Plan for the course

Lecture 1: big picture

- Why jets?
- $\gamma^* \to q\bar{q}g$: singularity structure
- Resummation and parton showers

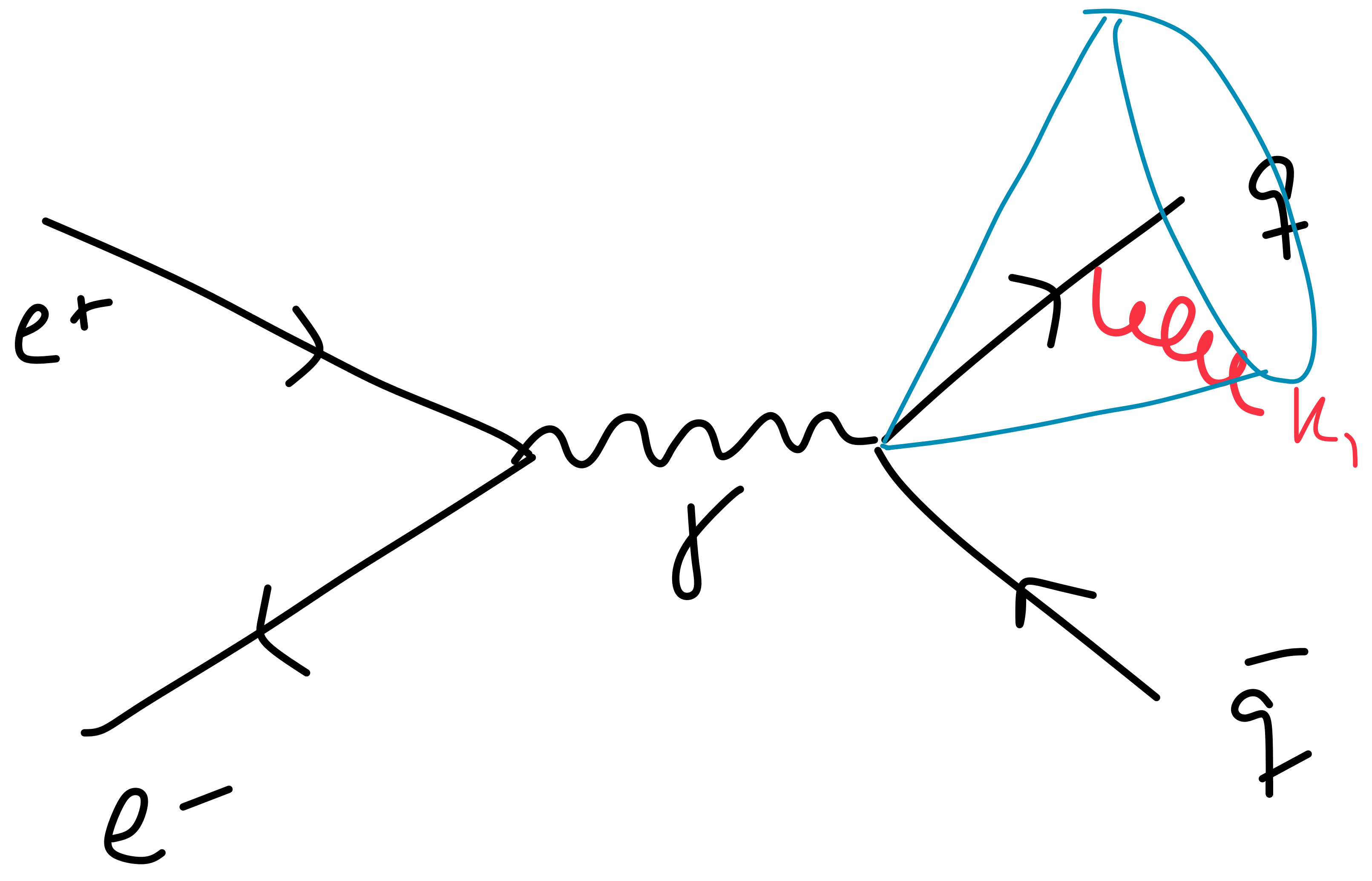
Lecture 2: jet algorithms

- Core ideas of jet reconstruction
- Sequential recombination algorithms
- Optimising jet parameters

Lecture 3: jet substructure

- The question of flavour
- Calculability: groomed jet mass
- Observables at the LHC

Quick recap: how did we define jets yesterday?



Definitions: $m^2 = \left(\sum_{i\in jet} k_i\right)^2$; $\Sigma(m^2) = \frac{1}{\sigma}\int_0^{m^2} dm'^2 \frac{d\sigma}{dm'^2} = 1+\alpha_s\Sigma^{(1)}+\mathcal{O}(\alpha_s^2)$

Soft limit: $|M_R|^2 = \frac{\alpha_s}{2\pi}(2C_F)\frac{k_1\cdot k_2}{(k_1\cdot k_3)(k_2\cdot k_3)}$

$k_1 = \frac{Q}{2}(1,0,0,1)$

$k_2 = \frac{Q}{2}(1,0,0,-1)$

$k_3 = \omega(1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

Phase-space: $\int d\Phi = \int_0^\infty \omega\, d\omega \int_{-1}^{1} d\cos\theta \int_0^{2\pi}\frac{d\phi}{2\pi}$

$$\alpha_s\Sigma^{(1)}(m^2) = \int_{-1}^{1} d\cos\theta \int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^{Q/2}\omega\, d\omega\, \frac{2C_F\alpha_s}{\pi}\,\frac{1}{\omega^2(1-\cos\theta)(1+\cos\theta)}$$

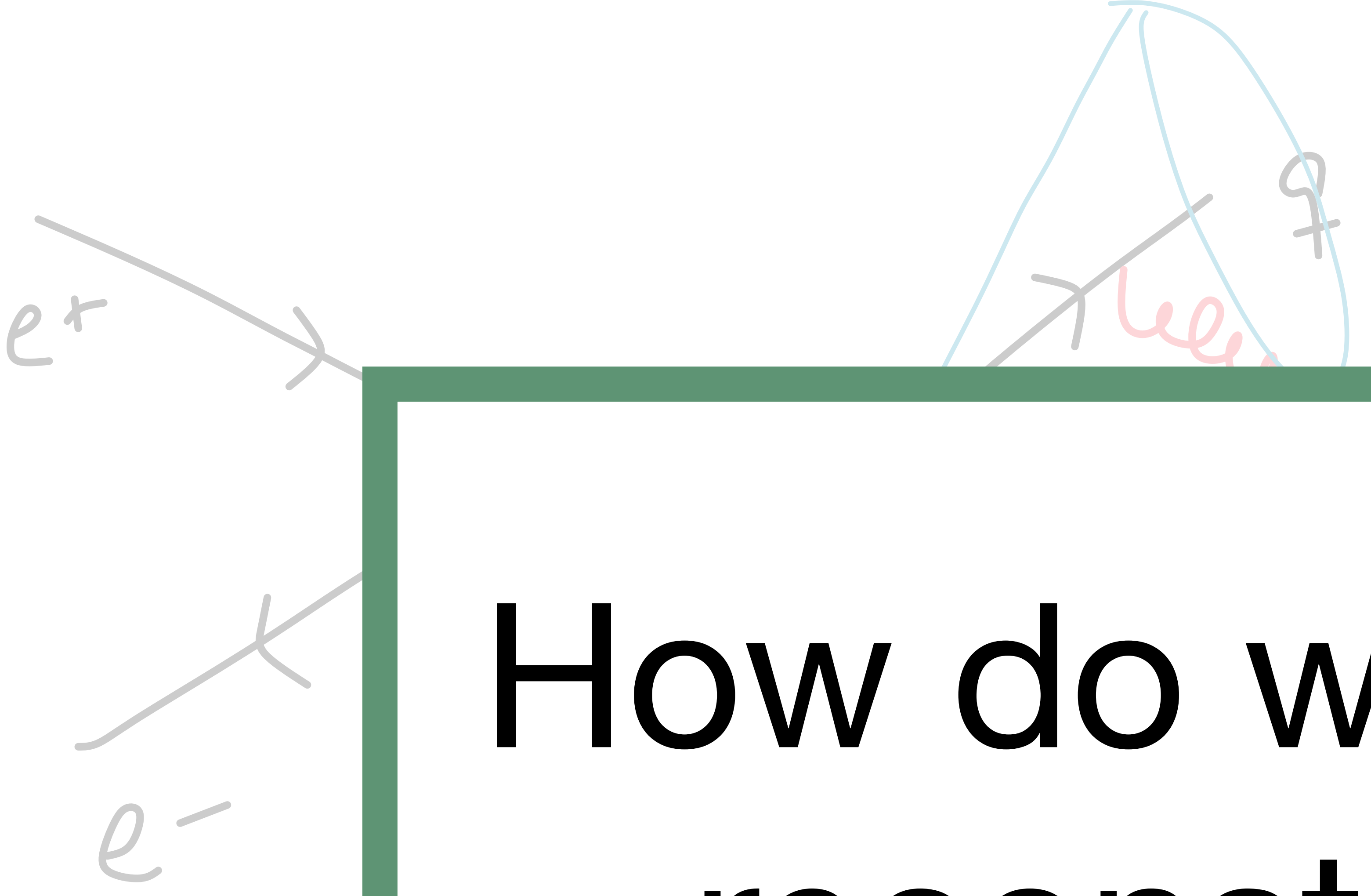
$$\times\left[\Theta_{\text{in-jet}}\,\Theta\left(\frac{2Q\omega}{2}(1-\cos\theta)<m^2\right)+\Theta_{\text{out-jet}}-1\right]$$

$\Theta_{\text{in-jet}}$: $\Theta(1-\cos\theta < 1-\cos R)$

$\Theta_{\text{out-jet}}$: $1-\Theta_{\text{in-jet}}$

-1: Virtual

Quick recap: how did we define jets yesterday?

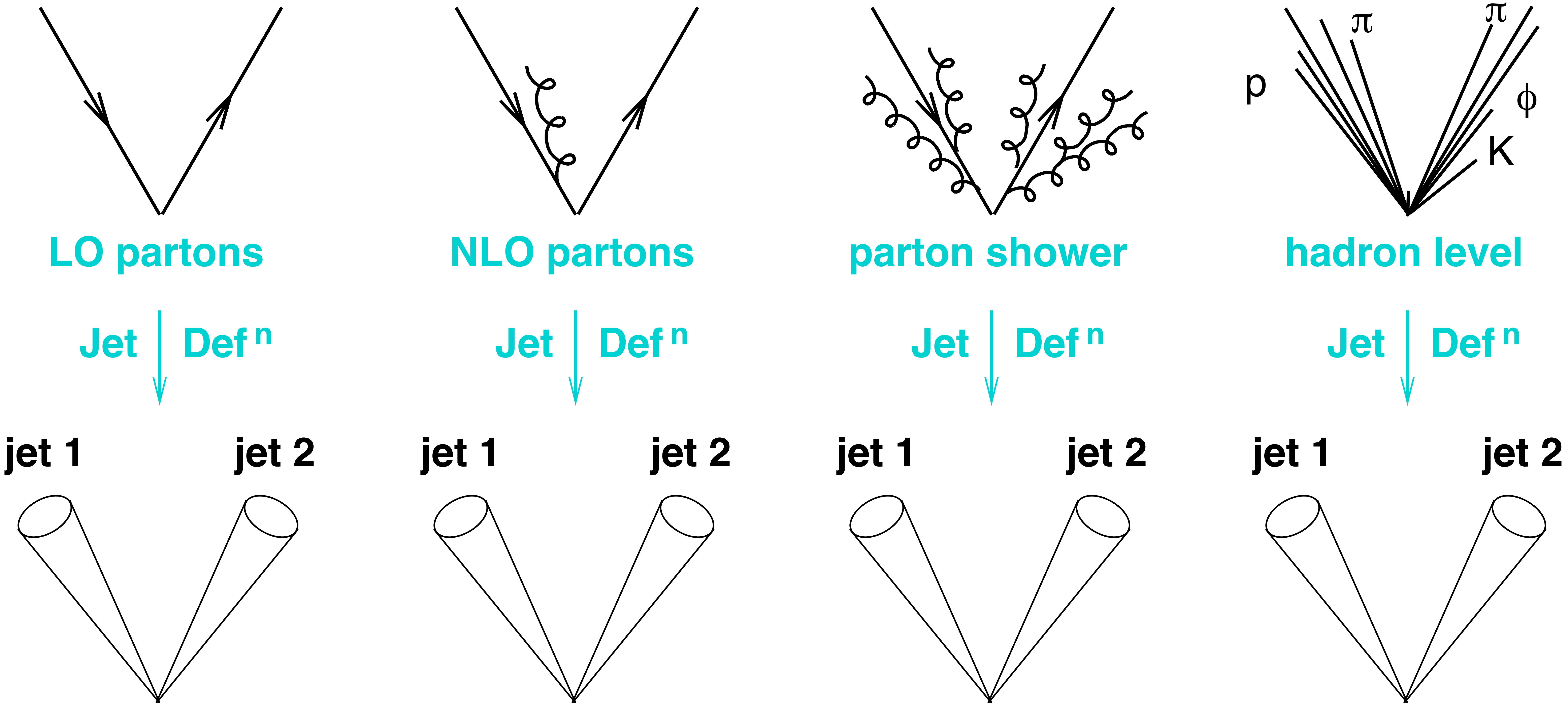


Definitions:

How do we implement $\theta < R$ when reconstructing collider events?

Some considerations: jet finding as a form of projection

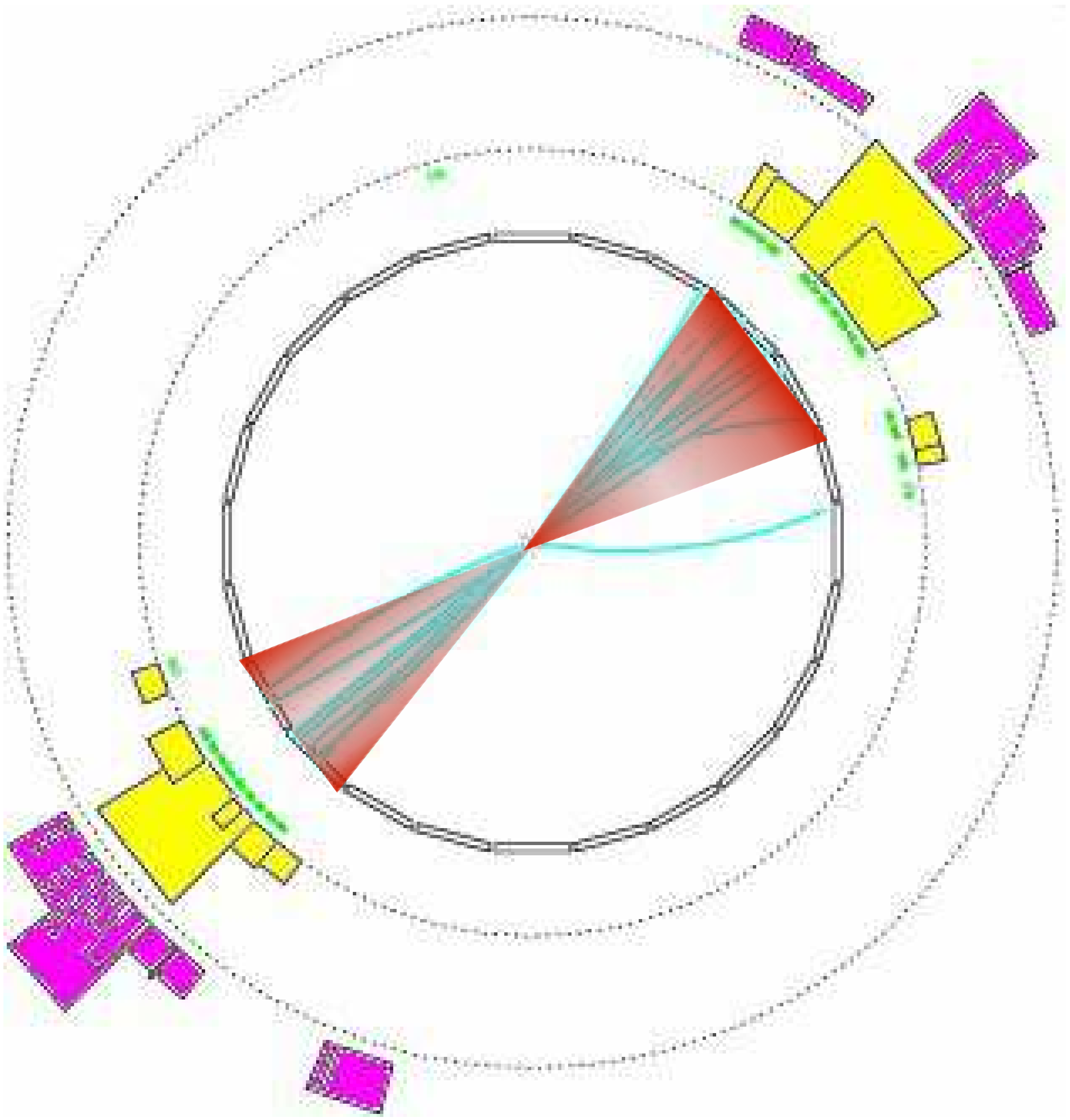
[Adapted from G.P. Salam]



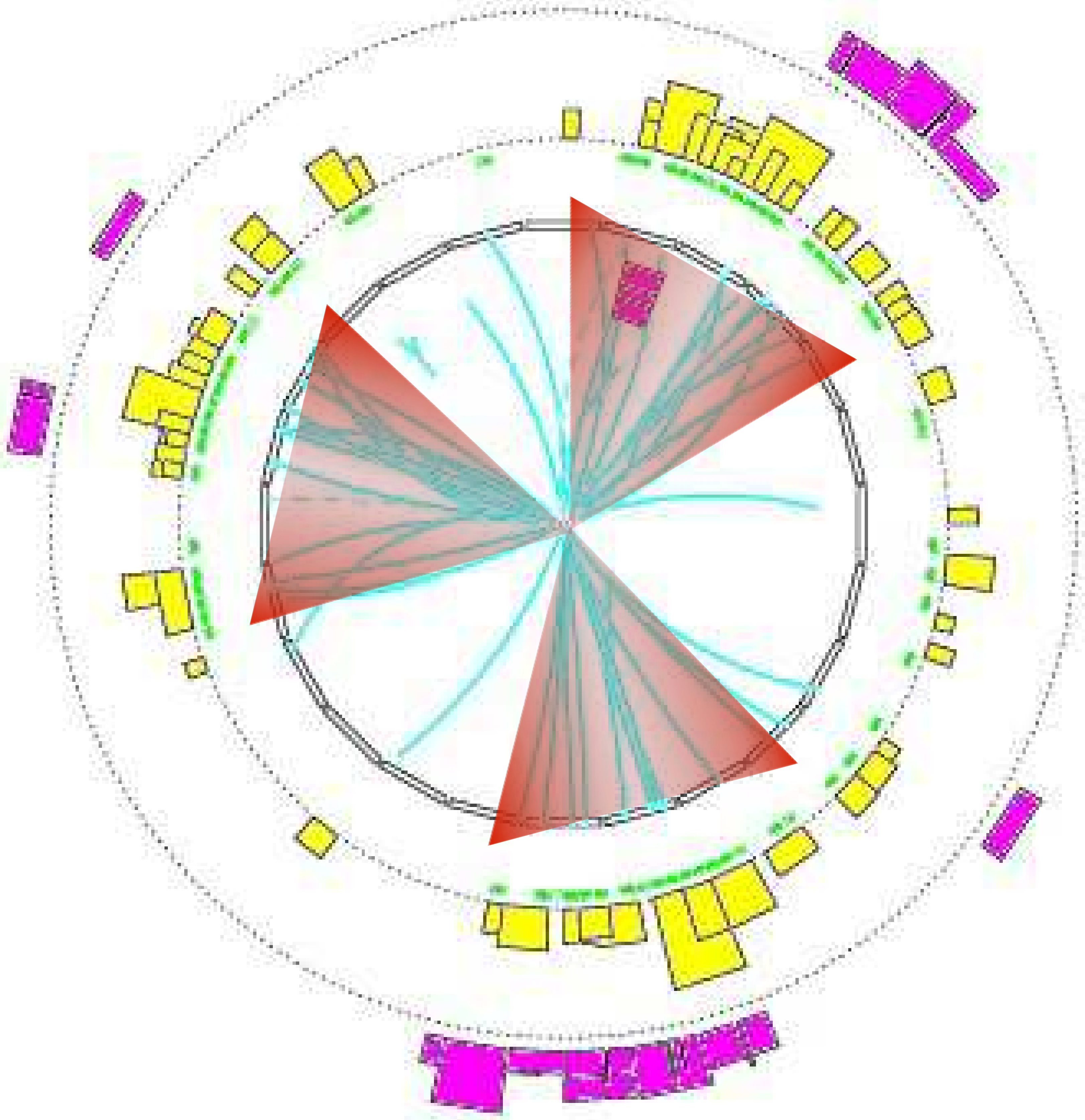
Projection to jets should be resilient to QCD effects

Some considerations: reconstructing jets is an ambiguous task

[Adapted from G.P. Salam]



