High-Energy QCD and Diffraction

part IV

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ANIVERSARIO PATRIMONIO MUNDIAL

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Lecture III Highlights

Matching NLL to NLO with JETHAD

¡ **Precision corrections** *expected* ⇔ *need* for an accurate NLL-to-NLO **Matching procedure** !

JETHAD Method → NLL/NLO Additive Matching (analytic: BFKL kernel + coefficient functions)

$$\underbrace{\mathrm{d}\sigma^{\mathrm{NLL/NLO^{-}}}(\Delta Y,\varphi,s)}_{\text{NLO POWHEG w/o PS}} = \underbrace{\mathrm{d}\sigma^{\mathrm{NLO}}(\Delta Y,\varphi,s)}_{\text{NLO fixed order}} + \mathrm{d}\sigma^{\mathrm{NLL^{-}}}(\Delta Y,\varphi,s) - \Delta\mathrm{d}\sigma^{\mathrm{NLL/NLO^{-}}}(\Delta Y,\varphi,s)$$

Highlights of Lecture III

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$$\underbrace{\qquad\qquad}_{\text{NLO POWHEG w/o PS}}$$

Matching NLL to NLO with JETHAD

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$$\underbrace{\mathrm{d}\sigma^{\mathrm{NLL/NLO^-}}(\Delta Y,\varphi,s)}_{\text{NLL/NLO}^-} = \underbrace{\mathrm{d}\sigma^{\mathrm{NLO}}(\Delta Y,\varphi,s)}_{\text{NLO fixed order}} + \underbrace{\mathrm{d}\sigma^{\mathrm{NLL^-}}(\Delta Y,\varphi,s)}_{\text{NLL}^- \text{ resum (HyF)}} - \underbrace{\Delta\mathrm{d}\sigma^{\mathrm{NLL/NLO^-}}(\Delta Y,\varphi,s)}_{\text{NLL}^- \text{ expanded at NLO}}$$

NLO POWHEG w/o PS

NLL⁻ JETHAD w/o NLO⁻ double counting

Highlights of Lecture III

Matching NLL to NLO with JETHAD

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$$\underbrace{\mathrm{d}\sigma^{\mathrm{NLL/NLO^{-}}}(\Delta Y,\varphi,s)}_{\textbf{NLL/NLO}^{-}\text{ matched}} = \underbrace{\mathrm{d}\sigma^{\mathrm{NLO}}(\Delta Y,\varphi,s)}_{\textbf{NLO}\text{ fixed order}} + \underbrace{\mathrm{d}\sigma^{\mathrm{NLL^{-}}}(\Delta Y,\varphi,s)}_{\textbf{NLL}^{-}\textbf{ resum (HyF)}} - \underbrace{\Delta\mathrm{d}\sigma^{\mathrm{NLL/NLO^{-}}}(\Delta Y,\varphi,s)}_{\textbf{NLL}^{-}\textbf{ expanded at NLO}}$$

$$\underbrace{\textbf{NLL/NLO}^{-}\text{ matched}}_{\textbf{NLO}\text{ POWHEG}\oplus\textbf{NLL}^{-}\text{ JETHAD}} \qquad \underbrace{\textbf{NLO}\text{ fixed order}}_{\textbf{NLO}\text{ POWHEG w/o PS}} \qquad \underbrace{\textbf{NLL}^{-}\textbf{ resum (HyF)} \qquad \textbf{NLL}^{-}\textbf{ expanded at NLO}}_{\textbf{NLL}^{-}\text{ JETHAD w/o NLO}^{-}\text{ double counting}}$$

HELL + ggHiggs

N^3LL_{lx}/LL_{sx}/N^3LO

Inclusive Higgs

[M. Bonvini, S. Marzani (2018)]

HEJ framework

NLL_{sx}^{-}/NLO

Higgs + jet(s)

[J. R. Andersen et al. (2022)]

RadISH + MCFM-8.3

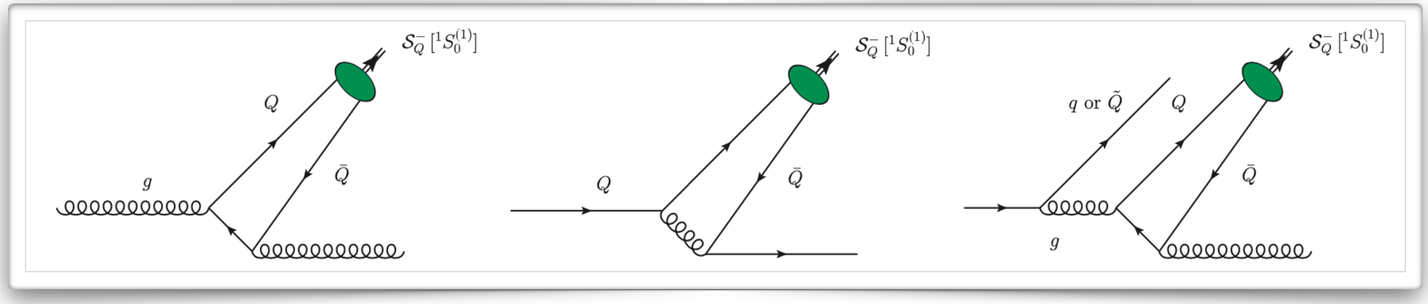
$NNLL_{TM}$/NLO

Higgs + jet

[P.F. Monni et al. (2020)]

Highlights of Lecture III

HF-NRevo: Initial-scale NRCQD inputs



Scalar η_c

HF-NRevo for $\eta_c[^1S_0^{(1)}]$ collinear fragmentation

Partonic hard factor

DGLAP AOevo

All-Order evolution (numeric)

DGLAP EDevo

Expanded Decoupled evolution

g
$\tilde{q}$
c
b
$2m_c$

NRQCD

SDCs

LDMEs

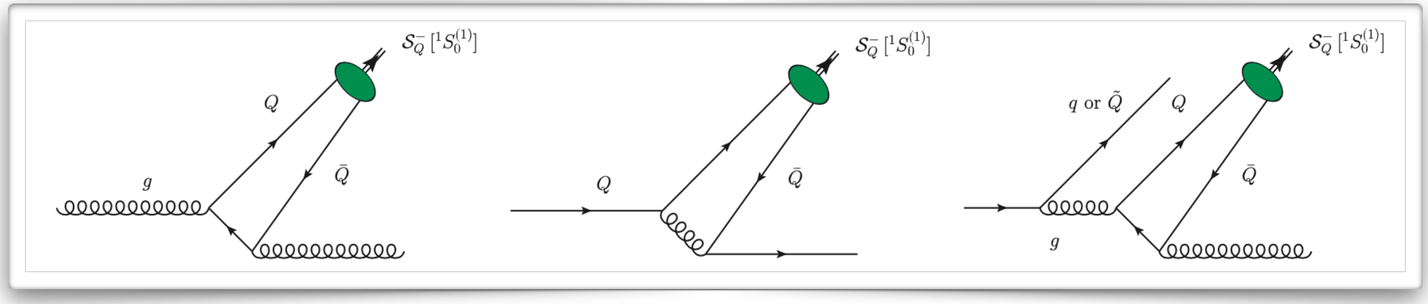
$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

Energy

Evolution-ready scale $Q_0 \equiv m_b + 2m_c$ $3m_c$ $m_{\tilde{q}} + 2m_c$

Time

HF-NRevo: Initial-scale NRCQD inputs



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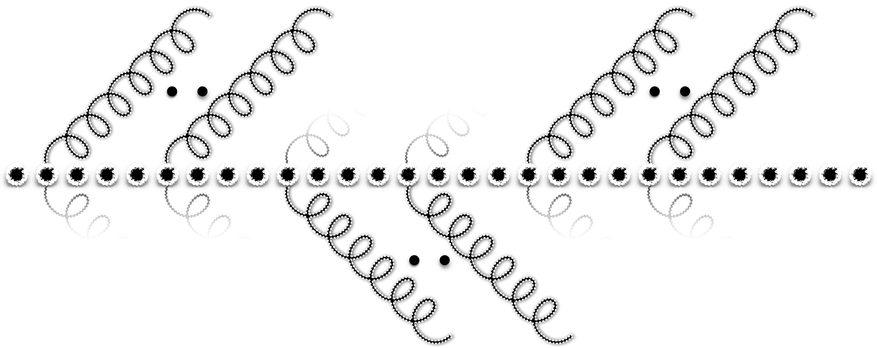

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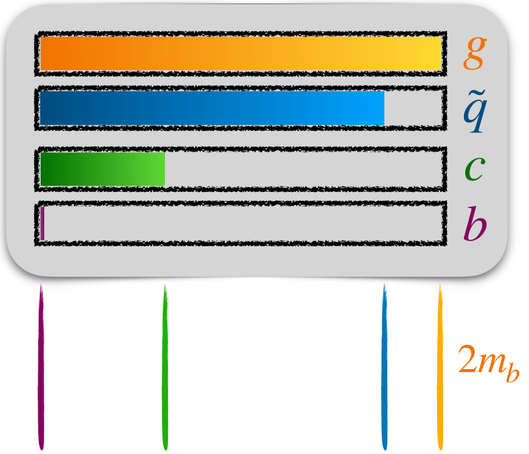
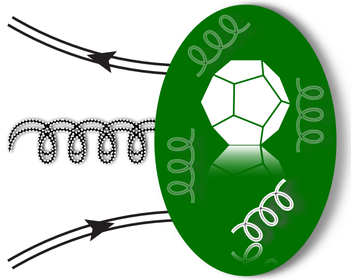


NRQCD

LDMEs

SDCs



$\mathcal{D}_i^{\mathcal{Q}}(z,[n]) \otimes \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$

Energy

Evolution-ready scale $Q_0 \equiv 3m_b$ $m_c + 2m_b$ $m_{\tilde{q}} + 2m_b$

Time

1

The BFKL UGD

Omnes viae small-x ducunt

Incomplete list of small-x formalisms → *linear* (BFKL) or *saturation* (BK/JIMWLK) effects embodied

Unintegrated parton densities

A (hybrid) high-energy factorization established

- **BFKL UGD**: pure small-x evolution, Reggeons
- HEF, CCFM, PRA **uPDFs**: BFKL + collinear matching

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Small-x improved collinear PDFs

DGLAP description improved via BFKL

- **ABF approach**: PDFs + small-x resummed splitting

[R.D. Ball, V. Bertone, M. Bonvini, S. Marzani, J. Rojo, L. Rottoli (2018)]

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Small-x improved gluon TMDs

Nonperturbative content via an enhanced spectator model

- **Pavia model**: initial-scale f_1^g and $h_{1\perp}^g$ matched to PDFs

[A. Bacchetta, F.G.C., M. Radici, P. Taels (2020)]

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Helicity and OAM at small-x

Need for sub-eikonal corrections, neglected by BFKL

- **BER**: DLA, flavor singlet and nonsinglet
- **KPS**: evolution via Wilson lines, saturation

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CGC/JIMWLK gluon TMDs

Gluon-recombination effects encoded

- **WW** vs **DP** gluon TMDs, **GTMDs**
- **iTMD**: interpolating between TMD and BFKL regimes

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CGC/JIMWLK gluon TMDs

Gluon-recomb

- **WW** vs **DP**
- **iTMD**: inte

All-Twist High-Energy Factorization

AT_HEF

TMD Factorization

CSS_TMD

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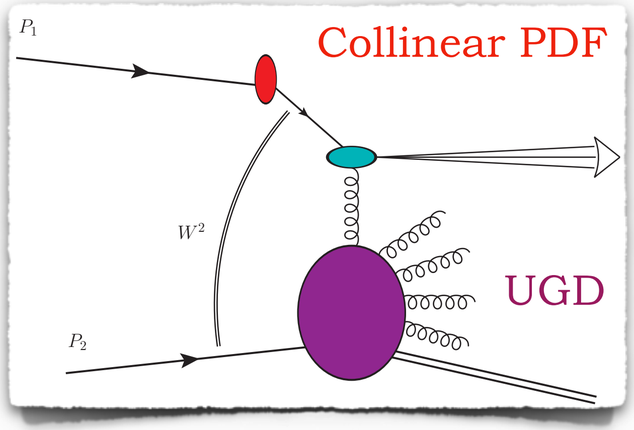
IV.1 The BFKL UGD

Hybrid or pure factorization?

Forward emissions

* *Asymmetric* config. ↔ fast parton + small-x gluon

* Hybrid **high-energy**/**collinear** factorization



* *Distinctive signals* of small-x dynamics **expected**

* Phenomenology:
  forward jet, Drell-Yan, Higgs or vector meson

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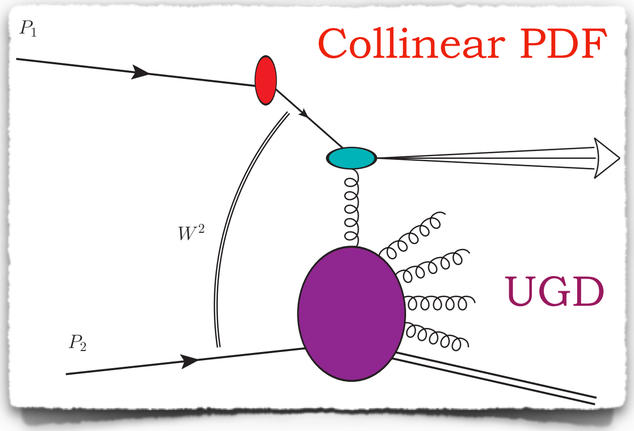

* *Distinctive signals* of small-x dynamics **expected**
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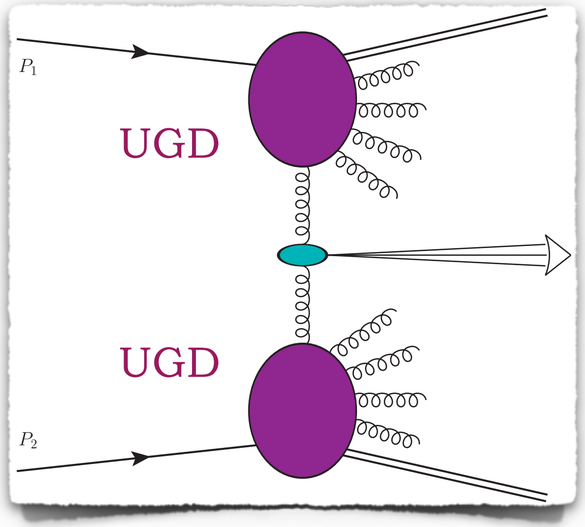
  forward jet, Drell-Yan, Higgs or vector meson

Central emissions

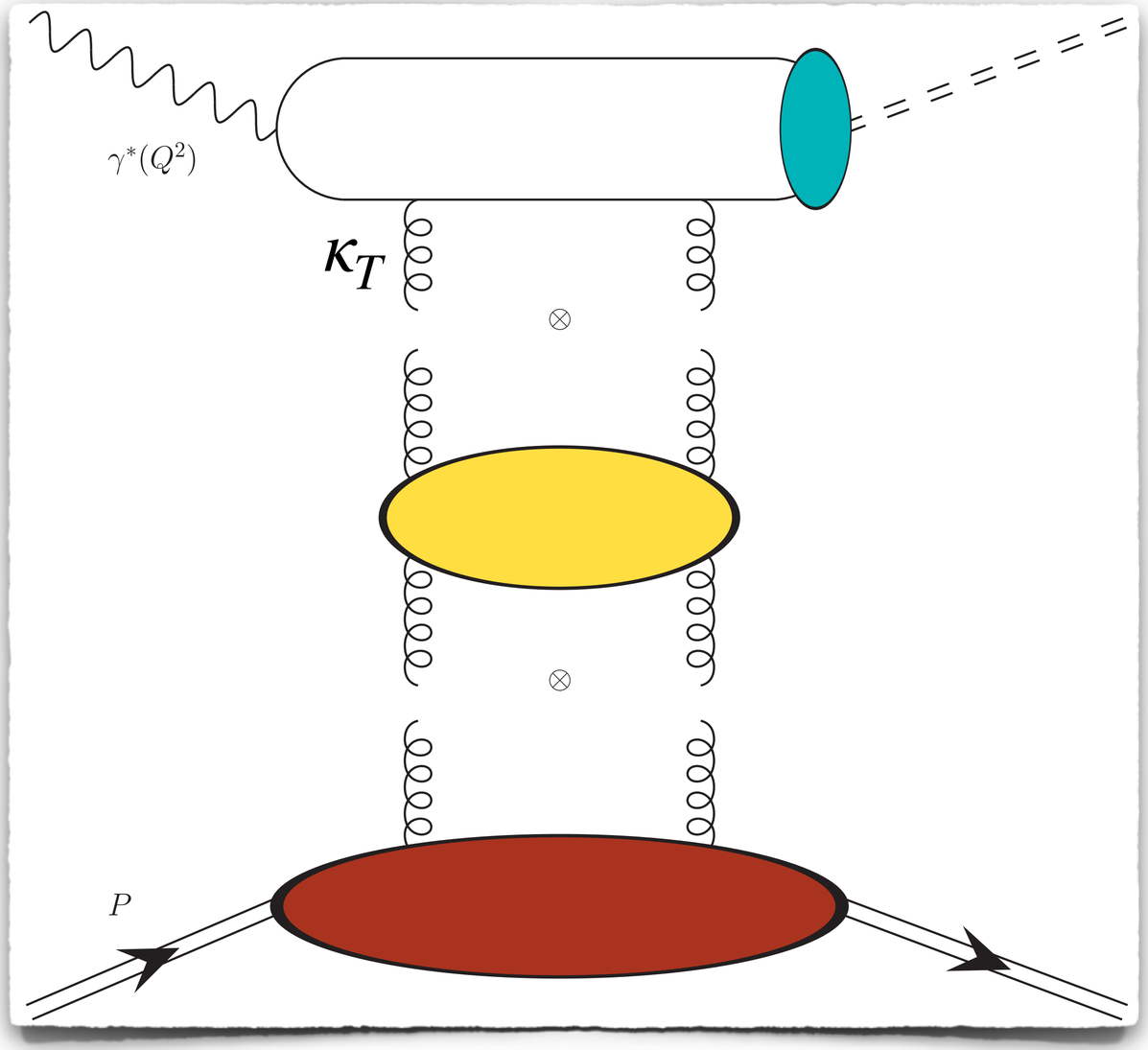
* *Gluon induced* ↔ small-x gluons
* Pure **high-energy** factorization



* Small-x dynamics to **enhance** f.o. description
* Phenomenology:
  central jet, Higgs or vector meson

Table complemented by *exclusive* counterparts and *lepto-hadronic* channels

Forward ρ-mesons: A factorization…of factorizations!



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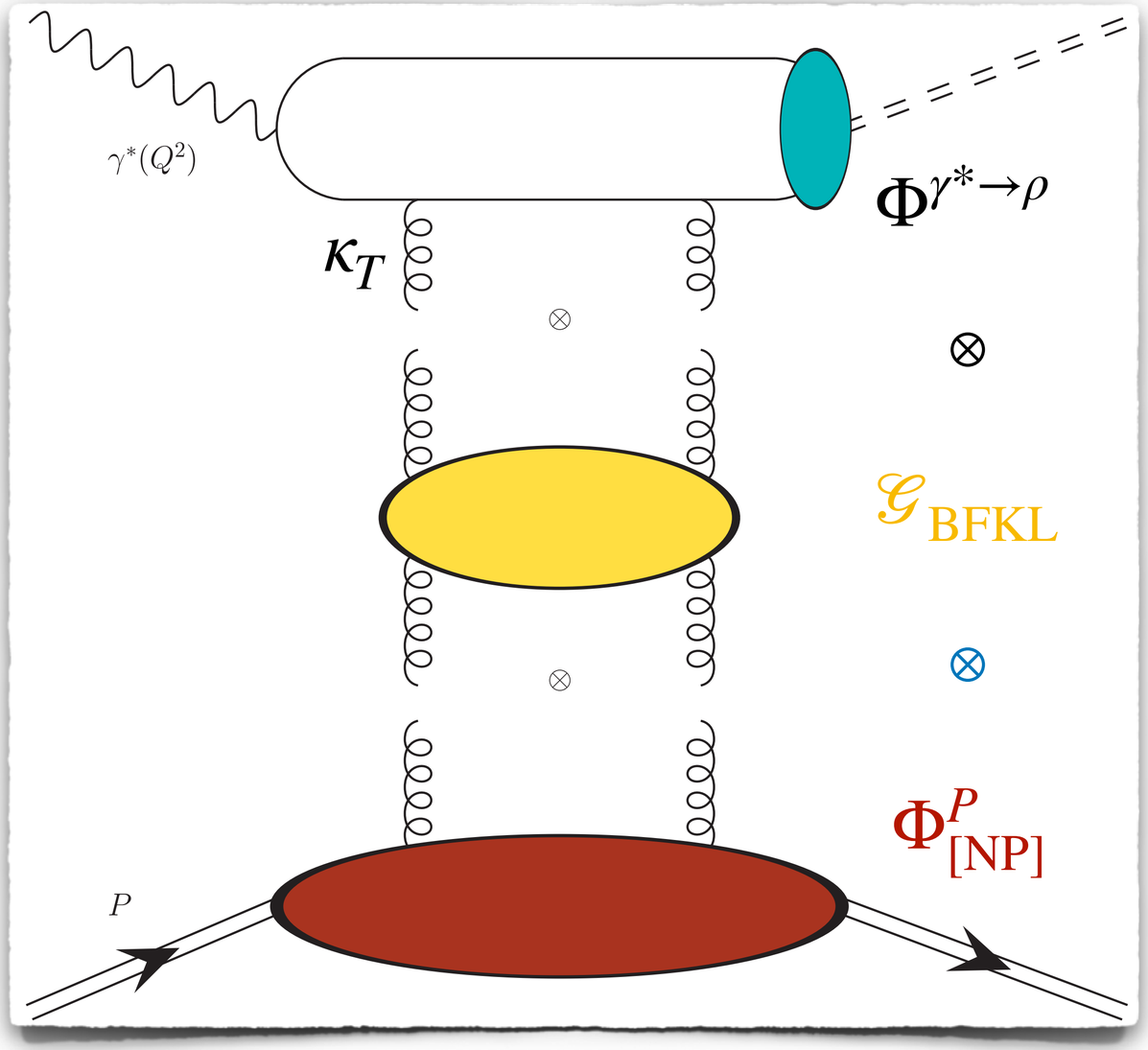

$\Phi^{\gamma^*\to\rho}$ $\otimes$ $\mathcal{G}_{\text{BFKL}}$ $\otimes$ $\Phi^{P}_{[\text{NP}]}$

BFKL factorization

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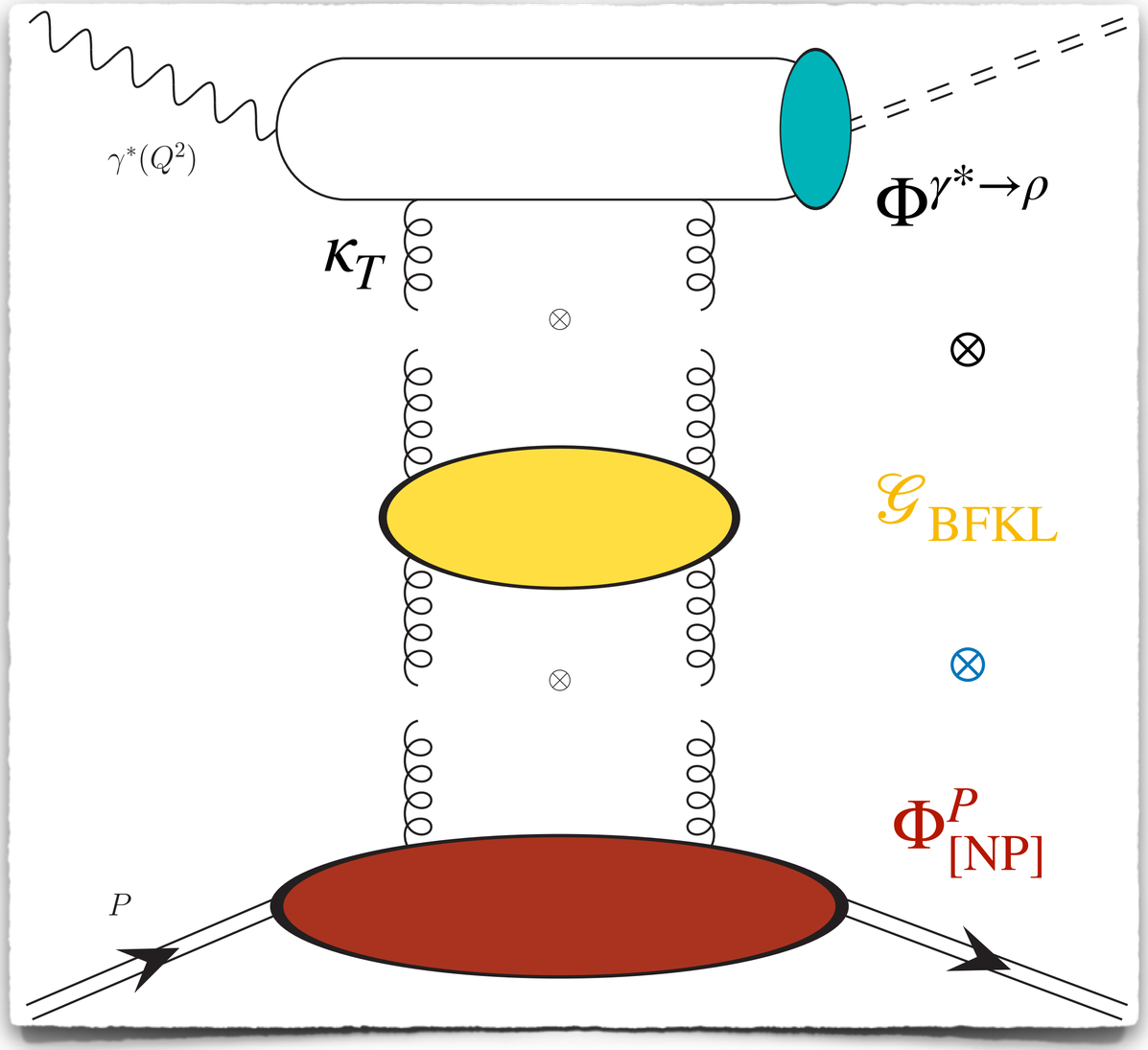
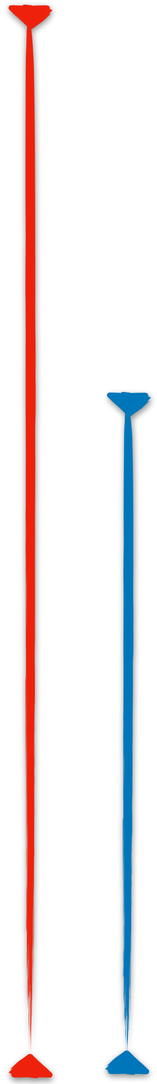


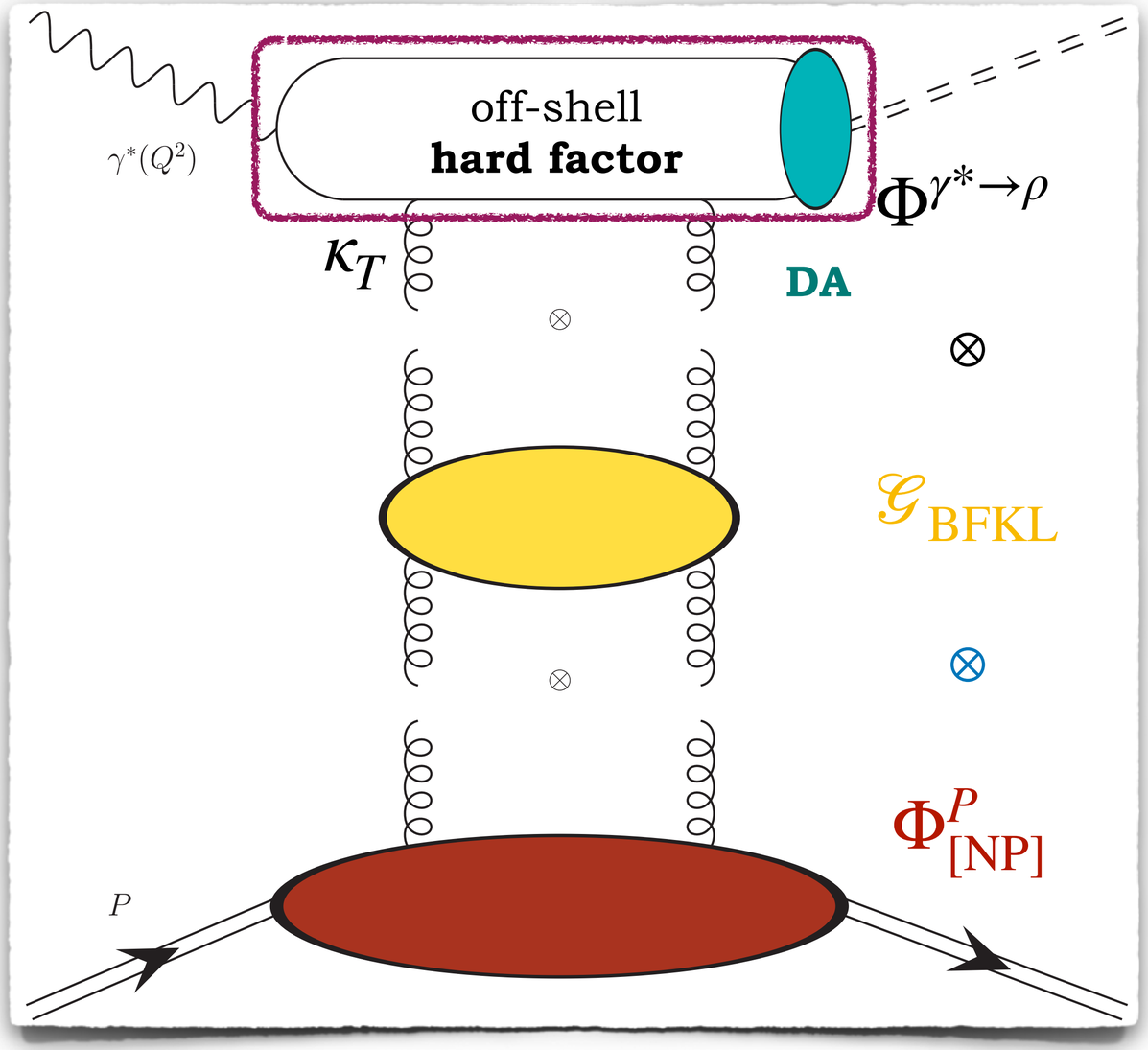


UGD

BFKL factorization

IV.1 The BFKL UGD

Forward ρ-mesons: A factorization…of factorizations!



$\gamma \to \rho$ impact factor

$$\Phi^{\gamma^*\to\rho} \propto \int_0^1 \mathrm{d}z\, T_H^{\gamma^*\to\rho}(z, \kappa_T, Q, \mu_R, \mu_F)\, \phi^{\lambda_\rho}(z, \mu_F)$$

Collinear convolution $\Leftrightarrow$ large κ_T (!)

UGD

BFKL factorization

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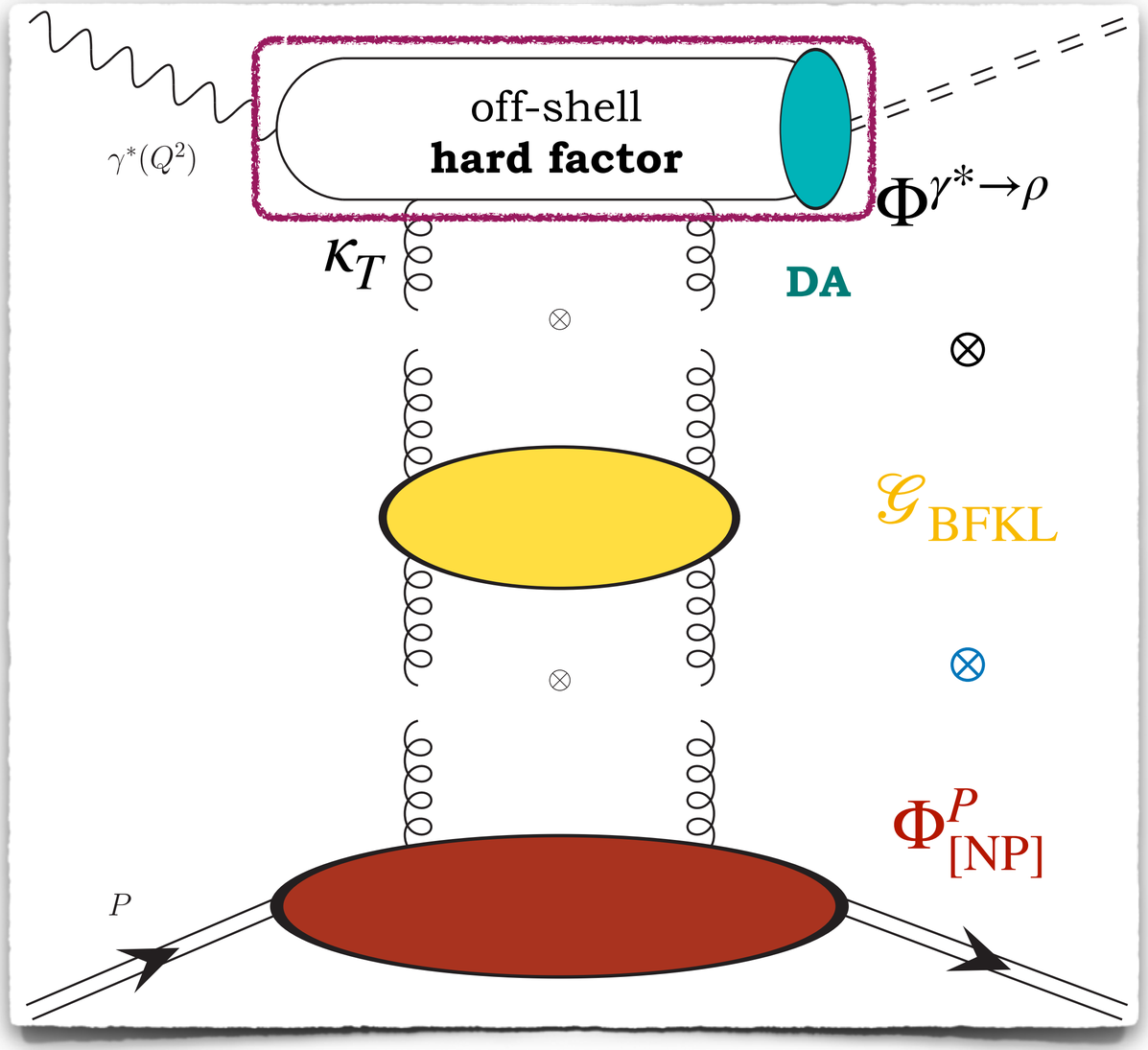

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* $\gamma_L^* \to \rho_L$ transition [twist-2]

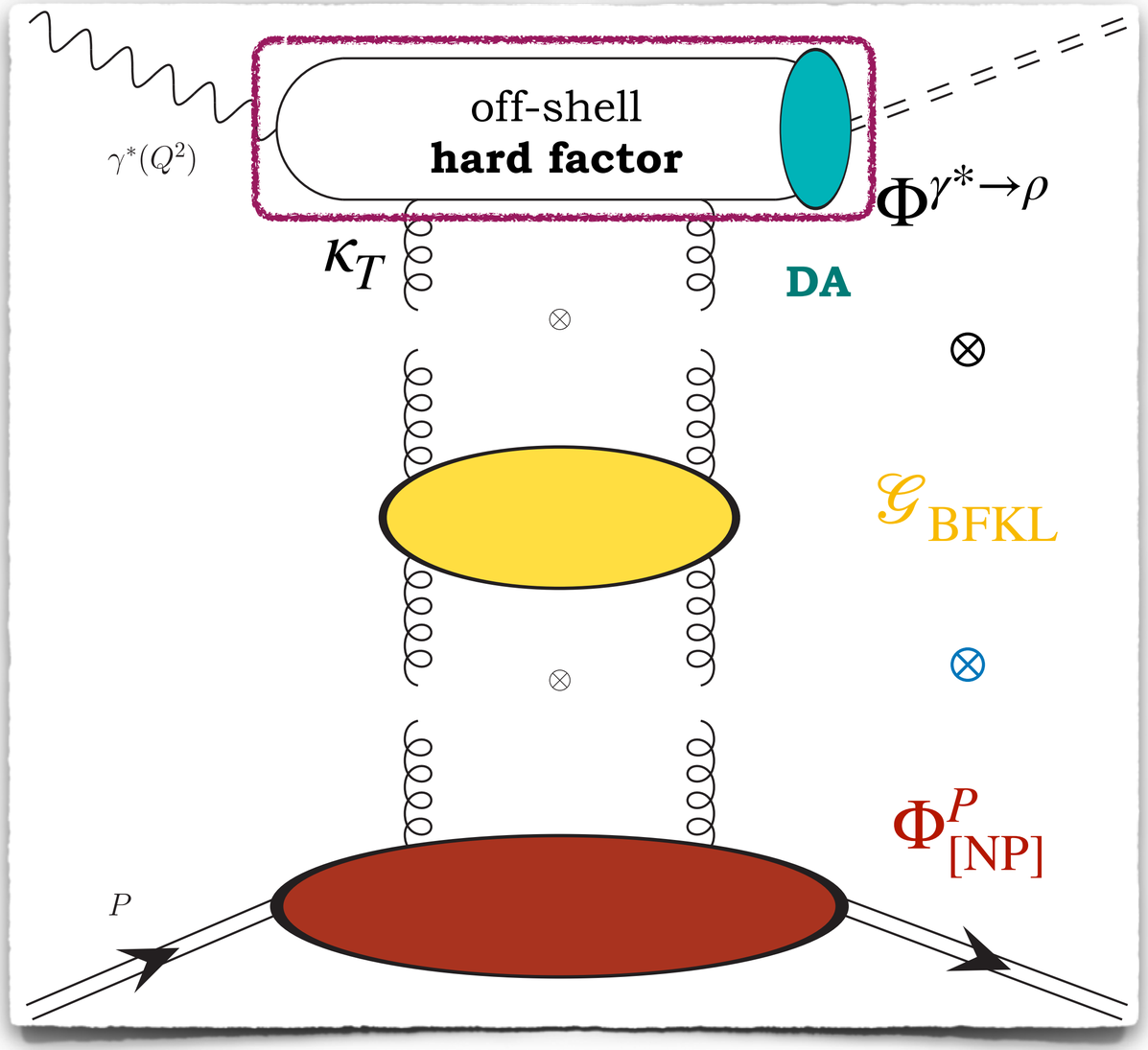
$$\frac{1}{\kappa_T^2}\Phi^{\gamma_L^*\to\rho_L} \underset{\kappa_T\to 0^+}{\sim} \text{constant}$$

UGD

BFKL factorization

IV.1 The BFKL UGD

Forward ρ-mesons: A factorization...of factorizations!



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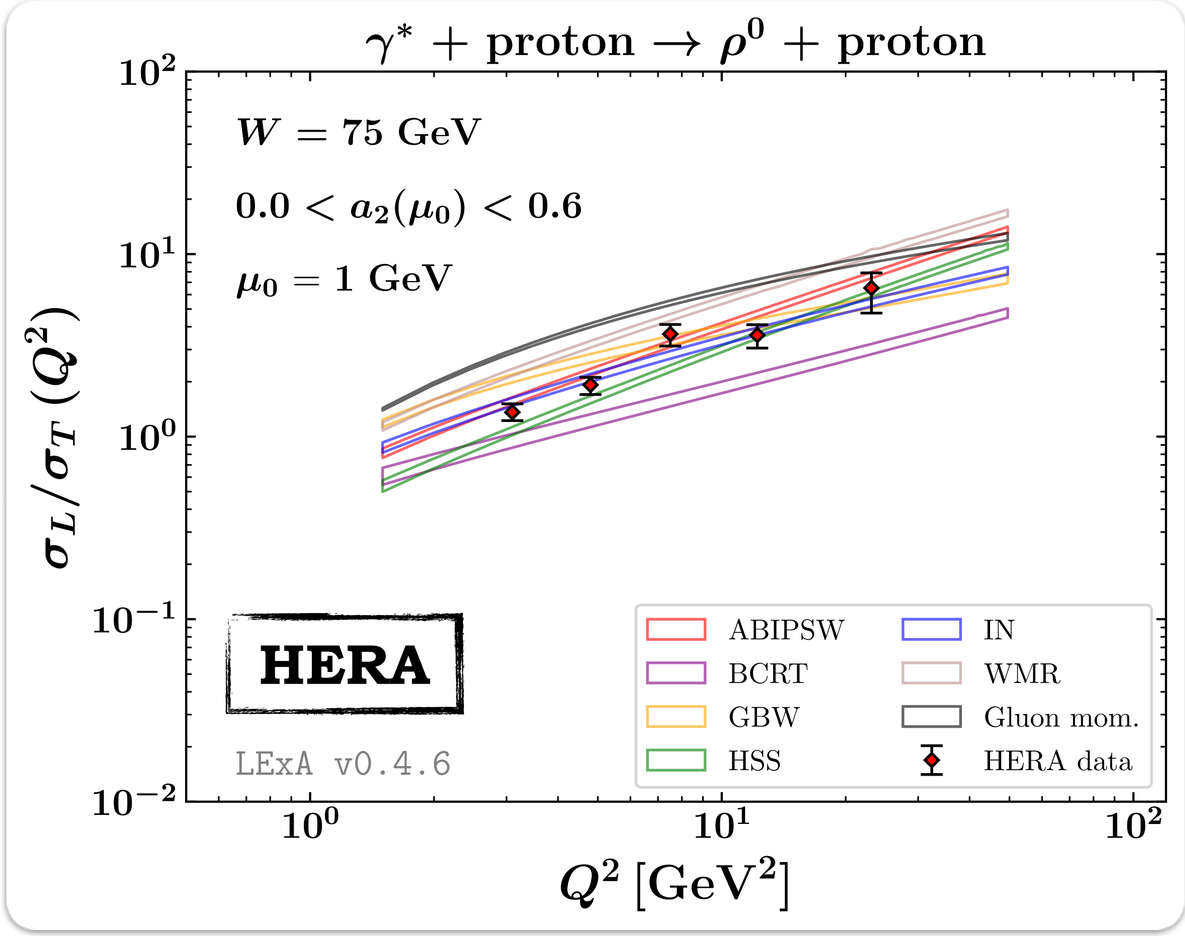
* $\gamma_T^* \to \rho_T$ transition [twist-3]

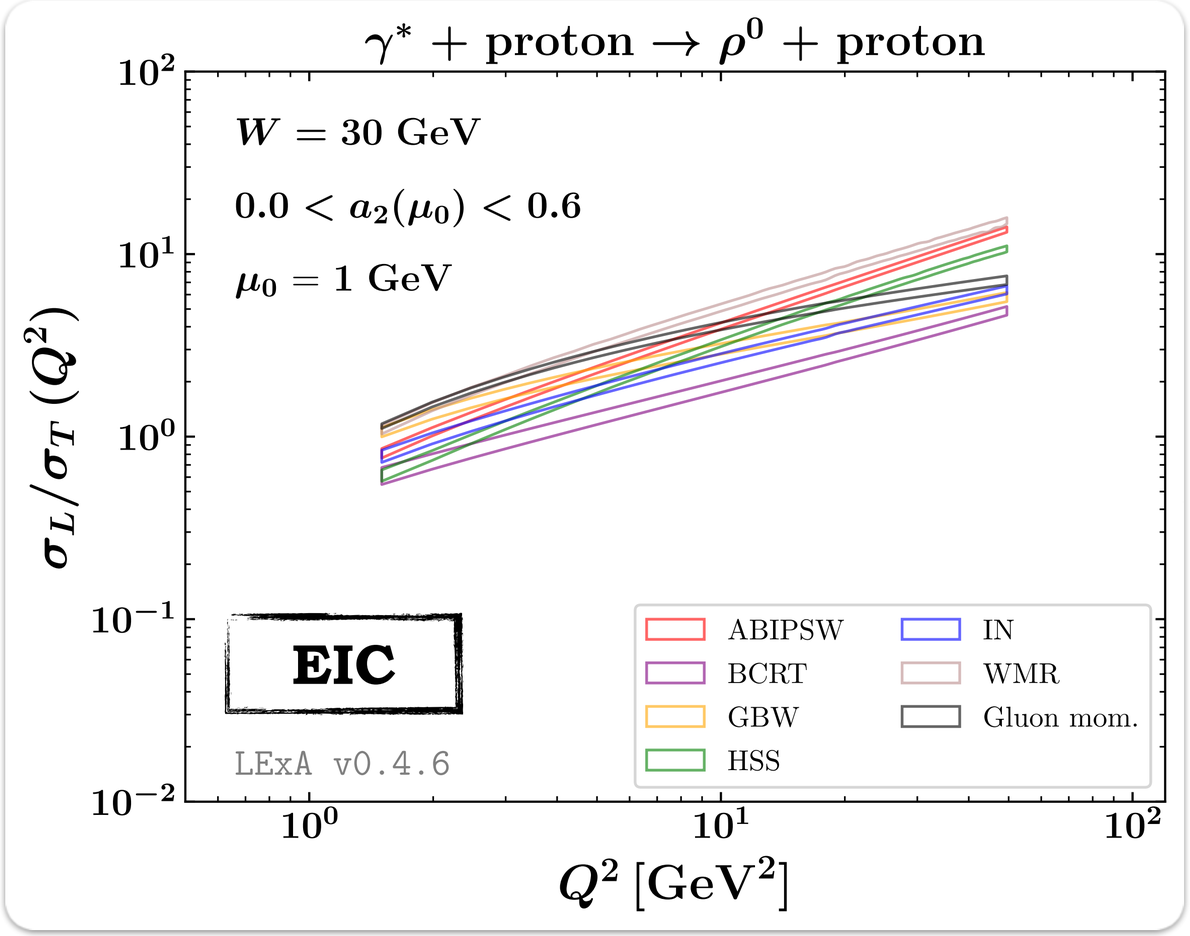
$$\frac{1}{\kappa_T^2}\Phi^{\gamma_T^*\to\rho_T} \underset{\kappa_T\to 0^+}{\sim} \ln\frac{\kappa_T^2}{Q^2}$$

UGD

BFKL factorization

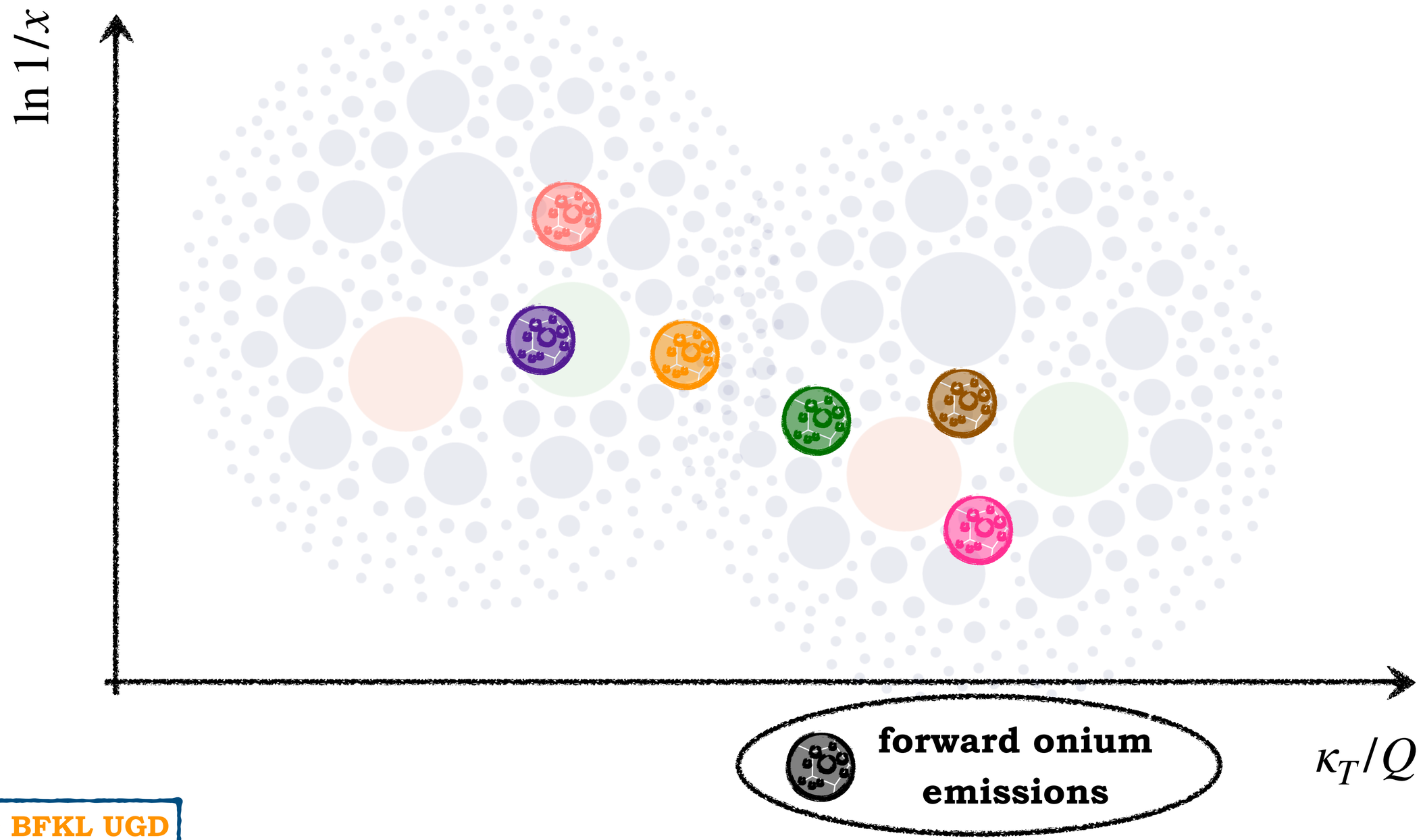
Forward ρ-mesons from HERA to the EIC: $\sigma_L/\sigma_T(Q^2)$





(saturation effects in ρ-meson polarization @HERA) 🔗 [A. Besse, L. Szymanowski, S. Wallon (2013)]
(ρ-meson helicity amplitudes @HERA) 🔗 [A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa (2018)]
(extension to ϕ-meson emissions) 🔗 [A.D. Bolognino, A. Szczurek, W. Schäfer (2020)]
(in this slide) 🔗 [A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa, W. Schäfer, A. Szczurek (2021)]

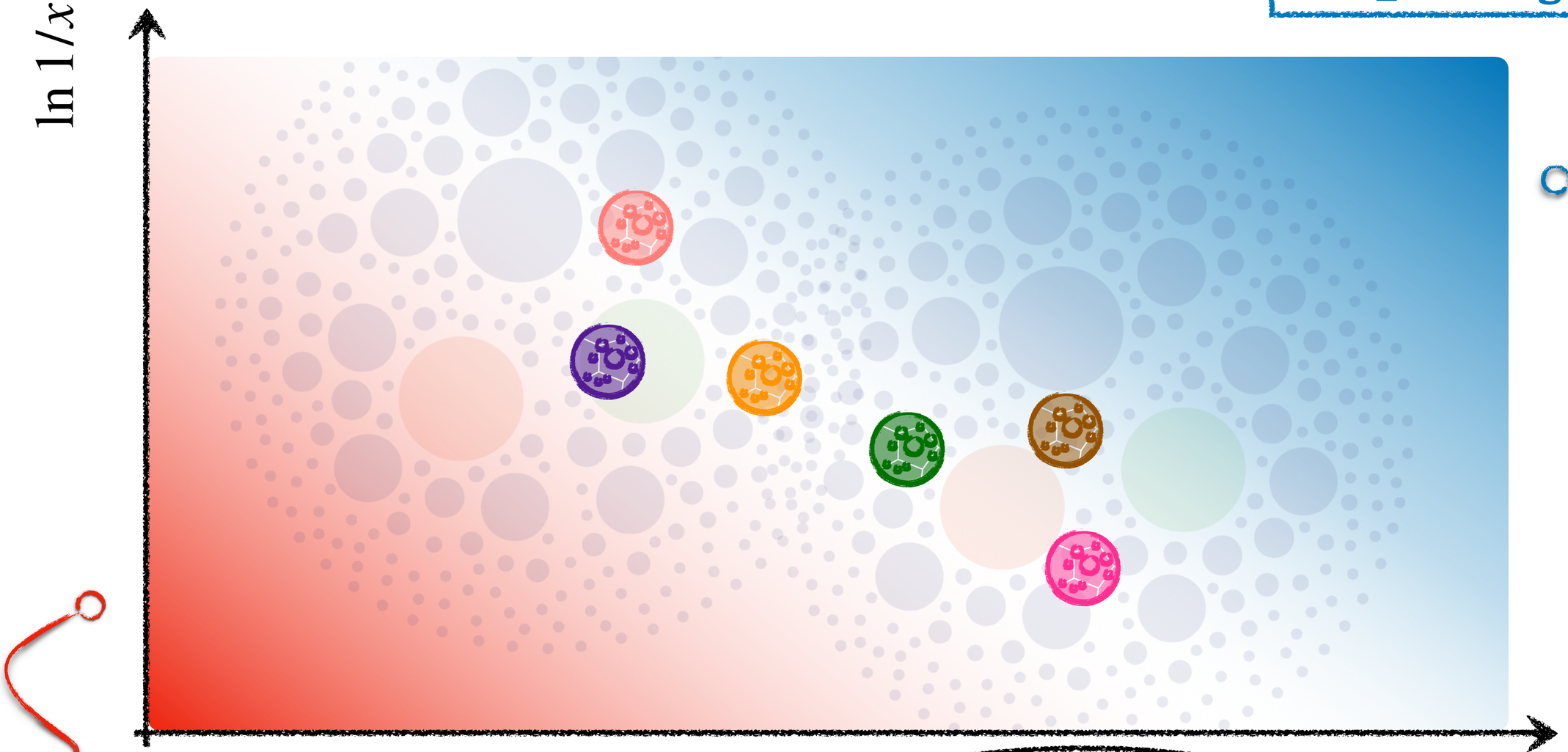
Hadronic structure & quarkonia



Hadronic structure & quarkonia

$x \ll 1;\ \kappa_T \approx Q \gg \Lambda_{\mathrm{QCD}}$

AT_HEF regime



$\kappa_T \ll Q$

CSS_TMD regime

forward onium emissions

Forward Drell-Yan production

- LHC, **forward region** $\rightarrow$ $(l^+ l^-)$ produced in the fragmentation region of h_2
  - Asymmetric configuration: $x_1 \gg x_2$, down to $x_2 \simeq 10^{-6}$

