

# Monte Carlo Event Generators

**Alan Price**

**Midsummer school in QCD 2024**

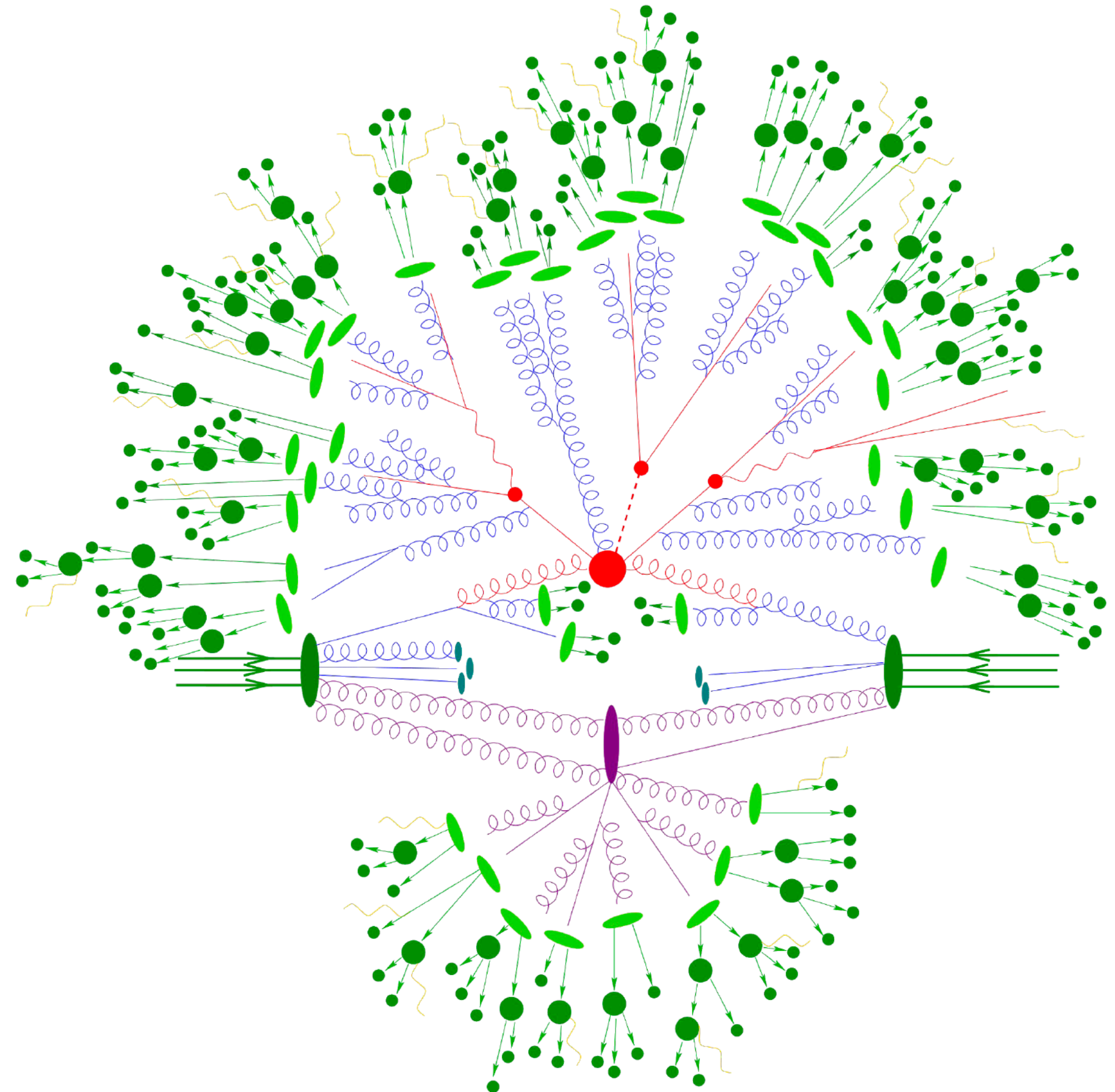
**Saariselkä**



JAGIELLONIAN UNIVERSITY  
IN KRAKÓW

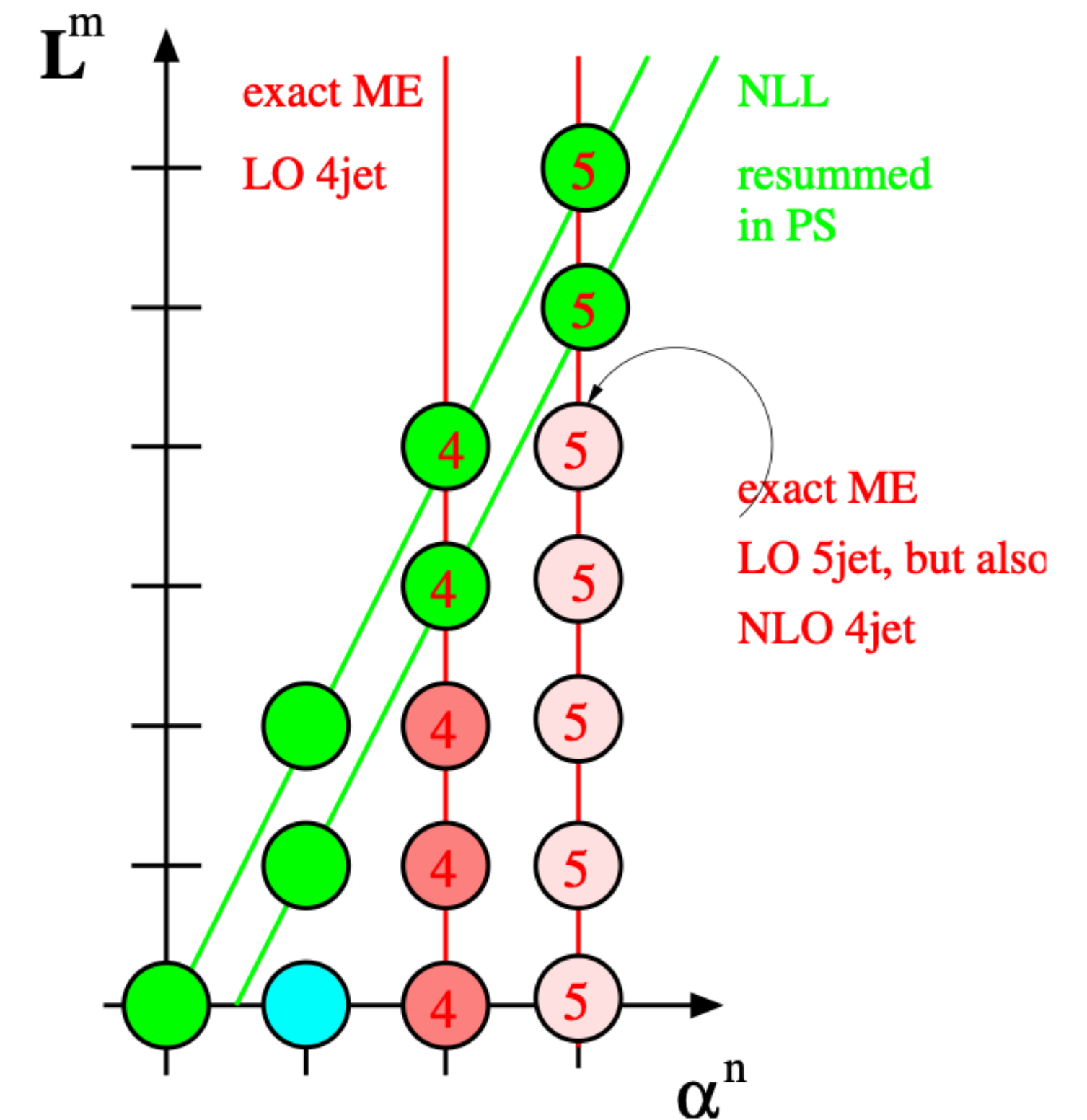
# Structure of LHC Events

- ❖ **Hard Interaction**
- ❖ **Radiative Corrections**
- ❖ **Hadronization**
- ❖ **Hadron Decays**
- ❖ **Underlying Event**



# Multijet Merging

- ❖ Parton shower resums logarithms and is a good description of collinear/soft emissions jet evolution (Large Logs)
- ❖ Matrix elements at given order is fair description of hard/large-angle emissions jet production (small logs)
- ❖ “Merge” both approaches







# Matching at LO

Matching at LO is essentially trivial

$$\langle O \rangle^{LO} = \int d\Phi_n B(\Phi_n) O(\Phi_n)$$



$$\langle O \rangle^{LOPS} = \int d\Phi_n B(\Phi_n) PS(t,0)$$

Trivial if you know how to calculate both LO ME and PS

# Matching at NLO

Matching at NLO?

$$\begin{aligned} \langle O \rangle^{NLOPS} = & \int d\Phi_n [B(\Phi_n) + V(\Phi_n) + I(\Phi_n)] PS(\Phi_n) \\ & + \int d\Phi_{n+1} [R(\Phi_{n+1})PS(\Phi_{n+1}) - S(\Phi_n \otimes \Phi_1)PS(\Phi_n)] \end{aligned}$$

First bracket seems fine but we have a problem in the second

$R$  and  $S$  have different PS contributions: IR subtraction will be spoilt

# Reminder

**NLO cross section:**

$$d\sigma_n^{NLO} = d\Phi_n \left[ B(\Phi_n) + V(\Phi_n) + I(\Phi_n) + \int d\Phi_{n+1} (R(\Phi_{n+1}) - S(\Phi_{n+1})) \right]$$
$$= d\Phi_n \bar{B}(\Phi_n)$$

**Splitting Kernels:**

$$d\Phi_1 \frac{R(\Phi_{n+1})}{B(\Phi_n)} \xrightarrow{\text{IR Limit}} d\Phi_1 \frac{\alpha_s}{2\pi} K_{ij,k}(\Phi_{ij,k})$$

$K_{ij,k}$  reproduces the process-independent behaviour of  $\frac{R(\Phi_{n+1})}{B(\Phi_n)}$  in the soft/collinear regions of phasespace

$$d\Phi_1 \frac{R(\Phi_{n+1})}{B(\Phi_n)} \longrightarrow d\Phi_1 \frac{\alpha_s}{2\pi} K_{ij,k}(\Phi_{ij,k})$$

**Define a modified Sudakov Form Factor**

$$\Delta_N^K(\mu_N^2, t_0) = \exp \left[ - \int_{t_0}^{\mu_N^2} d\Phi_1 K_{ij,k}(t, z, \phi) \right] \longrightarrow \Delta_N^{R/B}(\mu_N^2, t_0) = \exp \left[ - \int_{t_0}^{\mu_N^2} d\Phi_1 \frac{R(\Phi_{n+1})}{B(\Phi_n)} \right]$$

- ❖ Assumes factorization of phasespace  $\Phi_{n+1} = \Phi_n \otimes \Phi_1$
- ❖  $\alpha_s$  will typically vary with the shower scale



# POWHEG

## Generate Emissions with Sudakov Form Factor

$$d\sigma^{NLO} = d\Phi_n \bar{B}(\Phi_n) \left[ \Delta_n^{R/B}(\mu^2, t_0) + \int_{t_0}^{\mu_n^2} d\Phi_1 \frac{R(\Phi_{n+1})}{B(\Phi_n)} \Delta_n^{R/B}(\mu_n^2, t(\Phi_1)) \right]$$

- ❖ Square bracket again integrates to 1: **Unitarity of PS**
- ❖ Radiation pattern now follows the real ME correction