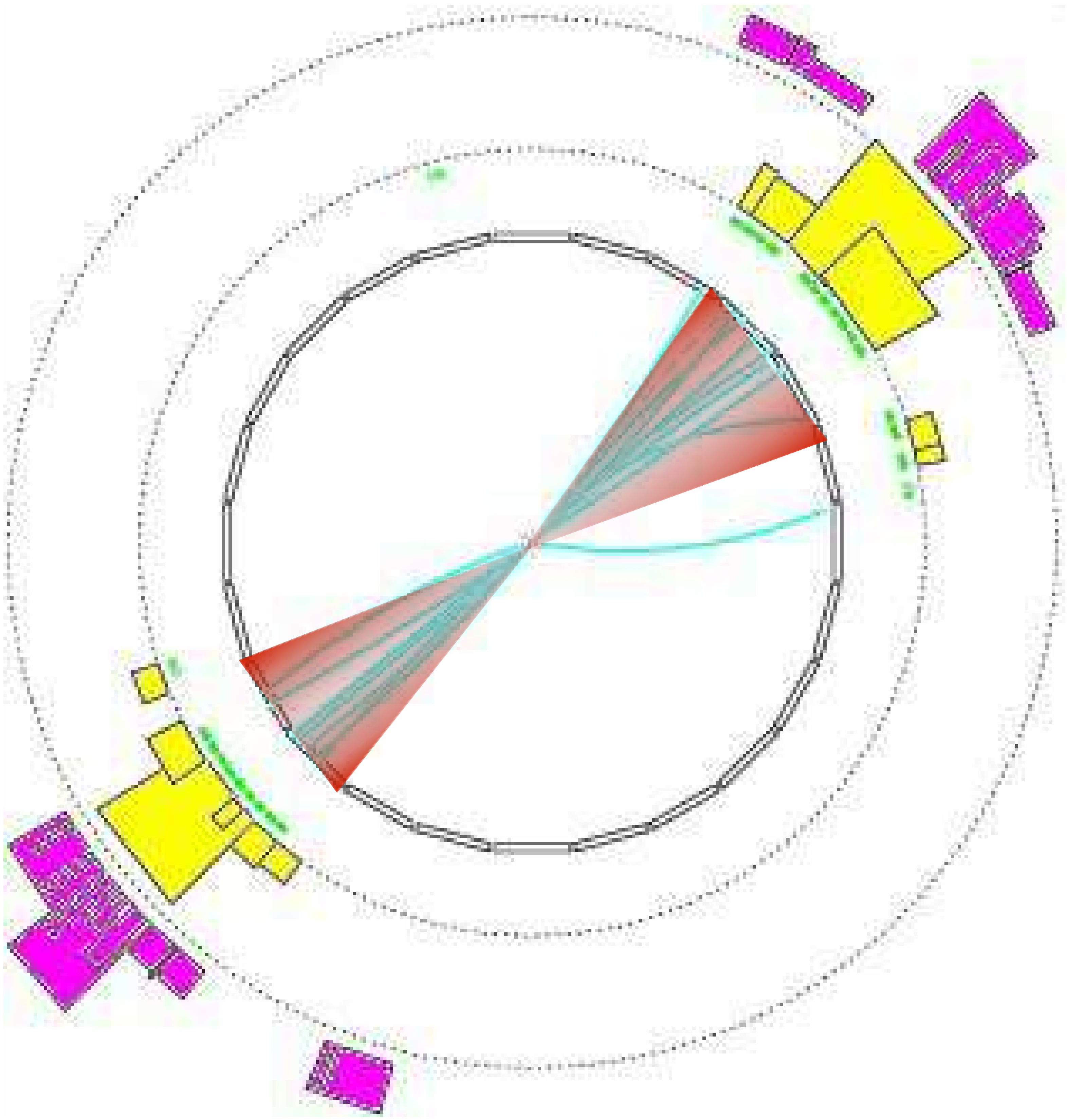
2 clear jets



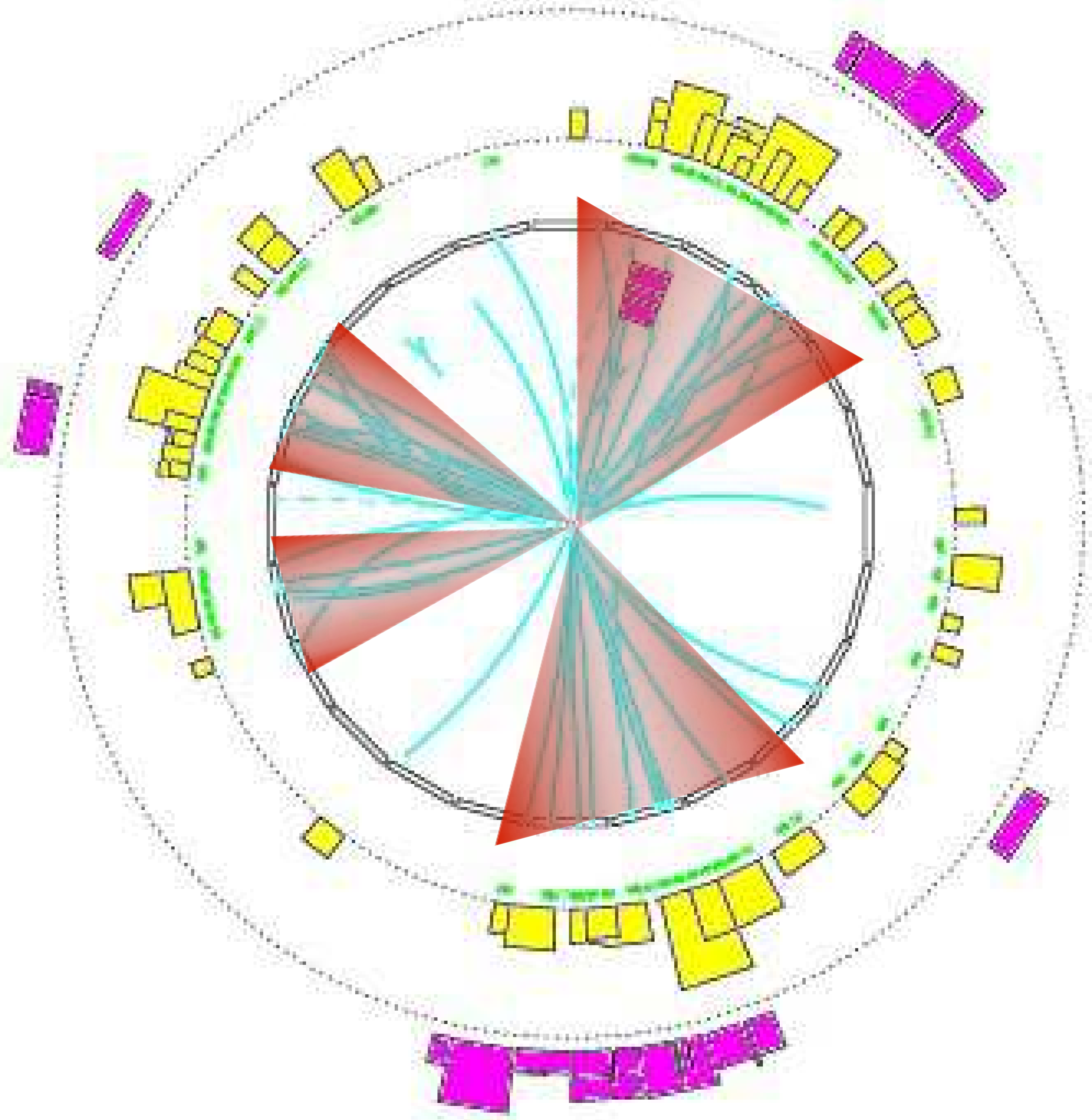
3 jets?

Some considerations: reconstructing jets is an ambiguous task

[Adapted from G.P. Salam]



2 clear jets



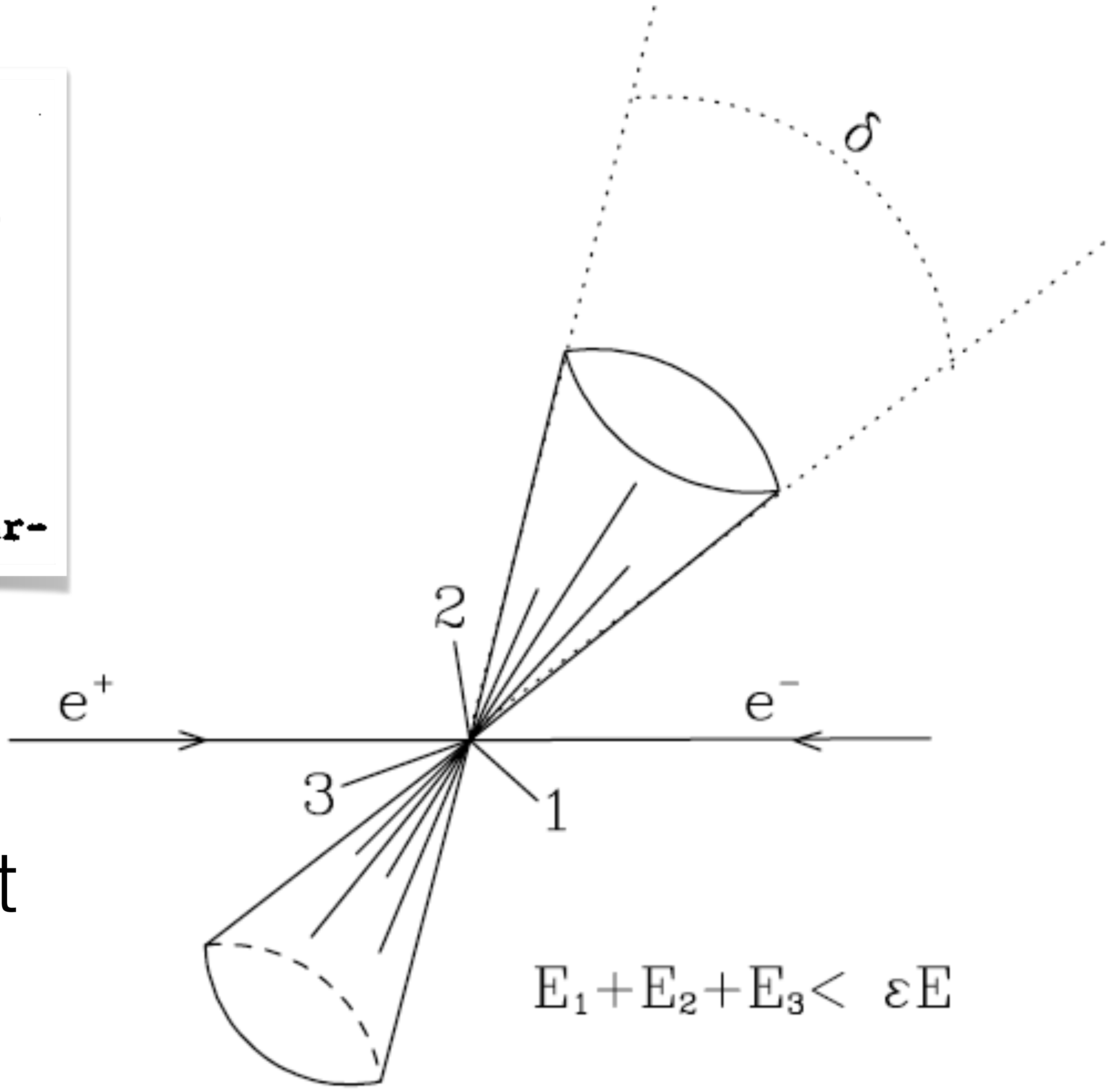
3 jets?
or 4 jets?

Jet definitions date back to the late 1970s

[Sterman and Weinberg, PRL 39 (1977) 1436]

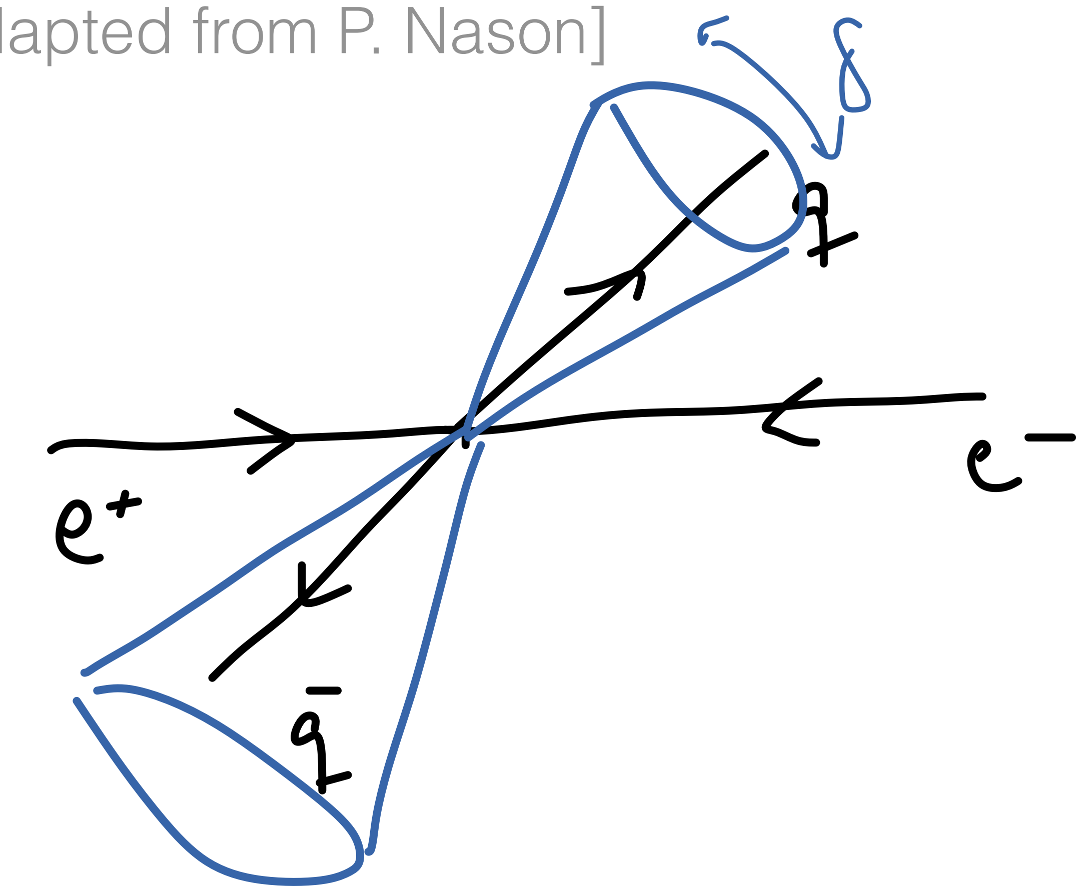
> To study jets, we consider the partial cross section $\sigma(E,\theta,\Omega,\varepsilon,\delta)$ for e^+e^- hadron production events, in which all but a fraction $\varepsilon \ll 1$ of the total e^+e^- energy E is emitted within some pair of oppositely directed cones of half-angle $\delta \ll 1$, lying within two fixed cones of solid angle Ω (with $\pi\delta^2 \ll \Omega \ll 1$) at an angle θ to the e^+e^- beam line. We expect this to be measur-

An event contributes to the jet x-section, if we can find 2 cones of opening angle δ that contain a fraction $1-\varepsilon$ of the total energy (i.e. most of the event's energy)

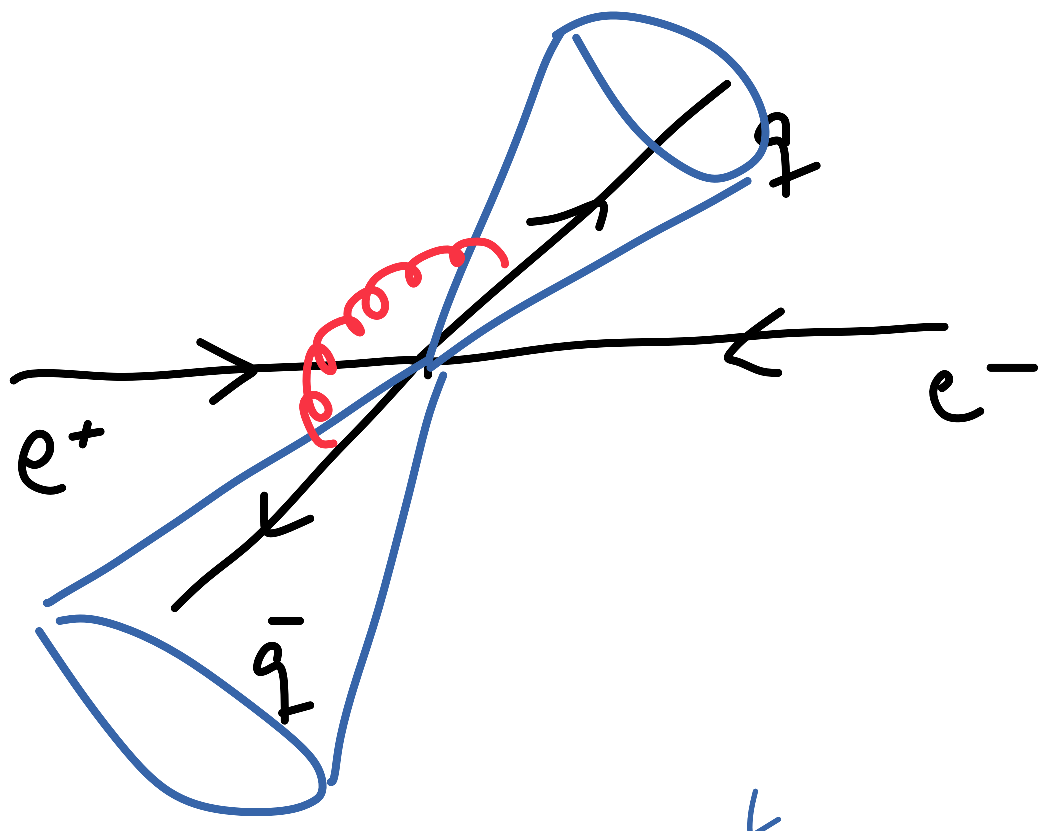


Sterman-Weinberg jet cross section

[Adapted from P. Nason]

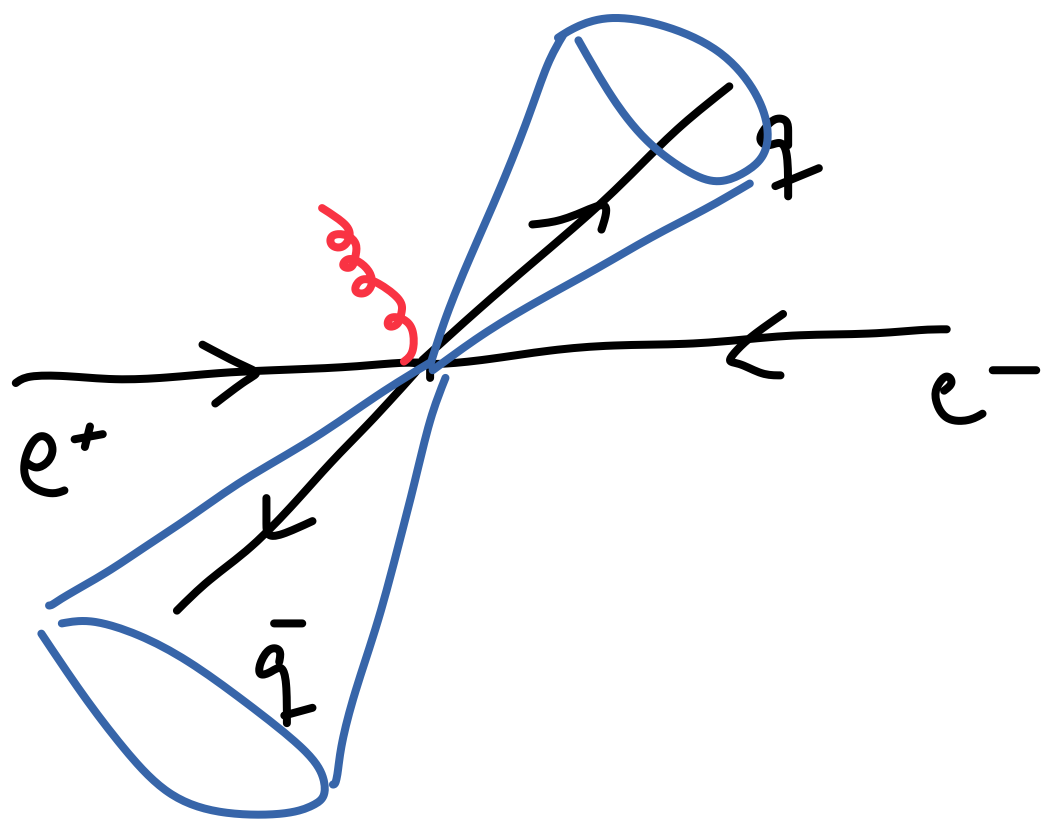


$= \text{Born} = \sigma_0$



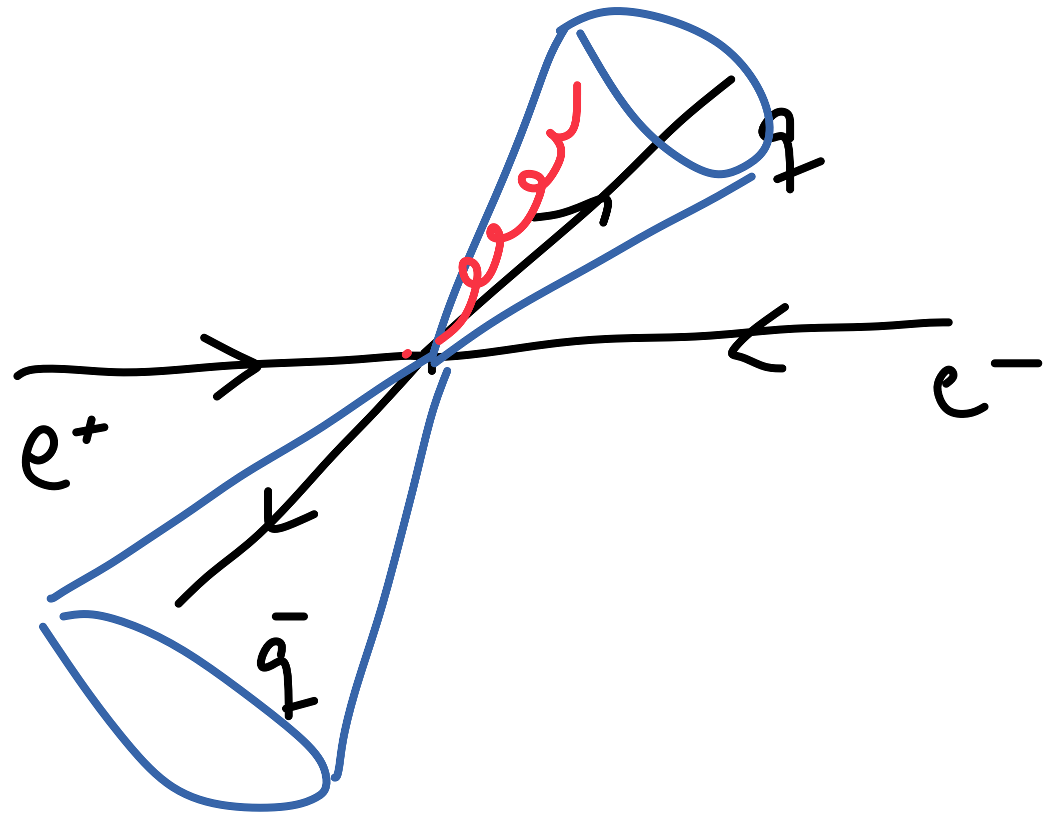
$\equiv \text{Virtual} = -\sigma_0 \alpha_s C_F \int_0^E \frac{dE_g}{E_g} \int_0^\eta \frac{d\cos\theta}{1-\cos\theta}$

