QCD and PDFs

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ERC adG YoctoLHC

These lectures will …

- explain main theoretical and experimental results leading to development of quantum chromodynamics (QCD) and outline its concepts
- teach you how to calculate scaling violations for parton distribution functions (PDFs)
- give a taste of rich phenomenology of PDFs

Plan of lectures:

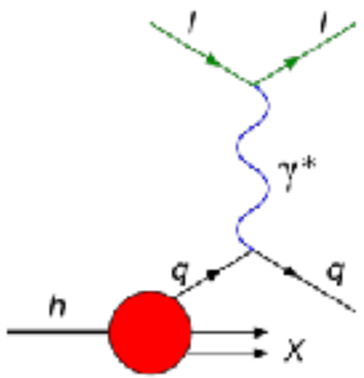
- **Lecture 1**: The quark model, deep inelastic scattering (DIS), the parton model, main concepts of quantum chromodynamics (QCD)
- **Lecture 2**: Scaling violations in QCD, DGLAP evolution equations, factorization theorem
- **Lecture 3**: Phenomenology of proton, nucleus and photon PDFs

Literature:

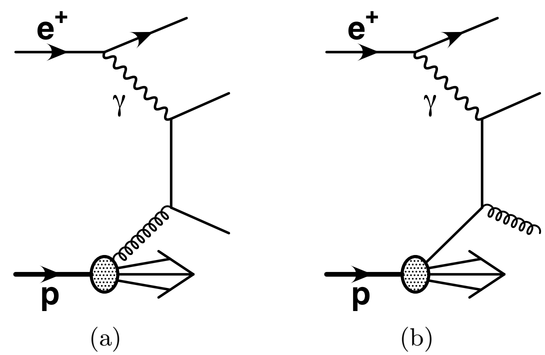
- **Lecture 1**: Halzen, Martin, Quarks and Leptons: An Introductory Course in Modern Particle Physics (1984); Kronfeld. Quigg, “Resource Letter: Quantum Chromodynamics”, arXiv:1002.5032 [hep-ph]; Gross, Klempt et al. “50 Years of Quantum Chromodynamics”, Eur. Phys. J C (2023) 1125
- **Lecture 2**: Dokshitzer, Diakonov, Troian, “Hard Processes in Quantum Chromodynamics”, Phys. Rept. 58 (1980) 269; Sterman et al., “Handbook of perturbative QCD”, Rev. Mod. Phys. 67 (1995) 157-248
- **Lecture 3**: Aschenauer, Thorne, Yoshida, “Structure functions”, Review of Particle Physics, Particle Data Group; Nisius, Phys. Rept. 332 (2000) 165-317 [arXiv:hep-ex/9912049].

Collinear factorization in QCD (1/5)

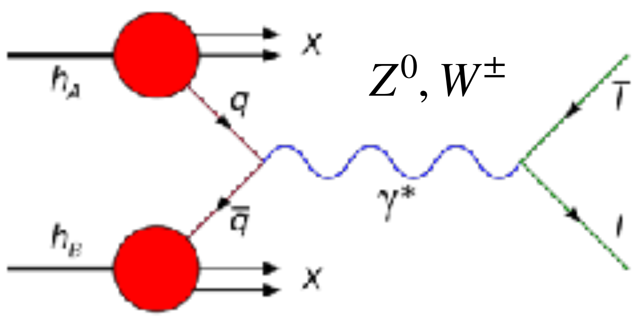
• **Collinear factorization** in perturbative QCD has been proven for many hard (large scale) processes in lepton-hadron (Jefferson Lab, HERA, EIC, FCC-eh) and hadron-hadron (Tevatron, RHIC, LHC) scattering



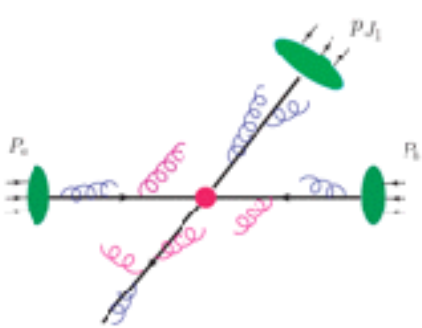
Inclusive DIS



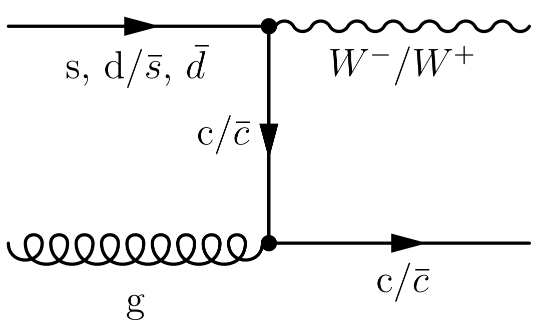
Jet electroproduction



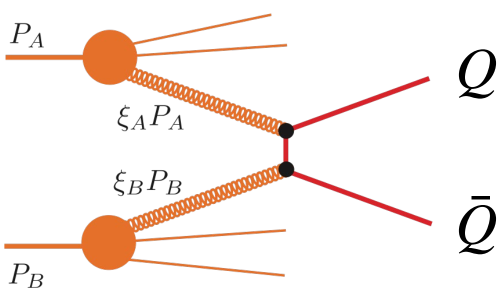
Drell-Yan process



Dijet hadroproduction in $pp \to jet + jet + X$



Gauge boson hadroproduction in $pp \to W + c + X$



Heavy quark production in $pp \to b\bar{b}(t\bar{t}) + X$

Collinear factorization in QCD (2/5)

- Structure functions and cross sections are convolutions of PDFs with the coefficient functions or partonic cross sections.

- DIS: $F_2(x,Q^2) = \sum_{i=q,\bar{q},g} \int_x^1 d\xi C_i\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) f_i(\xi,\mu^2)$
- Hadroproduction: $d\sigma(pp \to \ldots) = \sum_{i=q,\bar{q},g} \int d\xi_A \int d\xi_B\, f_{A/p}(\xi_A,\mu^2) f_{B/p}(\xi_B,\mu^2)\, d\hat{\sigma}_{AB\to\ldots}$

- The coefficient functions are process-specific and can be calculated order-by-order in perturbative QCD.

- Parton distribution functions (PDFs) are non-perturbative quantities:

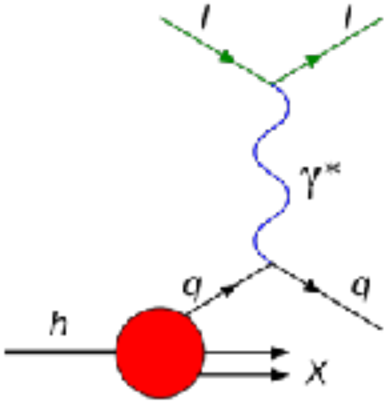
$$f_q(x,Q^2) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{-ixz^-p^+} \langle p\,|\,\bar{\psi}(x)\gamma^+\psi(0)\,|\,p\rangle_{z^+=\mathbf{z}_\perp=0}$$

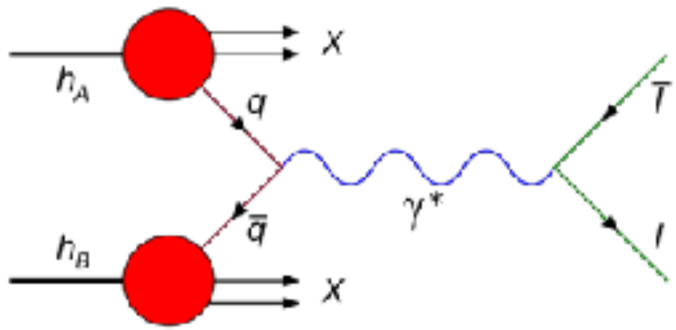
- → cannot be calculated from first principles (except for several first Mellin momenta calculated in lattice QCD, no access to interesting small-x region)

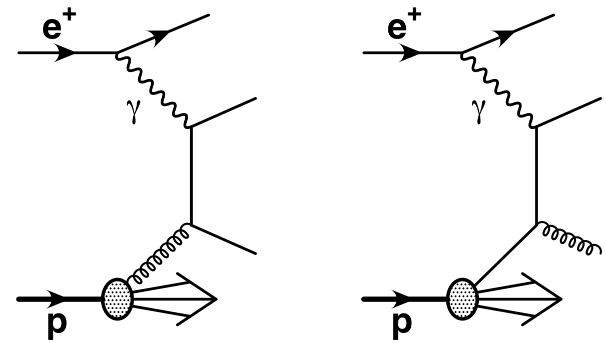
- can only be extracted from data taking advantage of universality of PDFs.

