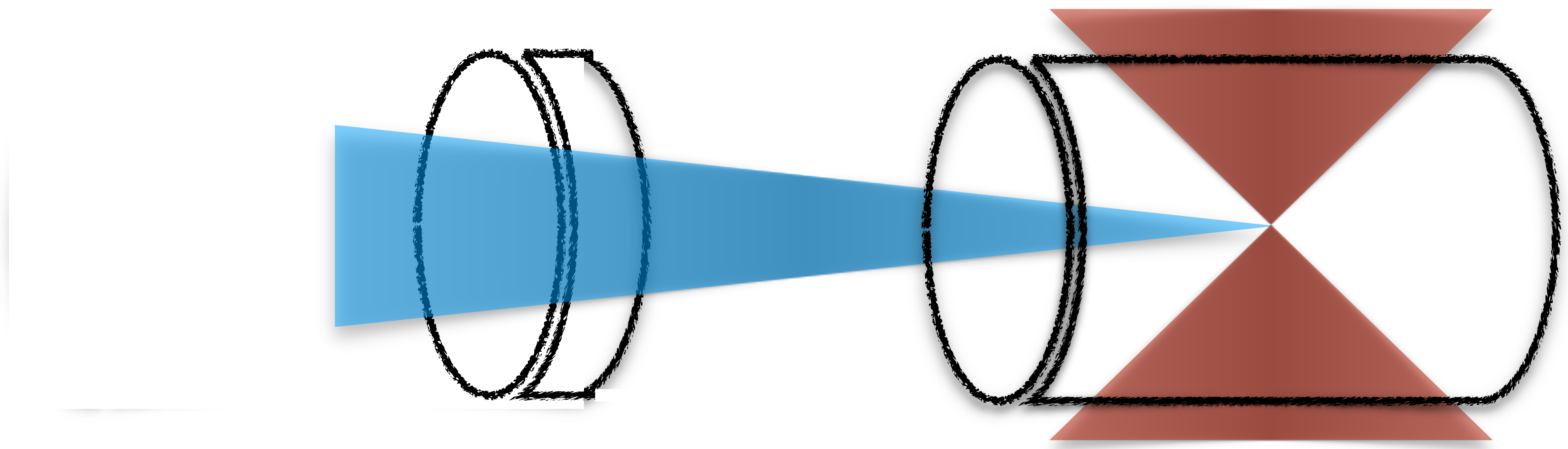
$|y_{\rm jet}| < 4.7$

CMS barrel + endcaps

$|y_{\rm hadron}| < 2.4$

CMS barrel

Forward CMS + ultra-backward CASTOR detections



$-6.6 < |y_{\rm jet}| < -5.2$

CASTOR

$|y_{\rm hadron}| < 2.4$

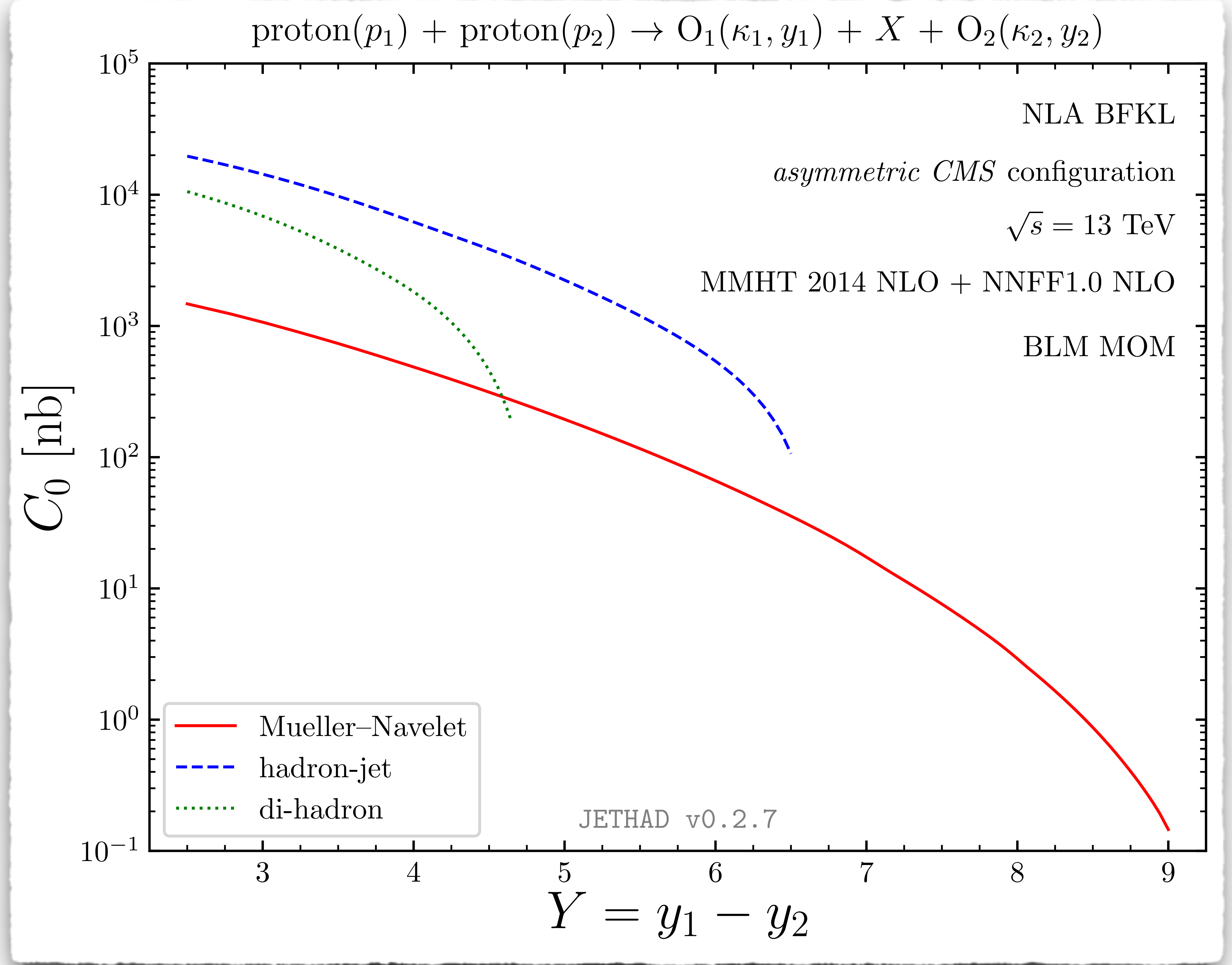
CMS barrel

(hadron + jet) 🔗 [F. G. C. et al., Eur. Phys. J. C 78 (2018) 9, 772]

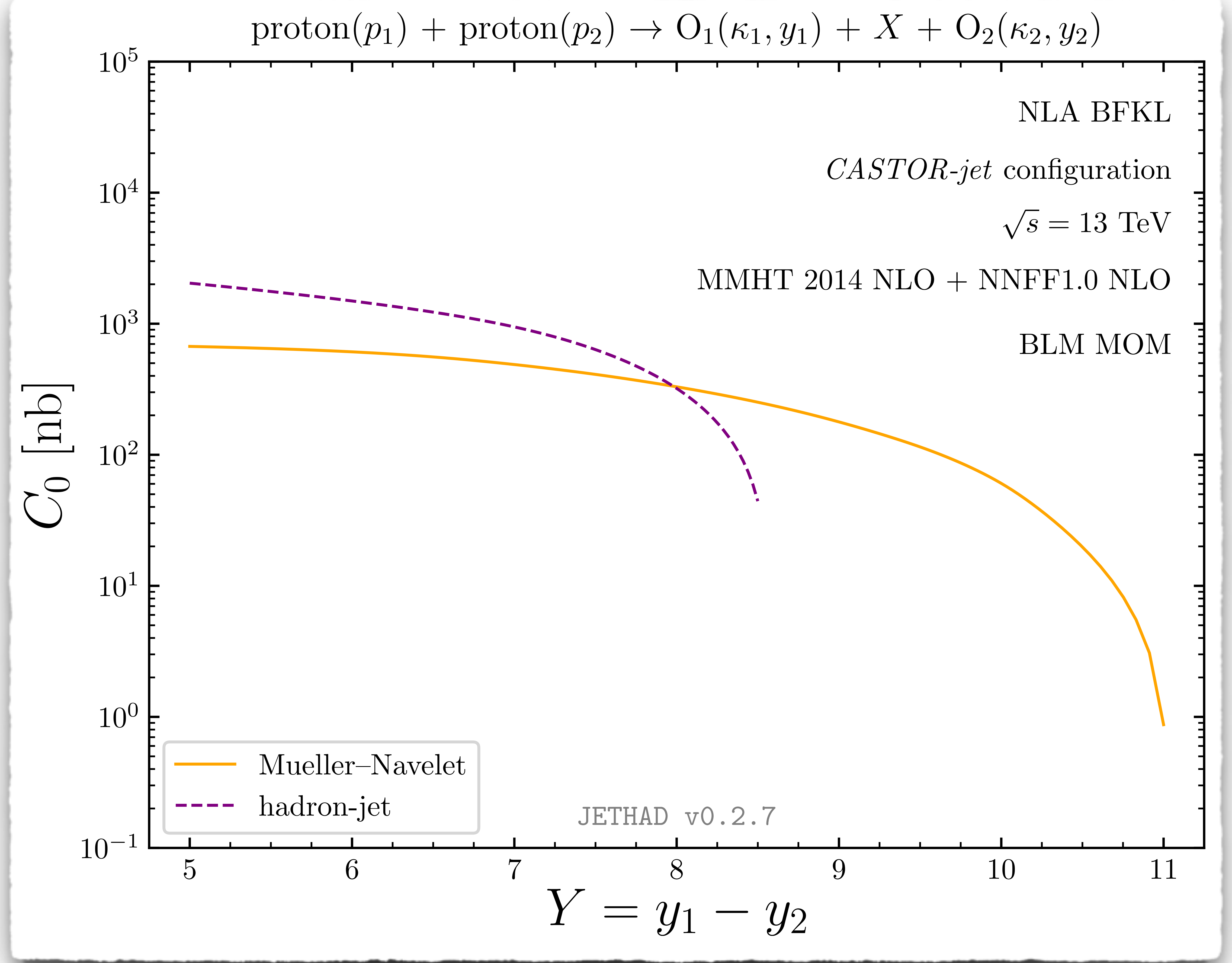
(Hunting BFKL) 🔗 [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

