Monte Carlo Event Generators

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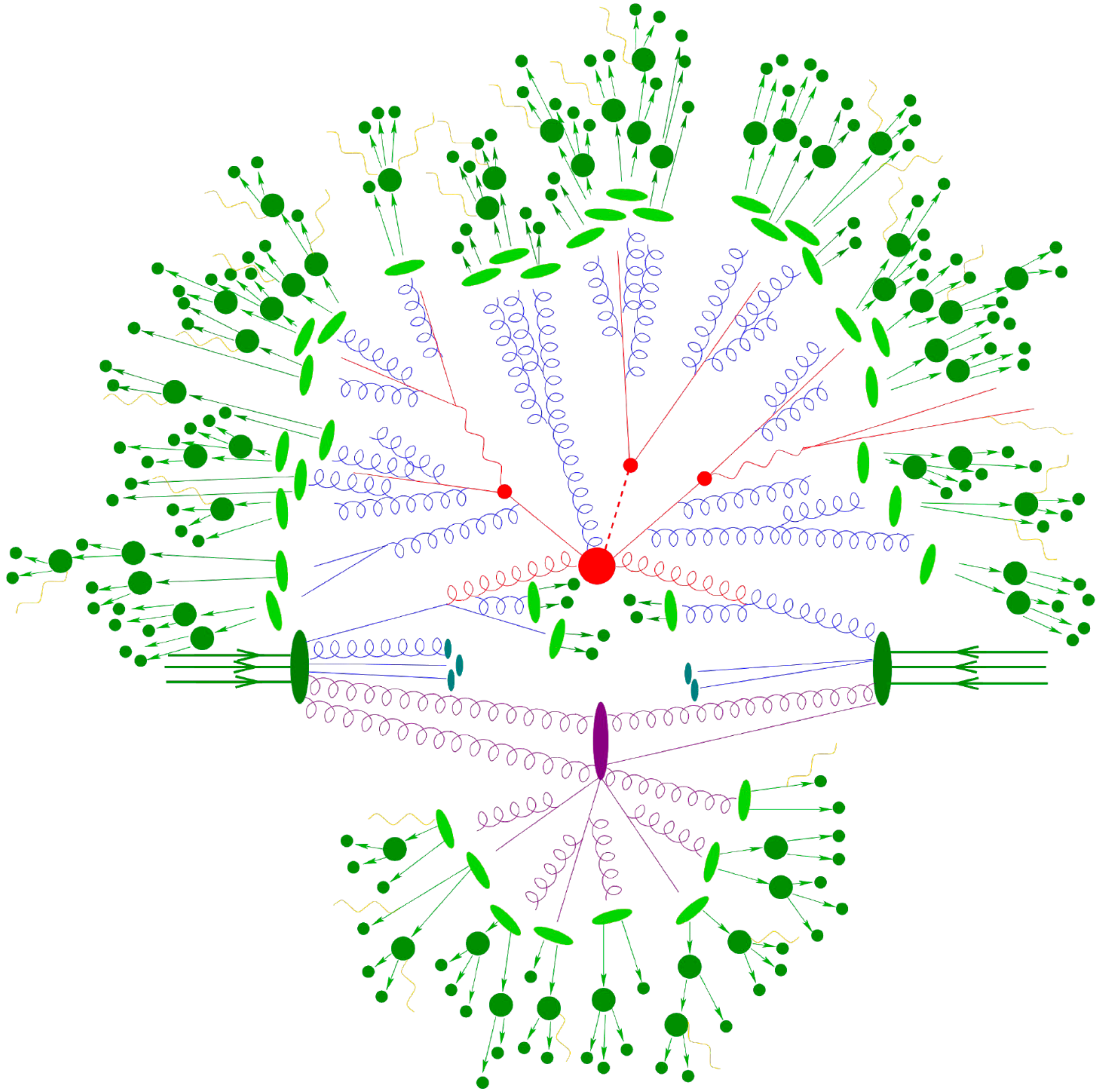
Structure of LHC Events

- Hard Interaction
- Radiative Corrections ★
- Hadronization
- Hadron Decays
- Underlying Event



Parton Showers

- Accelerated charges radiate: At the LHC we have large momentum transfer which leads to a lot of radiation
- **QED:** Electrically charge particles will emit photons, which can further split
- **QCD:** Quarks will emit gluons, gluons will split into quark pairs or into two gluons

This cascade of emissions is known as a parton shower

An analogy: Radioactive decays

- Consider radioactive decay of an unstable isotope with half-life τ
- "Survival" after time t

$$S(t) = \mathscr{P}(t) = e^{-\frac{t}{\tau}}$$

- Probability for an isotope to decay at time t

$$\frac{d\mathscr{P}_{\text{dec}}(t)}{dt} = -\frac{d\mathscr{P}_{\text{nodec}}(t)}{dt} = \frac{1}{\tau}e^{-\frac{t}{\tau}}$$

- Probability for isotope decay at any fixed time t determined by $\Gamma = \frac{1}{\tau}$

An analogy: Radioactive decays

- Add time dependence $\Gamma \to \Gamma(t)$

$$\frac{d\mathscr{P}_{\text{dec}}(t)}{dt} = \Gamma(t)\ \exp\left[-\int_0^t dt'\,\Gamma(t')\right] = \Gamma(t)\mathscr{P}_{\text{no-dec}}(t)$$

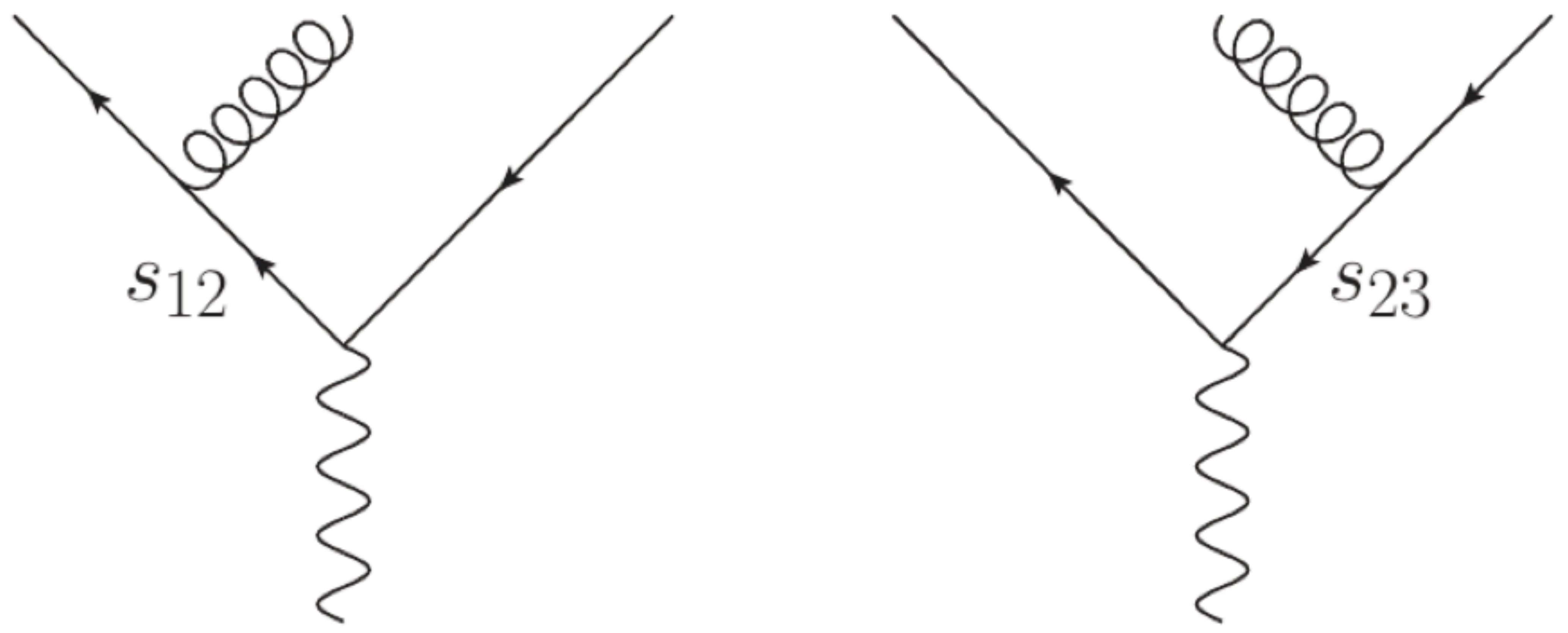
- First term is for the actual decay to happen
- The exponential suppresses a decay before t
- We will see later that this is so called **Sudakov Form Factor**