Collinear factorization in QCD (3/5)

- Different processes access different combinations of PDFs.







(a) (b)

Inclusive DIS: probes $q + \bar{q}$, gluons are via Q^2 scaling violations of $F_2(x, Q^2)$ + from longitudinal sf $F_L(x, Q^2)$

Drell-Yan process: probes $\bar{q}$

Jet production: probes both quarks and gluons at the same order of pQCD → sensitivity to gluons

- Neutral current and charged current (neutrino) DIS access different combinations of quarks → can be used for flavor-separation of quark PDFs.

Collinear factorization in QCD (4/5)

- Different processes access different combinations of PDFs.
- Scattering with fixed targets and in collider mode → different regions of x.

Process	Subprocess	Partons	x range
$\ell^\pm\,\{p,n\} \to \ell^\pm\,X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
$\ell^\pm\,n/p \to \ell^\pm\,X$	$\gamma^*\,d/u \to d/u$	d/u	$x \gtrsim 0.01$
$pp \to \mu^+\mu^-\,X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \to \mu^+\mu^-\,X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu})\,N \to \mu^-(\mu^+)\,X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu\,N \to \mu^-\mu^+\,X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu}\,N \to \mu^+\mu^-\,X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^\pm\,p \to e^\pm\,X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$10^{-4} \lesssim x \lesssim 0.1$
$e^+\,p \to \bar{\nu}\,X$	$W^+\,\{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
$e^\pm p \to e^\pm\,c\bar{c}X, e^\pm\,b\bar{b}X$	$\gamma^* c \to c,\ \gamma^* g \to c\bar{c}$	c, b, g	$10^{-4} \lesssim x \lesssim 0.01$
$e^\pm p \to$ jet$+X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p}, pp \to$ jet(dijet)$+X$	$gg, qg, qq \to 2j$	g, q	$0.00005 \lesssim x \lesssim 0.5$
$p\bar{p} \to (W^\pm \to \ell^\pm\nu)\,X$	$ud \to W^+, \bar{u}\bar{d} \to W^-$	$u, d, s, \bar{u}, \bar{d}, \bar{s}$	$x \gtrsim 0.05$
$pp \to (W^\pm \to \ell^\pm\nu)\,X$	$u\bar{d} \to W^+, d\bar{u} \to W^-$	$u, d, s, \bar{u}, \bar{d}, \bar{s}, g$	$x \gtrsim 0.001$
$p\bar{p}(pp) \to (Z \to \ell^+\ell^-)X$	$uu, dd, ..(u\bar{u}, ..) \to Z$	$u, d, s, ..(g)$	$x \gtrsim 0.001$
$pp \to W^- c,\ W^+\bar{c}$	$gs \to W^- c$	$s, \bar{s}$	$x \sim 0.01$
$pp \to (\gamma^* \to \ell^+\ell^-)X$	$u\bar{u}, d\bar{d}, .. \to \gamma^*$	$\bar{q}, g$	$x \gtrsim 10^{-5}$
$pp \to (\gamma^* \to \ell^+\ell^-)X$	$u\gamma, d\gamma, .. \to \gamma^*$	γ	$x \gtrsim 10^{-2}$
$pp \to b\bar{b}\,X,\ \ t\bar{t}X$	$gg \to b\bar{b},\ \ t\bar{t}$	g	$x \gtrsim 10^{-5}, 10^{-2}$
$pp \to t(\bar{t})\,X,$	$bu(\bar{b}d) \to td(\bar{t}u)$	$b, d/u$	$x \gtrsim 10^{-2}$
$pp \to$ exclusive $J/\psi,\ \Upsilon$	$\gamma^*(gg) \to J/\psi,\ \Upsilon$	g	$x \gtrsim 10^{-5}, 10^{-4}$
$pp \to \gamma\,X$	$gq \to \gamma q,\ g\bar{q} \to \gamma\bar{q}$	g	$x \gtrsim 0.005$

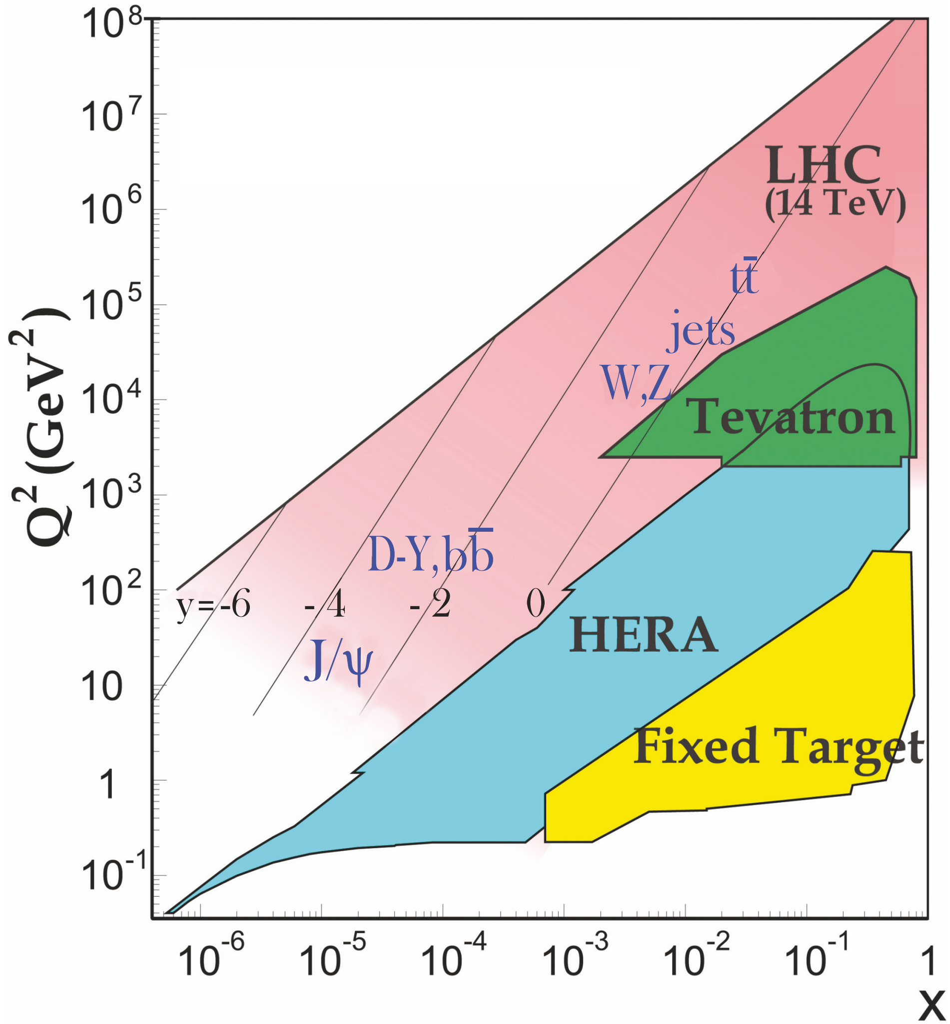
Fixed targets

$e^\pm p$ HERA

$p\bar{p}$ at Tevatron and pp at LHC

Collinear factorization in QCD (5/5)

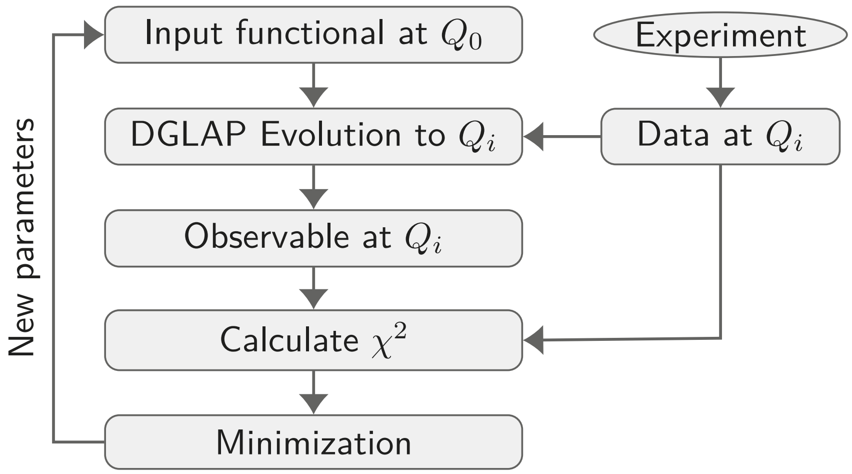
- These processes cover wide region on (x, Q^2) plane → sensitive to different combinations of PDFs: valence quarks at low Q_0^2 and sea quarks and gluons at large Q^2 → DGLAP Q^2 evolution connects low Q_0^2 and Q^2 regions.