MN jets, di-hadron and hadron + jet @13 TeV LHC

CMS-jet



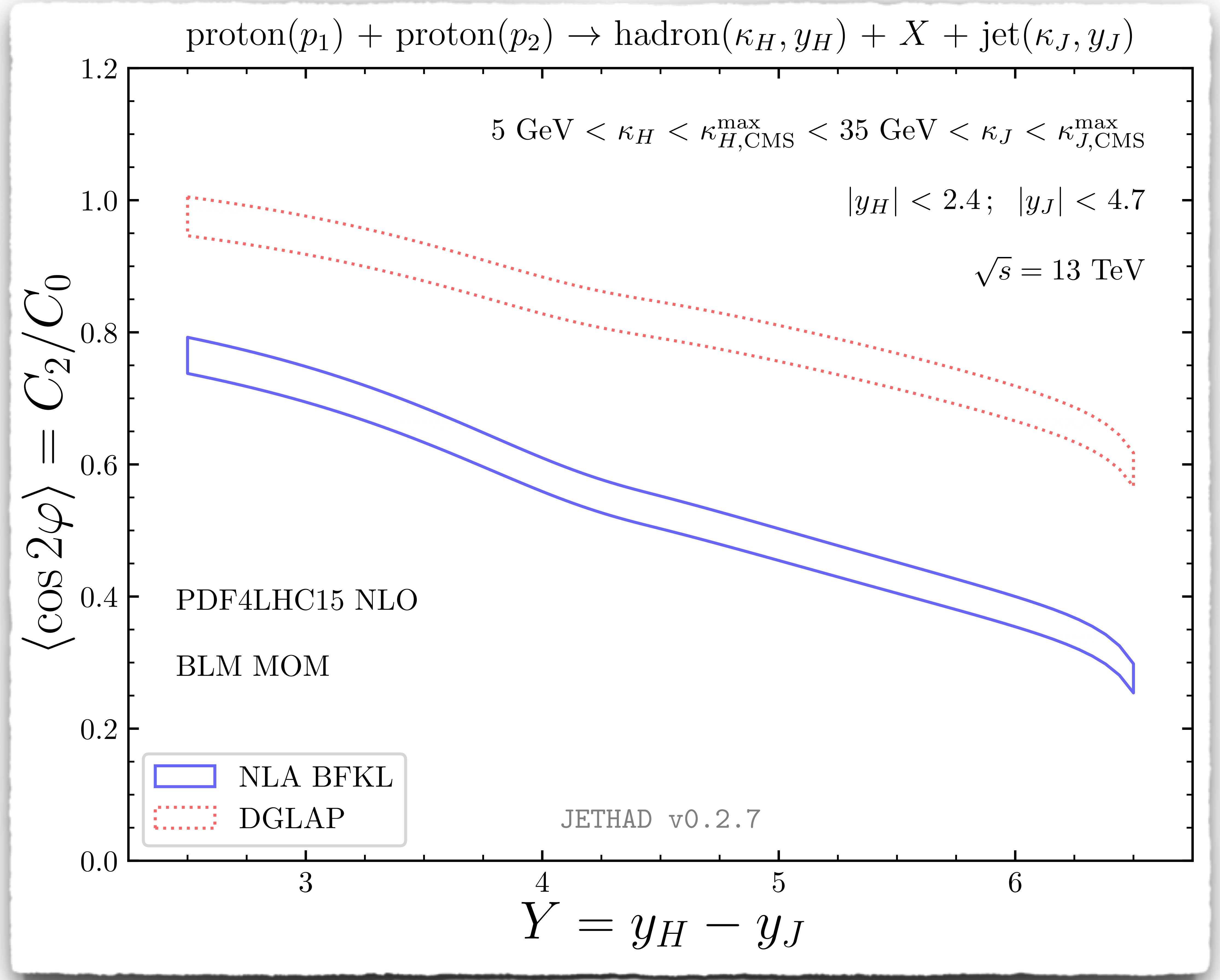
CASTOR-jet



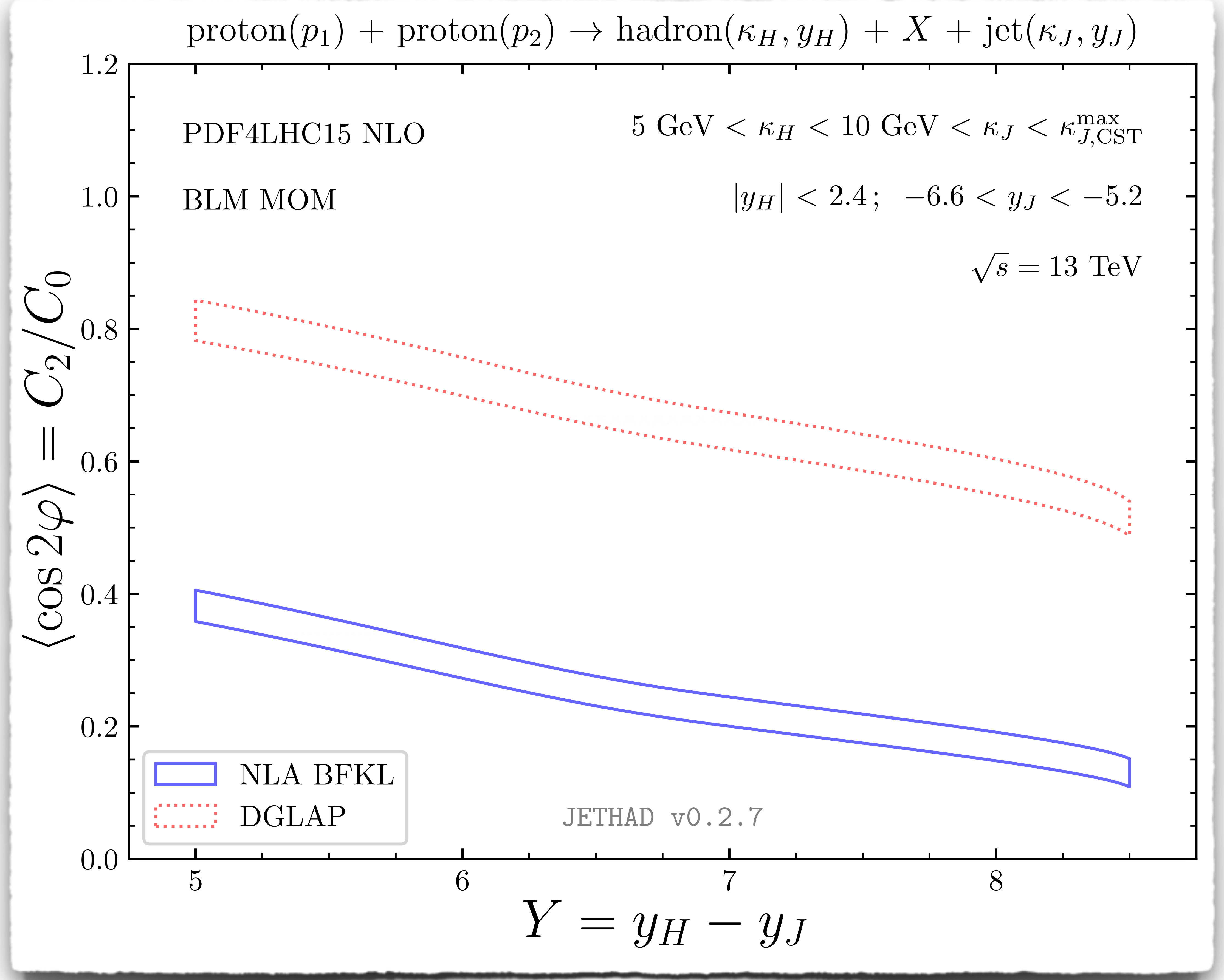
🔗 [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

MN jets, di-hadron and hadron + jet @13 TeV LHC

CMS-jet



CASTOR-jet



🔗 [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

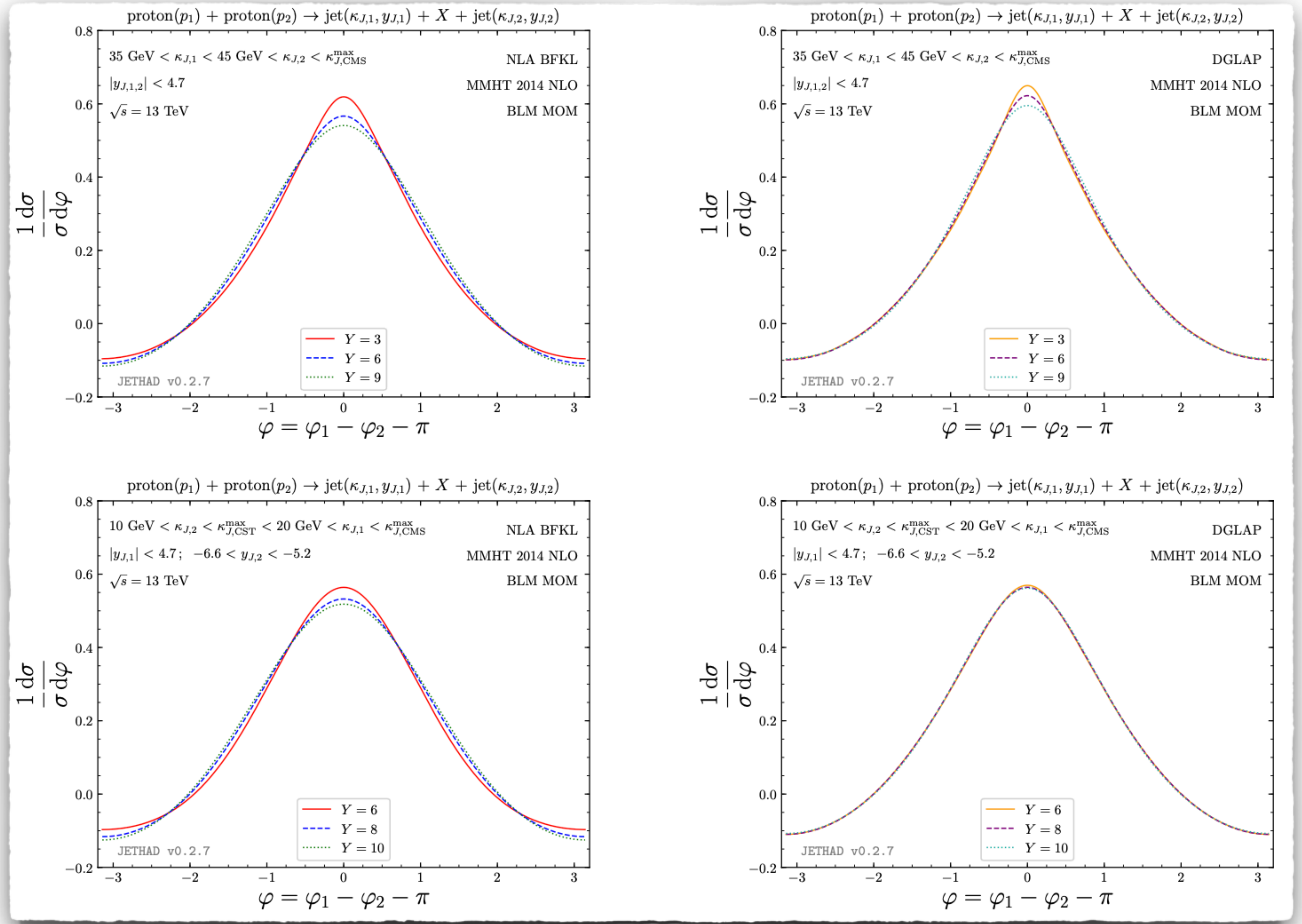
Hadron + jet: Azimuthal distributions @13 TeV LHC

NLL/NLO

HE-NLO

CMS-jet

CASTOR-jet



[F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

II.2 Light hadrons

3

NLL instabilities

Hadron + jet: Azimuthal distributions @13 TeV LHC

NLA BFKL corrections to cross section with opposite sign with respect to the leading order (LO) result and large in absolute value...

- ...call for some optimization procedure...
- ...choose scales to mimic the most relevant subleading terms

Hadron + jet: Azimuthal distributions @13 TeV LHC

NLA BFKL corrections to cross section with opposite sign with respect to the leading order (LO) result and large in absolute value...

- ◇ ...call for some optimization procedure...
- ◇ ...choose scales to mimic the most relevant subleading terms

- **BLM** [S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]
  - ✓ preserve the conformal invariance of an observable...
  - ✓ ...by making vanish its β_0-dependent part

* "Exact" BLM:

  suppress NLO IFs + NLO Kernel β_0-dependent factors

* Partial (approximated) BLM:

a) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \, \exp\left[2\left(1+\frac{2}{3}I\right) - f(\nu) - \frac{5}{3}\right]$ ← NLO IFs β_0

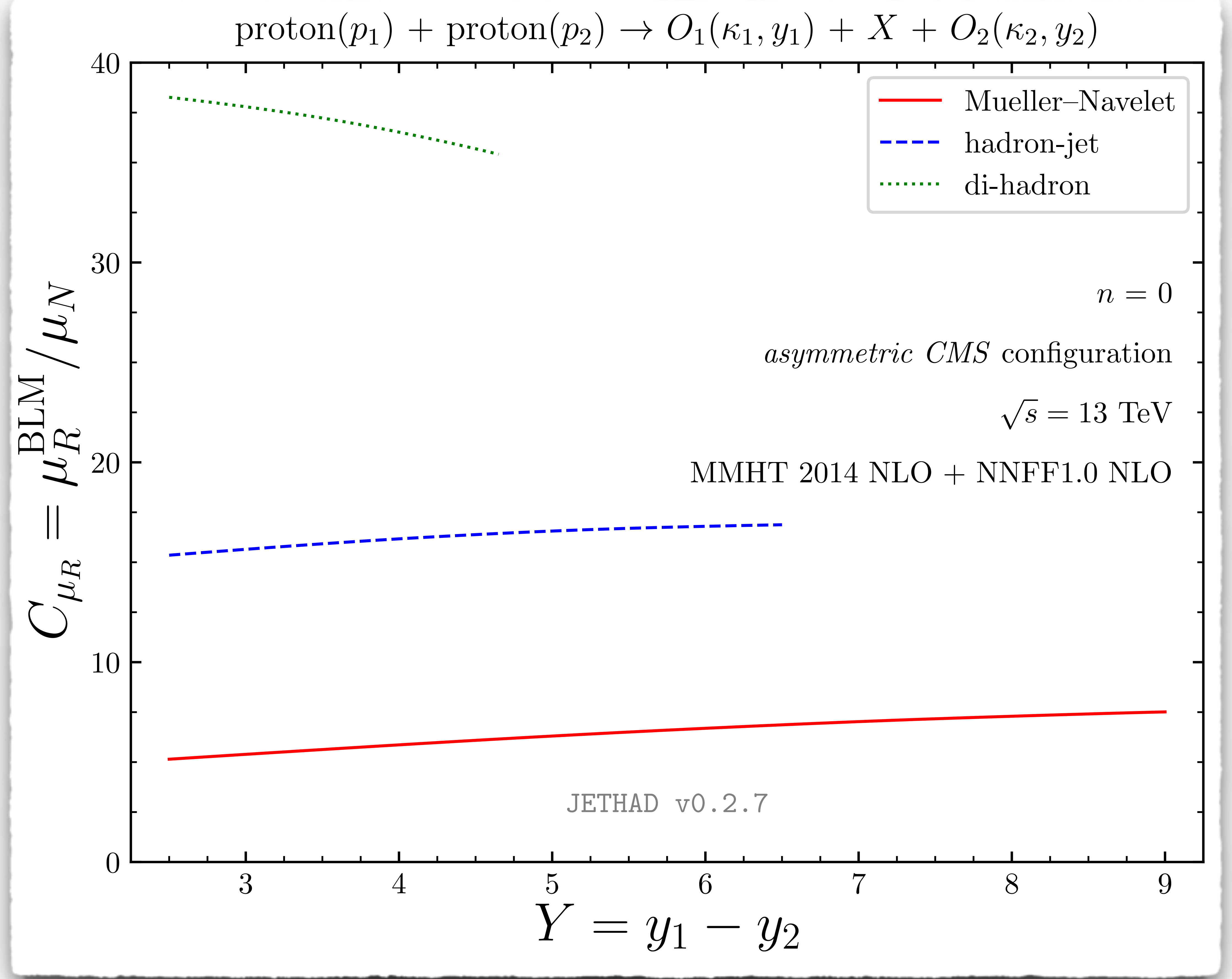
b) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \, \exp\left[2\left(1+\frac{2}{3}I\right) - 2f(\nu) - \frac{5}{3} + \frac{1}{2}\chi(\nu,n)\right]$ ← NLO Kernel β_0

$$\text{with} \quad i\frac{d}{d\nu}\ln\left(\frac{c_1}{c_2}\right) = 2\left[f(\nu) - \ln\left(\sqrt{\vec{k}_1^2\vec{k}_2^2}\right)\right]$$