The pattern of QCD radiation

Example: $e^+e^- \to q\bar{q}g$

$$\frac{d\sigma_{ee\to 3j}}{dx_1 dx_2} = \sigma_{ee\to 2j}\frac{C_F\alpha_s}{\pi}\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Divergent in the limits $x_{1/2} \to 1$



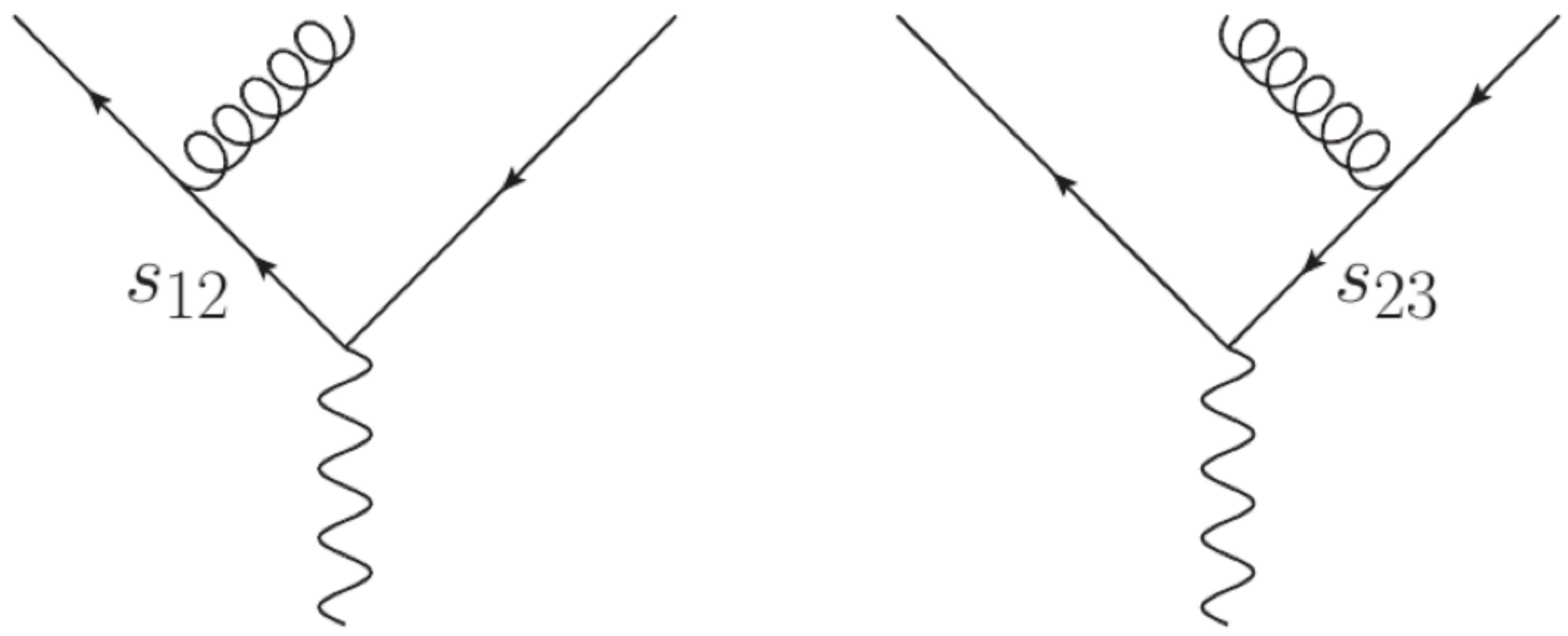
The pattern of QCD radiation

Example: $e^+e^- \to q\bar{q}g$

Rewrite in terms of opening angle and gluon fractional energy

$$\frac{d\sigma_{ee\to 3j}}{d\cos\theta_{qg}dx_3} = \sigma_{ee\to 2j}\frac{C_F\alpha_s}{\pi}\left[\frac{2}{\sin\theta_{qg}}\frac{1+(1-x_3)^2}{x_3} - x_3\right]$$

Singular for $x_3 \to 0$ (**soft)**
and $\sin\theta_{qg} \to 0$ (**Collinear)**



The pattern of QCD radiation

Example: $e^+e^- \to q\bar{q}g$

Re-express the singularities

$$\frac{2d\cos\theta}{\sin^2\theta} = \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\theta}{1+\cos\theta} \approx \frac{d\theta^2}{\theta} + \frac{d\bar{\theta}^2}{\bar{\theta}}$$

Independent evolution of two jets

$$d\sigma_3 \propto \sigma_2 \sum_{i=q,\bar{q}} C_F \frac{\alpha_s}{2\pi} \frac{d\theta_i^2}{\theta_i^2} P(z)$$

The pattern of QCD radiation

Any variable with the same limiting behaviour leads to the same equation

- Transverse momentum $k_\perp^2 = z^2(1-z)^2\theta^2E^2$
- Virtuality $q^2 = z(1-z)\theta^2E^2$

$$\frac{d\theta^2}{\theta^2} = \frac{dk_\perp^2}{k_\perp^2} = \frac{dq^2}{q^2} = \frac{dt}{t}$$

Universal Collinear Approximation

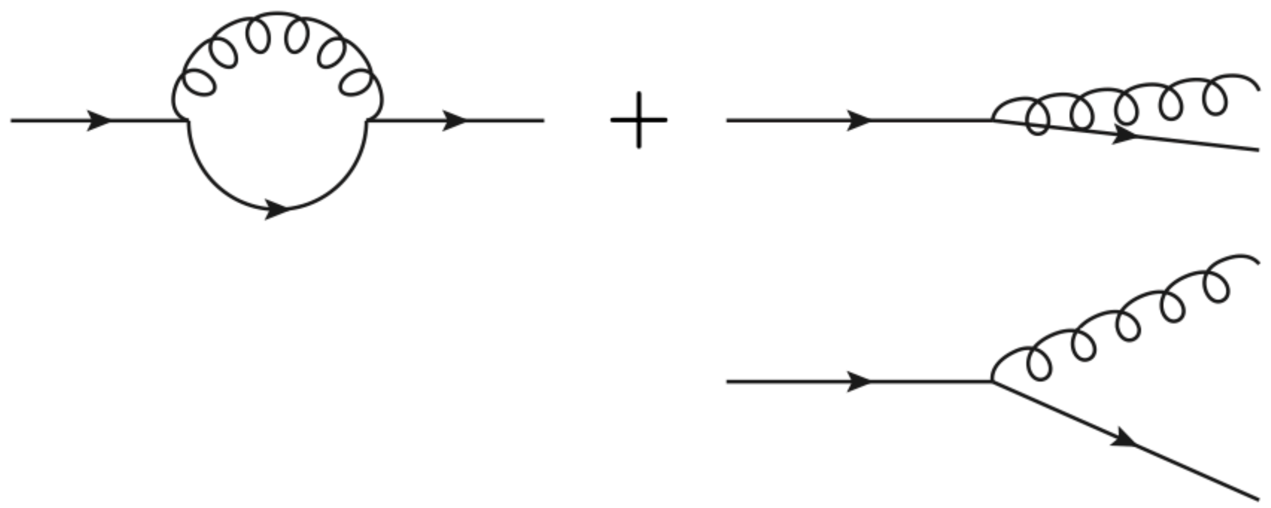
$P_{ab}(z)$ Splitting function

(See QCD lectures)

$$d\sigma_{n+1} \propto \sigma_n \sum_i \frac{\alpha_s}{2\pi}\frac{dt}{t}P_{ab}(z)$$

Parton Shower: Construction

- Collinear partons are not separately resolvable
- Introduce a "resolution criterion" which also acts an IR regulator
- **KLN** theorem again: Combine the virtual with unresolved to get IR finite result
- **Unitarity:**
  $\mathscr{P}(\text{resolved}) + \mathscr{P}(\text{unresolved}) = 1$



$$d\sigma_{n+1} \propto \sigma_n \sum_i \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{ab}(z)$$

Sudakov Form Factor

- Differential probability for emission between q^2 and $q^2 + dq^2$

$$d\mathscr{P}_{em} = \frac{dt}{t} \int dz \frac{\alpha_s}{2\pi} P_{ab}(z)$$

- Emissions can modelled as Poisson distribution

$$\mathscr{P}_{\text{no-em}}(t, t') = \exp\left\{ -\int_t^{t'} \frac{d\bar{t}}{\bar{t}} \int dz \frac{\alpha_s}{2\pi} P_{ab}(z) \right\}$$

