

# HIGH-ENERGY QCD AND DIFFRACTION

## PART III

**Francesco Giovanni Celiberto, UAH Madrid**

### Midsummer School in QCD 2024

24 June – 6 July 2024

Saariselkä, Finland



Madrid  
**UAH**



**talento**

cm

Programa de atracción  
de talento investigador  
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The BFKL UGD - 3D tomography with (un)polarized gluon TMDs

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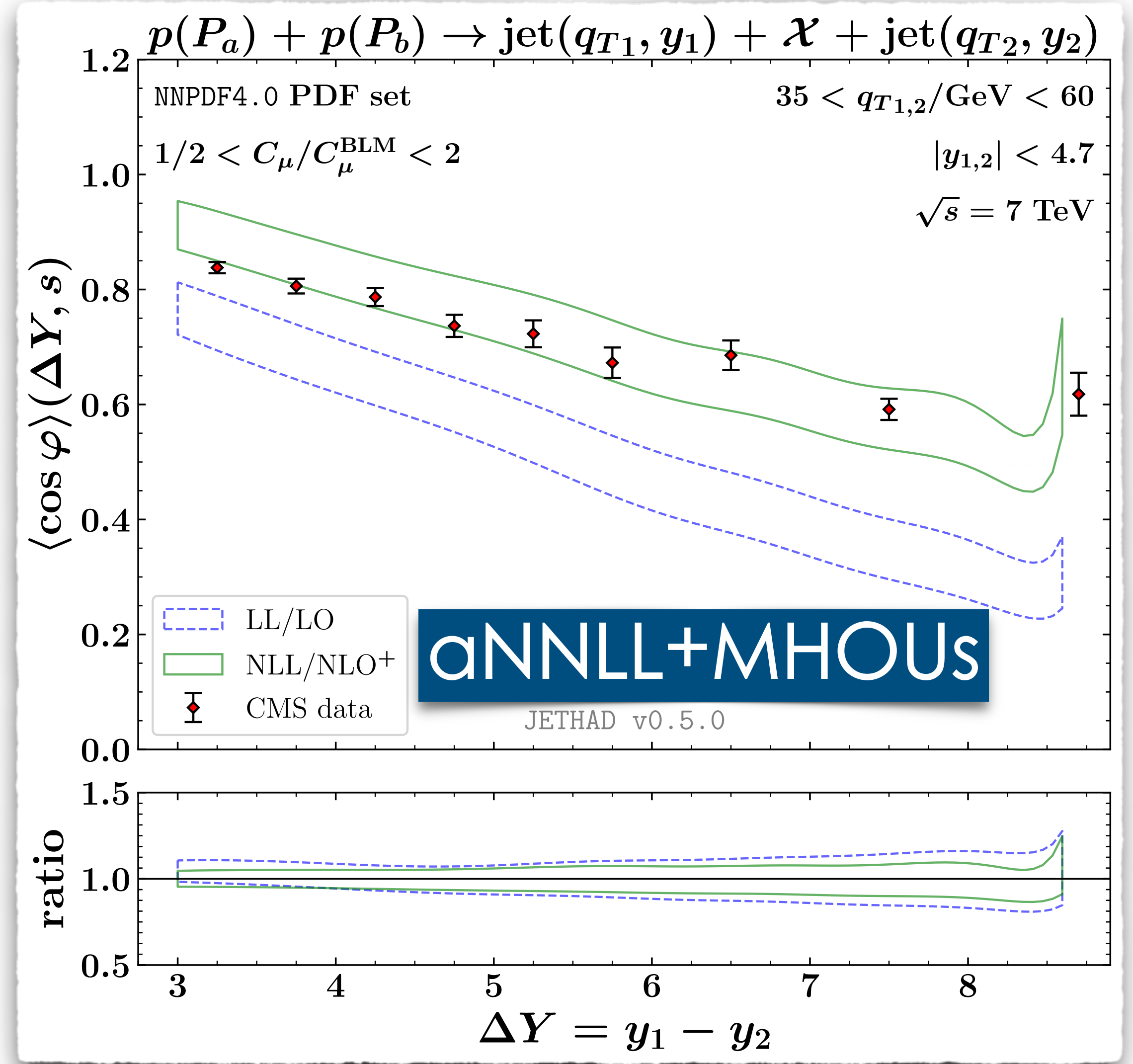
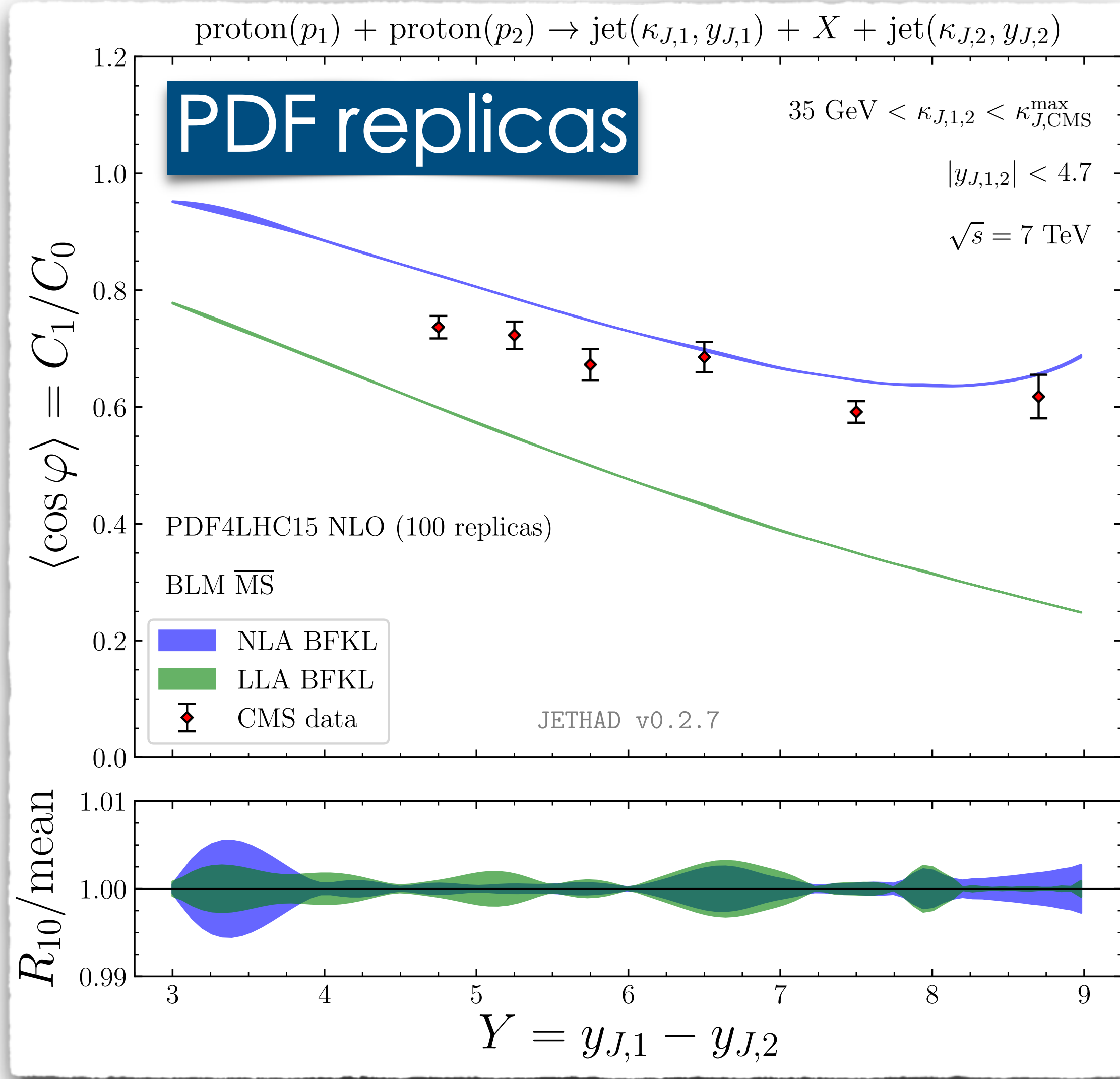
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# Lecture II Highlights


# MN jets: Hunting BFKL @7TeV CMS

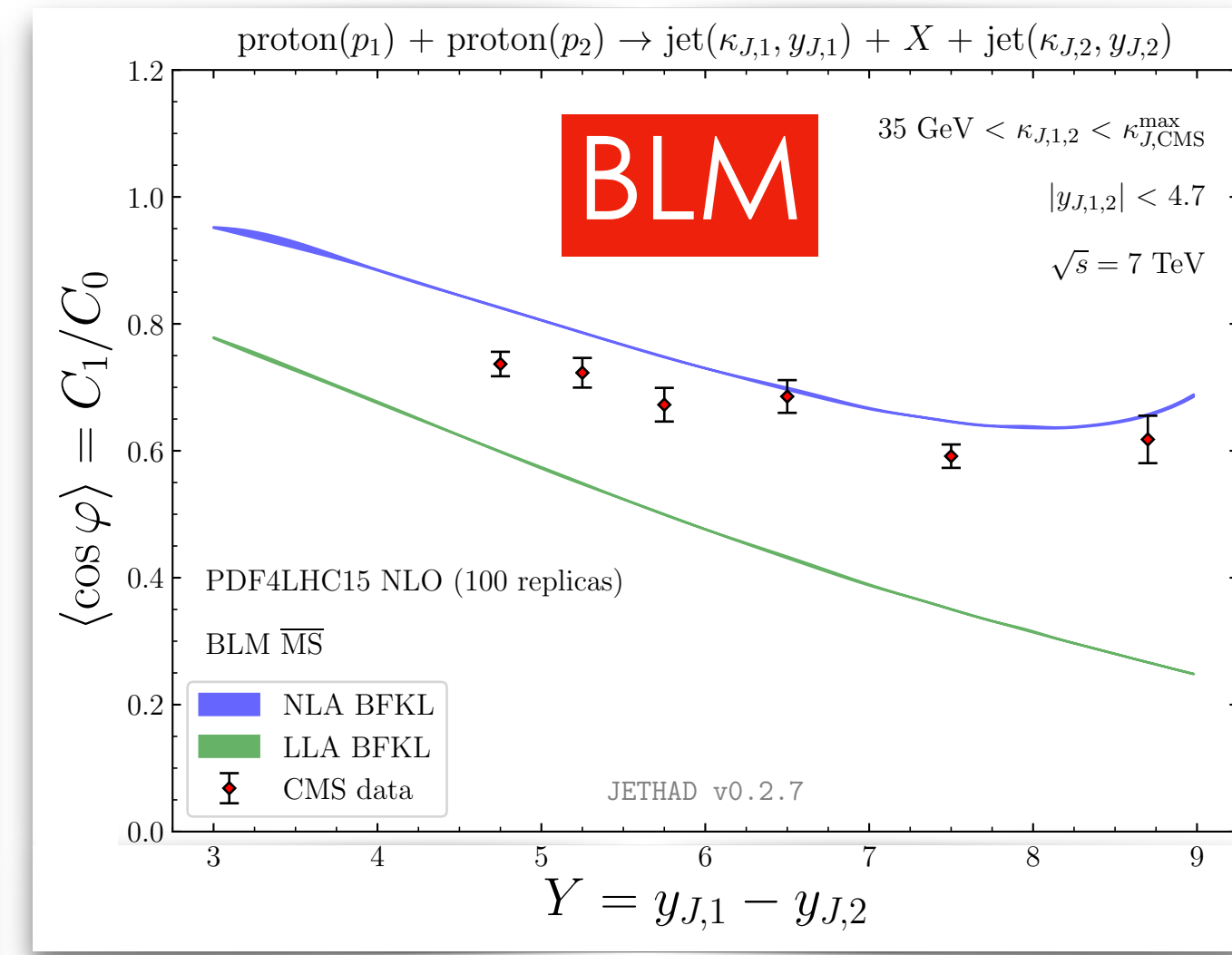
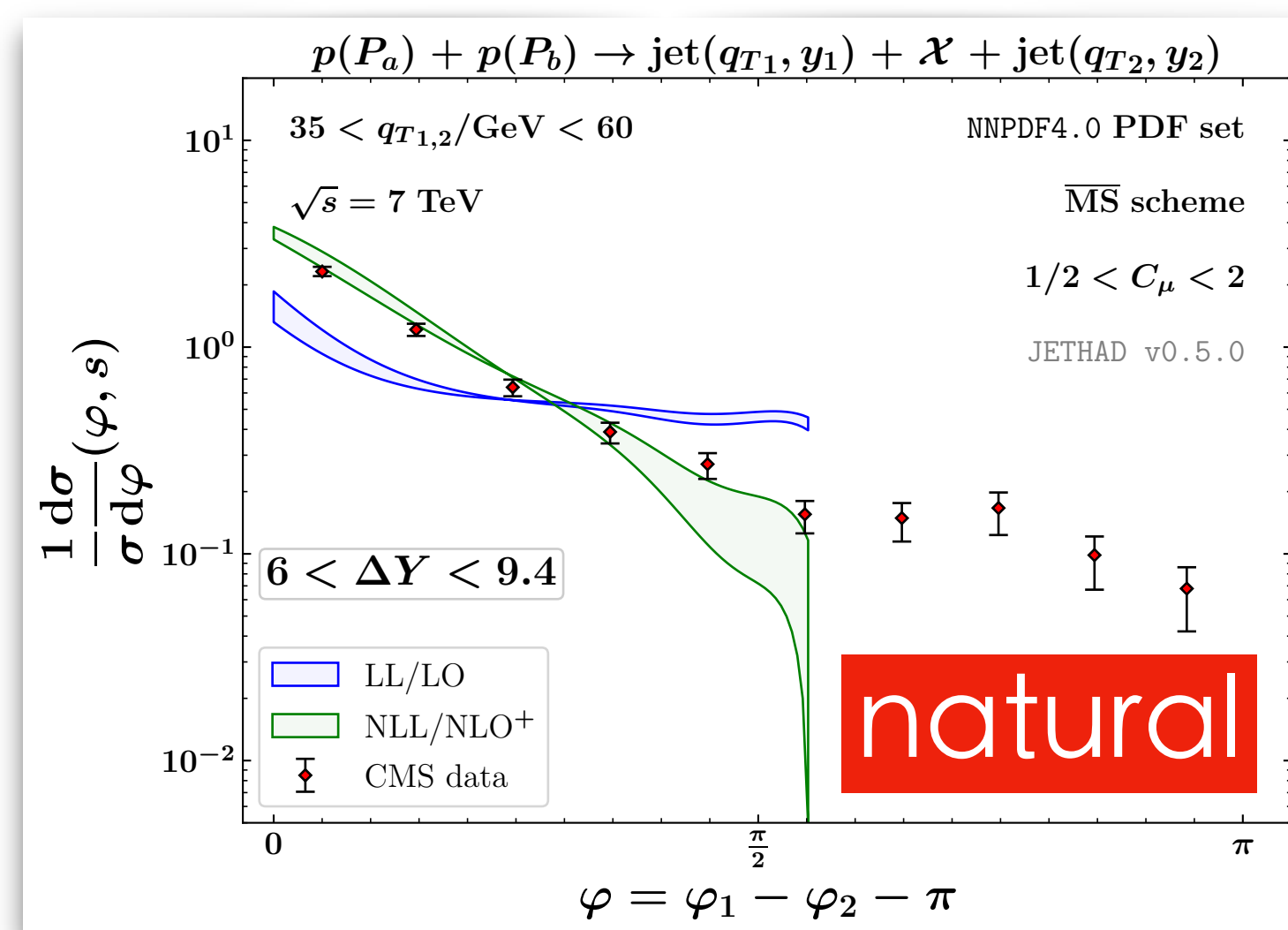





(left) [\[F. G. C., Eur. Phys. J. C 81 \(2021\) 8, 691\]](#)

(right) [\[F. G. C., A. Papa, Phys. Rev. D 106 \(2022\) 11, 114004\]](#)

# Mueller-Navelet jets @LHC & resummation instabilities


- BLM scales, theory vs experiment: CMS @7TeV with symmetric  $p_T$ -ranges 
- Strong manifestation of higher-order instabilities via scale variation (!)
- ! At natural scales: NLL/LL ratio large, no agreement with data, unphysical values !

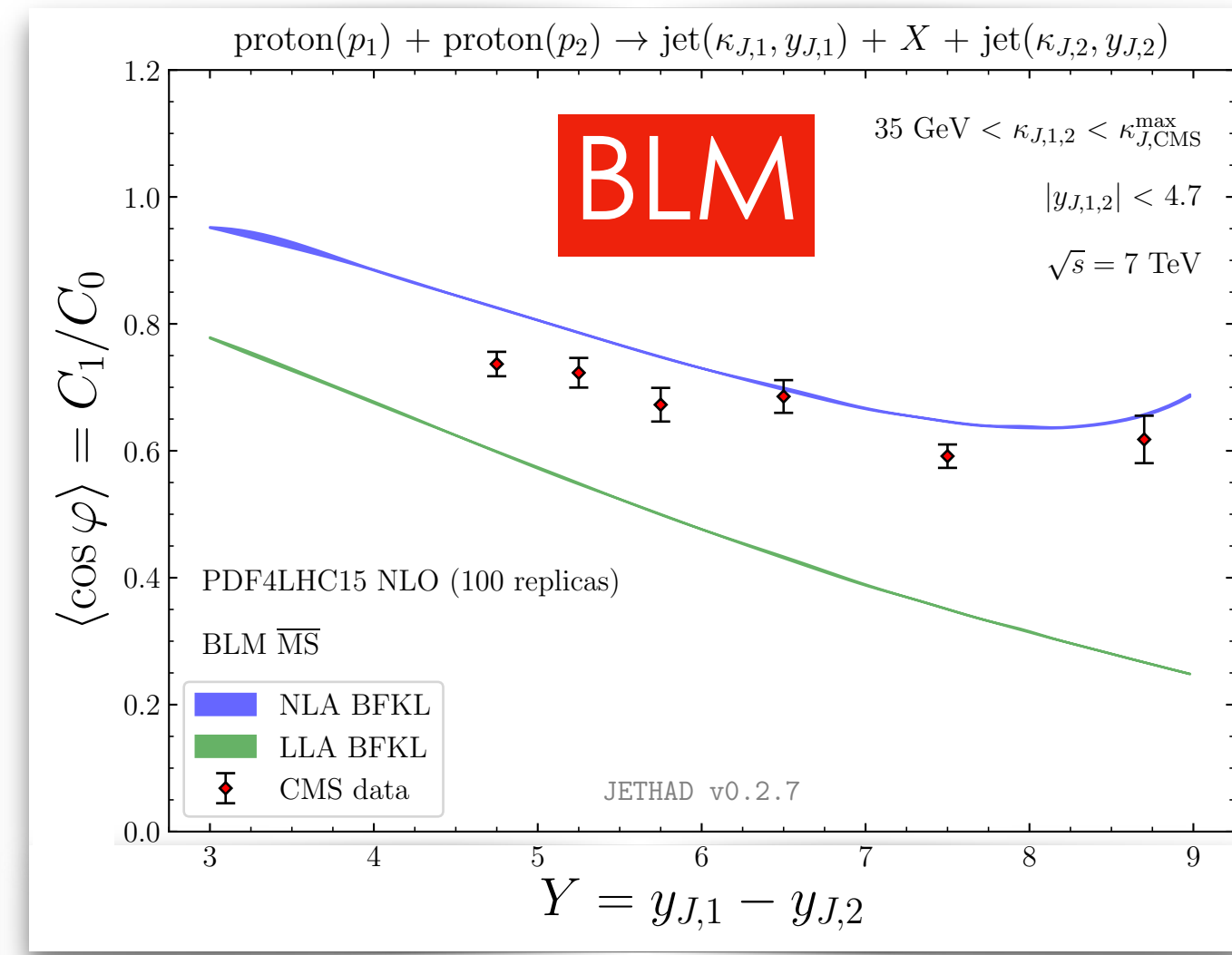
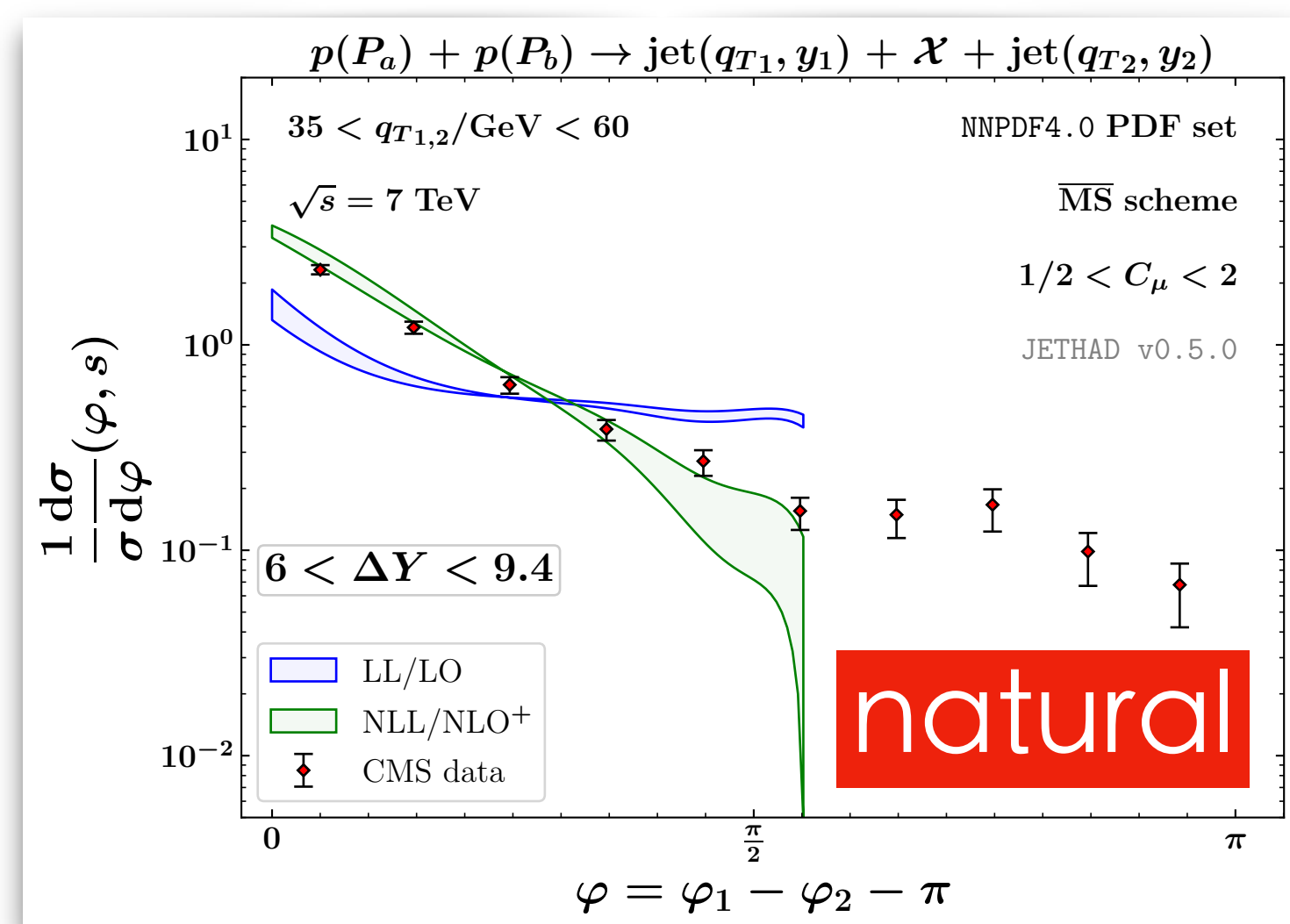


 [CMS Collaboration, JHEP 08 (2016) 139]  
 [B. Ducloué et al., Phys. Rev. Lett. 112 (2014) 082003]  
 [F. Caporale et al., Eur. Phys. J. C 74 (2014) 10, 3084]

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# Mueller-Navelet jets @LHC & resummation instabilities

- BLM scales, theory vs experiment: CMS @7TeV with symmetric p<sub>T</sub>-ranges 
- Strong manifestation of higher-order instabilities via scale variation (⚠️)
- ⚠️ At natural scales: NLL/LL ratio large, no agreement with data, unphysical values !



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$\mu_R^{\text{BLM}} \gg \mu_R^{\text{nat.}} \Rightarrow d\sigma^{\text{BLM}}/d\sigma^{\text{nat.}} \sim 10^{-(1\div 2)} \Rightarrow$  precision studies hampered



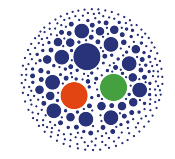
Unsuccessful scale optimization → processes featuring natural stability (⚠️?)

1

# Higgs + jet



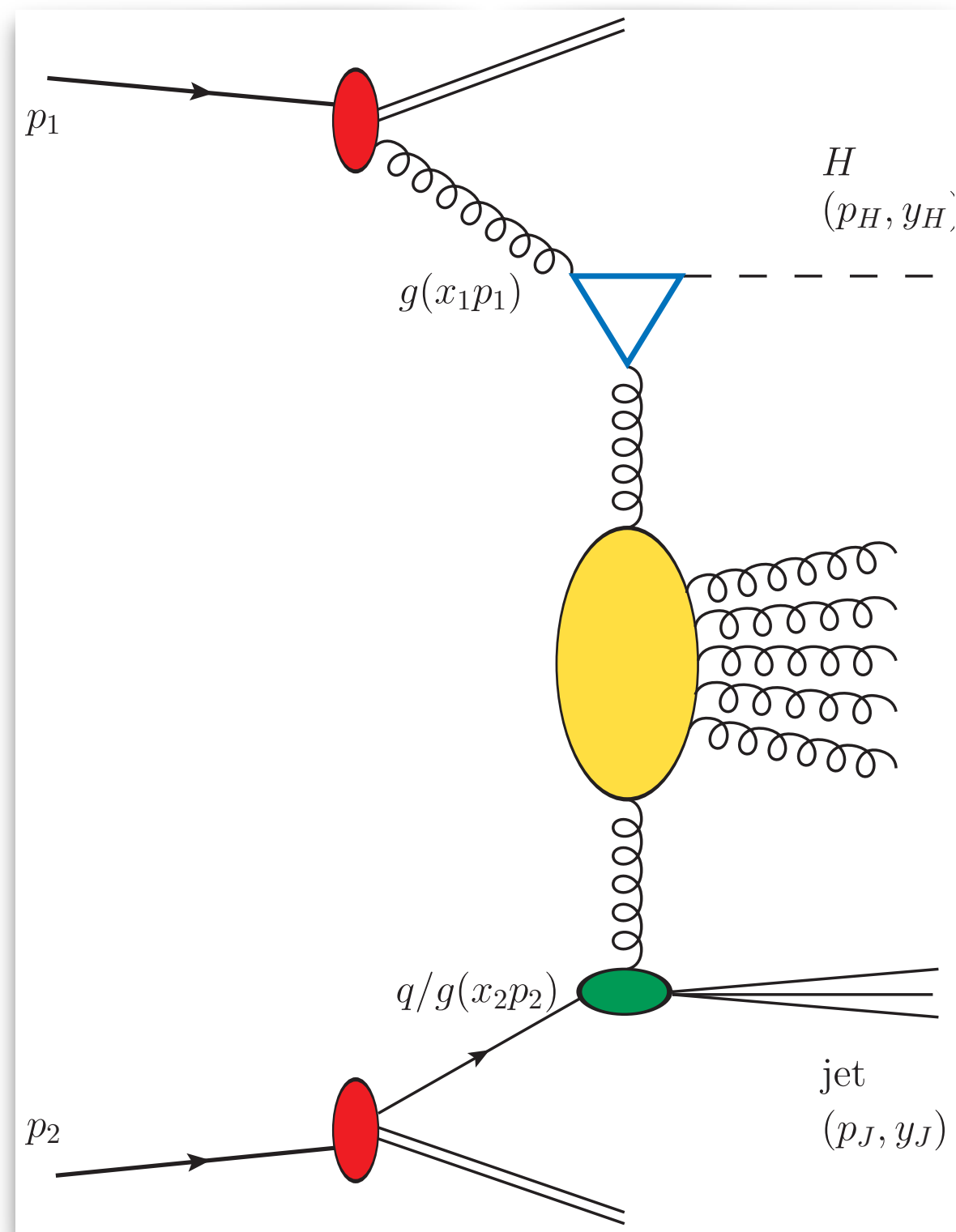
# From Mueller-Navelet to Higgs and heavy flavor



Pheno path: hunt for channels leading to a NLL **stabilization pattern** at **natural scales** (!)

## HIGGS BOSON

Stabilizers  $\Leftrightarrow$  large Higgs transverse masses



(Higgs + jet, NLL/NLO\*) [\[F. G. C. et al., Eur. Phys. J. C \(2021\) 8, 780\]](#)

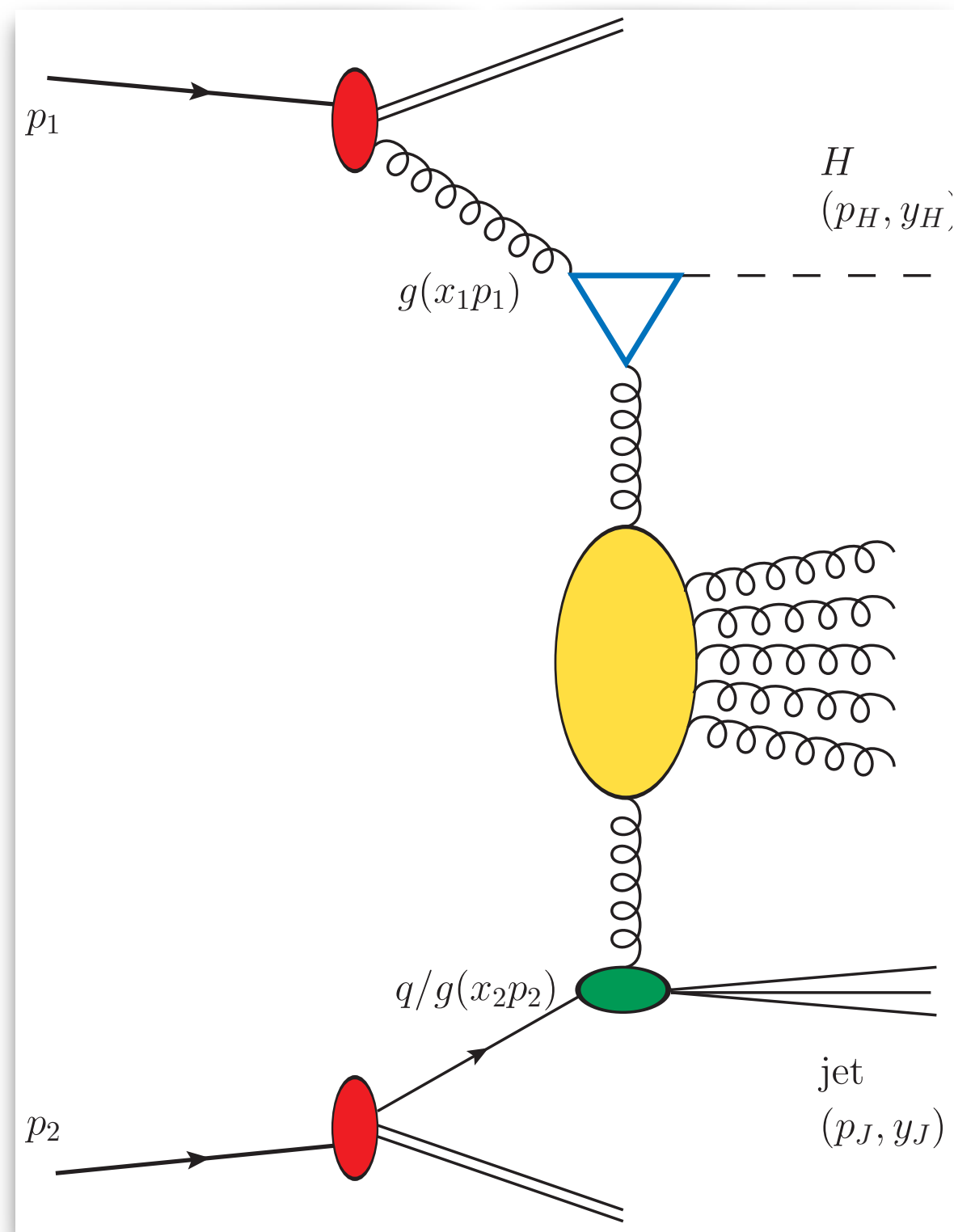
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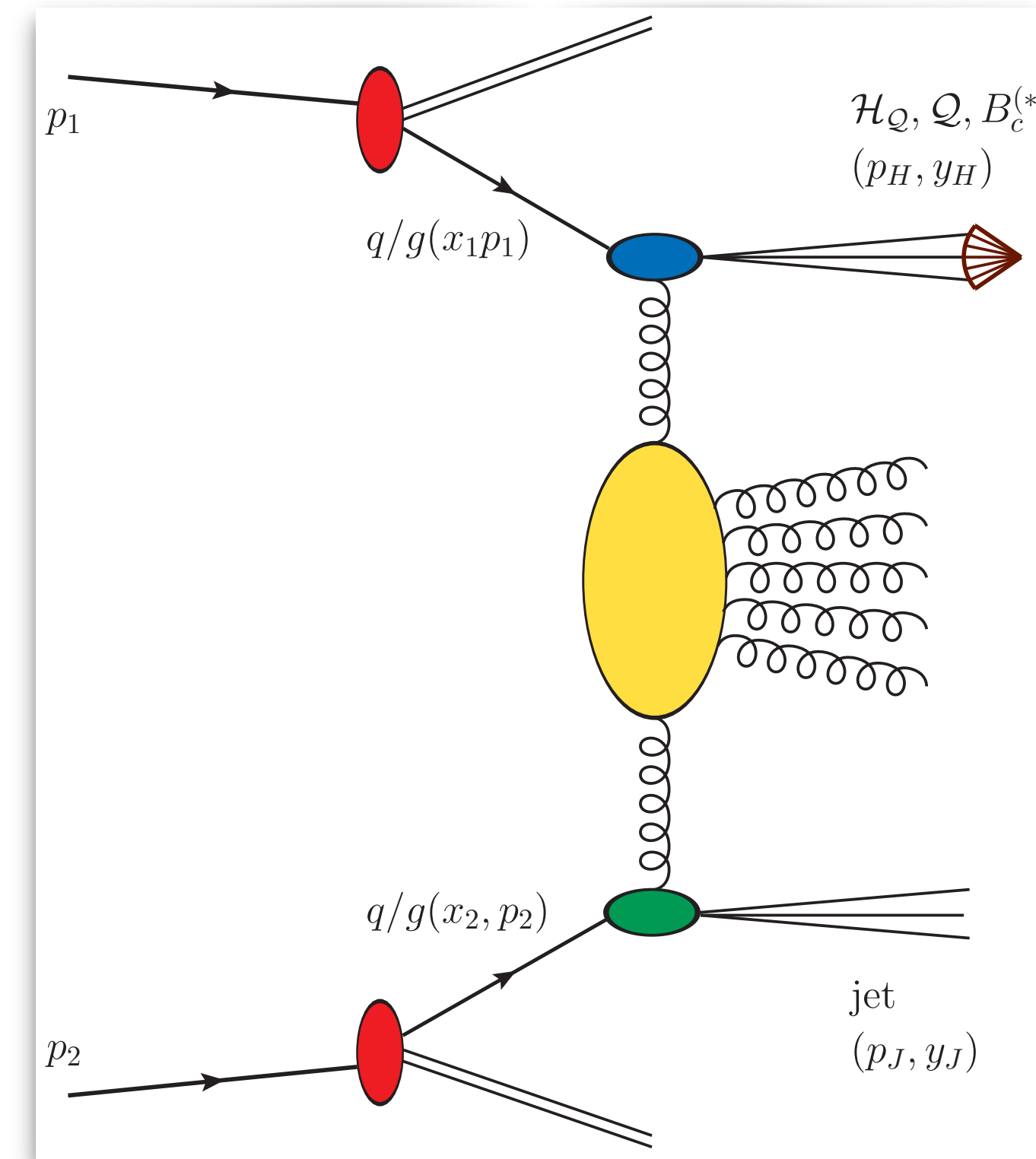


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## HEAVY FLAVOR AT LARGE $P_T$

Stabilizers  $\Leftrightarrow$  gluon fragmentation channels

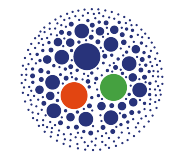


( $\Lambda_c^\pm$  baryons, NLL/NLO)  $\otimes$  [F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]

( $J/\psi$  or  $\Upsilon$ , NLL/NLO)  $\otimes$  [F. G. C. et al., Eur. Phys. J. C 82 (2022) 10, 929]

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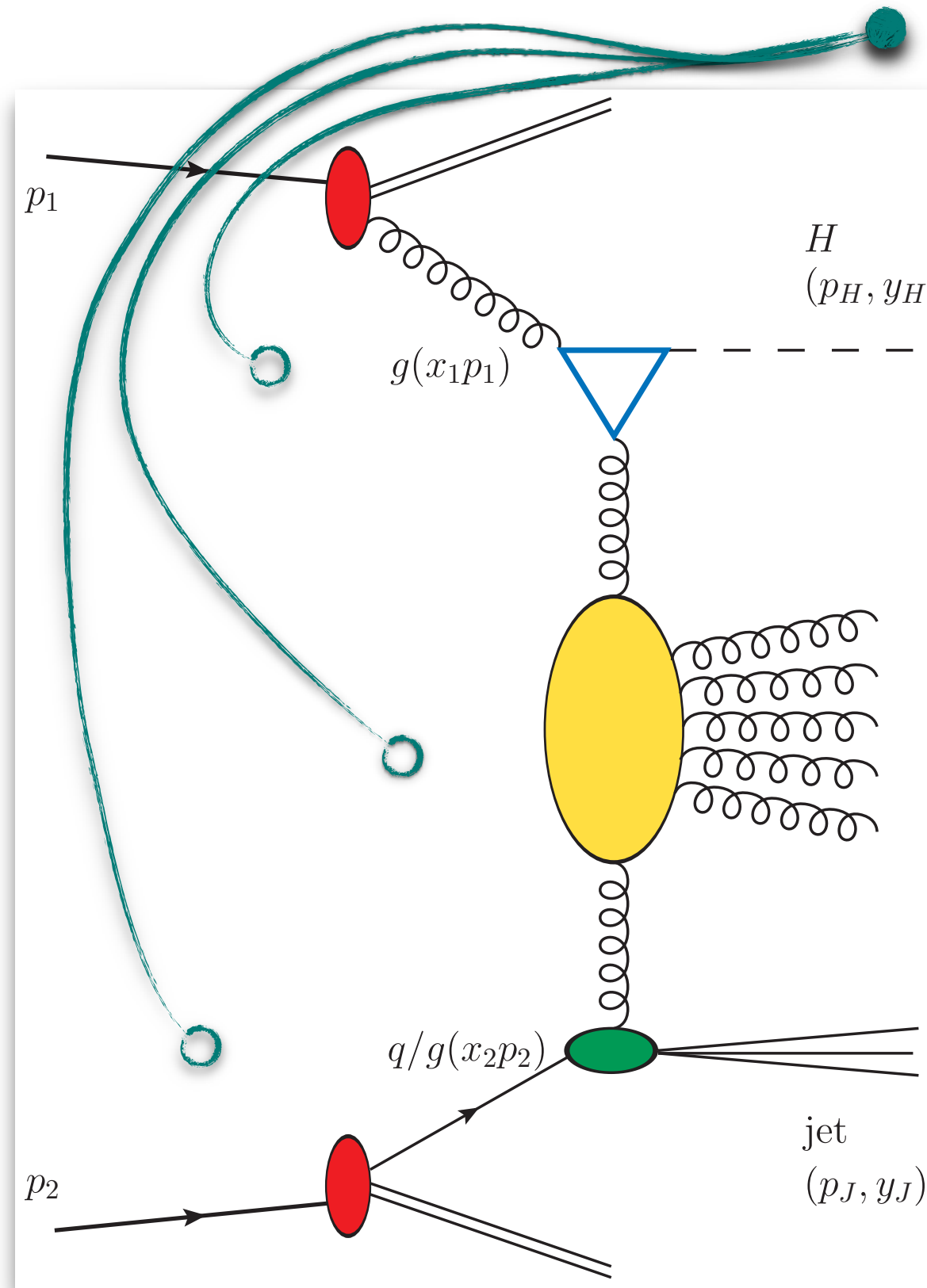
# From Mueller-Navelet to Higgs and heavy flavor



Pheno path: hunt for channels leading to a NLL stabilization pattern at natural scales (!)

## HIGGS BOSON

Stabilizers  $\Leftrightarrow$  large Higgs transverse masses



$$\mu_{F,R} \sim M_{H,\perp}$$

NLO\*

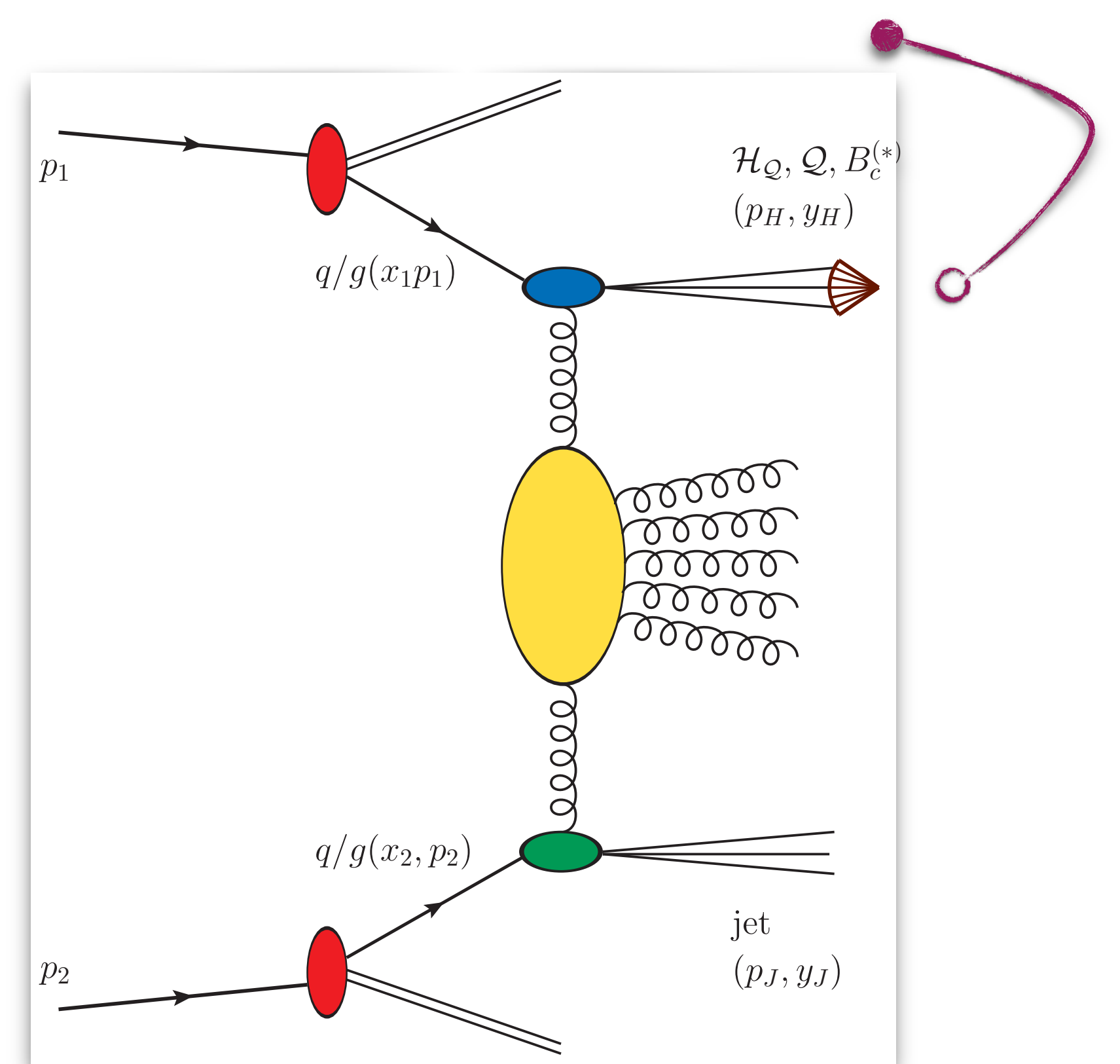
$$= \text{LO} + \text{NLO}_{\text{RG}}$$

NLL

NLO

## HEAVY FLAVOR AT LARGE $P_T$

Stabilizers  $\Leftrightarrow$  gluon fragmentation channels



NLO(+)

NLL

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(Higgs + jet, NLL/NLO\*) [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

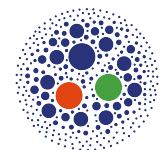
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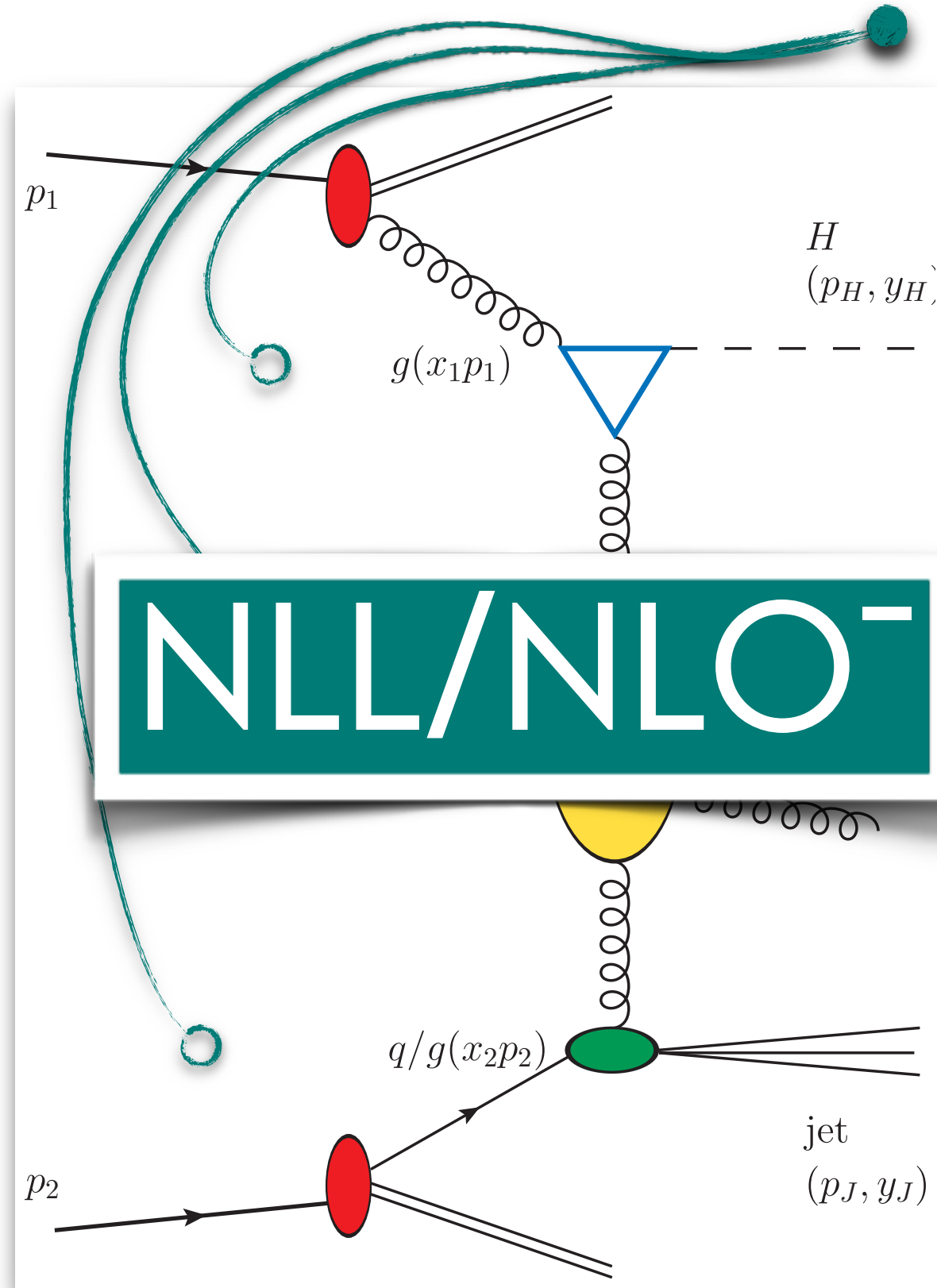
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Pheno path: hunt for channels leading to a NLL **stabilization pattern** at **natural scales** (!)

## HIGGS BOSON

Stabilizers  $\Leftrightarrow$  large Higgs transverse masses



**NLL/NLO<sup>-</sup>**

$$\mu_{F,R} \sim M_{H,\perp}$$

**NLO\***

$$= \text{LO} + \text{NLO}_{\text{RGE}}$$

**NLL**

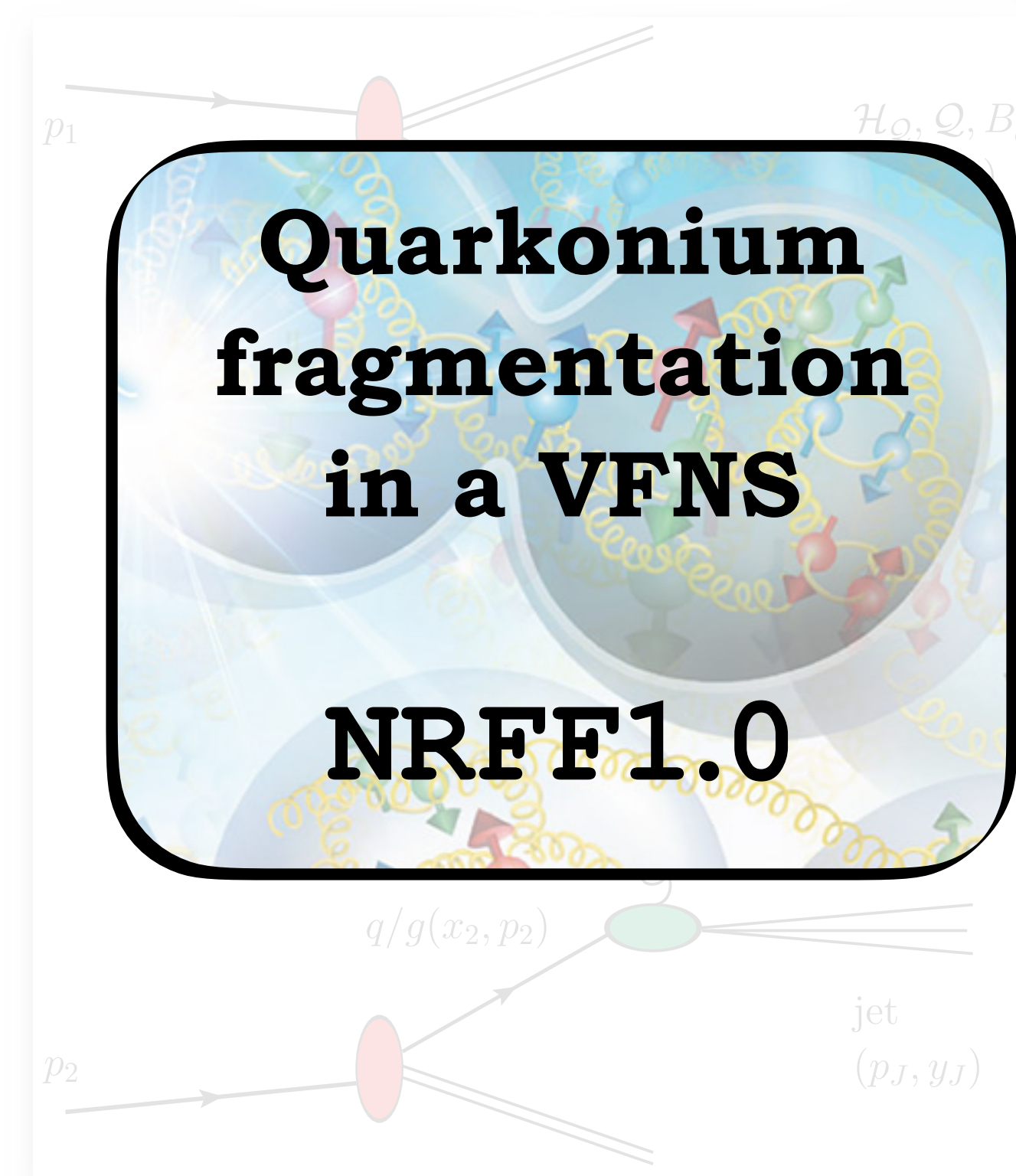
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## HEAVY FLAVOR AT LARGE $P_T$

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**Quarkonium fragmentation in a VFNS**

**NREF1.0**

**NLO(+)**

**NLL**

**NLO(+)**

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# Higgs production from LHC to FCC

PHYSICAL REVIEW LETTERS **120**, 202003 (2018)

## Double Resummation for Higgs Production

Marco Bonvini<sup>1,\*</sup> and Simone Marzani<sup>2,†</sup>

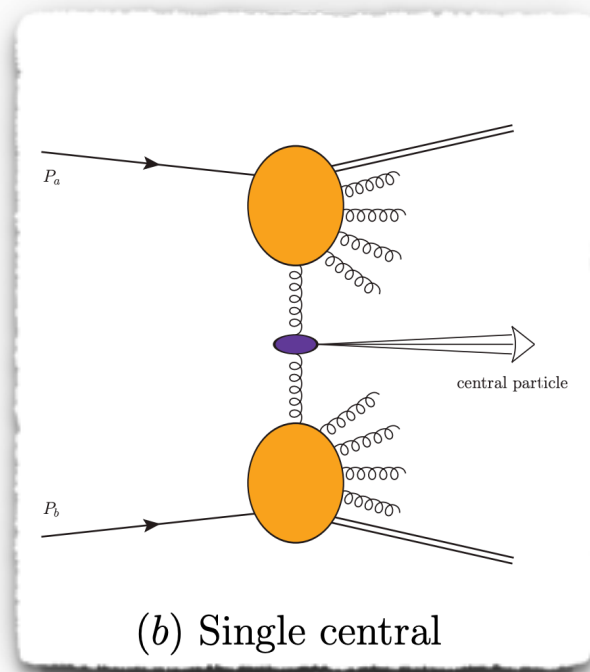
<sup>1</sup>INFN, Sezione di Roma 1, Piazzale Aldo Moro 5, 00185 Roma, Italy

<sup>2</sup>Dipartimento di Fisica, Università di Genova and INFN, Sezione di Genova, Via Dodecaneso 33, I-16146 Genova, Italy

 (Received 26 February 2018; published 16 May 2018)

We present the first double-resummed prediction of the inclusive cross section for the main Higgs production channel in proton-proton collisions, namely, gluon fusion. Our calculation incorporates to all orders in perturbation theory two distinct towers of logarithmic corrections which are enhanced, respectively, at threshold, i.e., large  $x$ , and in the high-energy limit, i.e., small  $x$ . Large- $x$  logarithms are resummed to next-to-next-to-next-to-leading logarithmic accuracy, while small- $x$  ones to leading logarithmic accuracy. The double-resummed cross section is furthermore matched to the state-of-the-art fixed-order prediction at next-to-next-to-next-to-leading accuracy. We find that double resummation corrects the Higgs production rate by 2% at the currently explored center-of-mass energy of 13 TeV and its impact reaches 10% at future circular colliders at 100 TeV.

DOI: [10.1103/PhysRevLett.120.202003](https://doi.org/10.1103/PhysRevLett.120.202003)



High-energy resummation (BFKL)  $\Rightarrow$  PDFs at small- $x$  

Altarelli-Ball-Forte  to stabilize the  $NLL_{sx}$  BFKL kernel 

$N^3LL_{lx}/LL_{sx}/N^3LO$  rapidity-inclusive coefficient functions

$$C_{ij}(x, \alpha_s) = C_{ij}^{fo}(x, \alpha_s) + \Delta C_{ij}^{lx}(x, \alpha_s) + \Delta C_{ij}^{sx}(x, \alpha_s)$$

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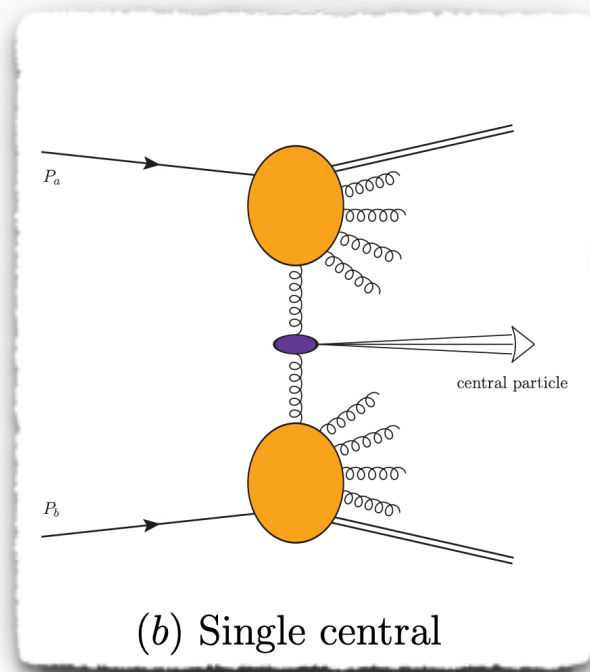
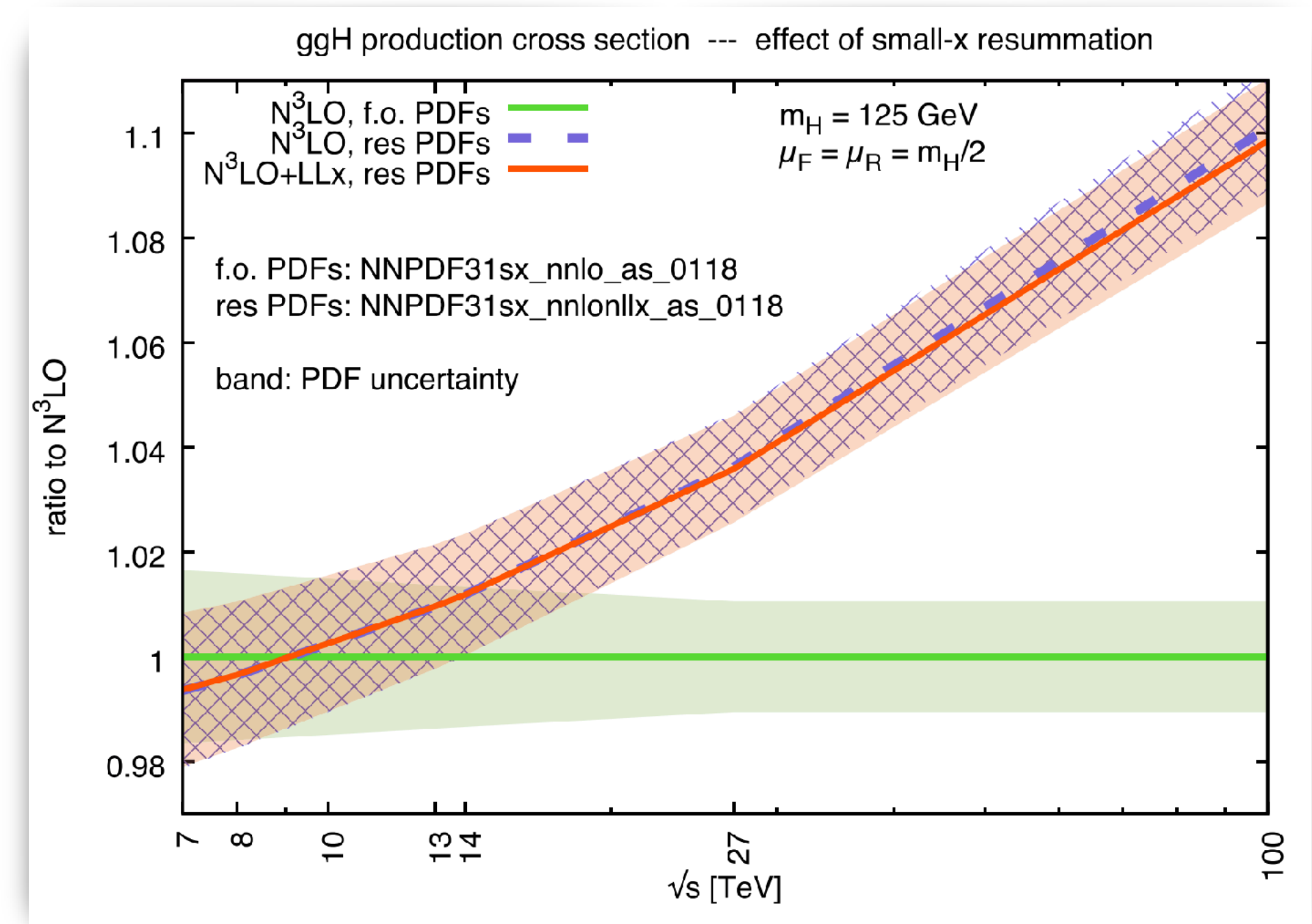
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(i!) 100 TeV EW physics  $\Leftrightarrow$  small- $x$  physics!

(c?) Can LHC physics be BFKL physics?

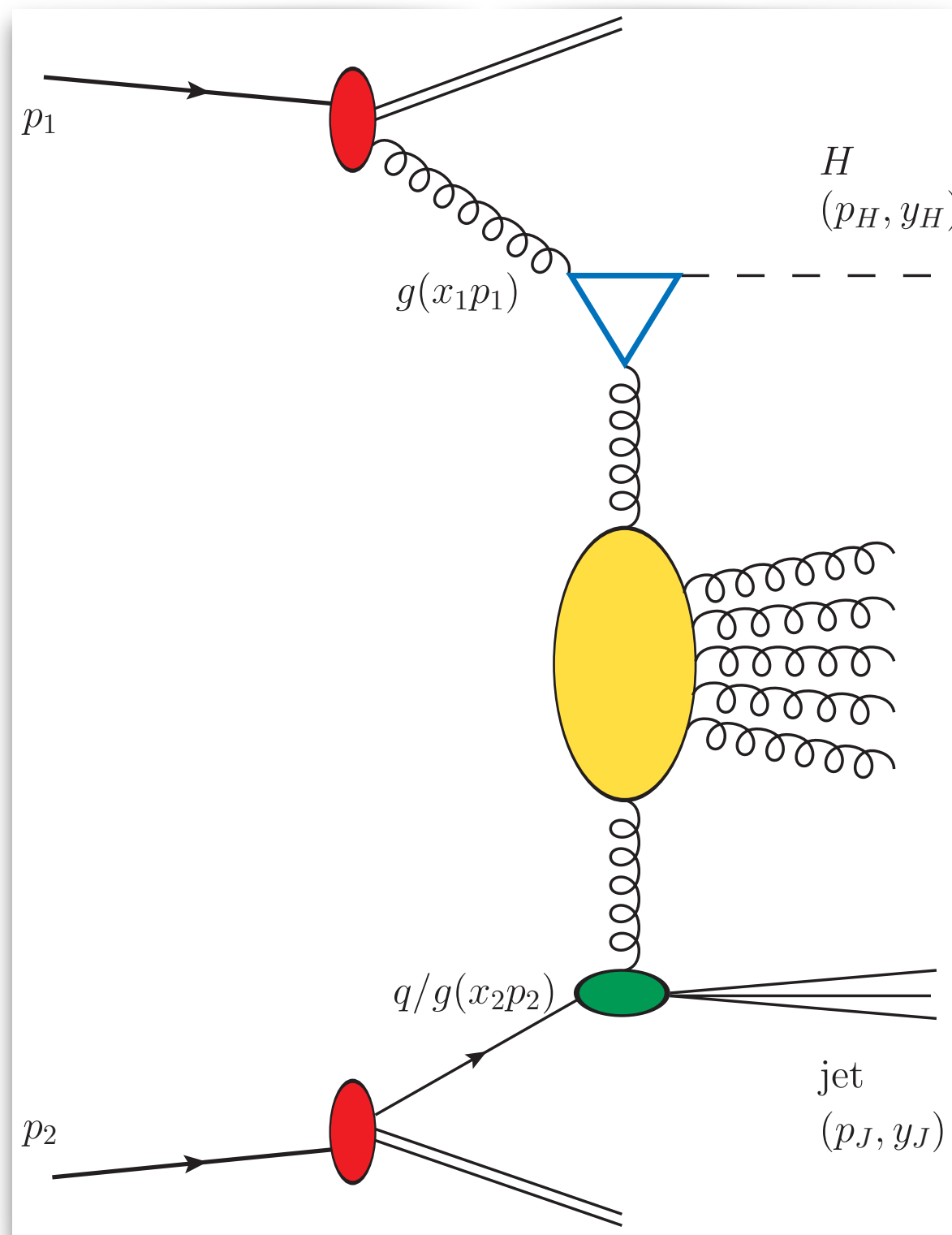
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# Inclusive Higgs + jet at the LHC

- Inclusive h.p. of a Higgs + jet system with high  $p_T$  and large rapidity separation,  $\Delta Y$
- Large energy scales expected to **stabilize** the high-energy resummed series

$$\frac{d\sigma}{dx_1 dx_2 d|\vec{p}_H| d|\vec{p}_J| d\varphi_H d\varphi_J} = \frac{1}{(2\pi)^2} \left[ \mathcal{C}_0 + \sum_{n=1}^{\infty} 2 \cos(n\varphi) \mathcal{C}_n \right]$$

$$\varphi = \varphi_H - \varphi_J - \pi$$



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**Higgs vertex**  
(off-shell coefficient function)

**jet vertex**  
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$$\text{NLO}^* = \text{LO} + \text{NLO}_{\text{RG}}$$

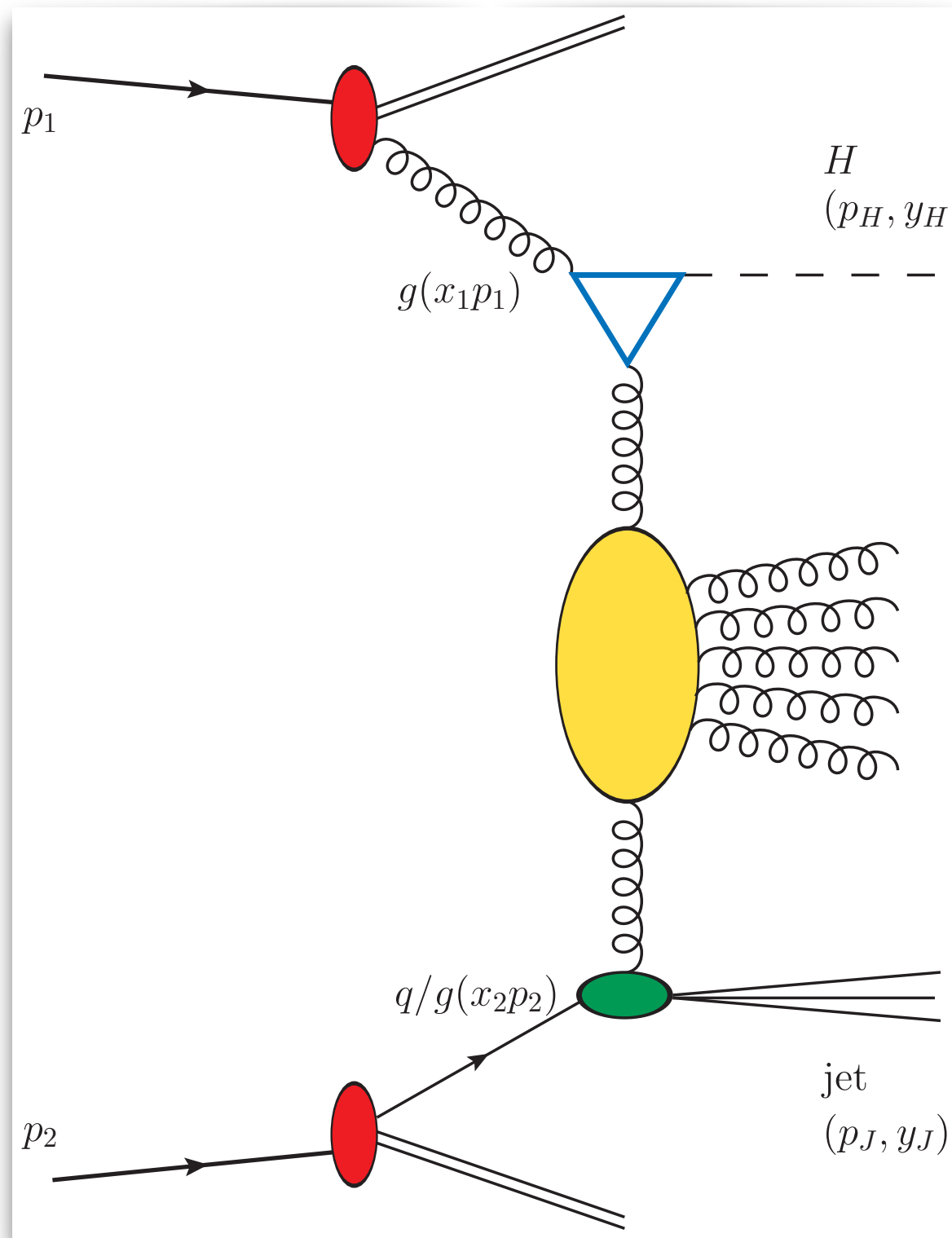
**NLO\***

**NLL**

**NLO(\*)**

$$\begin{aligned} \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_H dy_J d^2\vec{p}_H d^2\vec{p}_J} &= \frac{1}{(2\pi)^2} \\ &\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^2} \mathcal{V}_H^{(r)}(\vec{q}_1, s_0, x_1, \vec{p}_H) \\ &\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left( \frac{x_1 x_2 s}{s_0} \right)^\omega \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2) \\ &\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^2} \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{p}_J) \end{aligned}$$

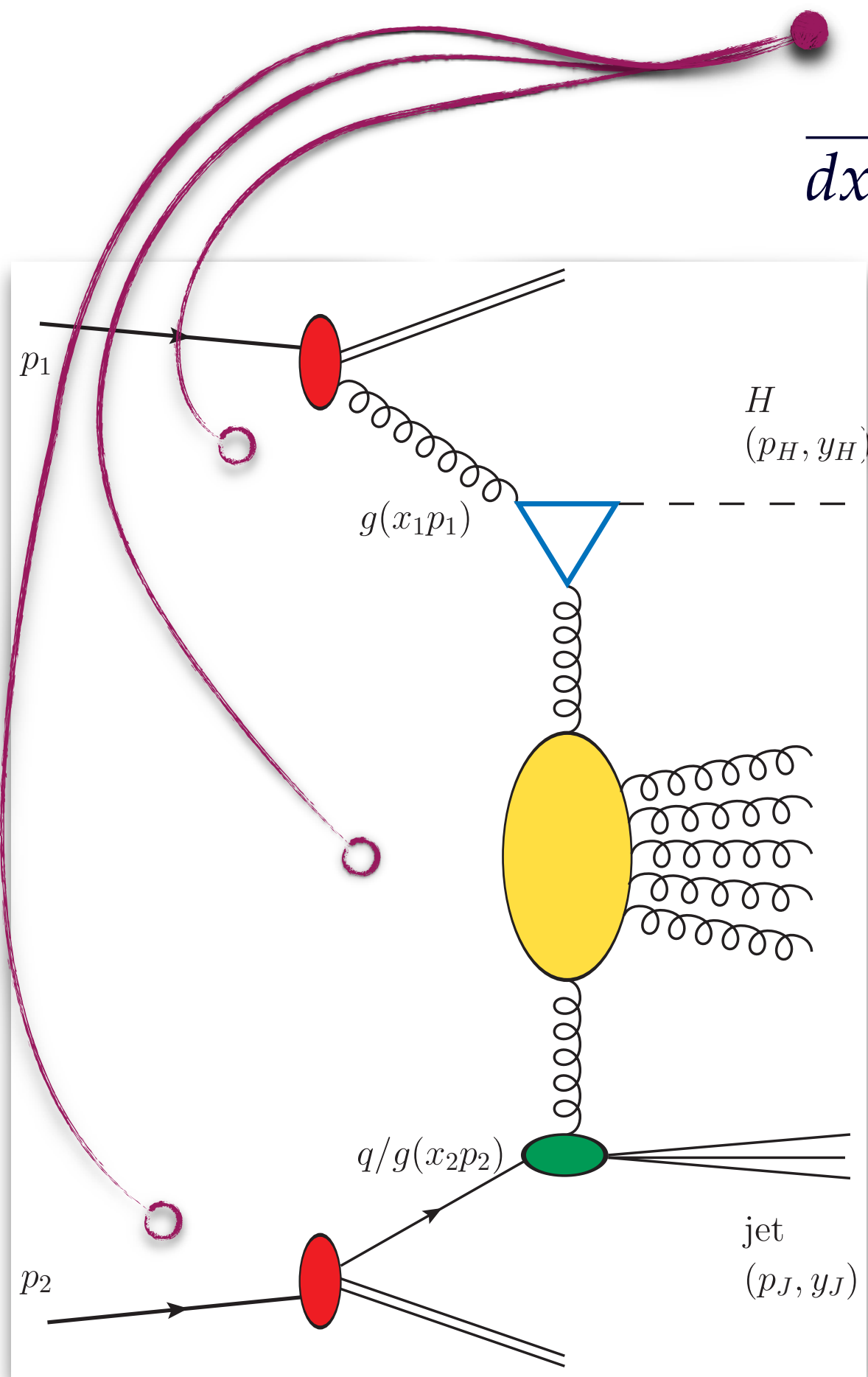
**BFKL Green's function**





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$$\mu_R \sim \sqrt{M_{H,\perp} P_J}$$

**NLO\***

**NLL**

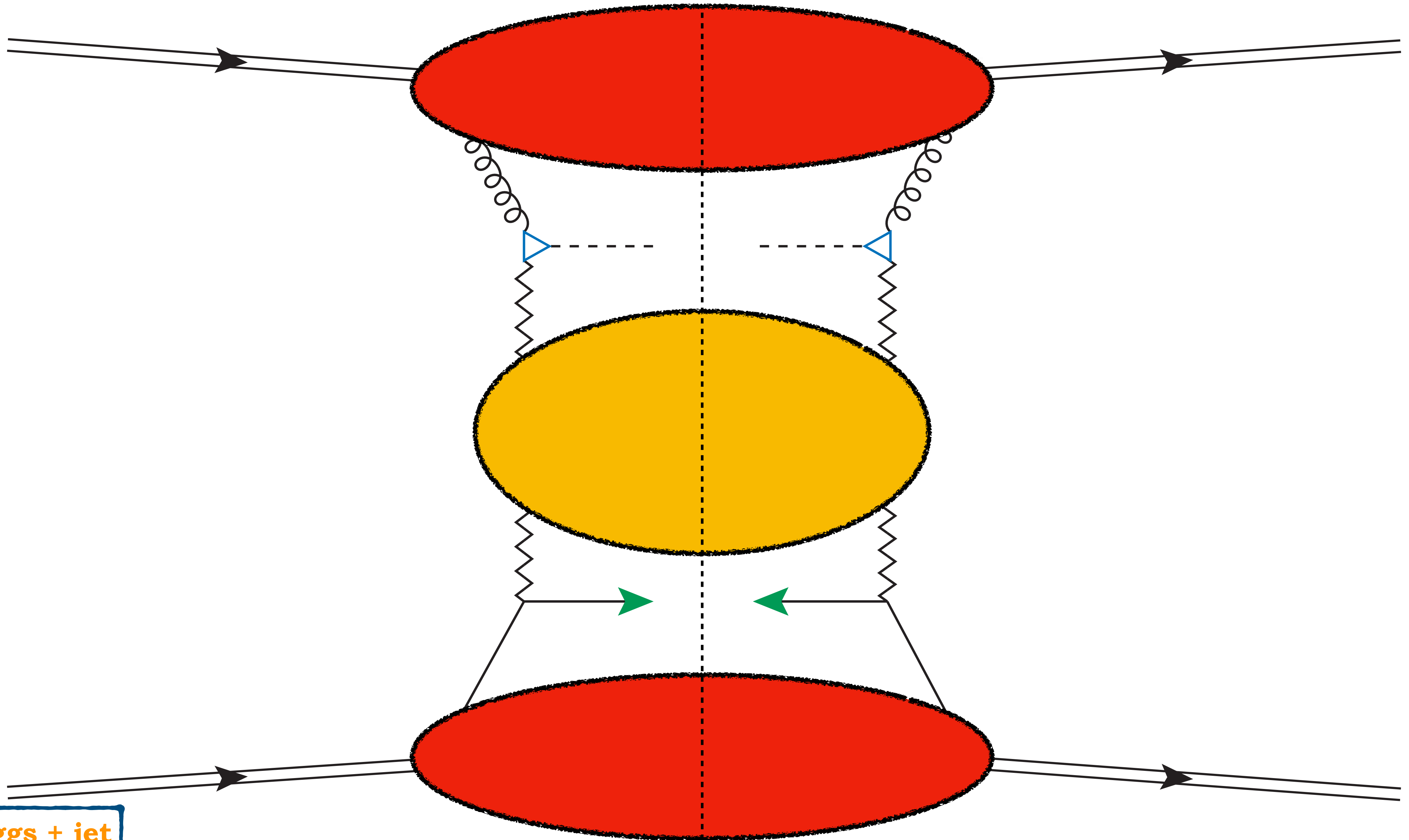
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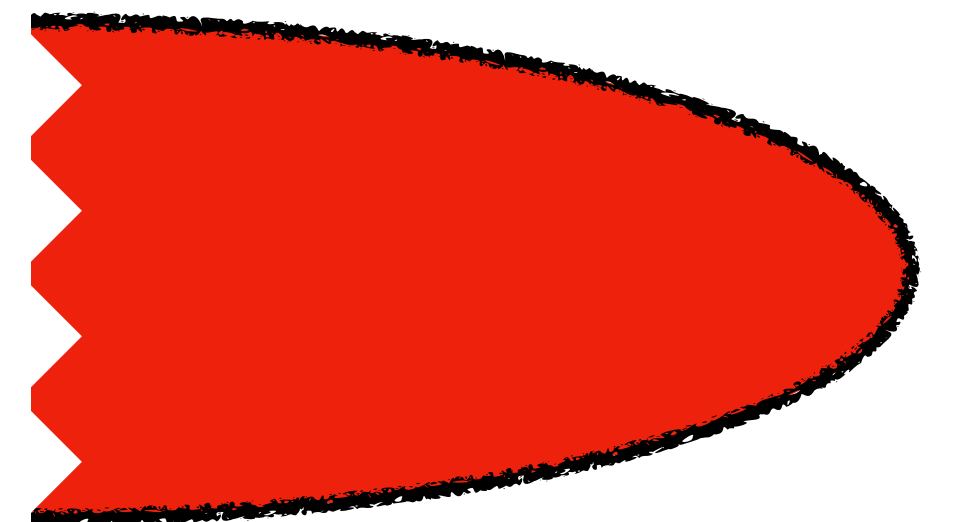
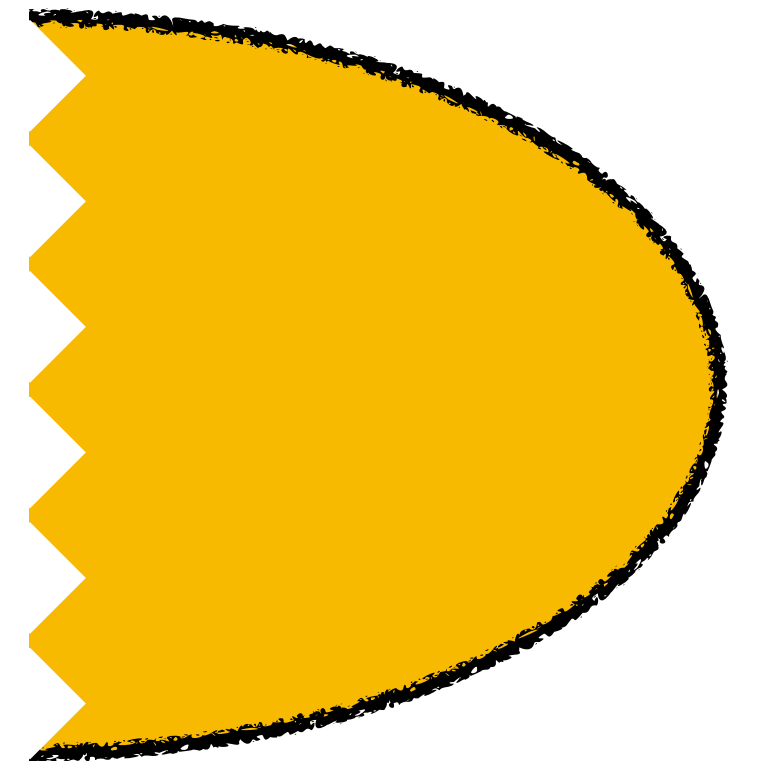
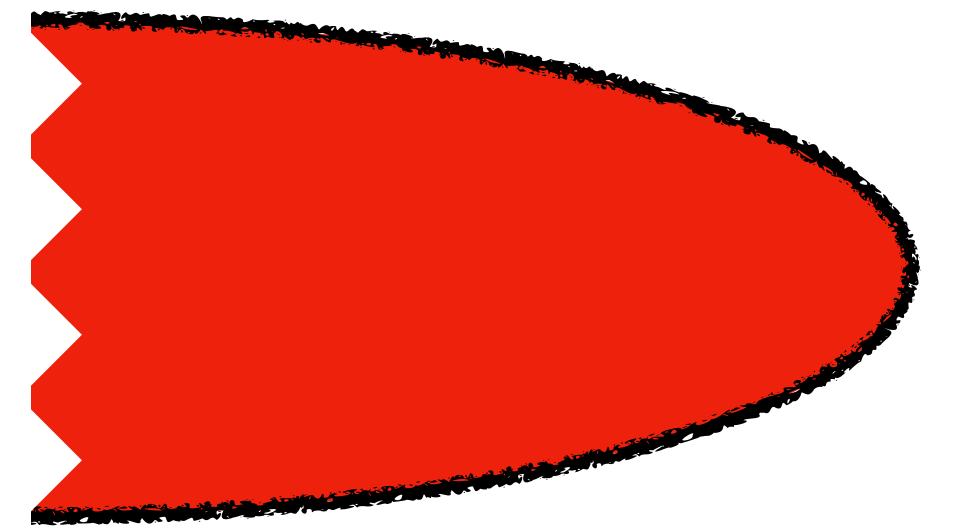
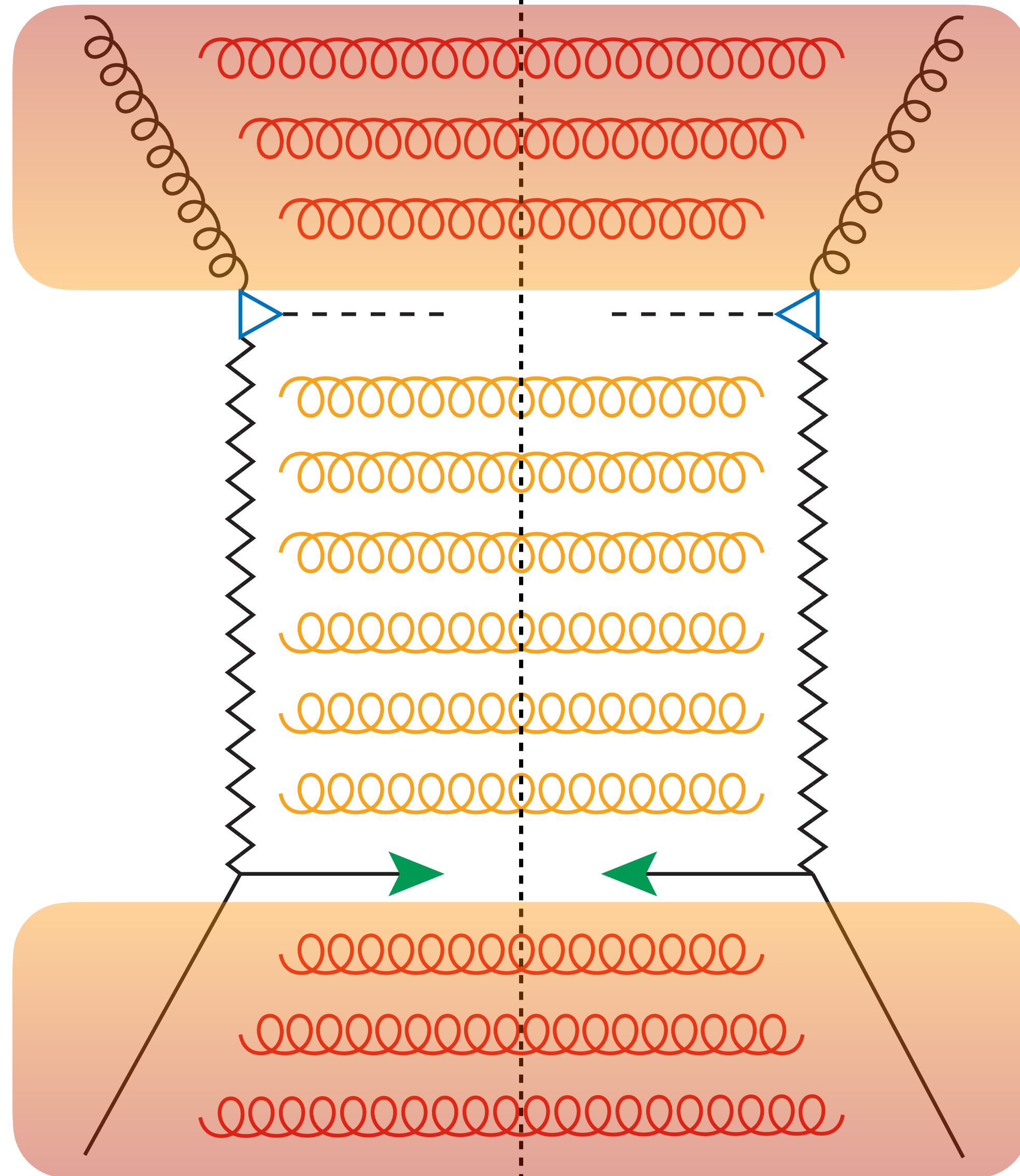
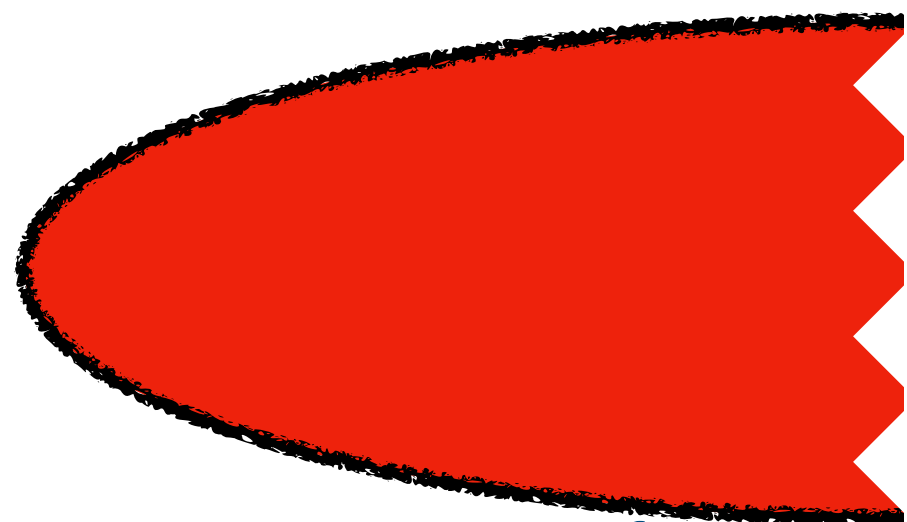
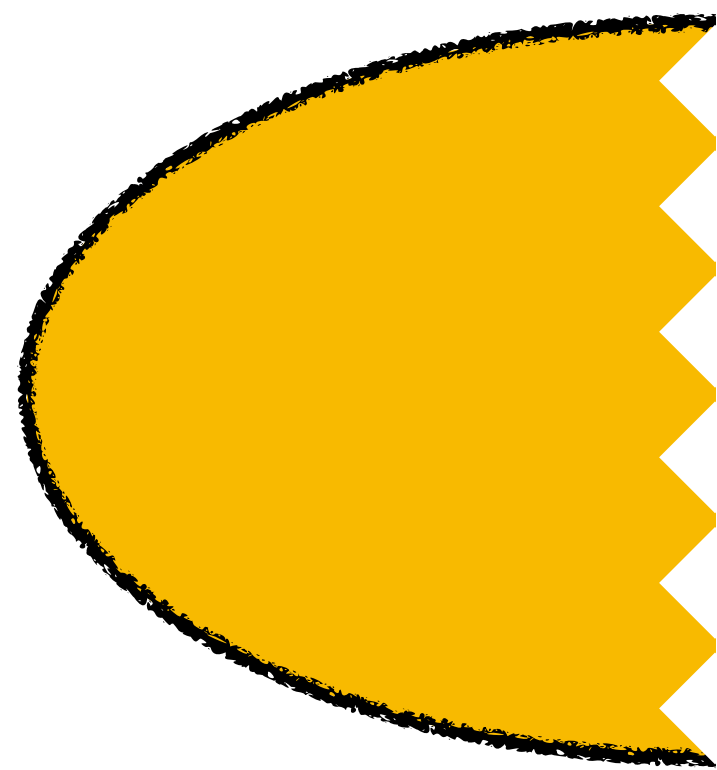
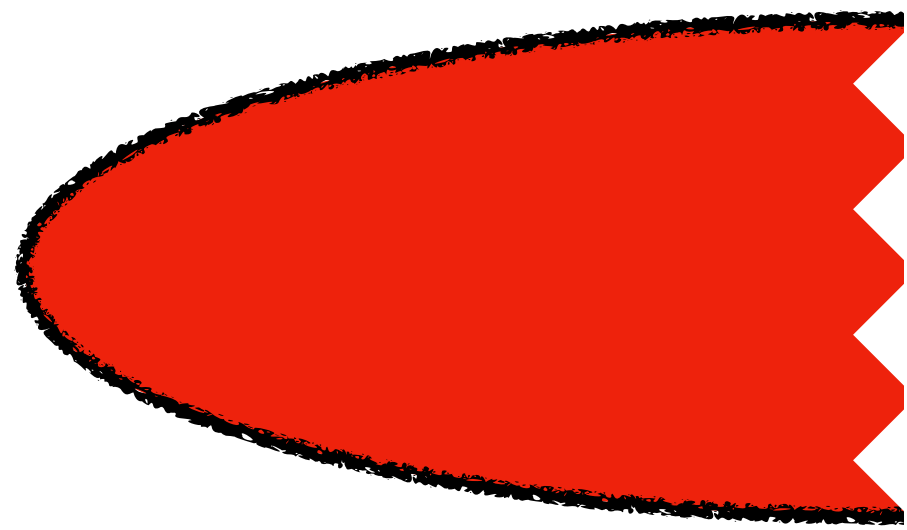
**BFKL Green's function**