Global analysis of proton PDFs (1/2)

- Parton distributions (PDFs) are determined from statistical fitting of the available data → called global QCD fits.
- State-of-the-art is NNLO accuracy → ongoing work toward N³LO.
- Assume a form of PDFs at input scale $Q_0 \approx 1-2$ GeV: $xf_i(x, Q_0^2) = x^{a_i}(1-x)^{b_i}F(c_i, d_i, \ldots)$, where $a_i, b_i, \ldots$ are free parameters (typically, 14-32 free parameters).
- Use DGLAP evolution equation to calculate $xf_i(x, Q^2 > Q_0^2)$ at Q^2 of the experiment.
- Using the evolved $xf_i(x, Q^2)$, calculate observables, e.g., the structure function $F_2(x, Q^2)$, the Drell-Yan and dijet cross section, …
- Compare to the data and find the free parameters by minimizing the χ^2 function:

$$\chi^2 = \sum_{i,j} (D_i - T_i)(C^{-1})_{ij}(D_j - T_j)$$

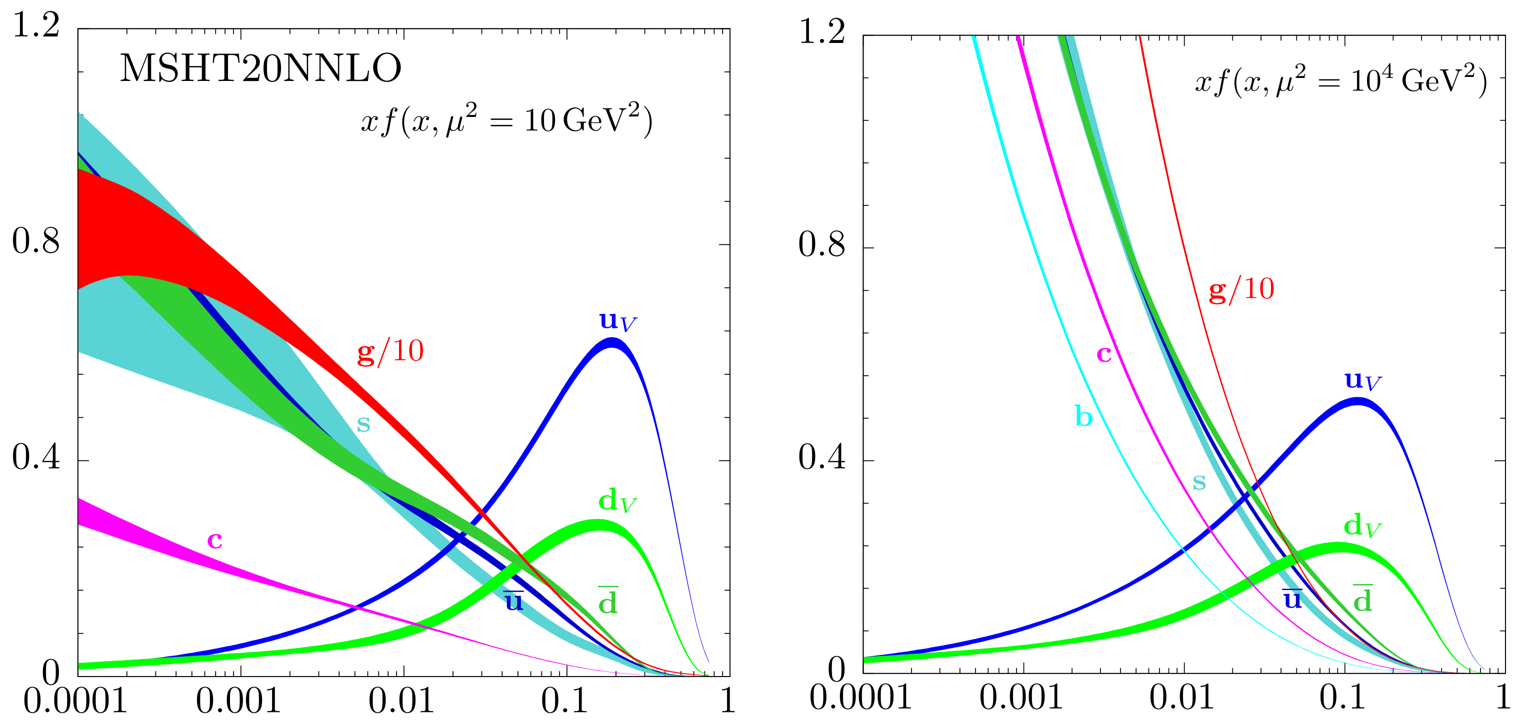


Global analysis of proton PDFs (2/2)

- Example: MSHT20 PDFs fitting ~5000 data points with $\chi^2/N_{\text{points}} \approx 1.2$, Bailey, Cridge, Harland-Lang, Martin, Thorn, Eur. Phys. J. C 81 (2021) 4, 341.

- The uncertainty bands from error PDFs using the Hessian method.

- Uncertainties decrease as Q^2 increases → consequence of DGLAP evolution from large x to low x due parton splitting.



- Alternative fitting strategy to avoid input bias → use neural networks (NNPDFs).

Nuclear parton distributions (1/4)

- Similarly to proton case, global QCD fits for nuclear PDFs, Klasen, Paukkunen, arXiv:2311.00450 [hep-ph]

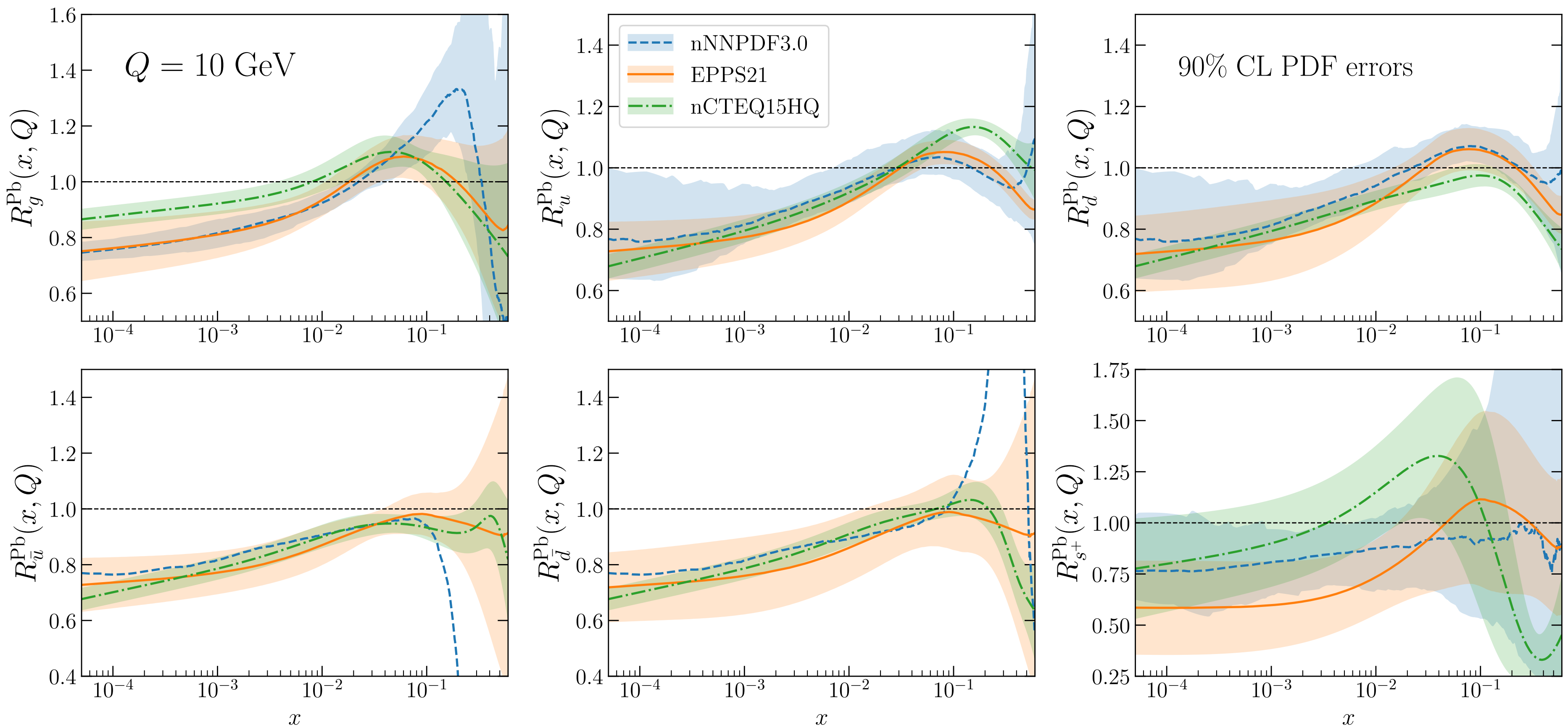
Table 1: Key features of recent global analyses of nuclear PDFs.