- **Sudakov Form Factor:** $\Delta(t, t') := \mathscr{P}_{\text{no-em}}(t, t')$

Compact Notation

- **Sudakov Form Factor**

$$\Delta^K_{ij,k}(t,t_0) = \exp\left[-\int_{t_0}^{t} \frac{dt}{t}\,\frac{\alpha_s}{2\pi}\int dz\,\frac{d\phi}{2\pi}\,K_{ij,k}(t,z,\phi)\right]$$

- $K_{ij,k}(t,z,\phi)$ is splitting kernel for $(ij) \to i,j$ with spectator k

- Evolution parameter t is defined by kinematics e.g angle or $p_\perp$

- Simplify $dt\,dz\,\dfrac{d\phi}{2\pi} \to d\Phi_1$

First Emission

- **Sudakov Form Factor** for emission of N partons

$$K_n(\Phi_1) = \frac{\alpha_s}{2\pi} \sum_{\{ij,k\}} K_{ij,k}(\Phi_{ij,k}) \qquad \Delta_n^K(t, t_0) = \exp\left[-\int_{t_0}^{t} d\Phi_1 K_n(\Phi_1)\right]$$

- Consider the first emission off the Born configuration

$$d\sigma_B = d\Phi_n\, B_n(\Phi_n)\left[\Delta_n^K(\mu^2, t_0) + \int_{t_0}^{\mu^2} d\Phi_1\, K_n(\Phi_1)\Delta_n^K(\mu^2, t(\Phi_1))\right]$$

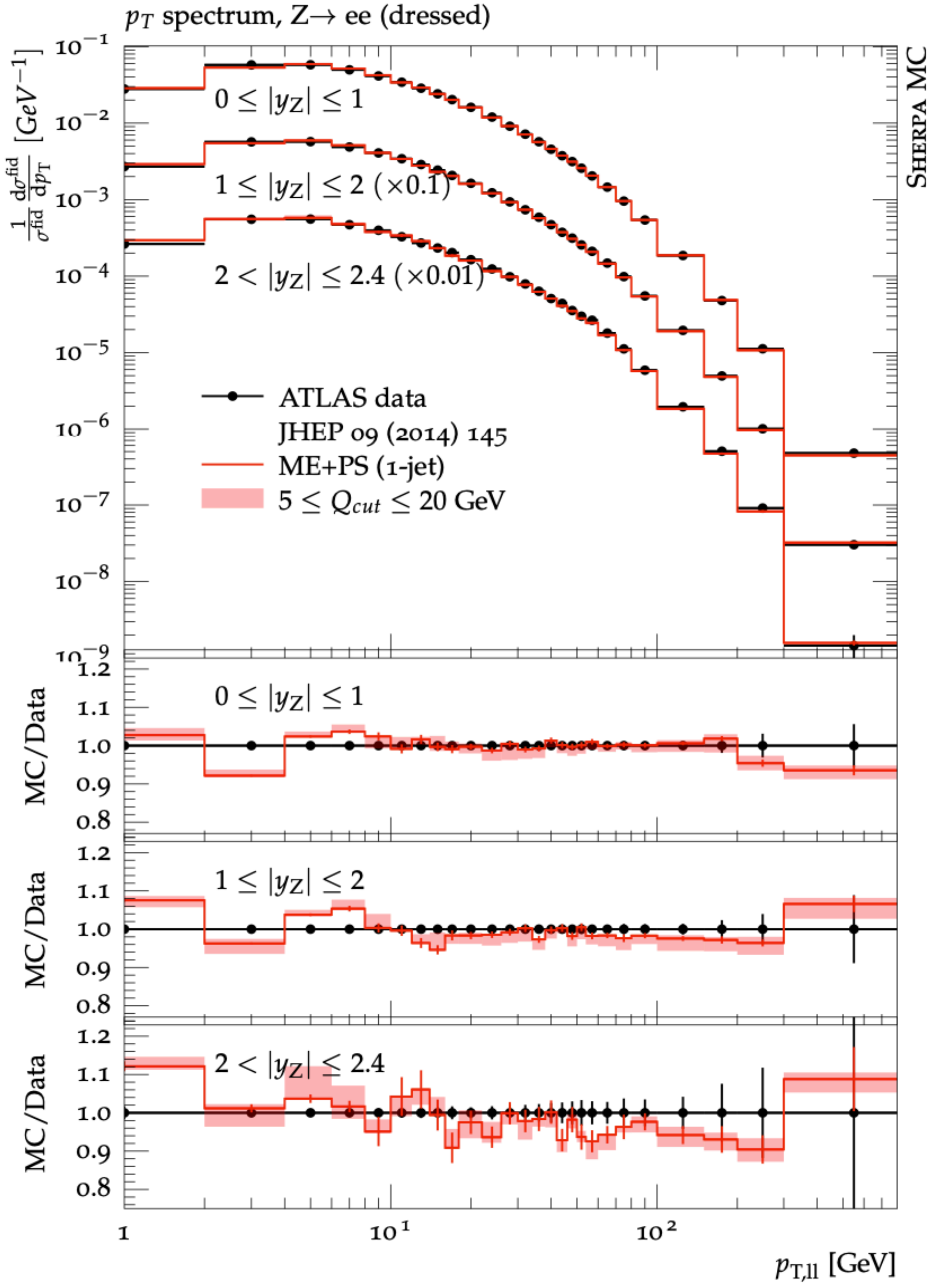
- Term in square brackets integrates to one: **Unitarity of the parton shower**