- ❖ Basic idea: divide  $R$  into **soft(S)** and **hard (H)** part

$$R = R^S + R^H = B \otimes dS_1 + H_n$$

- ❖ We can identify the subtraction term  $dS_1$  with the kernels  $dS_1 = \sum K_{ij,k}$

$$d\sigma^{NLO} = d\Phi_n \tilde{B}(\Phi_n) \left[ \Delta_n^{R/B}(\mu^2, t_0) + \int_{t_0}^{\mu_n^2} d\Phi_1 \frac{R(\Phi_{n+1})}{B(\Phi_n)} \Delta_n^{R/B}(\mu_n^2, t(\Phi_1)) \right] + d\Phi_{n+1} H(\Phi_{n+1})$$

$$\tilde{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + I(\Phi_n)$$

# MC@NLO

$$d\sigma^{NLO} = d\Phi_n \tilde{B}(\Phi_n) \left[ \Delta_n^{R/B}(\mu^2, t_0) + \int_{t_0}^{\mu_n^2} d\Phi_1 \frac{R(\Phi_{n+1})}{B(\Phi_n)} \Delta_n^{R/B}(\mu_n^2, t(\Phi_1)) \right] + d\Phi_{n+1} H(\Phi_{n+1})$$

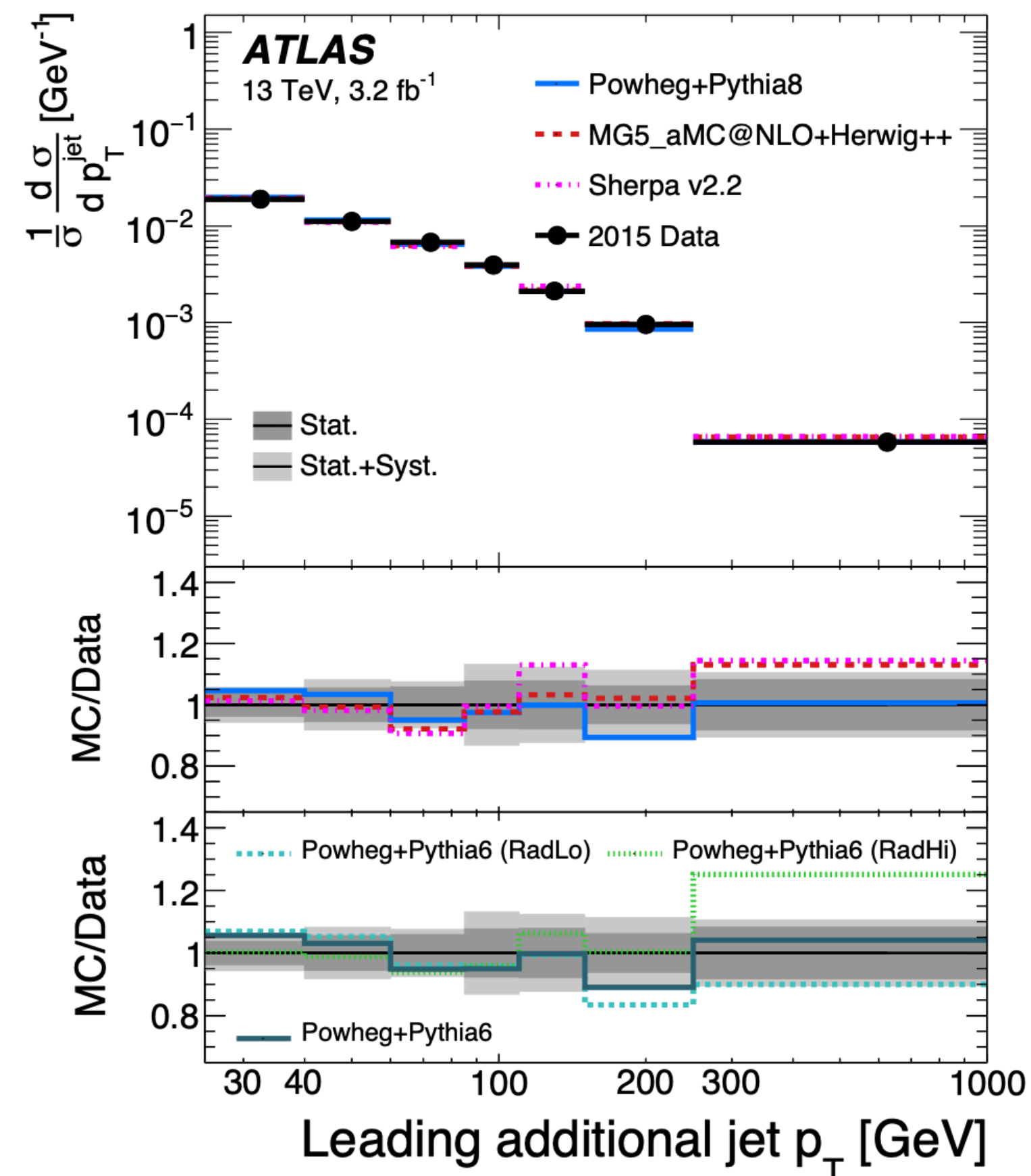
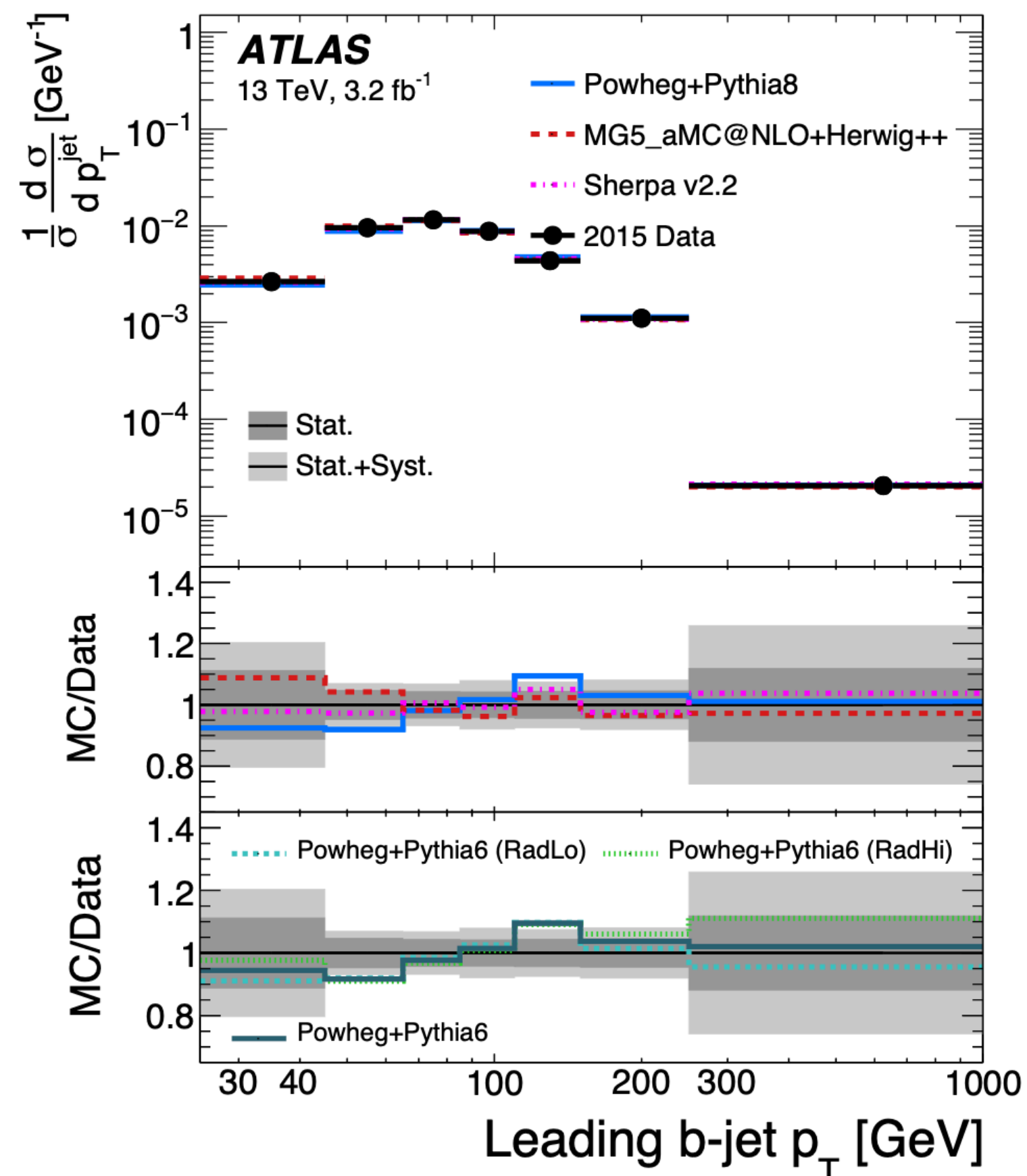
$$\tilde{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + I(\Phi_n)$$

- ❖ Essentially MC@NLO decomposes the real emission into a part driven by the shower and a hard remainder
- ❖ Virtual corrections are only applied to emissions generated by the shower
- ❖  $H(\Phi_{n+1})$  corrects for the hardest emission and fills regions of phase space inaccessible to the PS

