SOFT DIFFRACTION: THEORY OVERVIEW

Outline

Valery Khoze (IPPP, Durham)

- Introduction
- Studies of Soft Diffraction at the LHC
- s-channel viewpoint on Diffraction
- t-channel picture of diffraction
- How large is large?
- Long way to the asymptotics
- The Odderon

"Repetition is the MOTHER of all learning"

-old Russian Proverb

INTRODUCTION

Theory
Paul Hoyer

optics

***Diffraction of light** occurs when a **light** wave passes by a corner or through an opening or slit that is physically the approximate size of, or even smaller than that **light's** wavelength.*

Experiment

Paul Newman
Kenneth Osterberg
Christoph Royon

…, diffraction scattering of hadrons reveals features similar to the optical diffraction pattern when light is scattered by an obstacle; for example, alternating light (maximum) and dark (minimum) stripes on the screen. Hadron scattering also reveals a similar structure, namely, the dip , the first minimum following the diffraction peak. In principle, at higher transferred momenta other diffraction maxima/minima are possible.

Despite the enormous successes of Quantum Chromodynamics, there remain a number of deep questions to be answered in the domain of strong interaction physics. These concern first of all small momentum transfer processes which are generically called soft interactions.

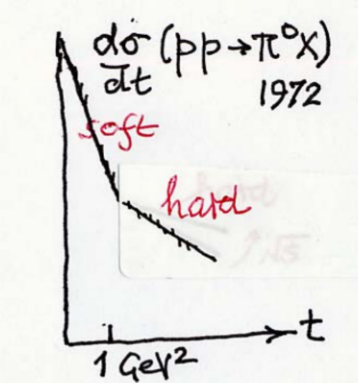
Welcome to the world of difficult physics

Soft and Hard HE interactions

Soft processes

have momentum transfer squared |t| less ~0.5 GeV^2, and have $d\sigma/dt \sim e^{-20t}$ at LHC, so v.few large |t| events.

Such processes described by Regge Field Theory. At high energies, Pomeron exch. dominates, and gives both LRGs & multi-pt events.



Hard processes

characterized by a large energy scale, |t| more ~ 2 GeV^2 – slower, power-like, fall-off with |t|, modulo logs. Here perturbative QCD is appropriate

Paul Hoer,
Francesco Giovanni Celiberto

Soft QCD is everywhere

- **Key area of SM where knowledge of fundamental processes is limited**
- **Theoretically:**
  - Beyond pQCD regime
  - Employ phenomenological models with tunable parameters
  - **Measurements are vital**
- **Crucial input for other LHC searches + measurements & beyond!**

Why it is important to study soft and diffractive processes

soft interactions give an underlying component to rare 'hard' events, from which we hope to extract signals for New Physics

Ways to estimate the probability that rapidity gaps, which occur in 'hard' diffractive events, survive rescattering effects,

Paul Newman

an understanding of diffractive processes is very important for the evaluation of pile-up backgrounds in high-luminosity *pp* collisions, which have a direct impact on various experimental measurements

studies of diffractive processes should help in the understanding of the structure of high-energy cosmic ray cascades, which requires detailed knowledge of the spectra of particles carrying a large fraction *x* of the incoming momentum in proton-air and nucleus-air interactions

- The LHC provides a significant lever-arm in providing constrains for hadronic Monte Carlos for UHECR

Tanguy Pierog

The cross-sections are (normally) large, and we do not need high luminosity.
Special (high β^*) optics is required.
Pile-up at high instantaneous luminosity.

At high collision energies soft interactions play a dominant role.

Unfortunately, soft interactions cannot be described in terms of PT QCD.

These are non-perturbative phenomena related to confinement which are generally considered in terms of S –matrix based on first principles, such as **analyticity, crossing symmetry and unitarity of partial waves.**

The most self-consistent way is the Regge approach.

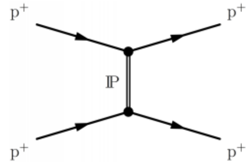
Paul Hoyer

It is based on singularities of scattering amplitudes in the complex angular momentum j-plane.

Grigorios Chachamis

This could be matched with PT QCD calculations at larger momentum transfer.

Within perturbative QCD there is a Pomeron: an even-signature singularity in the j-plane with vacuum quantum numbers.



Review of Particle Physics, Particle Data Group, *PTEP* 2022 (2022) 083C01

20. High Energy Soft QCD and Diffraction

V.A. Khoze , M.G. Ryskin and M. Taševský

Diffractive events have properties similar to those of the well-known from optics pattern of diffraction of a beam of light on an obstacle. By analogy, in high-energy physics, the corresponding processes are usually called diffractive.

There is no universally agreed definition of diffractive processes. Theoretically, diffraction is the effect caused by the absorption of the incoming plane-wave in some region of impact parameter, b. After a decomposition of the distorted plane-wave over the outgoing momentum, q, due to absorption we arrive at some set of plane-waves with non-zero transverse momentum, $q_t \neq 0$.

Theoretically, high-energy diffraction may be studied from either the *s*-channel or the *t*-channel viewpoint.

Studies of Soft Diffraction at the LHC

Fundamental interest.

Hopes to distinguish
between the different theoretical asymptotic scenarios for HE interactions.
(currently available data are still not decisive)

Rich testing ground for the dynamics of Soft Interactions

Practical interest.

Underlying events, triggers, calibration, number of interactions per bunch crossing..

In HE pp collisions about 40% of σ_{tot} comes from diffractive processes, like elastic scatt., SD, DD.
Need to study diffraction to understand the structure of σ_{tot} and the nature of the underlying events which accompany the sought-after rare hard subprocesses.
(Note the LHC detectors do not have 4π geometry and do not cover the whole rapidity interval. So minimum-bias events account for only part of total $\sigma_{inelastic}$.)

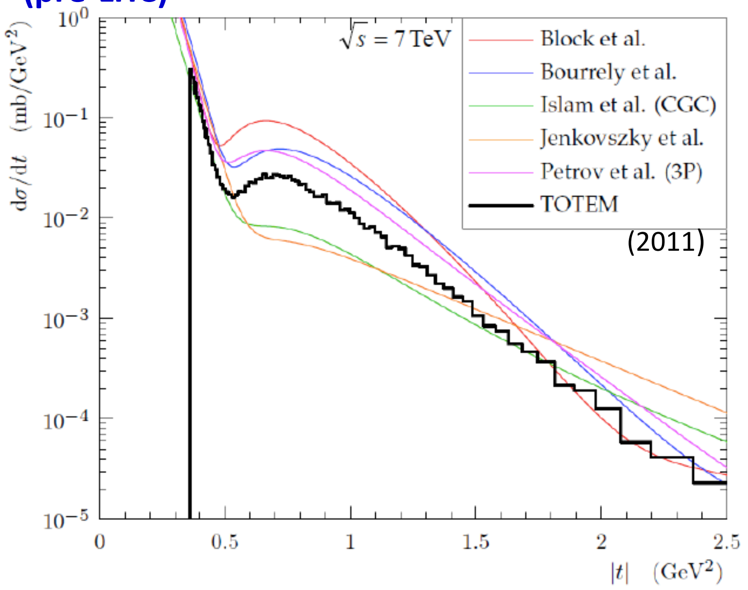
Diffraction at the LHC

• The LHC has allowed measurement of diffraction to be made out to unprecedented collider energies, with broad rapidity coverage and proton tagging.

• Already measurements of the elastic, total and diffractive cross sections in the first LHC runs have thrown up some interesting 'surprises' and a hard diffraction program is developing.

Model Comparisons

(pre-LHC)

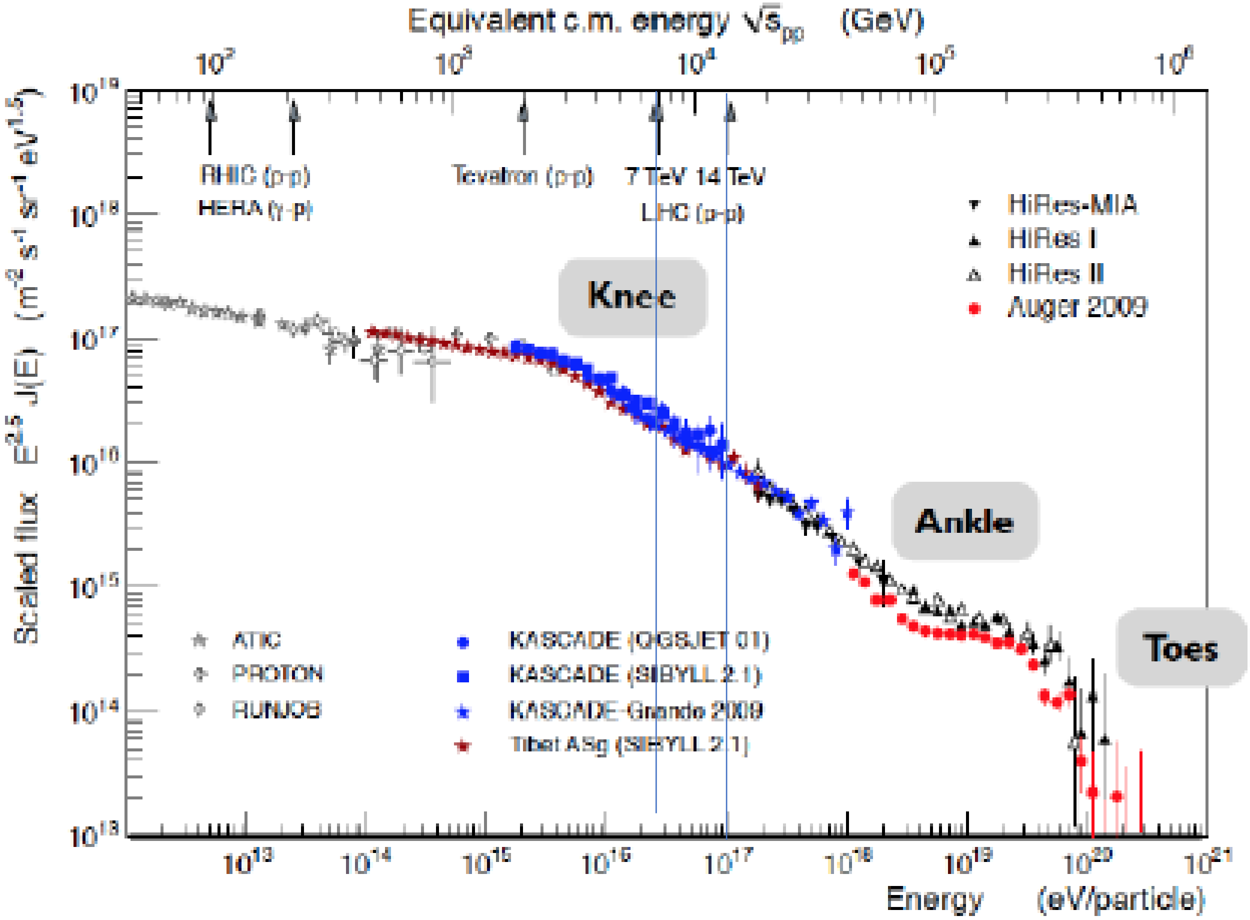


No theoretical / phenomenological model describes the TOTEM data completely.

Total Inelastic Cross Section

- Crucial quantity for understanding cosmic ray air showers

- Ingredient for modelling pile-up (and lumi) at LHC



Tanguy Pierog

No unique definition of diffraction

matter of convention

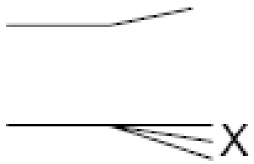
Paul Newman

1. Diffraction is elastic (or quasi-elastic) scattering caused, via s-channel unitarity, by the absorption of components of the wave functions of the incoming particles

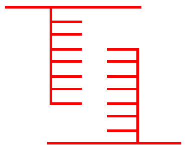
   e.g. pp→pp,

   pp→pX (single proton dissociation, SD),

   pp→XX (both protons dissociate, DD)



Good for quasi-elastic proc.
– but not high-mass dissoc^{n}



2. A diffractive process is characterized by a large rapidity gap (LRG), which is caused by t-channel Pomeron exch. (or, to be more precise, by the exchange corresponding to the rightmost singularity in the complex angular momentum plane with vacuum quantum numbers).

   Only good for very LRG events – otherwise Reggeon/fluctuation contaminations

DIFFRACTIVE PP scattering PROCESSES

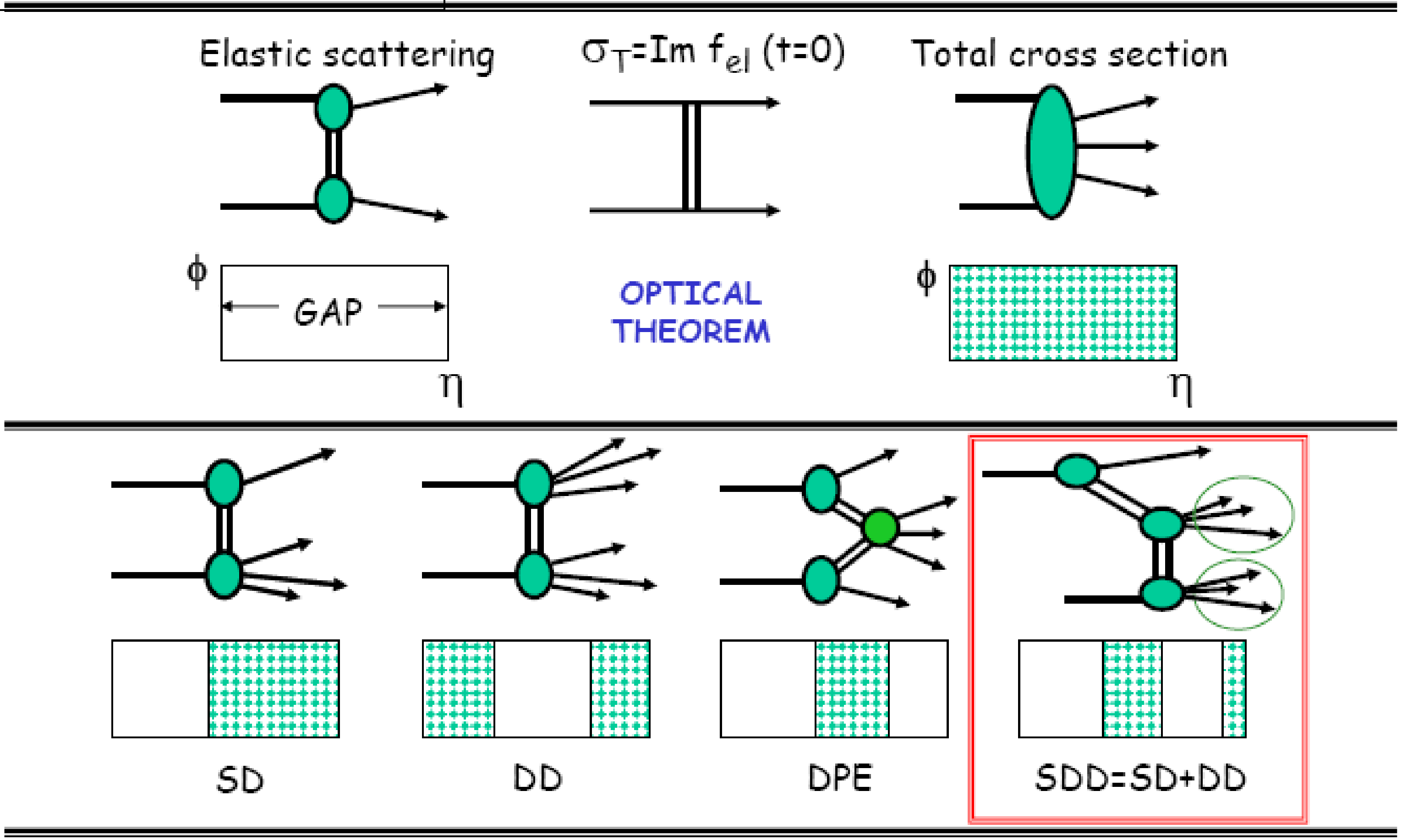
Experimental signature - presence of: (also EW exchanges)

- intact leading protons
- Large Rapidity Gaps (typically at least $\Delta\eta > 4$)

All these events have properties similar to those of the well-known from optics pattern of diffraction of a beam of light on an obstacle. By analogy, in high-energy physics, the corresponding processes are usually called diffractive.

MIND THE GAP

$M \ll \sqrt{s}$



rapidity

$$y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z}$$

(pseudorapidity)

$$(\eta = -\ln(\tan(\theta/2))$$

The large interval of rapidity is devoid of any hadronic activity-**LRG**

σ_{SD}, σ_{DD}, are of the order of 5–10 mb depending on the gap size.

Intact leading protons

one or both incoming particles stay intact after collision and are registered by the dedicated forward detectors placed a few hundred meters from the interaction point. The momentum loss of the initial particle, $\xi = 1 - x$, is typically smaller than 0.15.