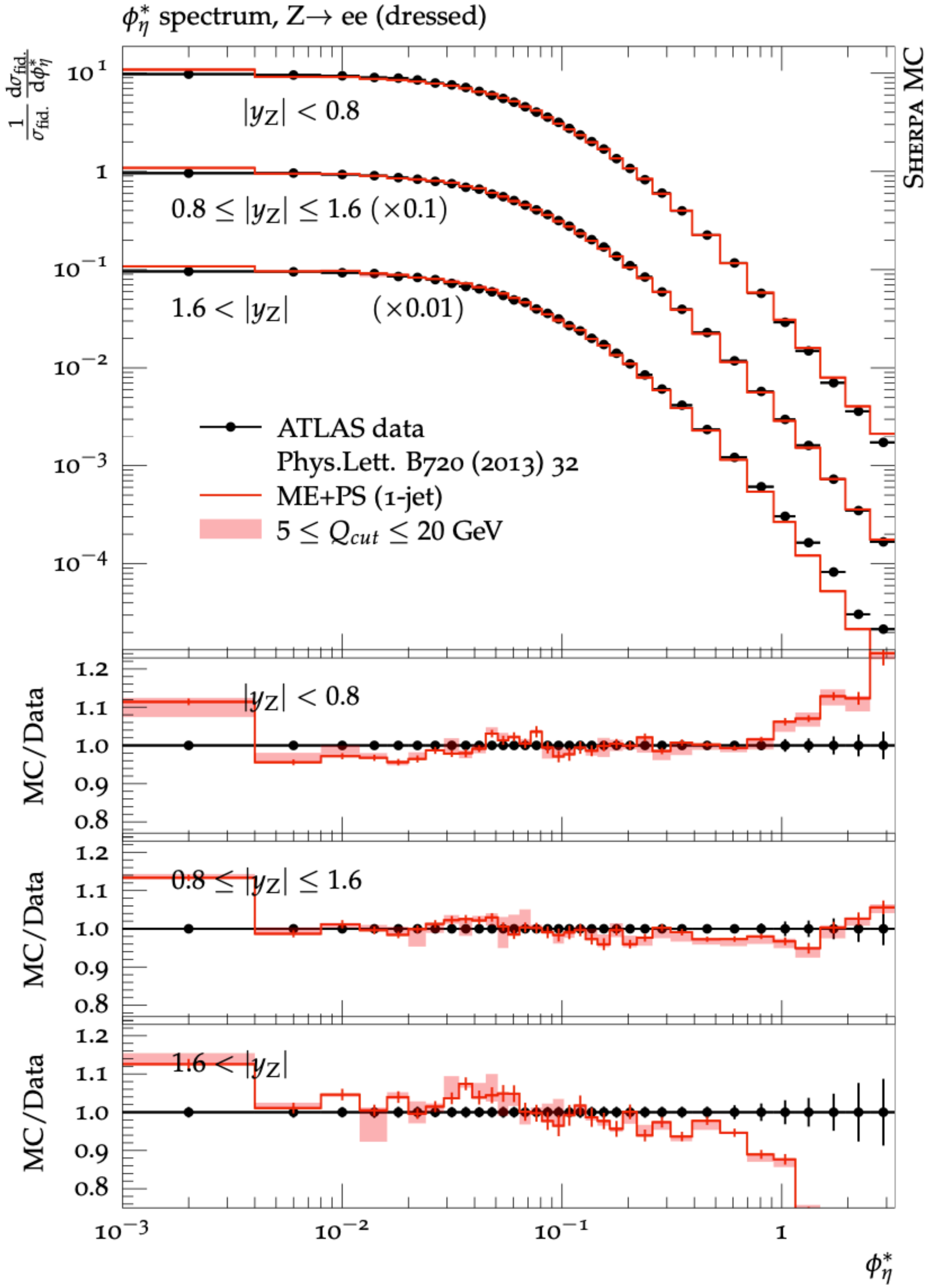
Further Emission

$$d\sigma_B = d\Phi_n\, B_n(\Phi_n) \left[\Delta_n^K(\mu^2, t_0) + \int_{t_0}^{\mu^2} d\Phi_1\, K_n(\Phi_1) \Delta_n^K(\mu^2, t(\Phi_1)) \right]$$

- Use a veto algorithm to generate a new scale t
- $t < \mu^2$: **Unresolved Region, no explicit emission**
- $t > \mu^2$**: Generate resolved emission at** t
- Construct the $n \rightarrow n+1$ kinematics in collinear approximation
- Continue until $t < \mu^2$

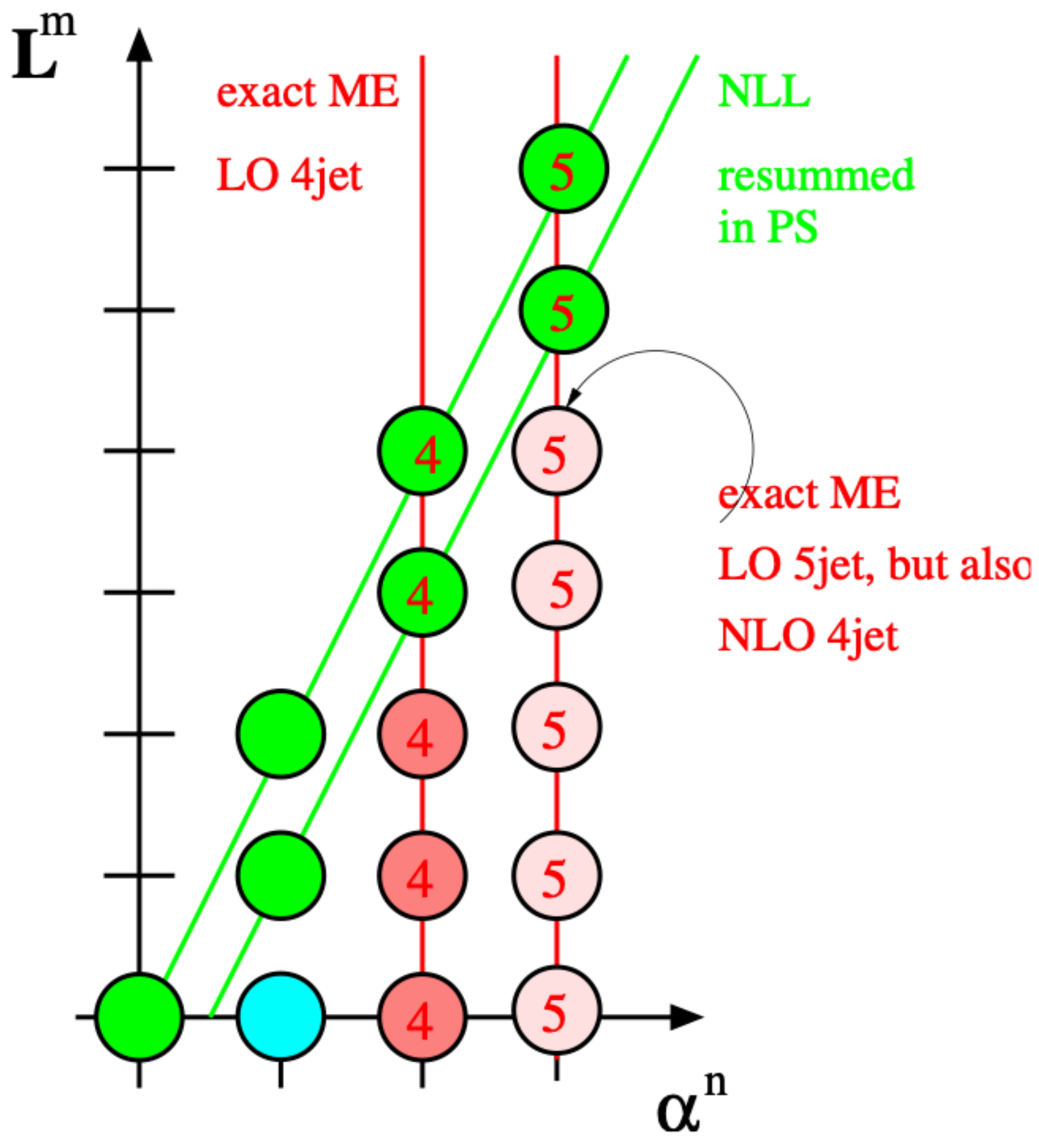
Example: ME+PS for Drell-Yan





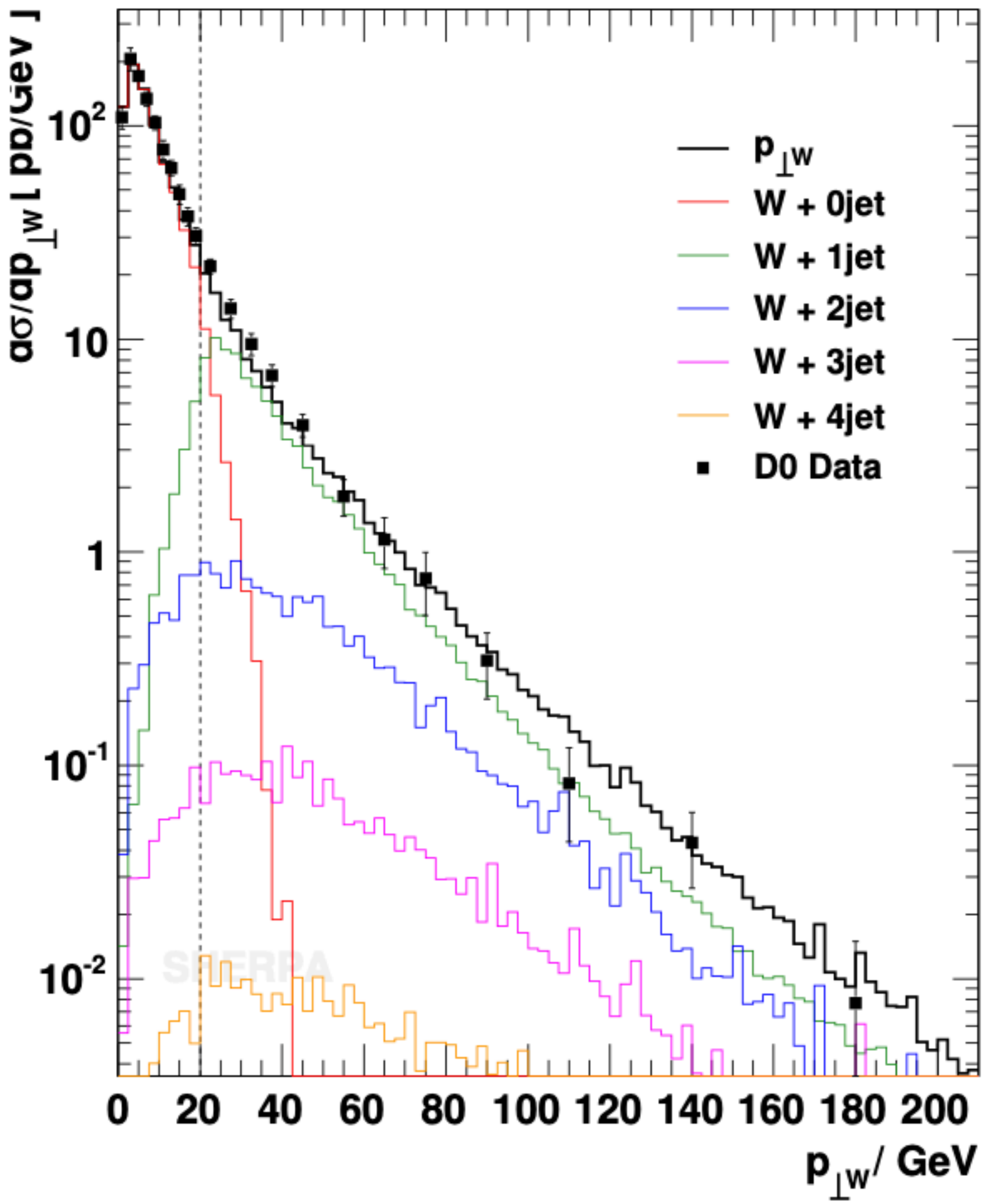
Multijet Merging

- Parton shower resums logarithms and is a good description of collinear/ soft emissions jet evolution (Large Logs)
- Matrix elements at given order is fair description of hard/large-angle emissions jet production (small logs)
- “Merge” both approaches