# POWHEG/MC@NLO

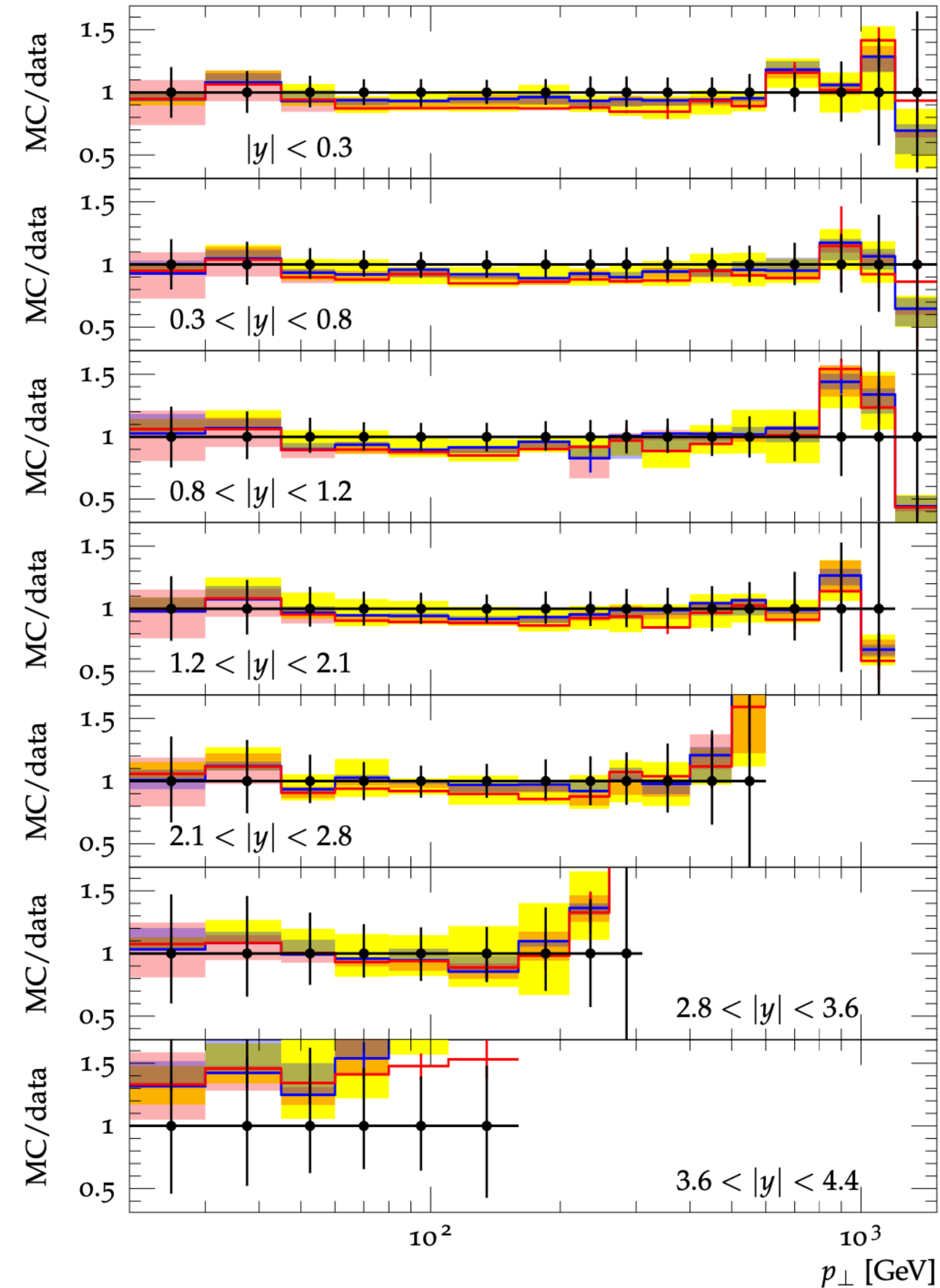
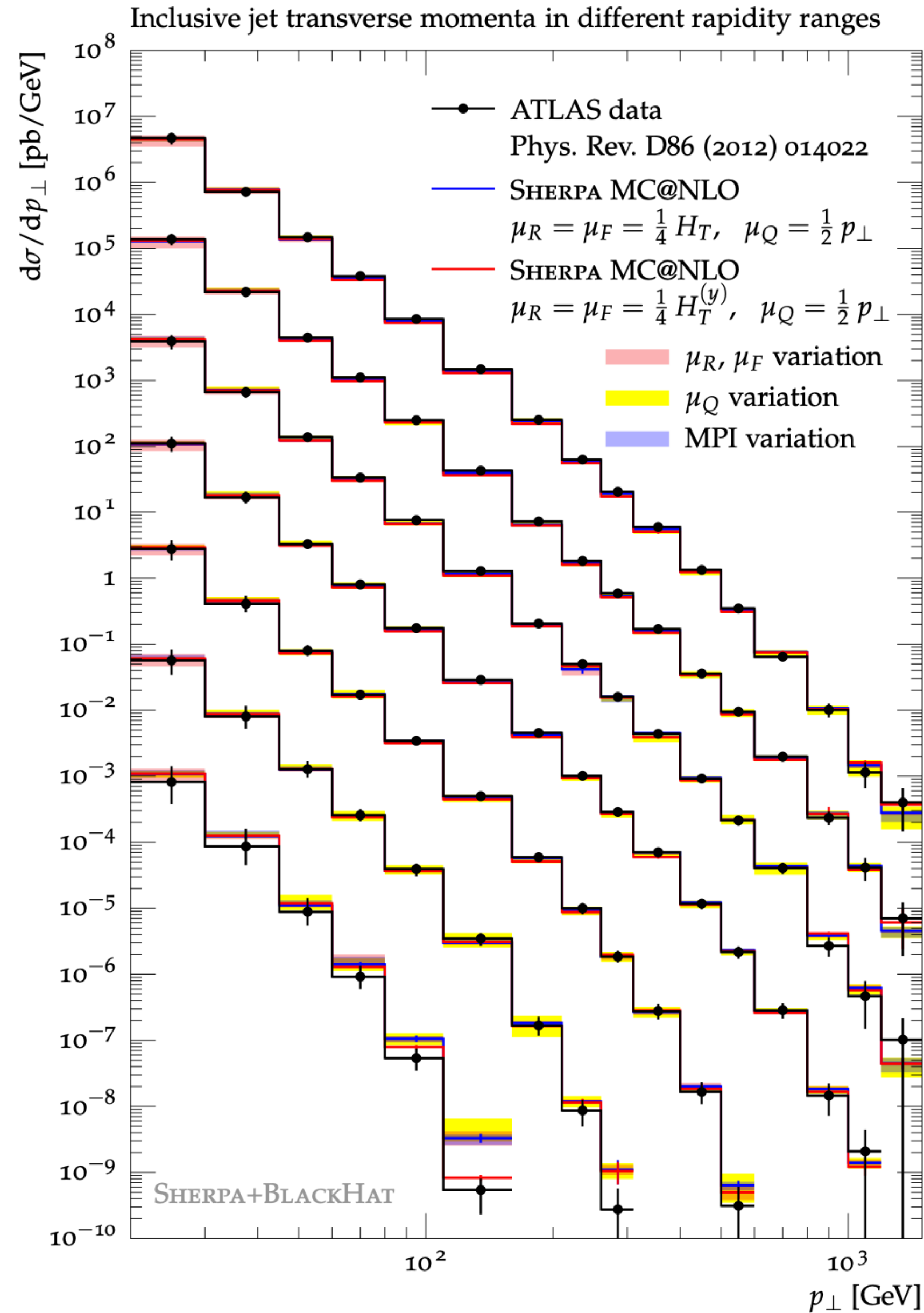
$t\bar{t}$  production

arXiv:1610.09978

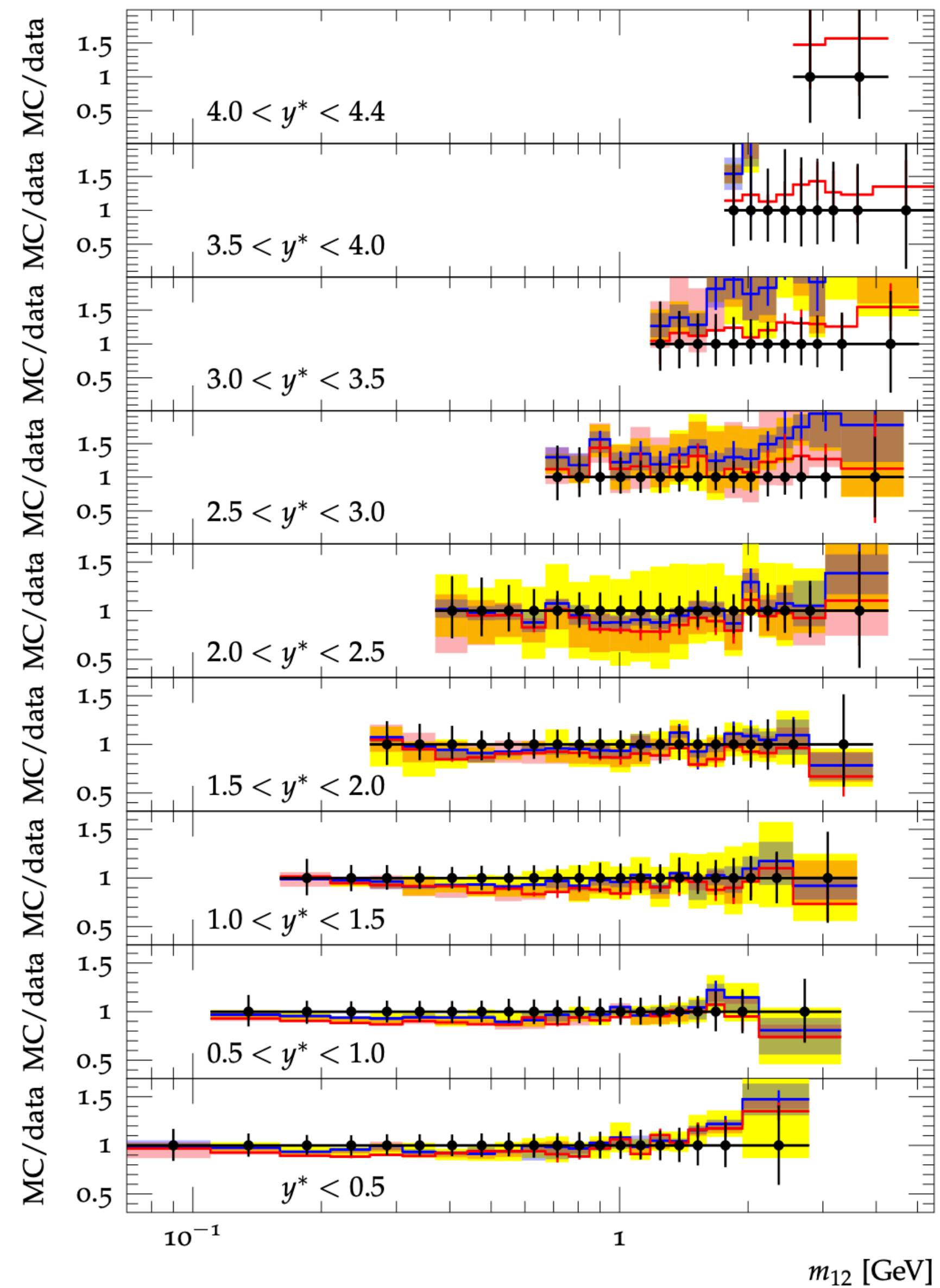
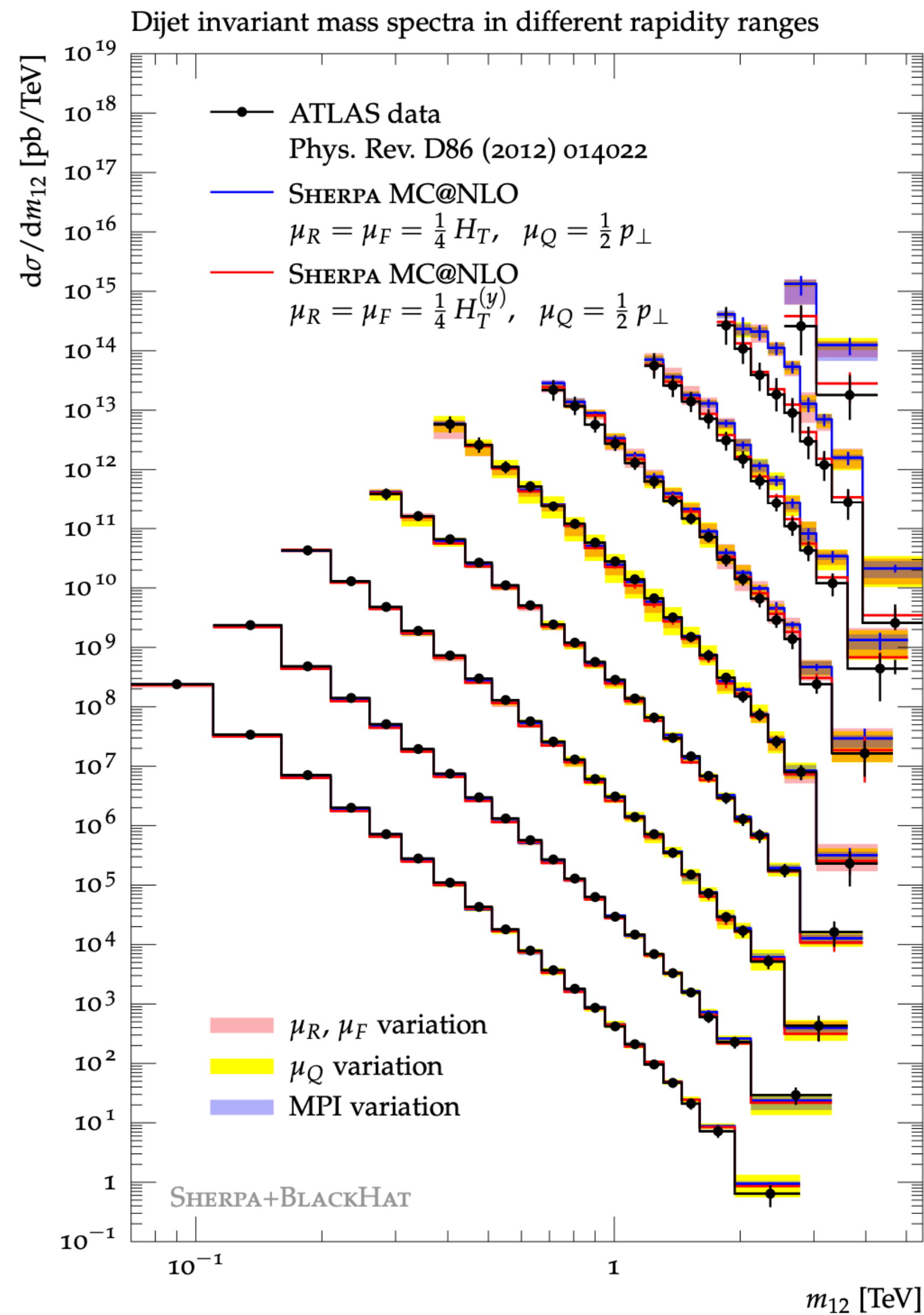