Single Diffraction: definitions

η - pseudorapidity

$\eta \equiv y|_{m=0} = -\ln \tan(\vartheta/2)$

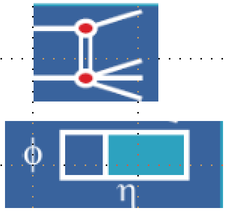
t - four-momentum transfer squared

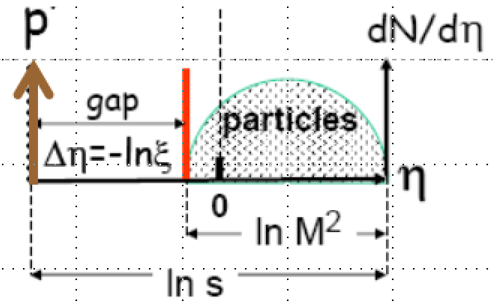
ξ - fractional momentum loss

M_X - mass of diffractive system X

$\xi = M_X^2/s$

$M \ll \sqrt{s}$





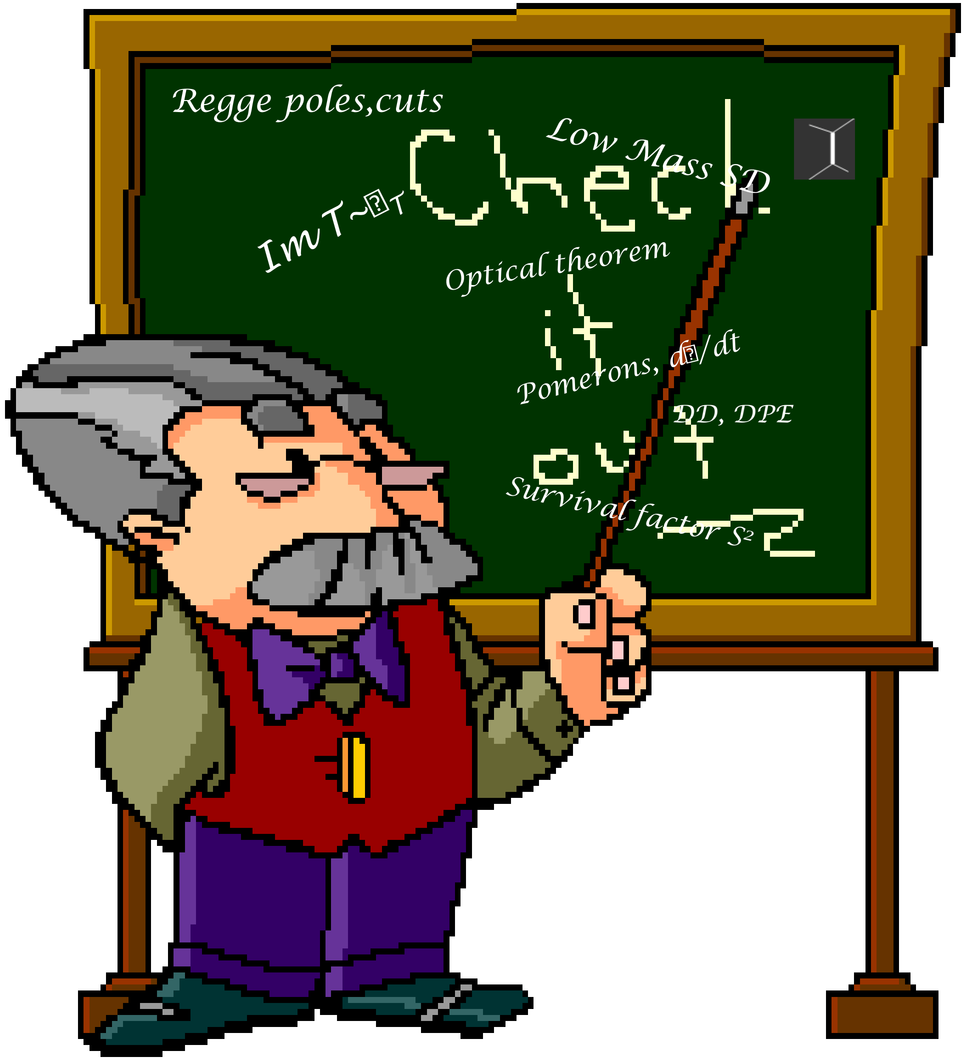
Typically at the LHC the integrated cross sections of diffractive dissociation, σ_{SD}, σ_{DD}, are of the order of 5–10 mb depending on the gap size.

Diffraction through the theorist's eyes.

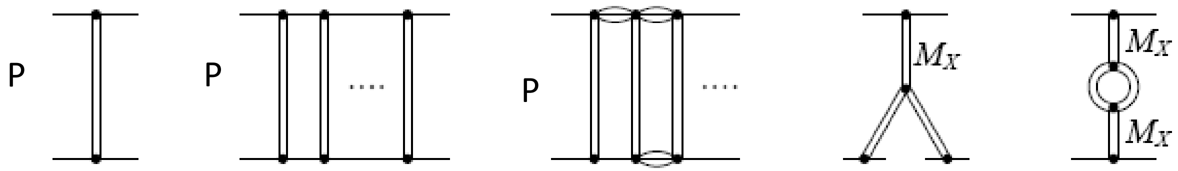
Current theoretical models for soft hadron interactions are still incomplete, and their parameters are not fixed, in particular, due to lack of HE data on Low-Mass diffraction.

Recent (RFT-based) models allow reasonable description of the data in the ISR-LHC range:

The differences between the results of other existing models wildly fluctuate.



Reggeon Field Theory, Gribov- 1986



charge-parity C-even amplitude

2-channel eikonal global fit to describe all high-energy $d\sigma_{el}/dt$, σ_{tot}, σ_{lowM}^{diff} pp data
(KMR 1806.05970)

11 parameters in total
4 for Pom: σ_0, $\alpha_P(0)$, α'_P, γ
7 for two GW eigenstates

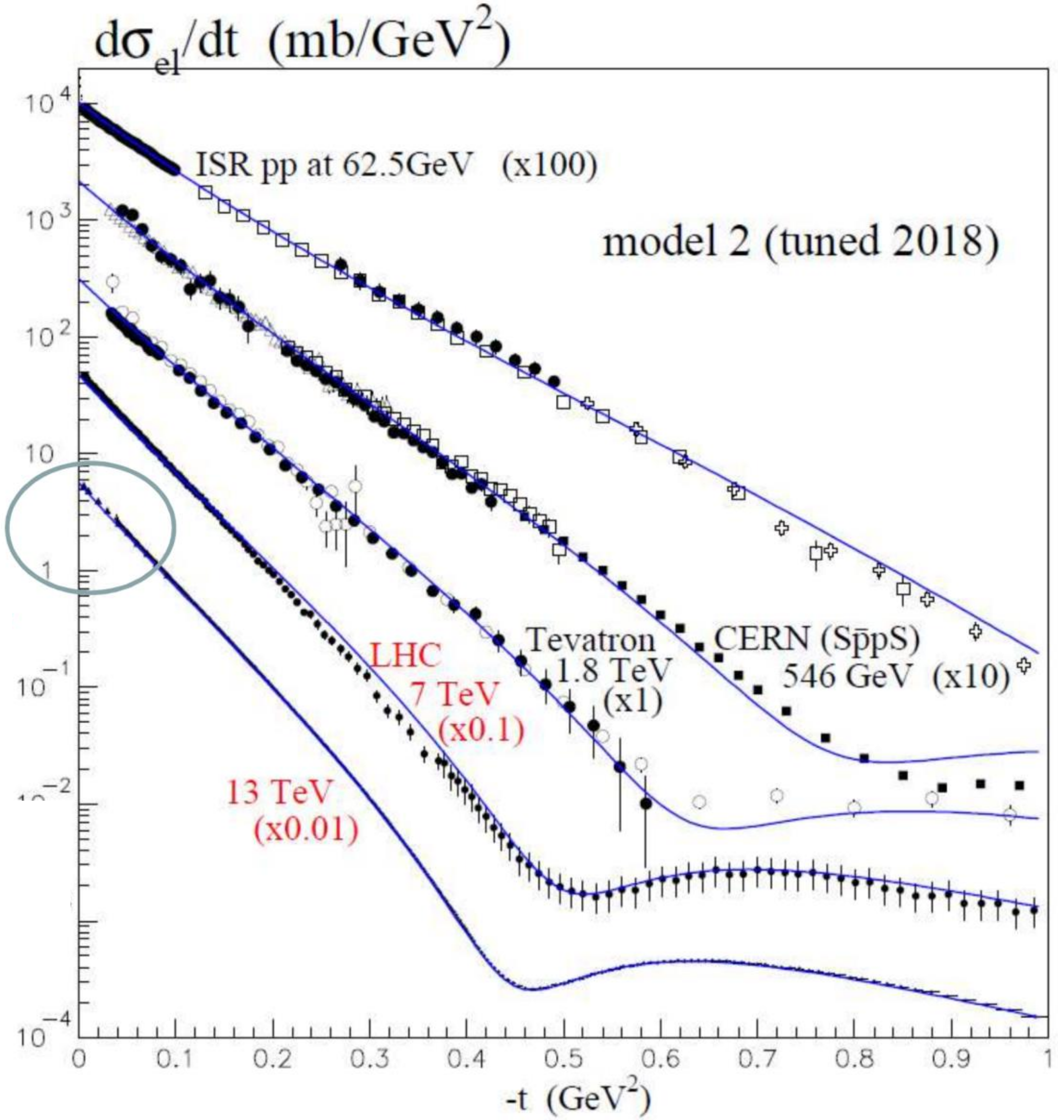
σ_{tot} 104.2 mb at 13 TeV

ALFA $\sigma_{tot} = 104.7 \pm 1.1$ mb, 2023

TOTEM σtot = (110.5±2.4)mb 2017

Good description of the low-t region

In agreement with the recent LRK-24 paper.

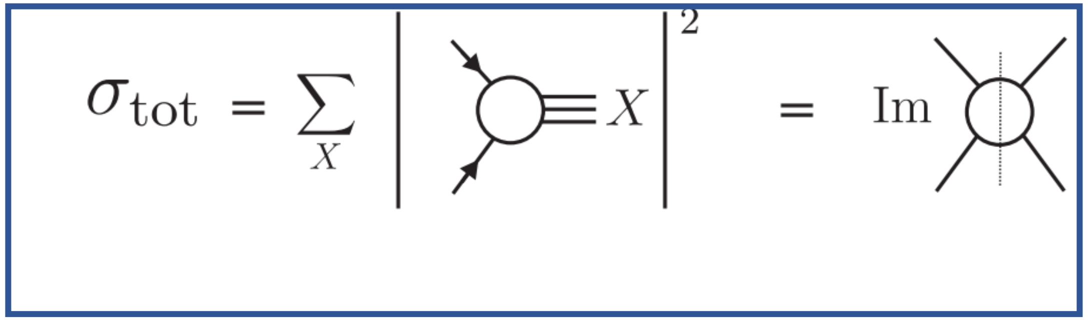


Let us start with the

s-channel viewpoint

Unitarity plays a central role in diffractive processes.

Unitarity gives us the optical theorem



Paul Hoyer

S matrix and the Optical Theorem

s-channel unitarity of the S-matrix

$$\sum_n P(i \to n) = 1 = \sum_n |\langle n|S|i\rangle|^2 = \sum_n \langle i|S^\dagger|n\rangle\langle n|S|i\rangle = \langle i|S^\dagger S|i\rangle = 1$$

true for any $|i\rangle$, so $S^\dagger S = I.$ Introduce trans matrix T: $S = I + iT$

$$(I - iT^\dagger)(I + iT) = I$$

$$i(T^\dagger - T) = T^\dagger T$$

$$i\langle f|T^\dagger - T|i\rangle = \sum_n \langle f|T^\dagger|n\rangle\langle n|T|i\rangle$$

$$2\ \mathrm{Im}T(i \to f) = \sum_n \langle n|T^*|f\rangle\langle n|T|i\rangle$$

put $f = i$, forward elastic scatt. $\to$ Optical theorem

$$2\ \mathrm{Im}T_{\mathrm{el}}(t = 0) = \sum_n |T(i \to n)|^2 = \sigma_{\mathrm{tot}}$$

$$disc\ T \equiv T - T^\dagger = iT^\dagger T$$

denotes a cut in s-channel between incoming and outgoing particles

the s-channel unitarity relation is diagonal in the *b-basis*

Eikonal (Ω(s,b)) parametrization

$$2\,\mathrm{Im}T_{el} = \sum_n |T(i\to n)|^2 = |T_{el}|^2 + G_{inel}$$

fixed b corresponds to a particular partial wave l, l=

best to work in b space, since at high energies the value of b is frozen

$$2\,\mathrm{Im}T_{el}(s,b) = |T_{el}(s,b)|^2 + G_{inel}(s,b)$$

$\vec{b}$

The general solution

$$T_{el}(b) = i(1 - e^{-\Omega(b)/2})$$

$$G_{inel}(s,b) = 1 - \mathrm{e}^{-\mathrm{Re}\Omega(b)} = 1 - P_{nointer}(s,b),$$

where G_{inel} is the sum over all inelastic intermediate states and $P_{nointer}$ is a probability to have no inelastic interactions. $G_{inel}(s,b)$ describes the b-profile of inelastic particle collisions. It satisfies $0 \le G_{inel} \le 1$

$$\sigma_{tot} = 2\int d^2b\,\mathrm{Im}T_{el}(s,b) = 2\int d^2b\,(1-e^{-\Omega/2})$$

$$\sigma_{el} = \int d^2b\,|T_{el}(s,b)|^2 = \int d^2b\,(1-e^{-\Omega/2})^2$$

$$\sigma_{inel} = \int d^2b\,[2\mathrm{Im}T_{el}(s,b) - |T_{el}(s,b)|^2] = \int d^2b\,(1-e^{-\Omega})$$

with $\mathrm{Re}\Omega \ge 0$. Amp ~ imag. at HE so eikonal Ω is real

Note $e^{-\Omega(s,b)}$ is prob. no inelastic inter^n occurs at b

$G_{inel} = 1$ for full absorption

$G_{inel} = 0$ the complete dominance of elastic scattering

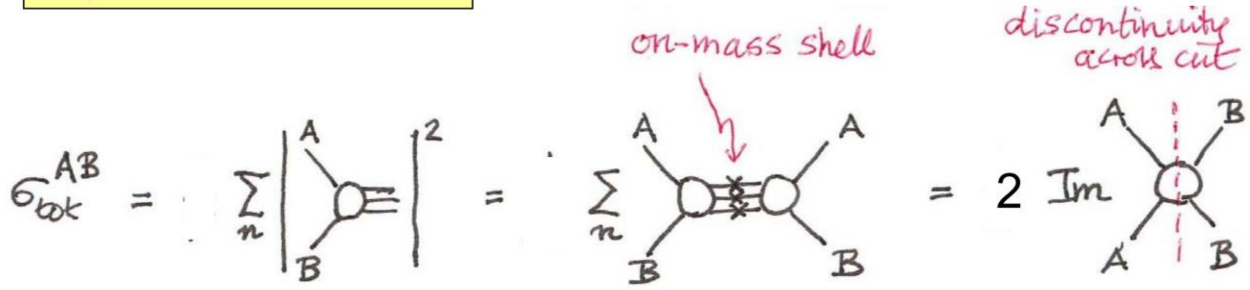
Ω ($\mathrm{Re}\Omega \ge 0$) is called the opacity (optical density) or eikonal

$S^2(b) \equiv e^{-\mathrm{Re}\Omega(b)} = P_{nointer}(b)$ so-called **survival factor**, which enables us to calculate the probability that the LRG survives soft rescattering.

At high energies, the *s*-channel unitarity relation is diagonal in the *b-basis*

Optical Theorem

$$\sigma_{\rm tot} \;=\; 2\,{\rm Im}T_{\rm el}(s,t=0)$$



1. The sum over all inelastic channels forms a “shadow”, which “generates” elastic scattering
   → diffraction → can generalise

2. As s increases Im $T_{el}(s,0)$ is the sum over increasing number of positive terms. No such constraint exists for Re T_{el}. $T_{el}(0)$ is predominantly imag. at HE.

3. Away from forward dir^n, phases in $2{\rm Im}T_{el} \sim T_{nf}^* \, T_{ni}$ vary. $T_{el}(s,t)$ rapidly decreases away from t=0.

disc T denotes a cut in the *s*-channel between incoming and outgoing particles as visualized by crosses scattering on a black disk, with $G_{\text{inel}} = 1$ for $b < R$, gives $\sigma_{\text{el}} = \sigma_{\text{inel}} = \pi R^2$ and $\sigma_{\text{tot}} = 2\pi R^2$

At HE the inelastic contribution, G_{inel}, dominates; $\Omega(s,b) \gg 1$.
In this so-called "black disk" limit $\text{Im}T_{\text{el}}(s,b) = 1$

Example: black disc of radius R

for $b<R$, $\Omega=\infty$ ($T_{el}=i$)

$$\sigma_{inel} = 2\pi\int_0^R (1-e^{-\Omega})\, b\, db = \pi R^2$$

total absorption

$\sigma_{el} = \pi R^2$ — shadow due to absorption leads to elastic scattering

$$\sigma_{tot} = 2\pi R^2$$

Since $$\frac{d\sigma_{\text{el}}}{dt} = |\text{Im}T_{\text{el}}(s,t)|^2\,(1+\rho^2)$$

$\rho \equiv \text{Re}T_{\text{el}}/\text{Im}T_{\text{el}}$

data directly determines $\text{Im}T_{\text{el}}(s,b)$

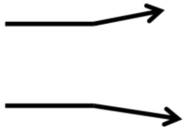
Fourier transform to b-space:

$(-t = q_T^2)$

$\vec{b} \longleftrightarrow \vec{q}_T$

wide narrow

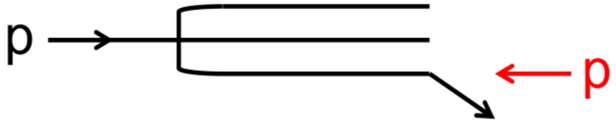
So far only discussed elastic diffraction



What about inelastic diffraction ?

to the $pp \to p + X$ and $pp \to X_1 + X_2$ processes where one or both protons are allowed tc dissociate into a system X with the quantum numbers of the proton.