$\Longrightarrow$ **possible small-x resummation effects expected!**

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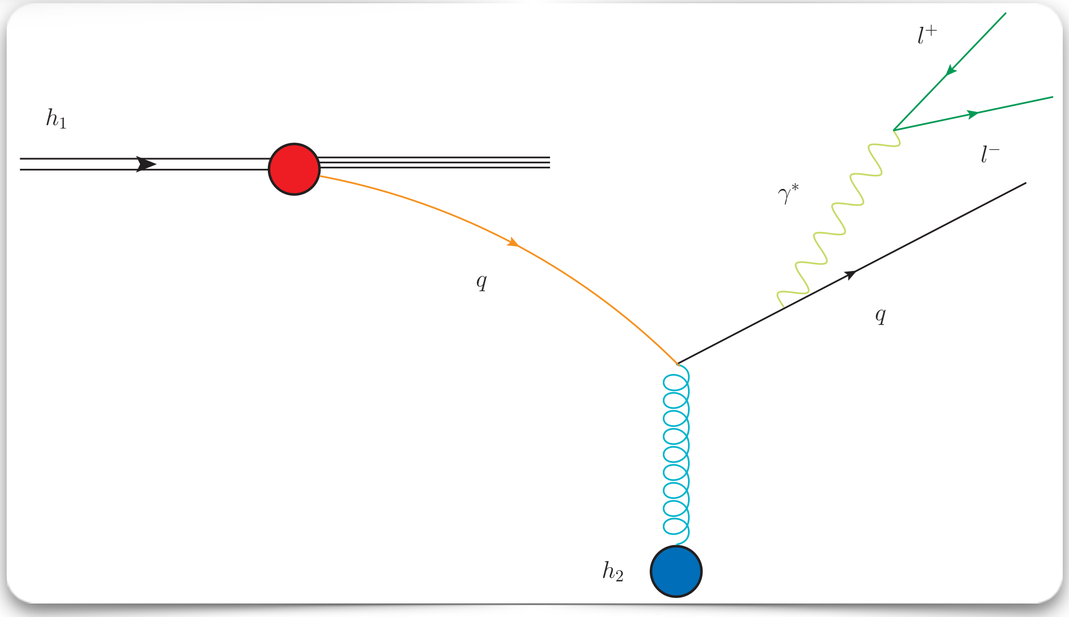
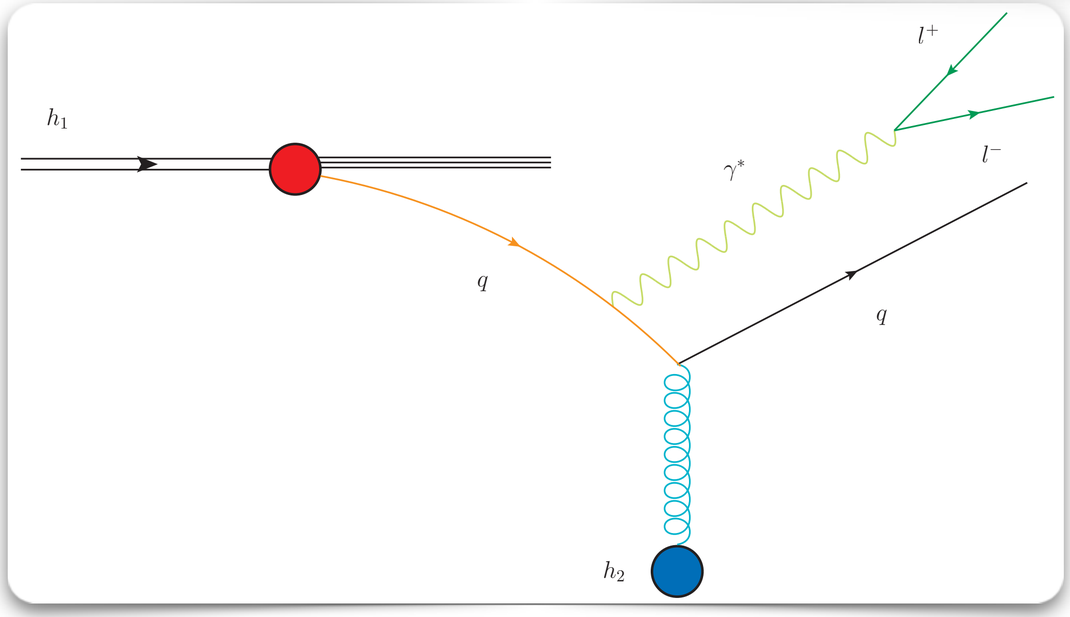


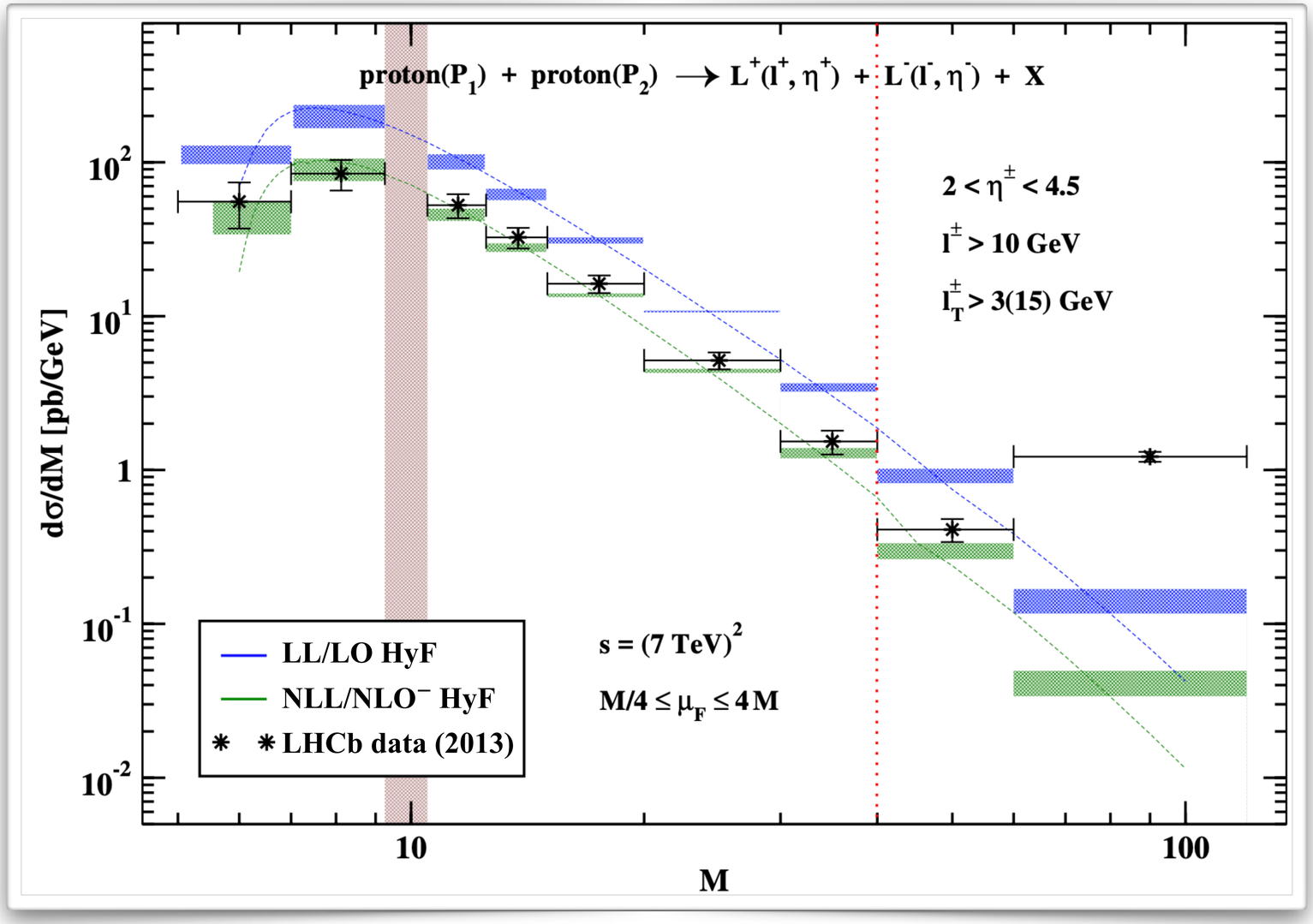


(LO DY emission functions + Mellin) [G. Golec-Biernat, E. Lewandowska, A. M. Stasto, Phys. Rev. D 82 (2010) 094010]

(LO DY emission functions + Mellin) [L. Motyka, M. Sadzikowski, T. Stebel, JHEP 05 (2015) 087]

(Twist decomposition + pheno) [D. Brzemiński, L. Motyka, M. Sadzikowski, T. Stebel, JHEP 01 (2017) 005]

Forward Drell-Yan production @LHC (2018)



NLL/NLO^{-} studies @7TeV LHCb

[F. G. C., D Gordo Gómez, A Sabio Vera, Phys. Lett. B 786 (2018) 201-206]

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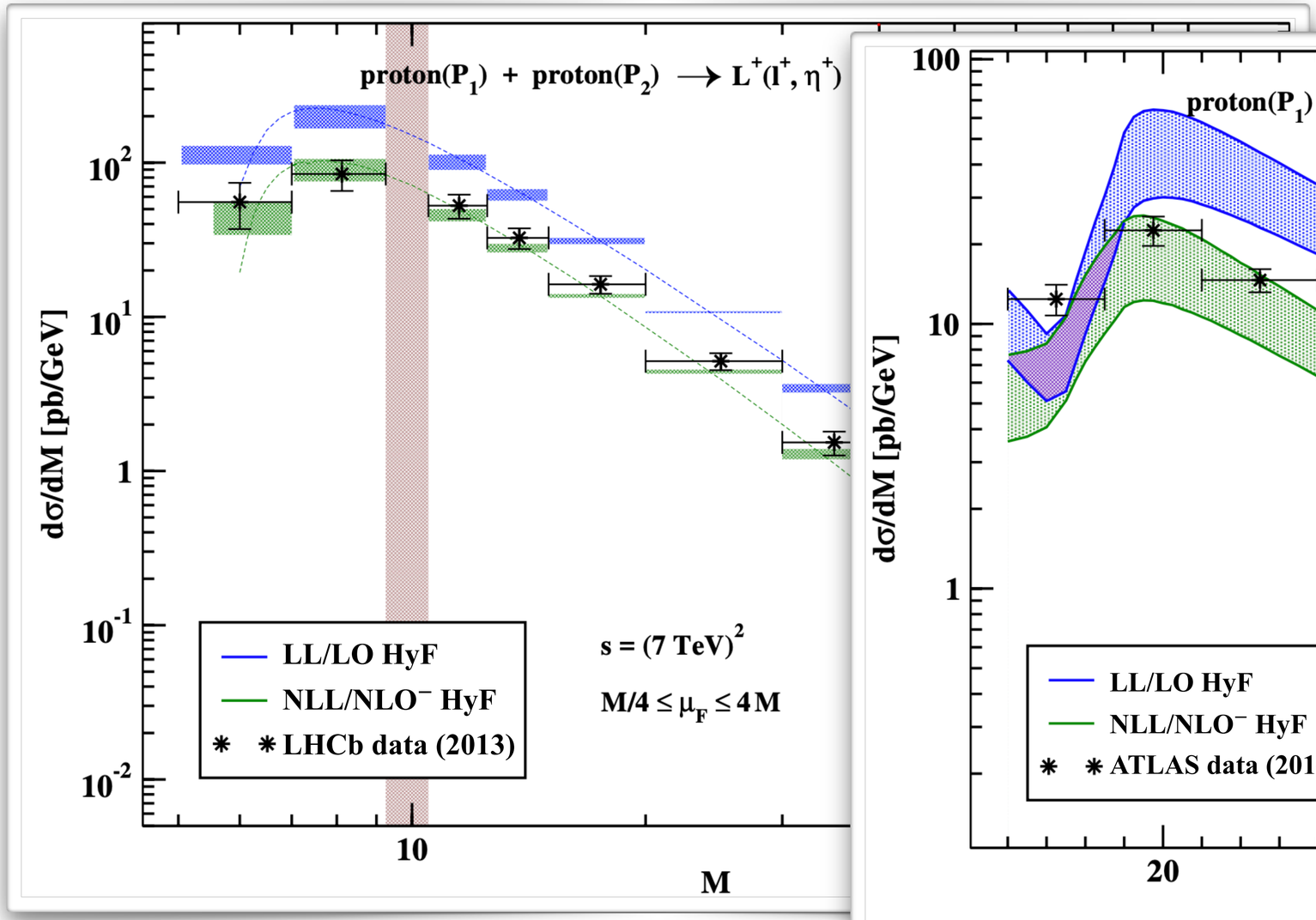

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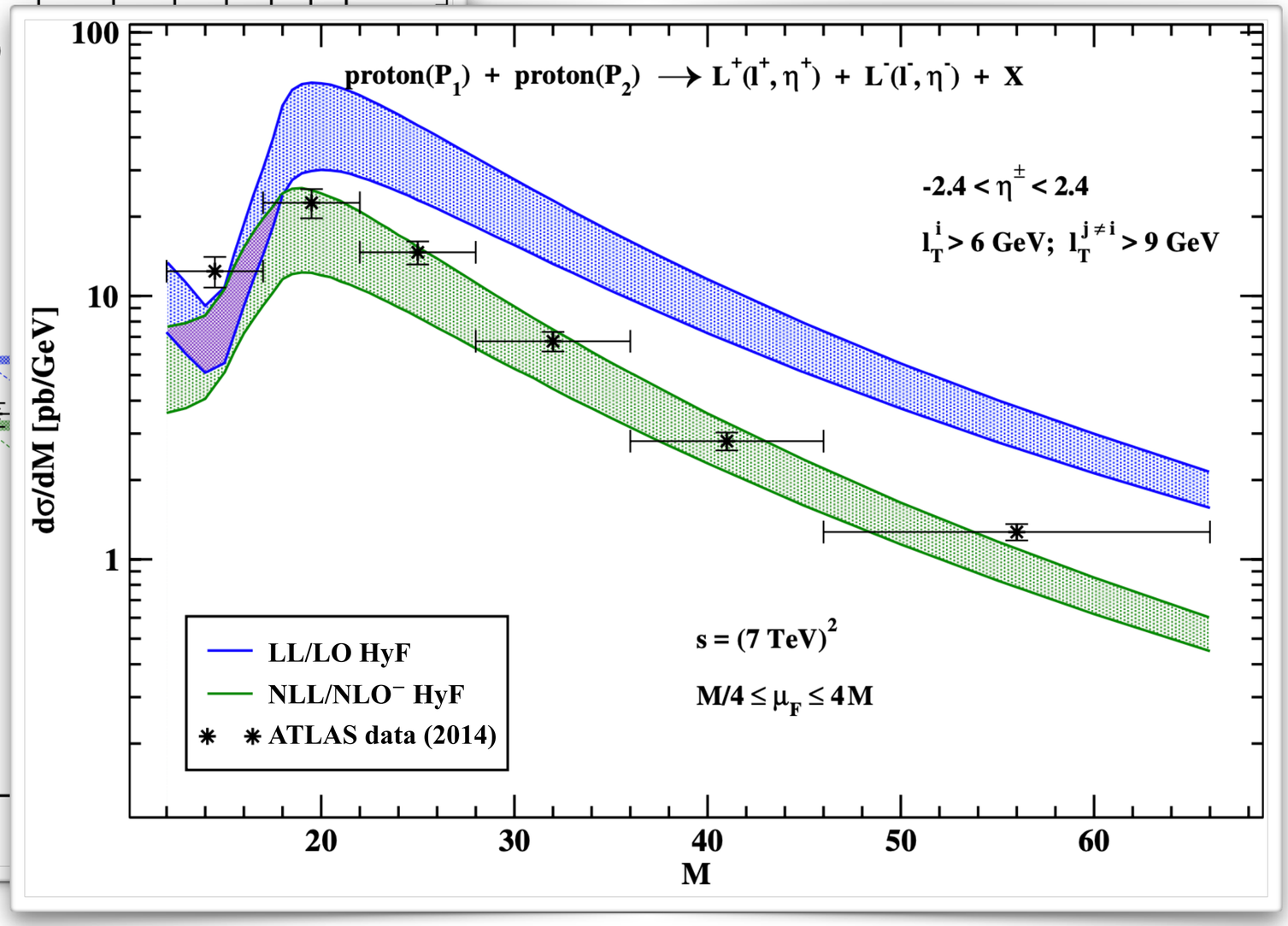

NLL/NLO^{-} studies @7TeV ATLAS

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2

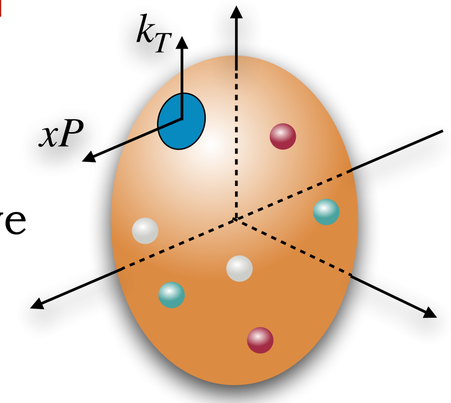
Gluon TMDs

Parton densities: An incomplete family tree

3D

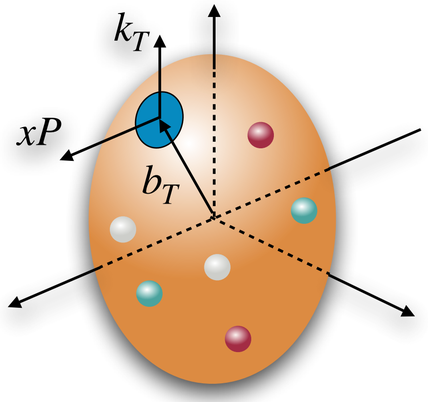
TMDs

(semi-)inclusive



5D

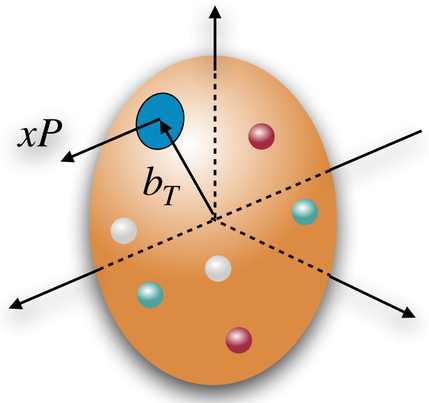
Wigner distributions



3D

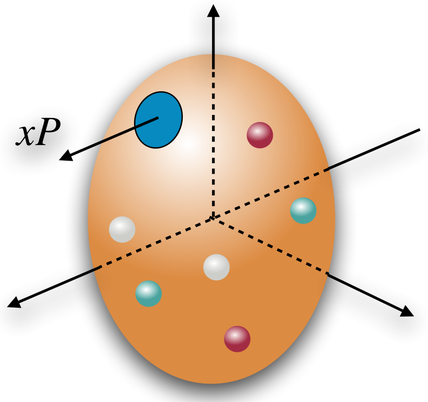
FT of GPDs

exclusive

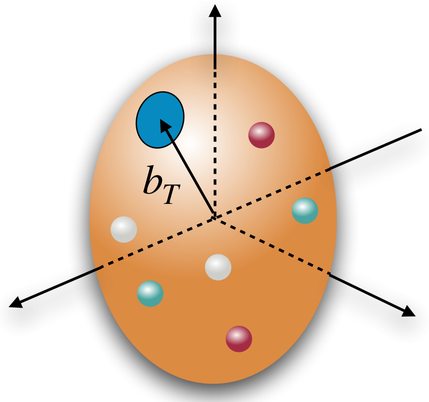


PDFs

(semi-)inclusive



FT of Form Factors



1D

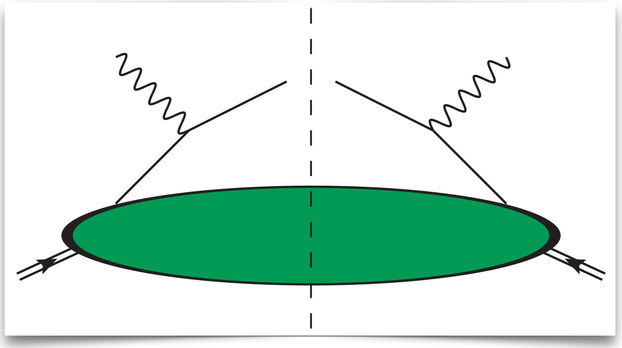
2D

$\vec{b}_\perp$ dependence

$\vec{k}_\perp$ dependence

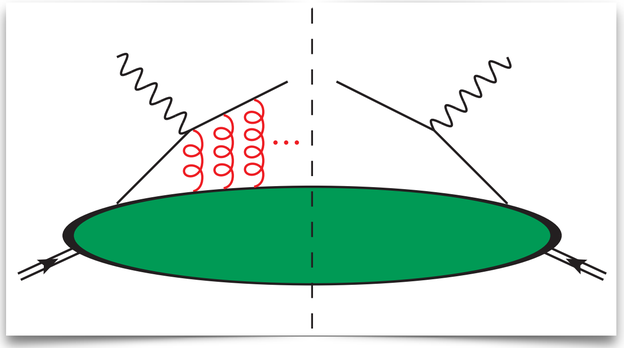
these two variables are NOT Fourier conjugate

Gauge links and processes dependence

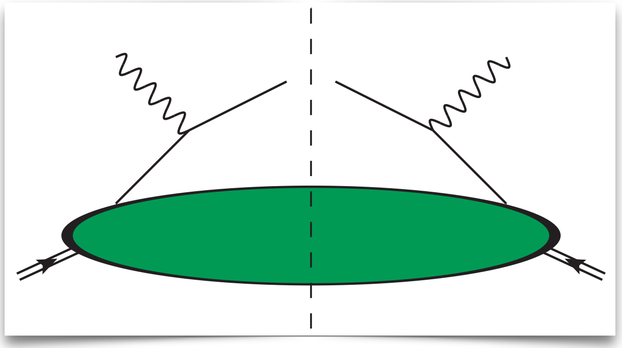


Gauge link (Wilson line)

Resummation of (calculable)
infinite gluon emissions



Gauge links and processes dependence



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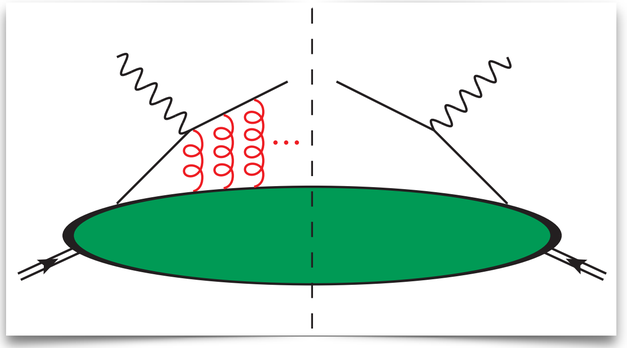
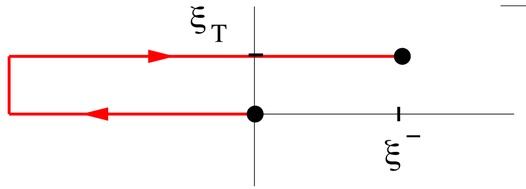
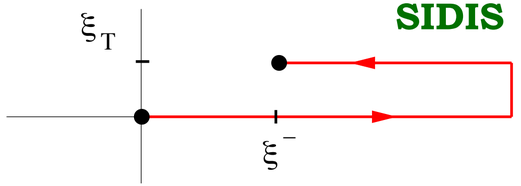


TMD PDFs

- *Transverse* gauge link not eliminated by gauge choice
- **Staple-like** gauge link (not unique!)
- ¡! **Process dependence**

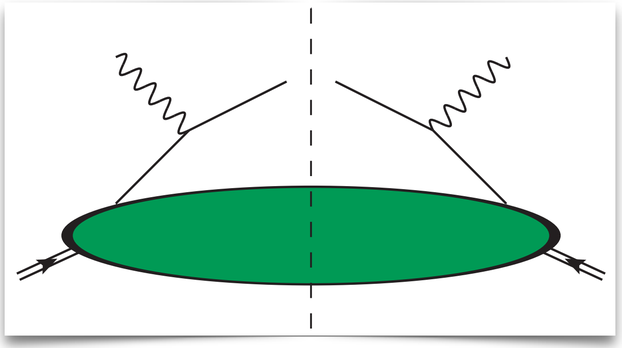


Drell-Yan



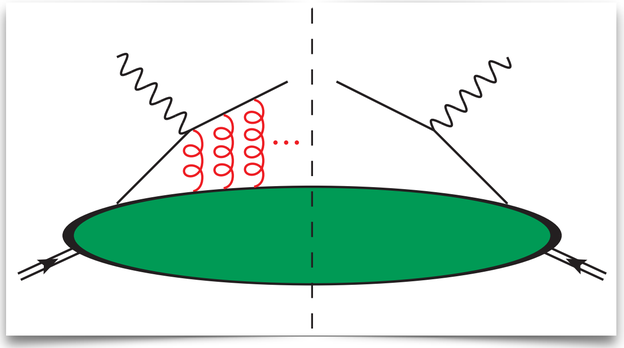
SIDIS

Gauge links and processes dependence



Gauge link (Wilson line)

Resummation of (calculable)
infinite gluon emissions



Link with precision QCD

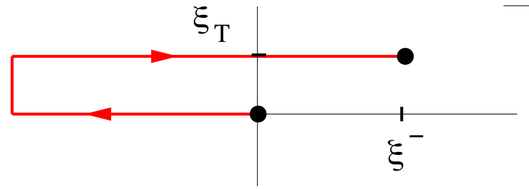
- *Subtraction* techniques
- WL → factorized **soft & collinear emissions**
- **Infrared-safe** observables

🙂 **Hadron structure** and **precision QCD**
speak the same language

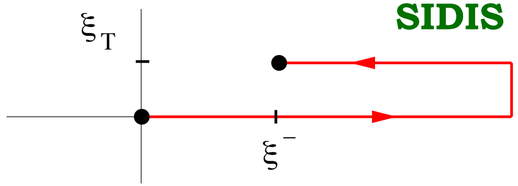
TMD PDFs

- *Transverse* gauge link not eliminated by gauge choice
- **Staple-like** gauge link (not unique!)

¡! **Process dependence**



Drell-Yan



SIDIS

Wilson lines

7/13 26.6.-24

Wilson line

$$\partial_t \mathcal{U} = igA^- \mathcal{U} \Rightarrow \mathcal{U}(x) = \mathbb{P}\, e^{ig\int_{-\infty}^{x^+} dz^+ A^-(x^-, z^+, x_\perp)}\, \mathcal{U}_0$$

$\mathbb{P}$ = path ordering

2. Wilson line $V(x_\perp)$

- $\mathcal{U}_0$ is d_R-component vector: color state of incoming particle
- $V(x_\perp)$ is $d_R \times d_R$ color matrix
- Path-ordering: $\mathbb{P}(A(x^+)B(z^+)) = \begin{cases} A(x^+)B(z^+) & \text{if } x^+ > z^+ \\ B(z^+)A(x^+) & \text{if } x^+ < z^+ \end{cases}$

  but better practical definition: $\mathbb{P}$ = defined by differential equation
- Eikonal $x_\perp$ does not change

  ↙ $p_\perp$ does change, but p^+ so large that $x_\perp$ does not have time to change

Wave out $\sim V(x_\perp) \cdot$ [Wave in] $\Rightarrow$ $V(x_\perp) - 1$ = scattering amplitude

Wilson line: The light-like Wilson line is the eikonal (= high energy) scattering amplitude in a color field

🔗 [Lectures by Tuomas Lappi]

Wilson lines

🔗 [Lectures by Edmond Iancu]

Wilson lines

- In the eikonal approx, the quark S-matrix reduces to a Wilson line

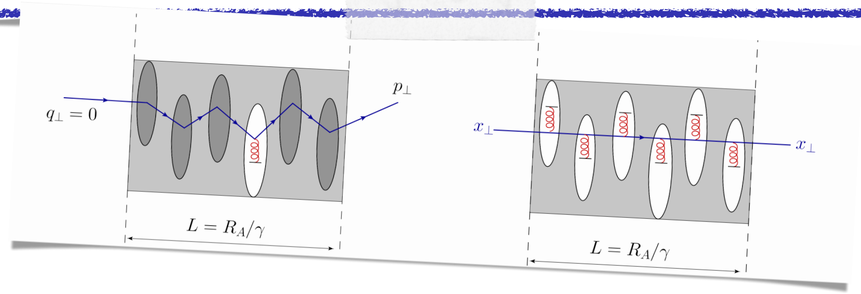
$$\hat{S}_q = \mathrm{T}\, e^{i\int d^4x\, \mathcal{L}_{\rm int}(x)} \quad \text{with} \quad \mathcal{L}_{\rm int}(x) = j_a^\mu(x) A_\mu^a(x)$$

  - in general, j_a^μ and A_a^μ are quantum operators; e.g. $j_a^\mu = g\bar\psi\gamma^\mu t^a\psi$
  - at high energy they can be replaced by their classical expectation values: their fluctuations are frozen
- The (classical) color current density of the right-moving quark: $v^\mu = \delta^{\mu+}$

$$j_a^\mu(x) = \delta^{\mu+} g t^a \delta(x^-)\delta^{(2)}(\boldsymbol{x}_\perp - \boldsymbol{x}_\perp^0) : \quad \text{indep. of LC time } x^+$$

$$\hat{S}_q \simeq V(\boldsymbol{x}_\perp^0) \quad \text{with} \quad V(\boldsymbol{x}_\perp) \equiv \mathrm{T}\exp\left\{ig\int dx^+ A_a^-(x^+,\boldsymbol{x}_\perp)t^a\right\}$$

- An exponential : multiple scattering is resummed to all orders
- A unitary matrix: $VV^\dagger = 1 \Longrightarrow$ a rotation of the quark color state



🔗 [Lectures by Tuomas Lappi]

Gluon TMD PDFs: Gauge links & modified universality

* Gluon TMDs → more complicated structure with respect to quark staple links
* Factorization-preserving processes → two main kinds of modified universality
* Different classes of processes → distinct gluon TMDs, not related to each other

Gluon TMD PDFs: Gauge links & modified universality

* Gluon TMDs → more complicated structure with respect to quark staple links
* Factorization-preserving processes → two main kinds of modified universality
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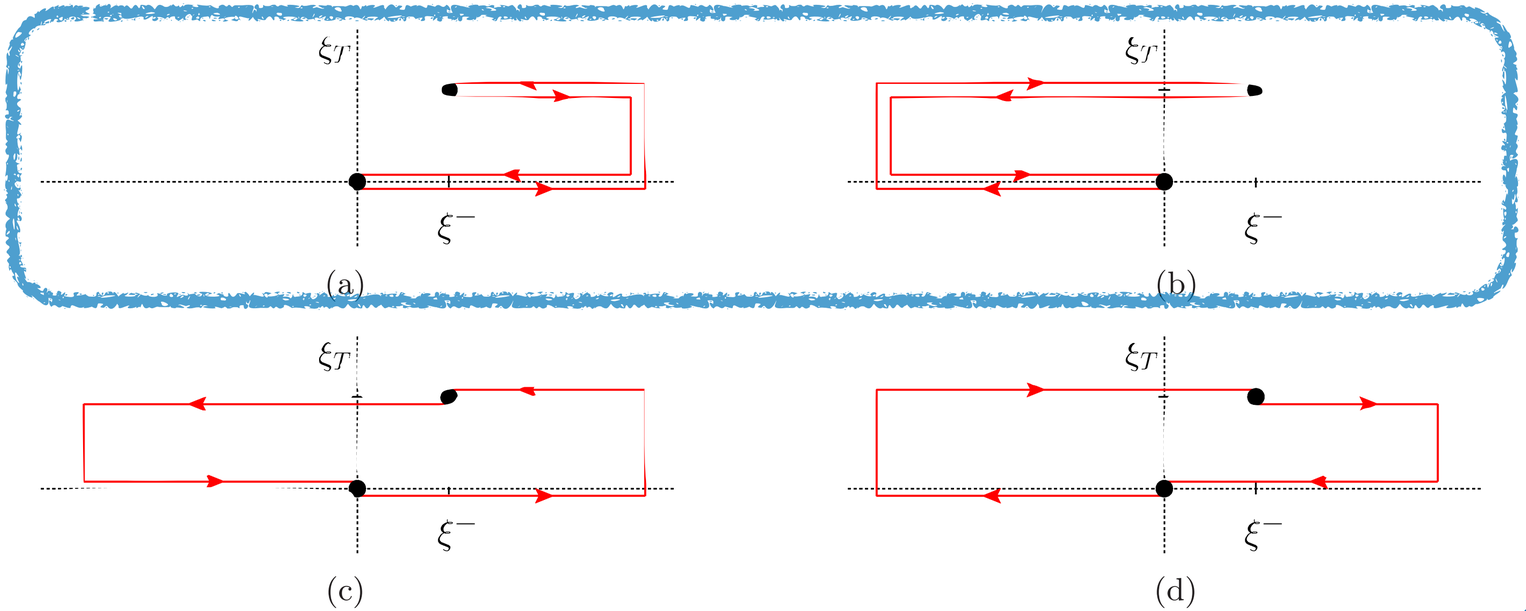
***f*-type (WW)**

(a) $[+,+]$ or (b) $[-,-]$



Gluon TMD PDFs: Gauge links & modified universality

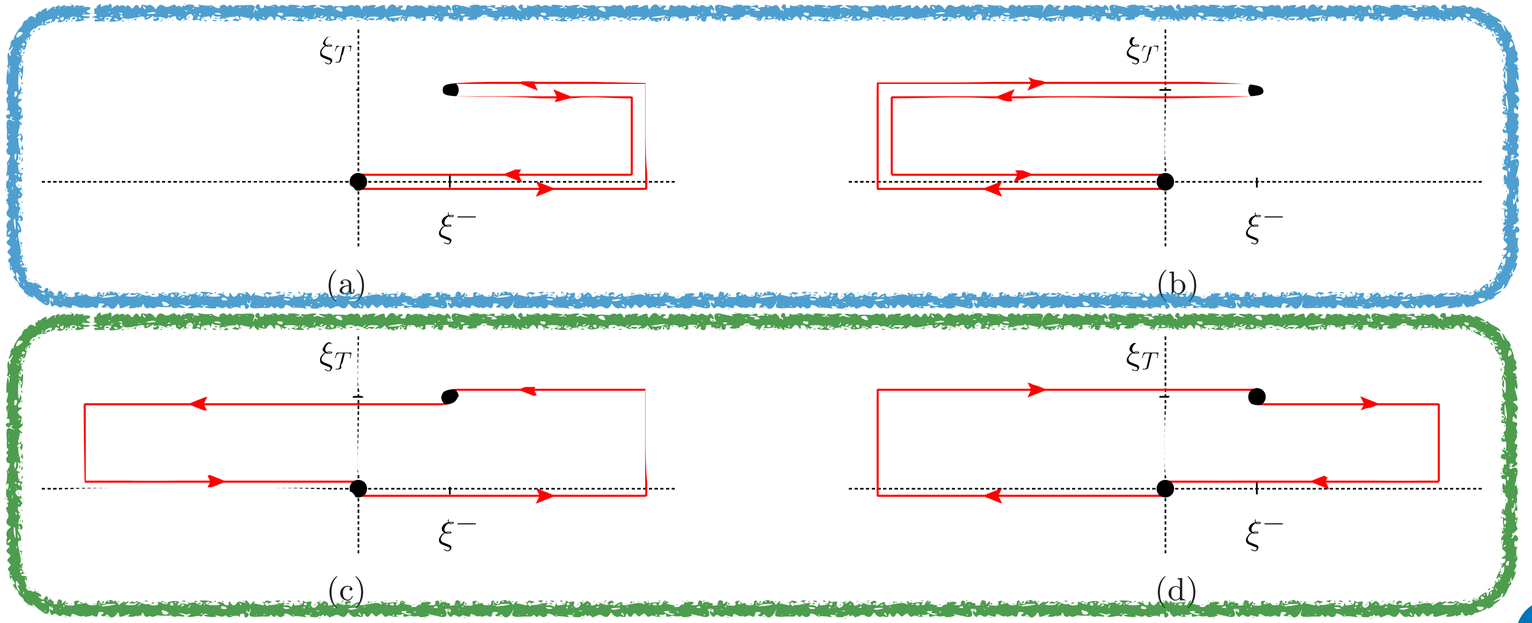
* Gluon TMDs → more complicated structure with respect to quark staple links
* Factorization-preserving processes → two main kinds of modified universality
* Different classes of processes → distinct gluon TMDs, not related to each other

f-type (WW)

(a) $[+,+]$ or (b) $[-,-]$

d-type (dipole)

(c) $[+,-]$ or (d) $[-,+]$



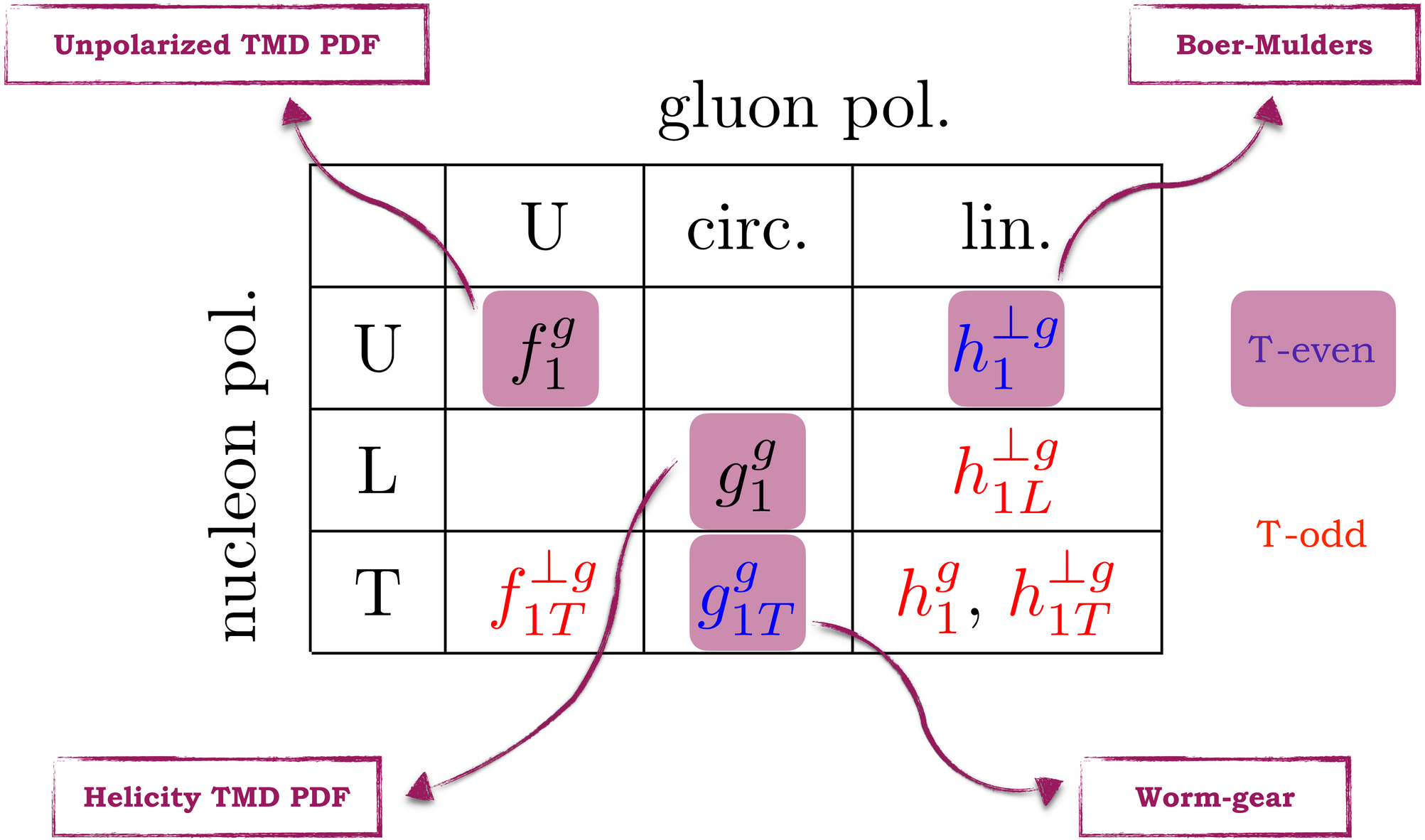
Gluon TMD PDFs at leading twist

nucleon pol. \ gluon pol.	U	circ.	lin.
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g,\ h_{1T}^{\perp g}$

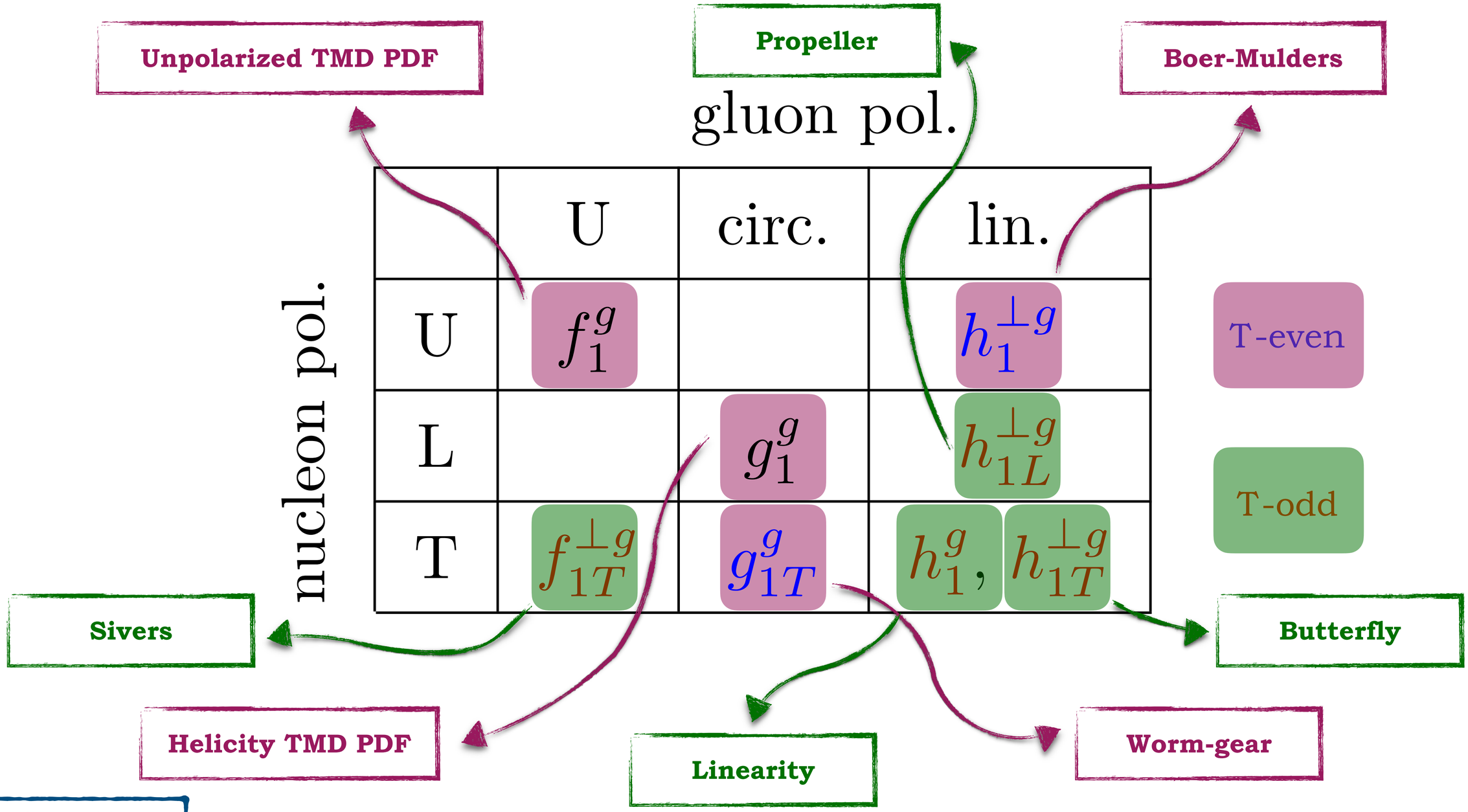
T-even

T-odd

Gluon TMD PDFs at leading twist



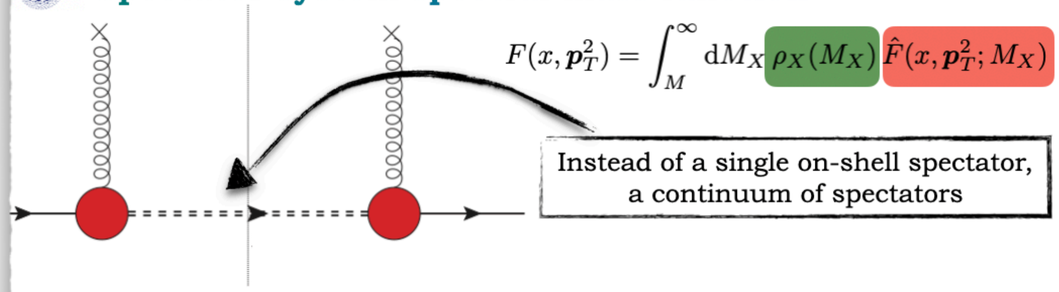
Gluon TMD PDFs at leading twist



Spectator-model gluon TMD PDFs

Our model at a glance

Spectator-system spectral-mass function



$$F(x, \boldsymbol{p}_T^2) = \int_M^\infty \mathrm{d}M_X \, \rho_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$$

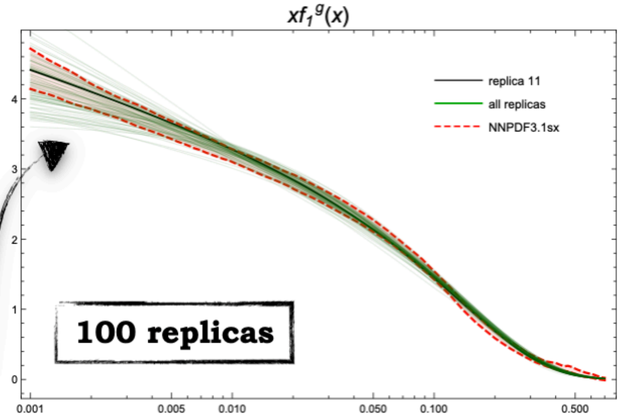
Instead of a single on-shell spectator, a continuum of spectators

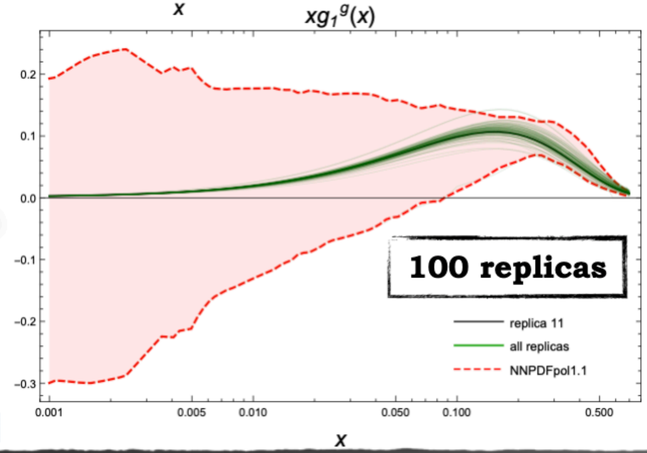
Spectral function **learns** small- and moderate-x info encoded in **NNPDF** collinear parametrizations (`NNPDF3.1sx + NNPDFpol1.1`)

- ☑ **Simultaneous fit** of f_1 and g_1 PDFs
- ☑ Inclusion of small-x resummation effects (**BFKL**)
- ☑ Calculation of all leading-twist T-even gluon TMDs

Link with collinear factorization

p_T-integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution

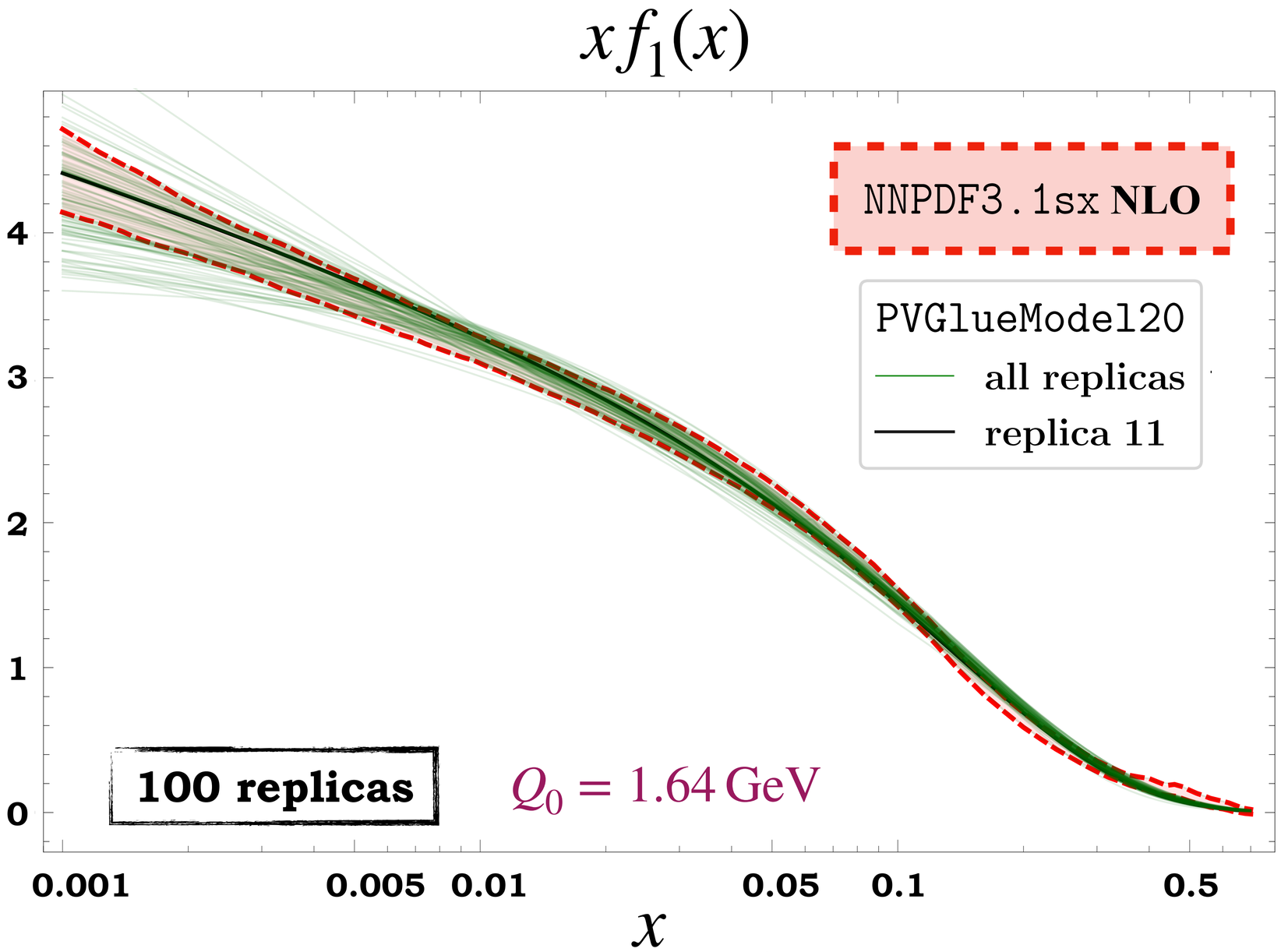




2.3 Modeling gluon TMDs 🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

🔗 [my talk at the EIC User Group Early Career Workshop 2022]