(E: total energy)



$= \text{Real} = \sigma_0 \alpha_s C_F \int_0^{\varepsilon E} \frac{dE_g}{E_g} \int_0^\eta \frac{d\cos\theta}{1-\cos\theta}$

(gluon takes at most εE; outside the jet cone)

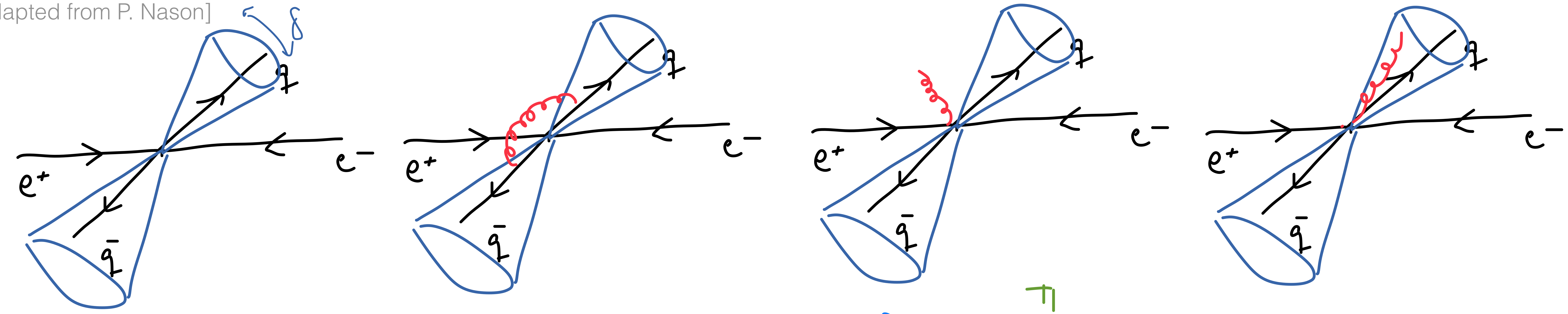


$= \text{Real} = \sigma_0 \alpha_s C_F \int_{\varepsilon E}^{E} \frac{dE_g}{E_g} \left[\int_0^\delta \frac{d\cos\theta}{1-\cos\theta} + \int_{\eta-\delta}^{\eta} \frac{d\cos\theta}{1-\cos\theta} \right]$

(no soft divergence; It falls outside the jet cones)

Sterman-Weinberg jet cross section

[Adapted from P. Nason]

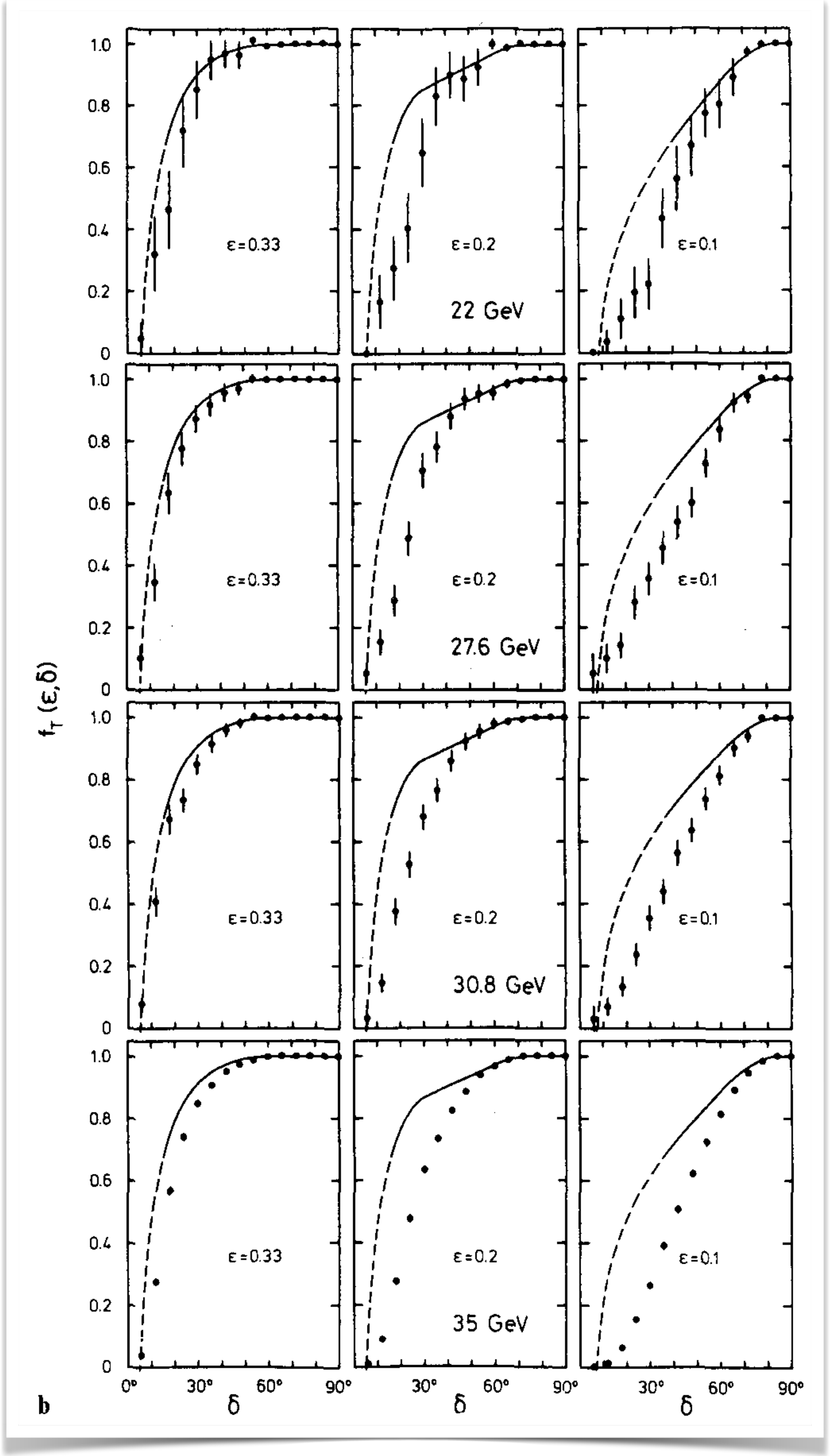


$$\sigma_{NLO} \simeq \sigma_0 - \sigma_0 \alpha_s C_F \int_{\varepsilon E}^{E} \frac{dE_g}{E_g} \int_{\theta=\delta}^{\pi-\delta} \frac{d\cos\theta}{1-\cos^2\theta}$$

$$= \sigma_0 \left(1 - \alpha_s C_F \ln\varepsilon \ln\delta\right)$$

At high energy (small α_s) most events are 2 jet events

Sterman-Weinberg jet cross section: theory-vs-data



Solid line: calculation in the previous slide

Dots: experimental data from PLUTO

[PLUTO Collab., Z.Phys.C 27 (1985) 167]

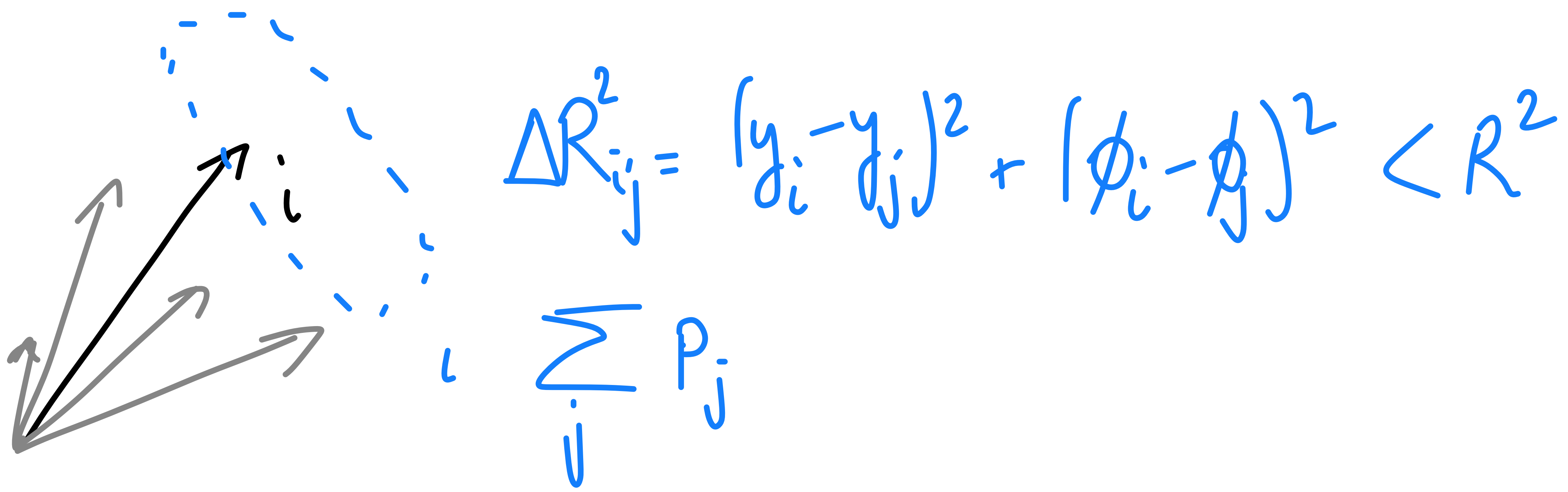
Key property of Sterman-Weinberg jets: calculable in QCD due to IRC safety

Not obvious how to extend Sterman-Weinberg jets to hadron colliders. Total energy? More than two jets?

Cone algorithms: top-down approach (widely used @Tevatron)

- A seed particle i (e.g. the hardest in the event) sets some initial direction

- Find all particles in the vicinity and sum their 4-momenta



- The direction of the resulting sum is then used as a new seed direction

- Iterate until the direction of the resulting cone is stable and call it a jet

> Underlying idea: momentum flow within a cone only marginally modified by QCD branching

Cone algorithms: top-down approach

[CDF Collab Phys.Rev.D 74 (2006) 071103]

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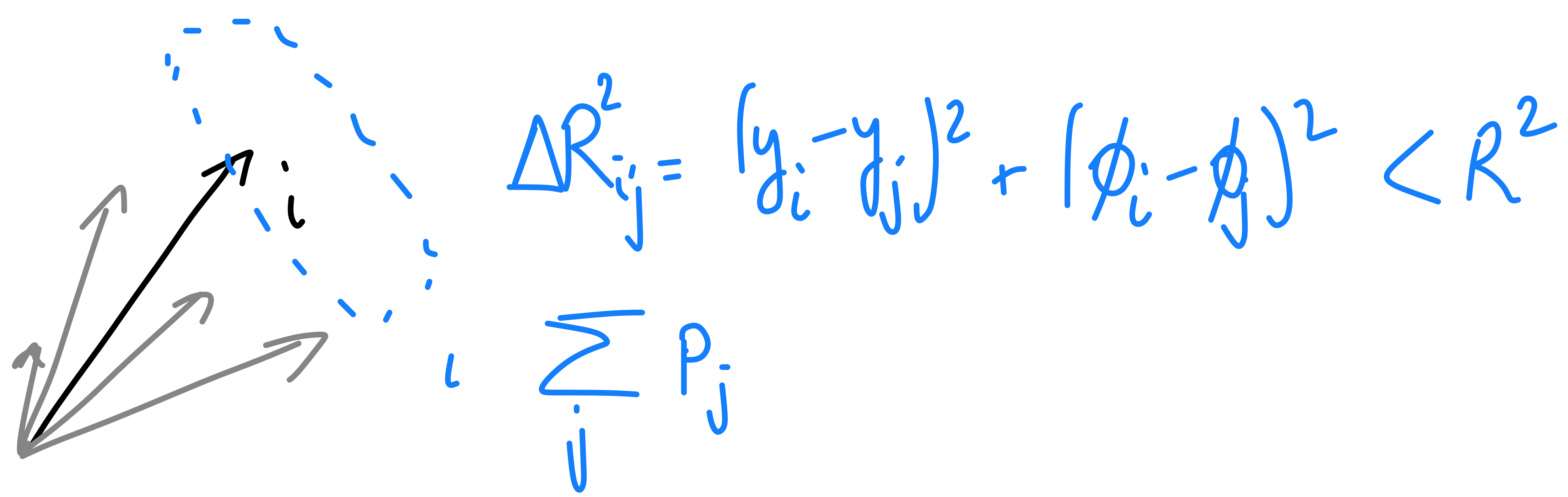
- Find all particles in the vicinity and sum their 4-momenta



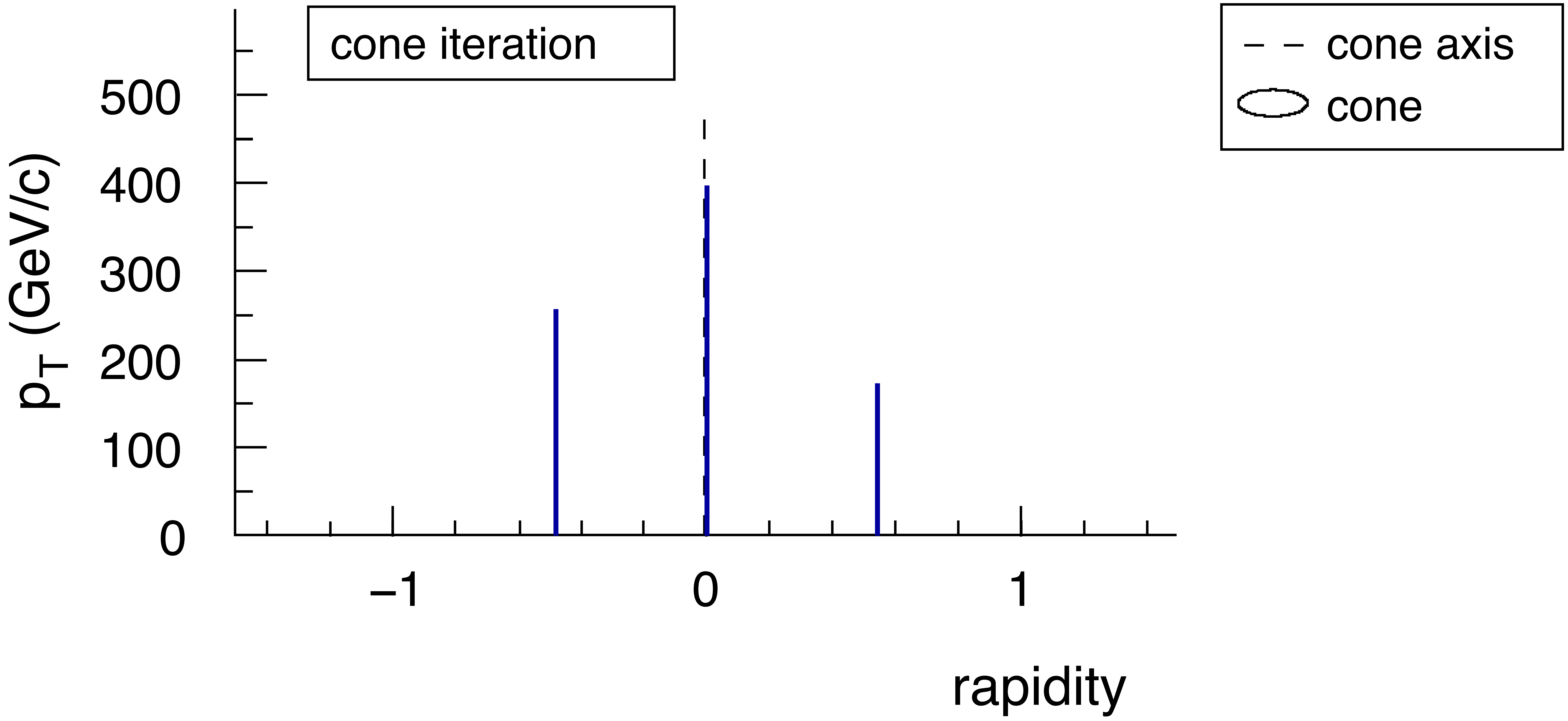
- The direction of the resulting sum is then used as a new seed direction
- Iterate until the direction of the resulting cone is stable and call it a jet

> What should one take as seed? What happens when cones share particles? Many answers, see *Towards Jetography*

Issues with cone algorithms: IRC unsafety (exception: SISCone)

[Adapted from G.P. Salam]

[SISCone: Salam, Soyez, JHEP 05 (2007) 086]



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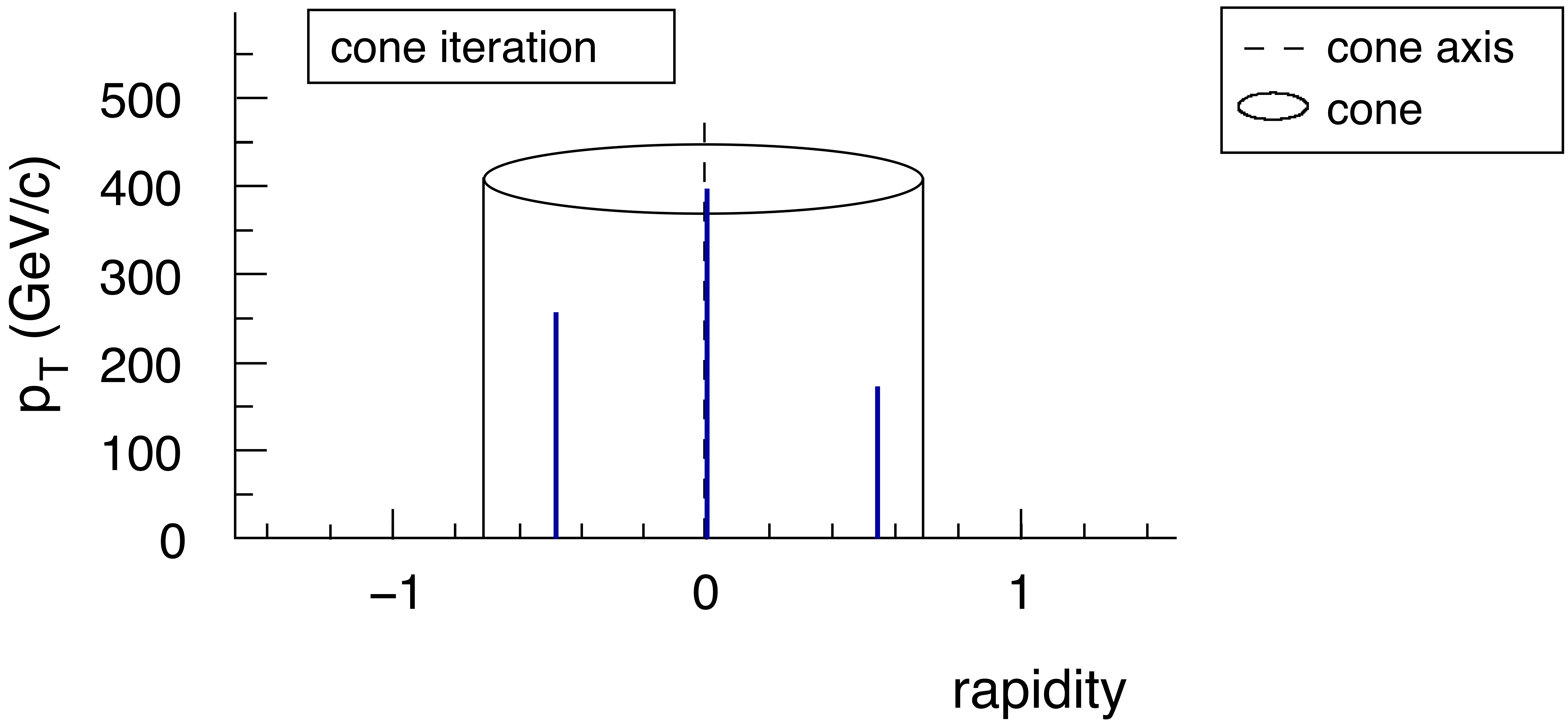