ANALYSIS	nCTEQ15HQ (50)	EPPS21 (51)	nNNPDF3.0 (52)	TUJU21 (78)	KSASG20 (79)
THEORETICAL INPUT:					
Perturbative order	NLO	NLO	NLO	NNLO	NNLO
Heavy-quark scheme	SACOT$-\chi$	SACOT$-\chi$	FONLL	FONLL	FONLL
Value of $\alpha_s(M_Z)$	0.118	0.118	0.118	0.118	0.118
Charm mass m_c	1.3 GeV	1.3 GeV	1.51 GeV	1.43 GeV	1.3 GeV
Bottom mass m_b	4.5 GeV	4.75 GeV	4.92 GeV	4.5 GeV	4.75 GeV
Input scale Q_0	1.3 GeV	1.3 GeV	1.0 GeV	1.3 GeV	1.3 GeV
Data points	1484	2077	2188	2410	4353
Independent flavors	5	6	6	4	3
Parameterization	Analytic	Analytic	Neural network	Analytic	Analytic
Free parameters	19	24	256	16	18
Error analysis	Hessian	Hessian	Monte Carlo	Hessian	Hessian
Tolerance	$\Delta\chi^2 = 35$	$\Delta\chi^2 = 33$	N/A	$\Delta\chi^2 = 50$	$\Delta\chi^2 = 20$
Proton PDF	~CTEQ6.1	CT18A	~NNPDF4.0	~HERAPDF2.0	CT18
Proton PDF correlations		✓	✓		
Deuteron corrections	(✓)[a,b]	✓[c]	✓	✓	✓
FIXED-TARGET DATA:					
SLAC/EMC/NMC NC DIS	✓	✓	✓	✓	✓
– Cut on Q^2	4 GeV^2	1.69 GeV^2	3.5 GeV^2	3.5 GeV^2	1.2 GeV^2
– Cut on W^2	12.25 GeV^2	3.24 GeV^2	12.5 GeV^2	12.0 GeV^2	
JLab NC DIS	(✓)[a]	✓			✓
CHORUS/CDHSW CC DIS	(✓/-)[b]	✓/-	✓/-	✓/✓	✓/✓
NuTeV/CCFR 2μ CC DIS	(✓/✓)[b]		✓/-		
pA DY	✓	✓	✓		✓
πA DY		✓			
COLLIDER DATA:					
Z bosons	✓	✓	✓	✓	
$W^\pm$ bosons	✓	✓	✓	✓	
Light hadrons	✓	✓[d]			
– Cut on p_T	3 GeV	3 GeV			
Jets		✓	✓		
Prompt photons			✓		
Prompt D^0	✓	✓	✓[e]		
– Cut on p_T	3 GeV	3 GeV	0 GeV		
Quarkonia (J/ψ, ψ', Υ)	✓				

[a] nCTEQ15HIX (26); [b] nCTEQ15ν (112); [c] through CT18A; [d] only π^0 in DAu; [e] only forward ($y > 0$).

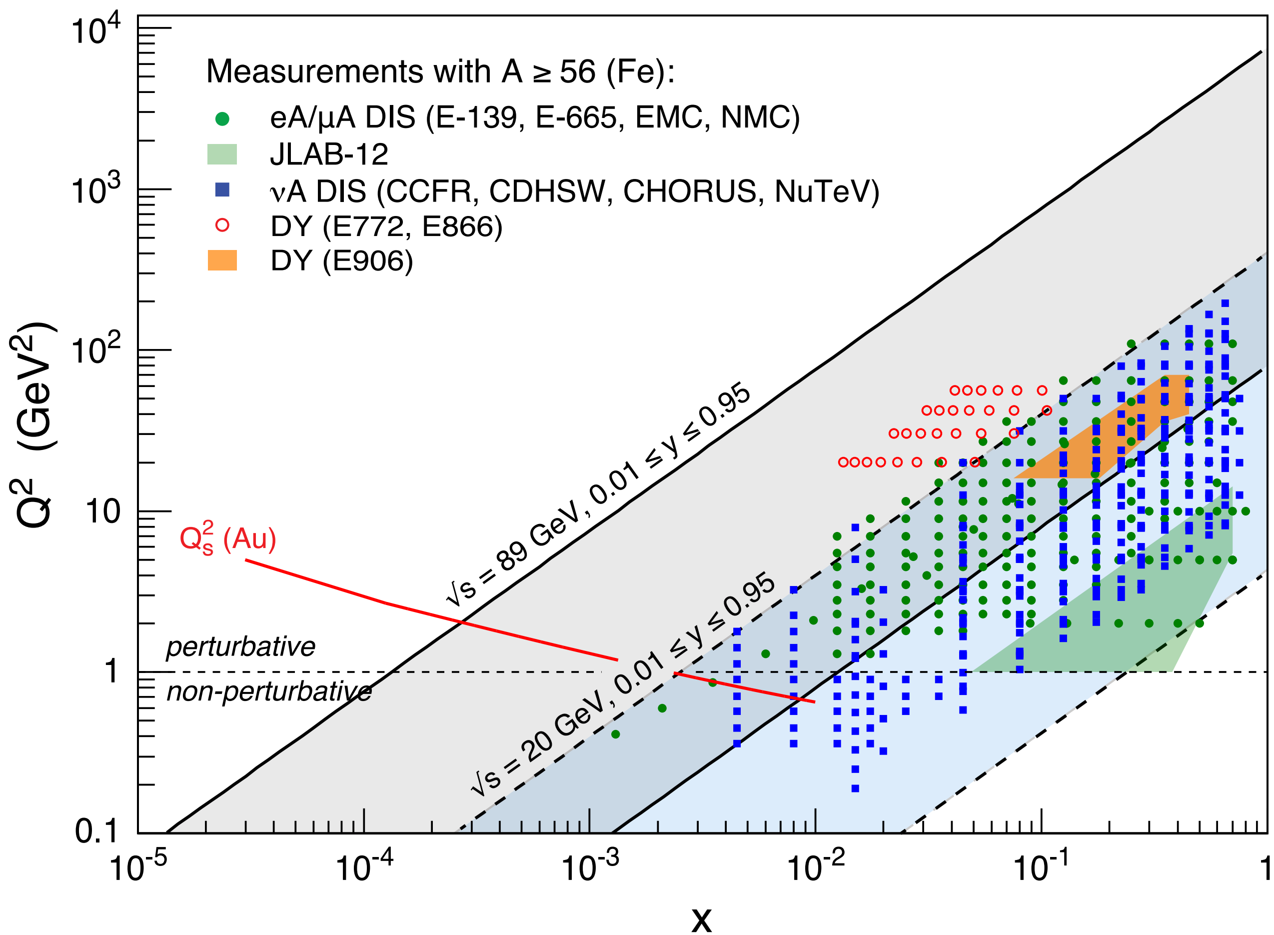
Nuclear parton distributions (2/4)

- While $\sqrt{Q^2}$ >> nuclear binding energy, $f_{i/A}(x, Q^2) \neq Z f_{i/p}(x, Q^2) + (A - Z) f_{i/n}(x, Q^2)$
- Nuclear modification factor: $R_i^A(x, Q^2) = \dfrac{f_{i/A}(x, Q^2)}{Z f_{i/p}(x, Q^2) + (A - Z) f_{i/n}(x, Q^2)}$
- Nuclear shadowing $(x < 0.05)$, nuclear anti-shadowing $(x \approx 0.1)$, EMC effect $(0.2 < x < 0.7)$, Fermi motion $(x > 0.7)$ $\rightarrow$ there are also quarks with $x_A > 1$.