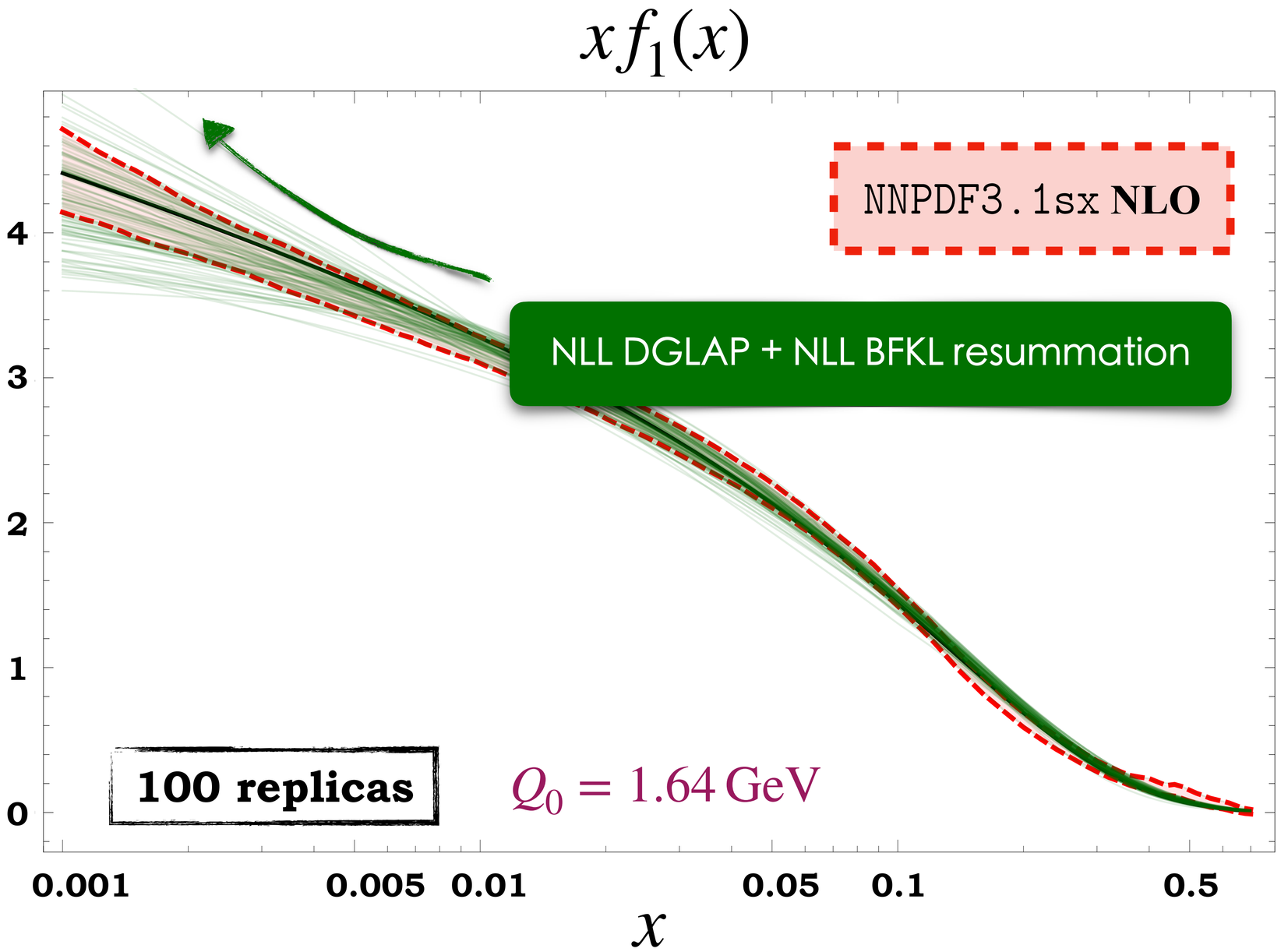
Unpolarized gluon collinear PDF



IV.2 Gluon TMDs

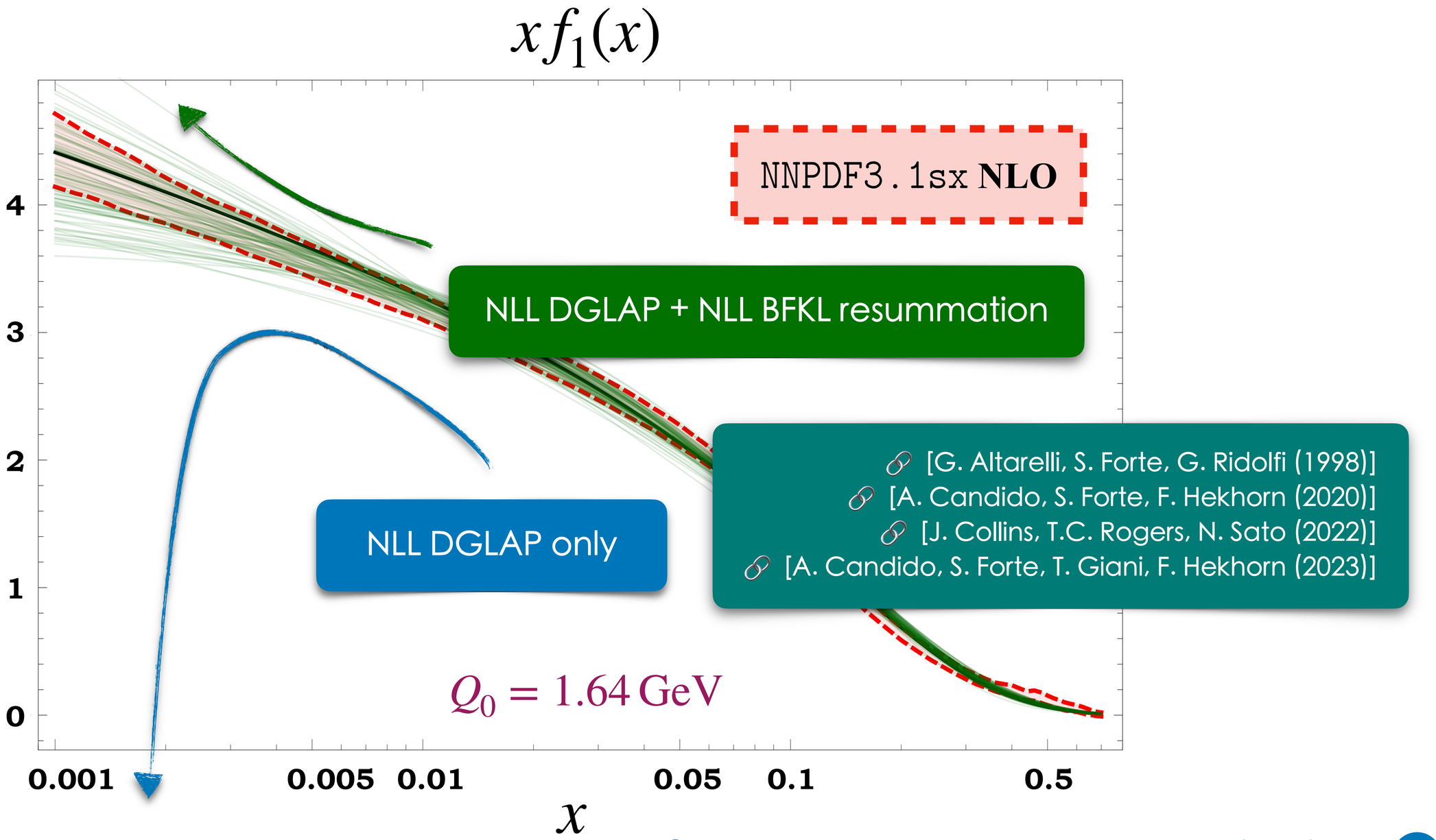
[A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

Unpolarized gluon collinear PDF



[A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

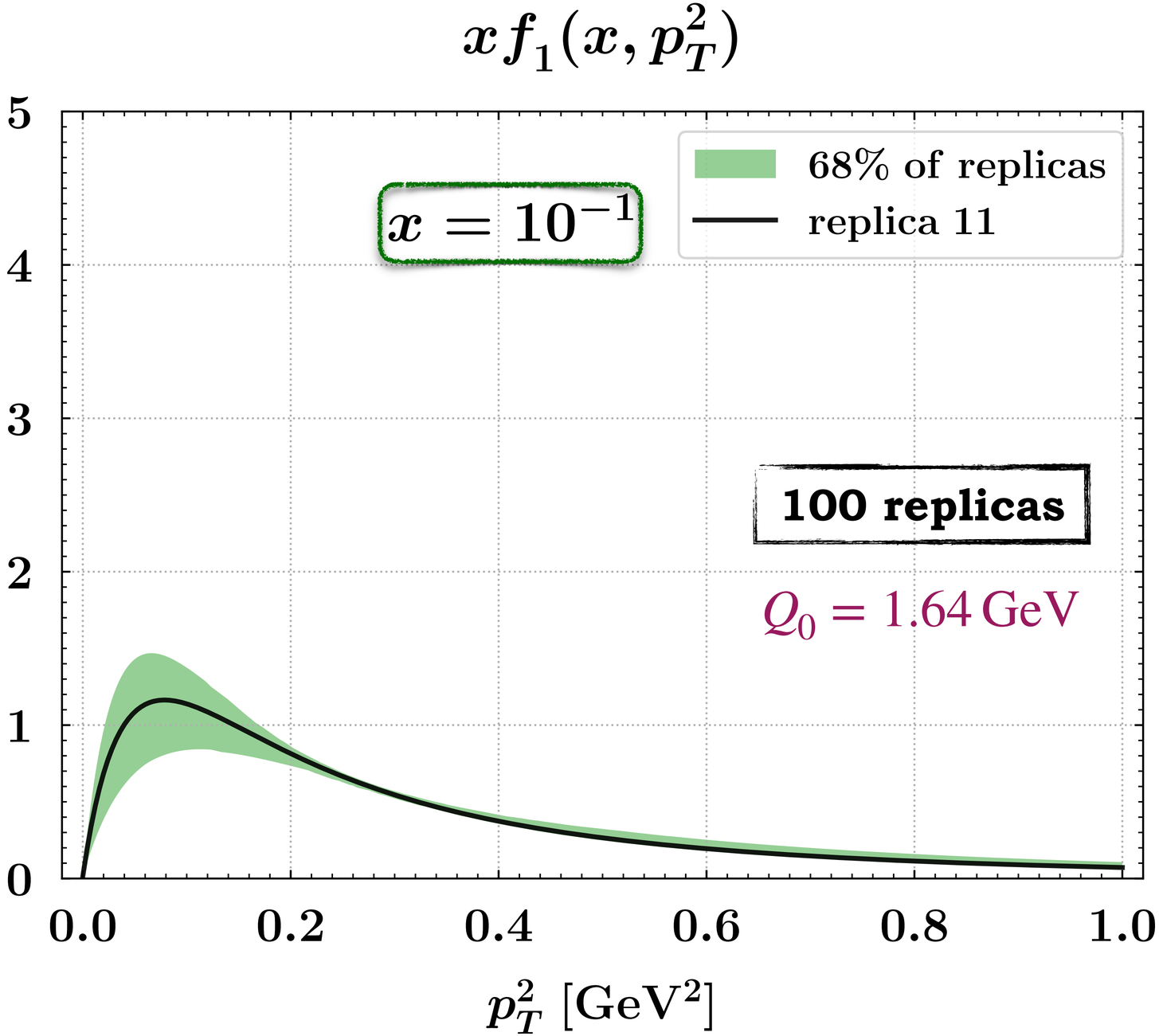
Unpolarized gluon collinear PDF



[A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

Unpolarized gluon TMD PDF

[A. Bacchetta, F.G.C., M. Radici, P. Taels (EPJC 2020)]



Unpolarized gluon TMD PDF

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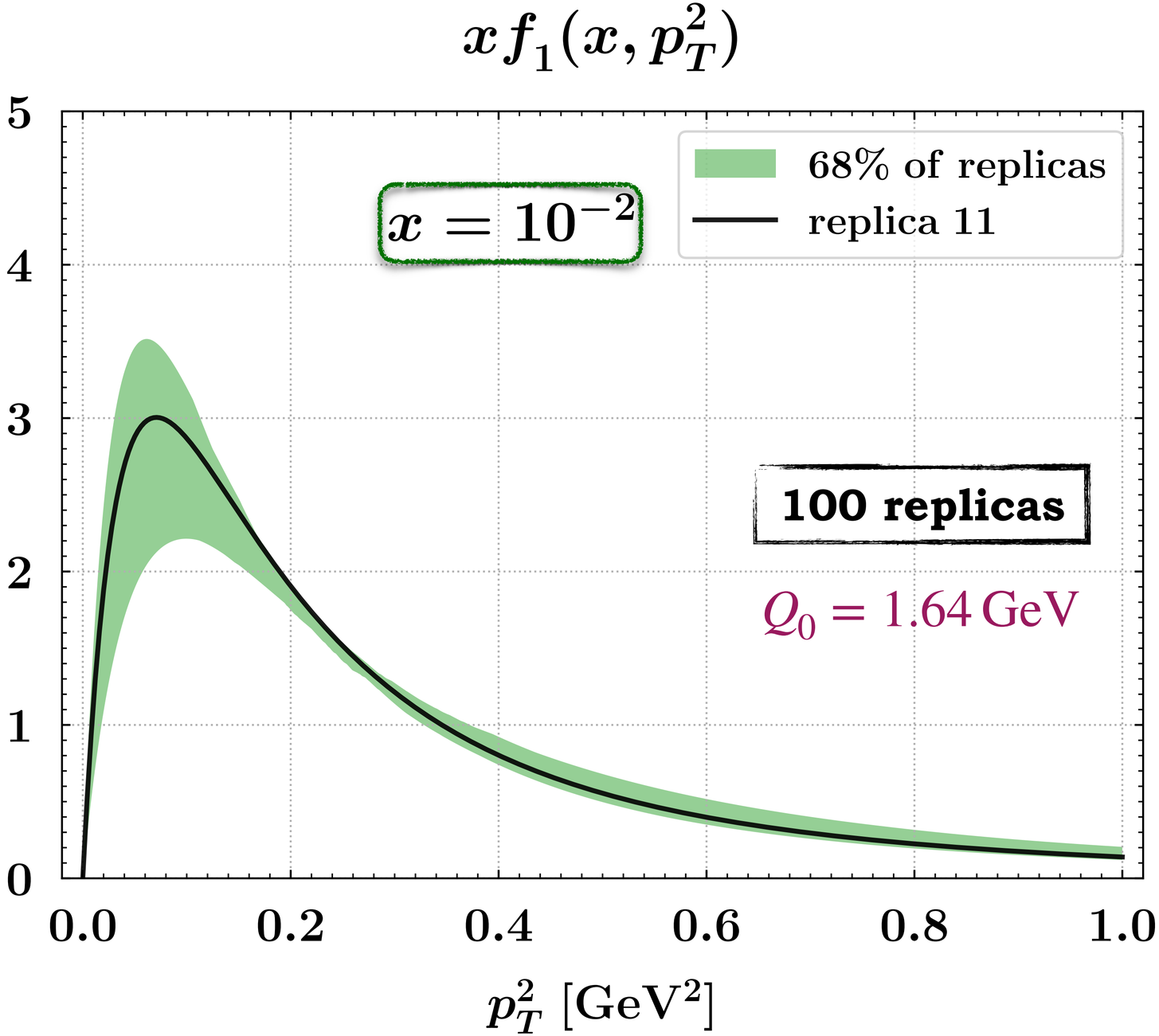

Unpolarized gluon TMD PDF

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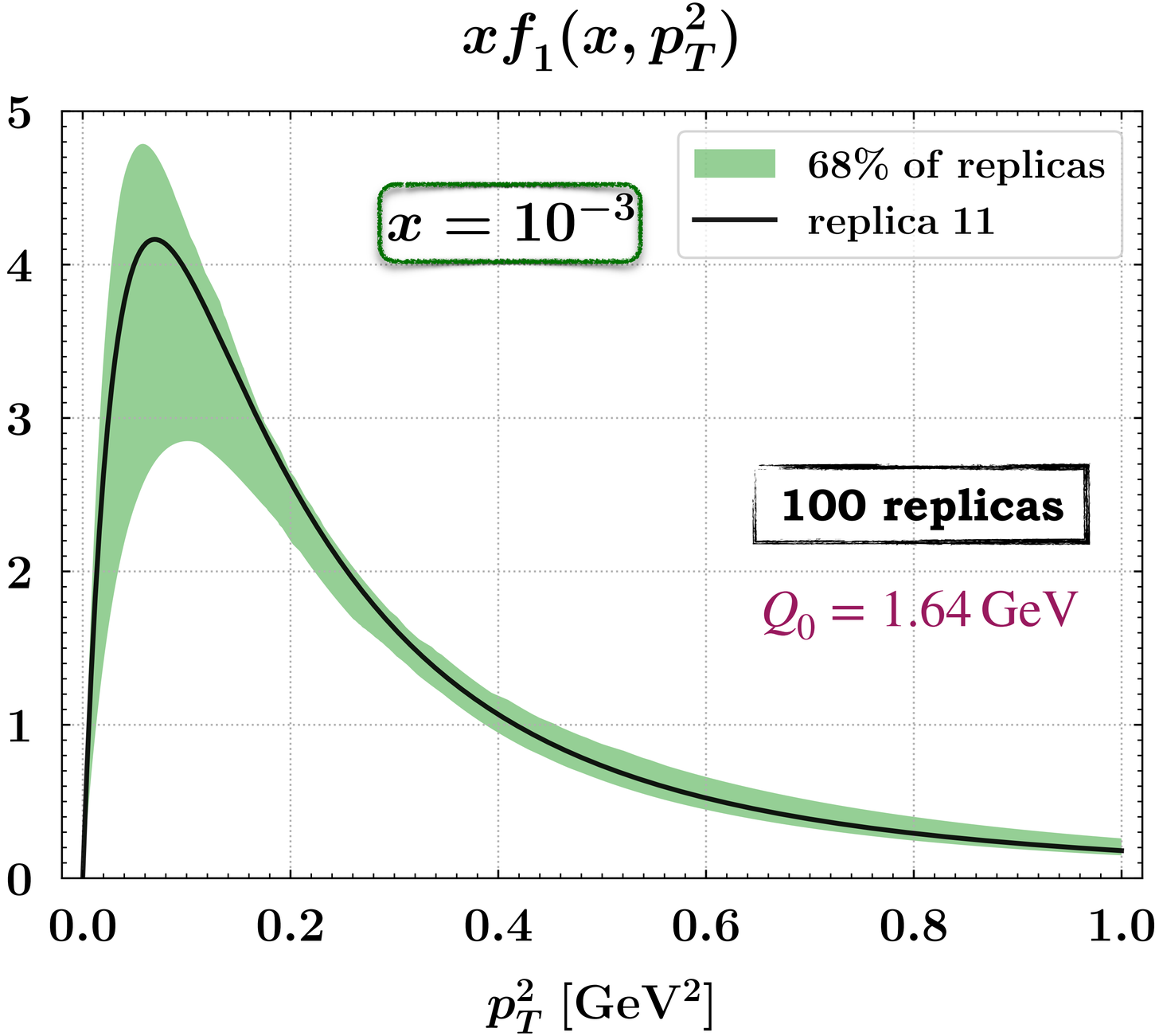
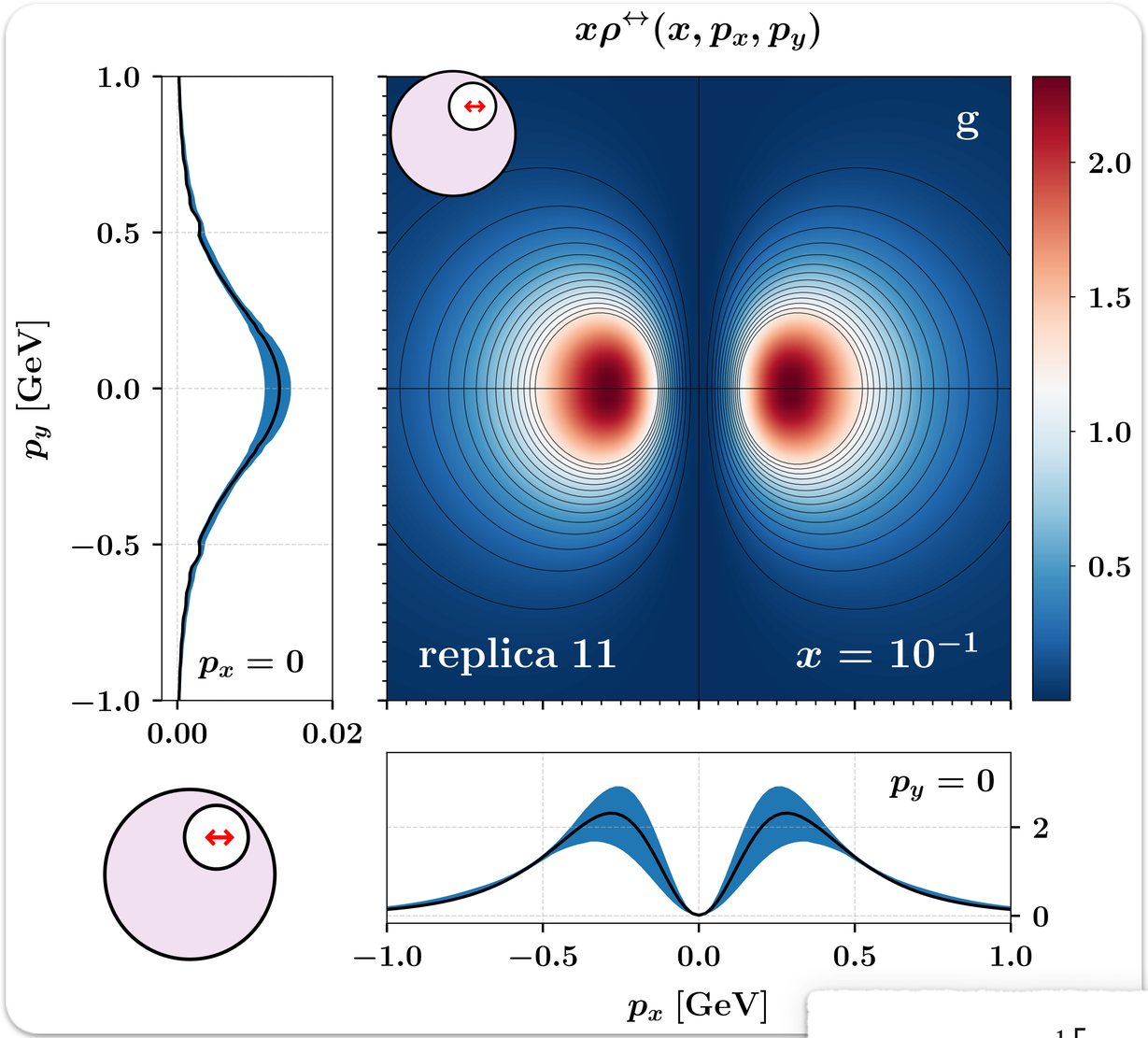


Gluon Boer-Mulders effect

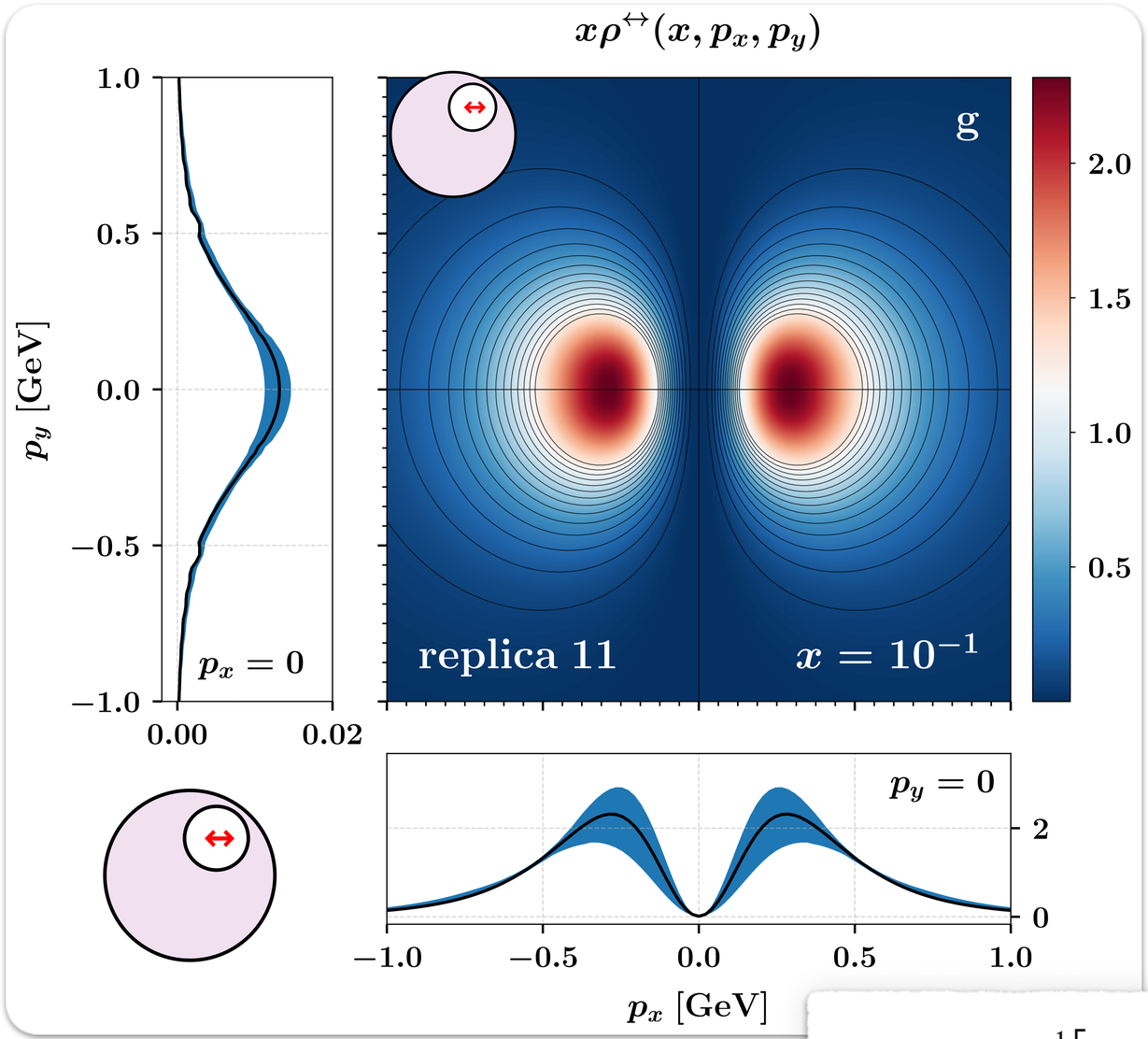
[A. Bacchetta, F.G.C., M. Radici, P. Taels (EPJC 2020)]

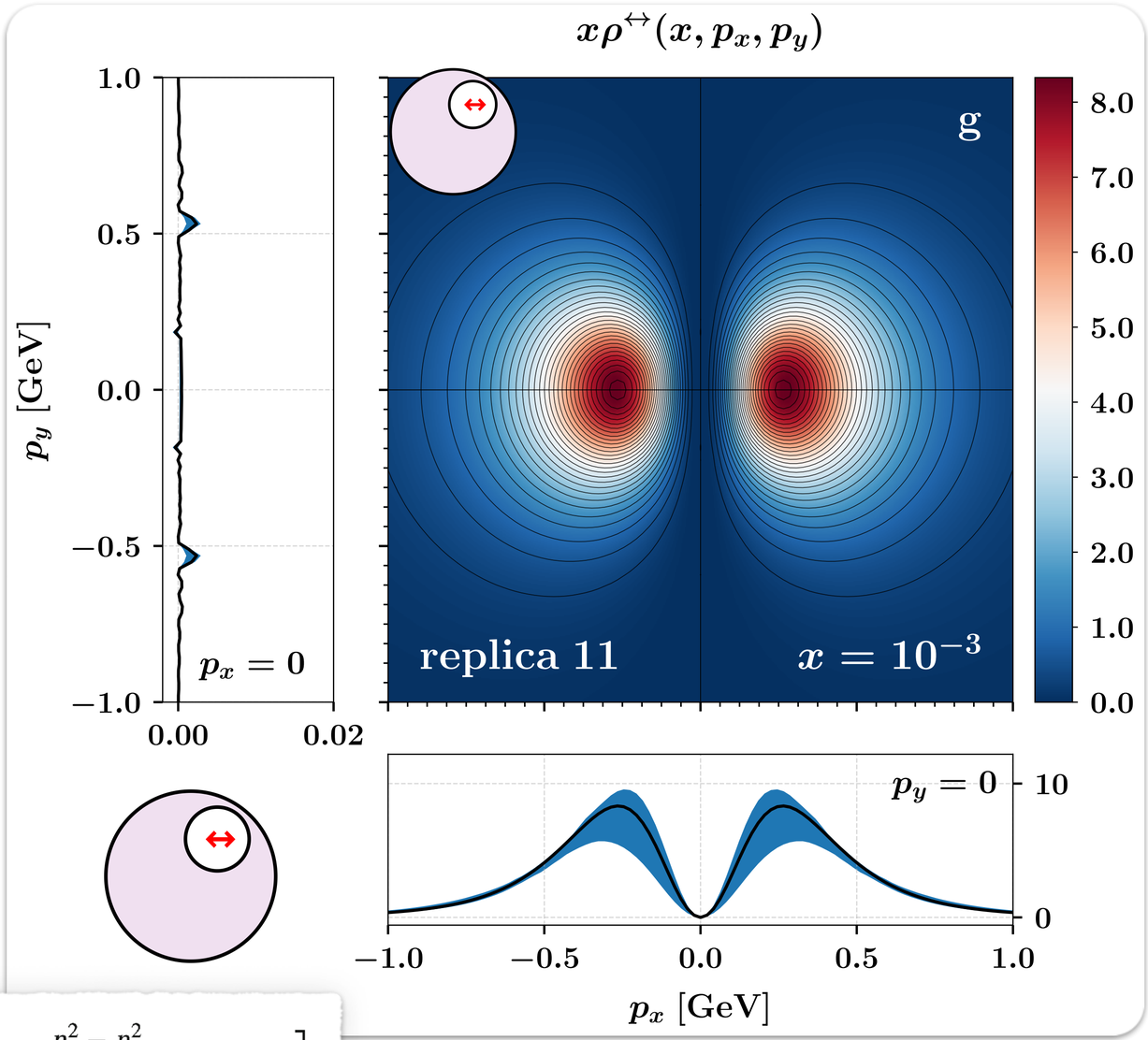


$$x\rho^{\leftrightarrow}(x, p_x, p_y) = \frac{1}{2}\left[x f_1^g(x, \boldsymbol{p}_T^2) + \frac{p_x^2 - p_y^2}{2M^2}\, x h_1^{\perp g}(x, \boldsymbol{p}_T^2)\right]$$

Gluon Boer-Mulders effect

[A. Bacchetta, F.G.C., M. Radici, P. Taels (EPJC 2020)]





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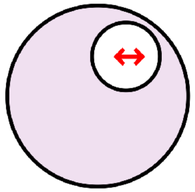
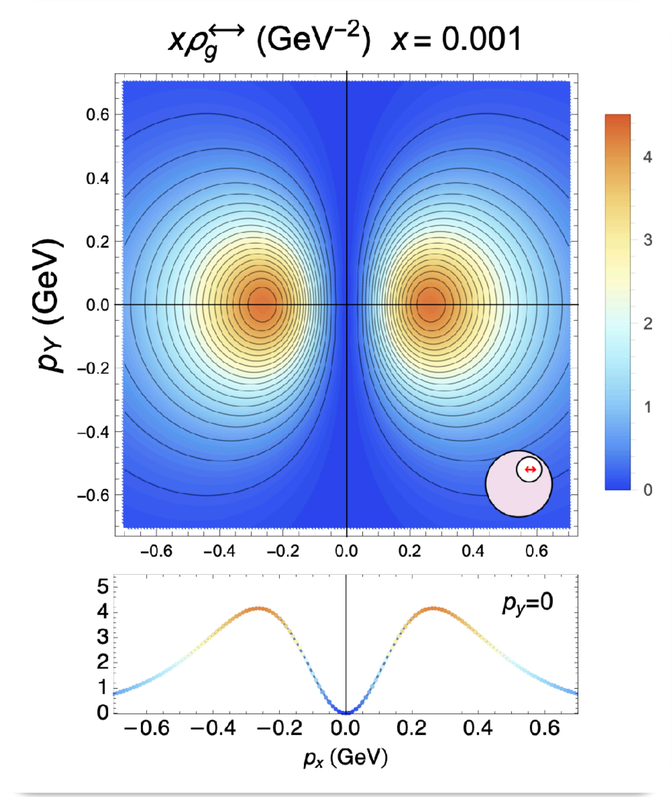
Boer-Mulders effect in unpolarized pp collisions



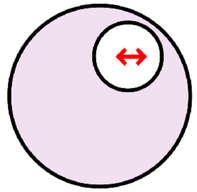
$$[\leftrightarrow/\mathbf{u}] \qquad f_1(x,p_x,p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x,p_x,p_y)$$

unpol. Boer-Mulders



🔗 [A. Bacchetta, F.G.C., M. Radici, P. Taels (2020)]

Boer-Mulders effect in unpolarized pp collisions



$$[\leftrightarrow/\mathbf{u}] \qquad f_1(x,p_x,p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^{\perp}(x,p_x,p_y)$$

unpol. Boer-Mulders

(Pseudo)scalar p_T-distributions: Higgs, $\eta_{c,b}$

$$\frac{d\sigma}{dq_T} \sim \Phi_A^U \, \Phi_B^U \, |\mathcal{M}|^2 \quad \text{at low transverse momentum for (pseudo)scalar state}$$

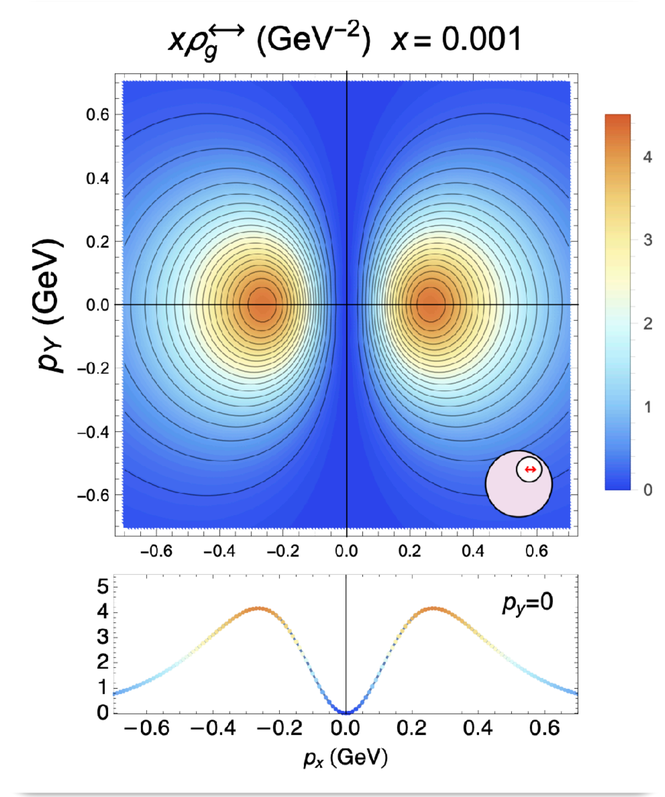
$$\sim \mathcal{C}[\, f_1^{g/A} \, f_1^{g/B} \,] \pm \mathcal{C}[\, h_1^{\perp g/A} \, h_1^{\perp g/B} \,]$$

$\mathcal{C}[\, f_1^{g/A} \, f_1^{g/B} \,]$: **unpolarized** gluons

$\mathcal{C}[\, h_1^{\perp g/A} \, h_1^{\perp g/B} \,]$: **lin. polarized** gluons

NRQCD

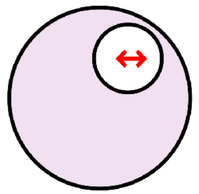
$$\frac{\text{CS}}{\text{CO}} \sim \frac{1}{v^4}$$



[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]
(Higgs+jet angular distributions) [D. Boer, C. Pisano (2015)]

[A. Bacchetta, F.G.C., M. Radici, P. Taels (2020)]

Boer-Mulders effect in unpolarized pp collisions



$$[\leftrightarrow/\mathbf{u}] \qquad f_1(x,p_x,p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^{\perp}(x,p_x,p_y)$$

unpol. — Boer-Mulders

(Pseudo)scalar p_T-distributions: Higgs, $\eta_{c,b}$

$$\frac{d\sigma}{dq_T} \sim \Phi_A^U \, \Phi_B^U \, |\mathcal{M}|^2 \quad \text{at low transverse momentum for (pseudo)scalar state}$$

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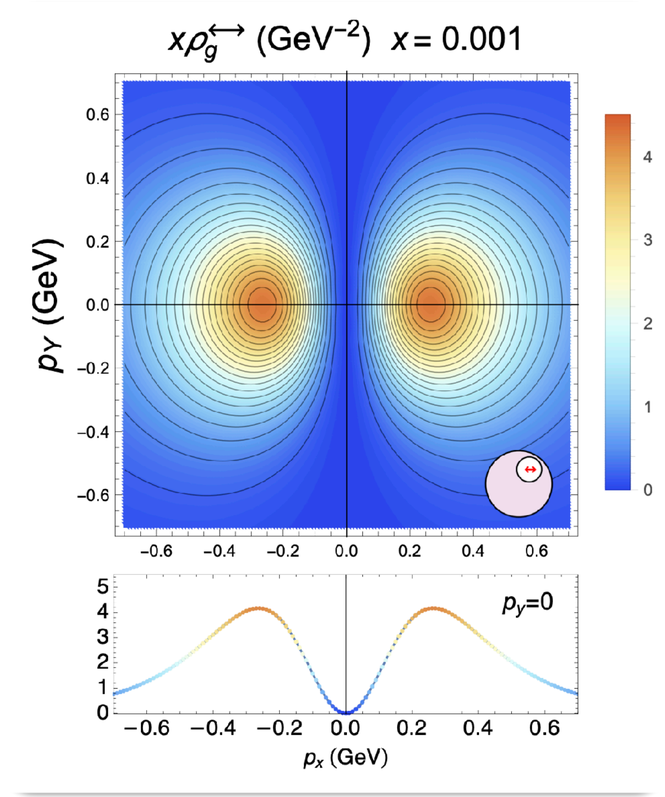
$\mathcal{C}[\, f_1^{g/A} \, f_1^{g/B} \,]$: **unpolarized** gluons

$\mathcal{C}[\, h_1^{\perp g/A} \, h_1^{\perp g/B} \,]$: **lin. polarized** gluons

NRQCD

$$\frac{\text{CS}}{\text{CO}} \sim \frac{1}{v^4}$$



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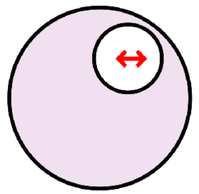
(Higgs+jet angular distributions) [D. Boer, C. Pisano (2015)]

[A. Bacchetta, F.G.C., M. Radici, P. Taels (2020)]

Model prediction at low x

$$\frac{f_1^g(x,p_T^2)}{h_1^{\perp g}(x,p_T^2)} \underset{x\to 0^+}{\sim} \text{constant}$$

Boer-Mulders effect in unpolarized pp collisions



$$[\leftrightarrow/\mathbf{u}] \qquad f_1(x,p_x,p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^{\perp}(x,p_x,p_y)$$

unpol. — Boer-Mulders

(Pseudo)scalar p_T-distributions: Higgs, $\eta_{c,b}$

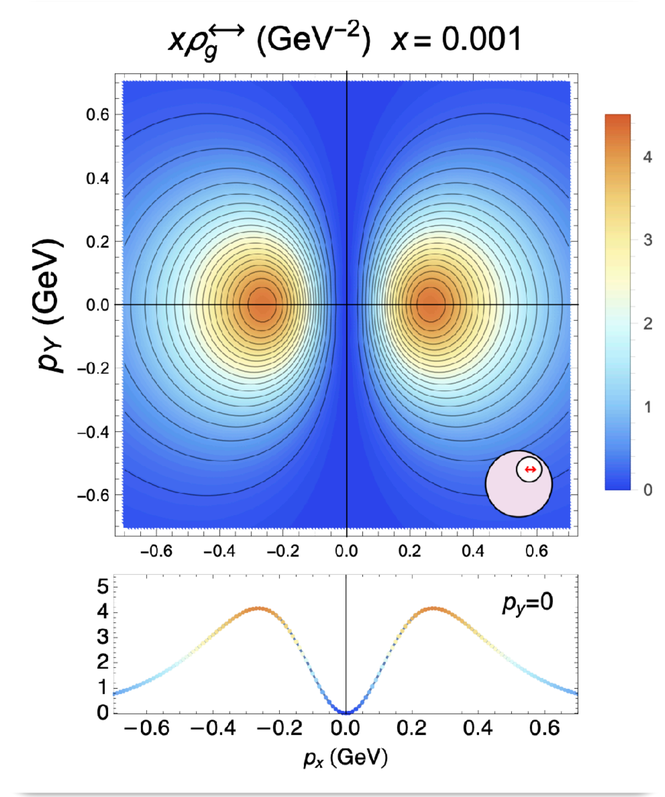
$$\frac{d\sigma}{dq_T} \sim \Phi_A^U \, \Phi_B^U \, |\mathcal{M}|^2 \quad \text{at low transverse momentum for (pseudo)scalar state}$$

$$\sim \mathcal{C}[\, f_1^{g/A} \, f_1^{g/B} \,] \pm \mathcal{C}[\, h_1^{\perp g/A} \, h_1^{\perp g/B} \,]$$

$\mathcal{C}[\, f_1^{g/A} \, f_1^{g/B} \,]$: **unpolarized** gluons; $\mathcal{C}[\, h_1^{\perp g/A} \, h_1^{\perp g/B} \,]$: **lin. polarized** gluons

NRQCD

$$\frac{\text{CS}}{\text{CO}} \sim \frac{1}{v^4}$$



[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]
(Higgs+jet angular distributions) [D. Boer, C. Pisano (2015)]

[A. Bacchetta, F.G.C., M. Radici, P. Taels (2020)]

Model prediction at low x

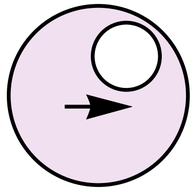
$$\frac{f_1^g(x,p_T^2)}{h_1^{\perp g}(x,p_T^2)} \underset{x\to 0^+}{\sim} \text{constant}$$

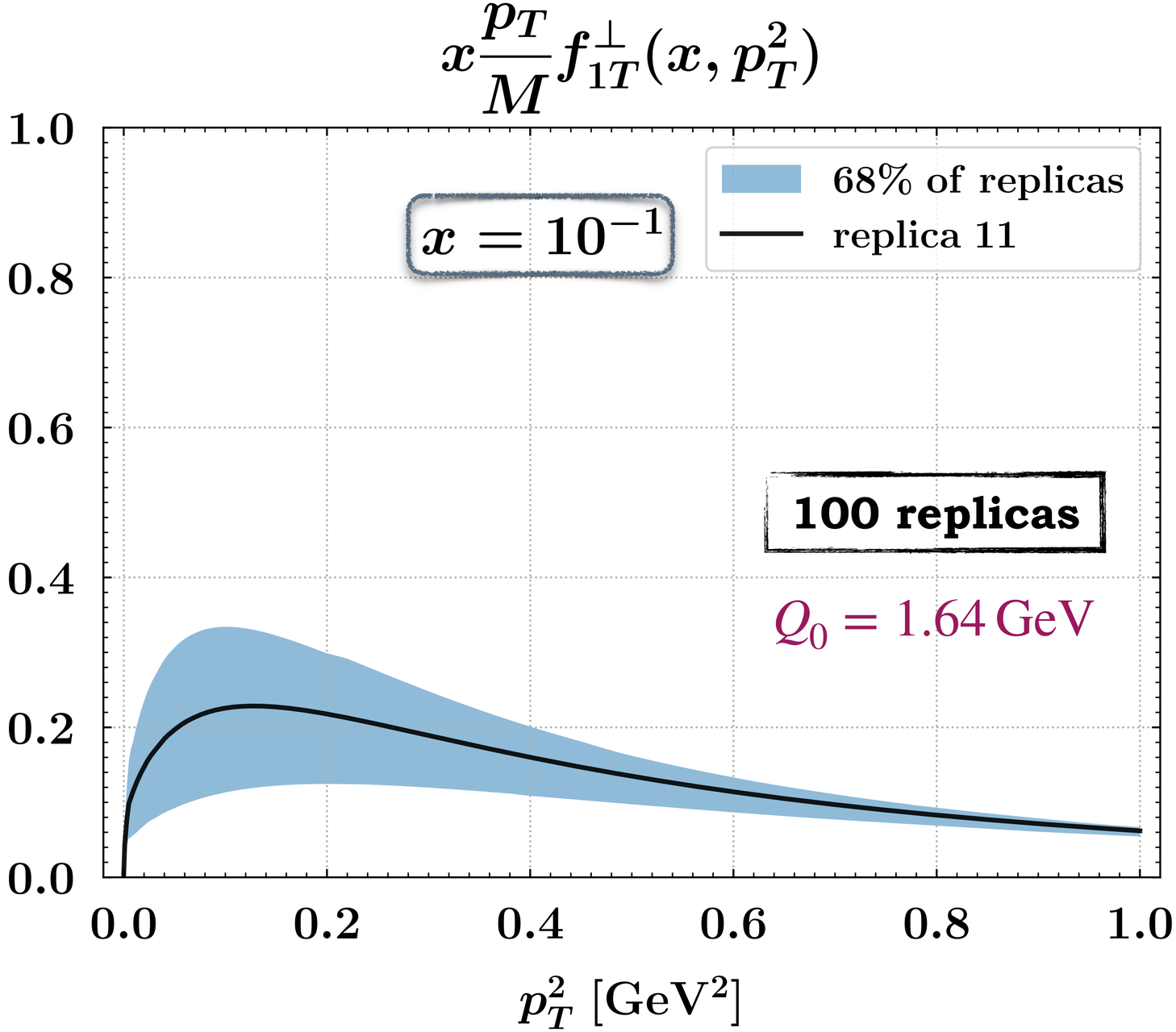
BFKL regime (linear low-x evolution)

$$f_1^g(x,p_T^2) = h_1^{\perp g}(x,p_T^2) + \text{higher twist}$$

f-type Sivers gluon TMD

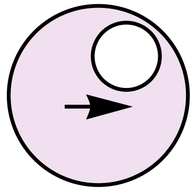
[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]

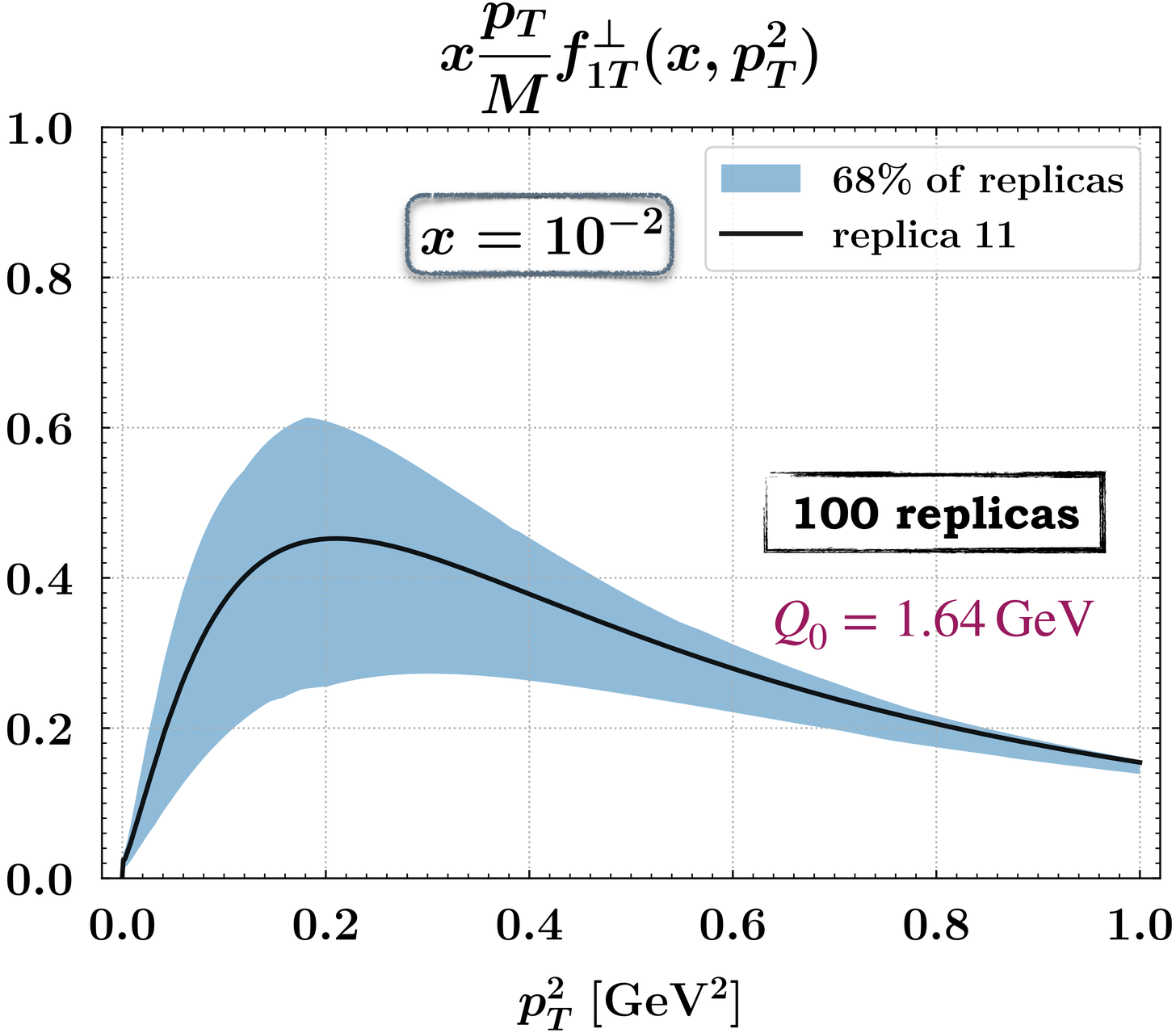




f-type Sivers gluon TMD

[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]