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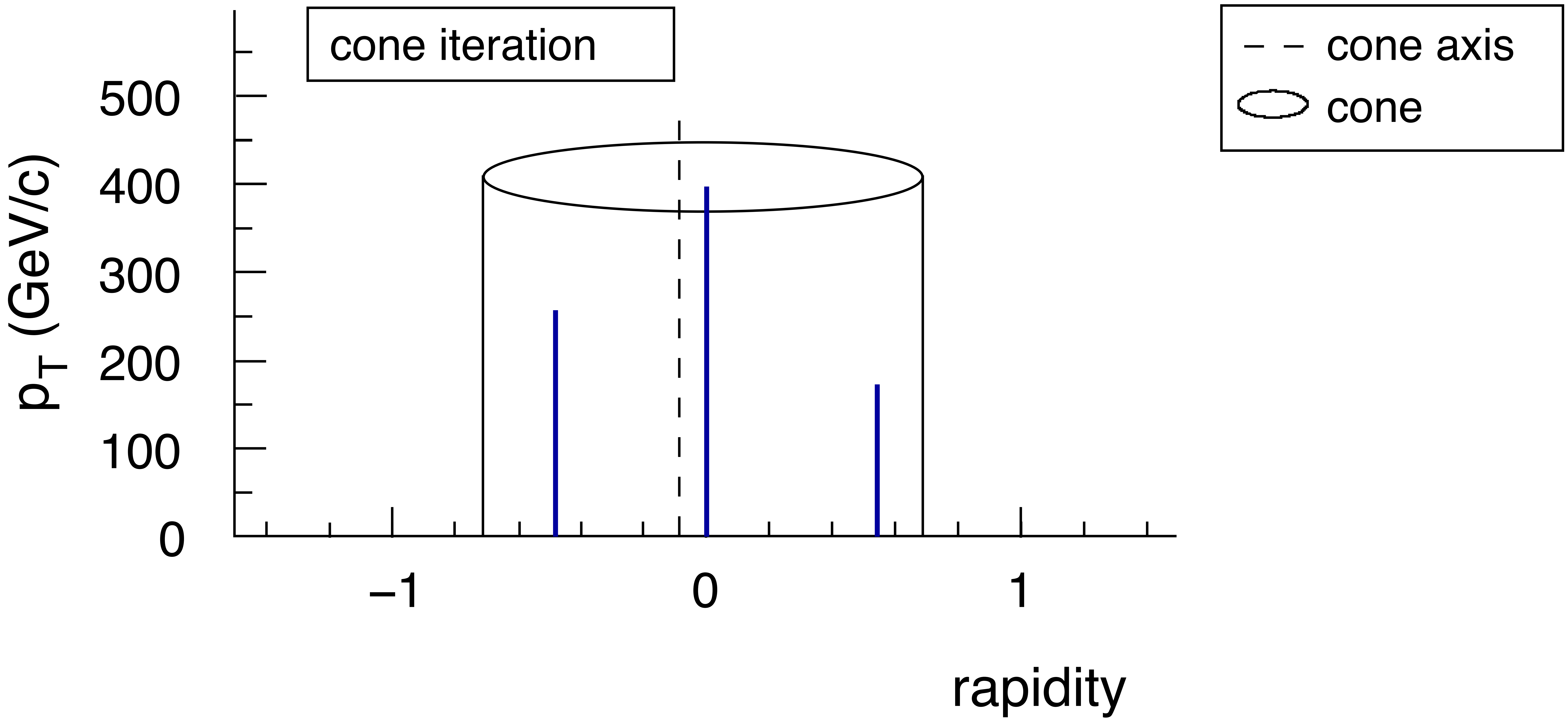

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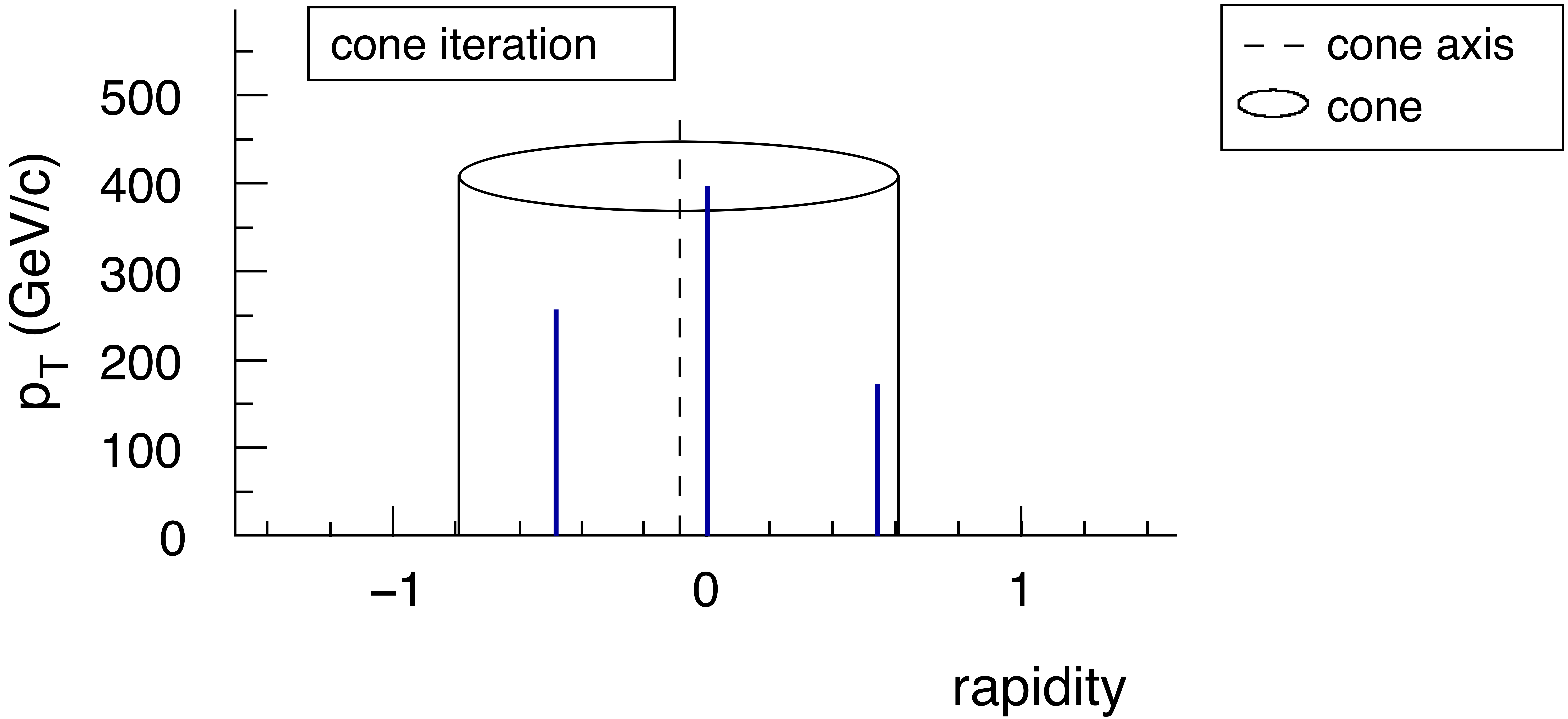

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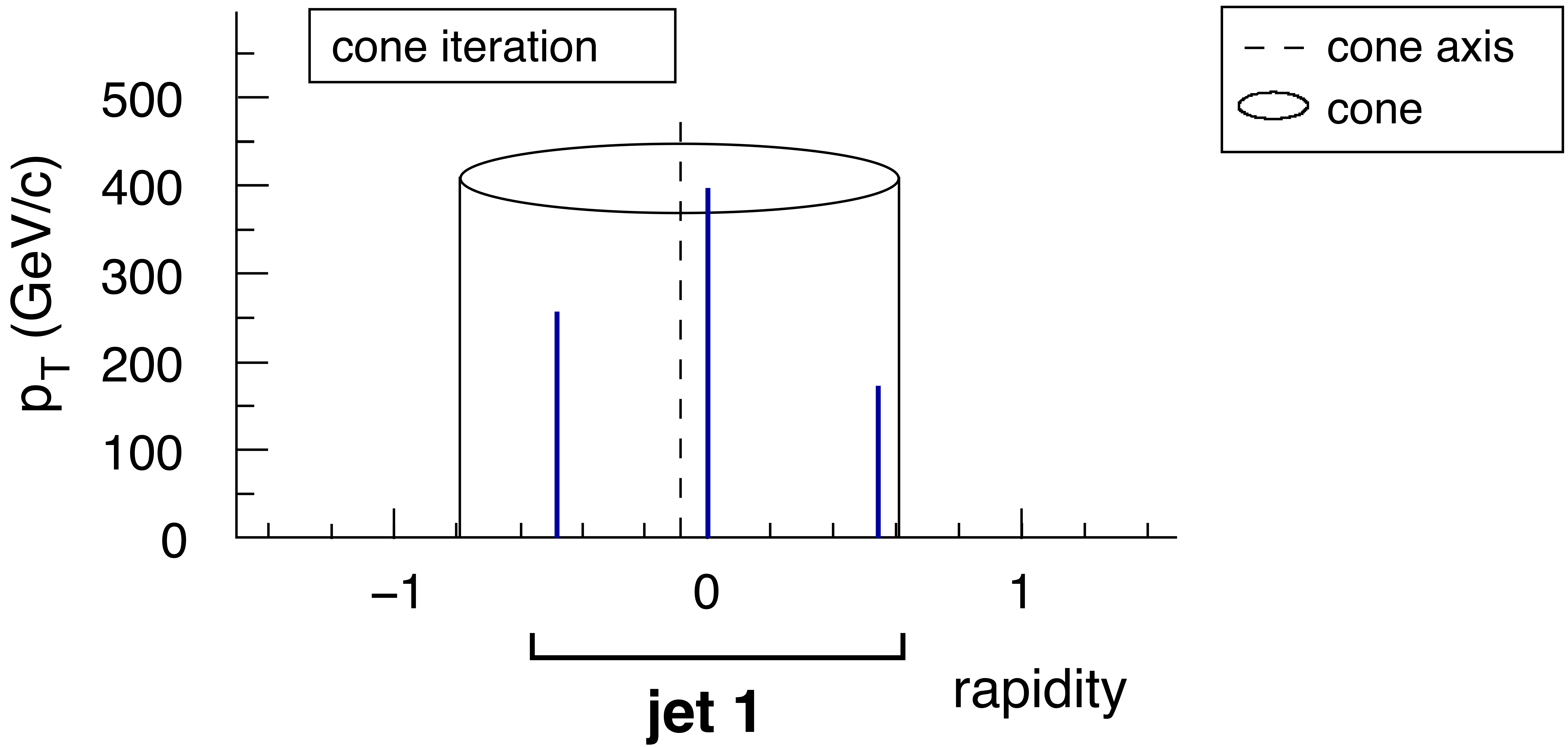

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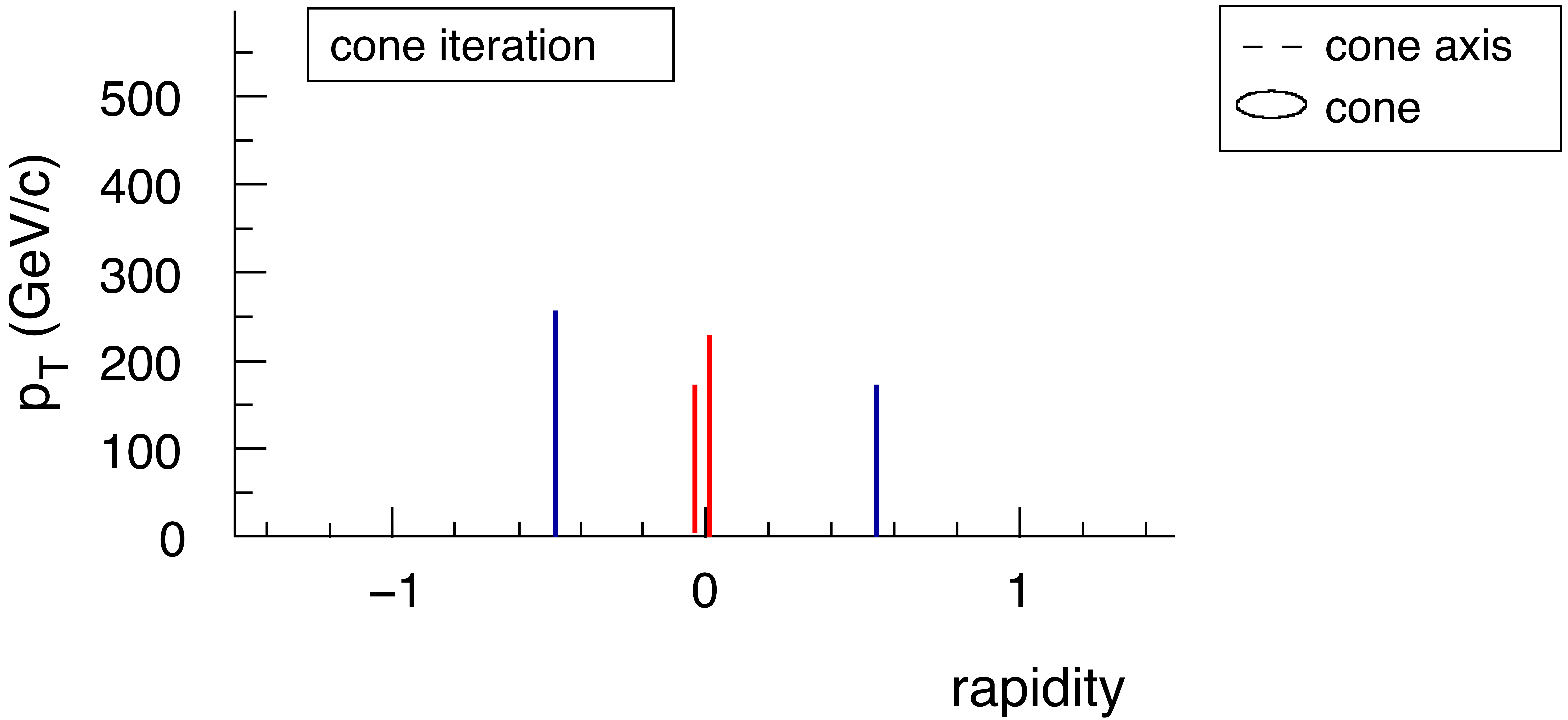

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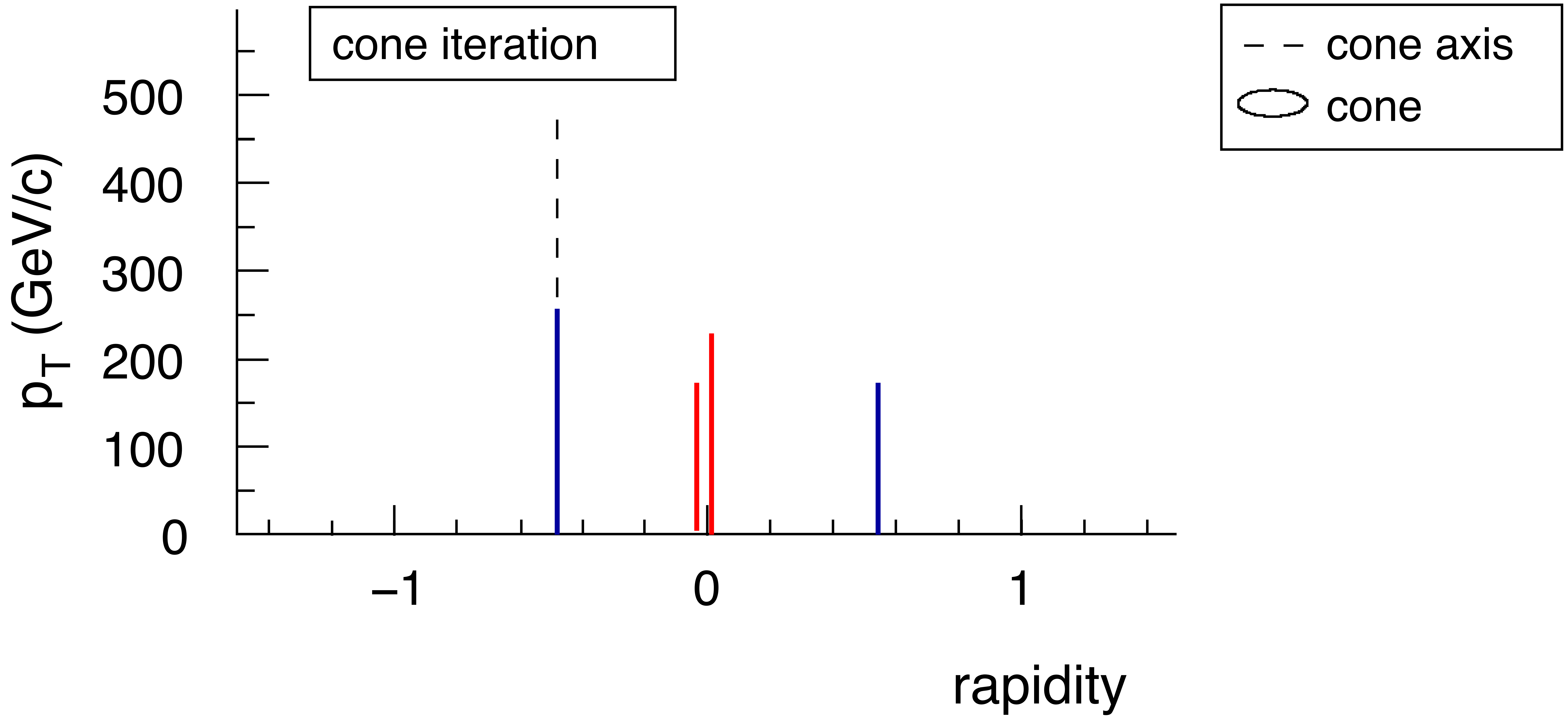

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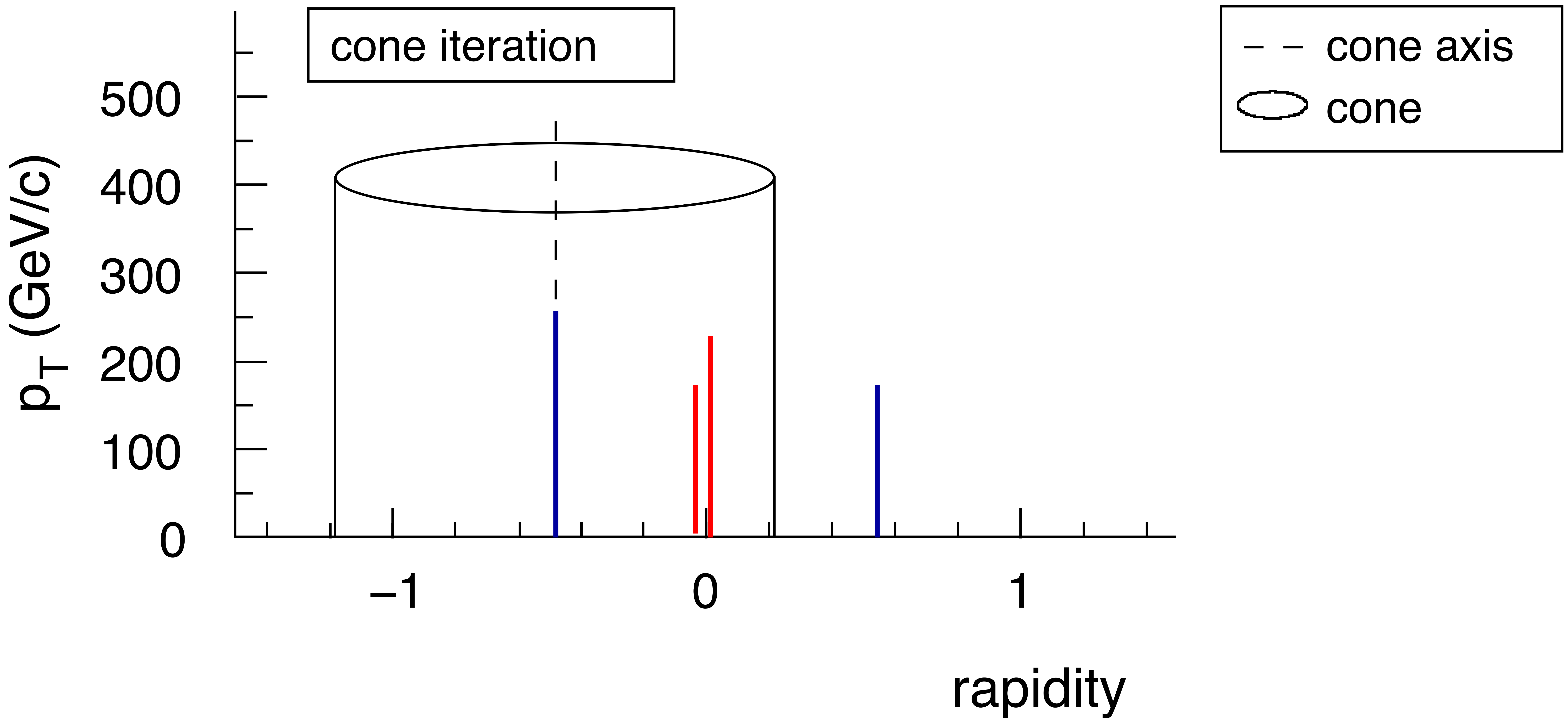