Inelastic diffraction is a consequence of internal struct of p



The lifetimes of each Fock component

$\tau \sim E/m^2$, are large

At HE fluctuations of p are frozen.

A constituent of p can scatter and destroy coherence of fluctuations

→ inelastic, as well as, elastic diffraction (single diffractive dissociation)

Diffraction dissociation - a quantum mechanical process caused by the fact that different components of the incoming hadron wave function have different probabilities for interaction with a target

Good-Walker formalism for low-mass diff^{ve} dissoc^{n}

We write $|p\rangle = \sum a_k |\phi_k\rangle$ where $|\phi_k\rangle$ diagonalise T

The $|\phi_k\rangle$ undergo "elastic-type" scatt $\langle\phi_j|T|\phi_k\rangle = 0 \quad (j \neq k)$

$|p\rangle \rightarrow$ diffractive eigenstates $|\phi_k\rangle \rightarrow$ multichannel eikonal

the proton case describes different $p \to N^*,\ N_a^* \to N_b^*$ transitions)

$\mathrm{Im}\,T = a F a^T$ where $\langle\phi_j|F|\phi_k\rangle = F_k\,\delta_{jk}$

orthogonal matrix **a**

Elastic amp. $\langle p|\mathrm{Im}\,T|p\rangle = \sum |a_k|^2 F_k = \langle F\rangle$

average of F over the initial prob. distrib. of diff. estates

Diffractive events are just the elastic scattering of 'Good-Walker' eigenstates

the individual components of the incoming proton wave function interact differently with the target

M. L. Good and W. D. Walker,(1960).

Each hadronic constituent can undergo a scattering with its own probability and thus destroys the coherence of the fluctuations. As a result, the outgoing superposition of states will be different from the incident particle, and will most likely contain multiparticle states, so we will have inelastic, as well as elastic scattering.

The amplitude is normalised to $\sigma_{\rm tot} = 2\int d^2b\ {\rm Im}A(b)$

$$\frac{d\sigma_{tot}}{d^2b} = 2\langle p|{\rm Im}T|p\rangle = 2\sum|a_k|^2 F_k = 2\langle F\rangle$$

$$\frac{d\sigma_{el}}{d^2b} = |\langle p|T|p\rangle|^2 = \left(\sum|a_k|^2 F_k\right)^2 = \langle F\rangle^2$$

$$\frac{d\sigma_{el+SD}}{d^2b} = \sum_k |\langle\phi_k|T|p\rangle|^2 = \sum_k |a_k|^2 F_k^2 = \langle F^2\rangle$$

Comments

1. $\frac{d\sigma_{SD}}{d^2b} = \langle F^2\rangle - \langle F\rangle^2$ statistical dispersion in absorp. prob. of diff. estates

2. If all compts. of incident proton absorbed equally then diffracted superposition = incident one. No inelastic diffraction e.g. Small b: $F_k \simeq 1$ (~black disc), so diffr prod ~ zero
→ diffraction mainly on periphery.

Here the average is taken over the components *k* of the incoming proton which dissociates

3. $0 \le F_k \le 1$, $F_k^2 \le F_k$, $\langle F^2 \rangle \le \langle F \rangle$

Pumplin bound: $\frac{d\sigma_{SD}}{d^2b} \le \frac{1}{2}\frac{d\sigma_{tot}}{d^2b} - \frac{d\sigma_{el}}{d^2b}$

4. Easy to allow both protons to dissociate; expand both $|p\rangle$'s in diffractive eigenstates

5. High-mass dissociation not included yet.

the average is taken over the components *k* of the incoming proton which dissociates.

If the averages are taken over the components of both of the incoming particles, then we arrive at the sum of the cross sections for SD and DD

Under the assumption that amplitudes F_k at high energies cannot exceed the black disk limit, Im $F_k < 1$

$$\frac{d\sigma_{el+SD_1+SD_2+DD}}{d^2b} \le \frac{1}{2}\frac{d\sigma_{tot}}{d^2b}\,.$$

(the Pumplin bound)

(Strictly speaking, the proof of the Pumplin bound is justified only for low mass dissociation, with no overlap of GW states)

- suppression of the amplitude at low values of b automatically increases the elastic slope B_{el}, since $B_{el} \propto R^2 = \langle b^2 \rangle$ where R is the interaction radius.

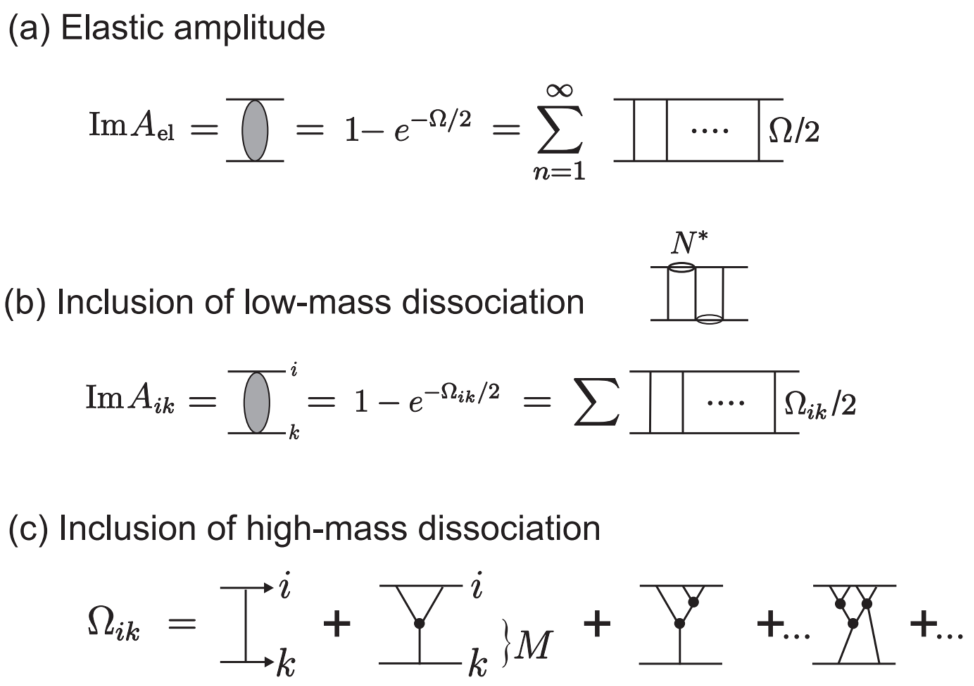


Fig. 1. (a) The single-channel eikonal description of elastic scattering; (b) the multichannel eikonal formula which allows for low-mass proton dissociations in terms of diffractive eigenstates $|\phi_i\rangle$, $|\phi_k\rangle$; and (c) the inclusion of the multi-Pomeron-Pomeron diagrams which allow for high-mass dissociation. In all these diagrams the exchanged lines represent Pomeron exchange.

In pre-QCD times, in order to describe the behaviour of scattering amplitudes at high energy, $\sqrt{s}$, and small momentum-transfer squared, $-t$, Regge theory was developed and successfully applied in a wide range of energies. The Regge approach | is based on the singularities of amplitudes in the complex angular momentum, j, plane.

Paul Hoyer

For instance, the measured $\pi^- p \to \pi^0 n$ amplitude behaves as $T_{\pi p}(s,t) \propto s^{\alpha_\rho(t)}$,

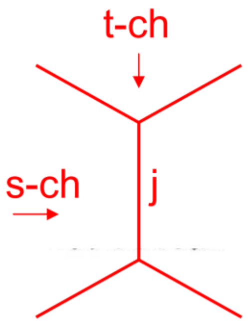
where the process is described by the exchange of the ρ-trajectory, $j = \alpha_\rho(t) \simeq 0.5 + 0.9t$ (with $t = (p_{\pi^-} - p_{\pi^0})^2$ in GeV^2). This trajectory passes through the spin-1 ρ-meson resonance in the 'crossed' t-channel $\pi^-\pi^0 \to \bar{p}n$; that is, $\alpha_\rho(t = m_\rho^2) = 1$. The corresponding cross section decreases with increasing s.

t-channel picture of Diffraction

First, v. brief overview of Regge Poles

partial wave expansion in t-ch: $T(s,t) \propto \sum_\ell (2\ell+1)\, a_\ell(t)\, P_\ell(\cos\theta_t)$

so exchange of particle of spin j in t-ch



$$T(s, t \sim M_j^2) \sim \frac{P_j(\cos\theta_t)}{M_j^2 - t} \to s^j \text{ as } s \to \infty$$

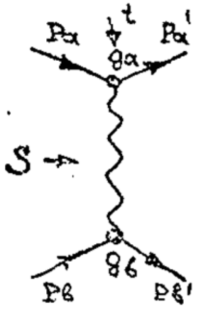
$$\cos\theta_t = 1 - \frac{s}{2k_t^2}$$

$$P_j(\cos\theta_t) \sim (\cos\theta_t)^j$$

whereas from unitarity

$$T(s, t=0) \lesssim c\, s \log^2 s$$

so s^j violates unitarity if $j > 1$.

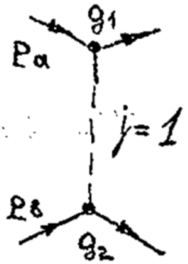


$$\sim g_a g_b \, s^{\alpha(t)}$$

$$s = (p_a + p_b)^2$$
$$t = (p_a - p_a')^2$$

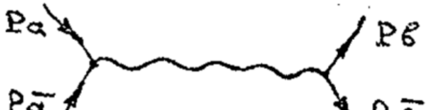
a)

$$j=0 \sim \frac{g_1 g_2}{t - \mu^2} \sim s^0$$



$$j=1 \sim \frac{g_1 g_2 (p_a p_b)}{t - \mu^2} \sim \frac{g_1 g_2}{t - \mu^2} s^1$$

b)



c)

Donnachie-Landshoff type simple Regge pole fit to

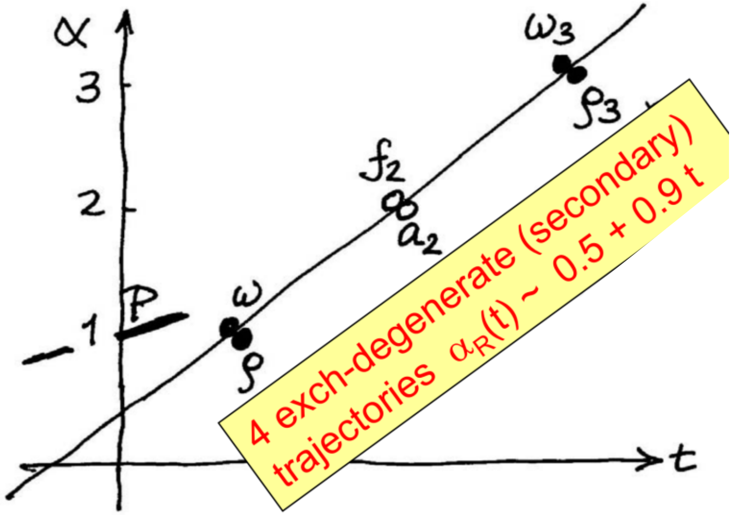
σ_{tot} and $d\sigma_{el}/dt$ for pp, $p\bar{p}$, πp, Kp,…

Good description up to Tevatron energies with

$\alpha_P^{eff}(t) \sim 1.08 + 0.25\,t$

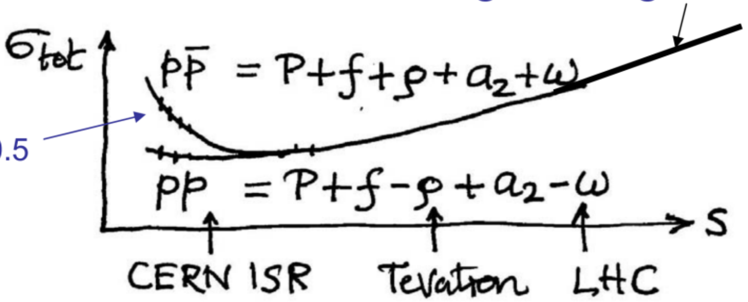
$\alpha_R(t) \sim 0.5 + 0.9\,t$

$\sigma_{tot} \sim s^{\alpha(t=0)}/s$



$s^{\alpha(0)-1} \sim s^{0.08}$

$s^{-0.5}$



$$\mathcal{A}(s,t) = \sum_i \eta_i(t)\gamma_i(t)s^{\alpha_i(t)}$$

Simplest singularities- isolated poles at $j = \alpha(t)$

$$\sigma_{tot}(s) = \sum_i 4\pi g_i s^{\alpha_i(0)-1}$$

where $g_i \equiv \gamma_i(0)\mathrm{Im}\{\eta_i(0)\}$

- above Tevatron energies, the secondary Reggeon contributions (which all have intercepts $\alpha(0) \simeq 0.5$) are highly suppressed, which enables us to study the properties of the Pomeron only.

Paul Hoyer

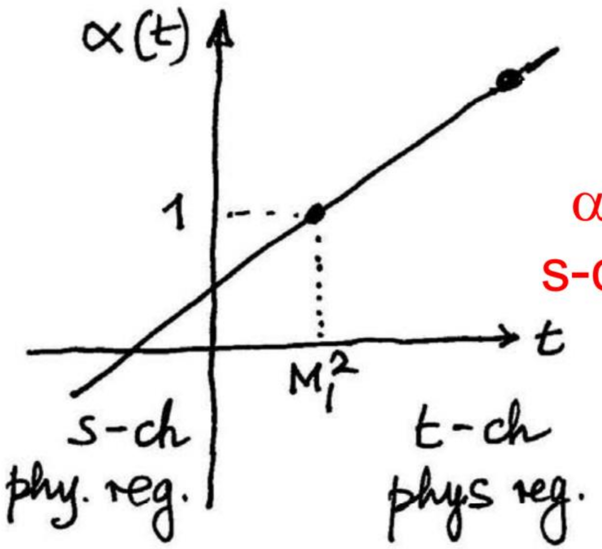
Consider particles lying on a single linear Regge trajectory

$$\alpha(t) = \alpha_0 + \alpha' t$$

pole in ℓ^{th} (t-ch) partial wave

$$a_\ell(t) \simeq \frac{\beta(t)}{\ell - \alpha(t)} = \frac{\beta(t)/\alpha'}{M_\ell^2 - t}$$

$$(\ell = \alpha_0 + \alpha' M_\ell^2)$$



OK
α<1 in
s-ch reg.