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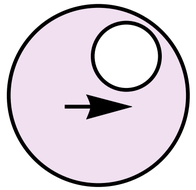
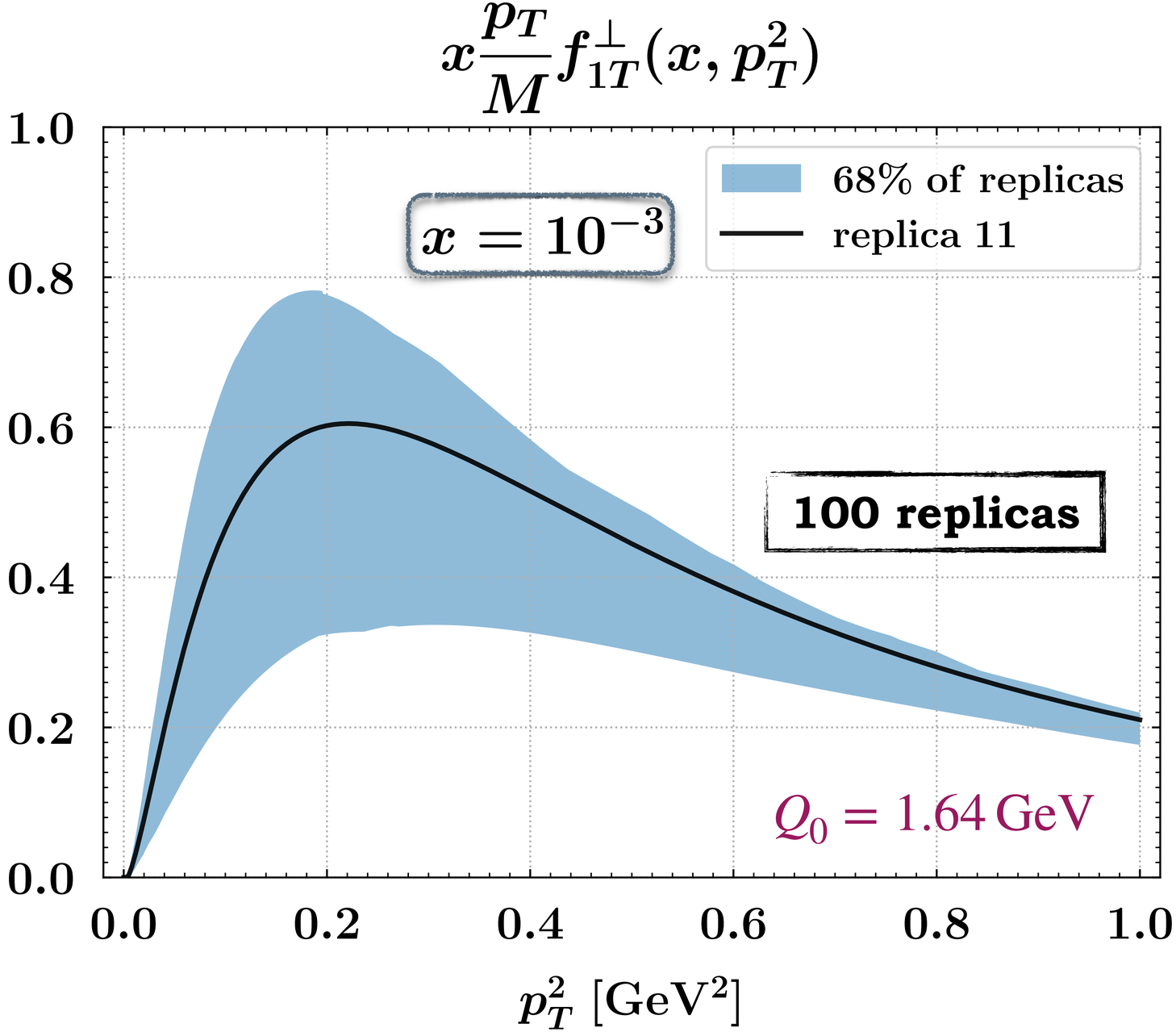
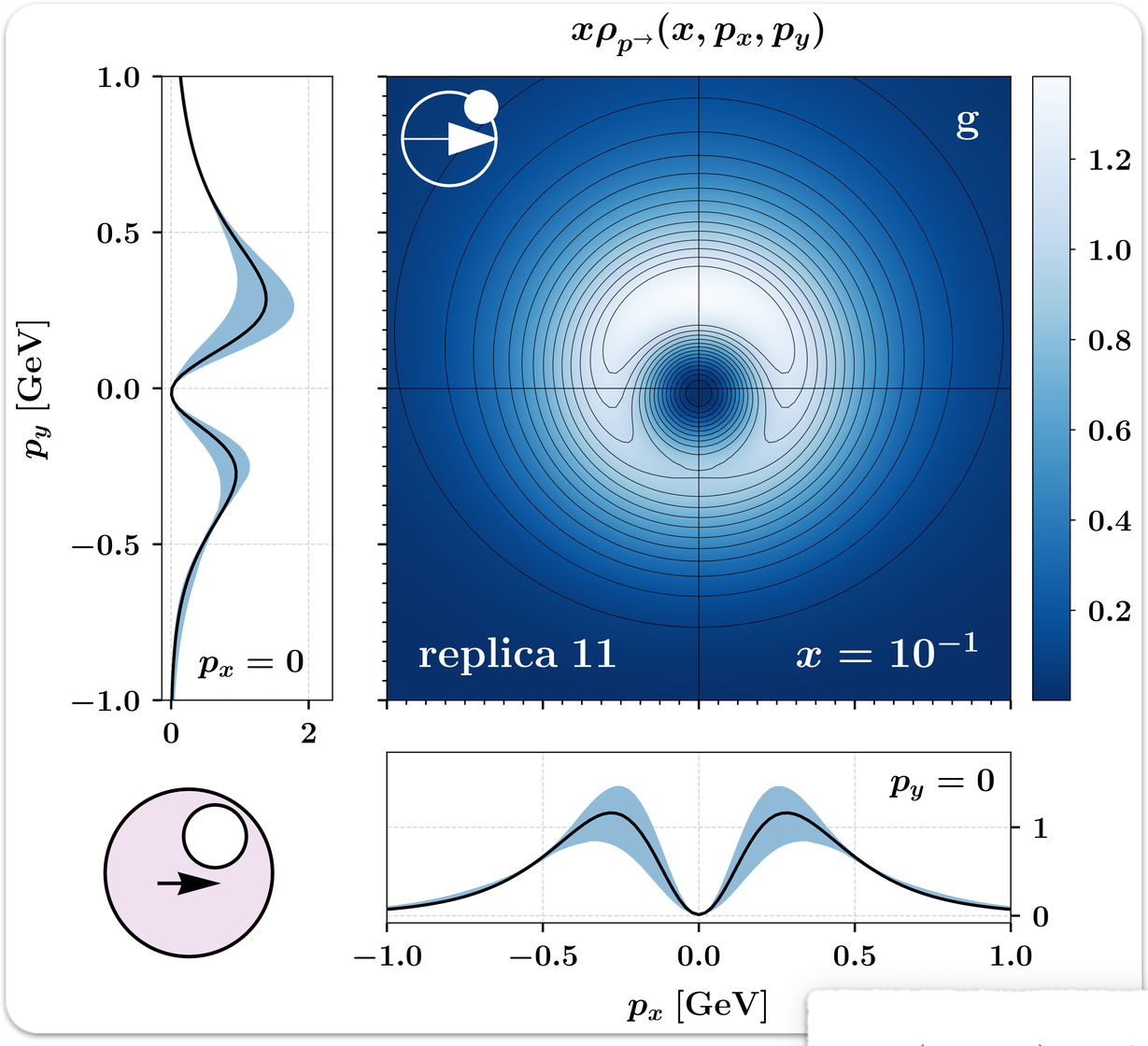
[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]





Gluon Sivers effect

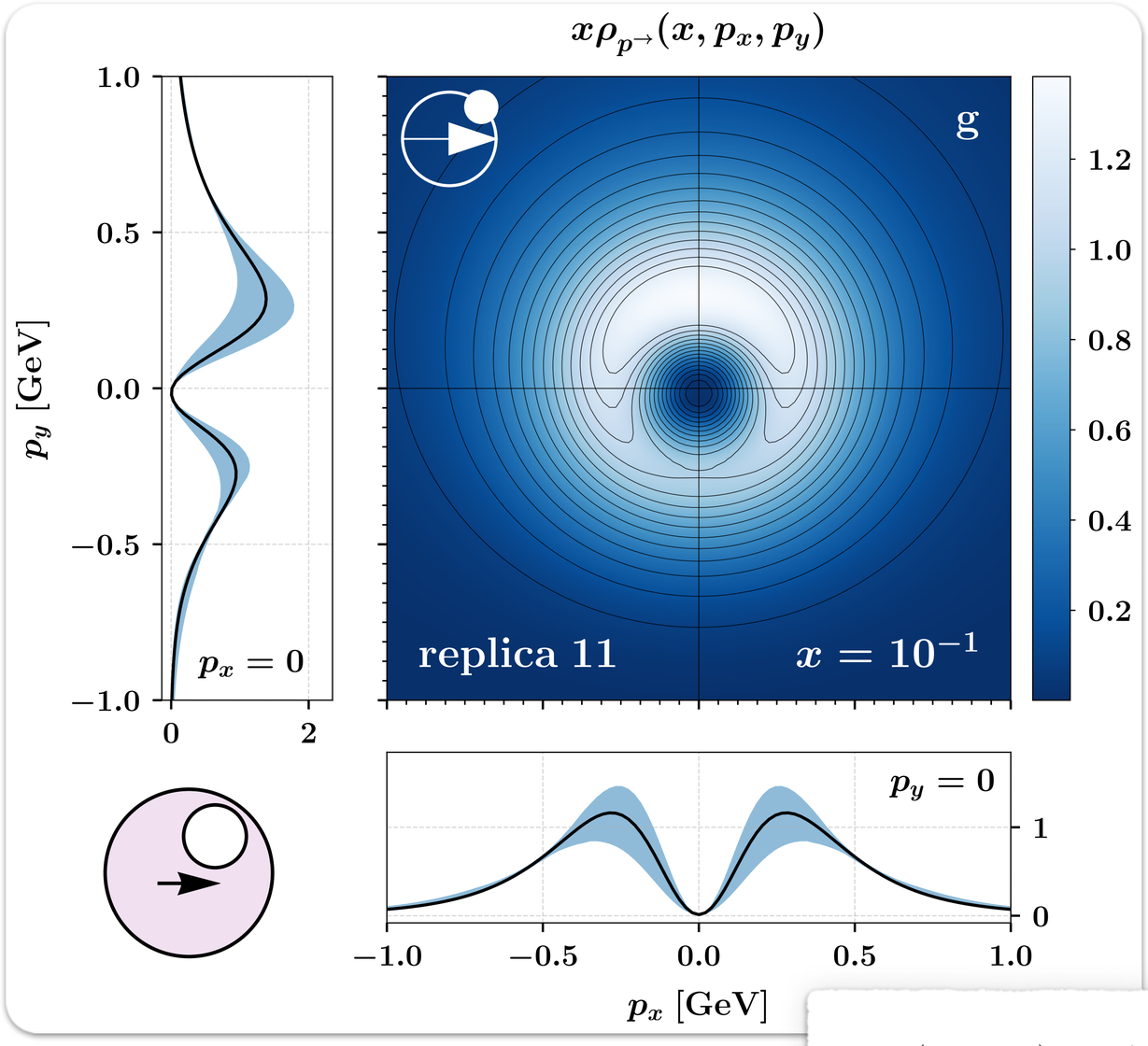
🔗 [A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]

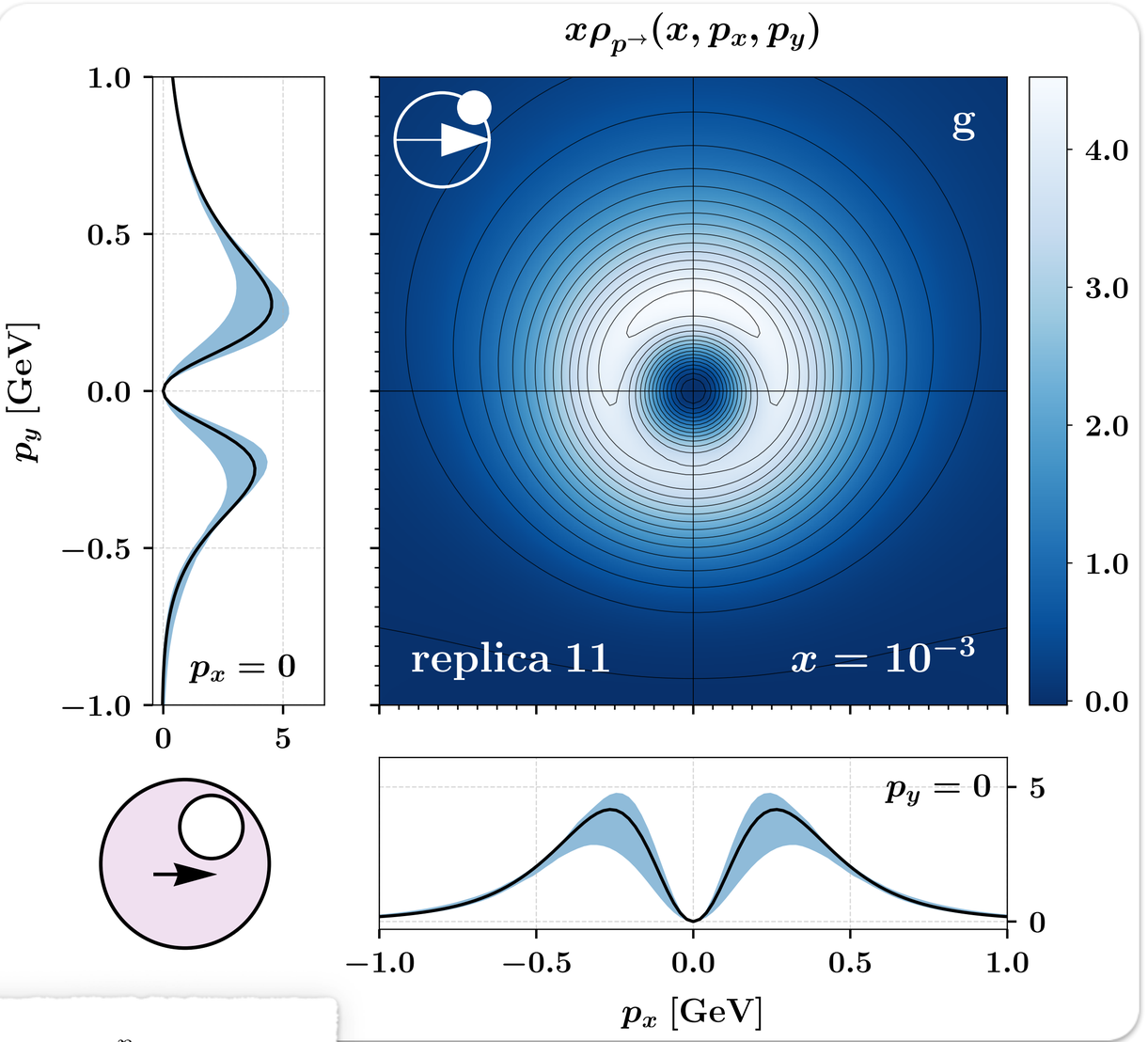


$$x\rho_{p^\rightarrow}(x, p_x, p_y) = x\, f_1(x, \boldsymbol{p}_T^2) + x\frac{p_y}{M}\, f_{1T}^\perp(x, \boldsymbol{p}_T^2)$$

Gluon Sivers effect

[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]

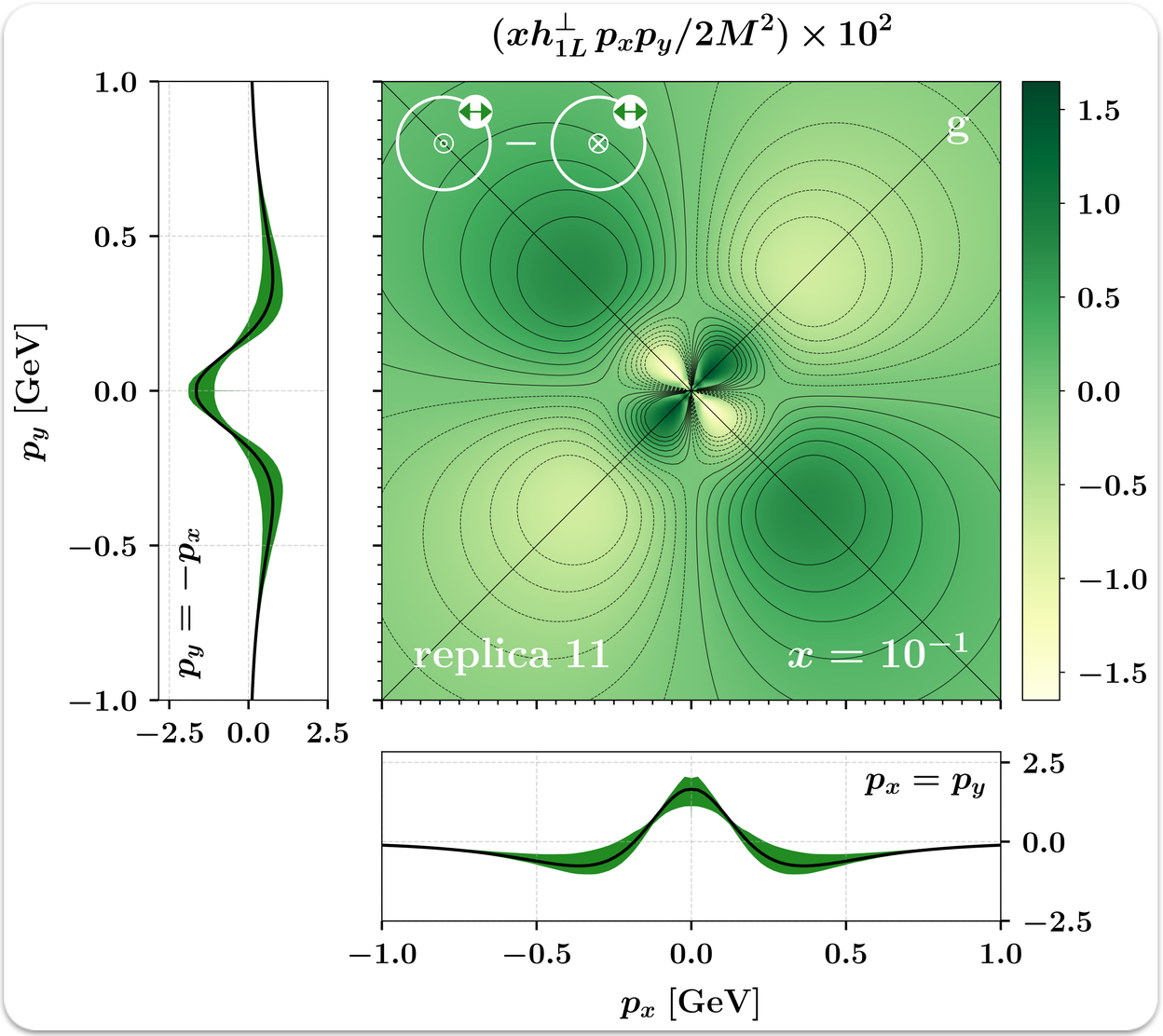


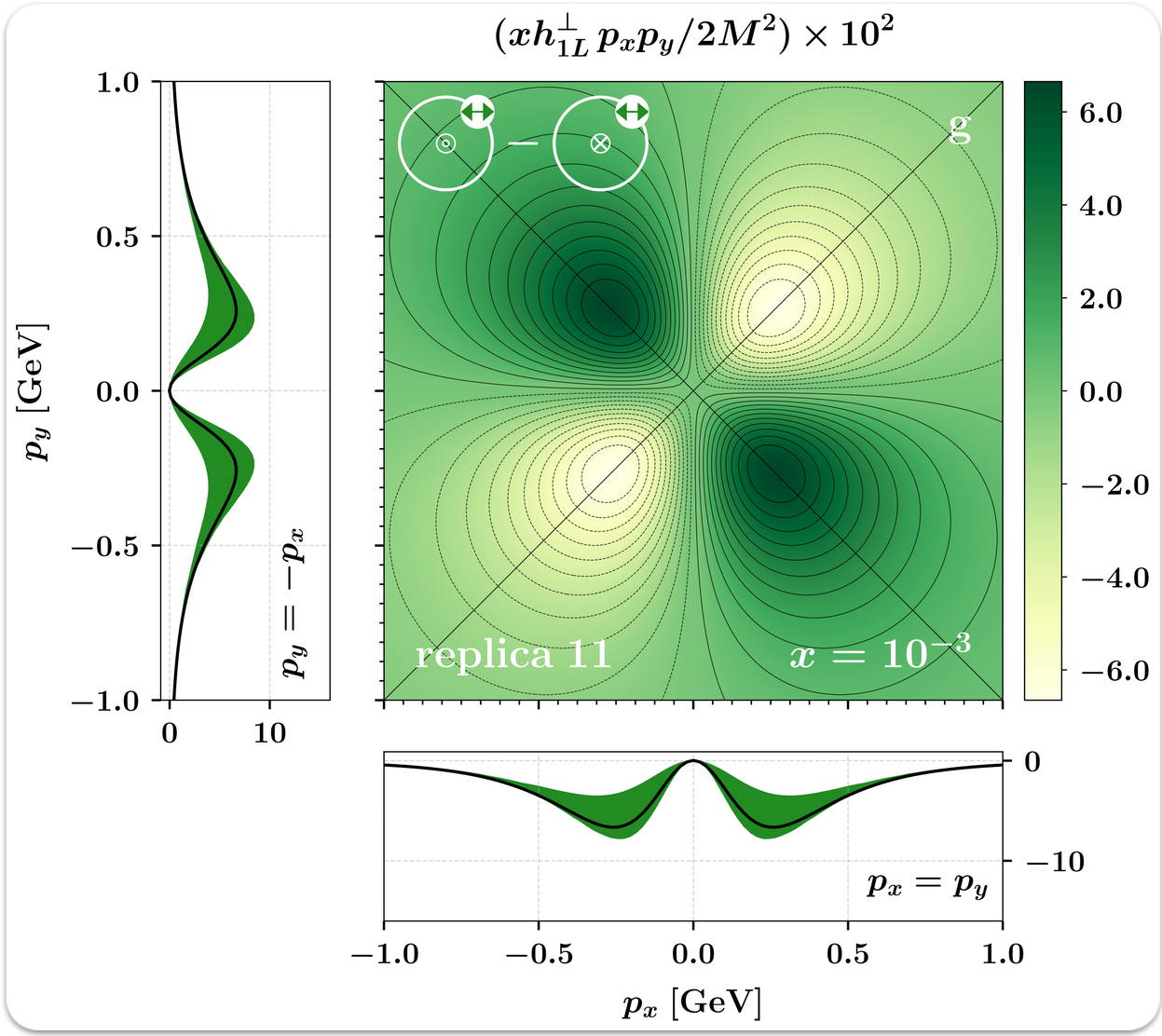


$$x\rho_{p^\rightarrow}(x, p_x, p_y) = x\, f_1(x, \boldsymbol{p}_T^2) + x\frac{p_y}{M}\, f_{1T}^\perp(x, \boldsymbol{p}_T^2)$$

Gluon propeller effect

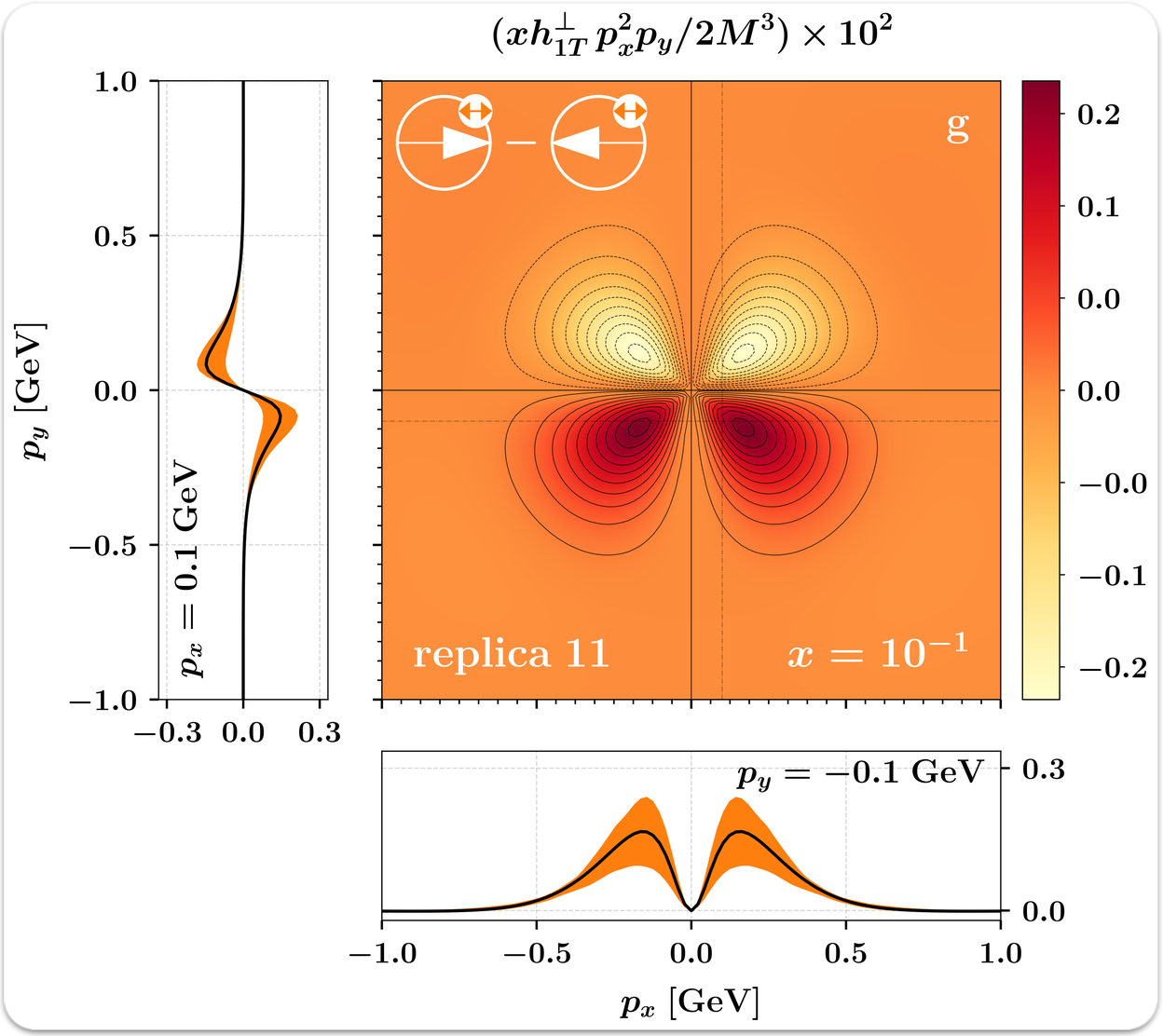
🔗 [A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]

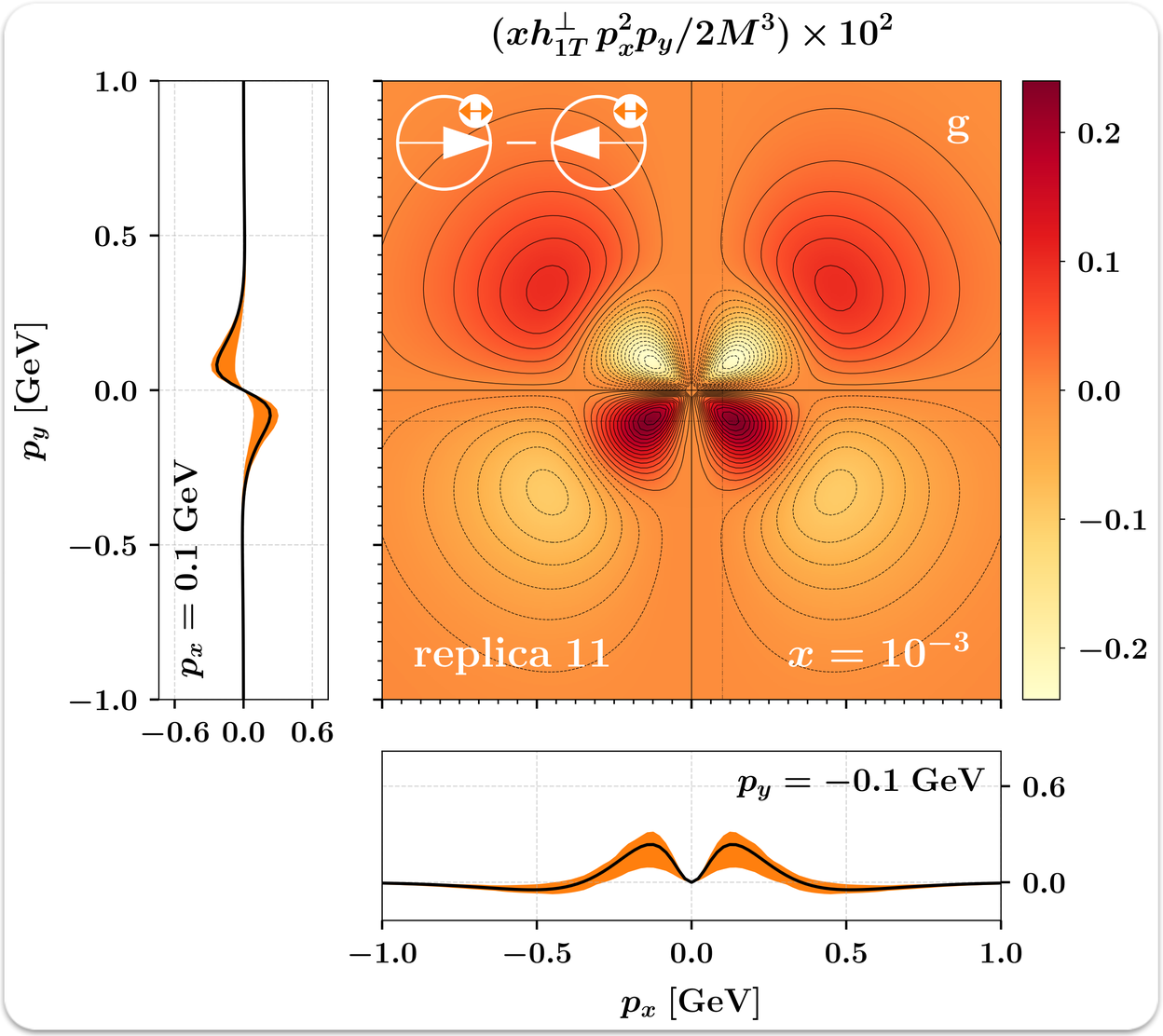




Gluon butterfly effect

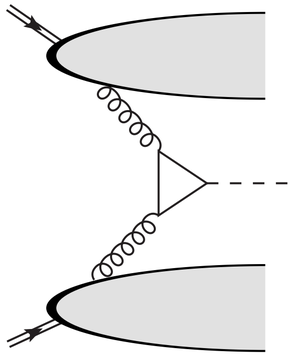
[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]





Gluon TMD pheno from the LHC to the EIC

Higgs in gluon fusion



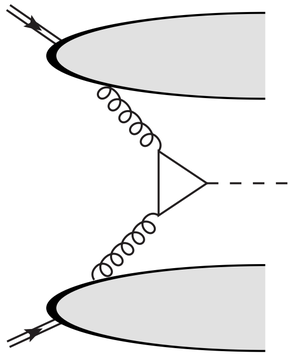
✓ No color entanglement
TMD factorization

¡! 🔗 Large low-p_T bin @CMS
More data @HL-LHC

LHC

Gluon TMD pheno from the LHC to the EIC

Higgs in gluon fusion



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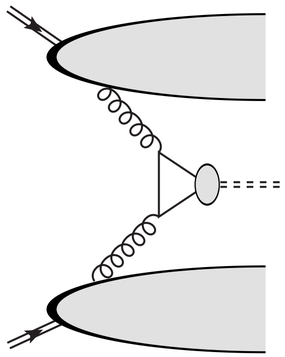
Single quarkonium



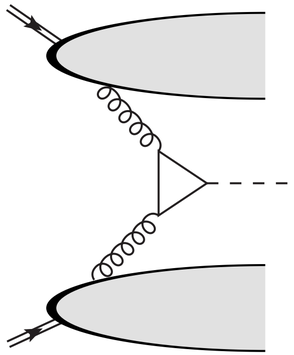
✓ $\eta_{c,b}$ ✗ $J/\psi, \Upsilon$
TMD factorization
C-parity selection rules

¡! 🔗 Large-p_T data @LHCb
More data @LHCspin

LHC

Gluon TMD pheno from the LHC to the EIC

Higgs in gluon fusion



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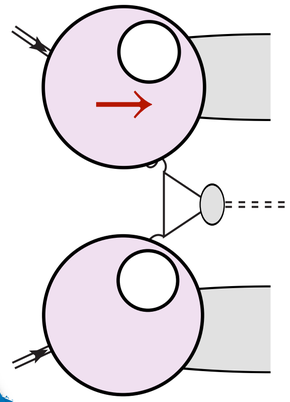


✓ ✗ TMD factorization
C-parity selection rules

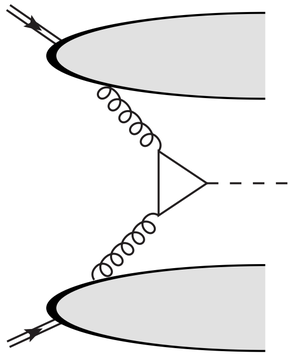
$\eta_{c,b}$ $J/\psi, \Upsilon$

¡! 🔗 Large-p_T data @LHCb
More data @LHCspin

LHC

Gluon TMD pheno from the LHC to the EIC

Higgs in gluon fusion



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TMD factorization

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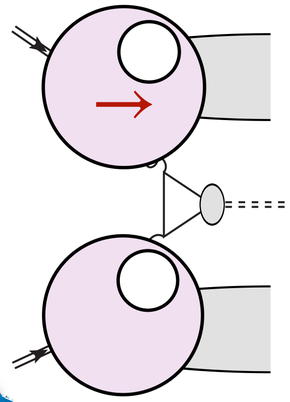
More data @HL-LHC

Single quarkonium



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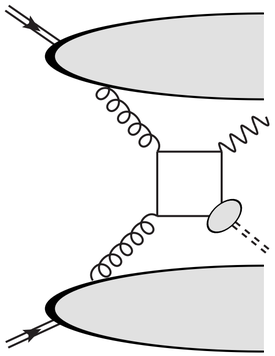
C-parity selection rules

¡! 🔗 Large-p_T data @LHCb
More data @LHCspin

LHC

$J/\psi + \gamma^{(*)}$

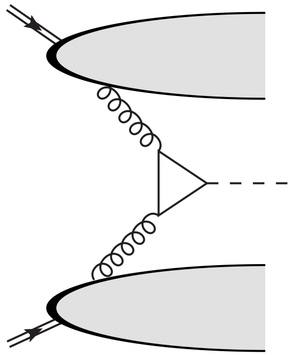


¡✓ Color entanglement
Potential TMD violation (CO)
Back-to-back suppresses CO

✓! Possible studies @HL-LHC
Currently no low-p_T data

Gluon TMD pheno from the LHC to the EIC

Higgs in gluon fusion



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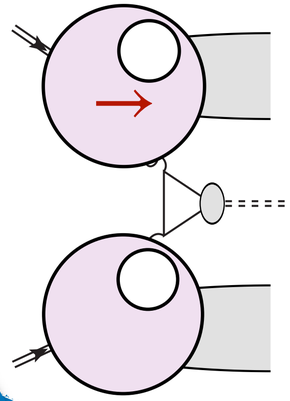
TMD factorization

¡! 🔗 Large low-p_T bin @CMS
More data @HL-LHC

Single quarkonium



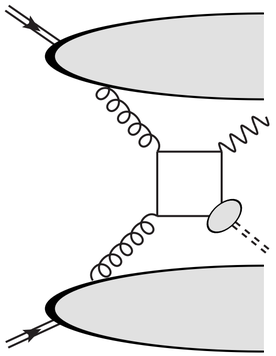
✓ ✗ TMD factorization
C-parity selection rules

$\eta_{c,b}$ $J/\psi, \Upsilon$

¡! 🔗 Large-p_T data @LHCb
More data @LHCspin

LHC

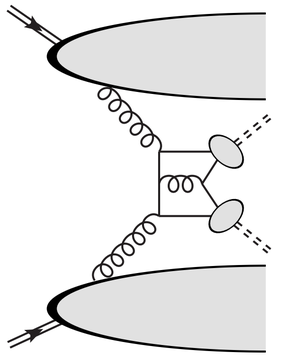
$J/\psi + \gamma^{(*)}$



¡✓ Color entanglement
Potential TMD violation (CO)
Back-to-back suppresses CO

✓! Possible studies @HL-LHC
Currently no low-p_T data

$J/\psi + J/\psi$

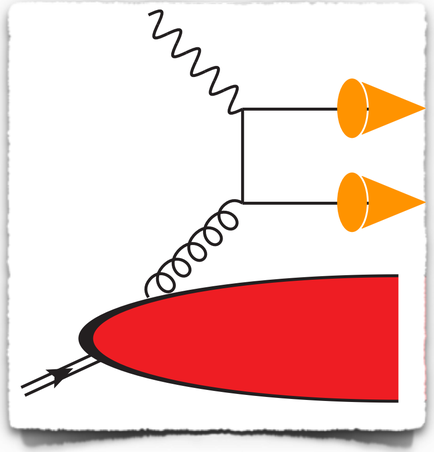


¡✓ No color entanglement
TMD factorization (CSM)

✓ 🔗 Low-p_T data @LHCb
Opportunities @HL- & @LHCSpin

Gluon TMD pheno from the LHC to the EIC

Two-jet SIDIS



jet function

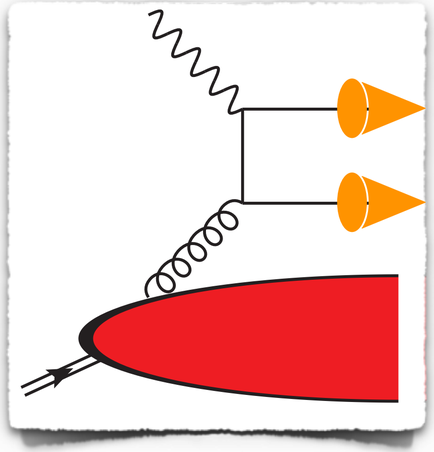
jet function

TMD PDF

EIC

Gluon TMD pheno from the LHC to the EIC

Two-jet SIDIS



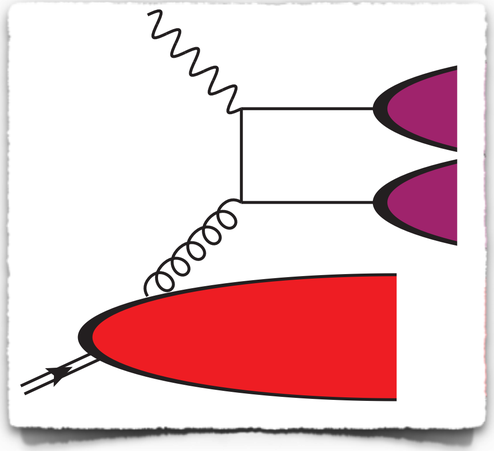
jet function

jet function

TMD PDF

EIC

Double D meson



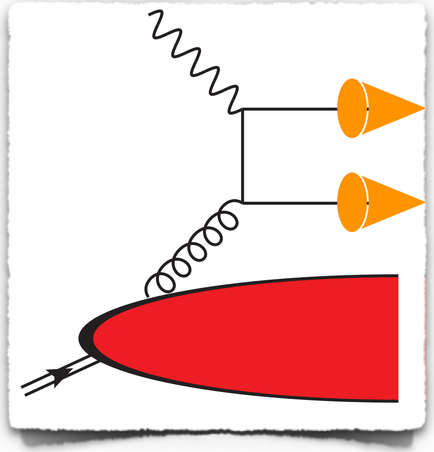
TMD FF

TMD FF

TMD PDF

Gluon TMD pheno from the LHC to the EIC

Two-jet SIDIS

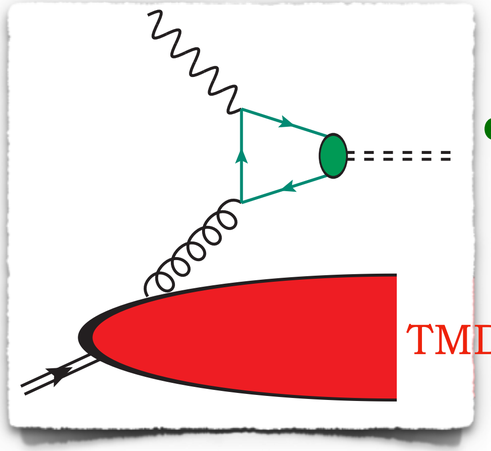


jet function

jet function

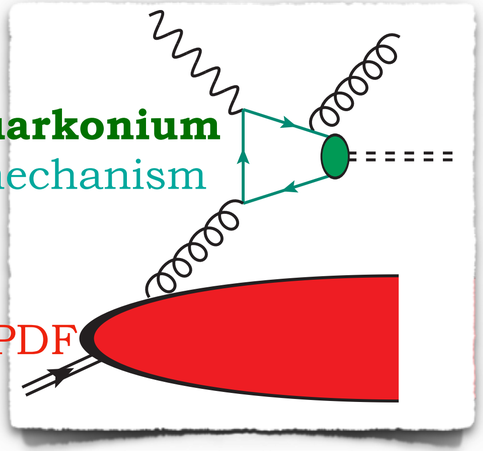
TMD PDF

Quarkonia



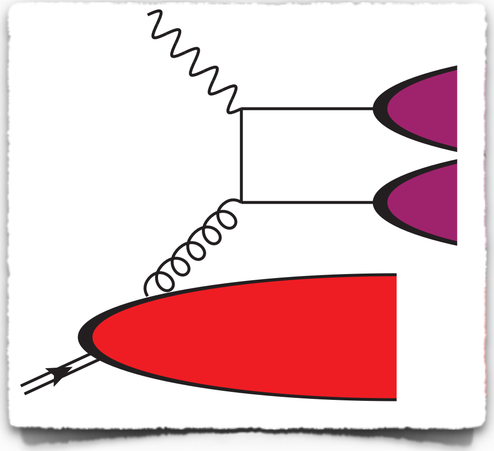
TMD PDF

quarkonium mechanism



EIC

Double D meson



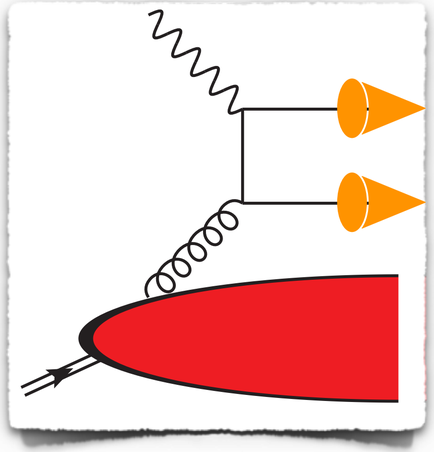
TMD FF

TMD FF

TMD PDF

Gluon TMD pheno from the LHC to the EIC

Two-jet SIDIS

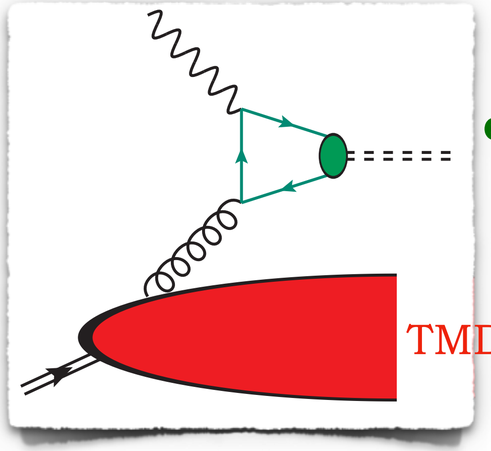


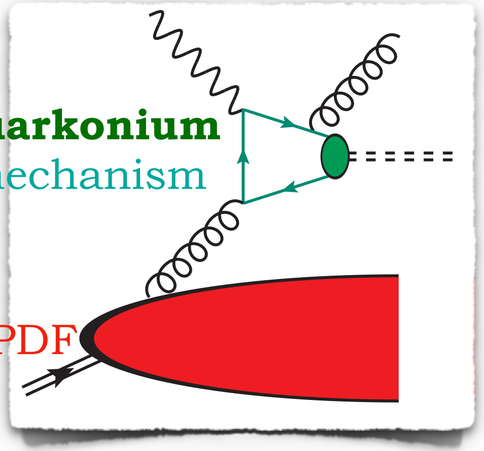
jet function

jet function

TMD PDF

Quarkonia



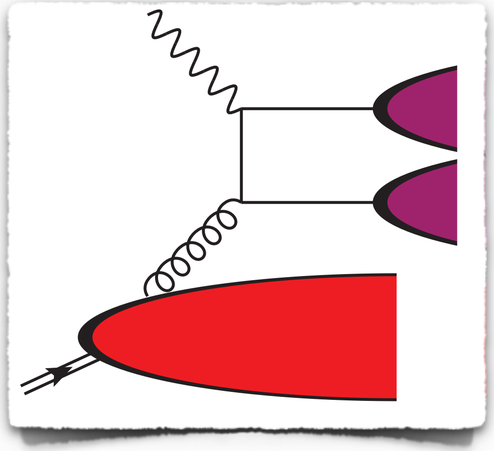


quarkonium mechanism

TMD PDF

EIC

Double D meson

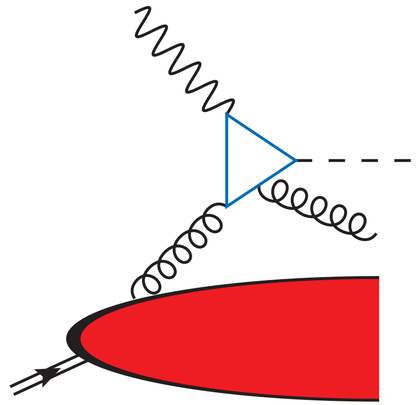


TMD FF

TMD FF

TMD PDF

...a theorist's dream

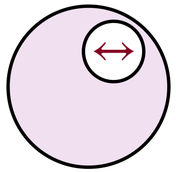


Higgs

TMD PDF

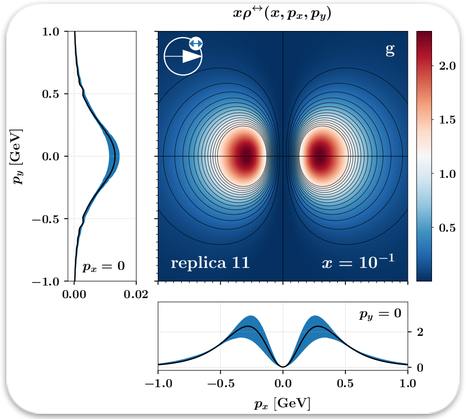
Quarkonia: Precision & Exploration

TMD exploration: Assets



EIC, LHCb, LHCspin

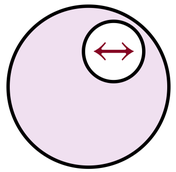
Boer-Mulders



🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (EPJC 2020)]

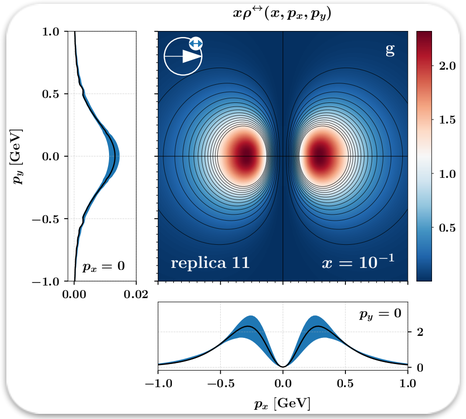
Quarkonia: Precision & Exploration

TMD exploration: Assets



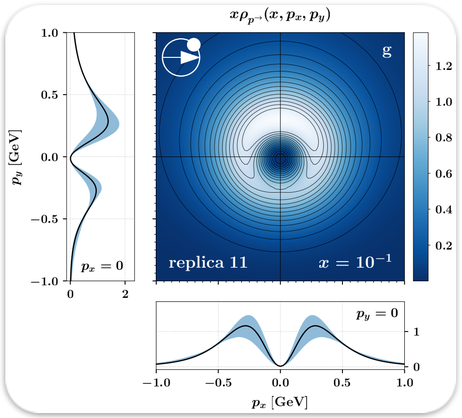
EIC, LHCb, LHCspin

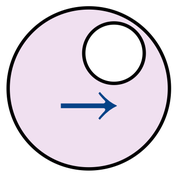
Boer-Mulders



[A. Bacchetta, F.G. C., M. Radici, P. Taels (EPJC 2020)]

[A. Bacchetta, F.G. C., M. Radici (EPJC 2024)]





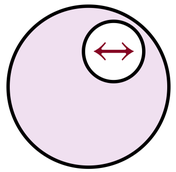
EIC, LHCspin

Sivers

IV.2 Gluon TMDs

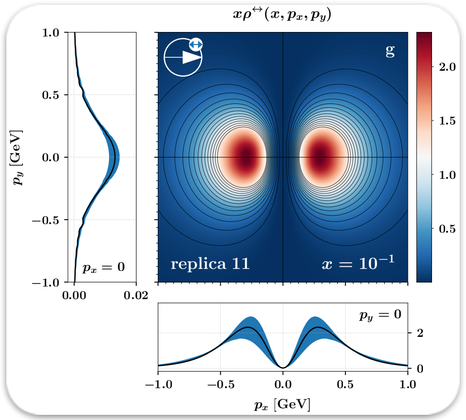
Quarkonia: Precision & Exploration

TMD exploration: Assets



EIC, LHCb, LHCspin

Boer-Mulders



🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (EPJC 2020)]
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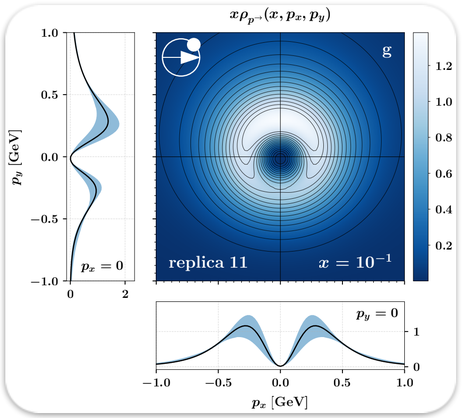
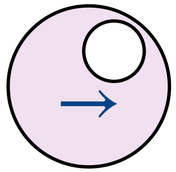


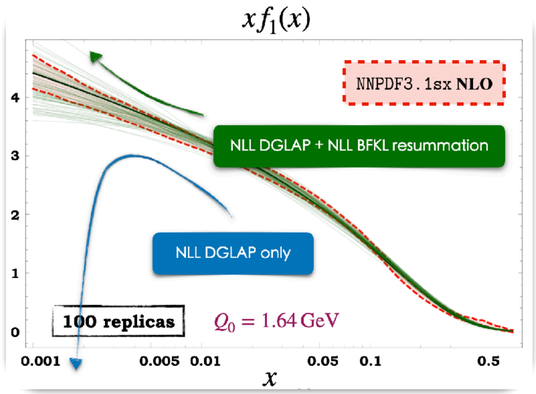


EIC, LHCspin

Sivers

Precision QCD: Challenges

Forward Onia ⇒ Gluon PDF & TMD Positivity



Low x and low Q^2

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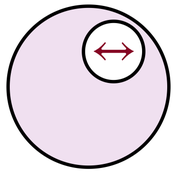
🔗 [A. Candido et al. (2020)]
🔗 [J. Collins et al. (2022)]
🔗 [A. Candido et al. (2023)]

Quarkonia: Precision & Exploration

TMD exploration: Assets



EIC, LHCb, LHCspin

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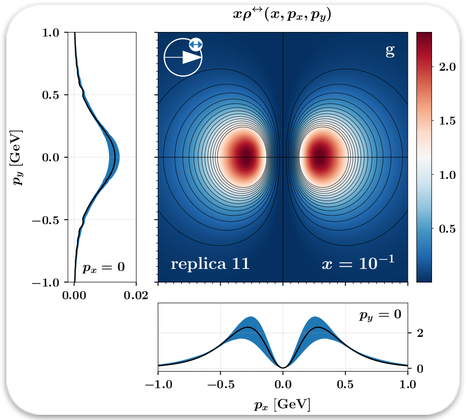

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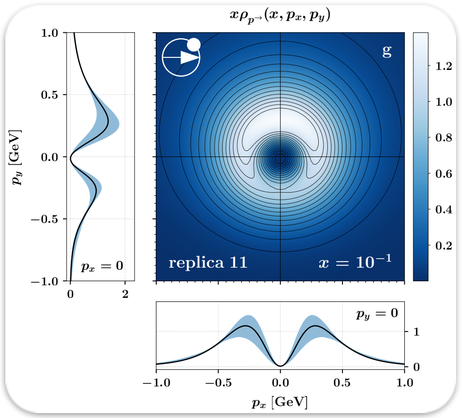

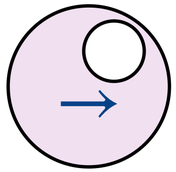

EIC, LHCspin

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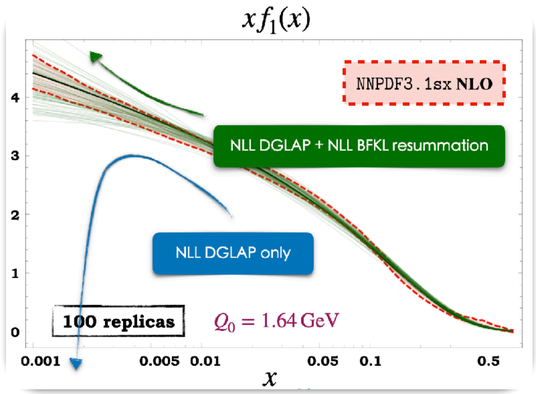