# Anatomy of Higgs + jet in hybrid factorization (HyF)



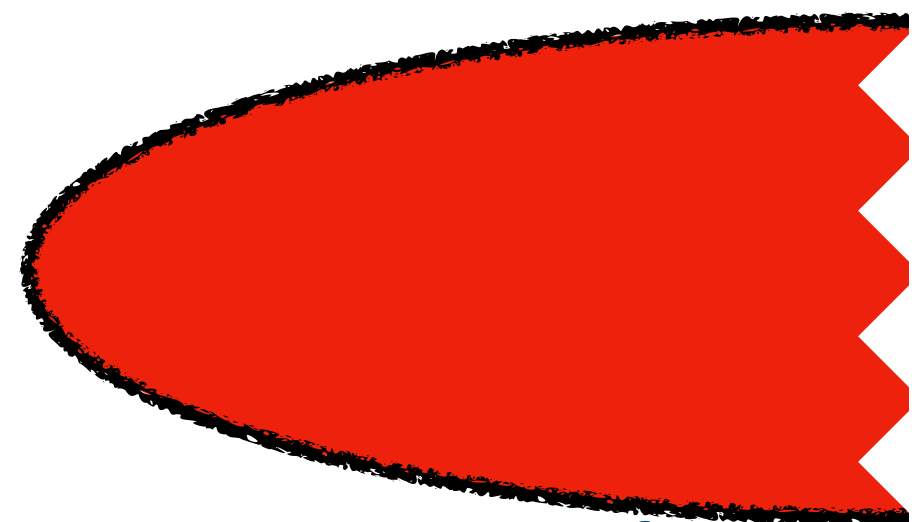
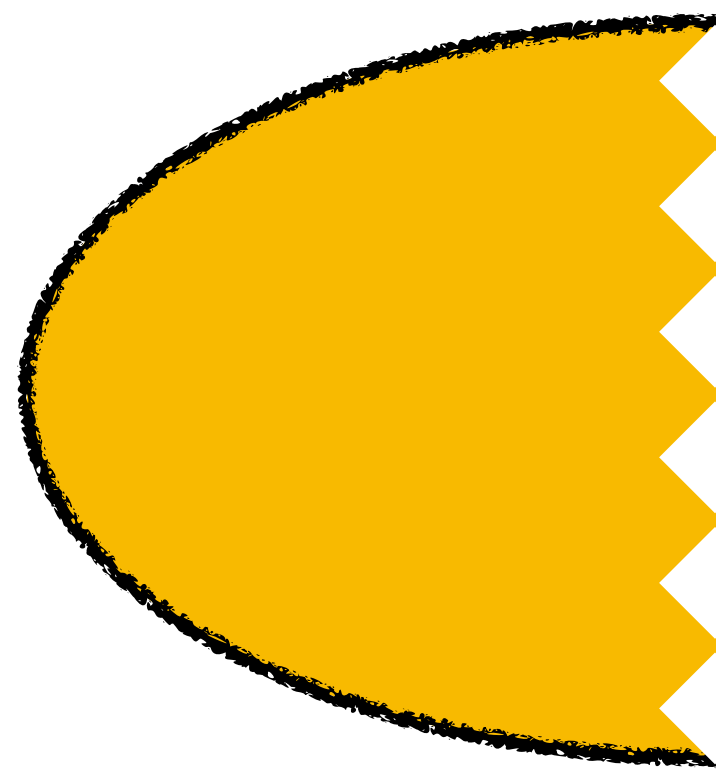
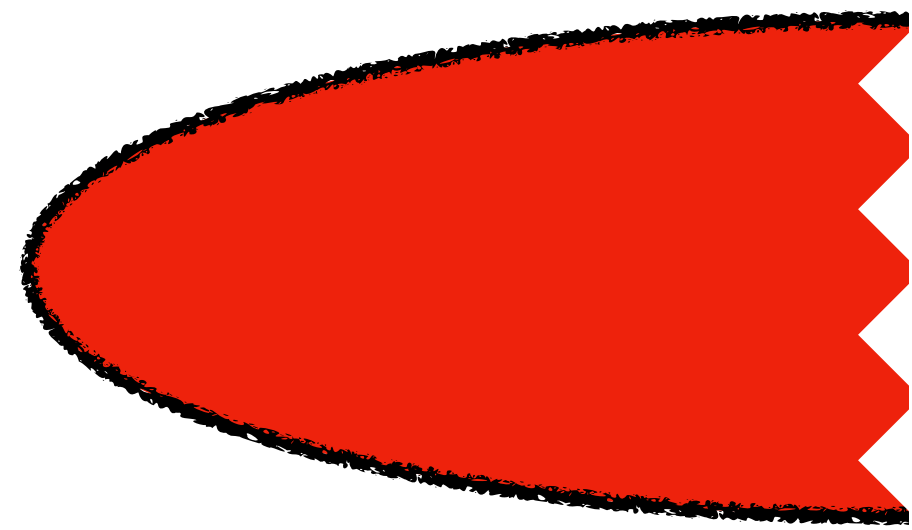
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FORWARD REGION



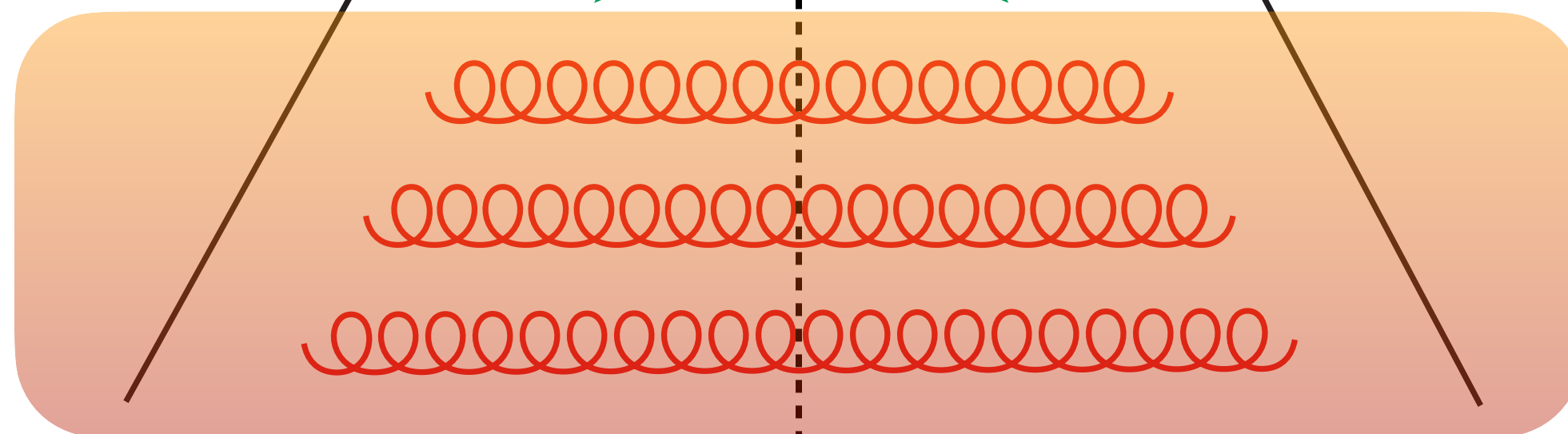
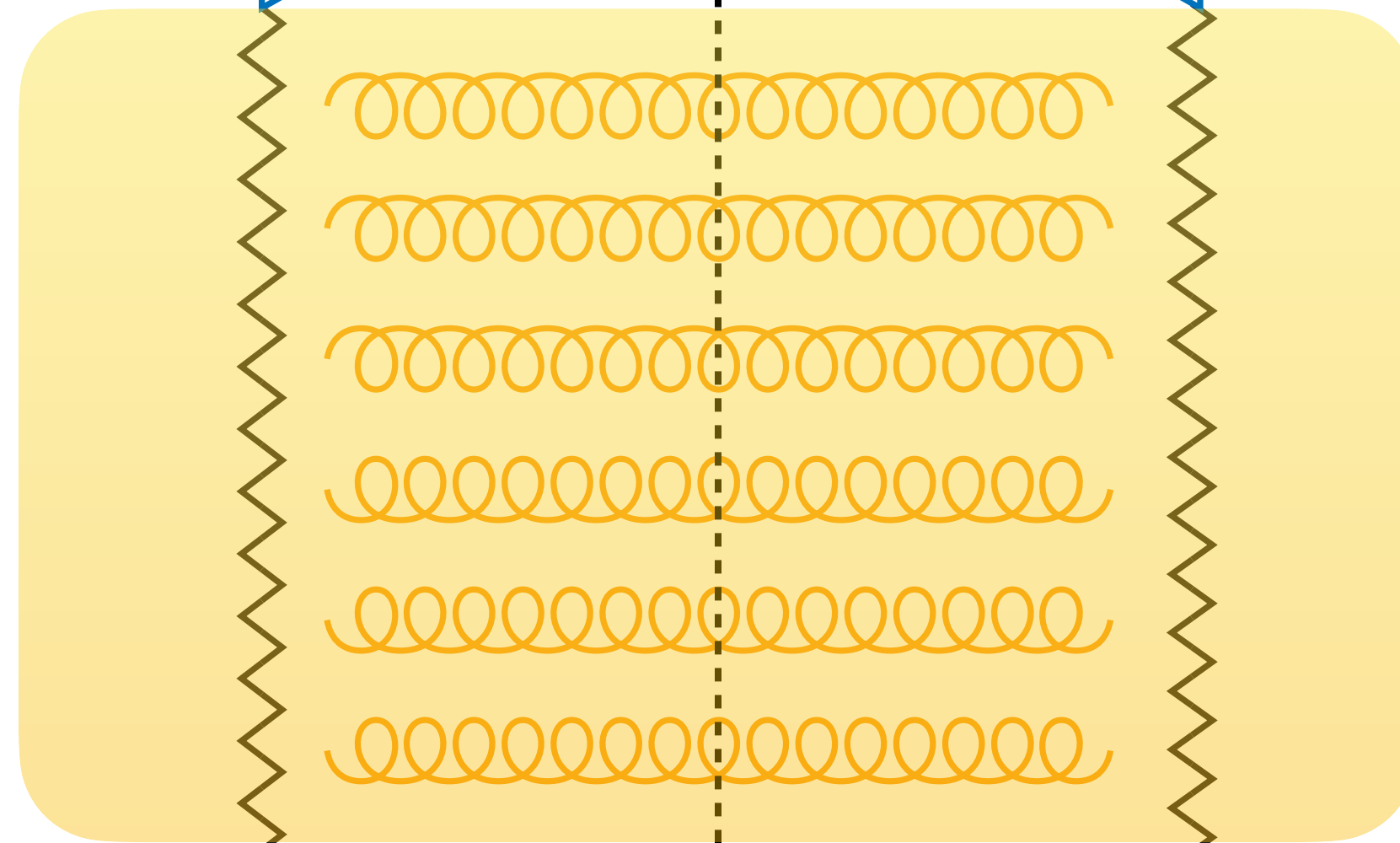
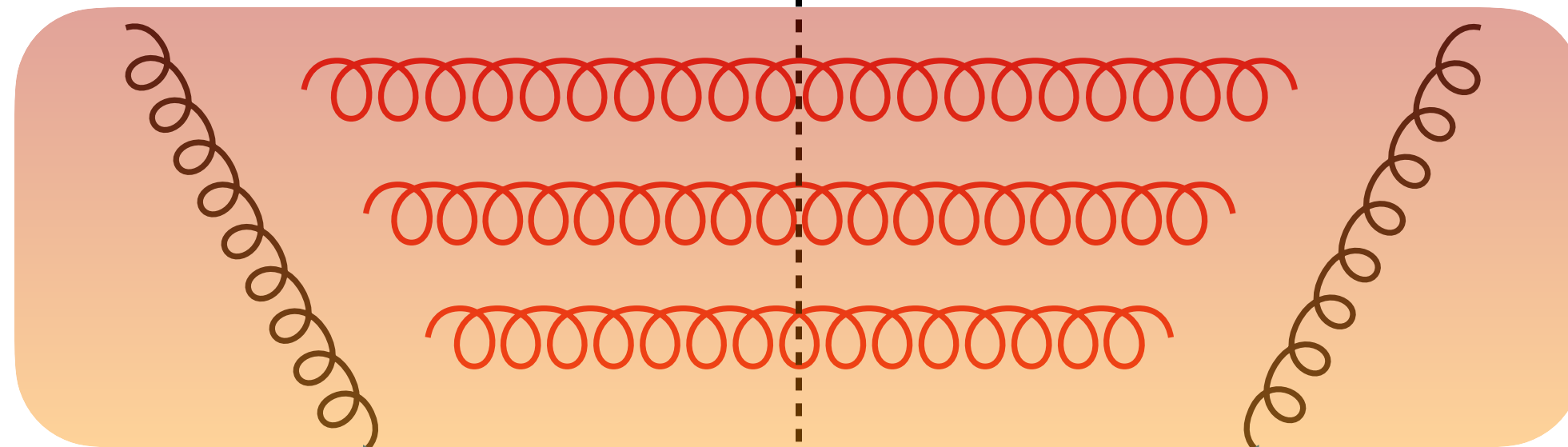
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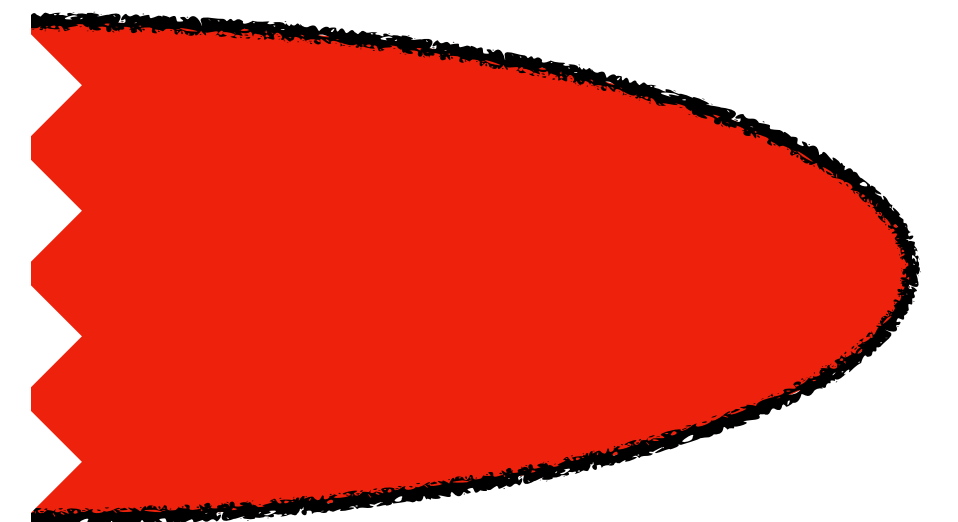
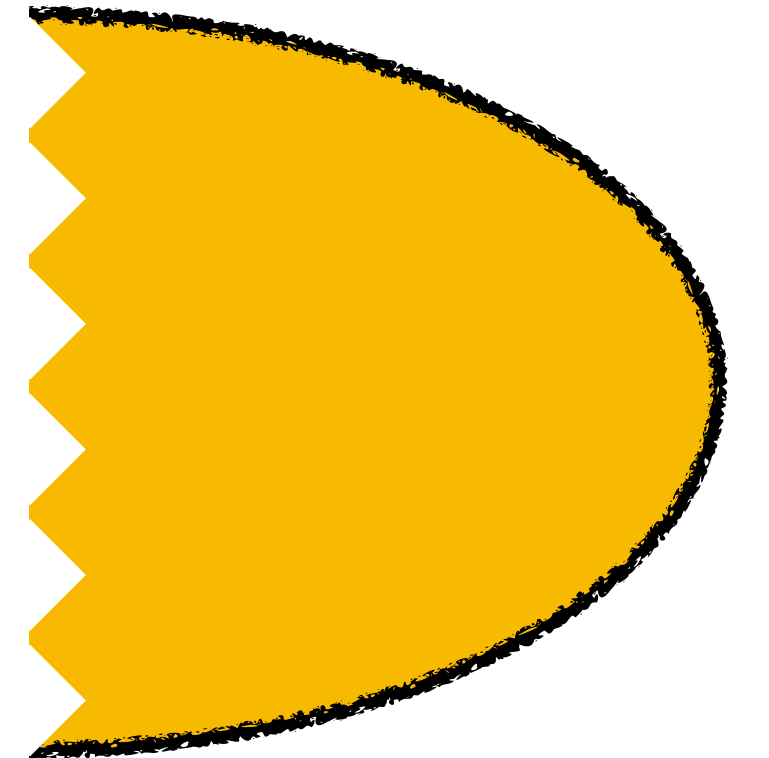
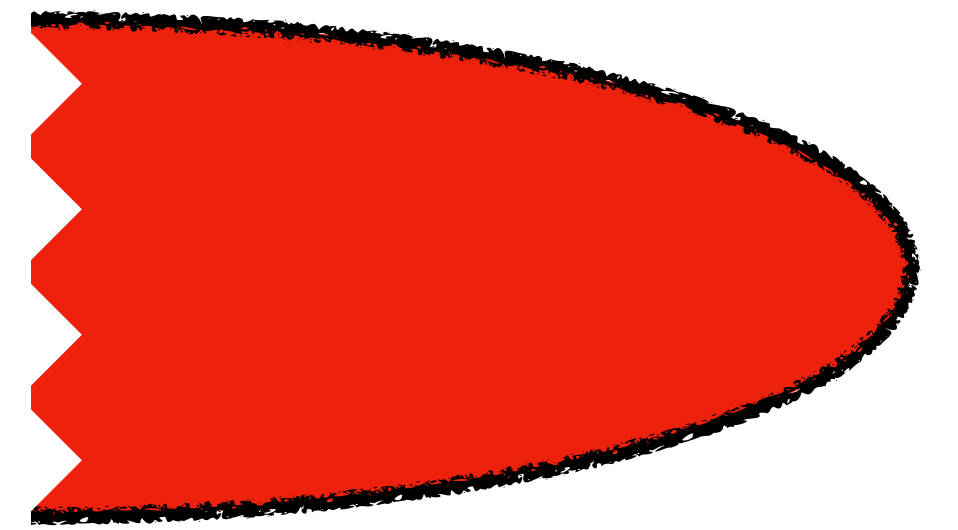
FORWARD REGION

KT  
RESUM



RESUM

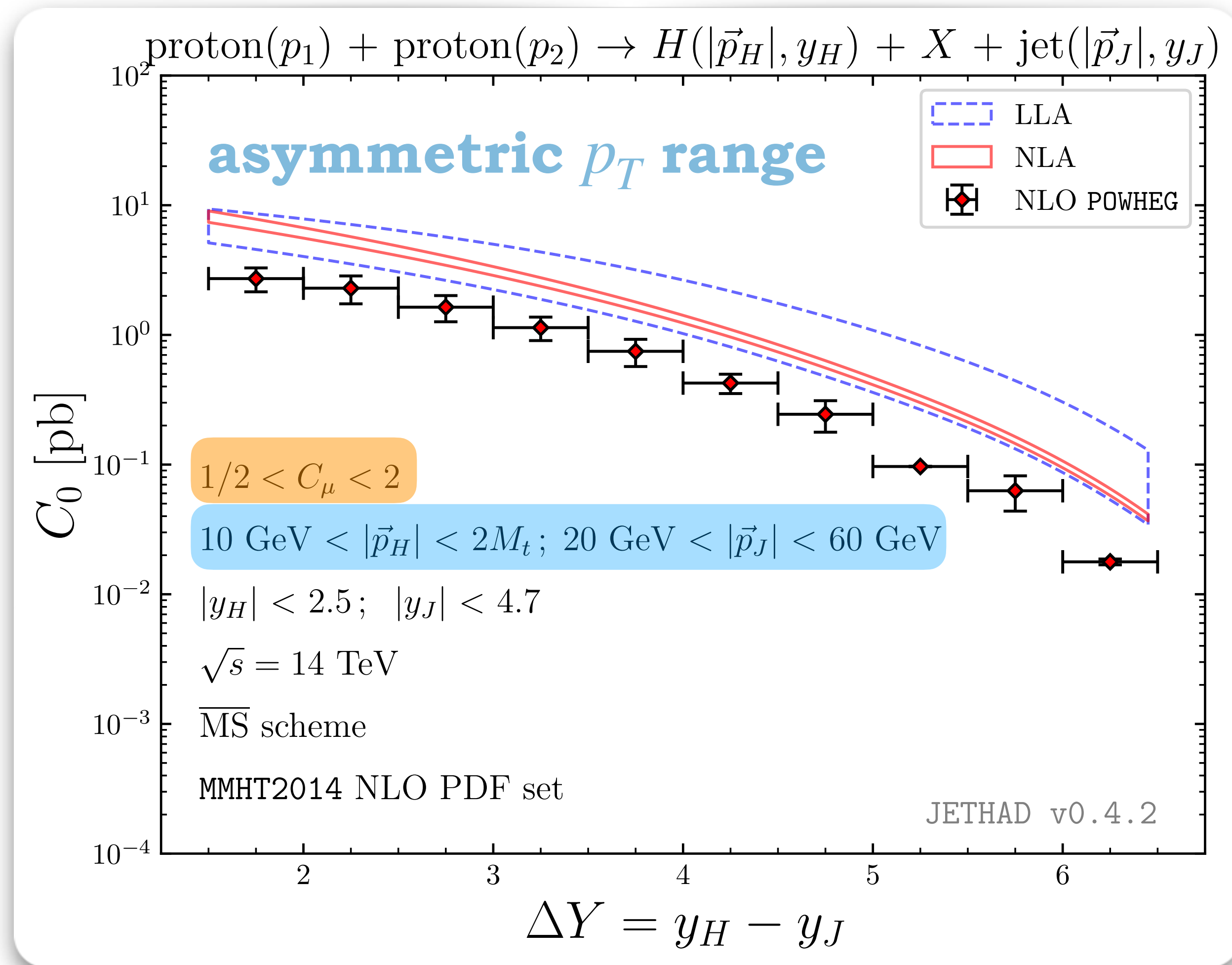
KT



FORWARD REGION

# $\Delta Y$ -distribution

$$C_n(\Delta Y, s) = \int_{p_H^{\min}}^{p_H^{\max}} d|\vec{p}_H| \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \int_{y_H^{\min}}^{y_H^{\max}} dy_H \int_{y_J^{\min}}^{y_J^{\max}} dy_J \delta(y_H - y_J - \Delta Y) C_n$$



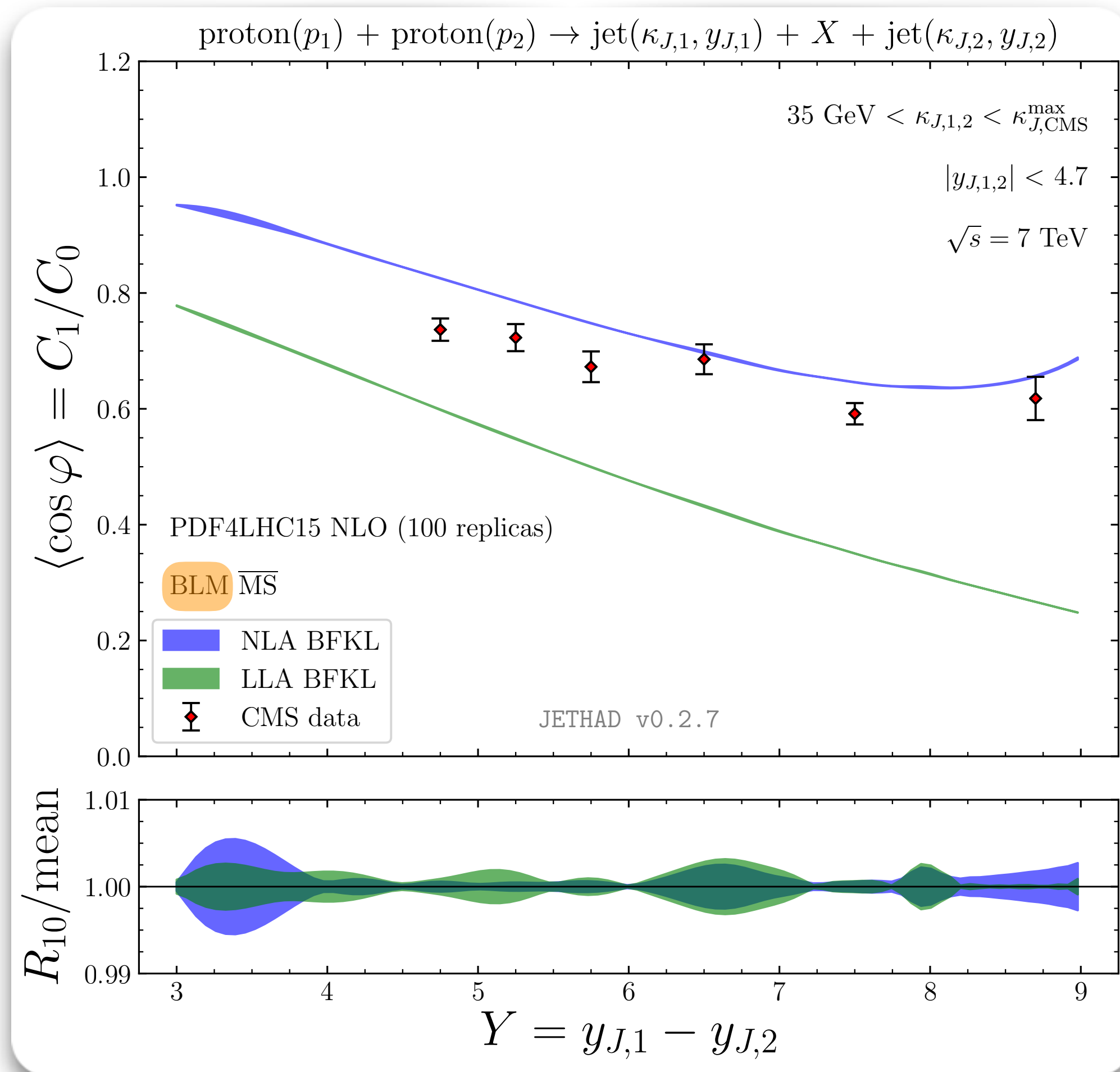
# Angular correlations

$$R_{n0}(\Delta Y, s) = C_n/C_0 \equiv \langle \cos n\varphi \rangle$$

## MUELLER-NAVELET JETS

[B. Ducloué, L. Szymanowski, S. Wallon, Phys.Rev.Lett. 112 (2014) 082003]

(figure below) [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

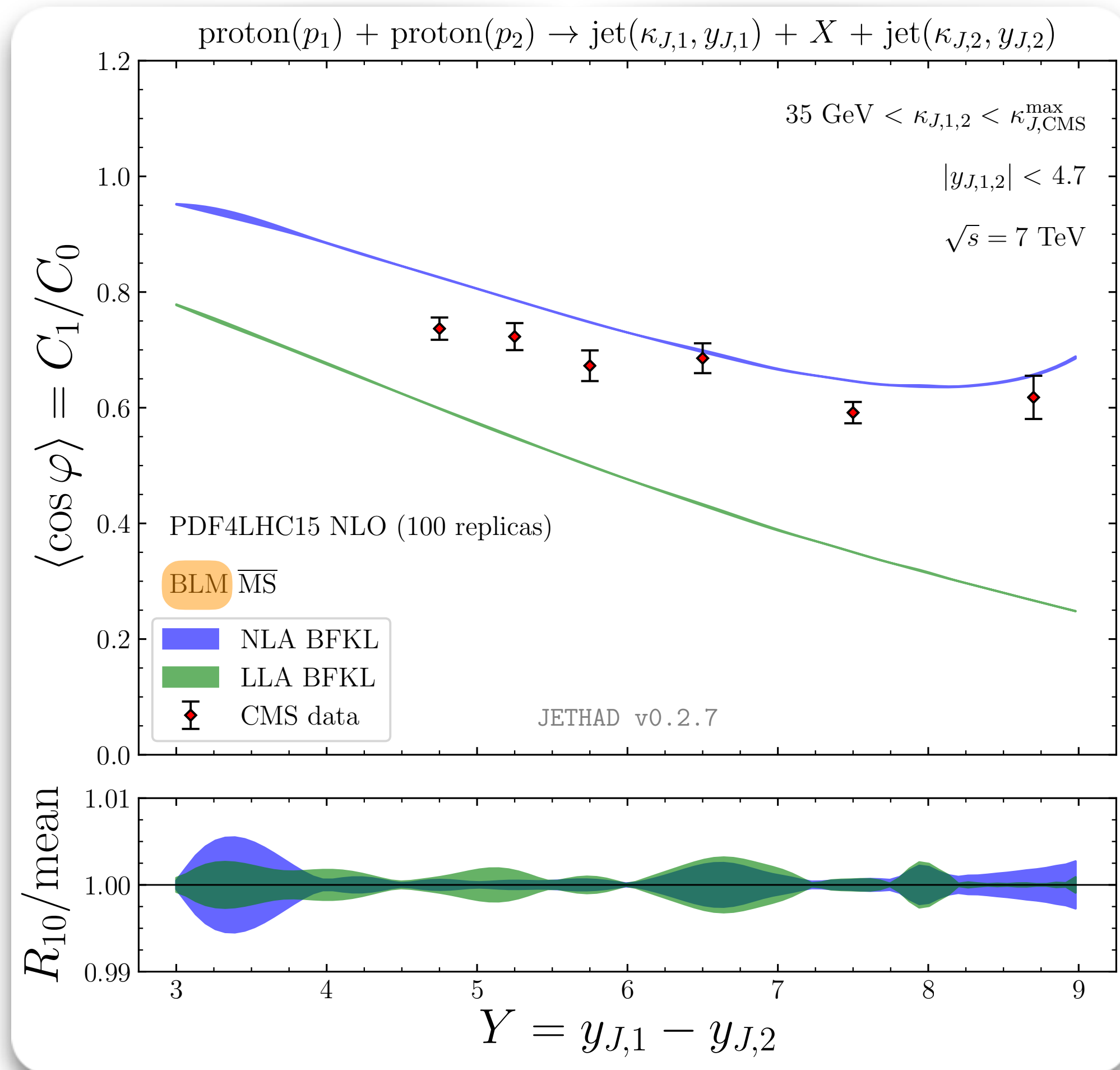


# Angular correlations

$$R_{n0}(\Delta Y, s) = C_n/C_0 \equiv \langle \cos n\varphi \rangle$$

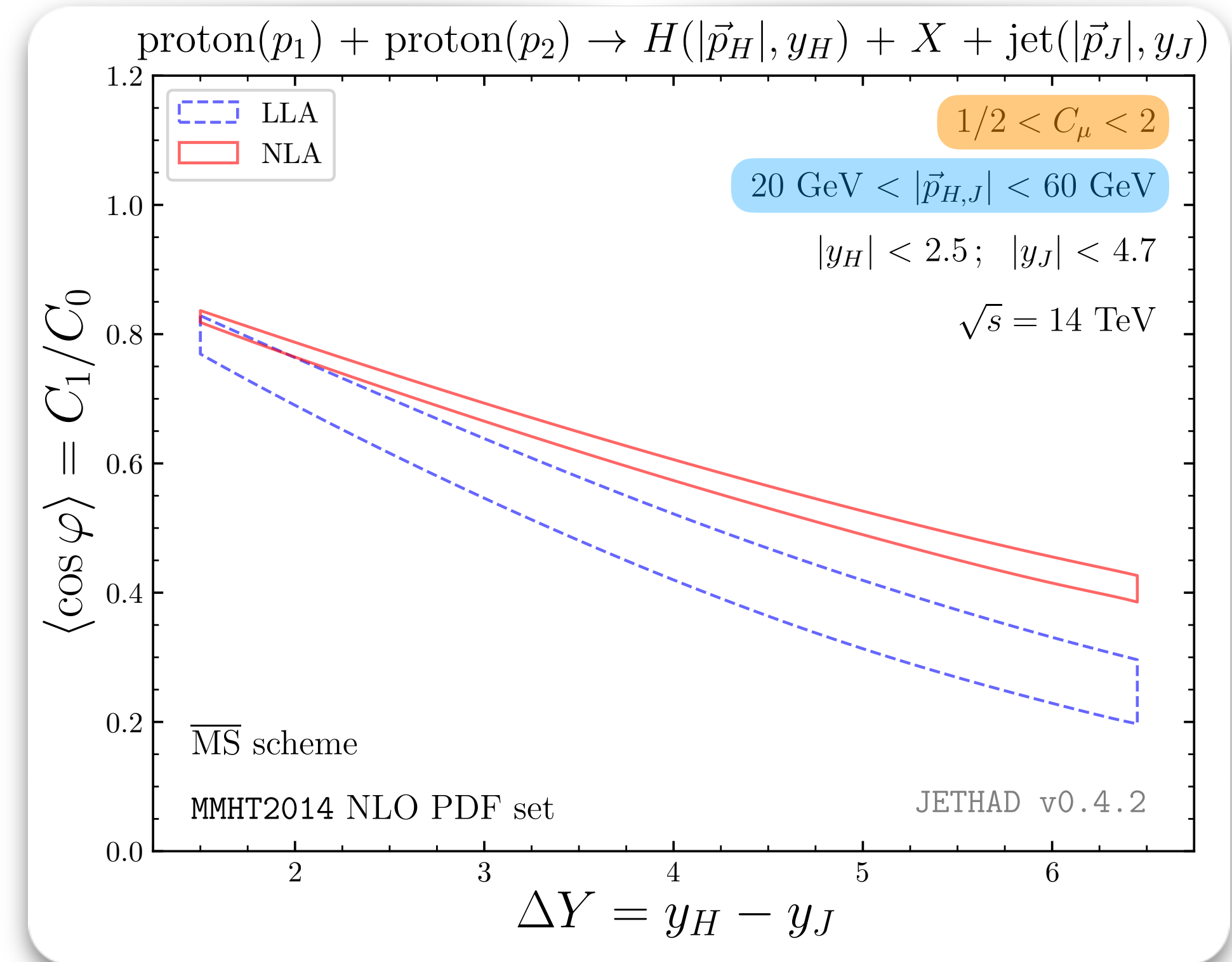
## MUELLER-NAVELET JETS

[B. Ducloué, L. Szymanowski, S. Wallon, Phys.Rev.Lett. 112 (2014) 082003]  
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## HIGGS + JET

(figure below) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 4, 293]  
 (NLO Higgs coefficient function) [F. G. C. et al., JHEP 08 (2022) 092]



natural scales

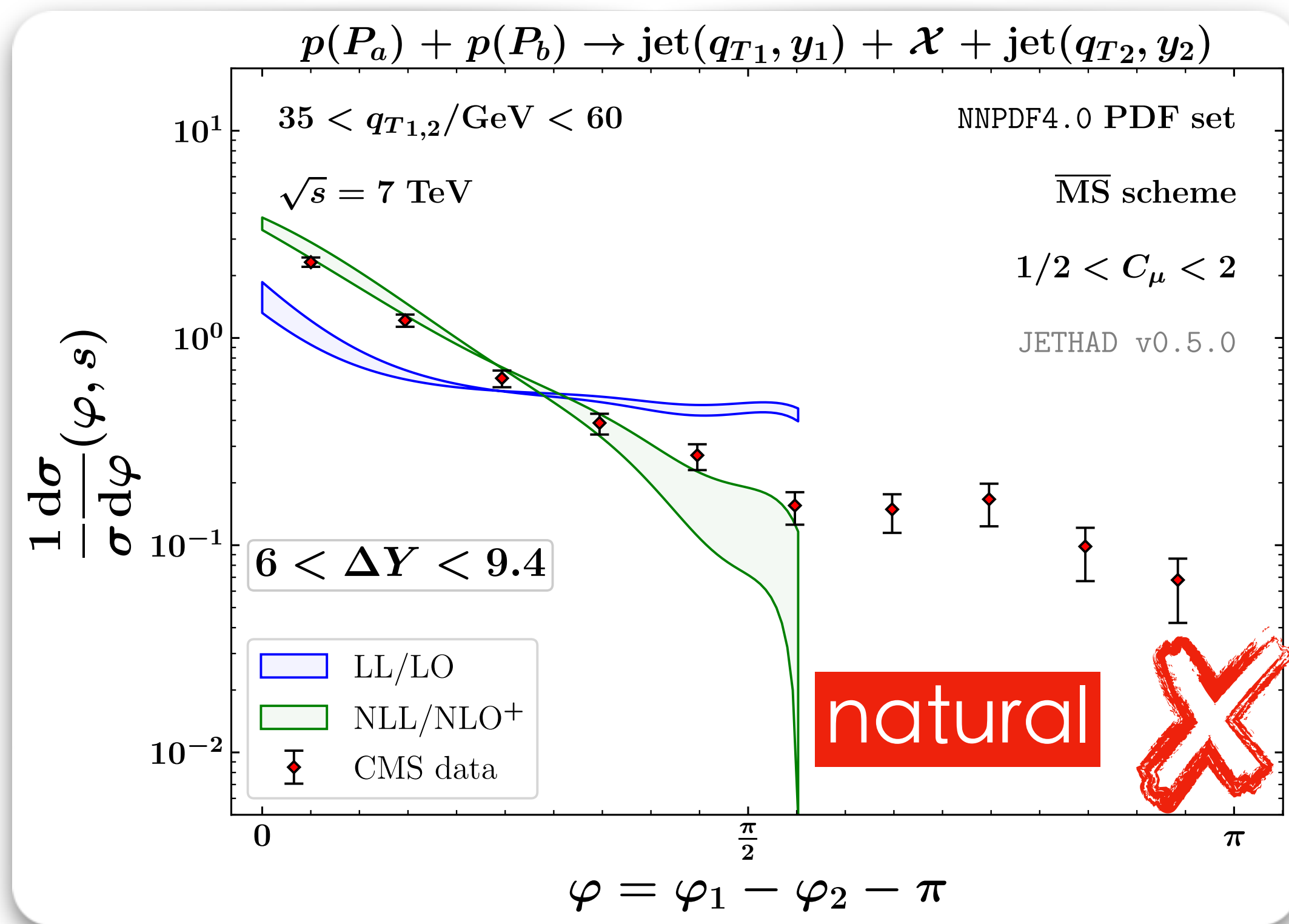
symmetric  $p_T$  range

# Azimuthal-angle multiplicity

$$\frac{1}{\sigma} \frac{d\sigma(\Delta Y, s)}{d\varphi} = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) \langle \cos(n\varphi) \rangle \right\}$$

## MUELLER-NAVELET JETS

- [\[B. Ducloué, L. Szymanowski, S. Wallon, Phys. Rev. Lett. 112 \(2014\) 082003\]](#)  
 (figure below) [\[F. G. C., A. Papa, Phys. Rev. D 106 \(2022\) 11, 114004\]](#)





# Azimuthal-angle multiplicity

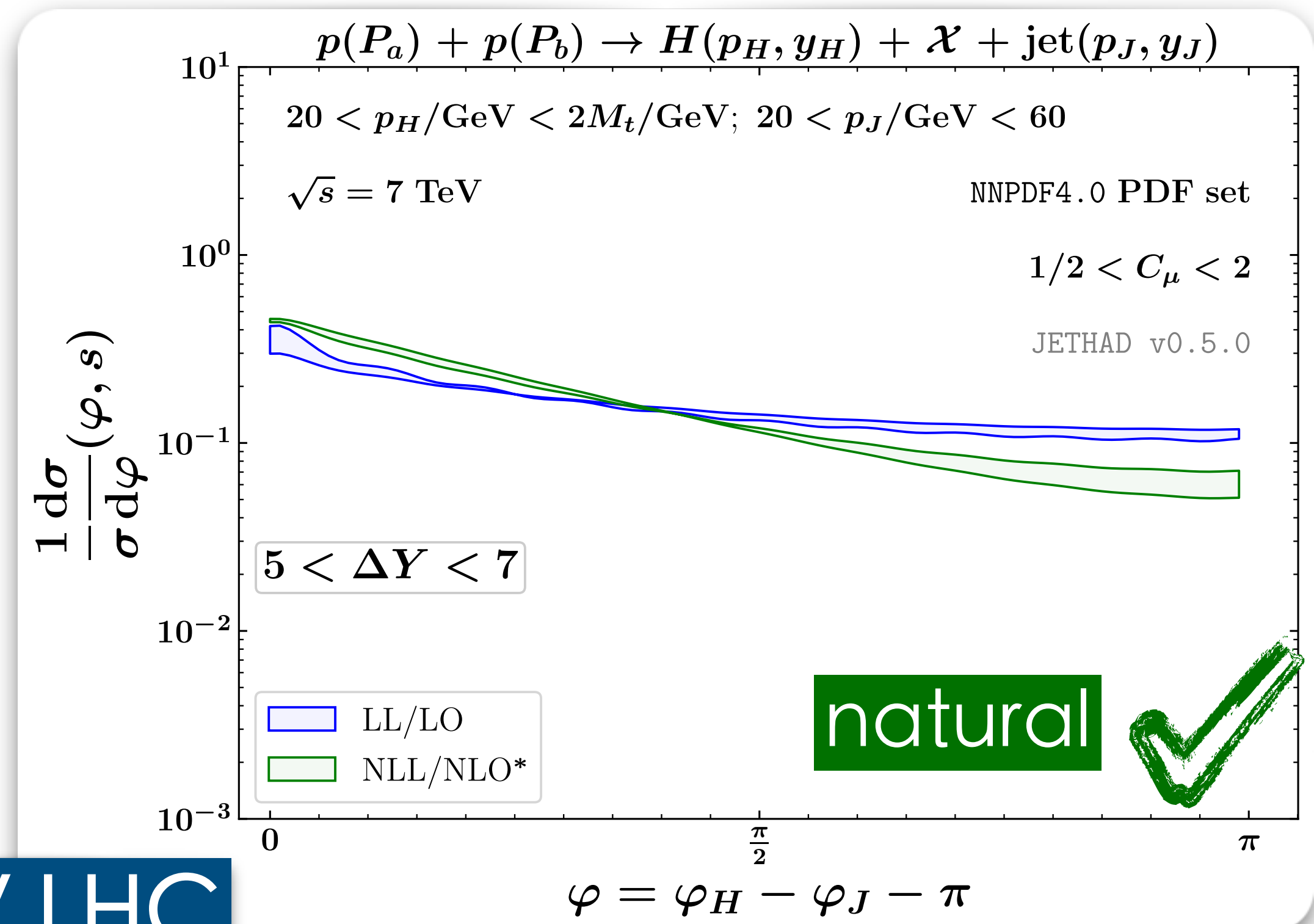
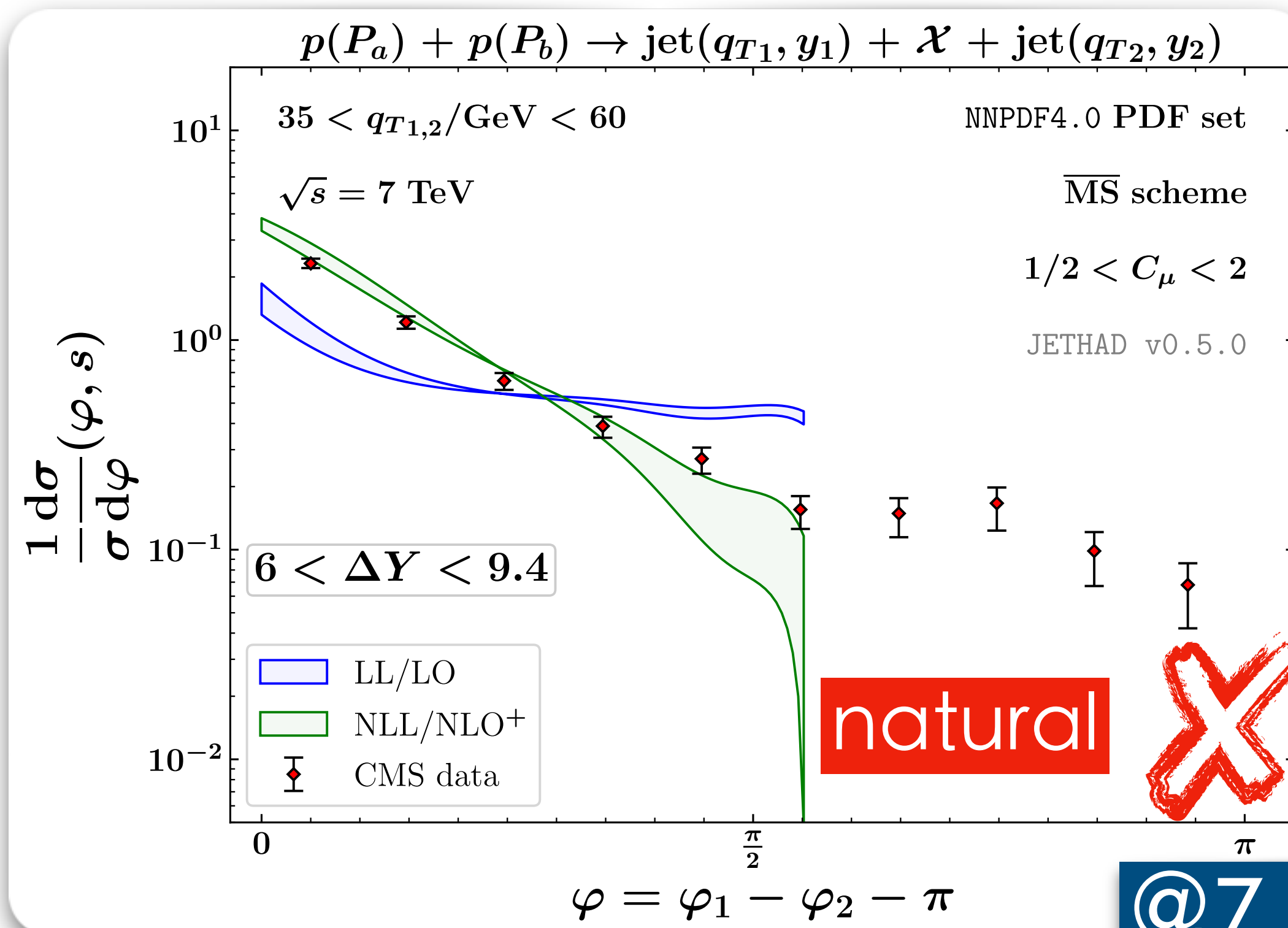
$$\frac{1}{\sigma} \frac{d\sigma(\Delta Y, s)}{d\varphi} = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) \langle \cos(n\varphi) \rangle \right\}$$

## MUELLER-NAVELET JETS

## HIGGS + JET

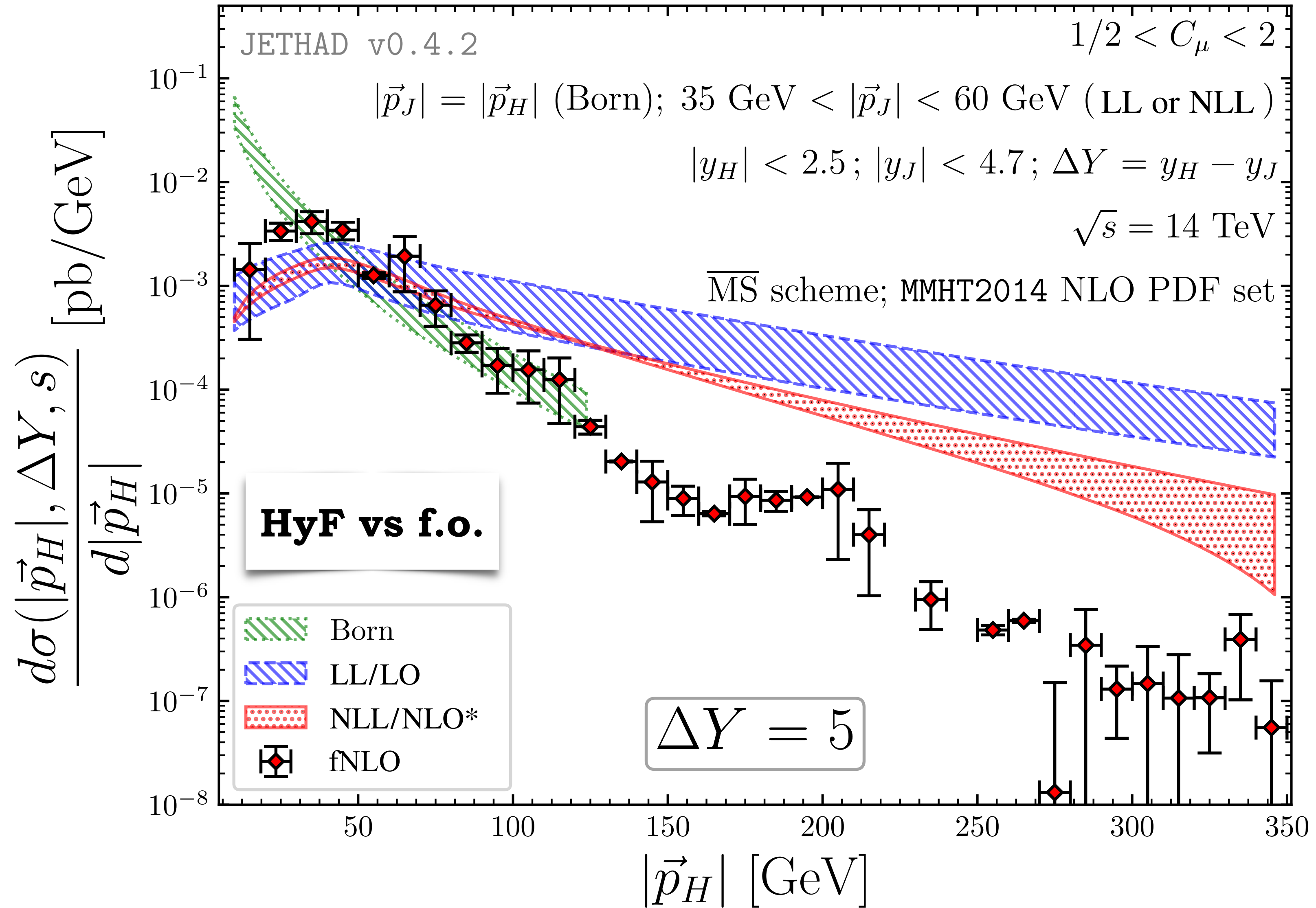
[\[B. Ducloué, L. Szymanowski, S. Wallon, Phys. Rev. Lett. 112 \(2014\) 082003\]](#)  
 (figure below) [\[F. G. C., A. Papa, Phys. Rev. D 106 \(2022\) 11, 114004\]](#)

(figure below) [\[F. G. C. et al., Eur. Phys. J. C 81 \(2021\) 4, 293\]](#)  
 (NLO Higgs coefficient function) [\[F. G. C. et al., JHEP 08 \(2022\) 092\]](#)



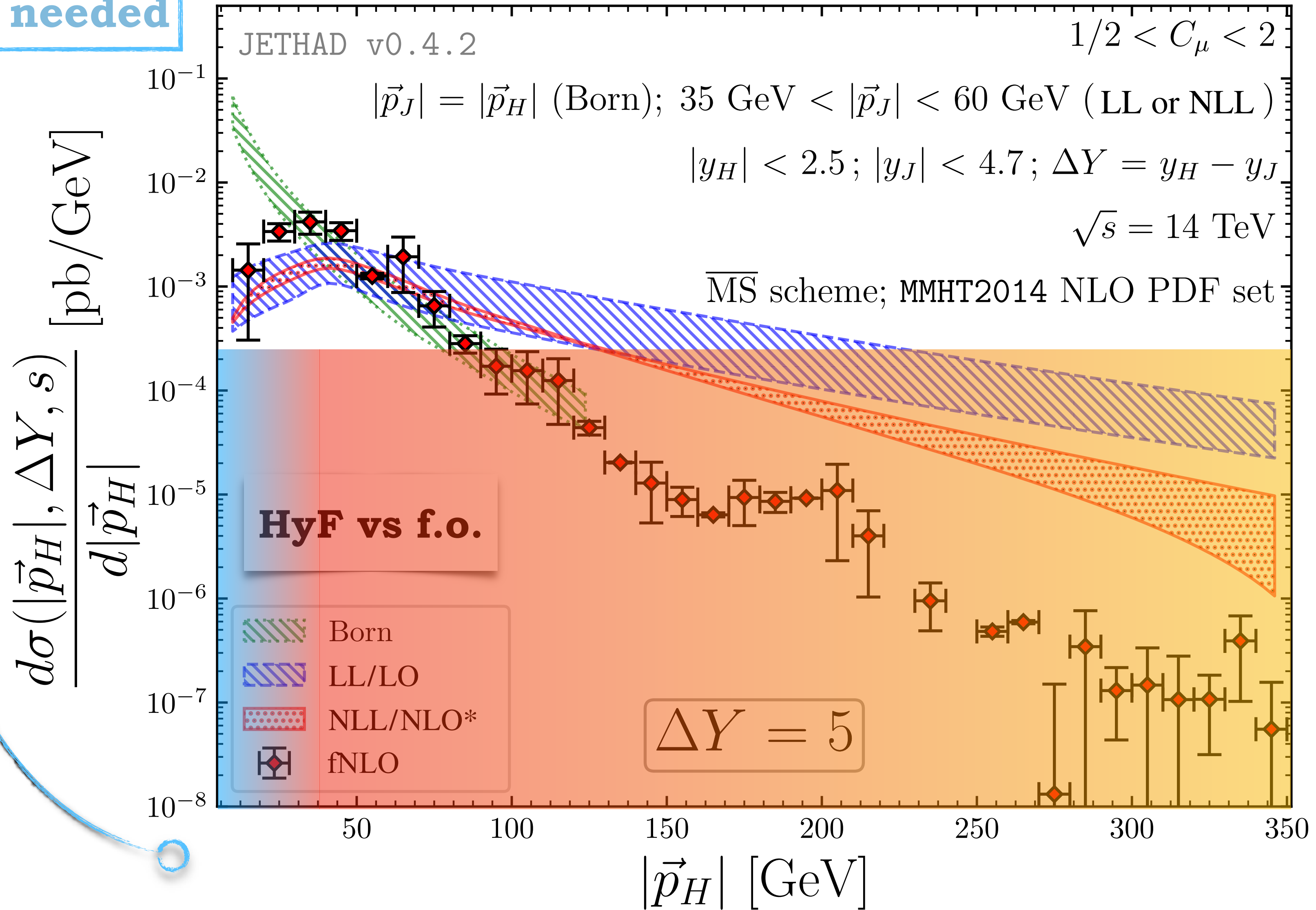
@7 TeV LHC

proton( $p_1$ ) + proton( $p_2$ )  $\rightarrow$   $H(|\vec{p}_H|, y_H) + X + \text{jet}(|\vec{p}_J|, y_J)$



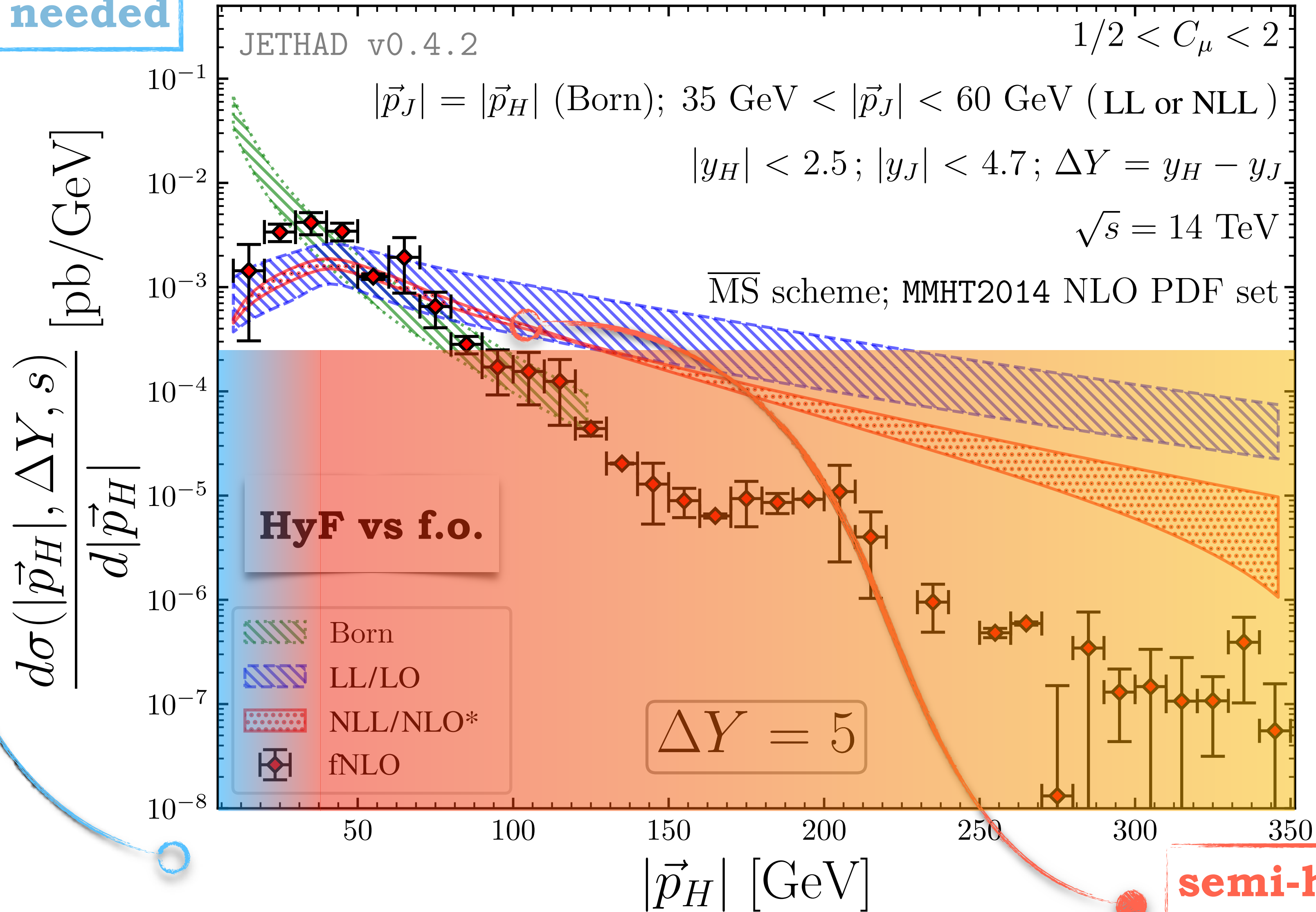
large  $p_T$  logs  
 $p_T$ -resum. needed

$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow H(|\vec{p}_H|, y_H) + X + \text{jet}(|\vec{p}_J|, y_J)$$



large  $p_T$  logs  
 $p_T$ -resum. needed

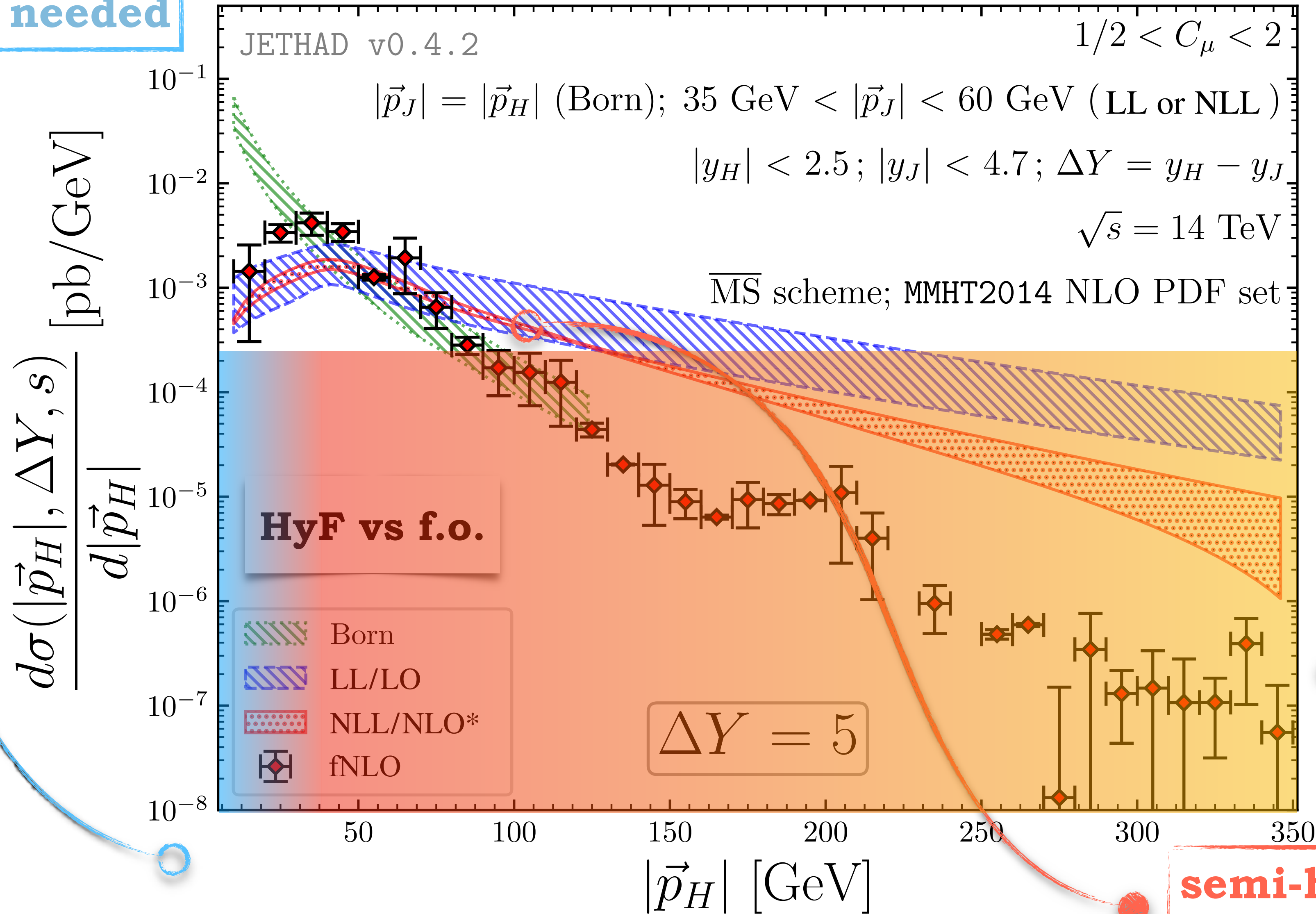
$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow H(|\vec{p}_H|, y_H) + X + \text{jet}(|\vec{p}_J|, y_J)$$



**DGLAP-type + large- $x$  threshold logs  $\rightarrow$  BFKL decoupling**

large  $p_T$  logs  
 $p_T$ -resum. needed

$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow H(|\vec{p}_H|, y_H) + X + \text{jet}(|\vec{p}_J|, y_J)$$

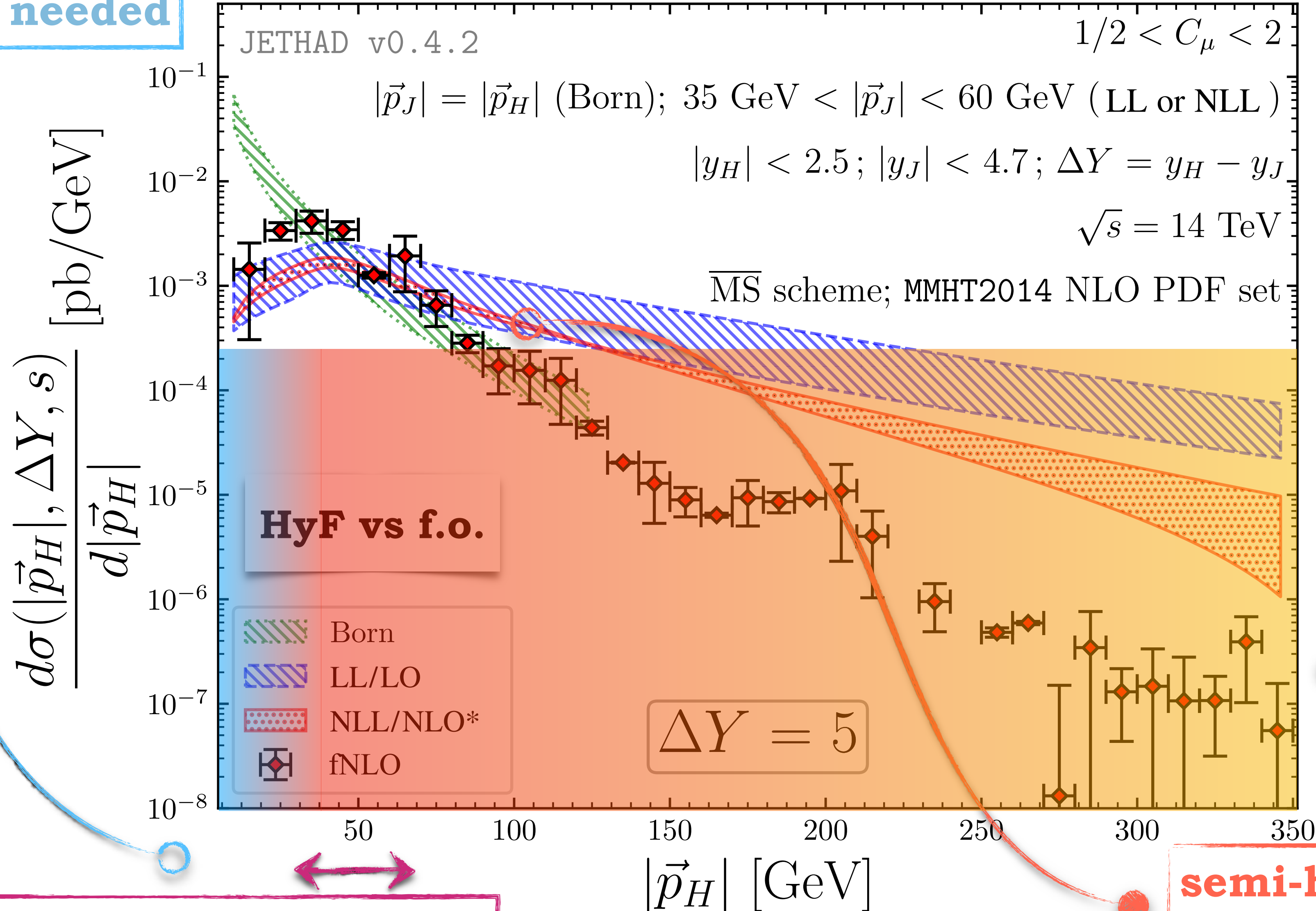


**semi-hard regime  
 BFKL expected**

**DGLAP-type + large- $x$  threshold logs  $\rightarrow$  BFKL decoupling**

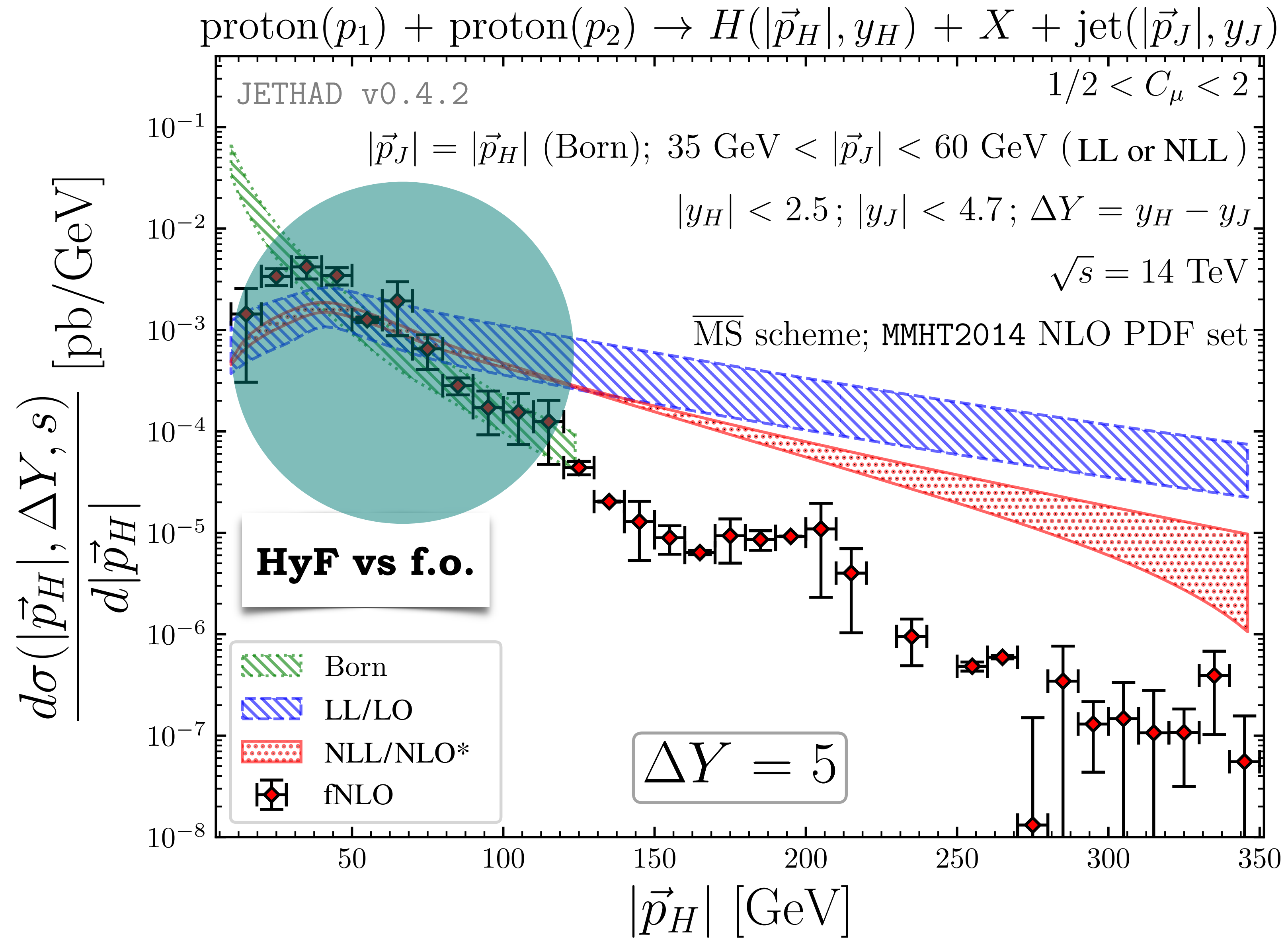
large  $p_T$  logs  
 $p_T$ -resum. needed

$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow H(|\vec{p}_H|, y_H) + X + \text{jet}(|\vec{p}_J|, y_J)$$

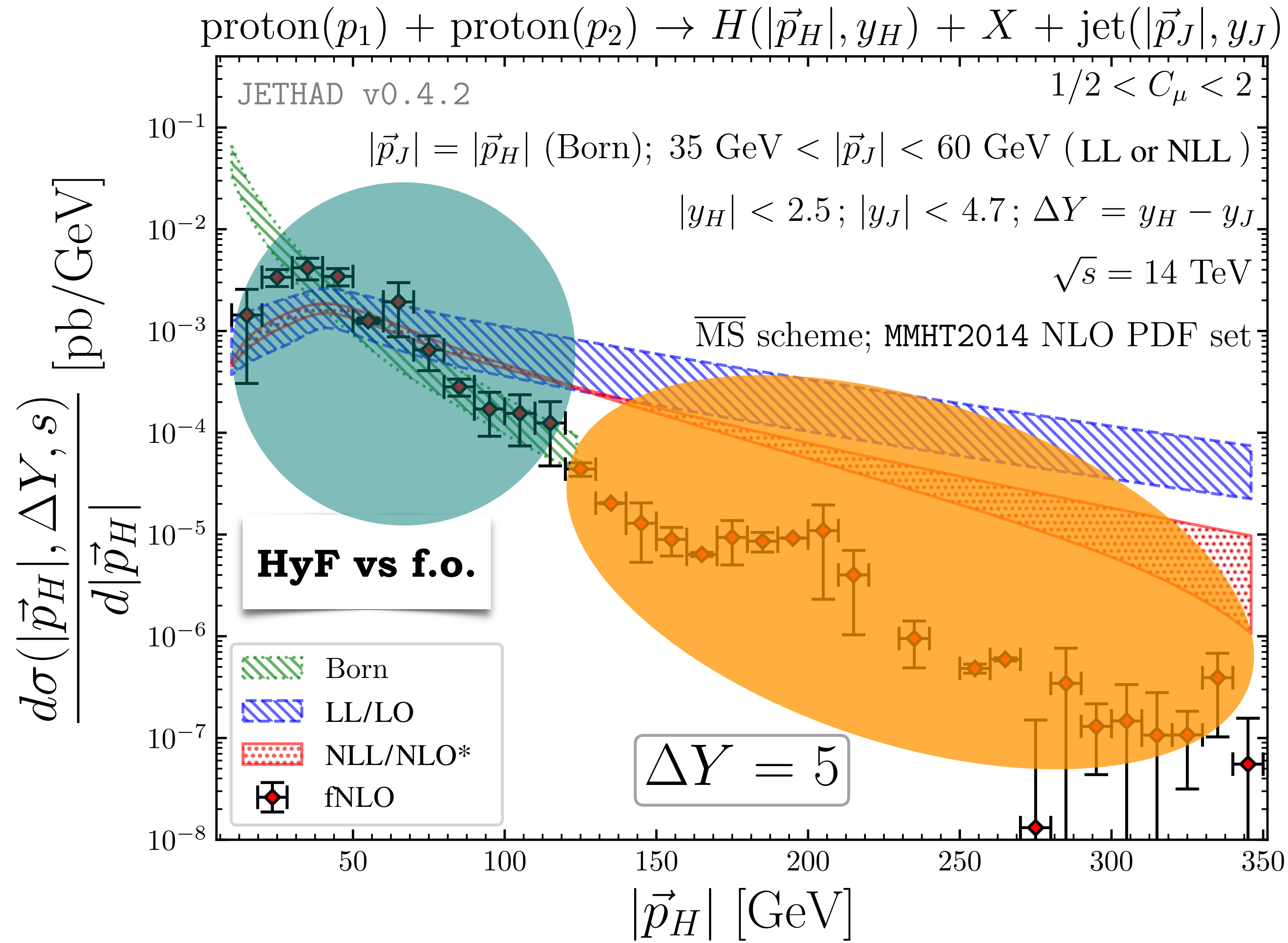


almost back-to-back emissions  
 large imbalance double logs

semi-hard regime  
 BFKL expected



**;** Precision corrections *expected*



! Precision corrections expected, but **HyF** predicts large deviations from **f.o.** !



¿ How to properly combine  
BFKL with the fixed order?



# Matching NLL to NLO with JETHAD

; **Precision corrections** *expected*  $\Leftrightarrow$  *need* for an accurate NLL-to-NLO **Matching procedure** !

 JETHAD Method  $\rightarrow$  NLL/NLO **Additive Matching** (analytic: BFKL kernel + coefficient functions)

$$\underbrace{d\sigma^{\text{NLL/NLO}^-}(\Delta Y, \varphi, s)} = \underbrace{d\sigma^{\text{NLO}}(\Delta Y, \varphi, s)}_{\text{NLO fixed order}} + d\sigma^{\text{NLL}^-}(\Delta Y, \varphi, s) - \Delta d\sigma^{\text{NLL/NLO}^-}(\Delta Y, \varphi, s)$$

$\underbrace{\hspace{10em}}_{\text{NLO POWHEG w/o PS}}$

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# Matching NLL to NLO with JETHAD

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 JETHAD Method  $\rightarrow$  NLL/NLO **Additive Matching** (analytic: BFKL kernel + coefficient functions)

$$\underbrace{d\sigma^{\text{NLL/NLO}^-}(\Delta Y, \varphi, s)}_{\text{NLO POWHEG w/o PS}} = \underbrace{d\sigma^{\text{NLO}}(\Delta Y, \varphi, s)}_{\text{NLO fixed order}} + \underbrace{d\sigma^{\text{NLL}^-}(\Delta Y, \varphi, s)}_{\text{NLL}^- \text{ resum (HyF)}} - \underbrace{\Delta d\sigma^{\text{NLL/NLO}^-}(\Delta Y, \varphi, s)}_{\text{NLL}^- \text{ expanded at NLO}}$$

**NLL<sup>-</sup>** JETHAD w/o NLO<sup>-</sup> double counting

# Matching NLL to NLO with JETHAD

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 JETHAD Method  $\rightarrow$  **NLL/NLO Additive Matching** (analytic: BFKL kernel + coefficient functions)

$$\underbrace{d\sigma^{\text{NLL/NLO}^-}(\Delta Y, \varphi, s)}_{\text{NLL/NLO}^- \text{ matched}} = \underbrace{d\sigma^{\text{NLO}}(\Delta Y, \varphi, s)}_{\text{NLO fixed order}} + \underbrace{d\sigma^{\text{NLL}^-}(\Delta Y, \varphi, s)}_{\text{NLL}^- \text{ resum (HyF)}} - \underbrace{\Delta d\sigma^{\text{NLL/NLO}^-}(\Delta Y, \varphi, s)}_{\text{NLL}^- \text{ expanded at NLO}}$$

NLO POWHEG  $\oplus$  NLL<sup>-</sup> JETHAD     
 NLO POWHEG w/o PS     
 NLL<sup>-</sup> JETHAD w/o NLO<sup>-</sup> double counting

HELL + ggHiggs  
 N<sup>3</sup>LL<sub>ix</sub>/LL<sub>sx</sub>/N<sup>3</sup>LO  
 Inclusive Higgs  
 [M. Bonvini, S. Marzani (2018)]

# Matching NLL to NLO with JETHAD

; **Precision corrections** *expected*  $\Leftrightarrow$  *need* for an accurate NLL-to-NLO **Matching procedure** !

 JETHAD Method  $\rightarrow$  **NLL/NLO Additive Matching** (analytic: BFKL kernel + coefficient functions)

$$\underbrace{d\sigma^{\text{NLL/NLO}^-}(\Delta Y, \varphi, s)}_{\text{NLL/NLO}^- \text{ matched}} = \underbrace{d\sigma^{\text{NLO}}(\Delta Y, \varphi, s)}_{\text{NLO fixed order}} + \underbrace{d\sigma^{\text{NLL}^-}(\Delta Y, \varphi, s)}_{\text{NLL}^- \text{ resum (HyF)}} - \underbrace{\Delta d\sigma^{\text{NLL/NLO}^-}(\Delta Y, \varphi, s)}_{\text{NLL}^- \text{ expanded at NLO}}$$

NLO POWHEG  $\oplus$  NLL<sup>-</sup> JETHAD
     
 NLO POWHEG w/o PS
     
 NLL<sup>-</sup> JETHAD w/o NLO<sup>-</sup> double counting

HELL + ggHiggs  
 N<sup>3</sup>LL<sub>ix</sub>/LL<sub>sx</sub>/N<sup>3</sup>LO  
 Inclusive Higgs  
 [M. Bonvini, S. Marzani (2018)]

HEJ framework  
 NLL<sub>sx</sub><sup>-</sup>/NLO  
 Higgs + jet(s)  
 [J. R. Andersen et al. (2022)]

# Matching NLL to NLO with JETHAD

; **Precision corrections** *expected*  $\Leftrightarrow$  *need* for an accurate NLL-to-NLO **Matching procedure** !

 JETHAD Method  $\rightarrow$  **NLL/NLO Additive Matching** (analytic: BFKL kernel + coefficient functions)

$$\underbrace{d\sigma^{\text{NLL/NLO}^-}(\Delta Y, \varphi, s)}_{\text{NLL/NLO}^- \text{ matched}} = \underbrace{d\sigma^{\text{NLO}}(\Delta Y, \varphi, s)}_{\text{NLO fixed order}} + \underbrace{d\sigma^{\text{NLL}^-}(\Delta Y, \varphi, s)}_{\text{NLL}^- \text{ resum (HyF)}} - \underbrace{\Delta d\sigma^{\text{NLL/NLO}^-}(\Delta Y, \varphi, s)}_{\text{NLL}^- \text{ expanded at NLO}}$$

NLO POWHEG  $\oplus$  NLL<sup>-</sup> JETHAD     
 NLO POWHEG w/o PS     
 NLL<sup>-</sup> JETHAD w/o NLO<sup>-</sup> double counting

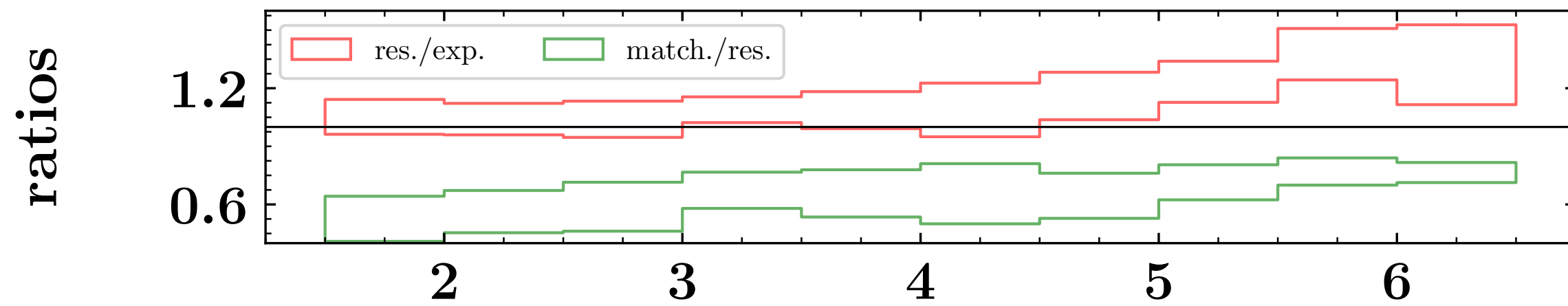
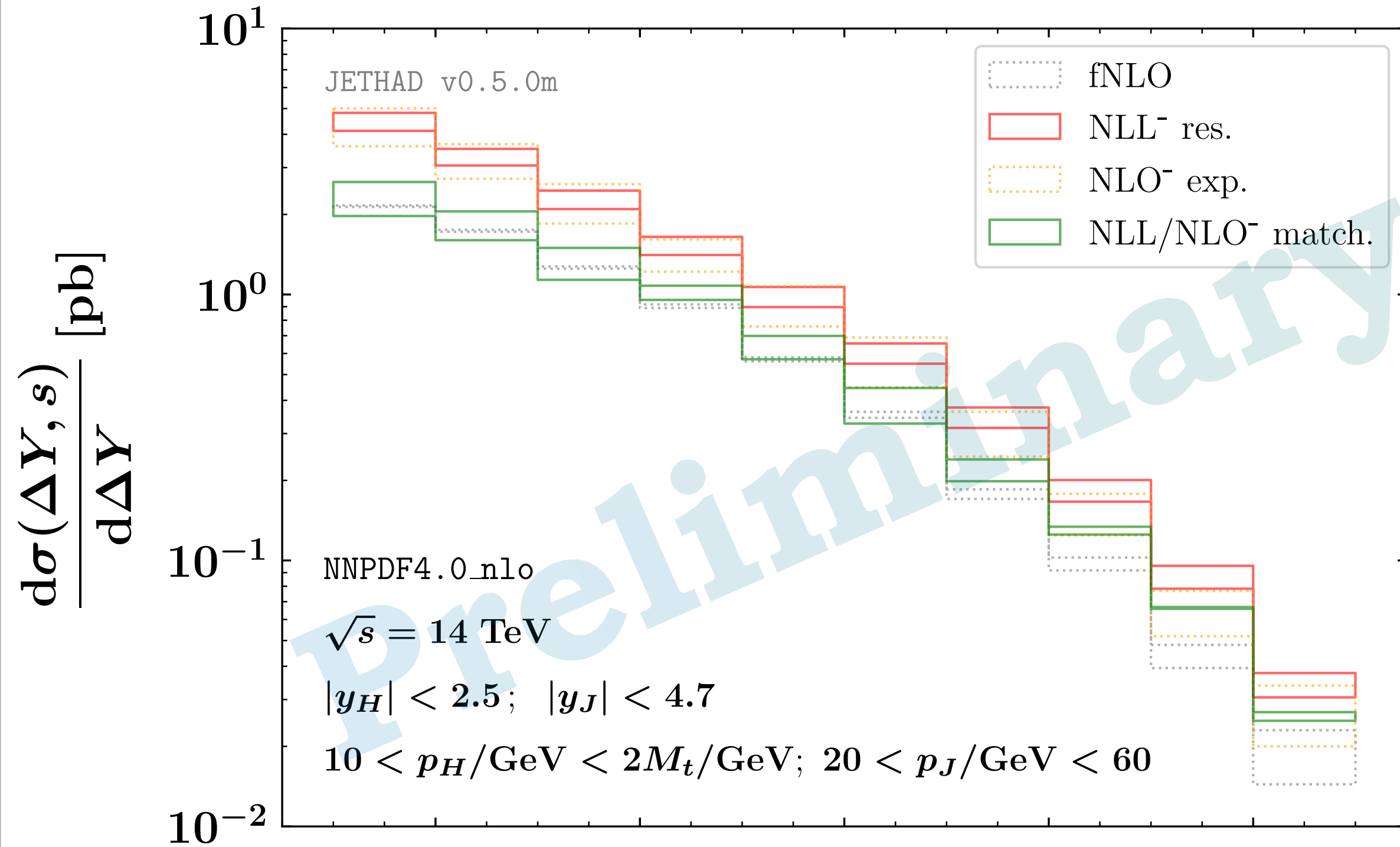
HELL + ggHiggs  
 N<sup>3</sup>LL<sub>ix</sub>/LL<sub>sx</sub>/N<sup>3</sup>LO  
 Inclusive Higgs  
 [M. Bonvini, S. Marzani (2018)]

HEJ framework  
 NLL<sub>sx</sub><sup>-</sup>/NLO  
 Higgs + jet(s)  
 [J. R. Andersen et al. (2022)]

RadISH + MCFM-8.3  
 NNLL<sub>TM</sub>/NLO  
 Higgs + jet  
 [P.F. Monni et al. (2020)]

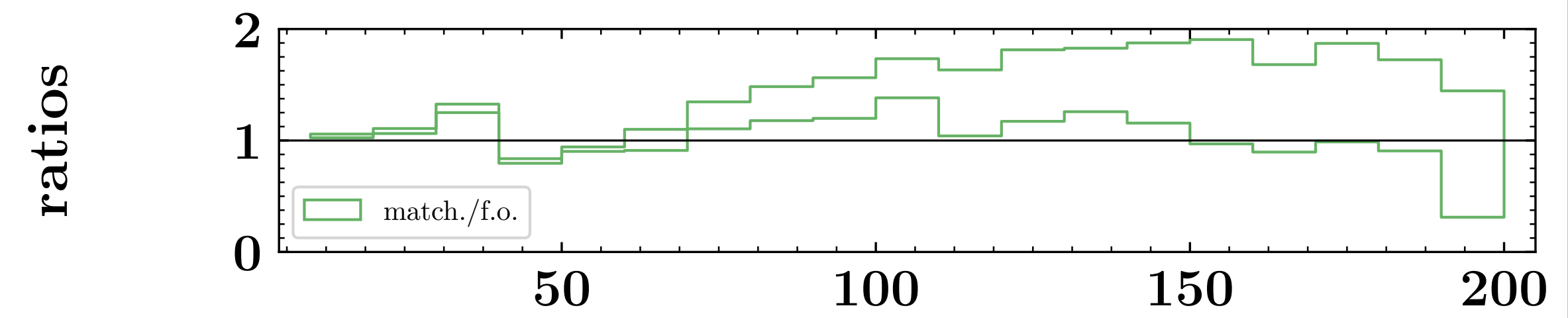
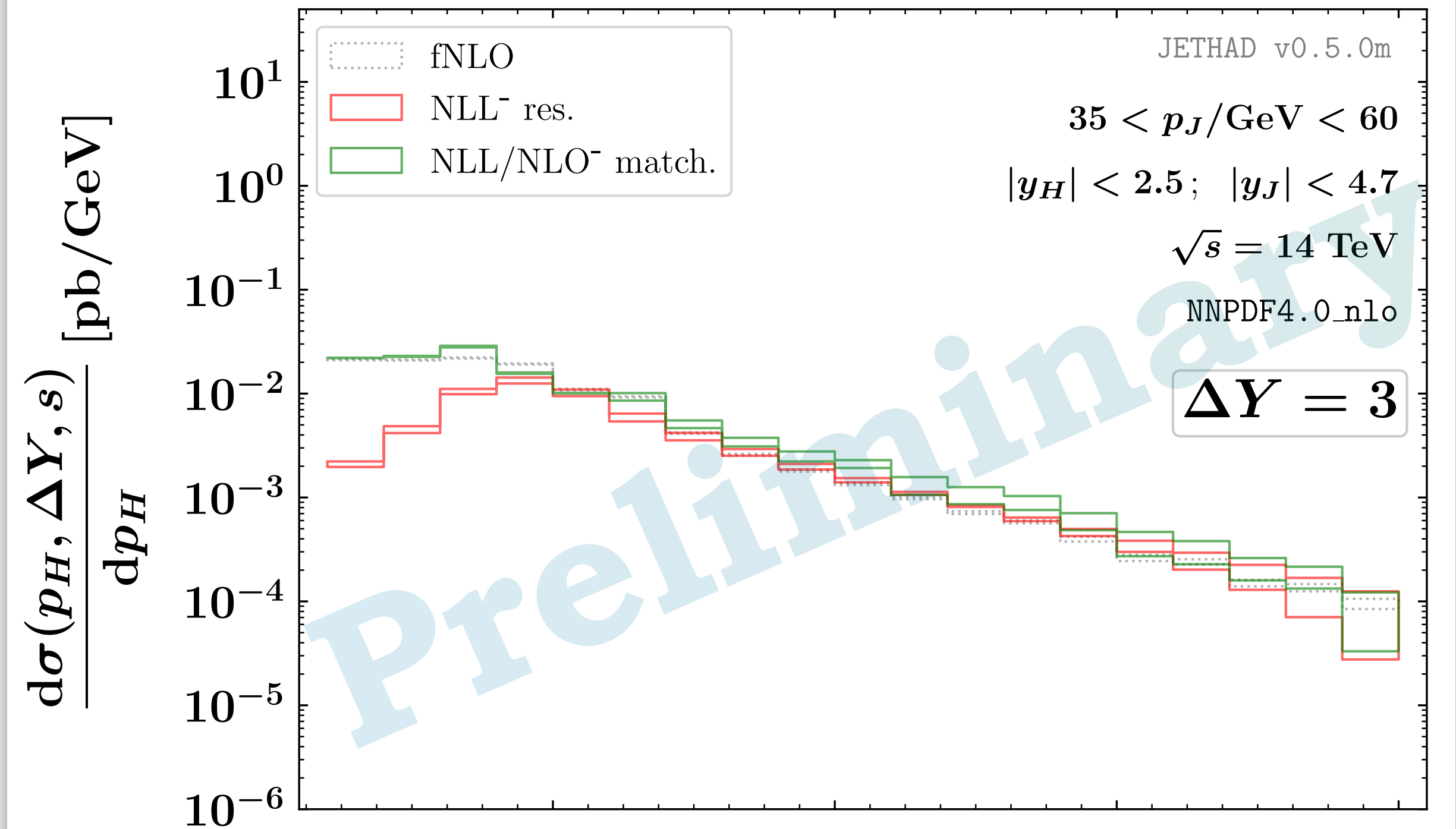
# The Higgs + jet spectrum from POWHEG + JETHAD

$$p(P_a) + p(P_b) \rightarrow H(p_H, y_H) + \mathcal{X} + \text{jet}(p_J, y_J)$$



$$\Delta Y = y_H - y_J$$

$$p(P_a) + p(P_b) \rightarrow H(p_H, y_H) + \mathcal{X} + \text{jet}(p_J, y_J)$$



$$p_H \text{ [GeV]}$$

$\Delta Y$  spectrum

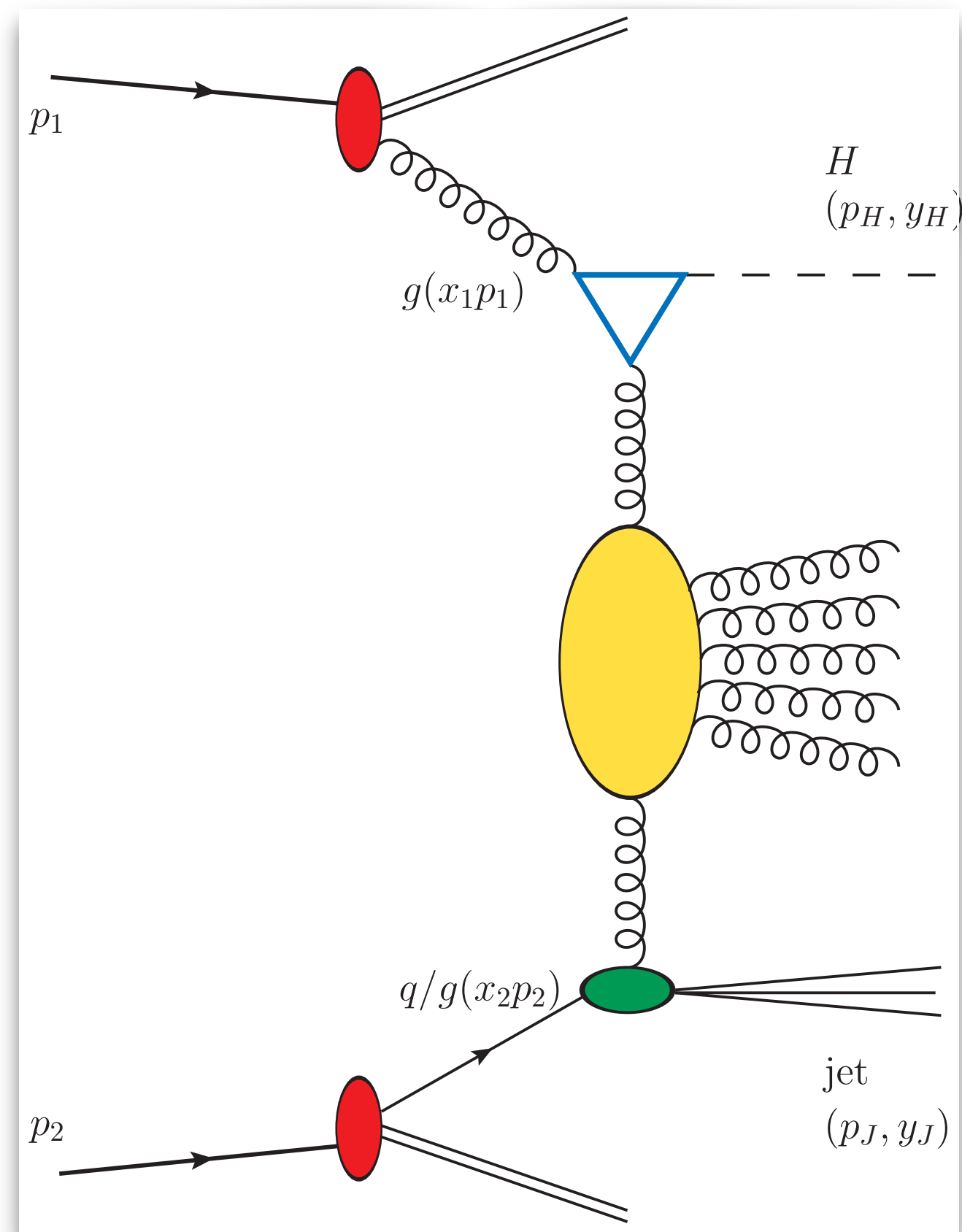
@14 TeV LHC

$p_H$  spectrum



# High-energy resummation for electroweak physics

## HIGGS BOSON + JET

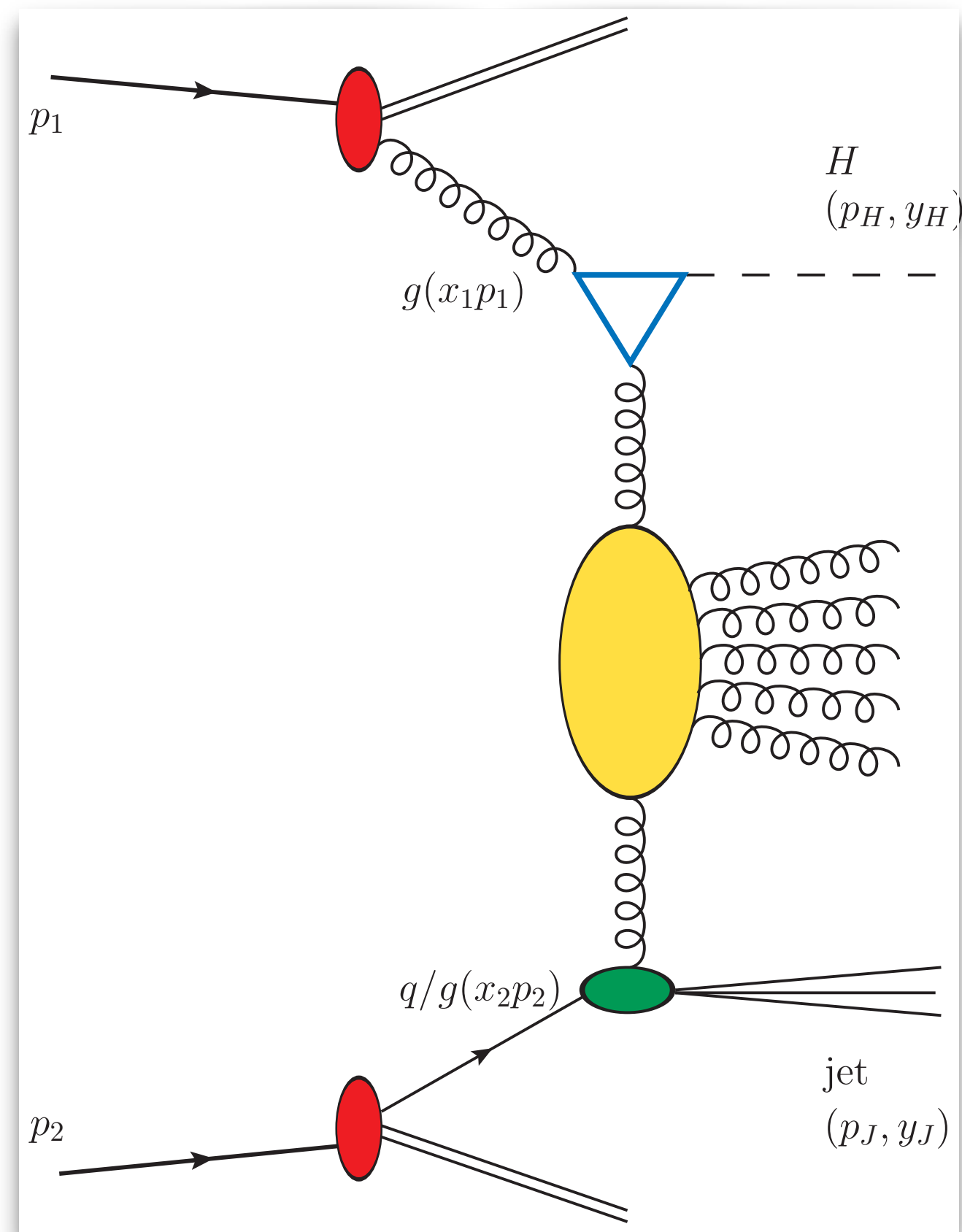


(Higgs + jet, NLL/NLO\*) [\[F. G. C. et al., Eur. Phys. J. C \(2021\) 8, 780\]](#)