# Inclusive jet production at the LHC



# Inclusive jet production at the LHC



# POWHEG/MC@NLO

## POWHEG

- ❖ Positive Weights (Nearly always) :)
- ❖ Resummation is modified :(
- ❖ **POWHEG-Box**
- ❖ **HERWIG**

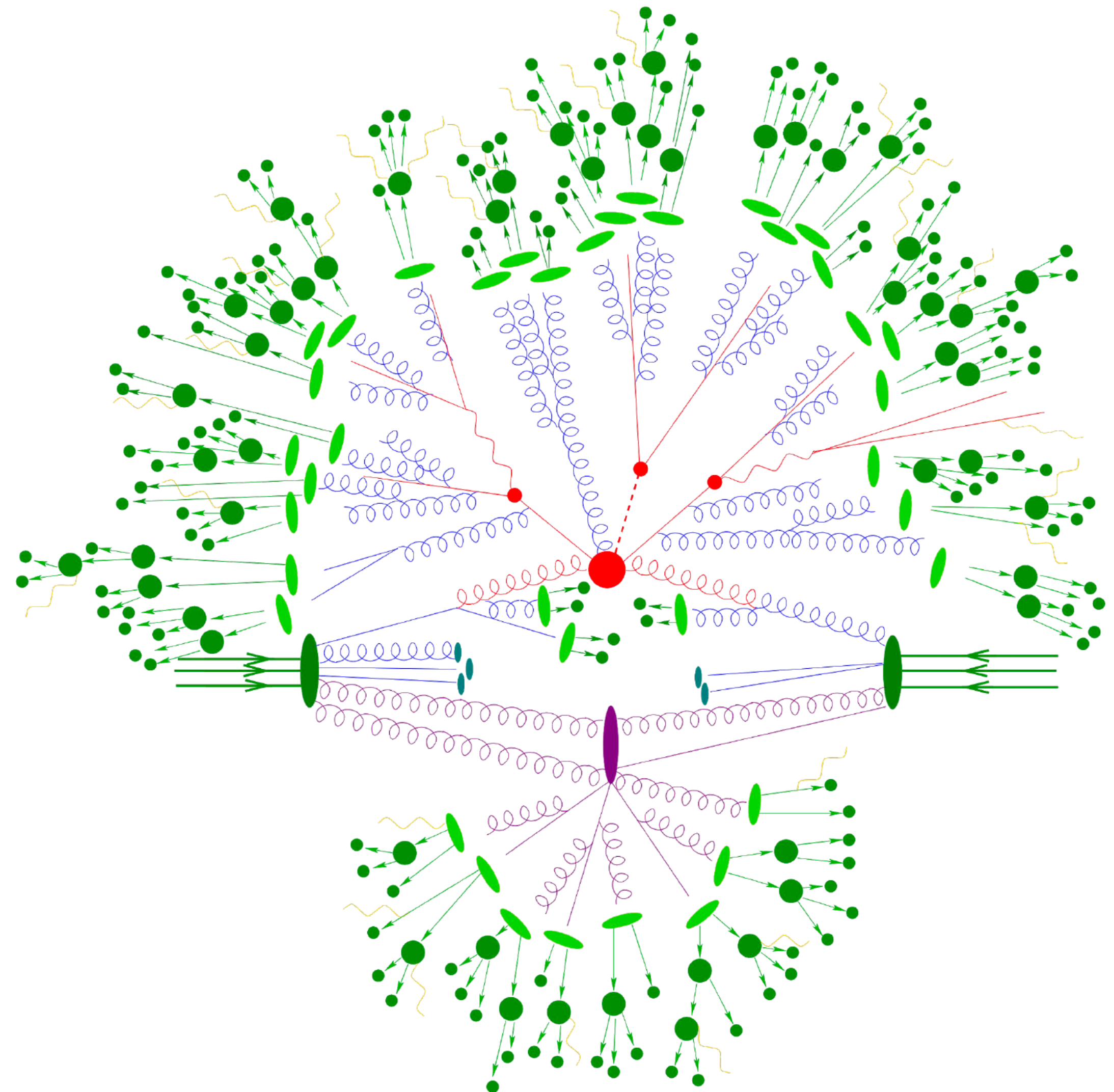
## MC@NLO

- ❖ Resummation is unchanged :)
- ❖ Negative Weights :(
- ❖ **MC@NLO**
- ❖ **SHERPA**
- ❖ **aMC@NLO**
- ❖ **HERWIG**



# Structure of LHC Events

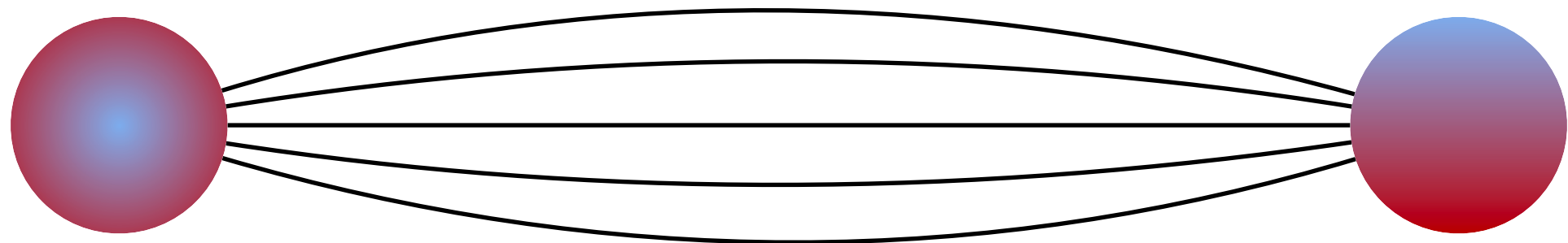
- ❖ **Hard Interaction**
- ❖ **Radiative Corrections**
- ❖ **Hadronization** ★
- ❖ **Hadron Decays**
- ❖ **Underlying Event**



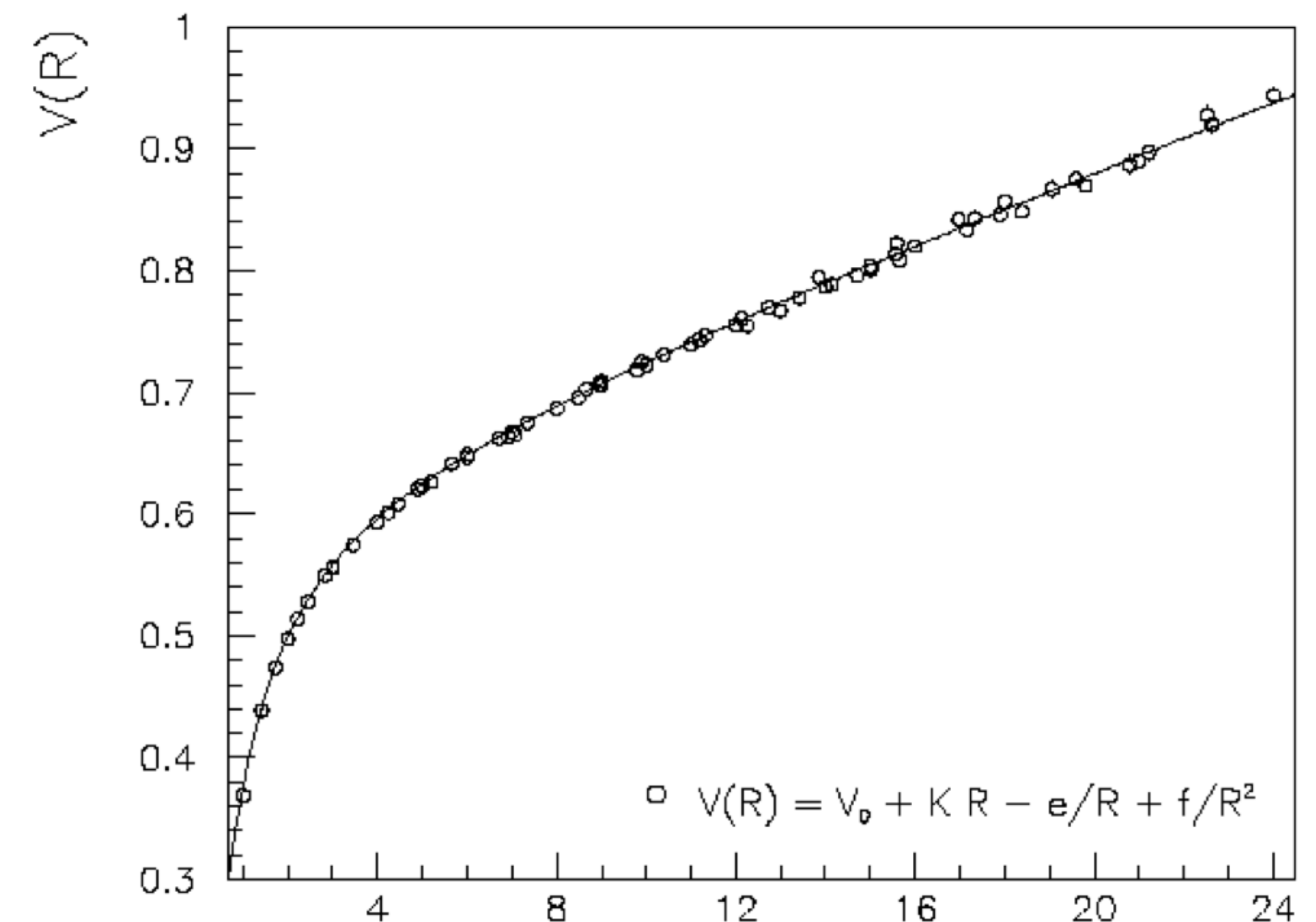


# Hadronization

- ❖ At short distances  $q\bar{q}$  are relatively free
- ❖ At long distances the strong forces starts to dominate
- ❖ From lattice we see that the QCD potential is approximately linear