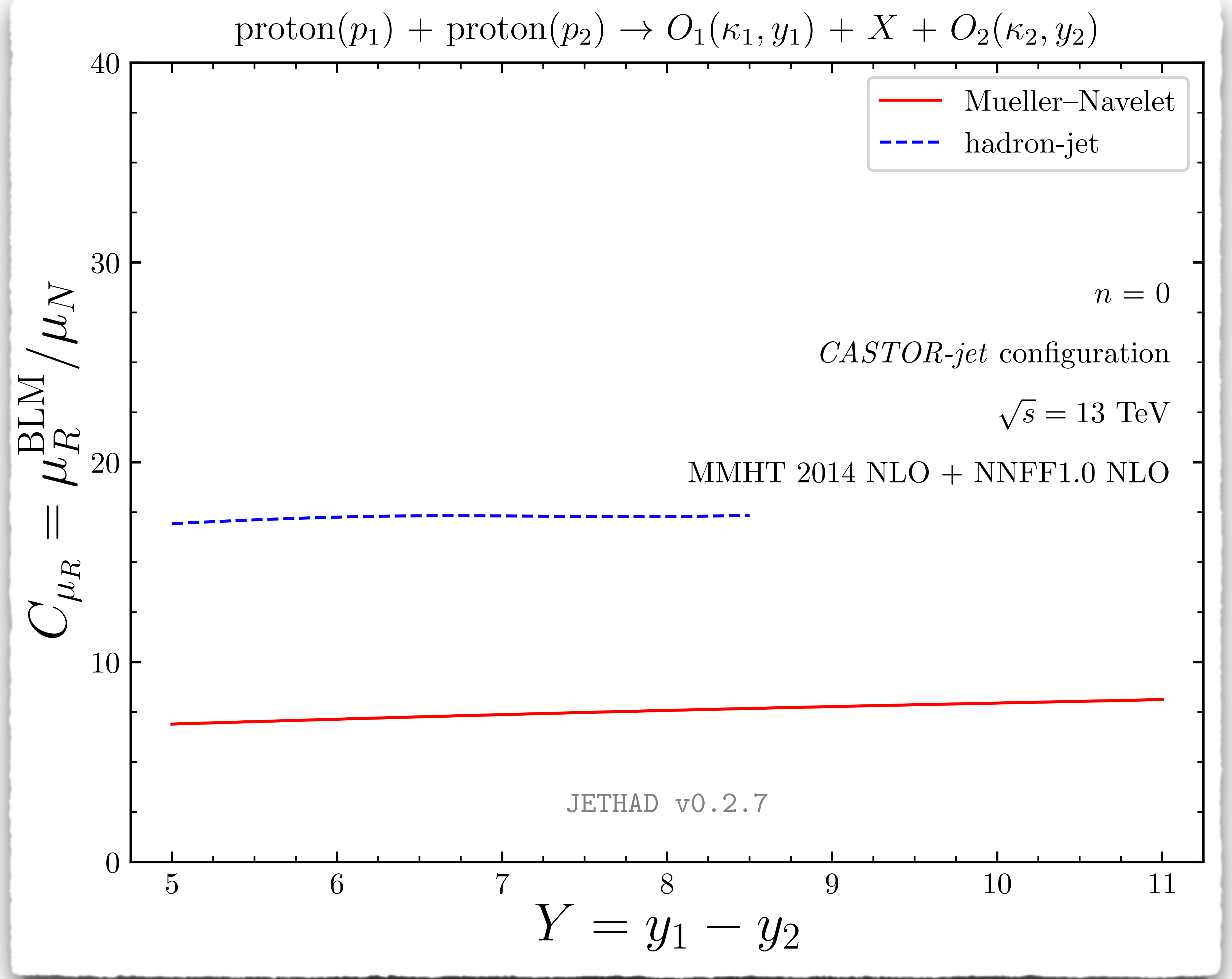
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]

Hadron + jet: BLM scales

CMS-jet



CASTOR-jet

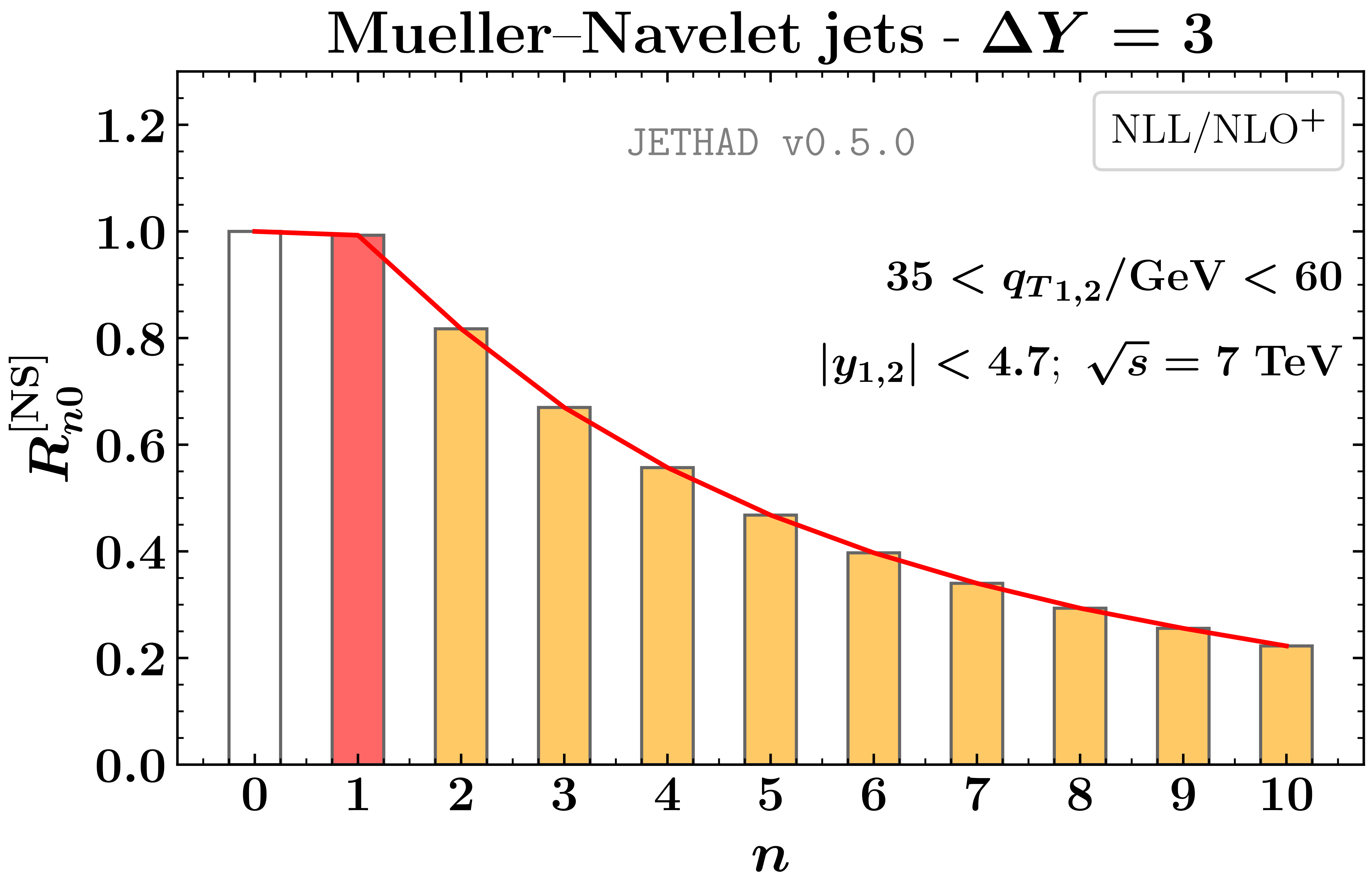


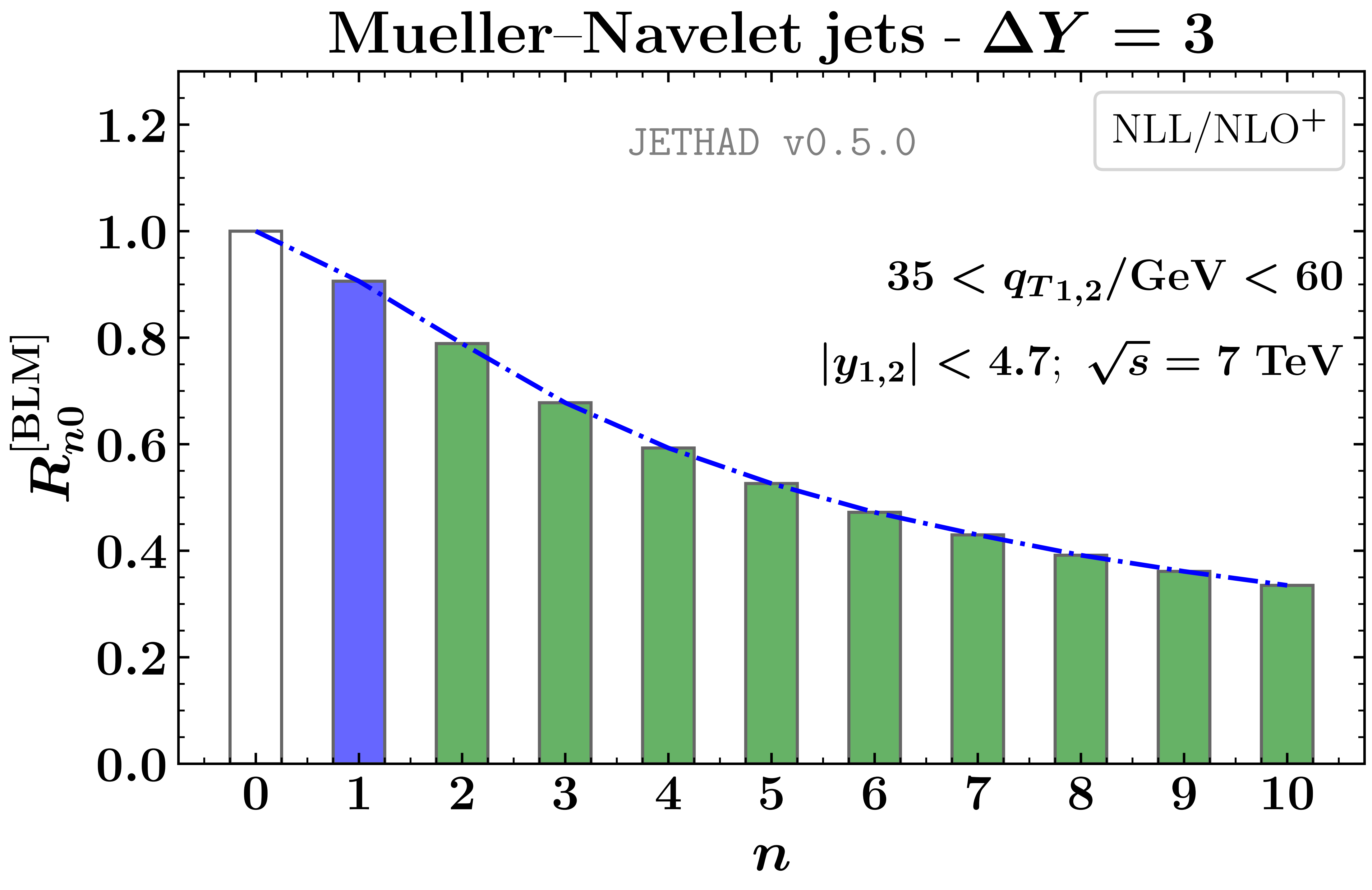
🔗 [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