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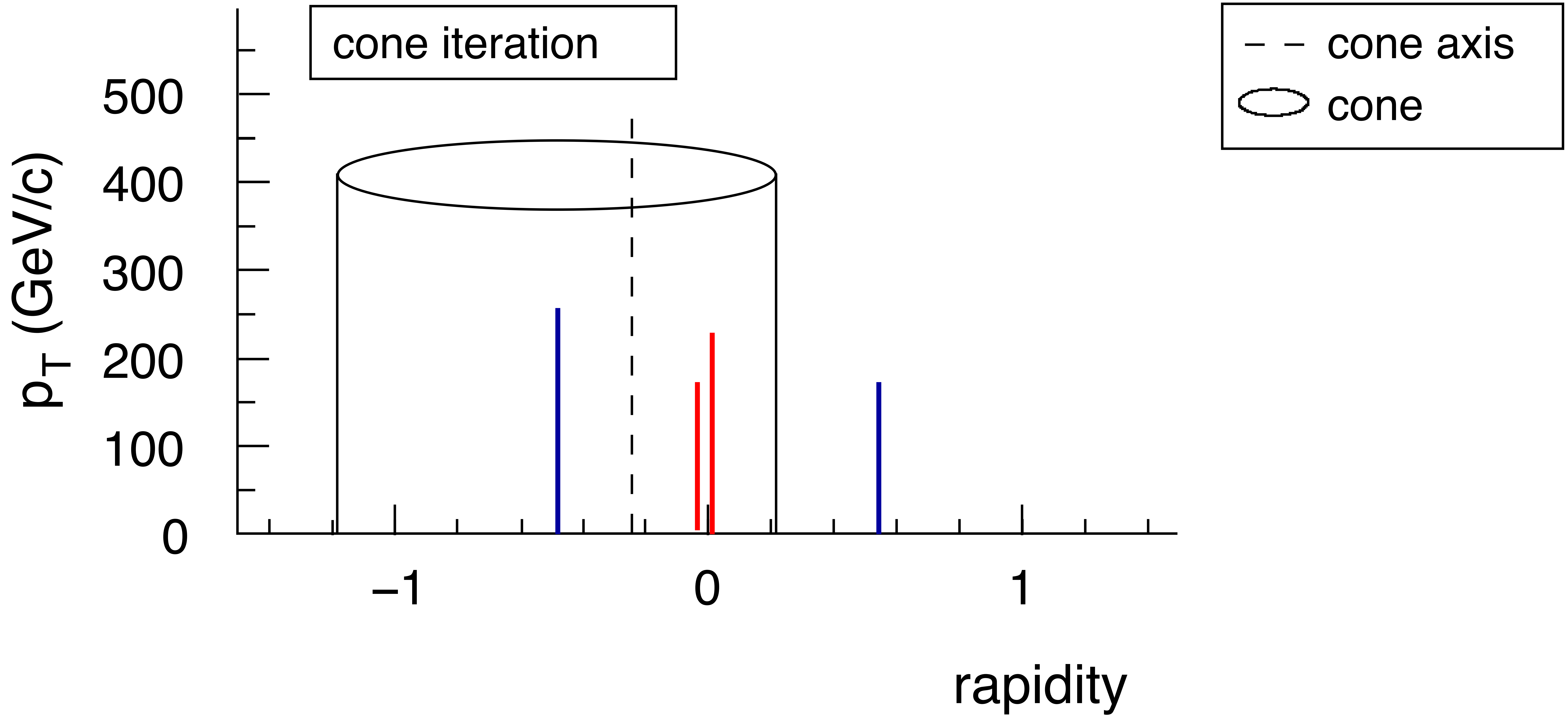

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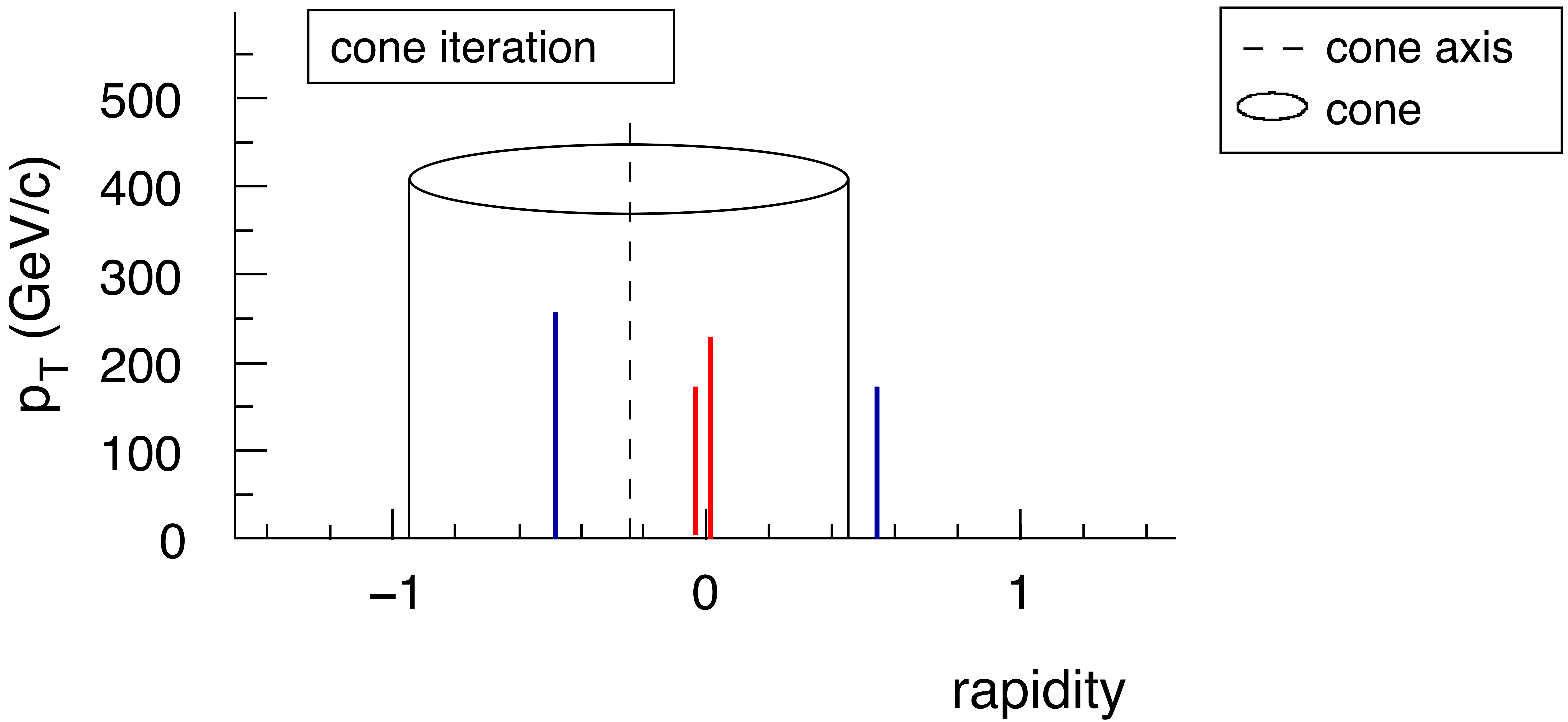

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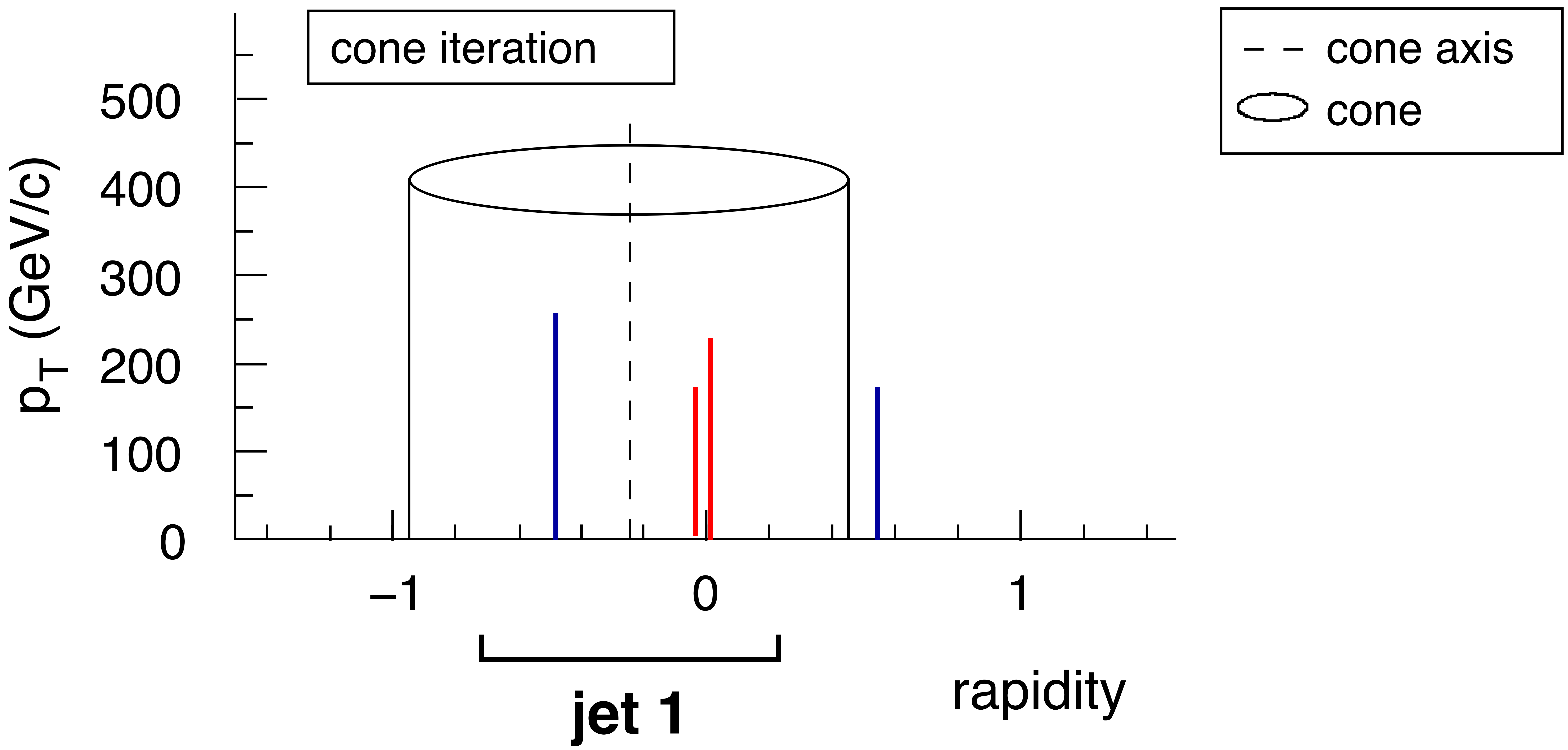

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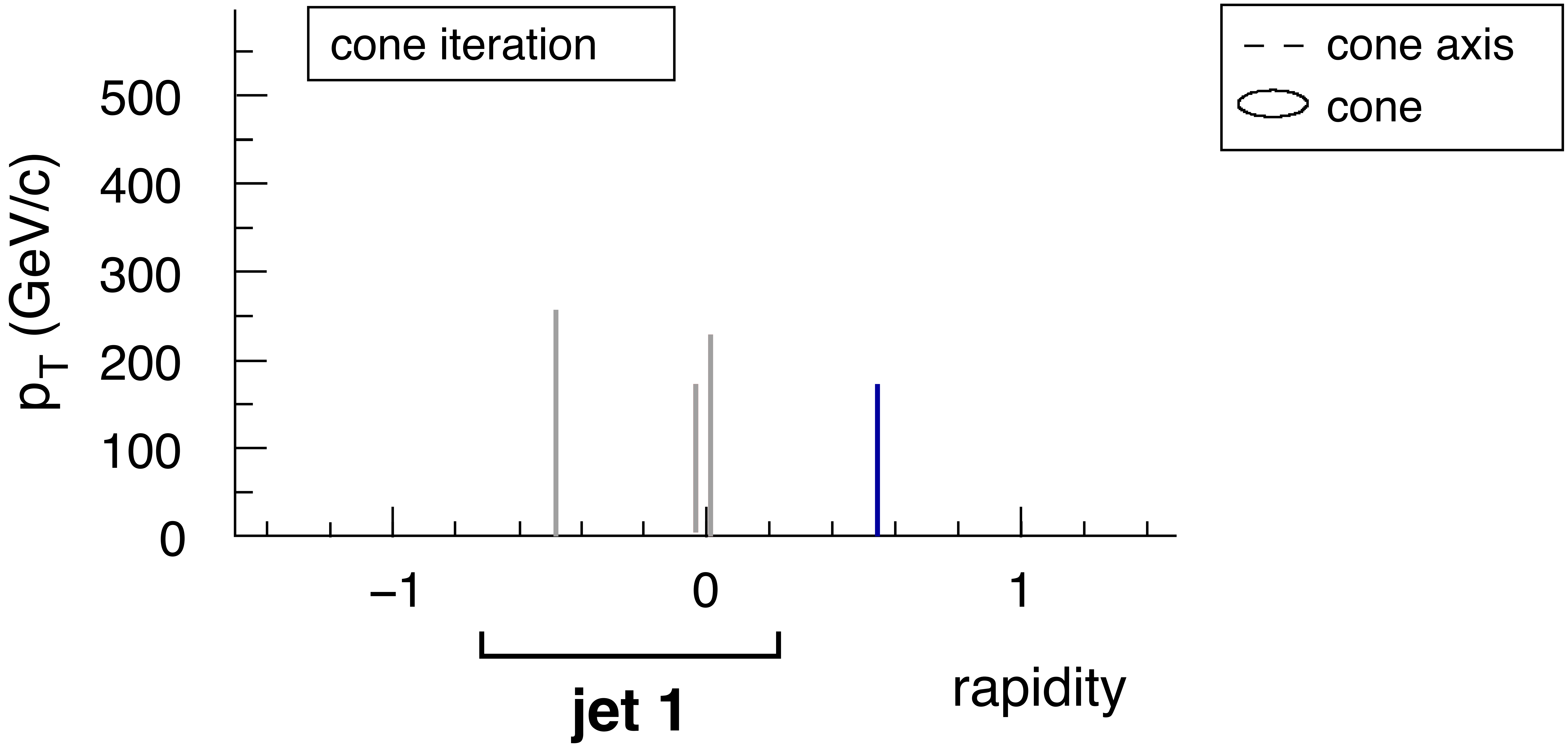

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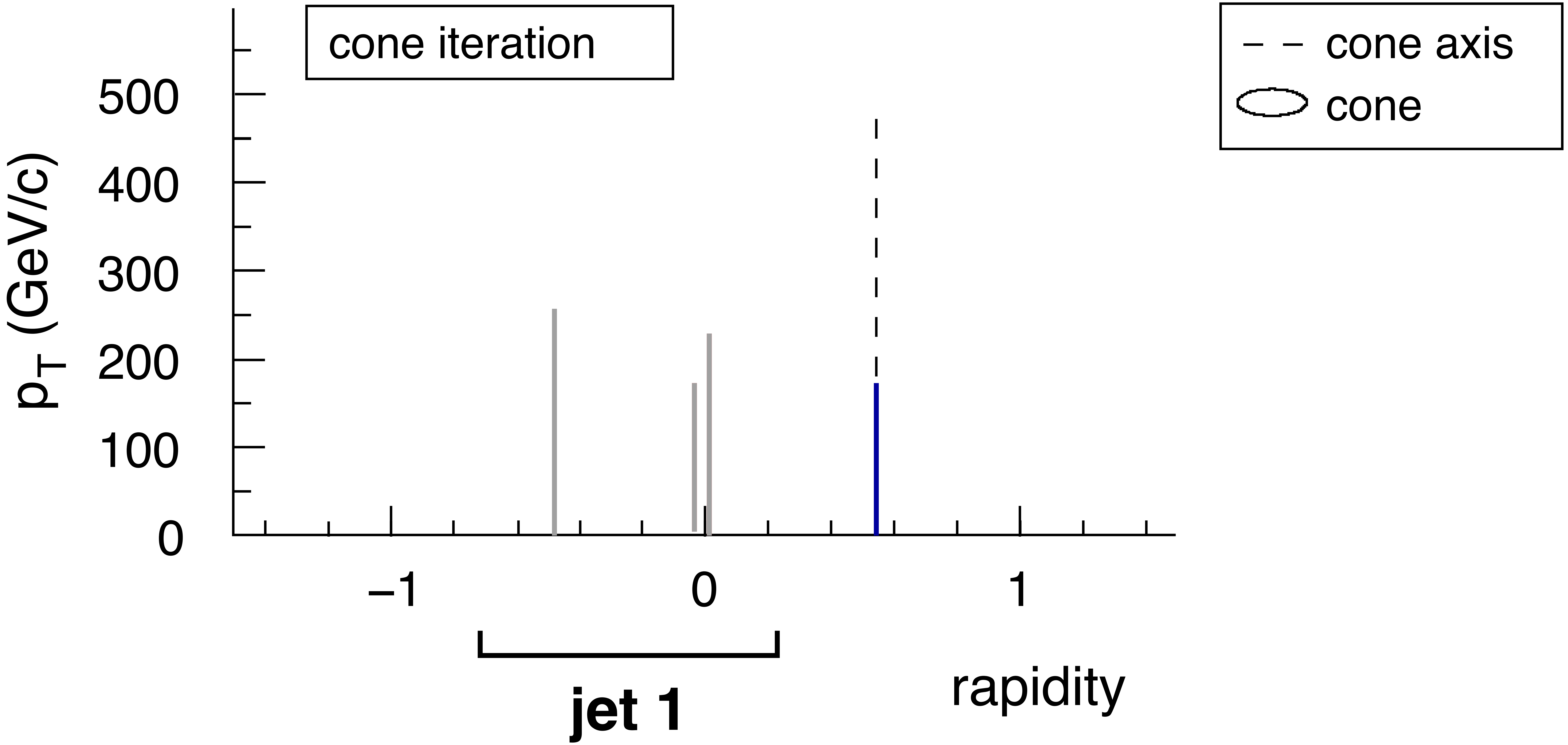

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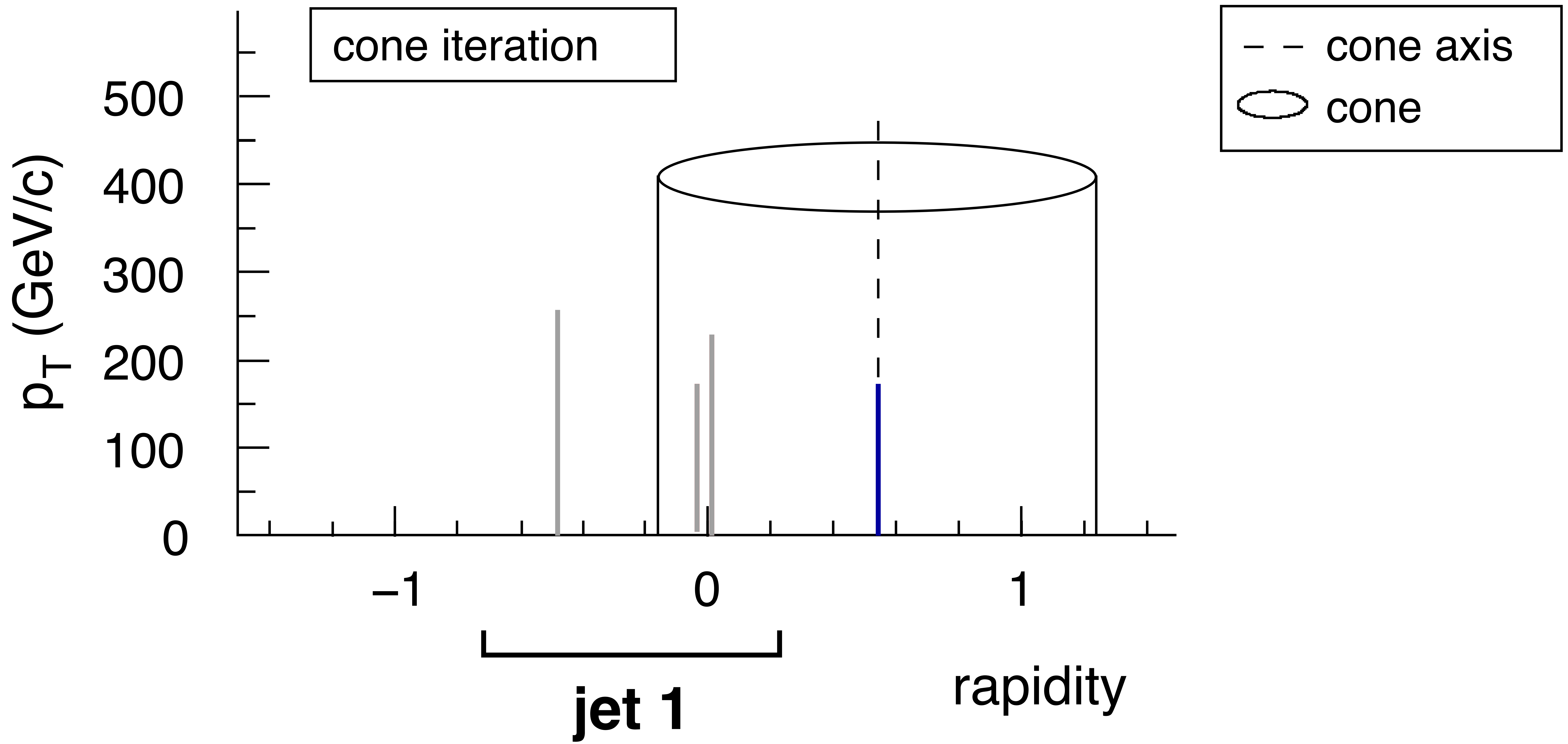