(NLO emission function) [\[F. G. C., M. Fucilla et al., JHEP 08 \(2022\) 092\]](#)

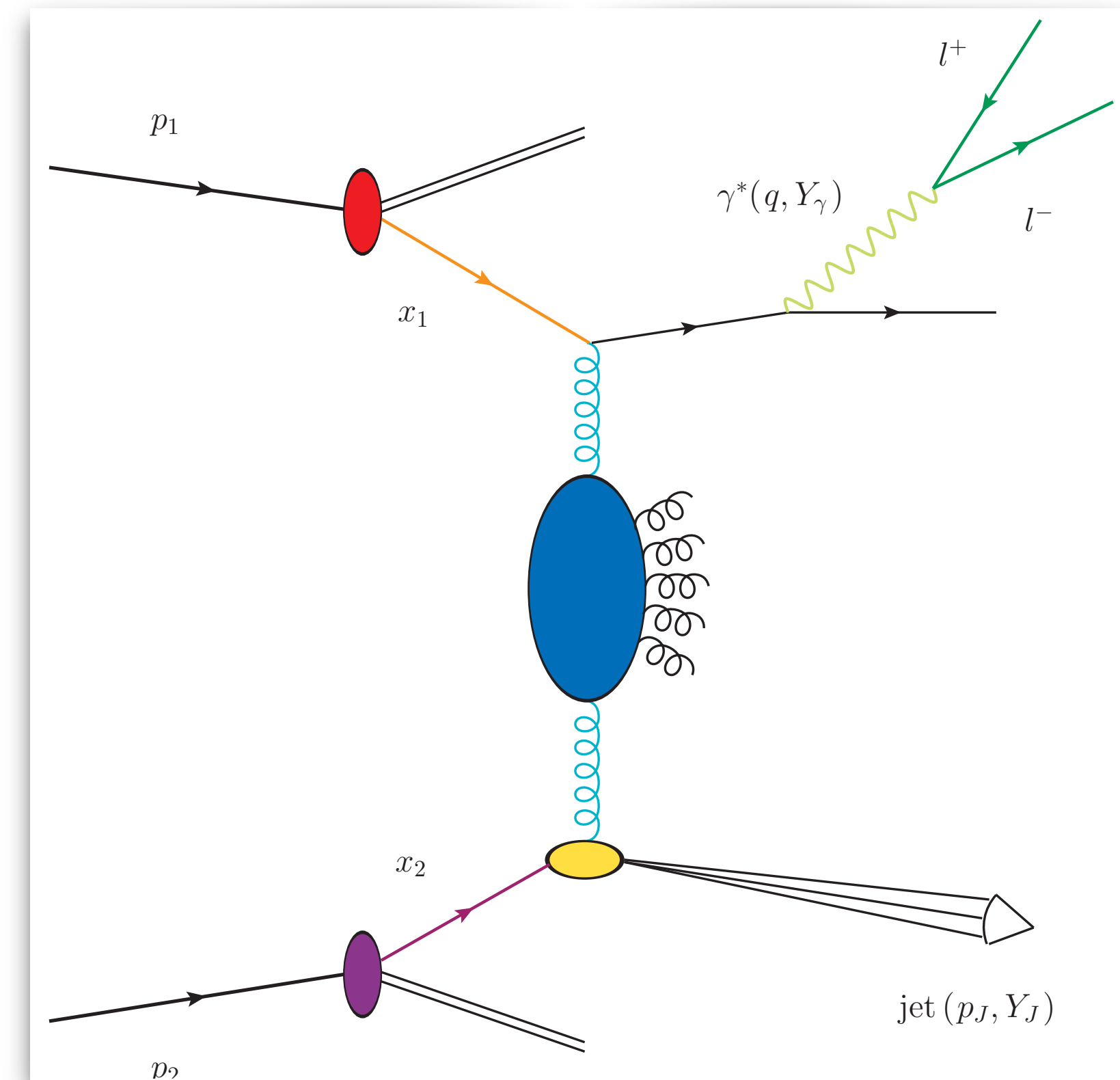
# High-energy resummation for electroweak physics

## HIGGS BOSON + JET



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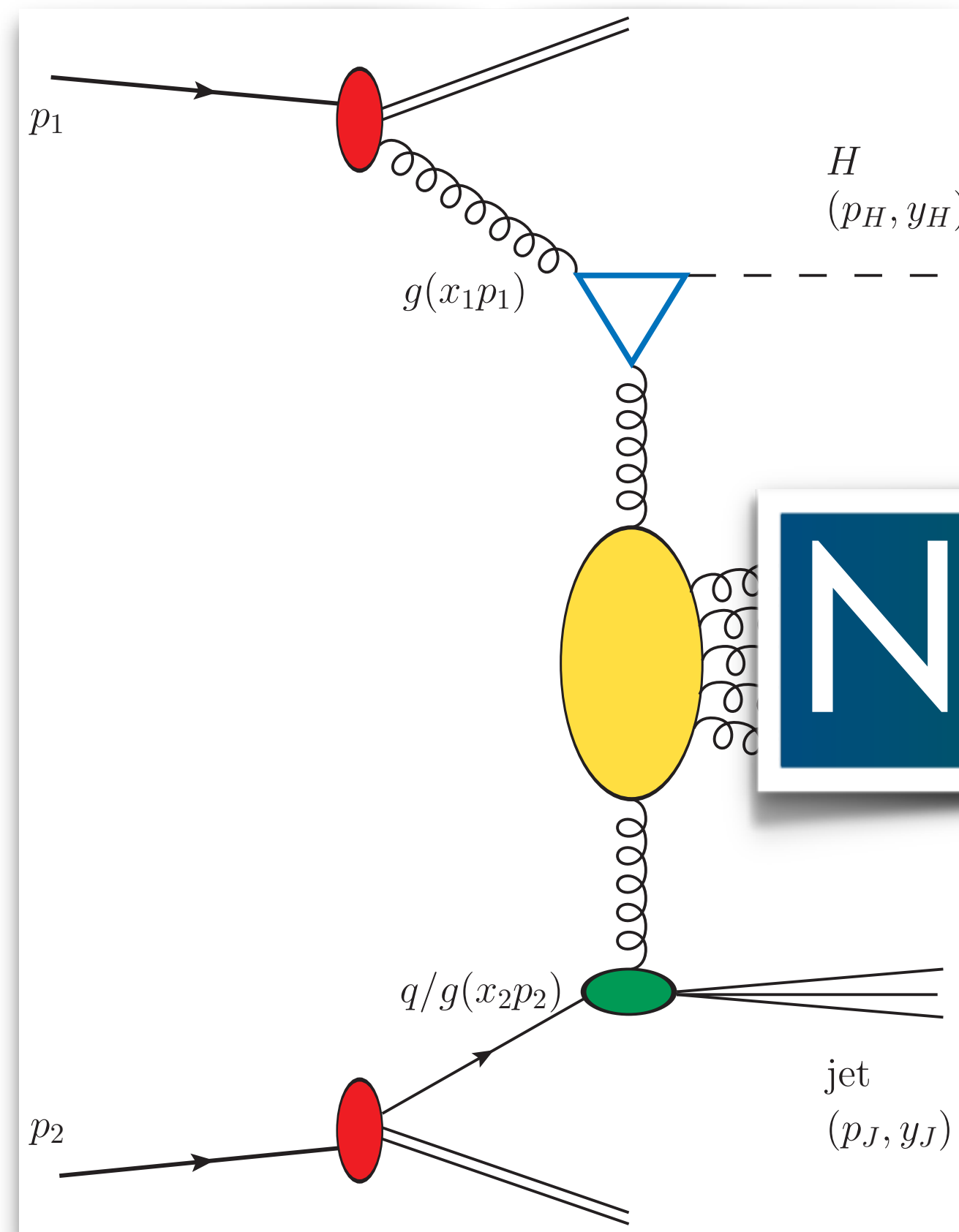
## Z BOSON + JET



(DY + jet, LL/LO+) [L. Motyka et al., JHEP 12 (2018) 091]  
(DY and Z + jet, NLL/NLO-) [F. G. C. et al. (in progress)]

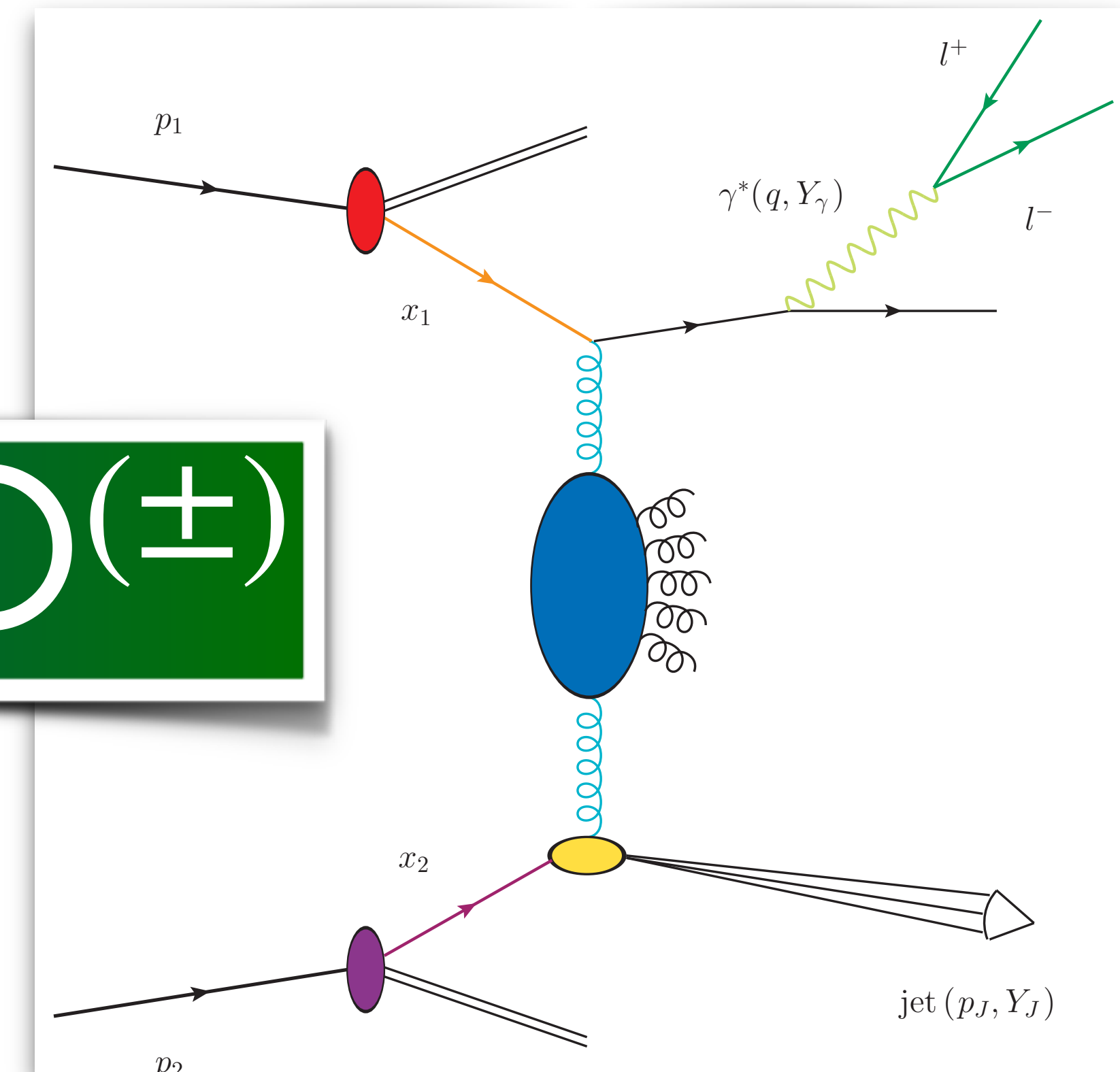
# High-energy resummation for electroweak physics

## HIGGS BOSON + JET



**NLL/NLO( $\pm$ )**

## Z BOSON + JET



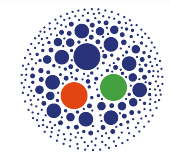
(Higgs + jet, NLL/NLO\*) [\[F. G. C. et al., Eur. Phys. J. C \(2021\) 8, 780\]](#)  
 (NLO emission function) [\[F. G. C., M. Fucilla et al., JHEP 08 \(2022\) 092\]](#)

(DY + jet, LL/LO<sup>+</sup>) [\[L. Motyka et al., JHEP 12 \(2018\) 091\]](#)  
 (DY and Z + jet, NLL/NLO<sup>-</sup>) [\[F. G. C. et al. \(in progress\)\]](#)

2

Open heavy flavor

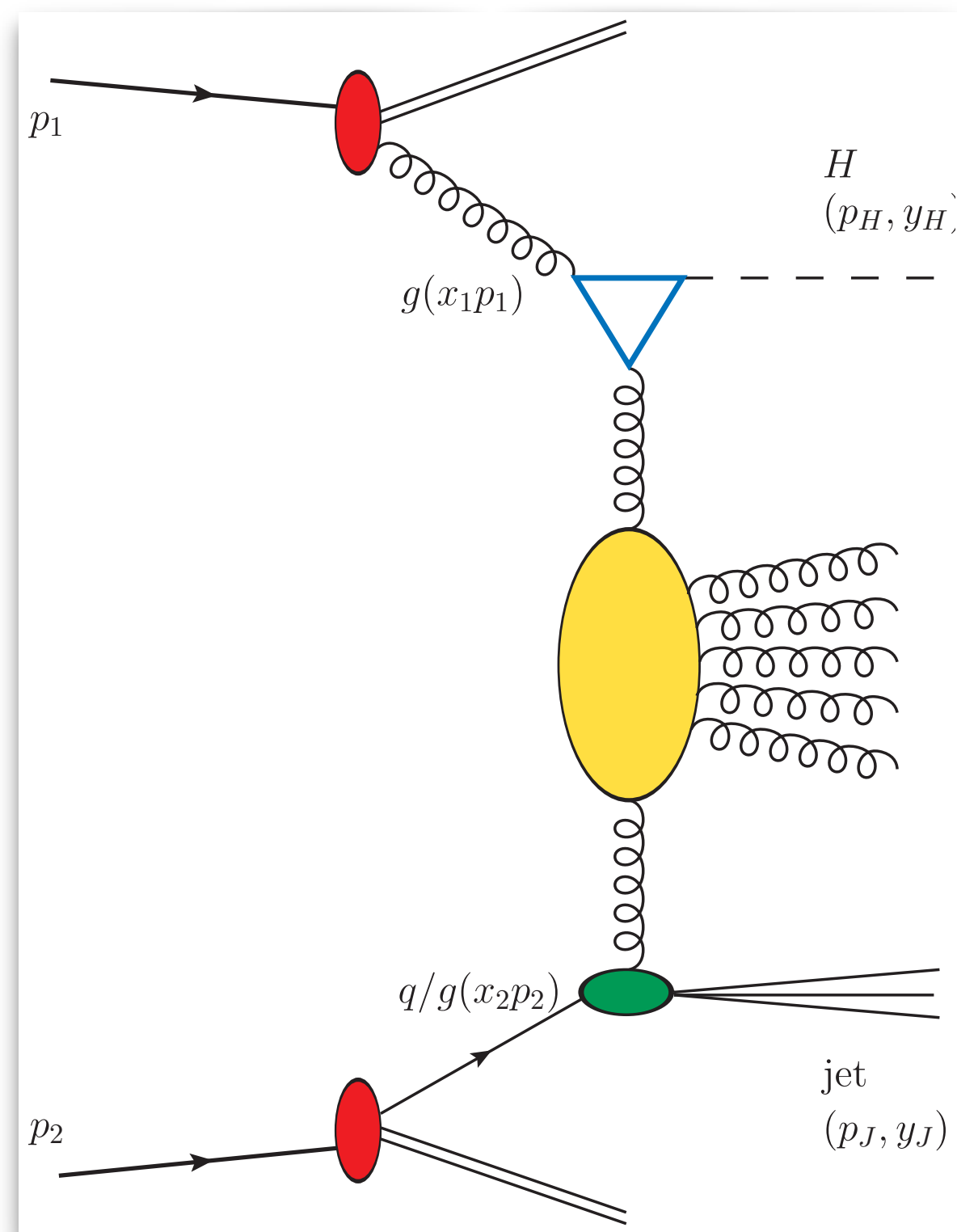
# From Mueller-Navelet to Higgs and heavy flavor



Pheno path: hunt for channels leading to a NLL **stabilization pattern** at **natural scales** (!)

## HIGGS BOSON

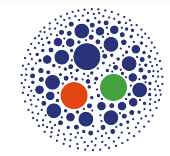
Stabilizers  $\Leftrightarrow$  large Higgs transverse masses



(Higgs + jet, NLL/NLO\*) [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

(NLO Higgs coeff. function) [F. G. C., M. Fucilla et al., JHEP 08 (2022) 092]

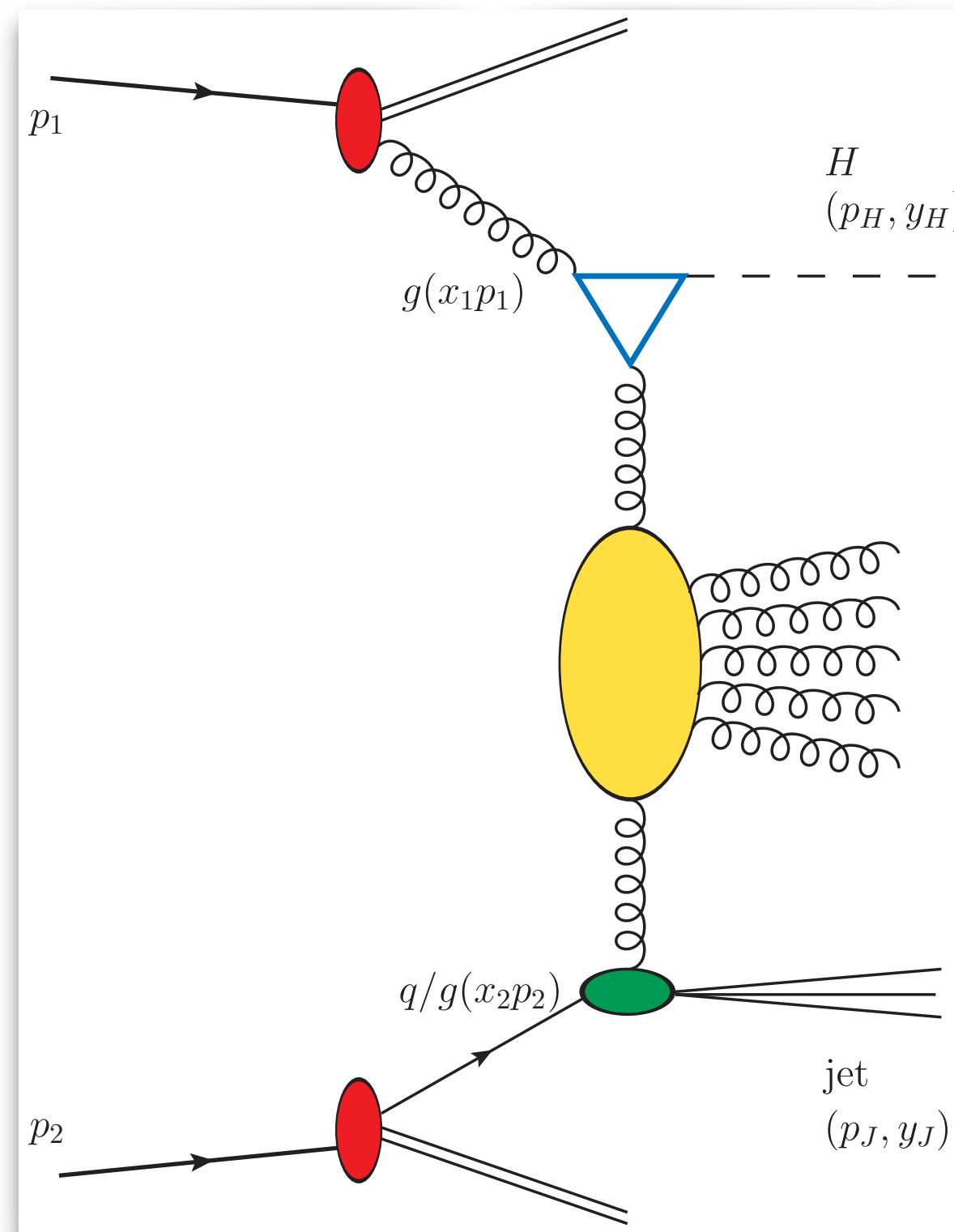
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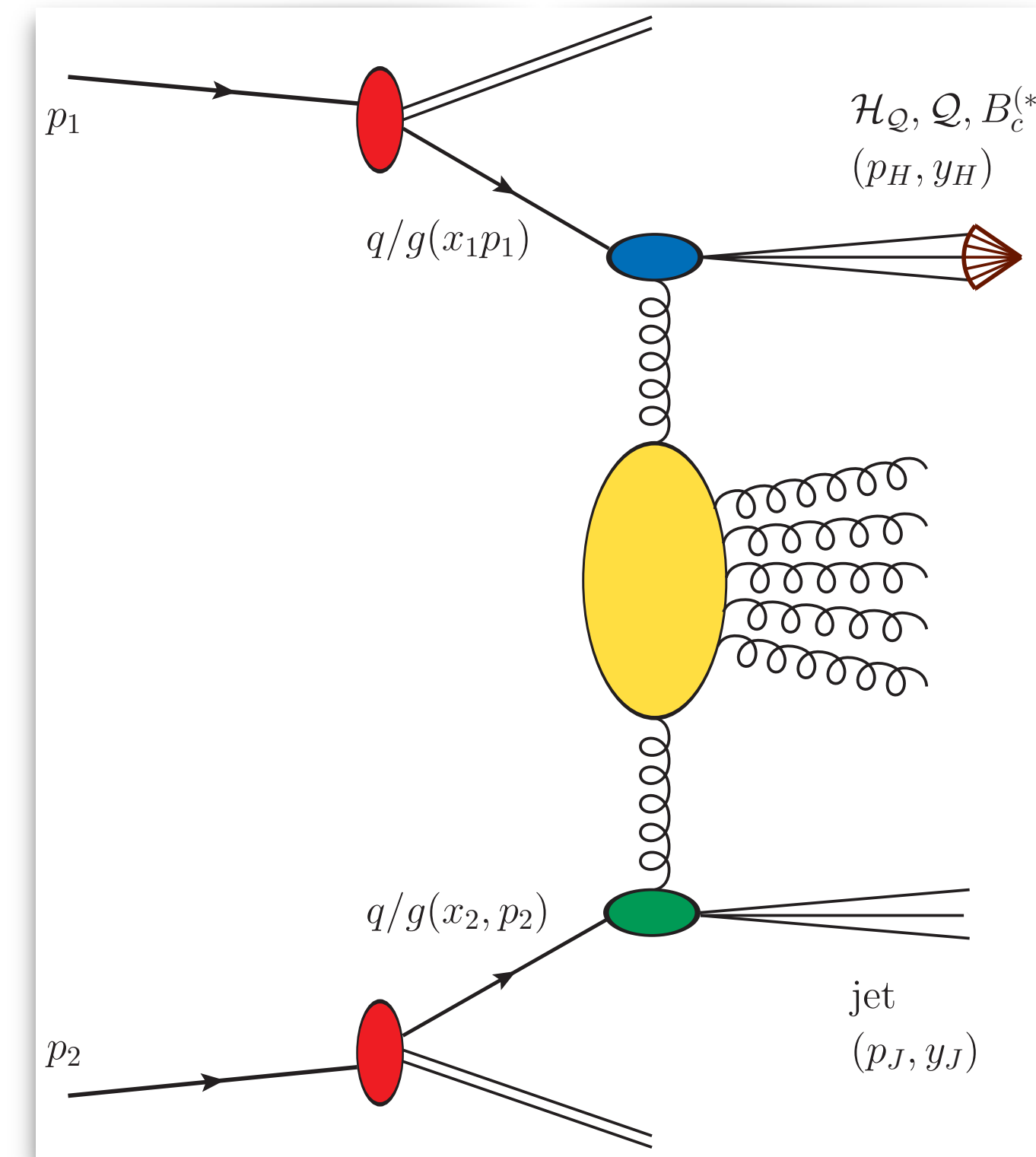
## HIGGS BOSON

Stabilizers  $\Leftrightarrow$  large Higgs transverse masses



## HEAVY FLAVOR AT LARGE $P_T$

Stabilizers  $\Leftrightarrow$  gluon fragmentation channels



(Higgs + jet, NLL/NLO\*)  $\otimes$  [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

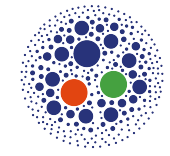
(NLO Higgs coeff. function)  $\otimes$  [F. G. C., M. Fucilla et al., JHEP 08 (2022) 092]

( $\Lambda_c^\pm$  baryons, NLL/NLO)  $\otimes$  [F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]

( $J/\psi$  or  $\Upsilon$ , NLL/NLO)  $\otimes$  [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

( $B_c^\pm(1S_0)$  or  $B_c^{*\pm}(3S_1)$ , NLL/NLO)  $\otimes$  [F. G. C., Phys. Lett. B 835 (2022) 137554]

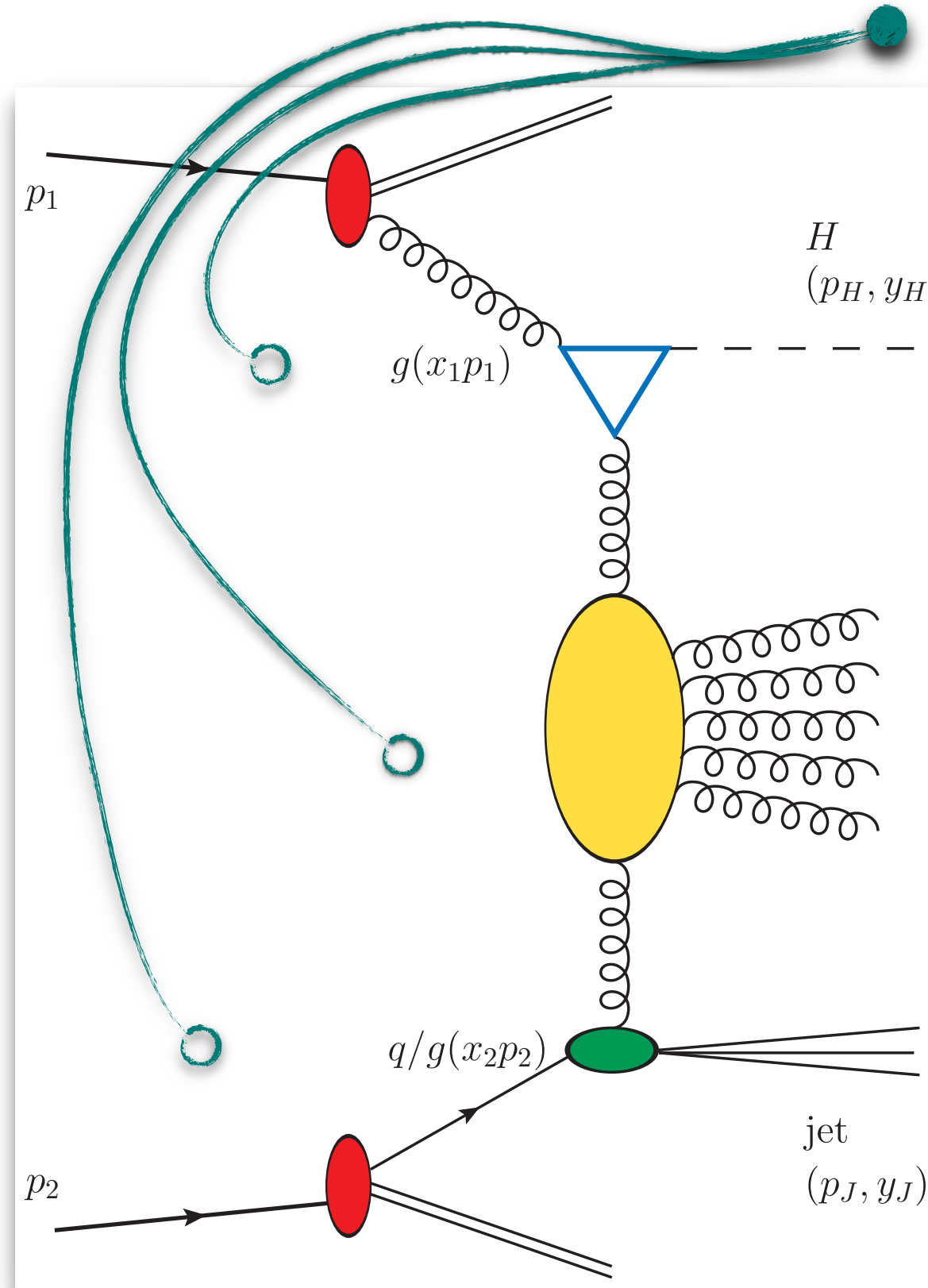
# From Mueller-Navelet to Higgs and heavy flavor



Pheno path: hunt for channels leading to a NLL **stabilization pattern** at **natural scales** (!)

## HIGGS BOSON

Stabilizers  $\Leftrightarrow$  large Higgs **transverse masses**



$$\mu_{F,R} \sim M_{H,\perp}$$

**NLO\***

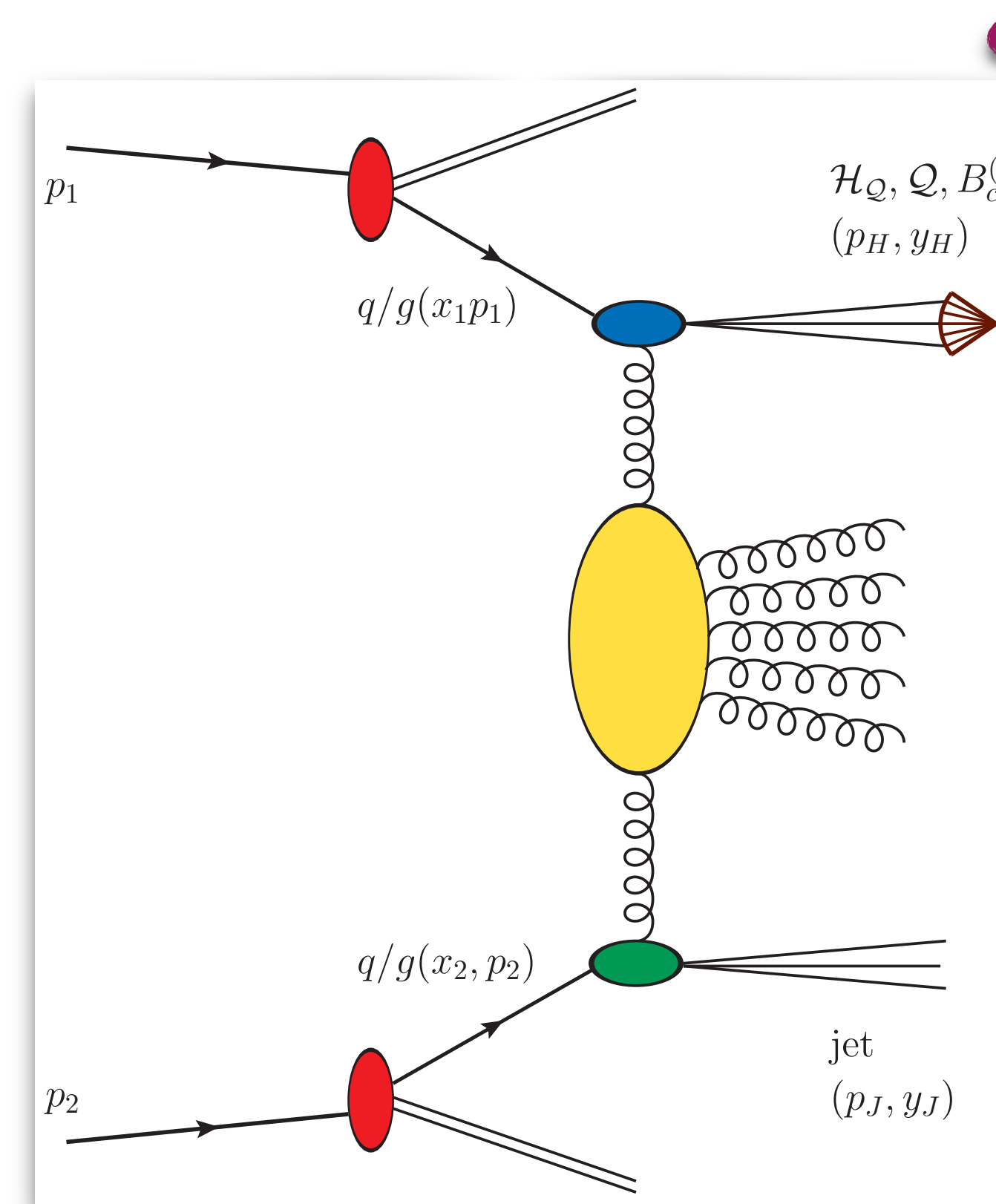
$$= \text{LO} + \text{NLO}_{\text{RGE}}$$

**NLL**

**NLO**

## HEAVY FLAVOR AT LARGE $P_T$

Stabilizers  $\Leftrightarrow$  **gluon fragmentation** channels



**NLO(+)**

**NLL**

**NLO(+)**

(Higgs + jet, NLL/NLO\*)  $\otimes$  [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

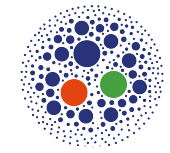
(NLO Higgs coeff. function)  $\otimes$  [F. G. C., M. Fucilla et al., JHEP 08 (2022) 092]

( $\Lambda_c^\pm$  baryons, NLL/NLO)  $\otimes$  [F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]

( $J/\psi$  or  $\Upsilon$ , NLL/NLO)  $\otimes$  [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

( $B_c^\pm(1S_0)$  or  $B_c^{*\pm}(3S_1)$ , NLL/NLO)  $\otimes$  [F. G. C., Phys. Lett. B 835 (2022) 137554]

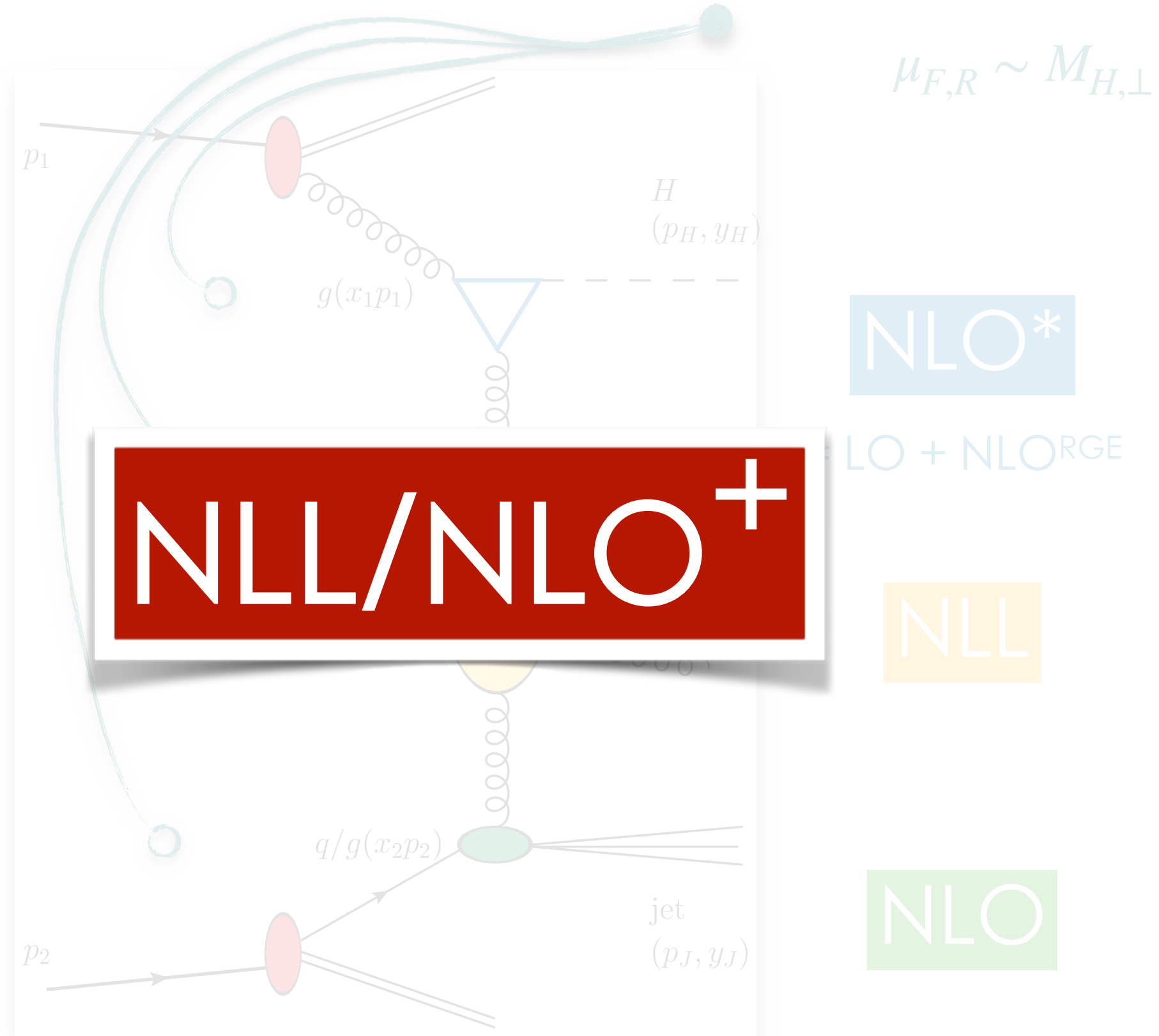
# From Mueller-Navelet to Higgs and heavy flavor



Pheno path: hunt for channels leading to a NLL **stabilization pattern** at **natural scales** (!)

## HIGGS BOSON

Stabilizers  $\Leftrightarrow$  large Higgs transverse masses

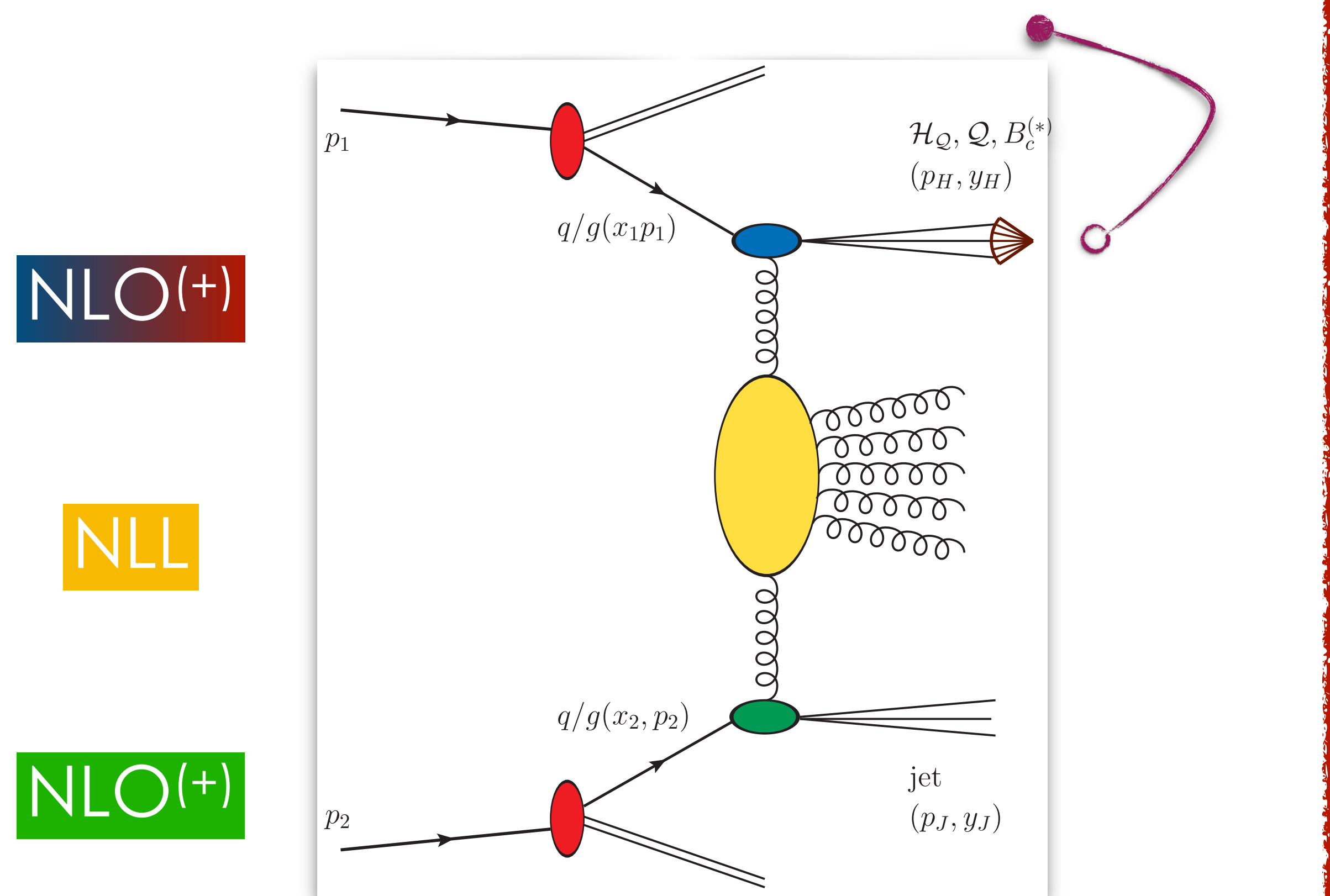


(Higgs + jet, NLL/NLO\*)  $\otimes$  [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

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## HEAVY FLAVOR AT LARGE $P_T$

Stabilizers  $\Leftrightarrow$  gluon fragmentation channels



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( $J/\psi$  or  $\Upsilon$ , NLL/NLO)  $\otimes$  [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

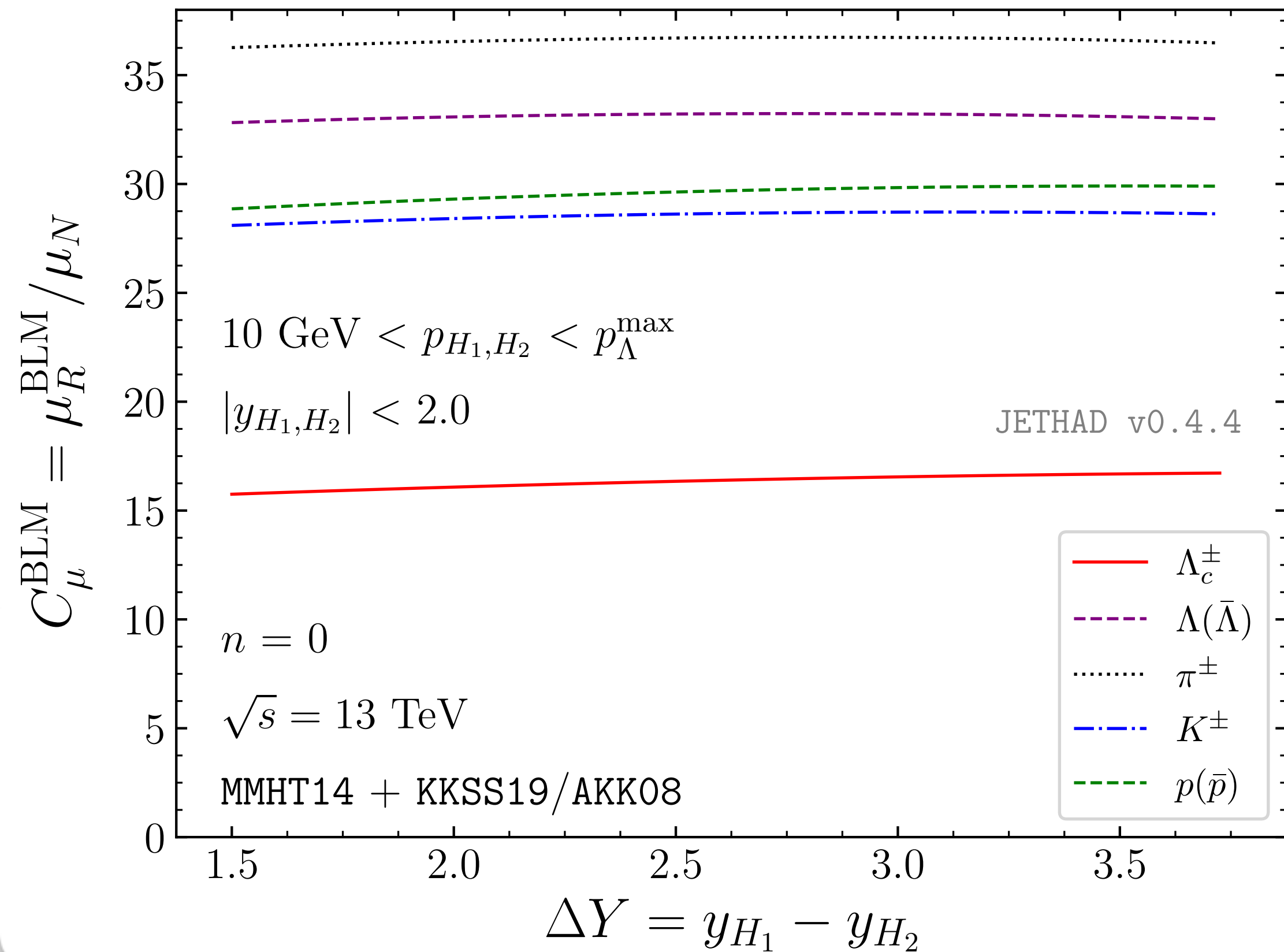
( $B_c^\pm(1S_0)$  or  $B_c^{*\pm}(3S_1)$ , NLL/NLO)  $\otimes$  [F. G. C., Phys. Lett. B 835 (2022) 137554]



# Heavy flavor at the LHC: BLM scales

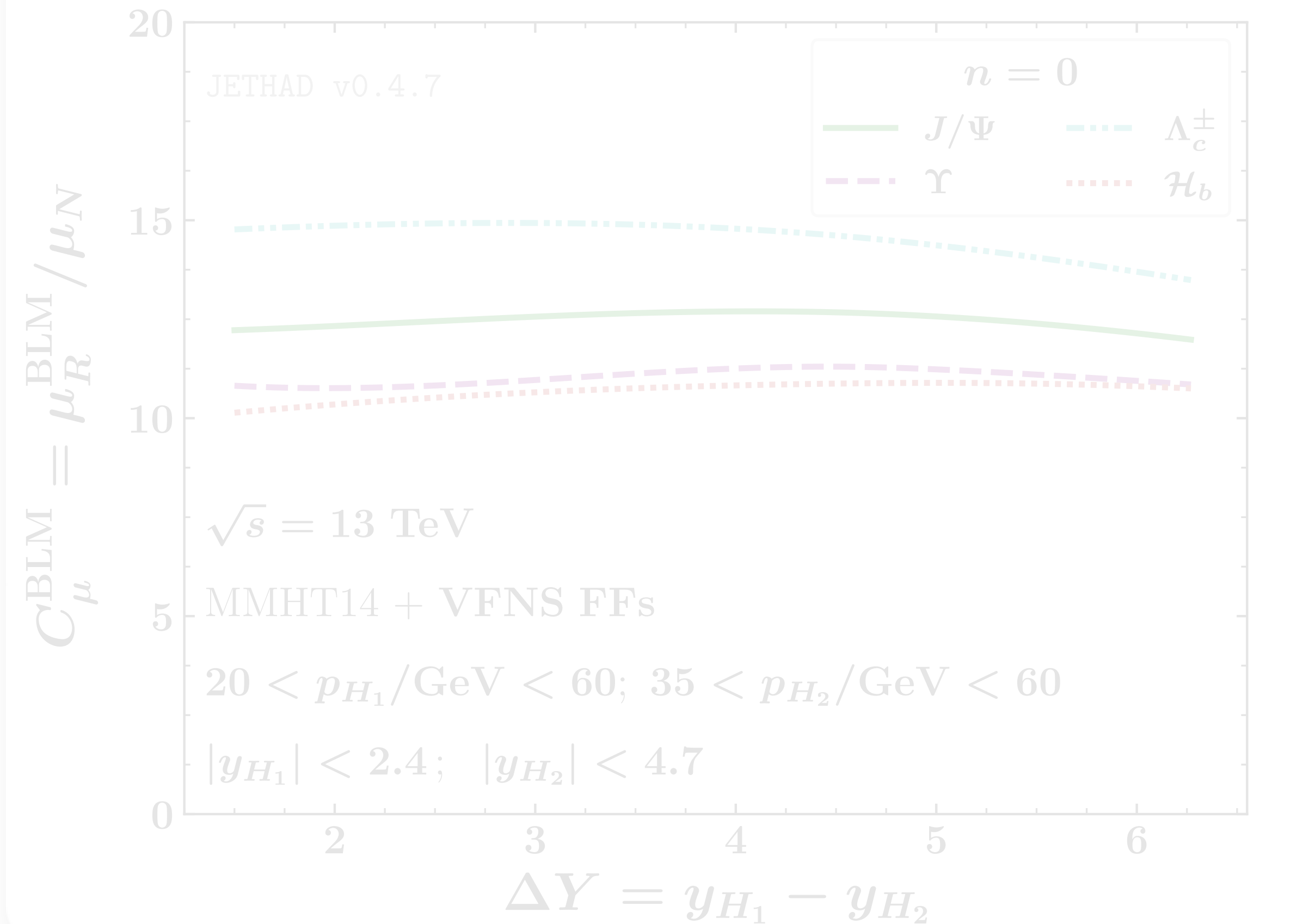
## Heavy-light hadrons

$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow H(p_{H_1}, y_{H_1}) + X + H(p_{H_2}, y_{H_2})$$



## Vector quarkonia

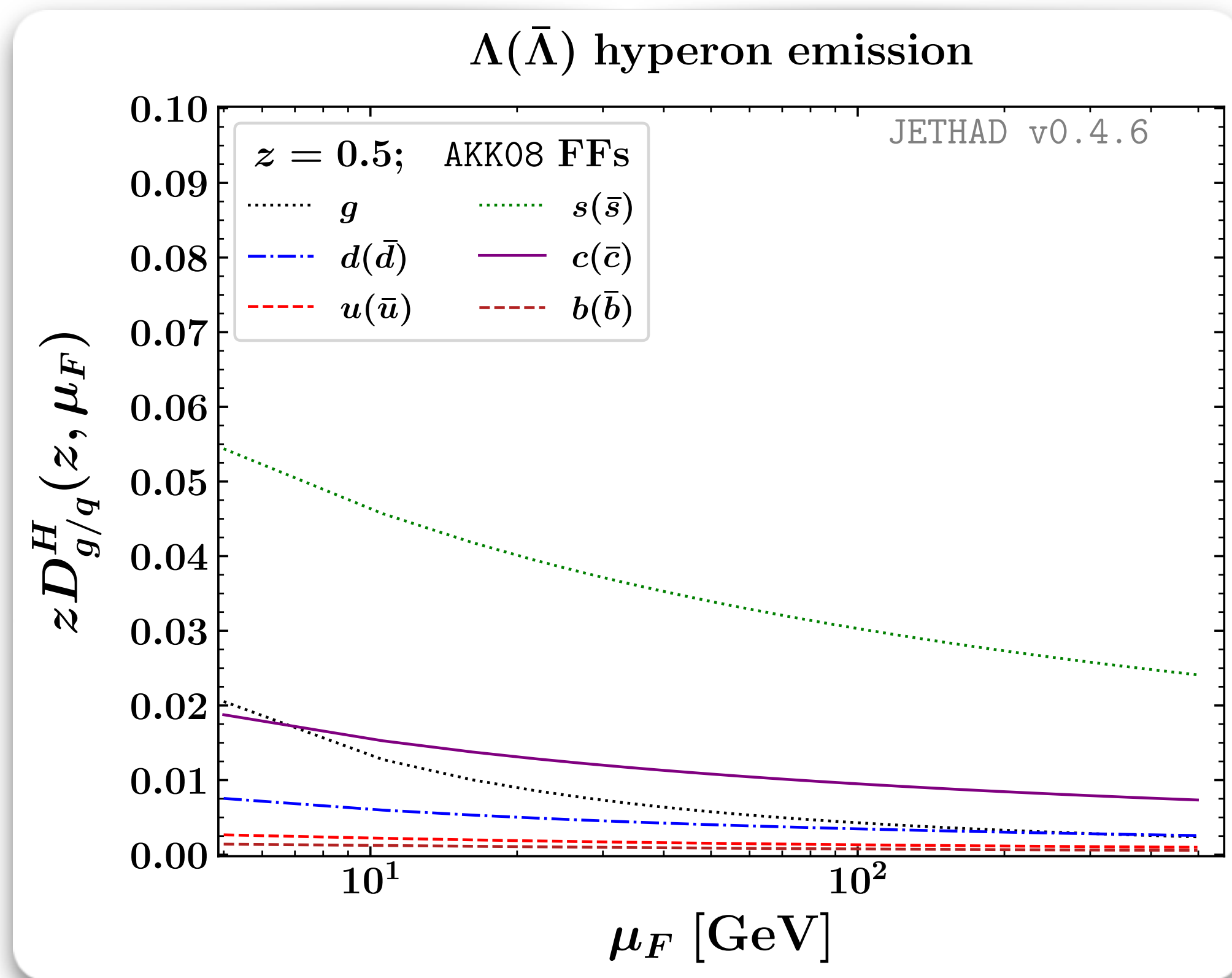
$$p(P_a) + p(P_b) \rightarrow H(p_{H_1}, y_{H_1}) + X + H(p_{H_2}, y_{H_2})$$



# Stabilizing effects of heavy-flavor fragmentation

 **AKK08** VFNS collinear FFs for  $\Lambda$  hyperon:  $|uds\rangle$

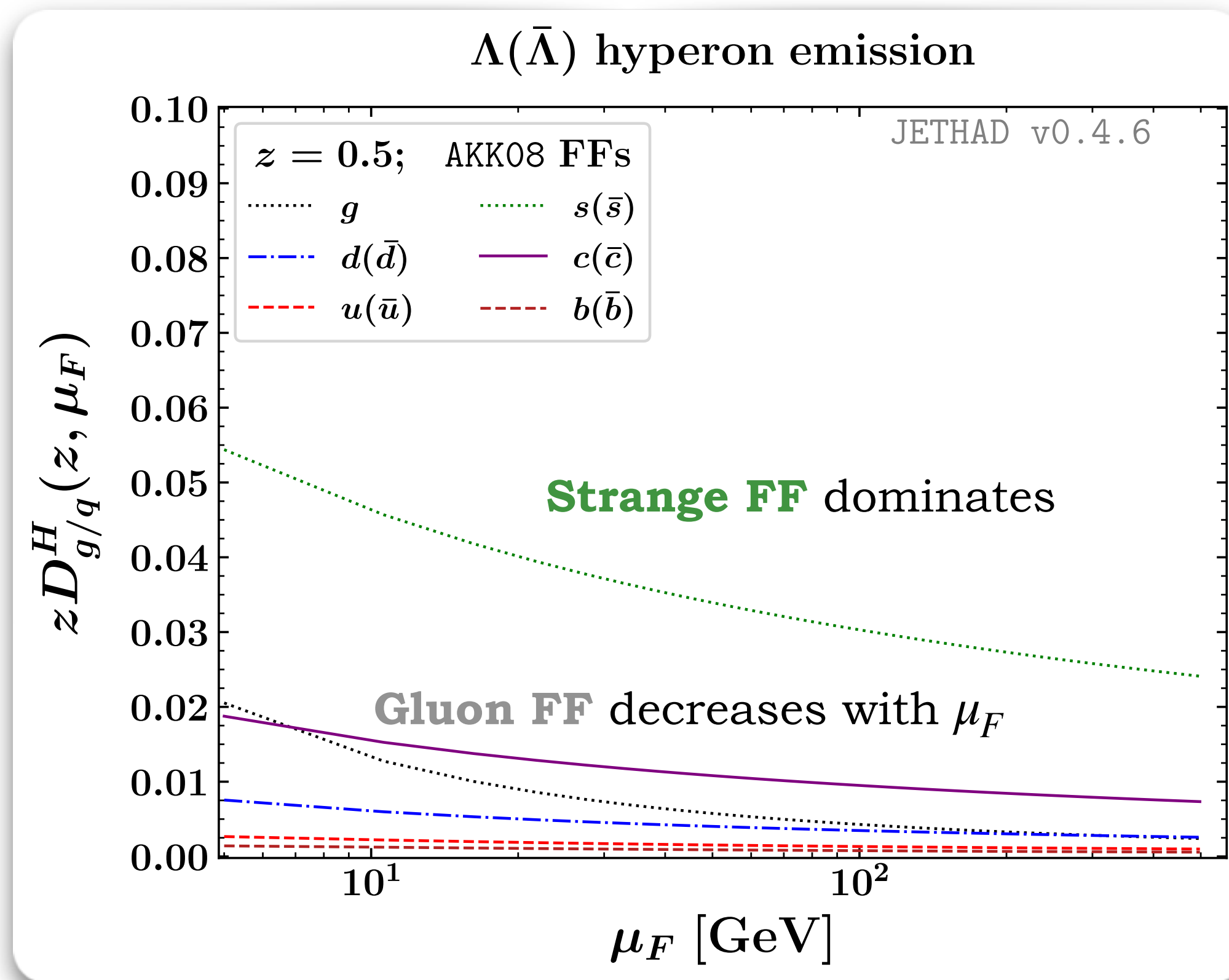
 [S. Albino et al., Nucl. Phys. B 803 (2008) 42-104]



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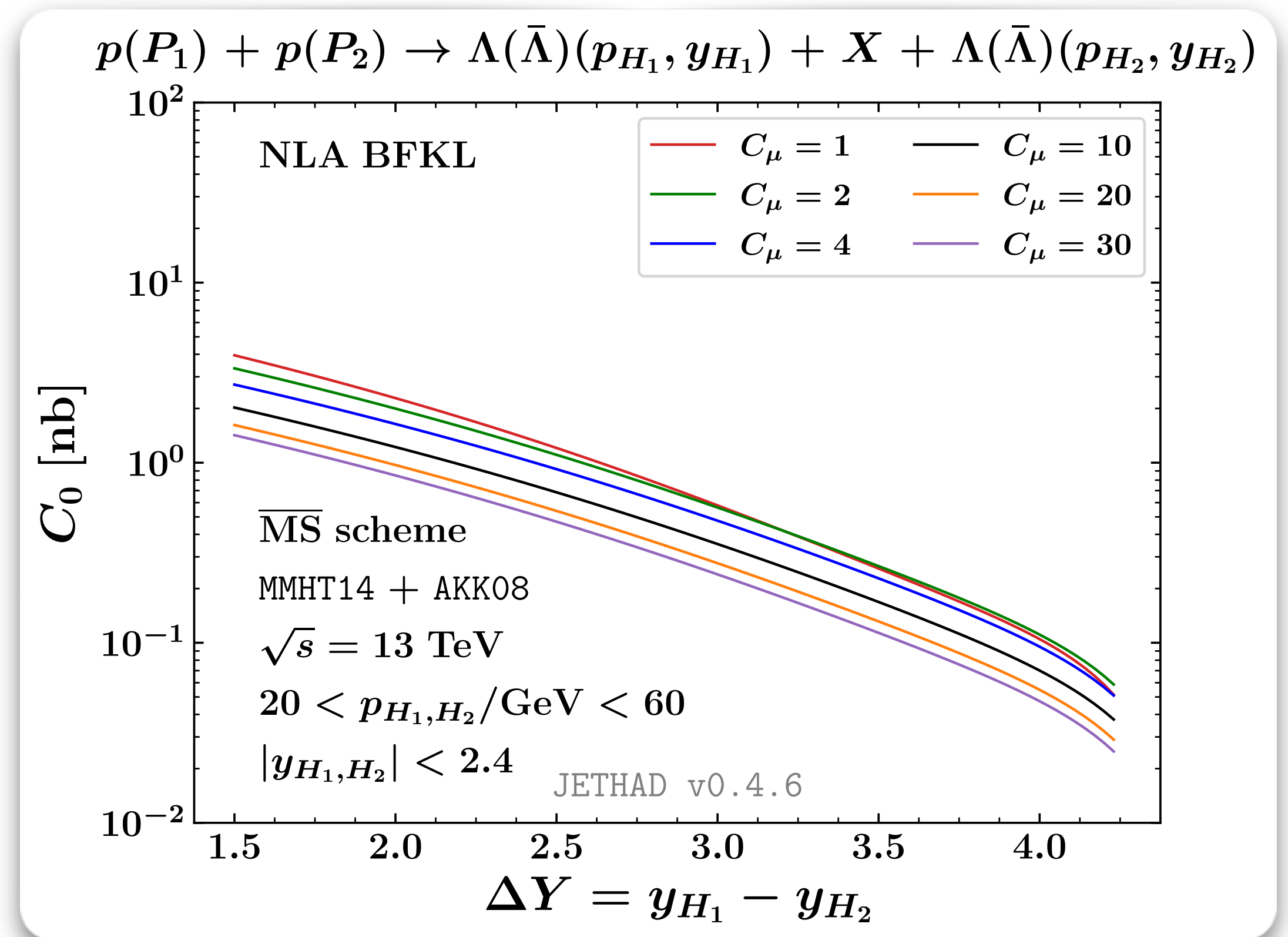
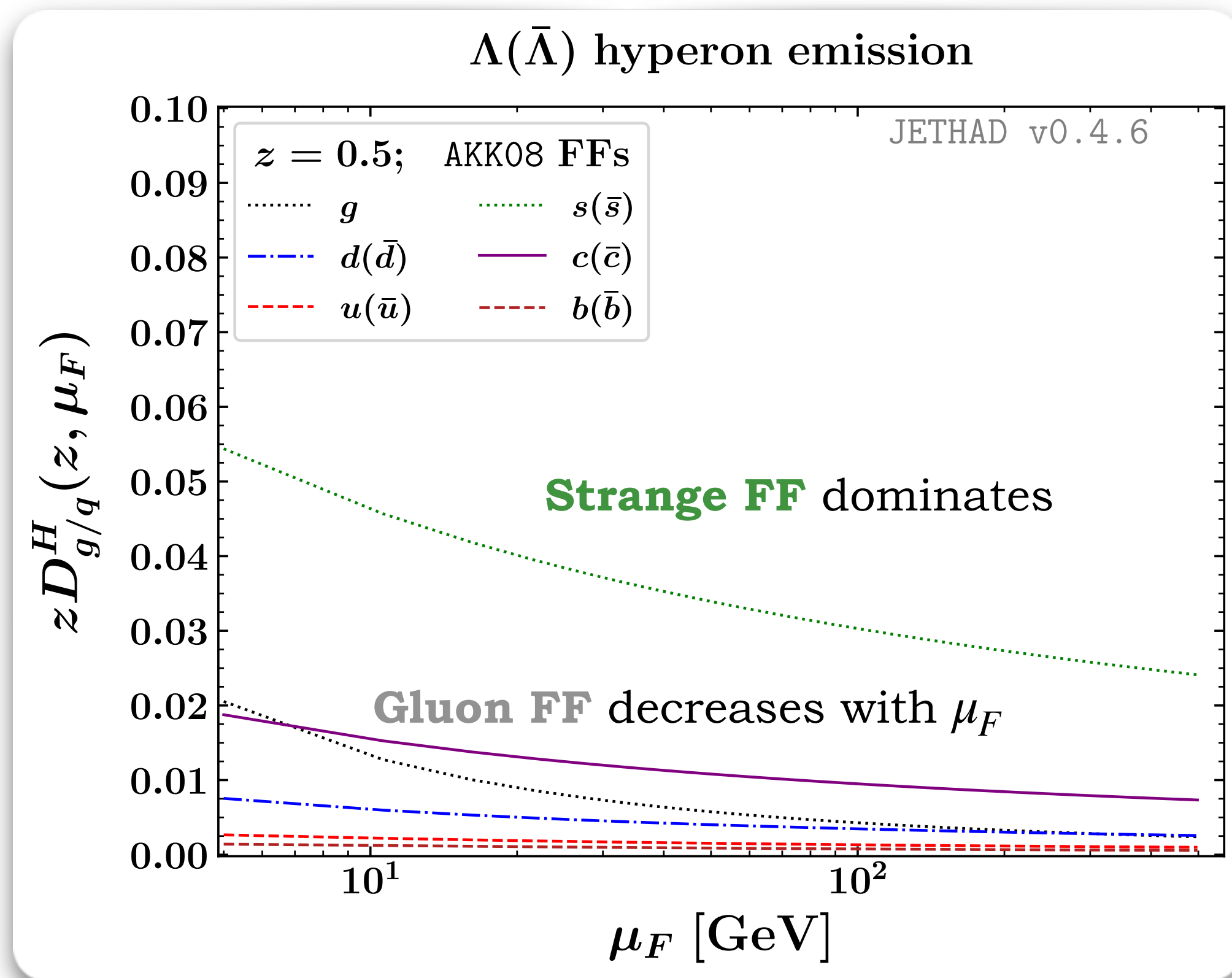
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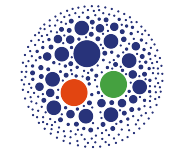
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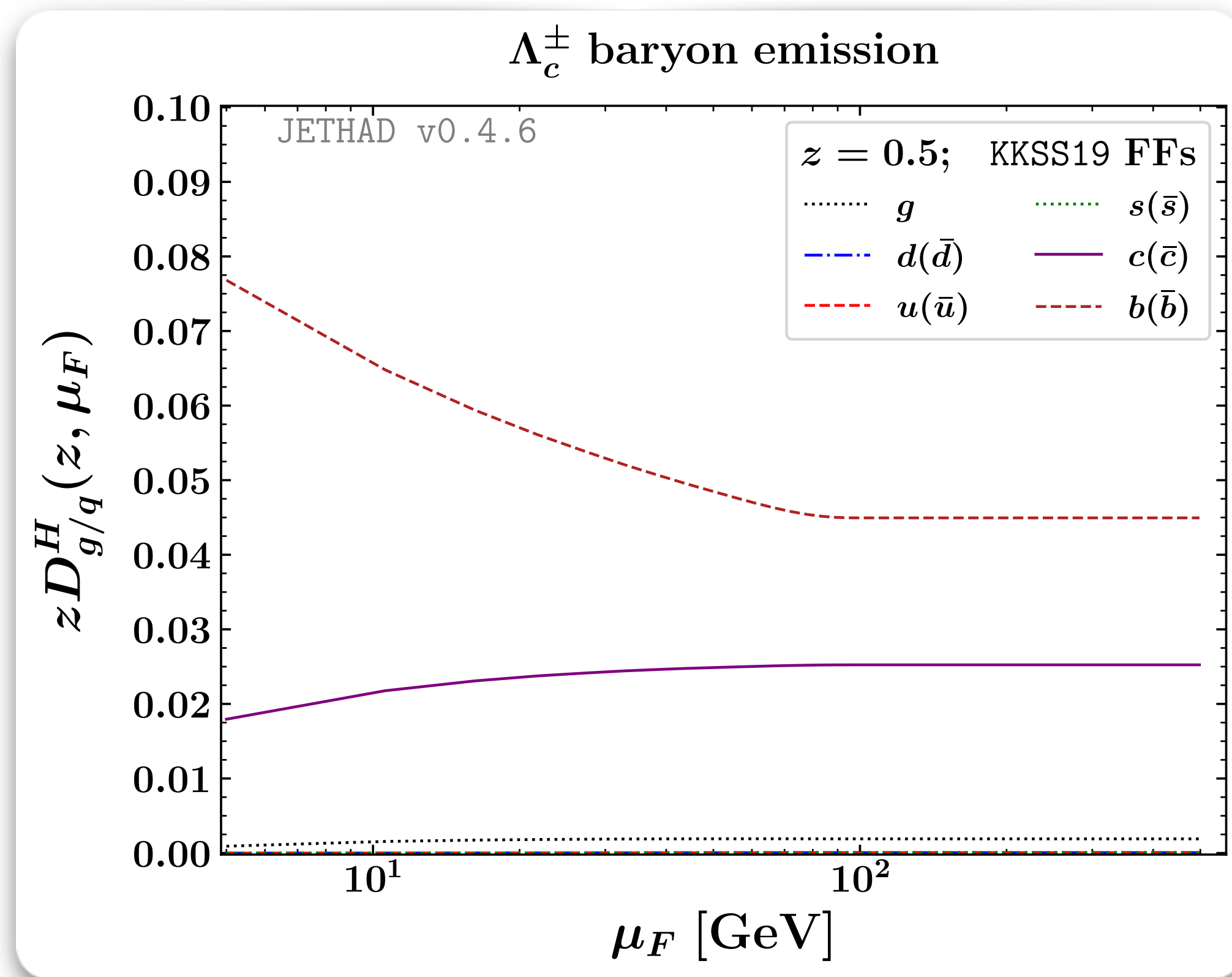
 Rapidity distribution **sensitive** to scale variations

# Stabilizing effects of heavy-flavor fragmentation

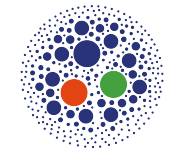


**KKSS19** VFNS collinear FFs for  $\Lambda_c$  baryons:  $|udc\rangle$

[\[B. A. Kniehl et al., Phys. Rev. D 101 \(2020\) 11, 114021\]](#)

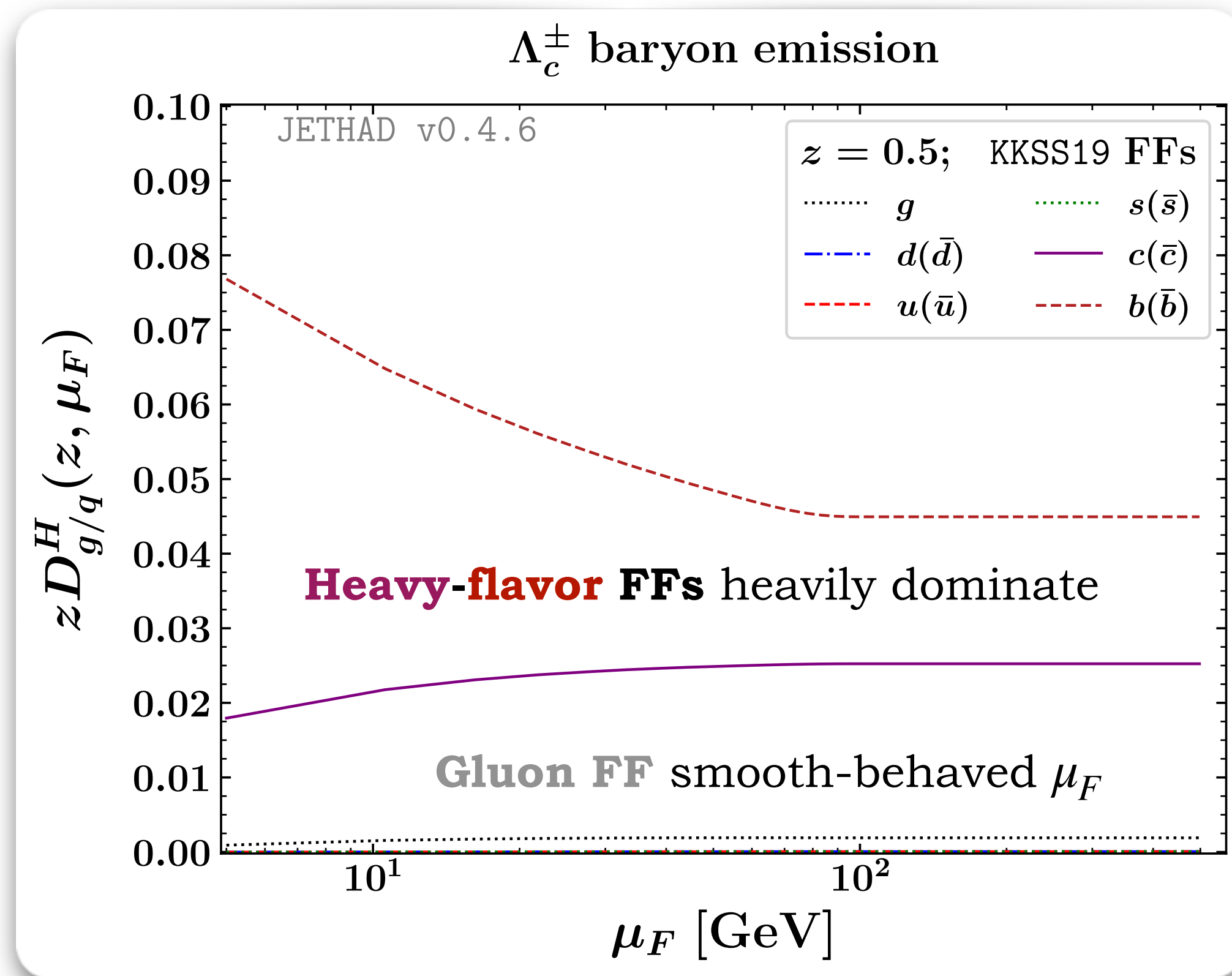


# Stabilizing effects of heavy-flavor fragmentation

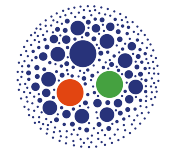


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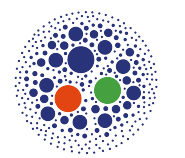
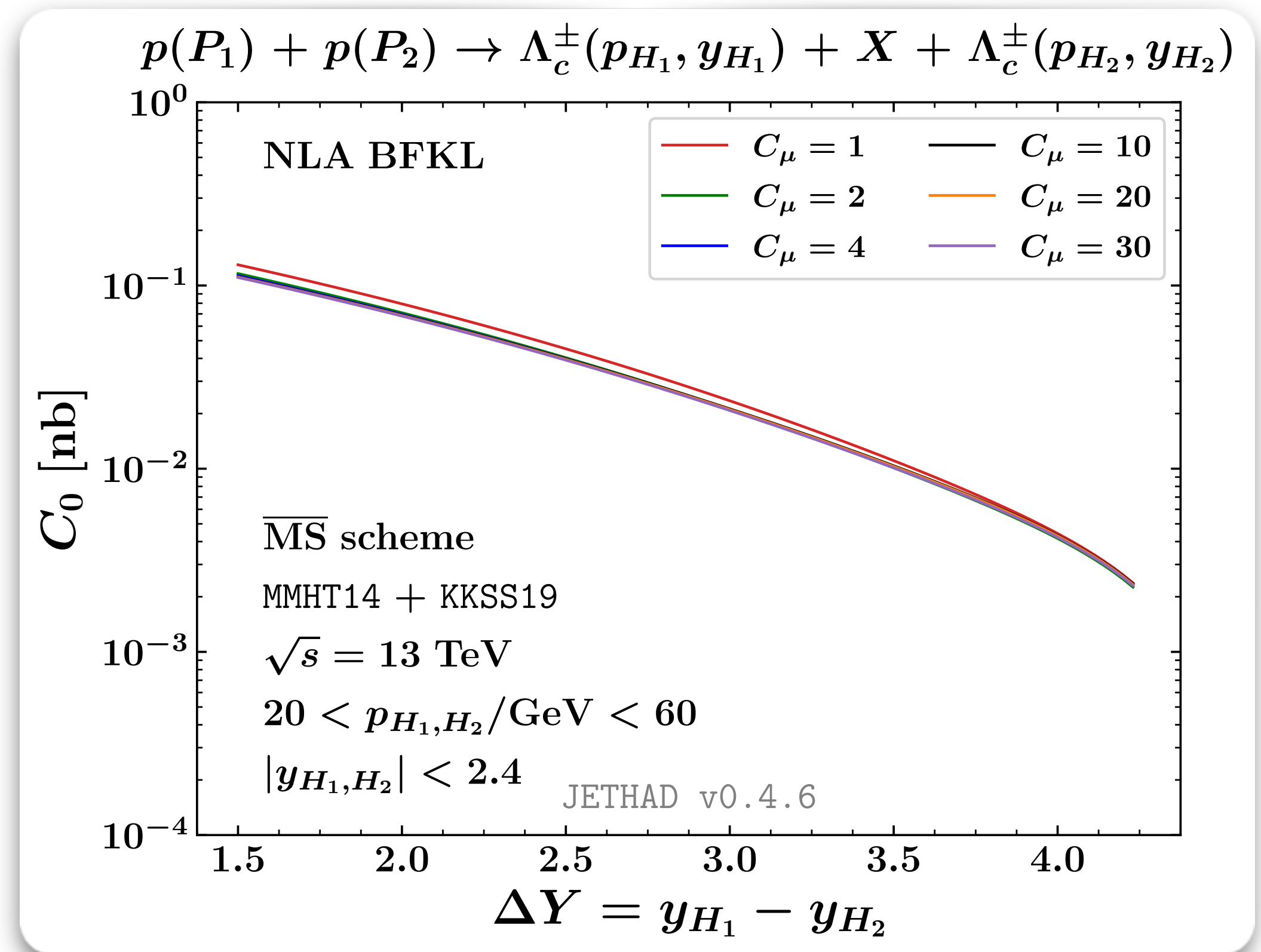
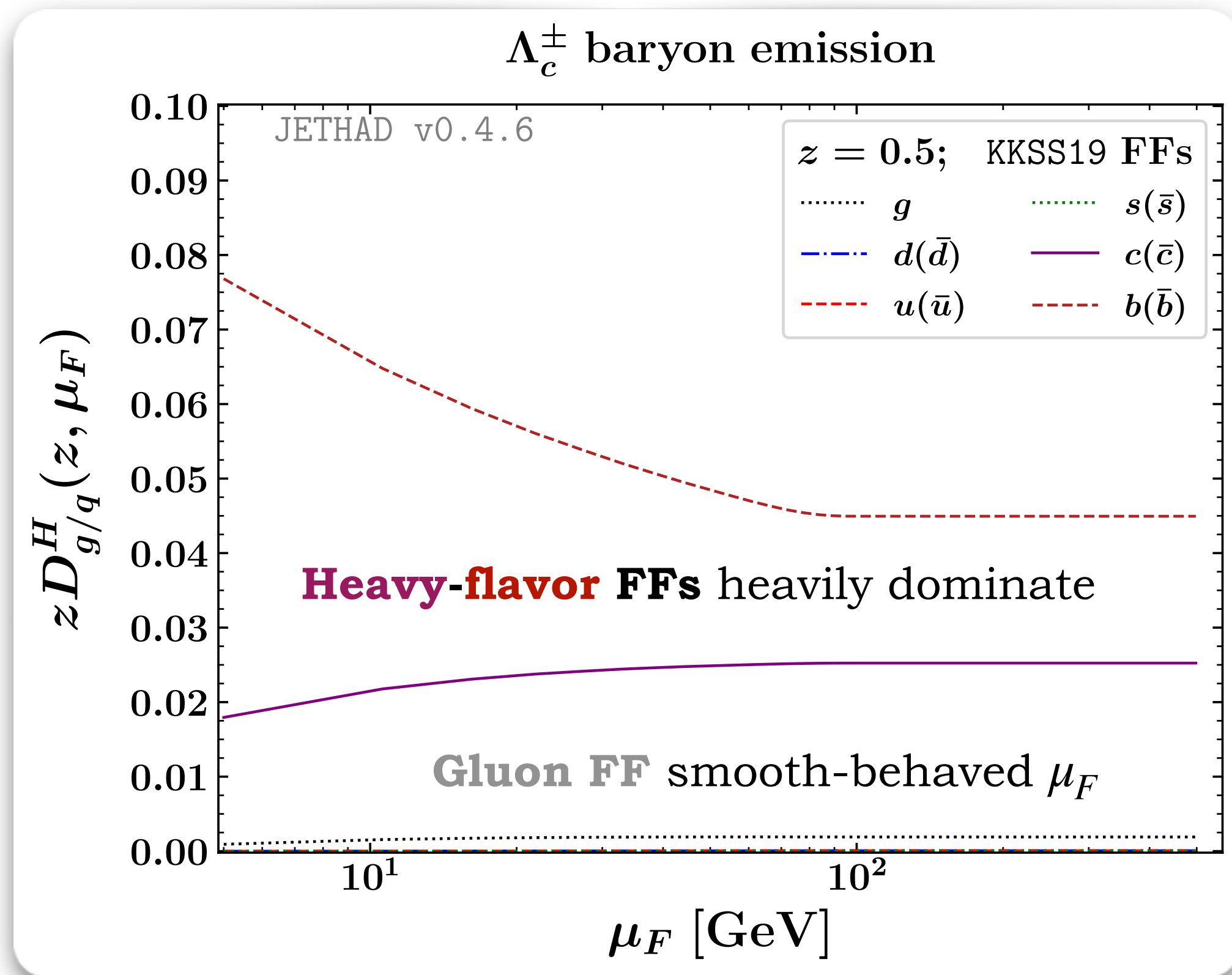


# Stabilizing effects of heavy-flavor fragmentation



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Rapidity distribution **stable** under scale variations

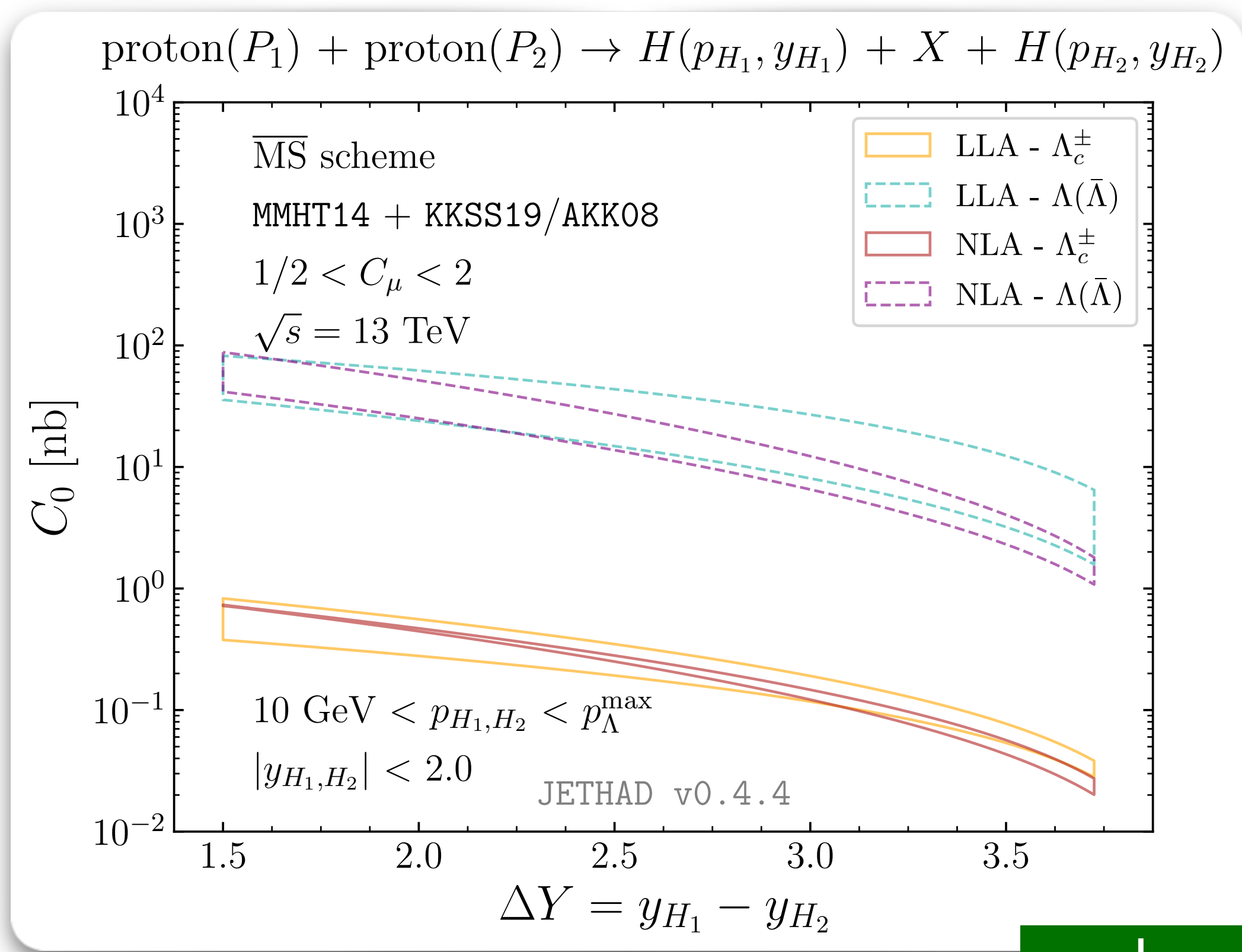
( $B_c^{(*)}$  hadrons) [F. G. C., Phys. Lett. B 835 (2022) 137554]

( $\Lambda_c$  baryons, in this slide) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 8, 780]

( $H_b$  hadrons) [F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]

# Stability under scale variations & NLL corrections

Hybrid factorization @work:  $\Lambda_c$  baryons  $|udc\rangle$  versus  $\Lambda$  hyperons  $|uds\rangle$