MN jets: Hunting data with azimuthal distributions

Natural scales

BLM scales



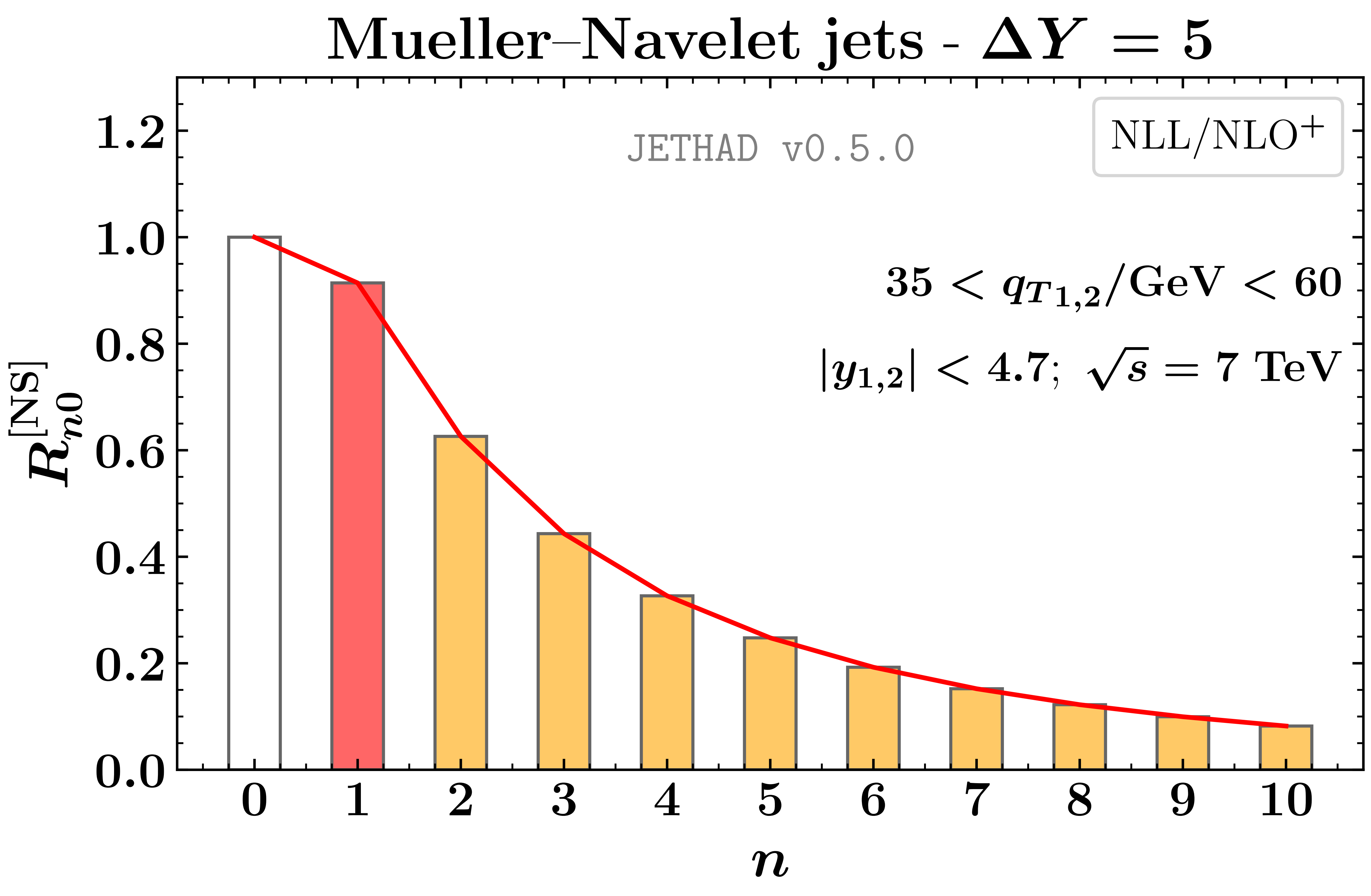


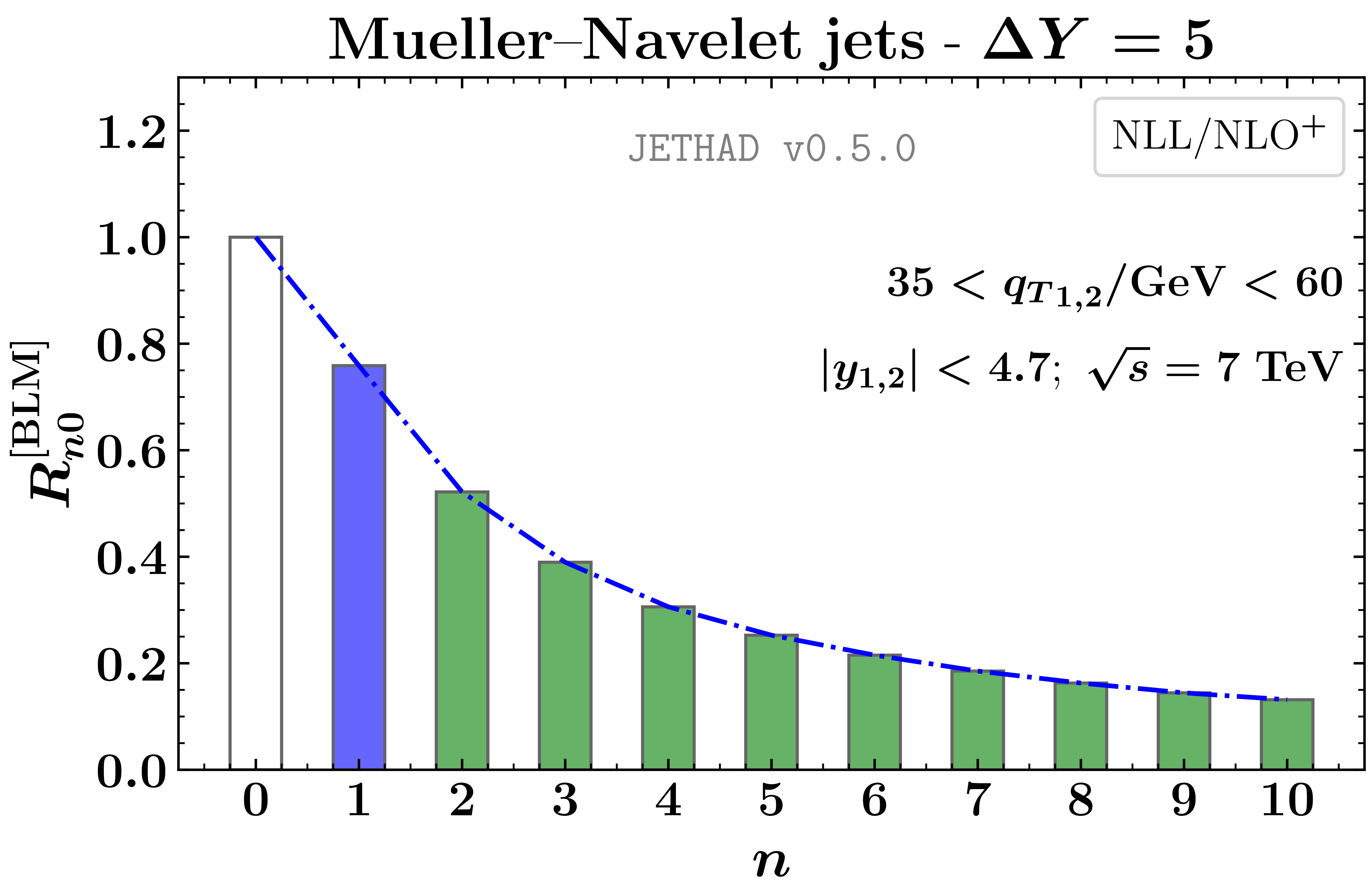
🔗 [F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]

MN jets: Hunting data with azimuthal distributions

Natural scales

BLM scales



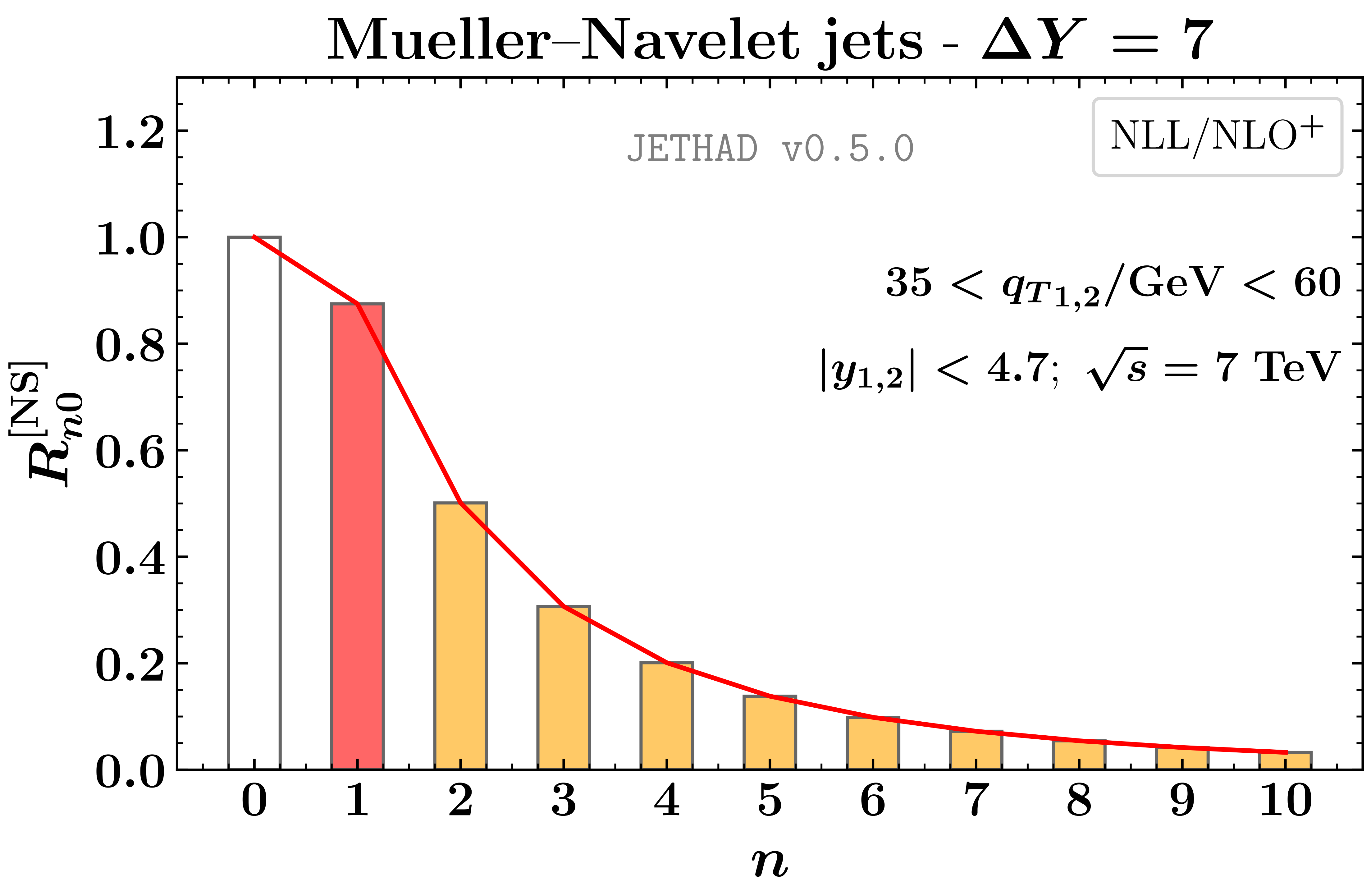


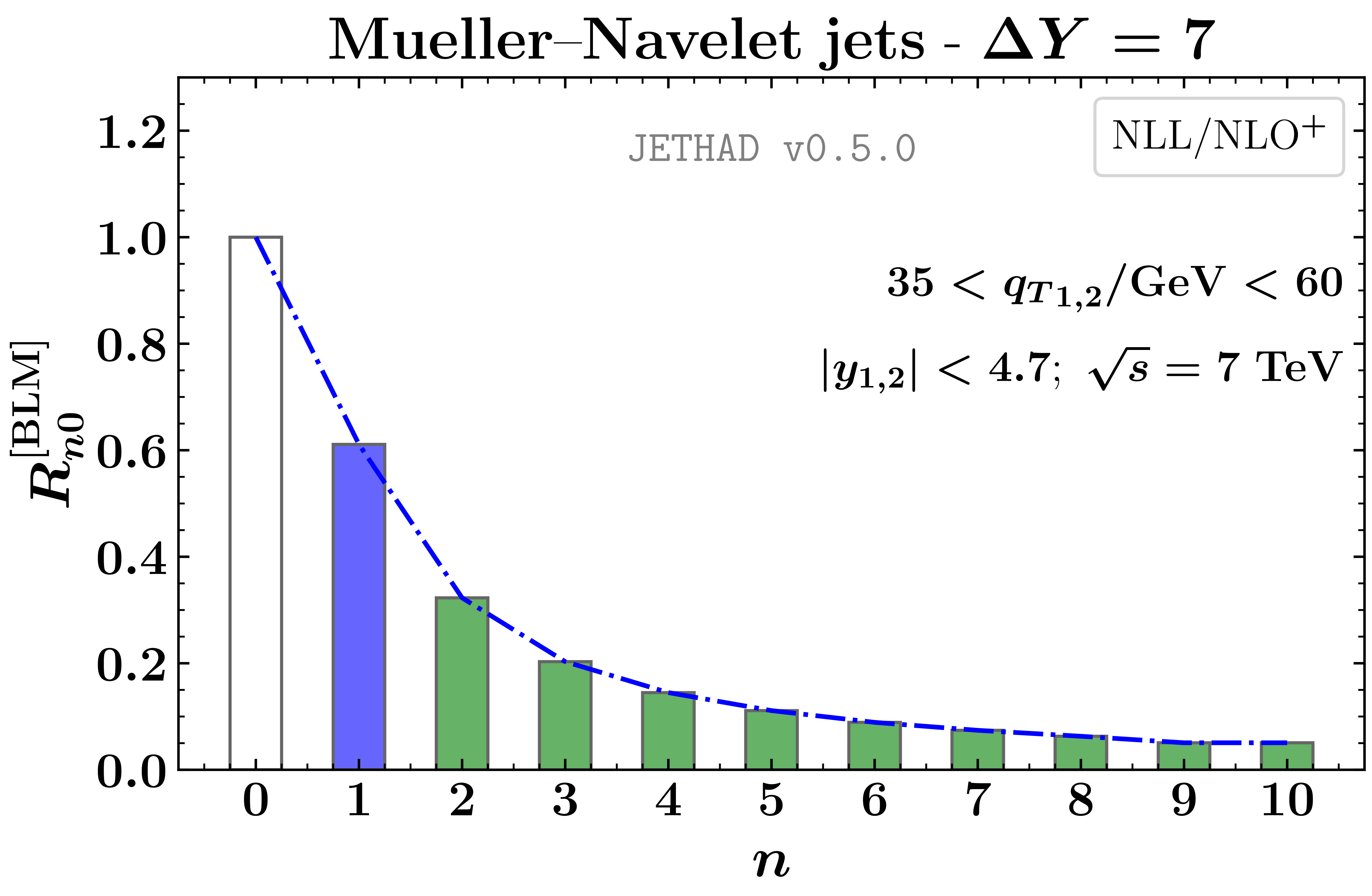
🔗 [F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]