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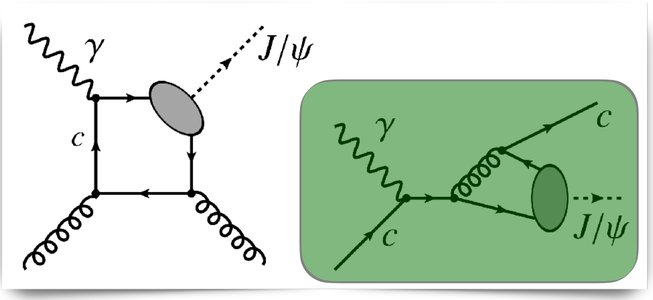
$\gamma + g \rightarrow J/\psi + c$ ⇒ Valence Intrinsic Charm @EIC

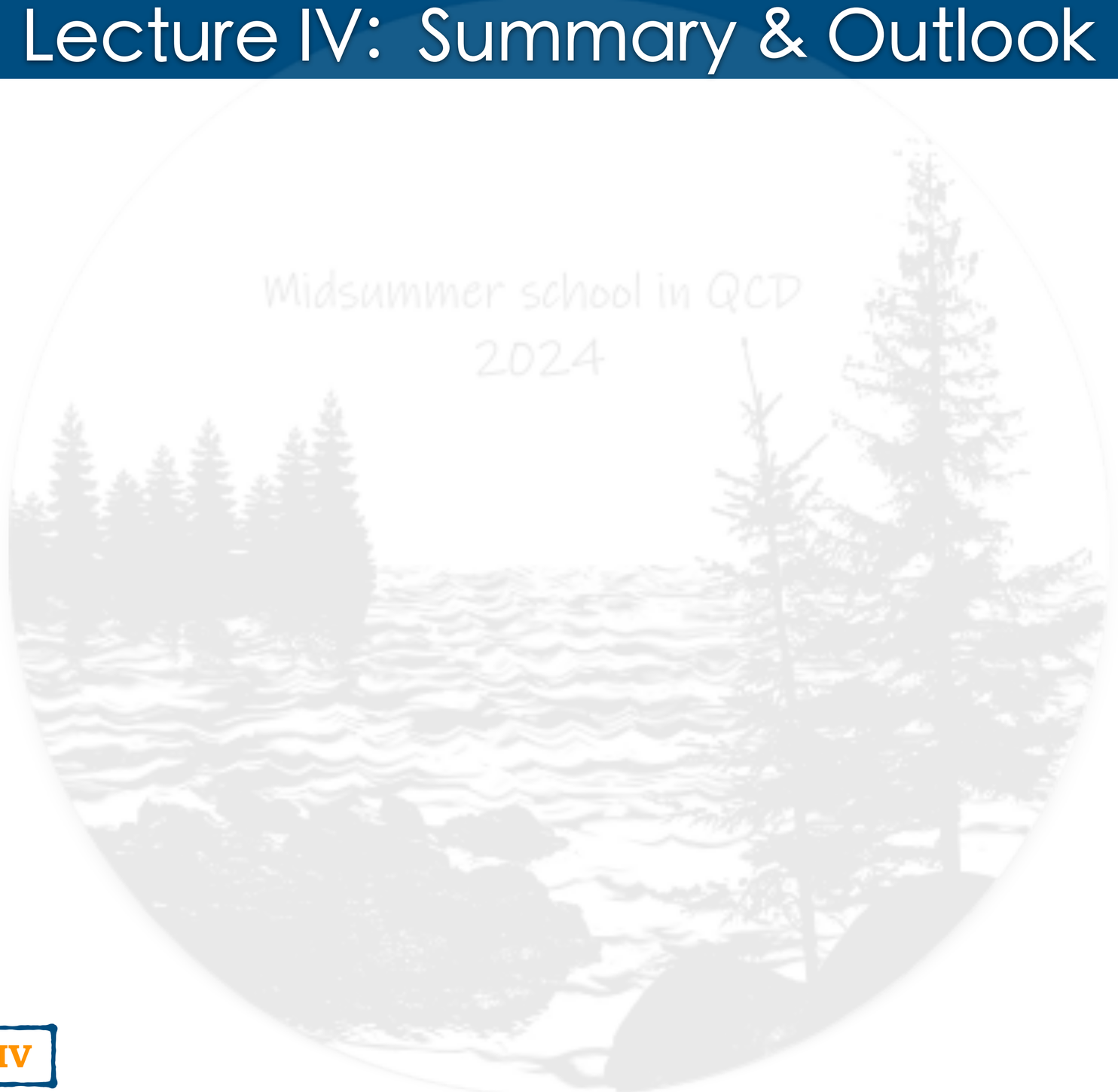
Large x
Moderate to large p_T



(J/ψ @EIC) 🔗 [C. Flore, J.-P. Lansberg, H.-S. Shao, Y. Yedelkina (2020)]
(Intrinsic Charm + valence) studies 🔗🔗 [NNPDF Collaboration (2022, 2023)]

Lecture IV Checkpoint

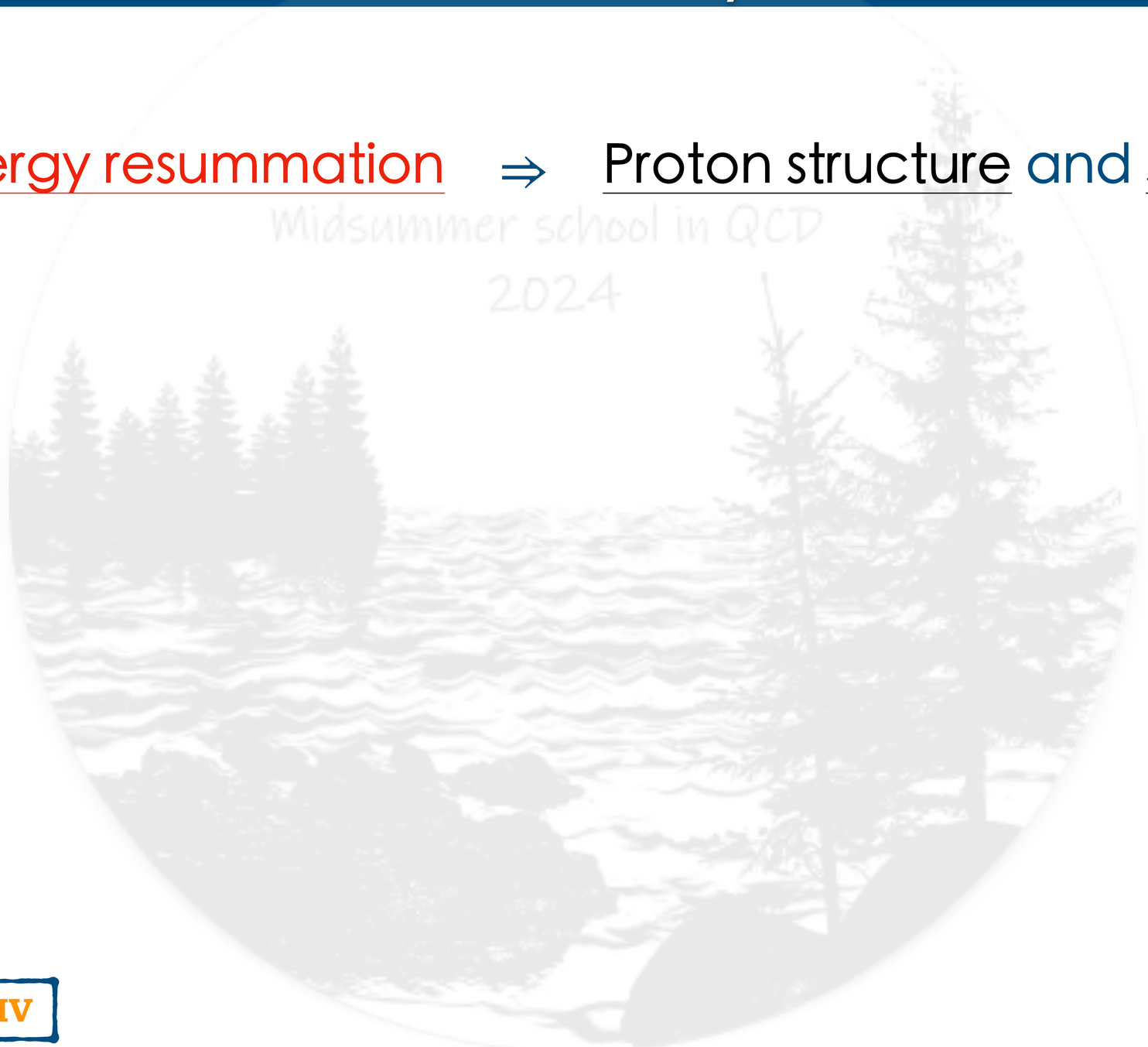
Lecture IV: Summary & Outlook



Checkpoint of Lecture IV

Lecture IV: Summary & Outlook

High-energy resummation ⇒ Proton structure and spin at small-x



Checkpoint of Lecture IV

Lecture IV: Summary & Outlook

- High-energy resummation ⇒ Proton structure and spin at small-x
- BFKL UGD ⇐ exclusive & inclusive forward emissions

Checkpoint of Lecture IV

Lecture IV: Summary & Outlook

- High-energy resummation ⇒ Proton structure and spin at small-x
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- Gluon TMDs ⇒ Proton 3D tomographic imaging + small-x effects

Checkpoint of Lecture IV

Lecture IV: Summary & Outlook

- High-energy resummation ⇒ Proton structure and spin at small-x
- BFKL UGD ⇐ exclusive & inclusive forward emissions
- Gluon TMDs ⇒ Proton 3D tomographic imaging + small-x effects
- ***¿ Is this the end of the story ?***

Checkpoint of Lecture IV

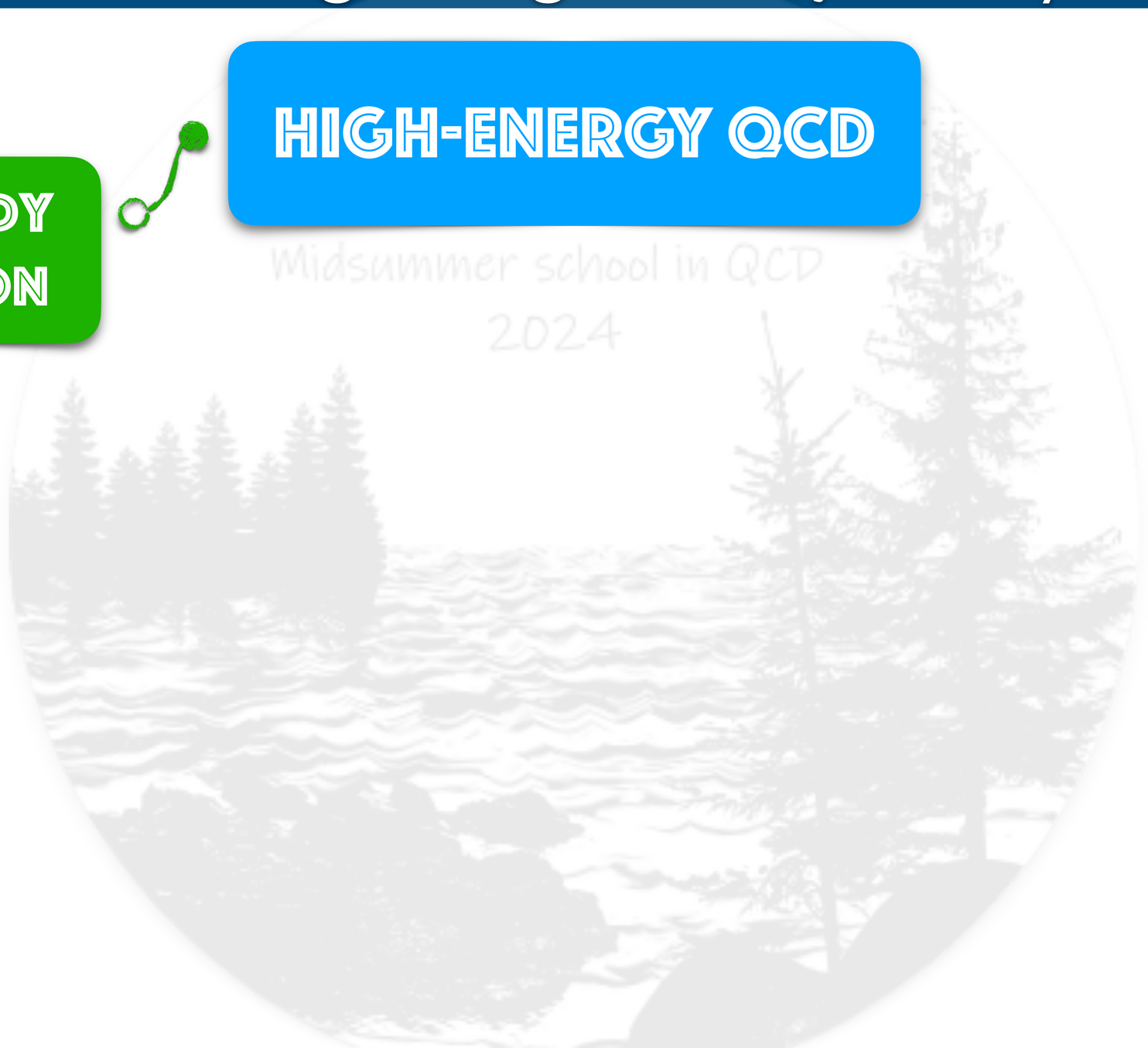
Final remarks

The beginning of the journey

HIGH-ENERGY QCD

FORMAL STUDY
RESUMMATION

Lecture I



The beginning of the journey

HIGH-ENERGY QCD

FORMAL STUDY RESUMMATION

Lecture I

JET & HADRON PHYSICS

Lecture II

The beginning of the journey

HIGH-ENERGY QCD

FORMAL STUDY RESUMMATION

Lecture I

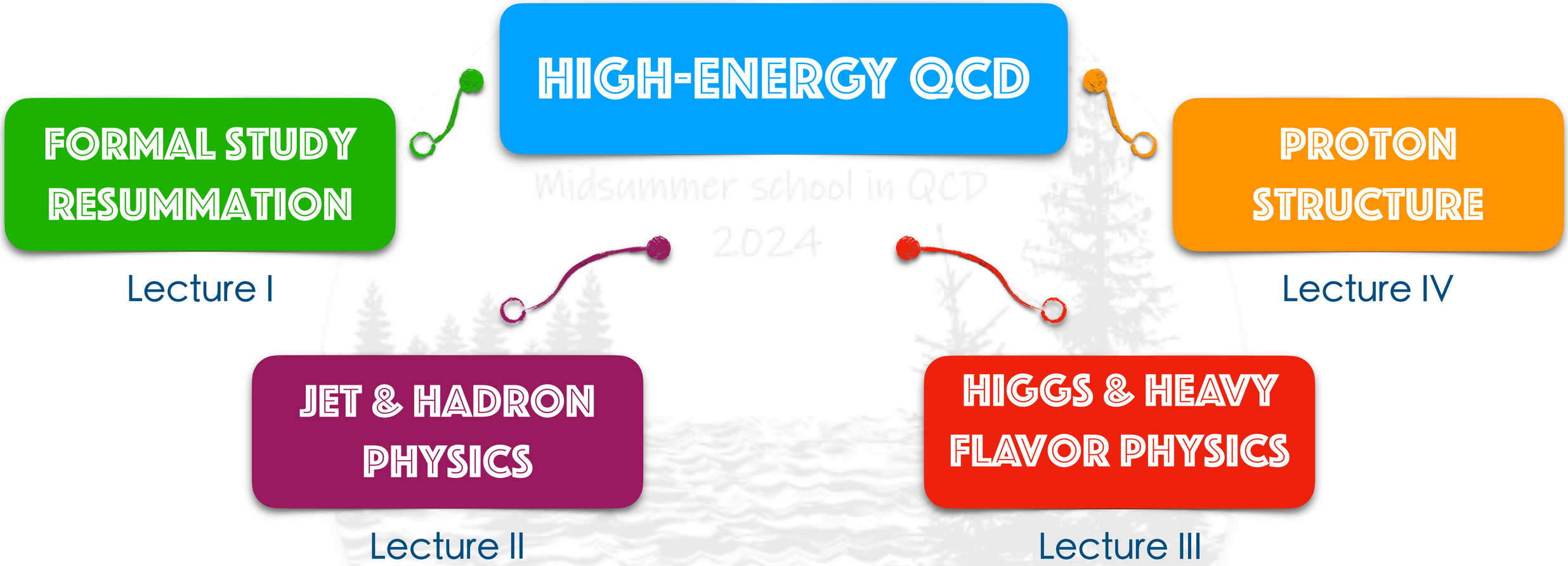
JET & HADRON PHYSICS

Lecture II

HIGGS & HEAVY FLAVOR PHYSICS

Lecture III

The beginning of the journey



The beginning of the journey

HIGH-ENERGY QCD

FORMAL STUDY RESUMMATION

Lecture I

PROTON STRUCTURE

Lecture IV

JET & HADRON PHYSICS

Lecture II

HIGGS & HEAVY FLAVOR PHYSICS

Lecture III

High-energy resummation in the QCD precision era...

...a fascinating path for realizing yourself as a scientist ***!***

High-energy QCD & the proton structure

High-energy physics

- Precision studies ⇐ SM and beyond
- Fixed-order perturbative calculations…
- …enhanced by resummations
- SM measurements: H, W, Z mass

Proton structure

- Inner structure ⇐ intrinsic parton motion
- Parton densities ⇒ nonperturbative nature
- Extracted from experiments via global fits
- Several types: 1D collinear, 3D TMD, and so on

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High-energy physics
assumes knowledge
of proton structure

Reduction of uncertainties
on parton densities
from high-energy studies

Perturbative and nonperturbative aspects ⇔ key ingredients to a joint search for New Physics

Towards the Electron-Ion Collider

PHYSICS AT THE EIC

based on drawing by Enki Bilal

Thomas Ullrich

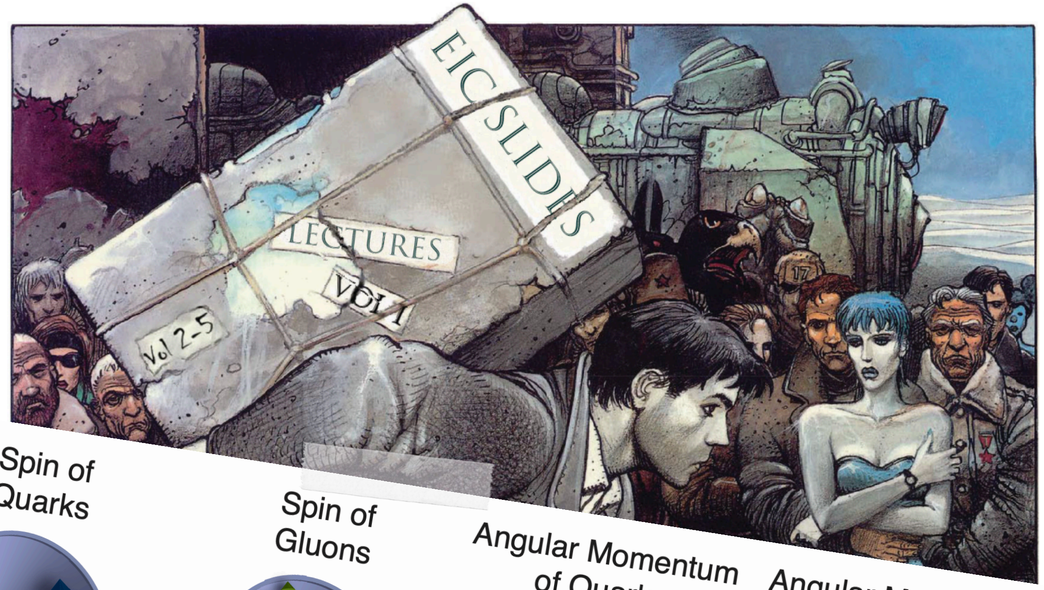
Midsummer School in QCD 2024
24 June - 6 July 2024
Saariselkä, Finland

Brookhaven National Laboratory

🔗 [Lectures by Thomas Ullrich]

Towards the Electron-Ion Collider

PHYSICS AT THE EIC



Thomas Ullrich
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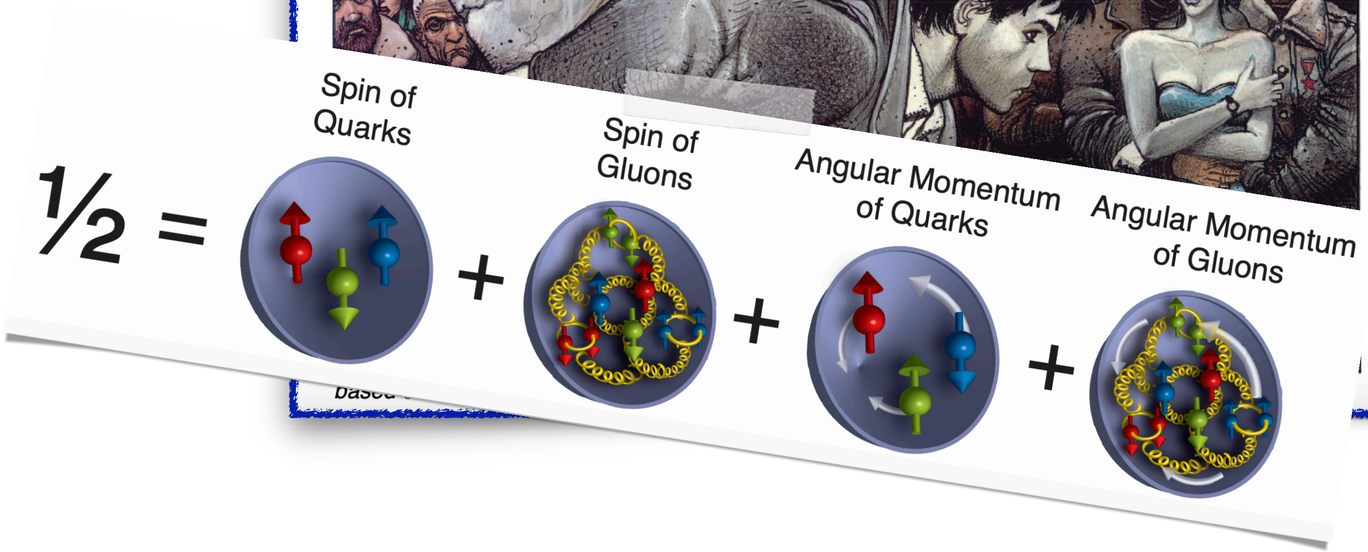
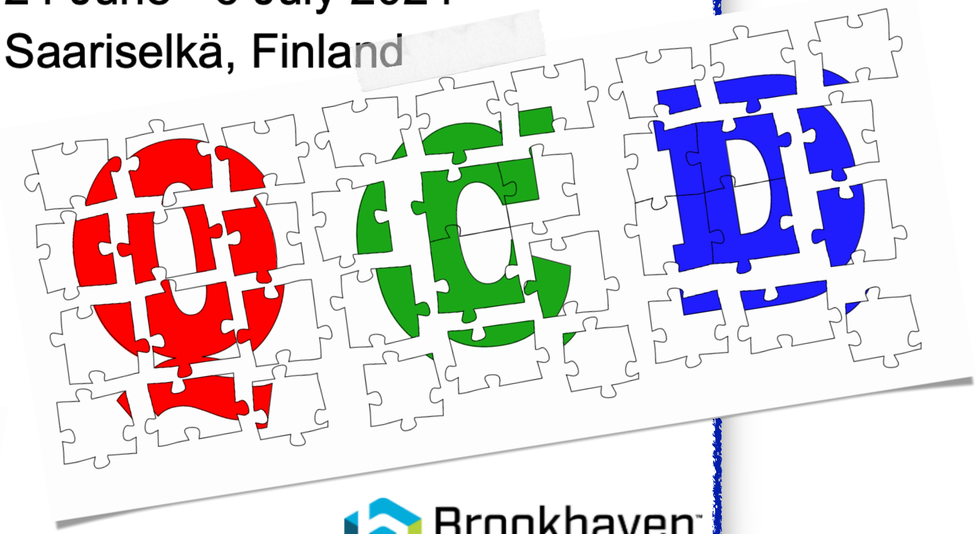
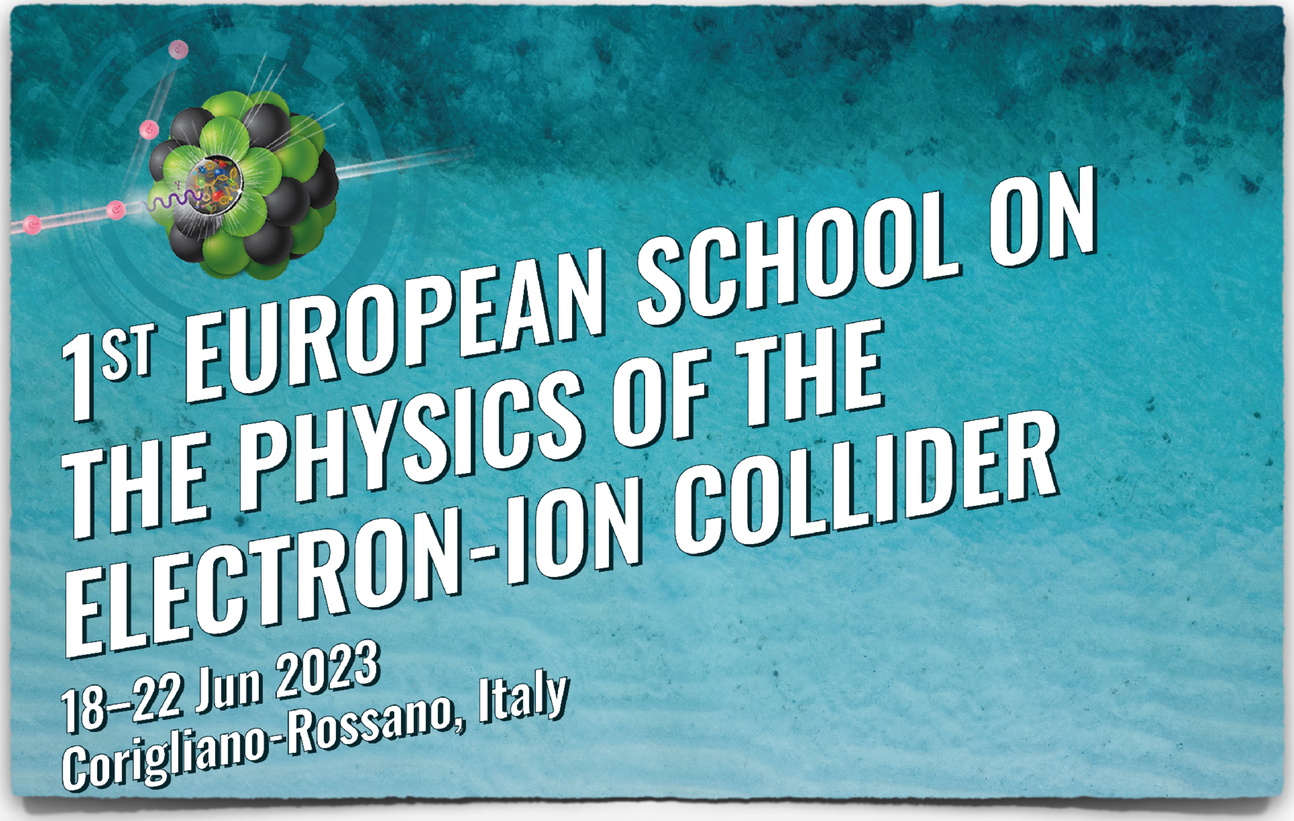




Brookhaven National Laboratory

🔗 [Lectures by Thomas Ullrich]

EIC EuroSchool 2023



1ST EUROPEAN SCHOOL ON THE PHYSICS OF THE ELECTRON-ION COLLIDER

18–22 Jun 2023
Corigliano-Rossano, Italy

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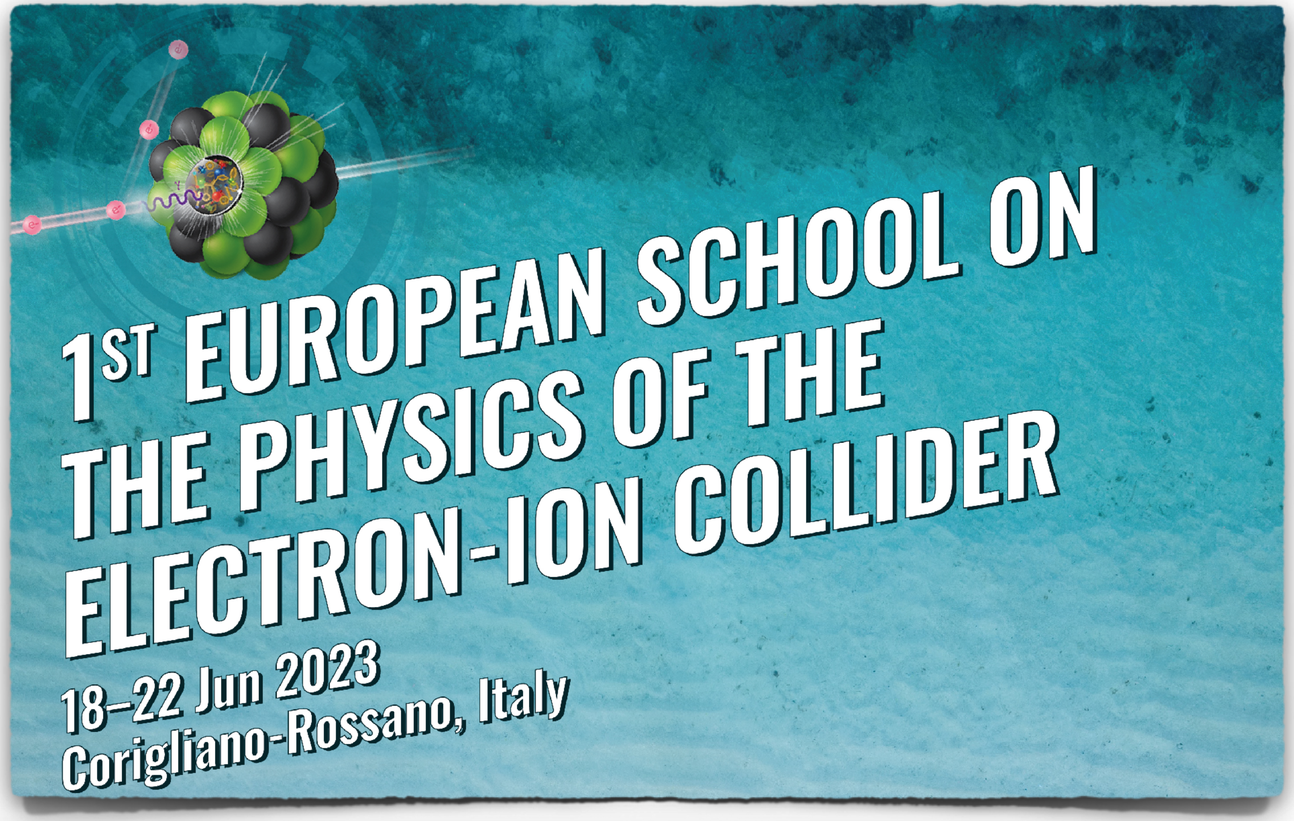

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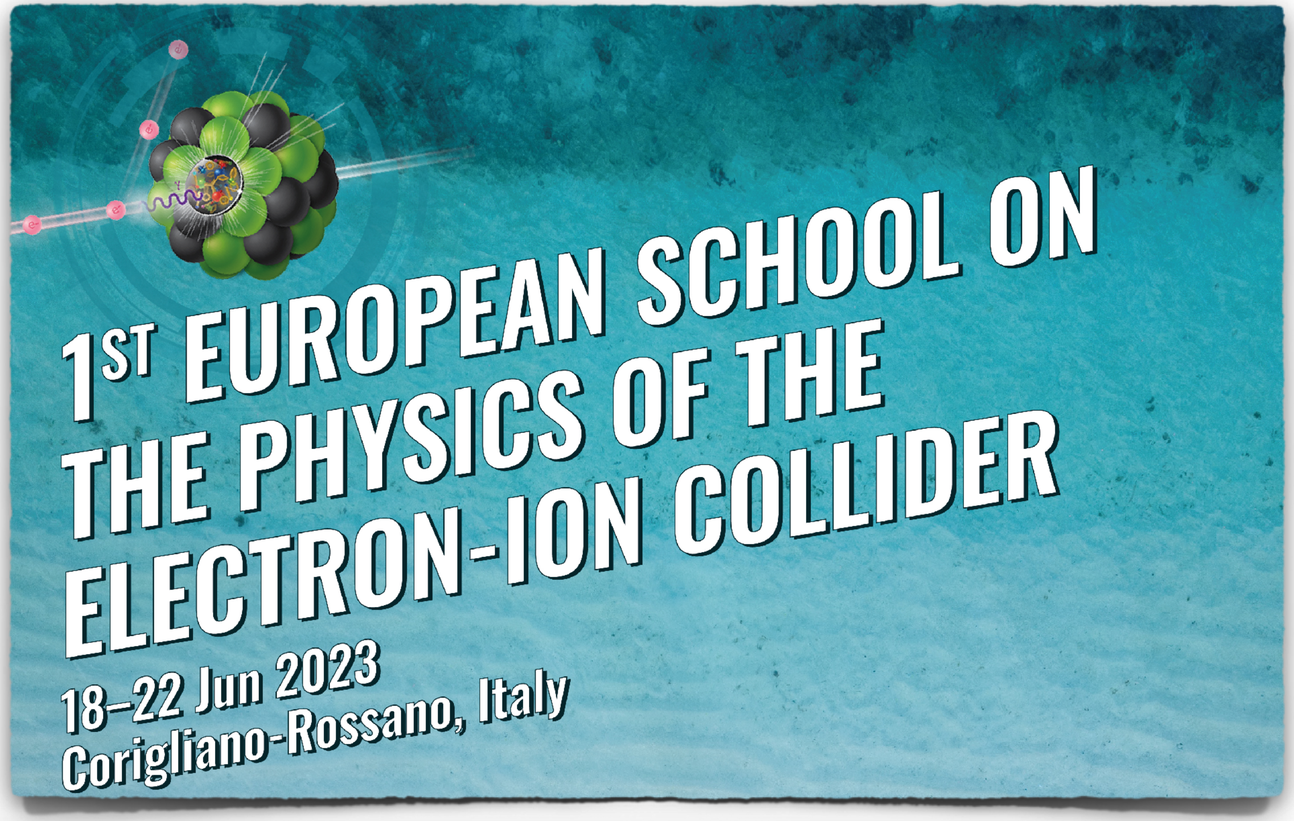

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EIC EuroSchool 2025

¡ Thanks for the attention !

Extras

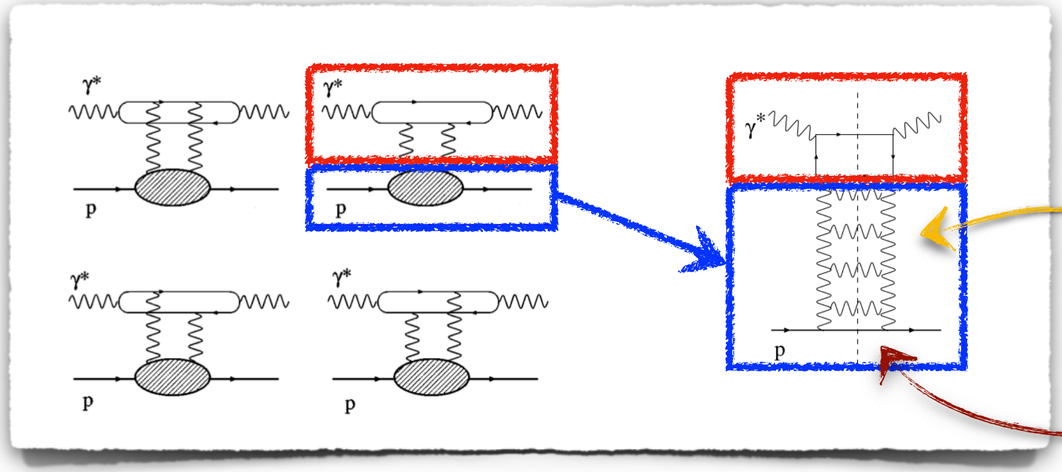
The BFKL UGD

High-energy factorization and the UGD

- example: **virtual photoabsorption** in **high-energy factorization**

$$\sigma_{\text{tot}}(\gamma^* p \to X) \propto \mathcal{I}m_s\{\mathcal{A}(\gamma^* p \to \gamma^* p)\} \equiv \Phi_{\gamma^* \to \gamma^*} \circledast \mathcal{F}(x, \kappa^2)$$

◇ $\mathcal{F}(x, \kappa^2)$ is the **unintegrated gluon distribution** (**UGD**) in the proton



▶ Small-x limit: **UGD** = [**BFKL gluon ladder**] $\circledast$ [**proton impact factor**]

◇ Takes into account the resummation of high-energy logs

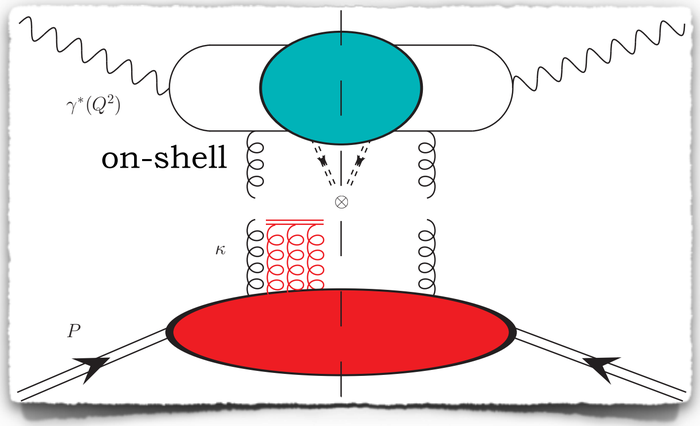
◇ Describes the coupling of the gluon Green's function to the proton

▶ Proton impact factor is non-perturbative $\Longrightarrow$ UGD needs to be modeled!

TMD versus high-energy factorization

TMD

* Semi-inclusive processes
* $\kappa_T \ll$ hardest scale
* Language of parton correlators
* Diagram: SIDIS onium



TMD
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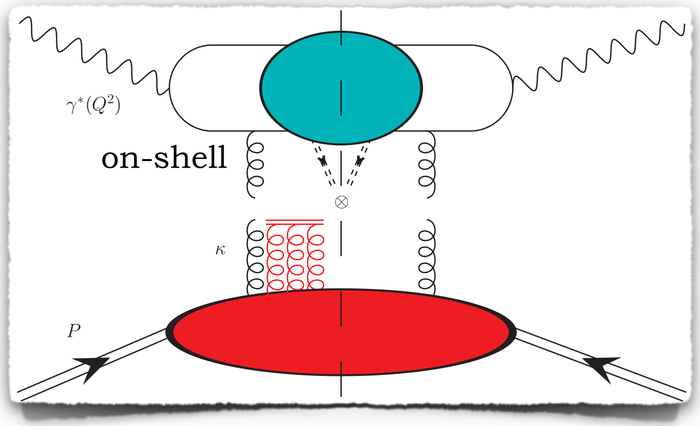
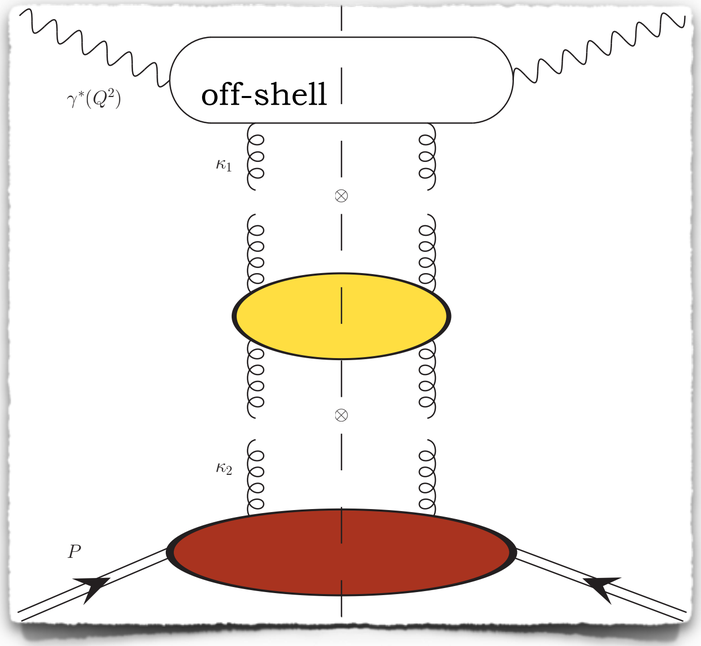


TMD PDF

HEF

* Inclusive or exclusive processes (!)
* Small x, large κ_T
* Language of Reggeized gluons
* Diagram: DIS



$$\Phi^{\gamma^*\to\gamma^*} \otimes \mathcal{G}_{\mathrm{BFKL}} \otimes \Phi^{P}_{[\mathrm{NP}]}$$

TMD versus high-energy factorization

TMD

IR-safe colorless $\{\Phi^{i\to o}\}$

(Fadin-Martin theorem)

[V.S. Fadin, A.D. Martin (1999)]

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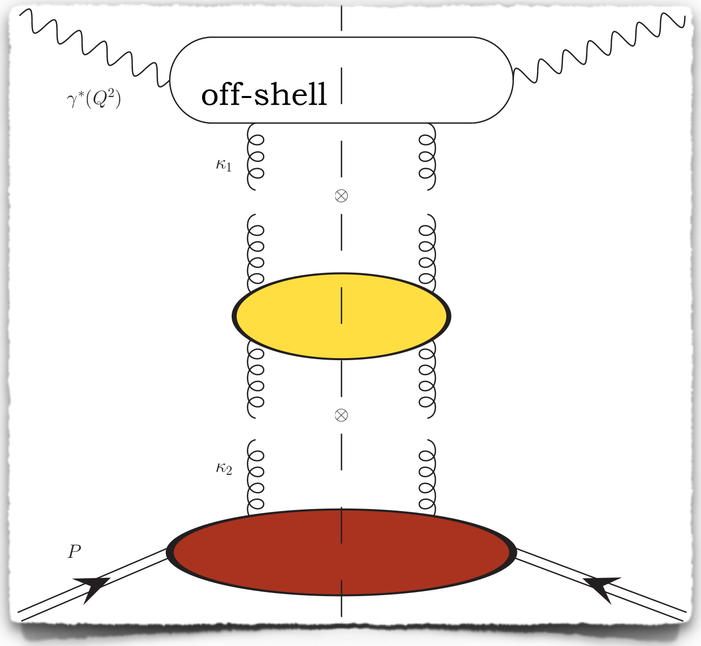

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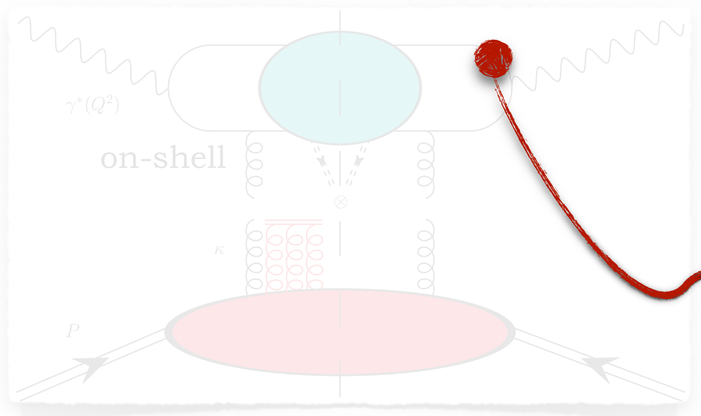
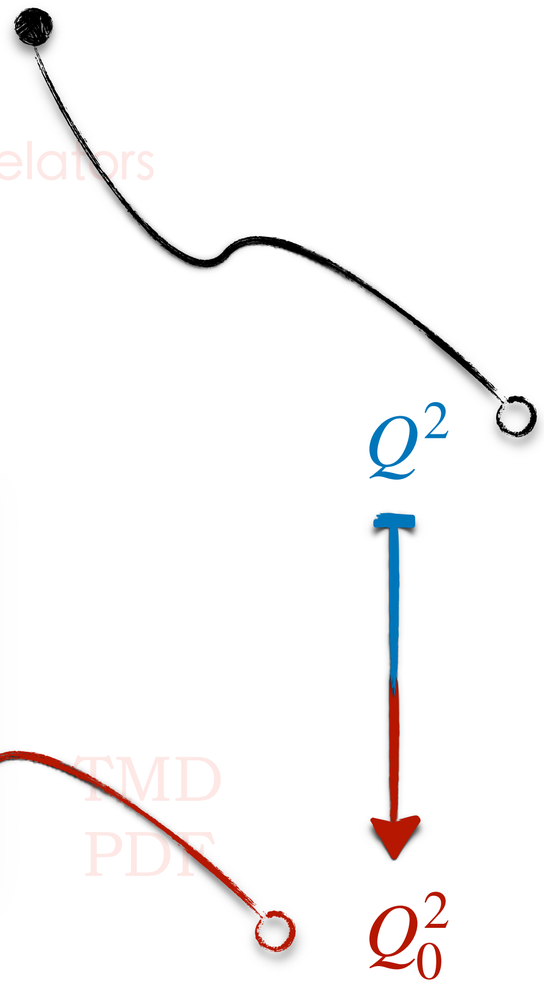
IR diffusion pattern

(Bartels' cigar)

[J. Bartels, H. Lotter (1993)]





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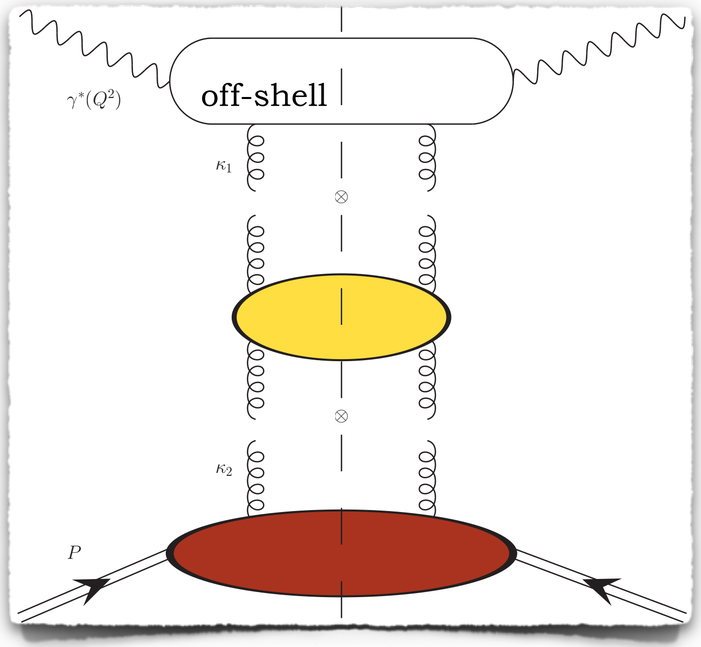
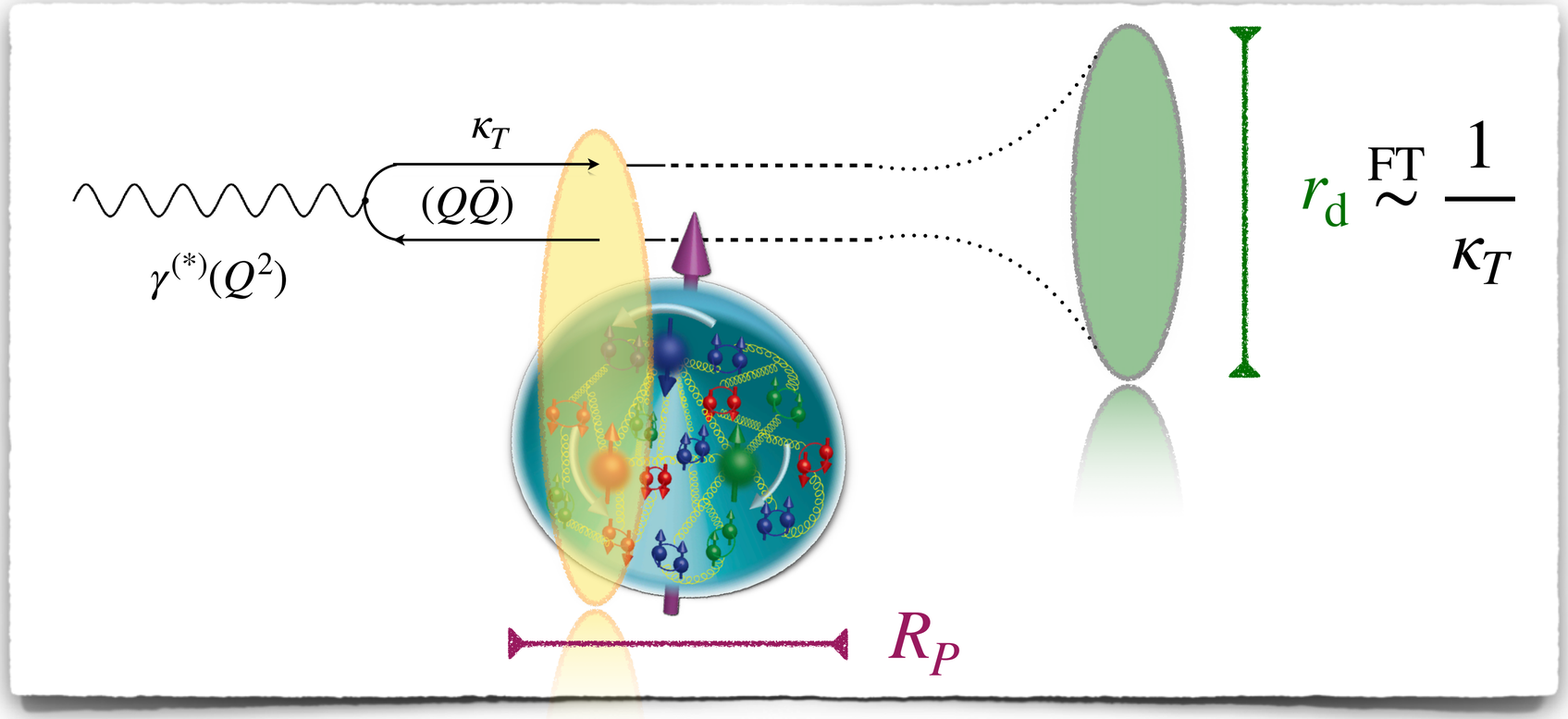


$$\Phi^{\gamma^*\to\gamma^*} \otimes \mathcal{G}_{\mathrm{BFKL}} \otimes \Phi^{P}_{[\mathrm{NP}]}$$

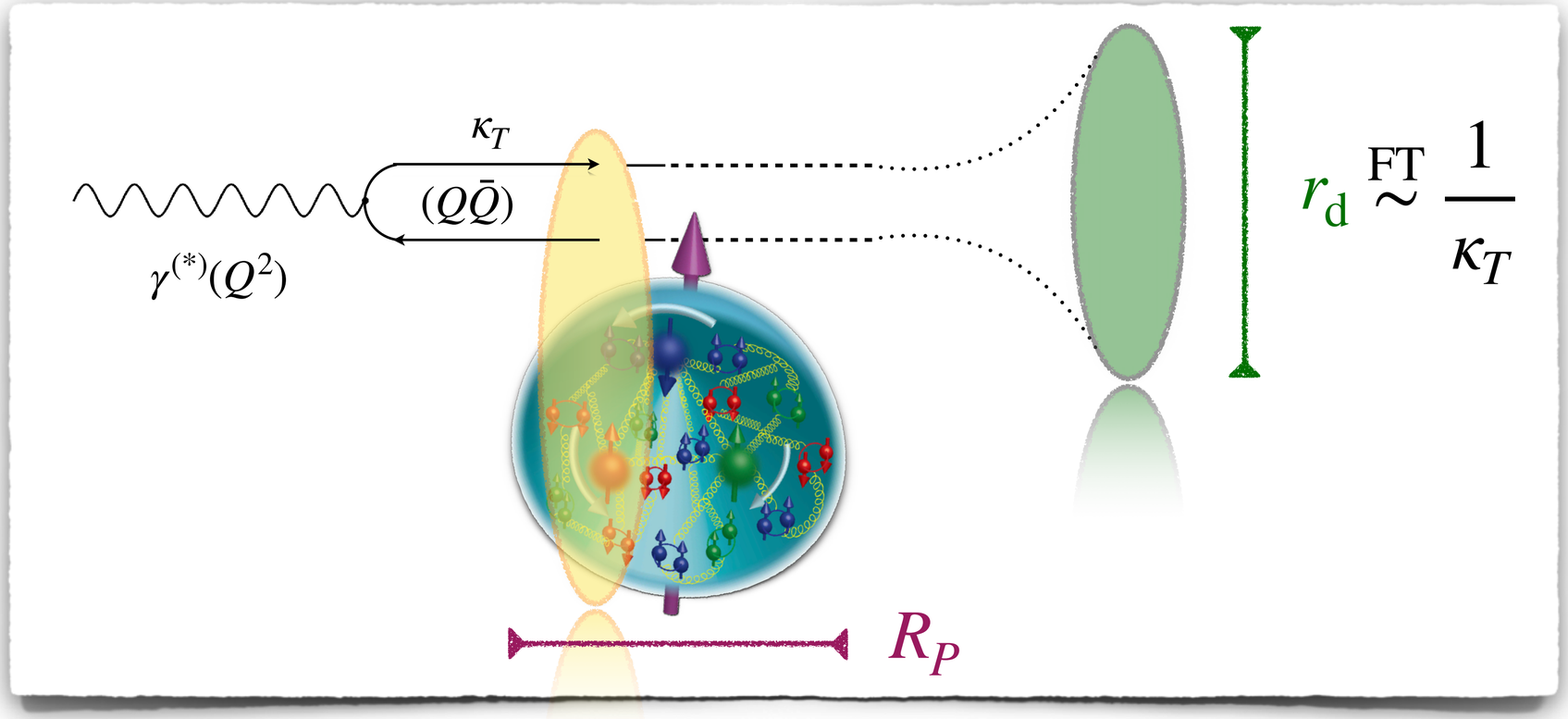
Diffractive $\gamma^* P$ scatterings and color dipoles



$$W_{\mu\nu} \propto \mathrm{Im}\left\{ i \int \mathrm{d}^4x\, e^{i\,q\cdot x} \langle P | \mathrm{T}\,[J_\mu(x)\, J_\nu(0)] | P \rangle \right\}$$

- Small-x $\Rightarrow$ Ioffe time $\gg R_P$
- At least one J_μ outside proton…
- …color dipole picture!