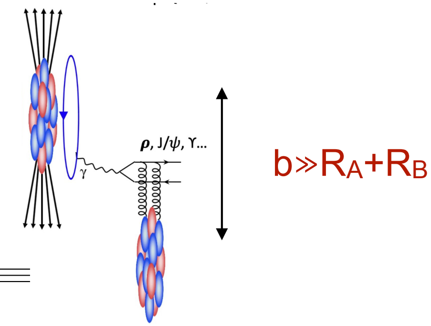
Nuclear parton distributions (3/4)

- How can one better determine nuclear PDFs?
- The planned Electron-Ion Collider at Brookhaven National Lab in USA:
  - wide $x - Q^2$ coverage
  - measurements of longitudinal $F_L^A(x, Q^2)$ directly sensitive to nuclear gluons
  - first ever measurement of nuclear diffractive structure functions



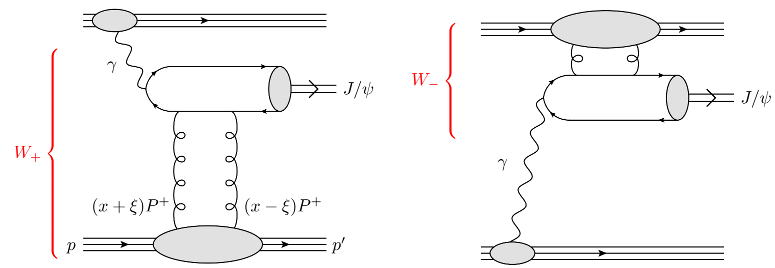
Nuclear parton distributions (4/4)

- Nuclear PDFs can be constrained in ultraperipheral collisions (UPCs) of heavy ions at LHC and RHIC.



- Coherent J/ψ production probes the nuclear gluon distribution:

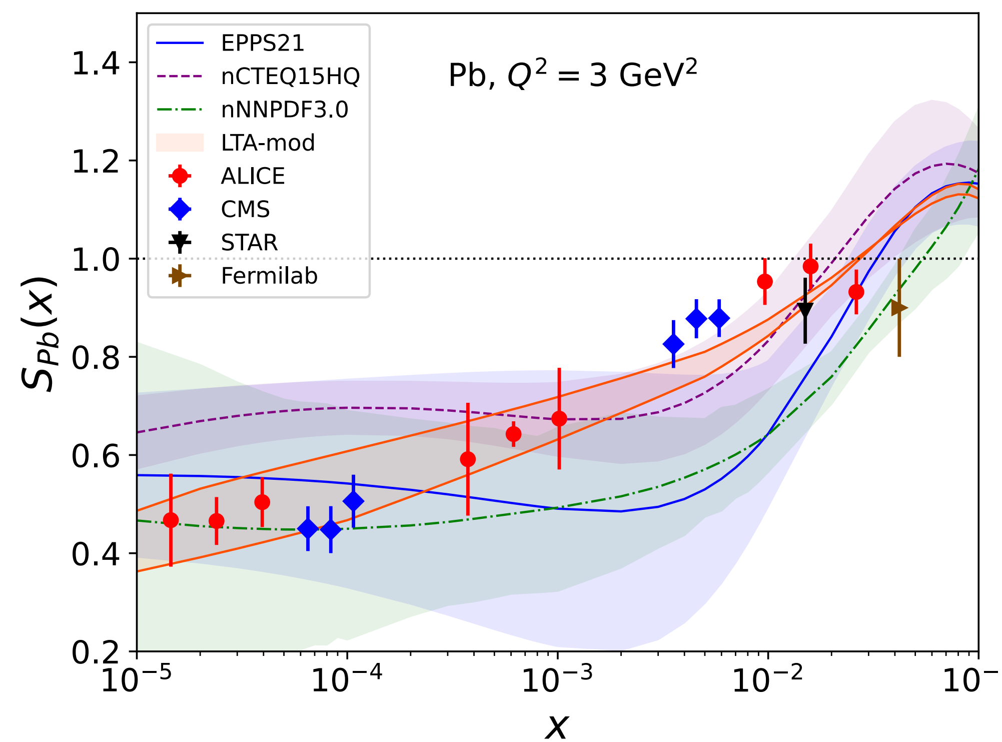
$$\sigma^{\gamma+A\to J/\psi+A}/dt \propto [xg_A(x,\mu^2)]^2$$



- Nuclear suppression factor

$$S_{Pb}(x) = [\sigma^{\gamma A\to J/\psi A}/\sigma_{IA}^{\gamma A\to J/\psi A}]^{1/2} = g_A(x,\mu^2)/[Ag_p(x,\mu^2)]$$

from UPC data → direct measure of nuclear gluon distribution at small x.

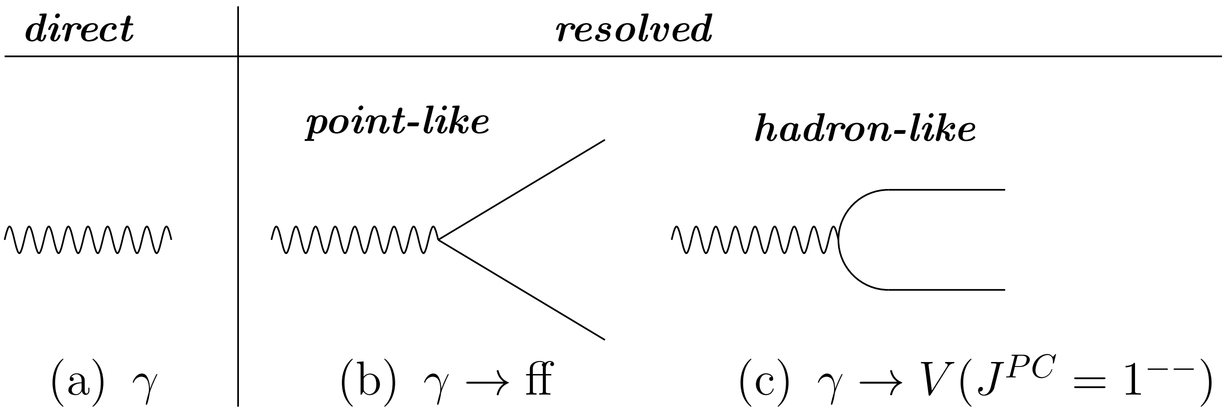


- Direct evidence of large gluon shadowing down to $x \approx 10^{-5}$!

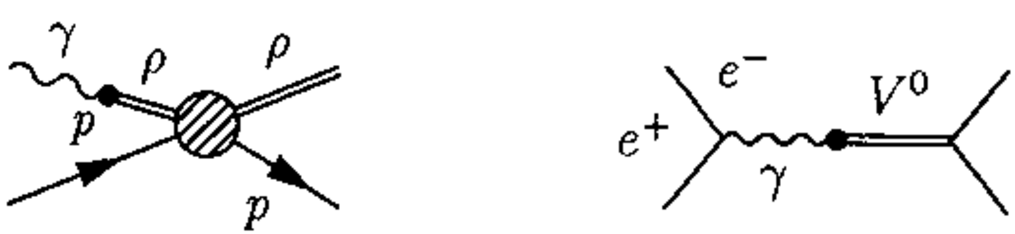
Photon PDFs (1/7)

- In QCD, the photon plays a dual role:
  - interacts directly with charged particles
  - interacts through fluctuations into $q\bar{q}$ pairs and vector mesons:

$$|\gamma\rangle = |\gamma\rangle_{\text{bare}} + \text{coeff}\,|q\bar{q}\rangle + g_\rho\,|\rho\rangle + \ldots$$

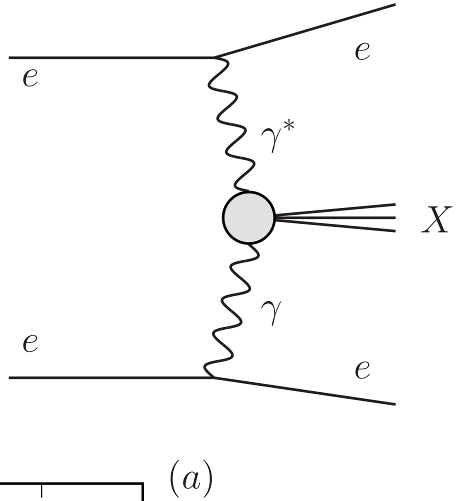


- Hadronic fluctuations in the form of vector mesons → vector meson dominance (VMD) model confirmed in γp scattering and e^+e^- annihilation:

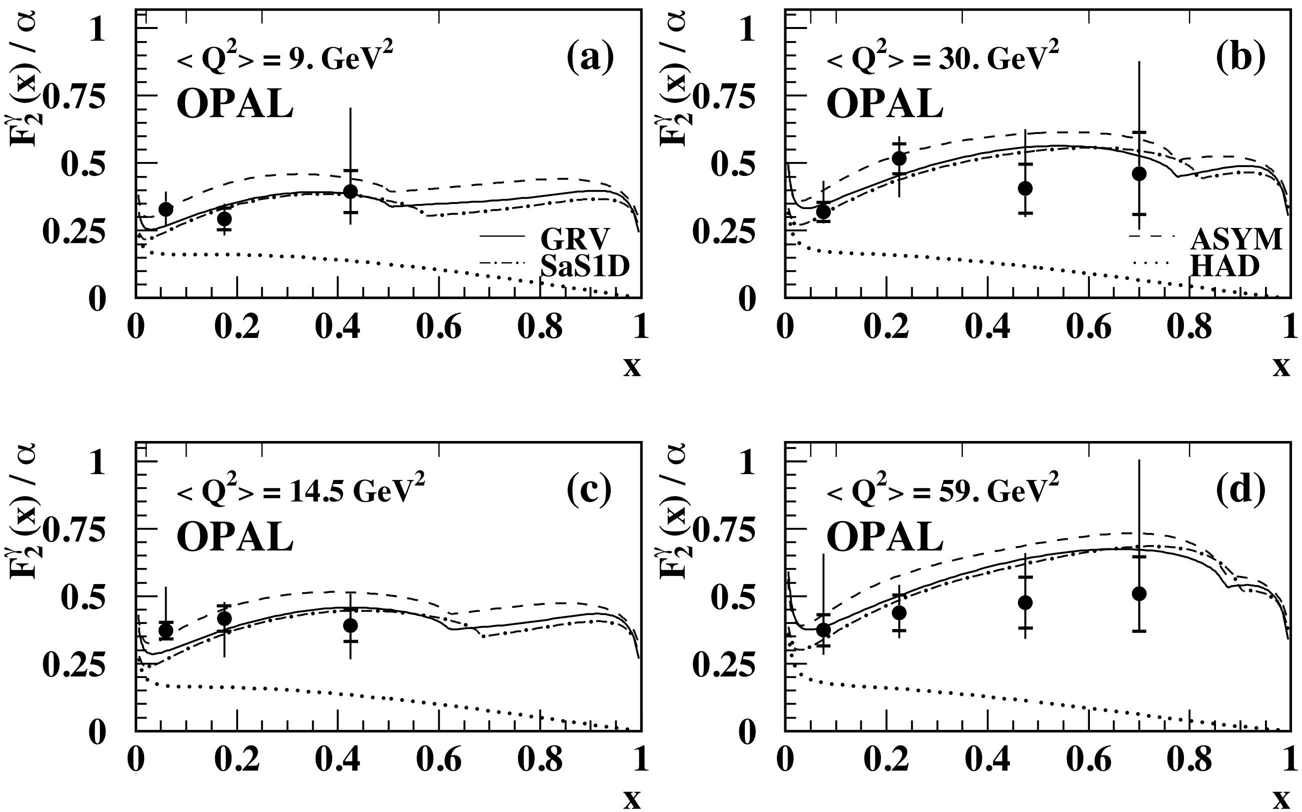


Photon PDFs (2/7)

- Similarly to the proton case, the partonic structure of the photon hadronic component using DIS on photon in e^+e^- annihilation.

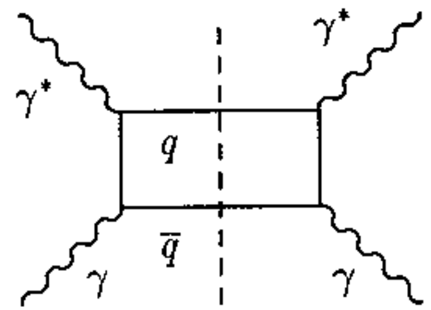


- $$\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[\left(1+(1-y)^2\right)F_2^\gamma(x,Q^2) - y^2 F_L^\gamma(x,Q^2)\right]$$



- Very different from the behavior of the proton $F_2(x,Q^2)$.

Photon PDFs (3/7)



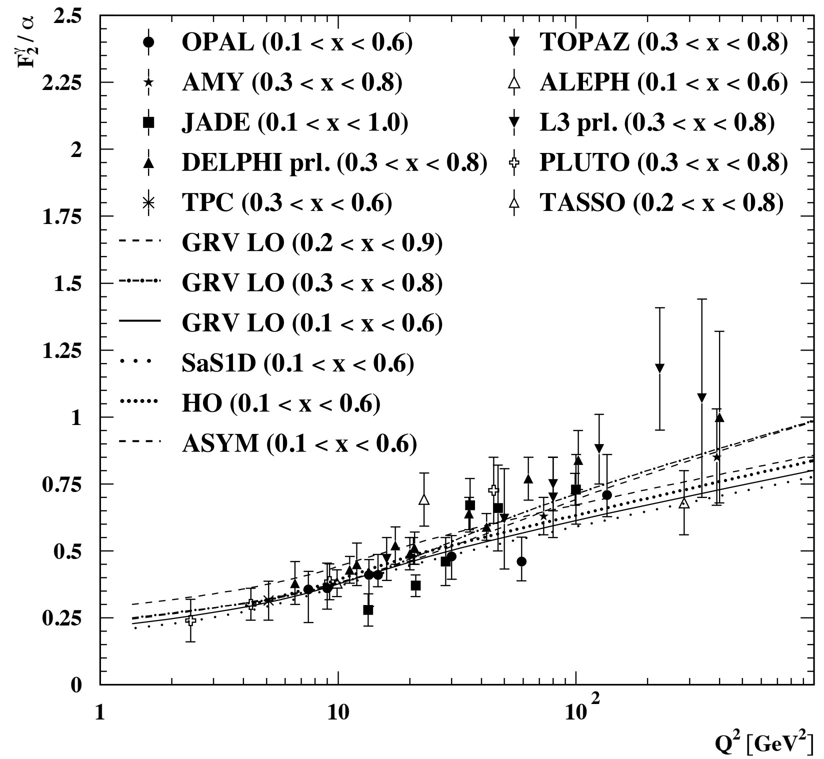
- In the quark parton model (QPM), one calculates $F_2^\gamma(x, Q^2)$ through the 'box' diagram (note we can restore its logarithmic term by recalling the gluon-quark splitting function $P_{qg}(z)$

$$\frac{F_2^\gamma(x)}{x} = \frac{N_c \alpha_{\text{e.m.}}}{\pi} \sum_q e_q^4 \left\{ (x^2 + (1-x)^2) \ln\left(\frac{Q^2}{m_q^2} \frac{1-x}{x} \right) + 8x(1-x) - 1 \right\}$$

- In contrast to proton, $F_2^\gamma(x, Q^2)$ manifests strong scaling violations, even without gluon radiation → scaling violations are positive for all x.

- Another difference is the x dependence: $F_2^\gamma(x, Q^2)$ increases and does not go to 0 as $x \to 1$.

- No Callan-Gross relation: $F_L^\gamma = F_2^\gamma(x) - 2xF_1^\gamma(x) \neq 0$



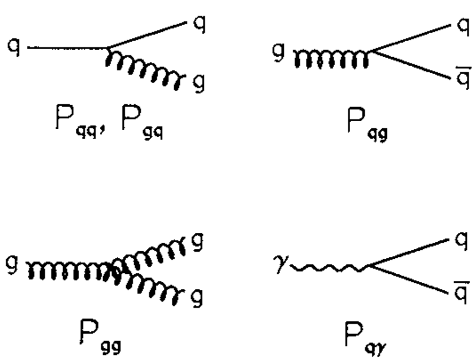
Photon PDFs (4/7)

- Similarly to proton, one can calculate corrections to the quark parton model due to parton emission → modified DGLAP evolution equations:

$$Q^2\frac{\partial}{\partial Q^2}\begin{pmatrix} f_q^\gamma(x,Q^2) \\ f_g^\gamma(x,Q^2)\end{pmatrix} = \frac{\alpha_{\mathrm{e.m.}}}{2\pi}\begin{pmatrix} k_q & 0 \\ 0 & k_g\end{pmatrix}\otimes\begin{pmatrix} f_q^\gamma(Q^2) \\ f_g^\gamma(Q^2)\end{pmatrix} + \frac{\alpha_s}{2\pi}\begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg}\end{pmatrix}\otimes\begin{pmatrix} f_q^\gamma(Q^2) \\ f_g^\gamma(Q^2)\end{pmatrix}$$

- In addition to qq, qg, gq and gg splittings, there is a $\gamma \to q\bar{q}$ splitting → inhomogeneous term in the evolution equations: $k_q = 3n_f\langle e^2\rangle 2(x^2+(1-x)^2)+\mathcal{O}(\alpha_s)$

- The gluon $k_g = \mathcal{O}(\alpha_s)$



- The $\gamma \to q\bar{q}$ splitting also contributes to the $F_2^\gamma(x,Q^2)$ structure function calculated in factorization framework:

$$F_2^\gamma(x,Q^2) = \sum_{i=q,\bar{q},g}\int_x^1 d\xi C_i\left(\frac{x}{\xi},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)f_i^\gamma(\xi,\mu^2) + \frac{\alpha_{\mathrm{e.m.}}}{4\pi}3n_f\langle e_q^4\rangle B_\gamma(x),$$ where

$$B_\gamma(x) = 4\left[(x^2+(1-x)^2)\ln\left(\frac{1-x}{x}\right) - 1 + 8x(1-x)\right]$$

Photon PDFs (5/7)

- One can present the solution of the evolution equations as $f_i^\gamma(x,Q^2)=f_{i,\mathrm{pl}}^\gamma(x,Q^2)+q_{i,\mathrm{had}}^\gamma(x,Q^2)$, but it is impractical because $B_\gamma<0$ for large x.