MN jets: Hunting data with azimuthal distributions

Natural scales

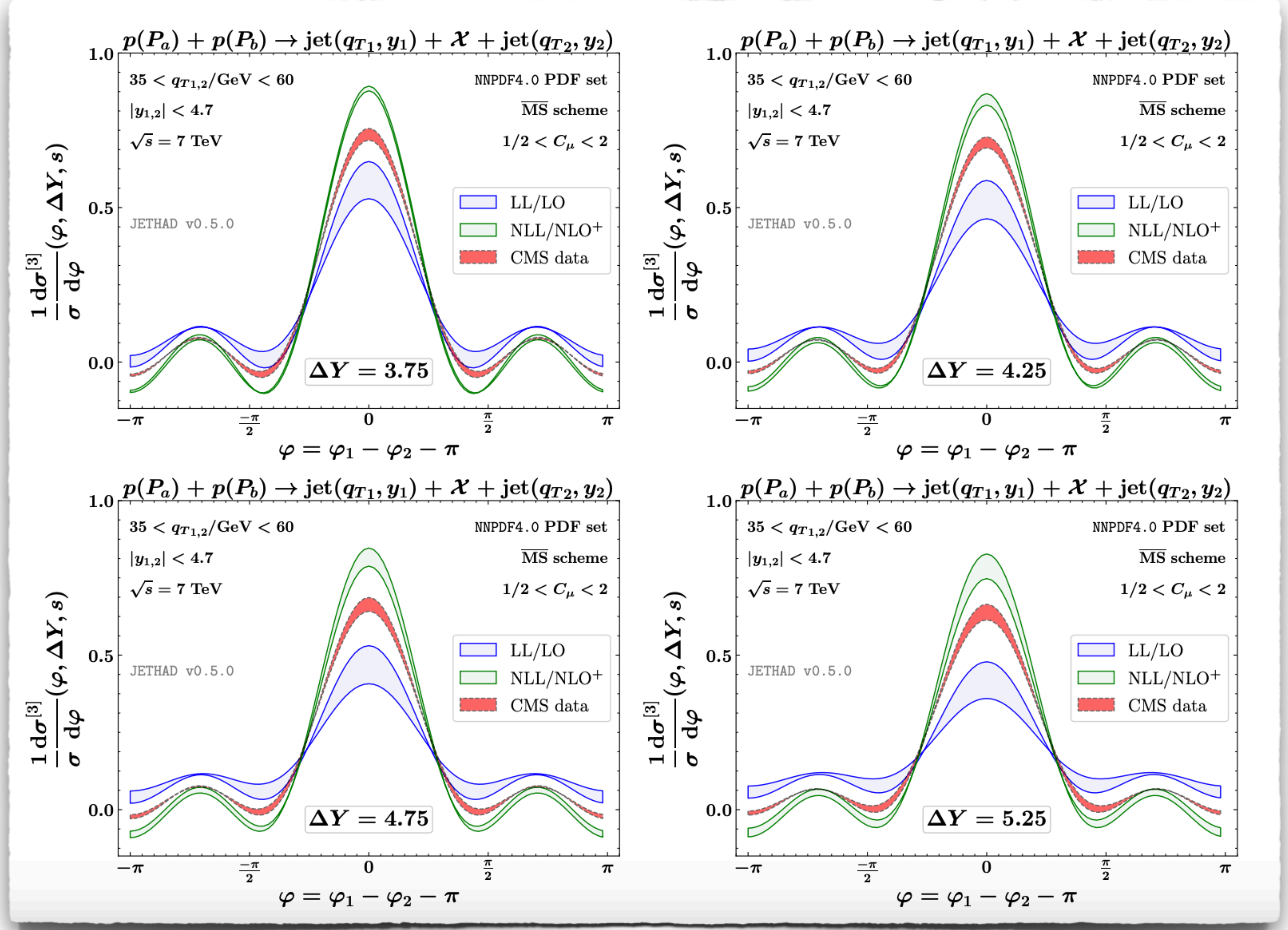
BLM scales





🔗 [F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]

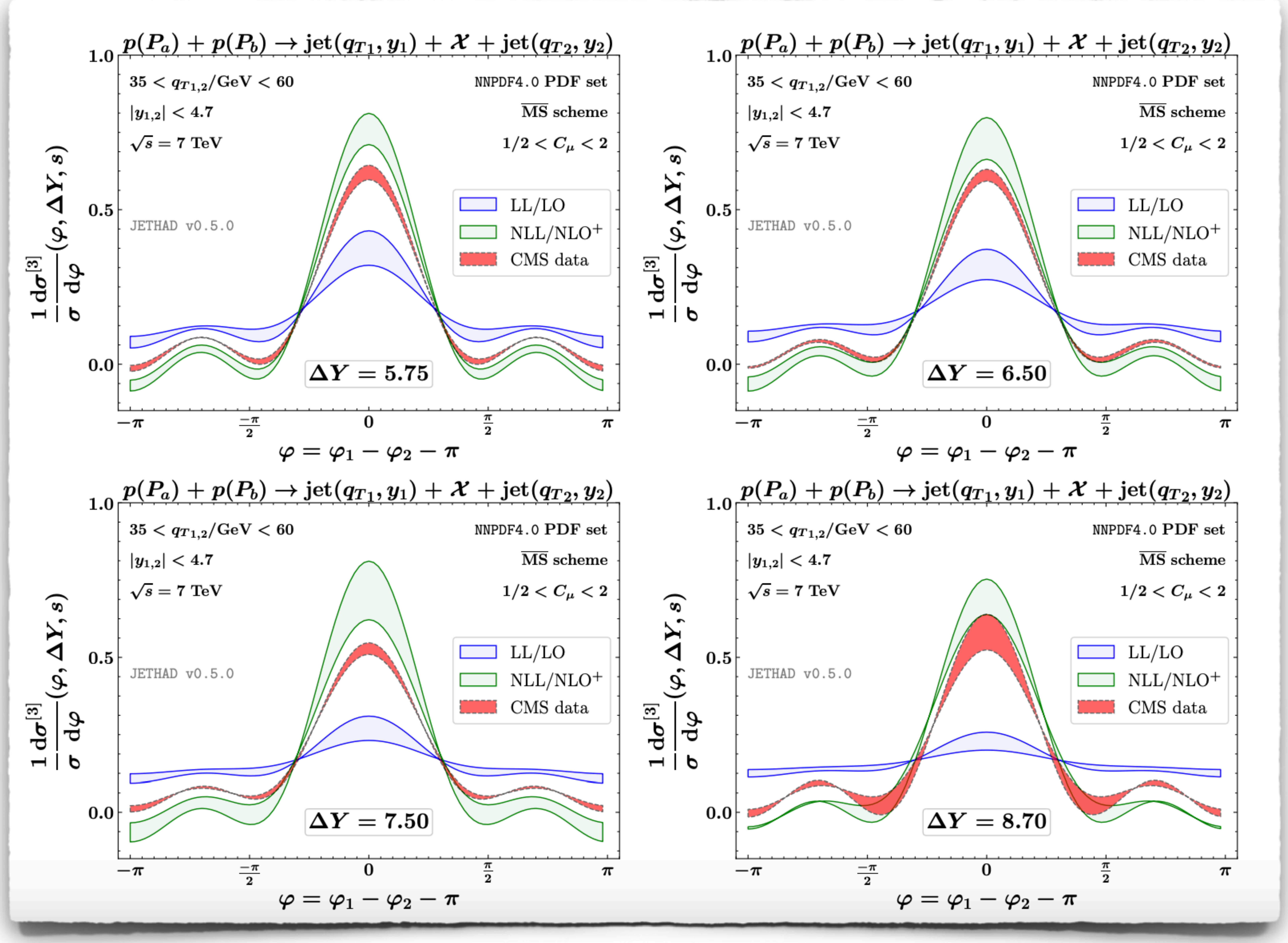
MN jets: Hunting data with azimuthal distributions



II.3 NLL instabilities

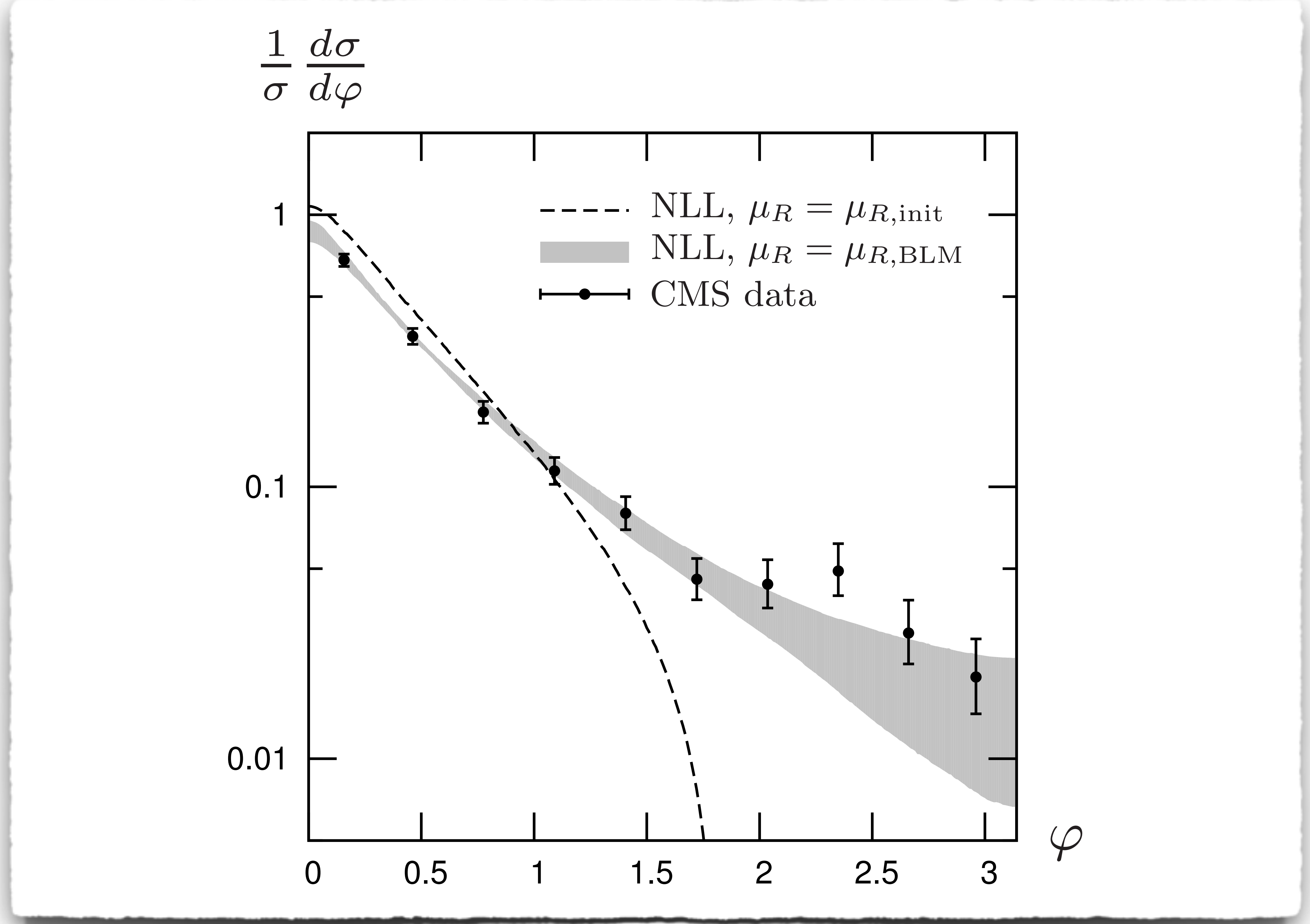
[F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]

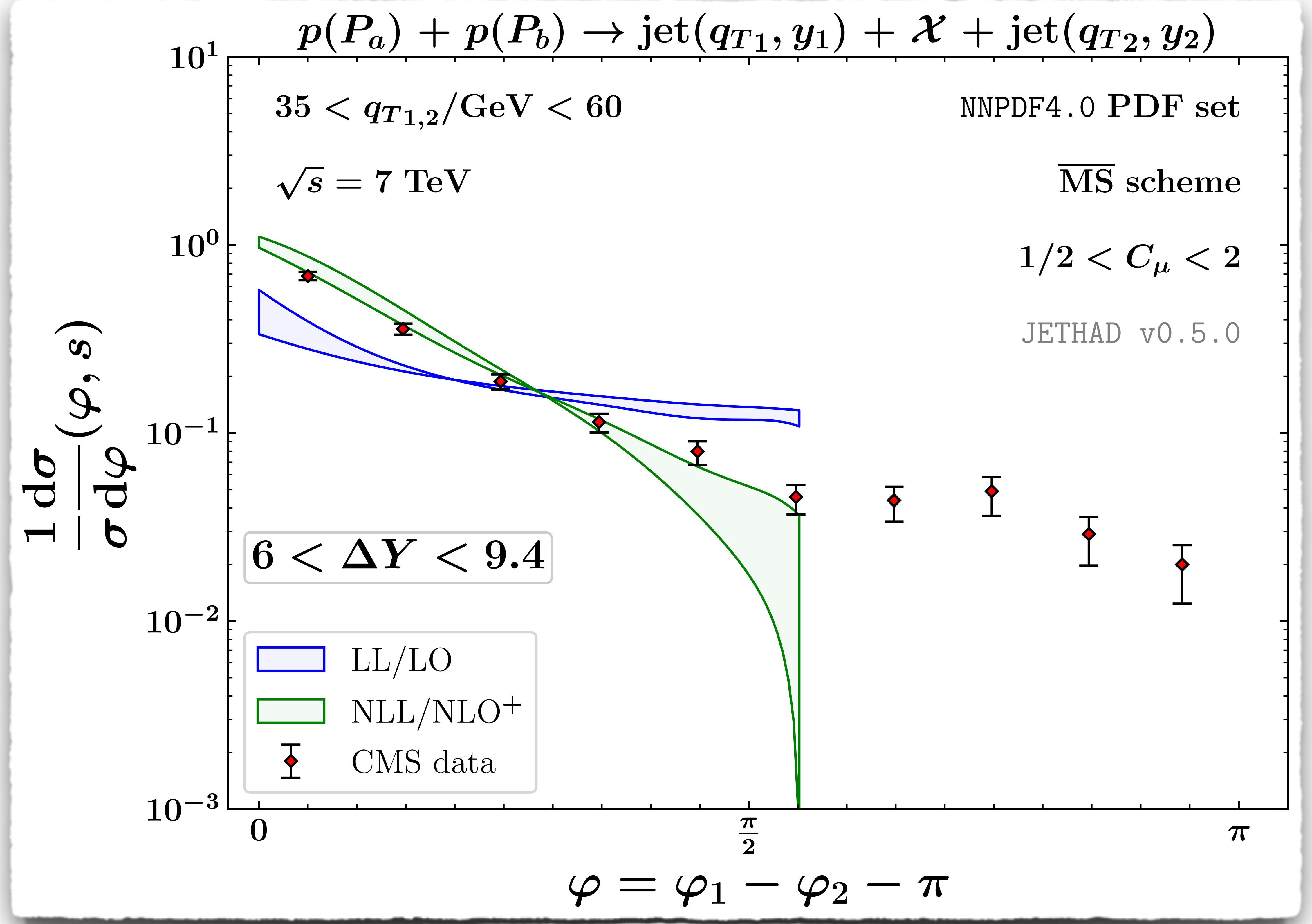
MN jets: Hunting data with azimuthal distributions