Diffractive $\gamma^* P$ scatterings and color dipoles



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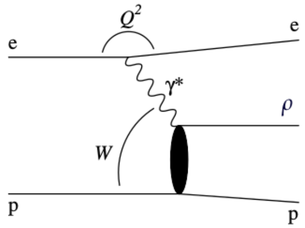
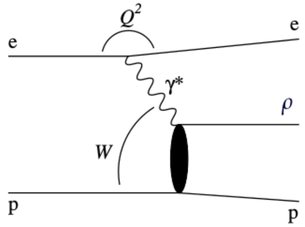
Exclusive forward ρ-meson leptoproduction



- High-energy regime:
  $s \equiv W^2 \gg Q^2 \gg \Lambda^2_{\mathrm{QCD}} \implies$ small $x = \frac{Q^2}{W^2}$
- photon virtuality Q is the **hard scale** of the process

▶ **Process solved in helicity** $\implies$ so far **unexplored testfield** for UGD

Exclusive forward ρ-meson leptoproduction



- High-energy regime:
  $s \equiv W^2 \gg Q^2 \gg \Lambda^2_{\rm QCD} \implies$ small $x = \frac{Q^2}{W^2}$
- photon virtuality Q is the **hard scale** of the process

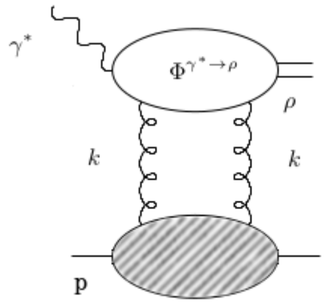
▶ **Process solved in helicity** $\implies$ so far **unexplored testfield** for UGD

Leading **helicity amplitudes** are known

Assumption:

- $\mathcal{I}m_s\{\mathcal{A}(\gamma^* p \to \rho\, p)\}$
- same W- and t-dependence for T_{11} and T_{00} $\implies$ high-energy factorization
  $\to$ same physical mechanism, scattering of small transverse size of dipole on the proton target, at work $\Rightarrow$ high-energy factorization

$$T_{\lambda_\rho \lambda_\gamma}(s; Q^2) = is \int \frac{d^2\kappa}{(\kappa^2)^2}\, \Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}(\kappa^2, Q^2)\, \mathcal{F}(x, \kappa^2)\,, \quad x = \frac{Q^2}{s}$$

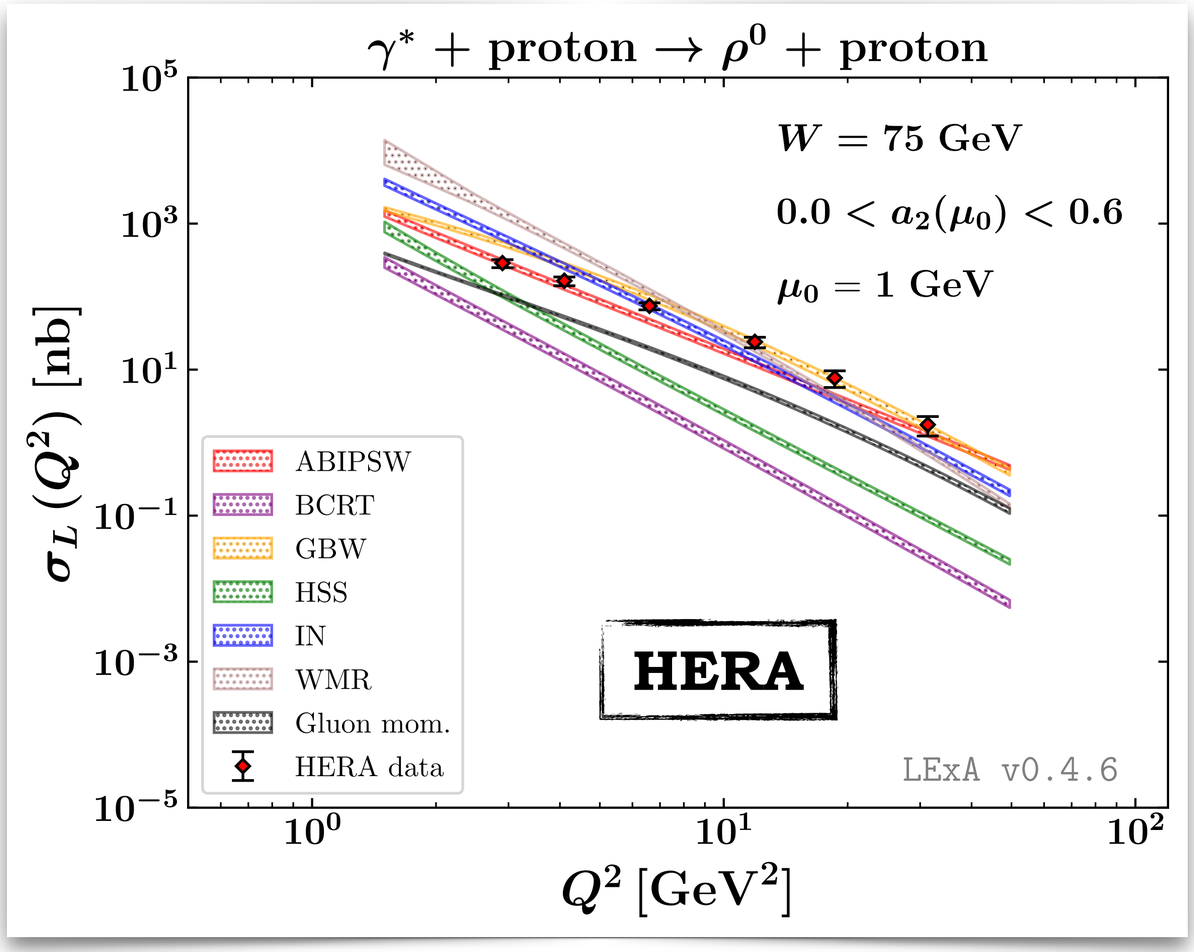


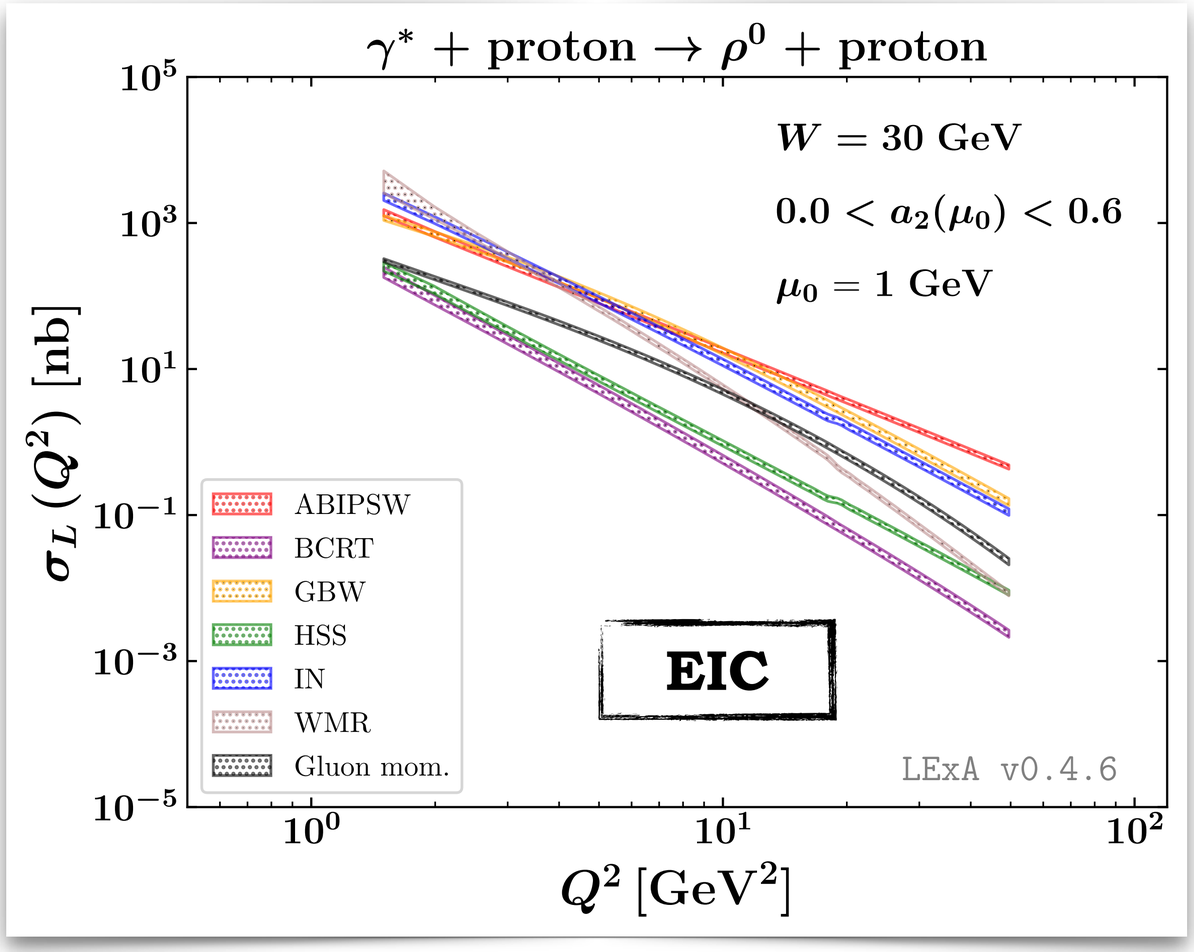
Interesting transitions:

- $\gamma^*_L \to \rho_L \xrightarrow{\text{encoded by}} \Phi^{\gamma^*_L \to \rho_L}$
- $\gamma^*_T \to \rho_T \xrightarrow{\text{encoded by}} \Phi^{\gamma^*_T \to \rho_T}$

$\implies$ **DAs** enter in $\Phi^{\gamma^* \to \rho}$

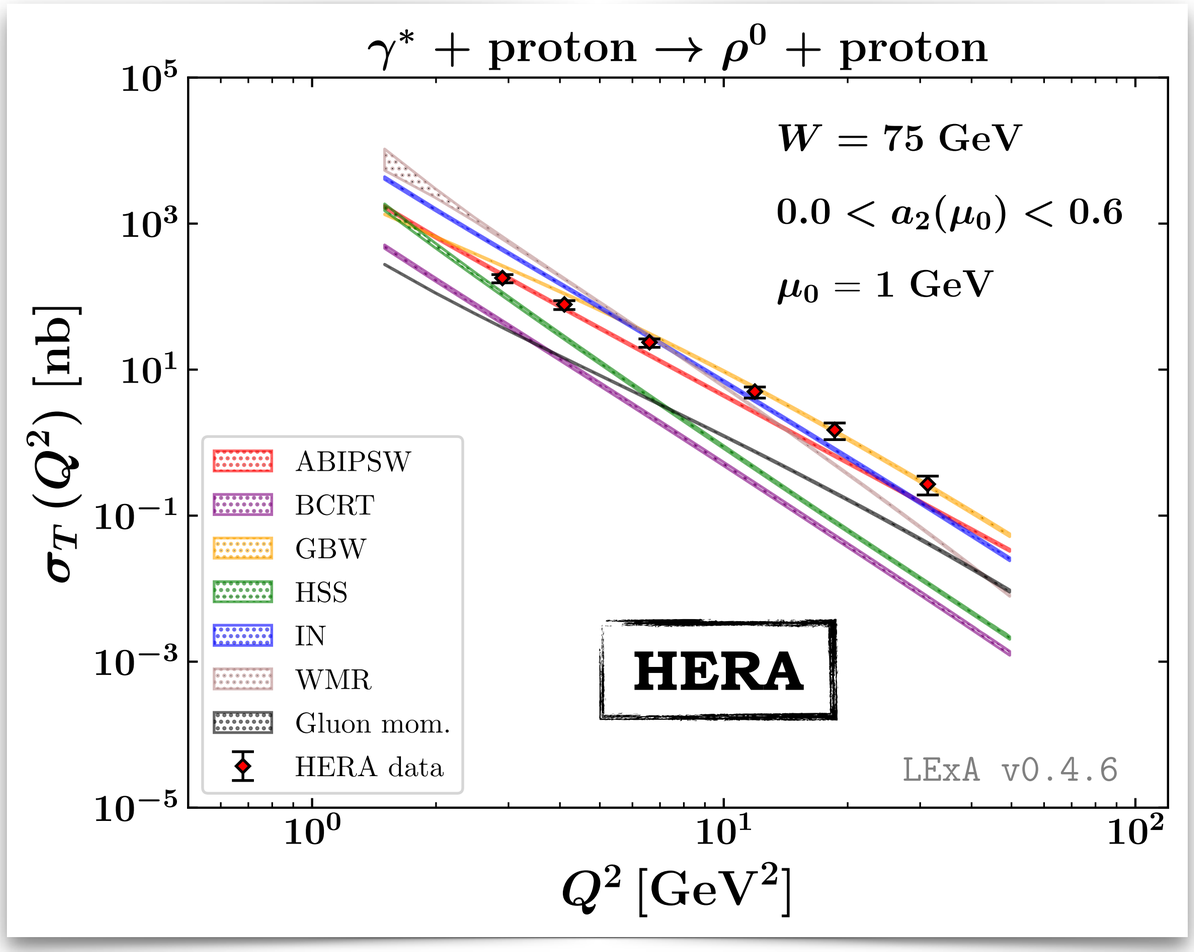
Forward ρ-mesons from HERA to the EIC: $\sigma_L(Q^2)$

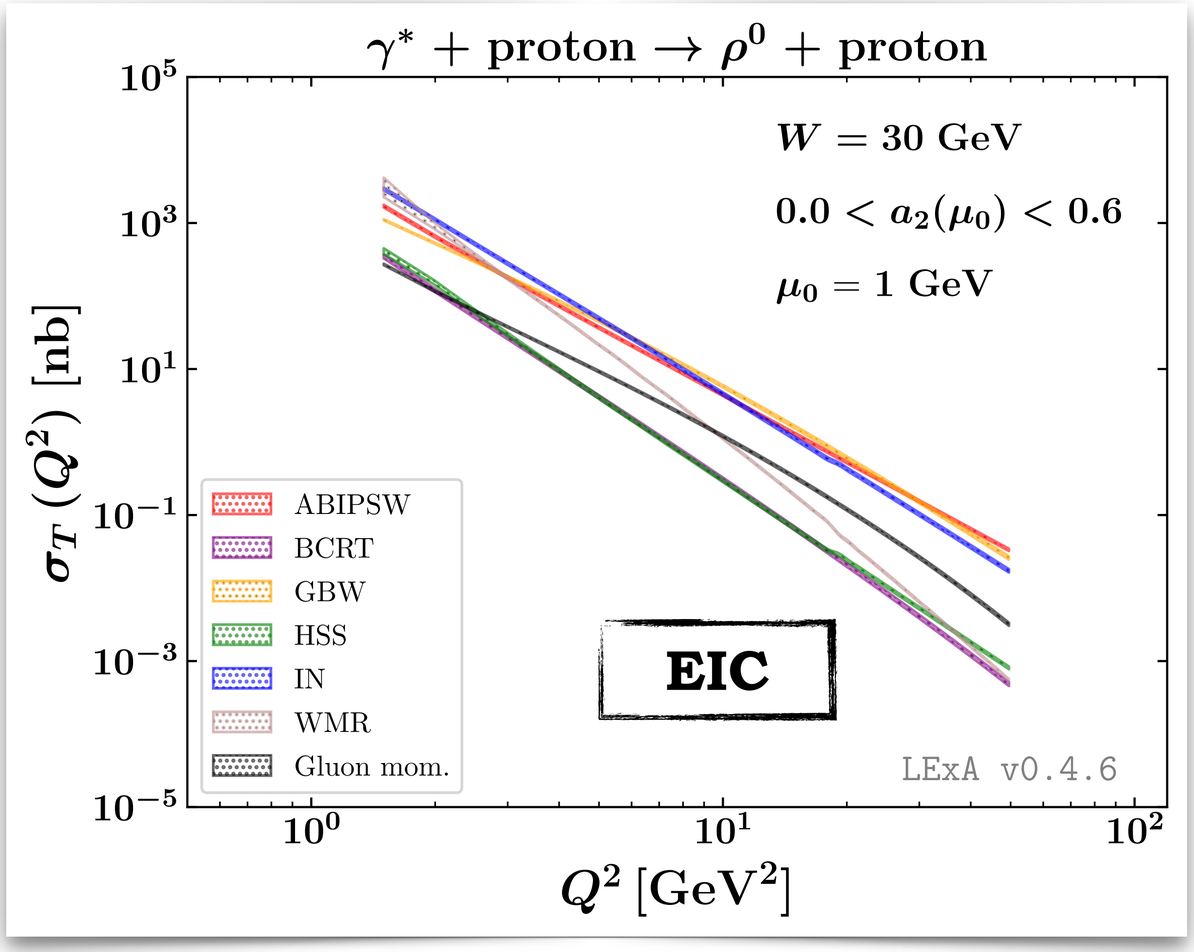




(saturation effects in ρ-meson polarization @HERA) [A. Besse, L. Szymanowski, S. Wallon (2013)]
(ρ-meson helicity amplitudes @HERA) [A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa (2018)]
(extension to ϕ-meson emissions) [A.D. Bolognino, A. Szczurek, W. Schäfer (2020)]
(in this slide) [A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa, W. Schäfer, A. Szczurek (2021)]

Forward ρ-mesons from HERA to the EIC: $\sigma_T(Q^2)$





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Single forward emissions

Exclusive light VM: ρ^0, ω, ϕ

* *Small-size* dipoles $\Rightarrow$ large κ_T
* Collinear description: twist-2/-3 LVM NP **DAs**

$$\Phi^{\gamma^* \to \rho} \propto \int_0^1 \mathrm{d}z \, T_H^{\gamma^* \to \rho}(z, \kappa_T, Q, \mu_R, \mu_F) \, \phi^{\lambda_\rho}(z, \mu_F)$$

* Significance of small κ_T under investigation…
* HERA indication: no large-r_d dynamics
* LVMs as tools: discrimination among UGD models
* LVMs as tools: UGD extraction $\Leftarrow$ HERA + EIC fits

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Quarkonia

* Size of dipoles $\Rightarrow$ wide range of κ_T
* Description: **NRQCD** (combined with LFWFs)

$$\left[\mathrm{LFWF} \otimes \mathscr{A}_{\mathrm{dip.}} \right] \overset{\text{dilute}}{\longleftrightarrow} \left[\Phi^{\gamma^* \to J/\Psi} \otimes \mathrm{UGD} \right]$$

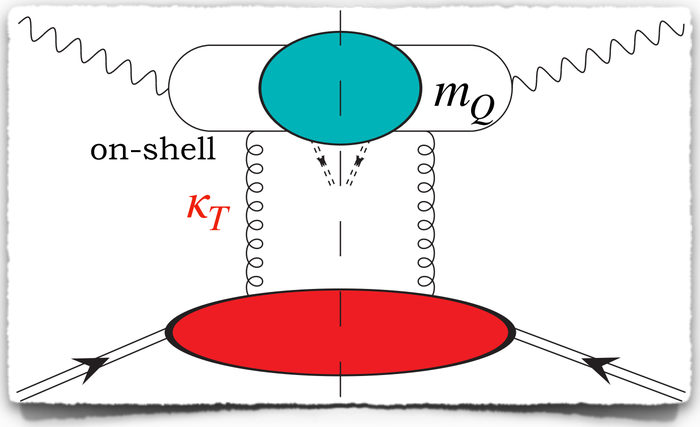
* Validity of *small-size* dipoles questionable…
* NRQCD: large-r_d dynamics for $\Psi(2s)$ ($\Upsilon(2s)$?)

🔗 [K. Suzuki *et al.* (2000)]; 🔗 [J. Cepila *et al.* (2019)]; 🔗 [M. Hentschinski *et al.* (2020)]

* Onia as tools: scan of TMD/HEF intersection range

Inclusive quarkonium production mechanisms

$$\kappa_T \ll Q$$



* Gluon TMD PDF
* Short-distance $(Q\bar{Q})$ + ShFs

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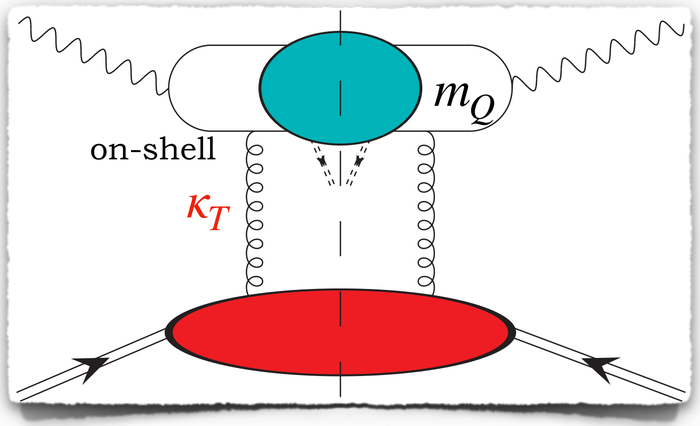

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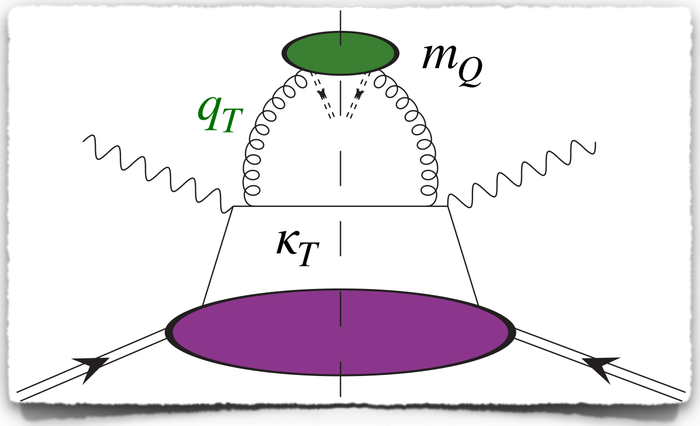
$\kappa_T \gg m_Q$



* Quark collinear PDF
* Onium in jet
* Single-quark TMD FF

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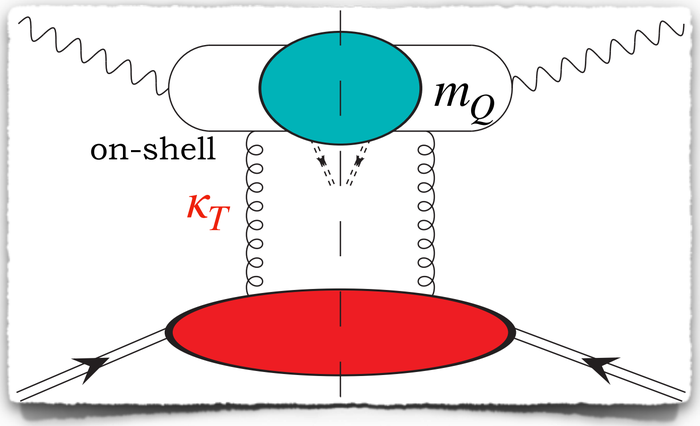

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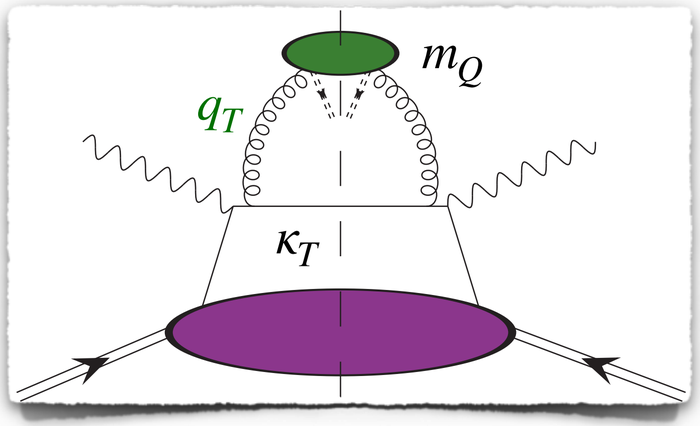

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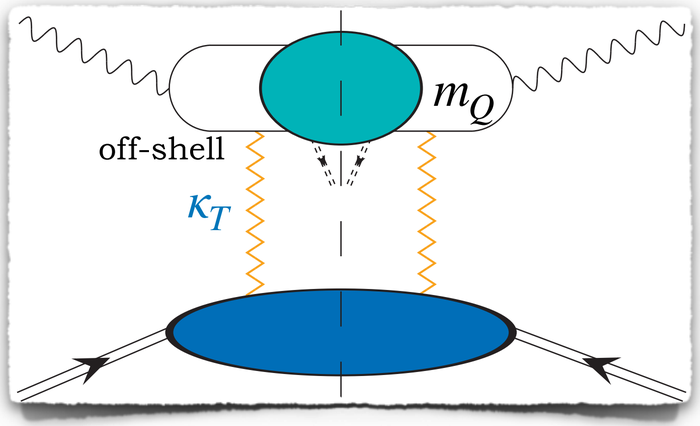
HEF



* BFKL UGD
* Reggeized gluons
* Dipole mechanism

Inclusive forward Drell-Yan di lepton production

- LHC, **forward region** $\rightarrow$ $(l^+ l^-)$ produced in the fragmentation region of h_2
  - ◇ Asymmetric configuration: $x_1 \gg x_2$, down to $x_2 \simeq 10^{-6}$

  $\Longrightarrow$ **possible small-x resummation effects expected!**

- **small-x** $\rightarrow$ evolution of sea $q\,(\bar{q})$ inside h_2 driven by gluon evolution
  - ◇ Dominance of sea $q\,(\bar{q})$ emerging in the last splitting (suppression of quark propagator at large rapidity)

- **high-energy factorization** $\rightarrow$ gluon exchange in the t-channel
  - ◇ collinear gluon PDF replaced by κ_T-UGD: $x\,g(x,\mu)$ $\rightarrow$ $\mathcal{F}(x,\kappa_T^2)$

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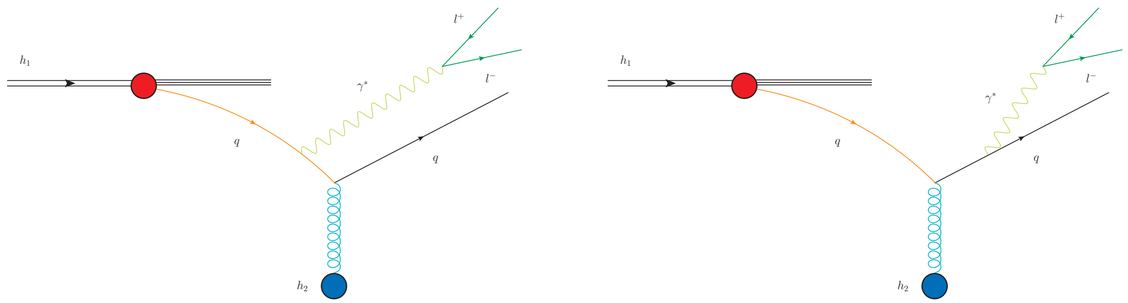

- Helicity structure functions in high-energy factorization:

$$\mathcal{W}_{[\lambda]} = \frac{2\pi M^2}{3} \int_{x_F}^1 \frac{dz}{z^2} \sum_{r=q,\bar{q}} f_r\left(\frac{x_F}{z},\mu_F\right) \int \frac{d\kappa_T\, d\phi_{\kappa_T}}{\left(\kappa_T^2\right)^2}\, \alpha_s(\mu_R)\, \mathcal{F}(x_g,\kappa_T^2)\, \Phi_{[\lambda]}(q_T,\vec{\kappa}_T,z)$$

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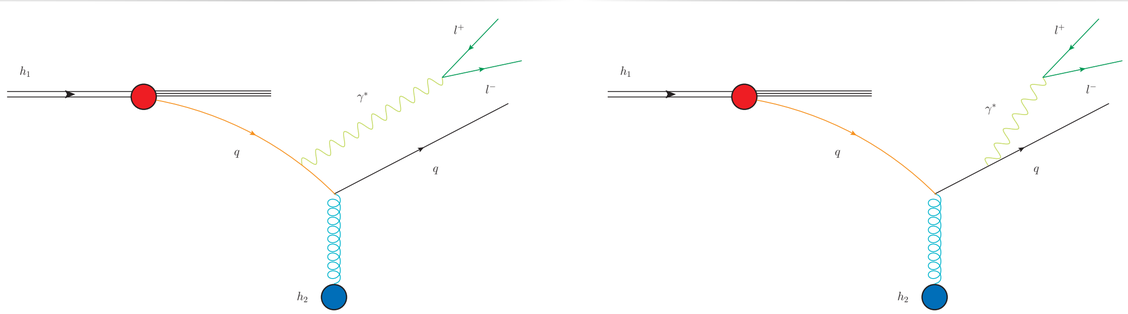

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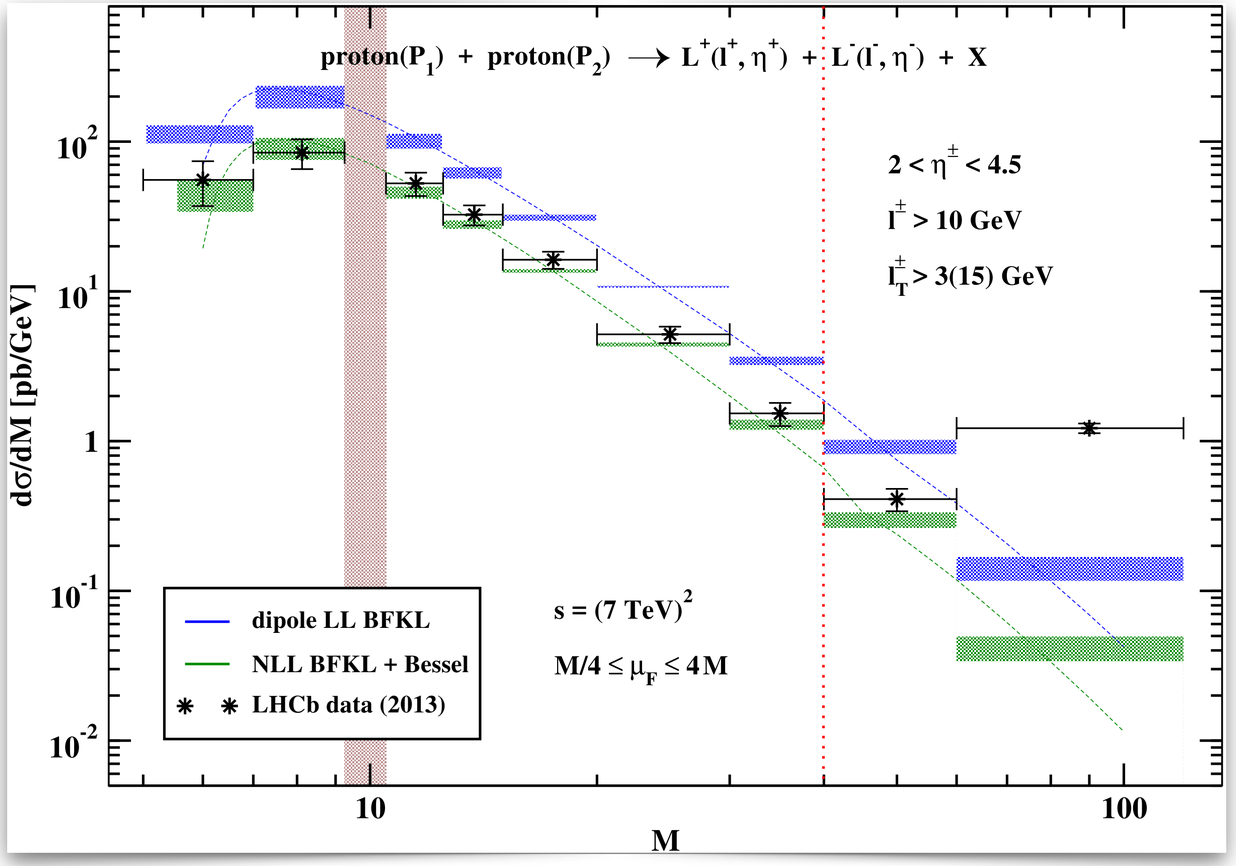

🔗 [D. Brzeminski, L. Motyka, M. Sadzikowski, T. Stebel (2017)]

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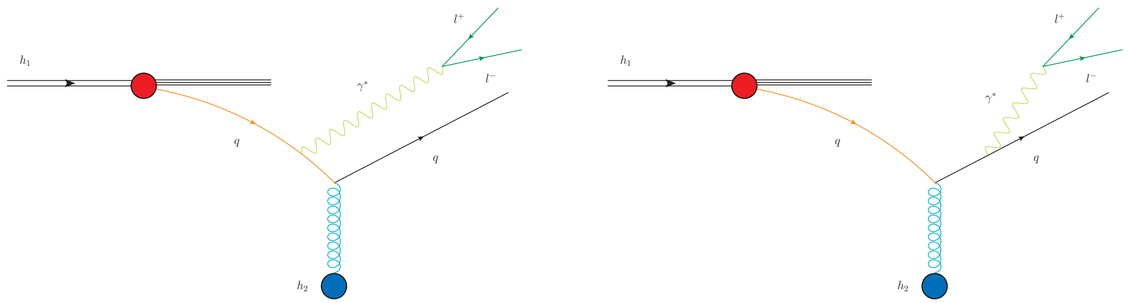

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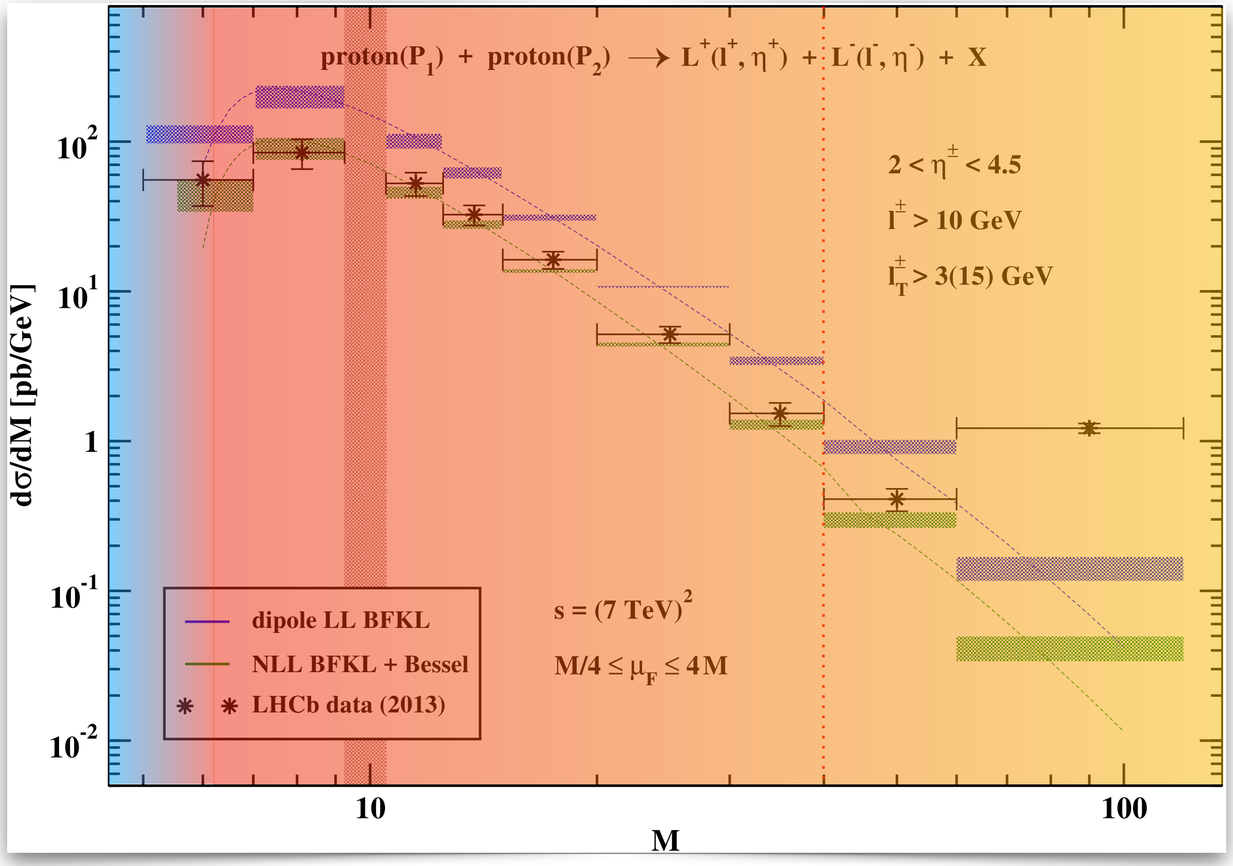
approaching limits of semi-hard regime



Z^0 contribution becoming relevant

UGD phenomenology: Partial conclusions

Summary...

Exclusive polarized ρ-leptoproduction

[A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa, arXiv:1808.02395, to appear in *Eur. Phys. J. C*]

- T_{11}/T_{00} helicity-amplitude ratio to constrain the **UGD** in the HERA range
- High sensitivity to **distinct UGD models**
- Low sensitivity to **region of small-κ_T values**

  effect of $\longrightarrow$ dominance of **small-size dipoles**

Forward Drell–Yan dilepton production

[F.G. C., D. Gordo Gómez, A. Sabio Vera, *Phys. Lett. B* **786** (2018) 201]

- Good description of $\mathrm{d}\sigma/\mathrm{d}M$ in the **BFKL approach** at the LHC
- Same observable well described also by **fixed-order** calculations

  [LHCb Coll.: LHCb-CONF-2012-013, CERN-LHCb-CONF-2012-013]; [ATLAS Coll.: G. Aad et al., *JHEP* **1406** (2014) 112]

- Future data for Drell–Yan production in **forward directions**

  peerless to $\longrightarrow$ gauge need for **high-energy resummation**

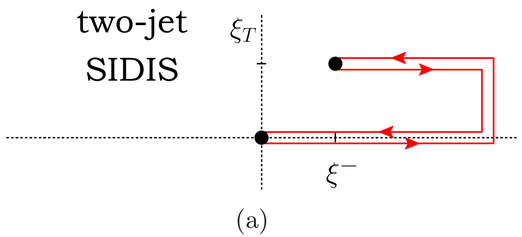
Gluon TMDs

Accessing f-type and d-type gluon TMD PDFs

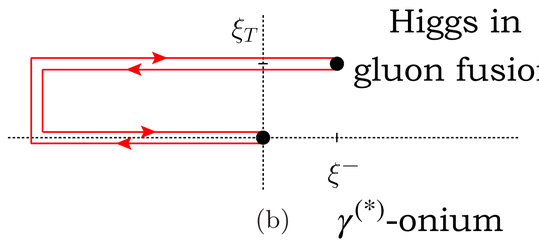
f-type (WW)

(a) $[+,+]$ or (b) $[-,-]$

two-jet
SIDIS



(a)



(b)

Higgs in
gluon fusion

$\gamma^{(*)}$-onium
hadroproduction

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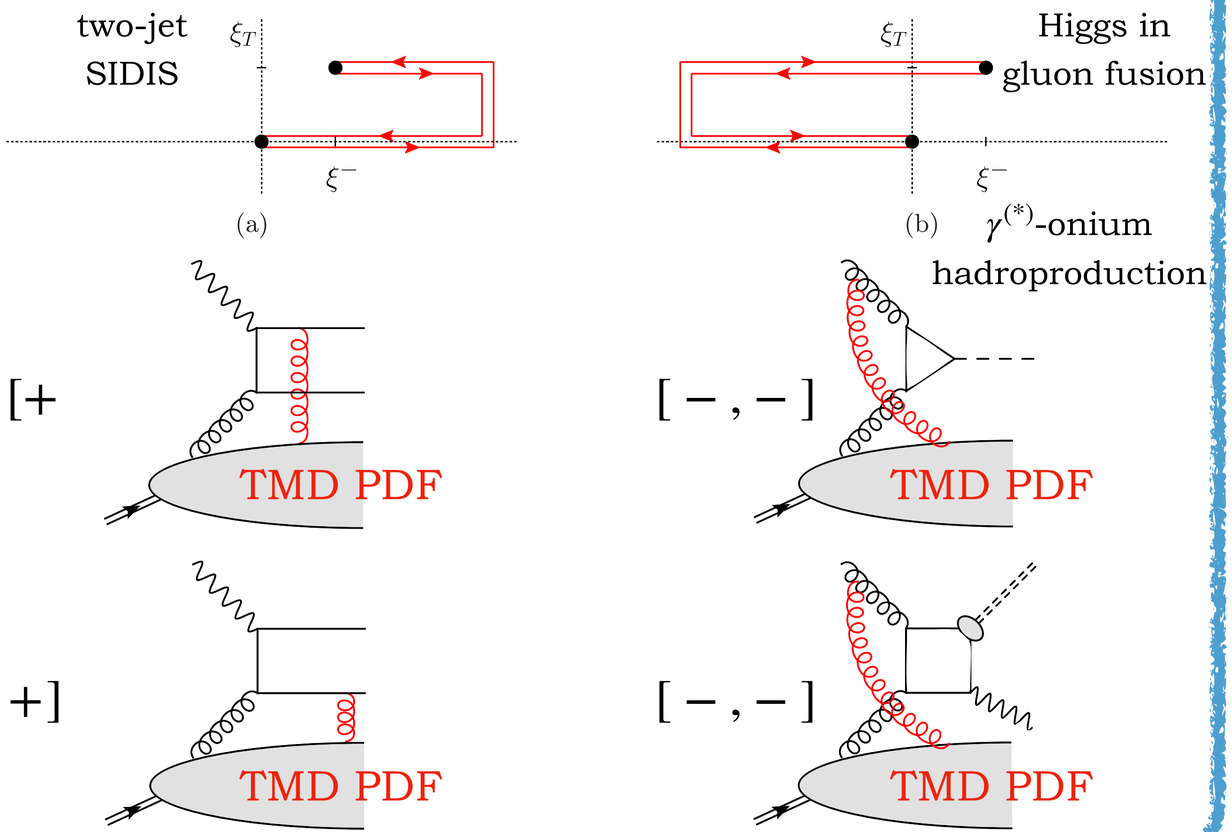

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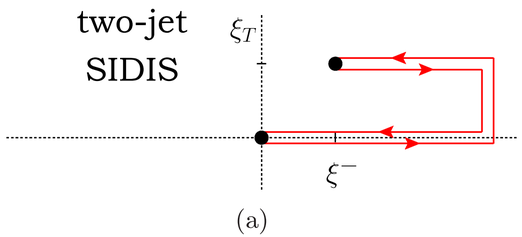
f-type (WW)

(a) [+ , +] or (b) [− , −]

two-jet SIDIS



(a)

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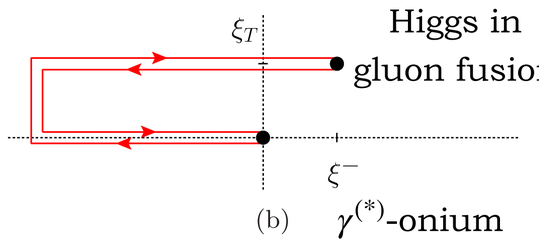
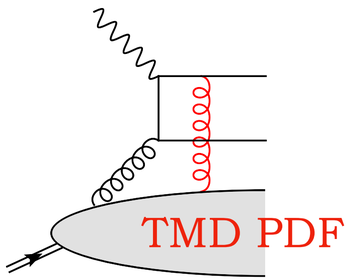


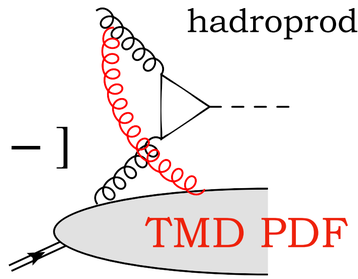
(b)

$\gamma^{(*)}$-onium hadroproduction

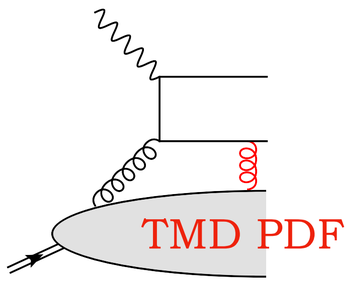
[+



[− , −]



+]



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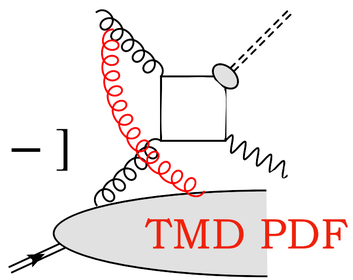
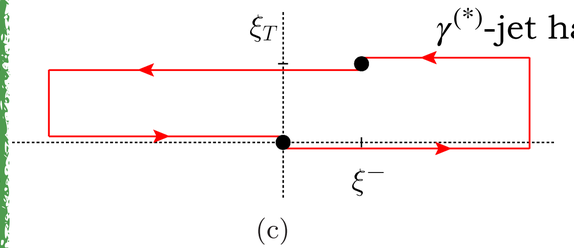


d-type (DP)

(c) [+ , −] or (d) [− , +]



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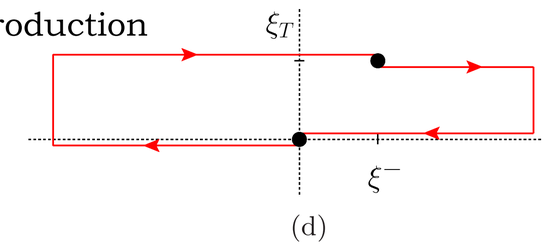

(d)

Accessing f-type and d-type gluon TMD PDFs

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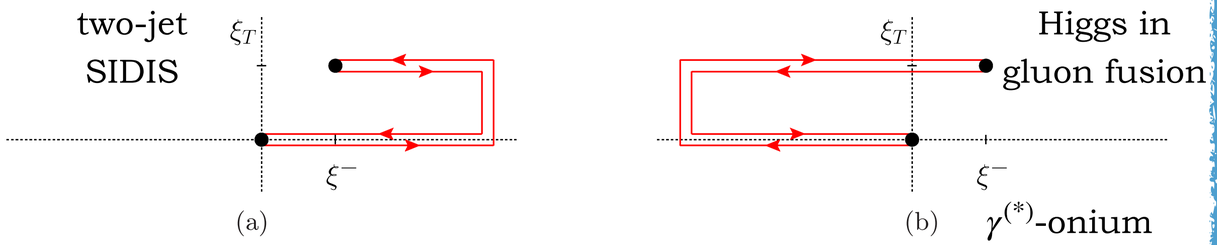

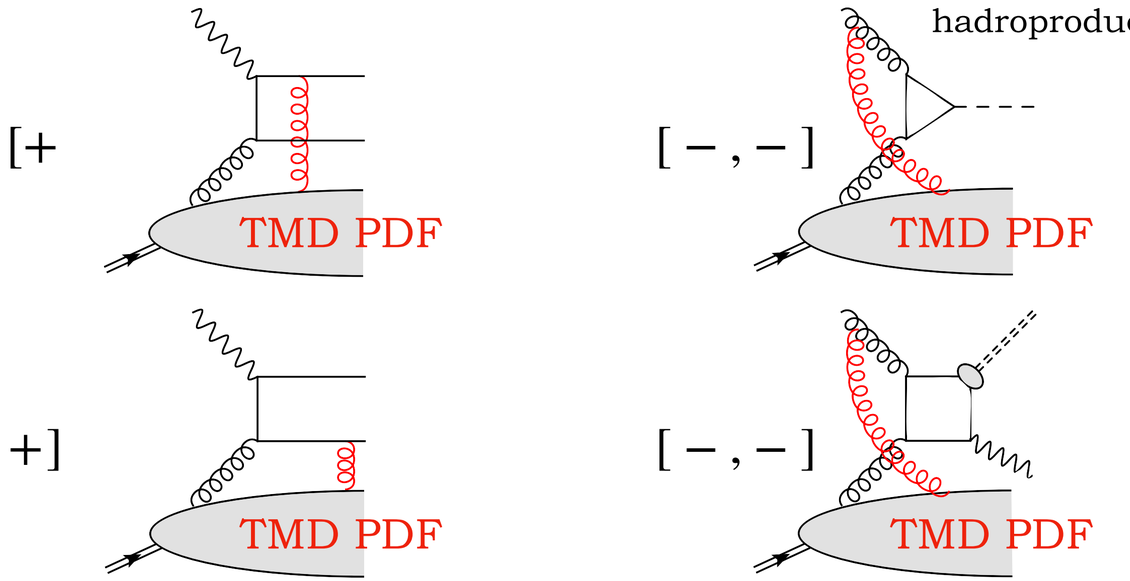

d-type (DP)

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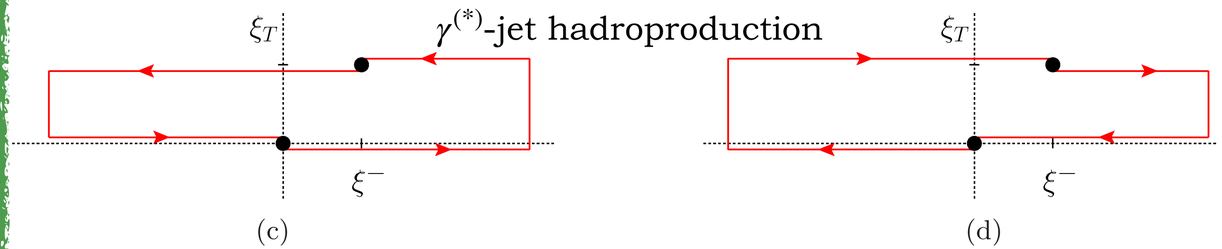

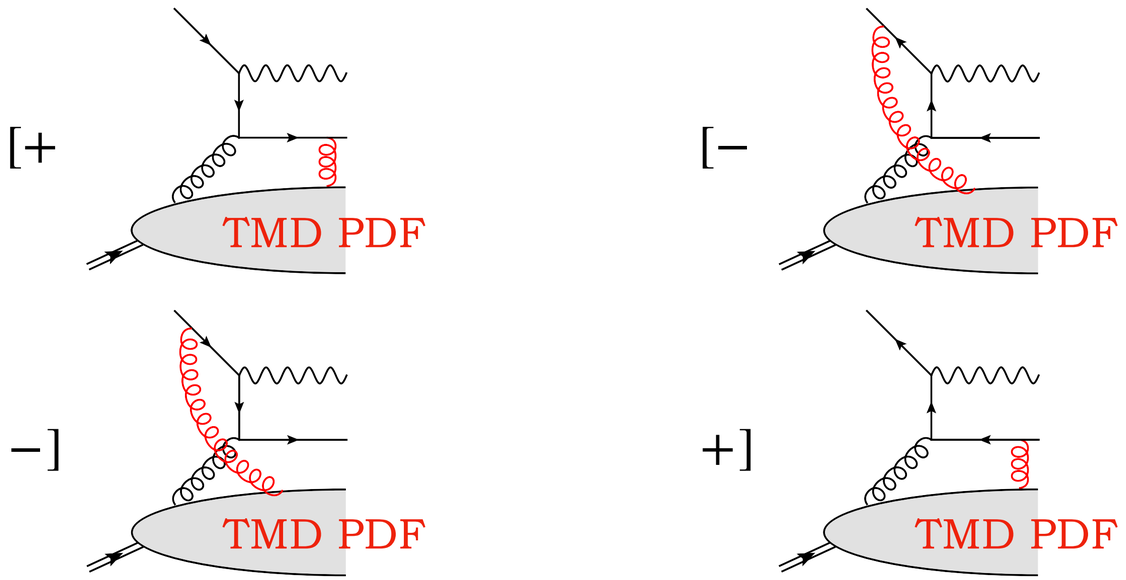

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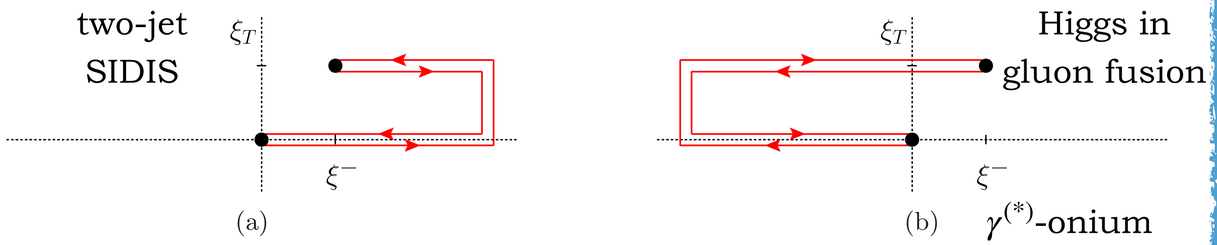
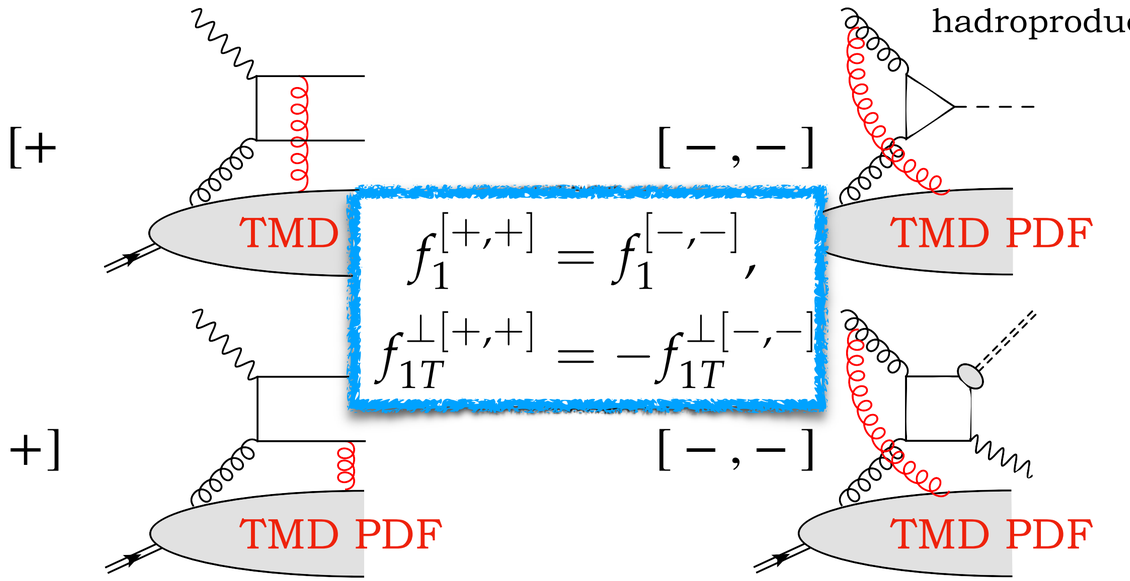
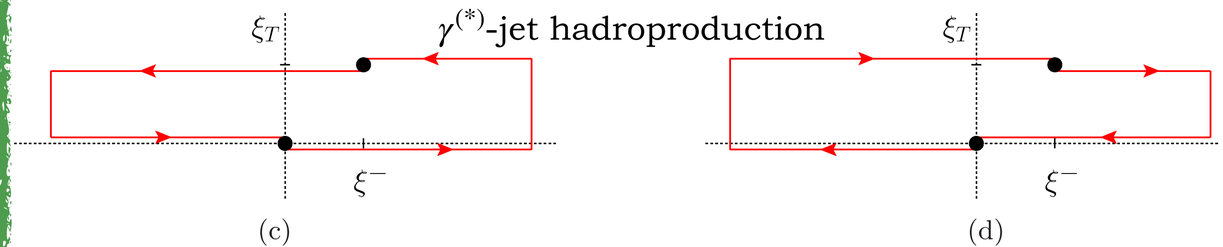


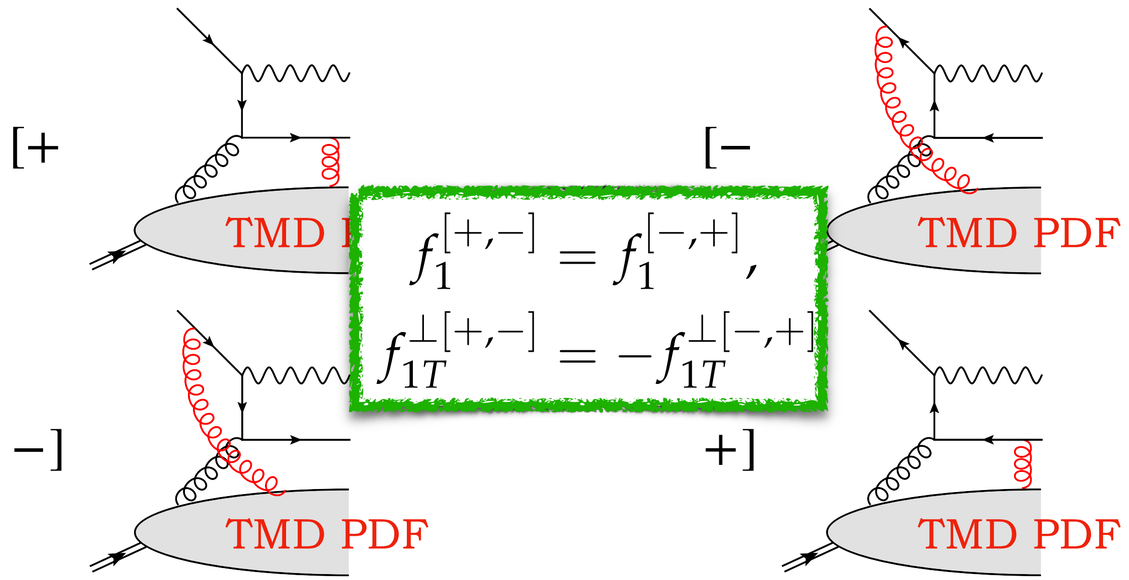


$$f_1^{[+,+]} = f_1^{[-,-]},$$
$$f_{1T}^{\perp[+,+]} = -f_{1T}^{\perp[-,-]}$$

d-type (DP)

(c) $[+,-]$ or (d) $[-,+]$





$$f_1^{[+,-]} = f_1^{[-,+]},$$
$$f_{1T}^{\perp[+,-]} = -f_{1T}^{\perp[-,+]}$$

¡ Gauge link → two main **independent** sets of TMD PDFs, **not related** to each other !