- To avoid numerical instabilities in global QCD analyses of photon PDFs, absorb the point-like contribution into the definition of PDFs
$\rightarrow$ DISγ factorization scheme:

$$(q^\gamma(x)+\bar{q}^\gamma(x))_{\mathrm{DIS}_\gamma}=(q^\gamma(x)+\bar{q}^\gamma(x))_{\overline{\mathrm{MS}}}+e_q^2\frac{3\alpha}{4\pi}B_\gamma(x) \quad ,$$

$$g^\gamma(x)_{\mathrm{DIS}_\gamma}=g^\gamma(x)_{\overline{\mathrm{MS}}}$$

- In this scheme, $F_2^\gamma(x,Q^2)$ has the form of proton $F_2(x,Q^2)\rightarrow$ one can use machinery of global QCD fits developed for proton.

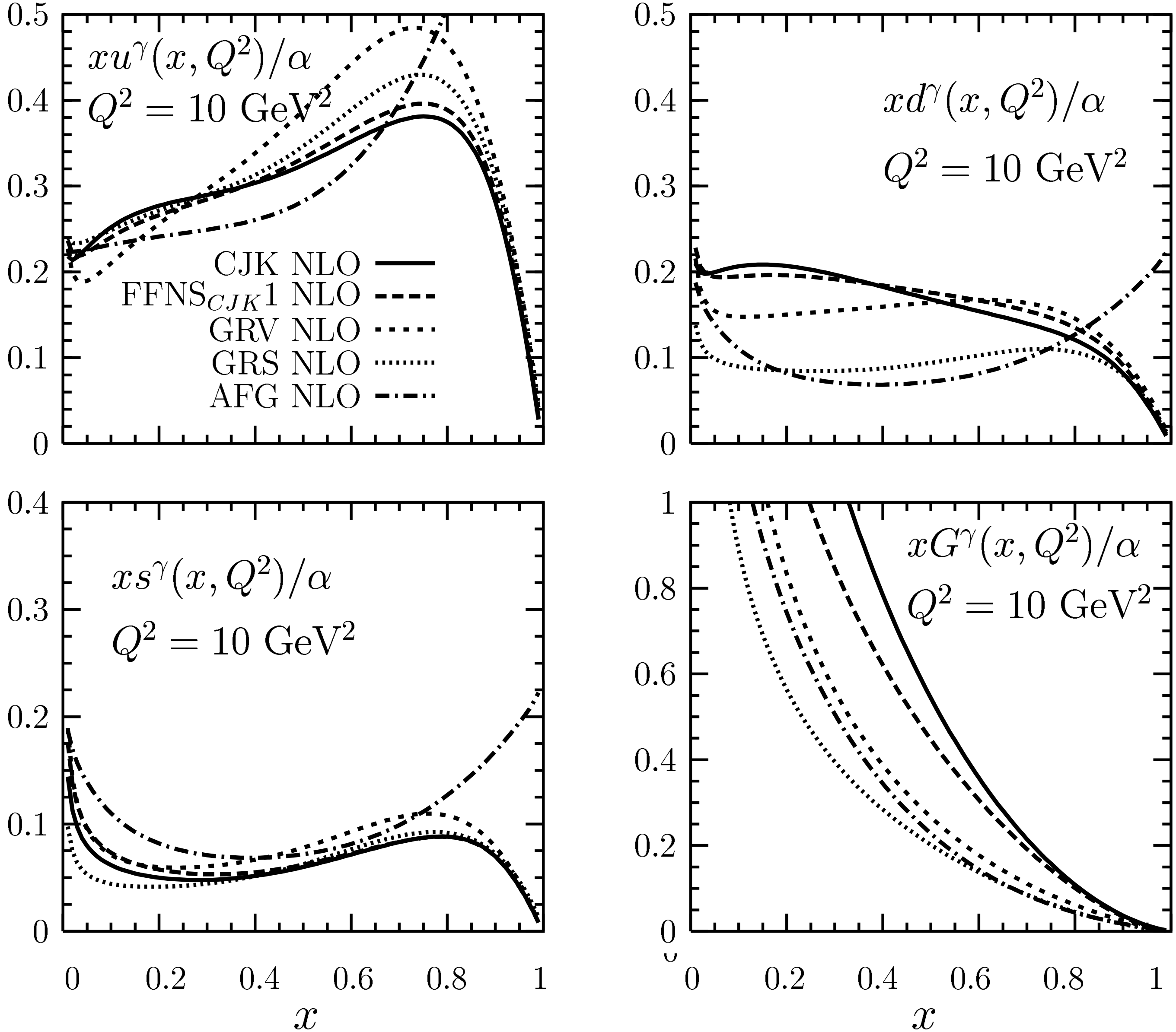
- Like in proton case, momentum sum rule, but it depends on Q^2

$$\int_0^1 dx\,x\left[\sum_q f_q^\gamma(x,Q^2)+f_g^\gamma(x,Q^2)+f_{\gamma/\gamma}(x,Q^2)\right]=1\rightarrow$$

$$\int_0^1 dx\,x\left[\sum_q f_q^\gamma(x,Q^2)+f_g^\gamma(x,Q^2)\right]=\frac{\alpha_{\mathrm{e.m.}}}{\pi}\sum_q e_q^2\log(Q^2/4\ \mathrm{GeV}^2)$$

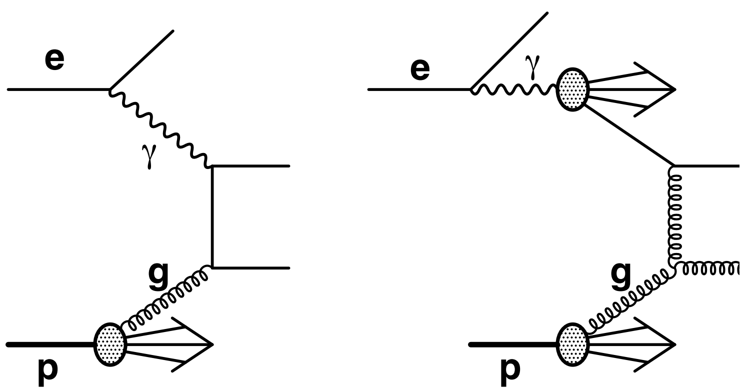
Photon PDFs (6/7)

- Global QCD fits to $F_2^{\gamma}(x, Q^2)$ data → photon PDFs at NLO accuracy, Nisius, Phys. Rept. 332 (2000) 165-317 [arXiv:hep-ex/9912049; Cornet, Jankowski, Krawczyk, PRD 70 (2004) 093004.