Reggeons-the same quantum numbers as resonances

$$T(s,t) = \sum_\ell (2\ell+1) \frac{\beta(t)}{\ell - \alpha(t)} P_\ell(\cos\theta_t) \underset{\substack{s\to\infty \\ \text{fixed } t}}{\sim} \sum_\ell \frac{\beta(t)\,(\cos\theta_t)^\ell}{\ell - \alpha(t)} \sim \beta(t)\,(\cos\theta_t)^{\alpha(t)}$$

$$\sim \beta(t)\, s^{\alpha(t)}$$

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2}|T(s,t)|^2 \sim F(t)\left(\frac{s}{s_0}\right)^{2\alpha(t)-2} \sim F(t)\left(\frac{s}{s_0}\right)^{2\alpha_0-2} \exp\left[2\alpha'\left(\log\frac{s}{s_0}\right)t\right]$$

so we have shrinkage of forward peak, exp(-Bt), as s increases

HE behaviour dominated by leading (highest) Regge-exch. trajector

σ_{tot}(hadron-hadron) → const. (actually slightly rising as s→infinity)

that is T(s, t=0) ~ s (actually $s^{1.08}$)

(In our discussion on Regge poles we use more usual normal^{n} of such optical theorem reads $2\text{Im}\, T_{el}(s,t=0)$ = flux σ_{tot} = 2s σ_{tot})

Implies Regge-pole exchange with $\alpha(0) = 1$

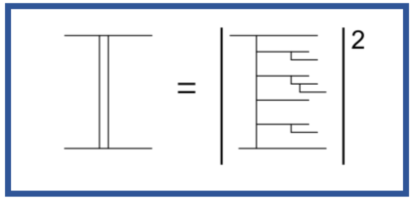
Grigorios Chachamis

The pole with the largest intercept

called the **Pomeron**

$$s\sigma_{\rm tot} = {\rm Im} T_{\rm el}(s, t=0)$$

trajectory with vacuum quantum numbers, $\sigma_{\rm tot} \propto s^{j-1}$ _ $T_{\rm el}(s,t) \propto s^{\alpha_{\mathbb{P}}(t)}$



the Pomeron is represented by gluon exchange – we need two gluons to form colourless exchange. But, for the moment, let us consider the Pomeron as a simple (effective) Regge pole

the opacity corresponding to the exchange of one Pomeron is

$$\Omega(s,b) = \int \frac{d^2 q_t}{4\pi^2}\, \Omega(s,q_t)\, e^{i\vec{q}_t\cdot\vec{b}}$$

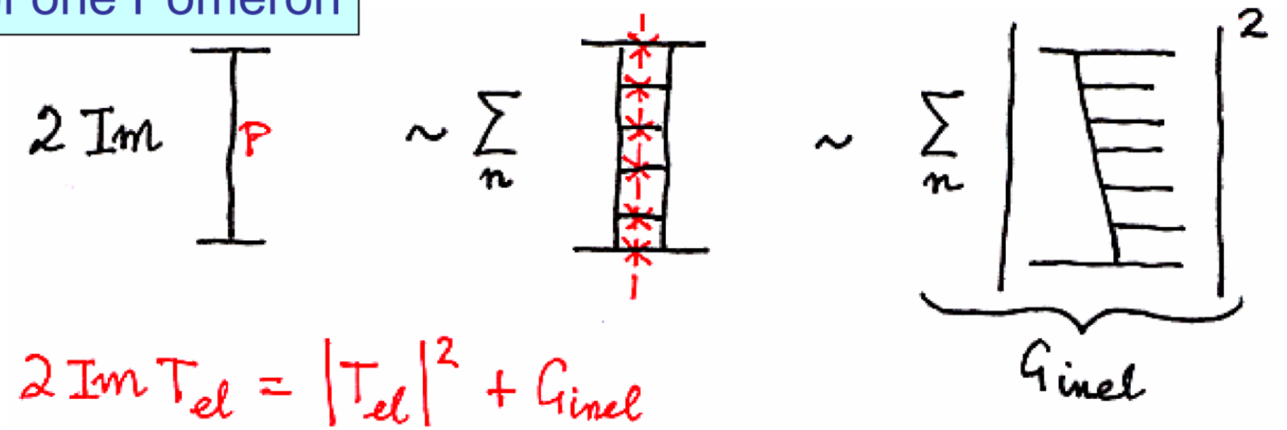
at HE the opacity has a Gaussian form in the *b*-space:

$$\Omega(s,b) = \frac{g_N^2(0)}{4\pi B}\left(\frac{s}{s_0}\right)^{\alpha_{\mathbb{P}}(0)-1} e^{-b^2/4B}$$

s-channel unitarity and Pomeron exchange

Unitarity relates the Im part of ladder diagrams (disc T = 2 Im T) to cross sections for multiparticle production

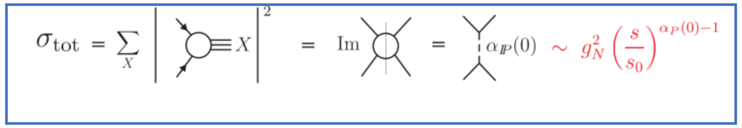
Exch. of one Pomeron



The coherence of ψ(beam) is destroyed by interaction of last exch. pt. with target. Leads, not only to inelastic high-multiplicity production, but also, via unitarity, to elastic scattering.
Elastic scattering is due to the absorption of an initial coherent component, and originates from the remaining part of ψ(beam) which preserves its coherence

Pomeron pole was named after I. Y. Pomeranchuk.

HE behaviour in the eikonal model

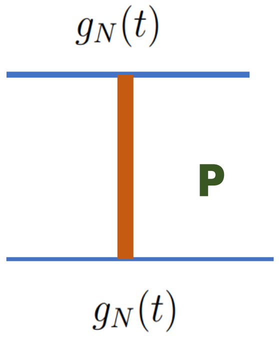


opacity given by the exchange of one pomeron with a linear trajectory of slope α'_P and intercept $\alpha_P(0) > 1$

$$\Omega(s,b) = \int \frac{d^2 q_t}{4\pi^2}\, \Omega(s,q_t)\, e^{i\boldsymbol{q}_t\cdot \boldsymbol{b}}$$

with

$$\Omega(s,q_t) = -i\eta_P(t) g_N(t) g_N(t) \left(\frac{s}{s_0}\right)^{\alpha_P(t)-1},$$



where $g_N(t)$ is the proton-pomeron coupling and where the conventional dimensionful scale $s_0 = 1$ GeV^2 is taken; finally η_P is the signature factor of the pomeron

$$\eta_P(t) = -\frac{1+\exp(-i\pi\alpha_P(t))}{\sin\pi\alpha_P(t)}.$$

Change in Reggeon contribution when **s** ⟶ **-s**

If we assume, as usual, an exponential t dependence of the coupling, $g_N(t) = g_N(0)\exp(b_0 t)$, then the opacity generated by one pomeron pole is

$$\text{Re}\ \Omega(s,q_t) = g_N(0) g_N(0) \left(\frac{s}{s_0}\right)^{\alpha_P(0)-1} \exp(Bt)$$

$$B = 2b_0 + \alpha'_P \ln\left(\frac{s}{s_0}\right)$$

with energy increasing the differential cross section becomes steeper

shrinkage of dif. cone

Single pomeron $\quad \sigma_{\rm tot} \propto s^{\Delta} \quad$ with $\quad \Delta \equiv \alpha_P(0) - 1$

at high energies as $\sqrt{\alpha'_P \ln(s/s_0)}$.

However, eikonal unitarization damps the power growth of the one pomeron exchange cross

$$\frac{\Omega(s,b)}{2} = \frac{g_N^2(0)}{8\pi B}\exp\left(\Delta\ln(s/s_0) - \frac{b^2}{4\alpha'\ln(s/s_0)}\right),$$
$$\gg 1 \quad \text{for } b^2 < R^2 = 4\Delta\alpha' \ln^2(s/s_0)$$

tends to the black disc limit for $b \lesssim R$.

A popular parameterization of the elastic pp-scattering amplitude by Donnachie-Landshoff (DL) is the Regge form

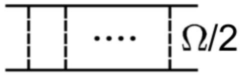
$$T_{\rm el}(s,t) = \eta_P \sigma_0 F_1^2(t) s^{\alpha_{\mathbb{P}}(t)}, \qquad \sigma_0 = 21.7 \text{ mb} \qquad \eta_P = \frac{1+\exp(-i\pi\alpha_{\mathbb{P}}(t))}{\sin(-\pi\alpha_{\mathbb{P}}(t))}$$

effective **Pomeron trajectory** $\alpha_{\mathbb{P}}(t) = 1 + \Delta + \alpha' t \simeq 1 + 0.0808 + 0.25t$

increasingly deficient at higher energies ➡ due to unitarity we have to take into account not only Regge poles but also multiple exchanges of Regge poles in the t-channel

Regge cuts

A powerful technique to evaluate Reggeon diagrams- Gribov (1968) Reggeon Field Theory (RFT)



$$i(1-e^{-\Omega(s,b)/2}) = i\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\Omega^n}{n!2^n},$$

$$\Omega(s,b) = \frac{g_N^2(0)}{4\pi B}\left(\frac{s}{s_0}\right)^{\alpha_{\mathbb{P}}(0)-1} e^{-b^2/4B}.$$

In terms of opacity the effective radius of interaction increases at high energies as $\sqrt{\alpha'_{\mathbb{P}}\ln(s/s_0)}$

This means that with energy increasing the differential cross section becomes steeper (the so called *shrinkage* of the diffractive peak).

The eikonal unitarization reduces the power growth of the one-Pomeron exchange cross-section

➡ amplitude $\text{Im } T_{\rm el}(s,b) = 1 - e^{-\Omega/2} < 1.$

That is for $\Delta = 0.1$ and $\alpha'_{\mathbb{P}}$=0.25 GeV^{-2} we may expect that the cross section increases as

$$\sigma_{\rm tot} = 2\pi R^2 \simeq c \cdot \ln^2 s,$$

with $c = 8\pi\Delta\alpha'_{\mathbb{P}} = 0.24$ mb. This value is close to that obtained by the COMPETE parameterization ($c = 0.27$ mb) but much smaller than the Froissart-Lukaszuk-Martin (FLM) bound With $c^{\rm FLM} = \pi/m_\pi^2 \simeq 60$ mb,

$$\sigma_{\rm tot} \leq \frac{\pi}{m_\pi^2}\ln^2\left(\frac{s}{s_0}\right).$$

even at the LHC we are very far from true high-energy asymptotics

If we assume an exponential t-dependence of the coupling, $g_N(t) = g_N(0)\exp(B_0 t)$, and neglect the Pomeron phase, then the opacity is

$$\Omega(s, q_t) = g_N(0) g_N(0) \left(\frac{s}{s_0}\right)^{\alpha_{\mathbb{P}}(0)-1} e^{Bt},$$

with the t-slope given by

$$B = 2B_0 + \alpha'_{\mathbb{P}} \ln\left(\frac{s}{s_0}\right).$$

$B = \mathrm{d}[\ln(\mathrm{d}\sigma_{\mathrm{el}}/\mathrm{d}t)]/\mathrm{d}t$

At high energies the opacity has a Gaussian form in the b-space:

$$\Omega(s, b) = \frac{g_N^2(0)}{4\pi B} \left(\frac{s}{s_0}\right)^{\alpha_{\mathbb{P}}(0)-1} e^{-b^2/4B}.$$

In terms of opacity the effective radius of interaction increases at high energies as $\sqrt{\alpha'_{\mathbb{P}} \ln(s/s_0)}$. This means that with energy increasing the differential cross section becomes steeper (the so called *shrinkage* of the diffractive peak).

To correct for unitarity: eikonalize amplitude

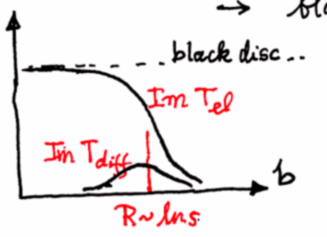
i.e. $\mathrm{Im}\, T_{el} = (1 - e^{-\Omega/2})$

with $$\frac{\Omega}{2} = \frac{\beta\eta}{B}\left(\frac{s}{s_0}\right)^{\Delta} \exp\left(-\frac{b^2}{4B}\right)$$

$$\underset{HE}{\sim} \frac{\beta\eta}{B} \exp\left(\Delta \ln\left(\frac{s}{s_0}\right) - \frac{b^2}{4\alpha' \ln\left(\frac{s}{s_0}\right)}\right)$$

$\gg 1$ for $b^2 < R^2 = 4\alpha' \Delta \ln^2 \frac{s}{s_0}$

$\rightarrow$ black disc for $b \lesssim R$



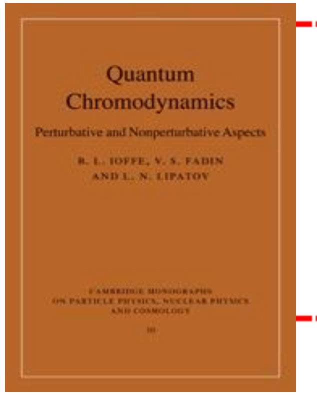
...re $B(s) = R_c^2 + \alpha'\left[\ln\left(\frac{s}{s_0}\right) - i\frac{\pi}{2}\right]$

$$T(s,b) = \frac{\beta(0)\eta}{B}\left(\frac{s}{s_0}\right)^{\alpha(0)-1} \exp\left(-\frac{b^2}{4B}\right)$$

Ladder structure of the Pomeron after QCD

Shortly after the discovery of QCD it was proposed that (colourless) two-gluon exch. had properties of Pomeron exch:
vacuum quantum no's, singularity at j=1

(BFKL-1975-78)



--Later, using the BFKL formalism, in which the t-ch gluons (rather than hadrons) become Reggeized, it was found possible (for sufficiently large k_T) to describe HE (low x) interactions in pQCD.

--BFKL sum up the leading $(\alpha_s \log 1/x)^n$ contributions and build up the hard/pQCD/BFKL Pomeron.

--The Pomeron, is not a pole, but a branch cut in the complex angular momentum plane, plus more complicated cuts at HO

Grigorios Chachamis

"Soft" and "Hard" Pomerons ?

A vacuum-exchange object drives soft HE interactions. Not a simple pole, but an enigmatic non-local object. Rising σ_{tot} means multi-Pom diags (with Regge cuts) are necessary to restore unitarity. σ_{tot}, $d\sigma_{el}/dt$ data, described, in a limited energy range, by eff. pole $\alpha_P^{eff} = 1.08 + 0.25t$

Sum of ladders of Reggeized gluons with, in LLx BFKL, a singularity which is a cut and not a pole. When HO are included the intercept of the BFKL/hard Pomeron is $\alpha_P^{bare}(0) \sim 1.3$

$$\Delta = \alpha_P(0) - 1 \sim 0.3$$

NNL BFKL Pomeron

$\alpha_P^{eff} \sim 1.08 + 0.25\ t$

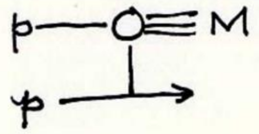
up to Tevatron energies

$(\sigma_{tot} \sim s^{\Delta})$

$\alpha_P^{bare} \sim 1.3 + 0\ t$

with absorptive (multi-Pomeron) effects

Recall low M diffraction



Let $|p\rangle, |N^*\rangle, \ldots \Rightarrow \sum a_{ik} |\phi_k\rangle$

$i = 1, \; 2, \ldots$

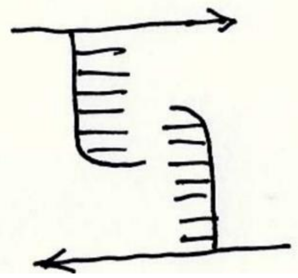
diffractive eigenstates only undergo "elastic-type" scatt.

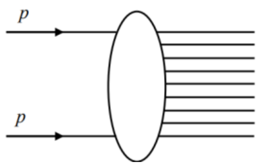
$i=1,2$ sufficient
two-channel eikonal

High M diffraction ?

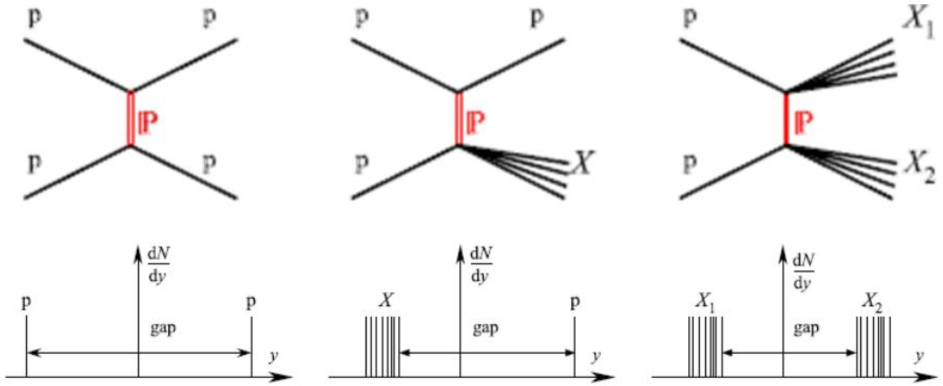
Enlarge no. of $|\phi_k\rangle$'s ?