Multijet Merging

- Separate regions of jet production and jet evolution with jet measure Q_j
- Populate hard region with Matrix elements
- Parton Shower will dominate the soft



Multijet Merging: First Emission

$$d\sigma_B = d\Phi_n\, B_n(\Phi_n)\left[\Delta_n^K(\mu^2, t_0) + \int_{t_0}^{\mu_n^2} d\Phi_1\, K_n(\Phi_1)\Delta_n^K(\mu_n^2, t_{n+1})\Theta(Q_J - Q_{n+1})\right]$$

$$+d\Phi_{n+1}B_{n+1}\Delta_n^K(\mu_{n+1}^2, t_{n+1})\Theta(Q_{n+1} - Q_J)$$

- Can be iterated to $n+2, n+3\ldots$
- Potential different starting scales
- **Unitarity** violation: Solved in UMEPS formalism

L. Lonnblad & S. Prestel, JHEP 1302 (2013) 09

Di-photons @ ATLAS

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Multijet-Merging at NLO

- Identical idea to LO: towers of MEs with increasing jet multi (but this time at NLO)
- Combine them all into one sample and consistently remove double-counting

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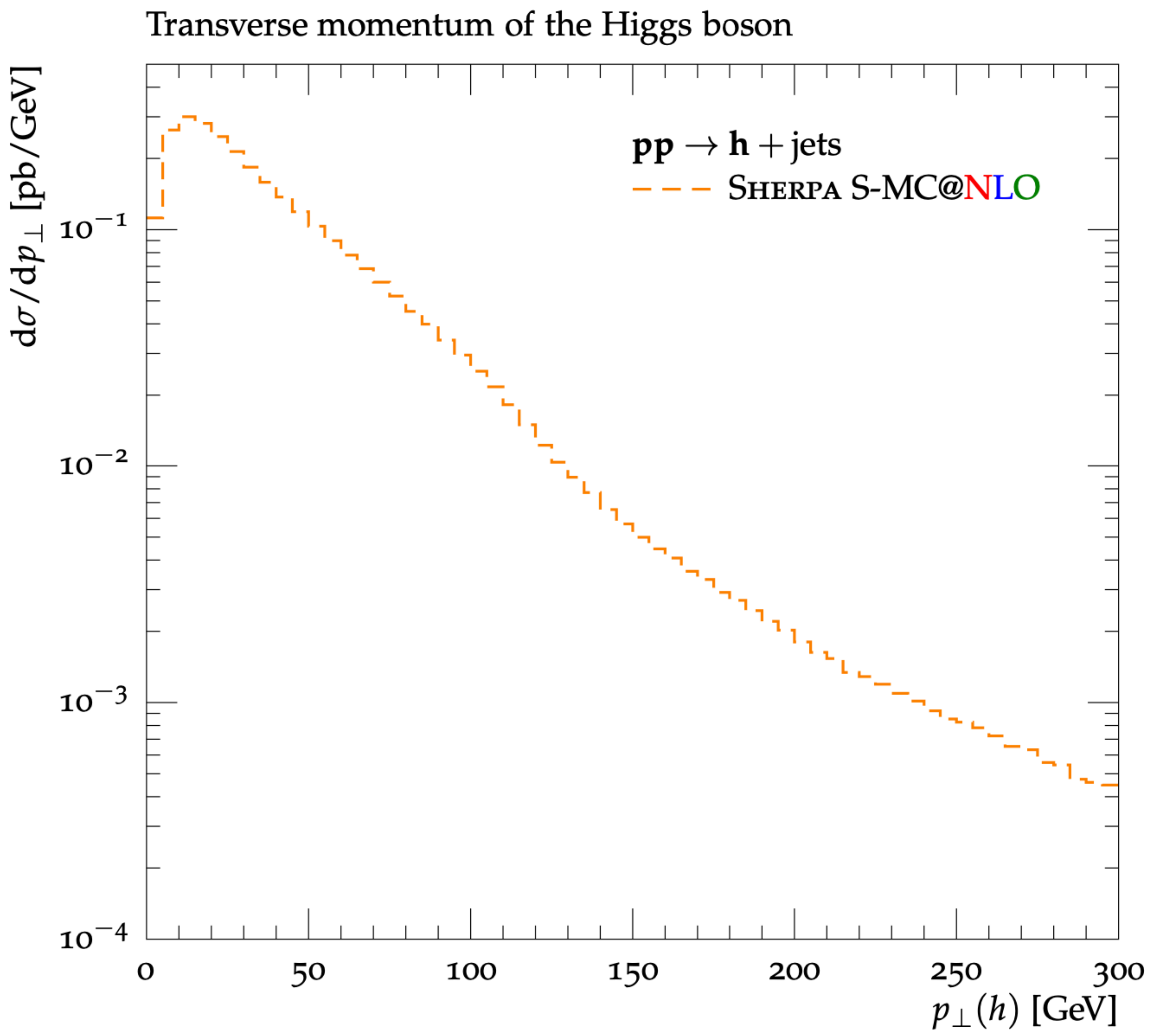

- First emission generated MC@NLO

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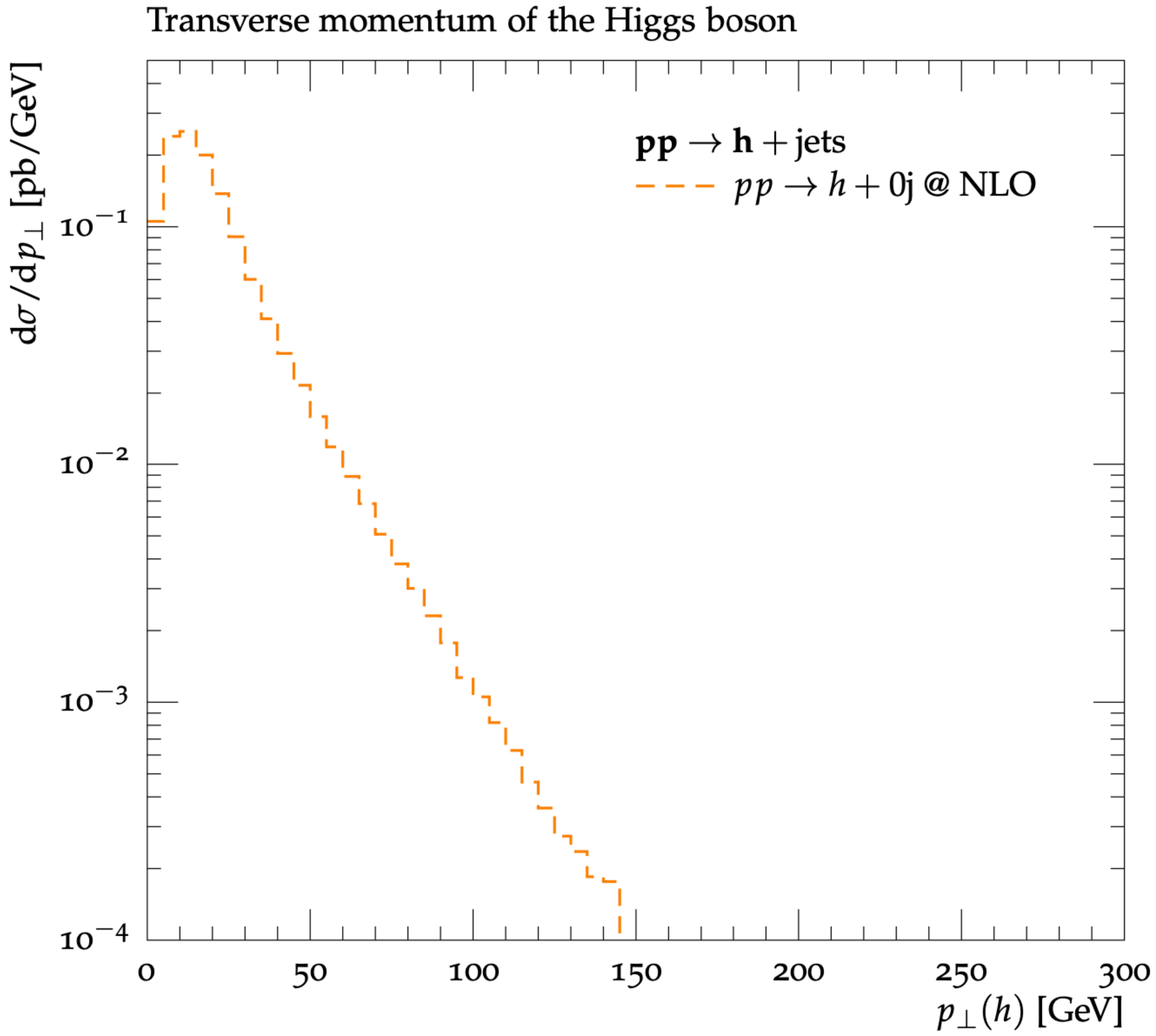