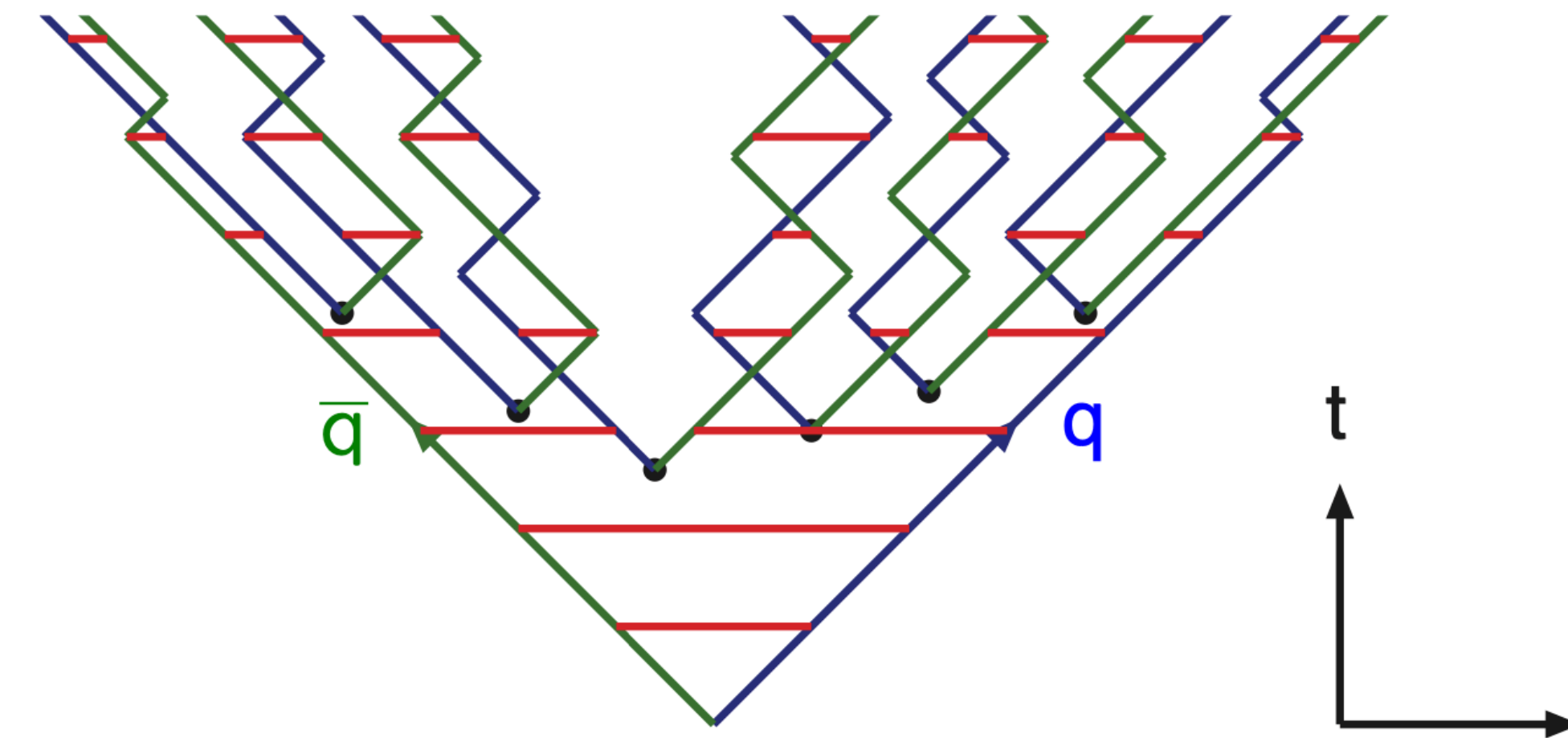
## Linear Potential = Confinement



# Lund String Model

[Andersson, Gustafson, Ingelman, Sjöstrand PR97\(1983\)31](#)

- ❖ Assume that light quarks are connected by “Strings”
- ❖ New  $q\bar{q}$  are created by tunnelling with prob  $\approx e^{-\frac{\pi m_q^2}{\sigma}}$ ,  
where  $\sigma$  is the string tension
- ❖  $q\bar{q}$  pairs can “yo-yo” allowing them to be more dynamical
- ❖ Hadrons are then formed from the multiple breakup of strings to more  $q\bar{q}$  pairs



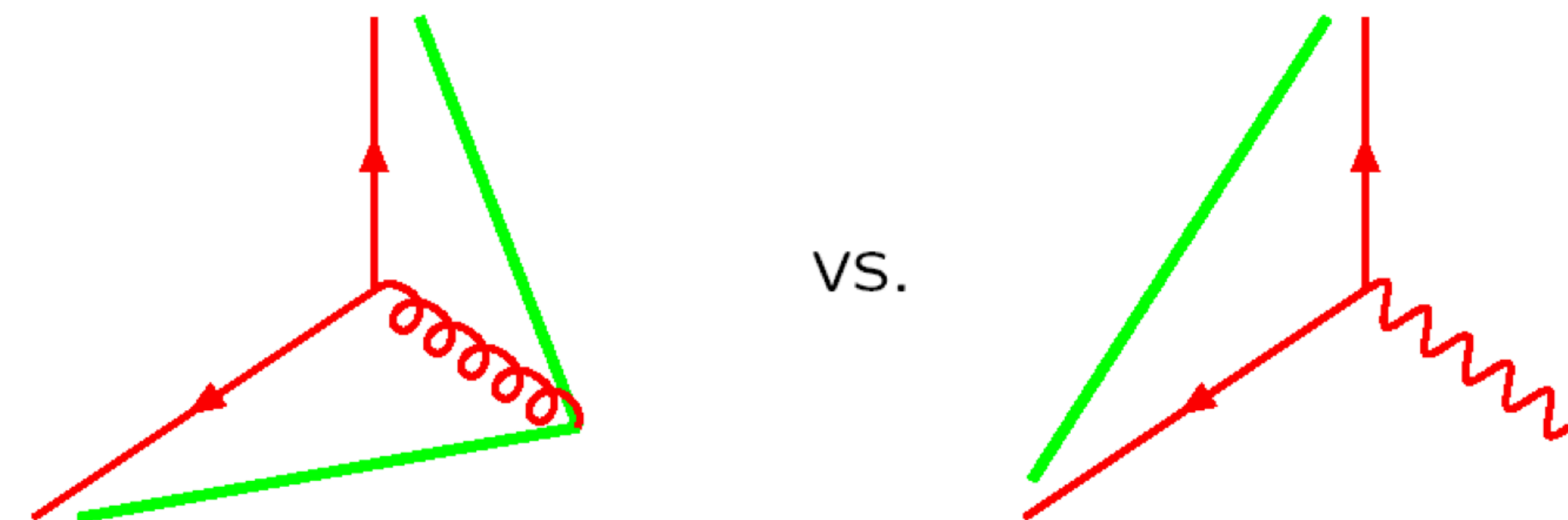
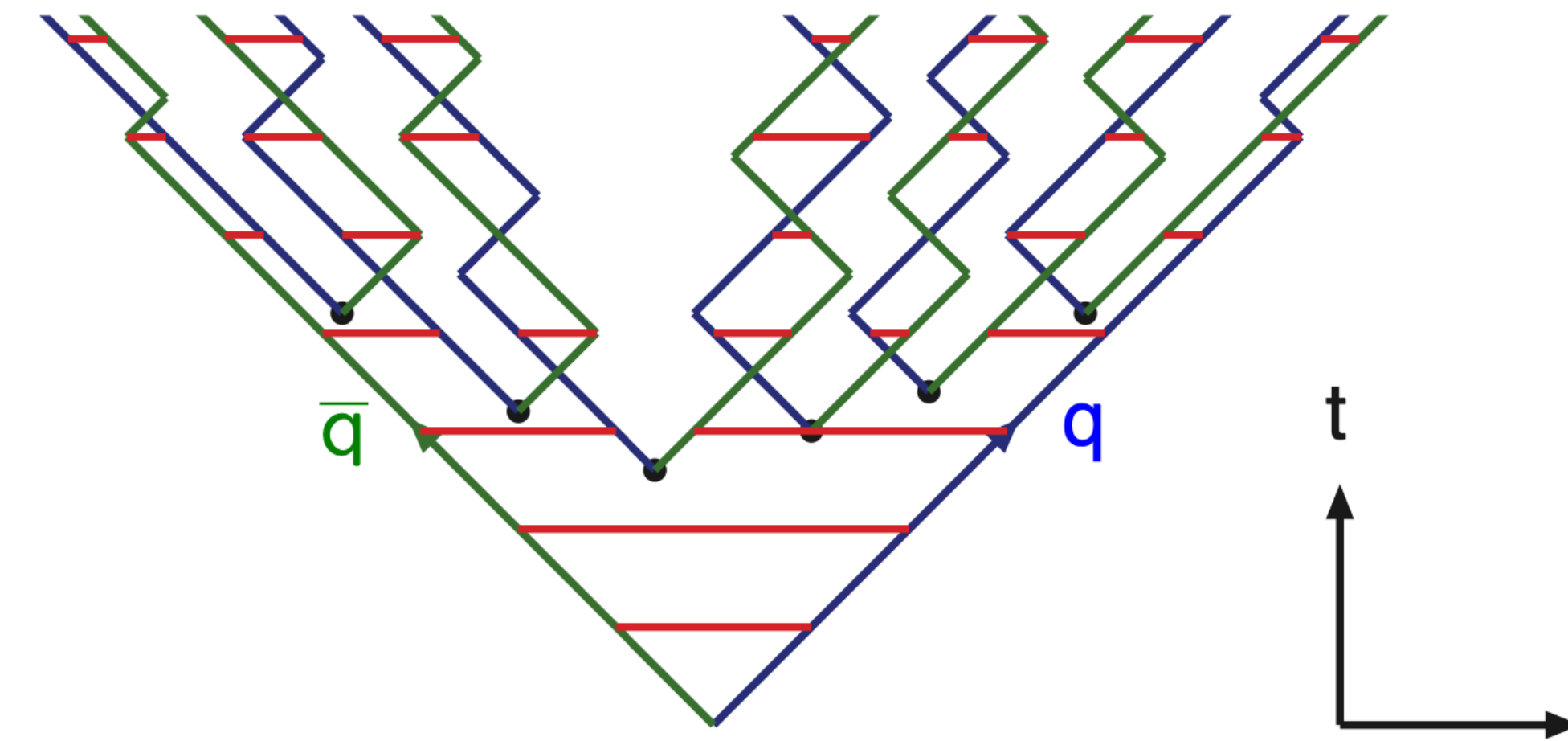
# Lund String Model

[Andersson, Gustafson, Ingelman, Sjöstrand PR97\(1983\)31](#)