Even if practical, have the problem of overlapping particle production for central rapidities





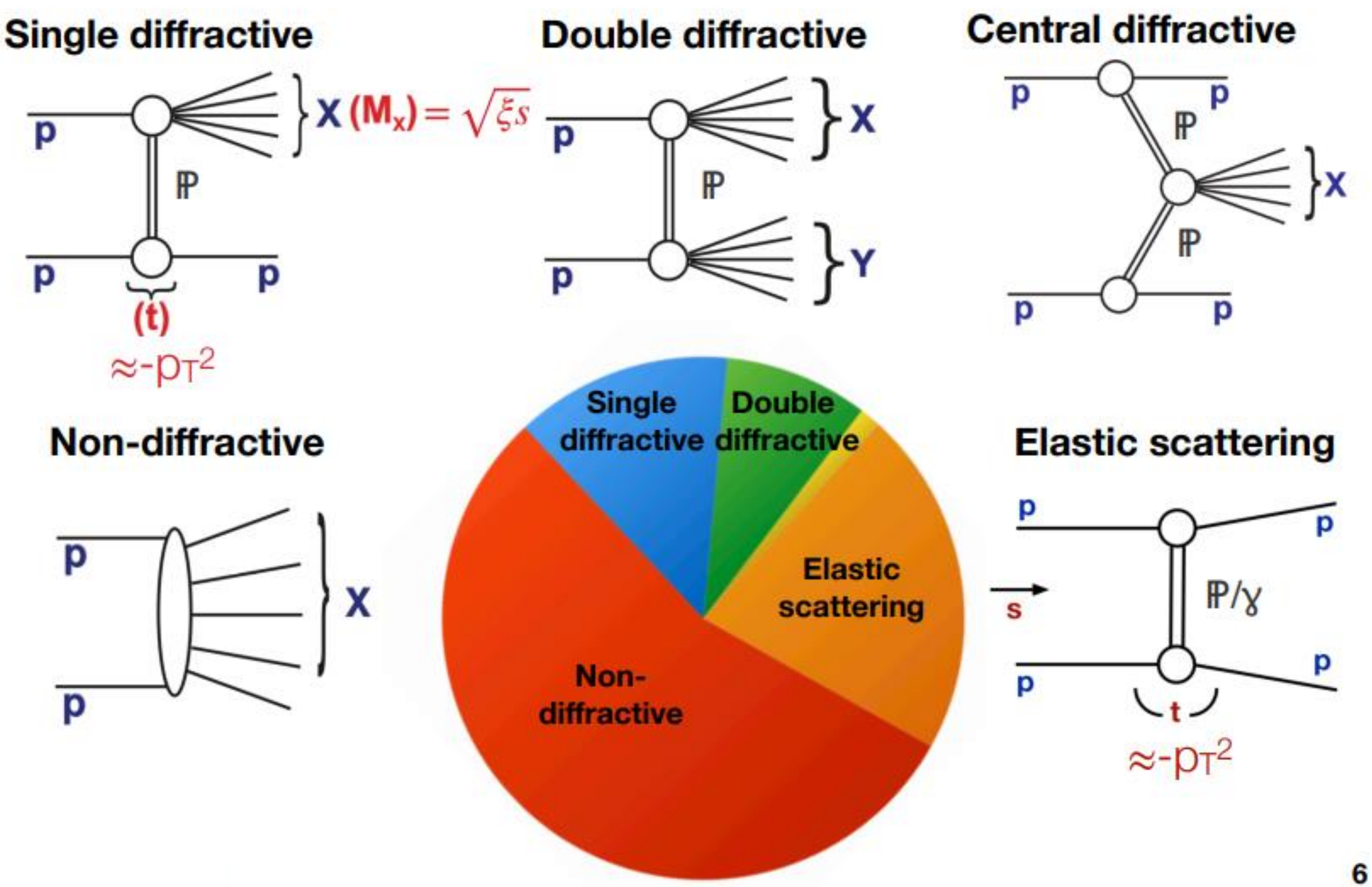
non-diffractive processes,



Feynman diagrams for different diffractive topologies. $I\!P$ stands for Pomeron and p for proton while X represents the diffractive systems. Below each diagram is also shown the corresponding rapidity distribution of the outgoing particles. Figure

LHC Strong interactions

ℙ = Pomeron

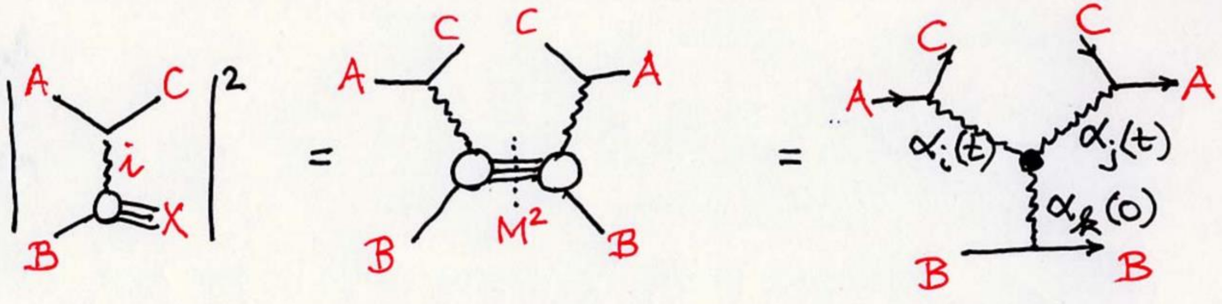


$pp \to X + p$ and $pp \to X_1 + X_2$, where the + sign denotes the presence of a LRG

Triple Regge region

$M^2 \to \infty, \quad \frac{s}{M^2} \to \infty$

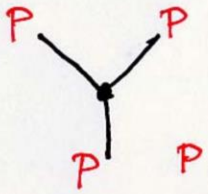
high mass diff^{ve} dissoc^{n}



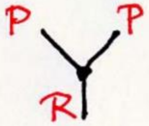
$$f = \frac{1}{s} \beta^{i}_{AC}(t) \beta^{j}_{AC}(t) \left(\frac{s}{M^2}\right)^{\alpha_j(t)+\alpha_i(t)} \mathrm{Disc}_{M^2} (\alpha_i B \to \alpha_j B)$$

$$= \frac{1}{s} \beta_{ijk}(t) \left(\frac{s}{M^2}\right)^{\alpha_j(t)+\alpha_i(t)} (M^2)^{\alpha_k(0)}$$

$$= \beta_{ijk}(t)\ s^{\alpha_j(t)+\alpha_i(t)-1} (M^2)^{\alpha_k(0)-\alpha_j(t)-\alpha_i(t)}$$



$f(s, t=0, M^2) \sim \frac{s}{M^2}$ if $\alpha_P(0) = 1$



$f(s, t=0, M^2) \sim s (M^2)^{-3/2}$ if $\alpha_R(0) = 0.5$

$$\Delta y = \ln\left(\frac{s}{M^2}\right) = \ln\left(\frac{1}{\xi}\right)$$

$\xi = 1 - x$

usually the condition $M^2 \ll s$

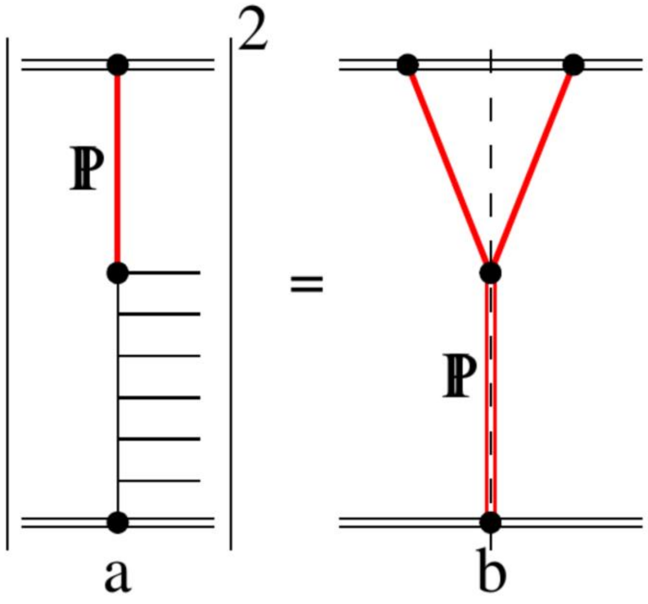


Figure 1: Schematic diagrams of Single Diffractive (SD) processes in Pomeron-proton collisions.

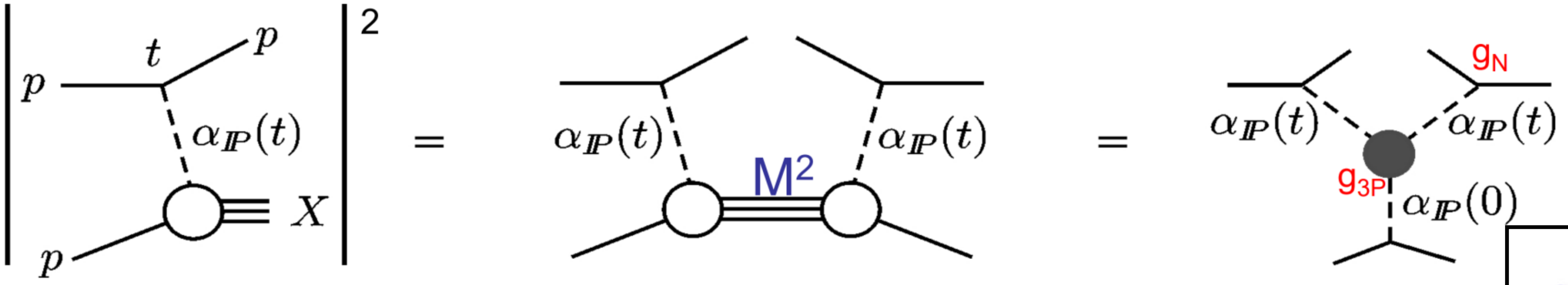
where $g_{3\mathbb{P}}(t)$ is the triple-Pomeron coupling. The value of the coupling $g_{3\mathbb{P}}$ is usually obtained from a triple-Regge analysis of lower energy data (KKPT-1973- LKMR-2009)

→ $g_{\rm eff} \;=\; g_{3\mathbb{P}}\,\langle S^2\rangle$

find $g_{3P}=\lambda\, g_N$ $\lambda\sim 0.2$

Optical theorem

naïve argument without absorptive effects:



$\Delta y = \ln\left(\frac{s}{M^2}\right) = \ln\left(\frac{1}{\xi}\right)$

triple-Pomeron diag

$g_N^3 g_{3P} \left(\frac{M^2}{s_0}\right)^{\alpha_{I\!P}(0)-1}\left(\frac{s}{M^2}\right)^{2\alpha_{I\!P}(t)-2}$

but screening even more important

$M^2 d\sigma_{SD}/dM^2 \sim g_N^3\, g_{3P} \sim \lambda\sigma_{el}$ $\ln s/M_0^2$ $(\sigma_{el} \sim g_N^4)$

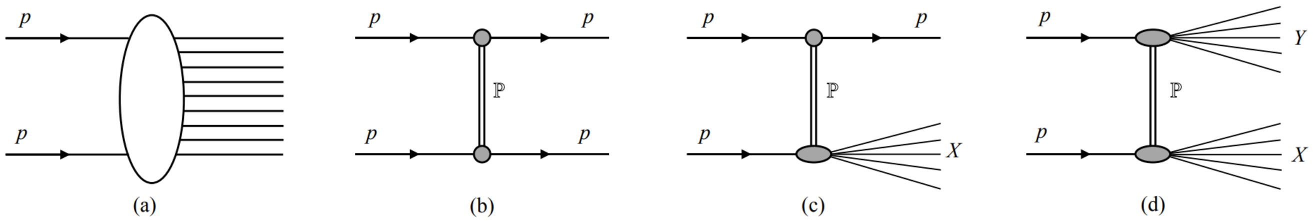
$$\sigma_{\rm SD} \;=\; \int \frac{M^2 d\sigma_{\rm SD}}{dM^2}\,\frac{dM^2}{M^2} \;\sim\; \lambda \ln(s/M_0^2)\,\sigma_{el}$$

grossly oversimplified

(intermediate energy range)

so at collider energies $\sigma_{SD}(\text{large } M) \sim \sigma_{el}$

SD is “enhanced” by larger phase space available at HE.



Schematic diagrams of soft pp processes. (a) non-diffractive processes, (b) elastic scattering, (c) single dissociation and (d) double dissociation. The double line corresponds to the Pomeron exchange.

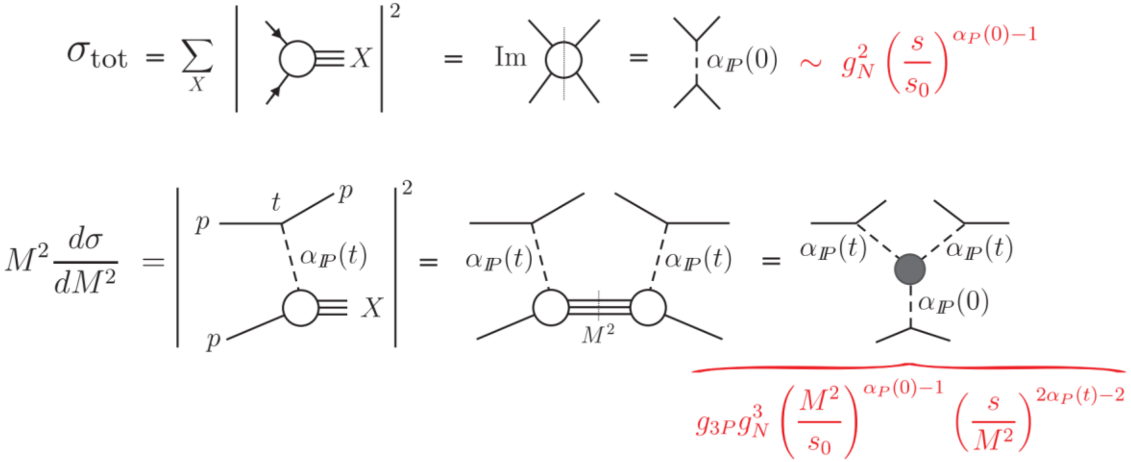
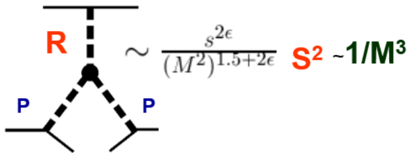


Figure 20.2: Illustration of the optical theorem for the total cross section and for high-mass diffractive dissociation in the absence of absorptive corrections.

Low mass diffractive dissociation

introduce diff^{ve} estates ϕ_i, ϕ_k (comb^{ns} of p,p*,..) which only undergo "elastic" scattering (Good-Walker)

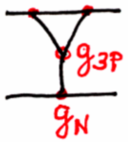
dual to



PPR-diagram

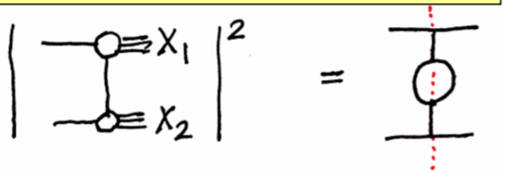
Screening is very important.- absorption ...

Single dissocation: $\frac{d\sigma^{SD}}{dM^2 dt} \sim \frac{1}{s}\, g_N^3\, g_{3P} \left(\frac{s}{M^2}\right)^{2\alpha_P(t)-1} (M^2)^{\alpha_P(0)-1}$



t is the momentum squared transferred through the LRG

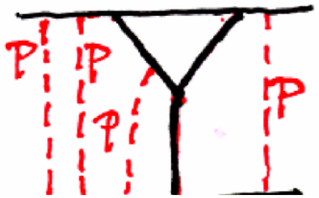
Double diffractive dissociation



$$\frac{d\sigma^{DD}}{dM_1^2\, dM_2^2\, dt} \sim \frac{1}{s}\, g_N^2\, g_{3P}^2 \left(\frac{s}{M_1^2 M_2^2}\right)^{2\alpha_P(t)-1} (M_1^2)^{\alpha_P(0)-1} (M_2^2)^{\alpha_P(0)-1}$$

So far Pomeron regarded as an exchanged particle. But Pomeron with "intercept" $\Delta = \alpha_P(0) - 1 > 0$ leads to a violation of unitarity as s→ infinity: $\sigma_{tot} \sim s^{\Delta}$, $\sigma_{SD,DD} \sim s^{2\Delta}$

Multi-Pomeron exch[s] suppress this growth and restore s-ch unitarity. Called unitarity/screening/abs corr[ns]



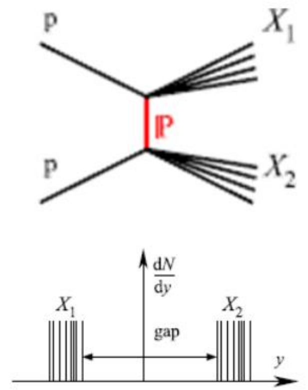
due to unitarity we have to take into account not only Regge poles, but also the cuts in the j-plane, which correspond to the multiple exchanges of Regge poles in the *t*-channel,

$\xi = 1 - x$ and x is the initial momentum fraction

$$\frac{\xi \mathrm{d}\sigma_{\mathrm{SD}}}{\mathrm{d}t\mathrm{d}\xi} = \frac{M^2 \mathrm{d}\sigma_{\mathrm{SD}}}{\mathrm{d}t\mathrm{d}M^2} = \frac{g_{3\mathbb{P}}(t) g_N(0) g_N^2(t)}{16\pi^2} \left(\frac{s}{M^2}\right)^{2\alpha_{\mathbb{P}}(t)-2} \left(\frac{M^2}{s_0}\right)^{\alpha_{\mathbb{P}}(0)-1}$$

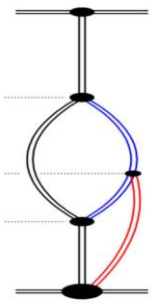
=1-x

$g_{3\mathbb{P}}(t)$ is the triple-Pomeron coupling.