Gauge invariance

- Quark fields in $x_{1,2}$ $\rightarrow$ matrix element **bilocal** $\rightarrow$ correlator **not** gauge invariant
- Densities parameterizing correlators *need* to be gauge invariant
- Gauge invariance *restored* by **Wilson line**, or **gauge link**

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$$\psi'_\beta(x(t)) = \mathbb{P}\exp\left\{ -ig\int_0^t ds \frac{dx^\mu}{ds} A^a_\mu(x(s)) T^a_{\beta\alpha} \right\} \psi_\alpha(x(0))$$
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path-ordered exponential factor involving *gauge connection*

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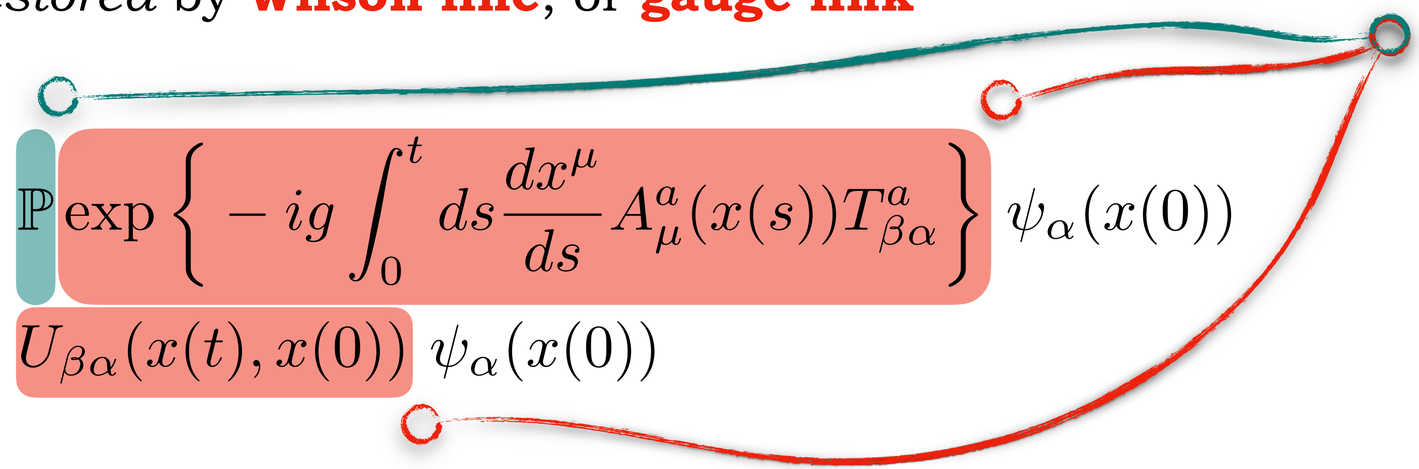

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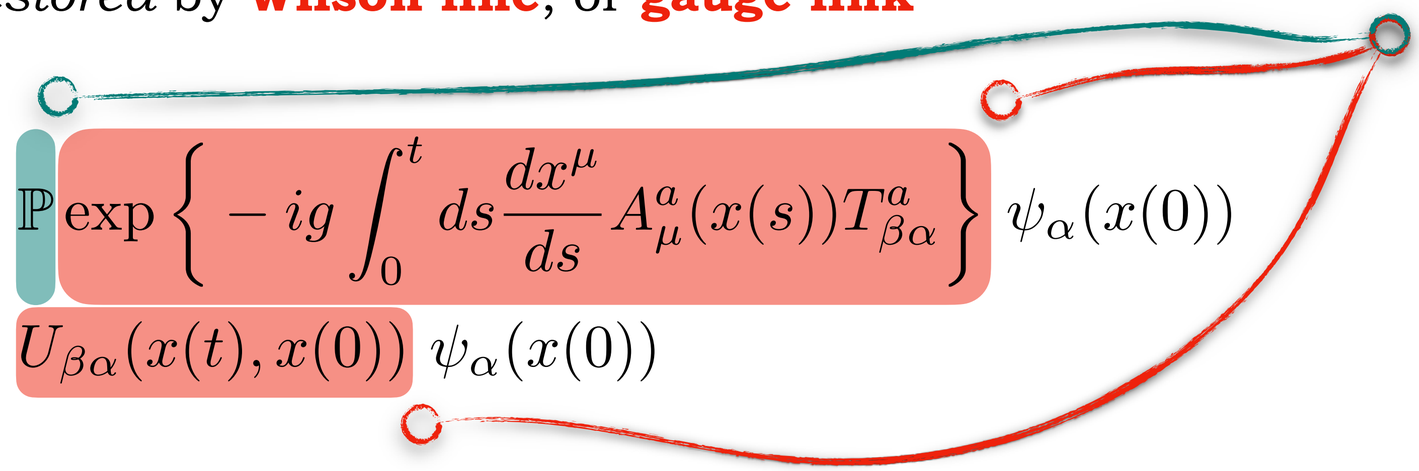
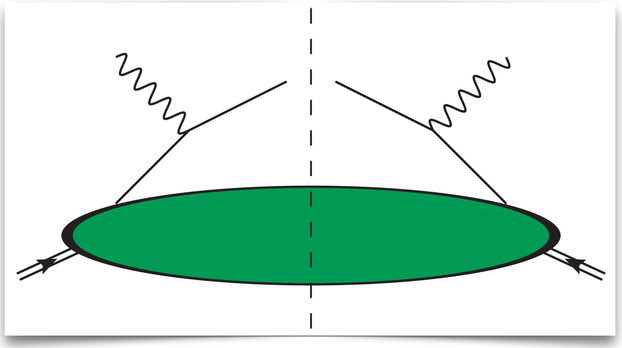


- Result → $\bar{\psi}_j(0)\,U(0,\xi)\,\psi_i(\xi)$ operator invariant under $SU(3)$ gauge transformations
- Physical interpretation → **resummation** of (calculable) **infinite gluon emissions**

Gauge links and processes dependence



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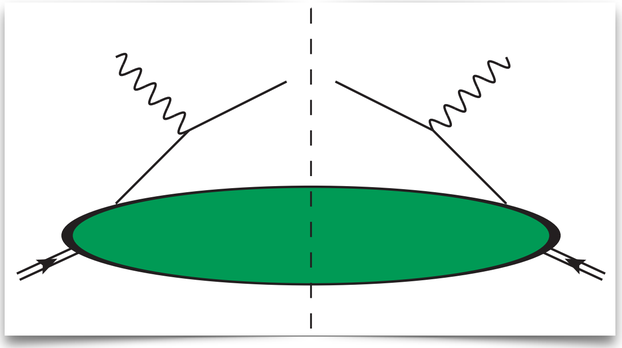
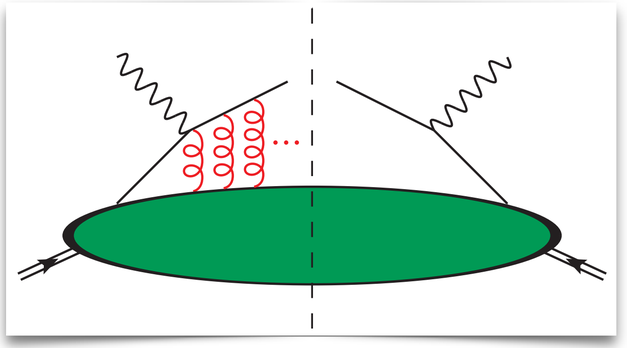


$$\bar{\psi}_j(0)\ U(0,\xi)\ \psi_i(\xi)$$

Gauge link (Wilson line)

Resummation of (calculable)
infinite gluon emissions



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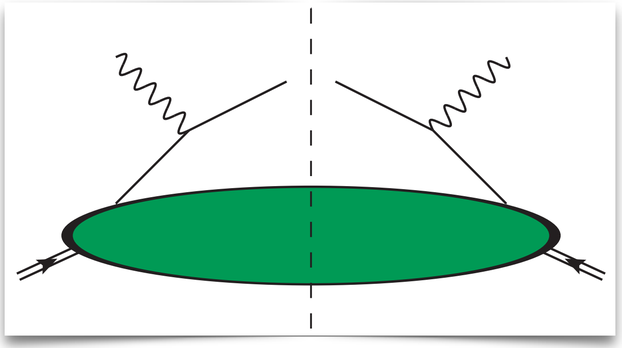

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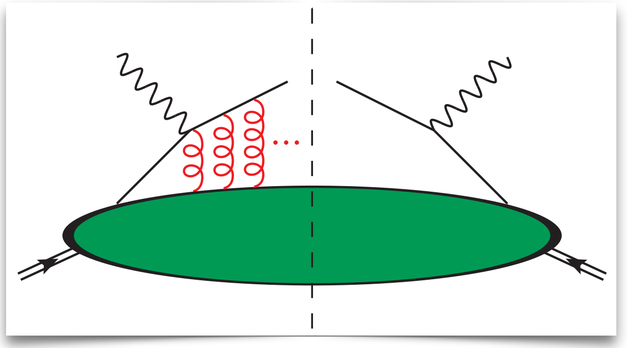
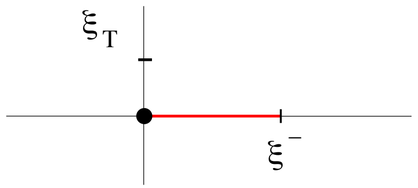


Collinear PDFs

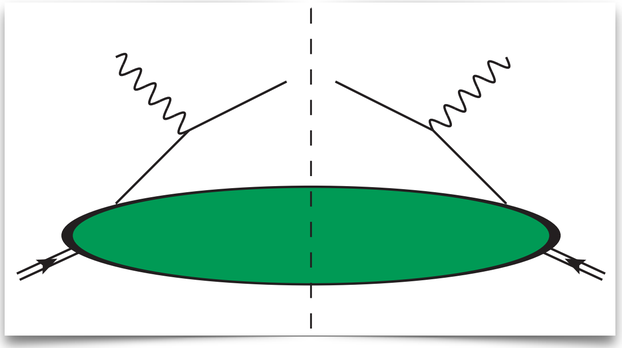
$$\Phi_{ij}(x) \doteq \int d^2\boldsymbol{p}_T \Phi_{ij}(x,\boldsymbol{p}_T) = \int \frac{d\xi^-}{2\pi} e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0)\psi_i(\xi)|P\rangle_{|\xi^+=0,\xi_T=0}$$



- Light-cone: $\xi^+ = 0$, $\boldsymbol{\xi} = 0$
- **Straight** gauge link (unique!)
- $(A^+ = 0)$ light-cone: WL $= \hat{1}$

Universality warranted

Gauge links and processes dependence



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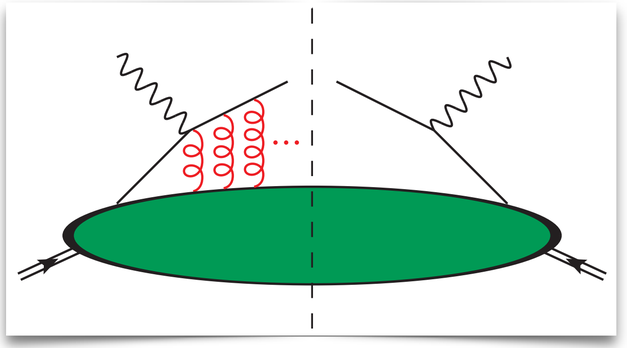

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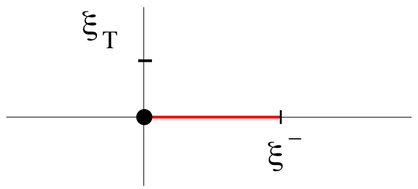

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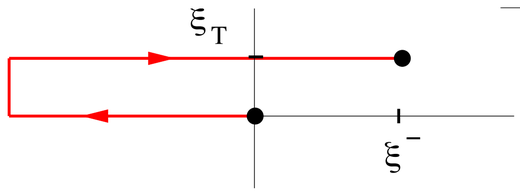
☑ **Universality warranted**

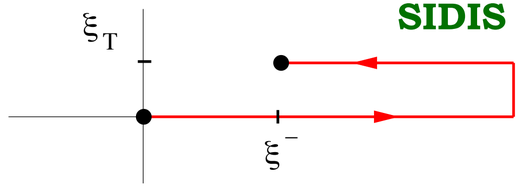
TMD PDFs

- *Transverse* gauge link not eliminated by gauge choice
- **Staple-like** gauge link (not unique!)

¡! **Process dependence**



Drell-Yan

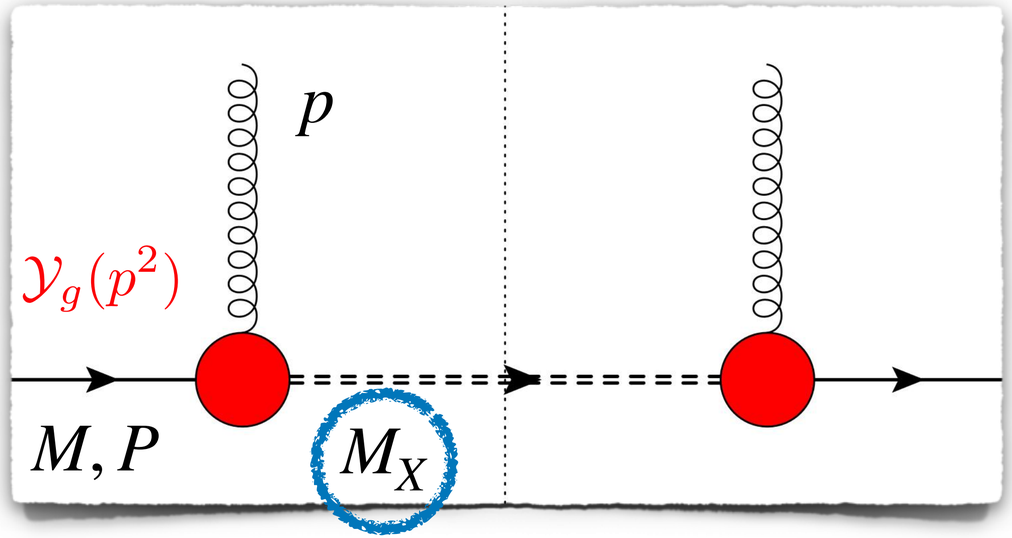


SIDIS

Spectator-model gluon TMD PDFs

Spin-1/2 spectator

Lowest Fock state:
tri-quark spectator
on-shell and
with mass M_X



[A. Bacchetta, F.G.C., M. Radici, P. Taels (EPJC 2020)]

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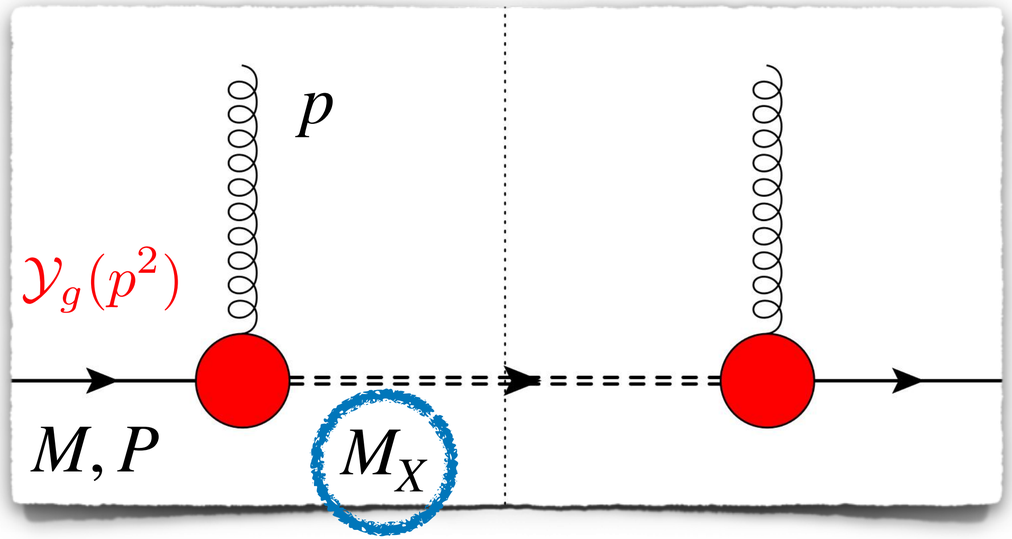

Nucleon-gluon-spectator vertex

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} Tr\left[(\not{P}+M)\frac{1+\gamma^5\not{S}}{2}G^*_{\mu\rho}(p)G^{\nu\sigma}(p)\mathcal{Y}_g^{\rho *}\mathcal{Y}_{g\sigma}(\not{P}-\not{p}+M)\right]$$

$$\mathcal{Y}_g^\mu = g_1(p^2)\gamma^\mu + i\frac{g_2(p^2)}{2M}\sigma^{\mu\nu}p_\nu$$

mimics proton form factors
(conserved EM current
of a free nucleon)

🔗 [A. Bacchetta, F.G.C., M. Radici, P. Taels (EPJC 2020)]

Spectator-model gluon TMD PDFs

Link with collinear factorization

1. p_T-integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution
2. TMDs and PDFs *decouple* due to evolution

🔗 [A. Bacchetta, F.G.C., M. Radici, P. Taels (EPJC 2020)]

Spectator-model gluon TMD PDFs

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Dipolar form factor(s)

$$g_{1,2}(p^2) = \kappa_{1,2}\,\frac{p^2}{|p^2 - \Lambda_X^2|^2}$$

1. Cancels singularity of gluon propagator
2. Suppresses effects of high p_T
3. Compensates log divergences arising from p_T-integration
4. Adds three more parameters: $\kappa_{1,2}$ and Λ_X

[A. Bacchetta, F.G.C., M. Radici, P. Taels (EPJC 2020)]

Assumptions of the model

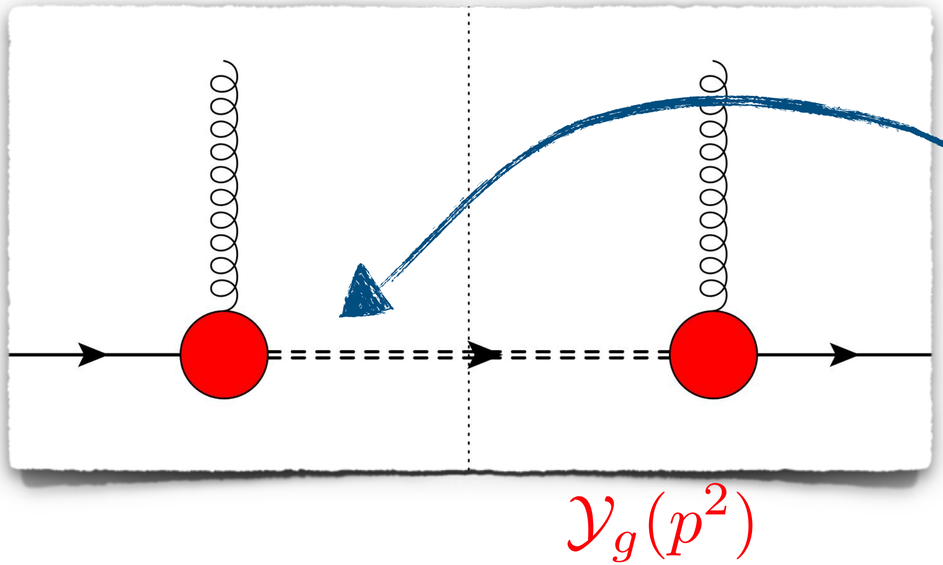
Spectator-system spectral-mass function

$$F(x, \boldsymbol{p}_T^2) = \int_M^\infty \mathrm{d}M_X \, \rho_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$$

spectral-mass function

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



Instead of a single on-shell spectator, a continuum of spectators

[A. Bacchetta, F.G.C., M. Radici, P. Taels (EPJC 2020)]

Assumptions of the model

🔗 [A. Bacchetta, F.G.C., M. Radici, P. Taels (EPJC 2020)]

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spectator-model TMD

🔗 [Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]

$$\rho_X\left(M_X; \{X^{(\mathrm{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\}\right) = \mu^{2a}\left[\frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}}\right]$$

low-x (high-μ^2) tail $\propto (a - b)$

$q\bar{q}$ contributions energetically available at large M_X

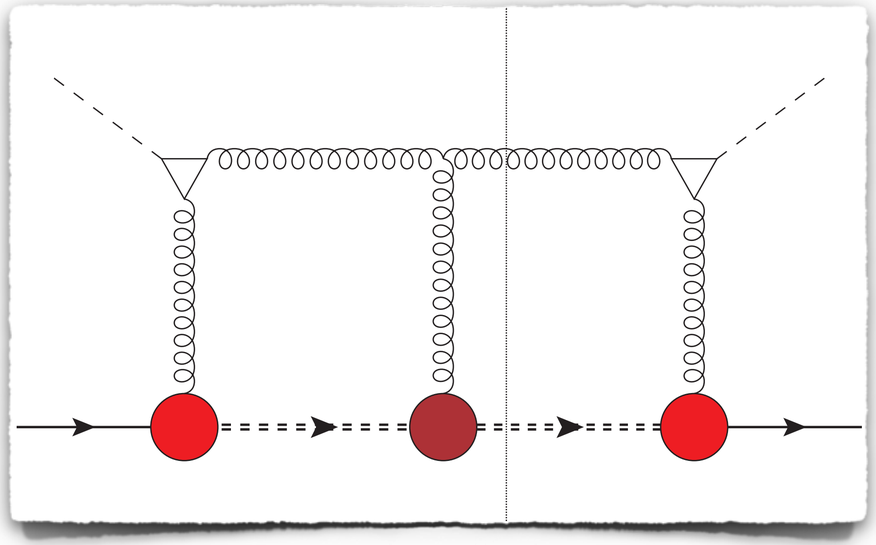
$$\mu^2 = M_X^2 - M^2$$

moderate-x trend

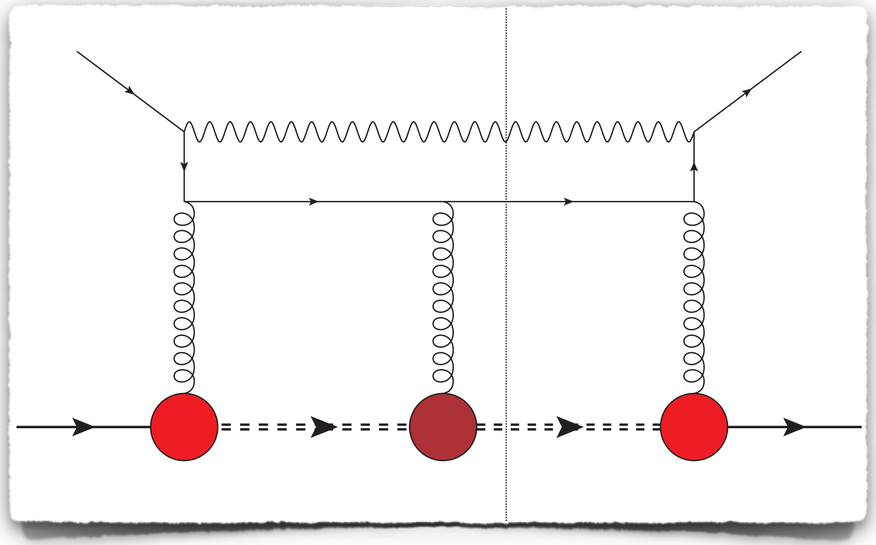
pure tri-quark contribution at low M_X

T-odd gluon TMD PDFs in a spectator model

Higgs-gluon fusion ⇒ f-type [+ , +]



Photon-jet emission ⇒ d-type [+ , –]



nucleon-gluon-spectator

spectator-gluon-spectator

$$\mathcal{Y}^{\mu}_{bc}(p^2) = \delta_{bc}\left[g_1(p^2)\,\gamma_\mu + g_2(p^2)\,\frac{i}{2M}\,\sigma^{\mu\nu}\,p_\nu\right]$$

$$\mathcal{X}^{\mu}_{abc}(p^2) = f^{abc}\left[g^f_1(p^2)\,\gamma^\mu + g^f_2(p^2)\,\frac{i}{2M}\,\sigma^{\mu\nu}\,p_\nu\right] - i\,d^{abc}\left[g^d_1(p^2)\,\gamma^\mu + g^d_2(p^2)\,\frac{i}{2M}\,\sigma^{\mu\nu}\,p_\nu\right]$$

T-odd gluon TMD PDFs in a spectator model

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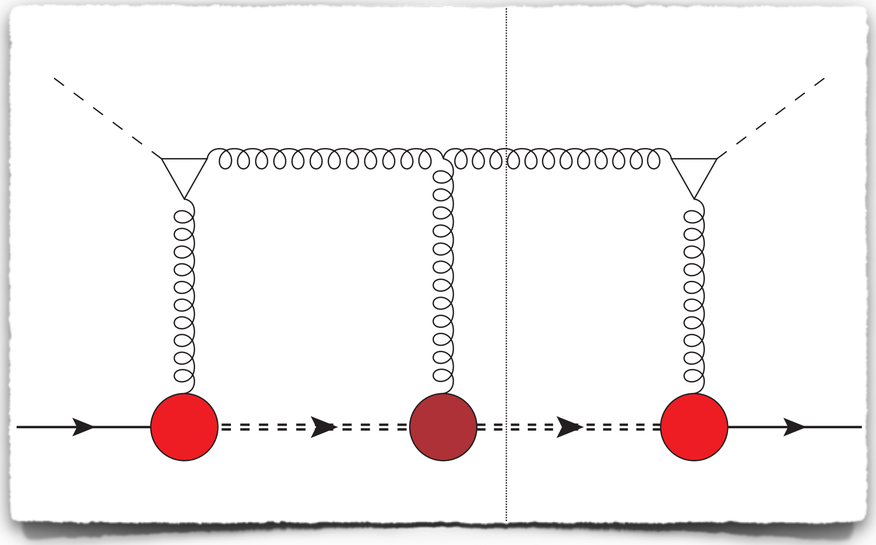

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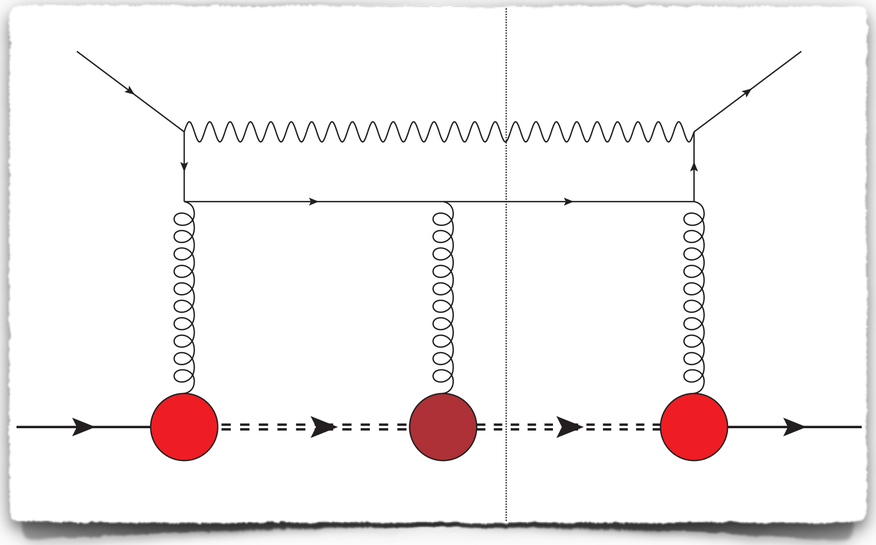

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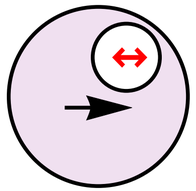
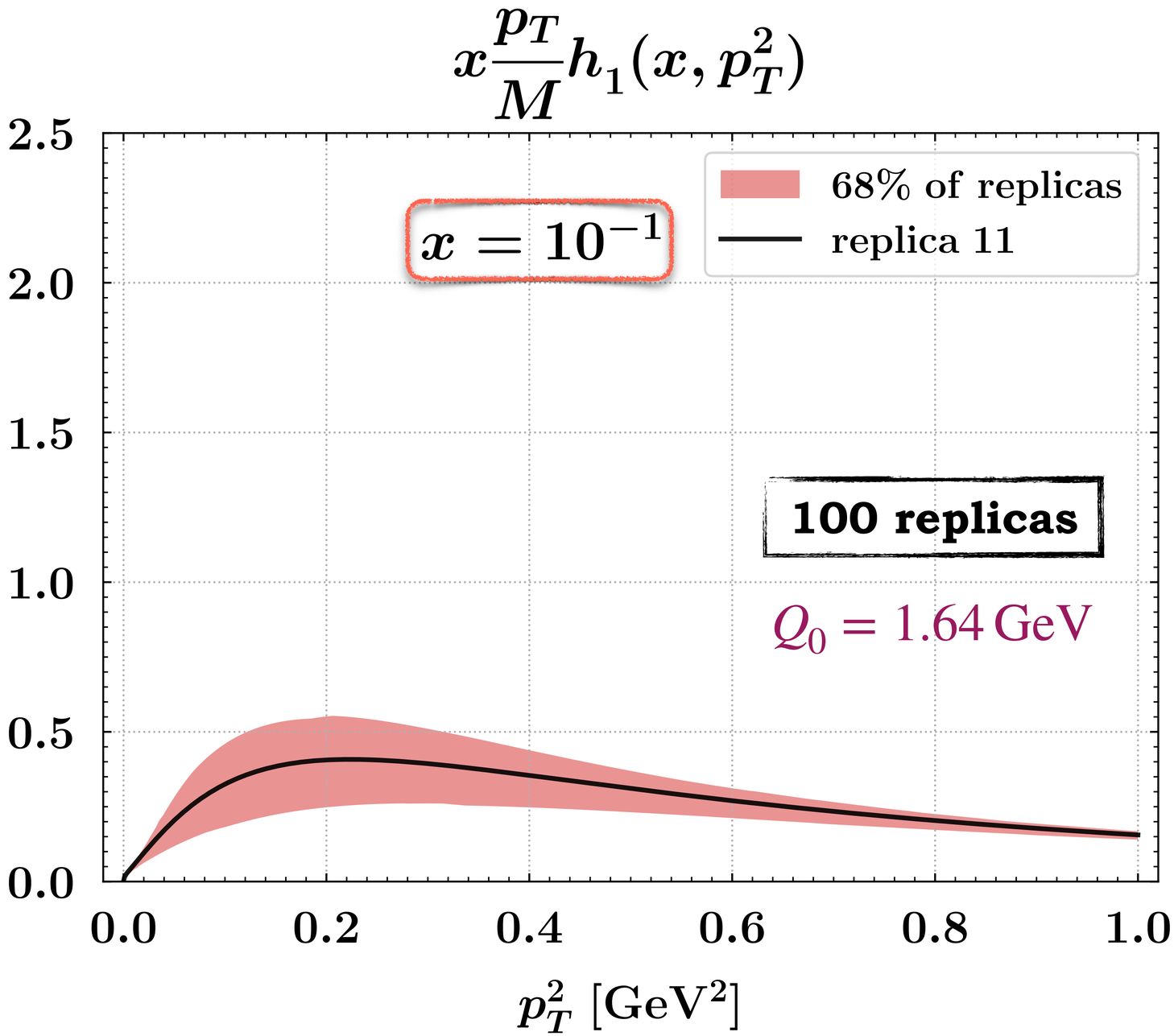
Assumption: $g^d_{1,2}(p^2) = g^f_{1,2}(p^2) \equiv g_{1,2}(p^2)$ $\Leftrightarrow$ $f_{1T}^{\perp[+,-]} = \frac{c_{[+,-]}}{c_{[+,+]}} f_{1T}^{\perp[+,+]} \equiv \frac{10}{18} f_{1T}^{\perp[+,+]}$

f-type linearity gluon TMD

[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]





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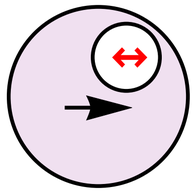

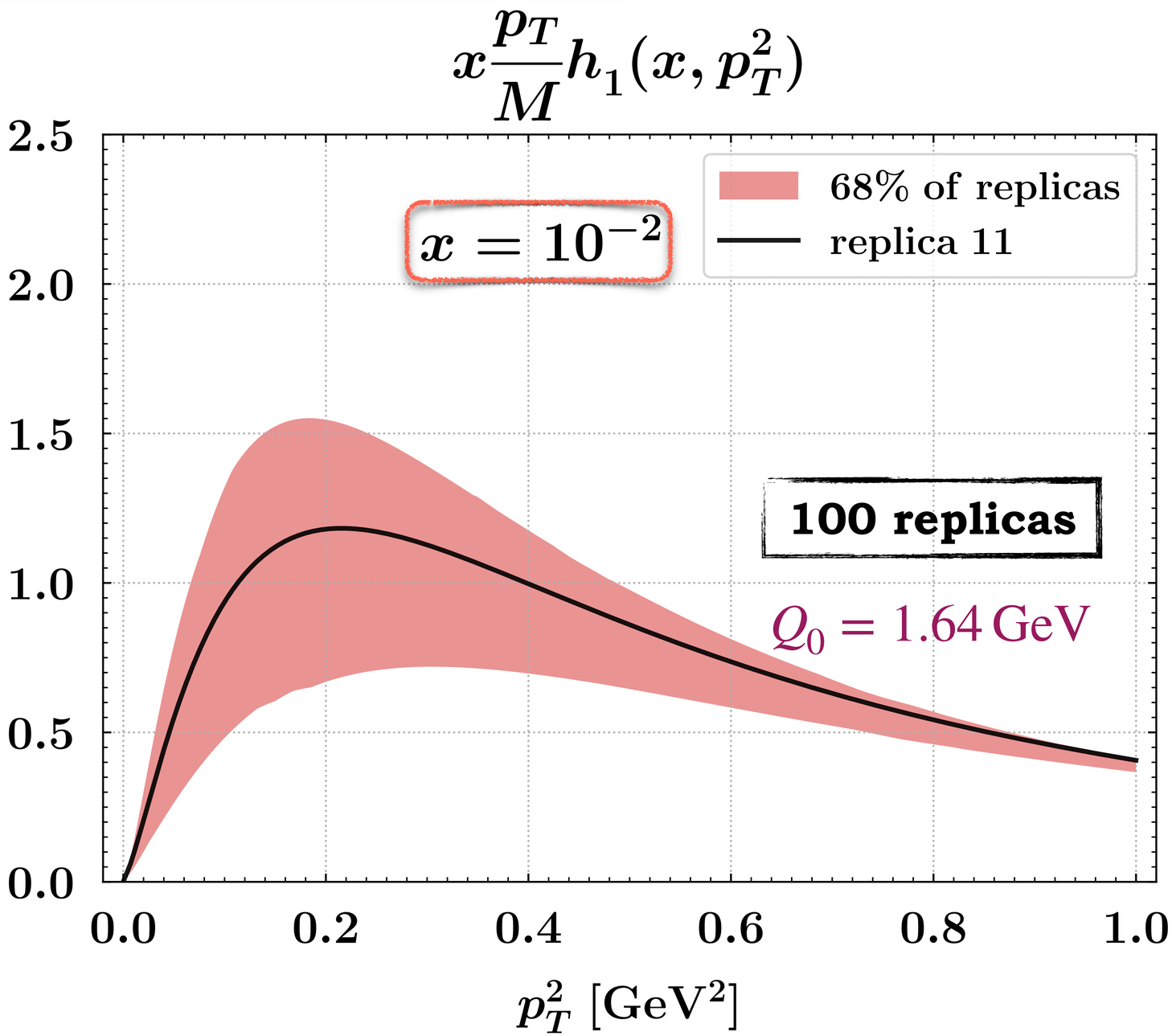

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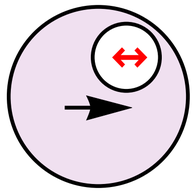

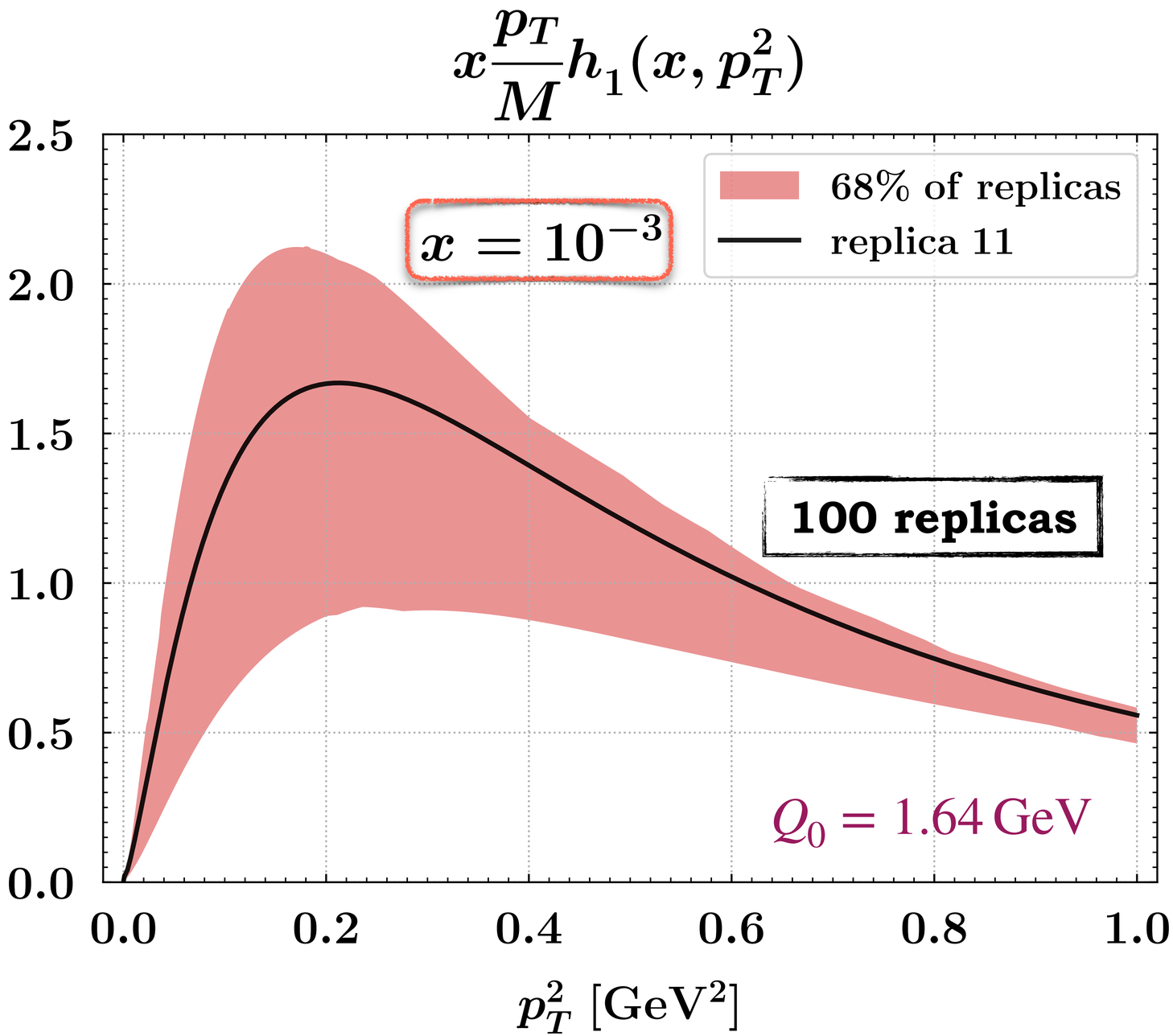

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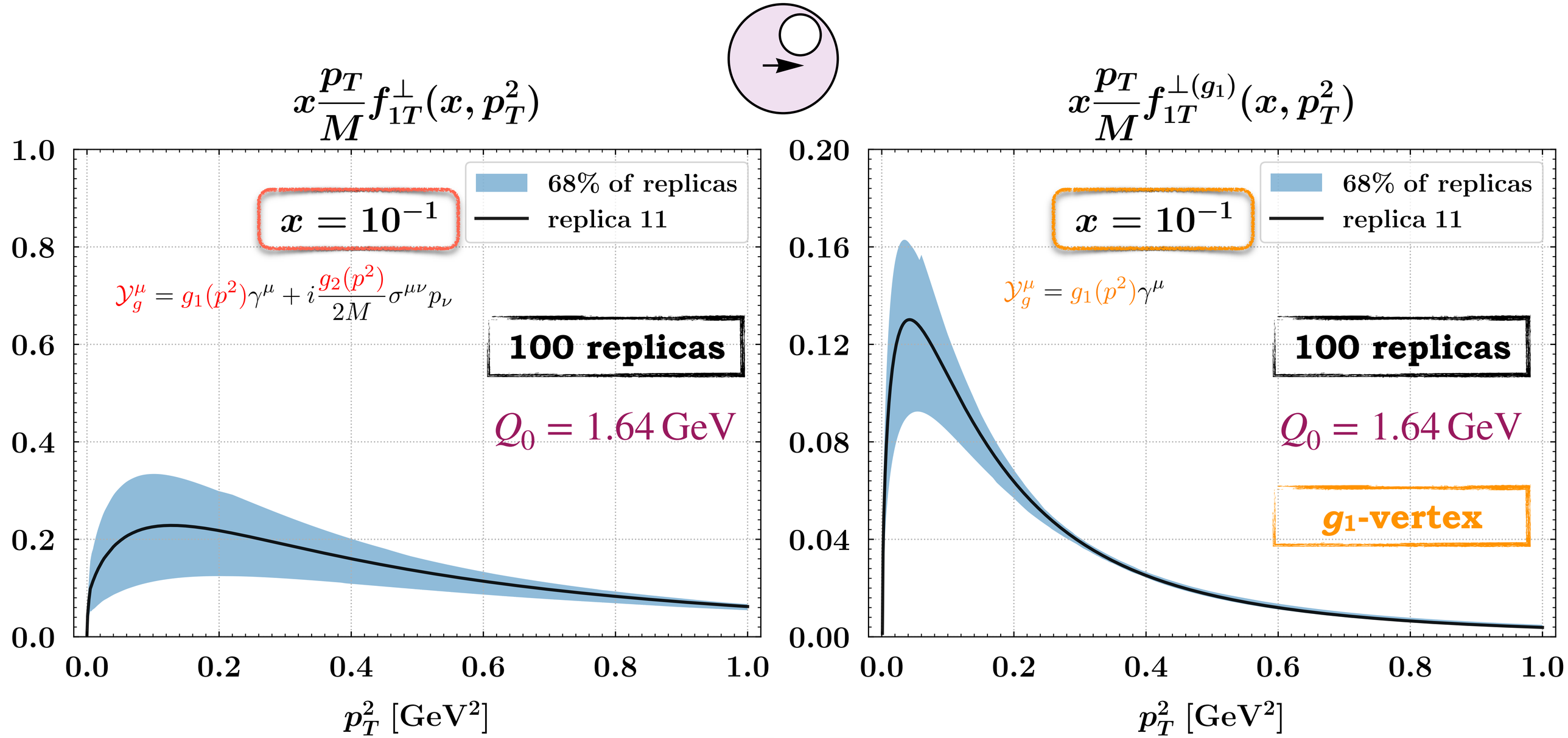
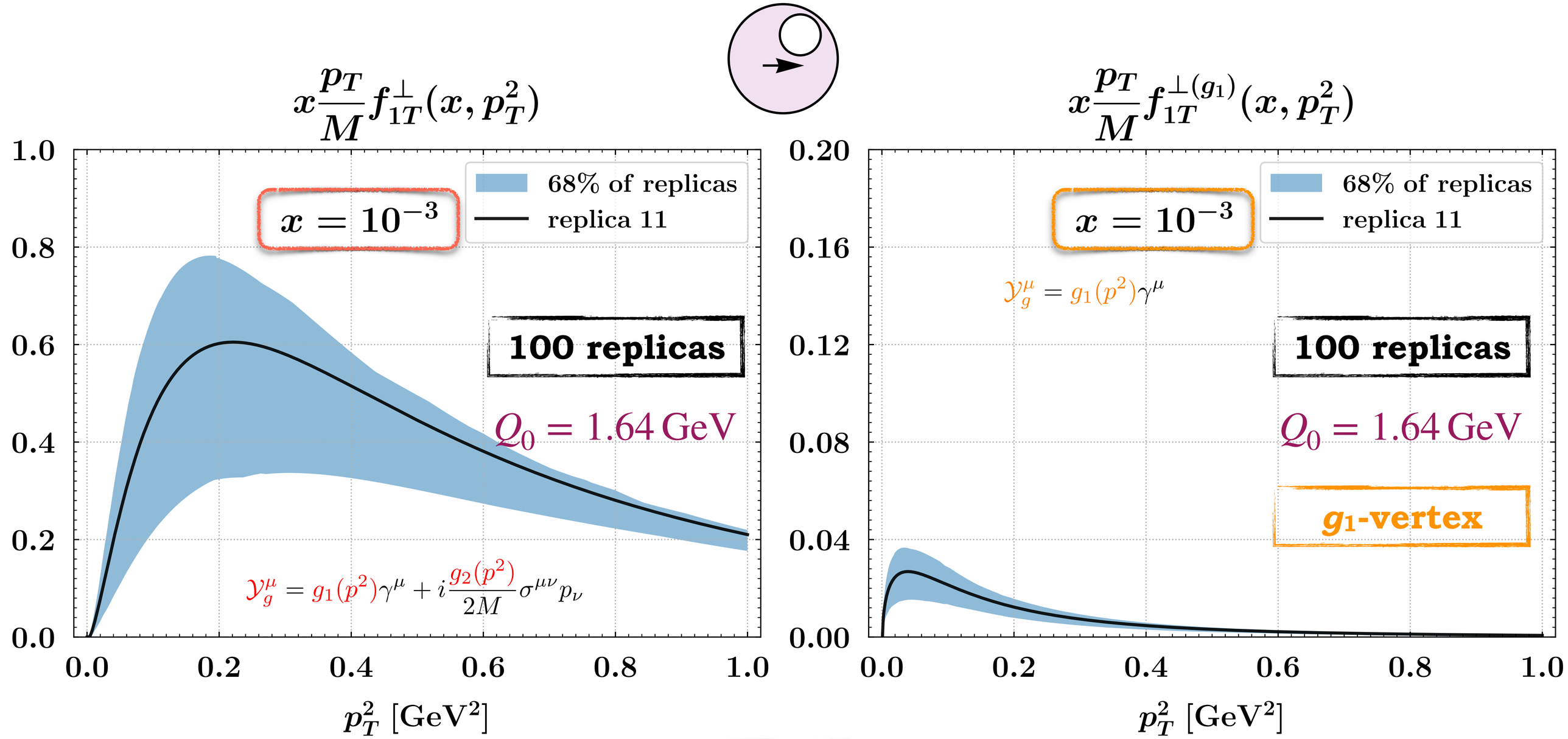
[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]