Photon PDFs (7/7)

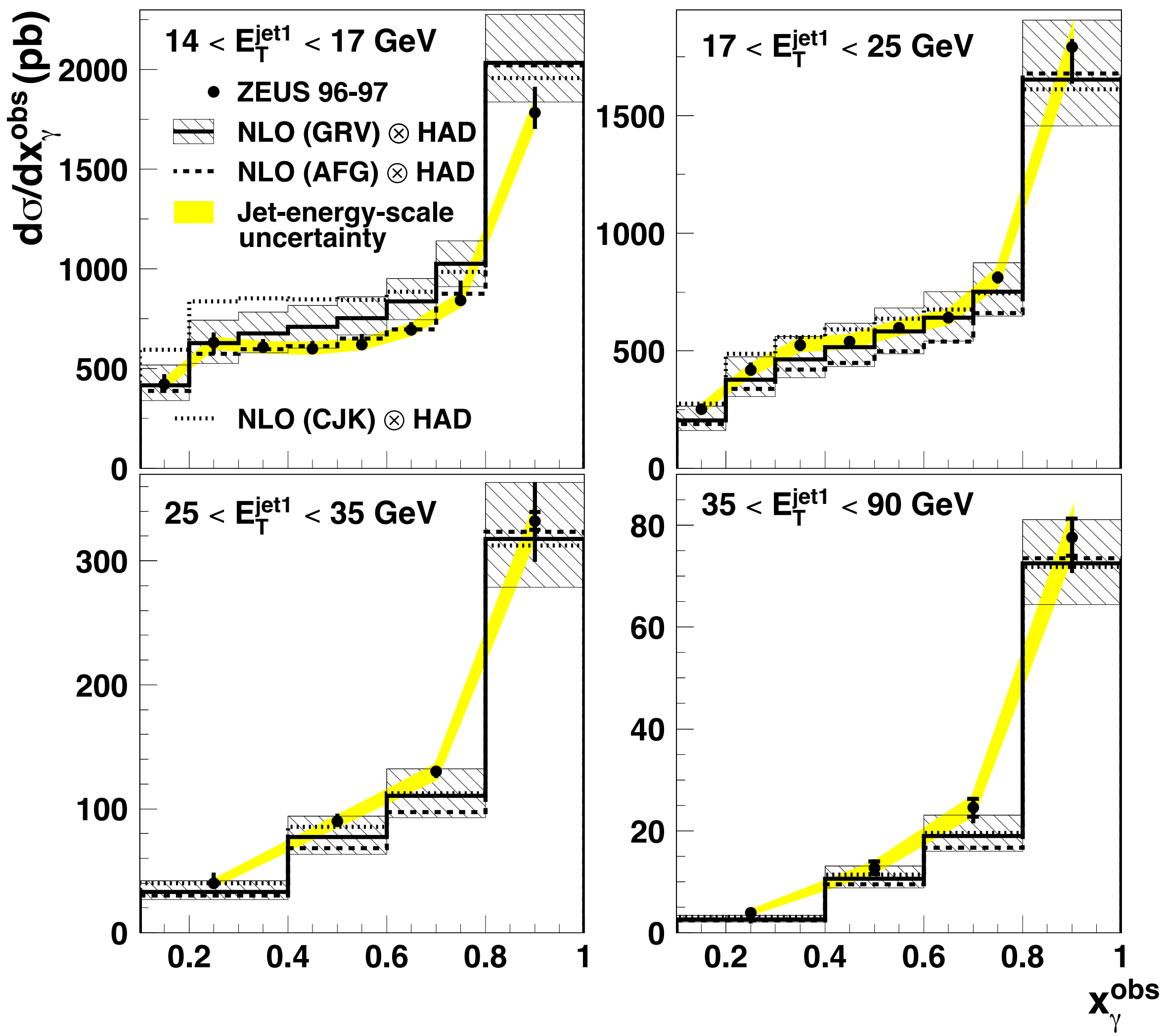
• Photon PDFs for the resolved-photon contribution for dijet photoproduction in ep scattering HERA $\rightarrow$ also in UPCs at LHC and eA scattering at EIC.



• Direct photon and resolved photon contributions → can be separated using dependence on

$$x_\gamma = \frac{1}{2E_e y}\left(E_{T,1}e^{-\eta_1} + E_{T,2}e^{-\eta_2}\right)$$

HERA dijet photoproduction

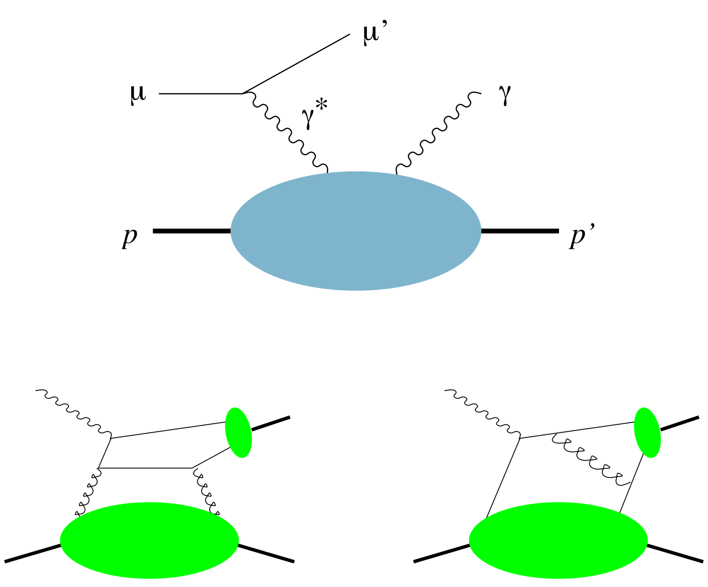


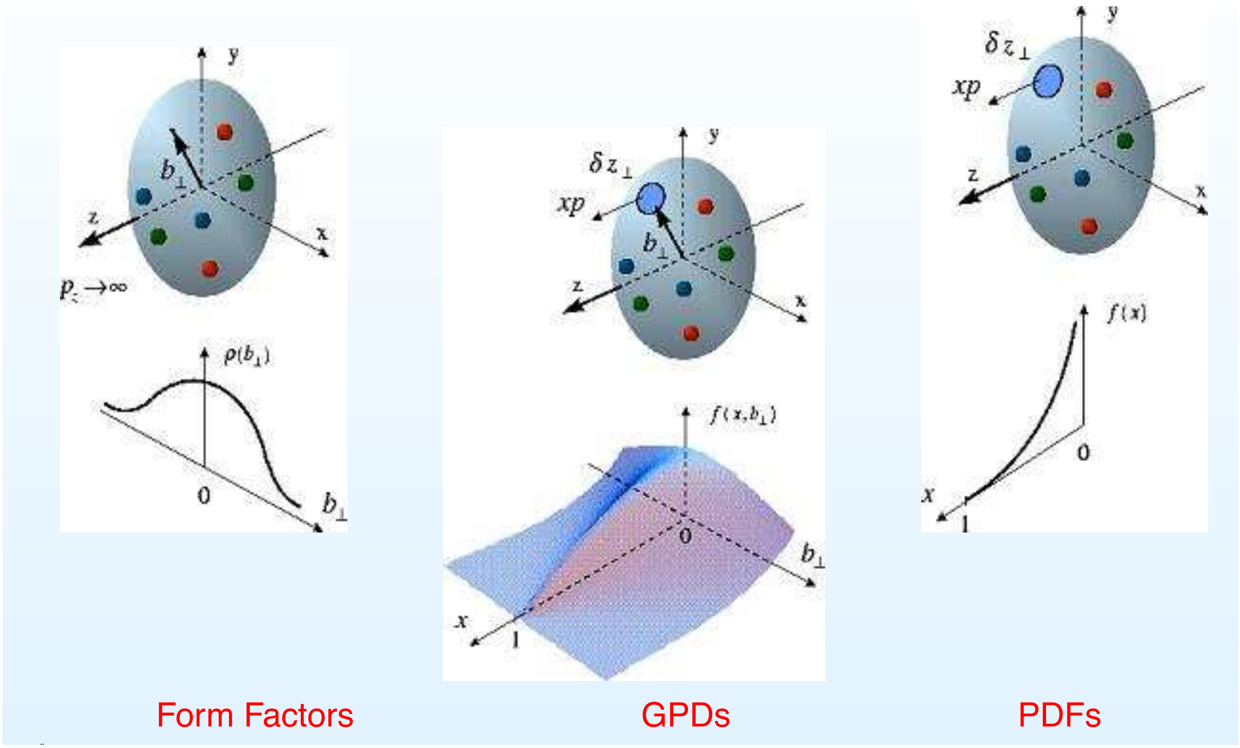
Instead of Summary (1/2)

• With these lectures, I just scratched the surface of a vast and active field of PDFs in QCD. The field is evolving in three directions:

• **Precision**: work toward N^{3}LO global QCD fits, use of neural networks, and elaborate methods of statistical analysis.

• **Imaging**: generalized parton distributions (GPDs) from deeply virtual Compton scattering (DVCS) and exclusive meson production → GPDs contain info on elastic form factors and PDFs → 3D image of the nucleon/nucleus.





Instead of Summary (2/2)

- **Inclusion of elements of BFKL physics.** E.g., small-x resummation in coefficient functions and parton splitting function in global QCD fits of proton PDFs → extension of applicability for low x, Ball, Bertone, Bonvini, Marzani, Rojo, Rottoli, EPJ C (2018) 78:321