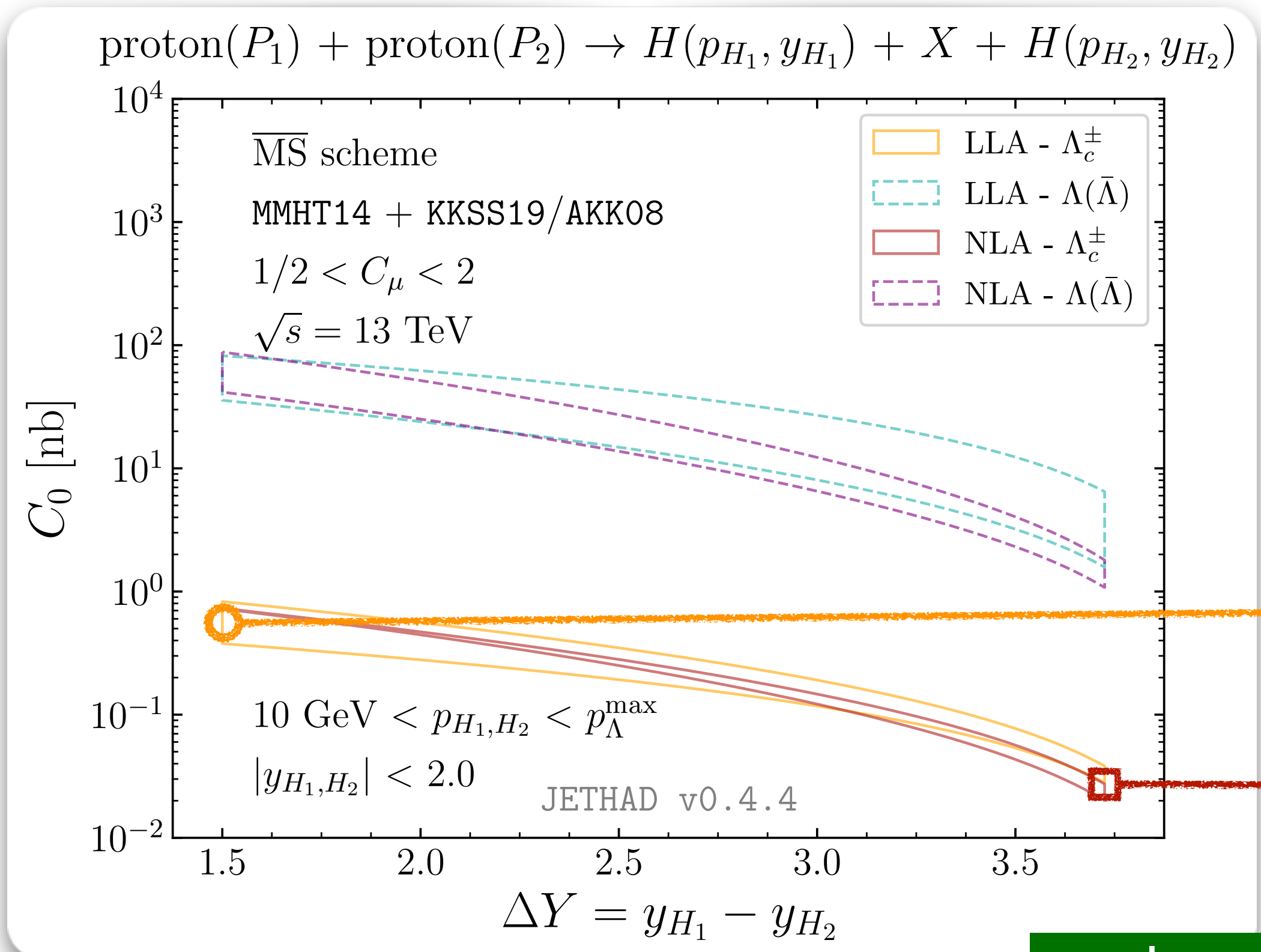


natural



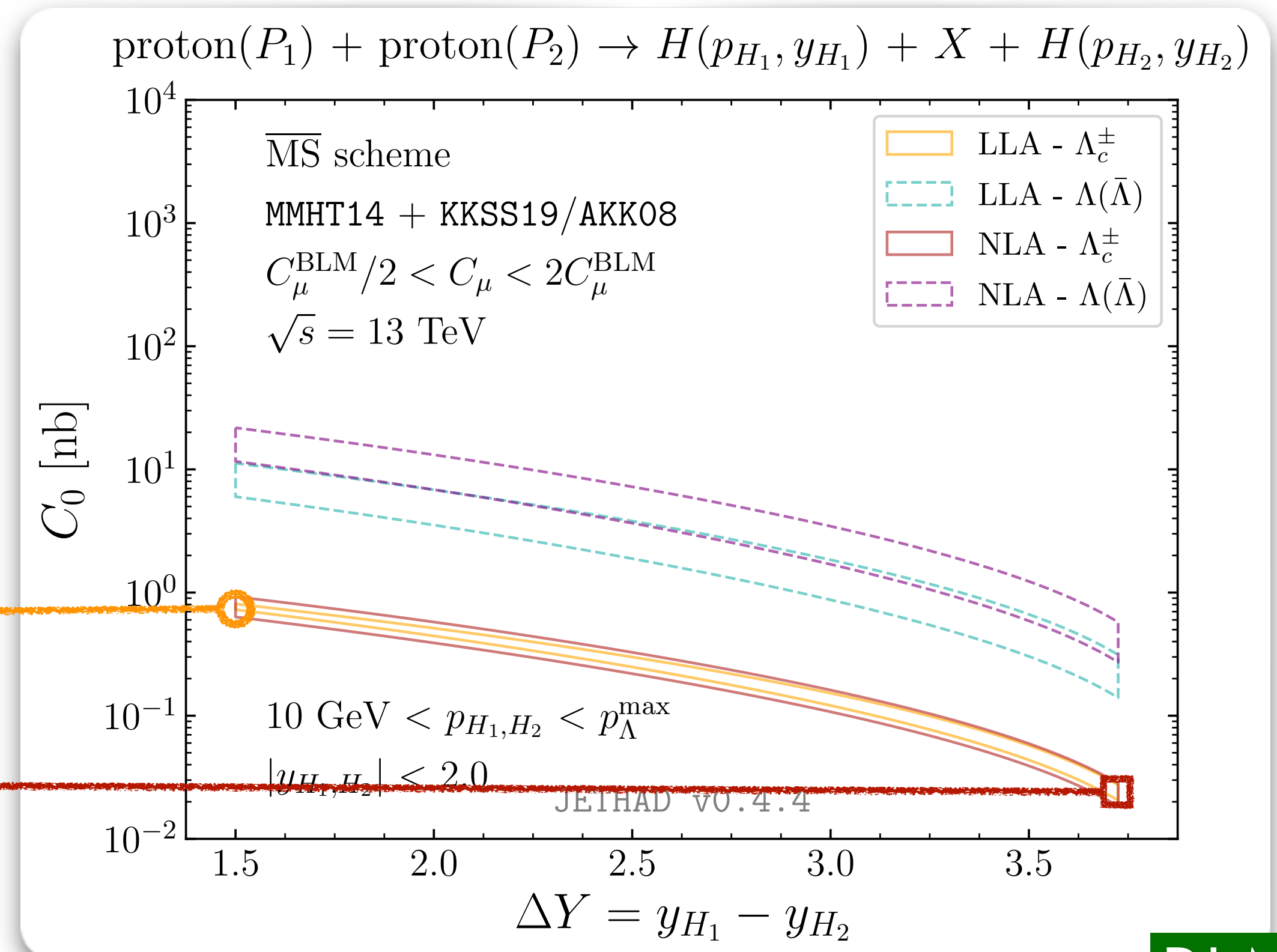
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Hybrid factorization @work:  $\Lambda_c$  baryons  $|udc\rangle$  versus  $\Lambda$  hyperons  $|uds\rangle$



LL  $\Lambda_c$   
NLL  $\Lambda_c$

natural



BLM

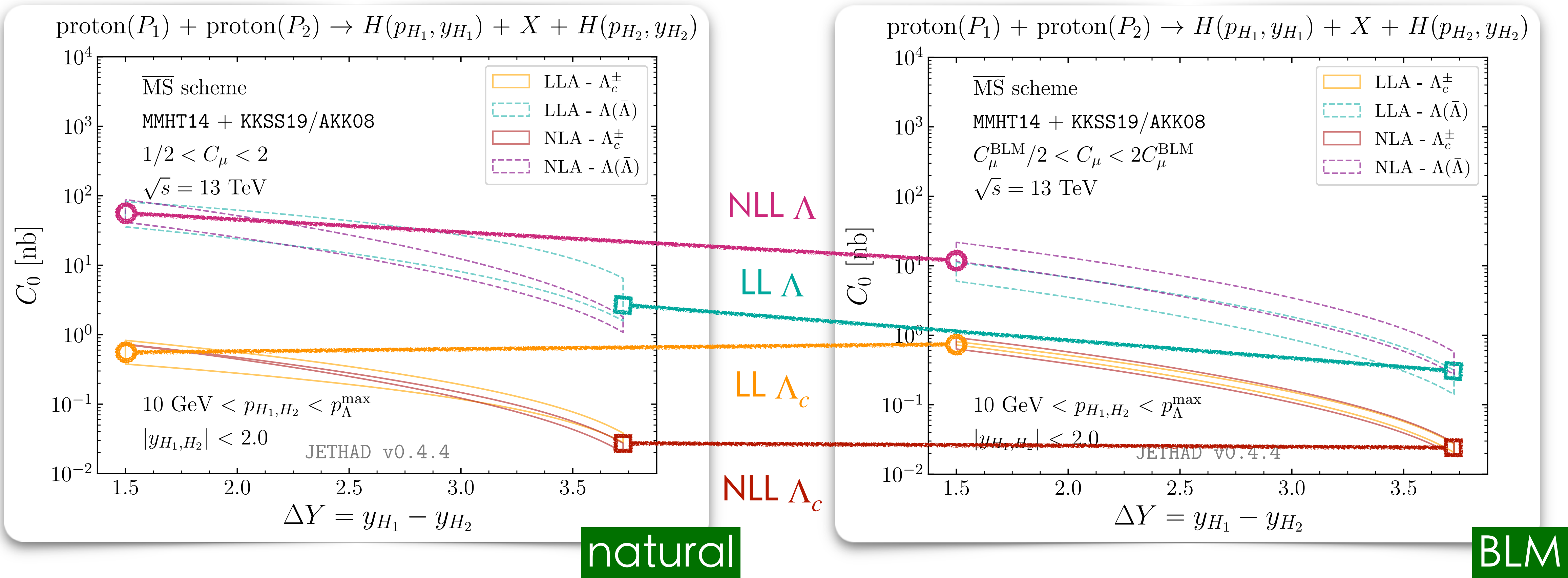
NLL corrections: rapidity distribution **stable** for  $\Lambda_c$

( $\Lambda_c$  baryons, in this slide) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 8, 780]

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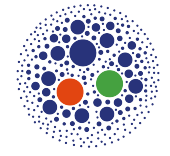


NLL corrections: rapidity distribution **stable** for  $\Lambda_c$ , loses  $\sim 10^1$  magnitude for  $\Lambda$

( $\Lambda_c$  baryons, in this slide) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 8, 780]

( $H_b$  hadrons) [F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]

# Stabilizing effects of heavy-flavor fragmentation

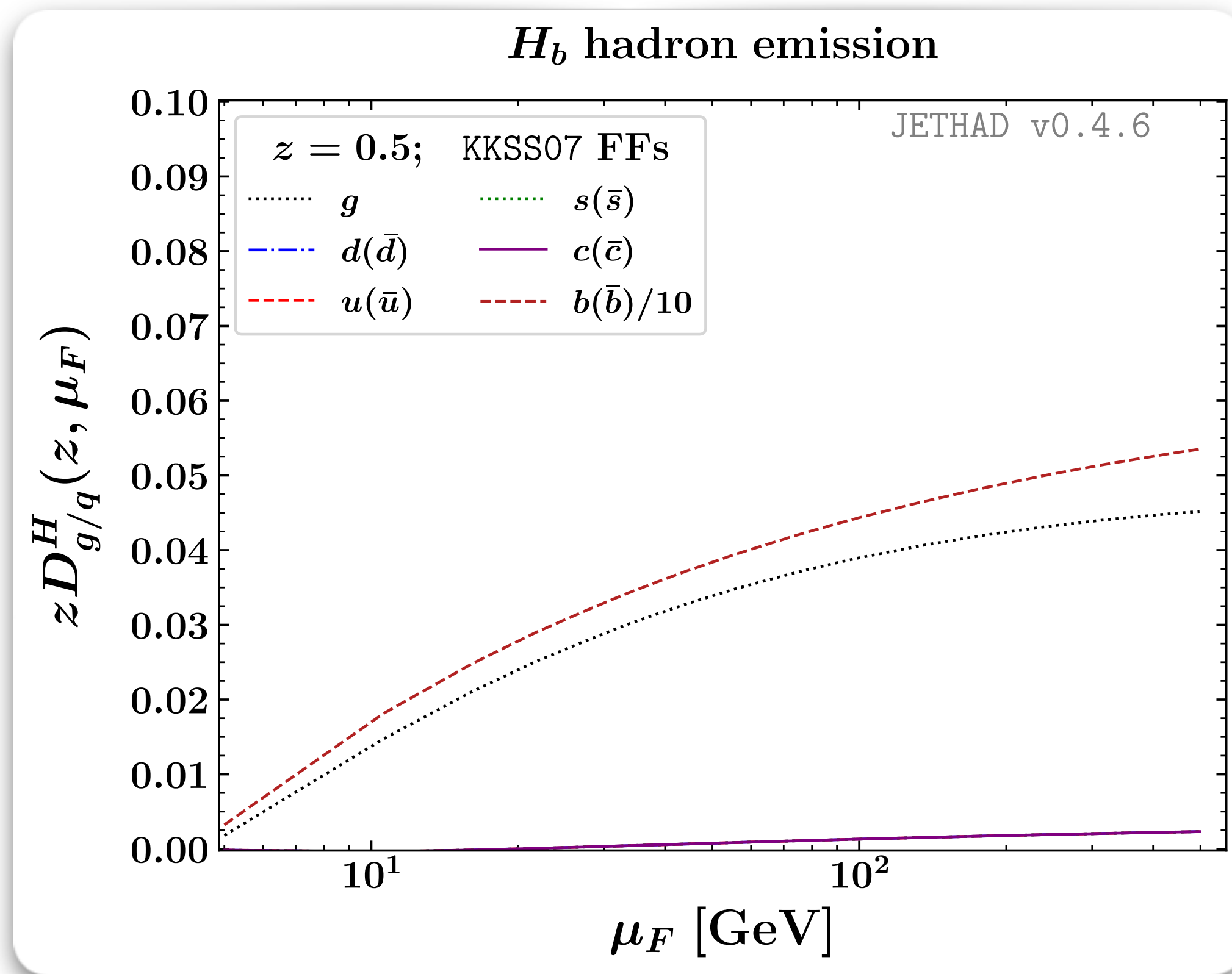


**KKSS07** VFNS collinear FFs for:

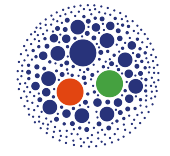
$$H_b = B^\pm, B^0, B_s^0, \Lambda_b$$

[\[B. A. Kniehl, H. Spiesberger, Phys. Rev. D 98 \(2018\) 11, 114010\]](#)

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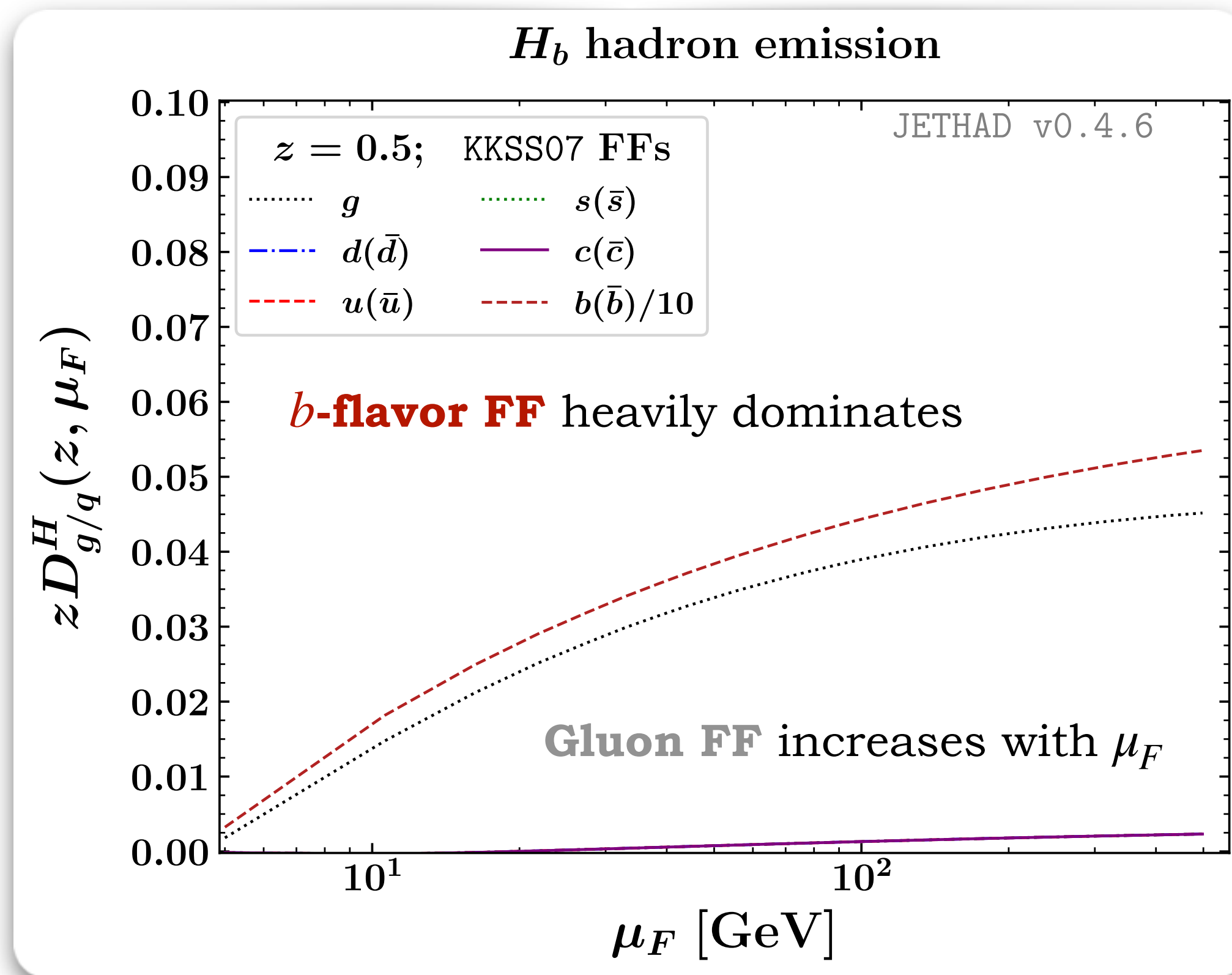


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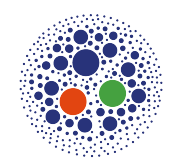
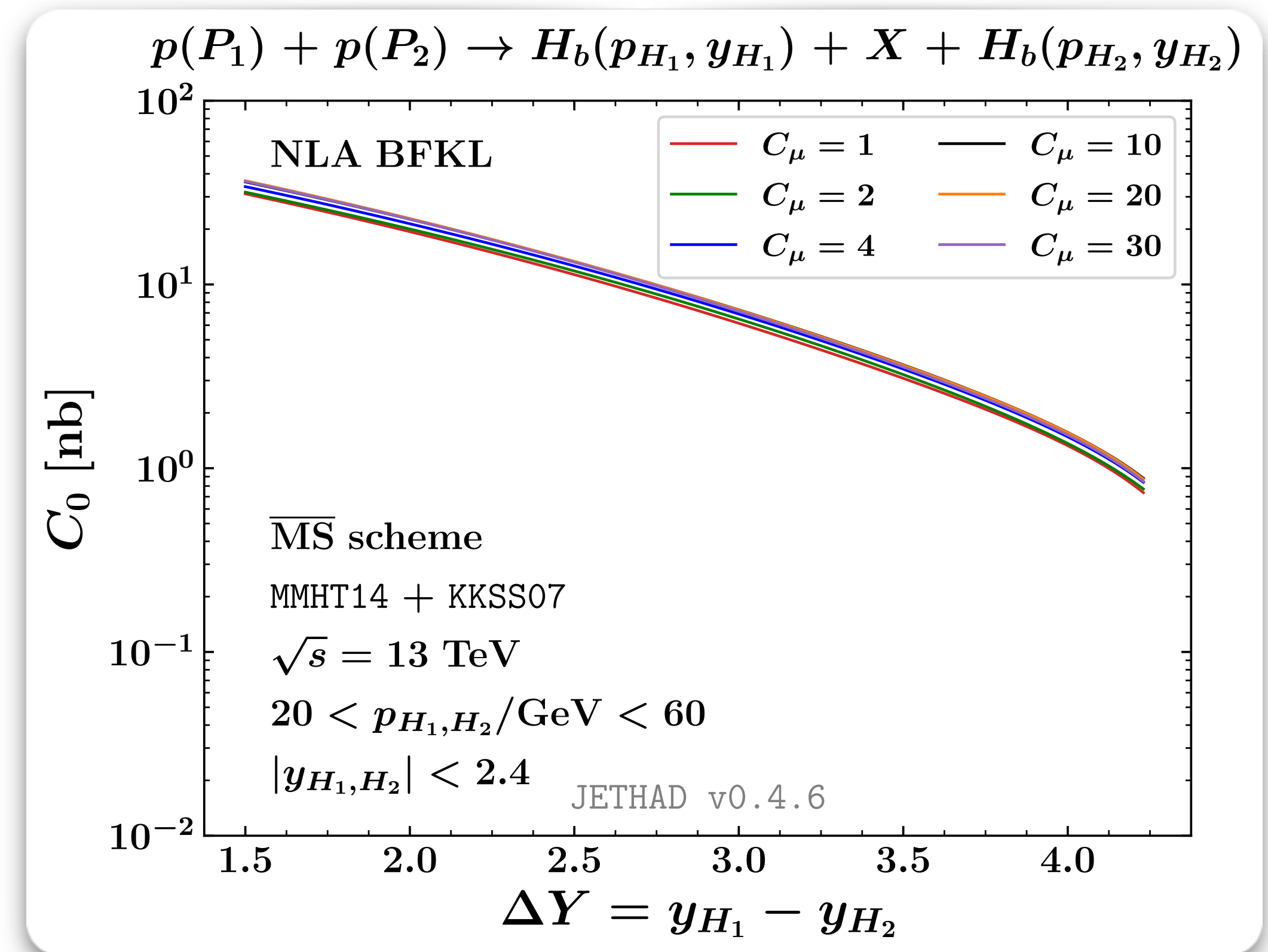
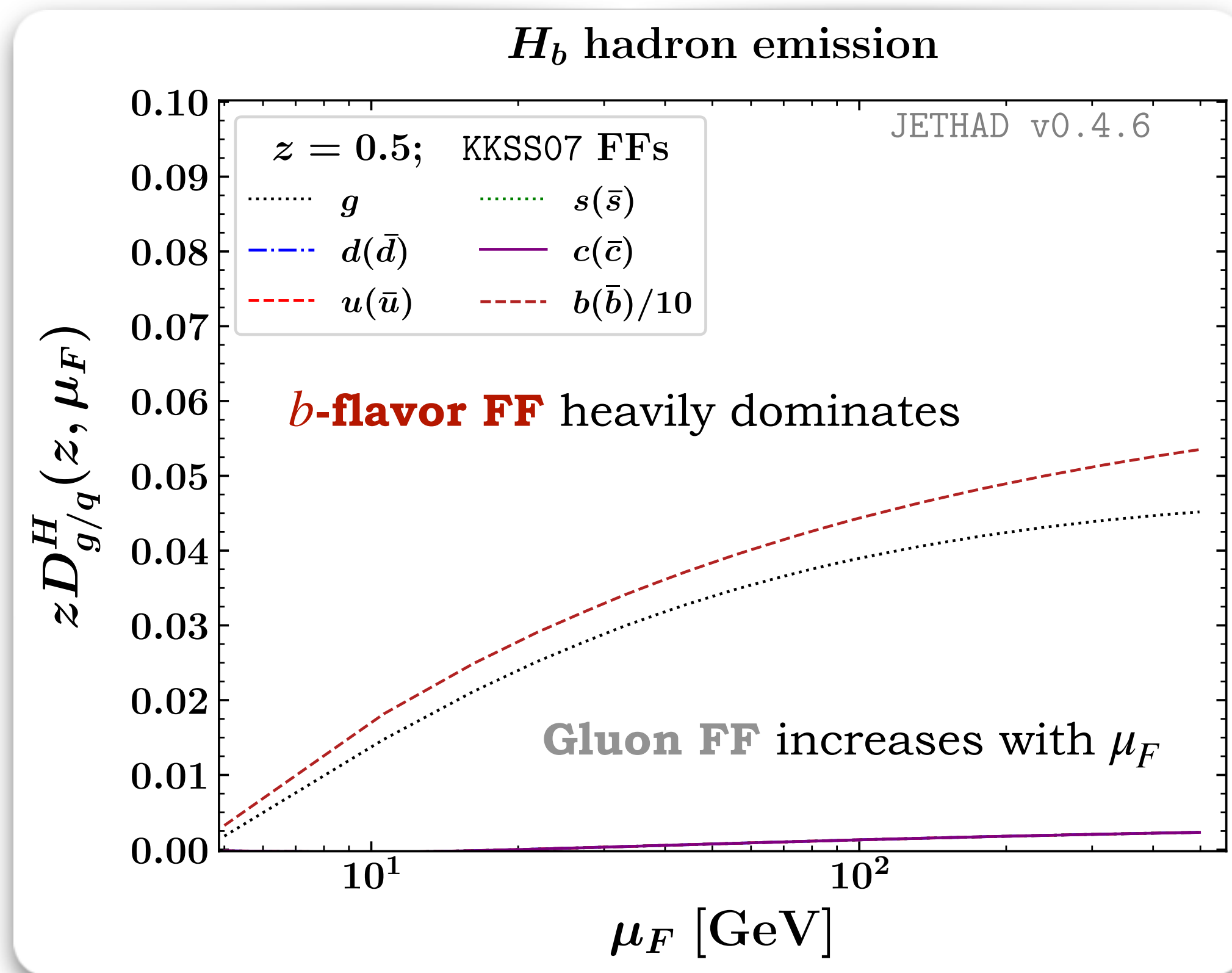


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Rapidity distribution **very stable** under scale variations

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# Stabilizing effects of heavy-flavor fragmentation

 Stabilization mechanism encoded in the heavy-flavor **gluon FF**

 Forward-hadron LO impact factor  $\Rightarrow$  **gluon FF** enhanced by **gluon PDF** in collinear convolution

$$c_{\Lambda}(n, \nu, |\vec{p}|, x) = 2\sqrt{\frac{C_F}{C_A}} (|\vec{p}|^2)^{i\nu-1/2} \int_x^1 \frac{dz}{z} \left(\frac{z}{x}\right)^{2i\nu-1} \left[ \frac{C_A}{C_F} f_g(z) D_g^{\Lambda} \left(\frac{x}{z}\right) + \sum_{a=q, \bar{q}} f_a(z) D_a^{\Lambda} \left(\frac{x}{z}\right) \right]$$

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Forward-hadron NLO impact factor  $\Rightarrow$  a **non-diagonal heavy-flavor** channel open...

$$c_1^{(1)}(n, \nu, |\vec{k}_1|, \alpha_1) = 2\sqrt{\frac{C_F}{C_A}} (\vec{k}_1^2)^{i\nu-\frac{1}{2}} \frac{1}{2\pi} \int_{\alpha_1}^1 \frac{dx}{x} \int_{\frac{\alpha_1}{x}}^1 \frac{d\zeta}{\zeta} \left(\frac{x\zeta}{\alpha_1}\right)^{2i\nu-1} \\ \times \left[ \frac{C_A}{C_F} f_g(x) D_g^h \left(\frac{\alpha_1}{x\zeta}\right) C_{gg}(x, \zeta) + \sum_{a=q,\bar{q}} f_a(x) D_a^h \left(\frac{\alpha_1}{x\zeta}\right) C_{qq}(x, \zeta) \right. \\ \left. + D_g^h \left(\frac{\alpha_1}{x\zeta}\right) \sum_{a=q,\bar{q}} f_a(x) C_{qg}(x, \zeta) + \frac{C_A}{C_F} f_g(x) \sum_{a=q,\bar{q}} D_a^h \left(\frac{\alpha_1}{x\zeta}\right) C_{gq}(x, \zeta) \right] \quad \dots \text{but } |C_{gg}| \sim 50 \div 10^4 |C_{gq}|$$

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Gluon FF rises with energy  $\Rightarrow$  this **compensates** PDF and BFKL kernel decreasing behavior



¿ Is natural stability robust ?



# Is the natural stability robust?

(1) **KKSS07** and **KKSS19** VFNS collinear FFs share the same extraction technology

⚠ *Might natural stability be related to the given FF determination(s) ?*

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(1) **KKSS07** and **KKSS19** VFNS collinear FFs share the same extraction technology

⚠ *Might natural stability be related to the given FF determination(s) ?*

(2) **KKSS07** and **KKSS19** VFNS collinear FFs assume no initial-scale gluon, but evolution-driven

⚠ *Might natural stability be artificially generated by this Ansatz ?*

3

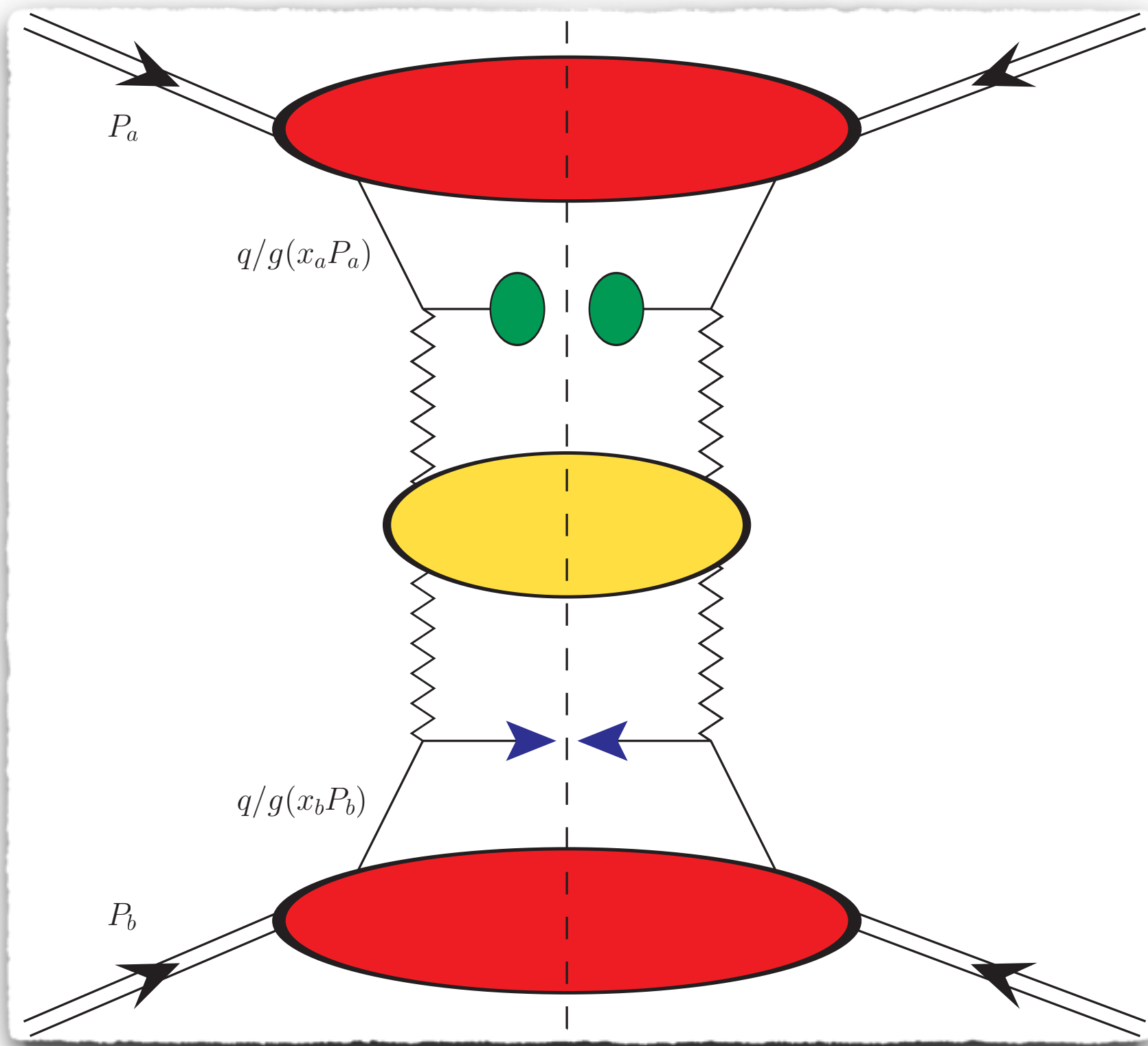
# Quarkonia

# Vector quarkonium from single-parton fragmentation

- (1) *! Let us consider  $J/\psi$  and  $\Upsilon$  at large  $p_T \rightarrow$  single-parton fragmentation from **NRQCD**!*

# Vector quarkonium from single-parton fragmentation

(1) **!** Let us consider  $J/\psi$  and  $\Upsilon$  at large  $p_T \rightarrow$  single-parton fragmentation from **NRQCD** !



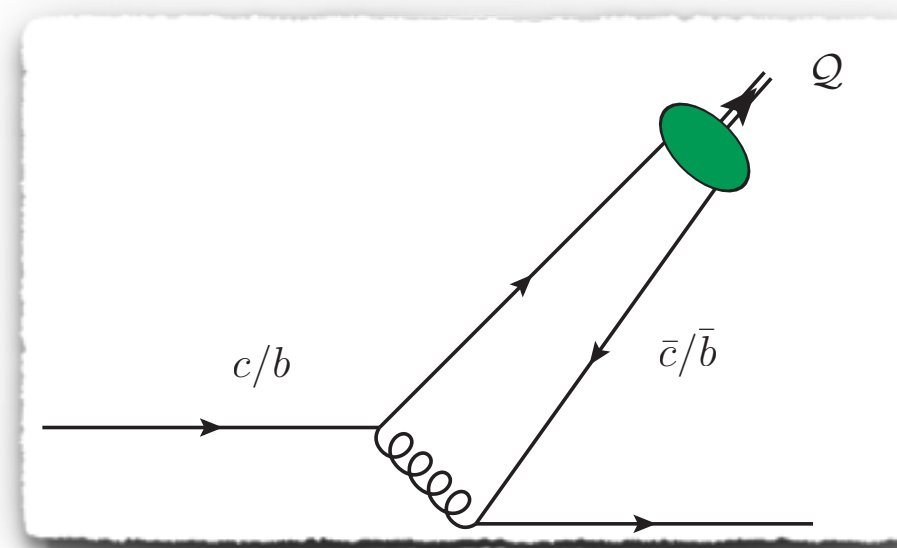
## 2.1 High-energy resummed cross section

The process under investigation is

$$p(P_a) + p(P_b) \rightarrow \mathcal{Q}(p_{\mathcal{Q}}, y_{\mathcal{Q}}) + X + \text{jet}(p_J, y_J), \quad (1)$$

where  $p(P_{a,b})$  stands for an initial proton with momentum  $P_{a,b}$ ,  $\mathcal{Q}(p_{\mathcal{Q}}, y_{\mathcal{Q}})$  is a  $J/\psi$  or a  $\Upsilon$  emitted with momentum  $p_{\mathcal{Q}}$  and rapidity  $y_{\mathcal{Q}}$ , the light jet is tagged with momentum  $p_J$  and rapidity  $y_J$ , and  $X$  denotes all the undetected products of the reaction. High observed transverse momenta,  $|\vec{p}_{\mathcal{Q},J}|$ , together with a large rapidity separation,  $\Delta Y = y_{\mathcal{Q}} - y_J$ , are required conditions to get a diffractive semi-hard configuration in the final state. Furthermore the transverse-momentum ranges need to be enough large to ensure the validity of description of the quarkonium production mechanism in terms of single-parton VFNS collinear fragmentation.

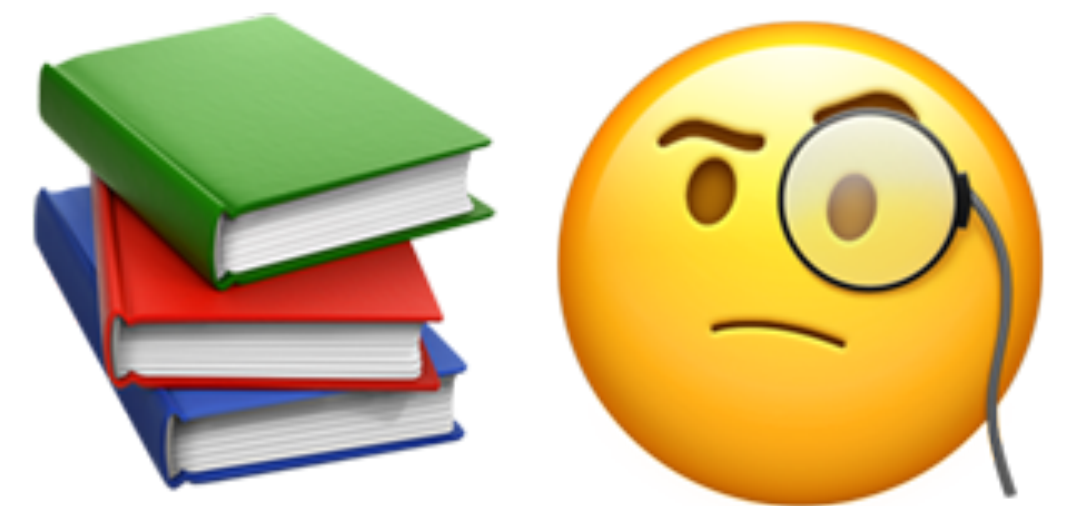
[\[F. G. C. et al., Eur. Phys. J. C 82 \(2022\) 10, 929\]](#)



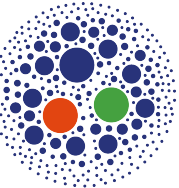
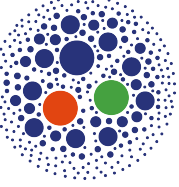
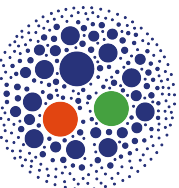
(LO) [\[E. Braaten et al., Phys. Rev. D 48 \(1993\) 4230-4235\]](#)

(NLO) [\[X. Zheng et al., Phys. Rev. D 100 \(2019\) 1, 014005\]](#)

¿ What is a quarkonium ?



# What is a quarkonium?

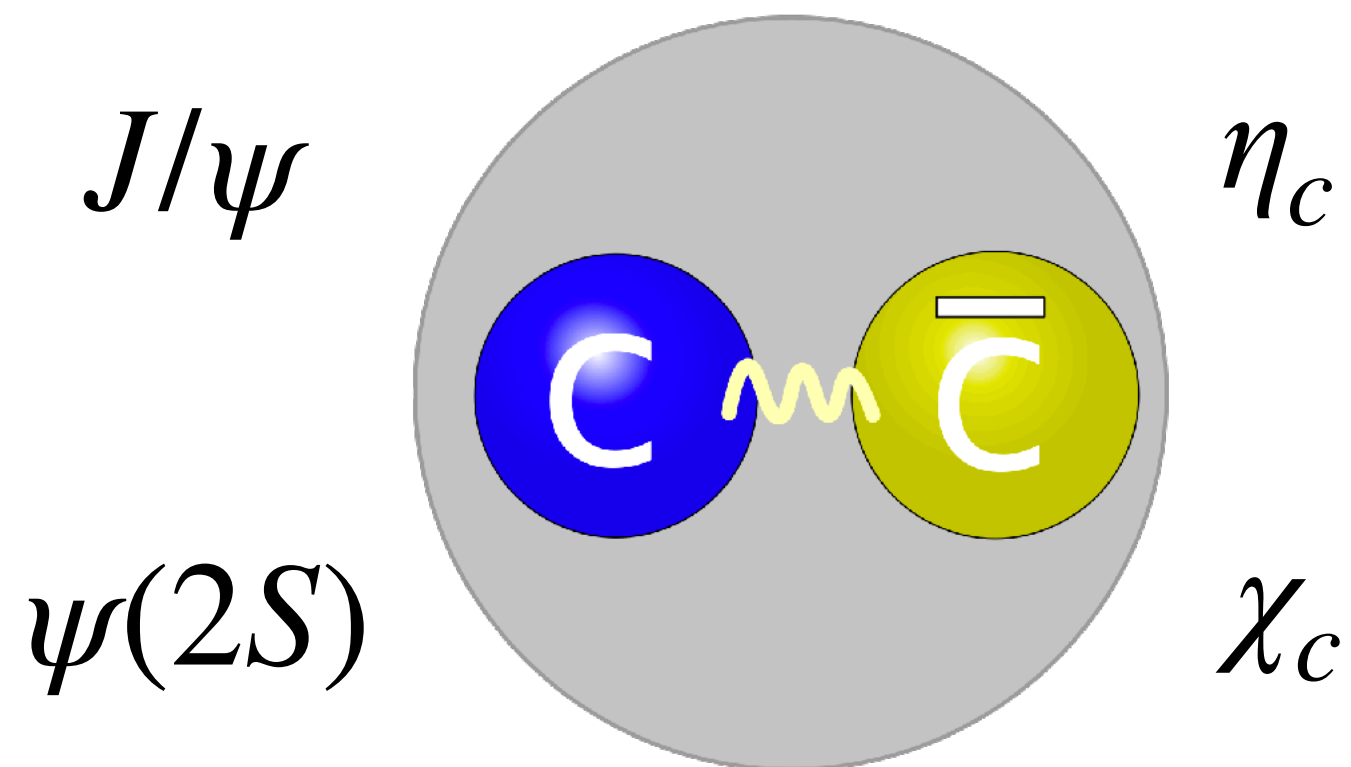
-  **Quarkonia** → mesons with a heavy quark & its antiquark →  $|Q\rangle \equiv |Q\bar{Q}\rangle$
-  They owe the name to the analogy with the **positronium** →  $|\mathcal{P}\rangle \equiv |e^+e^-\rangle$
-  A quarkonium is an electrically neutral particle & is also its own antiparticle



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## Charmonia

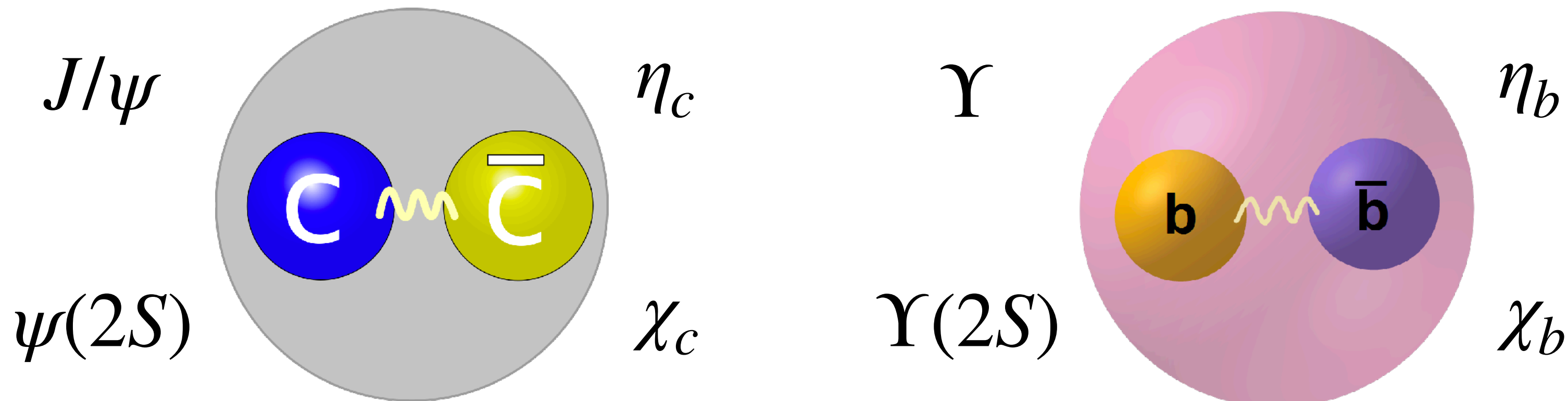


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## Charmonia

## Bottomonia



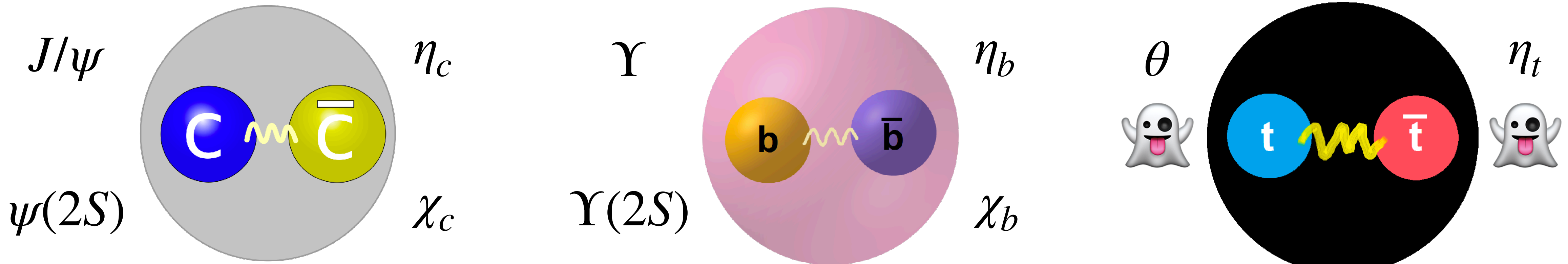
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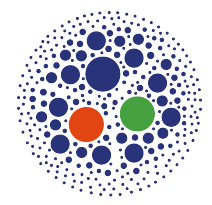
## Charmonia

## Bottomonia

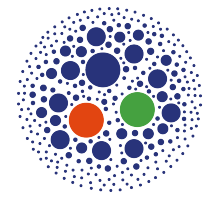
## Toponia



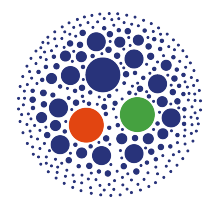
# What is a quarkonium?



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They owe the name to the analogy with the **positronium** →  $|\mathcal{P}\rangle \equiv |e^+e^-\rangle$

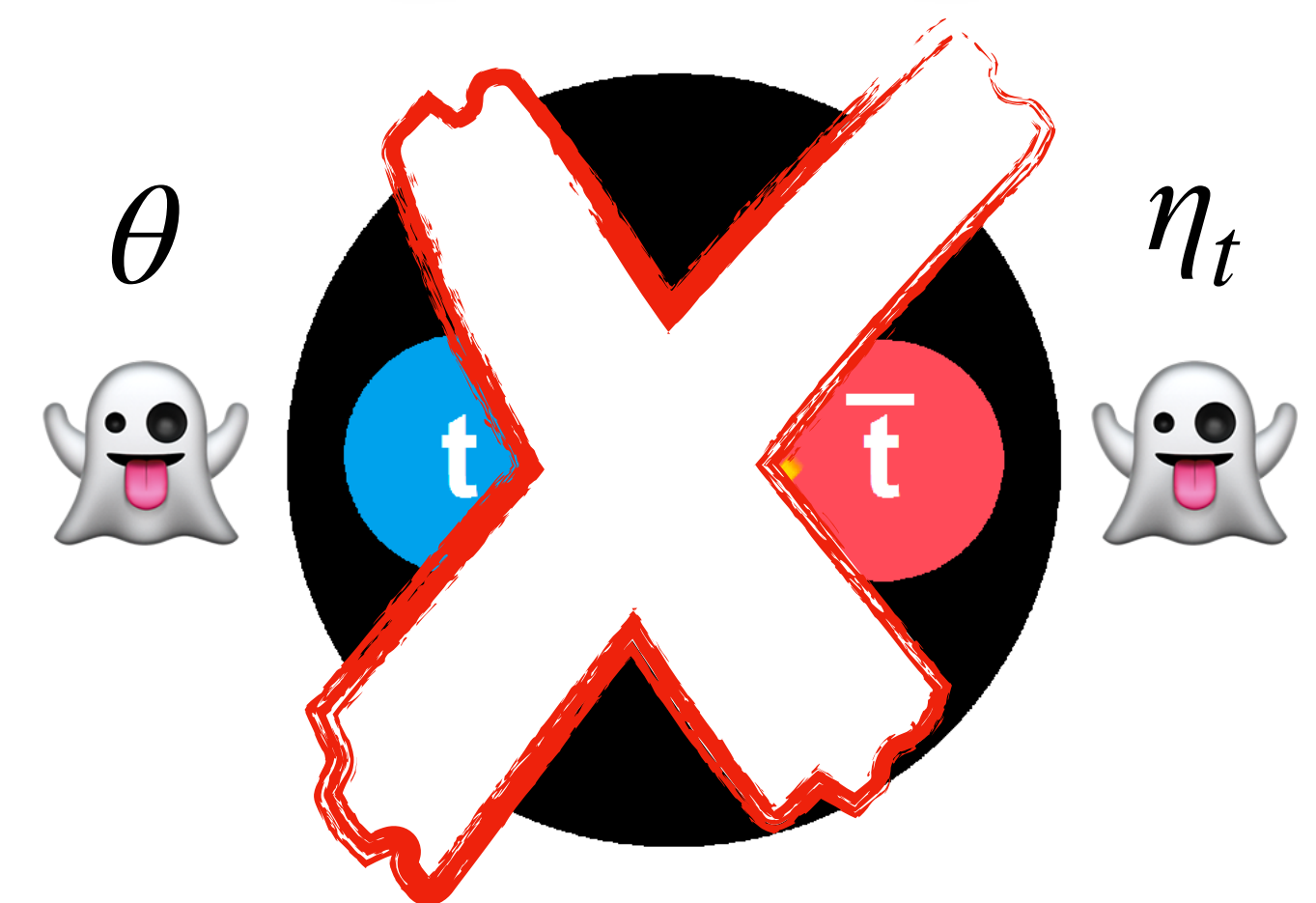
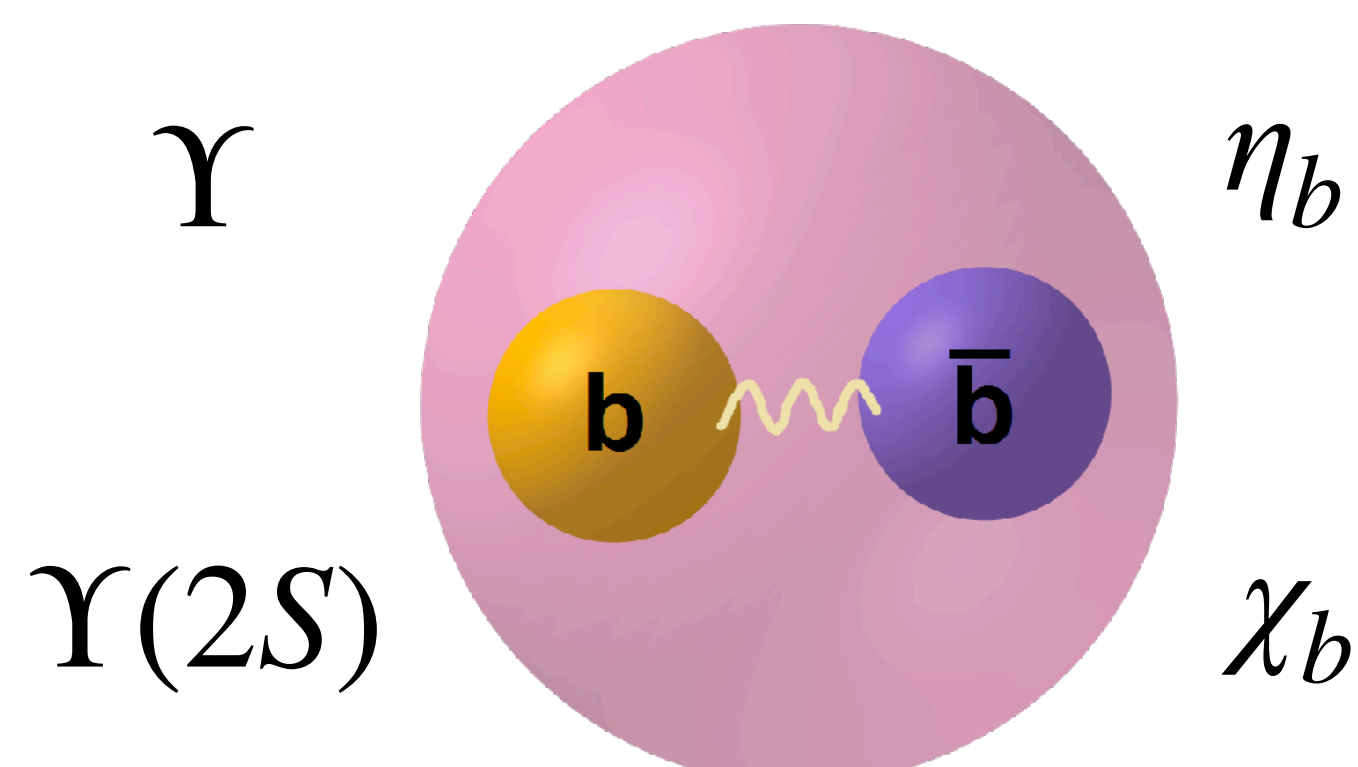
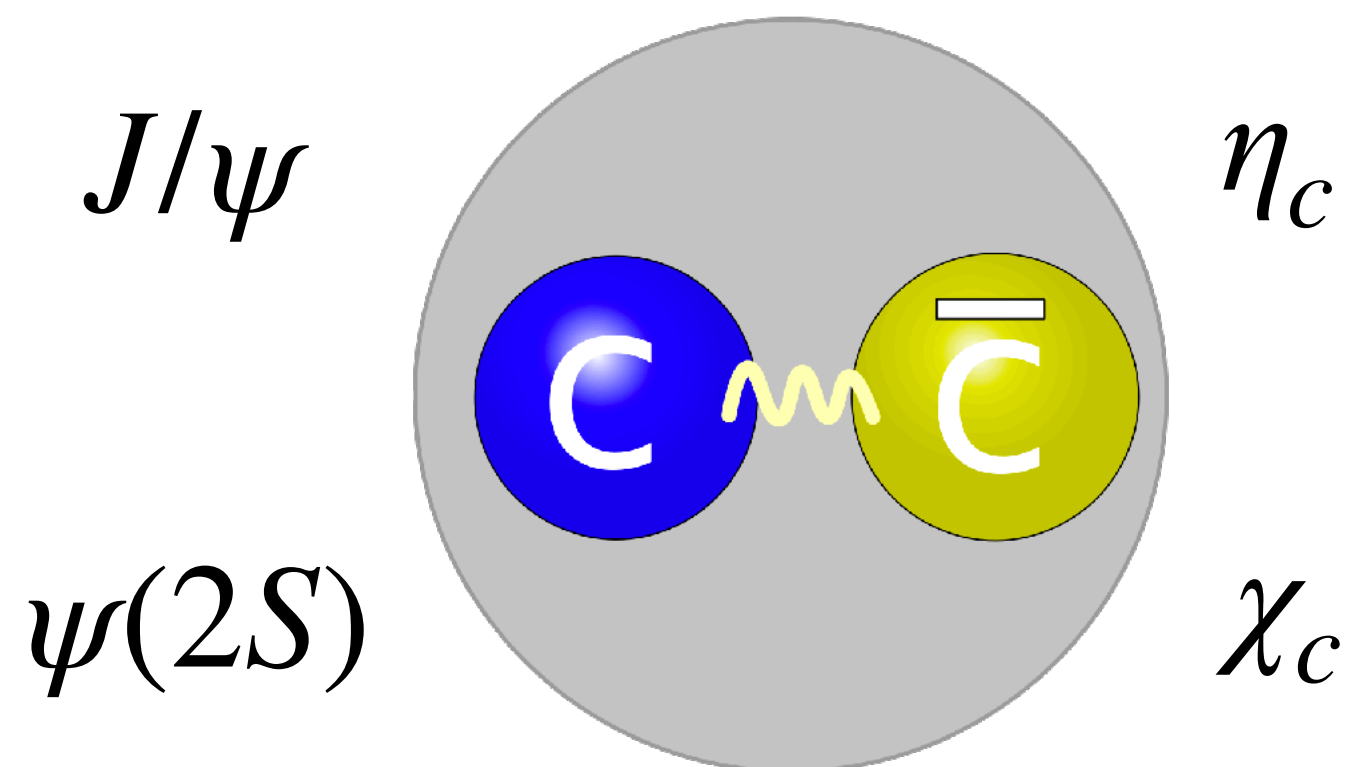


A quarkonium is an electrically neutral particle & is also its own antiparticle

## Charmonia

## Bottomonia

## Toponia



# Quarkonia as Tools: Precision & Exploration

Precision determinations  
of QCD running coupling

$\Upsilon(1S)$  hadronic inclusive decays

[CLEO Collaboration (2006)] [🔗](#)

[N. Brambilla et al. (2007)] [🔗](#)

[A. Deur et al. (2016)] [🔗](#)

[ $\alpha_s$ (2019) ECT\* Trento WG (2019)] [🔗](#)

**QUARKONIA**  
QCD Hydrogen Atoms

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[G. Altarelli et al. (1998)] [🔗](#)  
[A. Candido et al. (2020)] [🔗](#)  
[J. Collins et al. (2022)] [🔗](#)  
[A. Candido et al. (2023)] [🔗](#)

Forward inclusive reactions

Gluon PDF positivity  
at low  $x$  and low  $Q^2$

# Quarkonia as Tools: Precision & Exploration

## QUARKONIA QCD Hydrogen Atoms

Precision determinations  
of QCD running coupling

$\Upsilon(1S)$  hadronic inclusive decays

[CLEO Collaboration (2006)]  
[N. Brambilla et al. (2007)]  
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Quarkonia in nuclear matter

Confined matter to  
QGP transition

Forward inclusive reactions

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# Quarkonia as Tools: Precision & Exploration

Portals to exotic matter  
and BSM physics

Hadroquarkonium  
EW boson rare decays

- [S. Dubynskiy et al. (2008)]
- [F.-K. Guo et al. (2018)]
- [J. Ferretti et al. (2020)]
- [CMS Collaboration (2019)]

- [E. G. Ferreira (2015)]
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**QUARKONIA**  
QCD Hydrogen Atoms



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Forward inclusive reactions

Gluon PDF positivity  
at low  $x$  and low  $Q^2$



# Highlights of Non-Relativistic QCD

 **NRQCD** → effective field theory → nonrelativistic (NR)  $Q\bar{Q}$  system

 Heavy-quark spinor fields as NR DOFs in the NRQCD Lagrangian

$$\mathcal{L}_{\text{NRQCD}} = \psi^\dagger \left( iD_0 + \frac{\vec{D}^2}{2m_Q} \right) \psi + \chi^\dagger \left( iD_0 - \frac{\vec{D}^2}{2m_Q} \right) \chi + \mathcal{L}_{\text{light}} + \delta\mathcal{L}$$

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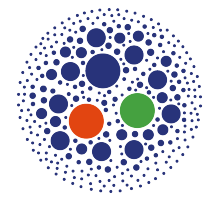
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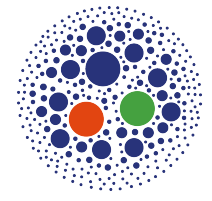
 [cross section] = [short-distance coeffs.]  $\otimes$  [long-distance matrix elements]

$$d\sigma(H + X) = \sum_n d\hat{\sigma}(Q\bar{Q}[n] + X) \langle \mathcal{O}^H[n] \rangle$$

# Highlights of Non-Relativistic QCD

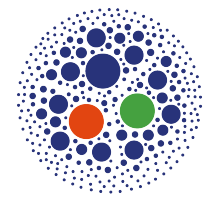


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$$d\sigma(H + X) = \sum_n d\hat{\sigma}(Q\bar{Q}[n] + X) \langle \mathcal{O}^H[n] \rangle$$

$$|\mathcal{Q}\rangle = \mathcal{O}(1) |Q\bar{Q}[^3S_1^{(1)}]\rangle + \mathcal{O}(v) |Q\bar{Q}[^3P_J^{(8)}g]\rangle + \mathcal{O}(v^2) |Q\bar{Q}[^1S_0^{(8)}g]\rangle \\ + \mathcal{O}(v^2) |Q\bar{Q}[^3S_1^{(1,8)}gg]\rangle + \mathcal{O}(v^2) |Q\bar{Q}[^3D_J^{(1,8)}gg]\rangle + \dots$$

# Highlights of Non-Relativistic QCD

NRQCD:

- an effective field theory

- Idea: start from the heavy quark Lagrangian

$\mathcal{L}_{\text{heavy}} = \bar{\Psi}(i\gamma^{\mu}D_{\mu} - m_Q)\Psi$  and expand in velocity

$$\begin{aligned} \rightarrow & \psi^{\dagger}(iD_t + \frac{1}{2m_Q}\bar{D}^2 + \dots)\psi \leftarrow \text{quark term} \\ & + \chi^{\dagger}(iD_t - \frac{1}{2m_Q}\bar{D}^2 + \dots)\chi \leftarrow \text{antiquark term} \end{aligned} \quad \Psi \sim \begin{pmatrix} \psi \\ \chi \end{pmatrix}$$

- This Lagrangian conserves heavy quarks and antiquarks

$\Rightarrow$  Add mixing terms like  $\mathcal{O} = \psi^{\dagger}\chi\chi^{\dagger}\psi$  to describe production and decay

$$\delta\mathcal{L} = \sum_i \frac{f_i}{m_Q^2} \mathcal{O}_i$$

$\mathcal{O} \sim$

[Lectures by [Jani Penttala](#)]

- expansion in the (relative) velocity  $v$  and  $\alpha_s \sim v$

- write particles in terms of different spin and color states

Example: spectroscopic notation for the spin and orbital angular momentum of the  $q\bar{q}$  pair

$$|0/\psi\rangle = \mathcal{O}(v^0) |^3S_1^{[1]}\rangle + \mathcal{O}(v) |^3P_1^{[8]}\rangle + \mathcal{O}(v^{3/2}) |^1S_0^{[8]}\rangle + \mathcal{O}(v^2) |^3D_1^{[1]}\rangle + \dots$$

$\downarrow$  color state of the  $q\bar{q}$  system

$\uparrow$  suppression in velocity

- describe particles in terms of universal long-distance matrix elements (LDMEs)

# Highlights of Non-Relativistic QCD

## NRQCD:

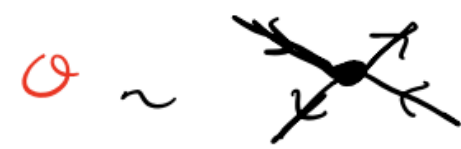
- an effective field theory
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- This Lagrangian conserves heavy quarks and antiquarks
- ⇒ Add mixing  $\theta$ -like  $\theta = \psi^{\dagger} \chi \chi^{\dagger} \psi$  to describe production a

$$\delta\mathcal{L} = \sum_i \frac{f_i}{m_Q^2}$$



Hadronization: NRQCD  
 Non-perturbative Long-Distance Matrix Elements (LDME) from experimental data

$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}) \rangle$    
  $\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}) \rangle$    
  $\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \rangle$    
  $\langle \mathcal{O}^{J/\psi}({}^3P_J^{[8]}) \rangle$

Chao et al. 1201.2675v4

[Lectures by [Jani Penttala](#)]

- expansion in the (relative) velocity  $v$  and  $\alpha_s \sim v$
  - write particles in terms of different spin and color states
- Example: spectroscopic notation for the spin and orbital angular momentum of the  $q\bar{q}$  pair

$$|J/\psi\rangle = \mathcal{O}(v^0) |{}^3S_1^{[1]}\rangle + \mathcal{O}(v) |{}^3P_1^{[8]}\rangle + \mathcal{O}(v^{3/2}) |{}^1S_0^{[8]}\rangle + \mathcal{O}(v^2) |{}^3D_1^{[8]}\rangle + \dots$$

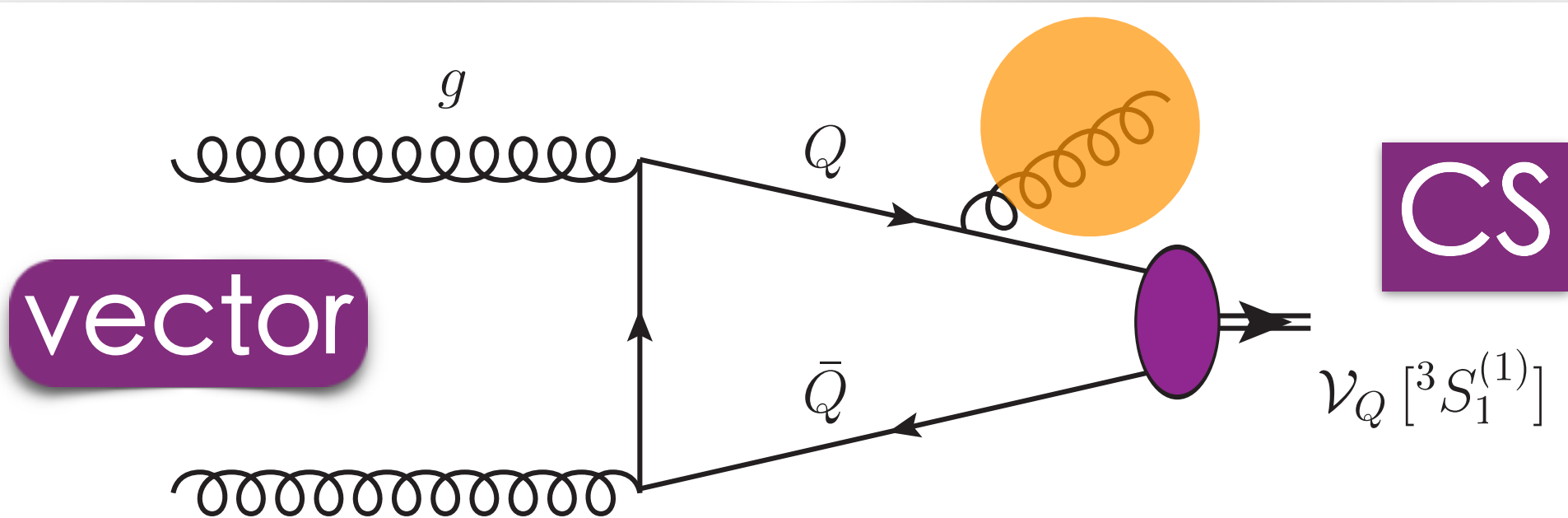
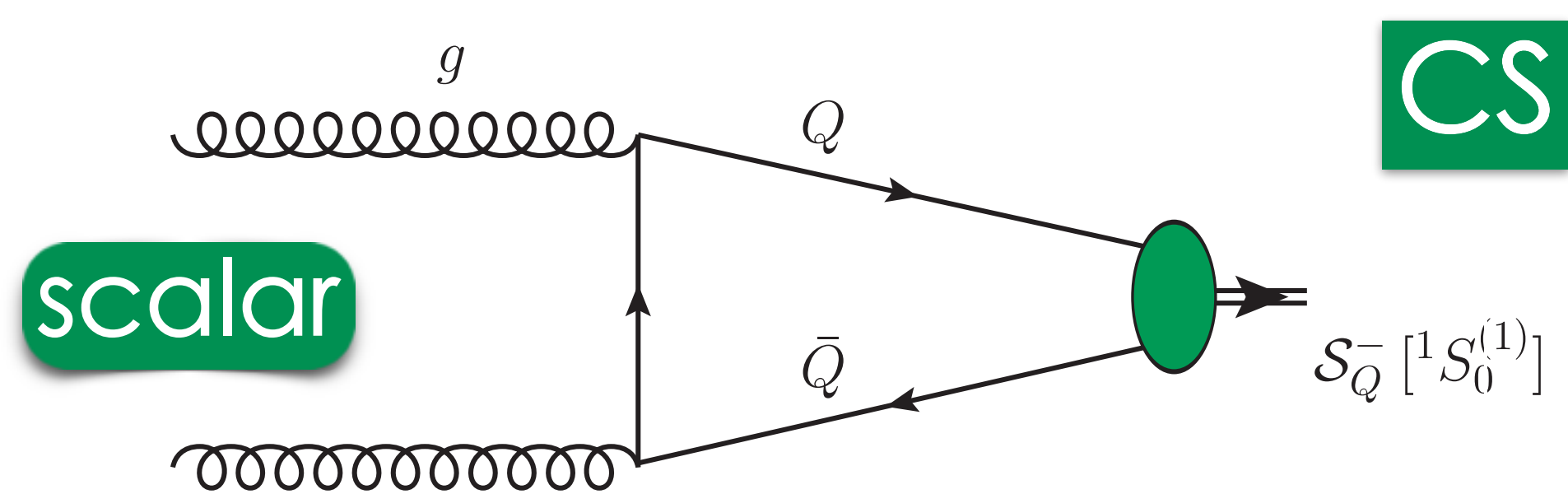
$\downarrow$  color state of the  $q\bar{q}$  system       $\uparrow$  suppression in velocity

- describe particles in terms of universal long-distance matrix elements (LDMEs)

[Talk by [Patricia Gimeno Estivill](#)]

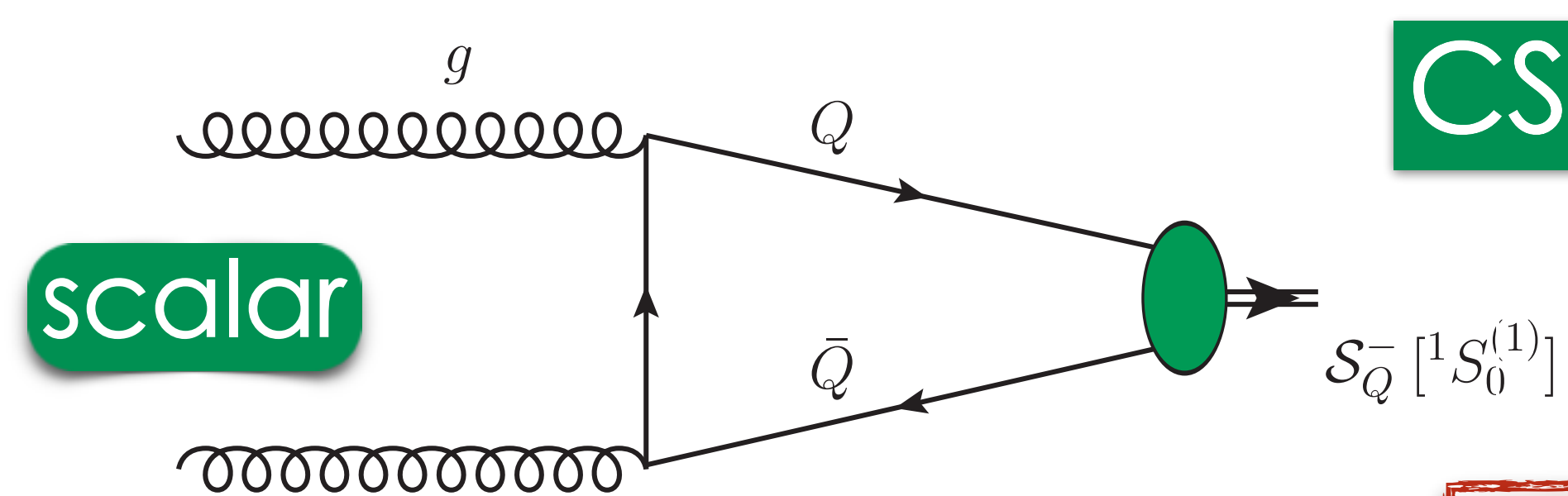
# From low to high energies

Low energy, low  $p_T$   
Short-distance production

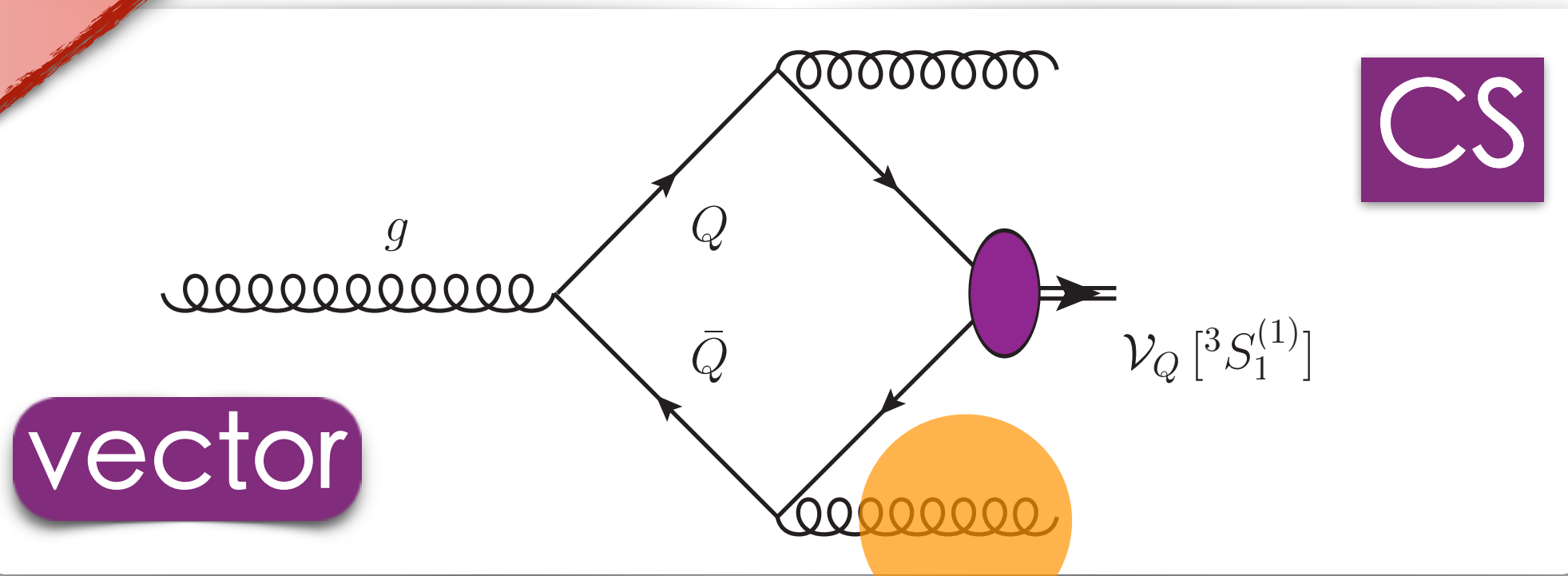
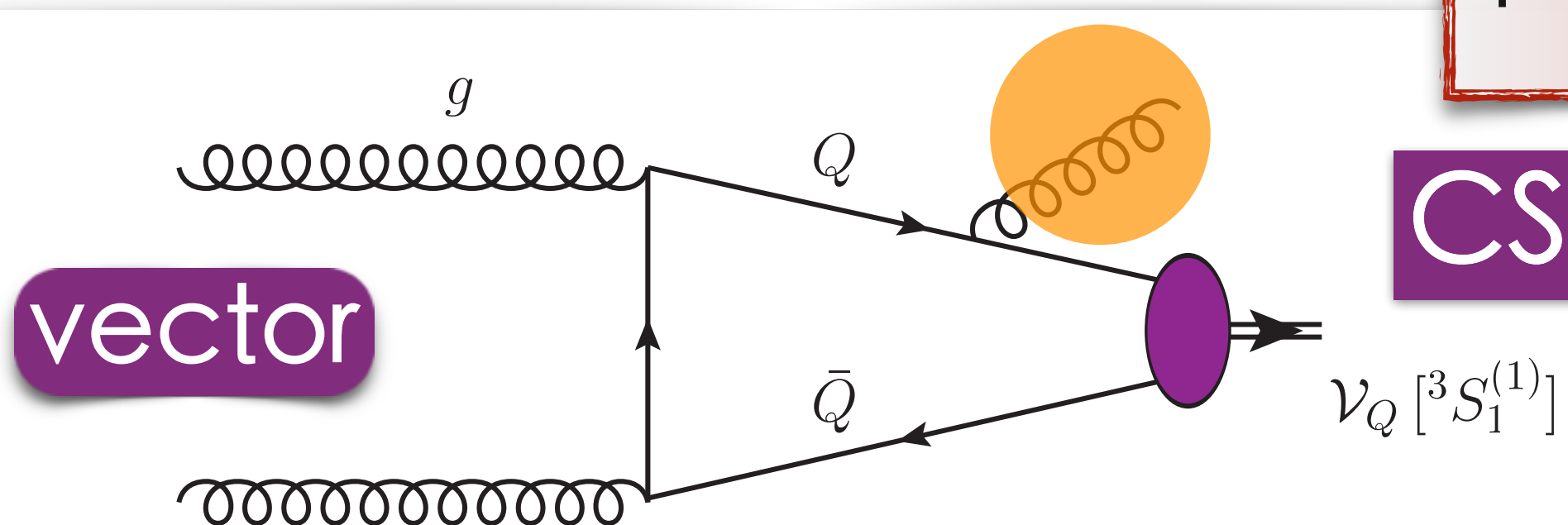
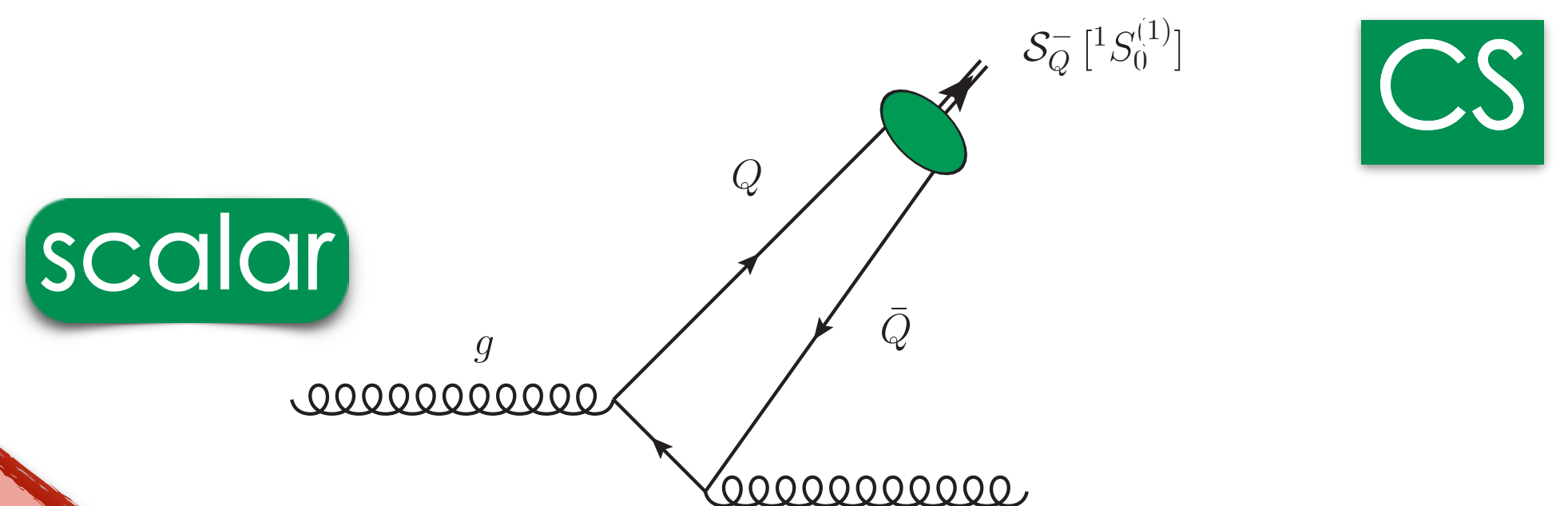


# From low to high energies

Low energy, low  $p_T$   
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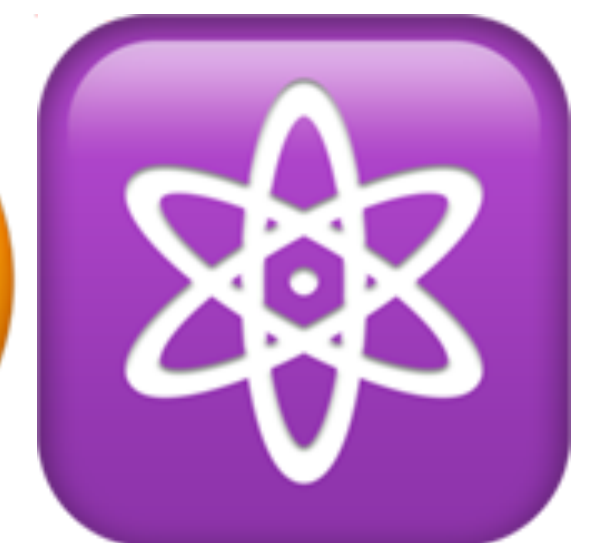
High energy, high  $p_T$   
Single-parton fragmentation



$p_T \gtrsim 10 \div 15 \text{ GeV}$   
(charmonia)

Extra gluon due to Landau-Yang selection rule

¿ How to address  
onium fragmentation ?





**HADRONIC STRUCTURE**  
**QUARKONIUM THEORY**

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**PRECISION QCD**  
**COLLINEAR FACTORIZATION**

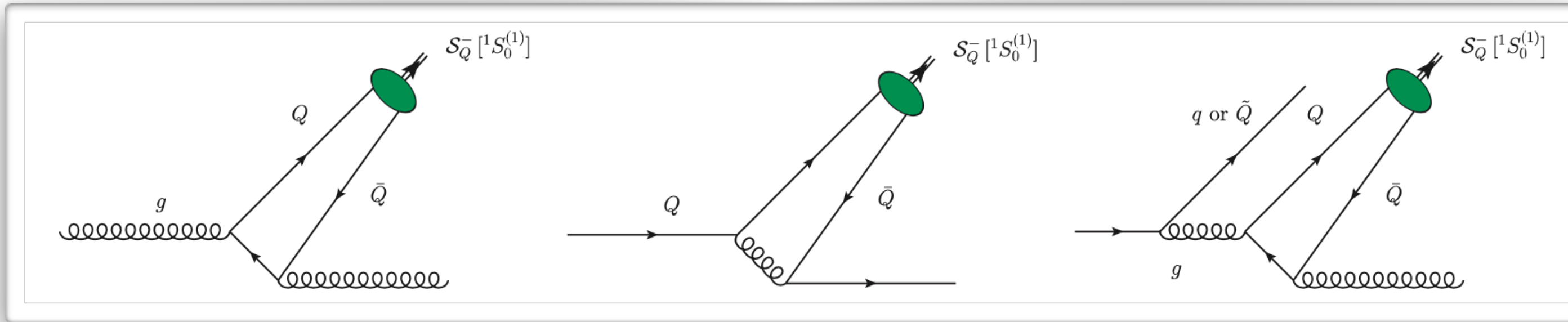
# Heavy-Flavor Non-Relativistic evolution

HADRONIC STRUCTURE  
QUARKONIUM THEORY

PRECISION QCD  
COLLINEAR FACTORIZATION

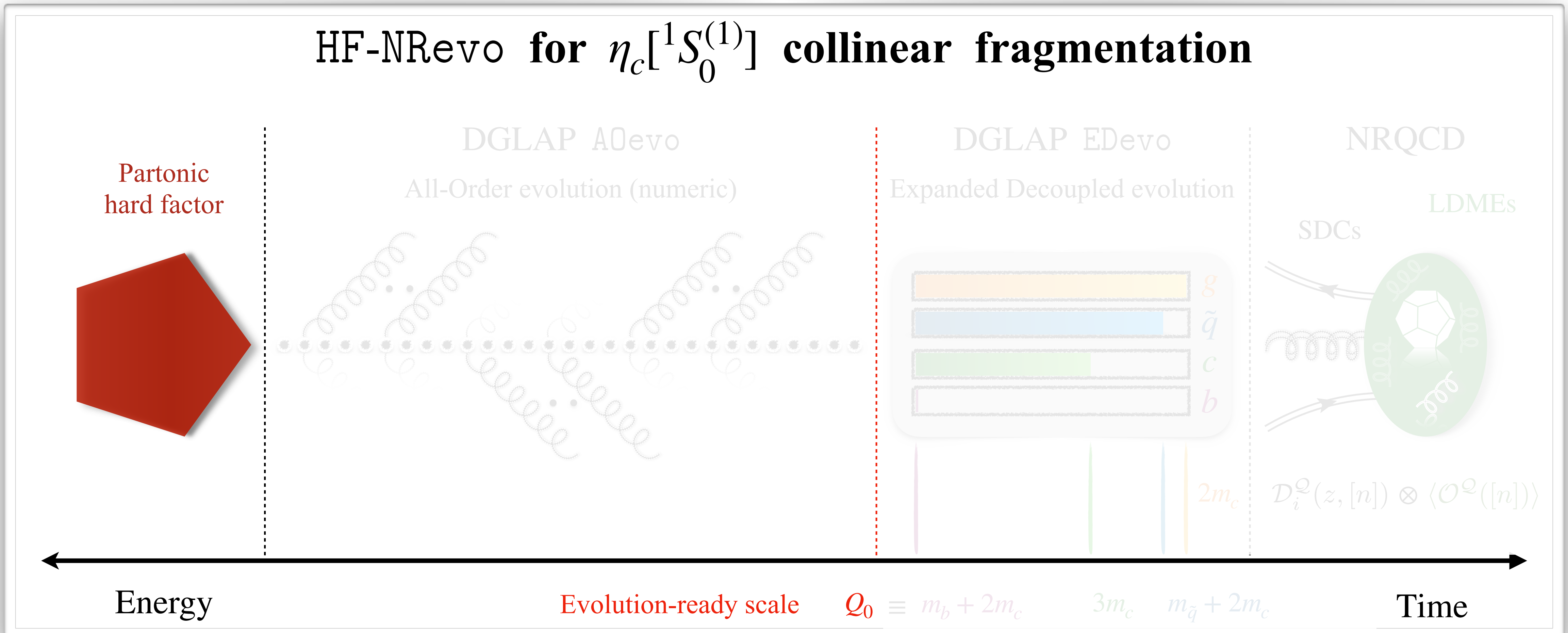
Guiding principle  $\Rightarrow$  ; Use the best of the Two Worlds as much as we can !