Backup

f-type Sivers gluon TMD

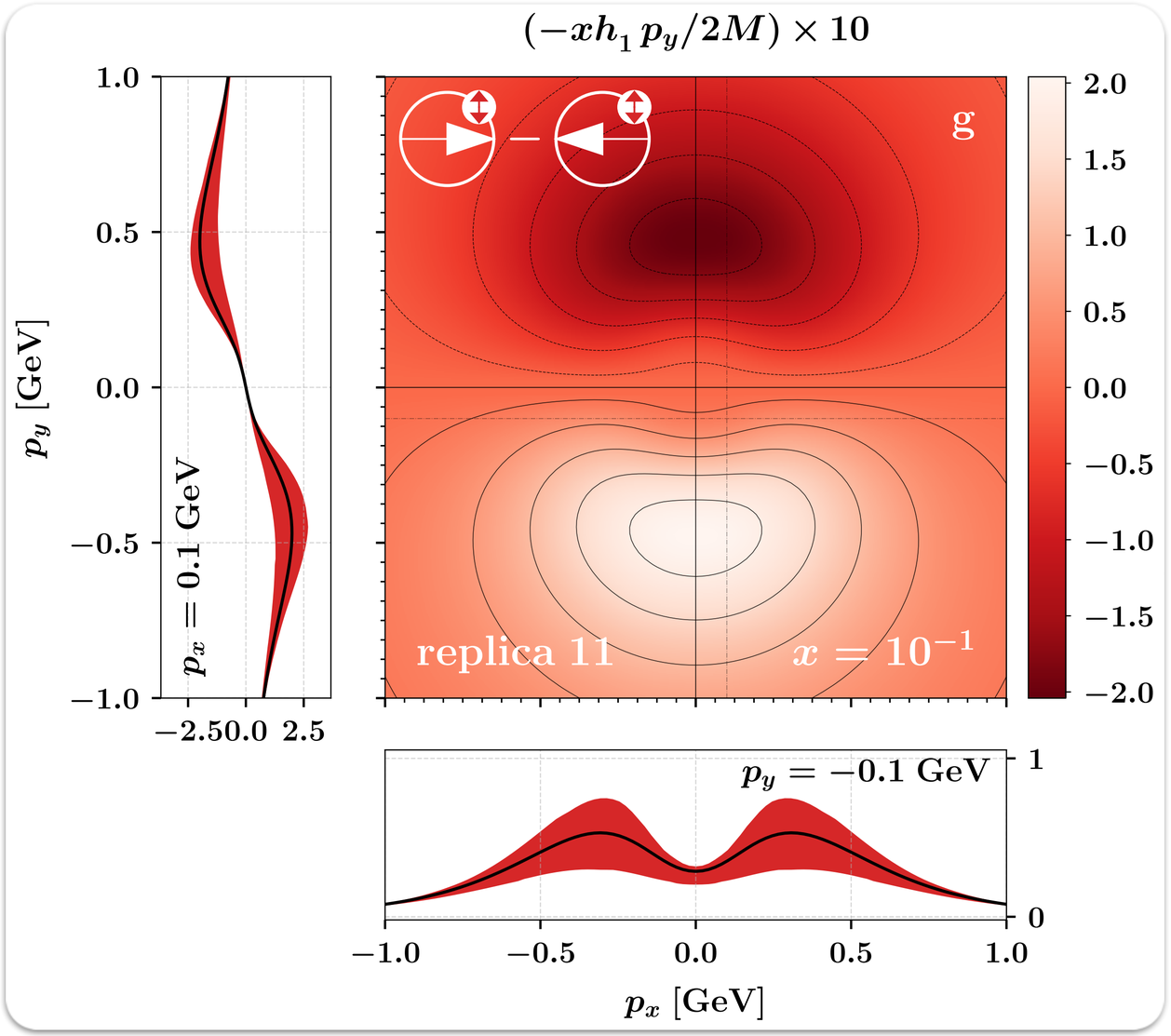
[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]

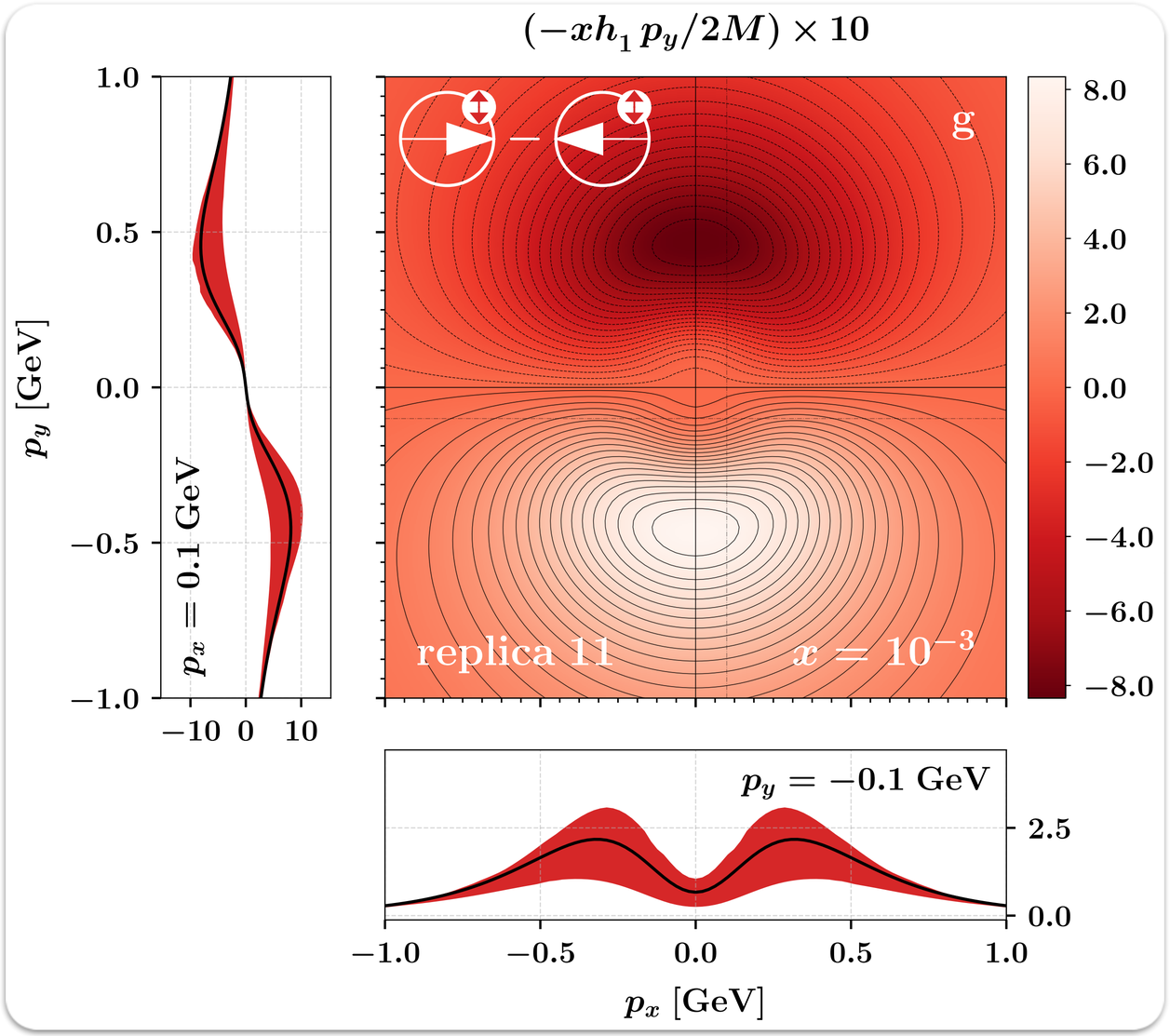


Backup

Gluon linearity effect

[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]





Backup

Anatomy of gluon TMD PDFs

matching coefficients

collinear PDF

nonperturbative Sudakov

nonperturbative TMD function

$$F(x, \mathbf{b}; \mu, \zeta) = \sum_j \left(C_j^{(F)} \otimes F^j \right)(x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu, \zeta)} e^{S_{\rm NP}(b)} F_{\rm NP}(x, b)$$

perturbative Sudakov

perturbative expansion in $\alpha_s(\mu)$

resummation of

$$L = \ln \frac{Q^2}{\mu_b^2}$$

define logarithmic ordering

slide adapted from C. Bissolotti

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perturbative expansion in $\alpha_s(\mu)$

resummation of $L = \ln \dfrac{Q^2}{\mu_b^2}$

Model input

define logarithmic ordering

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perturbative expansion in $\alpha_s(\mu)$

perturbative Sudakov

resummation of $L = \ln \dfrac{Q^2}{\mu_b^2}$

Model input

define logarithmic ordering

slide adapted from C. Bissolotti

$$f_1(x,\mathbf{b},\mu,\zeta) \;\to\; C_j^{(f_1)} \otimes f_1^j = \left[1 + \mathcal{O}(\alpha_s)\right]_j \otimes f_1^j$$

Anatomy of gluon TMD PDFs

matching coefficients

collinear PDF

nonperturbative Sudakov

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Model input

define logarithmic ordering

slide adapted from C. Bissolotti

$$f_1(x,\mathbf{b},\mu,\zeta) \;\to\; C_j^{(f_1)} \otimes f_1^j = \left[1+\mathcal{O}(\alpha_s)\right]_j \otimes f_1^j$$

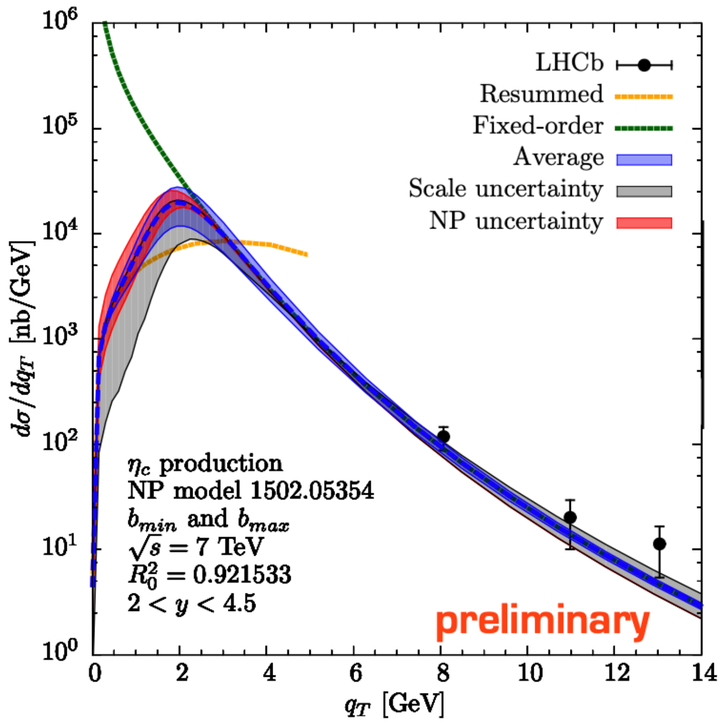
$$h_1^\perp(x,\mathbf{b},\mu,\zeta) \;\to\; C_j^{(h_1^\perp)} \otimes f_1^j = \left[\mathcal{O}(\alpha_s)\right]_j \otimes f_1^j$$

¿ Suppression of genuine NP effects ?

$\eta_{c,b}$ production @ 7TeV LHCb

η_c production at LHC

full transverse momentum spectrum:
low qT matched with high qT region



blue band: uncertainty from matching

grey band: scale uncertainty

red band: nonpert. uncertainty

$$S_{NP}(\bar{b}_T) = -\left[\frac{a_1}{2} + \frac{a_2}{2}\ \ln Q^2\right]\ \bar{b}_T^2$$

a_i = 0.5 GeV², var. 50%, envelope
both for unpolarized and
linearly polarized distributions

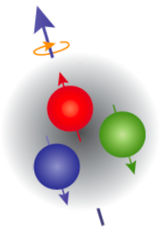
the formalism is in good shape!
we need the data at low qT

Jefferson Lab

25

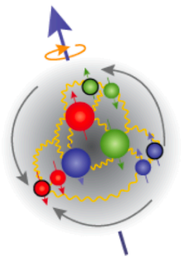
slide by Andrea Signori (INT 2017)

The proton spin crisis



quarks helicity

~ 30 %



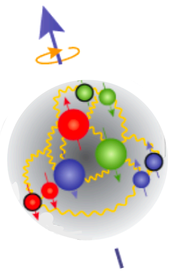
orbital angular momentum

¡ unknown !

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

gluons helicity

~ 20% – 50 % **?**

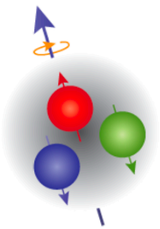


Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description…

🔗 (proton spin crisis) [EMC Collaboration, CERN (1987)]

slide adapted from C. Bissolotti

The proton spin crisis



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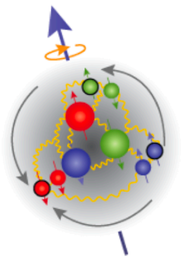

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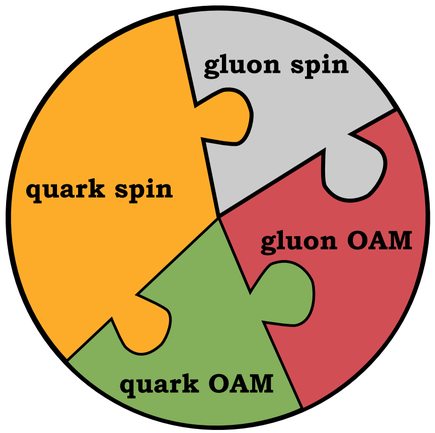

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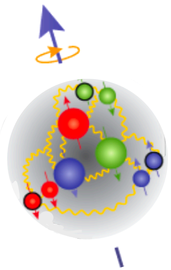
~ 20% – 50 % ?



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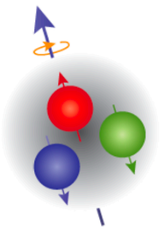

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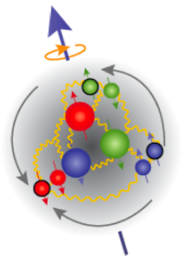

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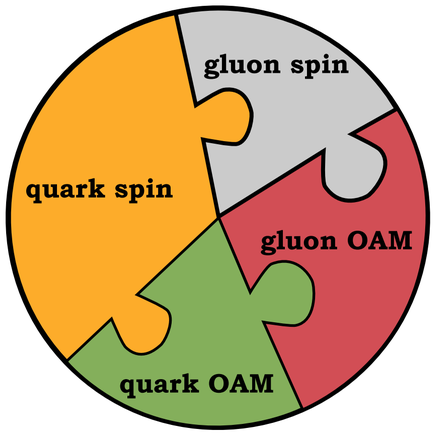

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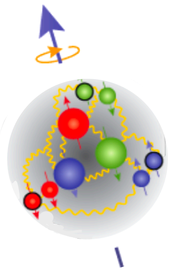

…many other effects in hadronic interactions cannot be understood in the purely collinear approach

Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description…

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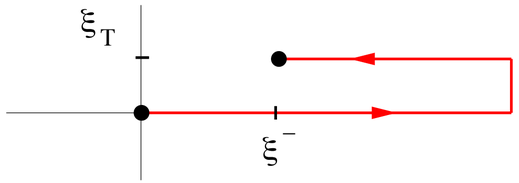
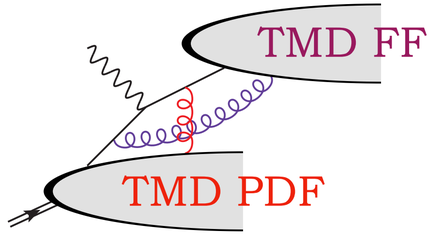
slide adapted from C. Bissolotti

Process dependence of quark TMD PDFs & FFs

SIDIS

[+] staple link





* PDF → color flow annihilated within final state

* FF → color flow from final to initial state

Process dependence of quark TMD PDFs & FFs

SIDIS

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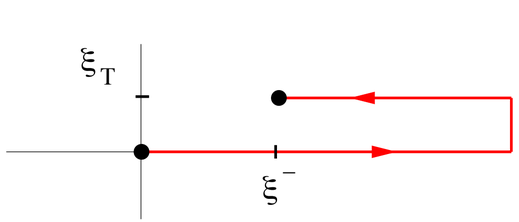

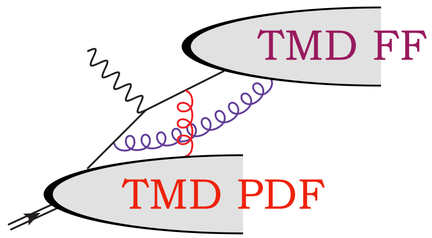

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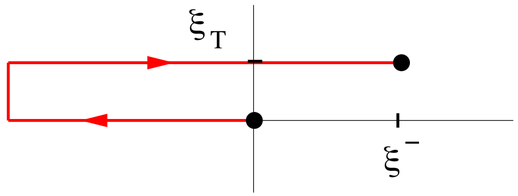
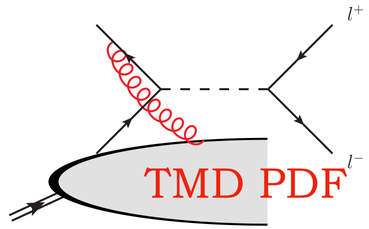
Drell-Yan

[–] staple link





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Process dependence of quark TMD PDFs & FFs

SIDIS

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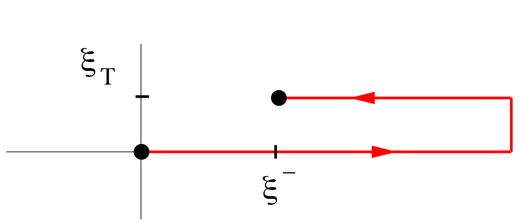

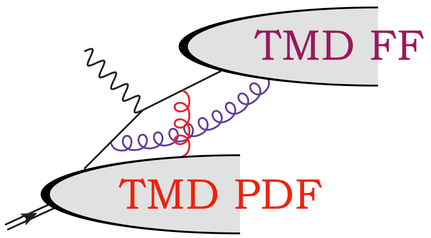

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Drell-Yan

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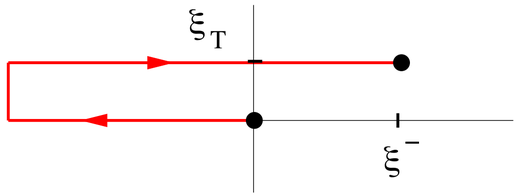

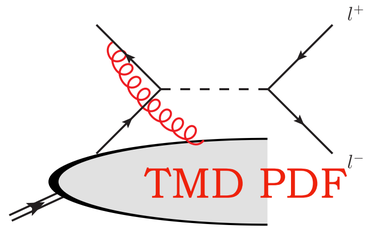

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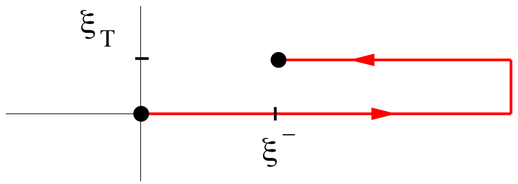
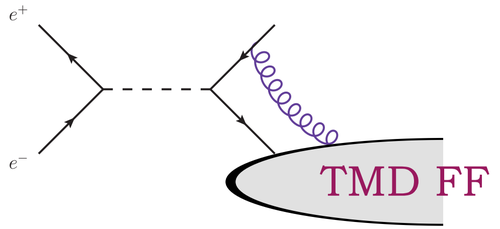
$e^+ + e^- \rightarrow$ hadrons

[+] staple link





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(Poles (do not) change with gauge-line direction) 🔗 [J. C. Collins, A. Metz (2004)]

Process dependence of quark TMD PDFs & FFs

SIDIS

[+] staple link

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Drell-Yan

[−] staple link

* PDF → color flow from final to initial state

Modified universality

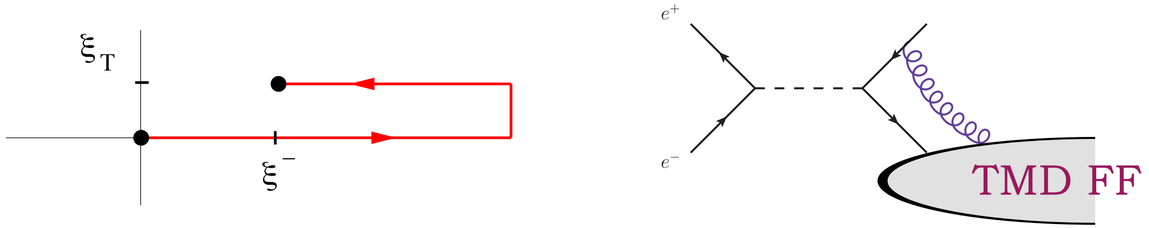
* PDFs → change of sign in T-odd densities

$$f_{1T}^{\perp\,[\mathrm{SIDIS}]} \equiv f_{1T}^{\perp\,[+]} = -f_{1T}^{\perp\,[-]} \equiv -f_{1T}^{\perp\,[\mathrm{DY}]}$$

* FFs → standard universality preserved

$e^+ + e^- \rightarrow$ hadrons

[+] staple link



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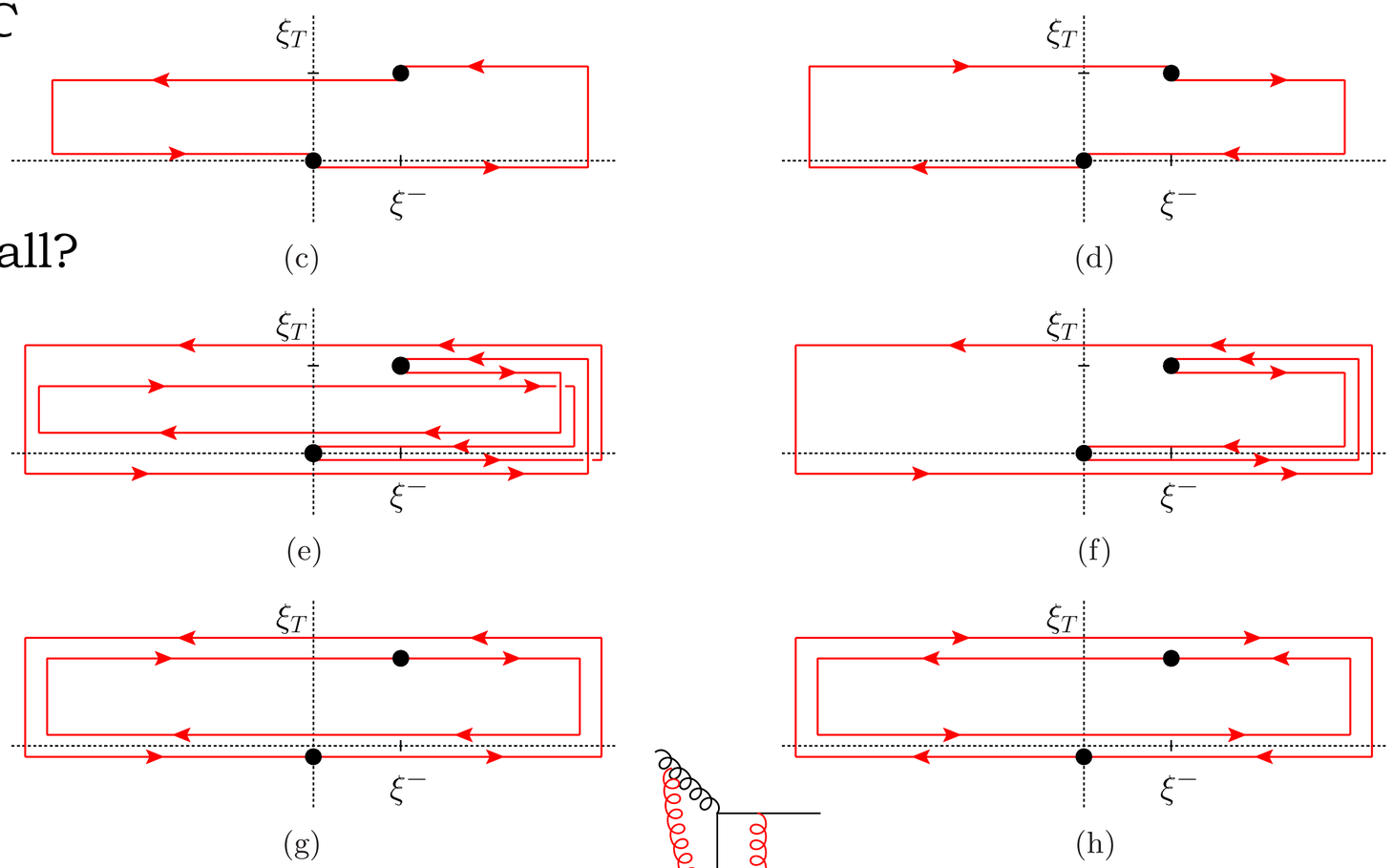
Dihadron hadroproduction & factorization breaking

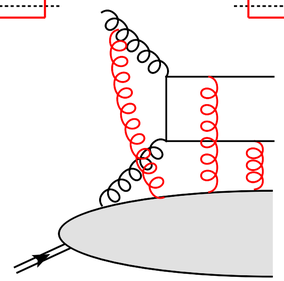
- Proof of factorization violation 🔗 [T. J. Rogers, P. J. Mulders (2010)]
- Assumed factorization in SCET and CGC
- Significance of low-x studies
- Size of factorization-breaking effects small?
- DP TMDs:

  (c) $[+,-]$ and (d) $[-,+]$
- Appearance of new gauge **loop links**:

  (e) $[+\square,+\square]$, (f) $[+,+\square]$,

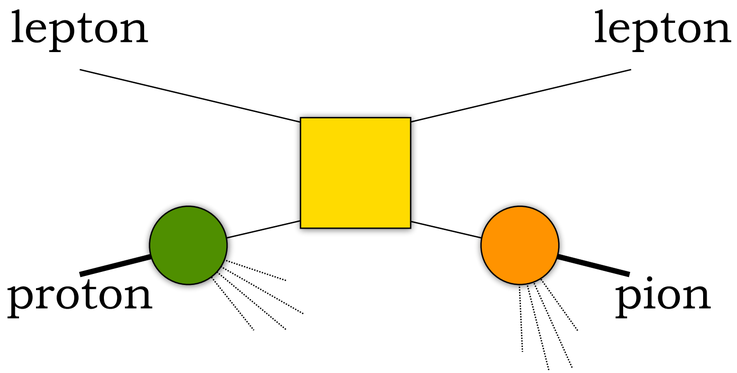
  (g) $[\square,\square]$, and (h) $[\square,\square]$





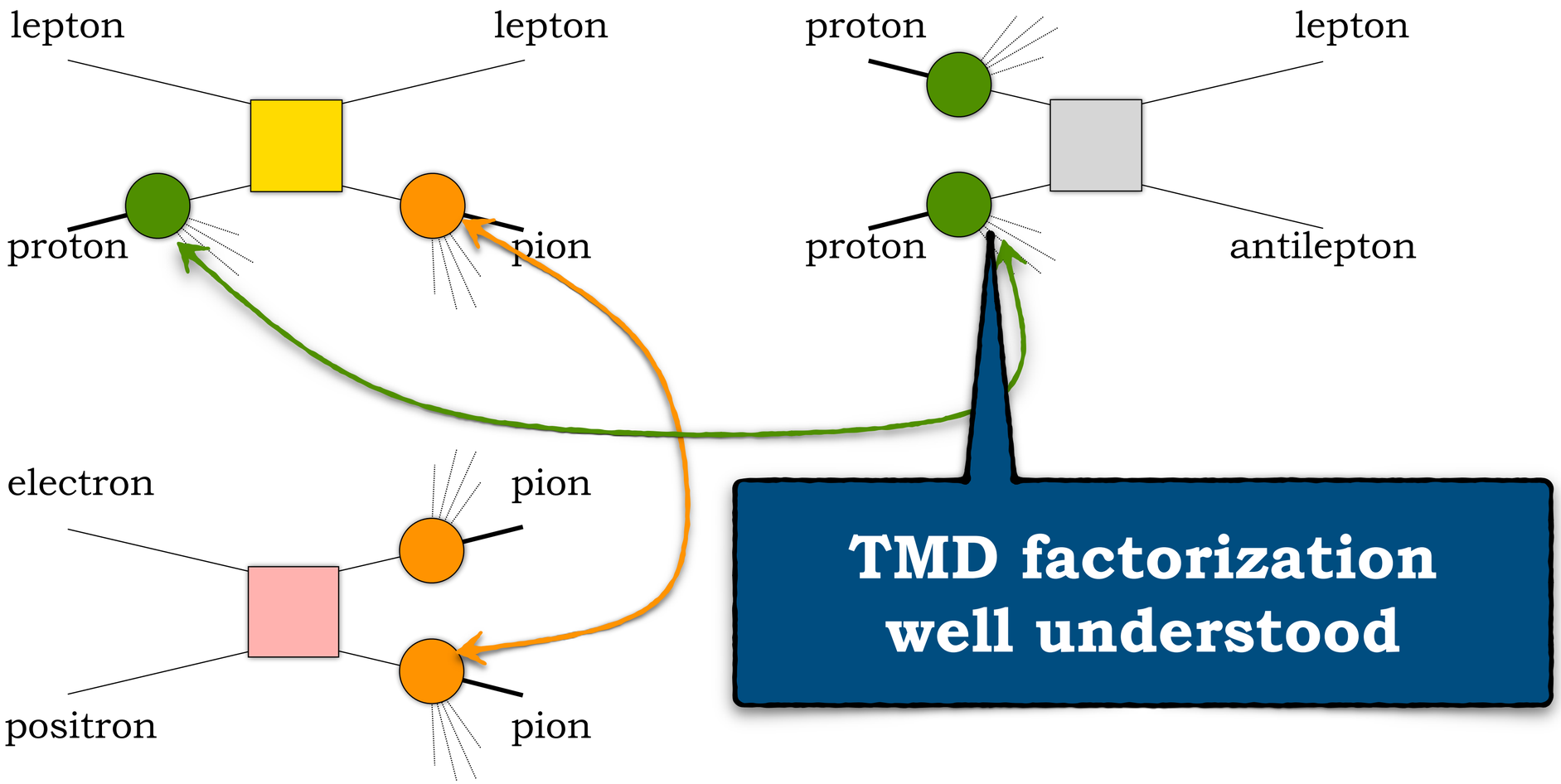
Factorization and universality

SIDIS



Factorization and universality

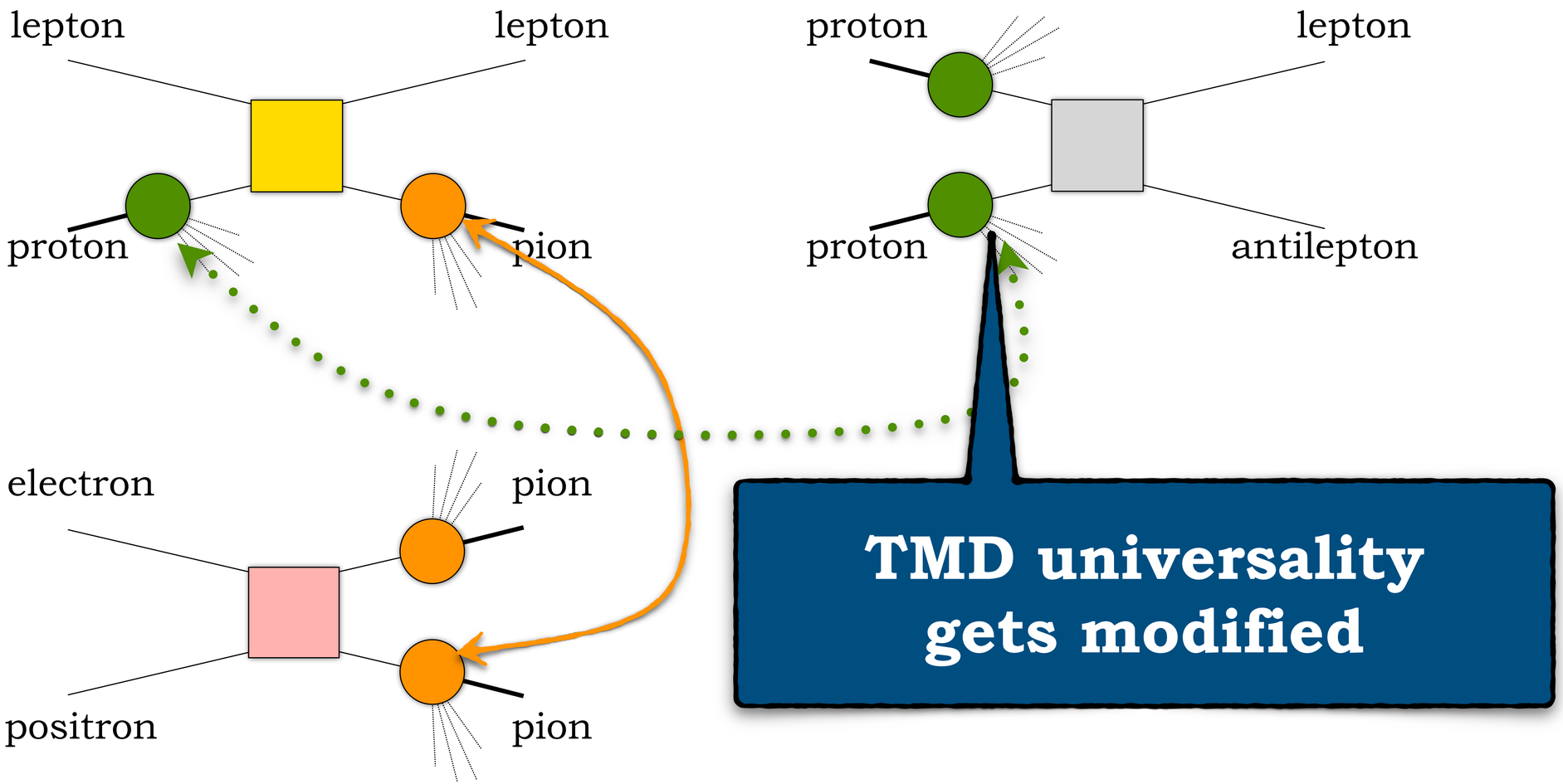
SIDIS **Drell-Yan**



$e^+ + e^- \rightarrow$ **hadrons**

Factorization & universality

SIDIS **Drell-Yan**



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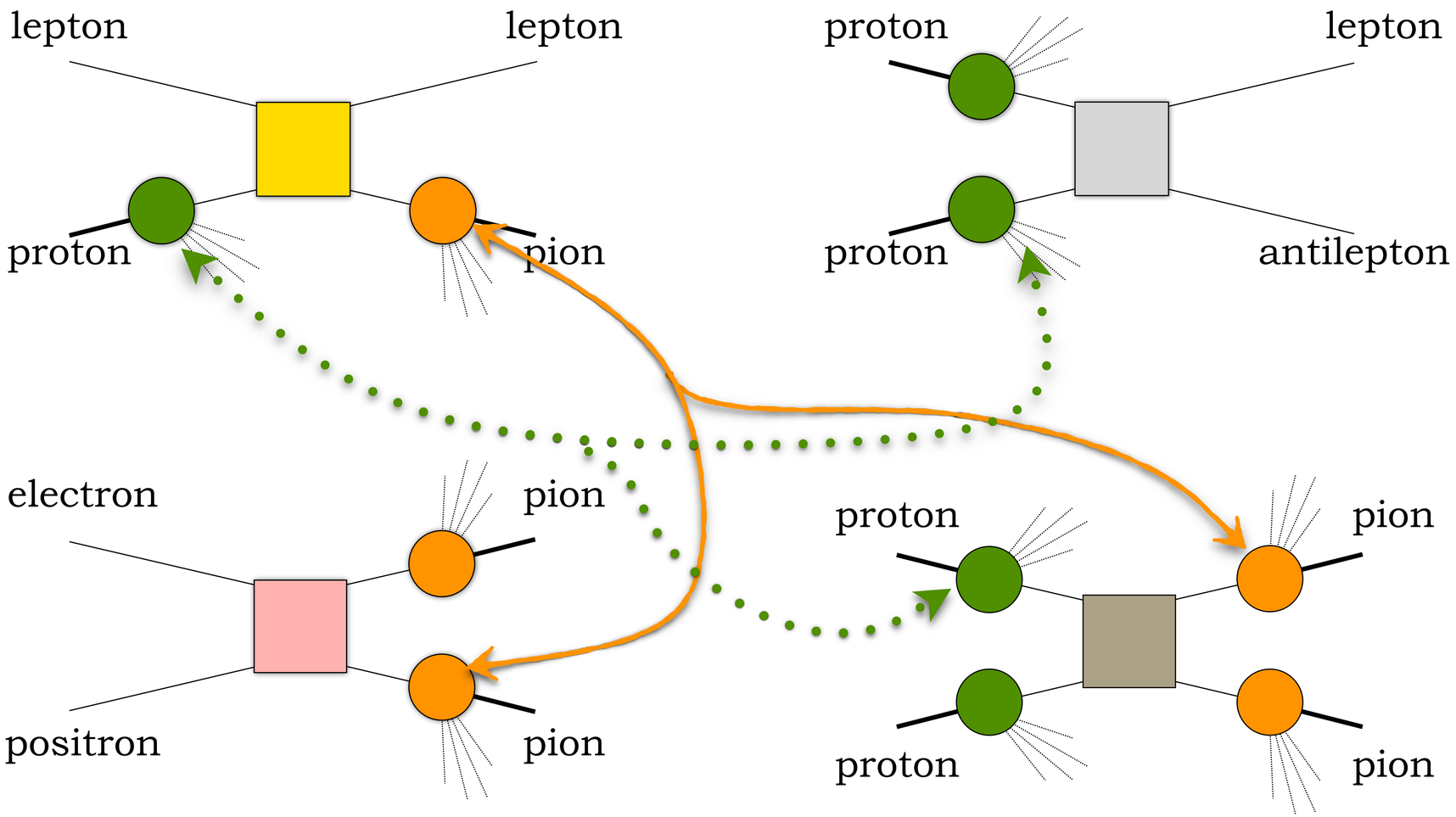
Factorization & universality

SIDIS

Drell-Yan



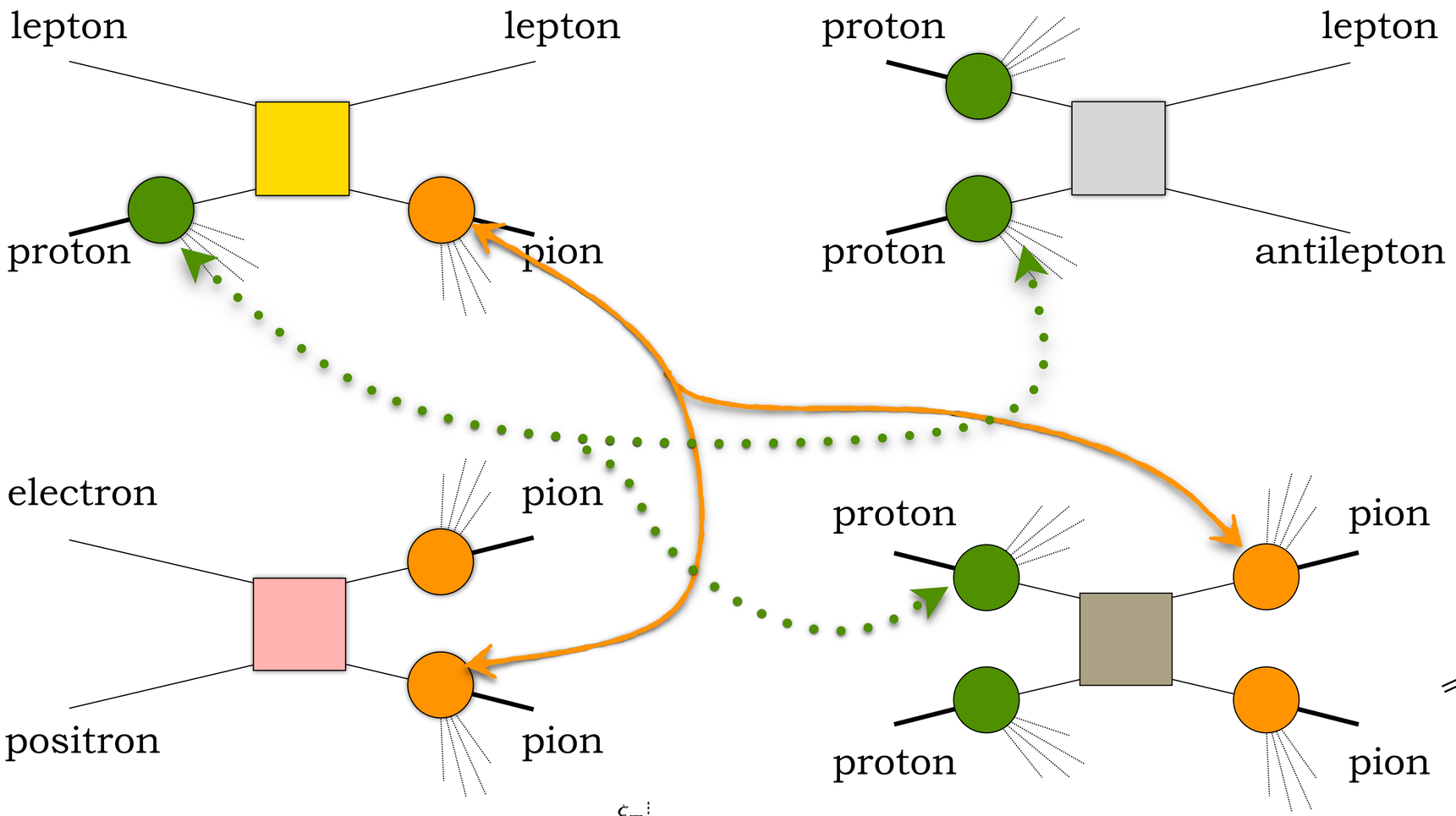
$e^+ + e^- \rightarrow$ **hadrons**

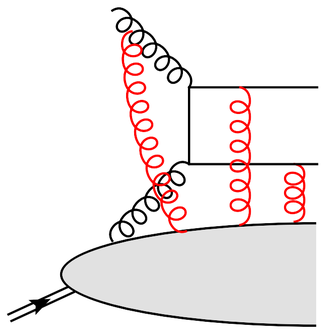
$p + p \rightarrow$ **hadrons**

Factorization & universality

SIDIS

Drell-Yan





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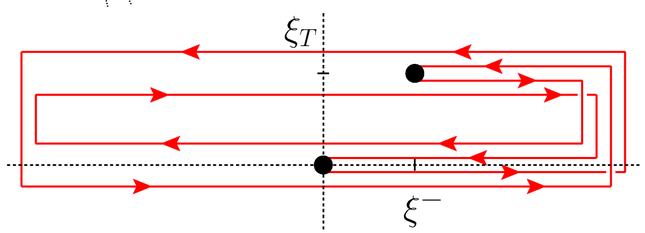
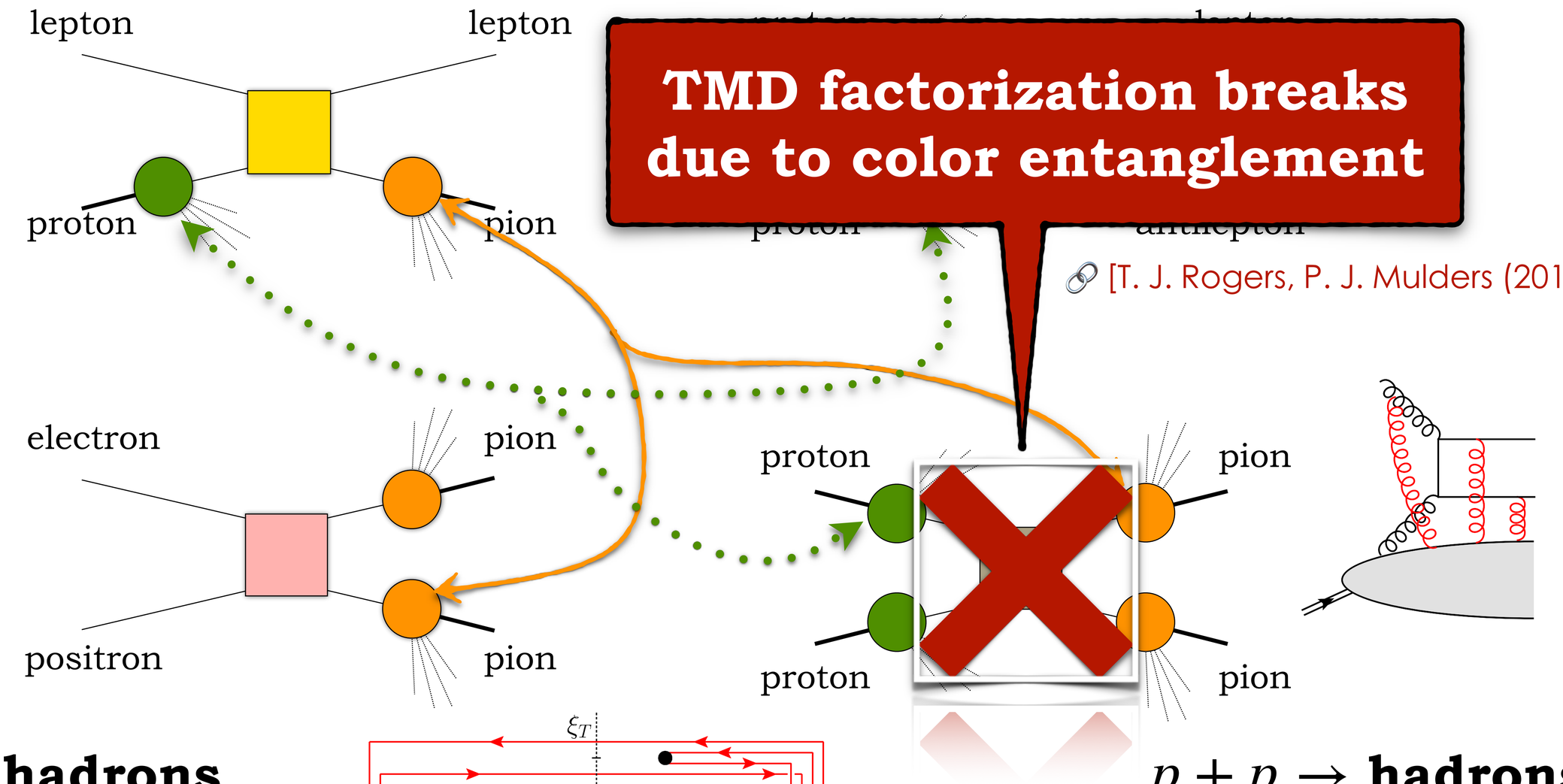


$p + p \rightarrow$ **hadrons**

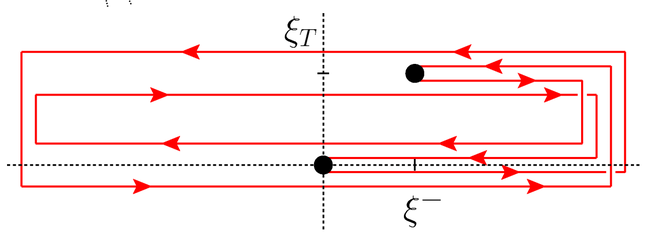
(also) gluon-induced channel

Factorization & universality

SIDIS **Drell-Yan**



$e^+ + e^- \to$ **hadrons**

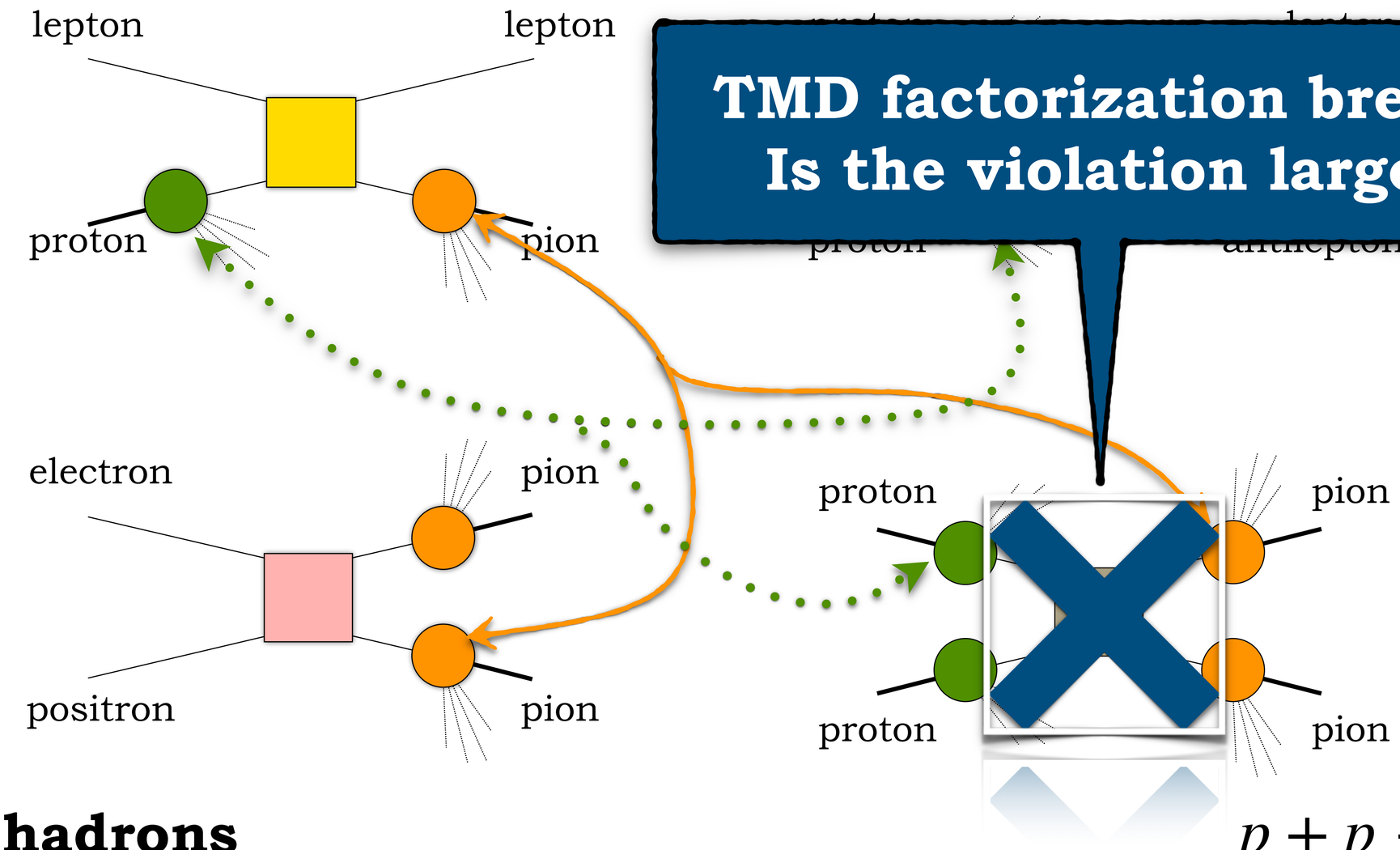


$p + p \to$ **hadrons**

(also) gluon-induced channel

Factorization % universality

SIDIS **Drell-Yan**



$e^+ + e^- \rightarrow$ **hadrons**

$p + p \rightarrow$ **hadrons**