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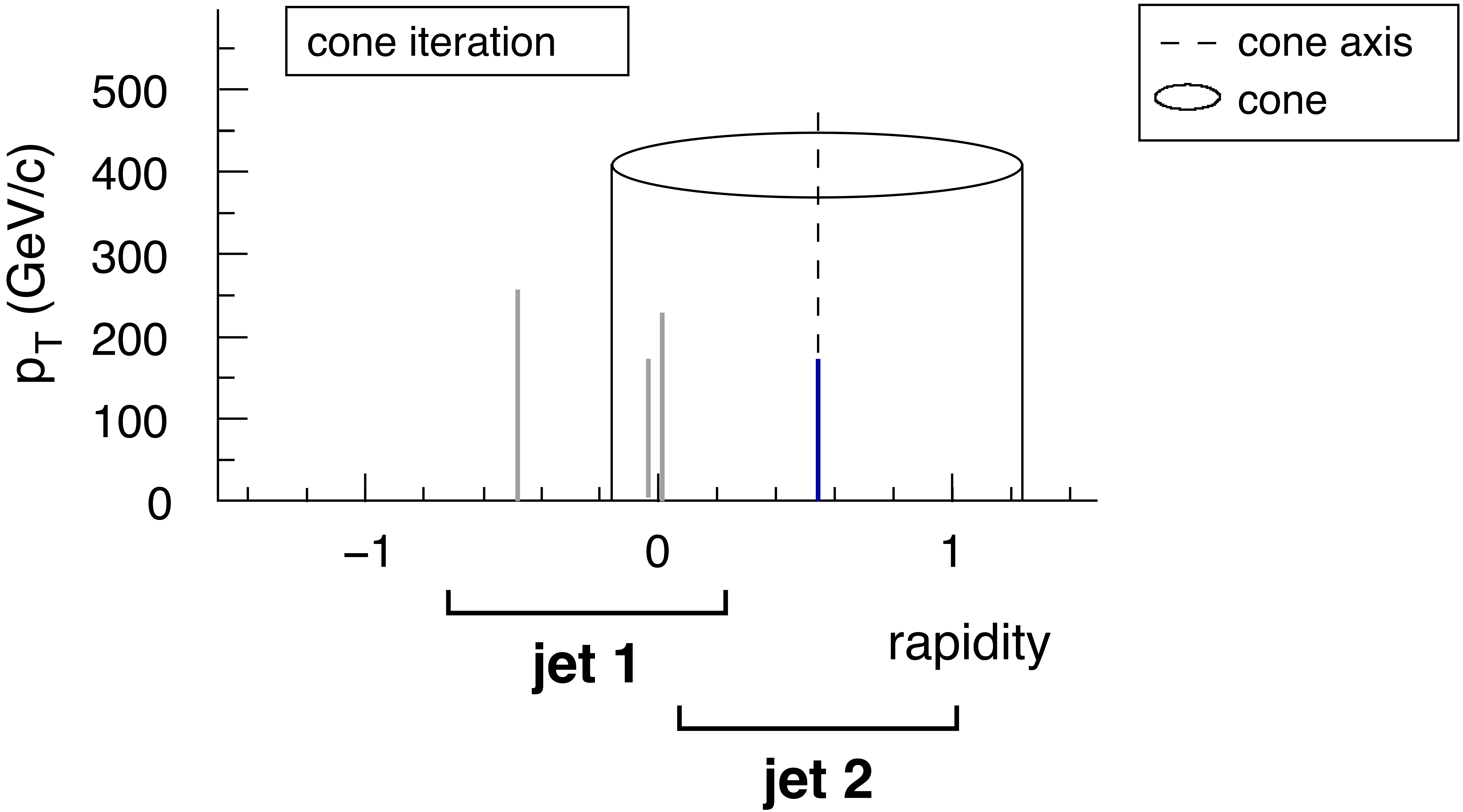

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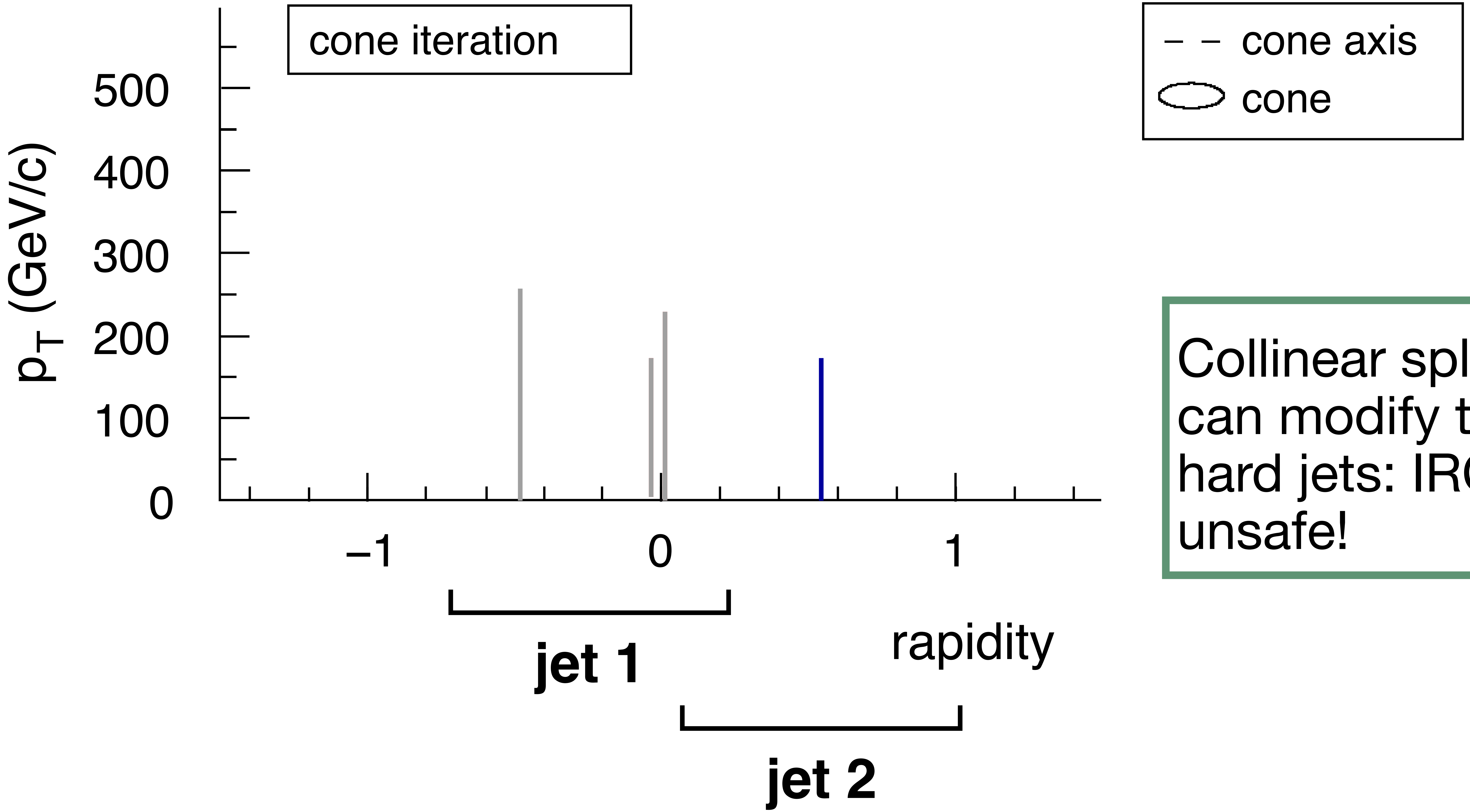
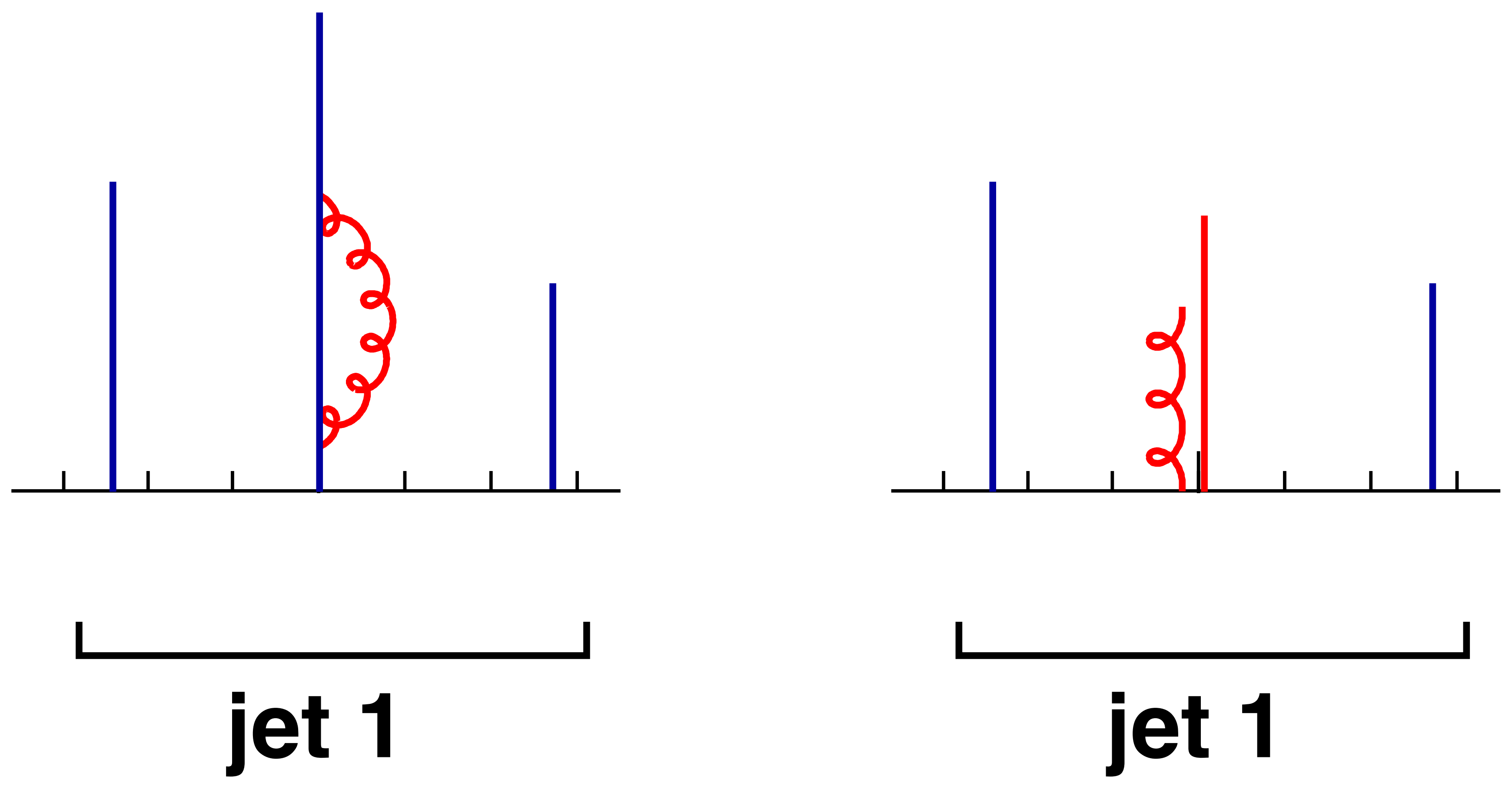


Collinear splitting can modify the hard jets: IRC unsafe!

IRC unsafety in a nutshell

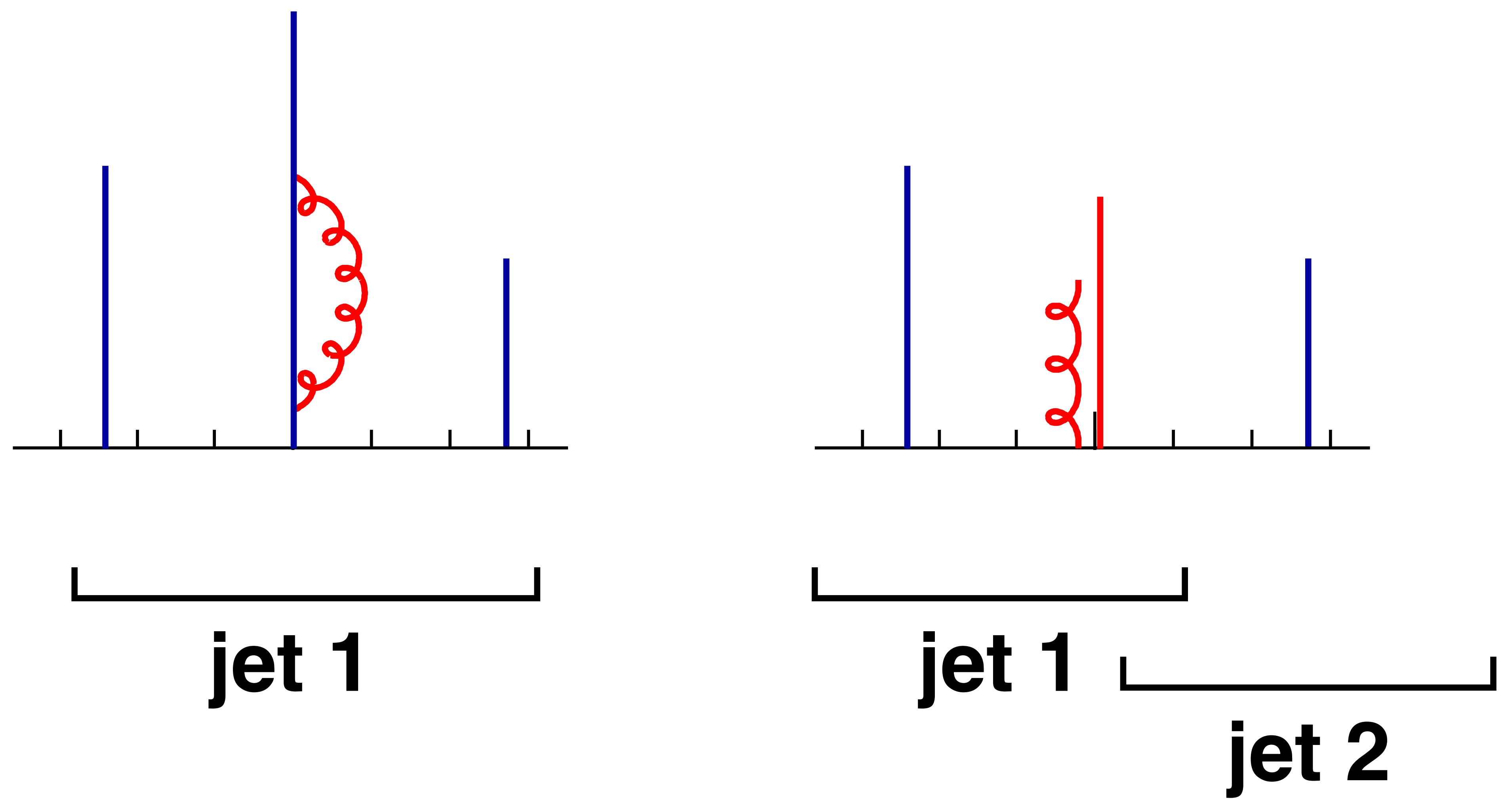
Collinear Safe



$\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$

Infinities cancel

Collinear Unsafe

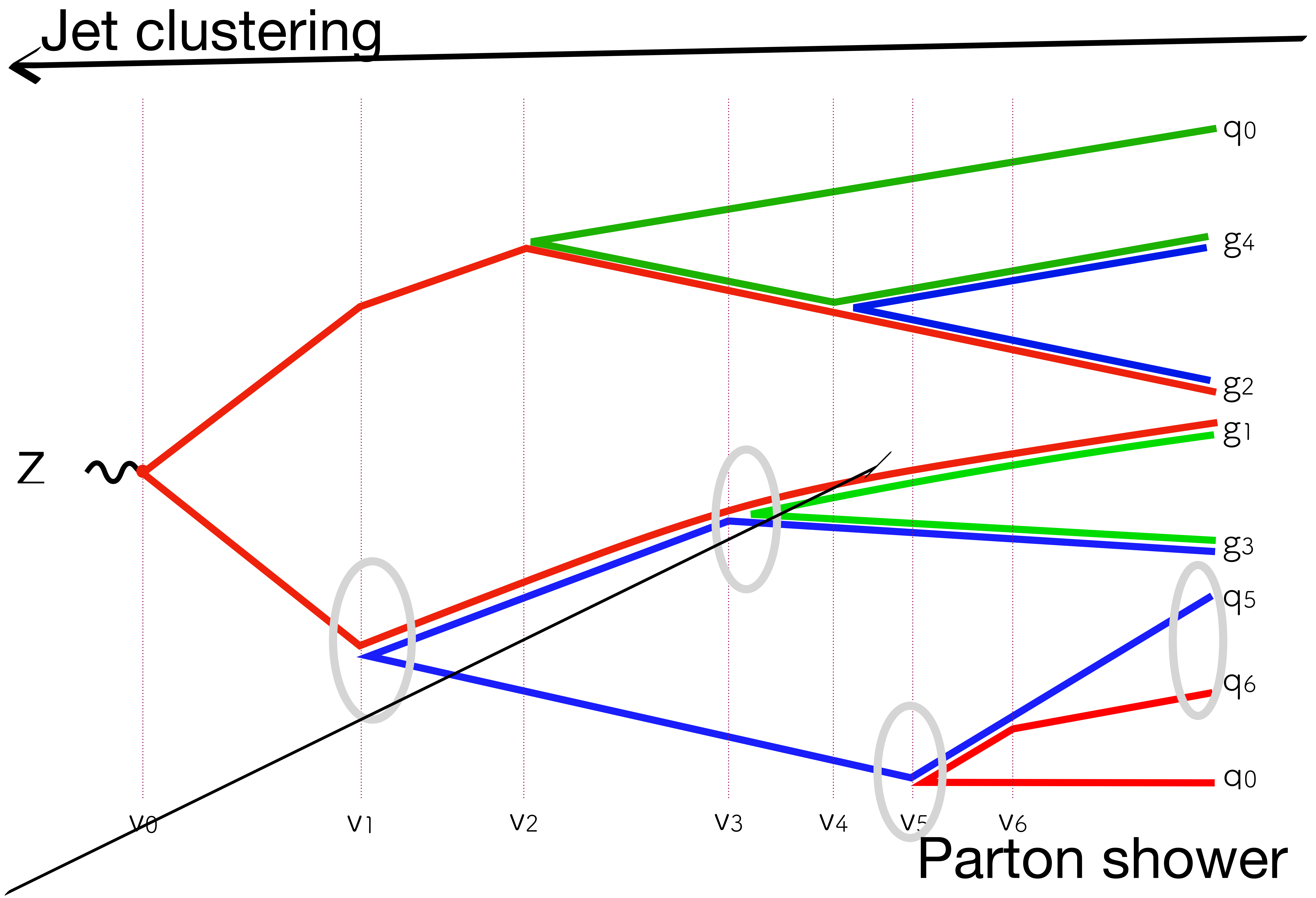


$\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$

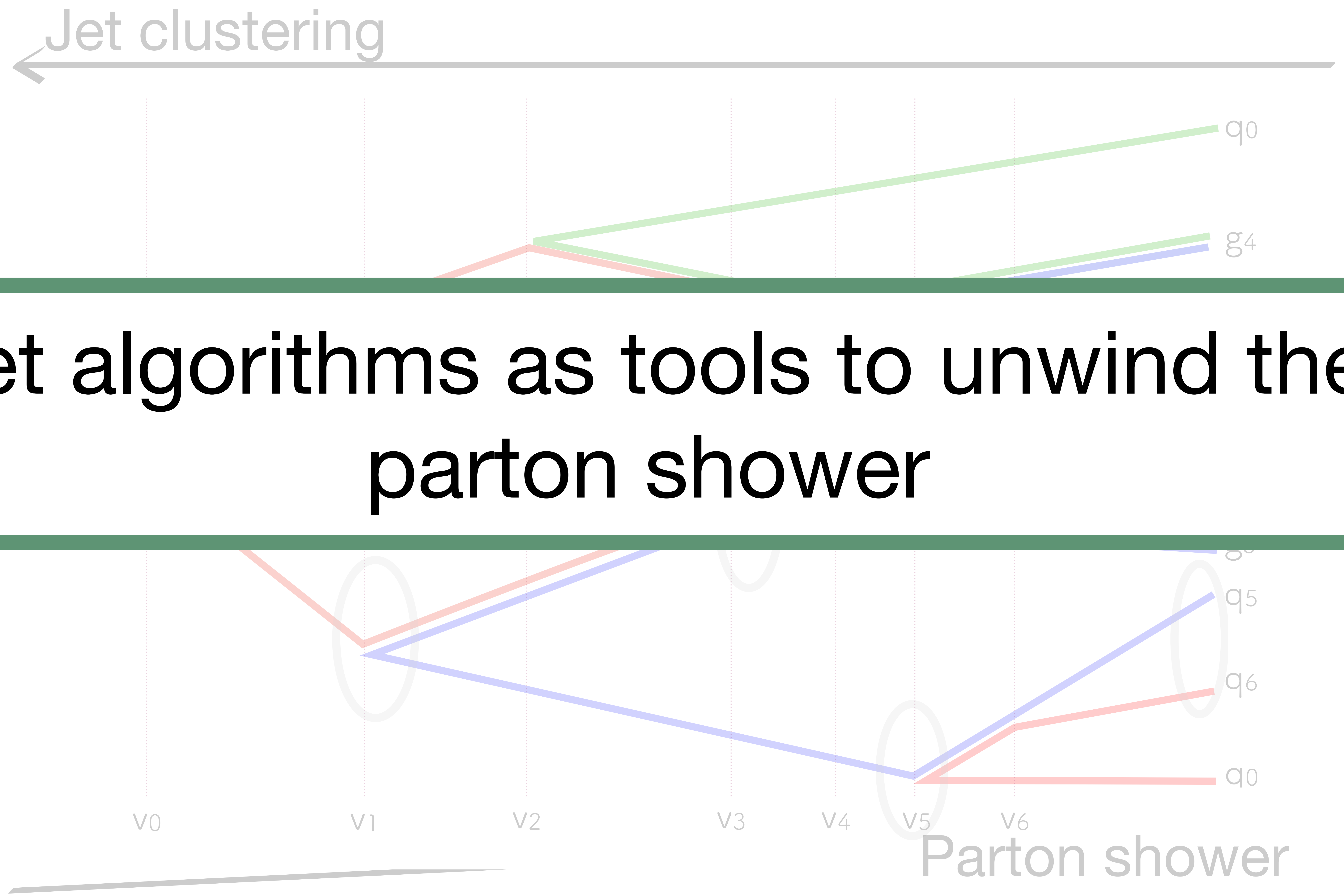
Infinities do not cancel

An IRC unsafe jet definition invalidates perturbation theory

Sequential recombination algorithms



Sequential recombination algorithms



Jet algorithms as tools to unwind the parton shower

Sequential recombination algorithms: bottom-up approach

For simplicity, let us begin with the e^+e^- k_t algorithm:

[Catani, Dokshitzer, Seymour, Webber Nucl.Phys.B 406 (1993) 187-224]