# HF-NRevo: Initial-scale NRCQD inputs

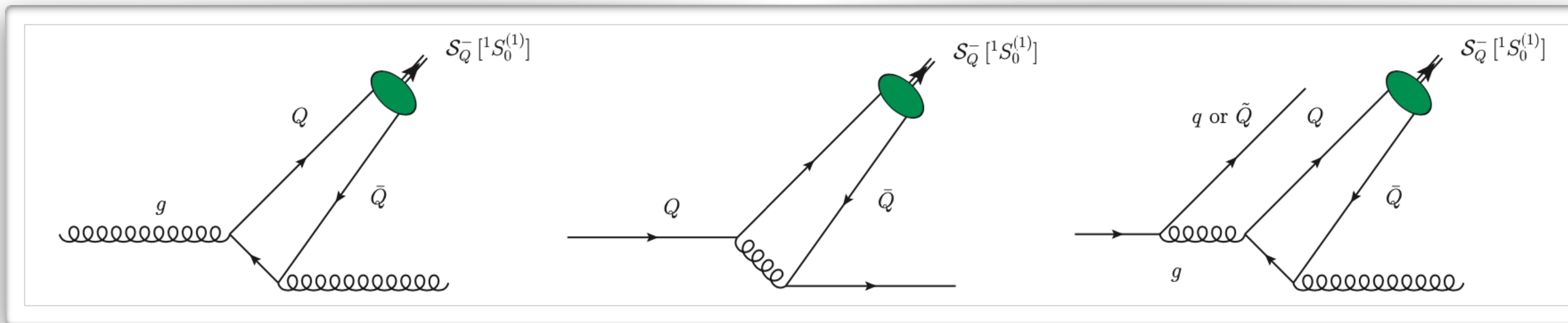


Scalar  $\eta_c$

## HF-NRevo for $\eta_c[{}^1S_0^{(1)}]$ collinear fragmentation

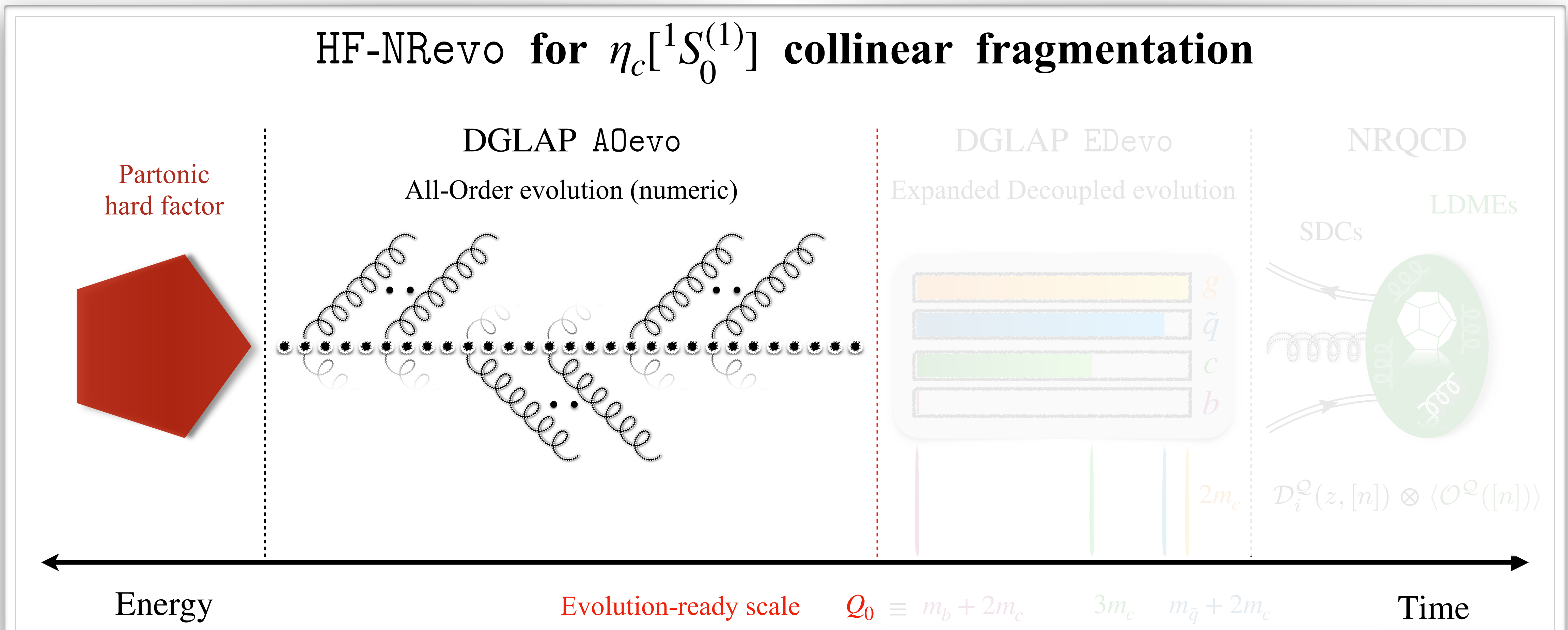


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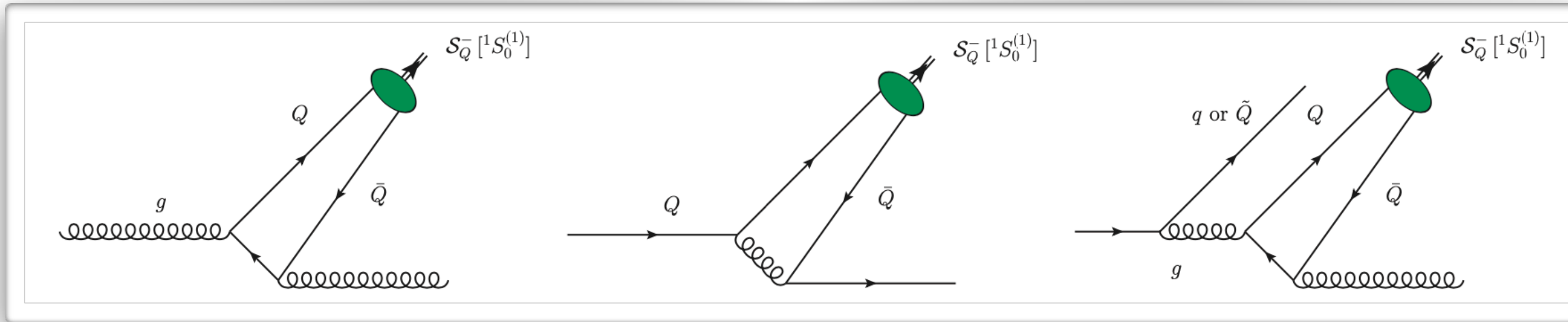


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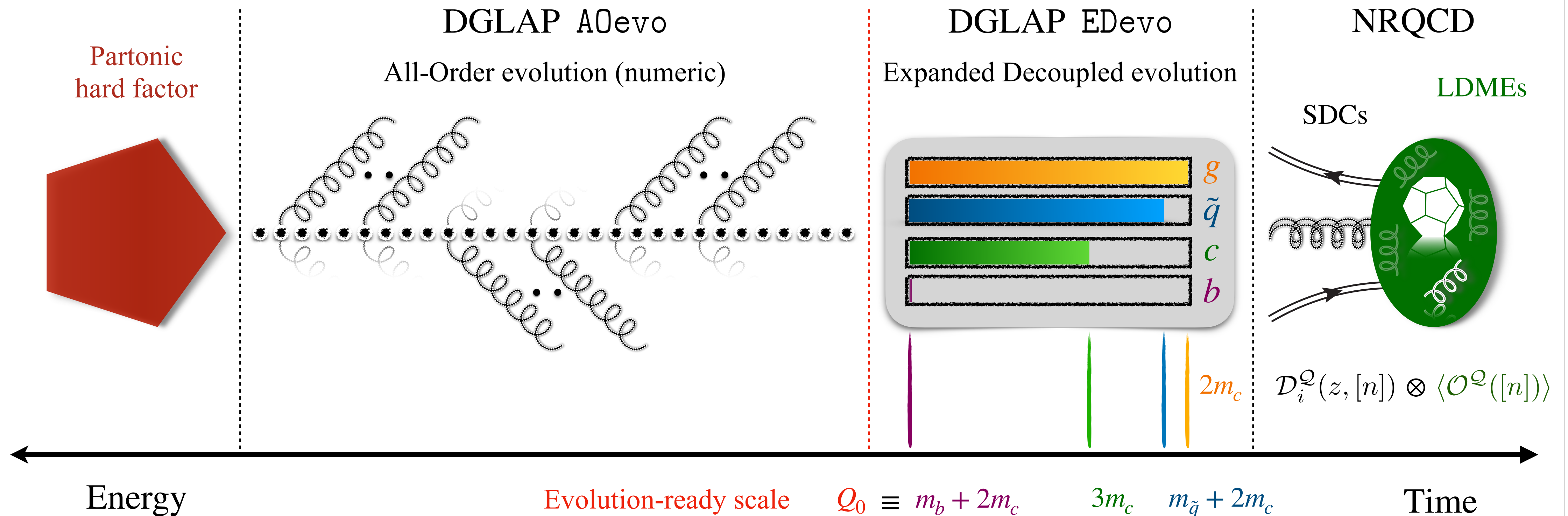


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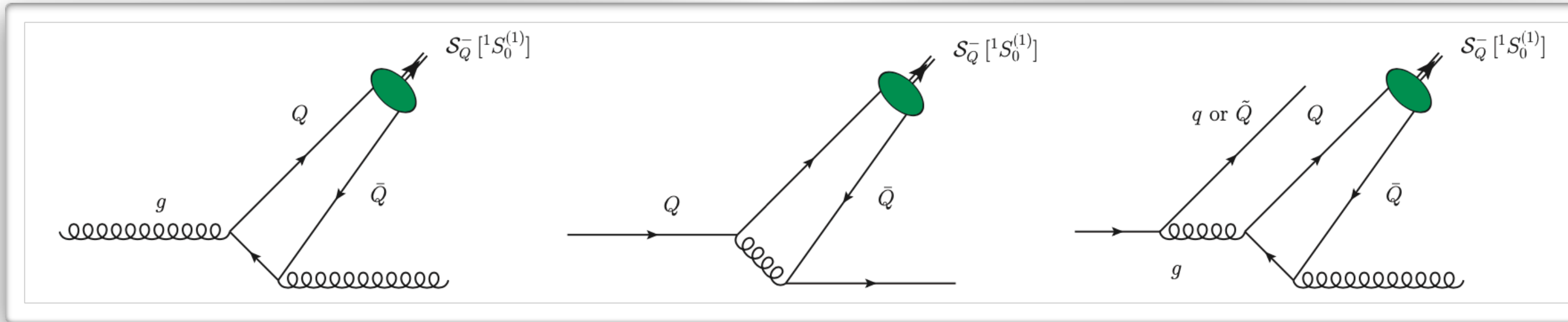


Scalar  $\eta_c$

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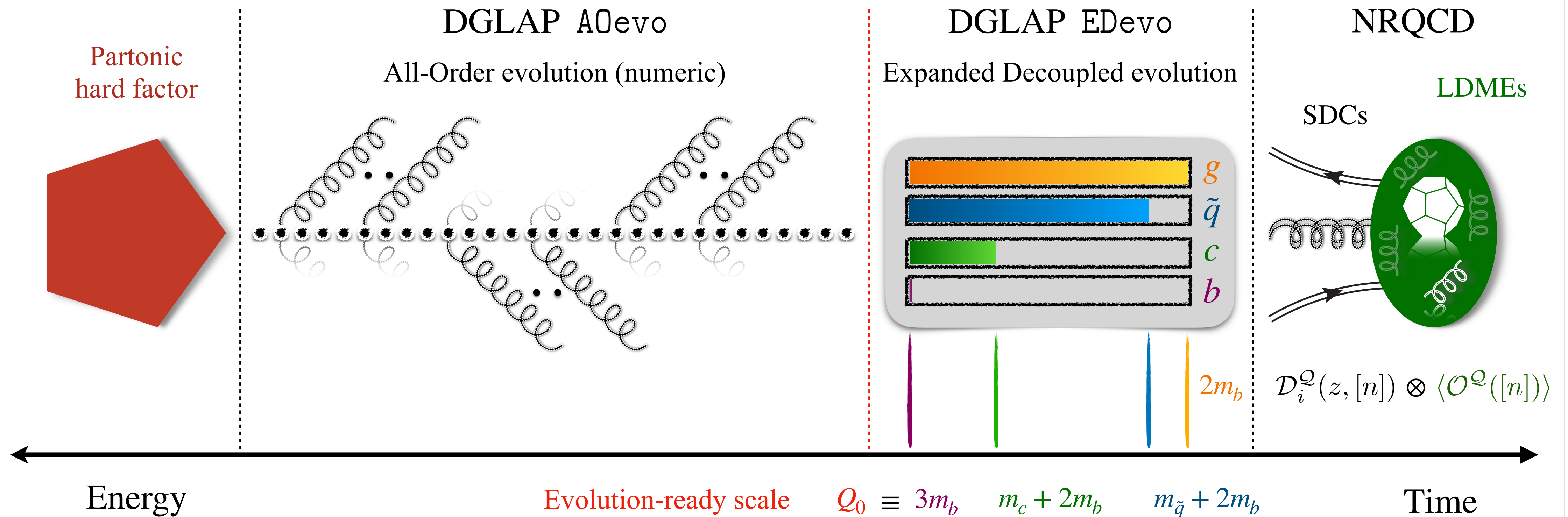


# HF-NRevo: Initial-scale NRCQD inputs



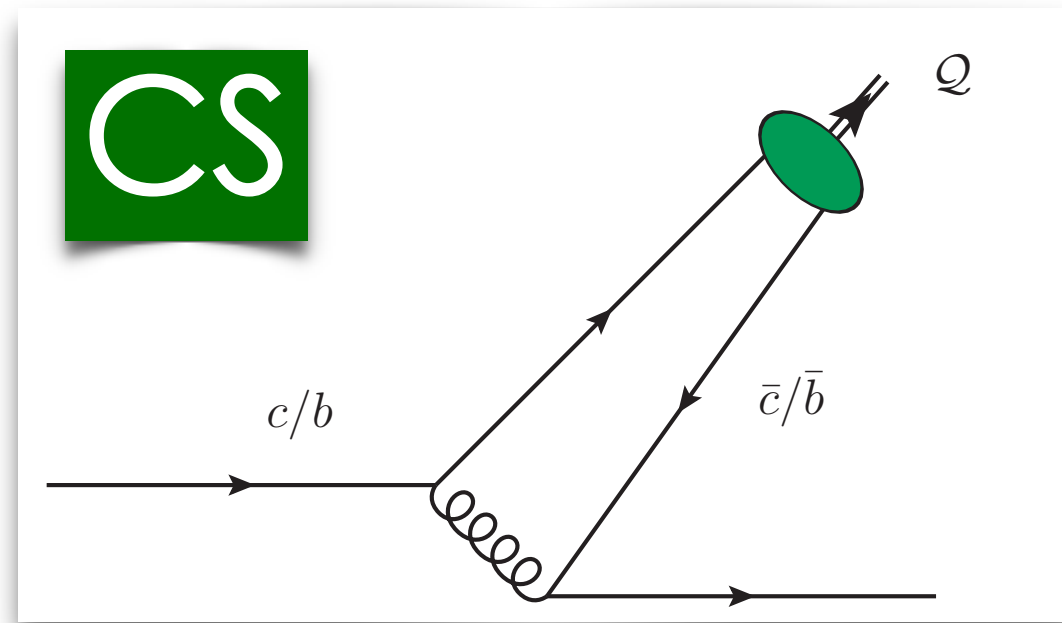
Scalar  $\eta_b$

## HF-NRevo for $\eta_b[{}^1S_0^{(1)}]$ collinear fragmentation



# Vector quarkonium from single-parton fragmentation

! Let us consider  $J/\psi$  and  $\Upsilon$  at large  $p_T \rightarrow$  initial-scale **heavy-quark** + **gluon** from **NRQCD**!



$$D_Q^Q(z, \mu_F \equiv \mu_0) = D_Q^{Q,LO}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3} |\mathcal{R}_Q(0)|^2 \Gamma^{Q,NLO}(z)$$

$(Q \rightarrow Q Q)$  at  $\mu_0 = 3m_Q$

(LO) [\[E. Braaten et al., Phys. Rev. D 48 \(1993\) 4230-4235\]](#)

(NLO) [\[X. Zheng et al., Phys. Rev. D 100 \(2019\) 1, 014005\]](#)

[\[F. G. C., M. Fucilla, Eur. Phys. J. C 82 \(2022\) 10, 929\]](#)

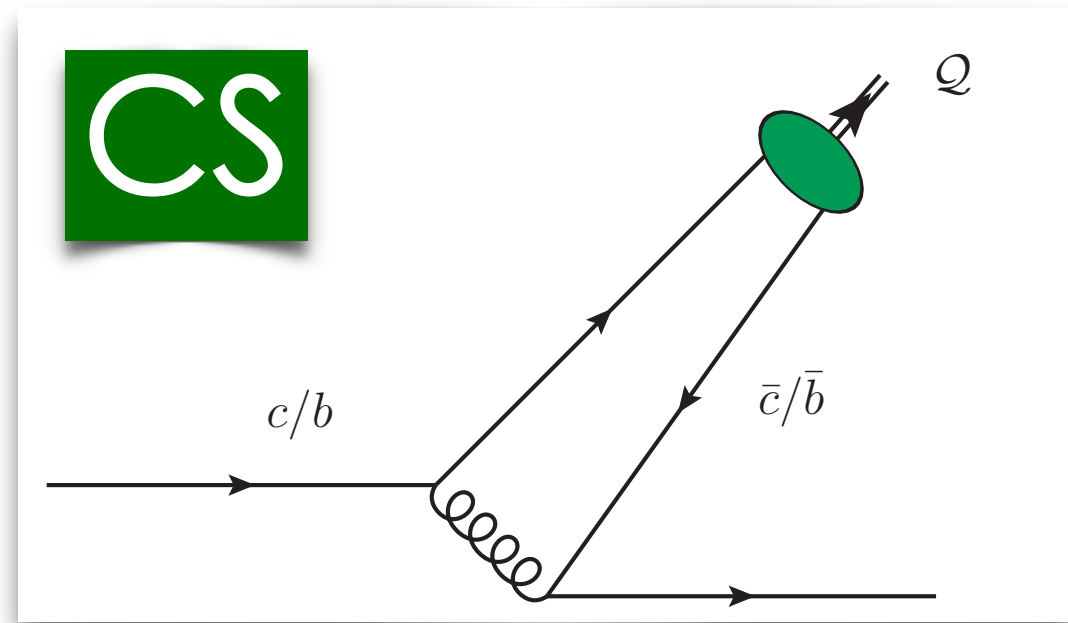
[\[My talk at the Higgs Centre, Edinburgh \(2023\)\]](#)

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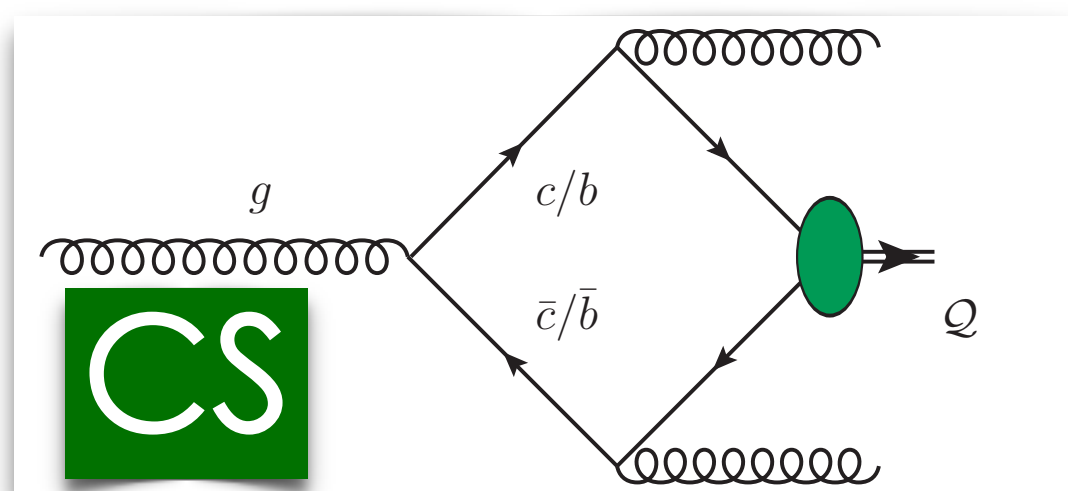
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$$D_g^Q(z, 2m_Q) = \frac{5}{36(2\pi)^2} \alpha_s^3(2m_Q) \frac{|\mathcal{R}_Q(0)|^2}{m_Q^3} \int_0^z d\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} d\tau \frac{1}{(1-\tau)^2(\tau-\xi)^2(\tau^2-\xi)^2}$$

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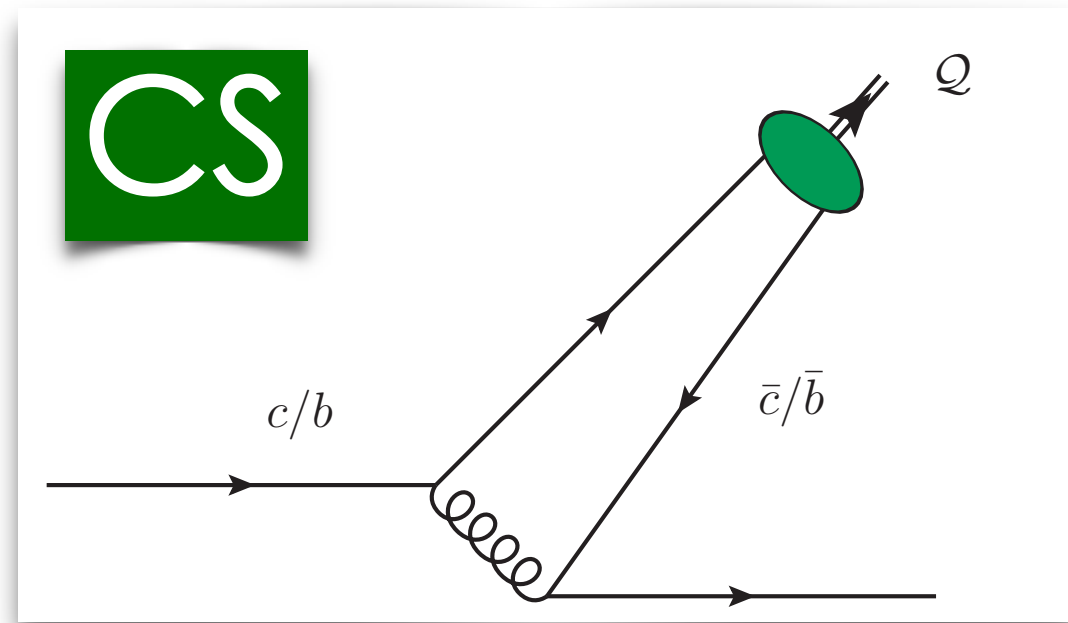
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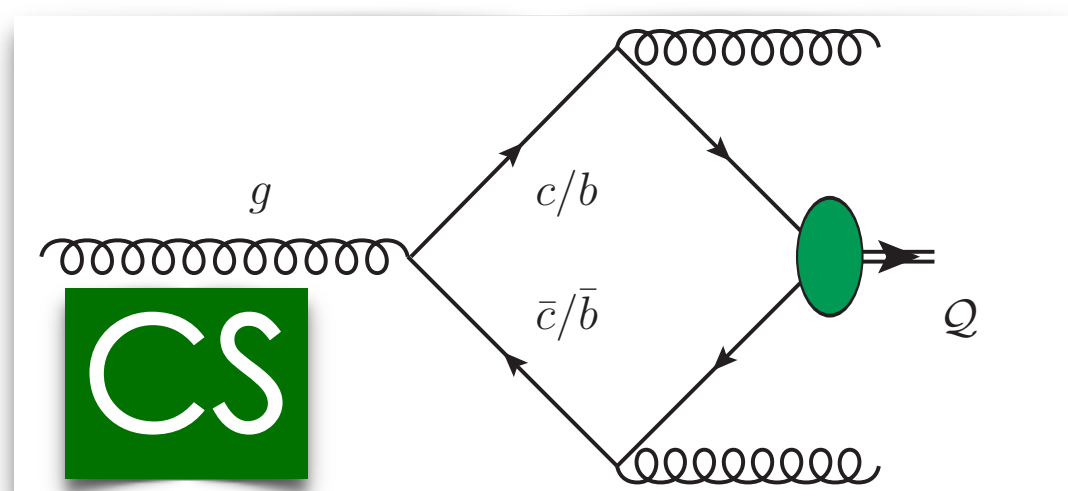
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[\[F. G. C., M. Fucilla, Eur. Phys. J. C 82 \(2022\) 10, 929\]](#)

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ZCW19+/NRF1.0

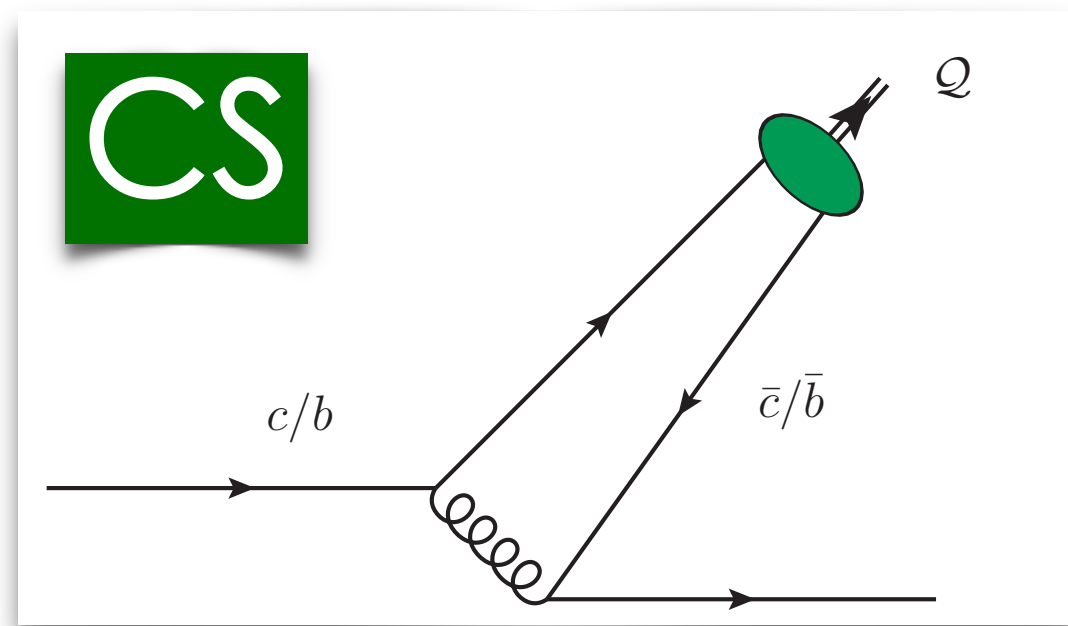
quarkonium FFs

=

APFEL++/EKO

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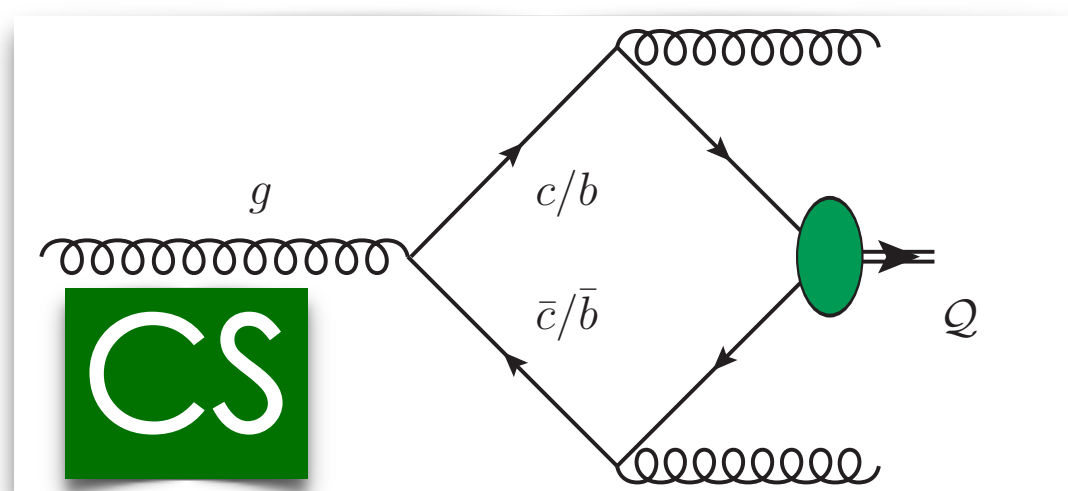


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+



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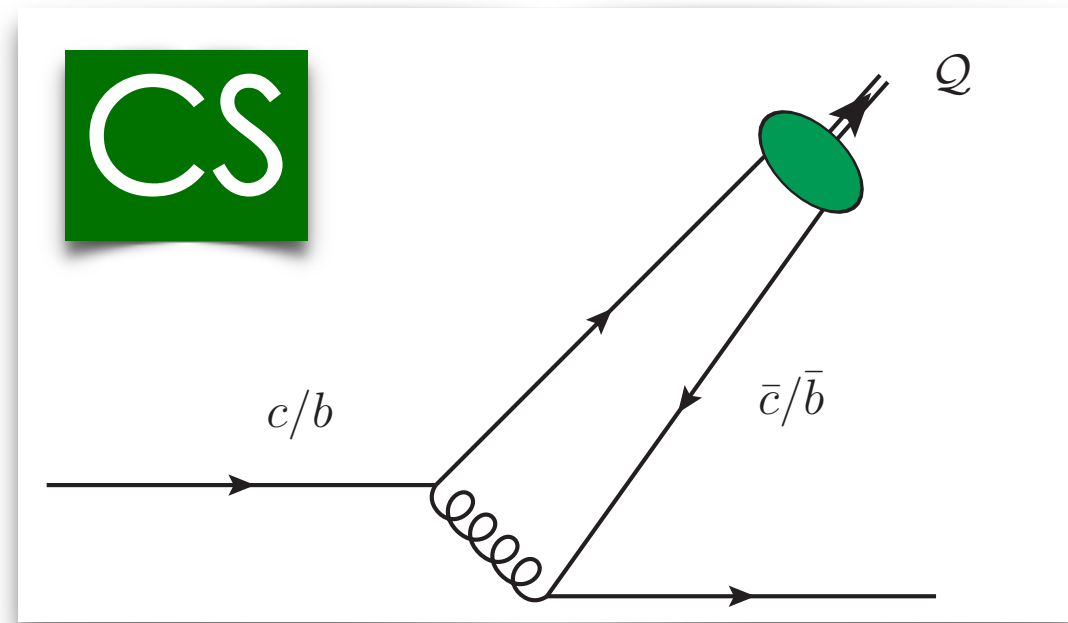
quarkonium FFs

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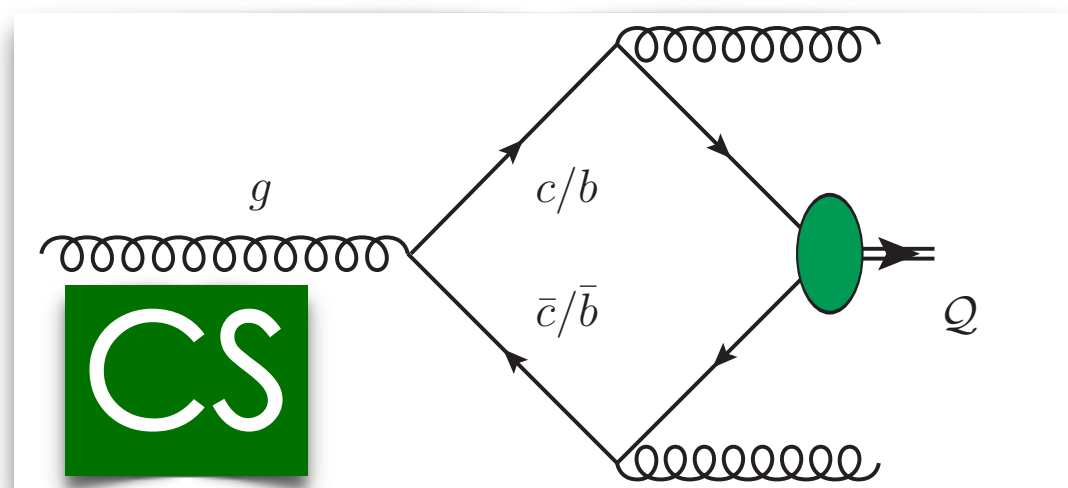


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+



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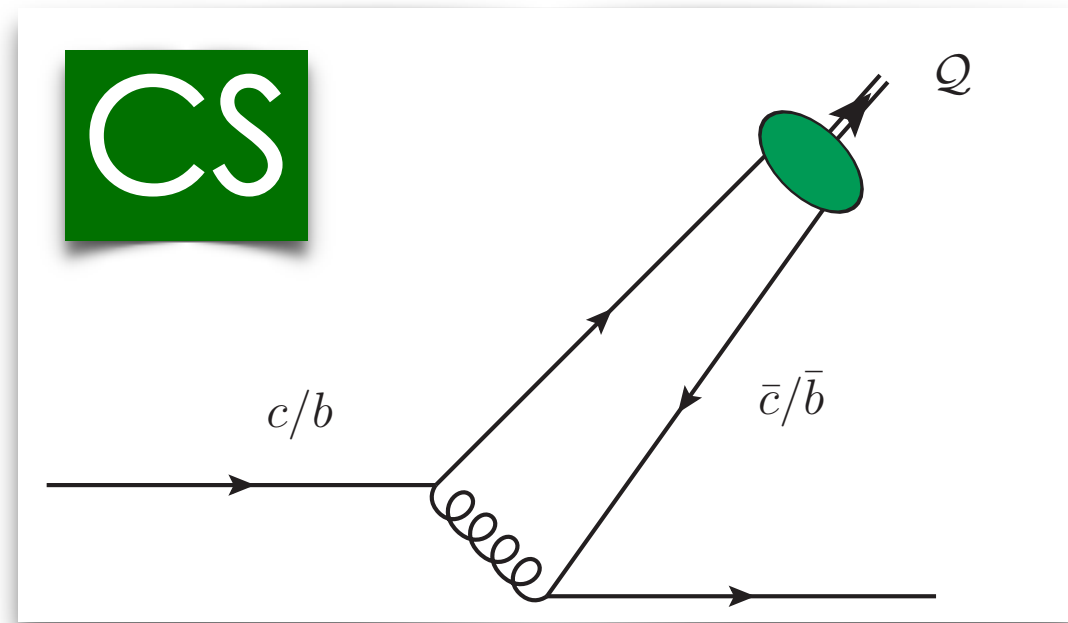
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APFEL++/EKO

# Vector quarkonium from single-parton fragmentation

! Let us consider  $J/\psi$  and  $\Upsilon$  at large  $p_T \rightarrow$  initial-scale **heavy-quark** + **gluon** from **NRQCD**!



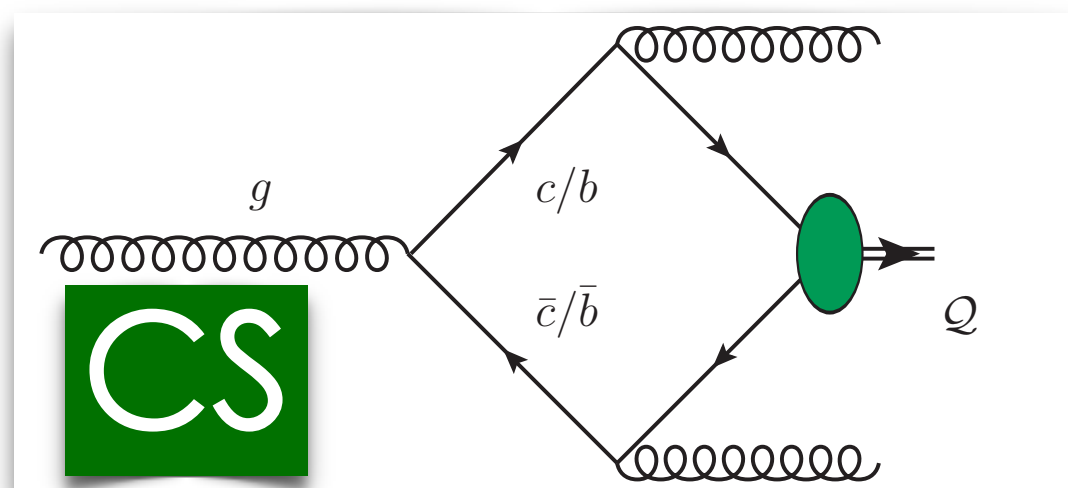
$$D_Q^Q(z, \mu_F \equiv \mu_0) = D_Q^{Q,LO}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3} |\mathcal{R}_Q(0)|^2$$

(LO) [E. Braaten et al., Phys. Rev. D 48 (1993) 4230-4234]  
 (NLO) [X. Zheng et al., Phys. Rev. D 100 (2019) 1, 014005]



$(Q \rightarrow Q Q)$  at  $\mu_0 = 3m_Q$

+



$$D_g^Q(z, 2m_Q) = \frac{5}{36(2\pi)^2} \alpha_s^3(2m_Q) \frac{|\mathcal{R}_Q(0)|^2}{m_Q^3} \int_0^z d\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} d\tau \frac{1}{(1-\tau)^2} \sum_{i=1}^2 z^i \left[ f_i^{(g)}(\xi, \tau) + g_i^{(g)}(\xi, \tau) \frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}} \ln \left( \frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}} \right) \right]$$

(LO) [A. Braaten, T.C Yuan, Phys. Rev. Lett. 71 (1993), 1673]



$(g \rightarrow Q gg)$  at  $\mu_0 = 2m_Q$



- [F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]
- [My talk at the Higgs Centre, Edinburgh (2023)]
- [My talk at Moriond QCD, La Thuile (2024)]



ZCW19+/NRFF1.0

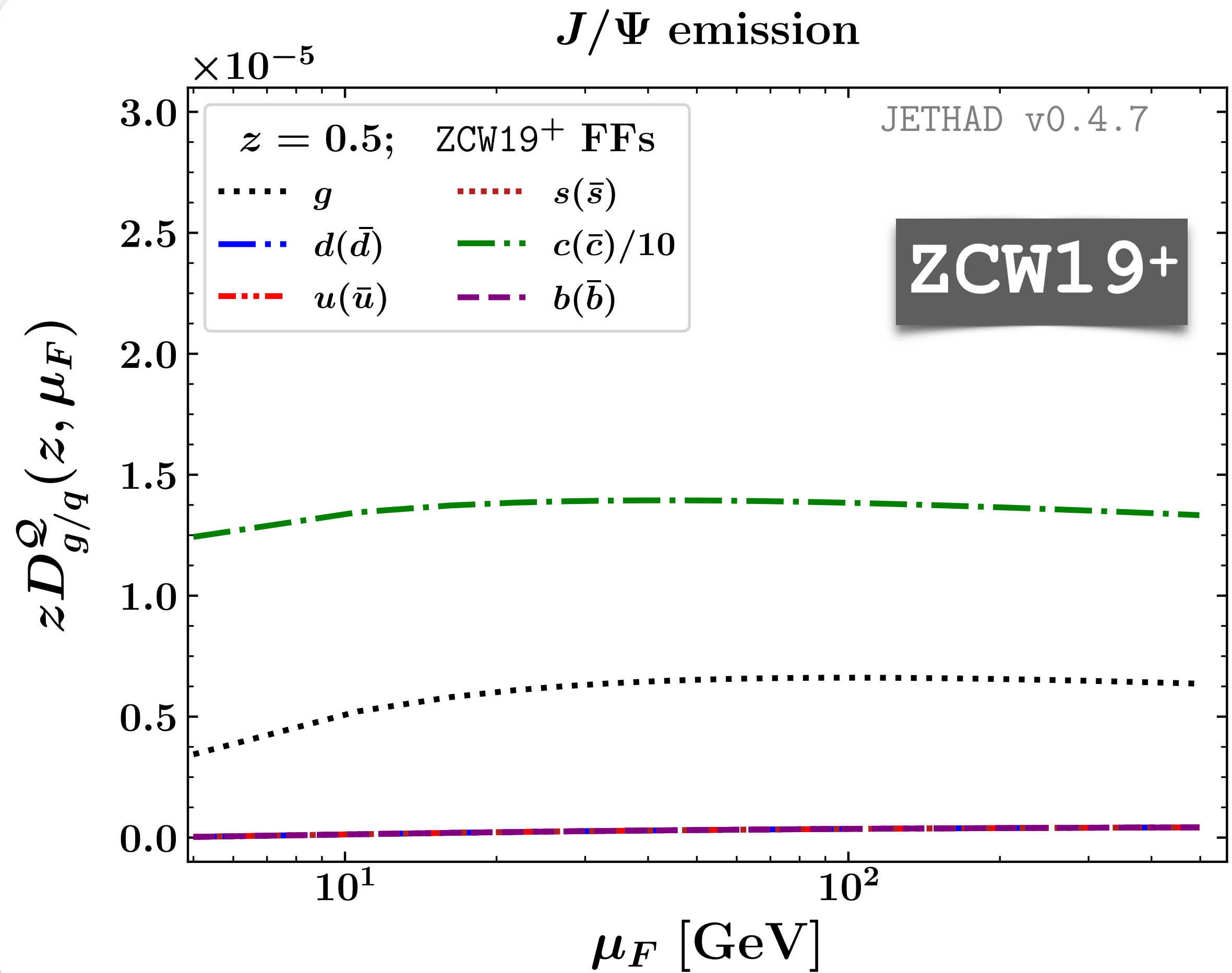
quarkonium FFs

=

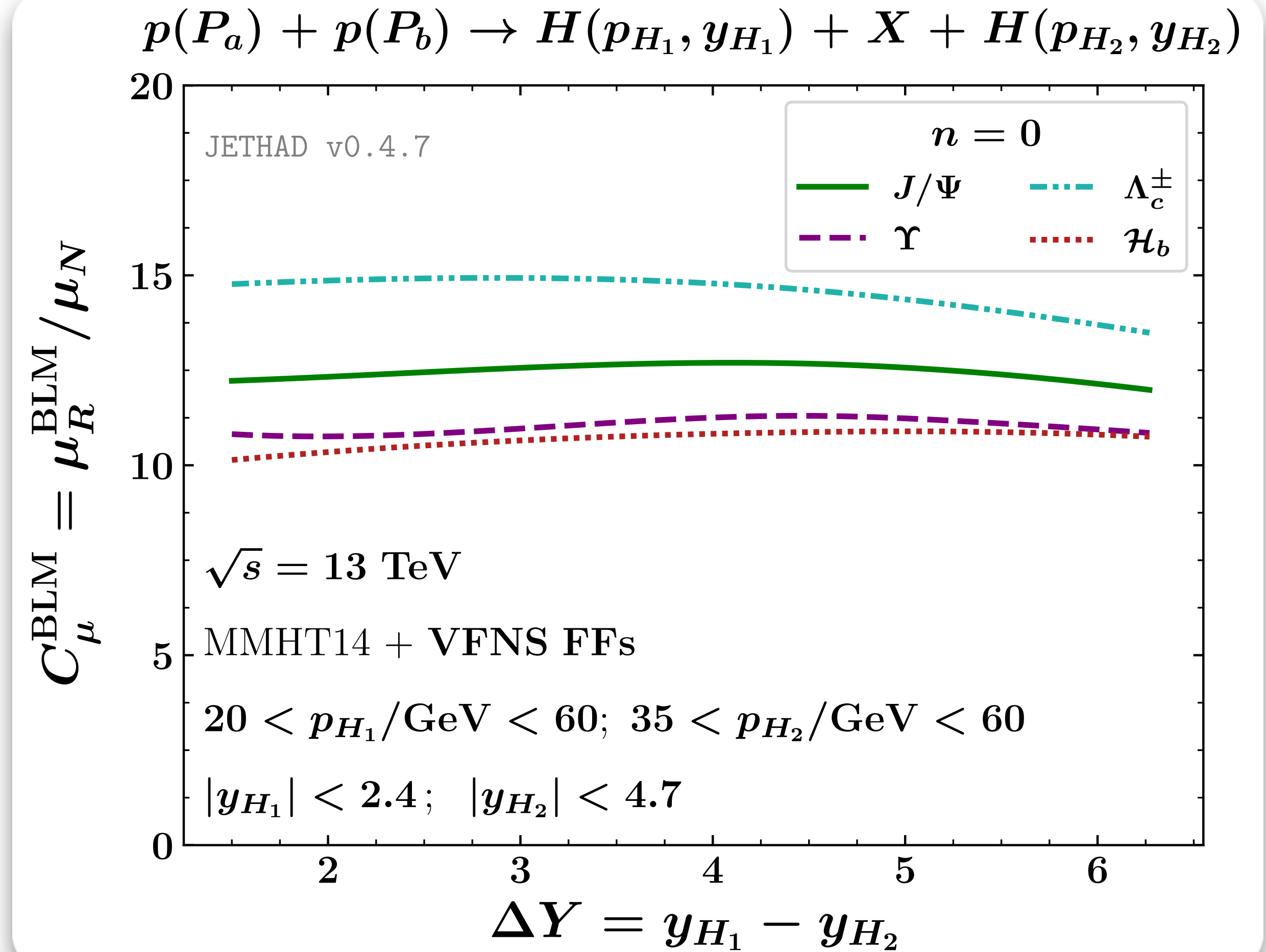
APFEL++/EKO

# $J/\psi$ + jet production at the LHC

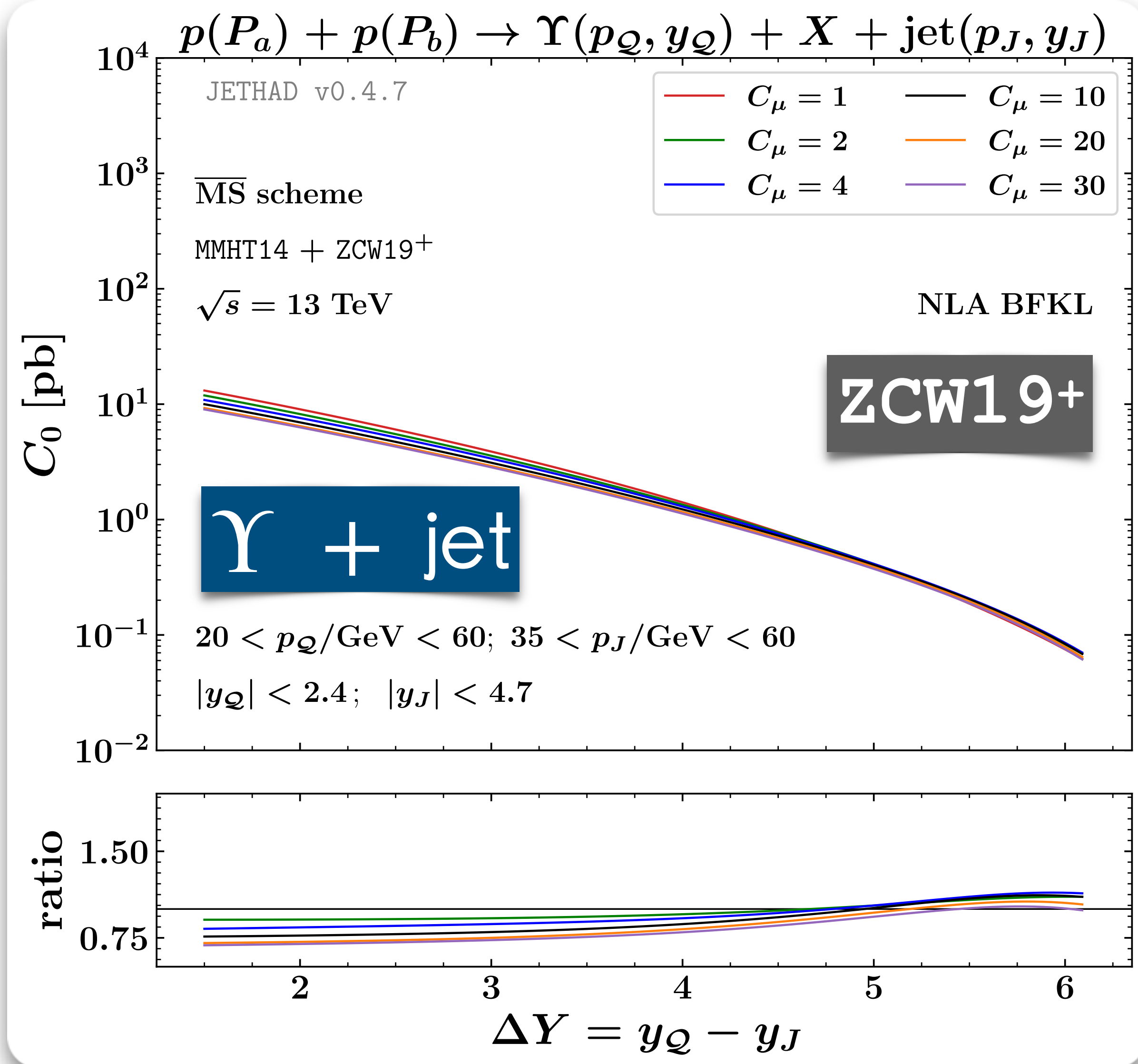
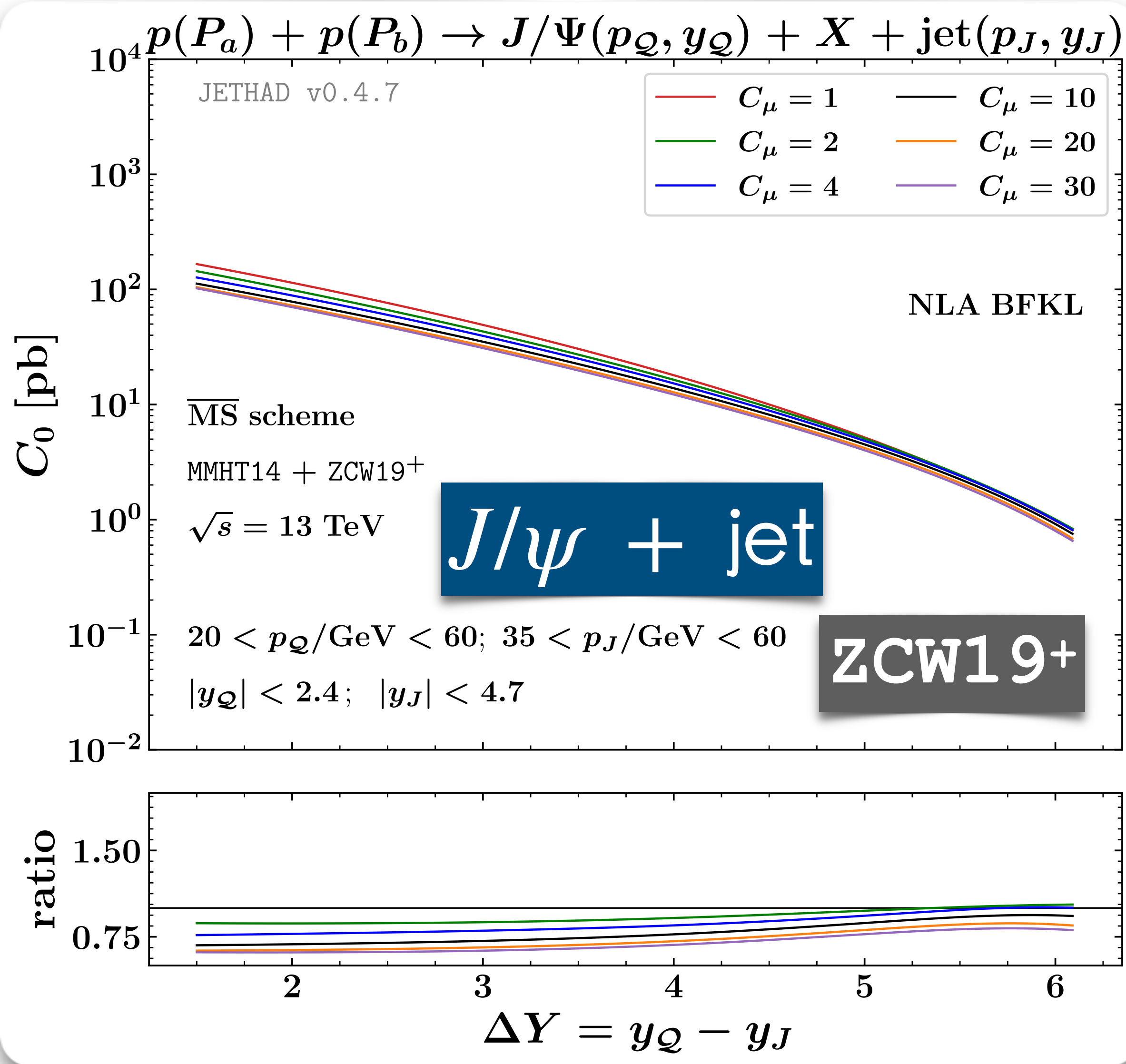
## Collinear FFs



## BLM scales



# Vector quarkonium + jet at the LHC



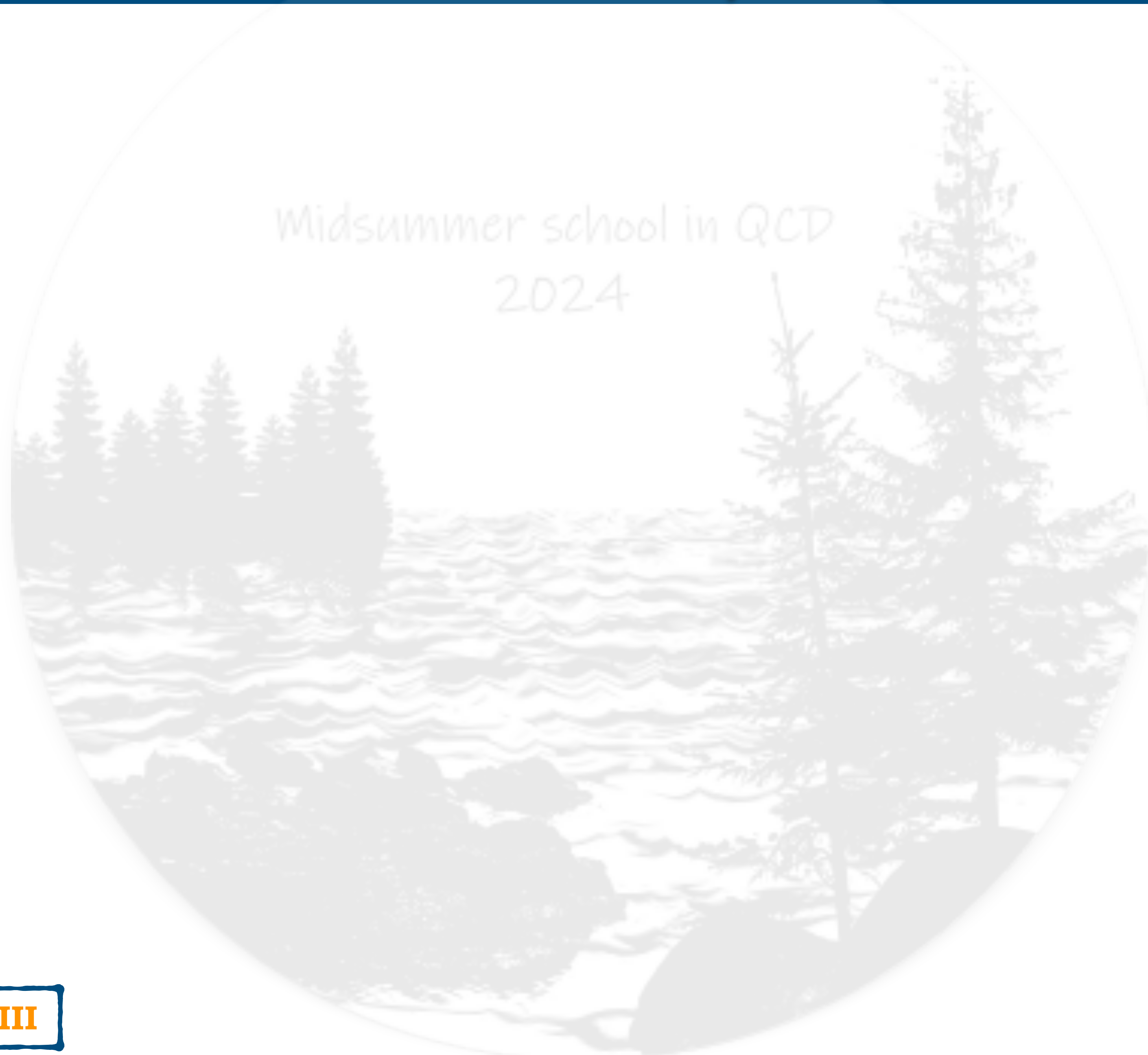
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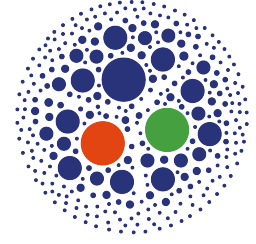
# Lecture III

# Checkpoint



# Lecture III: Summary & Outlook



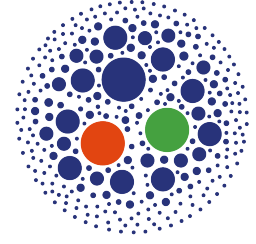


## Natural stability of the high-energy resummation

Midsummer school in QCD  
2024

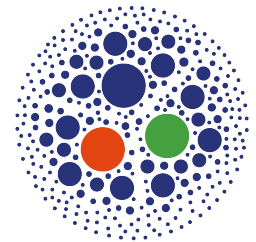


# Lecture II: Summary & Outlook



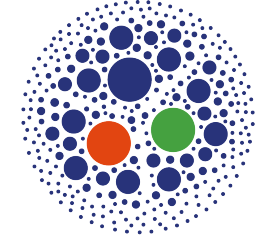
Natural stability of the high-energy resummation

Midsummer school in QCD  
2024



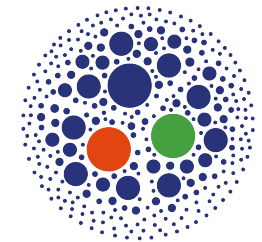
Higgs distributions  $\Rightarrow$  Tools for precision studies of high-energy QCD

# Lecture III: Summary & Outlook

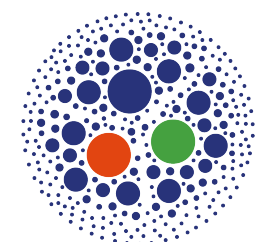


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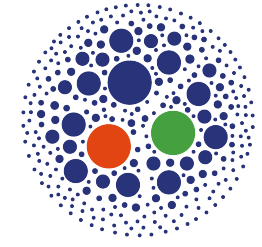


Higgs distributions  $\Rightarrow$  Tools for precision studies of high-energy QCD



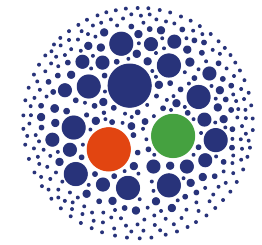
Heavy flavor FFs  $\Rightarrow$  Basis for explorations outside high-energy QCD

# Lecture III: Summary & Outlook

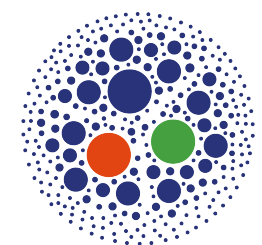


Natural stability of the high-energy resummation

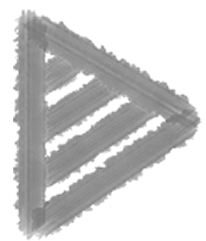
Midsummer school in QCD  
2024



Higgs distributions  $\Rightarrow$  Tools for precision studies of high-energy QCD



Heavy flavor FFs  $\Rightarrow$  Basis for explorations outside high-energy QCD

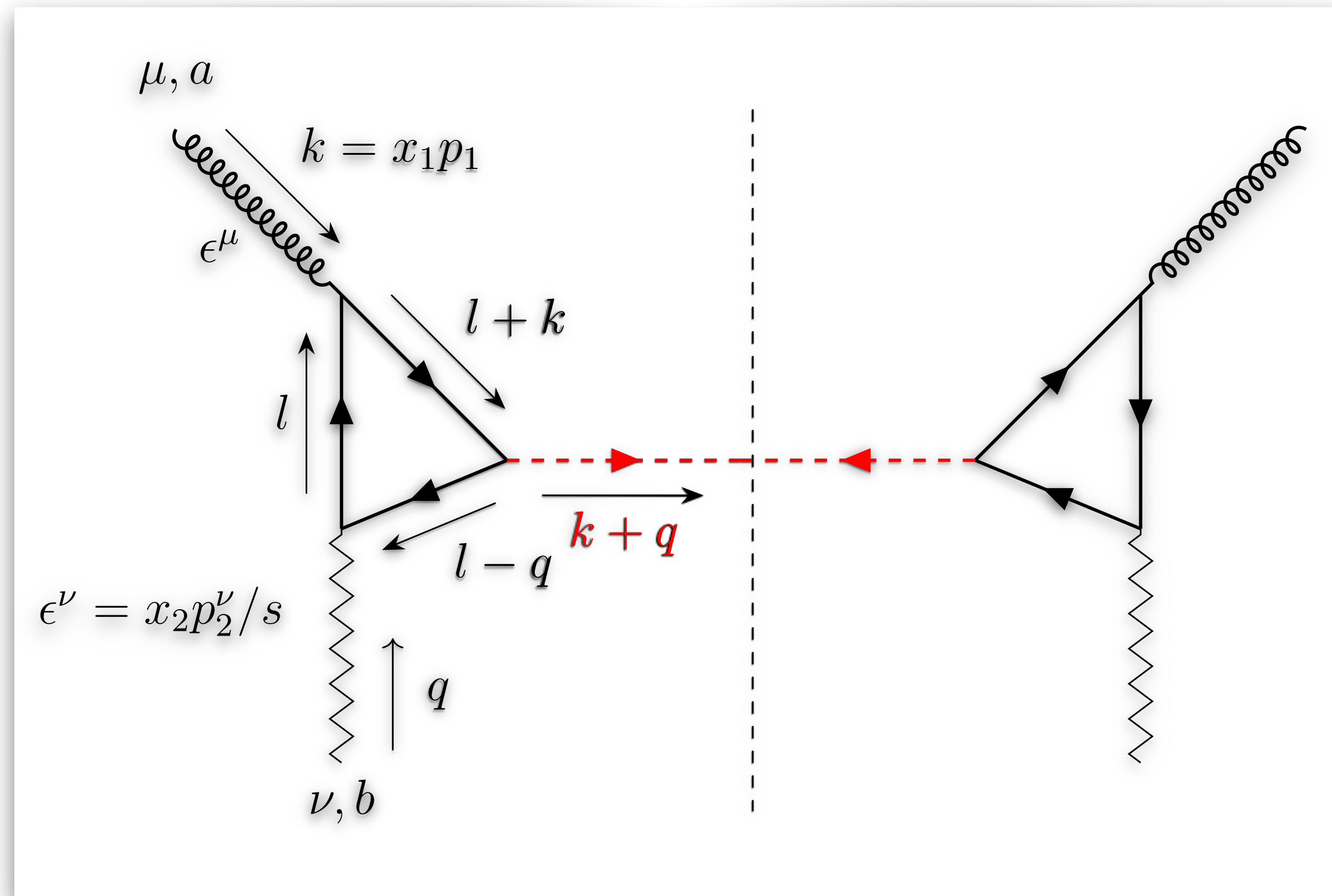


High-energy resummation  $\Rightarrow$  Proton structure and spin at small-x

**EXTRAS**

# HIGGS + JET

# Forward-Higgs LO impact factor



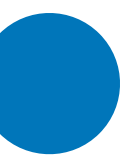
$$\frac{d\Phi_J^{(0)}(\nu, n)}{dx_J d^2\vec{p}_J} = 2\alpha_s \sqrt{\frac{C_F}{C_A}} (\vec{p}_J^2)^{i\nu-3/2} \left( \frac{C_A}{C_F} f_g(x_J) + \sum_{a=q\bar{q}} f_a(x_J) \right) e^{in\phi_J}$$





# Forward-Higgs NLO-RG impact factor

$$\begin{aligned} \tilde{c}_H^{(1)}(n, \nu, |\vec{p}_H|, x_H) = c_H(n, \nu, |\vec{p}_H|, x_H) & \left\{ \frac{\beta_0}{4N_c} \left( 2 \ln \frac{\mu_{R_1}}{|\vec{p}_H|} + \frac{5}{3} \right) + \frac{\chi(n, \nu)}{2} \ln \left( \frac{s_0}{M_{H,\perp}^2} \right) \right. \\ & + \frac{\beta_0}{4N_c} \left( 2 \ln \frac{\mu_{R_1}}{M_{H,\perp}} \right) \\ & \left. - \frac{1}{2N_c f_g(x_H, \mu_{F_1})} \ln \frac{\mu_{F_1}^2}{M_{H,\perp}^2} \int_{x_H}^1 \frac{dz}{z} \left[ P_{gg}(z) f_g \left( \frac{x_H}{z}, \mu_{F_1} \right) + \sum_{a=q, \bar{q}} P_{ga}(z) f_a \left( \frac{x_H}{z}, \mu_{F_1} \right) \right] \right\} \end{aligned}$$



# Forward-jet NLO-RG impact factor

$$\begin{aligned}
 \tilde{c}_J^{(1)}(n, \nu, |\vec{p}_J|, x_J) = & c_J(n, \nu, |\vec{p}_J|, x_J) \left\{ \frac{\beta_0}{4N_c} \left( 2 \ln \frac{\mu_{R_2}}{|\vec{p}_J|} + \frac{5}{3} \right) + \frac{\chi(n, \nu)}{2} \ln \left( \frac{s_0}{|\vec{p}_J|^2} \right) \right. \\
 & \left. - \frac{1}{2N_c \left( \frac{C_A}{C_F} f_g(x_J, \mu_{F_2}) + \sum_{a=q, \bar{q}} f_a(x_J, \mu_{F_2}) \right)} \ln \frac{\mu_{F_2}^2}{|\vec{p}_J|^2} \right. \\
 & \times \left( \frac{C_A}{C_F} \int_{x_J}^1 \frac{dz}{z} \left[ P_{gg}(z) f_g \left( \frac{x_J}{z}, \mu_{F_2} \right) + \sum_{a=q, \bar{q}} P_{ga}(z) f_a \left( \frac{x_J}{z}, \mu_{F_2} \right) \right] \right. \\
 & \left. \left. + \sum_{a=q, \bar{q}} \int_{x_J}^1 \frac{dz}{z} \left[ P_{ag}(z) f_g \left( \frac{x_J}{z}, \mu_{F_2} \right) + P_{aa}(z) f_a \left( \frac{x_J}{z}, \mu_{F_2} \right) \right] \right) \right\} .
 \end{aligned}$$



# Inclusive Higgs + jet: NLL/NLO\* azimuthal coefficients

$$\begin{aligned}
 C_n &= \frac{e^{\Delta Y}}{s} \frac{M_{H,\perp}}{|\vec{p}_H|} \\
 &\times \int_{-\infty}^{+\infty} d\nu \left( \frac{x_J x_H s}{s_0} \right)^{\bar{\alpha}_s(\mu_{R_c})} \left\{ \chi(n, \nu) + \bar{\alpha}_s(\mu_{R_c}) \left[ \bar{\chi}(n, \nu) + \frac{\beta_0}{8N_c} \chi(n, \nu) \left[ -\chi(n, \nu) + \frac{10}{3} + 4 \ln \left( \frac{\mu_{R_c}}{\sqrt{|\vec{p}_H \vec{p}_J|}} \right) \right] \right] \right\} \\
 &\quad \times \left\{ \alpha_s^2(\mu_{R_1}) c_H(n, \nu, |\vec{p}_H|, x_H) \right\} \left\{ \alpha_s(\mu_{R_2}) [c_J(n, \nu, |\vec{p}_J|, x_J)]^* \right\} \\
 &\quad \times \left\{ 1 + \bar{\alpha}_s(\mu_{R_1}) \frac{\tilde{c}_H^{(1)}(n, \nu, |\vec{p}_H|, x_H)}{c_H(n, \nu, |\vec{p}_H|, x_H)} + \bar{\alpha}_s(\mu_{R_2}) \left[ \frac{\tilde{c}_J^{(1)}(n, \nu, |\vec{p}_J|, x_J)}{c_J(n, \nu, |\vec{p}_J|, x_J)} \right]^* \right\} .
 \end{aligned}$$



# Higgs + jet highlights from the FCC Week 2022

The high-energy QCD dynamics from Higgs+jet correlations at FCC

Francesco G. Celiberto<sup>1,2,3</sup> and Alessandro Papa<sup>4,5</sup>

FCC Week 2022, Sorbonne Université, France

## Hors d'œuvre

- Higgs sector → SM benchmarks, BSM portals
- Gluon fusion → key ingredient for precision QCD
- Fixed-order ← improved by resummations
- FCC energies ↔ high-energy (HE) resummation
- Higgs+jet → golden channel to hunt for HE signals

## NLL/NLO differential cross section

$$\frac{d\sigma}{dy_1 dy_2 d^2k_1 d^2k_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2 f_r(x_1, \mu_F) f_s(x_2, \mu_F) \frac{d\hat{\sigma}_{rs}(x_1, x_2, s, \mu_F)}{dy_1 dy_2 d^2k_1 d^2k_2}$$

$$\frac{d\hat{\sigma}_{rs}(x_1, x_2, s, \mu)}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}} = \frac{1}{(2\pi)^2} \times \int \frac{d^2\vec{q}_1}{q_1^2} V_H^{(r)}(\vec{q}_1, s_0, x_1, \vec{p}_{T1}) \times \int_{s-i\infty}^{s+i\infty} \frac{d\omega}{2\pi i} \left( \frac{x_1 x_2 s}{s_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2) \times \int \frac{d^2\vec{q}_2}{q_2^2} V_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{p}_{T2})$$

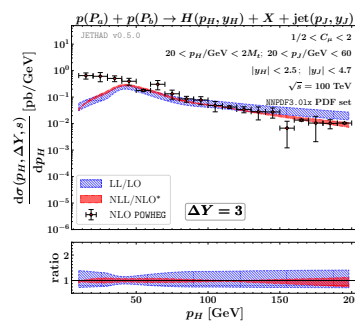
PDFs with threshold

NLO Higgs vertex

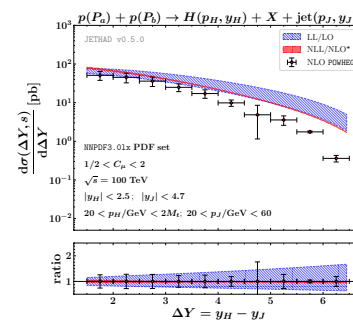
NLL BFKL kernel

NLO Jet vertex

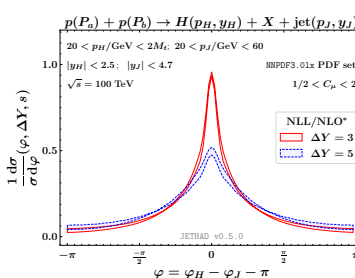
## Hybrid high-energy and collinear factorization at work



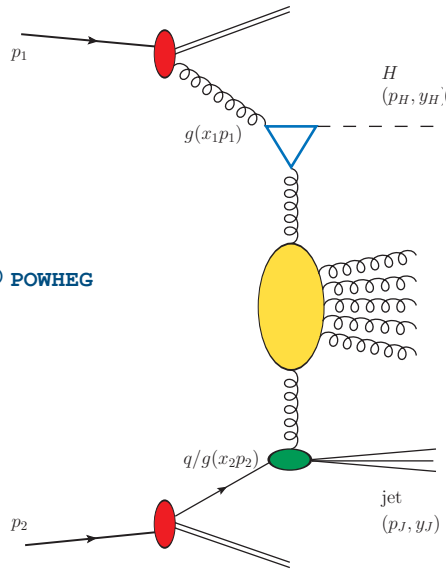
Higgs  $p_T$  distribution: NLL/NLO JETHAD vs NLO POWHEG



Rapidity distribution: NLL/NLO JETHAD vs NLO POWHEG



Azimuthal distribution at NLL/NLO



HE resummation from JETHAD

Large-x NNPDF3.0Lx PDFs with threshold

Comparison with fixed-order from POWHEG

Distributions stable under NLL corrections

## A path towards precision

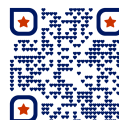
- ✓ NLL bands nested inside LL ones → solid stability
- ✓ HE signal clearly disengaged from NLO background
- ✓ Way toward precision studies of HE QCD (1!)
- Multilateral formalism → encode other resummations
- A window on proton structure at small-x (2?)

## Further information

- ECT\*, I-38123 Villazzano, Trento, Italy
- Fondazione Bruno Kessler (FBK), I-38123 Povo, Trento, Italy
- INFN-TIFPA, I-38123 Povo, Trento, Italy
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Contact: fceliberto@ectstar.eu

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PDFs with threshold

NLO Higgs vertex

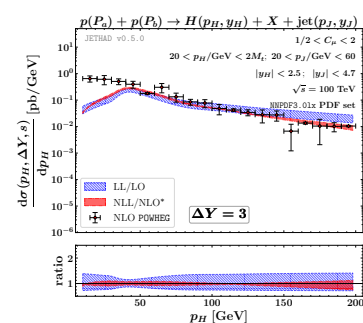
NLL BFKL kernel

NLO Jet vertex

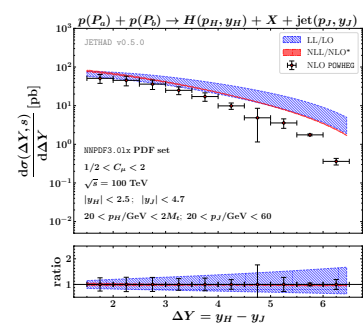
$$C_n(\Delta Y, s) = \int_{p_H^{\min}}^{p_H^{\max}} d|\vec{p}_H| \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \int_{y_H^{\min}}^{y_H^{\max}} dy_H \int_{y_J^{\min}}^{y_J^{\max}} dy_J \delta(y_H - y_J - \Delta Y) C_n$$

## Rapidity distribution: NLL/NLO\* JETHAD vs NLO POWHEG

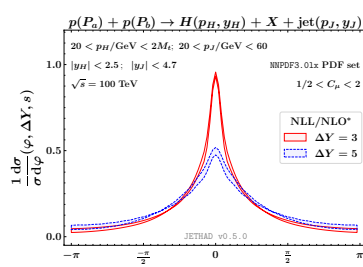
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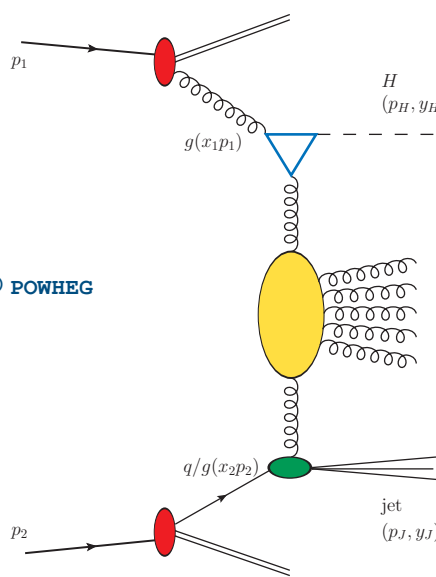
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Azimuthal distribution at NLL/NLO



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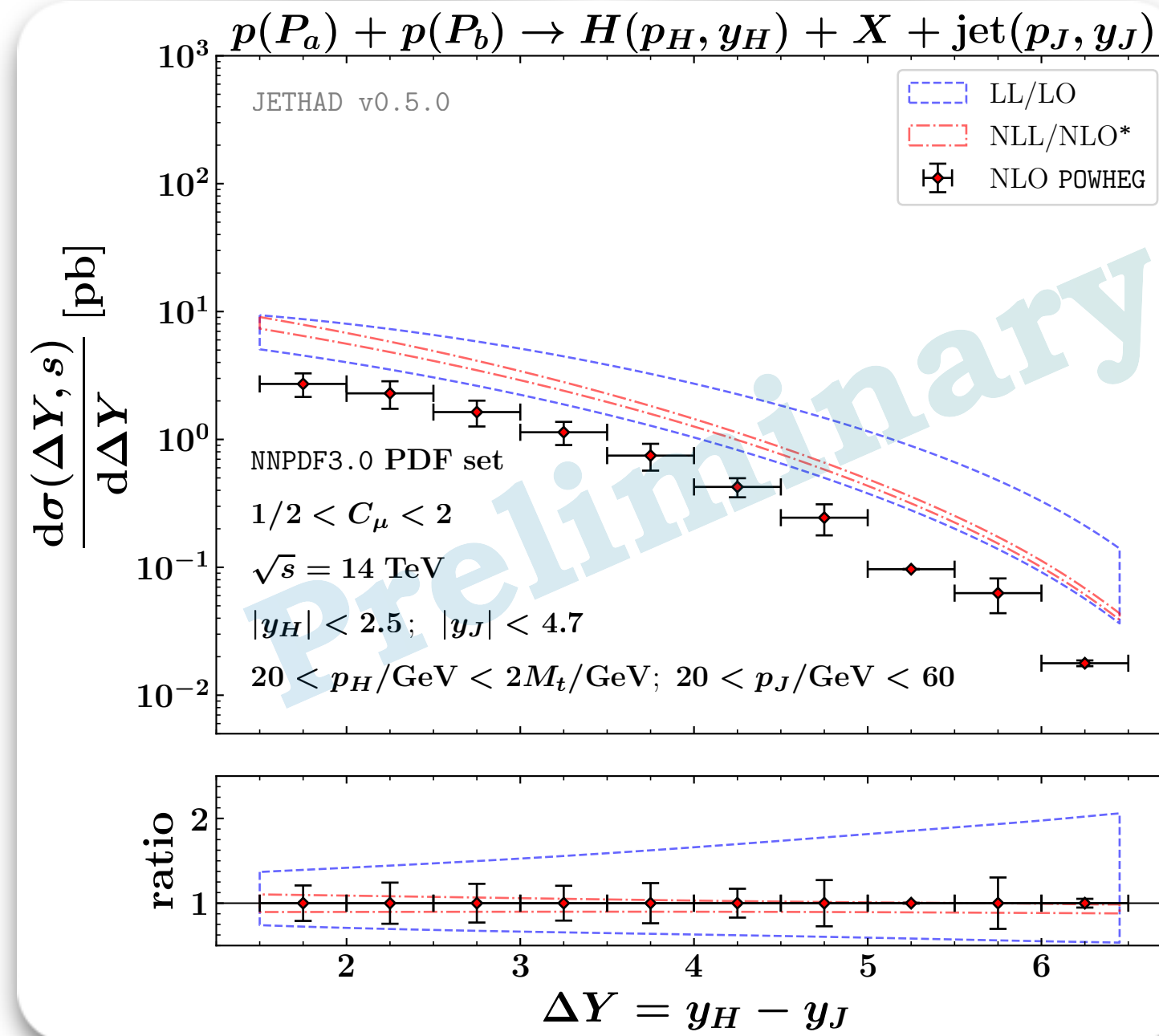
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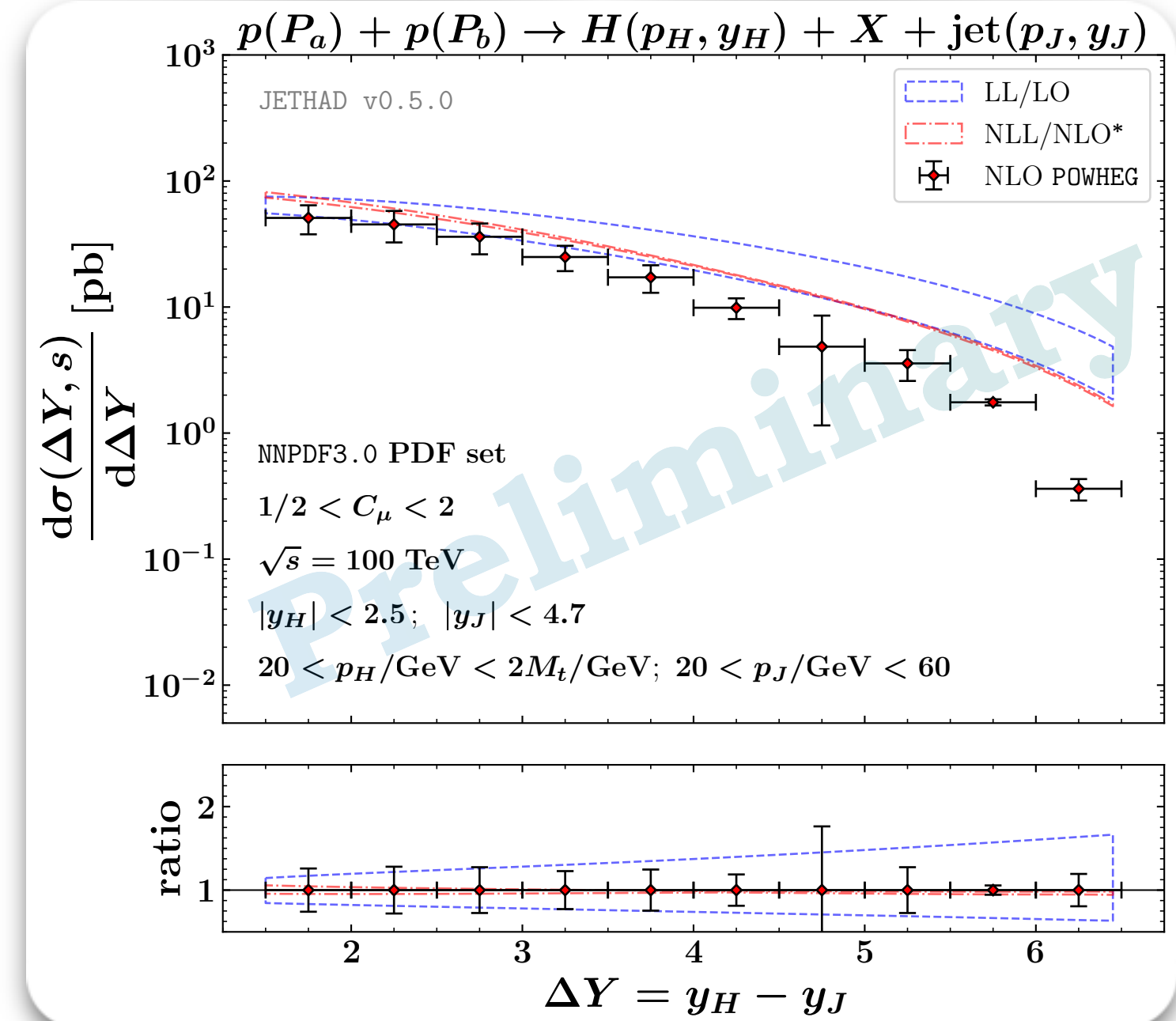
- ECT\*, I-38123 Villazzano, Trento, Italy
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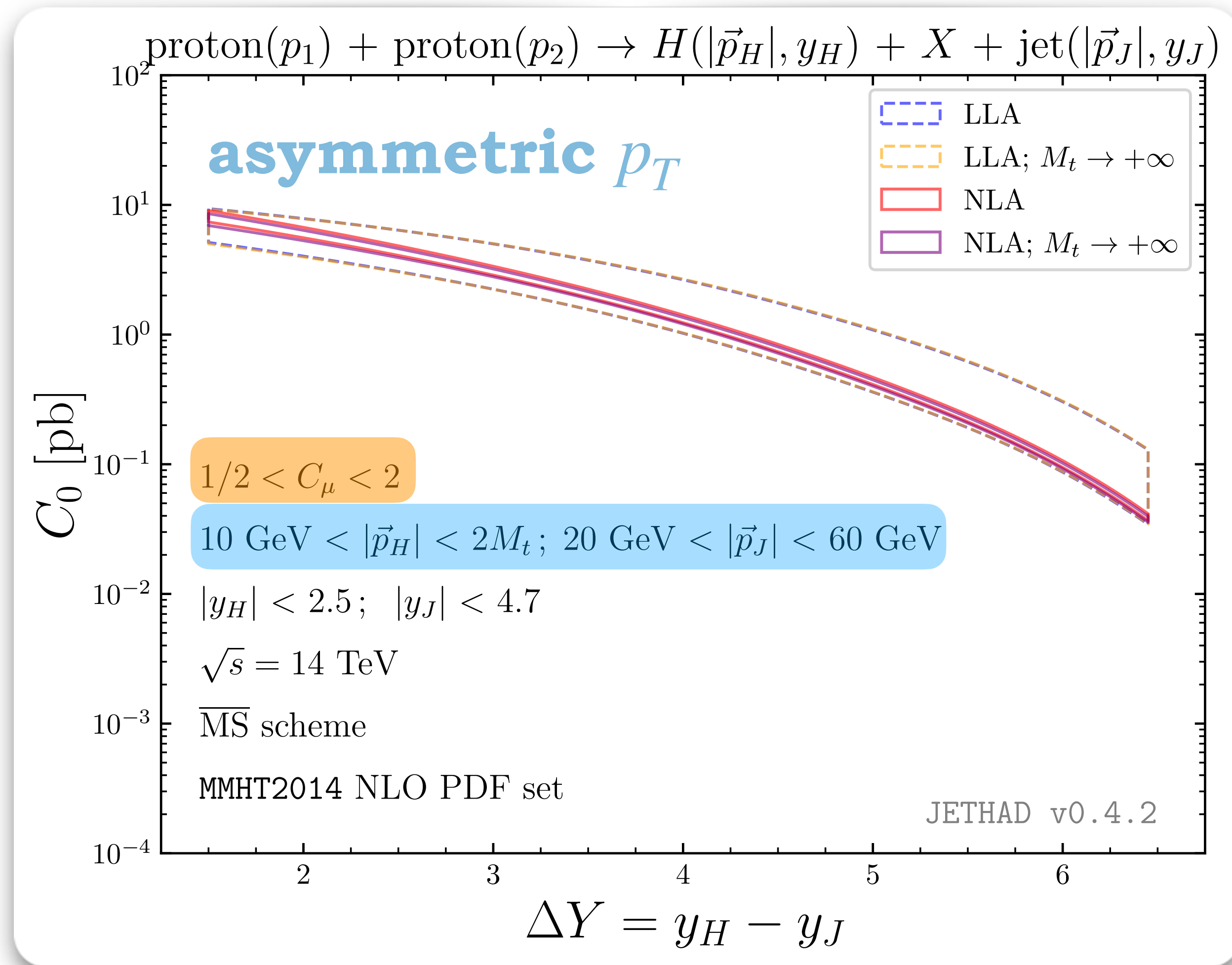
14 TeV LHC



100 TeV FCC

# $\Delta Y$ -distribution in the infinite top-mass limit

$$C_n(\Delta Y, s) = \int_{p_H^{\min}}^{p_H^{\max}} d|\vec{p}_H| \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \int_{y_H^{\min}}^{y_H^{\max}} dy_H \int_{y_J^{\min}}^{y_J^{\max}} dy_J \delta(y_H - y_J - \Delta Y) C_n$$



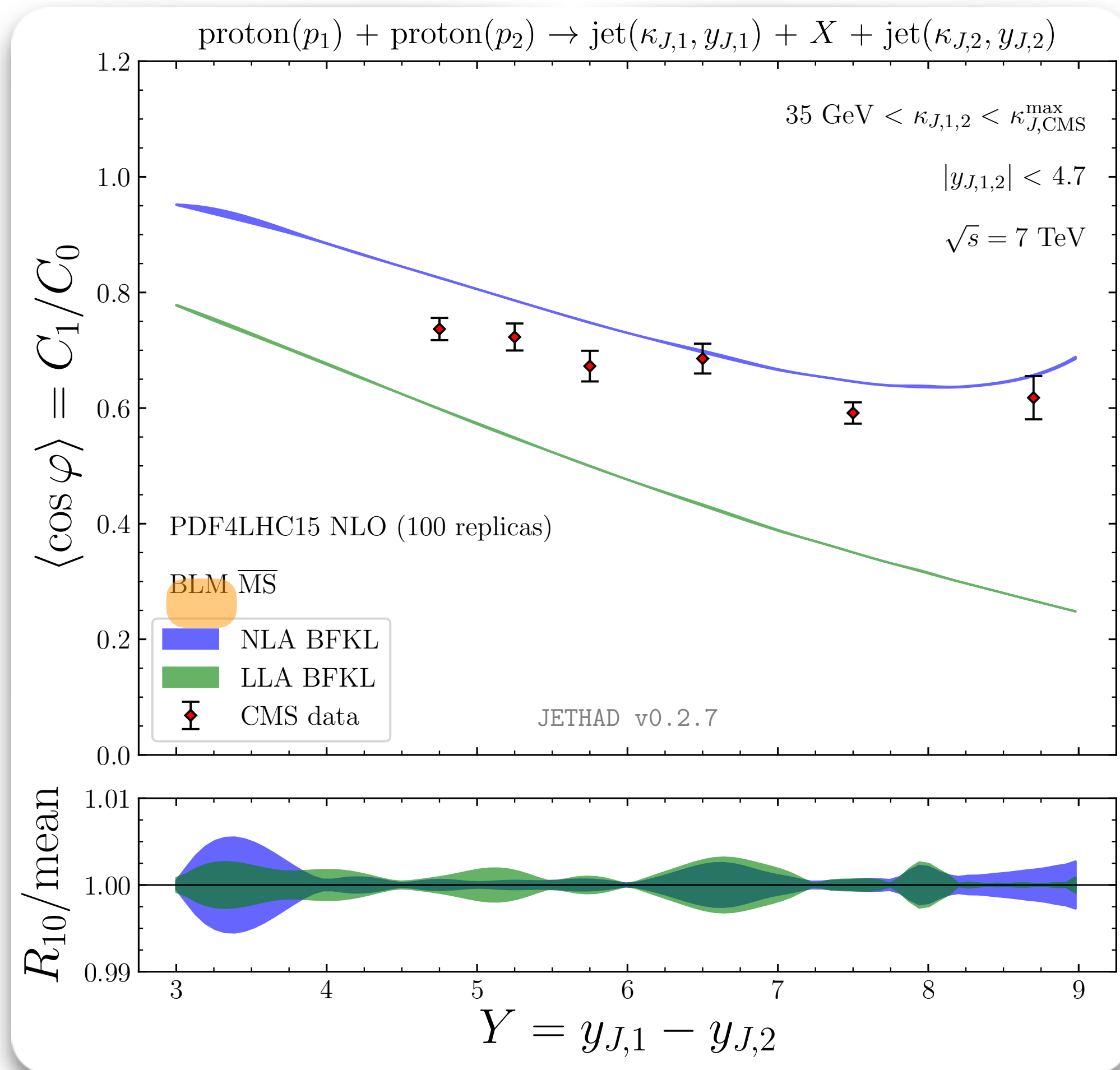
# Angular correlations in the infinite top-mass limit

$$R_{n0}(\Delta Y, s) = C_n/C_0 \equiv \langle \cos n\varphi \rangle$$

## Mueller-Navelet jets

[\[B. Ducloué, L. Szymanowski, S. Wallon, Phys.Rev.Lett. 112 \(2014\) 082003\]](#)

(figure below) [\[F. G. C., Eur. Phys. J. C 81 \(2021\) 8, 691\]](#)



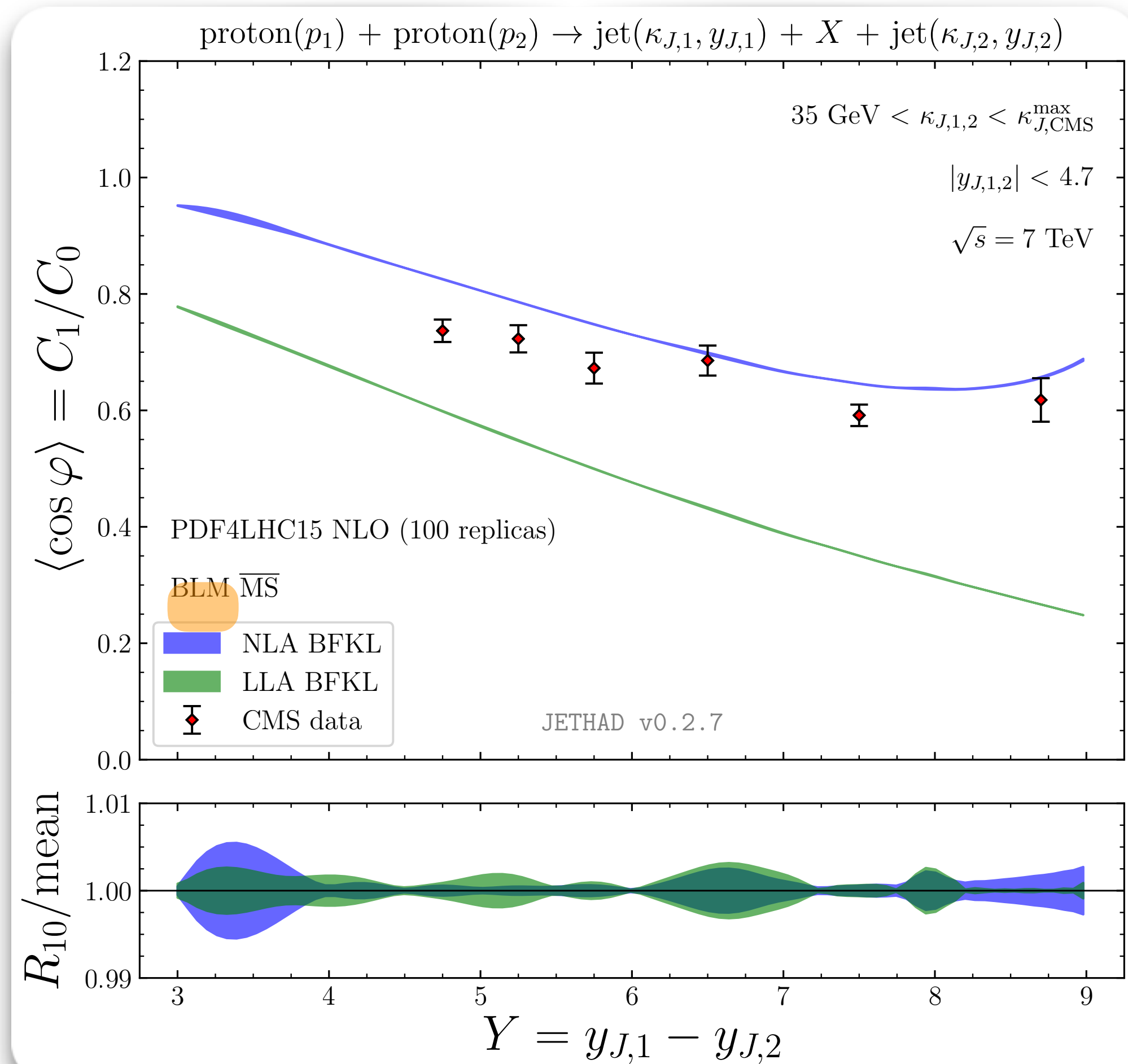
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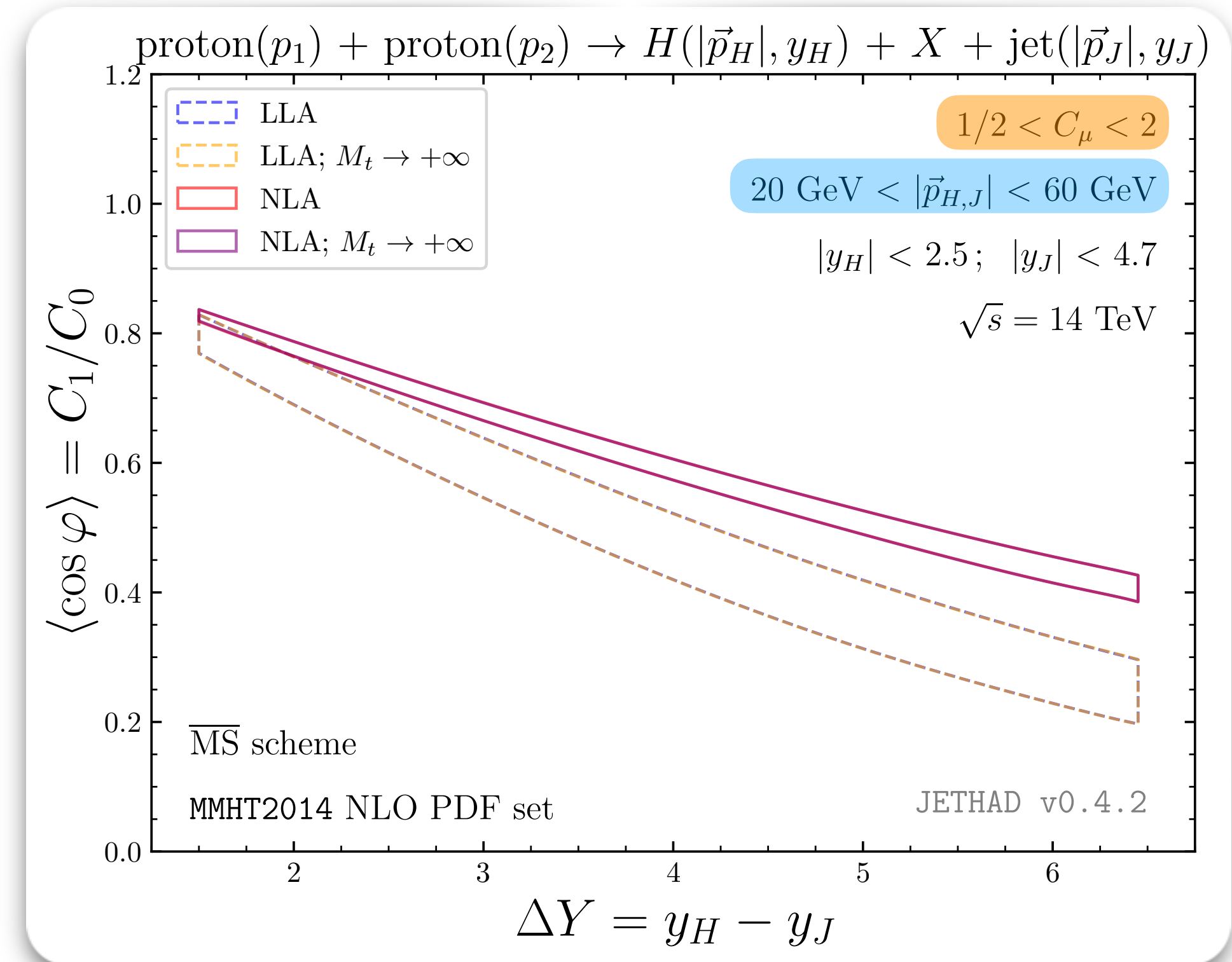
(figure below) [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]



## Higgs + jet

(figure below) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 4, 293]

(NLO Higgs impact factor) [F. G. C. et al., under review (2022)]