- First emission generated MC@NLO, restrict to $Q_{n+1} < Q_{cut}$

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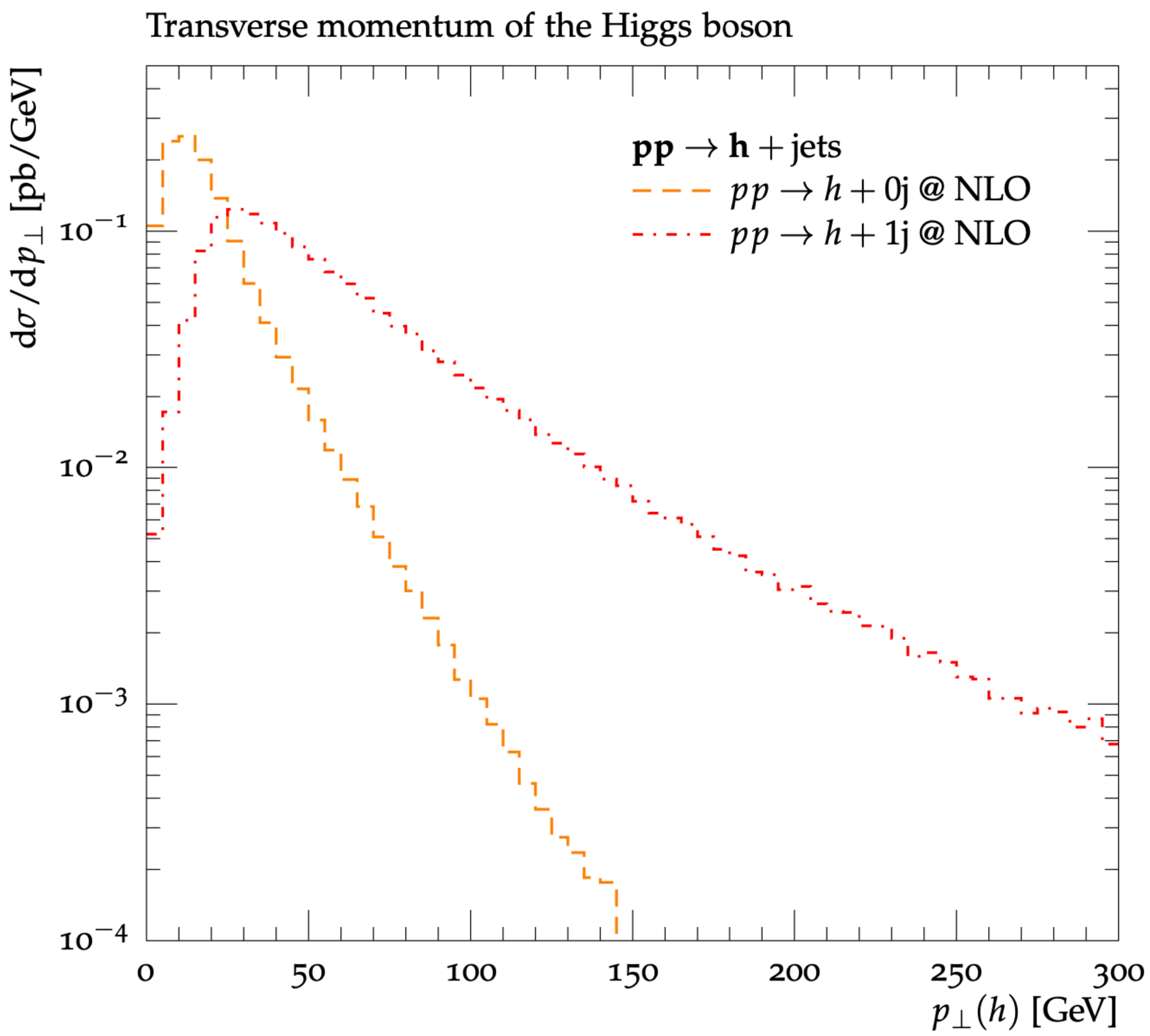

- First emission generated MC@NLO, restrict to $Q_{n+1} < Q_{cut}$
- Generate $pp \to h + j$ for $Q_{n+1} > Q_{cut}$

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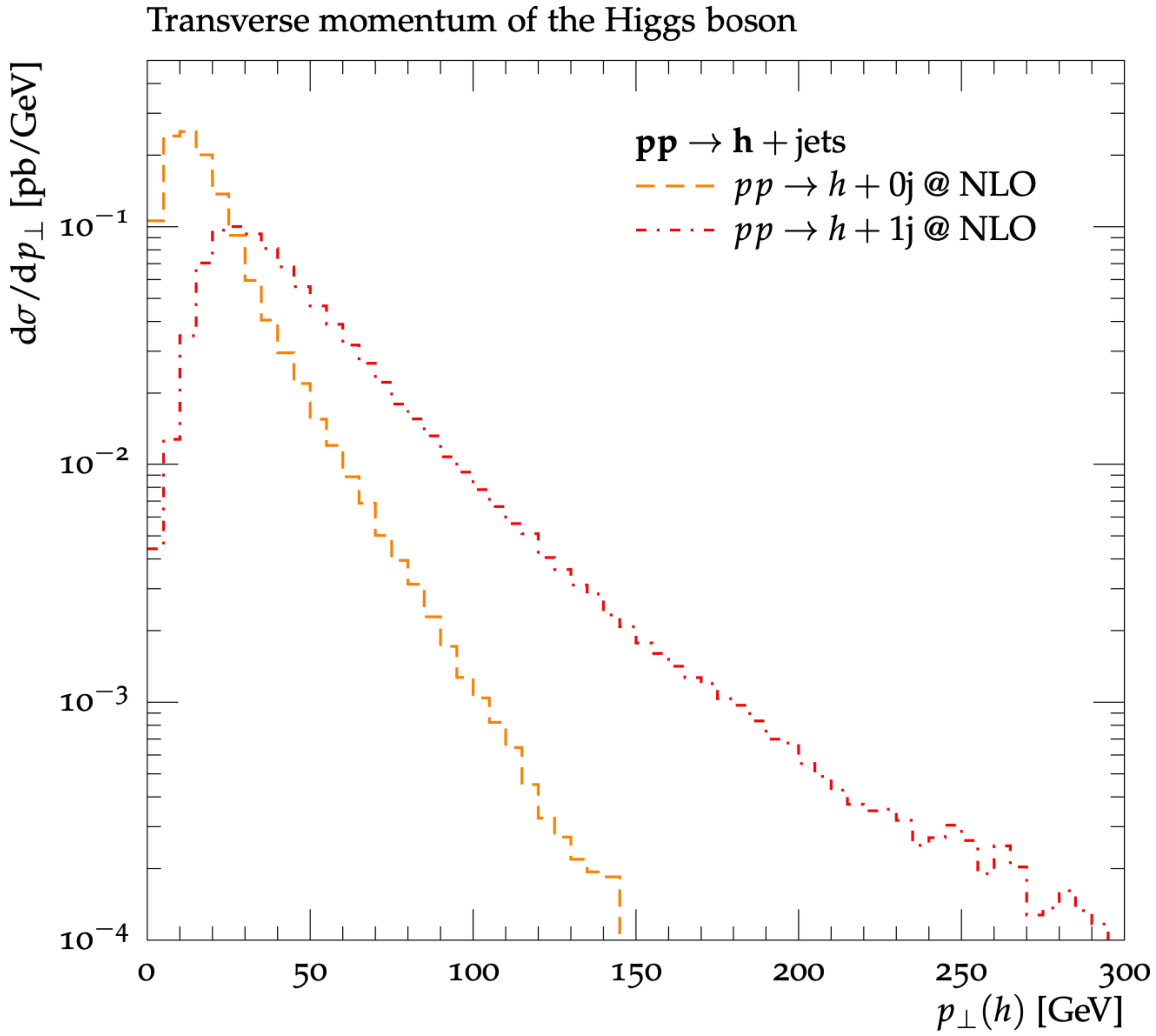