In an analogous way the cross section for double dissociation reads

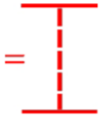
$$\frac{\xi_1 \xi_2 \mathrm{d}\sigma_{\mathrm{DD}}}{\mathrm{d}t\mathrm{d}\xi_1\xi_2} = \frac{M_1^2 M_2^2 \mathrm{d}\sigma_{\mathrm{DD}}}{\mathrm{d}t\mathrm{d}M_1^2\mathrm{d}M_2^2} = \frac{g_{3\mathbb{P}}^2(t) g_N^2(0)}{16\pi^3} \left(\frac{s s_0}{M_1^2 M_2^2}\right)^{2\alpha_{\mathbb{P}}(t)-2} \left(\frac{M_1^2 M_2^2}{s_0^2}\right)^{\alpha_{\mathbb{P}}(0)-1}$$



Semi-enhanced diagram

Elastic amp. $T_{el}(s,b)$

bare amp. $\Omega/2$ =

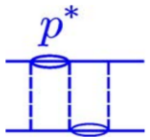


Survival effects

$$\mathrm{Im}\ T_{\rm el} = [\text{blob}] = 1 - e^{-\Omega/2} = \sum_{n=1}^{\infty} [\text{ladder}]\ \Omega/2$$

(s-ch unitarity)

(-20%)



Low-mass diffractive dissociation →multichannel eikonal

introduce diff[ve] estates ϕ_i, ϕ_k (comb[ns] of p,p*,..) which only undergo "elastic" scattering (Good-Walker)

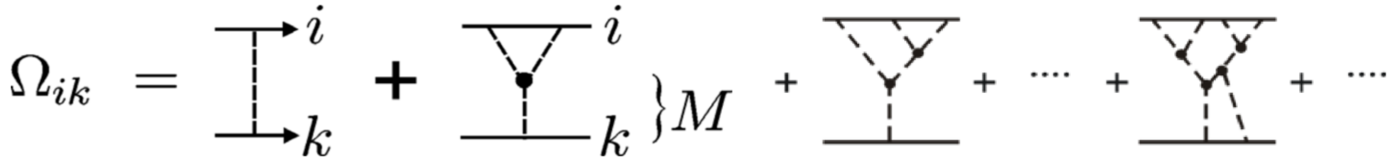
$$\mathrm{Im}\ T_{ik} = [\text{blob}]_{k}^{i} = 1 - e^{-\Omega_{ik}/2} = \sum [\text{ladder}]\ \Omega_{ik}/2$$

(-40%)

Black disc with a sharp edge:
dissociation is completely screened (
only elastic and inelastic channels
dif. dissociation comes from the edge of the disc

include high-mass diffractive dissociation

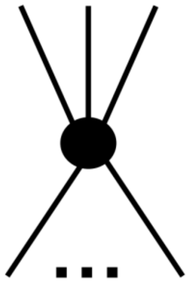
(SD -80%)



Multi-Pomeron couplings

So far, considered only triple-Pomeron coupling → leads to σ_{tot} which decreases at asymptotic energies.

More reasonable to include
m → n Pomeron vertices

$g^m{}_n =$ 

Absorption due to eikonal multi-pomeron exchanges

so-called enhanced diagrams with multi-Pomeron vertices, g_m^n, which couple m to n Pomerons

see EPJ**C71**(2011)1617 for more discussion.

the sum of all the enhanced diagrams
QGSJET Monte Carlo

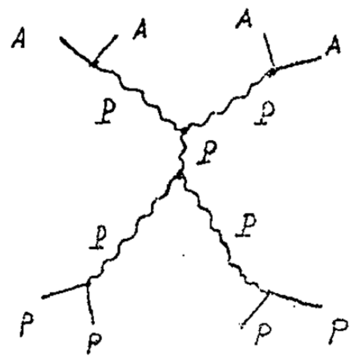
other approaches: analytical KMR, GLM...

Central Diffractive processes

Processes $pp \to p + X + p$, where an object X, produced in the central rapidity region, is separated from the outgoing protons by a LRG on each side, are called Central Exclusive Production (CEP). They are described by the double Pomeron exchange (DPE) diagrams. When the mass of the central system, M_X, is large and the interaction in the M_X region can be described by Pomeron exchange, the corresponding cross section reads

$$\frac{\xi_1 \xi_2 \mathrm{d}\sigma^{\mathrm{CEP}}}{\mathrm{d}\xi_1 \mathrm{d}t_1 \mathrm{d}\xi_2 \mathrm{d}t_2} = \frac{g_N^2(t_1) g_N^2(t_2)}{(16\pi^2)^2} \left(\frac{1}{\xi_1}\right)^{2\alpha_{\mathbb{P}}(t_1)-2} \left(\frac{1}{\xi_2}\right)^{2\alpha_{\mathbb{P}}(t_2)-2} g_{3\mathbb{P}}^2(0) \left(\frac{M_X^2}{s_0}\right)^{\alpha_{\mathbb{P}}(0)-1} \qquad S^2$$

$$\Delta y = \ln\left(\frac{s}{M^2}\right) = \ln\left(\frac{1}{\xi}\right)$$



"beetle-gram"

Summary of t-channel viewpoint

Regge formalism appropriate for HE (large s) and
forward scattering (t~0) --- for "soft" HE hadron inter^{ns}

Constant or increasing $\sigma_{tot}(s)$ with s → **Pomeron →**

→ (a) processes with large rapidity gaps
→ valuable exclusive HE data

(b) soft multiparticle production
→ vital to understand the underlying event to rare New Physics processes at the LHC

Triple Regge needed for high-mass dissociation

Importance of absorption (unitarity corrections)
→ multi-Pomeron exchanges

Energy dep. of σ_{el}, σ_{tot} controlled by intercept and slope of "effective" pomeron trajectory

Diffractive dip and $\sigma_{low\ M}$ controlled by properties of GW eigenstates

High-mass diss^n driven by multi-pomeron effects

Calculation of S^2

average over diff. estates i,k

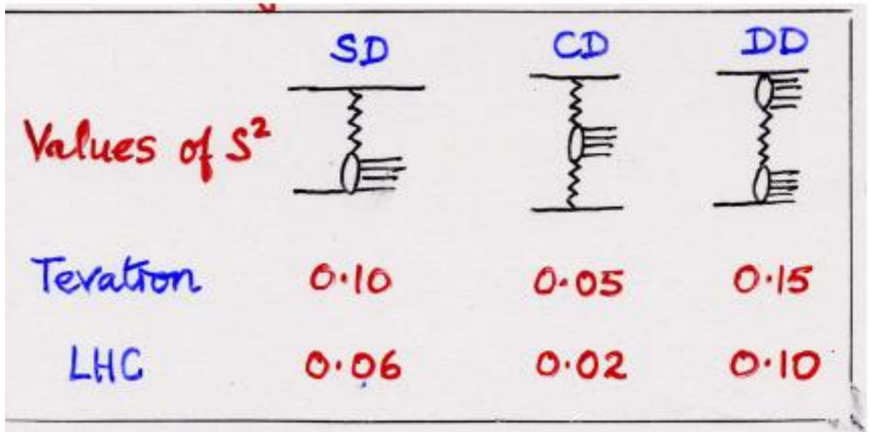
over b

prob. of proton to be in diffractive estate i

hard m.e. i k → H

$$\overline{S^2} = \frac{\sum_{i,k} \int d^2b \, |a_{pi}|^2 \, |a_{p'k}|^2 \, |\mathcal{M}_{ik}|^2 \, \exp(-\Omega_{ik}(s,b))}{\sum_{i,k} \int d^2b \, |a_{pi}|^2 \, |a_{p'k}|^2 \, |\mathcal{M}_{ik}|^2}$$

survival factor w.r.t. soft i-k interaction. Recall that $e^{-\Omega}$ is the prob. of no inelastic scatt. (which would otherwise fill the gap)



Values of S^2	SD	CD	DD
Tevatron	0·10	0·05	0·15
LHC	0·06	0·02	0·10

Since the opacity Ω increases with energy, at large Ω the number of multiple interactions grows $N \propto \Omega$, leading to a smaller S^2.

Because the QCD Pomeron is built mainly from gluons, it is natural to search for glueballs in double Pomeron exchange processes, particularly in CEP.

The t-slope and dip in the elastic cross-section

At small one- Pomeron amplitude the two-Pomeron contribution → Ω^2 term in the expansion of the eikonal $1 - \exp(-\Omega/2)$

the momentum transferred, $q_t = \sqrt{|t|}$, is divided between the two Pomerons → each Pomeron carries about a momentum $q_t/2$.

Since the two-Pomeron contribution has an opposite sign in comparison with the one-Pomeron exchange, their interference will result in the appearance of the first diffractive minimum which moves to smaller $|t|$ with energy increasing. Such interference effects are largely responsible for the zero in the imaginary part of the amplitude (with the minimum filled by the real part).

'two-Pomeron' amplitude will be $\exp(2B(t/4)) = \exp(Bt/2)$

in terms of the impact parameter, b, the domain of a **smaller** radius.

↓

an exchange of *n* Pomerons will rapidly decrease with *n* increasing.

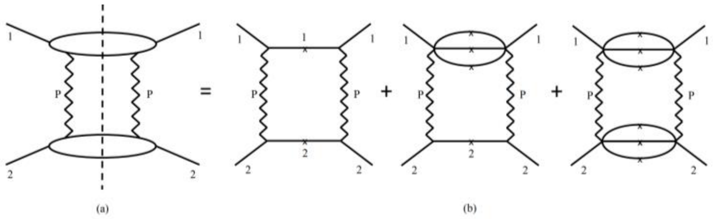


Figure 20.3: Two-Pomeron exchange in the t channel expressed as a sum over all diffractive intermediate states in the s-channel. The crosses indicate that the particles are on the mass shell.

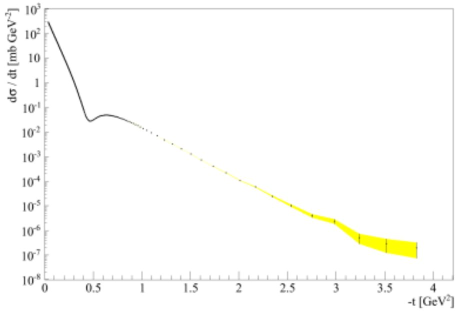
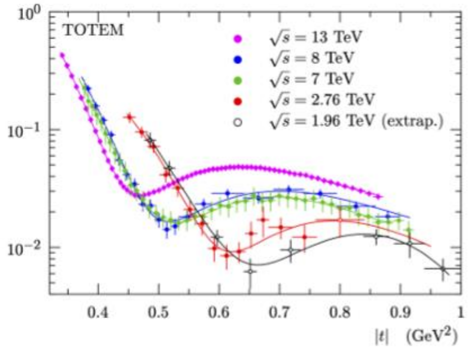


Fig. 8: (color) Differential elastic cross-section $d\sigma/dt$ at $\sqrt{s} = 13$ TeV. The statistical and $|t|$-dependent correlated systematic uncertainty envelope is shown as a yellow band.



$$A_{el} = \mathbb{P} - \mathbb{P}\mathbb{P} + \mathbb{P}\mathbb{P}\mathbb{P} - \mathbb{P}\mathbb{P}\mathbb{P}\mathbb{P} + \ldots$$

$\sqrt{s}$ =13TeV

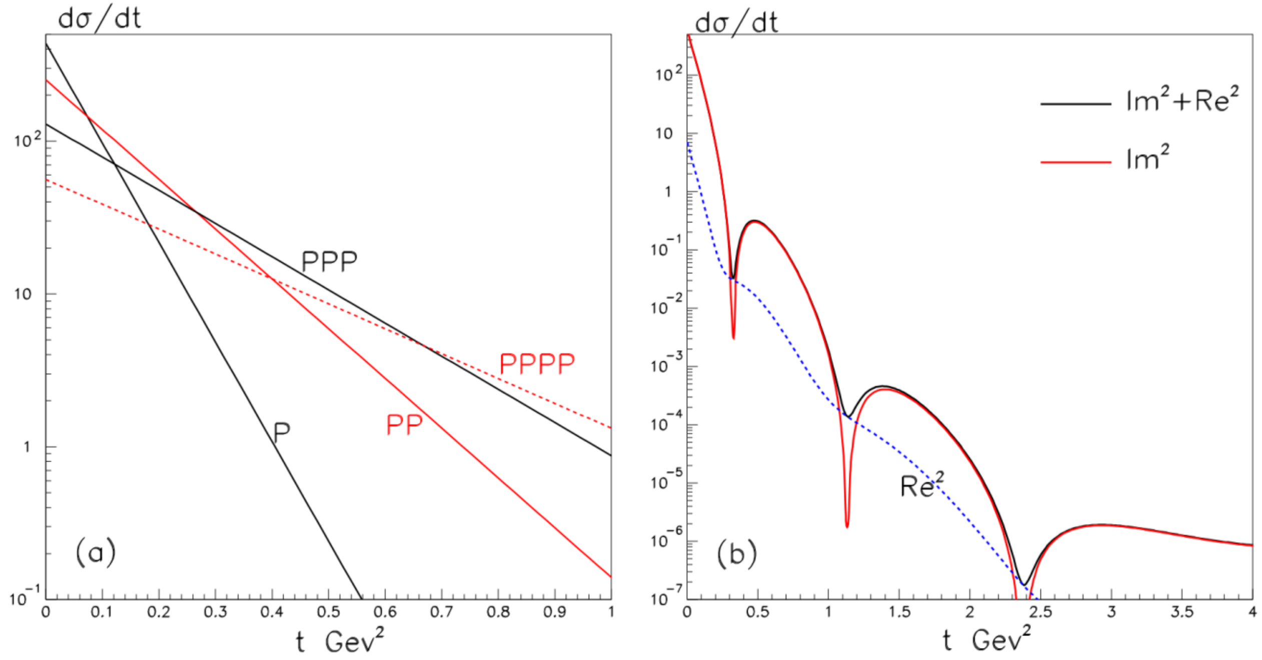


Figure 1: (a) The contribution of the individual multi-Pomeron diagrams to the elastic cross section; negative contributions are shown in red. (b) The elastic cross section generated by the sum of multi-Pomeron diagrams. The contribution of the imaginary part of amplitude is shown in red, while the contribution of the real part is show by the blue dashed curve.

for illustration purposes -assuming pure exponential behaviour of P-exchange amplitude

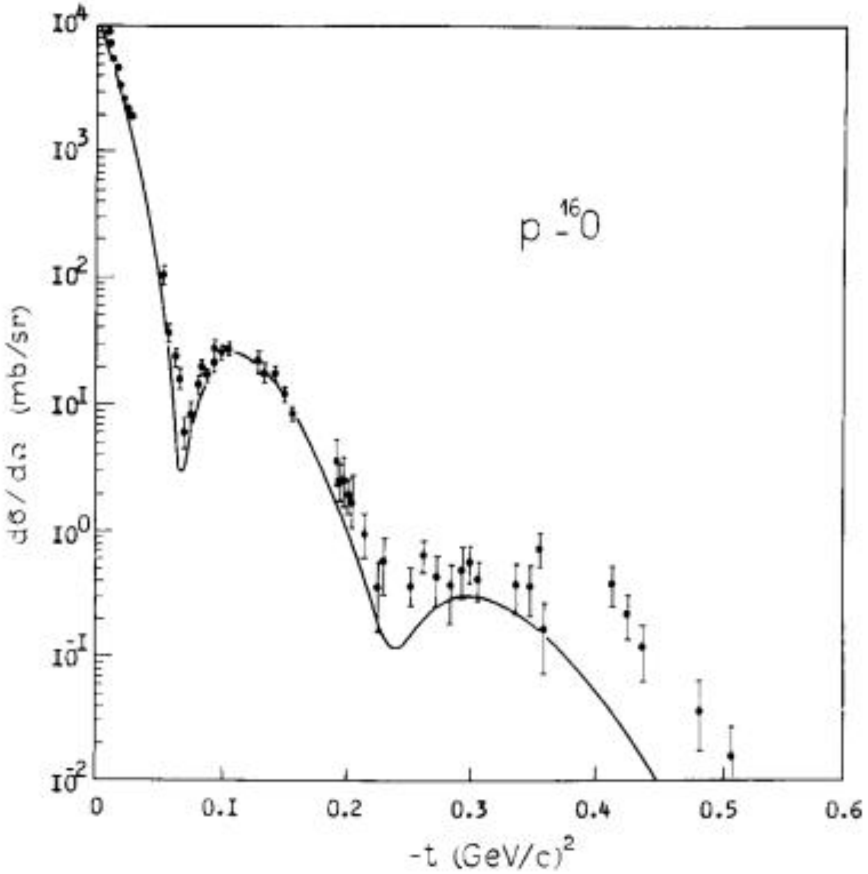


Fig. 1. The differential cross section for 1 GeV elastic proton scattering from ^{16}O nuclei measured in Brookhaven [1]. The theoretical curve is the result of calculations [4] within the framework of Glauber theory.

(possibility of proton rescattering on the different nucleons in the nucleus)

Summary of the s-channel viewpoint

- s-channel unitarity plays a key role.
- Impact parameter representation best.
- Inelastic scattering generates elastic amp.
- Eikonal formalism preserves unitarity.
- Slow approach to black disc limit at small b.
- Multichannel eikonal necessary for proton dissociation.
- Diffraction mainly in the periphery (large b).
- Need t-channel approach for high-mass dissociation.

How Large is Large ?

Diffraction is any process caused by Pomeron exchange.