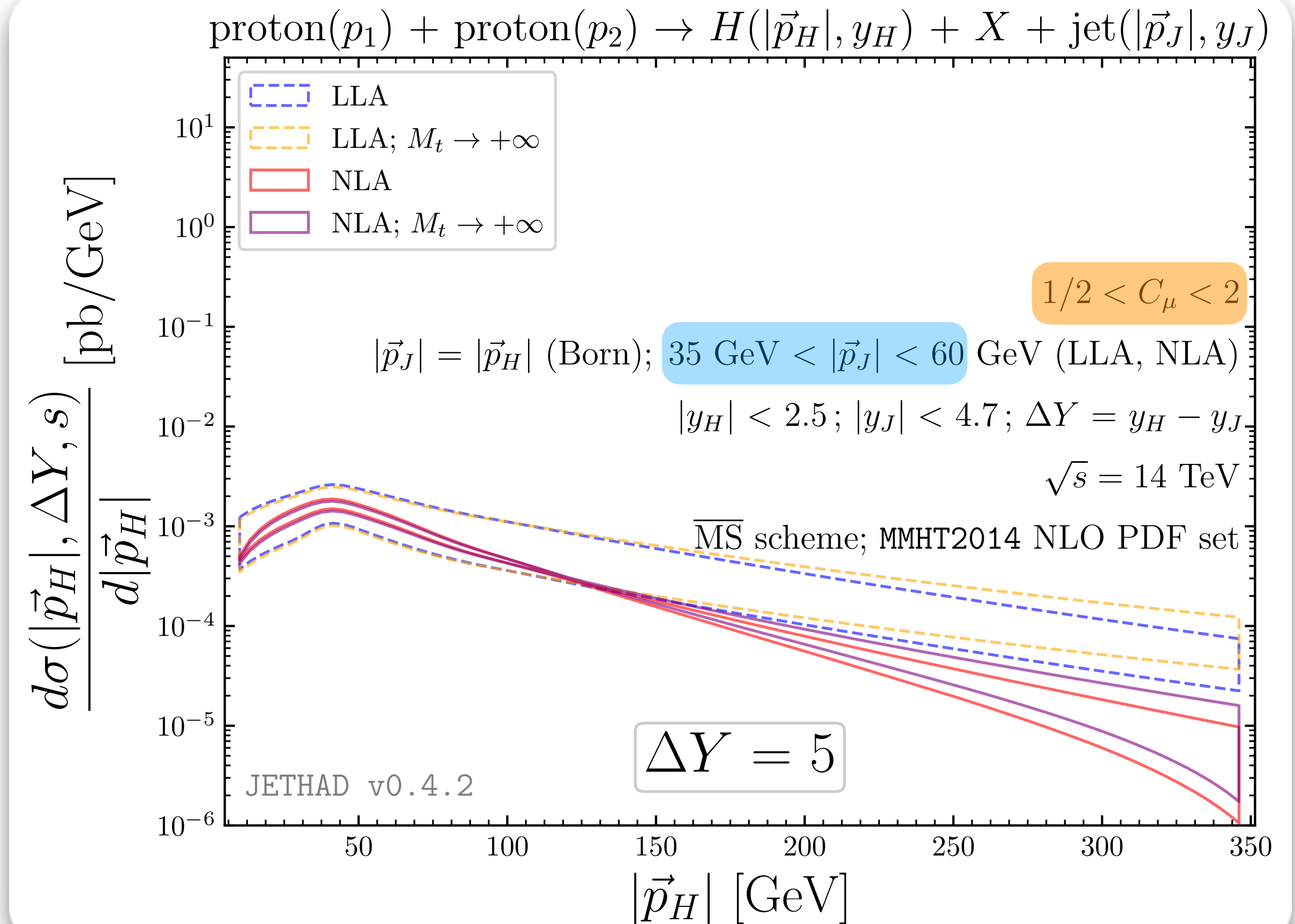
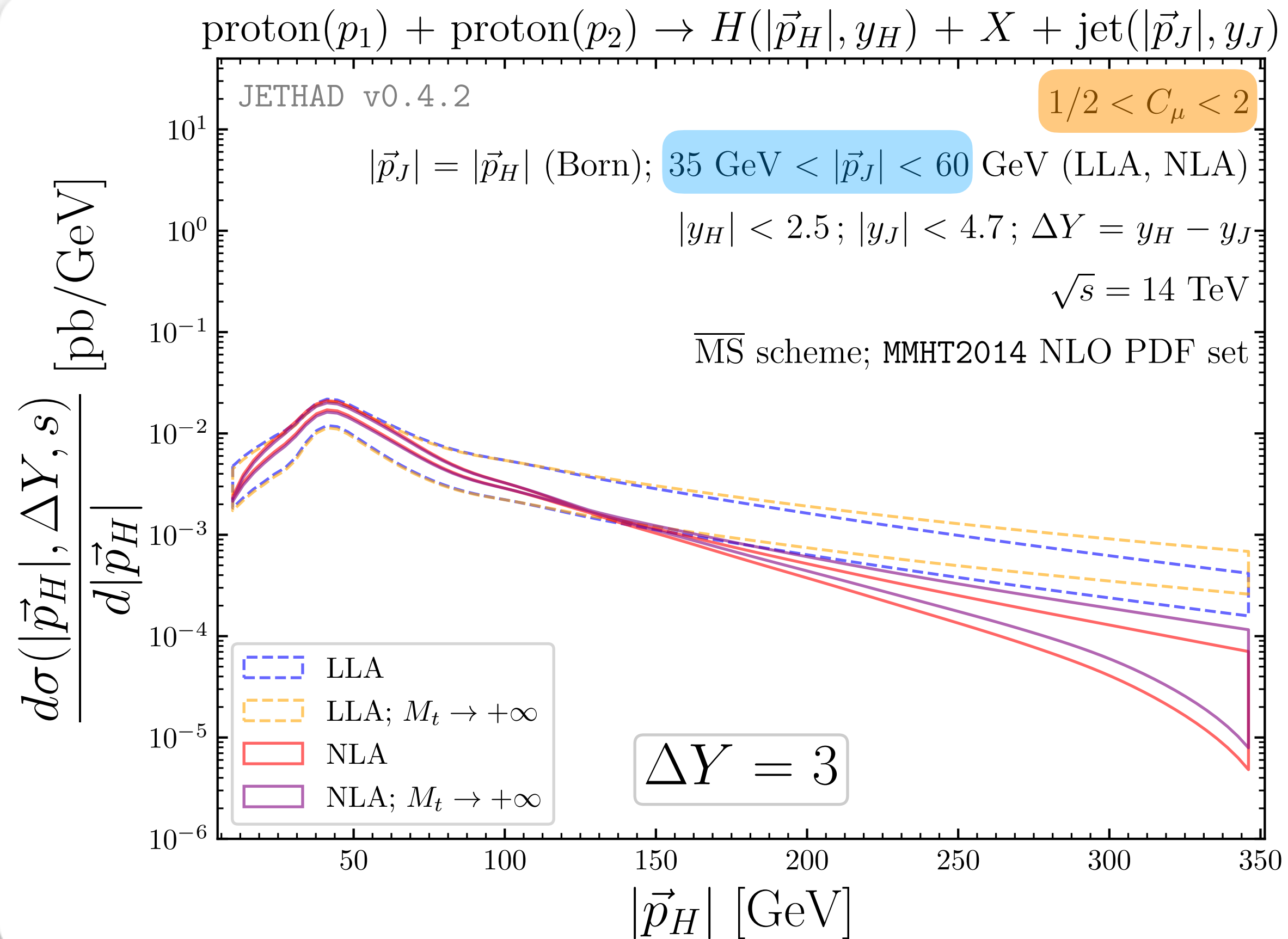


natural scales

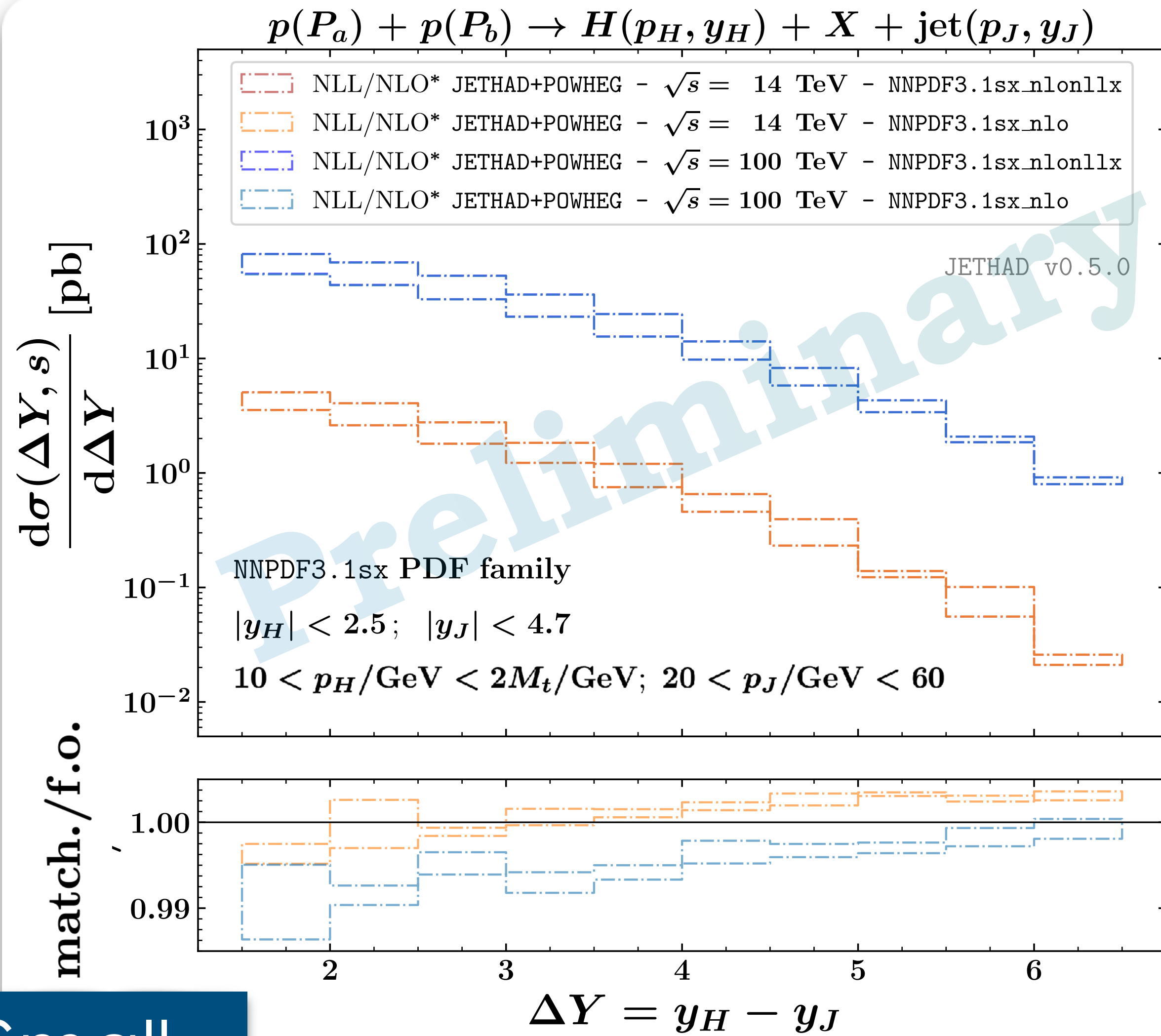
symmetric  $p_T$  range



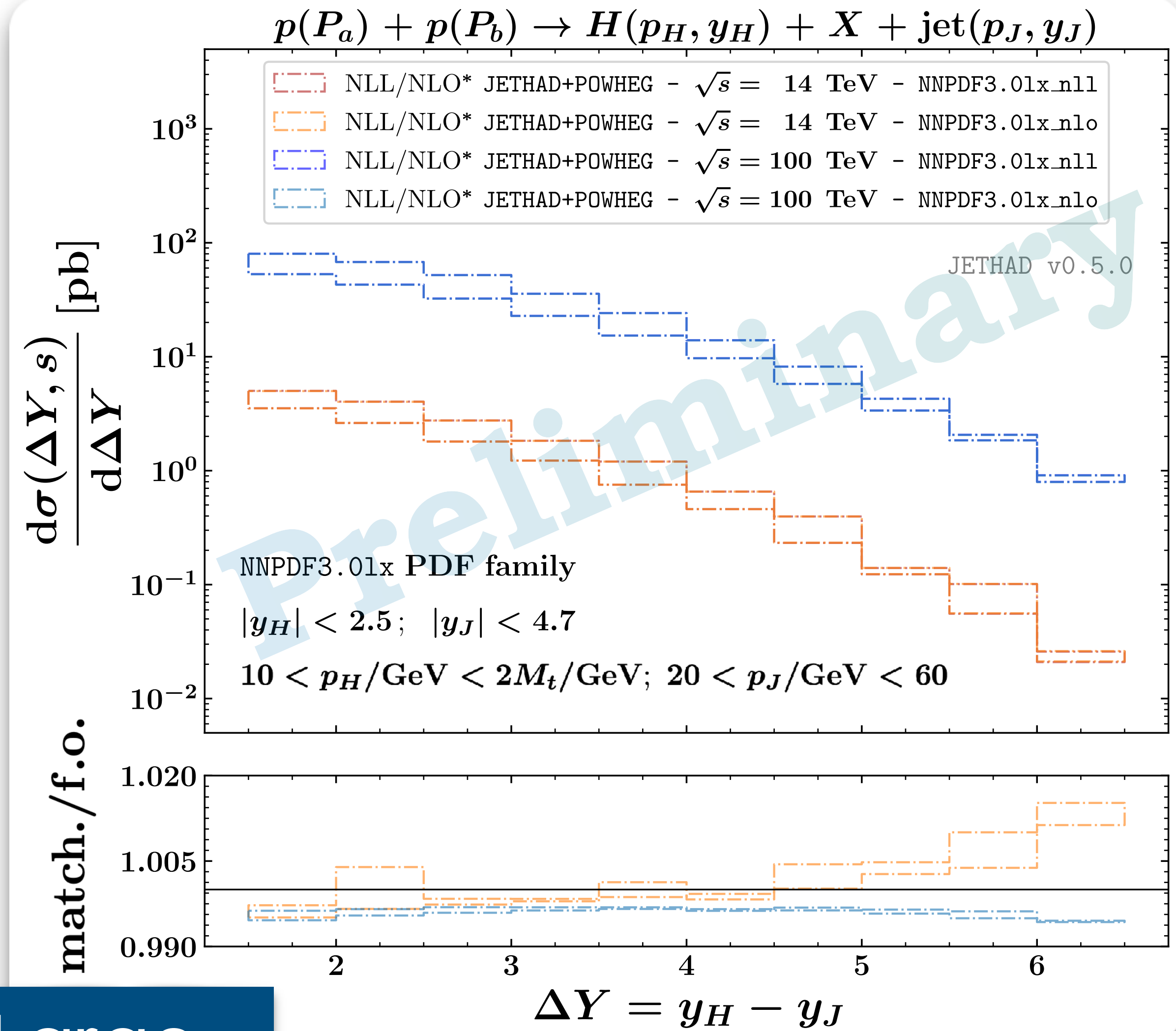
# Higgs transverse-momentum distribution for $(M_t \rightarrow +\infty)$



# Small- $x$ and large- $x$ enhancement from PDFs



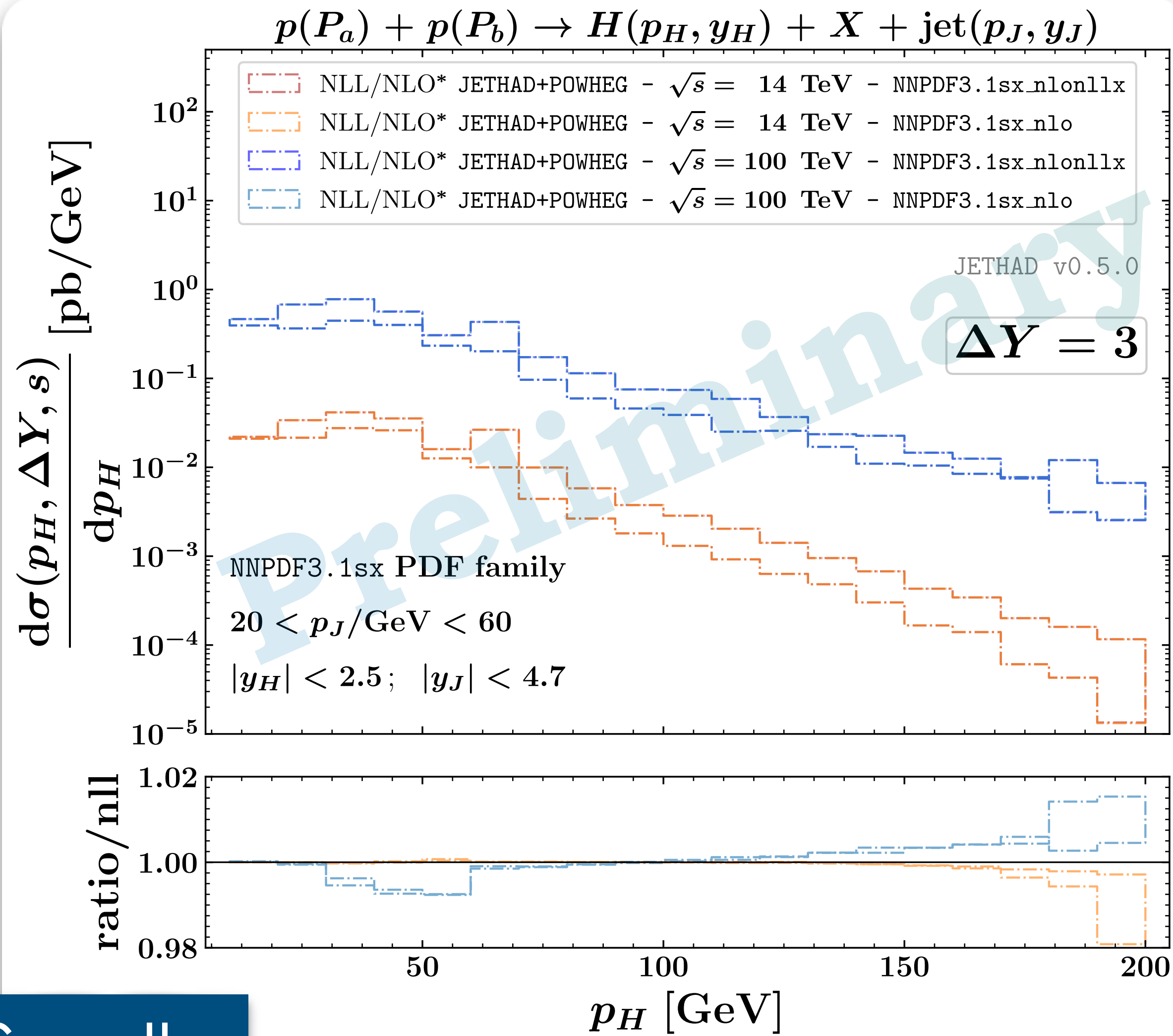
Small- $x$



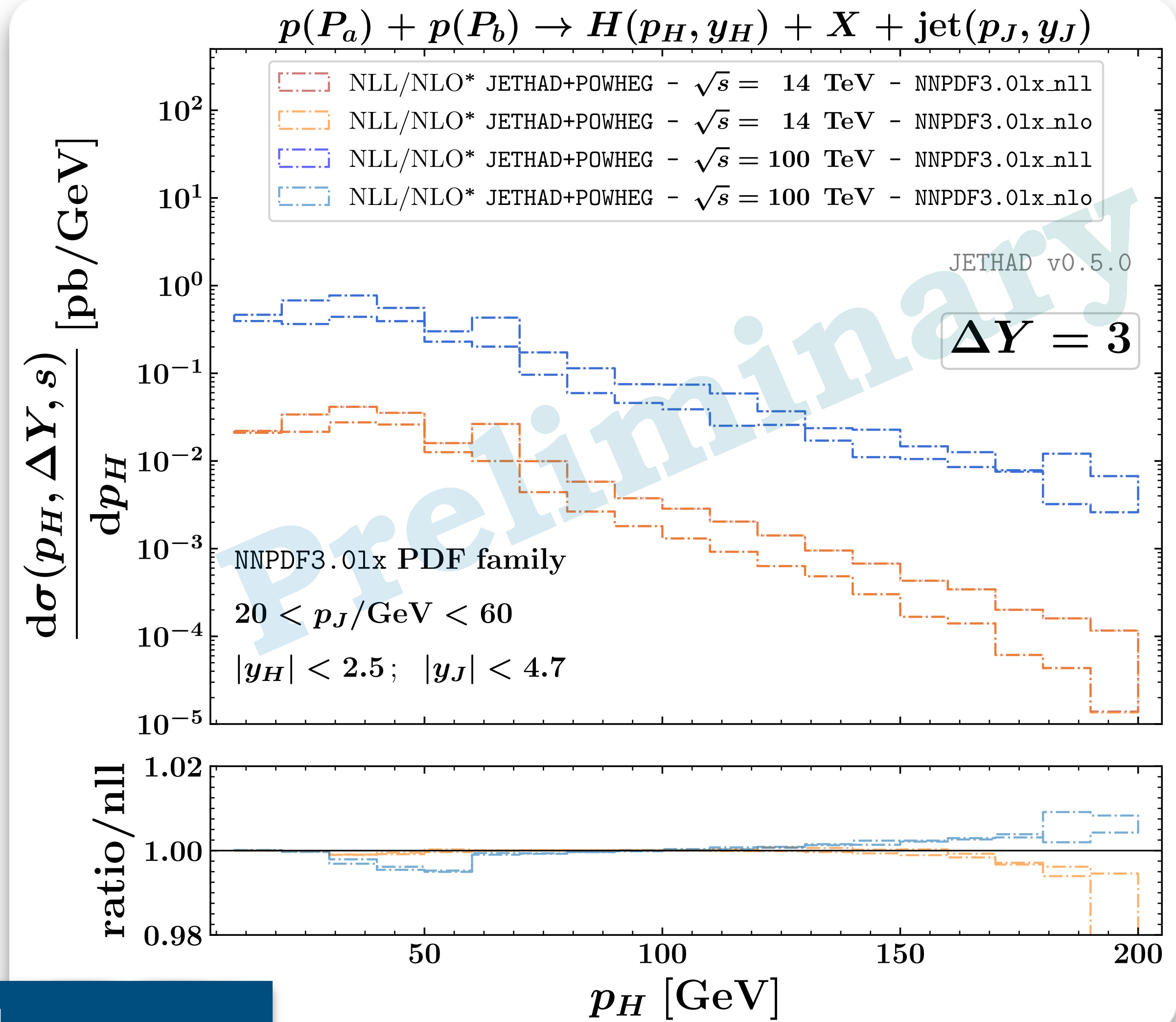
Large- $x$

Impact of large- $x$  threshold logs on NLO emission functions to be gauged

# Small- $x$ and large- $x$ enhancement from PDFs



Small- $x$

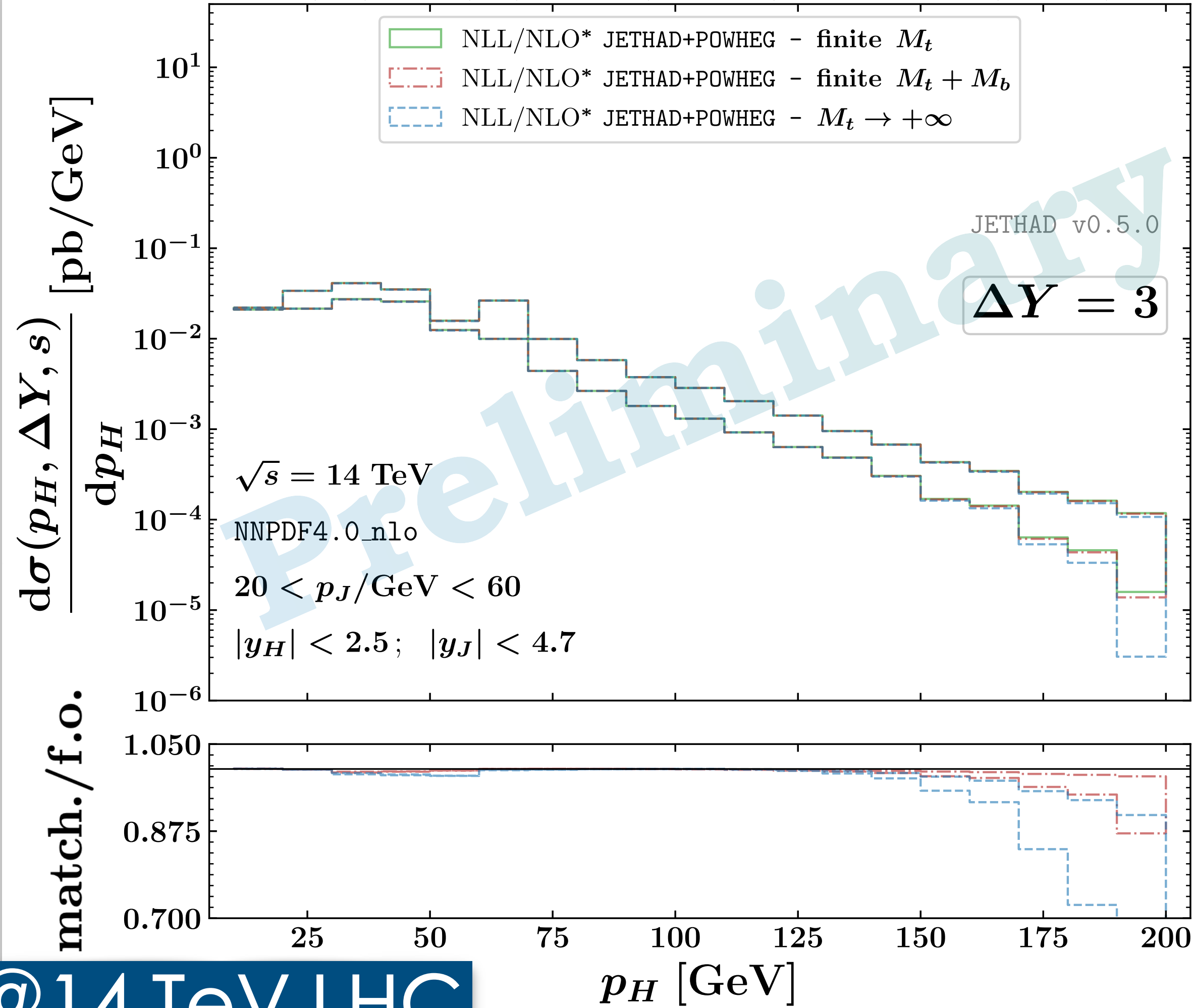


Large- $x$

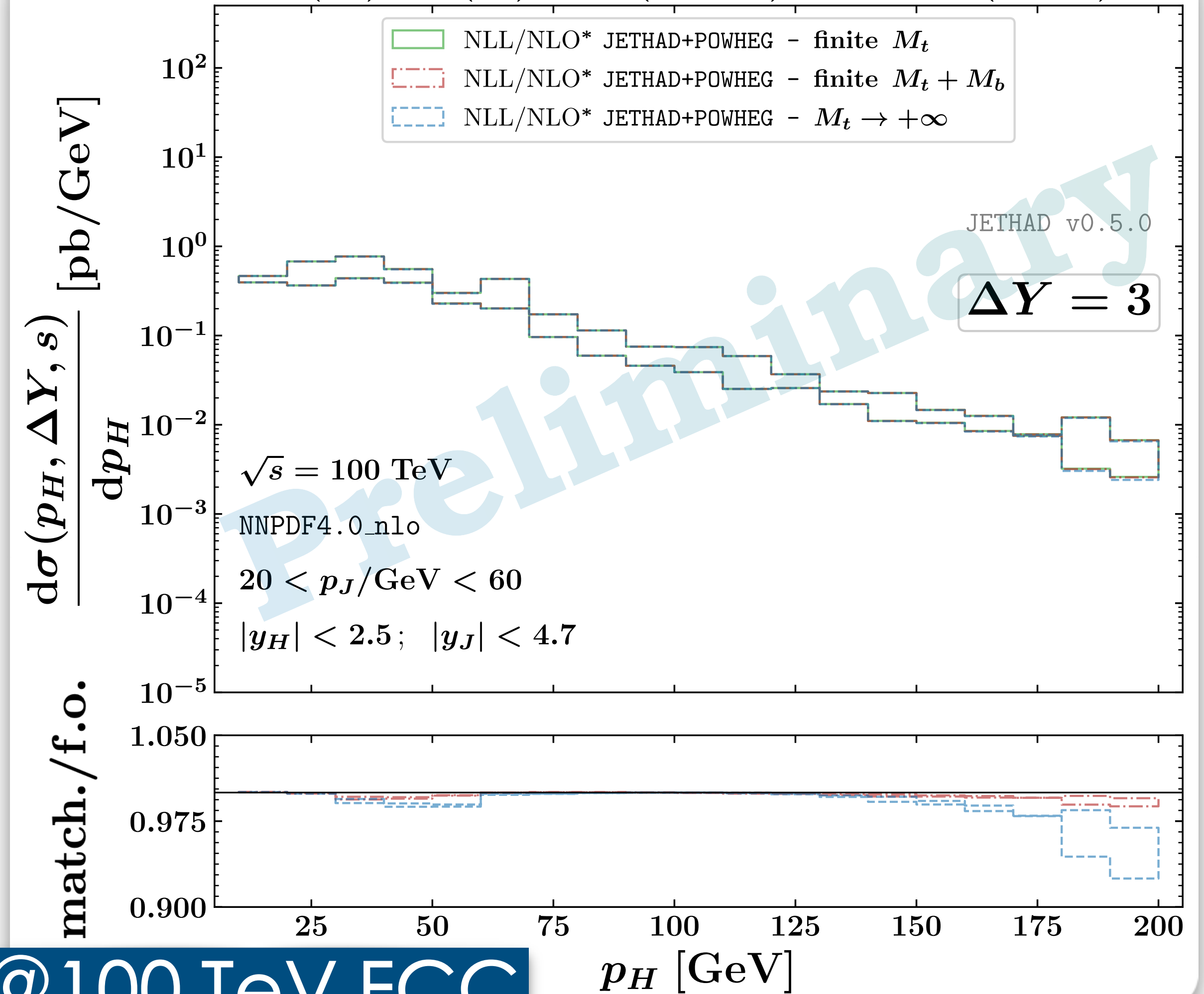
Impact of large- $x$  threshold logs on NLO emission functions to be gauged

# Finite top- and bottom-mass corrections

$$p(P_a) + p(P_b) \rightarrow H(p_H, y_H) + X + \text{jet}(p_J, y_J)$$

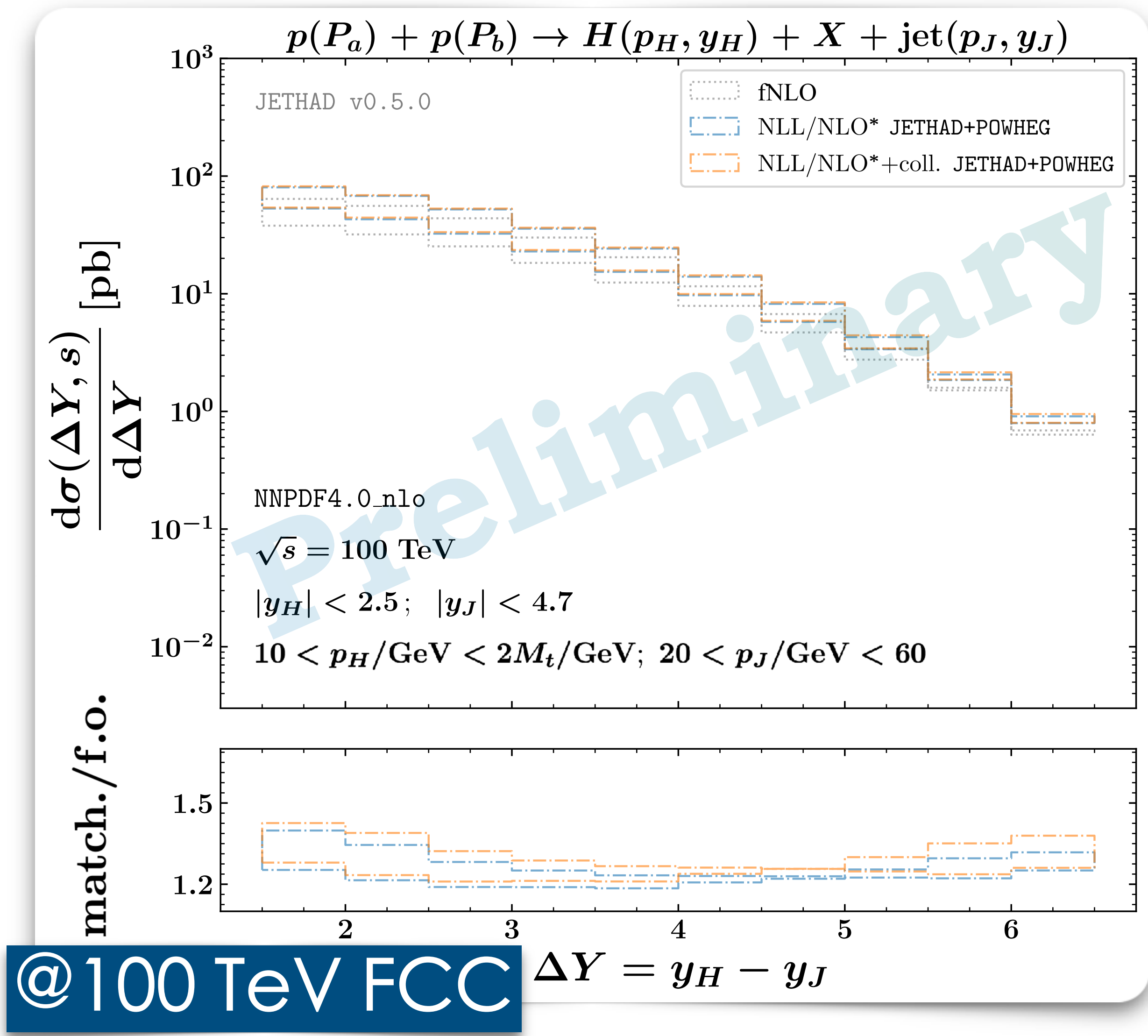
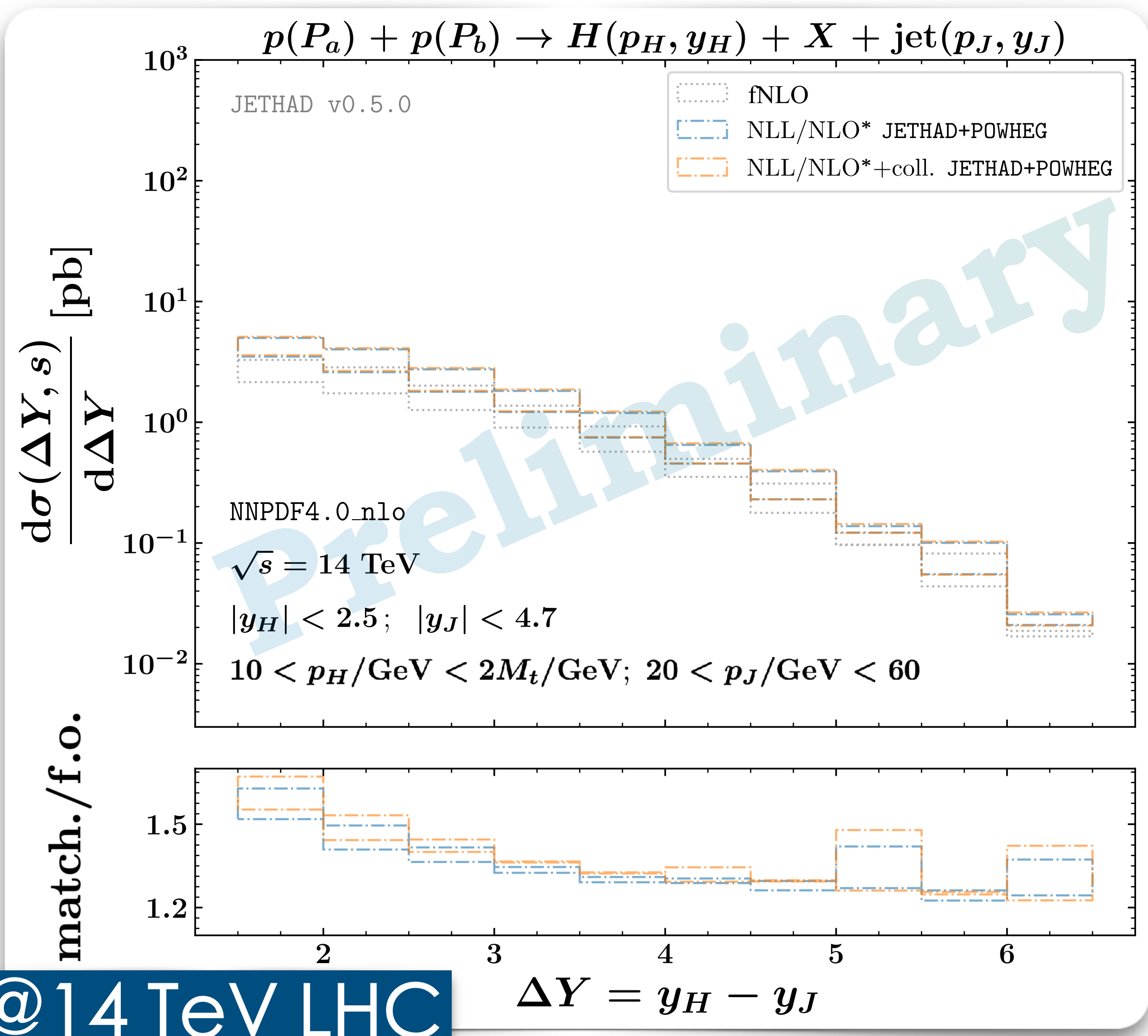


$$p(P_a) + p(P_b) \rightarrow H(p_H, y_H) + X + \text{jet}(p_J, y_J)$$



Effect of finite heavy-quark masses on NLO emission fns. to be gauged

# Effect of collinear improvement on NLL BFKL kernel

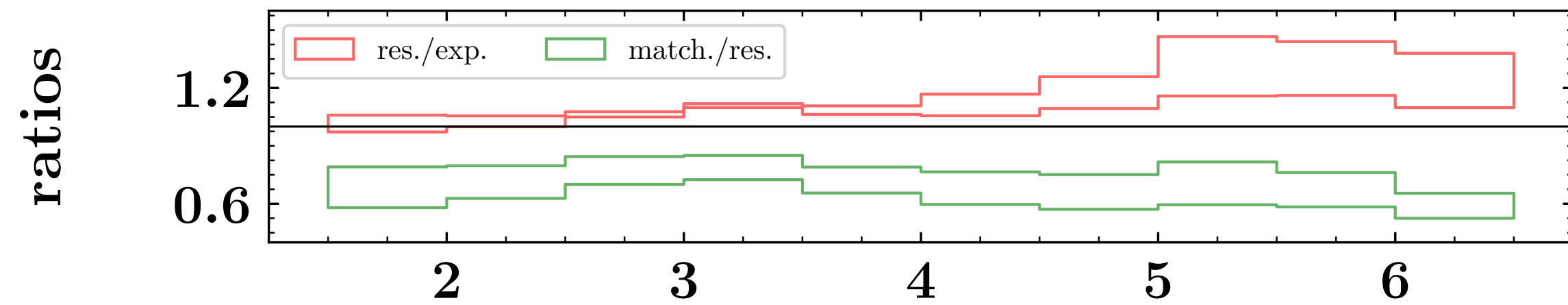
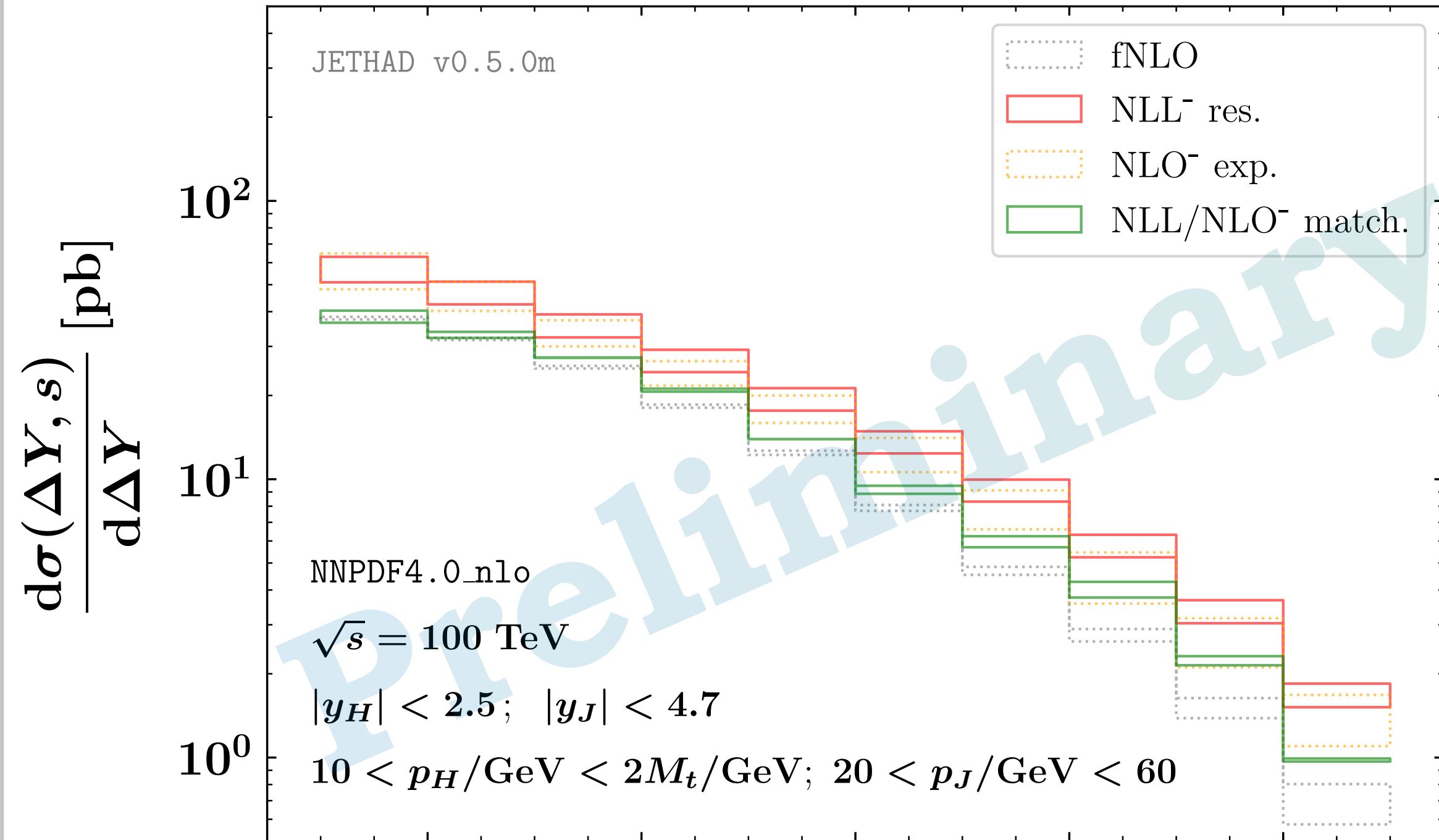


(Collinear improvement) [\[G.P. Salam, JHEP 07 \(1998\) 019\]](#); [\[M. Ciafaloni et al., Phys.Lett.B 587 \(2004\) 87-94\]](#); [\[A. Sabio Vera, Nucl.Phys.B 722 \(2005\) 65-80\]](#)

 Effect of ABF-stabilized kernel [\[to be gauged\]](#)

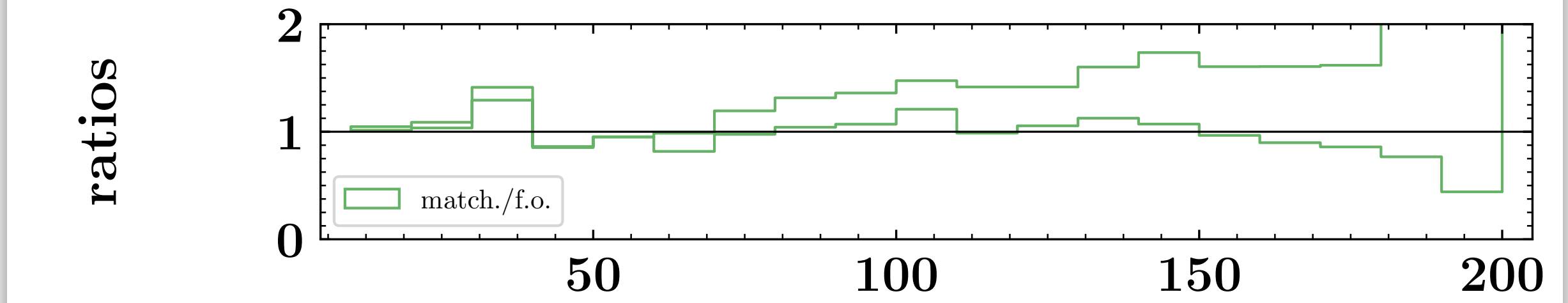
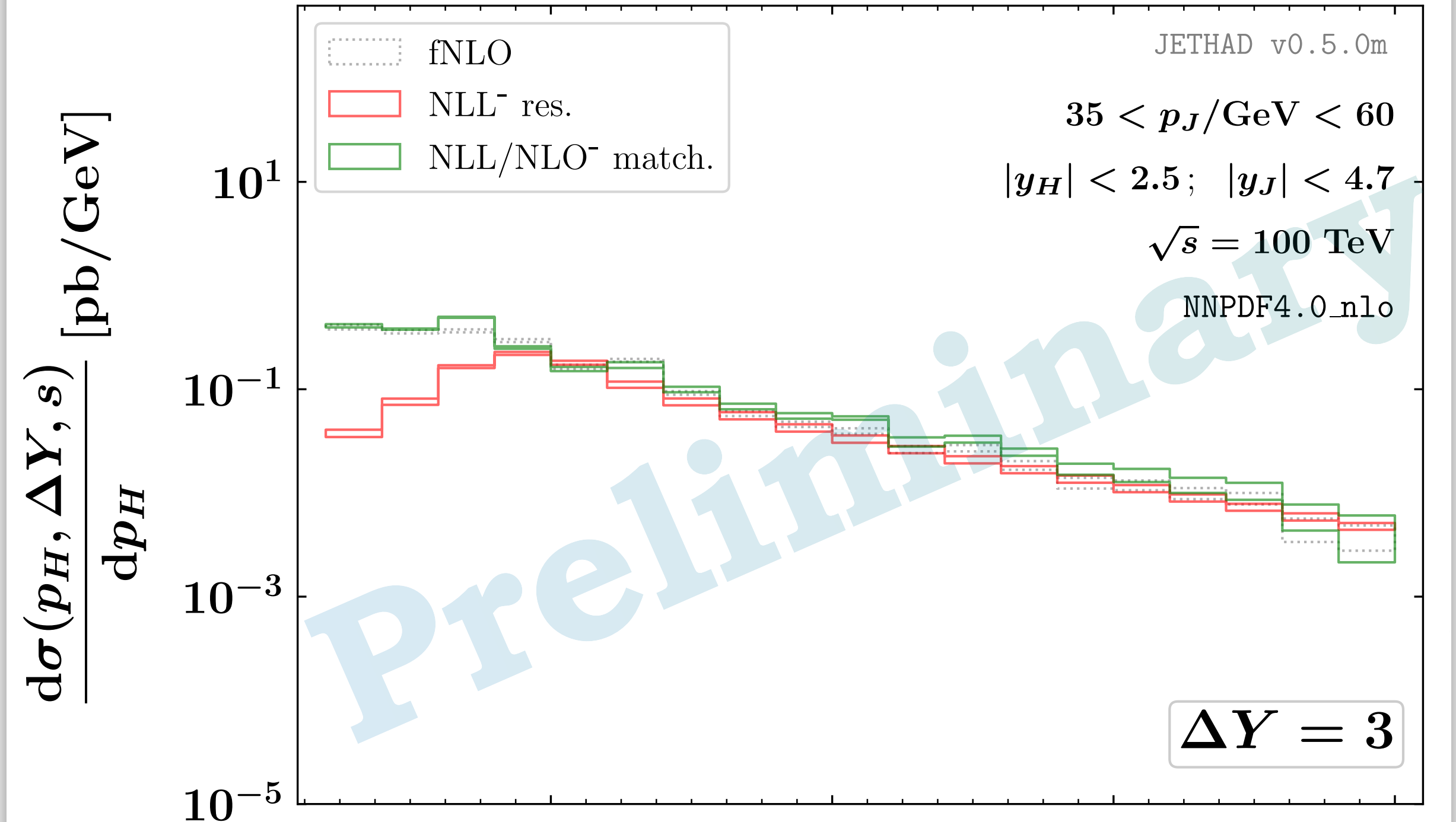
# The Higgs + jet spectrum from POWHEG + JETHAD

$$p(P_a) + p(P_b) \rightarrow H(p_H, y_H) + \mathcal{X} + \text{jet}(p_J, y_J)$$



$\Delta Y = y_H - y_J$

$$p(P_a) + p(P_b) \rightarrow H(p_H, y_H) + \mathcal{X} + \text{jet}(p_J, y_J)$$



$p_H [\text{GeV}]$

$\Delta Y$  spectrum

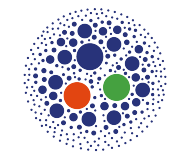
@ 100 TeV FCC

$p_H$  spectrum

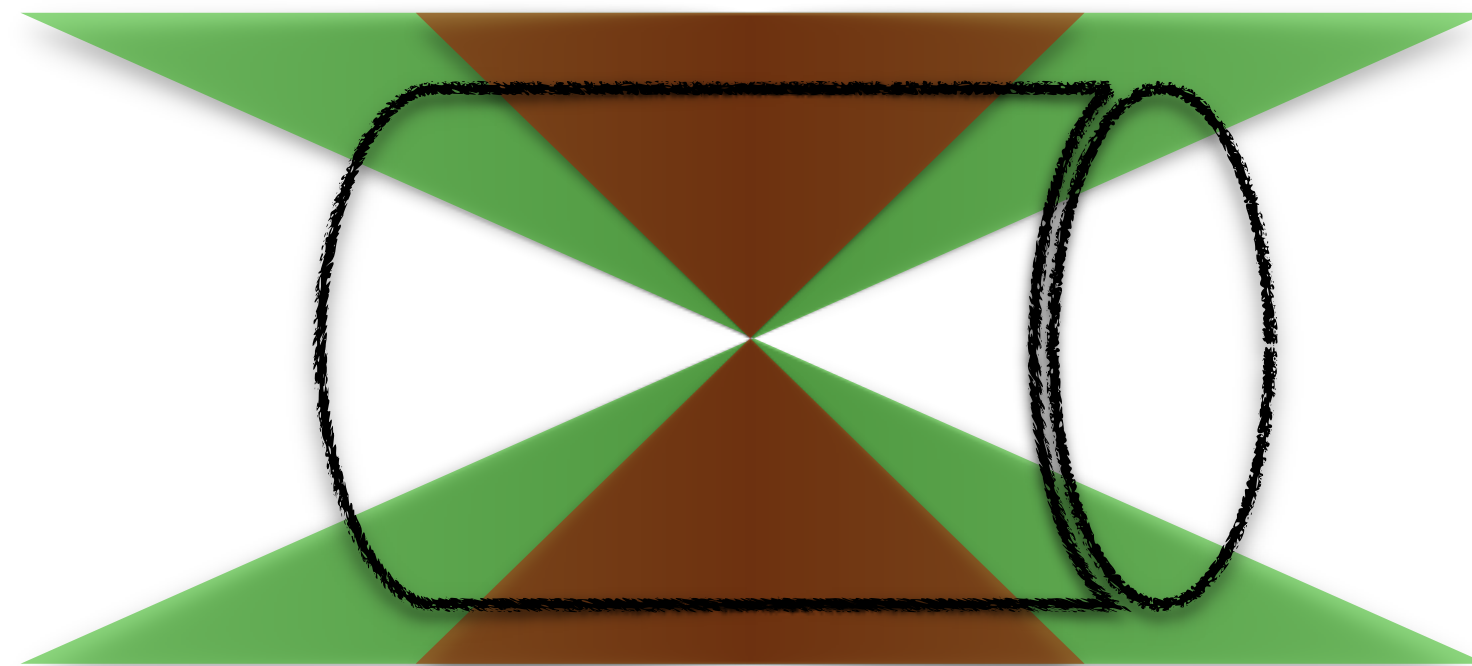
NLL matched to NLO fixed-order JETHAD + POWHEG (in progress)



# High-energy QCD at new-gen Forward Facilities



Forward + backward CMS detections: Mueller-Navelet, hadron-jet, di-hadron

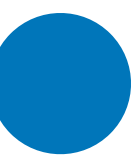


$$|y_{\text{jet}}| < 4.7$$

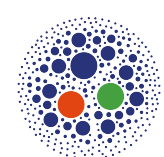
barrel + endcap

$$|y_{\text{hadron}}| < 2.4$$

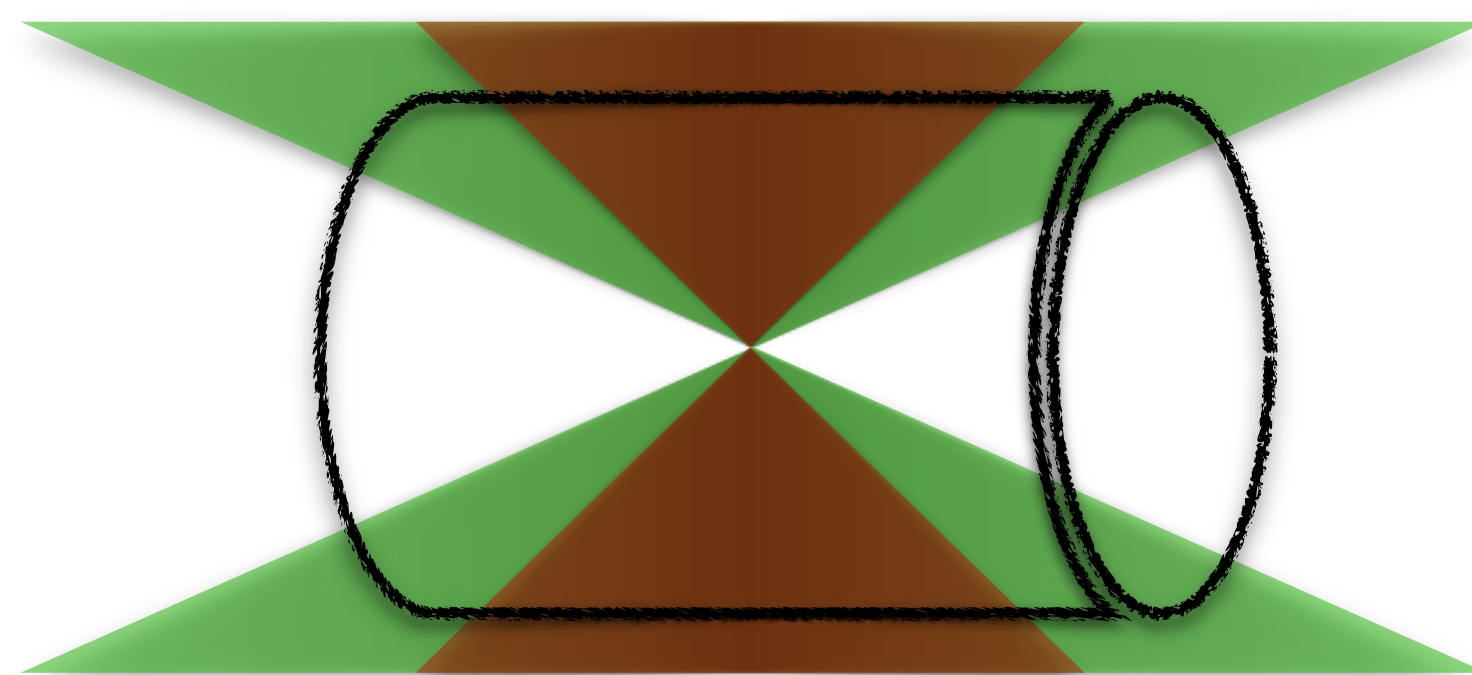
barrel



# High-energy QCD at new-gen Forward Facilities



Forward + backward CMS detections: Mueller-Navelet, hadron-jet, di-hadron

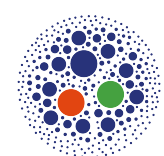


$$|y_{\text{jet}}| < 4.7$$

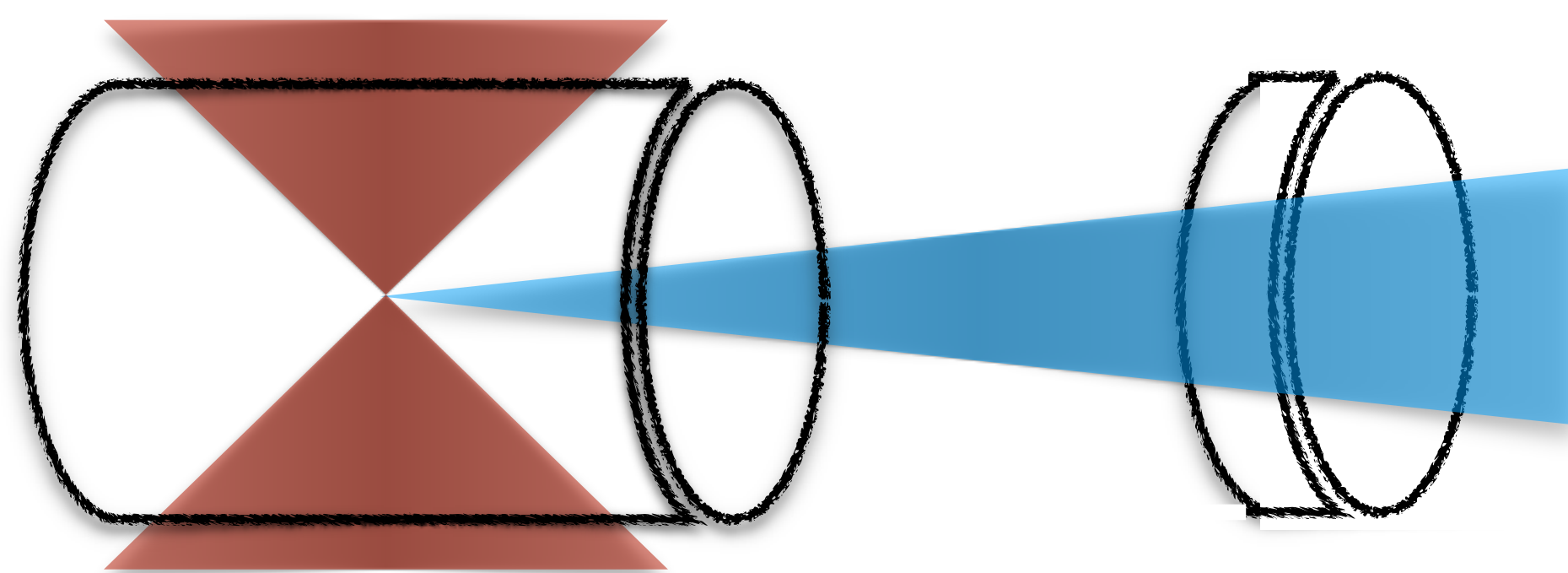
barrel + endcap

$$|y_{\text{hadron}}| < 2.4$$

barrel



Ultra-forward FPF + central ATLAS detections: single-charmed hadrons + Higgs



$$5 < |y_{D^*, \Lambda_c}| < 7$$

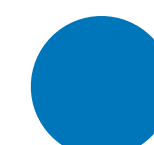
FPF

$$|y_{\text{Higgs}}| < 2.5$$

ATLAS barrel

(charm + Higgs)  [F. G. C. et al., Phys. Rev. D 105 (2022) 11, 114056]

(light mesons + heavy flavor)  [F. G. C., Phys. Rev. D 105 (2022) 11, 114008]





# High-energy QCD in ultraforward directions

Physics Reports 968 (2022) 1–50

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## The Forward Physics Facility: Sites, experiments, and physics potential

Luis A. Anchordoqui<sup>1</sup>, Akitaka Ariga<sup>2,3</sup>, Tomoko Ariga<sup>4</sup>, Weidong Bai<sup>5</sup>, Kinco Balazs<sup>6</sup>, Brian Batell<sup>7</sup>, Jamie Boyd<sup>6</sup>, Joseph Bramante<sup>8</sup>, Mario Campanelli<sup>9</sup>, Adrian Carmona<sup>10</sup>, **Francesco G. Celiberto**<sup>11,12,13</sup>, Grigorios Chachamis<sup>14</sup>, Matthew Citron<sup>15</sup>, Giovanni De Lellis<sup>16,17</sup>, Albert De Roeck<sup>6</sup>, Hans Dembinski<sup>18</sup>, Peter B. Denton<sup>19</sup>, Antonia Di Crescenzo<sup>16,17,6</sup>, Milind V. Diwan<sup>20</sup>, Liam Dougherty<sup>21</sup>, Herbi K. Dreiner<sup>22</sup>, Yong Du<sup>23</sup>, Rikard Enberg<sup>24</sup>, Yasaman Farzan<sup>25</sup>, Jonathan L. Feng<sup>26,\*</sup>, Max Fieg<sup>26</sup>, Patrick Foldenauer<sup>27</sup>, Saeid Foroughi-Abari<sup>28</sup>, Alexander Friedland<sup>29</sup>, Michael Fucilla<sup>30,31</sup>, Jonathan Gall<sup>32</sup>, Maria Vittoria Garzelli<sup>33,\*</sup>, Francesco Giuliani<sup>34</sup>, Victor P. Goncalves<sup>35</sup>, Marco Guzzi<sup>36</sup>, Francis Halzen<sup>37</sup>, Juan Carlos Helo<sup>38,39</sup>, Christopher S. Hill<sup>40</sup>, Ahmed Ismail<sup>41</sup>, Ameen Ismail<sup>42</sup>, Richard Jacobsson<sup>6</sup>, Sudip Jana<sup>43</sup>, Yu Seon Jeong<sup>44</sup>, Krzysztof Jodłowski<sup>45</sup>, Kevin J. Kelly<sup>46</sup>, Felix Kling<sup>29,47,\*\*</sup>, Fnu Karan Kumar<sup>20</sup>, Zhen Liu<sup>48</sup>, Rafał Maciuła<sup>49</sup>, Roshan Mammen Abraham<sup>41</sup>, Julien Manshanden<sup>33</sup>, Josh McFayden<sup>50</sup>, Mohammed M.A. Mohammed<sup>30,31</sup>, Pavel M. Nadolsky<sup>51</sup>, Nobuchika Okada<sup>52</sup>, John Osborne<sup>6</sup>, Hidetoshi Otono<sup>4</sup>, Vishvas Pandey<sup>53,46</sup>, Alessandro Papa<sup>30,31</sup>, Digesh Raut<sup>54</sup>, Mary Hall Reno<sup>55</sup>, Filippo Resnati<sup>6</sup>, Adam Ritz<sup>28</sup>, Juan Rojo<sup>56</sup>, Ina Sarcevic<sup>57</sup>, Christiane Scherb<sup>58</sup>, Holger Schulz<sup>59</sup>, Pedro Schwaller<sup>60</sup>, Dipan Sengupta<sup>61</sup>, Torbjörn Sjöstrand<sup>62</sup>, Tyler B. Smith<sup>26</sup>, Dennis Soldin<sup>54</sup>, Anna Stasto<sup>63</sup>, Antoni Szczurek<sup>49</sup>, Zahra Tabrizi<sup>64</sup>, Sebastian Trojanowski<sup>65,66</sup>, Yu-Dai Tsai<sup>26,46</sup>, Douglas Tuckler<sup>67</sup>, Martin W. Winkler<sup>68</sup>, Keping Xie<sup>7</sup>, Yue Zhang<sup>67</sup>

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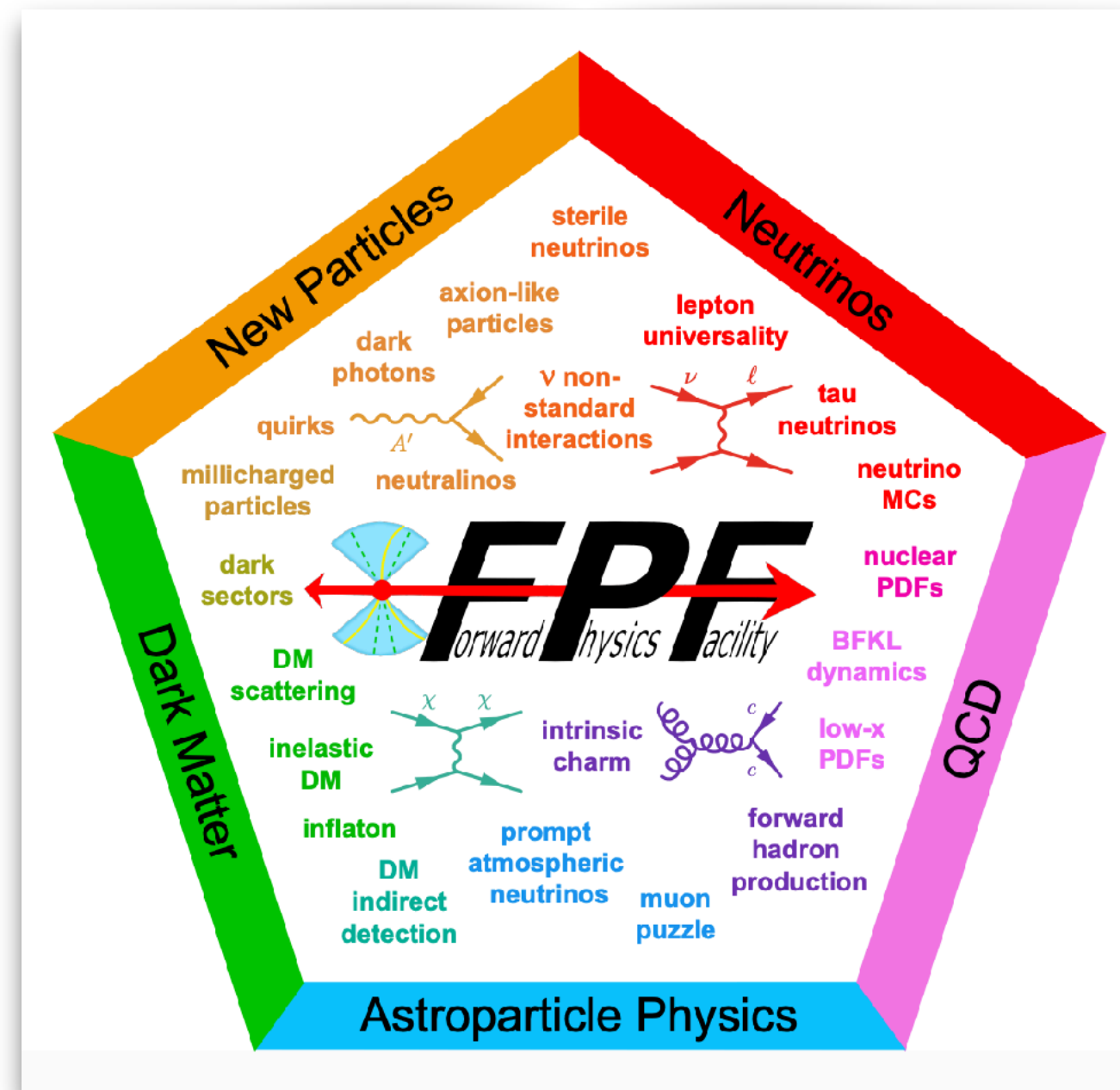
<sup>13</sup> INFN-TIFPA Trento Institute of Fundamental Physics and Applications, I-38123 Povo, Trento, Italy

\* Corresponding authors.

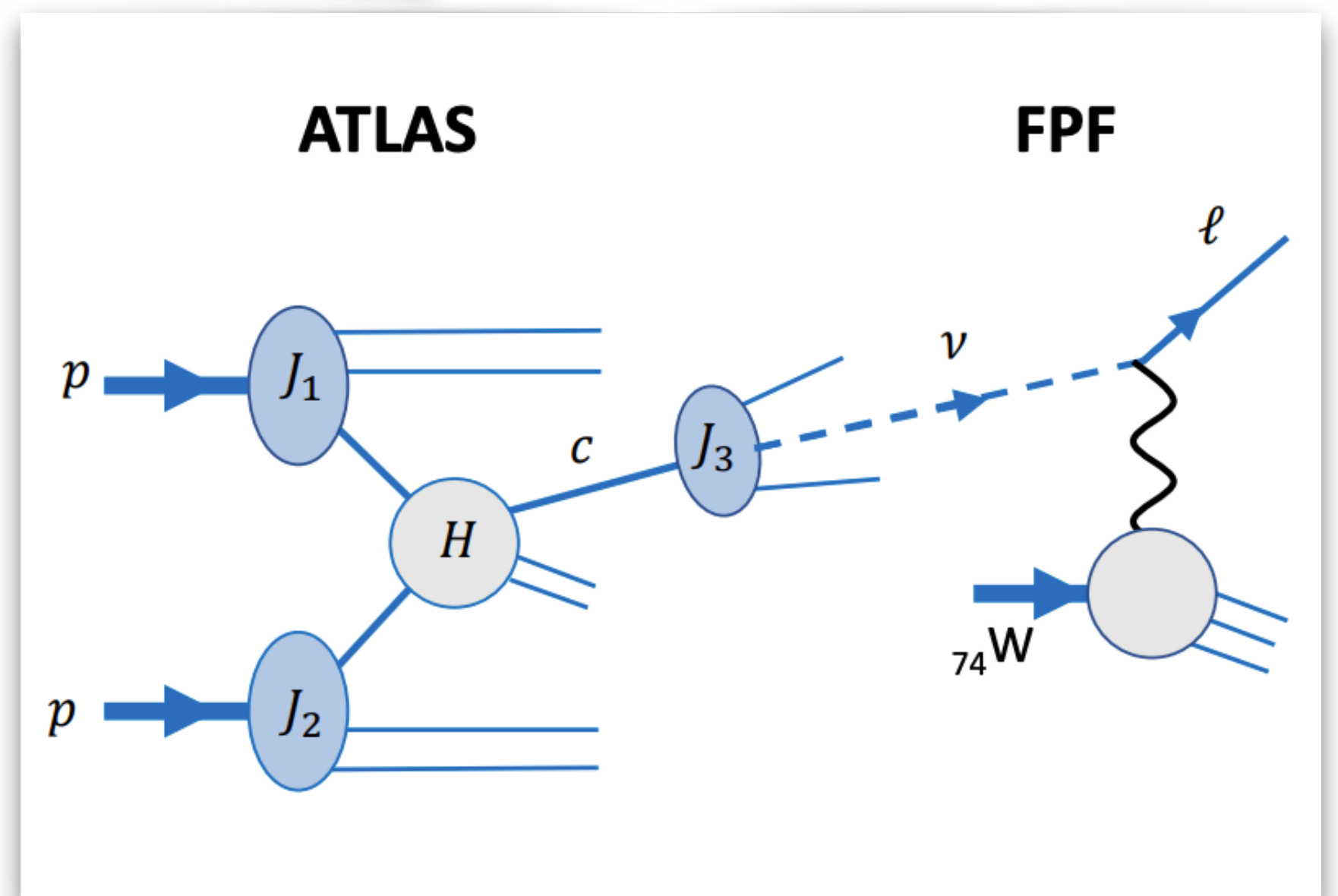
\*\* Corresponding author at: Deutsches Elektronen-Synchrotron DESY, Notkestrasse 85, 22607 Hamburg, Germany.  
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<https://doi.org/10.1016/j.physrep.2022.04.004>

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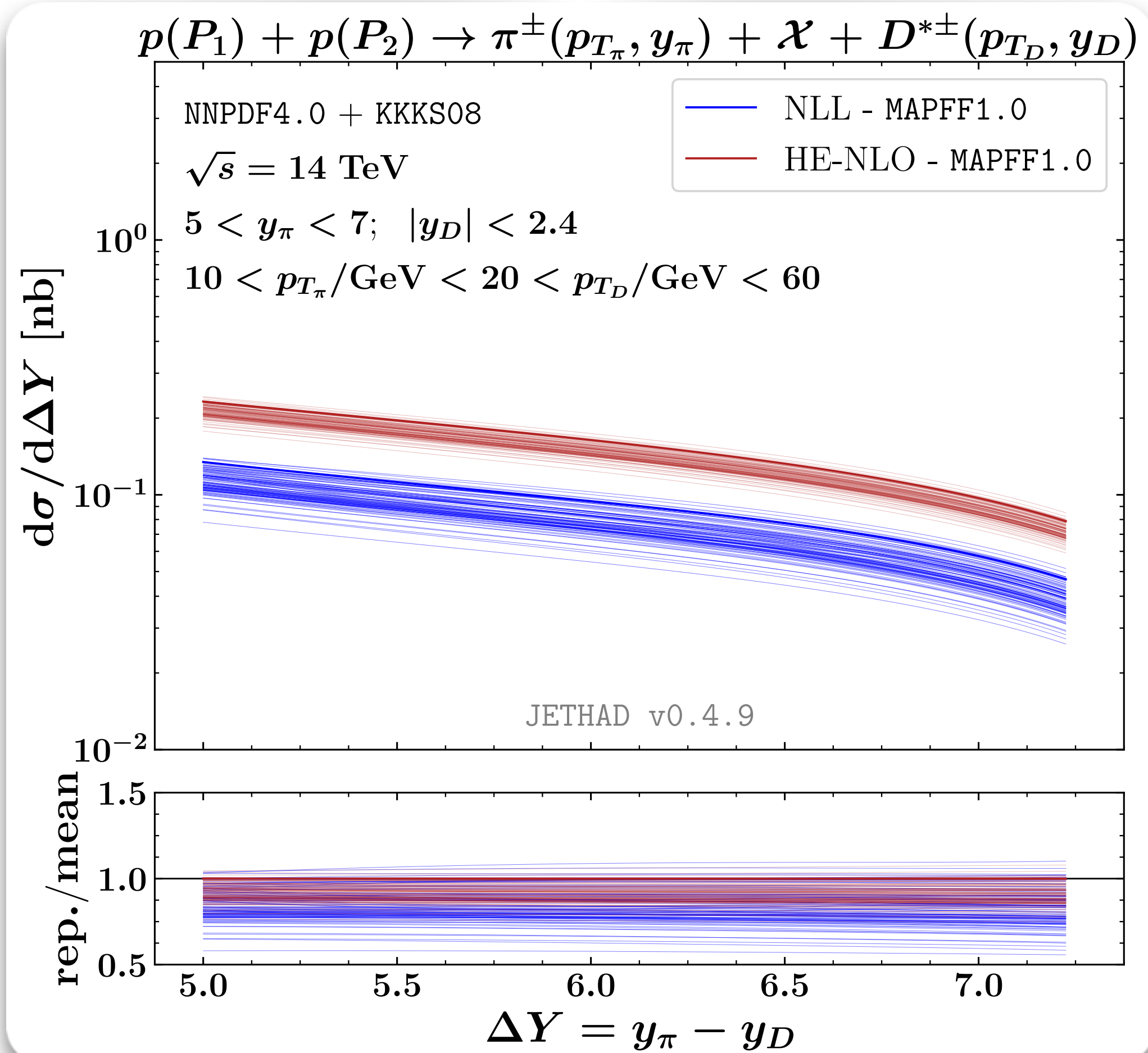
[FPF Snowmass Whitepaper]



# Rapidity distributions @FPF+ATLAS

**Inclusive  $\pi^\pm$  (FPF) +  $D^{*\pm}$  (ATLAS) production**

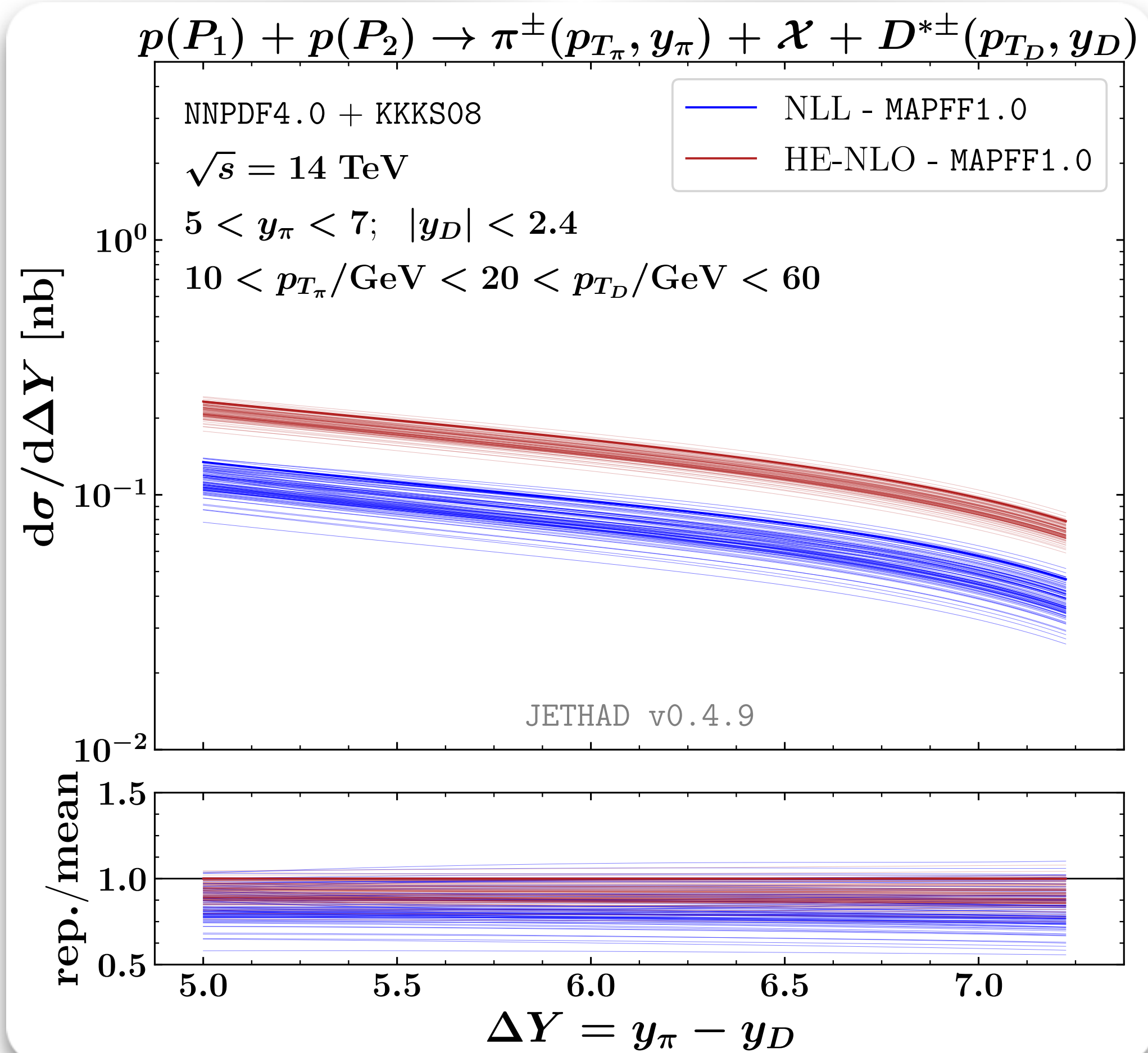
[\[FPF Snowmass Whitepaper\]](#)



# Rapidity distributions @FPF+ATLAS

**Inclusive  $\pi^\pm$  (FPF) +  $D^{*\pm}$  (ATLAS) production**

[\[FPF Snowmass Whitepaper\]](#)



\* Impact of collinear FFs on  $\Delta Y$ -distribution

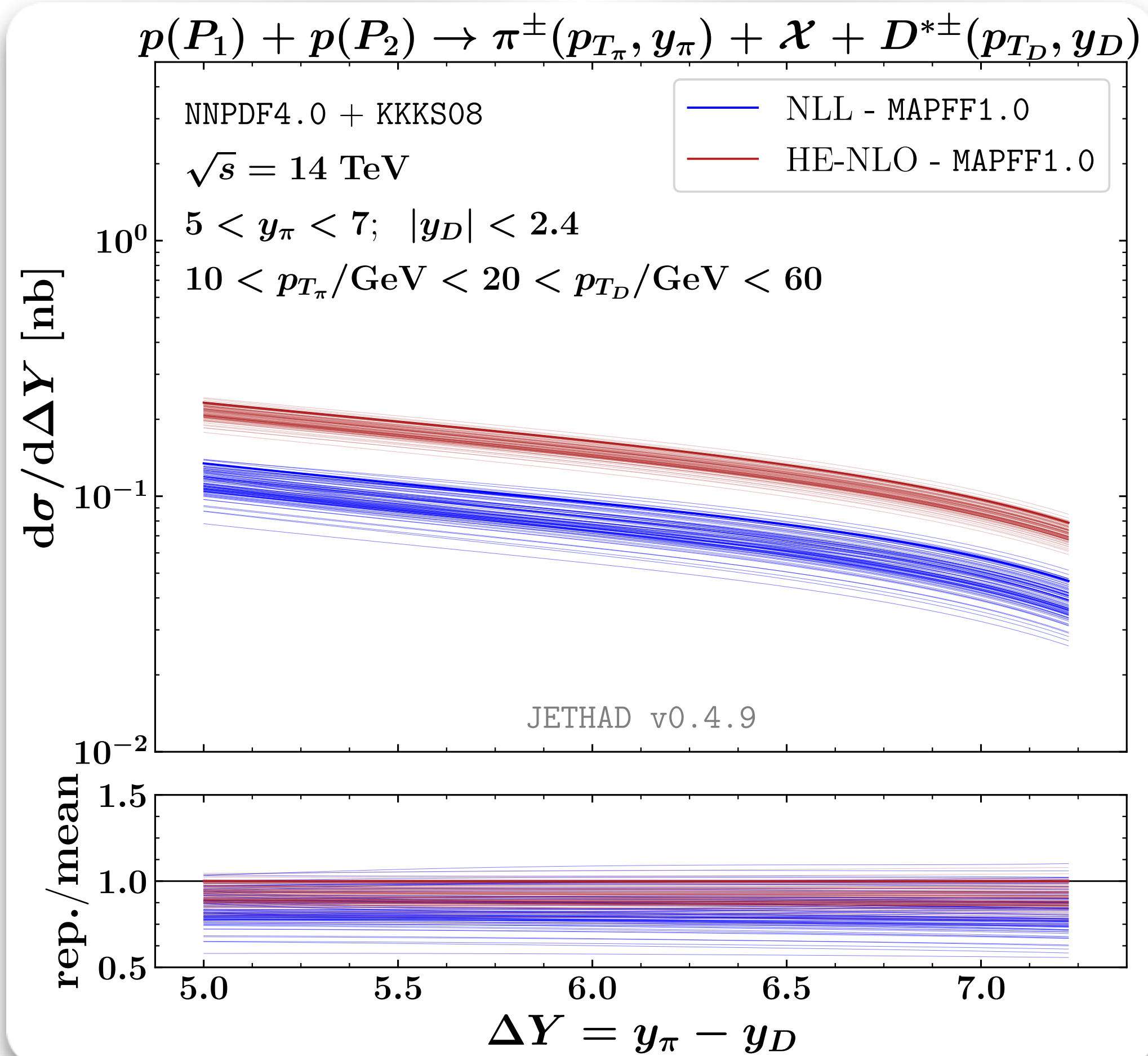
\* Replica method at work



# Rapidity distributions @FPF+ATLAS

## Inclusive $\pi^\pm$ (FPF) + $D^{*\pm}$ (ATLAS) production

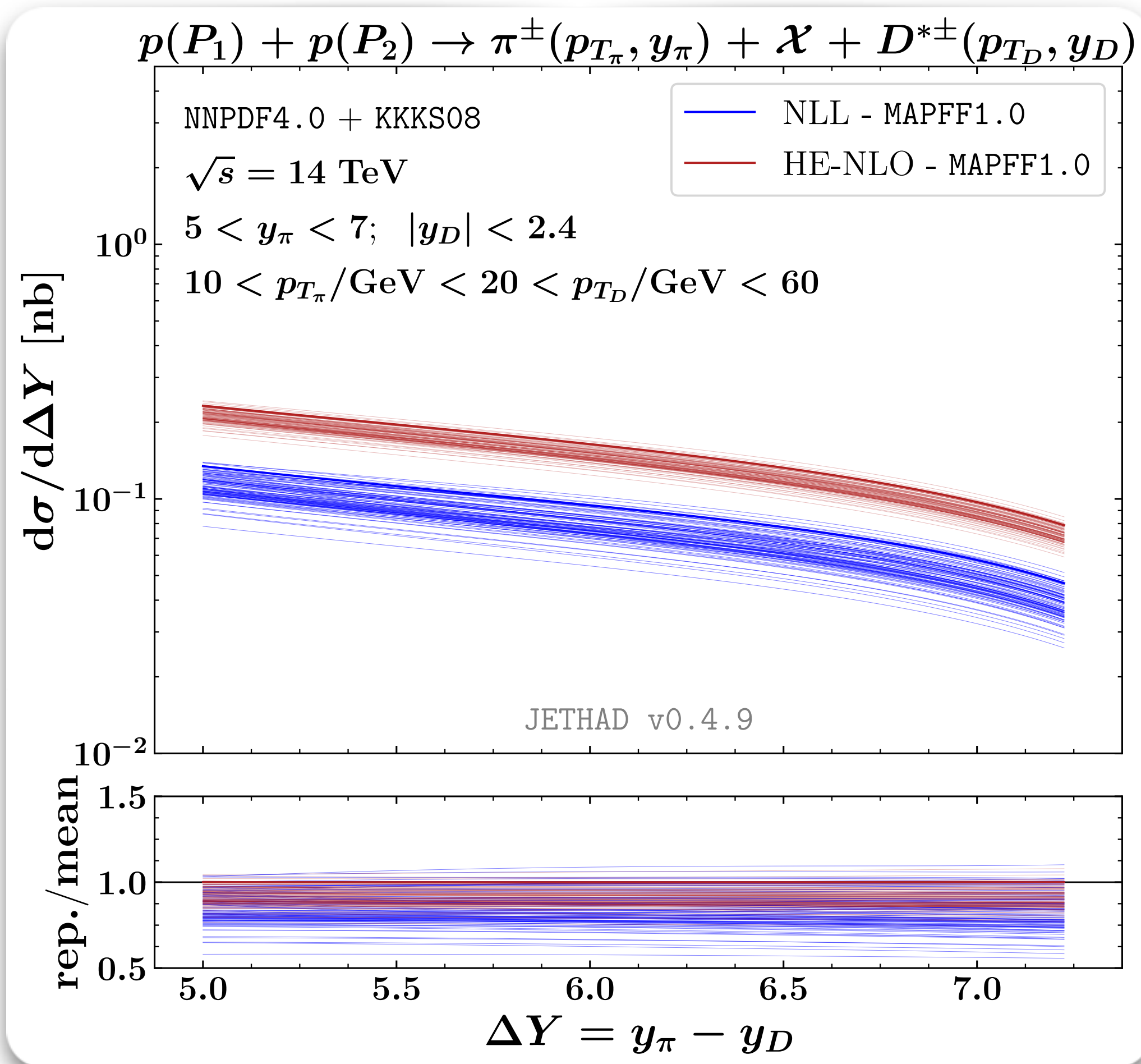
[FPF Snowmass Whitepaper]



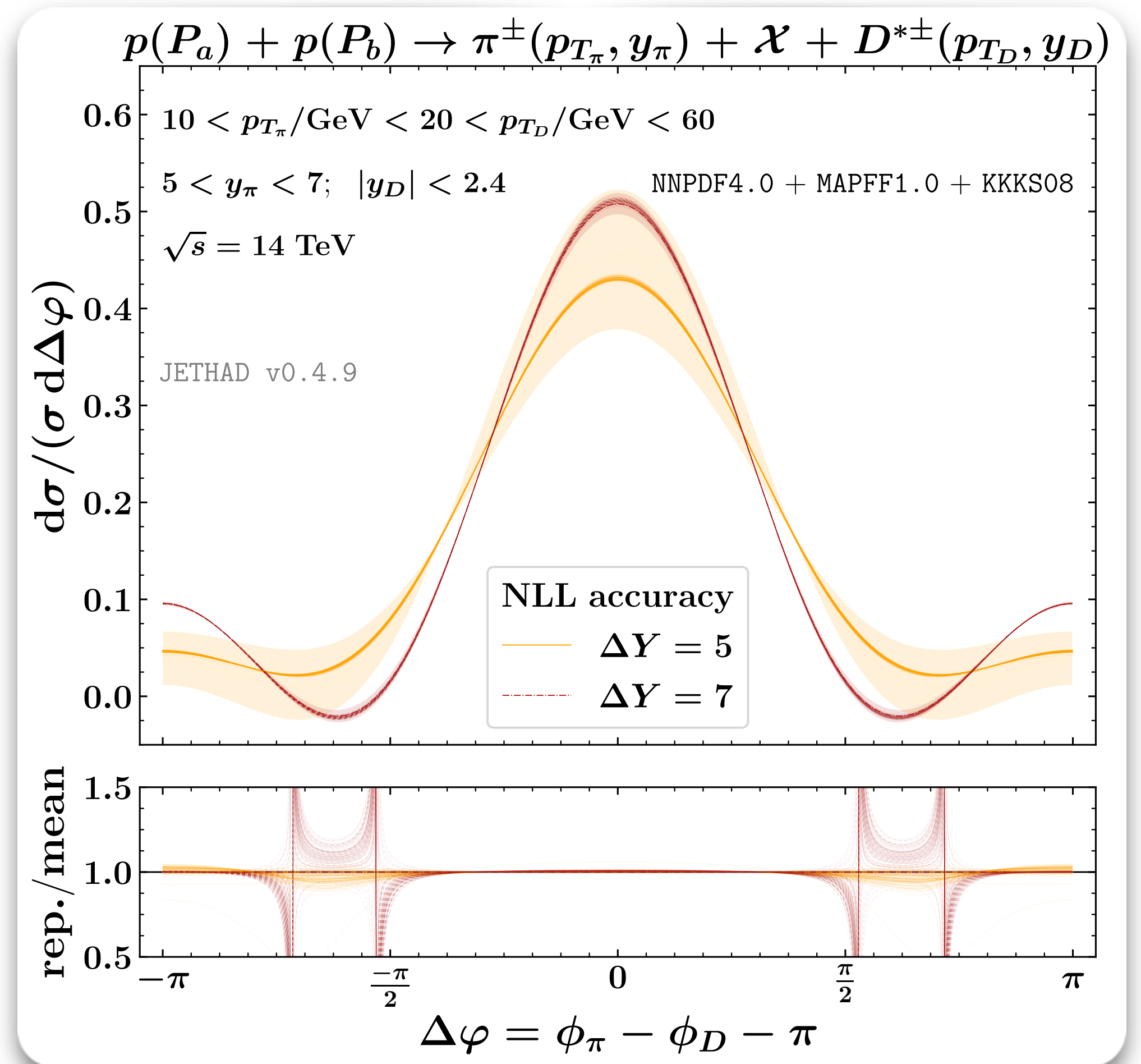
- \* Impact of collinear FFs on  $\Delta Y$ -distribution
- \* Replica method at work
- \* Larger spread of replicas at NLL
- \* Probe FFs in complementary ranges
  - Weight of FF replicas in the same set
  - Different sets via functional correlation?
- \* Complementary studies on FFs

# Rapidity distributions @FPF+ATLAS

**Inclusive  $\pi^\pm$  (FPF) +  $D^{*\pm}$  (ATLAS)  $\Delta Y$ -spectrum**

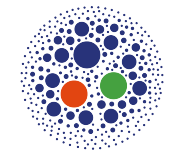


**Inclusive  $\pi^\pm$  (FPF) +  $D^{*\pm}$  (ATLAS)  $\Delta\phi$ -spectrum**



**OPEN HEAVY FLAVOR**

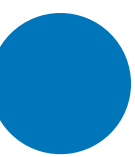
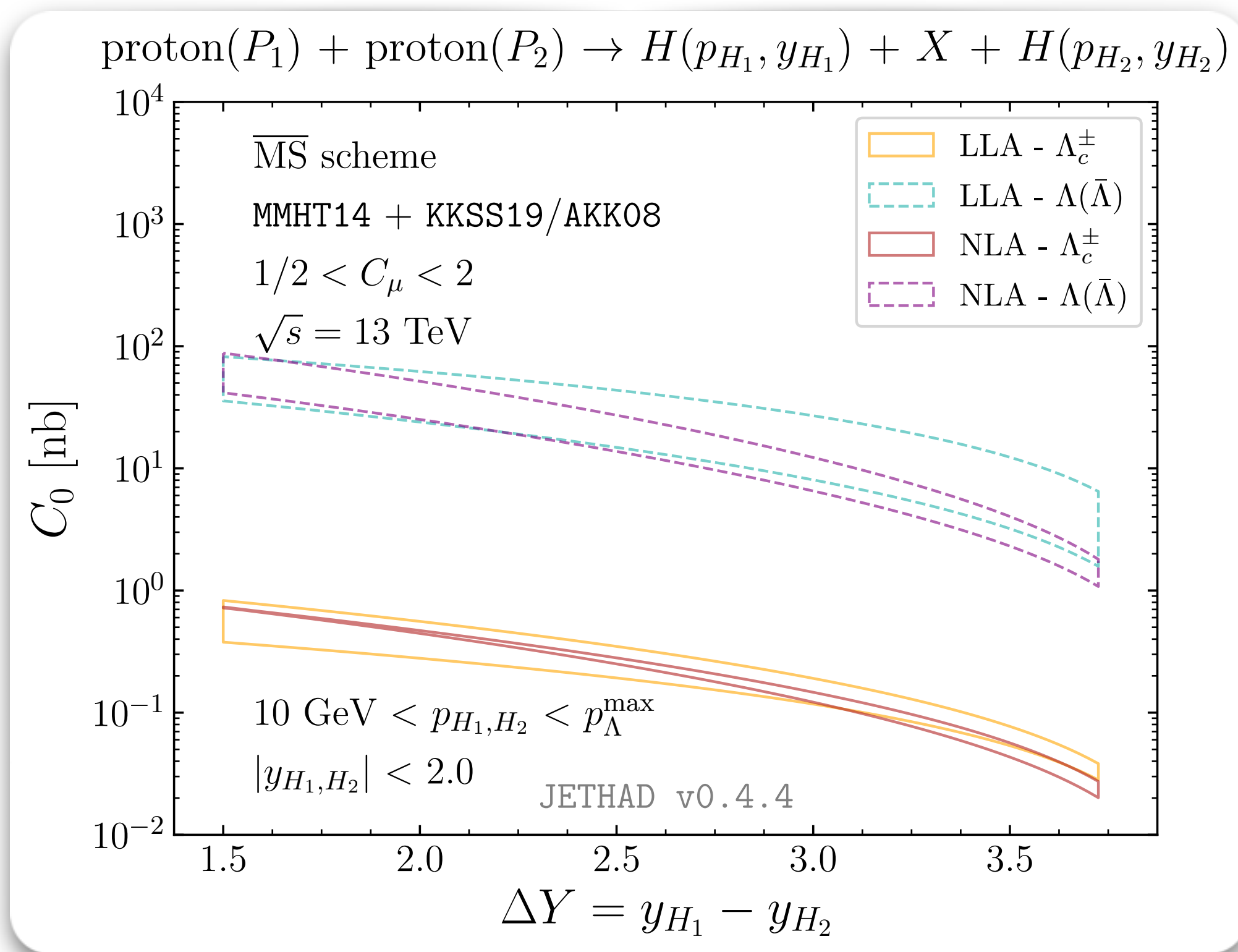
# From light to heavy-light bound states



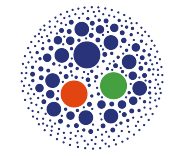
Light-hadron+jet: **higher-order instabilities** via scale variation, as in Mueller-Navelet (i!)