[F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]

MN jets: Hunting data with azimuthal distributions





🔗 [B. Ducloué et al., Phys. Rev. Lett. 112 (2014) 082003]

🔗 [F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]

Lecture II Checkpoint

Lecture II: Summary & Outlook

Checkpoint of Lecture II

Lecture II: Summary & Outlook

Mueller-Navelet jets ⇒ First comparison with LHC data

Checkpoint of Lecture II

Lecture II: Summary & Outlook

- Mueller-Navelet jets ⇒ First comparison with LHC data
- Hadron + jet ⇒ Hunt for BFKL signals & BFKL vs DGLAP

Checkpoint of Lecture II

Lecture II: Summary & Outlook

- Mueller-Navelet jets ⇒ First comparison with LHC data
- Hadron + jet ⇒ Hunt for BFKL signals & BFKL vs DGLAP
- NLL resummation instabilities ⇒ Precision studies hampered

Checkpoint of Lecture II

Lecture II: Summary & Outlook

- Mueller-Navelet jets ⇒ First comparison with LHC data
- Hadron + jet ⇒ Hunt for BFKL signals & BFKL vs DGLAP
- NLL resummation instabilities ⇒ Precision studies hampered

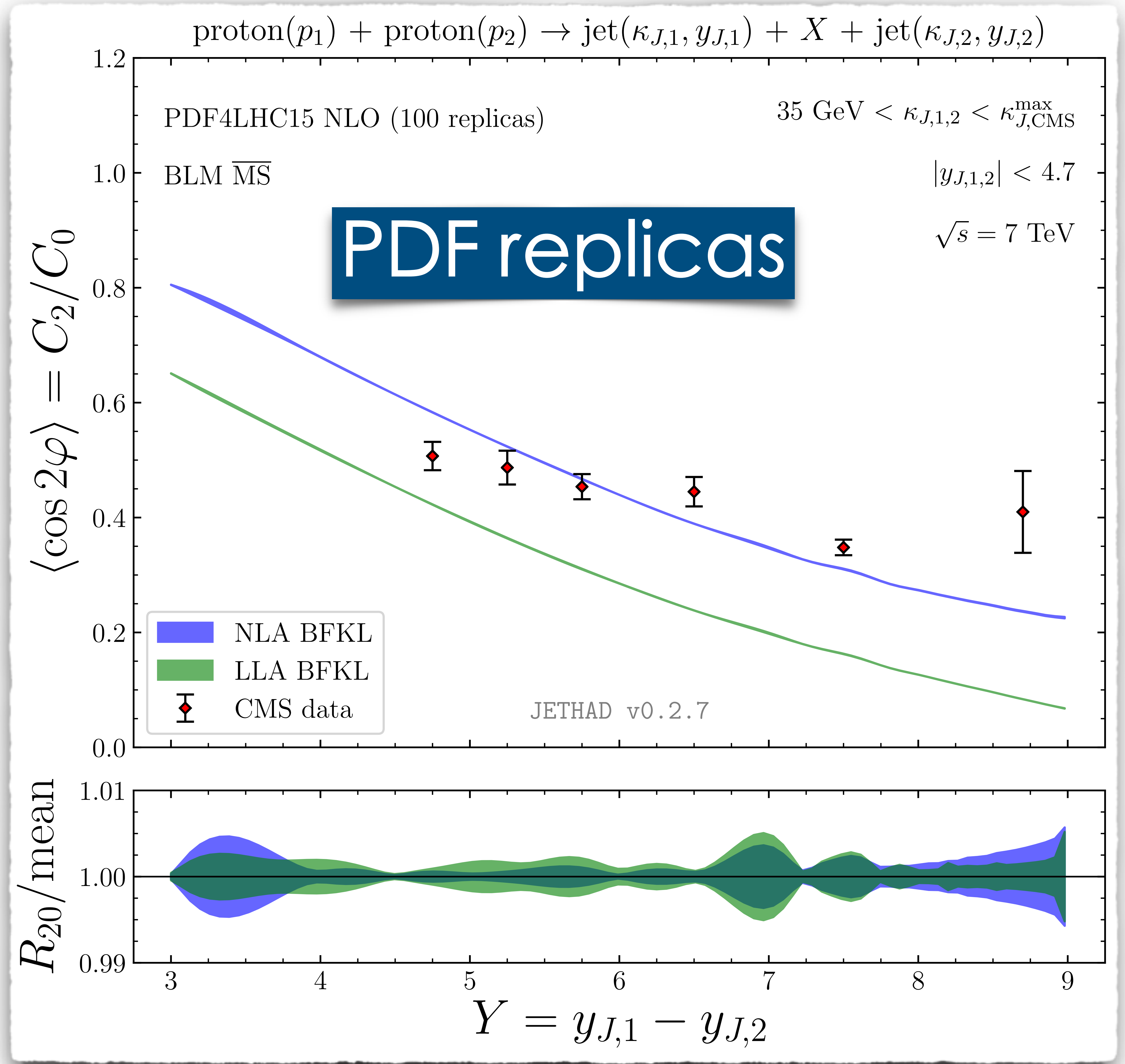
¡ Need for processes featuring a natural stabilization pattern !

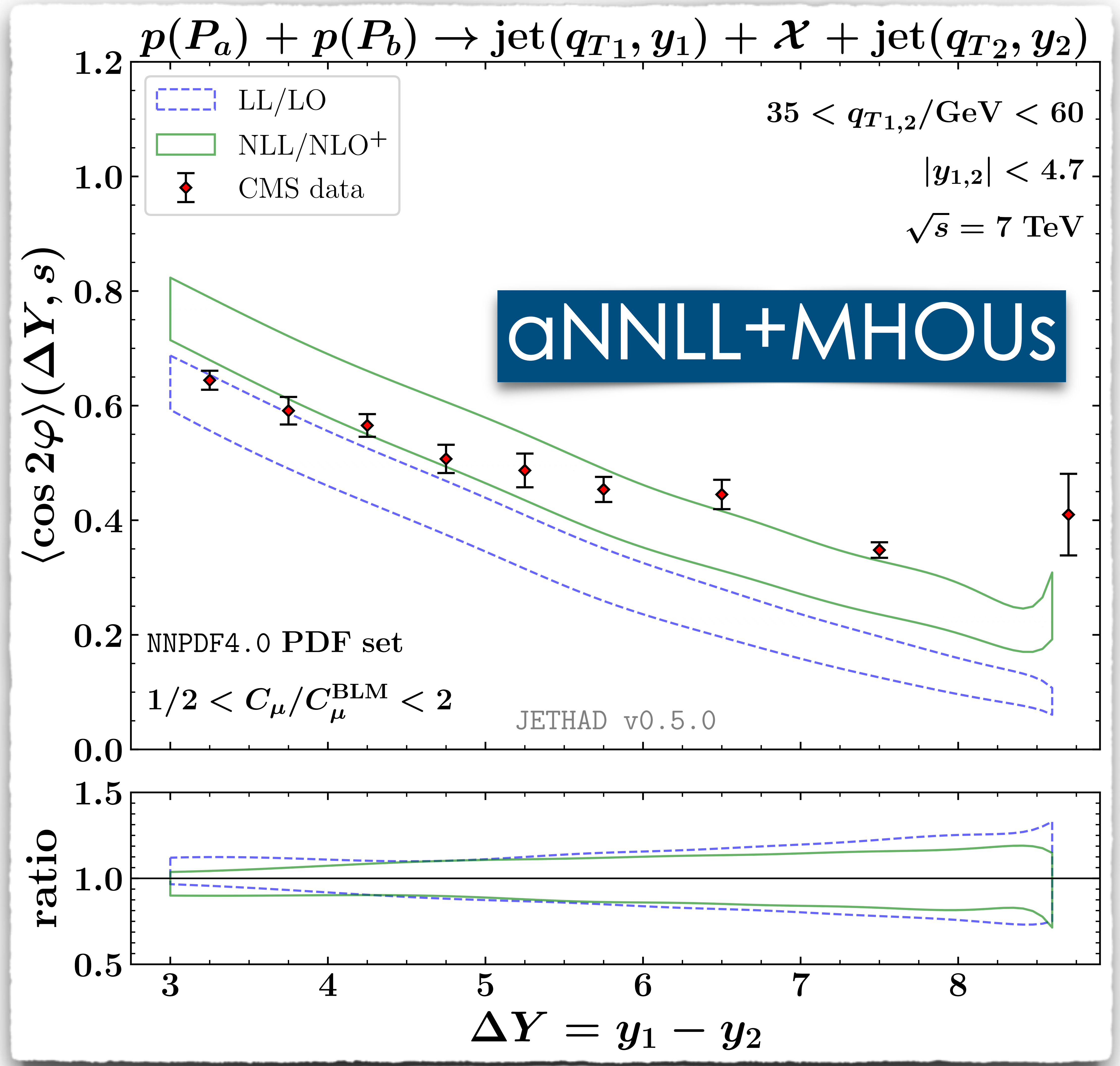
Checkpoint of Lecture II

Extras

Mueller-Navelet Jets

MN jets: Hunting BFKL @7TeV CMS





(left) 🔗 [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

(right) 🔗 [F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]

MN jets: Azimuthal coefficients

$$
\begin{aligned}
\mathcal{C}_n &= \int_{-\infty}^{+\infty} d\nu \left(\frac{x_1 x_2 s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\{\chi(n,\nu)+\bar{\alpha}_s(\mu_R)\mathcal{K}^{(1)}(n,\nu)\}} \\
&\times \frac{e^Y}{s}\,\alpha_s^2(\mu_R)\,c_1(n,\nu,|\vec{k}_1|,x_1)\,[c_2(n,\nu,|\vec{k}_2|,x_2)]^* \\
&\times \left\{1+\alpha_s(\mu_R)\left[\frac{c_1^{(1)}(n,\nu,|\vec{k}_1|,x_1)}{c_1(n,\nu,|\vec{k}_1|,x_1)}+\left[\frac{c_2^{(1)}(n,\nu,|\vec{k}_2|,x_2)}{c_2(n,\nu,|\vec{k}_2|,x_2)}\right]^*\right]\right. \\
&\left. + \bar{\alpha}_s^2(\mu_R)\ln\left(\frac{x_1 x_2 s}{s_0}\right)\frac{\beta_0}{4N_c}\chi(n,\nu)f(\nu)\right\}
\end{aligned}
$$

where

$$
\chi(n,\nu) = 2\psi(1)-\psi\left(\frac{n}{2}+\frac{1}{2}+i\nu\right)-\psi\left(\frac{n}{2}+\frac{1}{2}-i\nu\right)
$$

$$
\mathcal{K}^{(1)}(n,\nu) = \bar{\chi}(n,\nu)+\frac{\beta_0}{8N_c}\chi(n,\nu)\left[-\chi(n,\nu)+\frac{10}{3}+2\ln\left(\frac{\mu_R^2}{\sqrt{\vec{k}_1^2\vec{k}_2^2}}\right)\right]
$$

Light-flavored Hadrons

LO forward-jet and -hadron impact factors

- LO forward hadron impact factor in the (n, ν)-representation

$$c_H(n,\nu,|\vec{k}_H|,x_H) = 2\sqrt{\frac{C_F}{C_A}}(\vec{k}_H^2)^{i\nu-1/2}\int_{x_H}^{1}\frac{dx}{x}\left(\frac{x}{x_H}\right)^{2i\nu-1}$$

$$\times\left[\frac{C_A}{C_F}f_g(x)D_g^h\left(\frac{x_H}{x}\right)+\sum_{r=q,\bar{q}}f_r(x)D_r^h\left(\frac{x_H}{x}\right)\right]$$

- LO forward jet impact factor in the (n, ν)-representation

$$c_J(n,\nu,|\vec{k}_J|,x_J) = 2\sqrt{\frac{C_F}{C_A}}(\vec{k}_J^2)^{i\nu-1/2}\left(\frac{C_A}{C_F}f_g(x_J)+\sum_{s=q,\bar{q}}f_s(x_J)\right)$$

- $f(\nu)$ function

$$i\frac{d}{d\nu}\ln\left(\frac{c_H}{[c_J]^*}\right) = 2\left[f(\nu)-\ln\left(\sqrt{\vec{k}_H^2\vec{k}_J^2}\right)\right]$$

The BFKL BLM azimuthal coefficients

$$C_n^{\rm NLA} = \int_{y_1^{\rm min}}^{y_1^{\rm max}} dy_1 \int_{y_2^{\rm min}}^{y_2^{\rm max}} dy_2 \int_{k_1^{\rm min}}^{\infty} dk_1 \int_{k_2^{\rm min}}^{\infty} dk_2 \int_{-\infty}^{\infty} d\nu \frac{e^Y}{s} \left(\alpha_s^{\rm MOM}(\mu_R^{\rm BLM})\right)^2$$

$$\times\, e^{Y \bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM})\left[\chi(n,\nu)+\bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM})\left(\bar{\chi}(n,\nu)+\frac{T^{\rm conf}}{3}\chi(n,\nu)\right)\right]} c_1(n,\nu)[c_2(n,\nu)]^*$$

$$\times \left\{1 + \bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM}) \left[\frac{\hat{c}_1^{(1)}(n,\nu)}{c_1(n,\nu)} + \left[\frac{\hat{c}_2^{(1)}(n,\nu)}{c_2(n,\nu)}\right]^* + \frac{2T^{\rm conf}}{3}\right]\right\},$$

with the $\mu_R^{\rm BLM}$ scale chosen as the solution of the following integral equation...