- ❖ Gluons can be considered “kinks” in the string, with the gluon experiencing twice the string force
- ❖ Also see this from the associated Casimir operators

$$\frac{C_A}{C_F} = \frac{9}{4} \approx 2$$

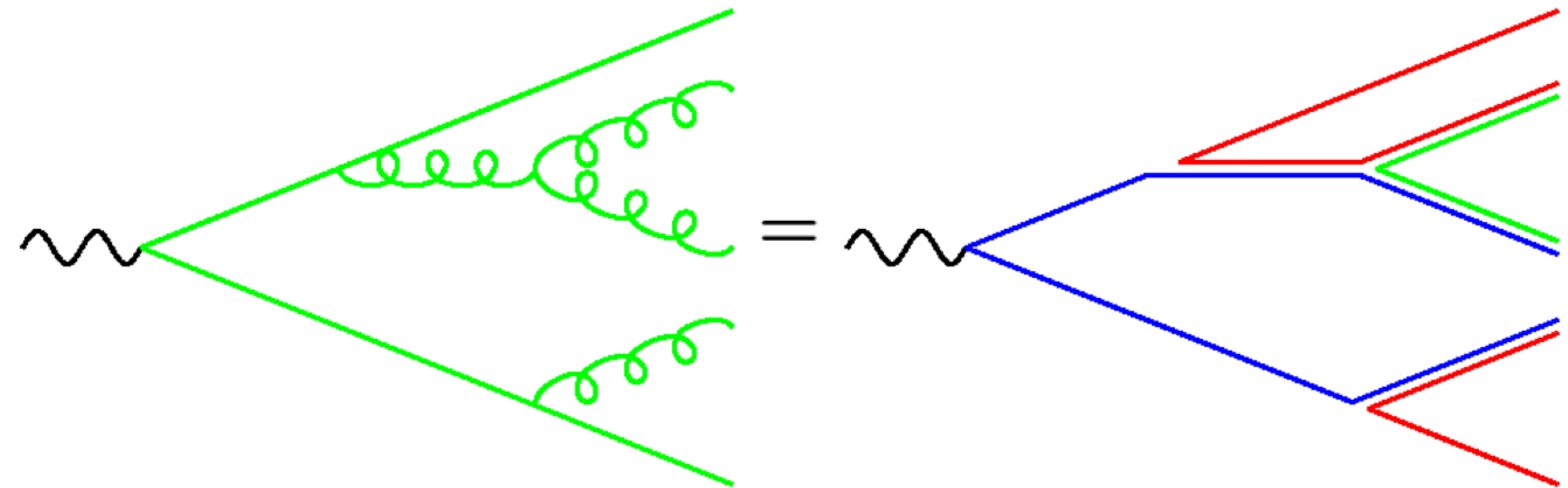
- ❖ This means hadron production is enhanced in regions dominated with  $qg$  or  $g\bar{q}$  pairs while suppressed in  $q\bar{q}$



# Cluster Model

[Webber] NPB238(1984)492

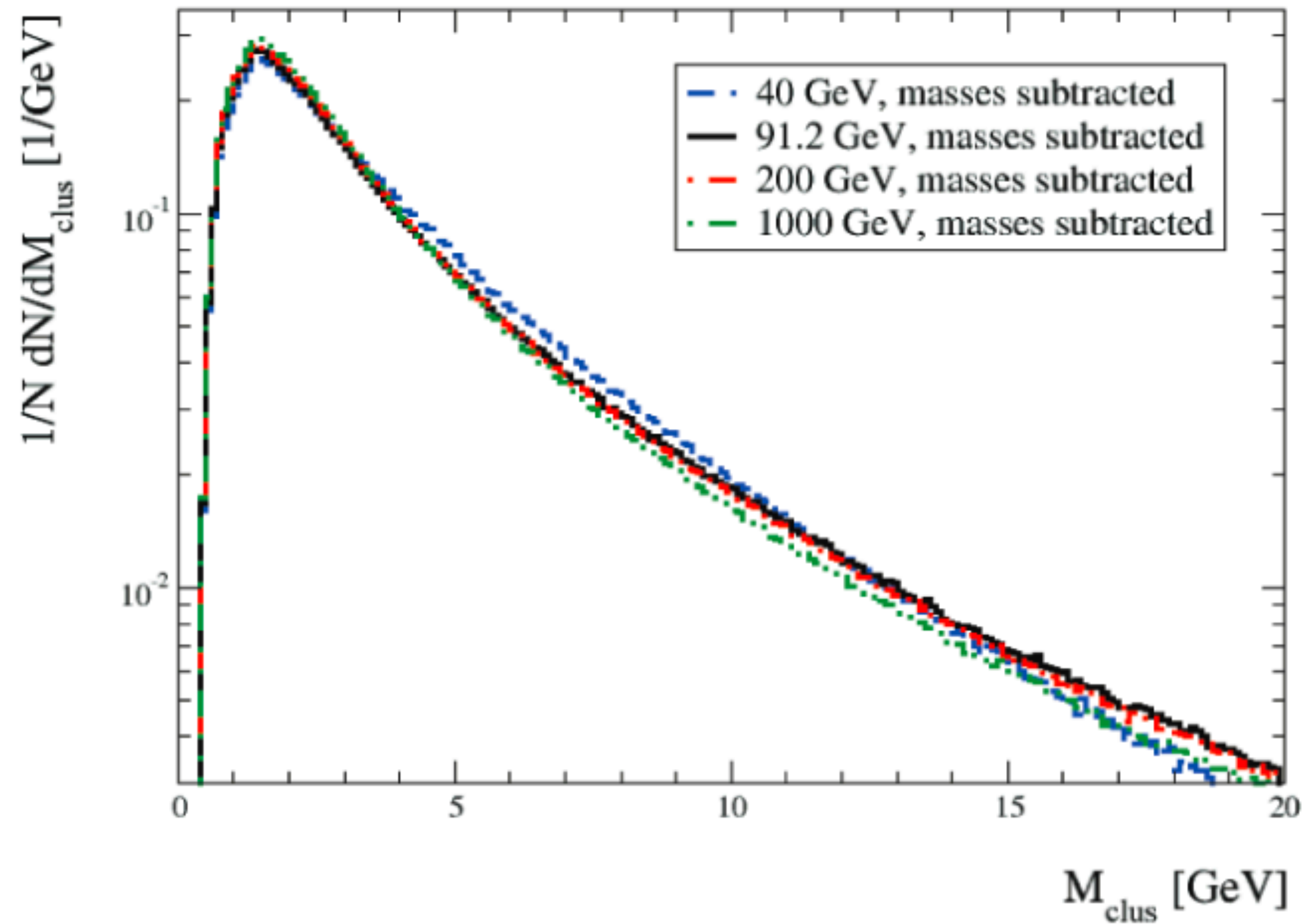
- ❖ Cluster models are based on the idea of **preconfinement**
- ❖ At the end of the Parton Shower all gluons are forced to split into  $q\bar{q}$  pairs.
- ❖ Color pairs end-up close together in phasespace
- ❖ Phasespace tells us how the clusters decay into hadrons





# Cluster Model

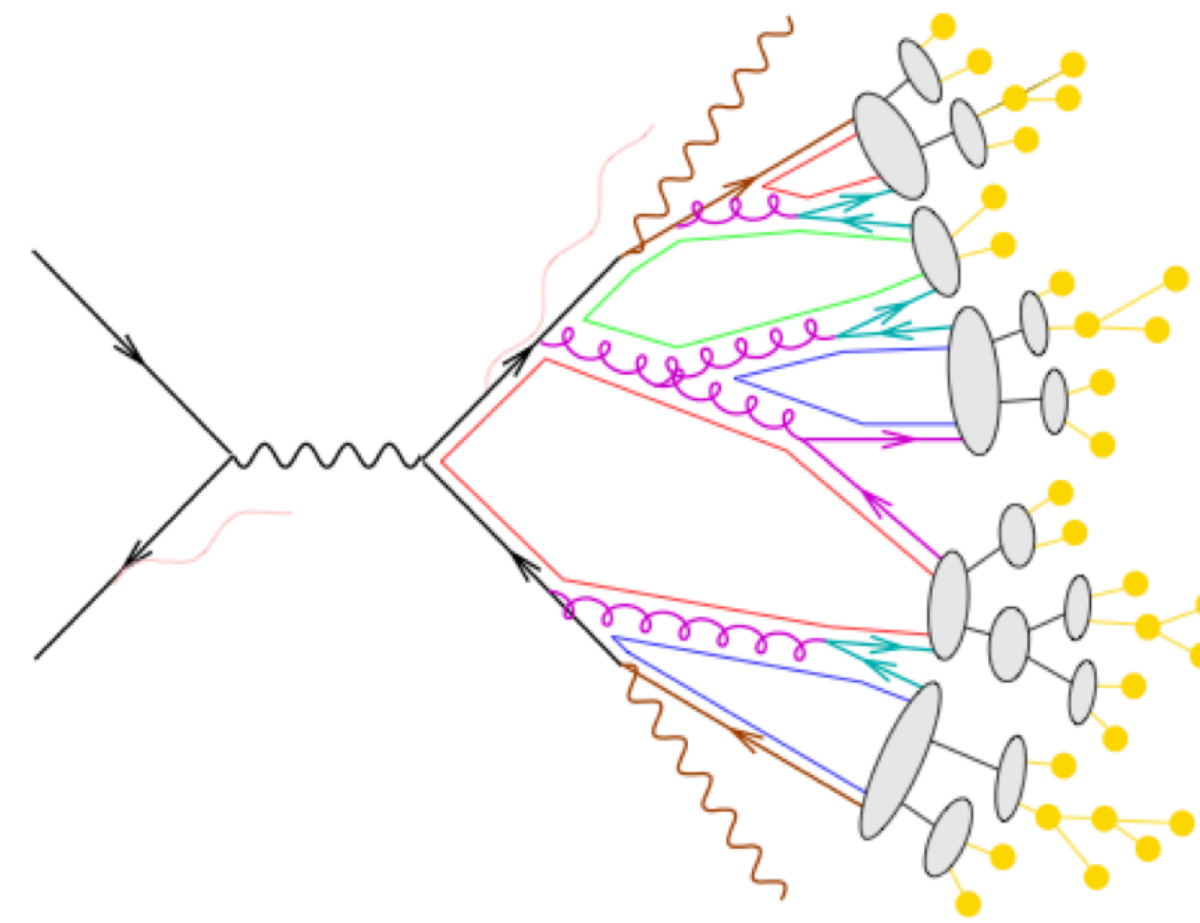
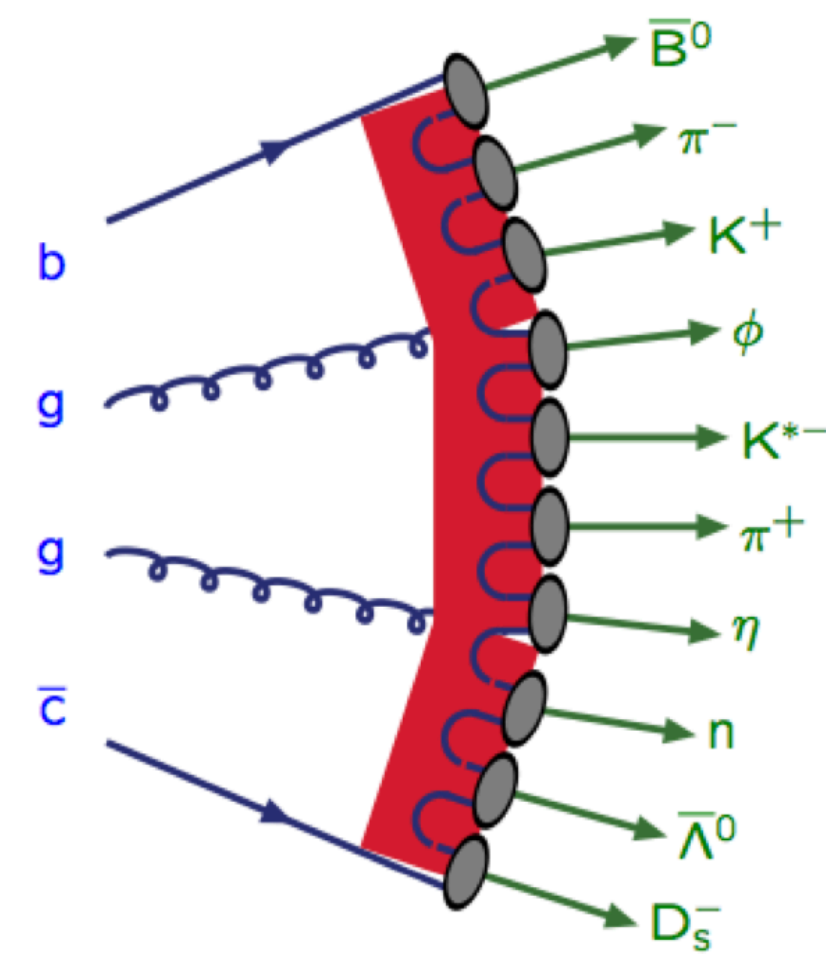
Cluster mass distribution  
 $e^-e^+ \rightarrow \text{hadrons}$  at various c.m. energies