$\Lambda_c$  baryons

[\[F. G. C. et al., Eur. Phys. J. C \(2021\) 8, 780\]](#)



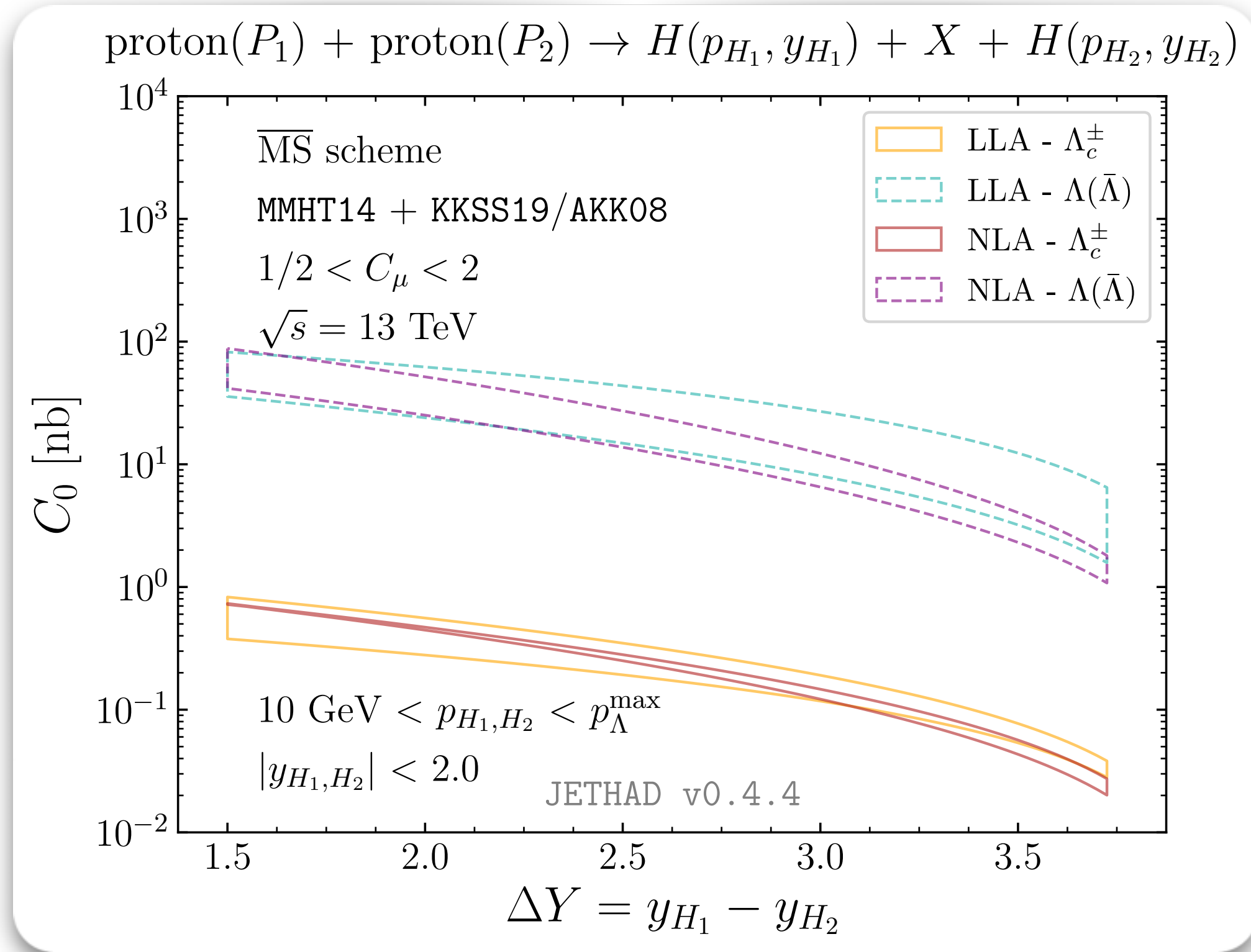
# From light to heavy-light bound states



Light-hadron+jet: **higher-order instabilities** via scale variation, as in Mueller-Navelet (!)

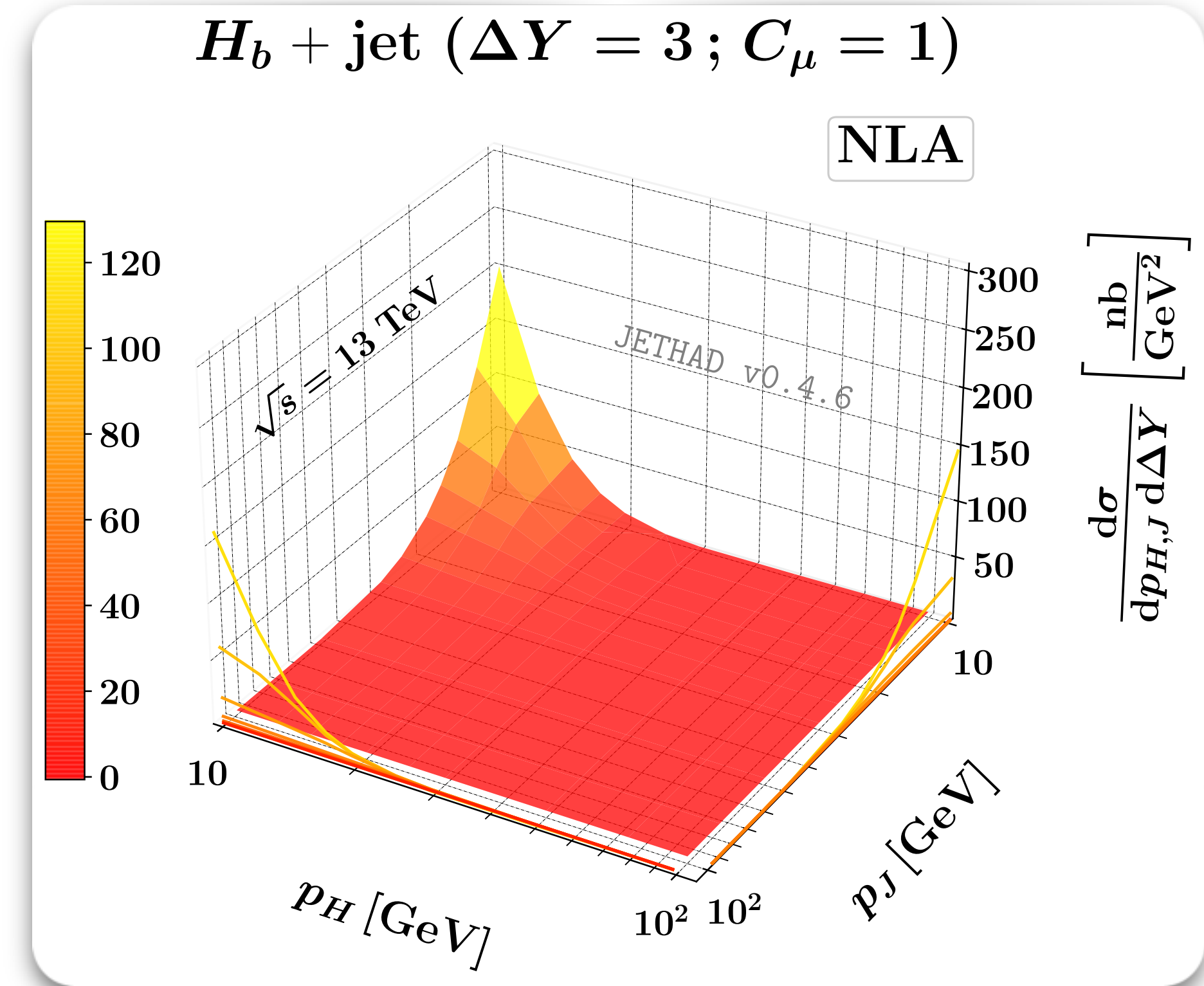
$\Lambda_c$  baryons

[F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]



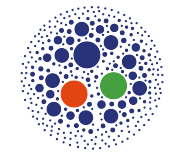
Bottom-flavored hadrons

[F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]





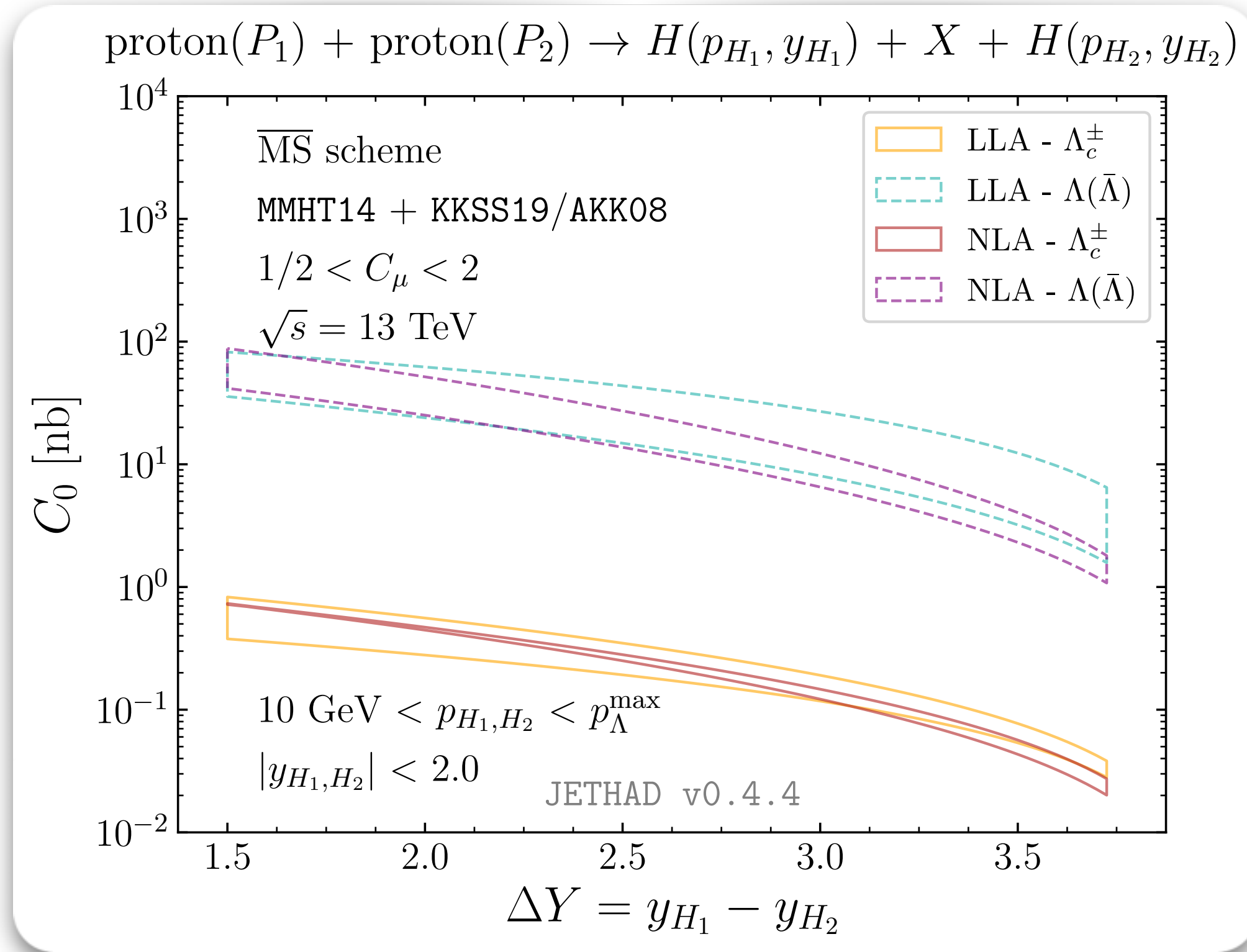
# From light to heavy-light bound states



Light-hadron+jet: **higher-order instabilities** via scale variation, as in Mueller-Navelet (!)

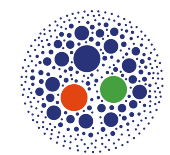
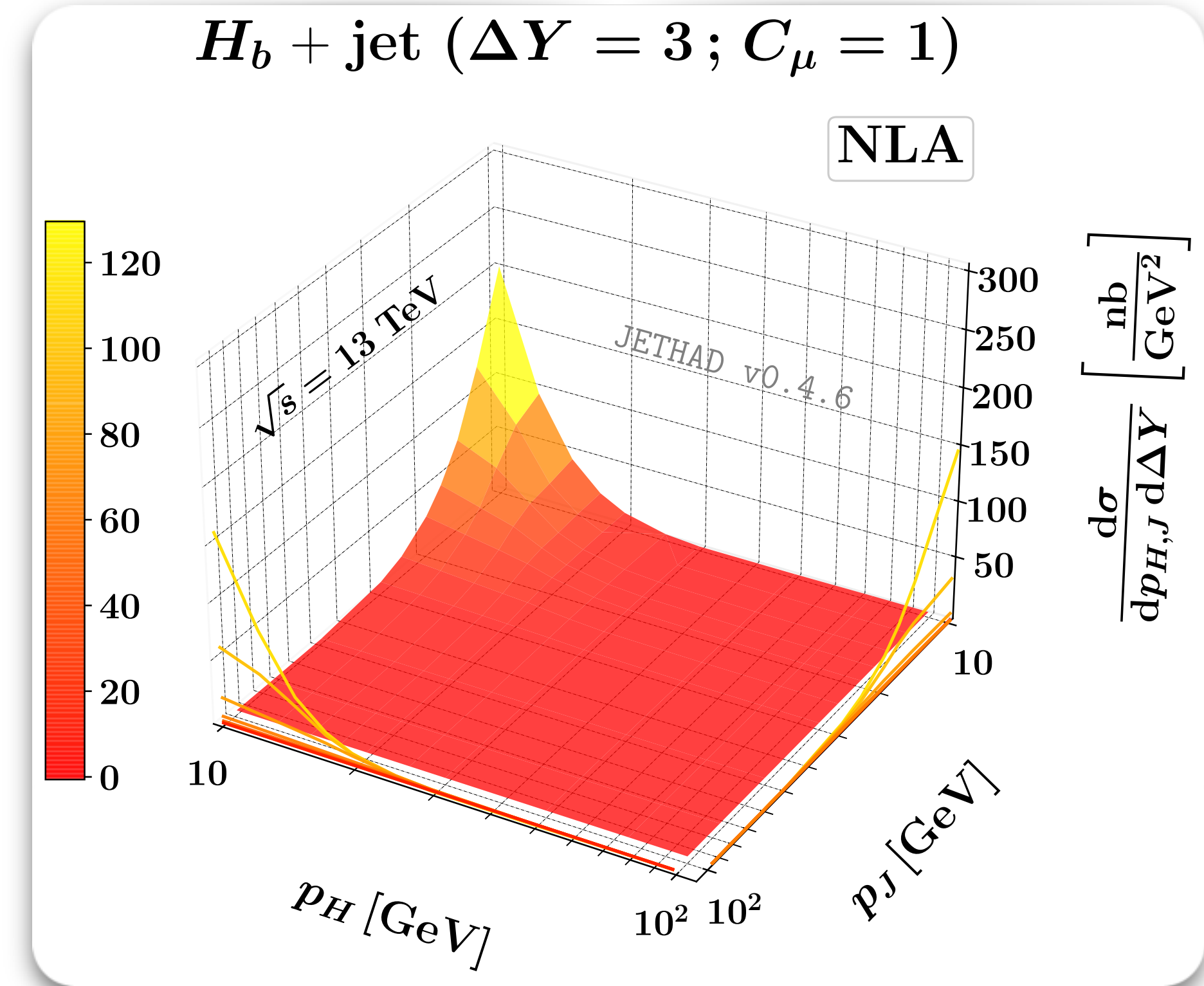
$\Lambda_c$  baryons

[F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]



Bottom-flavored hadrons

[F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]



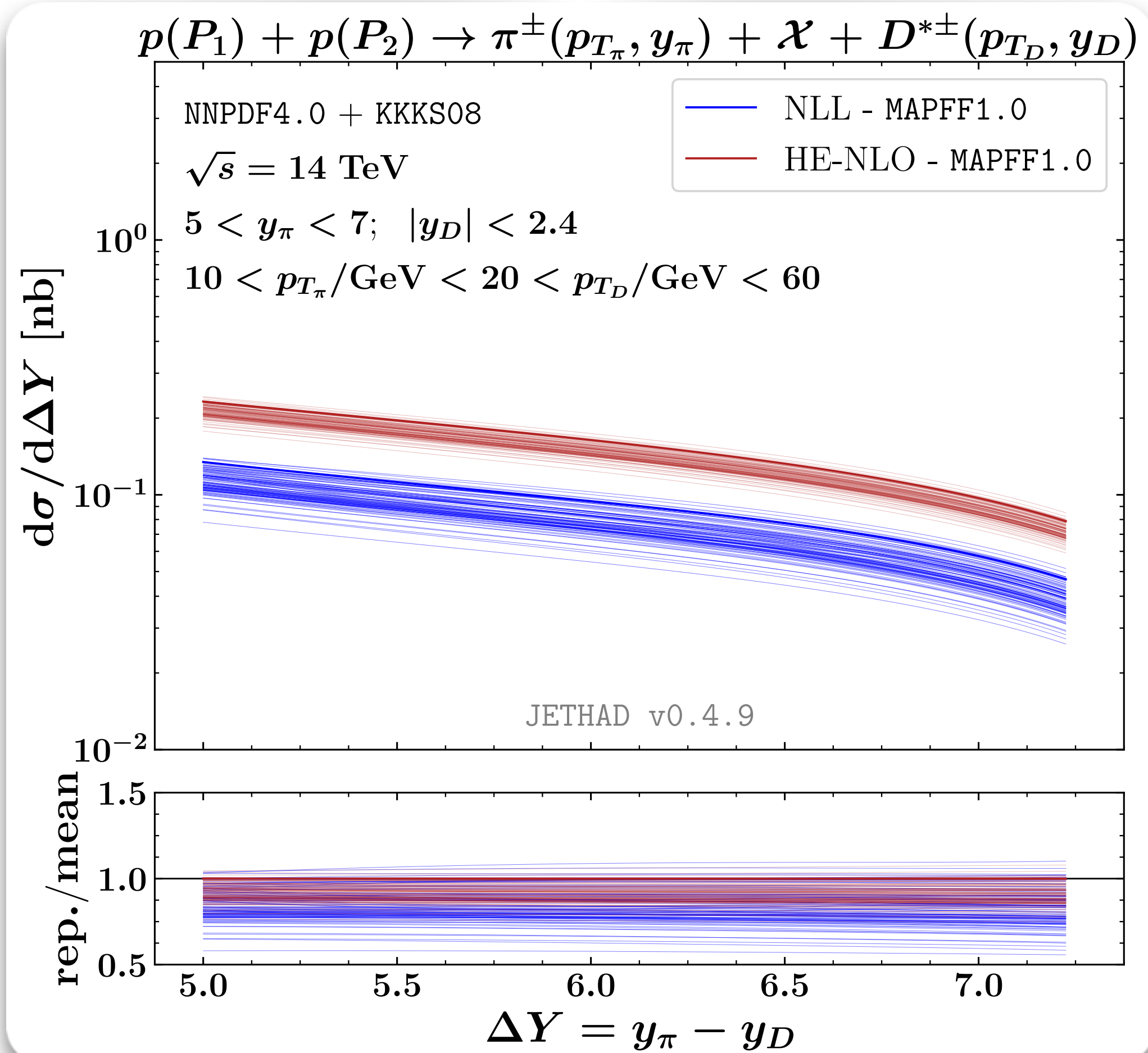
**Natural stability** as a tool to investigate high-energy dynamics of QCD



# Rapidity distributions @FPF+ATLAS

**Inclusive  $\pi^\pm$  (FPF) +  $D^{*\pm}$  (ATLAS) production**

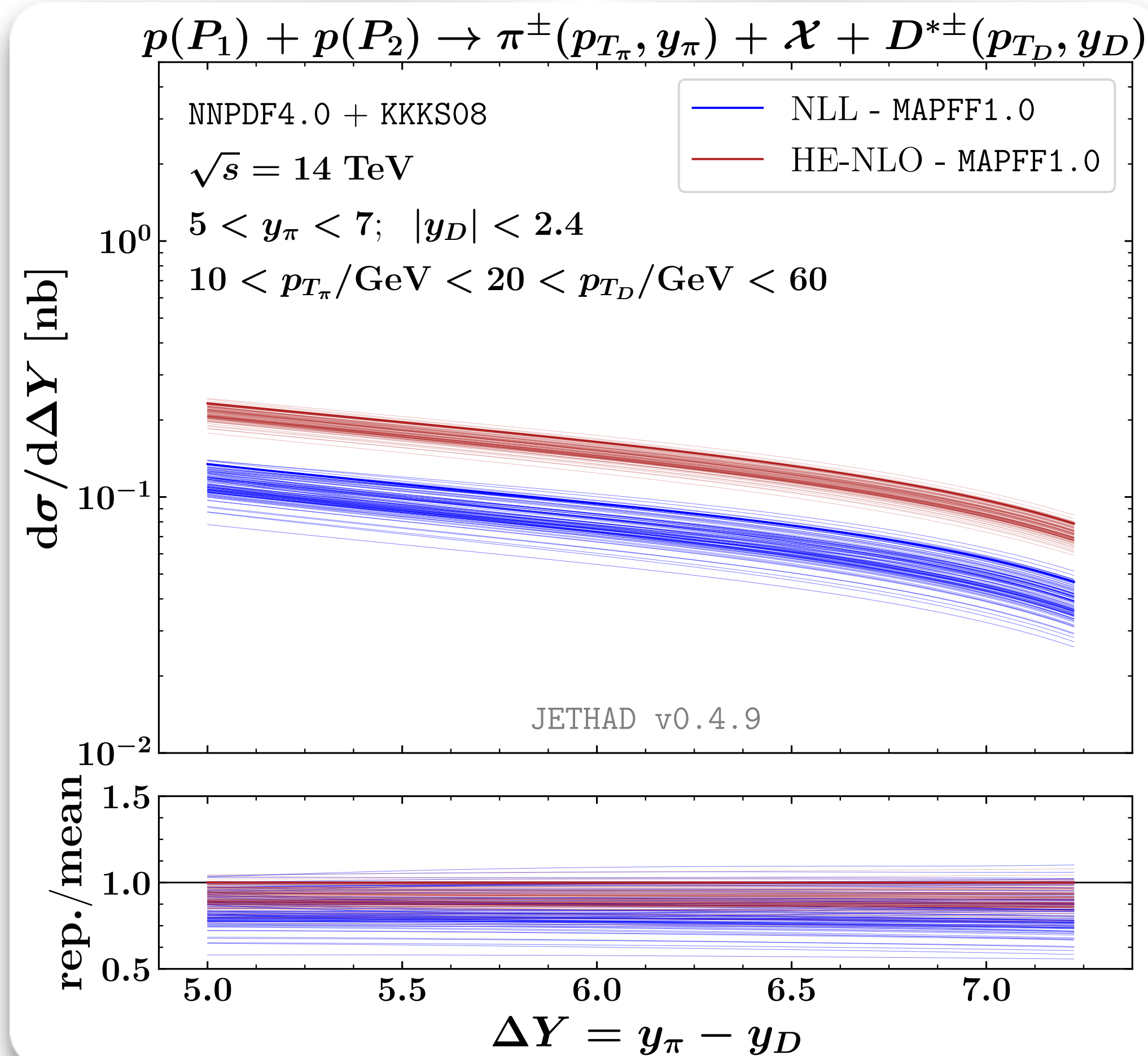
[FPF Snowmass Whitepaper]



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[FPF Snowmass Whitepaper]



\* Impact of collinear FFs on  $\Delta Y$ -distribution

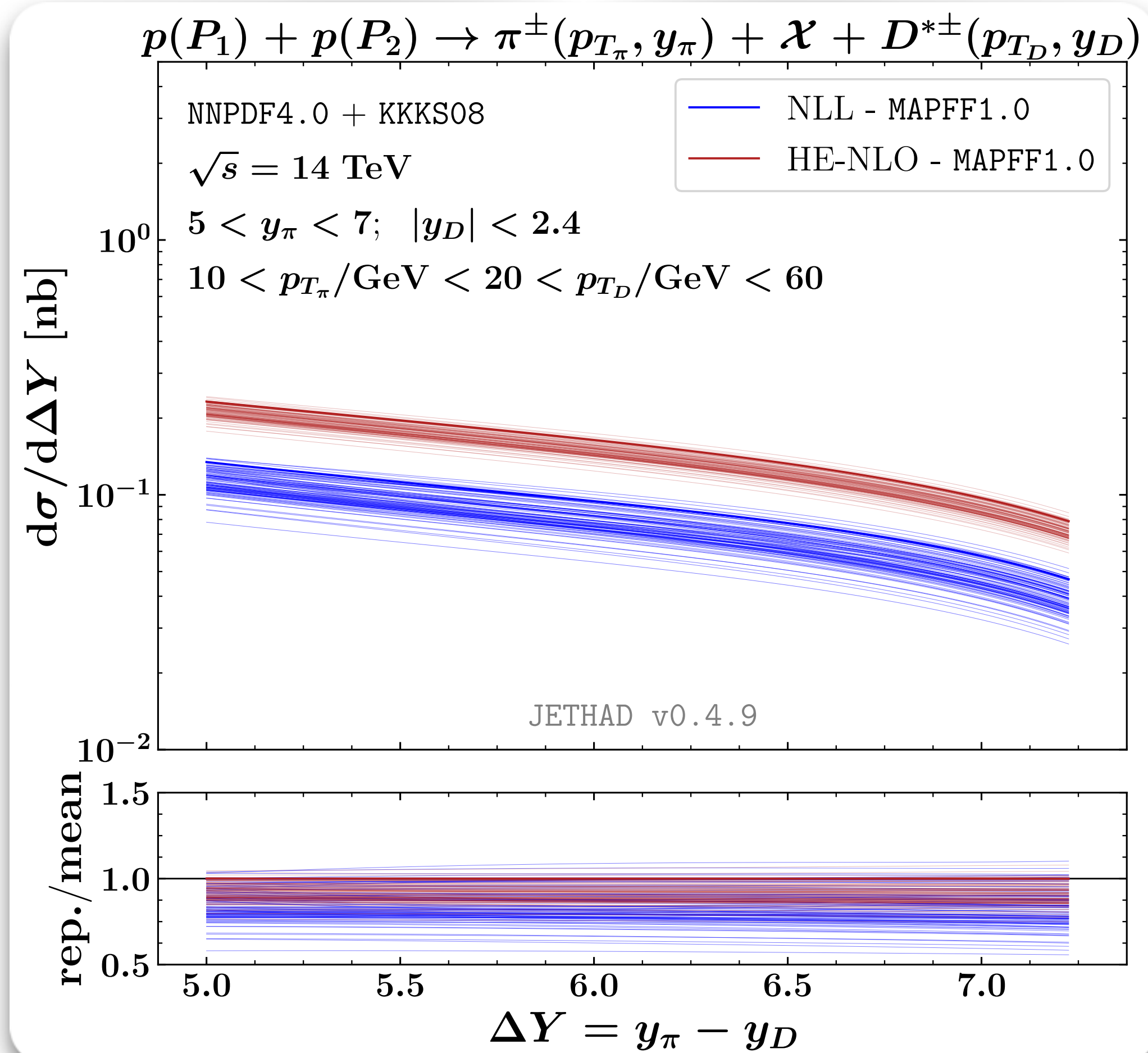
\* Replica method at work



# Rapidity distributions @FPF+ATLAS

## Inclusive $\pi^\pm$ (FPF) + $D^{*\pm}$ (ATLAS) production

[FPF Snowmass Whitepaper]



- \* Impact of collinear FFs on  $\Delta Y$ -distribution
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- \* Larger spread of replicas at NLL
- \* Probe FFs in complementary ranges
  - Weight of FF replicas in the same set
  - Different sets via functional correlation?
- \* Complementary studies on FFs

**QUARKONIA**

# Quarkonium studies: Progresses & challenges



Quarkonium discovery (1974): SLAC & BNL (🇺🇸), then Frascati (🇮🇹) →  $J/\psi$



# Quarkonium studies: Progresses & challenges



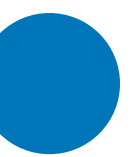
Quarkonium discovery (1974): SLAC & BNL (🇺🇸), then Frascati (🇮🇹) →  $J/\psi$



Proof of the existence of the **charm** quark ( $c$ )...



...and of quarks as real particles and not as mathematical artifacts



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Quarkonium discovery (1974): SLAC & BNL (🇺🇸), then Frascati (🇮🇹) →  $J/\psi$



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GIM mechanism (1970) → **charm** to explain electroweak flavor rotation



Proof of asymptotic freedom & confinement ←  $\psi(2S)$  resonance





# Quarkonium studies: Progresses & challenges



Quarkonium discovery (1974): SLAC & BNL (🇺🇸), then Frascati (🇮🇹) →  $J/\psi$



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...and of quarks as real particles and not as mathematical artifacts



GIM mechanism (1970) → **charm** to explain electroweak flavor rotation



Proof of asymptotic freedom & confinement ←  $\psi(2S)$  resonance



Quarkonia → easy to measure, difficult to understand



# Spectroscopic notation for quarkonia

Spin

Color: Singlet (CS) or Octet (CO)

$$[n] \equiv 2S+1 L (C) J$$

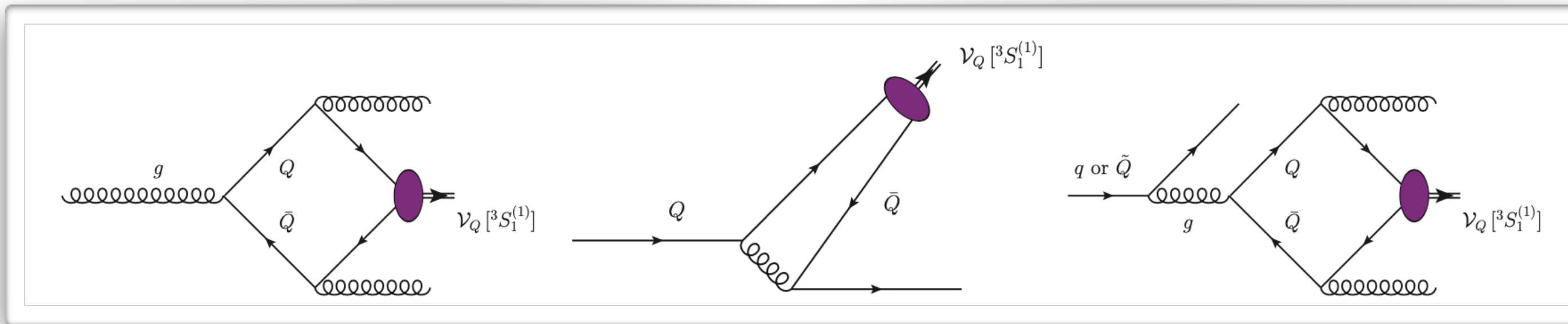
Orbital angular momentum

Total angular momentum

$$J = L + S$$

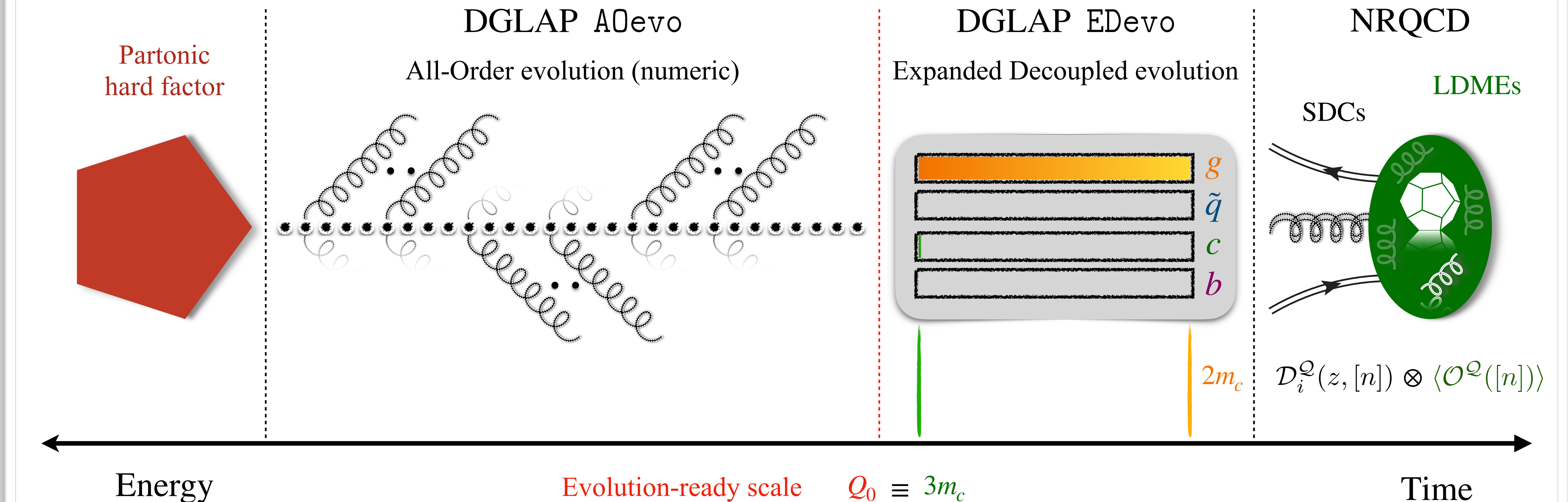


# HF-NRevo: Initial-scale NRCQD inputs

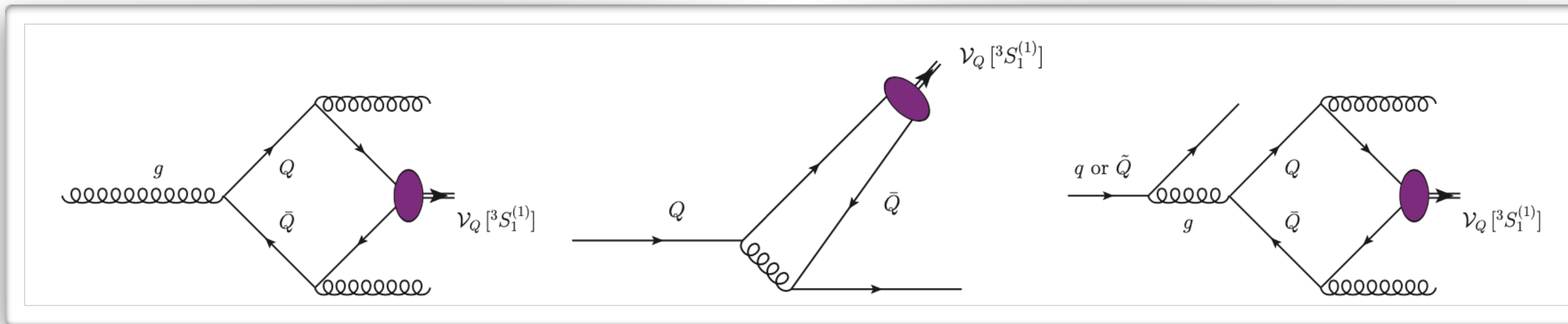


Vector  $J/\psi$

## HF-NRevo for $J/\psi[{}^3S_1^{(1)}]$ collinear fragmentation

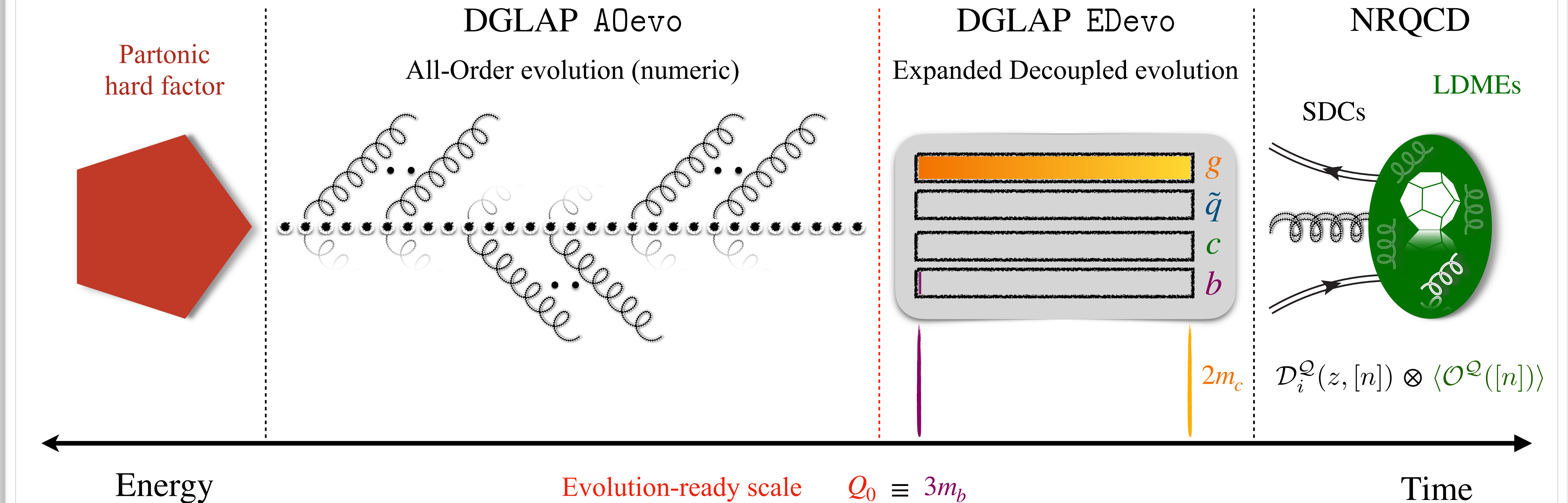


# HF-NRevo: Initial-scale NRCQD inputs



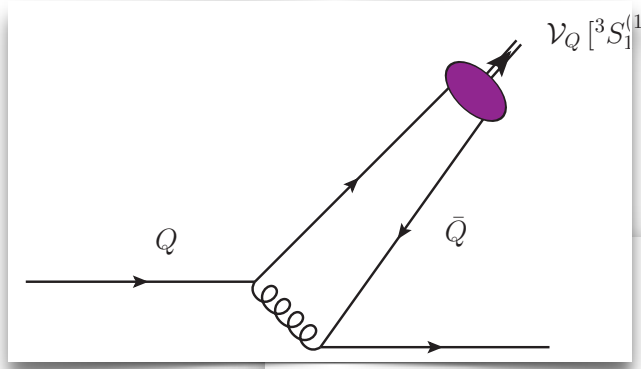
Vector  $\Upsilon$

## HF-NRevo for $\Upsilon[{}^3S_1^{(1)}]$ collinear fragmentation

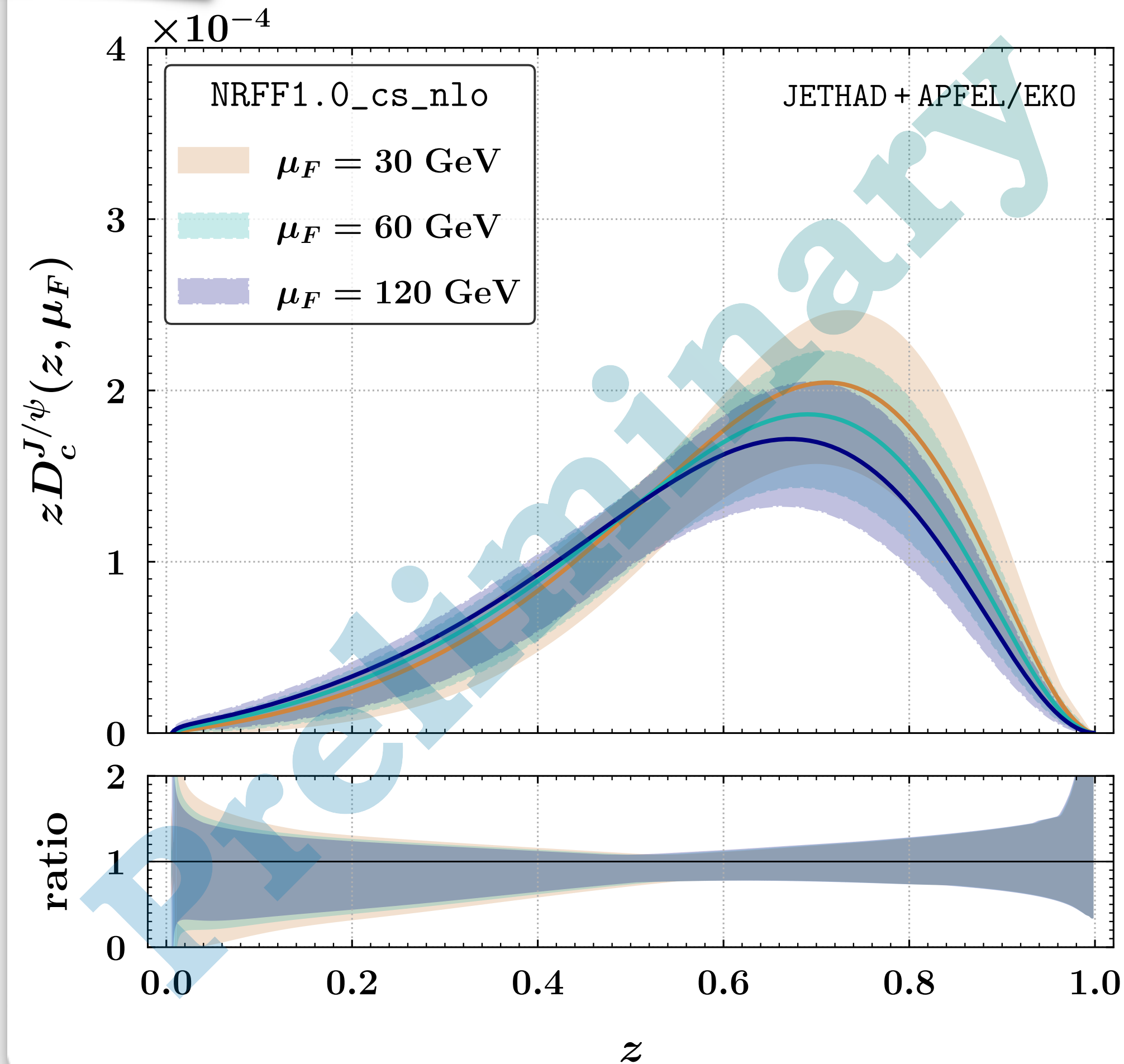


# NRFF1.0: Charm fragmentation to charmonia

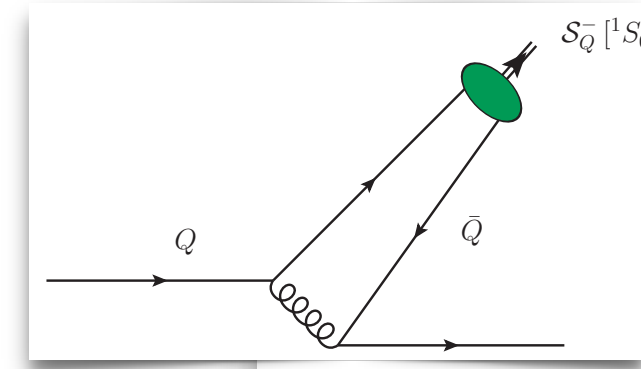
## Vector $J/\psi$



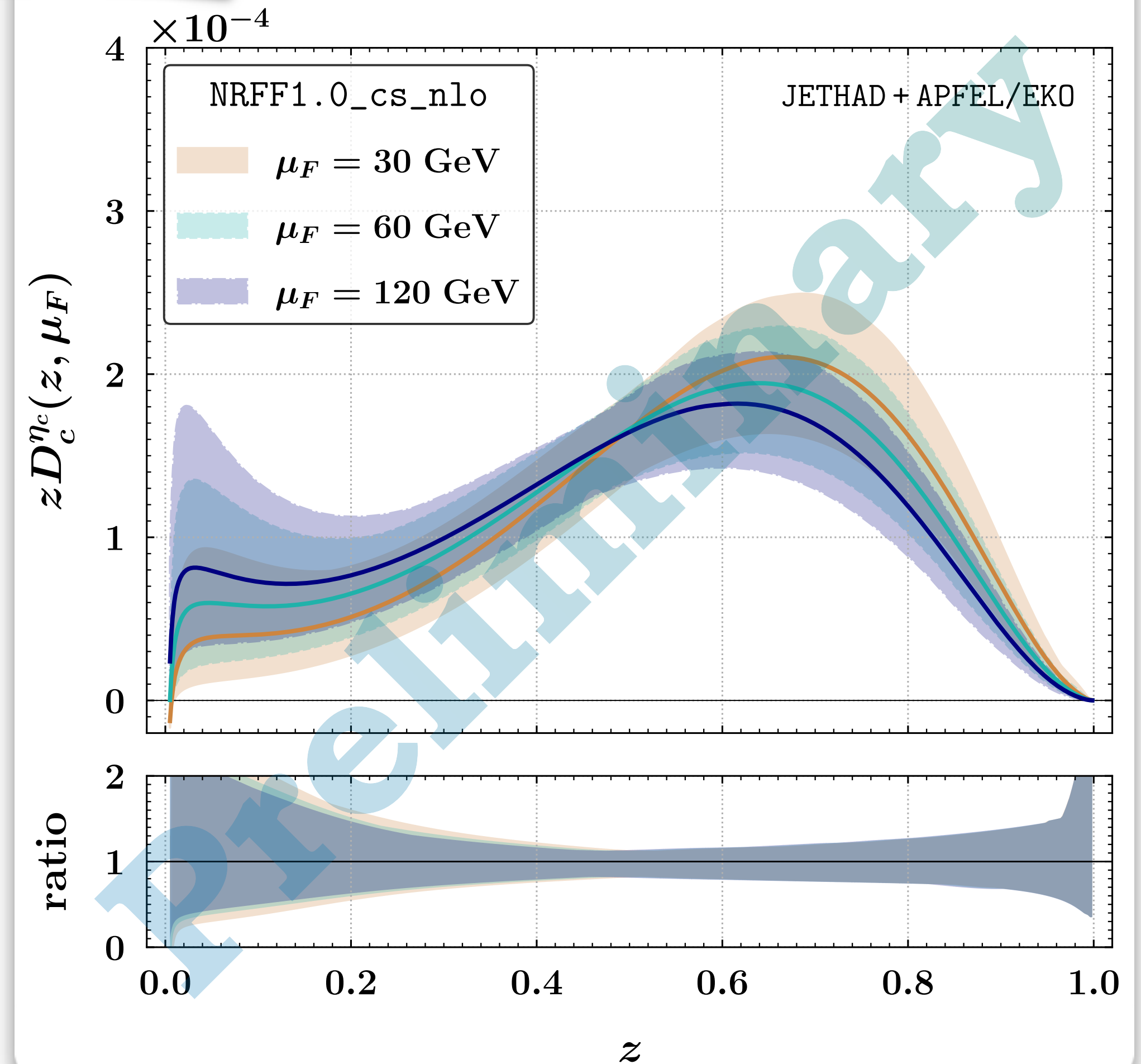
$(c \rightarrow J/\psi)$  fragmentation channel



## Scalar $\eta_c$

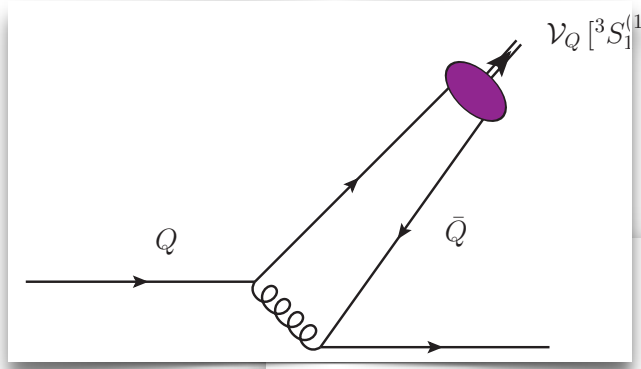


$(c \rightarrow \eta_c)$  fragmentation channel

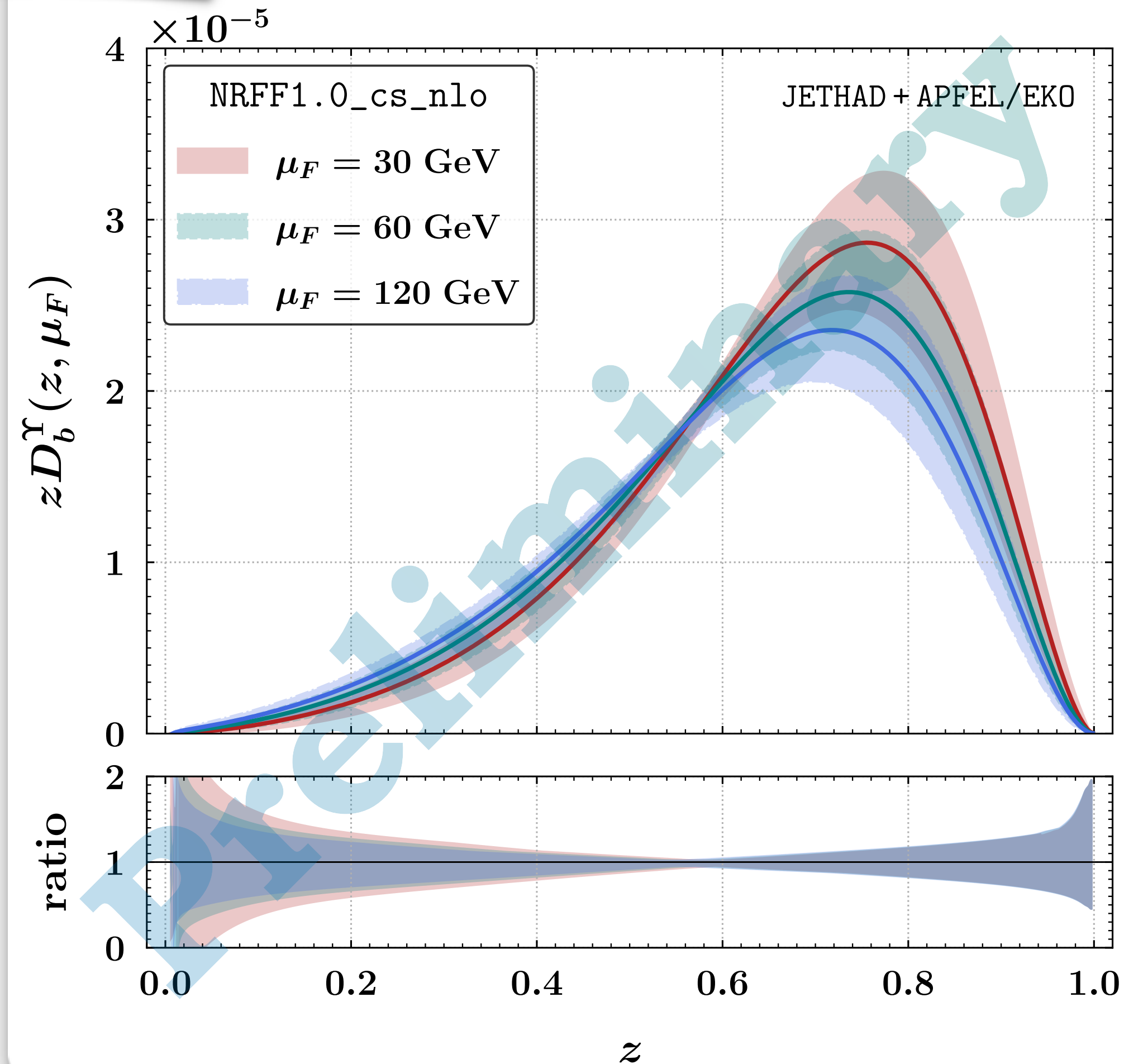


# NRFF1.0: Bottom fragmentation to bottomonia

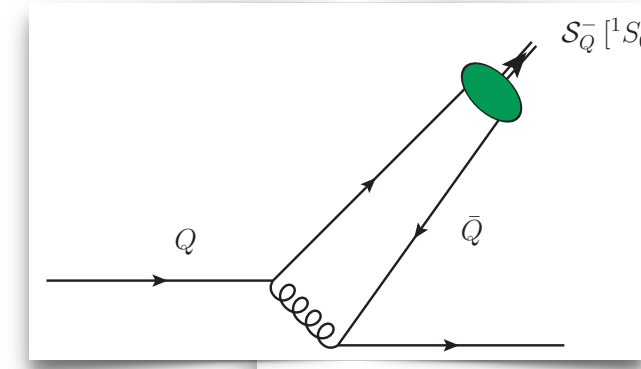
## Vector $\Upsilon$



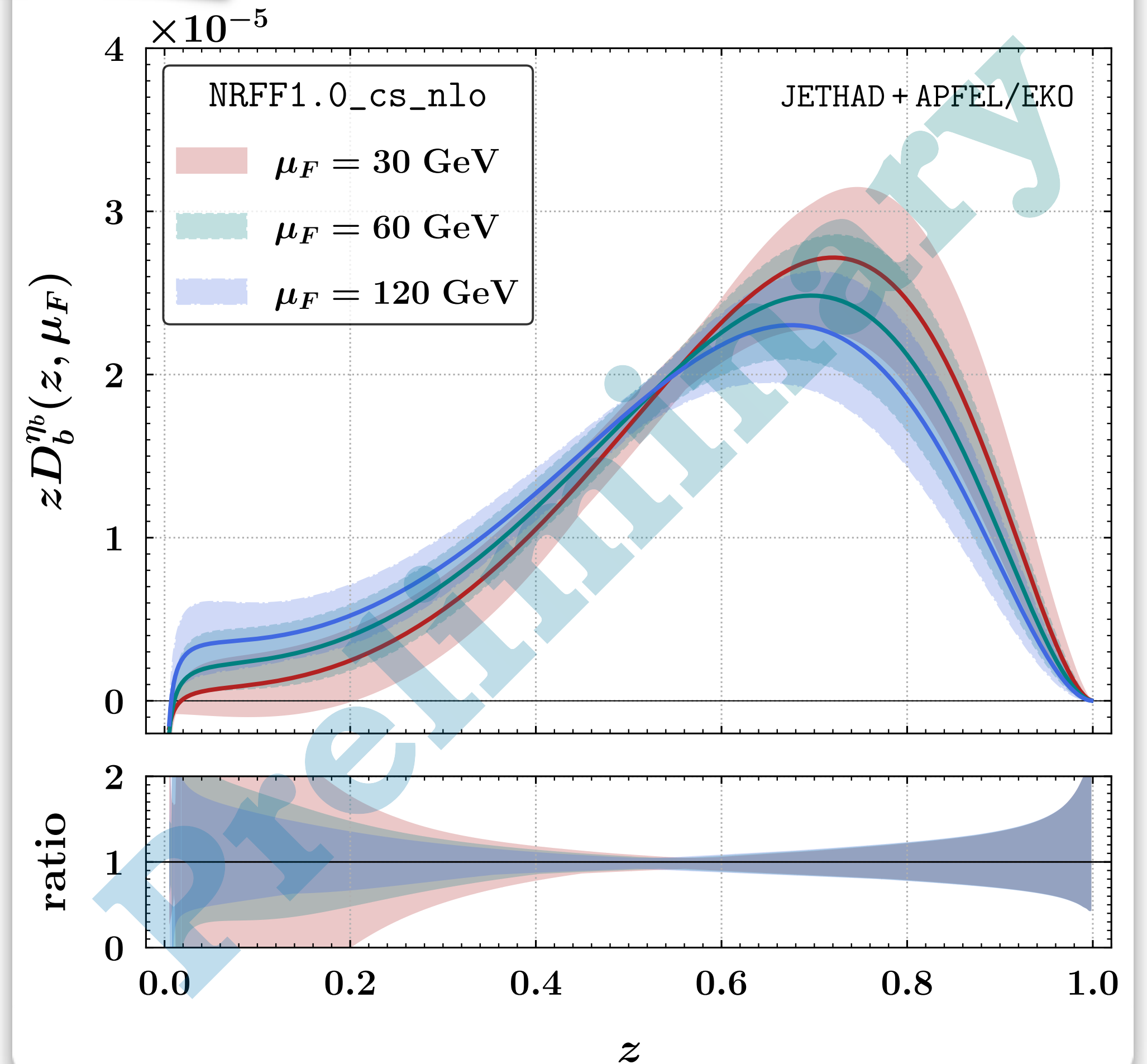
$(b \rightarrow \Upsilon)$  fragmentation channel



## Scalar $\eta_b$

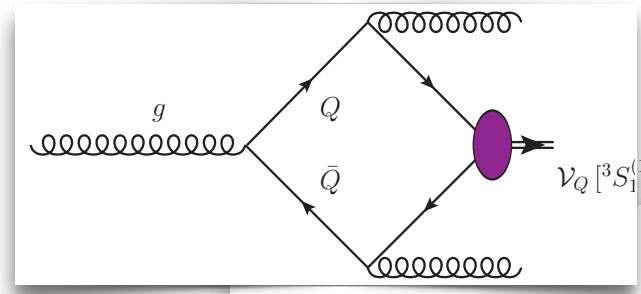


$(b \rightarrow \eta_b)$  fragmentation channel

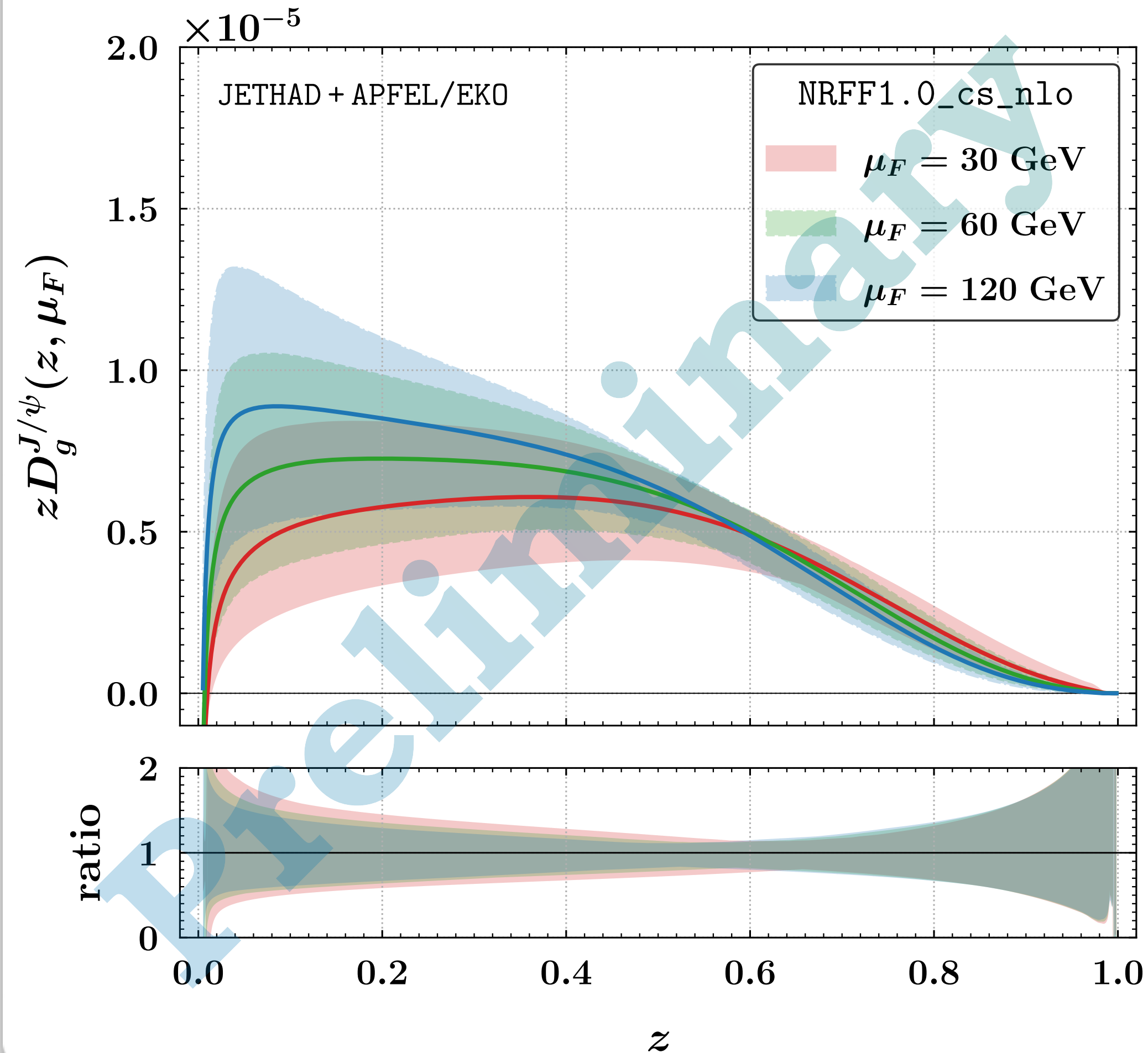


# NRFF1.0: Gluon fragmentation to charmonia

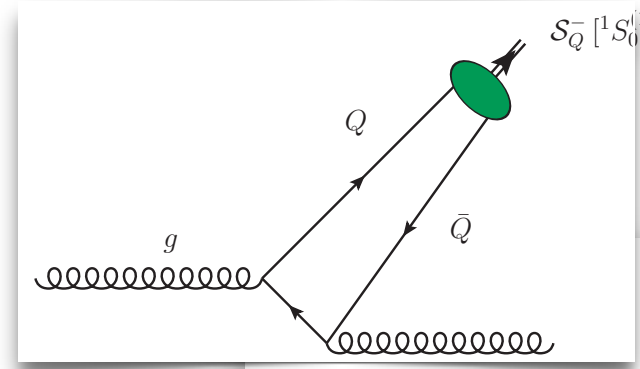
## Vector $J/\psi$



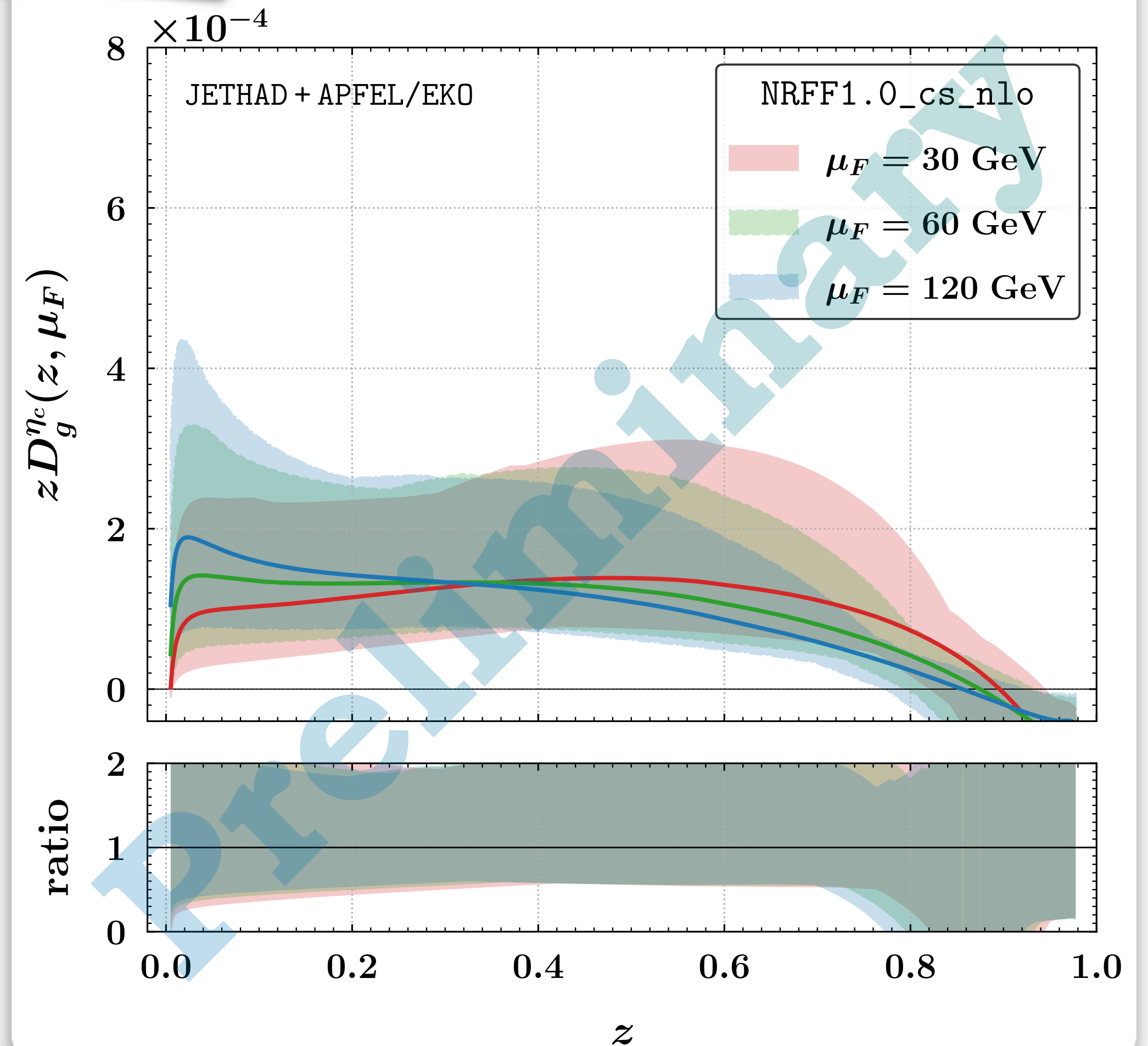
$(g \rightarrow J/\psi)$  fragmentation channel



## Scalar $\eta_c$

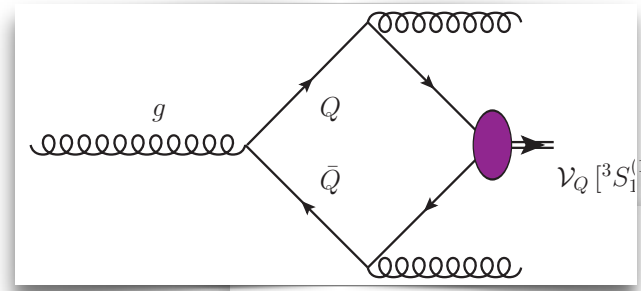


$(g \rightarrow \eta_c)$  fragmentation channel

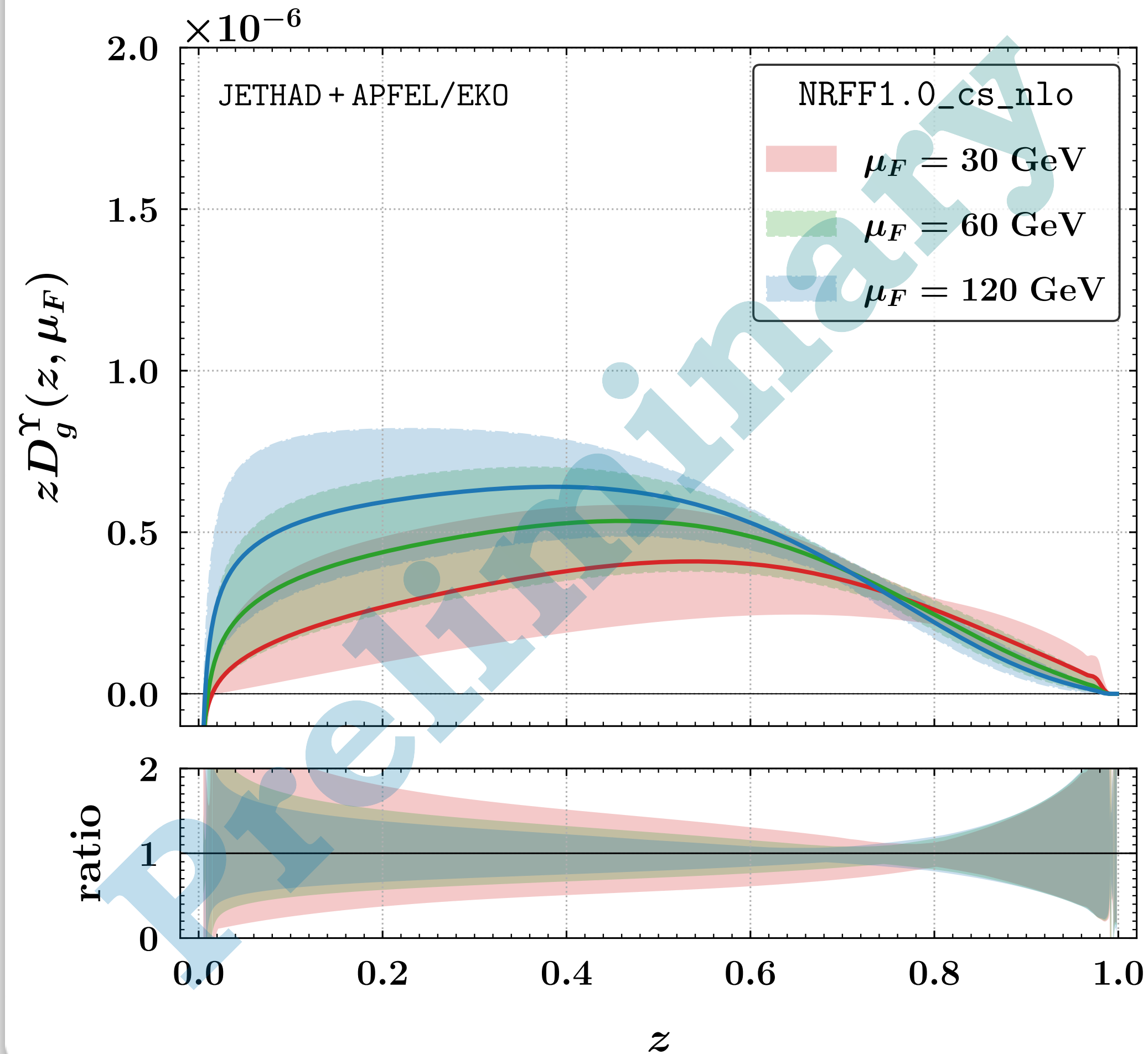


# NRFF1.0: Gluon fragmentation to bottomonia

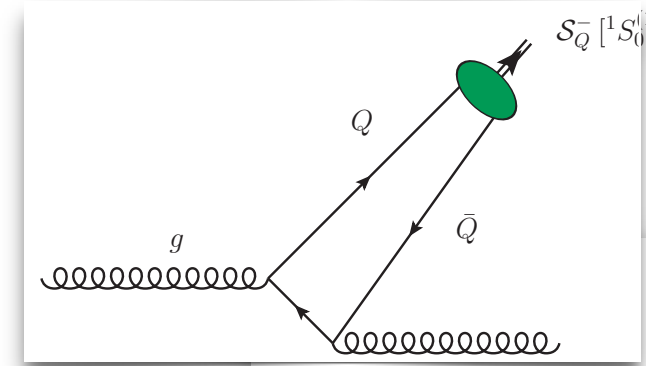
## Vector $\Upsilon$



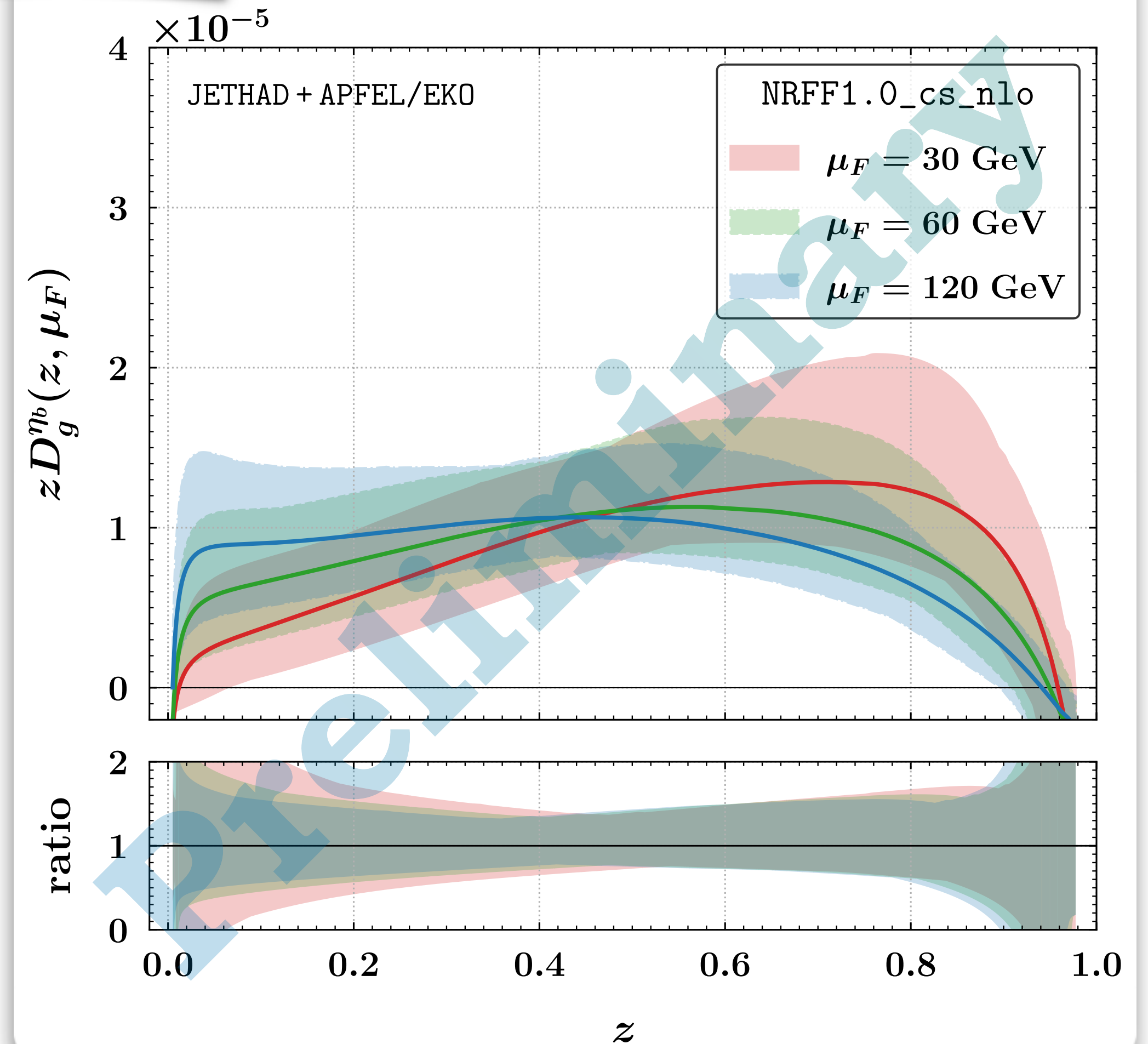
$(g \rightarrow \Upsilon)$  fragmentation channel



## Scalar $\eta_b$



$(g \rightarrow \eta_b)$  fragmentation channel

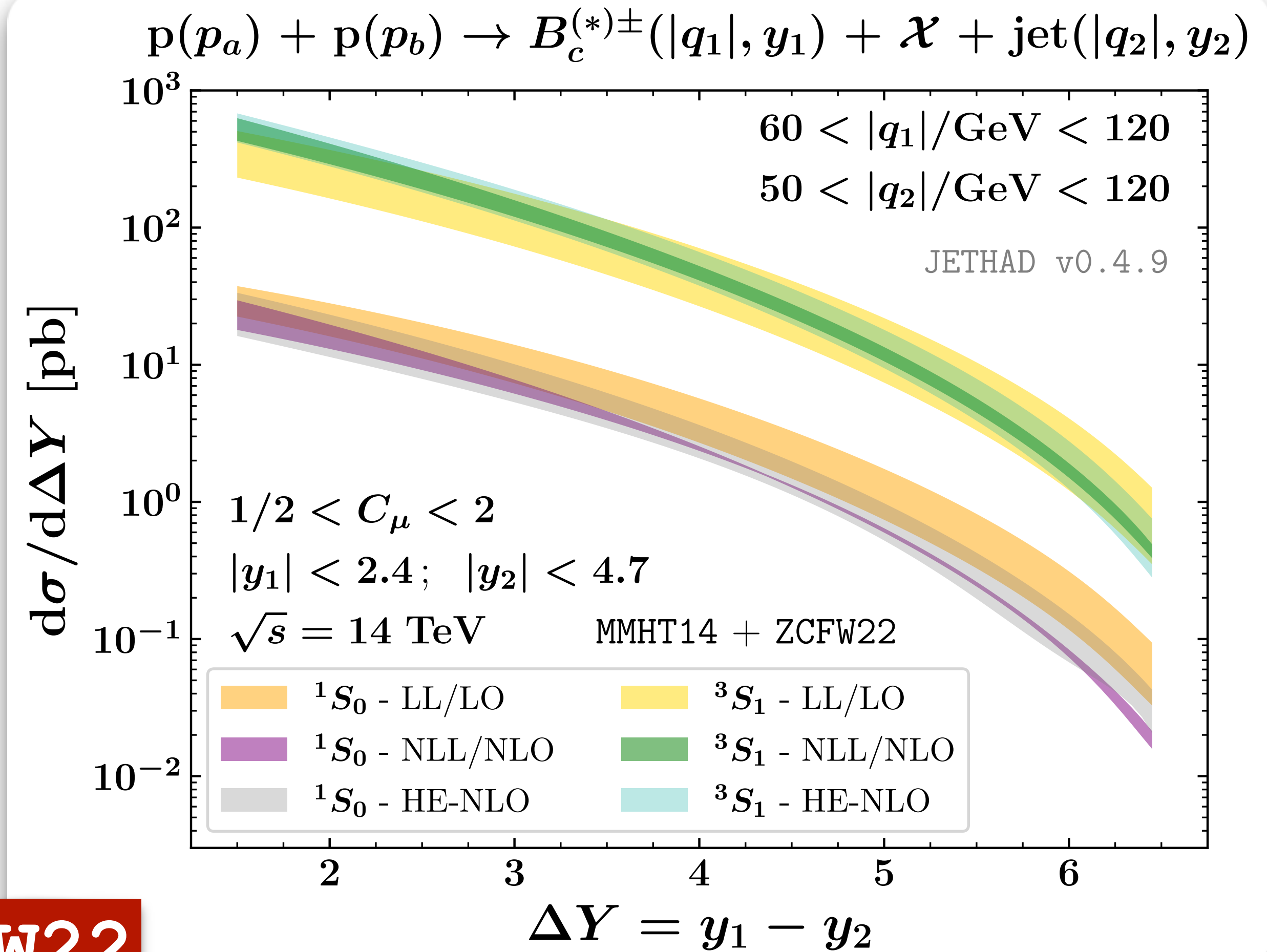
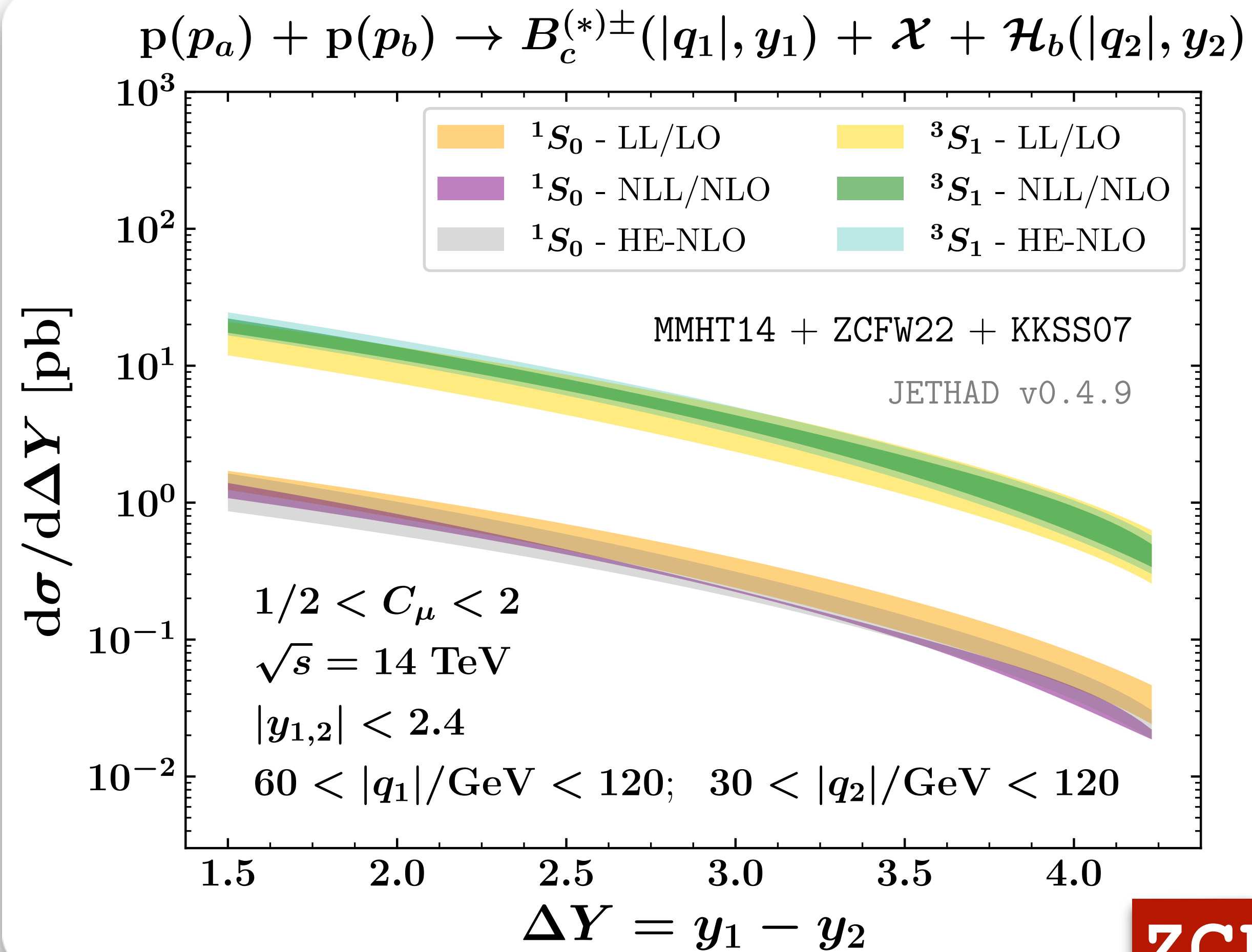




# Charmed $B$ -mesons + jet at the HL-LHC

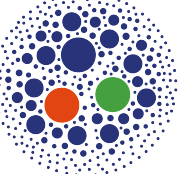
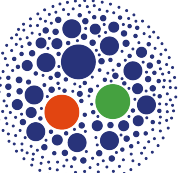
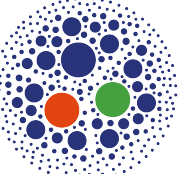
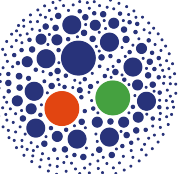
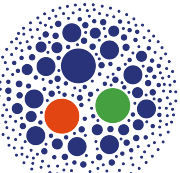
$B_c^\pm(^1S_0) + \text{b-hadron}$

$B_c^\pm(^3S_1) + \text{jet}$



**ZCFW22**

# ¿ What is the exotic matter ?

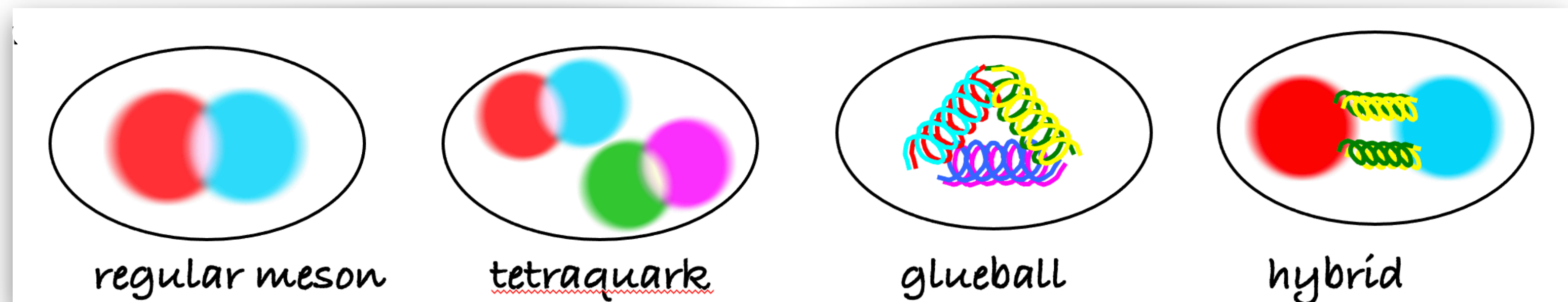
-  **Barions & mesons** → composite states →  $|q_1q_2q_3\rangle$  &  $|q_1\bar{q}_2\rangle$
-  **Ordinary hadrons** ← simplest valence-quark colorless states
-  QCD color neutrality → **tetraquarks, pentaquarks, hybrids** allowed
-  **Exotic hadrons** ← different quantum numbers than ordinary hadrons
-  First discovery of an exotic hadron at Belle (🇯🇵) in 2003 →  **$X(3872)$**



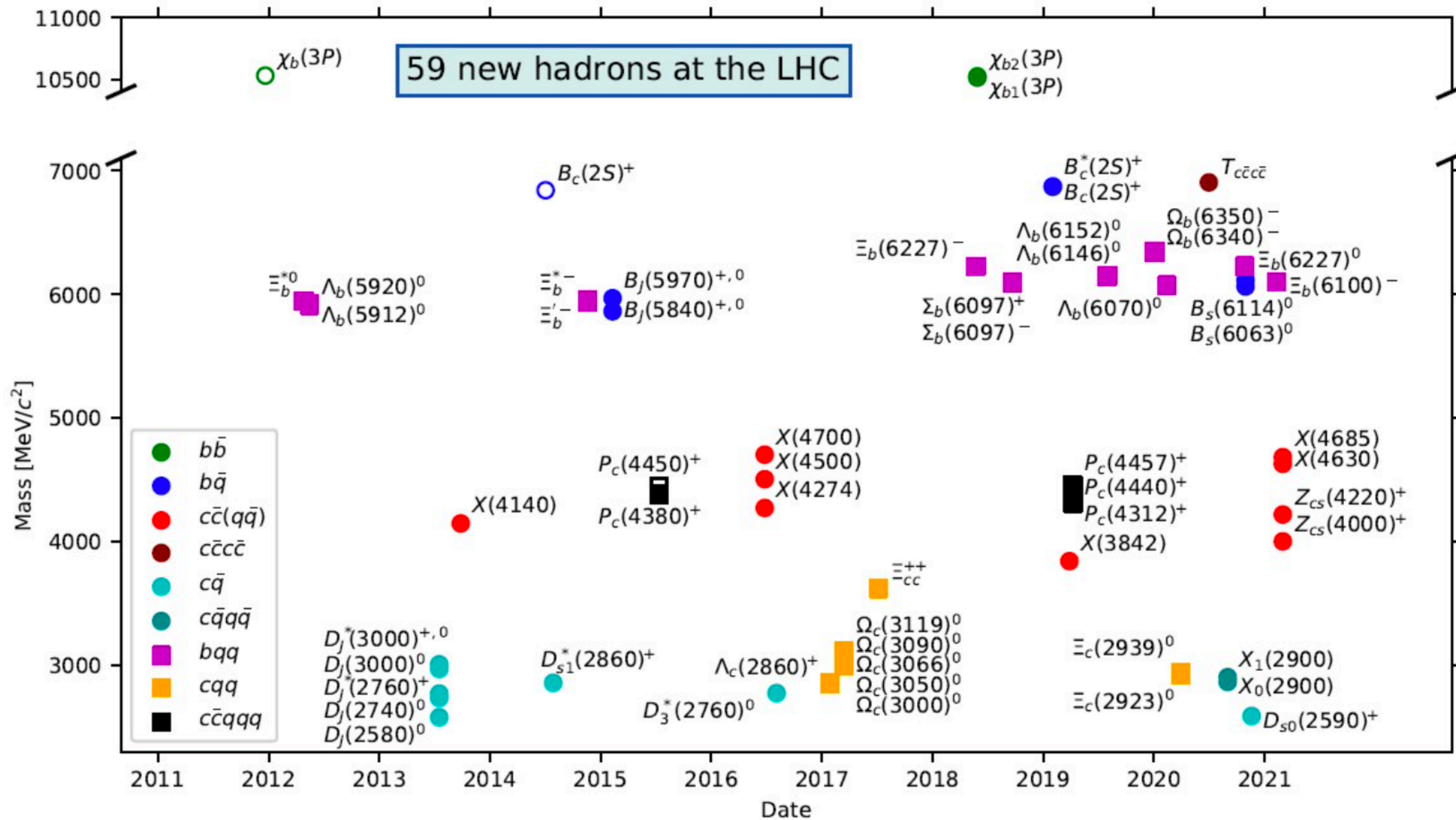
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- Barions & mesons → composite states →  $|q_1q_2q_3\rangle$  &  $|q_1\bar{q}_2\rangle$
- Ordinary hadrons ← simplest valence-quark colorless states
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- First discovery of an exotic hadron at Belle (🇯🇵) in 2003 →  $X(3872)$

## Hadron Spectroscopy



# Exotic hadrons at the LHC

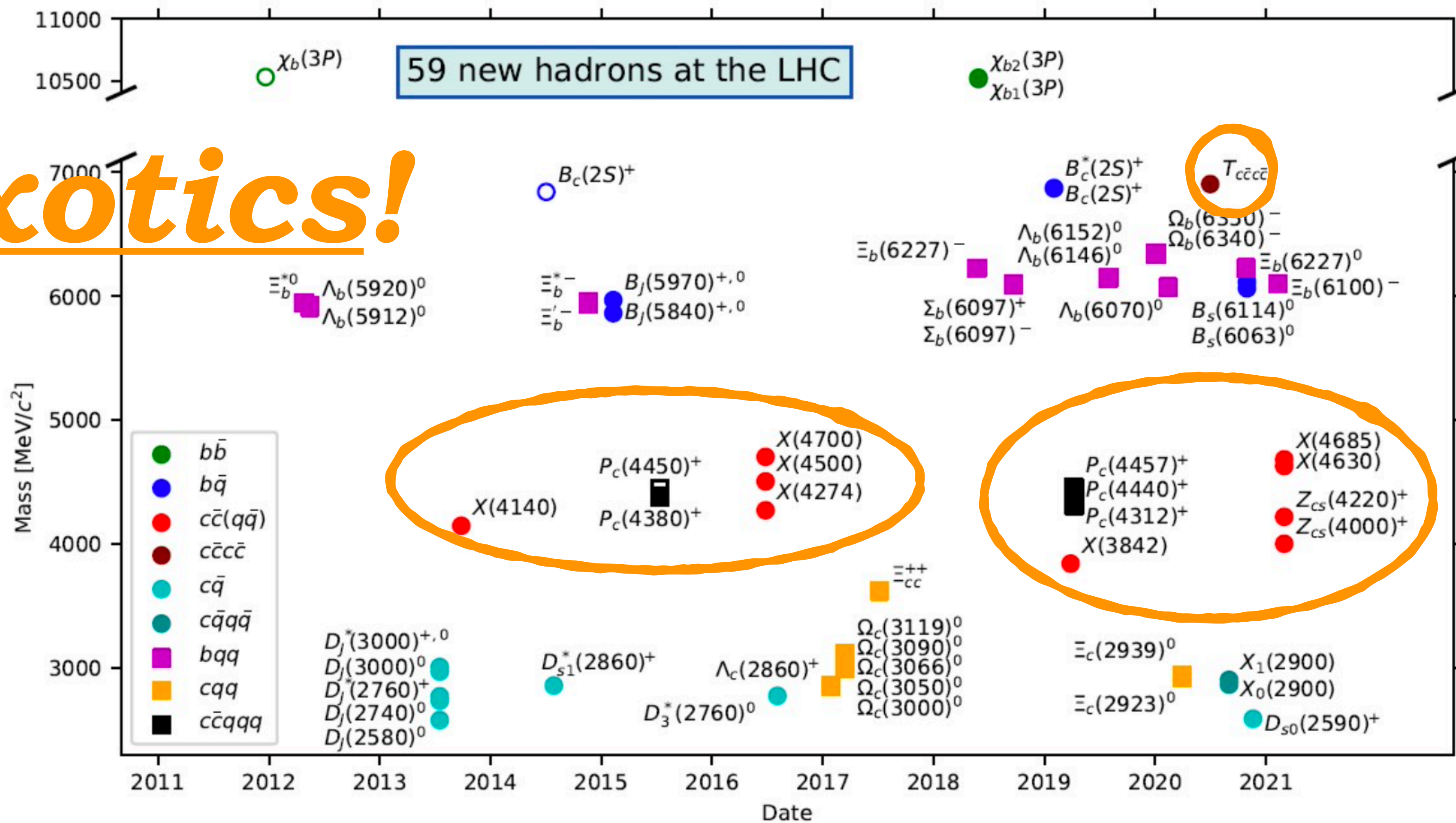


The full list of new hadrons found at the LHC, organised by year of discovery (horizontal axis) and particle mass (vertical axis). The colours and shapes denote the quark content of these states. (Image: LHCb/CERN)

<https://home.cern/news/news/physics/59-new-hadrons-and->

# Exotic hadrons at the LHC

*Exotics!*



The full list of new hadrons found at the LHC, organised by year of discovery (horizontal axis) and particle mass (vertical axis). The colours and shapes denote the quark content of these states. (Image: LHCb/CERN)