$$C_n^{\beta} \propto \int_{y_1^{\rm min}}^{y_1^{\rm max}} dy_1 \int_{y_2^{\rm min}}^{y_2^{\rm max}} dy_2 \int_{k_1^{\rm min}}^{\infty} dk_1 \int_{k_2^{\rm min}}^{\infty} dk_2 \int_{-\infty}^{\infty} d\nu\, e^{Y \bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM})\chi(n,\nu)} \left(\alpha_s^{\rm MOM}(\mu_R^{\rm BLM})\right)^3$$

$$c_1(n,\nu)[c_2(n,\nu)]^* \frac{\beta_0}{2N_c}\left[\frac{5}{3} + \ln\frac{(\mu_R^{\rm BLM})^2}{|\vec{k}_1||\vec{k}_2|} + f(\nu) - 2\left(1+\frac{2}{3}I\right)\right.$$

$$\left. + \bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM}) Y \frac{\chi(n,\nu)}{2}\left(-\frac{\chi(n,\nu)}{2} + \frac{5}{3} + \ln\frac{(\mu_R^{\rm BLM})^2}{|\vec{k}_1||\vec{k}_2|} + f(\nu) - 2\left(1+\frac{2}{3}I\right)\right)\right] \stackrel{!}{=} 0$$

The HE-NLO azimuthal coefficients

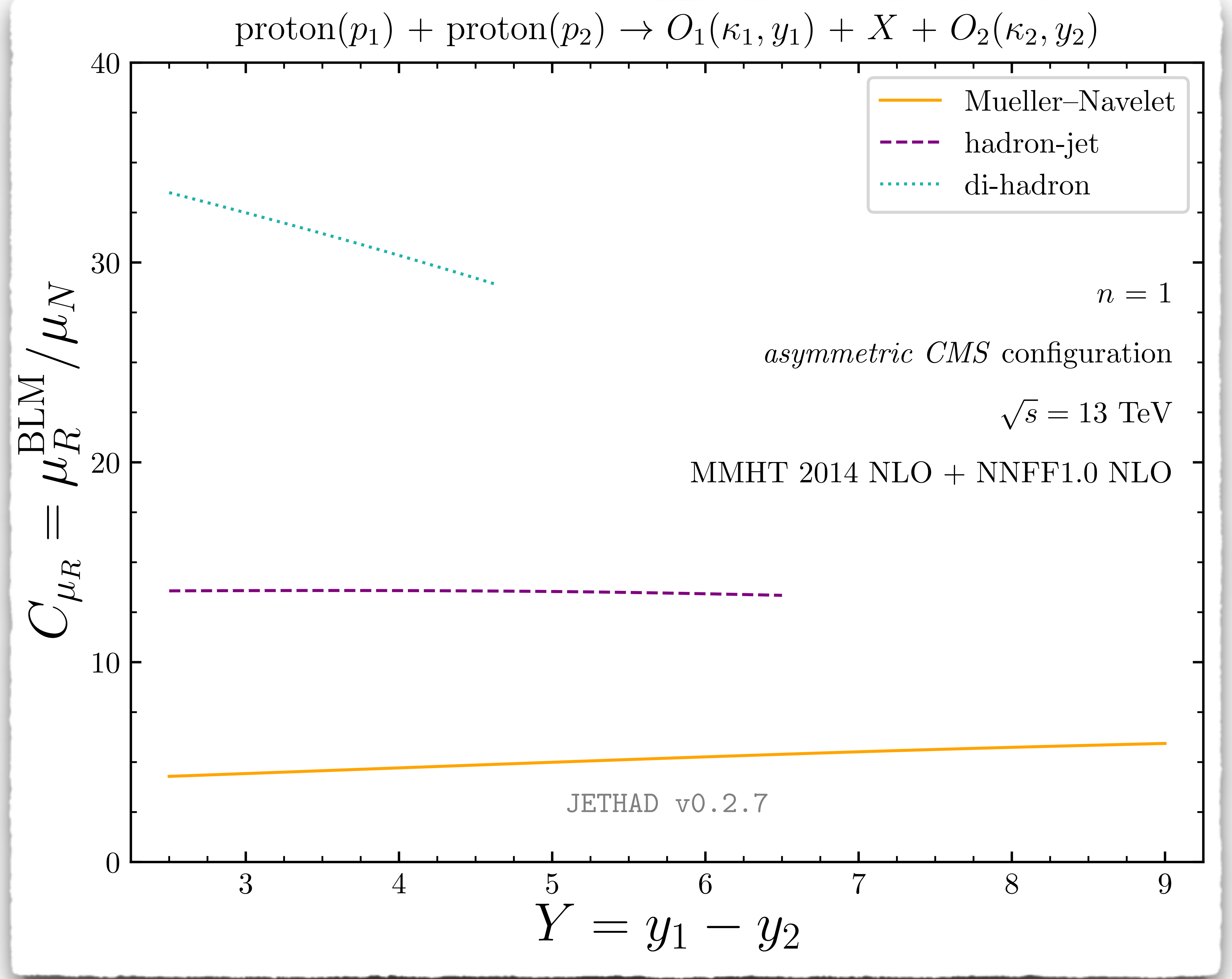
$$C_n^{\rm DGLAP} = \int_{y_1^{\rm min}}^{y_1^{\rm max}} dy_1 \int_{y_2^{\rm min}}^{y_2^{\rm max}} dy_2 \int_{k_1^{\rm min}}^{\infty} dk_1 \int_{k_2^{\rm min}}^{\infty} dk_2 \int_{-\infty}^{\infty} d\nu \, \frac{e^Y}{s} \left(\alpha_s^{\rm MOM}(\mu_R^{\rm BLM}) \right)^2$$
$$\times \, c_1(n,\nu)[c_2(n,\nu)]^*$$
$$\times \left\{ 1 + \bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM}) \left[Y \frac{C_A}{\pi} \chi(n,\nu) + \frac{\hat{c}_1^{(1)}(n,\nu)}{c_1(n,\nu)} + \left[\frac{\hat{c}_2^{(1)}(n,\nu)}{c_2(n,\nu)} \right]^* + \frac{2T^{\rm conf}}{3} \right] \right\} ,$$

◇ NLA BFKL expressions for the observables truncated to $\mathcal{O}\left(\alpha_s^3\right)$ **!**