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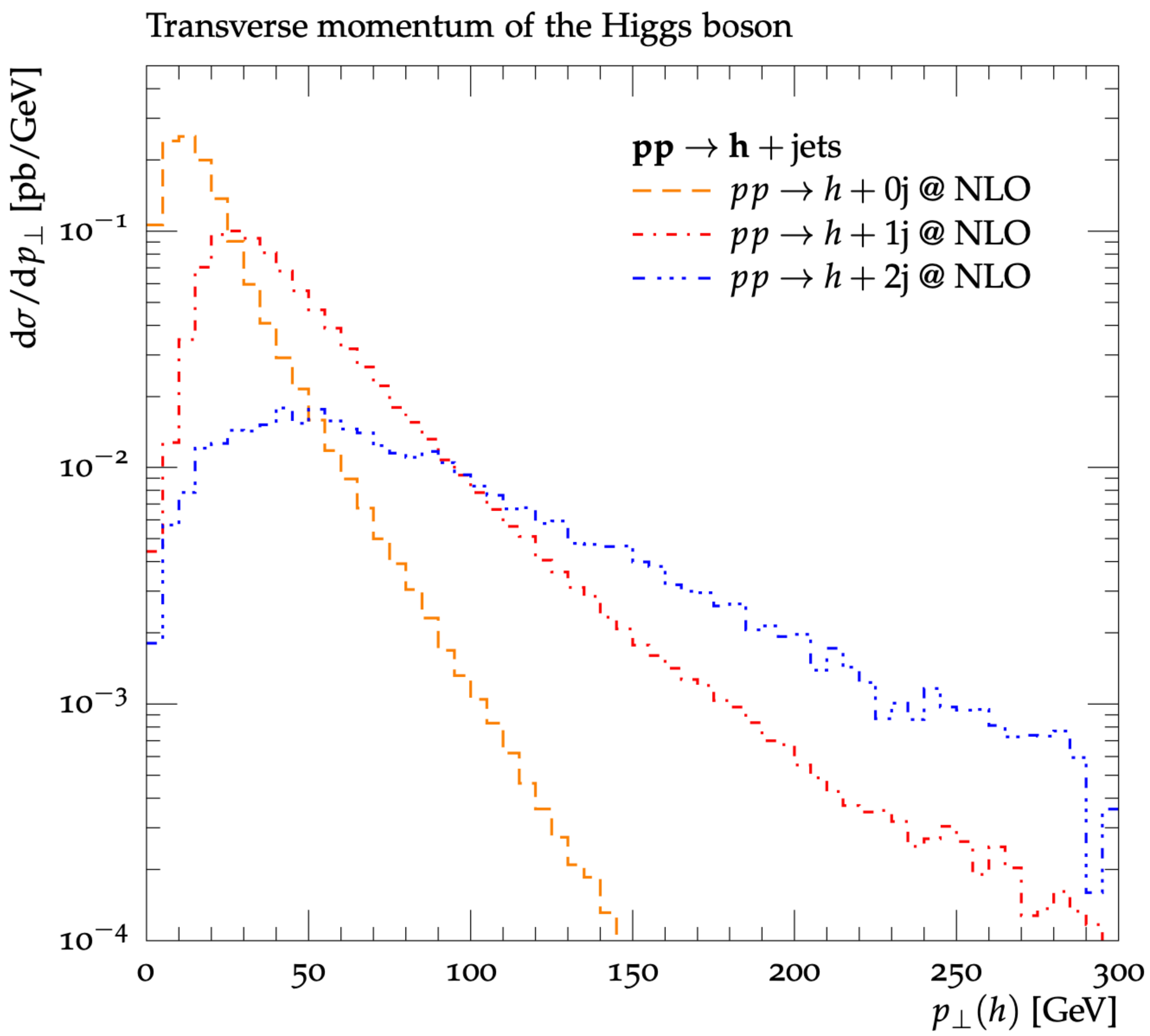

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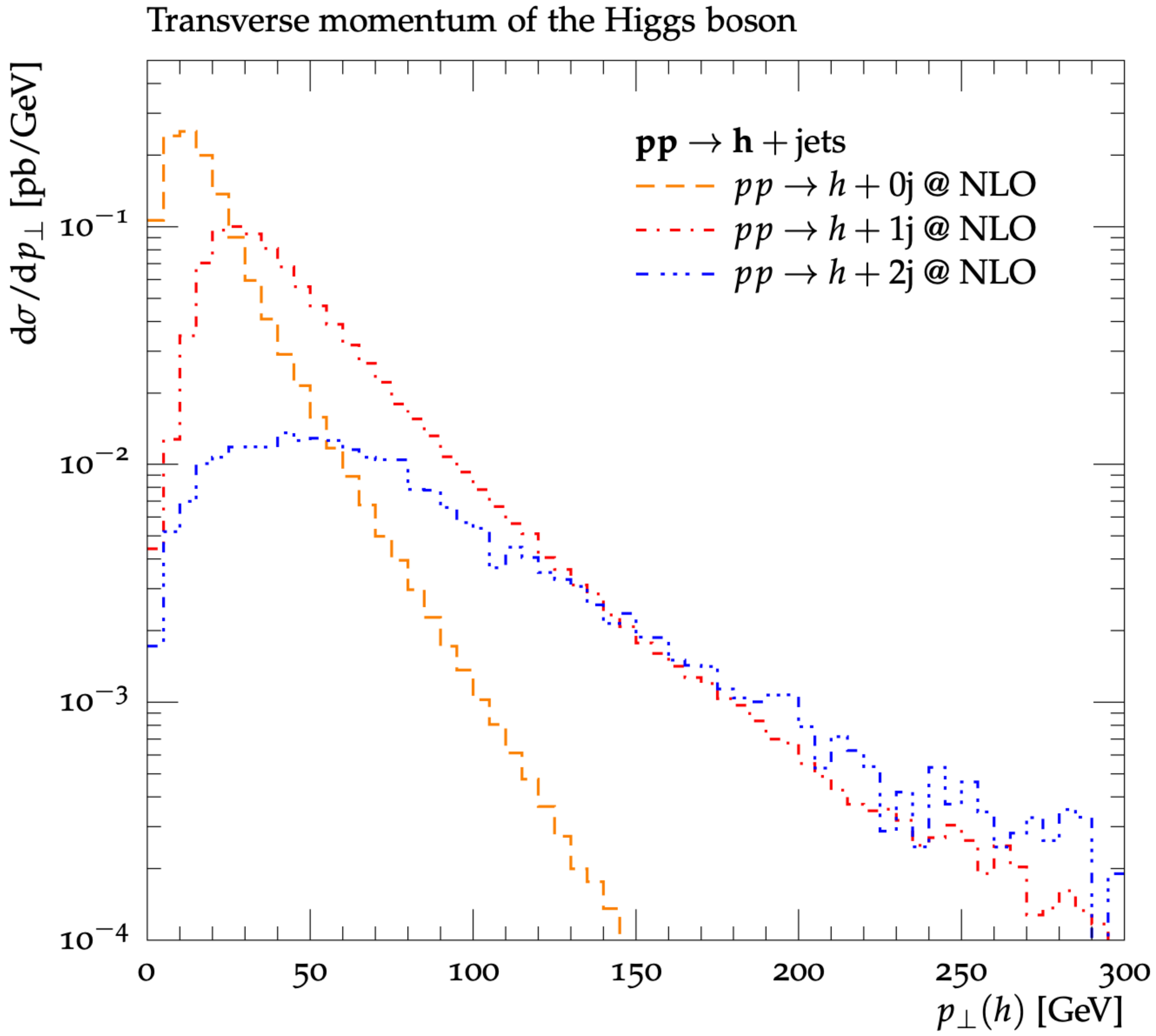