<https://home.cern/news/news/physics/59-new-hadrons-and->

# Interpretation of exotic states

Meson + continuum

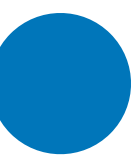
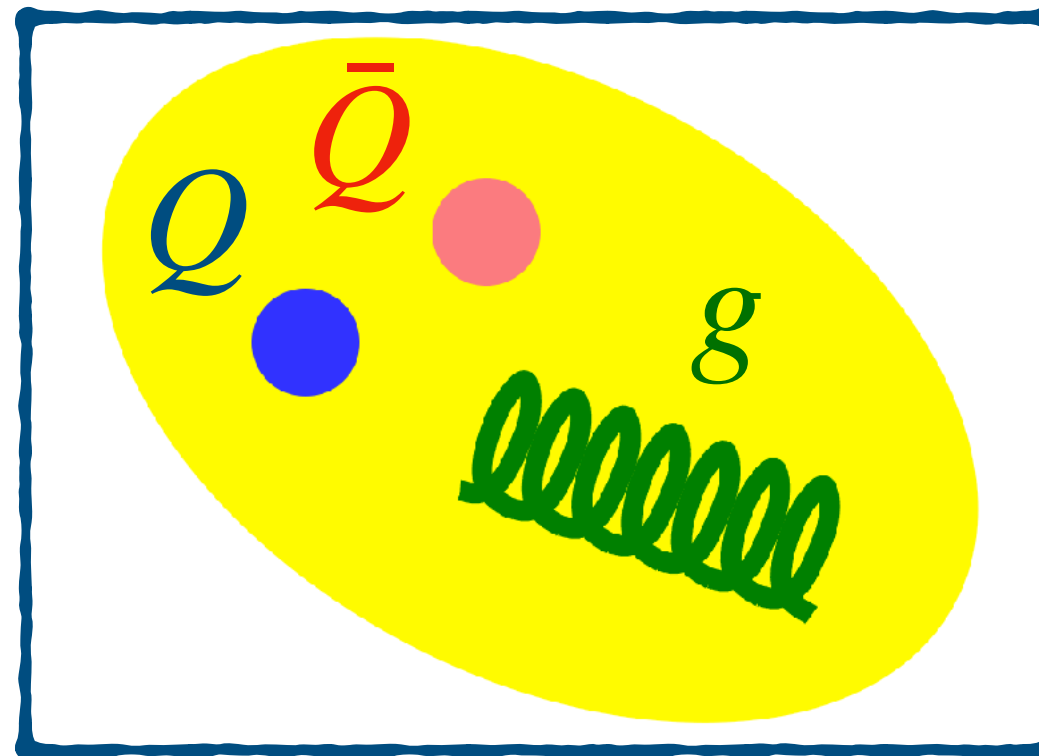


# Interpretation of exotic states

Meson + continuum

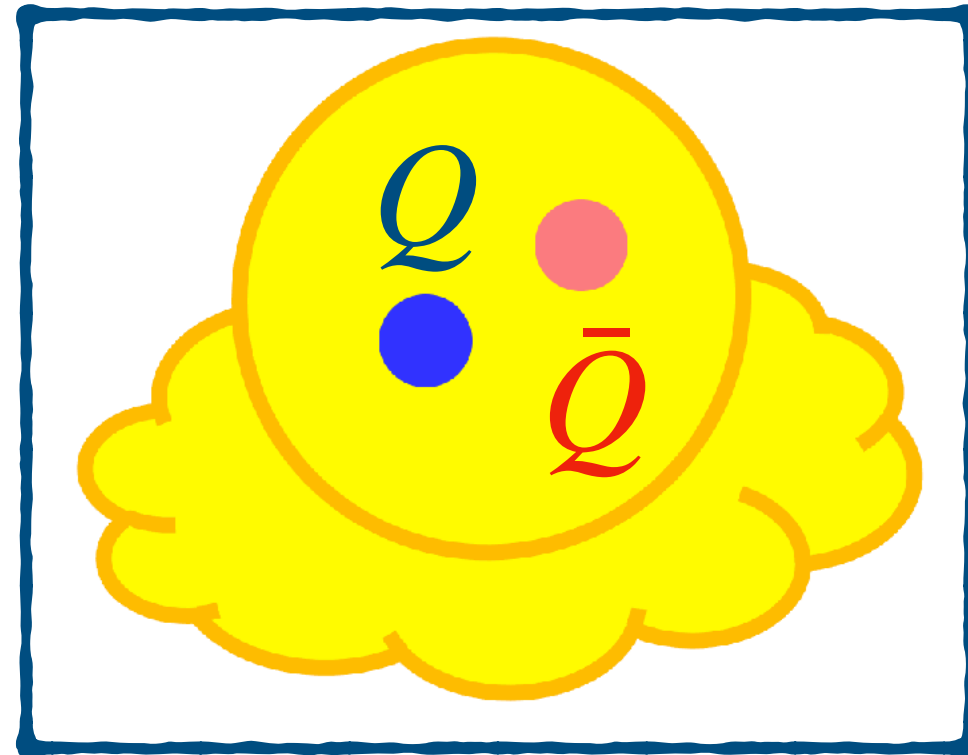


Hybrid

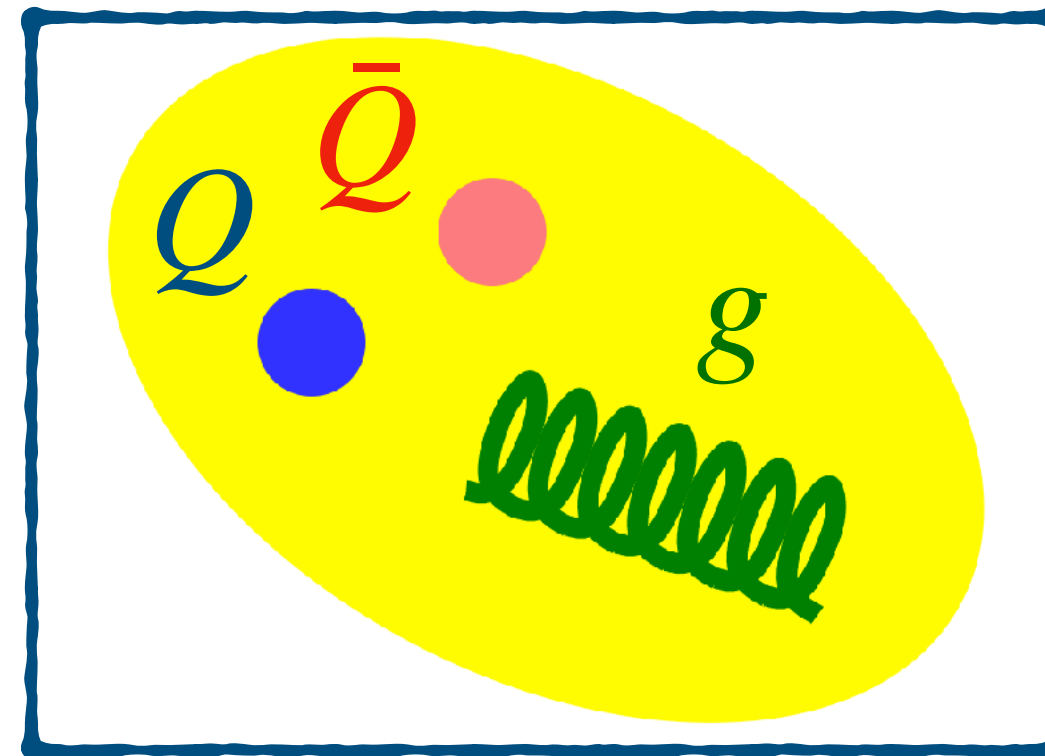


# Interpretation of exotic states

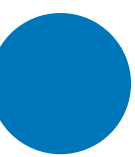
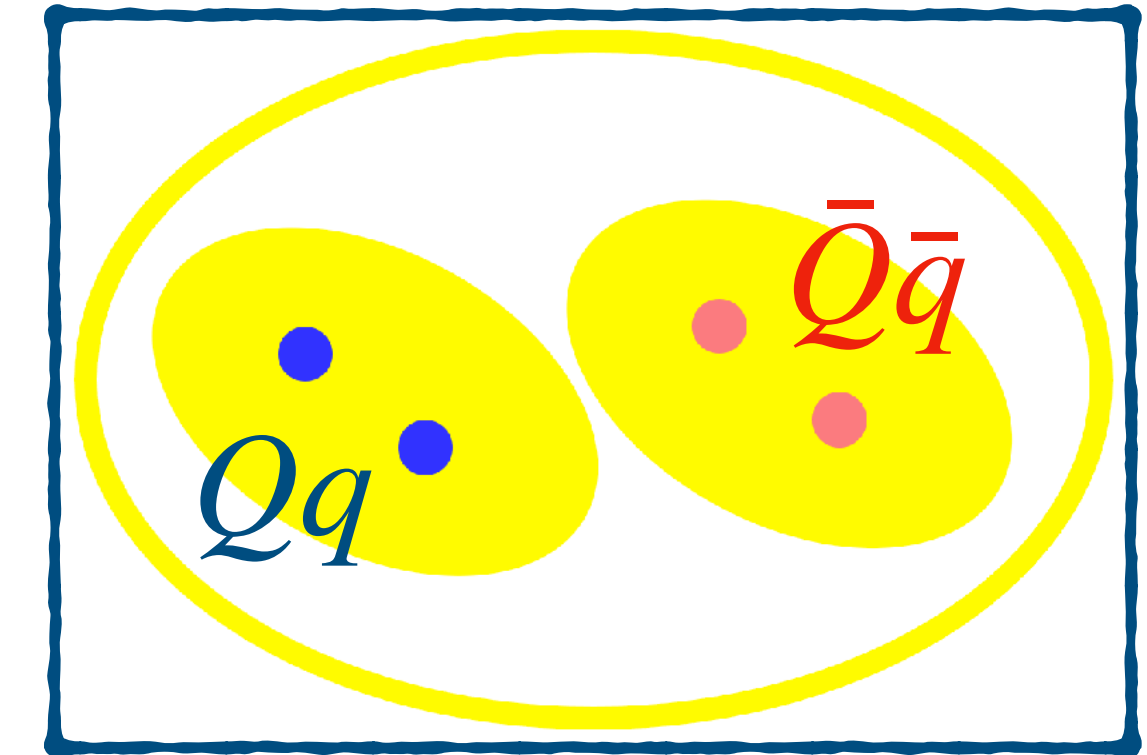
Meson + continuum



Hybrid



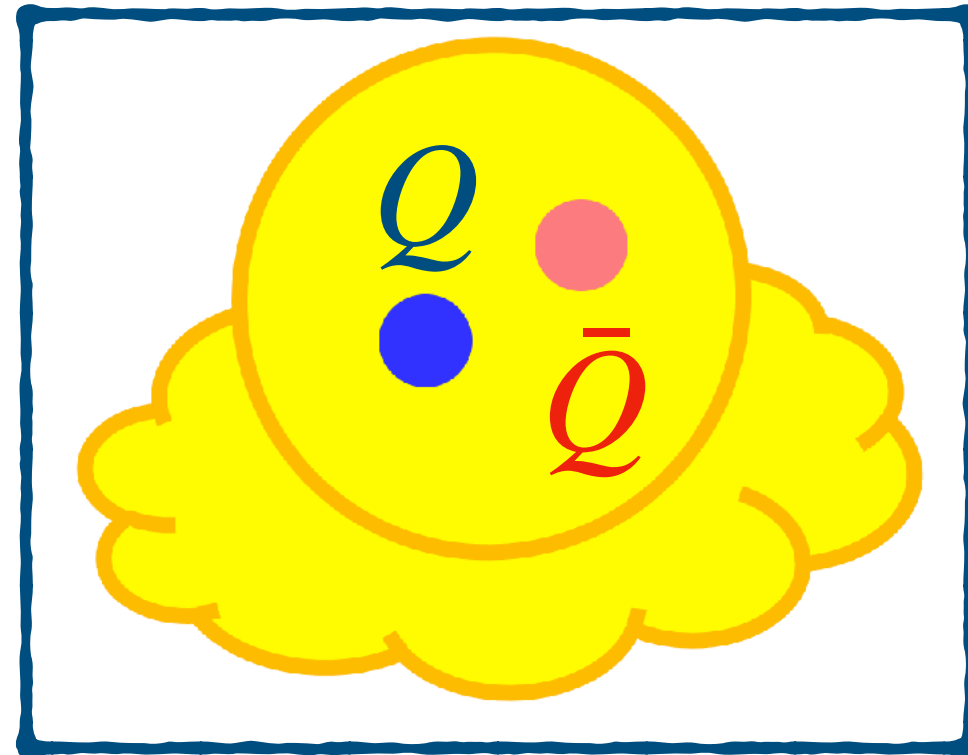
Diquark + antiquark



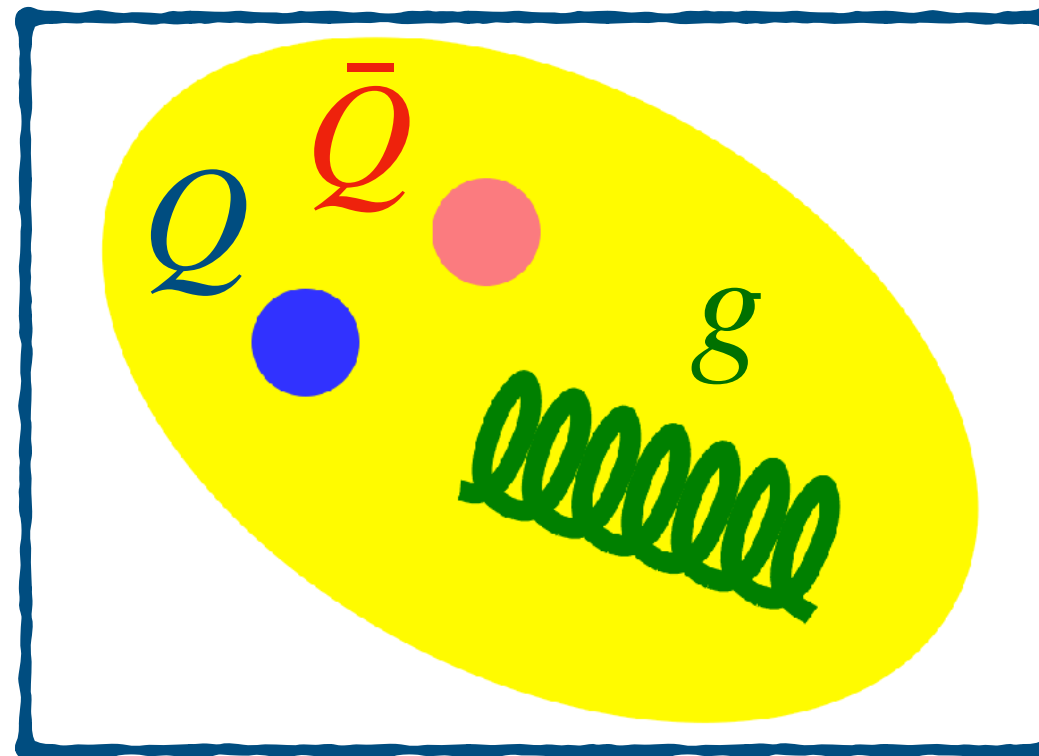


# Interpretation of exotic states

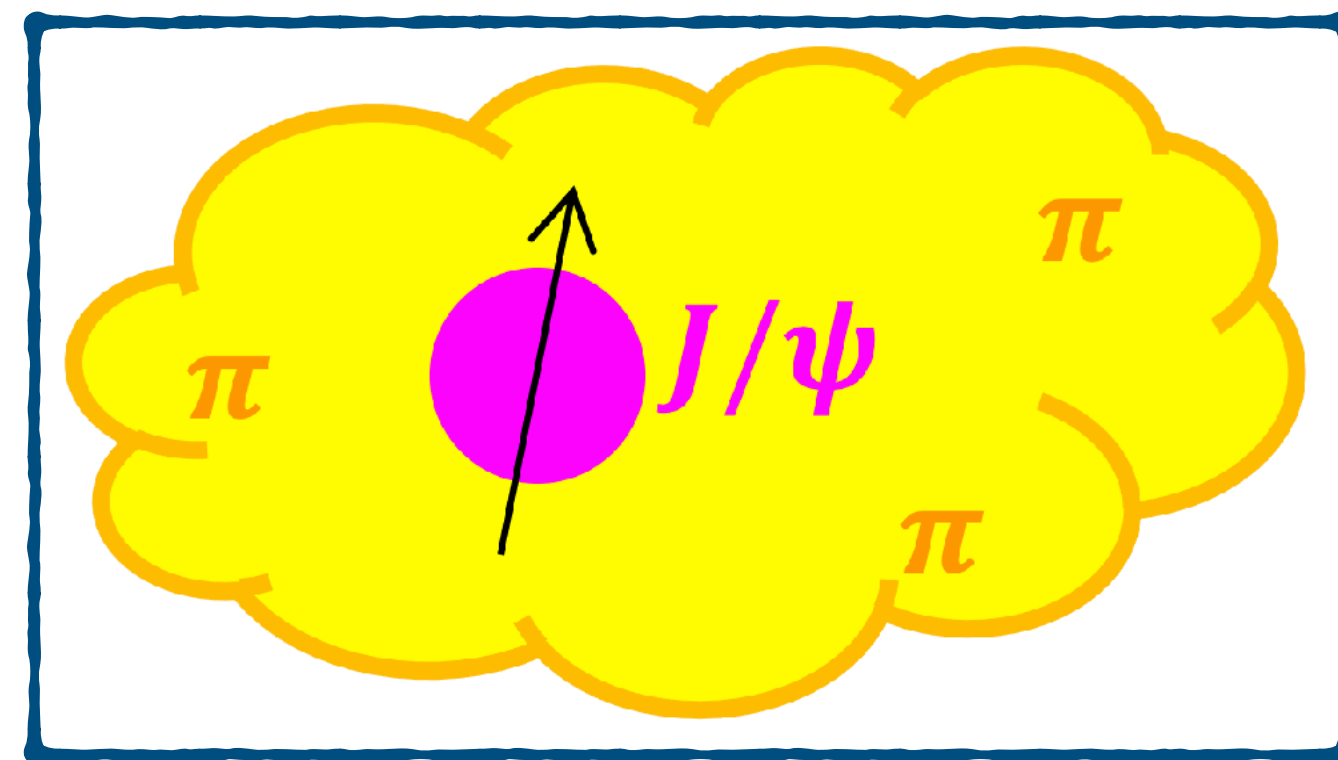
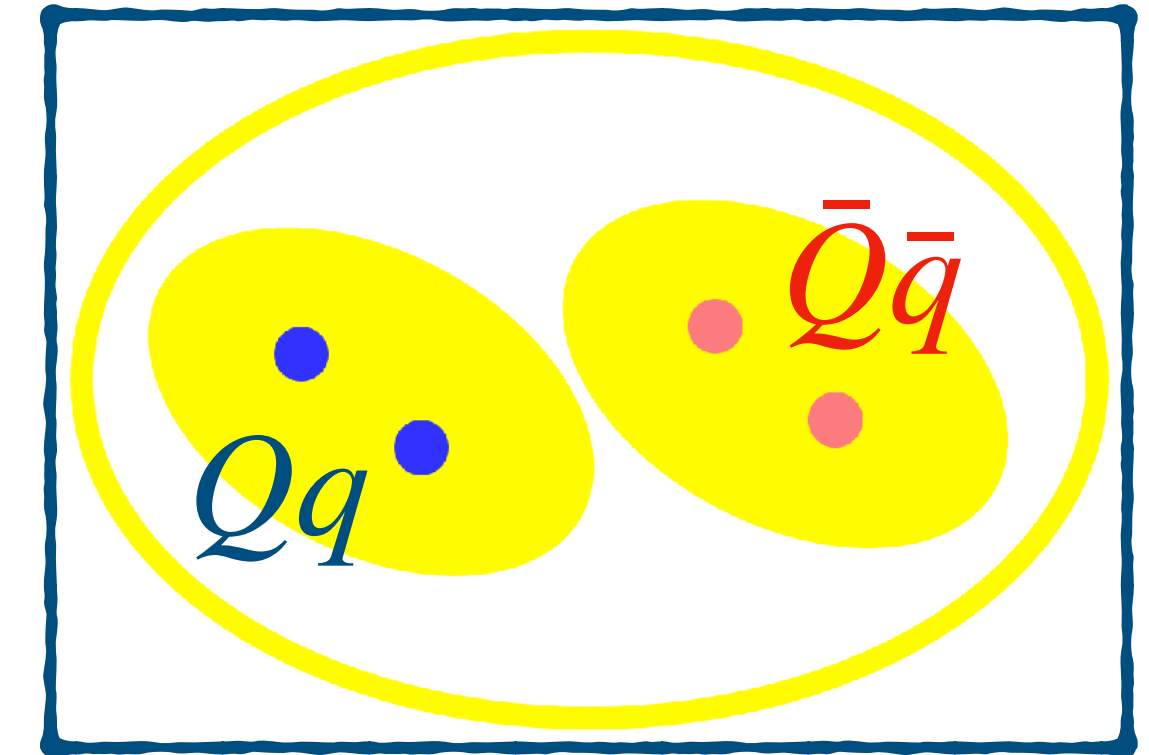
Meson + continuum



Hybrid



Diquark + antiquark

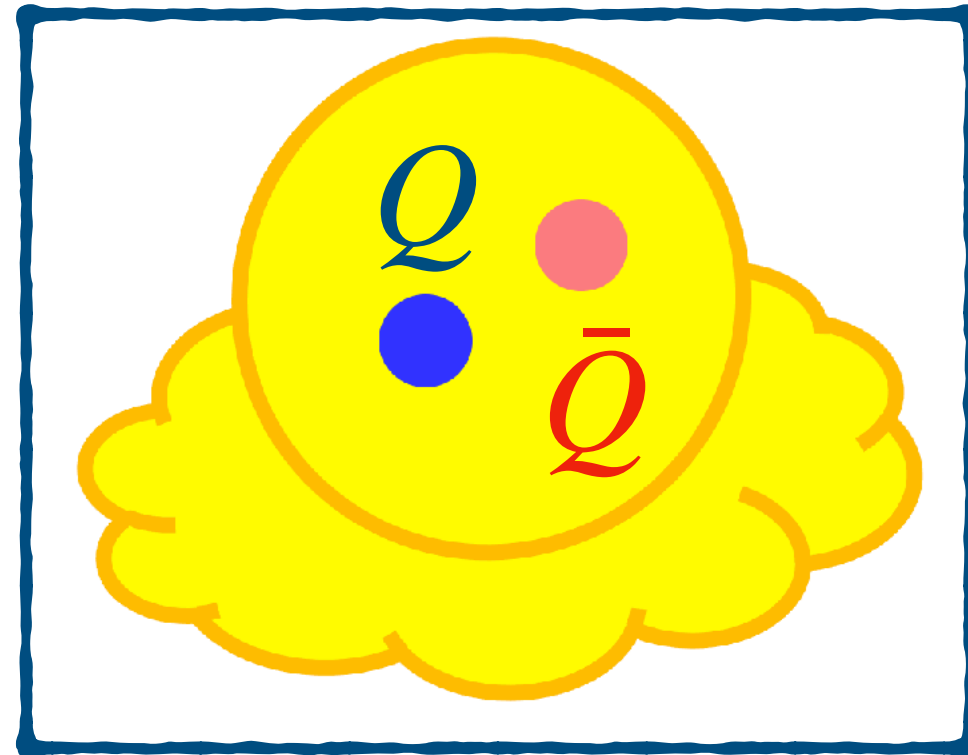


Hadroquarkonium

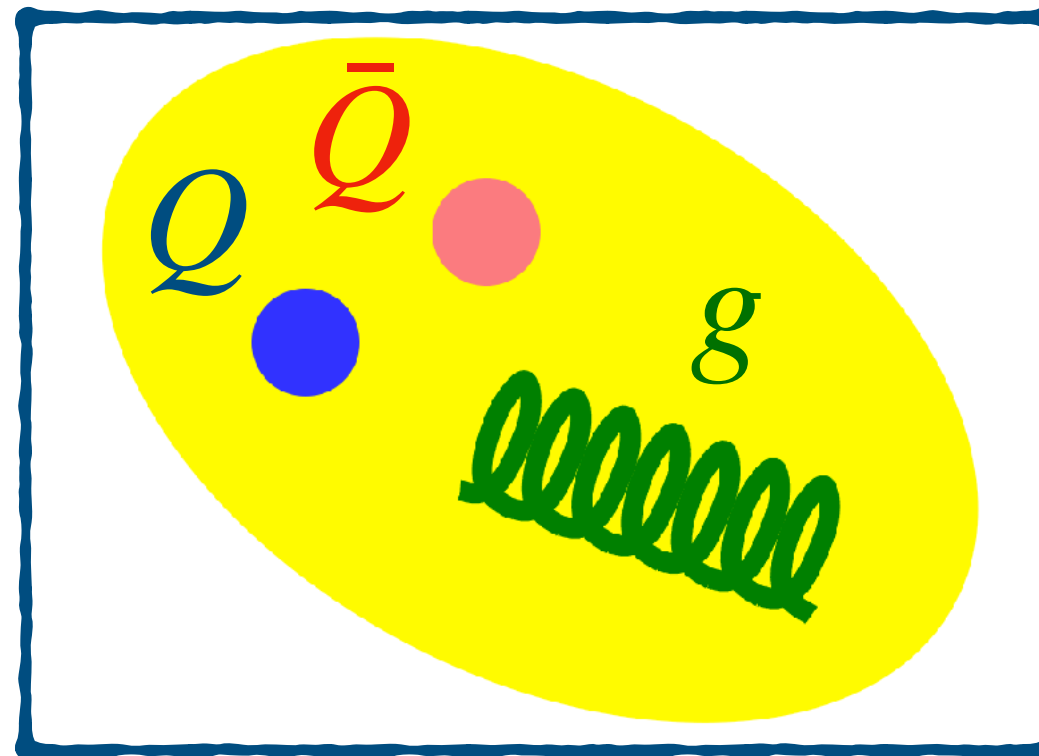


# Interpretation of exotic states

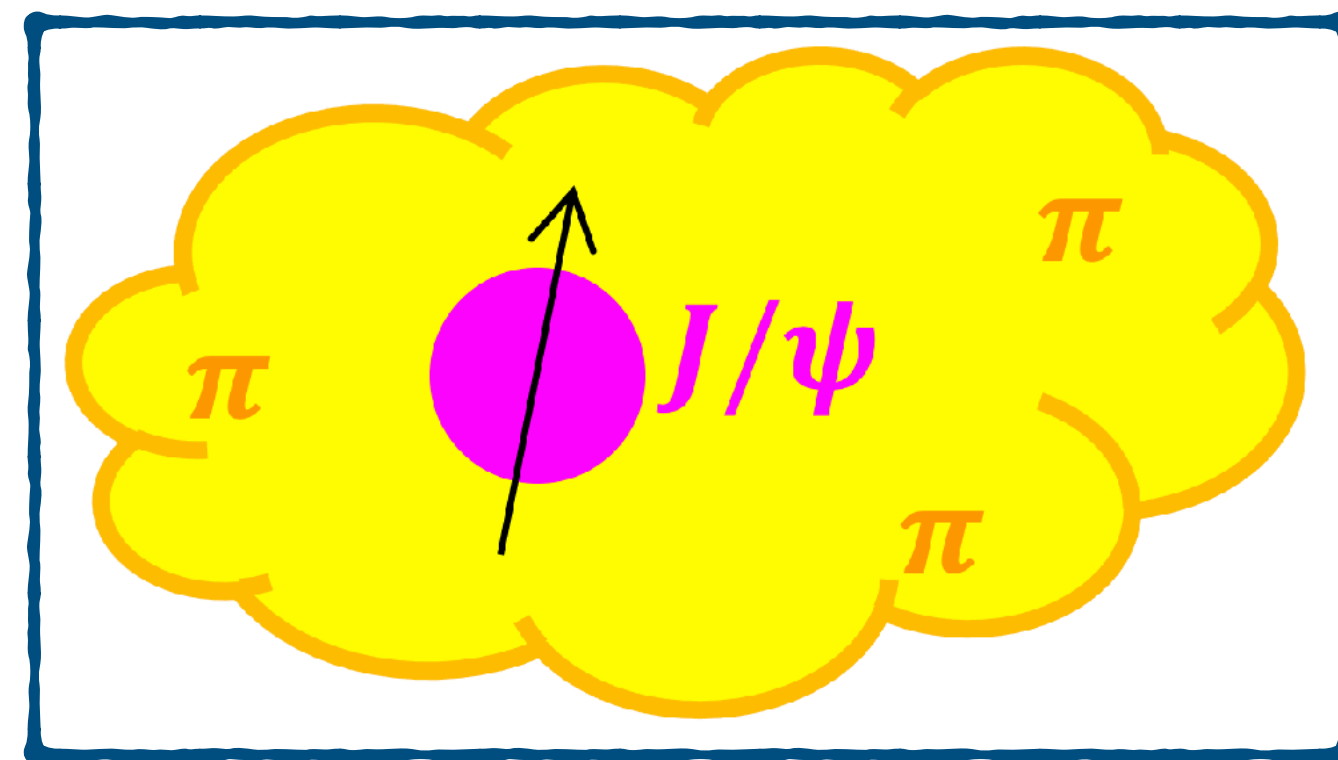
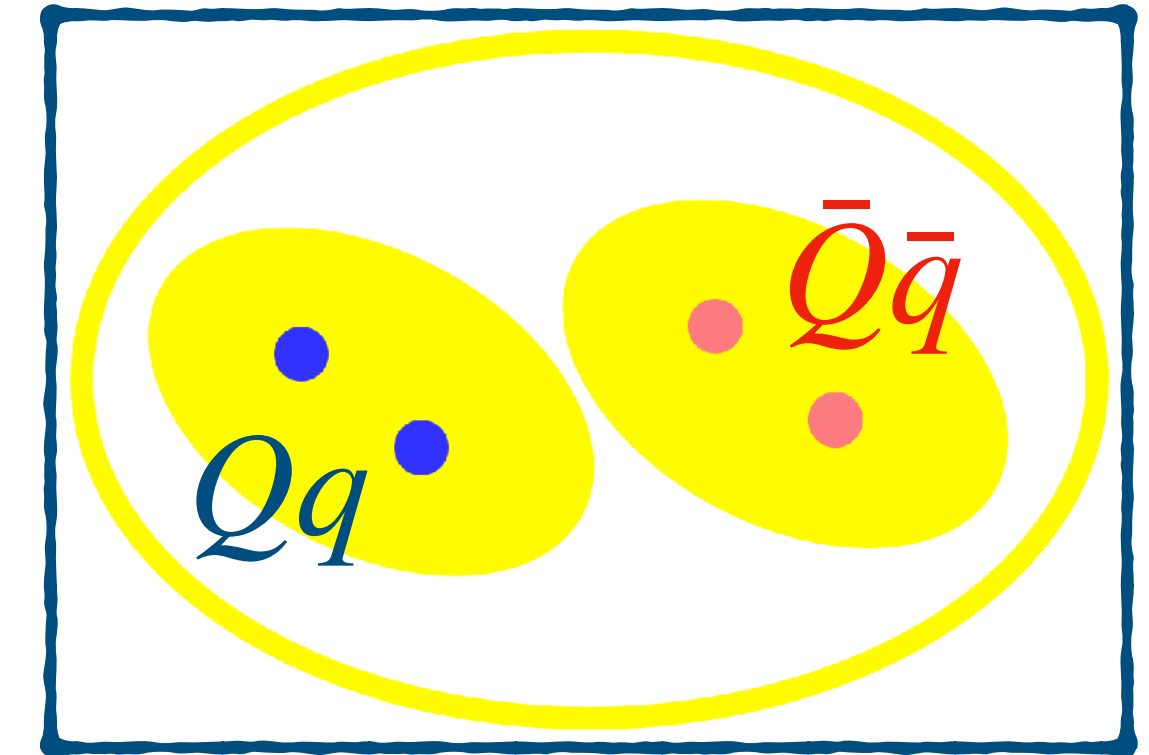
Meson + continuum



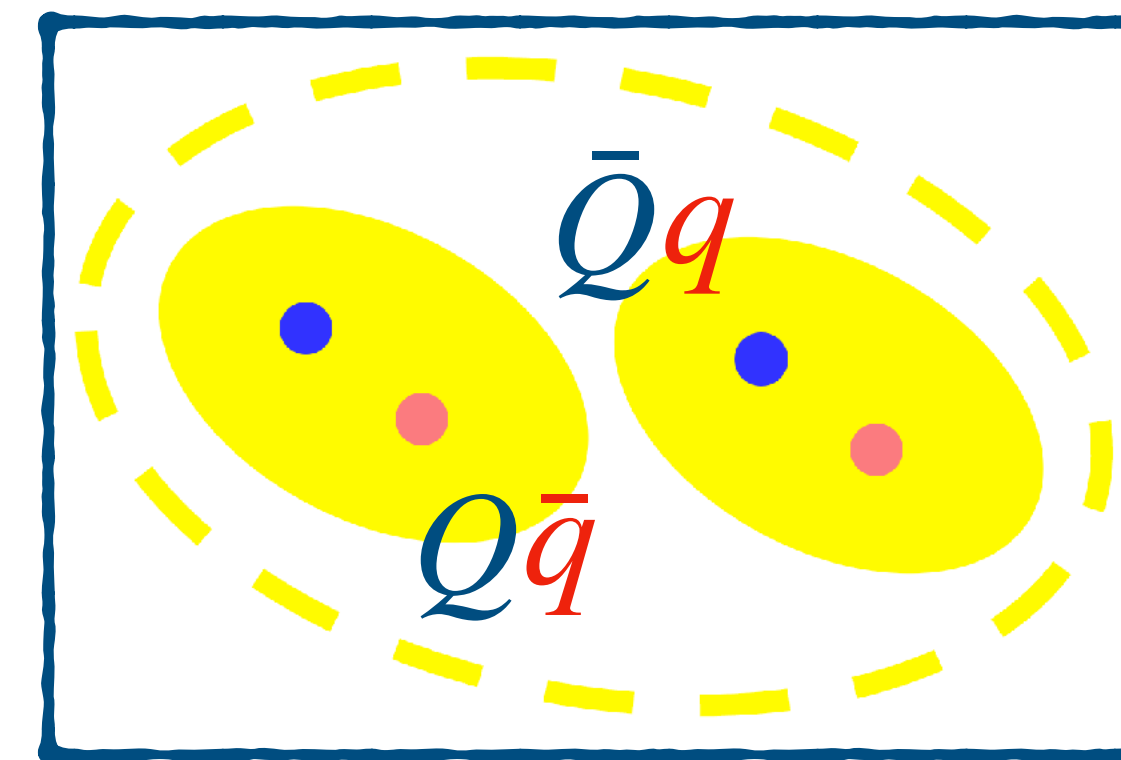
Hybrid



Diquark + antiquark



Hadroquarkonium

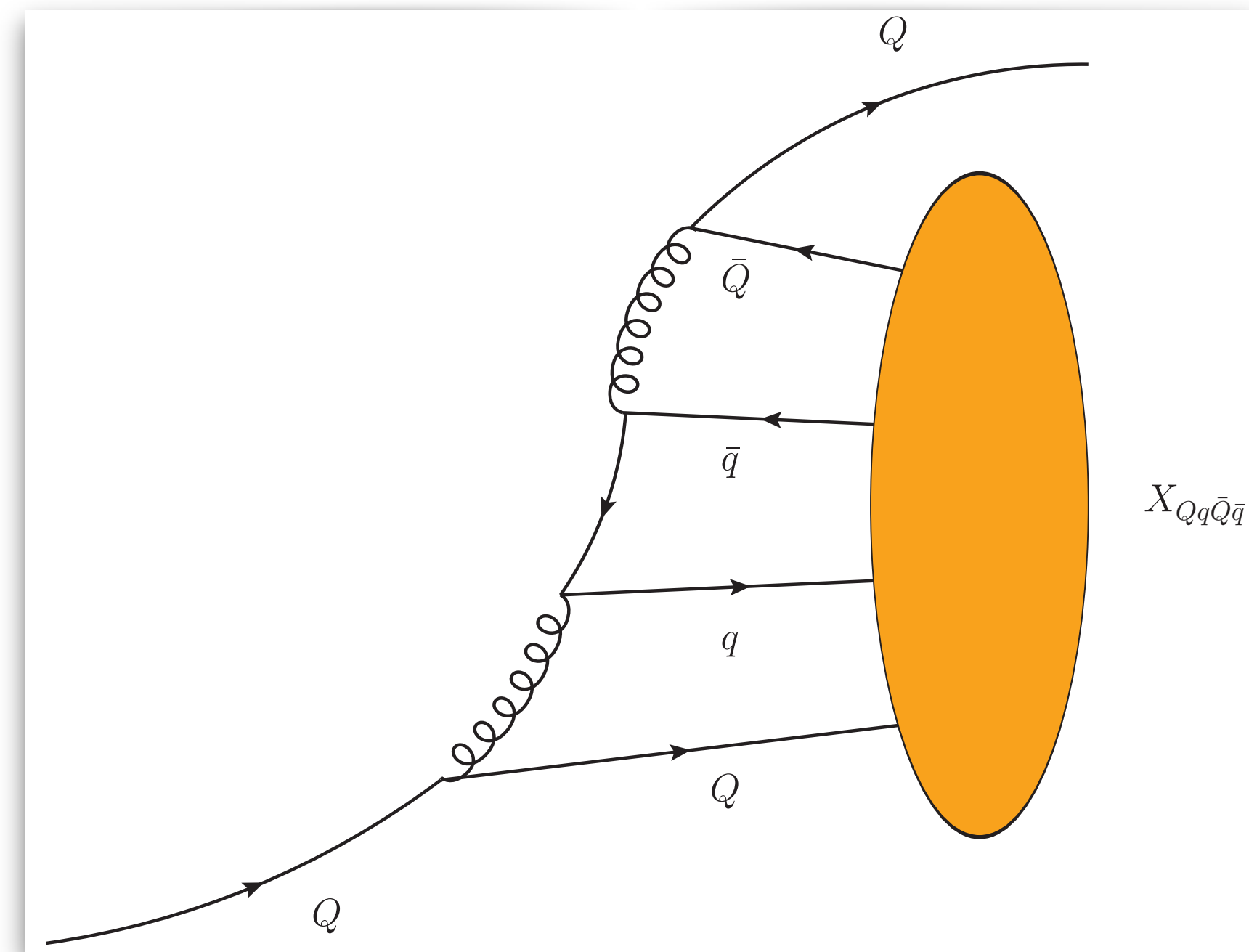
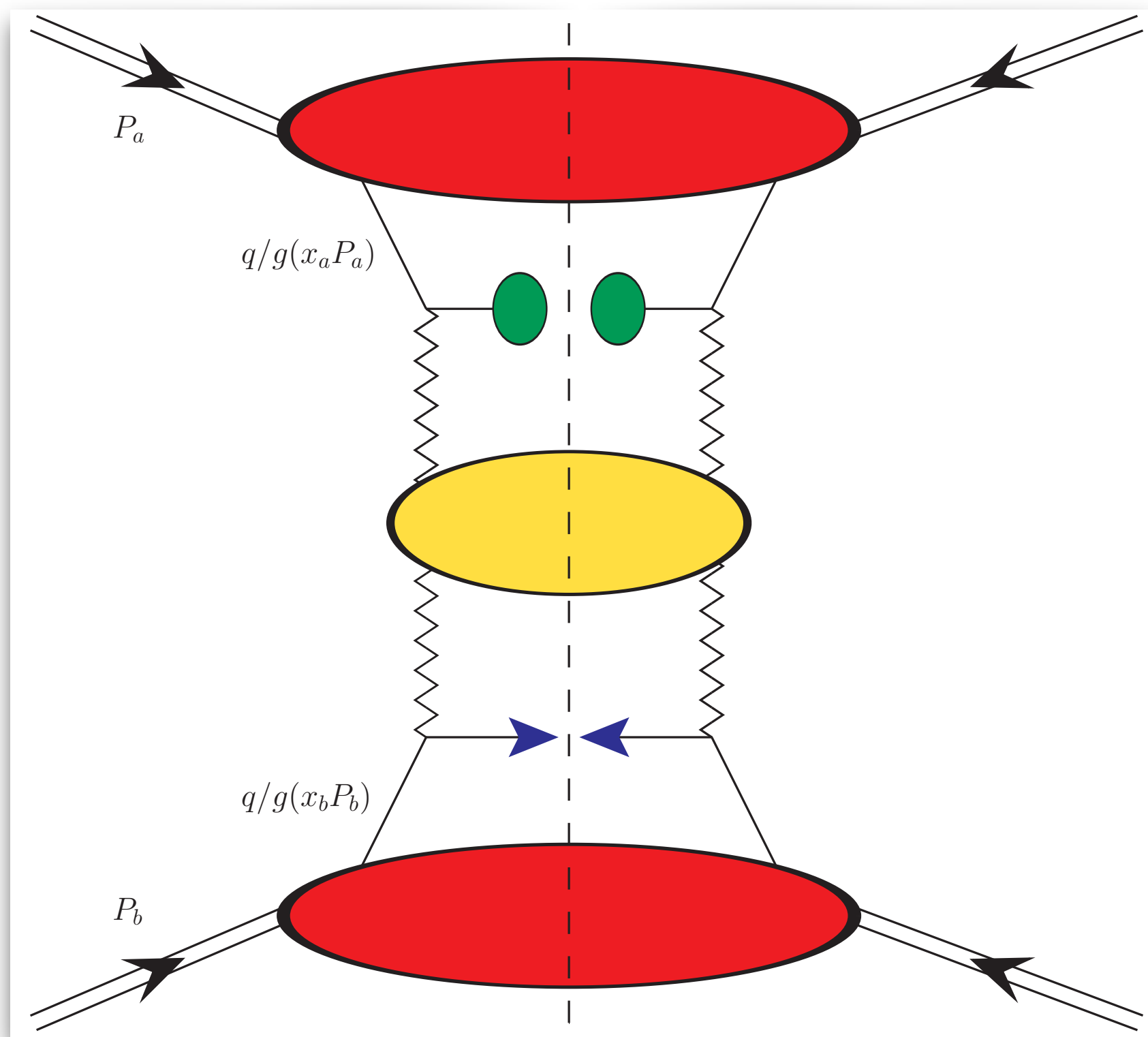


Molecule of mesons



# Heavy-light tetraquark from single-parton fragmentation

! Let us consider heavy-light  $X_{Qq\bar{Q}\bar{q}}$  tetraquarks at large  $p_T \rightarrow$  single-parton fragmentation !

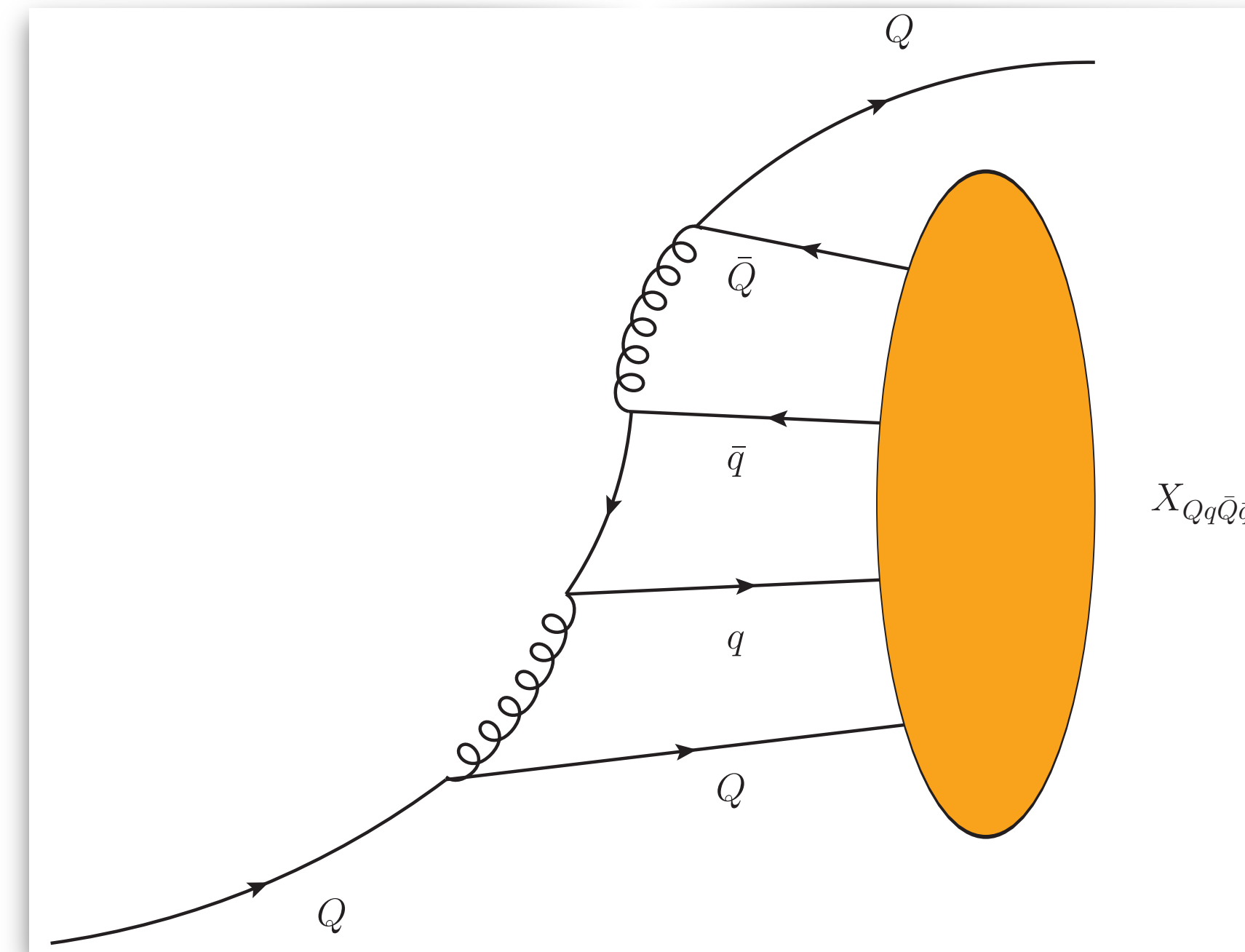
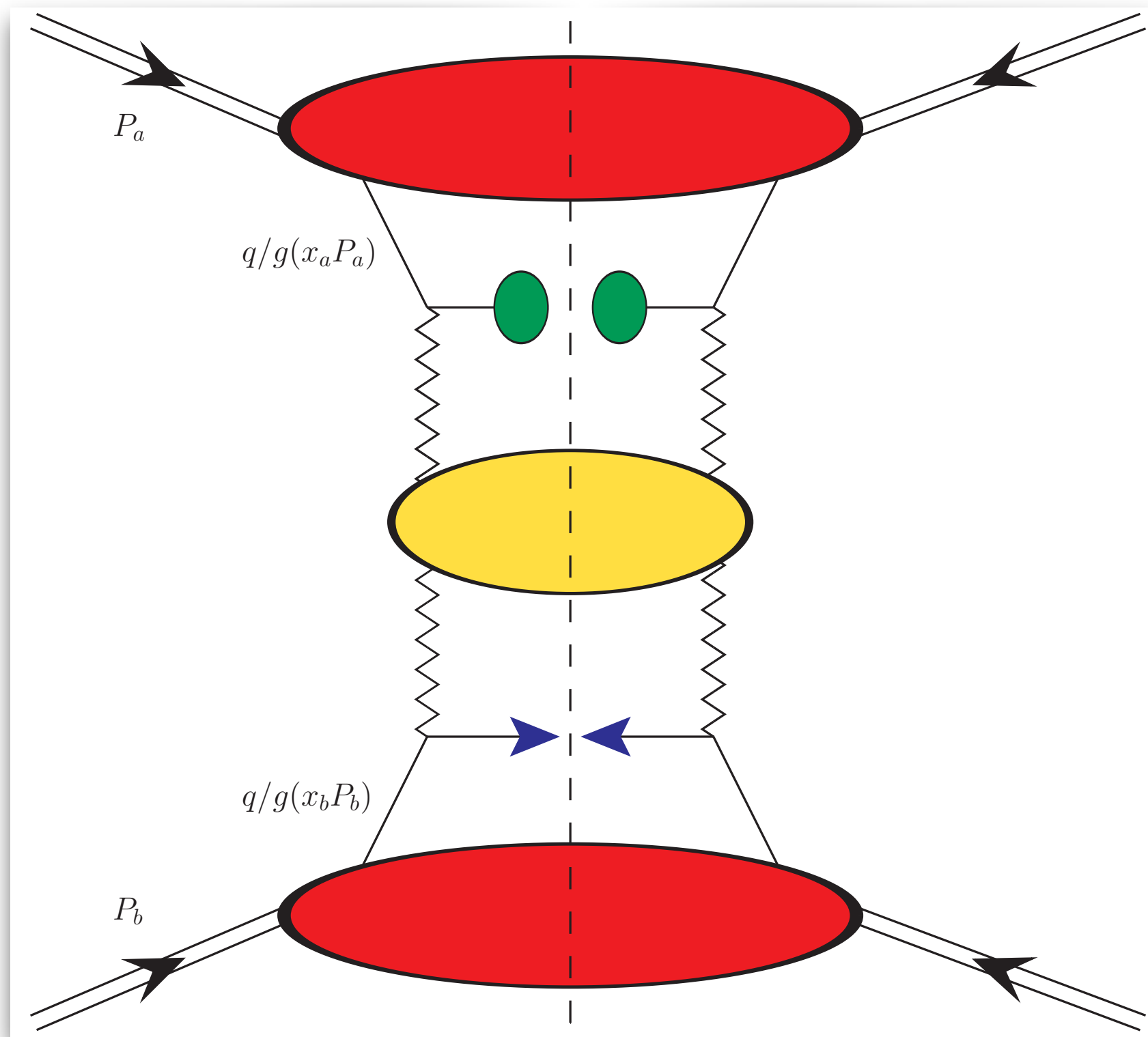


[F. G. C., A. Papa, Phys. Lett. B 848 (2024) 138406]



# Heavy-light tetraquark from single-parton fragmentation

Let us consider heavy-light  $X_{Qq\bar{Q}\bar{q}}$  tetraquarks at large  $p_T \rightarrow$  single-parton fragmentation !



[F. G. C., A. Papa, Phys. Lett. B 848 (2024) 138406]

S-wave

$$D_Q^X(z, \mu_0) = N \frac{z \times \Sigma_{\text{spin}} \Gamma \bar{\Gamma}}{(m_X^2 - 2m_Q^2 + 2p \cdot s')^2}$$

$$= N \frac{z \times \Sigma_{\text{spin}} \Gamma \bar{\Gamma}}{[m_X^2 - (m_Q^2 + \langle p_T^2 \rangle)(1 + z - \frac{1}{1-z})]^2}$$

**TQHL1.0** FFs:  $(Q \rightarrow X_{Qq\bar{Q}\bar{q}}) \otimes$  **APFEL++**  
 $[\mu_0 = m_X + m_Q]$

(LO) [S. M. Moosavi Nejad, Phys. Rev. D 05 (2022) 3, 034001]

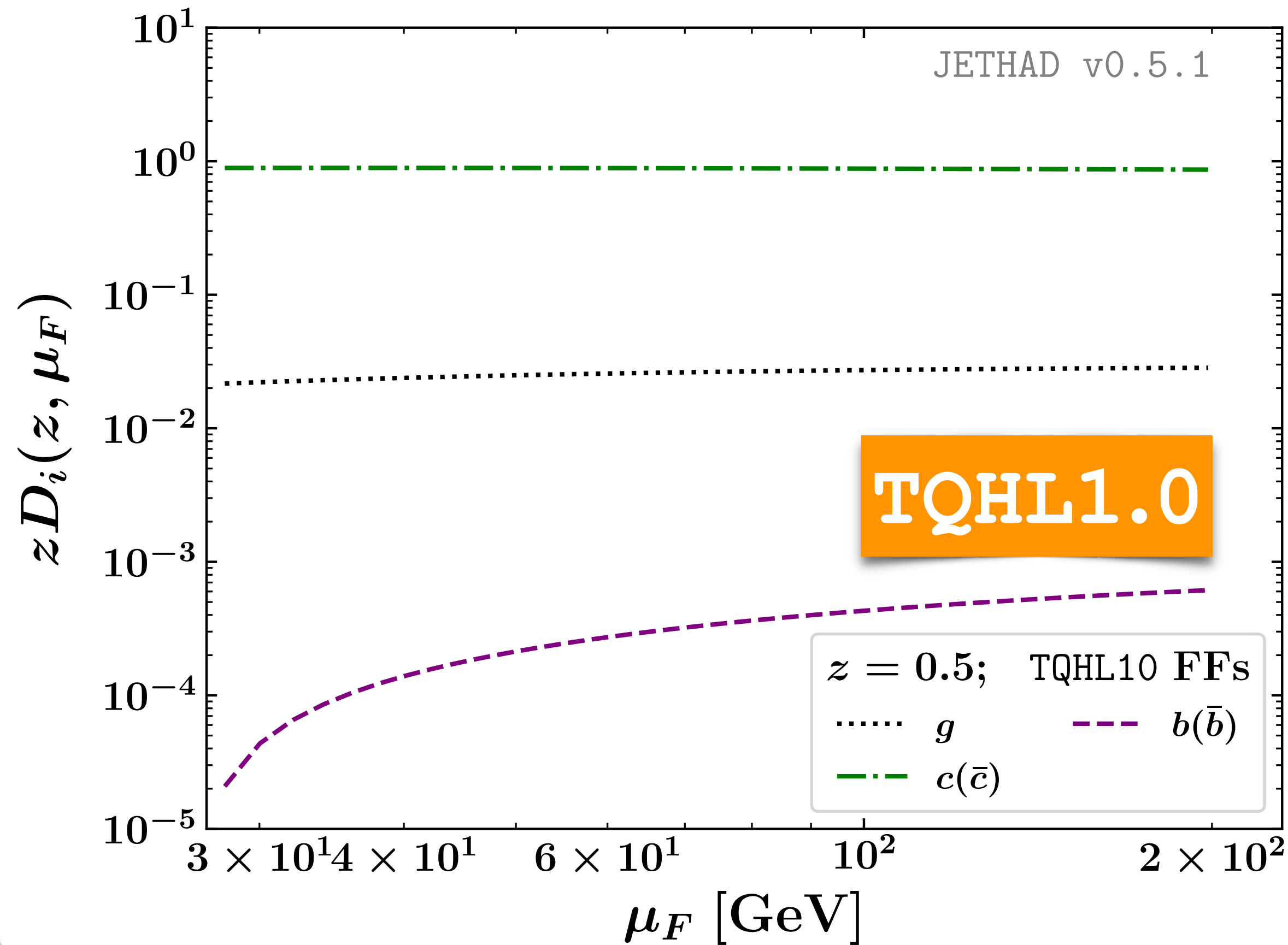
(framework) [M. Suzuki, Phys. Rev. D 33 (1986) 676]

# Heavy-light tetraquarks at the HL-LHC

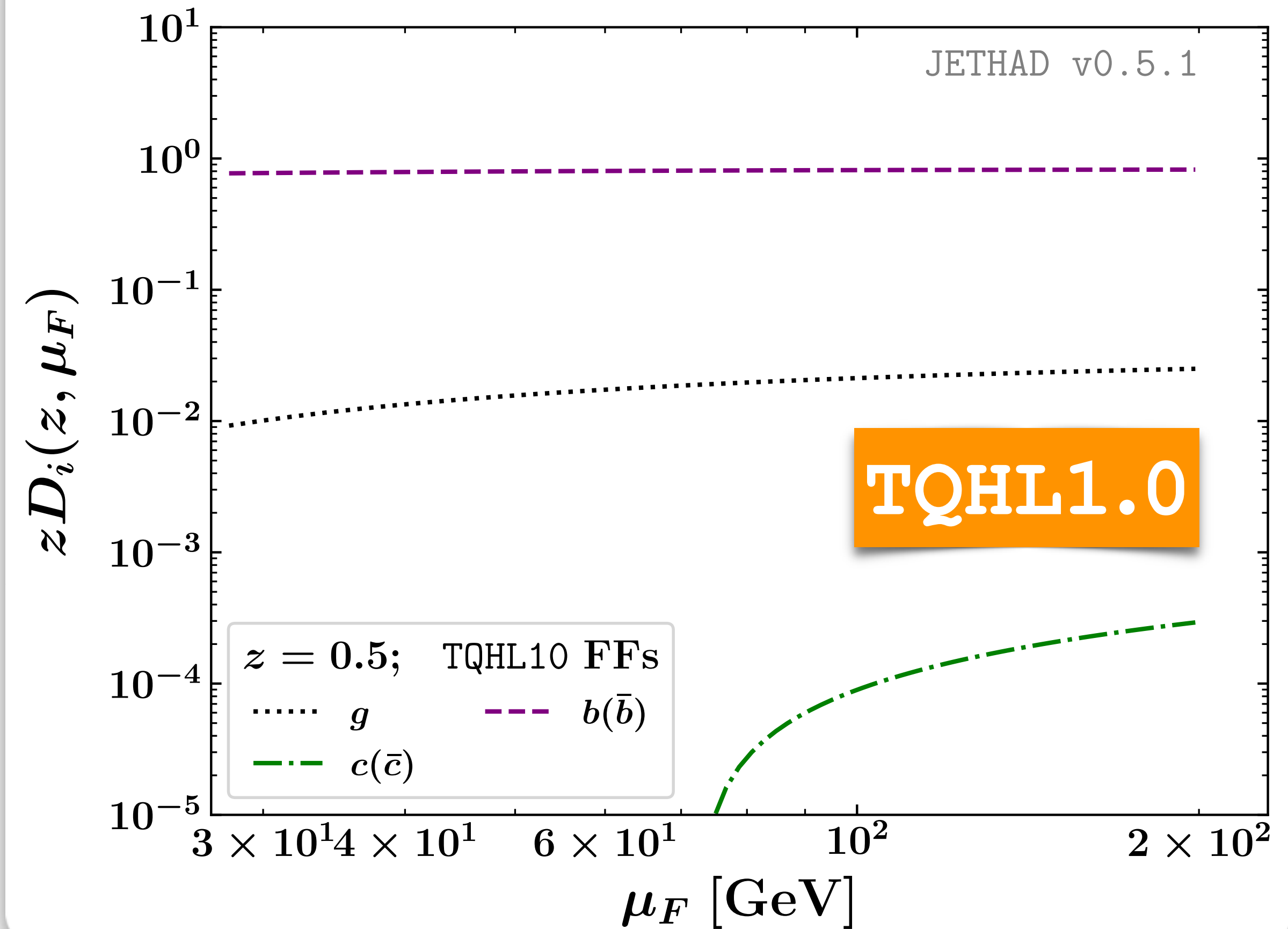
$X_{cu\bar{c}\bar{u}}$  collinear FFs

$X_{bs\bar{b}\bar{s}}$  collinear FFs

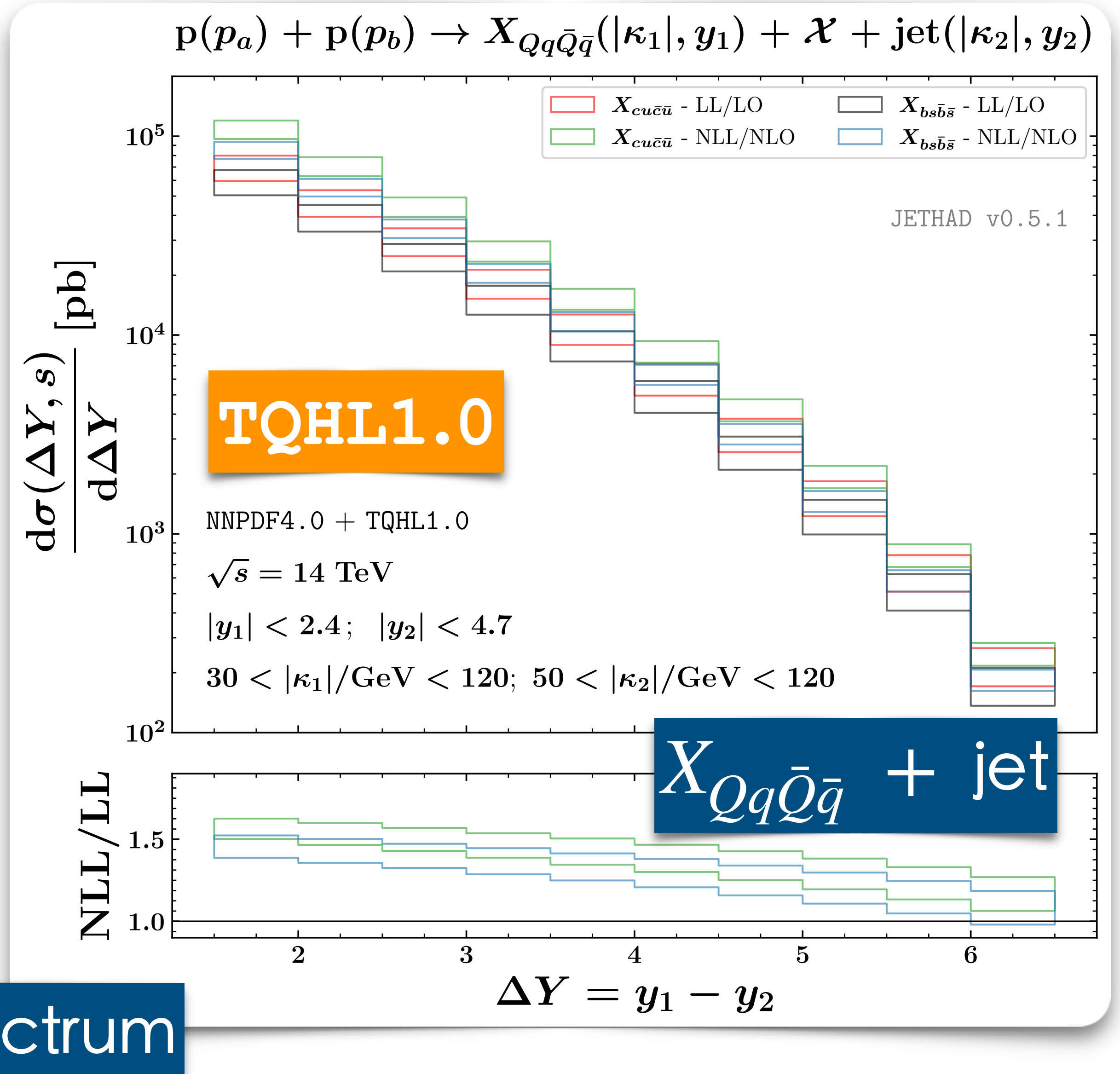
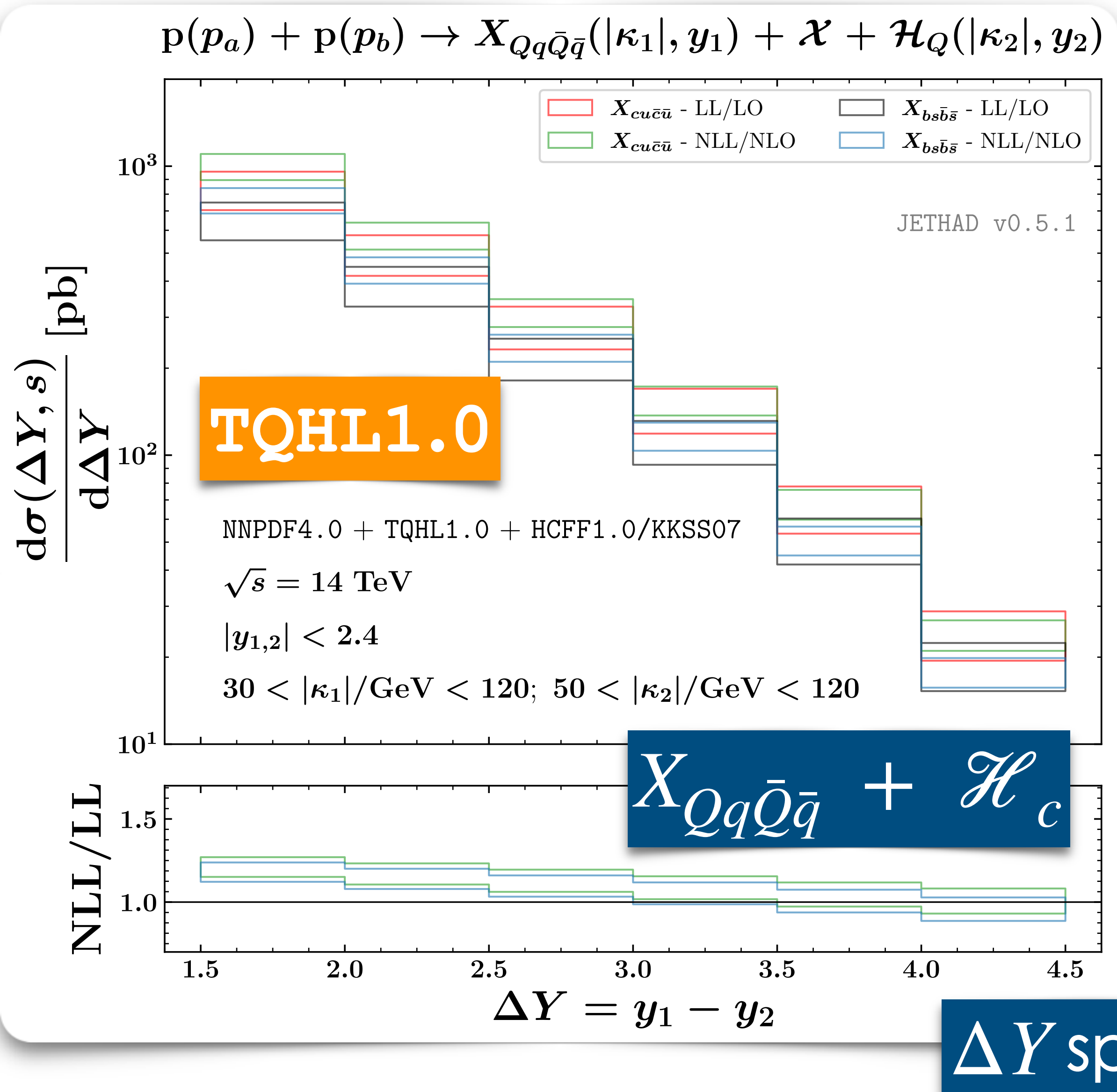
$X_{cu\bar{c}\bar{u}}$  tetraquark collinear FFs



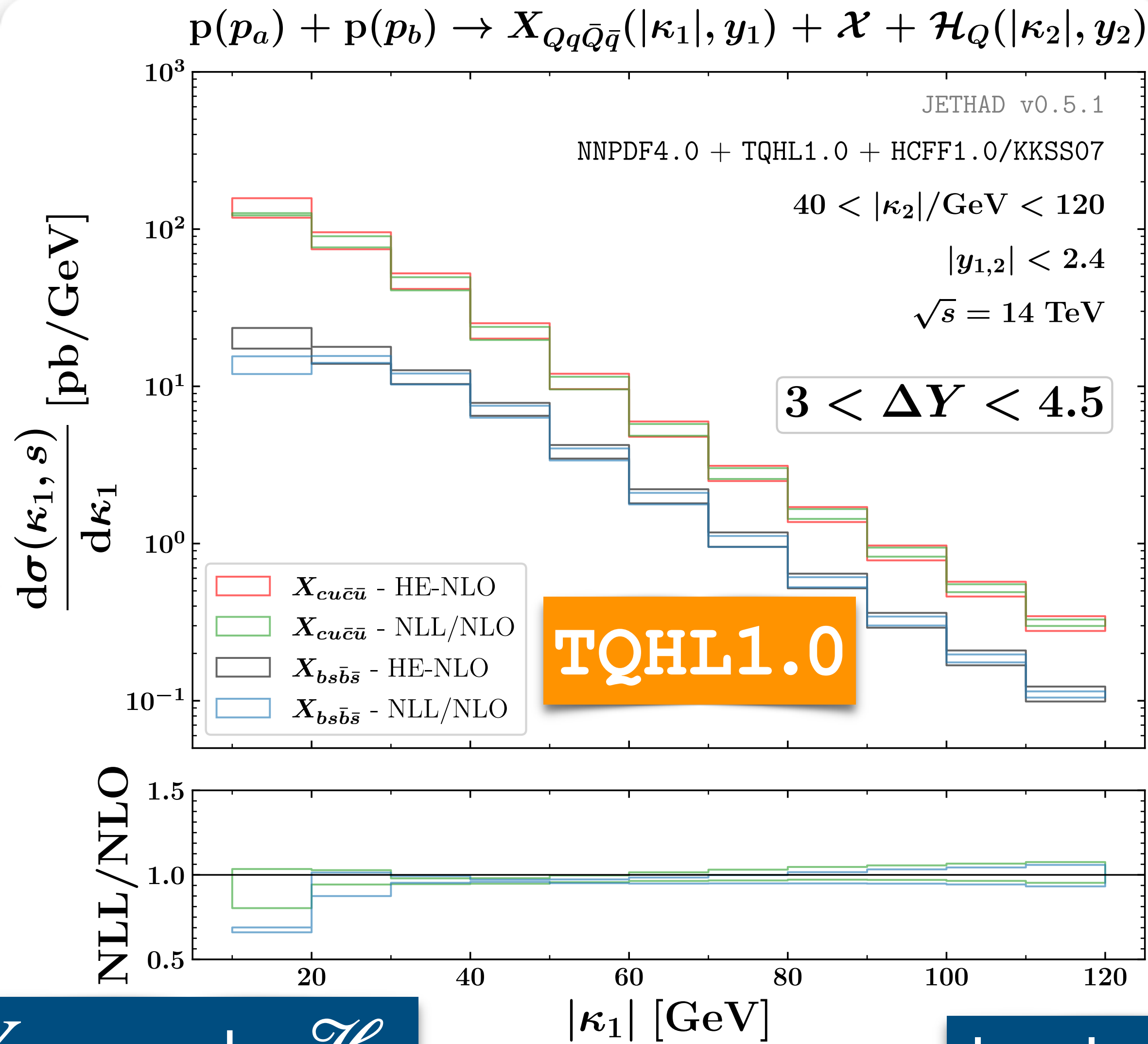
$X_{bs\bar{b}\bar{s}}$  tetraquark collinear FFs



# Heavy-light tetraquarks at the HL-LHC

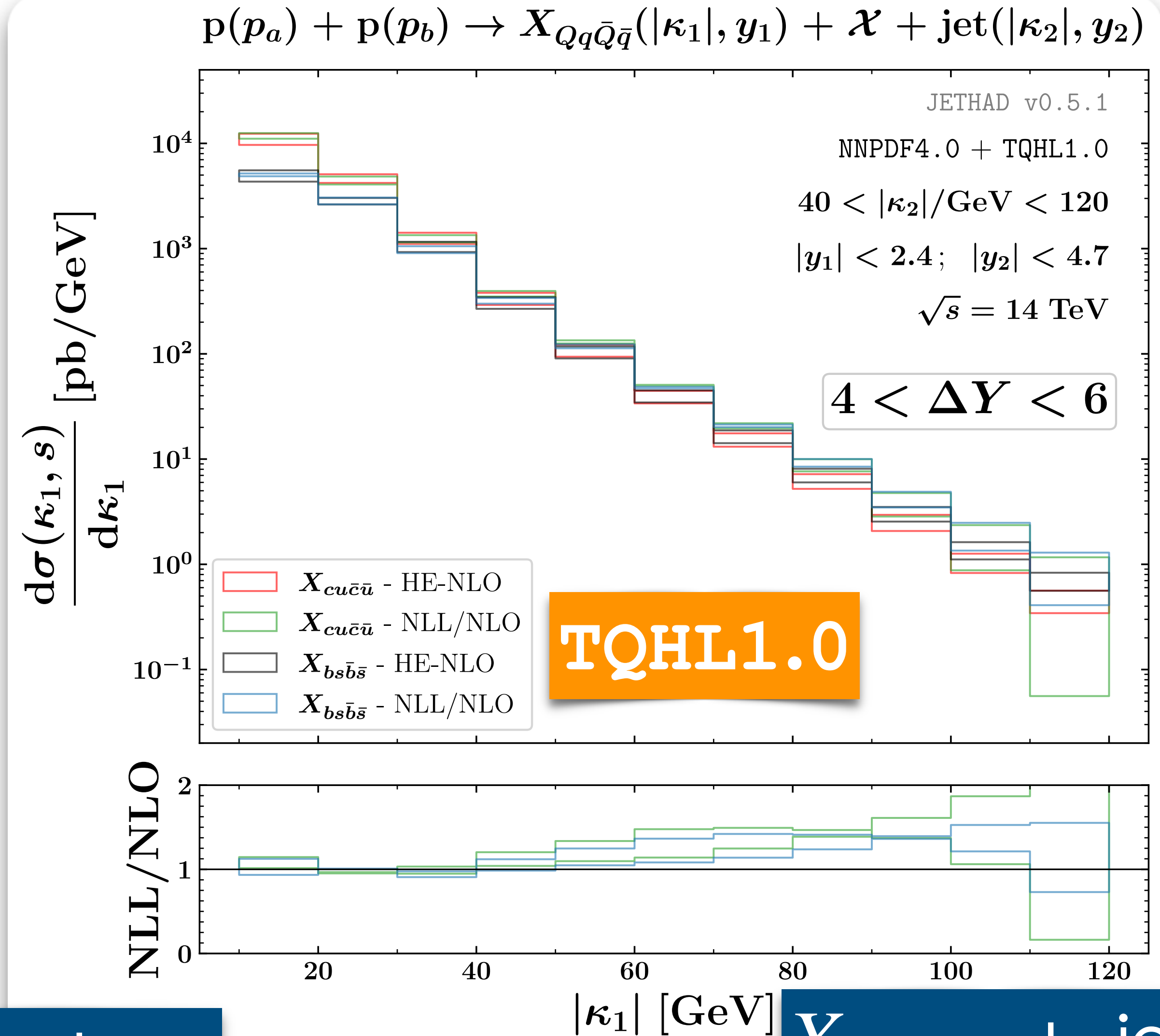


# Heavy-light tetraquarks at the HL-LHC



$X_{Qq\bar{Q}\bar{q}} + \mathcal{H}_c$

$|\kappa_1|$  spectrum



$X_{Qq\bar{Q}\bar{q}} + \text{jet}$

# Tetraquark fragmentation from a simpler HF-NRevo

Phys. Lett. B 848 (2024) 138406



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Letter

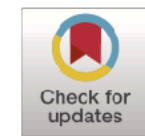
## A high-energy QCD portal to exotic matter: Heavy-light tetraquarks at the HL-LHC

Francesco Giovanni Celiberto<sup>a, ID, \*</sup>, Alessandro Papa<sup>b, c, ID</sup>

<sup>a</sup> Universidad de Alcalá (UAH), Departamento de Física y Matemáticas, Campus Universitario, Alcalá de Henares, E-28805, Madrid, Spain

<sup>b</sup> Dipartimento di Fisica, Università della Calabria, Ponte Pietro Bucci, Cubo 31C, Arcavacata di Rende, I-87036, Cosenza, Italy

<sup>c</sup> INFN, Gruppo Collegato di Cosenza, Ponte Pietro Bucci, Cubo 31C, Arcavacata di Rende, I-87036, Cosenza, Italy



### ARTICLE INFO

Editor: B. Grinstein

Dataset link: [https://github.com/FGCeliberto/Collinear\\_FF/](https://github.com/FGCeliberto/Collinear_FF/)

#### Keywords:

High-energy resummation  
HL-LHC phenomenology  
Heavy-light tetraquarks  
Hidden flavor  
Exotic matter

### ABSTRACT

By taking advantage of the natural stability of the high-energy resummation, recently discovered in the context of heavy-flavor studies, we investigate the inclusive hadroproduction of a neutral heavy-light, hidden-flavored tetraquark ( $X_{c\bar{c}u\bar{u}}$  or  $X_{b\bar{b}s\bar{s}}$  state), in association with a heavy (single  $c$ - or  $b$ -flavored) hadron or a light jet at the (HL-)LHC. We make use of the JETHAD multi-modular working package to provide predictions for rapidity, azimuthal-angle and transverse-momentum distributions calculated *via* the hybrid high-energy and collinear factorization, where the Balitsky–Fadin–Kuraev–Lipatov resummation of energy logarithms is supplemented by collinear parton densities and fragmentation functions. We rely upon the single-parton fragmentation mechanism, valid in the large transverse-momentum regime, to describe the tetraquark production. Our study represents a first attempt at bridging the gap between all-order calculations of high-energy QCD and the exotics.



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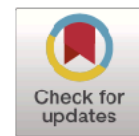


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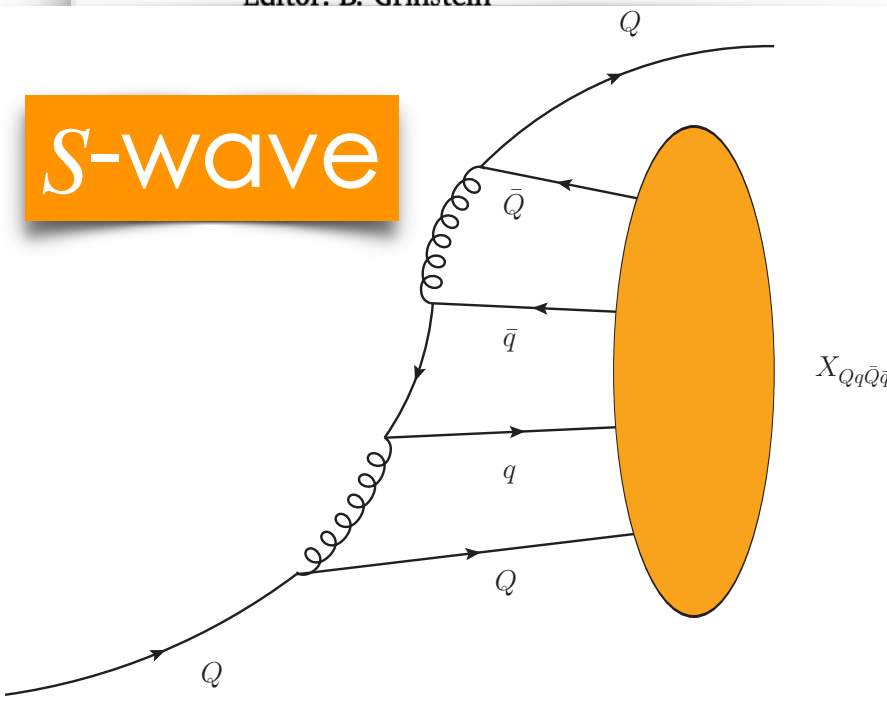
<sup>a</sup> Universidad de Alcalá (UAH), Departamento de Física y Matemáticas, Campus Universitario, Alcalá de Henares, E-28805, Madrid, Spain

<sup>b</sup> Dipartimento di Fisica, Università della Calabria, Ponte Pietro Bucci, Cubo 31C, Arcavacata di Rende, I-87036, Cosenza, Italy

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**TQHL1.0** FFs:  $(Q \rightarrow X_{QqQ\bar{q}}) \otimes$  **APFEL++**



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## A high-energy QCD portal to exotic matter: Heavy-light tetraquarks at HL-LHC

Francesco Giovanni Celiberto<sup>a, ID, \*</sup>, Alessandro Papa<sup>b, c, ID</sup>

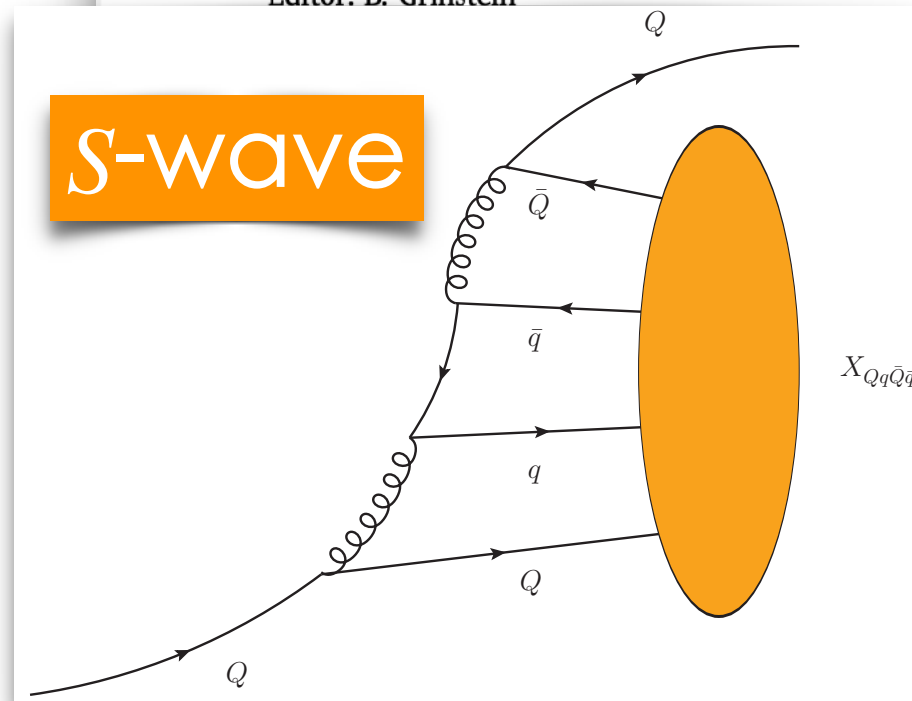
<sup>a</sup> Universidad de Alcalá (UAH), Departamento de Física y Matemáticas, Campus Universitario, Alcalá de Henares, E-28805, Madrid, Spain

<sup>b</sup> Dipartimento di Fisica, Università della Calabria, Ponte Pietro Bucci, Cubo 31C, Arcavacata di Rende, I-87036, Cosenza, Italy

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By taking advantage of the natural stability of the high-energy resummation, recent heavy-flavor studies, we investigate the inclusive hadroproduction of a neutral tetraquark ( $X_{c\bar{c}u\bar{u}}$  or  $X_{b\bar{b}s\bar{s}}$  state), in association with a heavy (single  $c$ - or  $b$ -flavor) quark, at the (HL-)LHC. We make use of the JETHAD multi-modular working package to provide the azimuthal-angle and transverse-momentum distributions calculated via the hybrid factorization, where the Balitsky–Fadin–Kuraev–Lipatov resummation of energy is implemented by collinear parton densities and fragmentation functions. We rely upon the hybrid factorization mechanism, valid in the large transverse-momentum regime, to describe the tetraquark production. This represents a first attempt at bridging the gap between all-order calculations of high-energy QCD.

## Fully charmed tetraquarks from LHC to FCC: Natural stability from fragmentation

Francesco Giovanni Celiberto<sup>1 † ID</sup>, Gabriele Gatto<sup>2,3 ‡</sup>, and Alessandro Papa<sup>2,3 § ID</sup>

<sup>1</sup> Universidad de Alcalá (UAH), E-28805 Alcalá de Henares, Madrid, Spain

<sup>2</sup> Dipartimento di Fisica, Università della Calabria, I-87036 Arcavacata di Rende, Cosenza, Italy

<sup>3</sup> Istituto Nazionale di Fisica Nucleare, Gruppo collegato di Cosenza, I-87036 Arcavacata di Rende, Cosenza, Italy

### Abstract

We study the inclusive detection of a fully charmed tetraquark,  $T_{4c}(0^{++})$  or its radial excitation  $T_{4c}(2^{++})$ , in association with a light jet in proton collisions. We build our study upon the collinear fragmentation of a single parton in a variable flavor number scheme, suited to describe the tetraquark formation mechanism at moderate to large transverse momenta. To this extent, we derive a novel set of DGLAP-evolving collinear FFs, named TQ4Q1.0 functions. They encode initial-scale inputs from both gluon and heavy-quark fragmentation channels, respectively defined in the context of quark potential and spin-physics inspired models. We work within the NLL/NLO<sup>+</sup> hybrid factorization and make use of the JETHAD numeric interface and the symJETHAD symbolic calculation plugin to provide predictions for observables sensitive to high-energy QCD dynamics at center-of-mass energies ranging from 14 TeV LHC to 100 TeV nominal FCC.

**TQHL1.0** FFs:  $(Q \rightarrow X_{QqQ\bar{q}}) \otimes$  **APFEL++**

# Tetraquark fragmentation from a simpler HF-NRevo

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## A high-energy QCD portal to exotic matter: Heavy-light tetraquarks at HL-LHC

Francesco Giovanni Celiberto<sup>a, ID, \*</sup>, Alessandro Papa<sup>b, c, ID</sup>

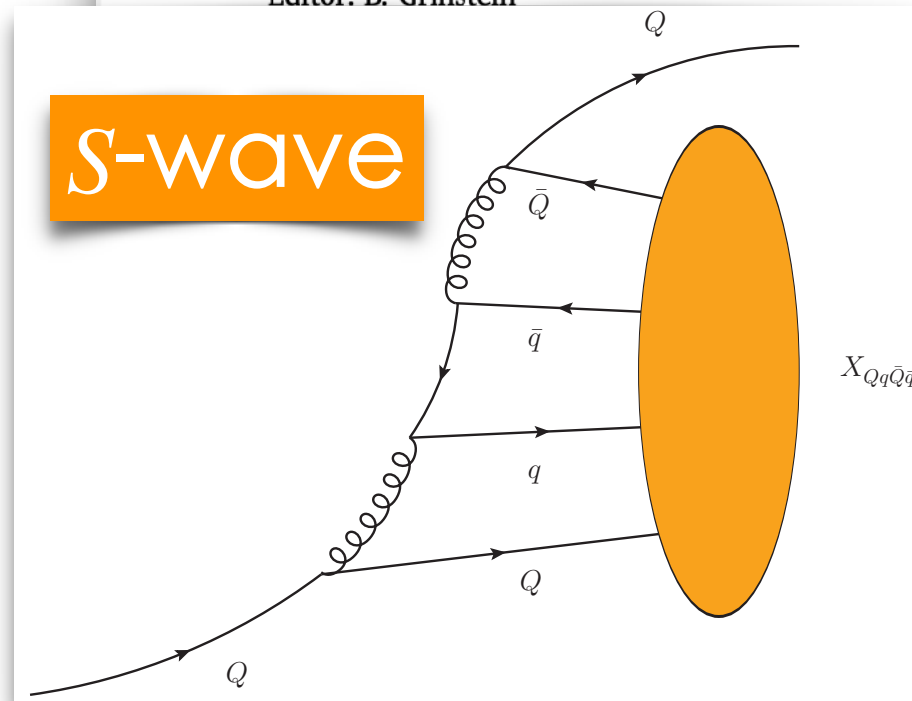
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### ABSTRACT

By taking advantage of the natural stability of the high-energy resummation, recent heavy-flavor studies, we investigate the inclusive hadroproduction of a neutral tetraquark ( $X_{c\bar{c}u\bar{u}}$  or  $X_{b\bar{b}s\bar{s}}$  state), in association with a heavy (single  $c$ - or  $b$ -flavor) jet at the (HL-)LHC. We make use of the JETHAD multi-modular working package to provide azimuthal-angle and transverse-momentum distributions calculated via the hybrid factorization, where the Balitsky–Fadin–Kuraev–Lipatov resummation of energy logarithms by collinear parton densities and fragmentation functions. We rely upon the hybrid factorization mechanism, valid in the large transverse-momentum regime, to describe the tetraquark formation mechanism. This work represents a first attempt at bridging the gap between all-order calculations of high-energy QCD and the current state-of-the-art fixed-order calculations.

**TQHL1.0** FFs:  $(Q \rightarrow X_{QqQ\bar{q}}) \otimes$  **APFEL++**

## Fully charmed tetraquarks from LHC to FCC: Natural stability from fragmentation

Francesco Giovanni Celiberto<sup>1 † ID</sup>, Gabriele Gatto<sup>2,3 ‡</sup>, and Alessandro Papa<sup>2,3 § ID</sup>

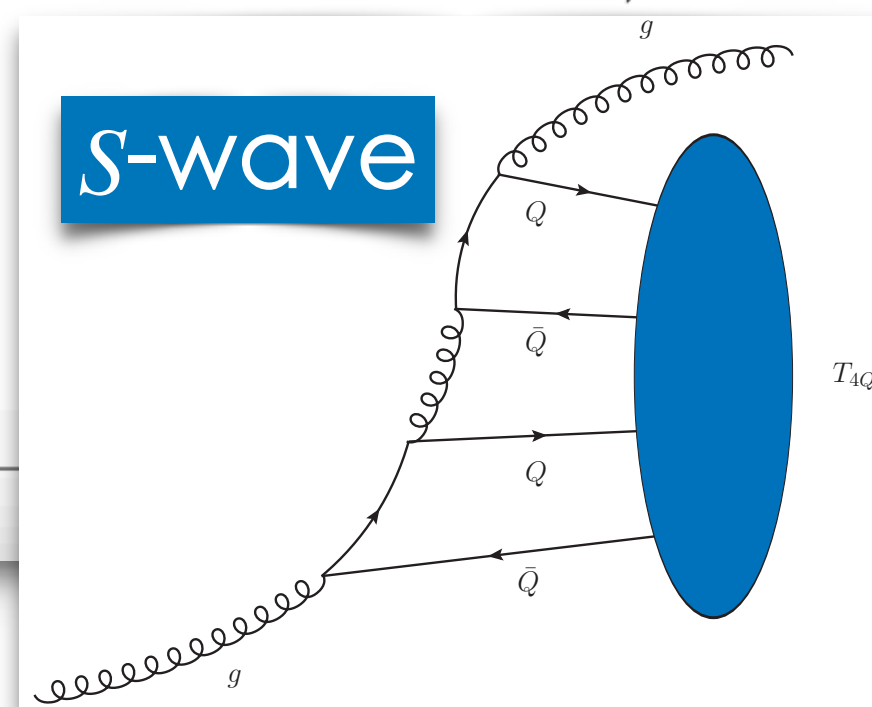
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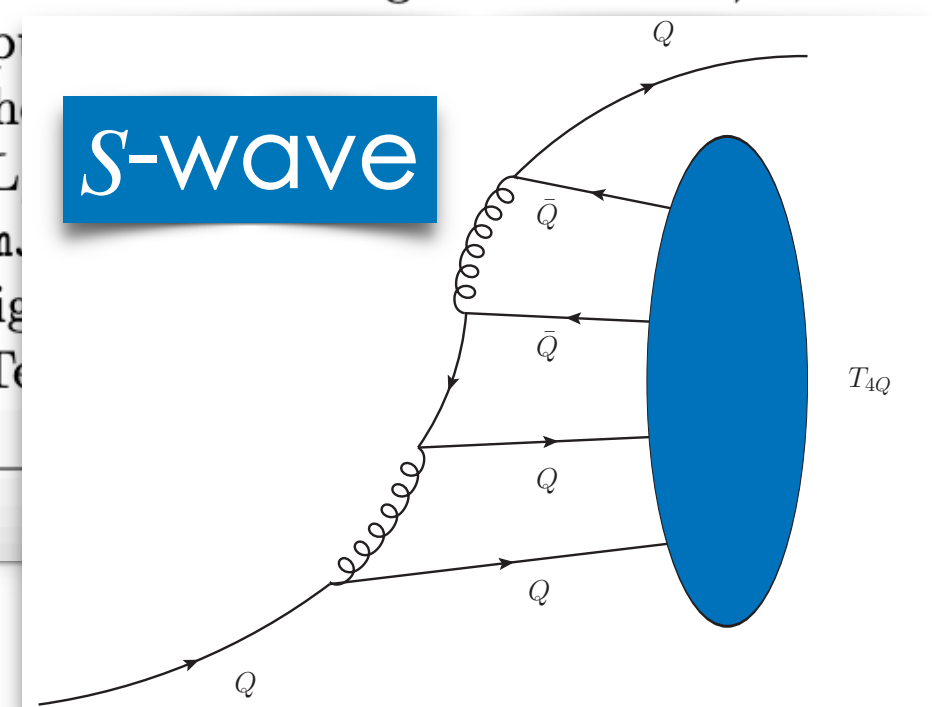
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### Abstract

We study the inclusive detection of a fully charmed tetraquark,  $T_{4c}(0^{++})$  or its radial excitation  $T_{4c}(2^{++})$ , in association with a light jet in proton collisions. We build our study upon the collinear fragmentation of a single parton in a variable flavor number scheme, suited to describe the tetraquark formation mechanism at moderate to large transverse momenta. To this extent, we derive a novel set of DGLAP-evolving collinear FFs, named

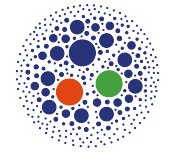


partial-scale invariants defined in the framework of the NLL evolution and the symmetry breaking sensitive to high-energy QCD from LHC to 100 TeV.



**TQ4Q1.0** FFs:  $(Q + g \rightarrow X_{QqQ\bar{q}}) \otimes$  **APFEL++**

# A high-energy QCD portal to exotic matter

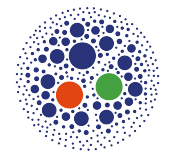


High-energy QCD **precision era** is at the **energy frontier(s)** of **new-generation** colliders

*; **Key ingredient**: interplay between **resummation(s)** & **production mechanisms** !*



# A high-energy QCD portal to exotic matter



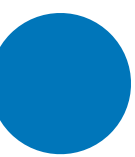
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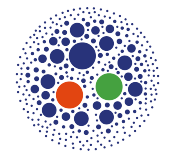
**High-energy  
physics**



**Hadronic  
structure**



# A high-energy QCD portal to exotic matter



High-energy QCD **precision era** is at the **energy frontier(s)** of **new-generation** colliders

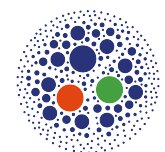
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**High-energy  
physics**



**Hadronic  
structure**

**⚠ Might high-energy QCD give us access to exotic matter ?**



Need for fragmentation mechanisms depicting **exotic** atoms / molecules / hadrons



[https://github.com/FGCeliberto/Collinear\\_FFs/](https://github.com/FGCeliberto/Collinear_FFs/)