- For each pair of particles i, j work out the distance

$$y_{ij} = \frac{2\min(E_i, E_j)(1-\cos\theta_{ij})}{Q}$$

Sequential recombination algorithms: bottom-up approach

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- For each pair of particles i, j work out the distance

$$y_{ij} = \frac{2\min(E_i, E_j)\,(1-\cos\theta_{ij})}{Q}$$

← inverse of the splitting probability $\frac{dP_{k\to ij}}{dE_i\, d\theta_{ij}} \sim \frac{\alpha_s}{\min(-}$

Sequential recombination algorithms: bottom-up approach

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- For each pair of particles i, j work out the distance

$$y_{ij} = \frac{2\min(E_i, E_j)(1-\cos\theta_{ij})}{Q}$$

- Find the minimum $y_{\min}$ of all the y_{ij}

sum their 4-momenta (E-scheme)

- If $y_{ij} < y_{\text{cut}}$, recombine i and j into a pseudo jet and repeat

Any soft/collinear particle will get recombined right at the start thus making the algorithm IRC safe, i.e. theory friendly

Sequential recombination algorithms: bottom-up approach

At hadron colliders one needs to introduce a couple of modifications

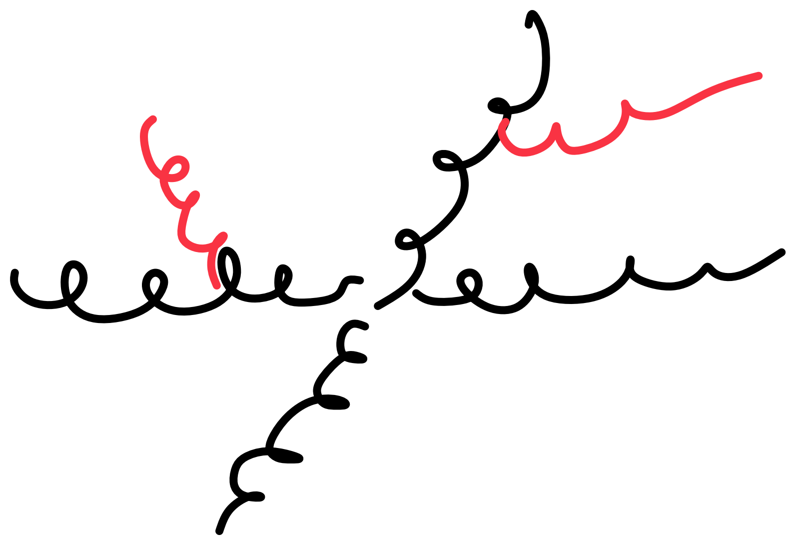
- Resolution parameter: jet radius

$$y_{cut} \to R$$

- Total energy unknown + boost invariance

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2)\frac{\Delta R_{ij}^2}{R^2} \quad \text{with } \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

- QCD divergences wrt the beam



$$d_{iB} = p_{ti}^2 \quad (\text{transverse momentum wrt beam})$$

Sequential recombination algorithms: bottom-up approach

- Work out all the d_{ij} and d_{iB}
- Find the minimum of the d_{ij}, d_{iB}
- If it is a d_{ij}, recombine i and j into a pseudo jet and repeat from 1
- If it is a d_{iB}, declare i to be a final state jet, and remove it from the list of particles. Return to step 1
- Stop when no particles remain

Generalised k_t family of sequential recombination algorithms

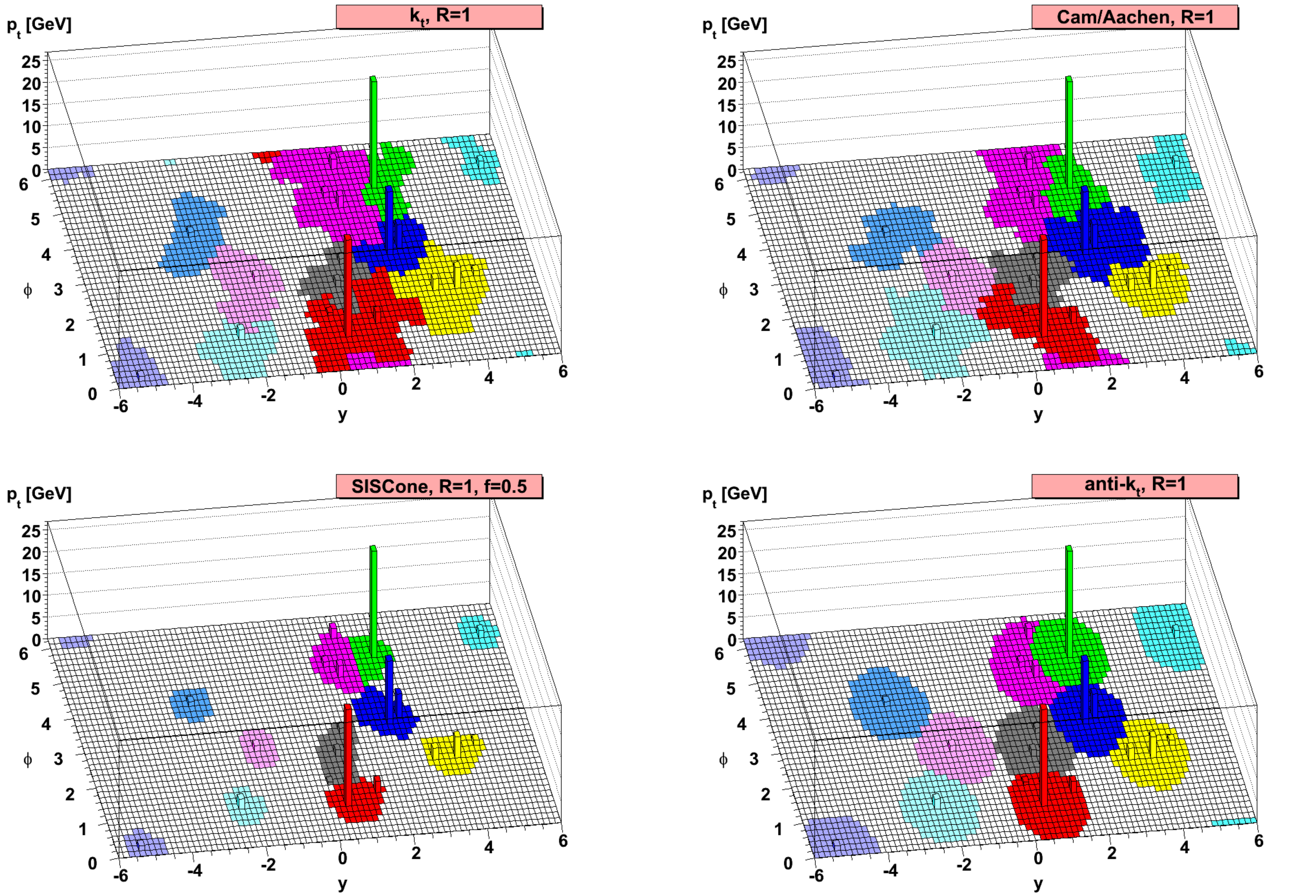
Introduce an additional free parameter, p, in the definition of the metric

$$d_{ij} = \min\left(p_{ti}^{2p}, p_{tj}^{2p}\right) \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = p_{ti}^{2p}$$

A few physically relevant choices for p:

- $p = 0$: Cambridge/Aachen algorithm. Hierarchical in angle
  [Dokshitzer, Leder, Moretti, Webber JHEP 08 (1997) 001]
- $p = 0.5$: τ algorithm. Hierarchical in mass/inverse formation time
  [Apolinario, Cordeiro, Zapp Eur.Phys.J.C 81 (2021) 6, 561]
- $p = 1$: k_t algorithm. Hierarchical in relative transverse momentum
  [Catani, Dokshitzer, Seymour, Webber Nucl.Phys.B 406 (1993) 187-224]
- $p = -1$: anti-k_t algorithm. Hierarchy meaningless
  [Cacciari, Salam, Soyez JHEP 04 (2008) 063]

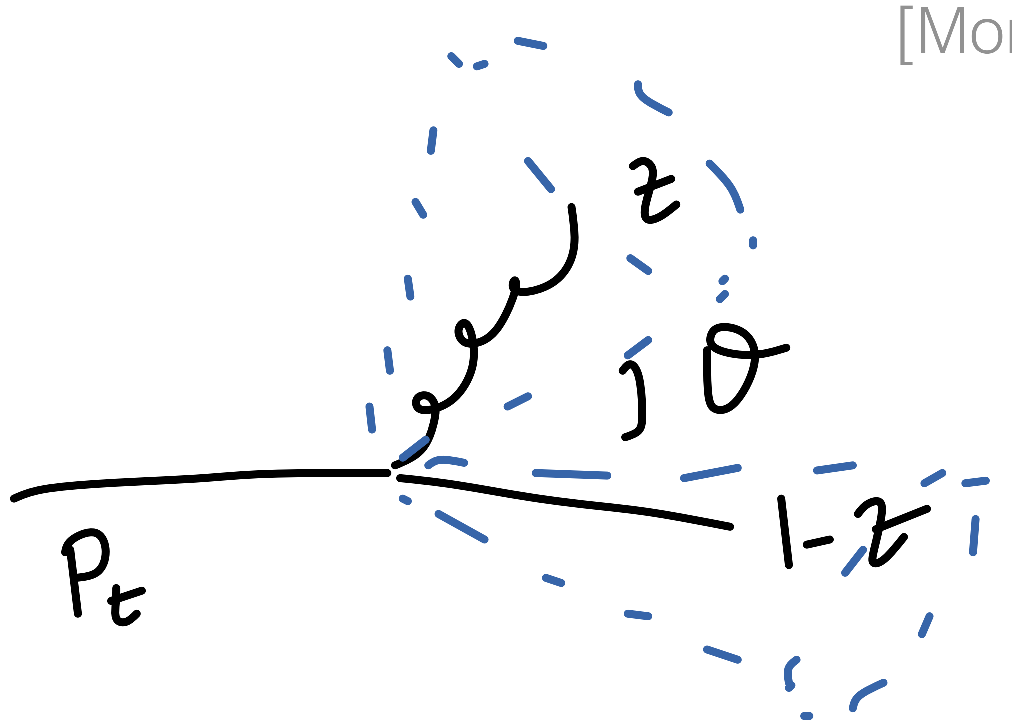
Generalised k_t family of sequential recombination algorithms



How close does a jet resemble a (MC) parton? Perturbative

[More details in 'Towards jetography']

Oversimplified



$$\langle \delta p_t \rangle = \int \frac{d\theta^2}{\theta^2} \int dz \, \frac{\alpha_s(z\theta p_t)}{2\pi} P(z)$$

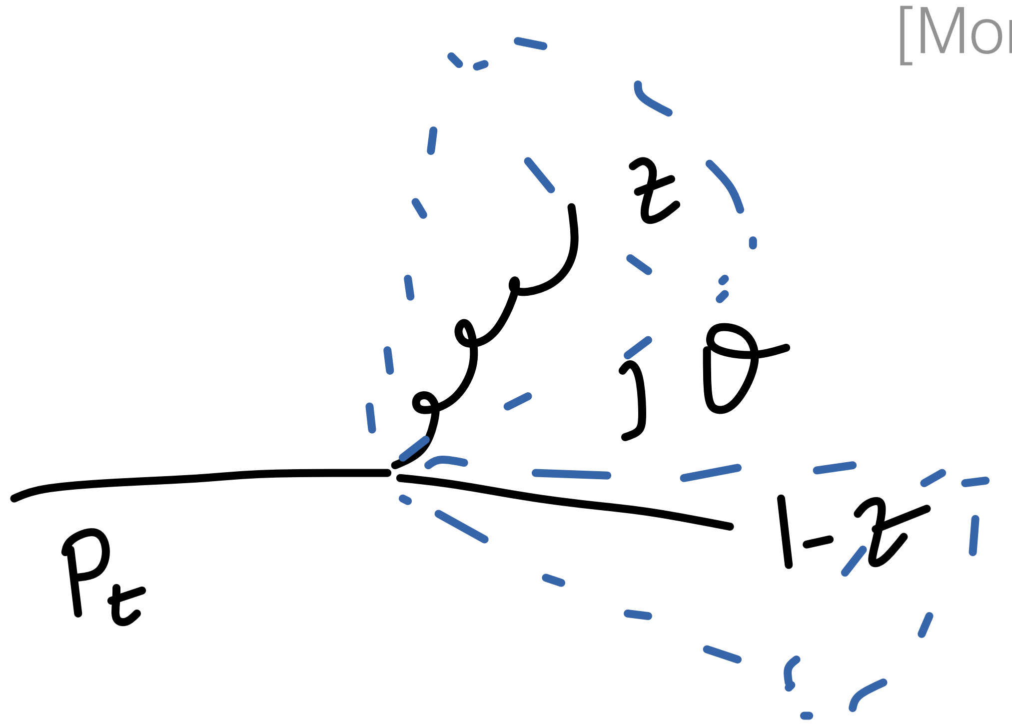
$$\times \underbrace{p_t \left[\max(z, 1-z) - 1 \right]}_{p_t\text{-loss}} \times \Theta\left(\theta > f_{alg}(z) R\right)$$

outside the jet

1 for k_t, anti-k_t, C/A

How close does a jet resemble a (MC) parton? Perturbative

[More details in ‘Towards jetography’]



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$$\times \underbrace{p_t\left[\max(z, 1-z) - 1\right]}_{p_t\text{-loss}} \times \Theta\left(\theta > f_{alg}(z) R\right)$$

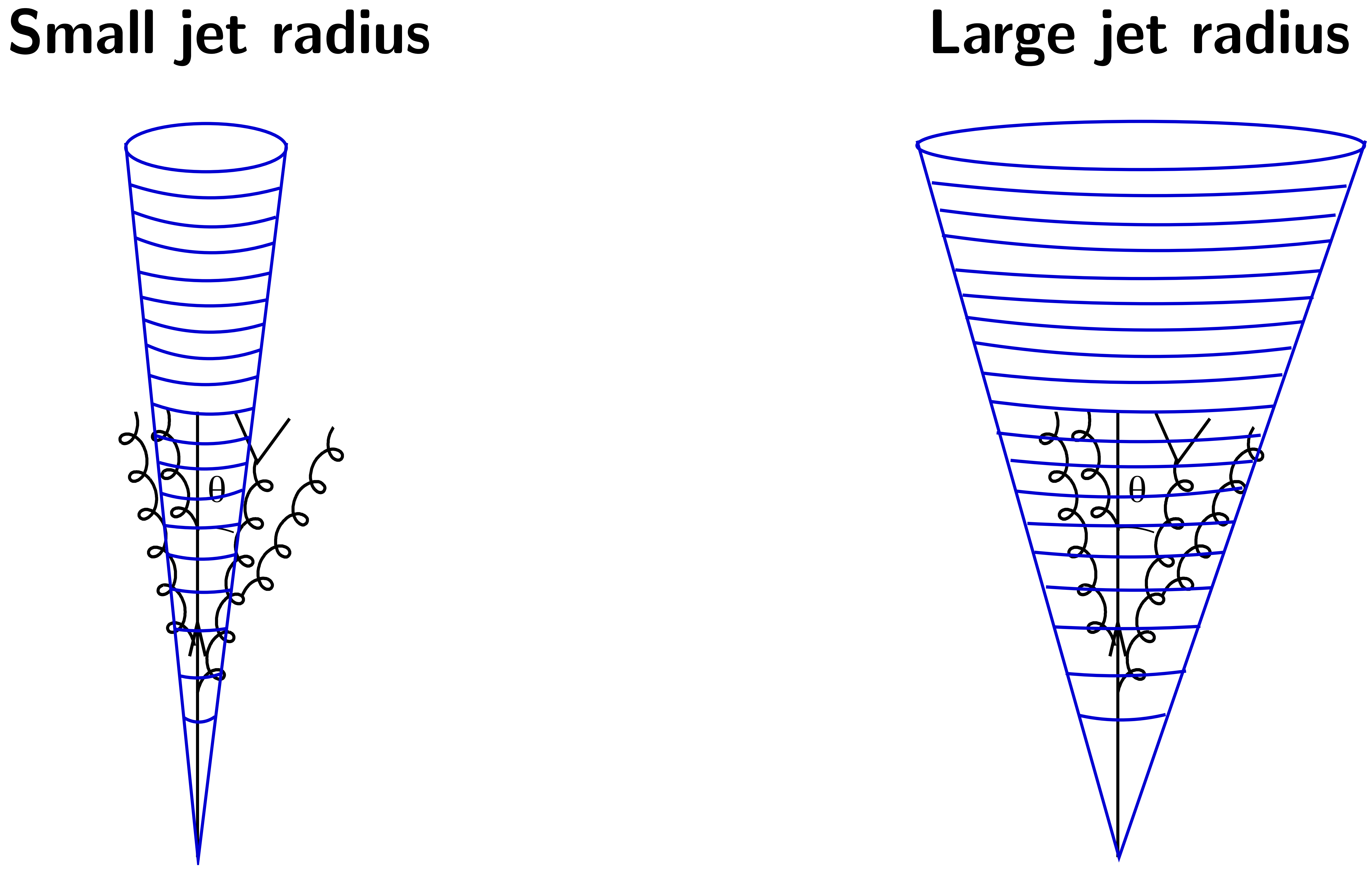
$\Theta(\ldots)$ ⇒ outside the jet

$f_{alg}(z)$ ← 1 for k_t, anti-k_t, C/A

$$\left.\frac{\langle \delta p_t \rangle}{p_t}\right|_{pert} = \frac{\alpha_s}{\pi} C_F \underbrace{\left(2\ln 2 - \frac{3}{8}\right)\ln R}_{<0} + \mathcal{O}(\alpha_s) + \mathcal{O}(R^2)$$

For R=0.4, a quark-induced jet looses $\sim 5\,\%$ of initiating parton’s p_t

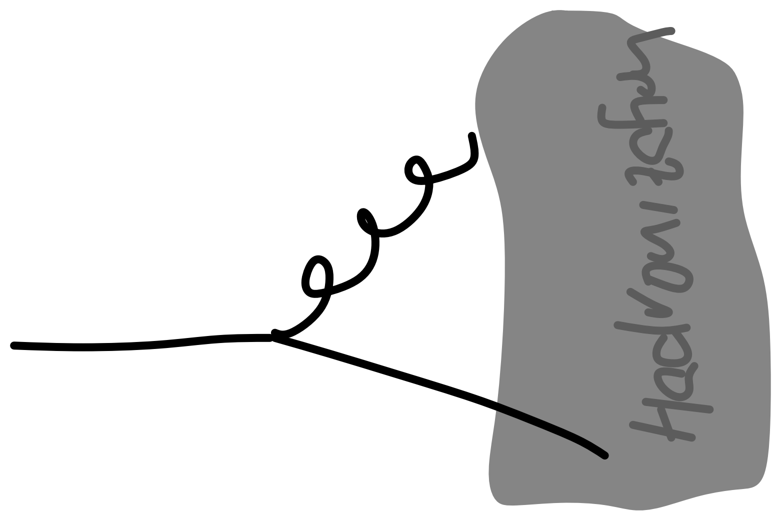
How close does a jet resemble a (MC) parton? Perturbative



Perturbative fragmentation: large radius better (it captures more)

How close does a jet resemble a (MC) parton? Non-perturbative

[More details in ‘Towards jetography’]



$$\langle \delta p_t \rangle \Big|_{NP} = \int \frac{d\theta^2}{\theta^2} \int dz \, \frac{\alpha_s^{NP}(z\theta p_t)}{2\pi} P(z)$$

$\alpha_s^{NP}(\mu) = \Lambda\delta(\mu-\lambda)$ ← Landau scale

$$\times \underbrace{p_t\left[\max(z,1-z)-1\right]}_{p_t\text{-loss}} \times \Theta\left(\theta > f_{alg}(z) R\right)$$

⇑ outside the jet

↑ 1 for k_t, anti-k_t, CA

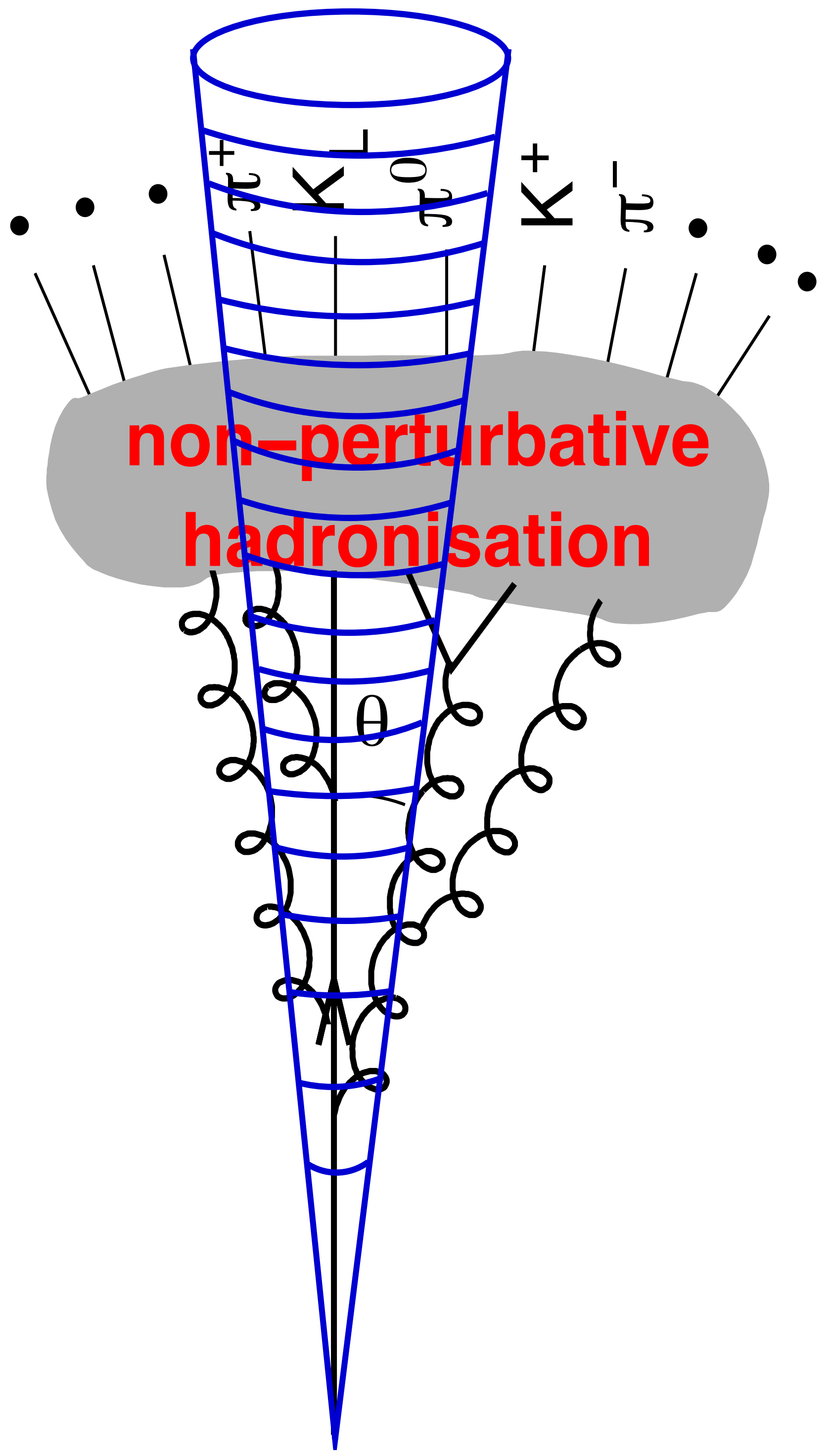
$$\left[\frac{\langle \delta p_t \rangle}{p_t}\Big|_{NP} \sim -\frac{2C_F}{\pi}\frac{\lambda}{R}\right]$$