(Old convention was any event with LRG of size $\delta\eta>3$, since Pomeron exchange gives the major contribution)

However LRG in the distribution of secondaries can also arise from

(a) Reggeon exchange
(b) fluctuations during the hadronization process

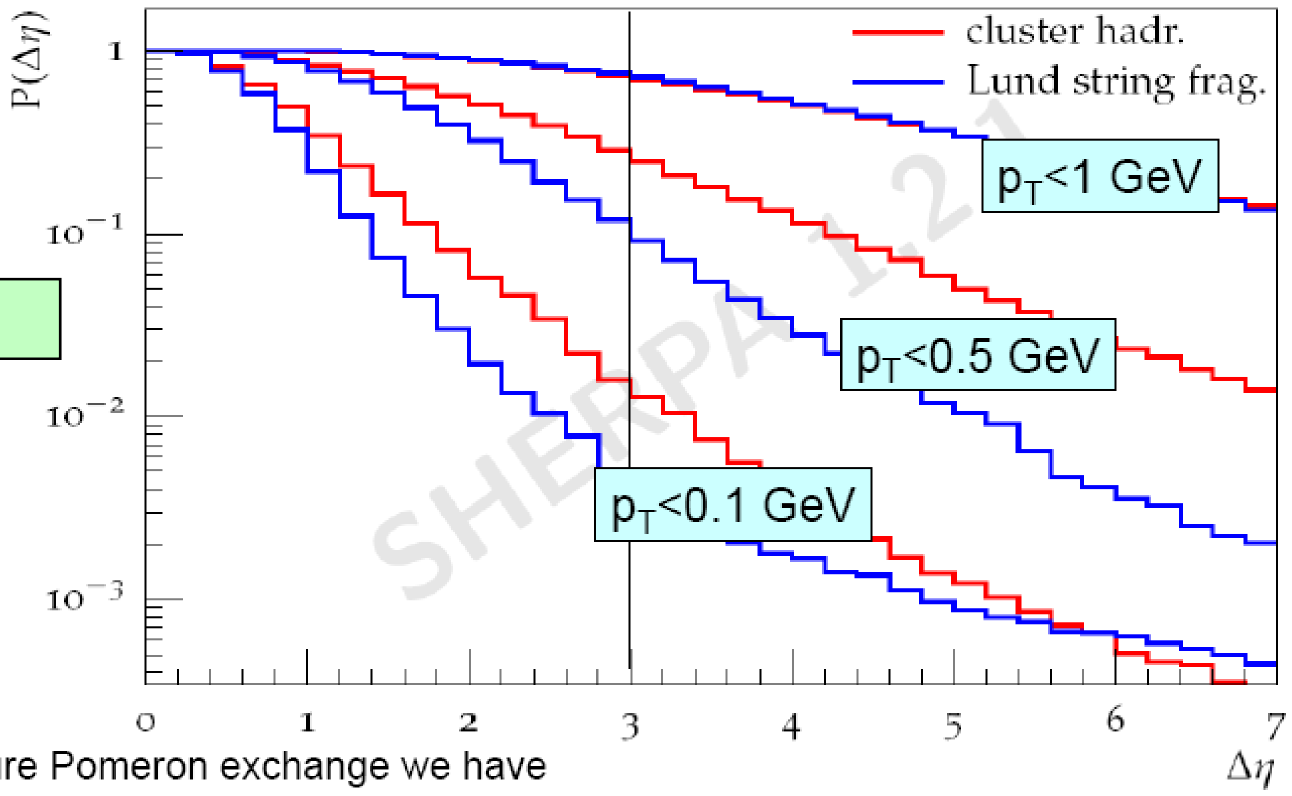
Indeed, at LHC energies LRG of size $\delta\eta>3$ do not unambiguously select diffractive events.

care must be taken in extracting the Pomeron contribution from LRG events.

Prob. of finding gap larger than Δη in inclusive event at 7 TeV due to fluctuations in hadronization

KKMRZ, arXiv:1005.4839

gap anywhere in $-5 < \eta < 5$, different threshold p_T



So to study pure Pomeron exchange we have

either to select much larger gaps

or to study the Δy dependence of the data, fitting so as to subtract the part caused by Reggeon and/or fluctuations.

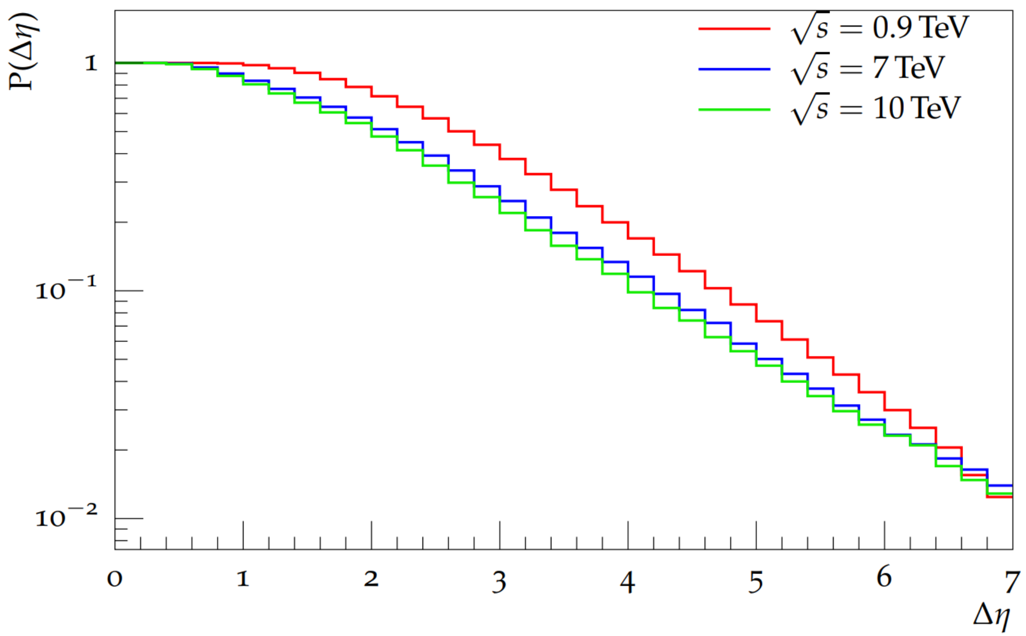


Fig. 5. Beam energy dependence of the probability for finding a rapidity gap (definition 'all') larger than $\Delta\eta$ in an inclusive QCD event ($p_{\perp,\text{cut}} = 0.5\,\text{GeV}$, no trigger condition, cluster hadronisation).

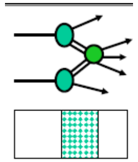
The gap probability decreases moderately with increasing beam energy (Fig. 5), since the multiplicity and mean gap probability decreases moderately with increasing beam energy (Fig. 5), since the multiplicity and mean $p_\perp$ increase with $\sqrt{s}$. '

This translates directly into a lower probability of large gaps caused by fluctuations.

with $p_{\perp cut}$ = 0.5 GeV the probability to have a gap $\delta\eta > 5$ is between 0.01 (string hadronisation) and 0.1 (cluster hadronisation). With the inelastic cross sec- tion $\sigma_{inel} \sim 50$ mb this leads to 0.5 − 5 mb of LRG caused by fluctuations

Conclusion of KKMRZ-2010

From a wider viewpoint, any process due to Pomeron exchange may be called *diffractive*. In general, such processes lead to Large Rapidity Gaps (LRG) in the distribution of secondary hadrons. However, the probability to obtain a gap without Pomeron exchange is not negligible; the gap can simply arise from fluctuations in the hadronization process. The Monte Carlo studies presented in this work show that, with the present rapidity acceptances and $p_\perp$ cuts of the LHC detectors, up to ~ 0.5 mb of the diffractive cross section can be mimicked by fluctuations[10] which have nothing to do with Pomeron exchange. This is not a serious background if the cross section of the diffractive process that we are studying is ~ 10 mb or larger, but it will pose a problem for studying the so-called double-Pomeron exchange (DPE) events, where the expected cross sections are $\sim 10\ \mu$b.



DPE

Long way to the asymptotics

$\sigma_{\rm tot}$, $\sigma_{\rm inel}$ could not be calculated from the first principles based on QCD- intimately related to the confinement of quarks and gluons
(some attempts within N=4 SYM).

Basic fundamental model-independent relations in the context of S-matrix: unitarity, crossing, analyticity, dispersion relations.
The Froissart-Lukaszuk-Martin bound $\sigma_{tot} \leq \text{Const} \ \ln^2 s.$
Important testable constraints

most models asympt. ~ $\ln^2 s.$

but not a Must

Phenomenological models- fit the data in the wide energy range and extrapolate to the higher energies.

Well-developed approaches based on Reggeon Field Theory with multi-Pomeron exchanges+ Good –Walker formalism to treat low mass diffractive dissociation: KMR-Durham, GLM- Tel-Aviv, Ostapchenko.
Differences/**Devil**  – in details

optical theorem:

$$d\sigma/dt = |T(t)|^2/16\pi s^2 \propto \exp(B_{el}t)$$

$$\operatorname{Im} T(s,t=0) = s\sigma_{tot}$$

There still is a freedom in the asymptotic behaviour

Different scenarios at $s \to \infty$

1. Weak coupling of the Pomerons $\sigma_{tot} \to \text{constant}$

2. Strong coupling of the Pomerons; $\sigma_{tot} \propto (\ln s)^{\eta}$ with $0 < \eta \leq 2,$

(V.N. Gribov,A.A.Migdal, -1969).

3. Asymptotically decreasing cross sections.

(P.Grassberger, K.Sundermeyer-1978; K.Boreskov-2001)

- All depends on the behaviour of the triple -(multi)-Pomeron vertices.

- Current data are usually described by scenario 2 with $\eta = 2$ (Froissart-Martin limit),

- To reach asymptotics we formally would need UH energies, when in the slope of elastic amplitude $\alpha'_P \ln(s) \gg B_0.$

How long is the way to asymptotics?

The high-energy behaviour of total hadronic cross sections has been one of the oldest problems of strong interactions over many decades, beginning from Heisenberg | W. Heisenberg, Z. Phys. **133**, 65 (1952)

$$\sigma_{\text{tot}} \leq \frac{\pi}{m_\pi^2} \ln^2\left(\frac{s}{s_0}\right).$$

M. Froissart, Phys. Rev. **123**, 1053 (1961).
A. Martin, Nuovo Cim. **A42**, 930 (1965).
L. Lukaszuk and A. Martin, Nuovo Cim. **A52**, 122 (1967).

$$c^{\text{FLM}} = \pi/m_\pi^2 \simeq 60 \text{ mb},$$

even at the LHC we are very far from true high-energy asymptotics

far from true high-energy asymptotics [6], and the observed growth of the cross section is driven by the interactions at relatively large transverse momenta $k_t \gg m_\pi$ rather than the smallest hadron mass m_π

It is interesting that the Froissart-type $\ln^2 s$ asymptotics of the pp total cross section are also supported by numerical results in lattice QCD |

It was also argued that in the case of an increasing (with energy) cross section the only regime consistent asymptotically with both the s- and the t-channel unitarities is that of a *black* disc whose radius increases as $R = c \cdot \ln s$ (i.e. $R \propto (\ln s)^\gamma$, with $\gamma = 1$ exactly).

KMR-2018

Finally, it is worth mentioning that the possibility that *asymptotically* the Pomeron intercept becomes smaller than 1, $\alpha_{\mathbb{P}}(0) < 1$, and at very high energies the total cross section starts to decrease with energy, though highly unlikely, is not yet completely rejected.

The Odderon

Properties of odd-signature high-energy amp studied in early 70's

Odderon first promoted in 1973 (Lukaszuk, Nicolescu) by Regge exchange for high-energy cross sections; for example pp and p$\bar{p}$

Paul Hoyer

$$A_{\pm} = A(pp) \pm A(p\bar{p})$$

simple poles $\alpha_{P,O}(0) \sim 1$

$A_+(pp) = A_+(p\bar{p}) \quad C = +1$ Pomeron --- dominantly imag

$A_-(pp) = -- A_-(p\bar{p}) \quad C = -1$ Odderon --- dominantly real

Christophe Royon
Kenneth Osterberg

1958 1. Pomeranchuk theorem $\Delta\sigma \equiv \sigma(\bar{p}p) - \sigma(pp) \sim \mathrm{Im}A_- \to 0 \quad \text{as} \quad s \to \infty$

2. Generalized Pomeranchuk th: $\dfrac{\sigma(\bar{p}p)}{\sigma(pp)} \to 1 \quad \text{as} \quad s \to \infty$

1 and 2 are not equivalent

$$\sigma(\bar{p}p) = A \ln^2 s + B \ln s + C$$

$$\sigma(pp) = A \ln^2 s + B' \ln s + C'$$

if $B \neq B'$ then satisfy 2, but not 1

In general $\Delta\sigma \leq c \ln s$

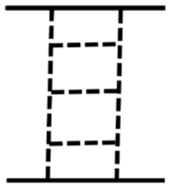
The Odderon exists in QCD

Need the existence of symmetric tensor d_{abc} of non-Abelian $SU(3)_{col}$ to form colourless ggg exchange with C=-1

Pomeron (gg)

Odderon (ggg)

BFKL eq.

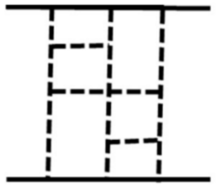


resum

$\alpha_P(0) > 1$

BKP eq.

Bartels; Kwiecinski, Praszalowicz 1980



resum

$\alpha_O(0) \approx 1$

Janik-Wosiek solution

Bartels-Lipatov-Vacca solution, 2000

THE ODDERON –is a firm prediction of PT QCD, but the amplitude is expected to be small compared to THE POMERON

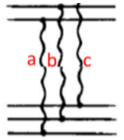
$A_{Odd} \sim \alpha_s^3 R_{min}^2$

$A_{Pom} \sim \alpha_s^2 R_{max}^2$

(M. Ryskin (1987))

- Odderon does not contribute to p-scattering.
- Odderon exchange in NN scattering comes from smaller distances than Pomeron.
  At acts as a single colour state
  Only for gluons with the Odderon contribution survives

For a two-gluon Pomeron q ~ 1/R had



Need the existence of symmetric tensor d_{abc} of non-Abelian $SU(3)_{col}$ to form colourless ggg exchange with C=-1

Pomeron counterpart, the odd-signature singularity placed at $j \simeq 1$ and formed by three t-channel gluons connected in colour space by the symmetric d^{abc} tensor of the colour $SU(3)$ group

The existence of the *C*-odd singularity with intercept $\alpha_{odd}(0) \cong 1$ is a firm prediction of QCD.

Odderon exchange amplitude has the opposite sign for *pp and pp¯* scatterings.

mainly real and is about 100 times smaller than the imaginary part of the Pomeron exchange amplitude

When calculating the elastic amplitude we have to replace the opacity Ω(b) by the sum $\Omega = \Omega_{even} + \Omega_{odd}$, where Ω_{even} is mainly real and Ω_{odd} is imaginary

Must include full Ω in amplitude

$$A(b) = i\left(1 - e^{-\Omega(b)/2}\right)$$

with $\Omega = \Omega_{\text{even}} + \Omega_{\text{odd}}$

→ Automatically accounts for absorptive effect caused by elastic rescattering

To get a clear confirmation of the Odderon effects it would be very instructive to have the $d\sigma_{el}/dt$ data for both *pp* and *pp¯* reactions at the **same** high energy 1 TeV (ideally in the same apparatus) and, in the ideal case, to study the energy dependence. At the moment we can only compare the *pp* cross section measured by TOTEM at √*s* = 2.76 TeV (TOTEM data)

in the TeV energy range the *ω*, *ρ* and *ωP*, *ρP* exchange contributions, which may be responsible for the difference between the *pp* and *pp¯* cross sections in are practically negligible.

Another way to search for the Odderon is to measure the real part of the elastic *pp* scattering amplitude via interference with the pure QED one-photon exchange. Since the one-photon exchange amplitude contribution is sizeable only at very small |*t*|, this way we can study the Odderon at or near to *t* = 0.