We see that the cluster mass peaks around 1 GeV, regardless of the CME

This shows the independence of the cluster at the end of the Parton shower from the hard scale

# String vs Cluster



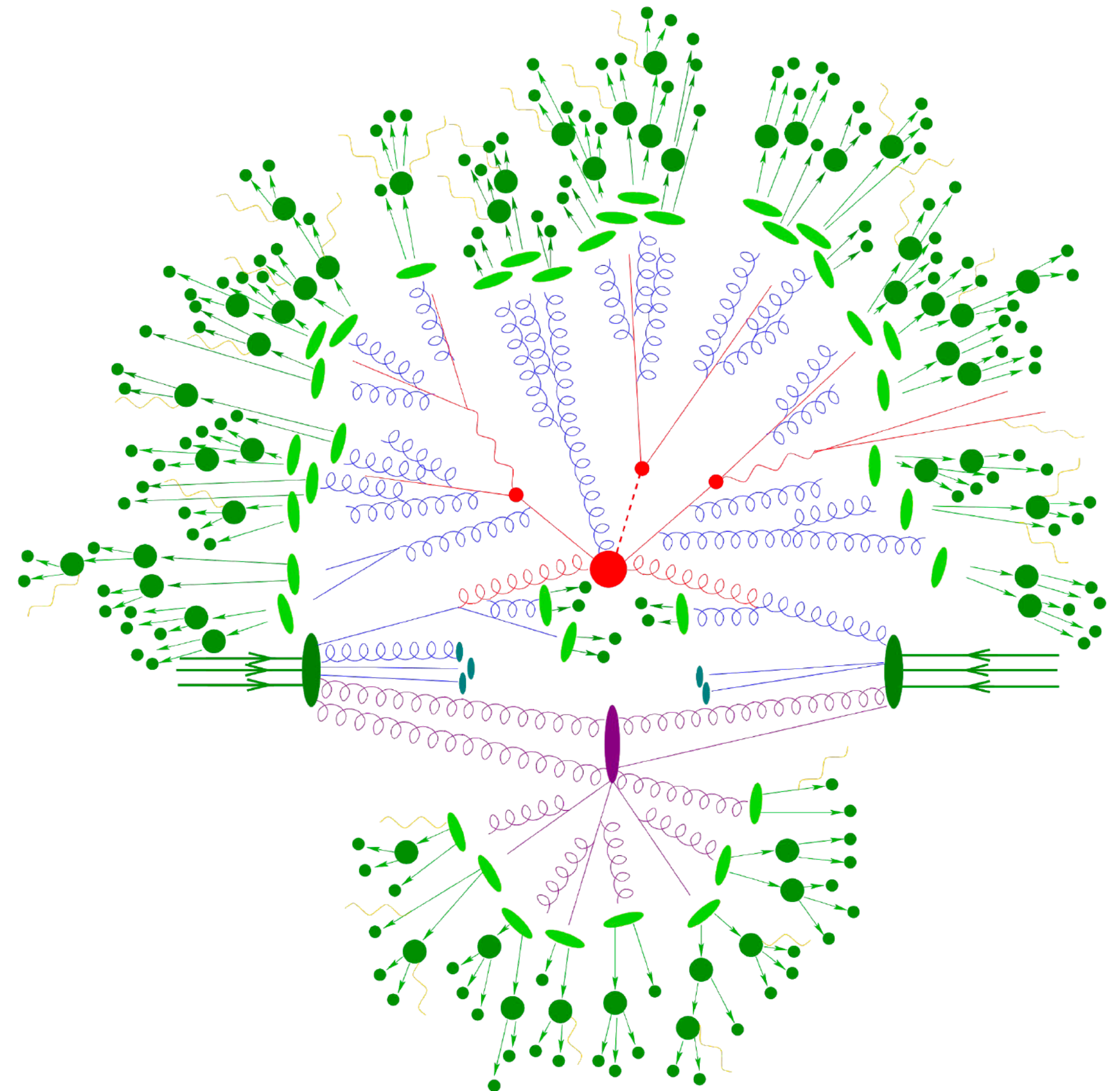
[T.Sjöstrand]

program	PYTHIA	Herwig
model	string	cluster
energy-momentum picture	powerful	simple
parameters	predictive	unpredictive
flavour composition	few	many
parameters	messy	simple
	unpredictive	in-between
	many	few

“There ain’t no such thing as a parameter-free *good* description”

# Structure of LHC Events

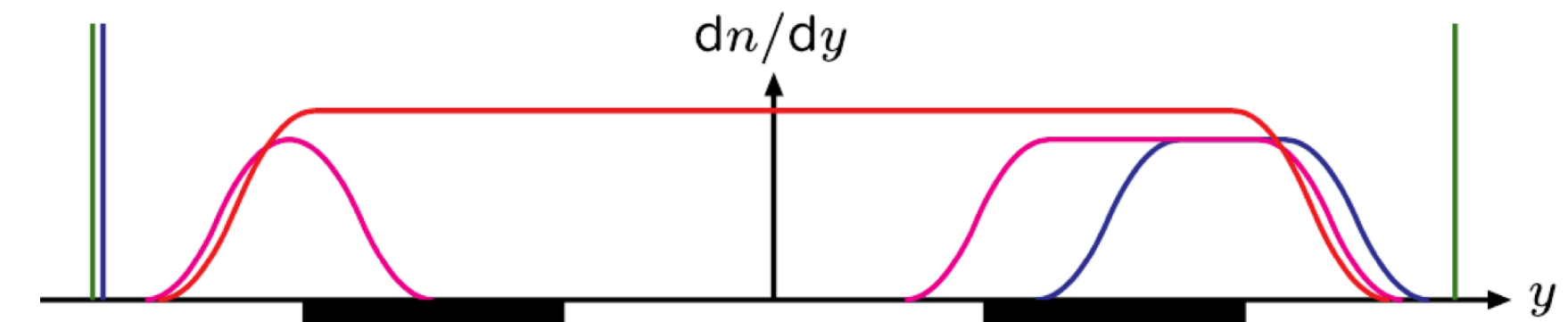
- ❖ Hard Interaction
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- ❖ Hadron Decays
- ❖ Underlying Event ★





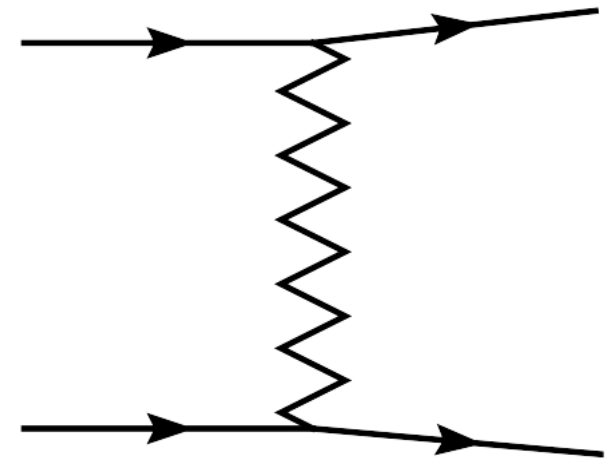
# Multi-Parton Interactions

$$\sigma_{\text{total}} = \sigma_{\text{Elastic}} + \sigma_{\text{Single Diffractive}} + \sigma_{\text{Double Diffractive}} + \sigma_{\text{Non Diffractive}}$$



# Underlying Event

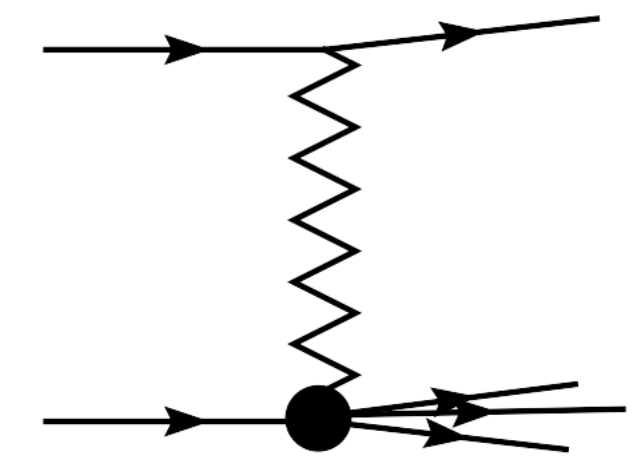
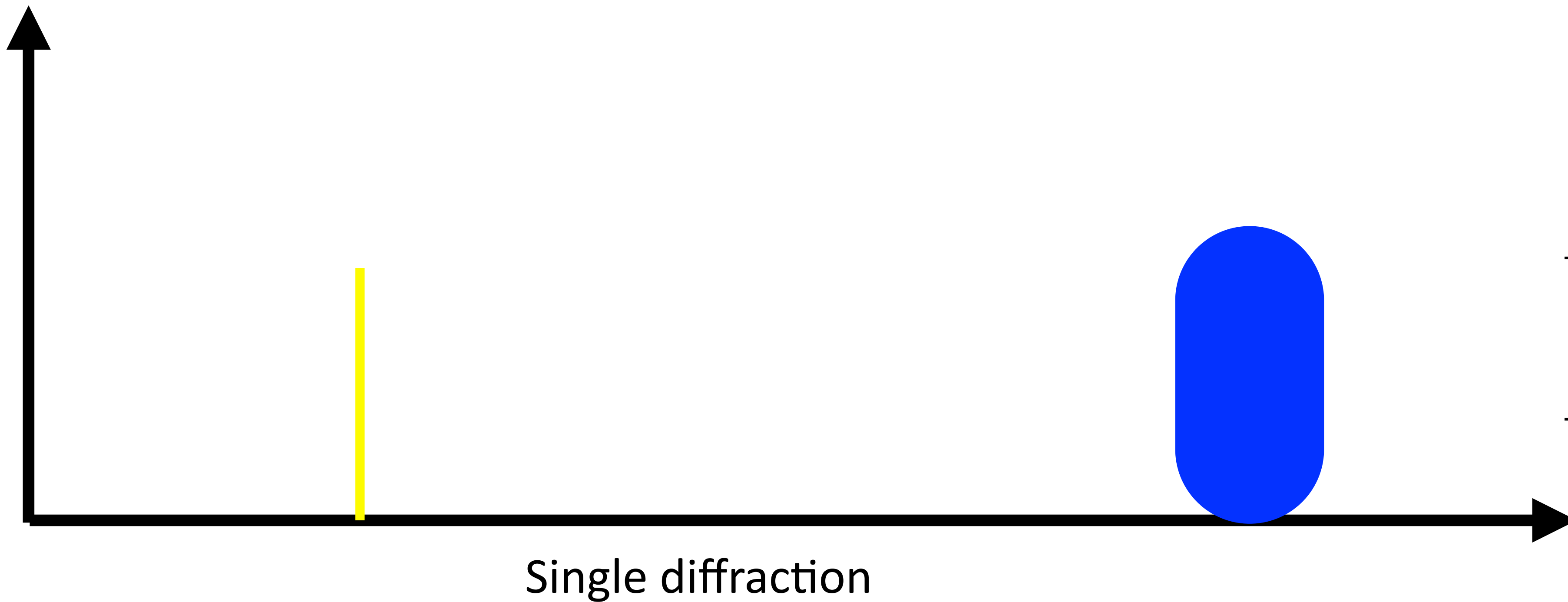
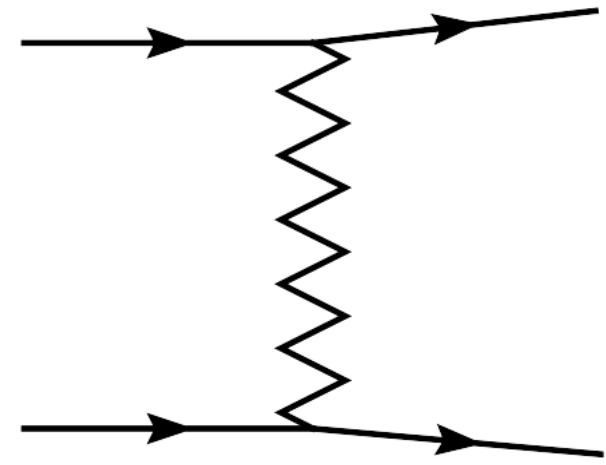
$$\sigma_{\text{total}} = \sigma_{\text{Elastic}} + \sigma_{\text{Single Diffractive}} + \sigma_{\text{Double Diffractive}} + \sigma_{\text{Non Diffractive}}$$