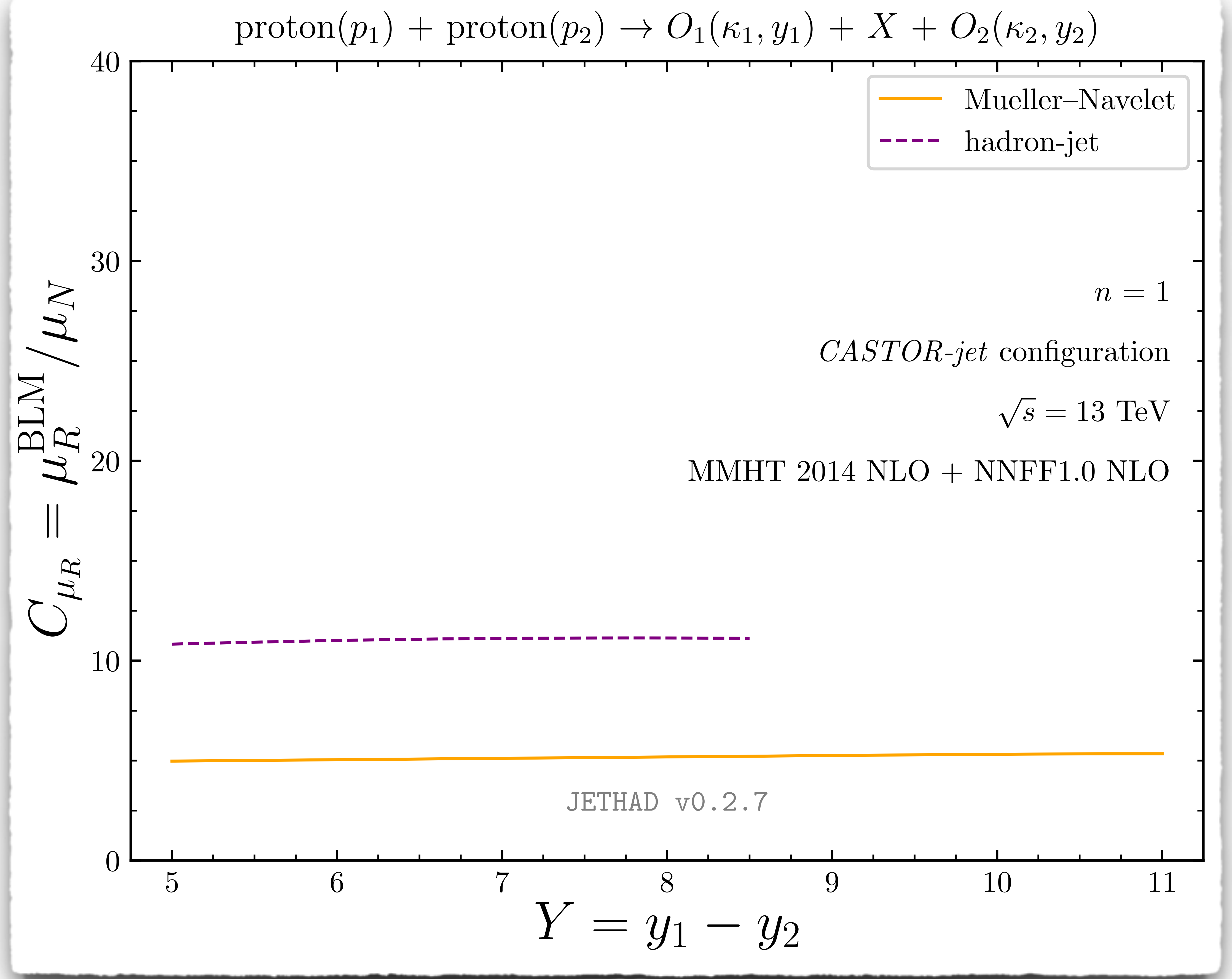
NLL Instabilities

Hadron + jet: BLM scales

CMS-jet



CASTOR-jet



🔗 [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

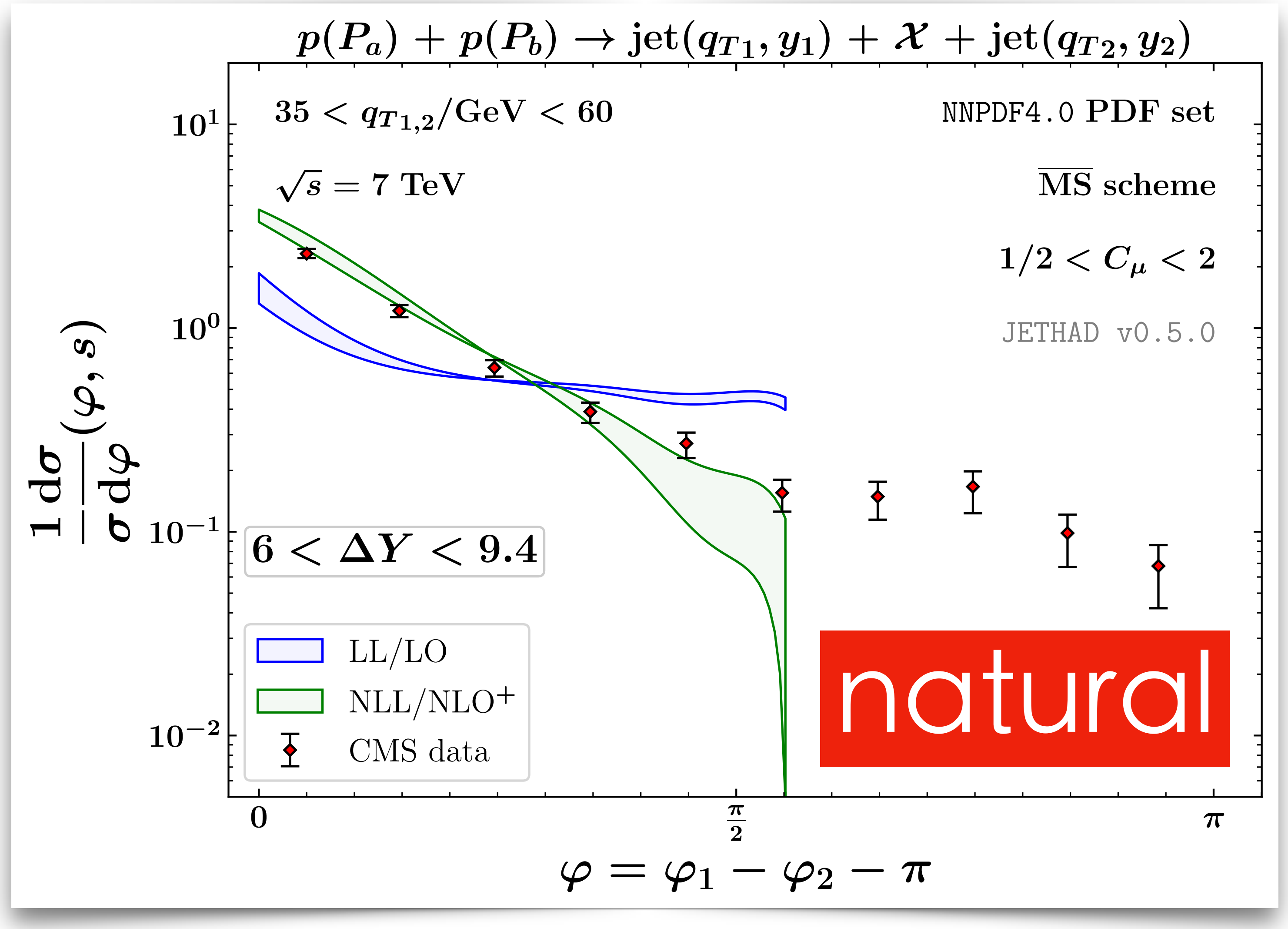
MN jets: NLL resummation instabilities

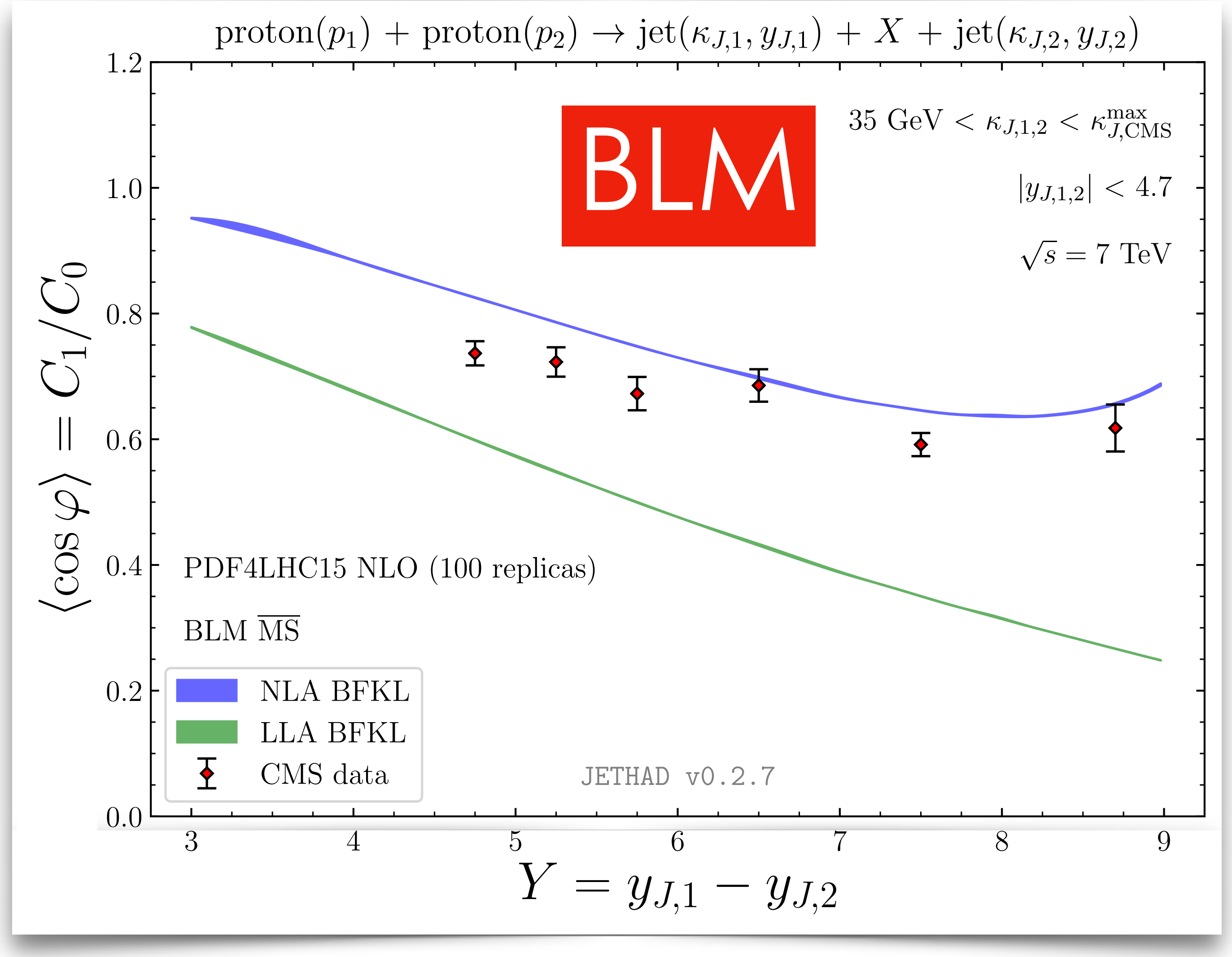
- Strong manifestation of higher-order instabilities via scale variation (¡!)
- ¡ At natural scales: NLL/LL ratio large, no agreement with data, unphysical values !
- BLM scales, theory vs experiment: CMS @7TeV with symmetric p_T-ranges

MN jets: NLL resummation instabilities

- Strong manifestation of higher-order instabilities via scale variation (¡!)
- ¡ At natural scales: NLL/LL ratio large, no agreement with data, unphysical values !
- BLM scales, theory vs experiment: CMS @7TeV with symmetric p_T-ranges

[CMS Collaboration, JHEP 08 (2016) 139]
[B. Ducloué et al., Phys. Rev. Lett. 112 (2014) 082003]
[F. Caporale et al., Eur. Phys. J. C 74 (2014) 10, 3084]



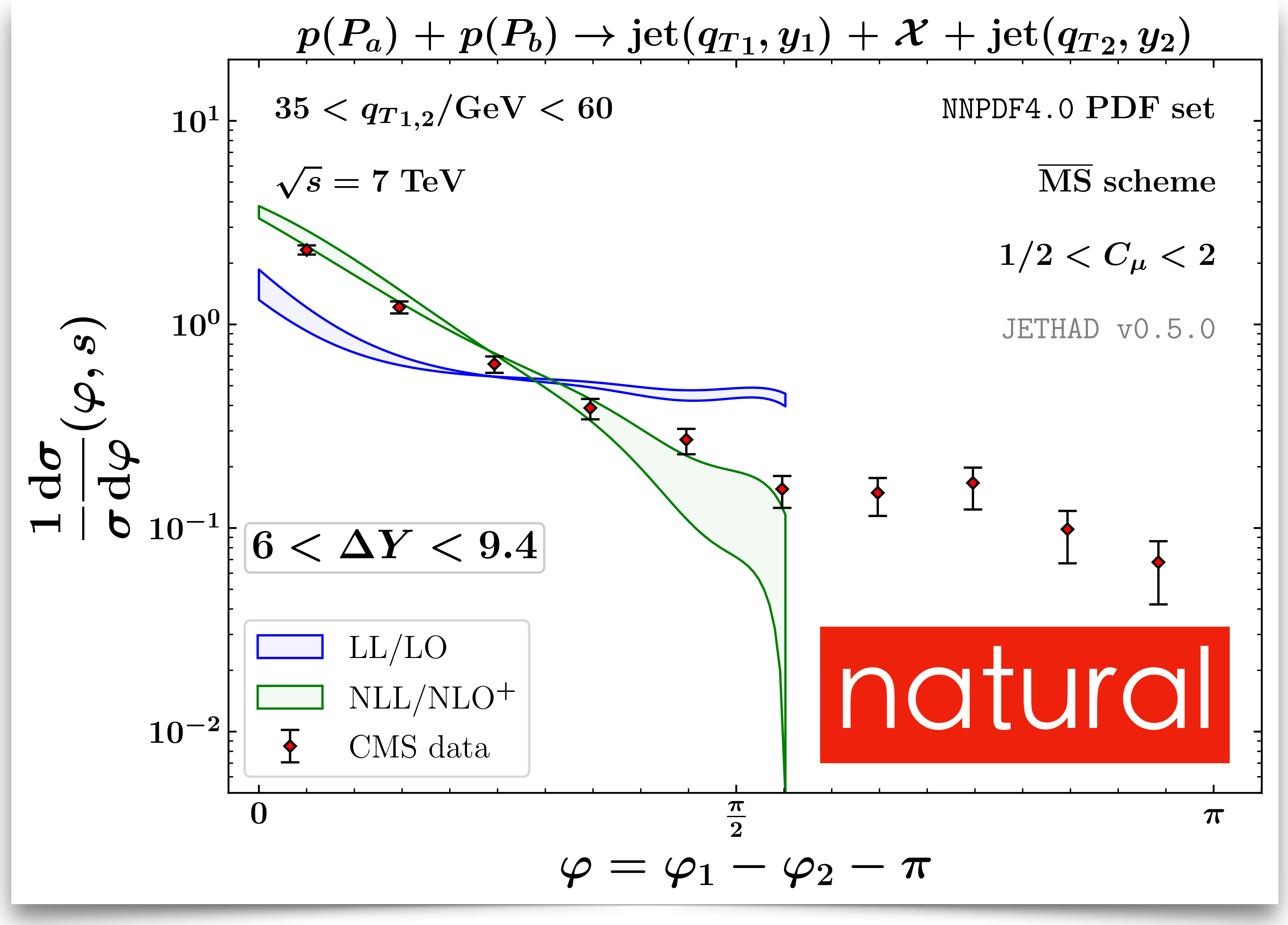


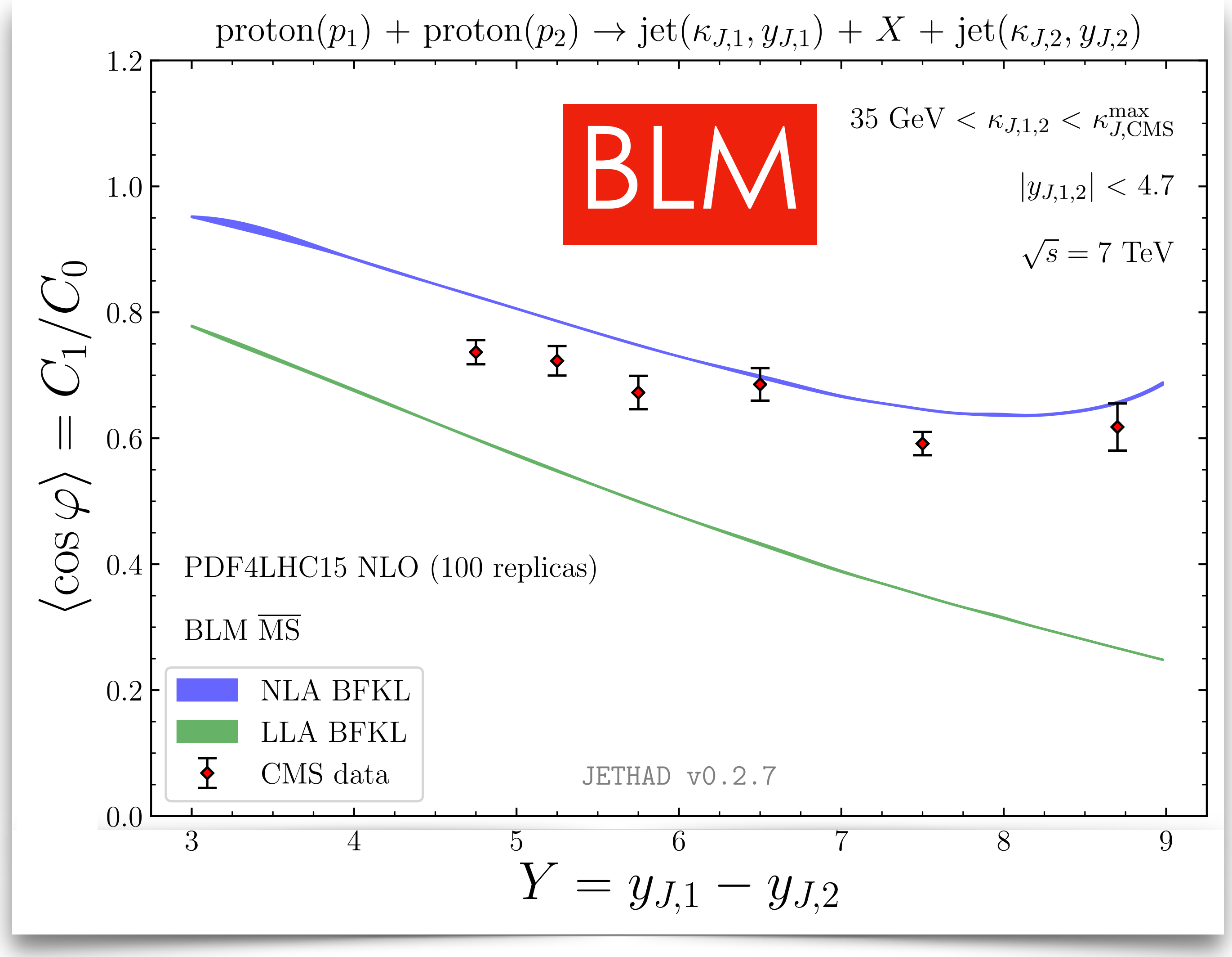
(left figure) [F. G. C., A. Papa (2022)]
(right figure) [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

MN jets: NLL resummation instabilities

- Strong manifestation of higher-order instabilities via scale variation (¡!)
- ¡ At natural scales: NLL/LL ratio large, no agreement with data, unphysical values !
- BLM scales, theory vs experiment: CMS @7TeV with symmetric p_T-ranges

[CMS Collaboration, JHEP 08 (2016) 139]
[B. Ducloué et al., Phys. Rev. Lett. 112 (2014) 082003]
[F. Caporale et al., Eur. Phys. J. C 74 (2014) 10, 3084]





(left figure) [F. G. C., A. Papa (2022)]
(right figure) [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

- $\mu_R^{\text{BLM}} \gg \mu_R^{\text{nat.}} \;\Rightarrow\; d\sigma^{\text{BLM}}/d\sigma^{\text{nat.}} \sim 10^{-(1\div 2)} \;\Rightarrow$ precision studies hampered ✗
- Unsuccessful scale optimization → processes featuring natural stability (¿?)