Extraction of the odderon coupling from the dip region- different t-range

Note that the Odderon contribution is strongly screened by the multi-Pomeron diagrams, which facilitate the falling-off of *ρ* with energy increasing

Properties of odd-signature high-energy amp studied in early 70's

Odderon first promoted in 1973 (Lukaszuk, Nicolescu) by Regge exchange for high-energy cross sections; for example pp and p$\bar{p}$

$$A_{\pm} = A(pp) \pm A(p\bar{p})$$

simple poles $\alpha_{P,O}(0) \sim 1$

$$A_+(pp) = A_+(p\bar{p}) \quad C = +1$$

$$A_-(pp) = -- A_-(p\bar{p}) \quad C = -1$$

Pomeron --- dominately imag

Odderon --- dominately real $0.96 - 1 \leq 1$

for $\alpha_{Odd} = 1$ C-odd amplitude A_{Odd} is real

1. Pomeranchuk theorem $\Delta\sigma \equiv \sigma(\bar{p}p) - \sigma(pp) \sim \mathrm{Im}A_- \to 0 \quad \text{as} \quad s \to \infty$

2. Generalized Pomeranchuk th: $\frac{\sigma(\bar{p}p)}{\sigma(pp)} \to 1 \quad \text{as} \quad s \to \infty$

At any impact parameter b

$$\mathrm{Im}A_{Odd}(s,b) \leq \mathrm{Im}A_{even}(s,b)$$

That is $\alpha_{Odd}(t=0) \leq \alpha_{even}(t=0)$ and $B_{Odd} \leq B_{even}$

Feynman's rules

Pomeron exchange is proportional to

$$i\left(\frac{-is}{s_0}\right)^{\alpha_{\mathbb{P}}(t)-1},$$

Odderon exchange is proportional to

$$\xi_{\mathbb{O}}\left(\frac{-is}{s_0}\right)^{\alpha_{\mathbb{O}}(t)-1}$$

$$\xi_{\mathbb{O}} = -1$$

TOTEM

LO PT-QCD (FK-1979)

$$\mathrm{Im}\,\mathcal{F}^{pp}_{\bar{p}p}(s) = I_{\mathbb{P}}(s) \pm I_{\mathbb{O}}(s) > 0$$

$$I_{\mathbb{P}}(s) = \frac{s}{2}\left[\sigma^{pp}_{tot}(s) + \sigma^{\bar{p}p}_{tot}(s)\right] > 0$$

$$I_{\mathbb{O}}(s) = \frac{s}{2}\left[\sigma^{pp}_{tot}(s) - \sigma^{\bar{p}p}_{tot}(s)\right]$$

not bound by the positivity requirements.

$$I_{\mathbb{P}}(s) > |I_{\mathbb{O}}(s)|$$

$$\beta_{\mathbb{O}}(t) = \beta_{\mathbb{O}}(0)\exp\left(\frac{r_{\mathbb{O}}t}{2}\right),$$

$$r_{\mathbb{O}} \leq r_{\mathbb{P}},$$

(σ (b) positivity in b-space)

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Feynman's rules

Pomeron exchange is proportional to

$$i\left(\frac{-is}{s_0}\right)^{\alpha_{\mathbb{P}}(t)-1},$$

Odderon exchange is proportional to

$$\xi_{\mathbb{O}}\left(\frac{-is}{s_0}\right)^{\alpha_{\mathbb{O}}(t)-1}$$

$\xi_{\mathbb{O}} = -1$ TOTEM

LO PT-QCD (FK-1979)

$$\beta_{\mathbb{O}}(t) = \beta_{\mathbb{O}}(0)\exp\left(\frac{r_{\mathbb{O}}t}{2}\right),$$

$$r_{\mathbb{O}} \le r_{\mathbb{P}},$$

$$\mathrm{Im}\,\mathcal{F}^{pp}_{\bar{p}p}(s) = I_{\mathbb{P}}(s) \pm I_{\mathbb{O}}(s) > 0$$

$$I_{\mathbb{P}}(s) = \frac{s}{2}\left[\sigma^{pp}_{tot}(s) + \sigma^{\bar{p}p}_{tot}(s)\right] > 0$$

$$I_{\mathbb{O}}(s) = \frac{s}{2}\left[\sigma^{pp}_{tot}(s) - \sigma^{\bar{p}p}_{tot}(s)\right]$$

not bound by the positivity requirements.

$$I_{\mathbb{P}}(s) > |I_{\mathbb{O}}(s)|$$

to exclude systematics we have to measure pp and $\bar{p}p$ in the SAME experiment (LHC at 900 GeV)

Ways to observe the Odderon

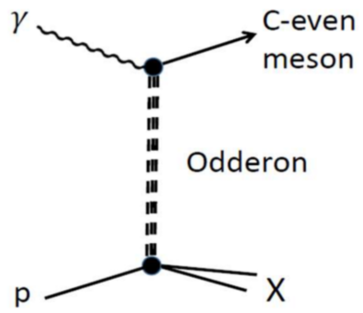
(1) To measure $d\Delta\sigma/dt$ in the dip region
a difference in pp and pp(bar) was seen at energy 53 GeV (ISR),
but cannot disentangle from b'ground due to the Pomeron-ω cut

TOTEM extrapolation 2.76TeV->1.96 TeV

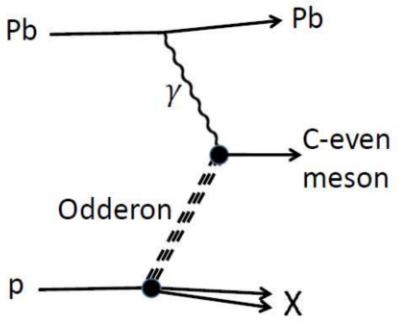
(2) Re A/Im A at t=0 in pp elastic scattering at the LHC

TOTEM (ALFA)

(3) photoproduction of C-even mesons: π^0, f_2, ... (HERA)
but dominated by,
γ exchange b'gd



(4) ultraperipheral
prod^n in p-Pb
collisions (LHC)
Z^2 in γ flux

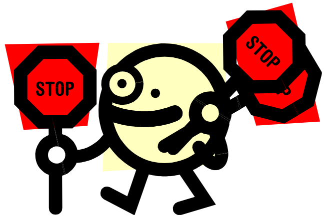


(5) K_L regeneration caused by the Odderon

(6) Energy dependence of C-even meson photoproduction EIC

Conclusion

- We firmly believe that a rich LHC diffractive programme will allow to impose strong 'restriction order' on the models of diffraction and provide a vital information on the dynamics of soft hadron interaction.



- A very promising start-up of diffractive studies at the LHC. More data & excitement to come soon.

LET THE DATA TALK !


QUESTIONS?

Dispersion relation

$$\mathrm{Re}A(s,t=0) \; = \; \frac{1}{\pi}\int_{-\infty}^{0} \frac{ds'\mathrm{Im}A(s',t)}{s'-s} + \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{ds'\mathrm{Im}A(s',t)}{s'-s}$$

$$\mathrm{Im}A(s,0) = \sigma_{tot}$$

$$\mathrm{Re}A(s,t=0) = \frac{1}{\pi}\int_{-\infty}^{0} \frac{ds'\sigma(p\bar{p})(-s'+4m^2)}{s'-s} + \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{ds'\sigma(pp)(s')}{s'-s}$$

for $\alpha_{Odd} \simeq 1$

$$\mathrm{Re}A_{Odd} \sim \ln s\cdot\mathrm{Im}A_{Odd} \text{ i.e. } \mathrm{Re}A_{Odd} >> \mathrm{Im}A_{Odd}$$

$$\mathrm{Re}A_{even} \; << \; \mathrm{Im}A_{even}$$

$$\mathrm{Re}A_{even}(s,t=0) \; \simeq \; \frac{2s}{\pi}\int_{4m^2}^{\infty} \frac{ds'\sigma(pp)}{s'^2-s^2} \; \simeq \; \frac{\pi}{2}\frac{\partial\sigma(s)}{\partial\ln s}$$

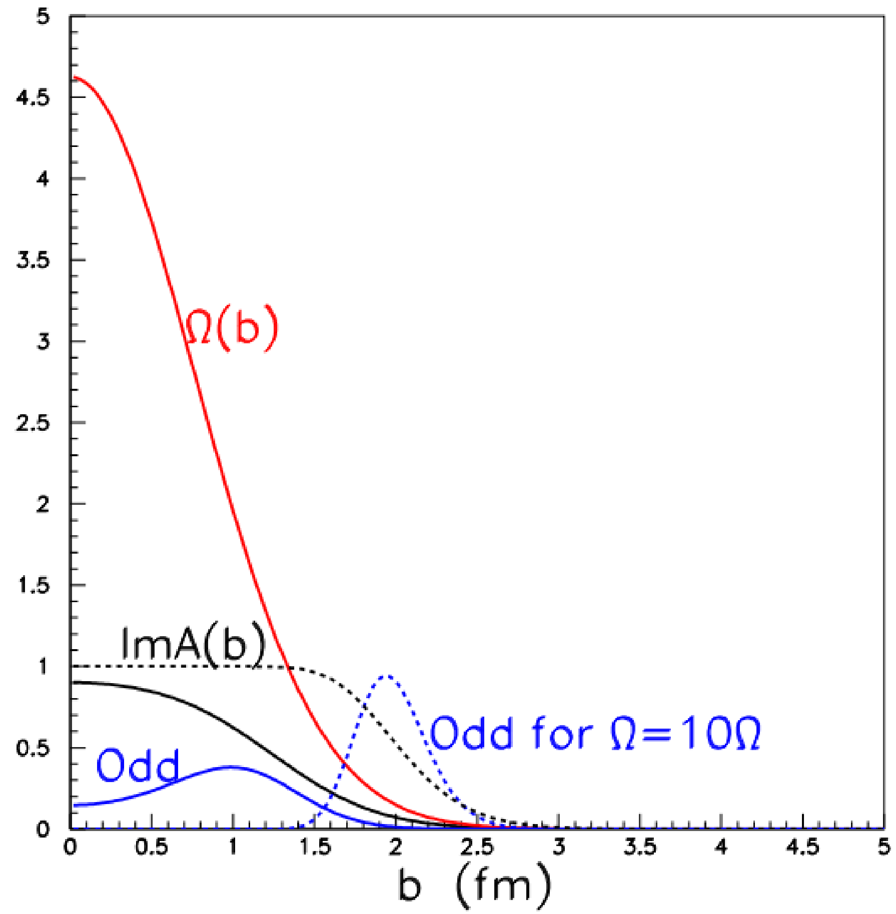
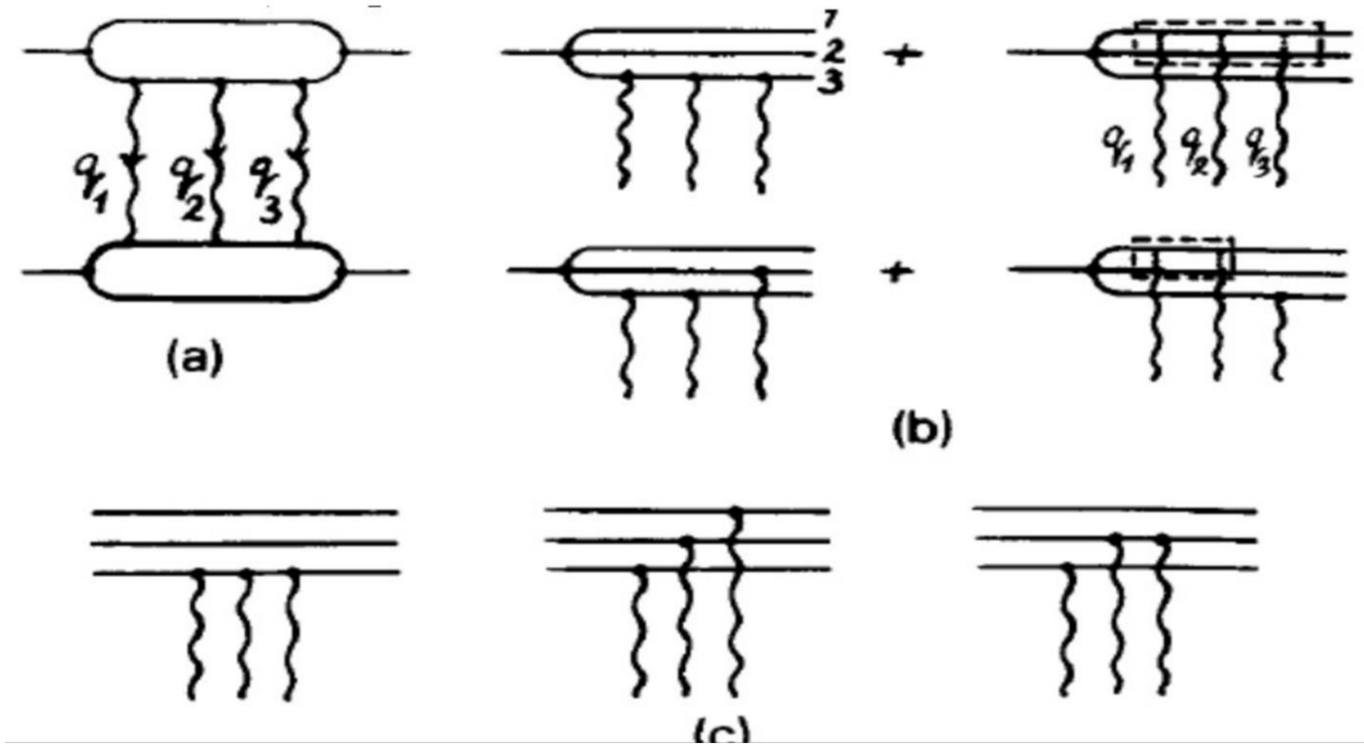


Figure 2: Impact parameter b dependence of opacity (upper red curve) and the elastic amplitude (continuous black curve). The amplitude in the case of 10 times larger opacity is shown by black

absorption is crucial at small b

10 times opacity increase → $\sqrt{s} \sim 2 \cdot 10^5$ TeV.

At the lowest α_s order (Born approx.)
Odderon = 3 gluon exchange



In terms of the opacity

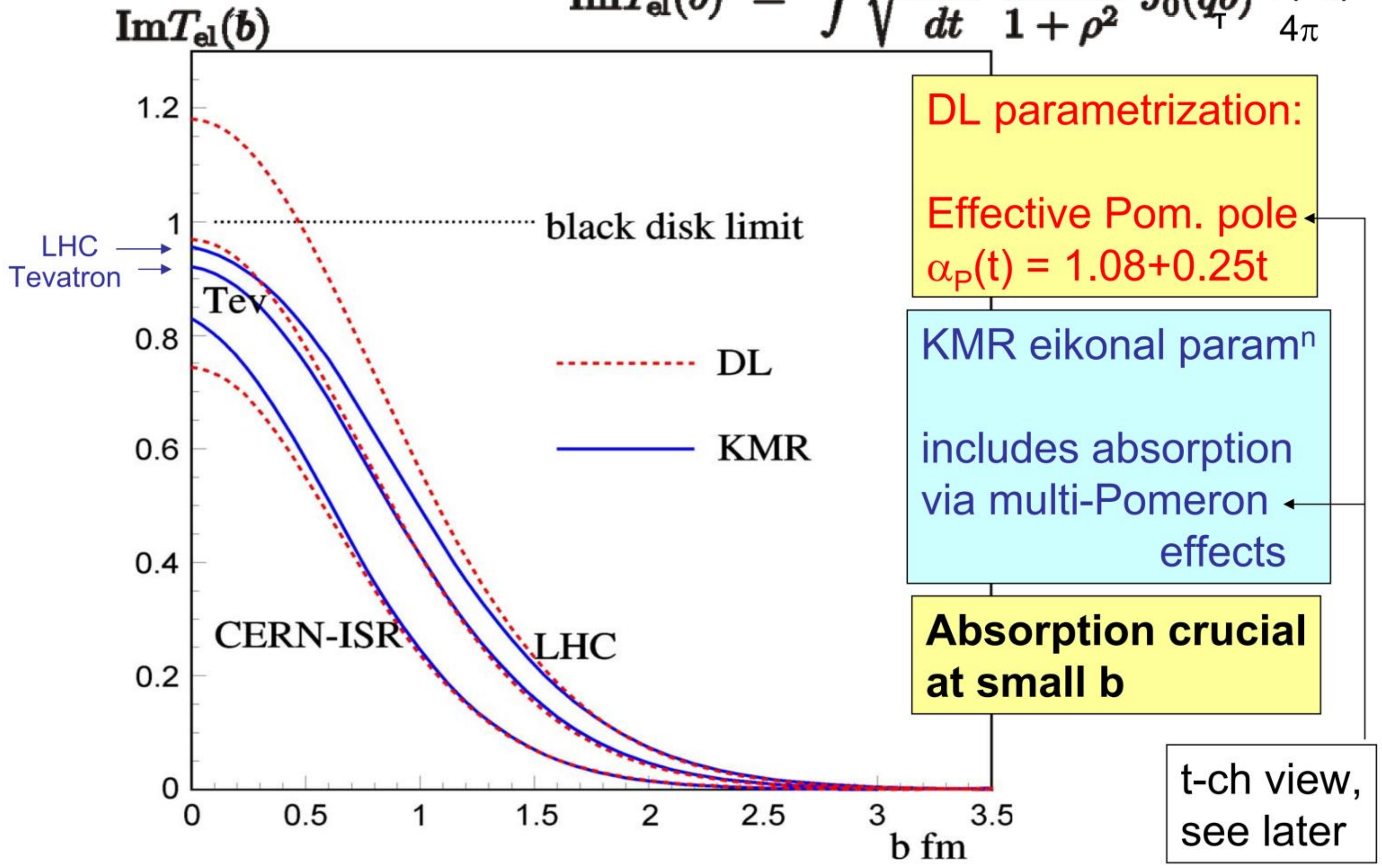
$$\frac{d\sigma_{el}}{dt} = \frac{1}{16\pi s^2}|T_{el}(s,t)|^2 = \frac{1}{4\pi}\left|\int d^2b\, e^{i\vec{q}_t\cdot\vec{b}}(1-e^{-\Omega(b)/2})\right|^2 = \pi\left|\int bdb\, J_0(q_t b)(1-e^{-\Omega(b)/2})\right|^2$$

$$q_t = \sqrt{|t|}$$

Fourier-Bessel transform from the experimental data

U. Amaldi, M. Jacob, and G. Matthiae, . *Ann.Rev.Nucl.Part.Sci.* **26** (1976) 385–456.

$$\mathrm{Im}T_{el}(b) = \int \sqrt{\frac{d\sigma_{el}}{dt}\frac{16\pi}{1+\rho^2}}\, J_0(q_T b)\,\frac{q_T dq_T}{4\pi}$$



$\rho \equiv \mathrm{Re}T_{el}/\mathrm{Im}T_{el}$

The value of ρ can be derived via the dispersion relation,

$$\frac{1}{s}\mathrm{Re}T_{el}(s) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{ds'}{s'-s}\sigma_{tot}(|s'|) = \frac{1}{\pi}\int_0^{\infty}\sigma_{tot}(s')\frac{2s ds'}{s'^2-s^2}.$$

C-even amplitude

a major contribution comes from $s' \sim s$.

$$\rho(t=0) \simeq \frac{\pi}{2}\frac{\partial \ln \sigma_{tot}(s)}{\partial \ln s}.$$

not set in stone