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- Iterate

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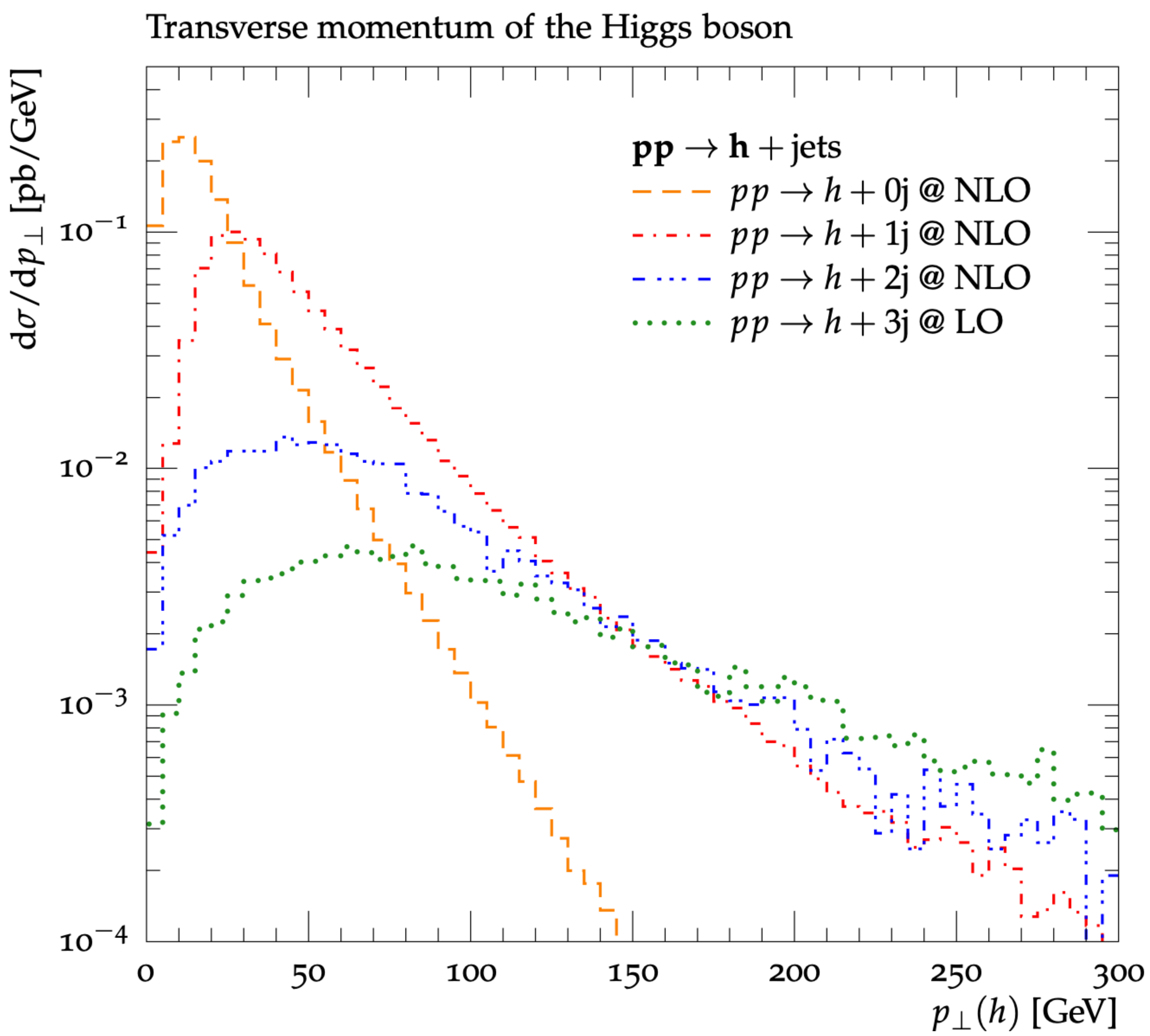

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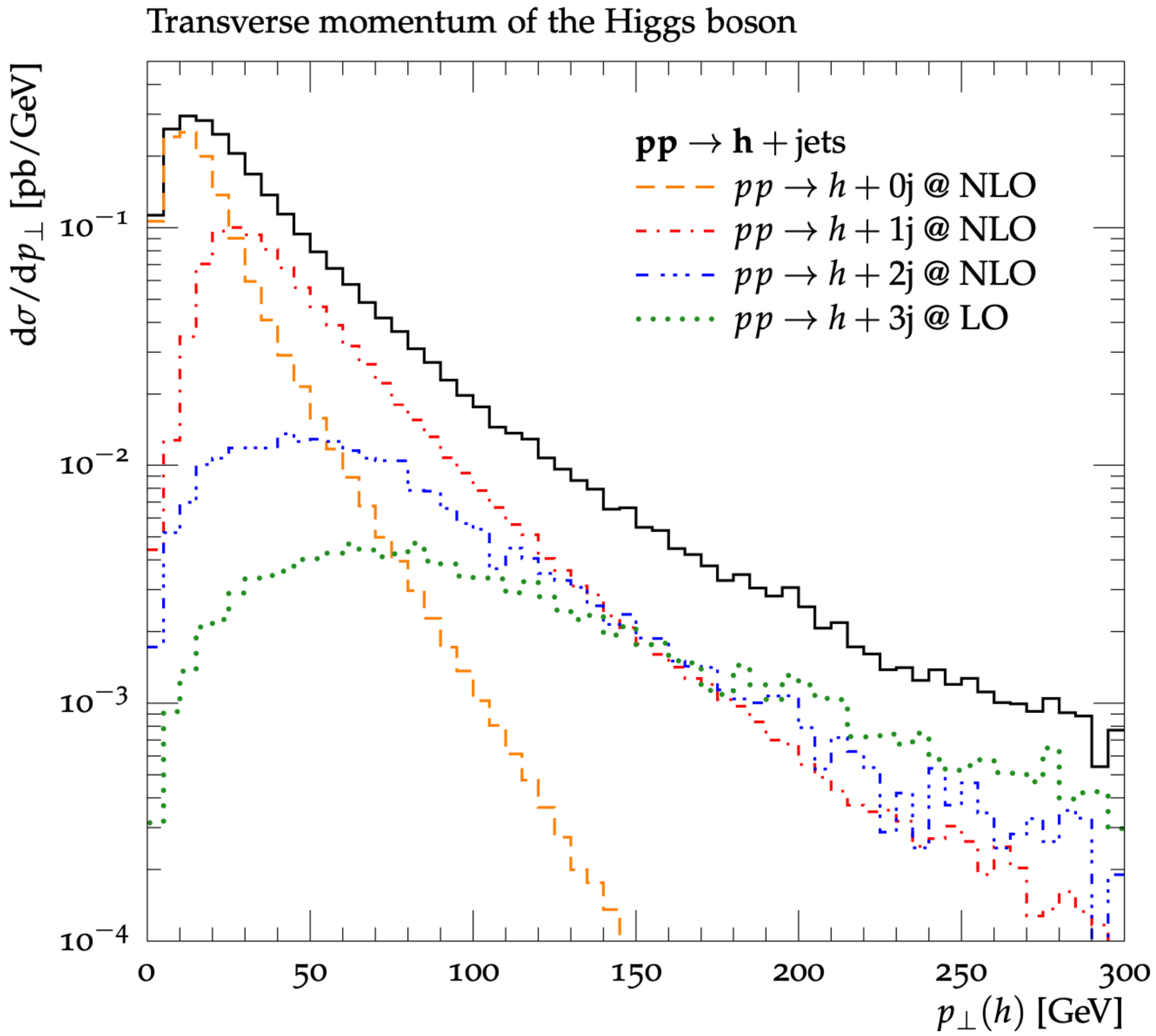

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- Generate $pp \to h + 2j$ for $Q_{n+2} > Q_{cut}$
- Iterate
- Sum all contributions

Questions?

Sherpa Tutorial

https://gitlab.com/aprice/midsummer-school-2024