Very oversimplified

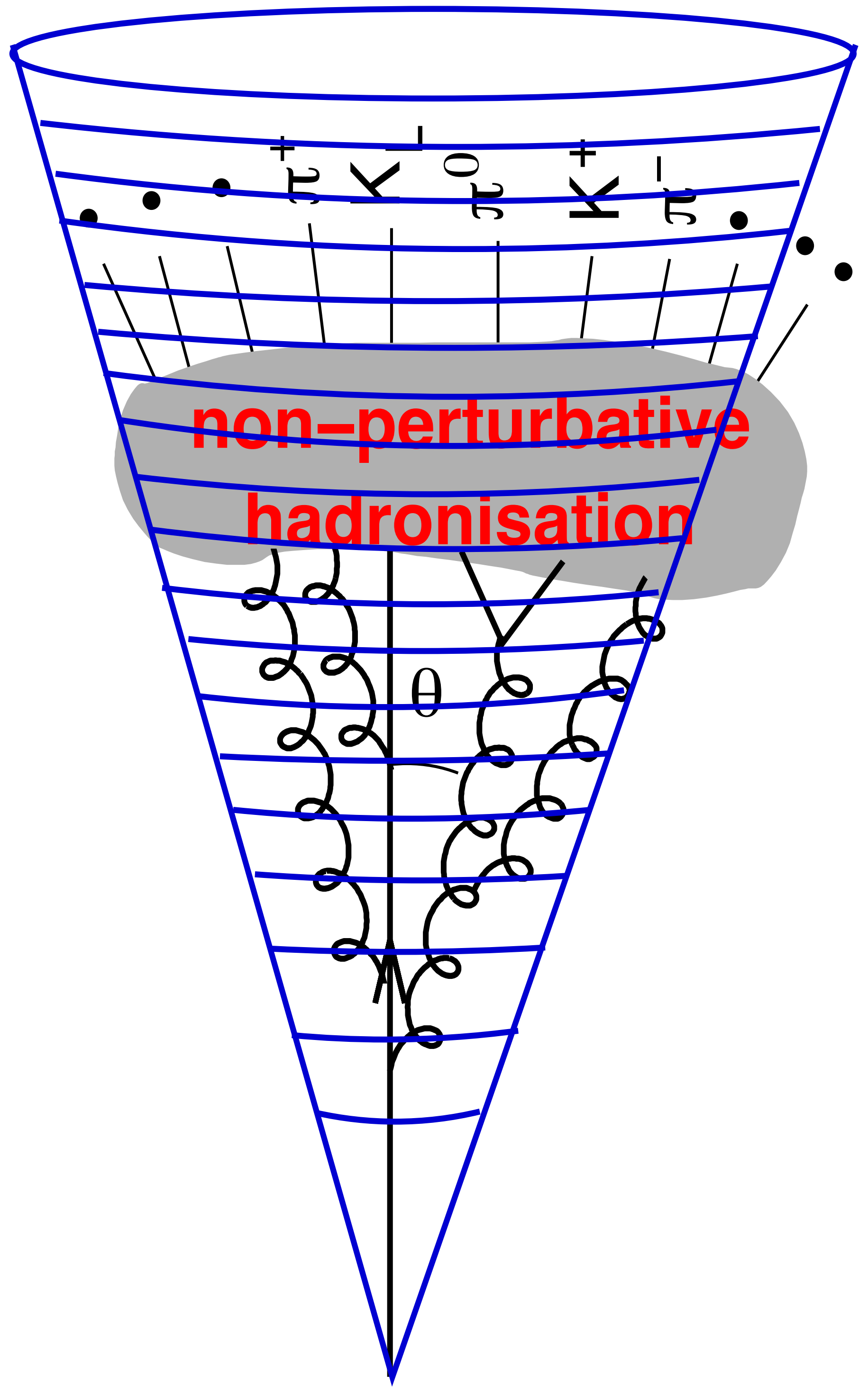
Hadronisation removes transverse momentum $\mathcal{O}(\lambda/R)$ from a jet

How close does a jet resemble a (MC) parton? Non-perturbative

Small jet radius



Large jet radius



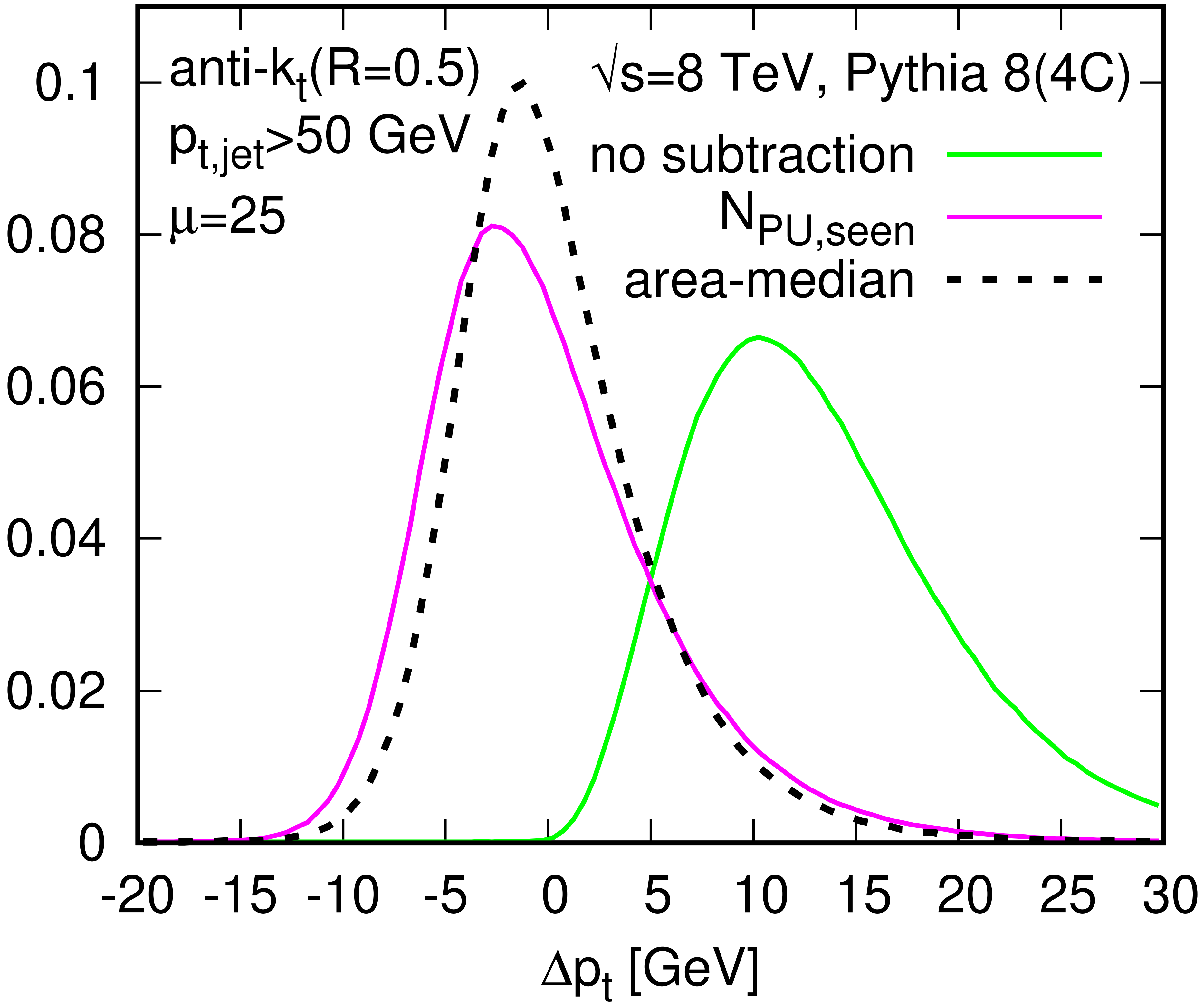
Non-perturbative fragmentation: large radius better (it captures more)

How close does a jet resemble a (MC) parton? Underlying event

Jets at hadron colliders sit on top of a QCD background (pileup, MPI)

Assuming a uniform distribution, it will induce an extra amount of p_t

[Soyez Phys.Rept. 803 (2019) 1-158]



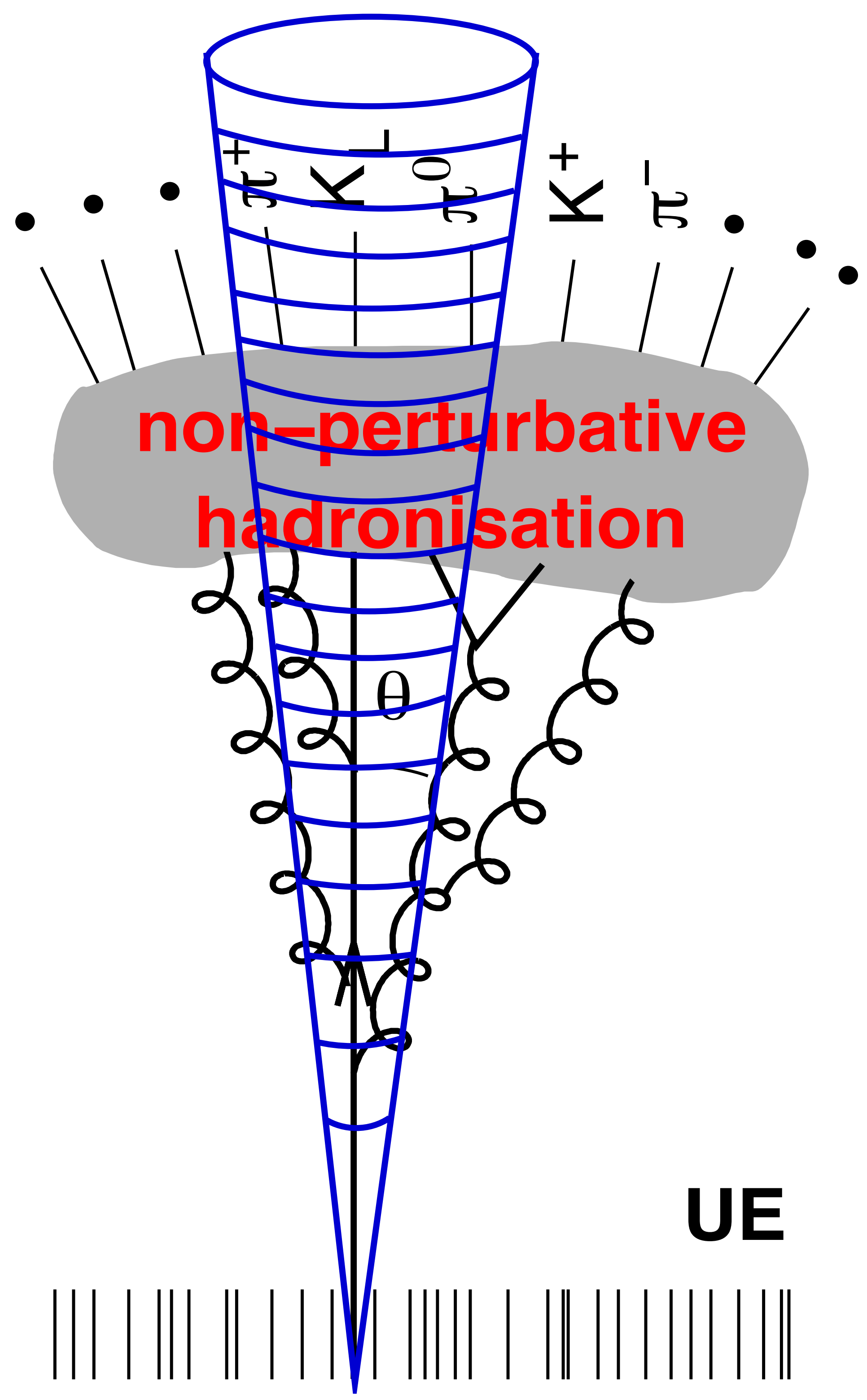
$$\langle \delta p_t \rangle_{UE} \sim \Lambda_{UE}\left(\frac{R^2}{2} - \frac{R^4}{8}\right)$$

$\Lambda_{UE} \sim 20\,\text{GeV}$ (in pp)

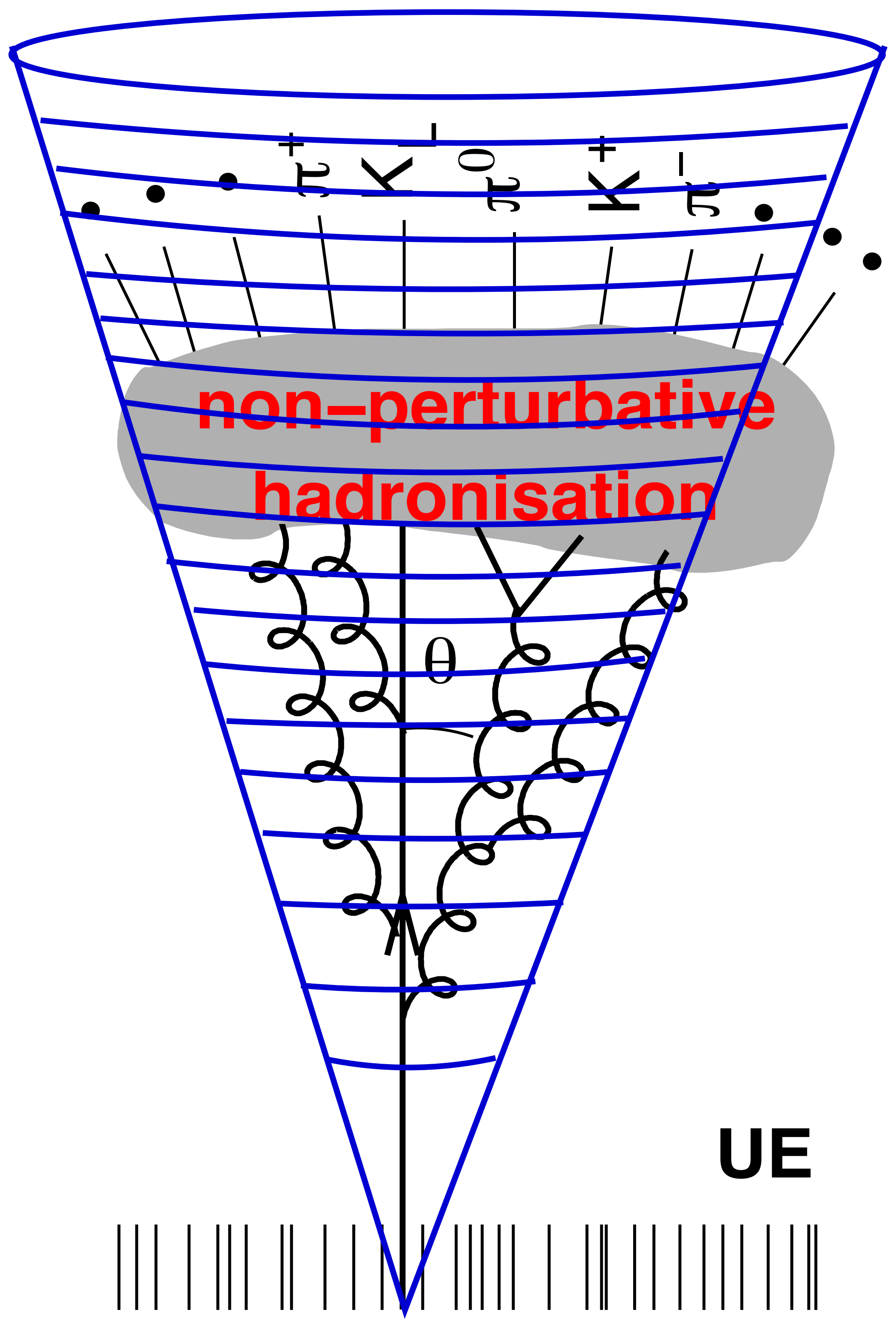
$\sim 250\,\text{GeV}$ (in PbPb)

How close does a jet resemble a (MC) parton? Underlying event

Small jet radius

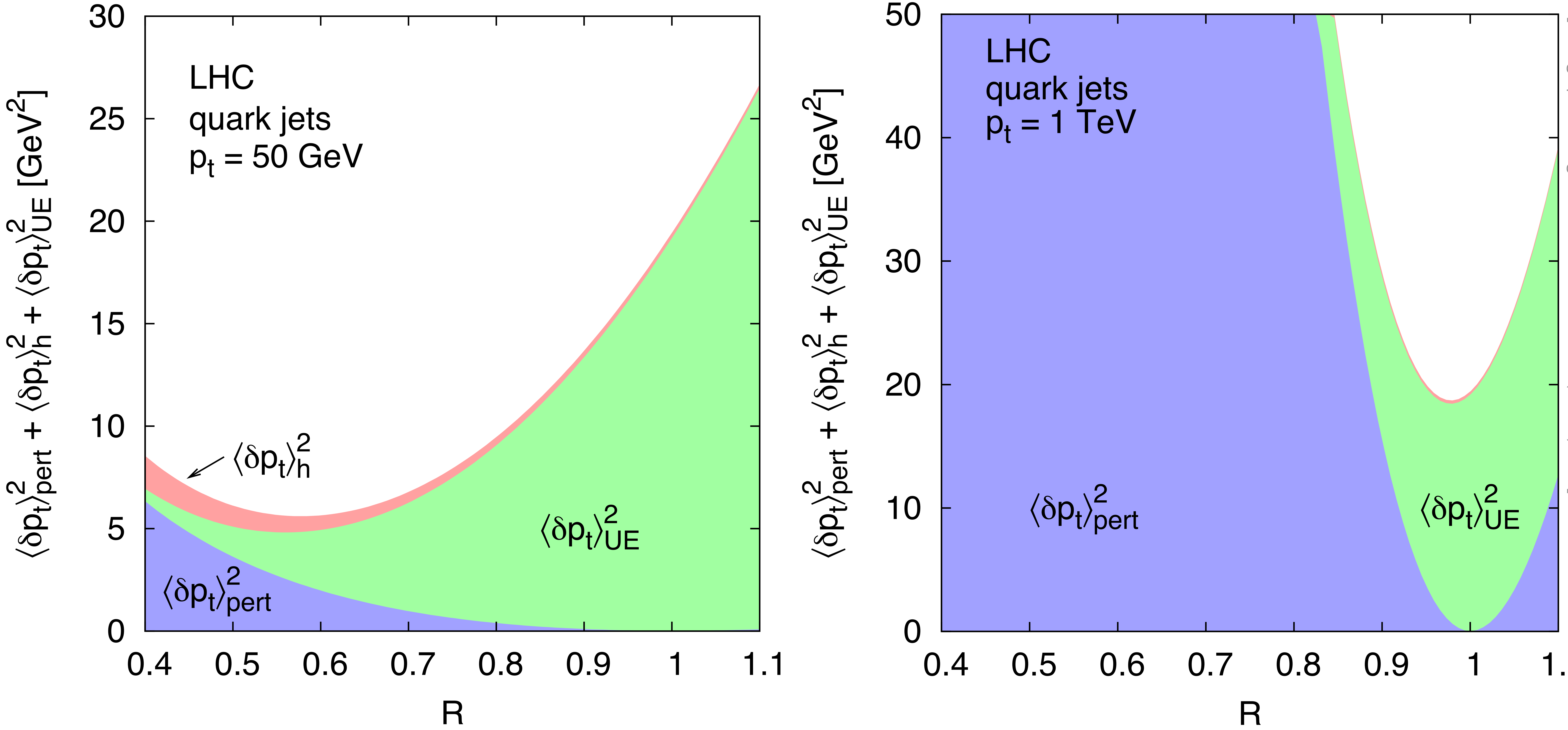


Large jet radius



UE and pileup: small radius better (it capture less)

Putting everything together



[Dasgupta, Magnea, Salam JHEP 02 (2008) 055]

At low p_t, small R limits UE impact. At high p_t, pQCD dominates

To conclude: standard LHC jet finding

- Uses the anti-k_t algorithm
- Uses a jet radius $R = 0.4$
- Uses a transverse momentum threshold that is typically at least 20 GeV (exact value depends on the analysis)
- Radius and p_t threshold choices give a good compromise between:
  - ability to resolve multi-jet physics
  - loss of radiation from jets
  - additional spurious jets
  - contamination from pileup