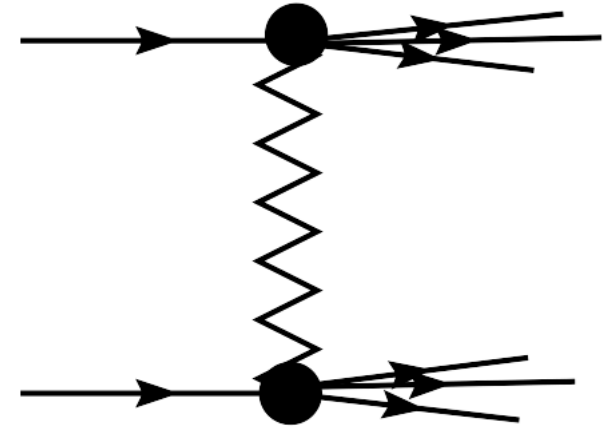
# Underlying Event

$$\sigma_{\text{total}} = \sigma_{\text{Elastic}} + \sigma_{\text{Single Diffractive}} + \sigma_{\text{Double Diffractive}} + \sigma_{\text{Non Diffractive}}$$

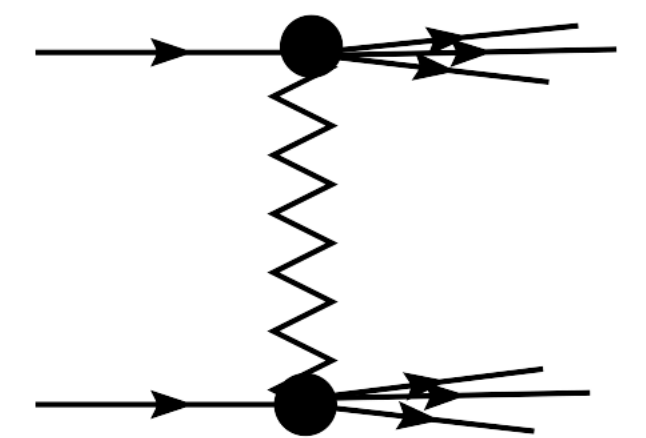


# Underlying Event

$$\sigma_{\text{total}} = \sigma_{\text{Elastic}} + \sigma_{\text{Single Diffractive}} + \sigma_{\text{Double Diffractive}} + \sigma_{\text{Non Diffractive}}$$

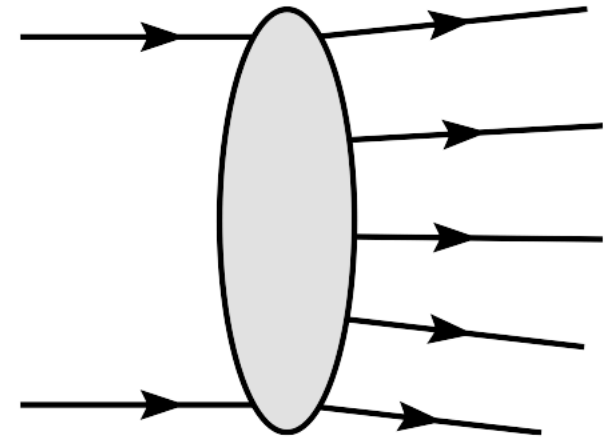


Double diffraction



# Underlying Event

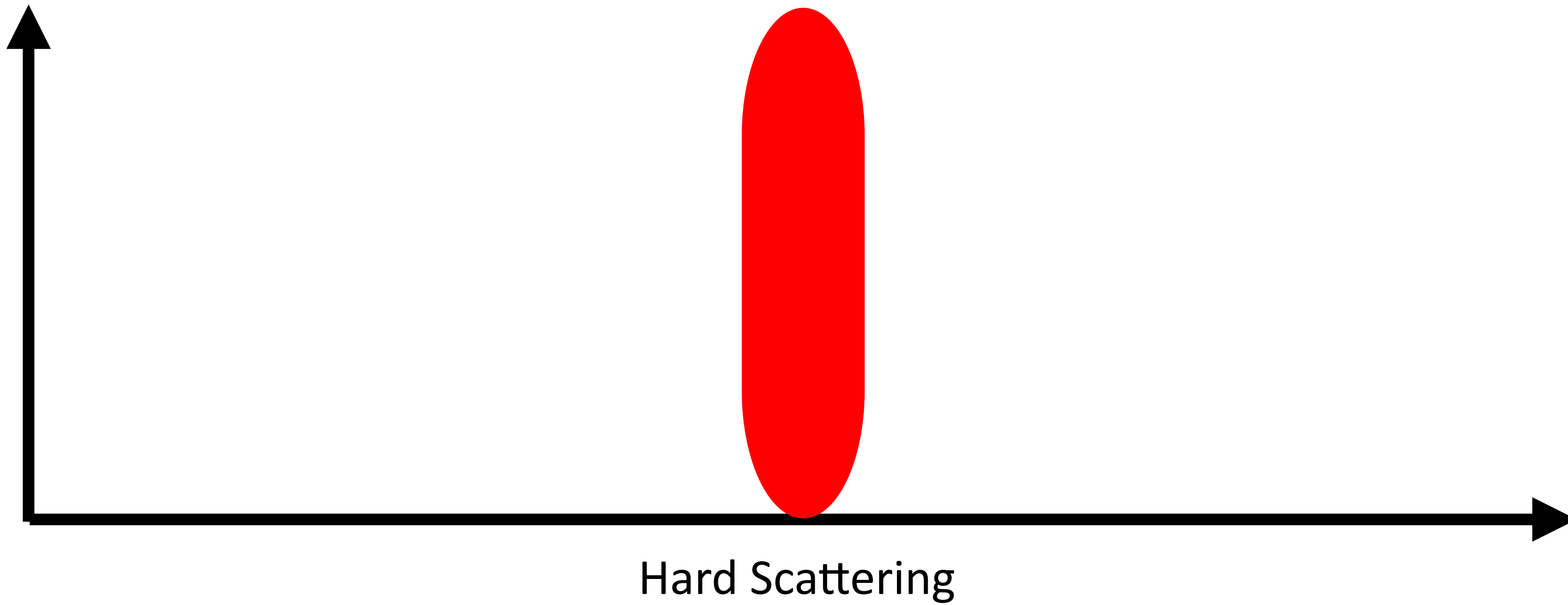
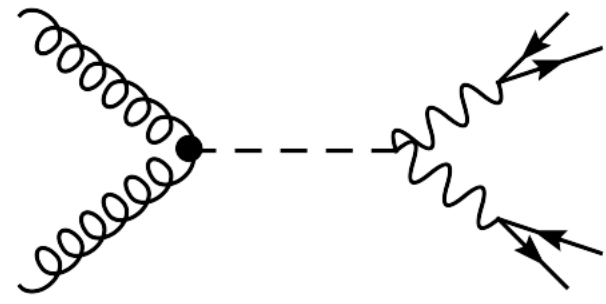
$$\sigma_{\text{total}} = \sigma_{\text{Elastic}} + \sigma_{\text{Single Diffractive}} + \sigma_{\text{Double Diffractive}} + \sigma_{\text{Non Diffractive}}$$



(multiple/soft) interactions

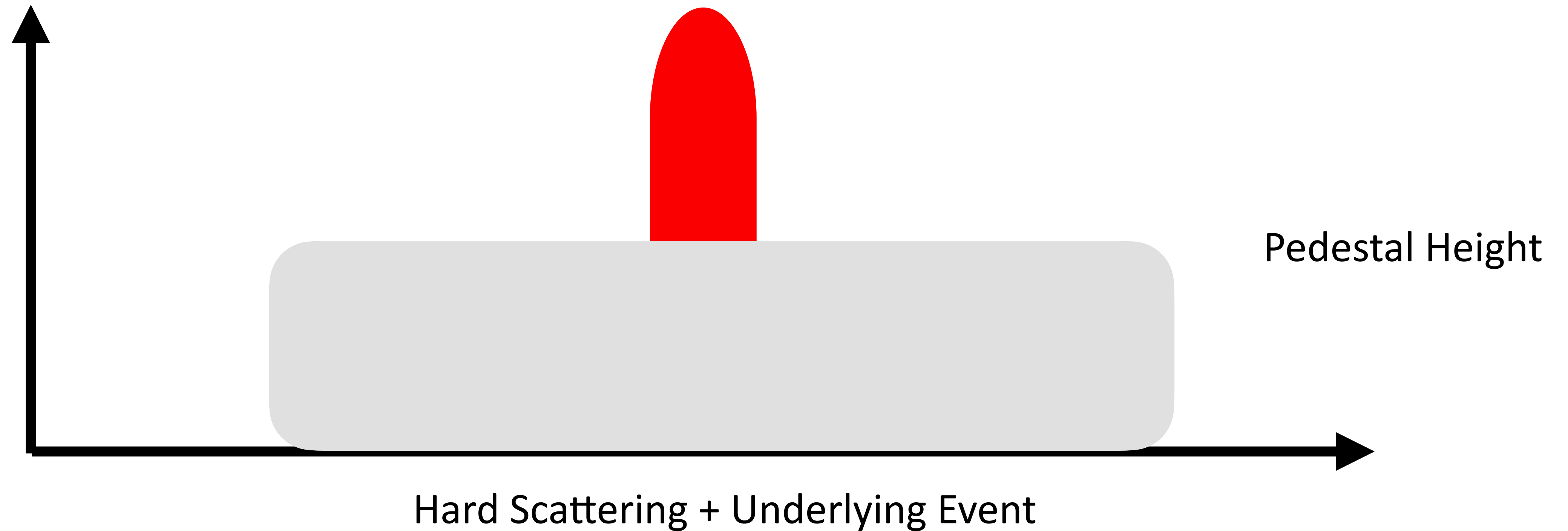
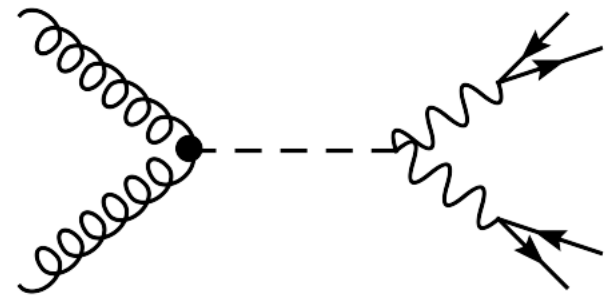
# Underlying Event

$$\sigma_{\text{total}} = \sigma_{\text{Elastic}} + \sigma_{\text{Single Diffractive}} + \sigma_{\text{Double Diffractive}} + \sigma_{\text{Non Diffractive}}$$



# Underlying Event

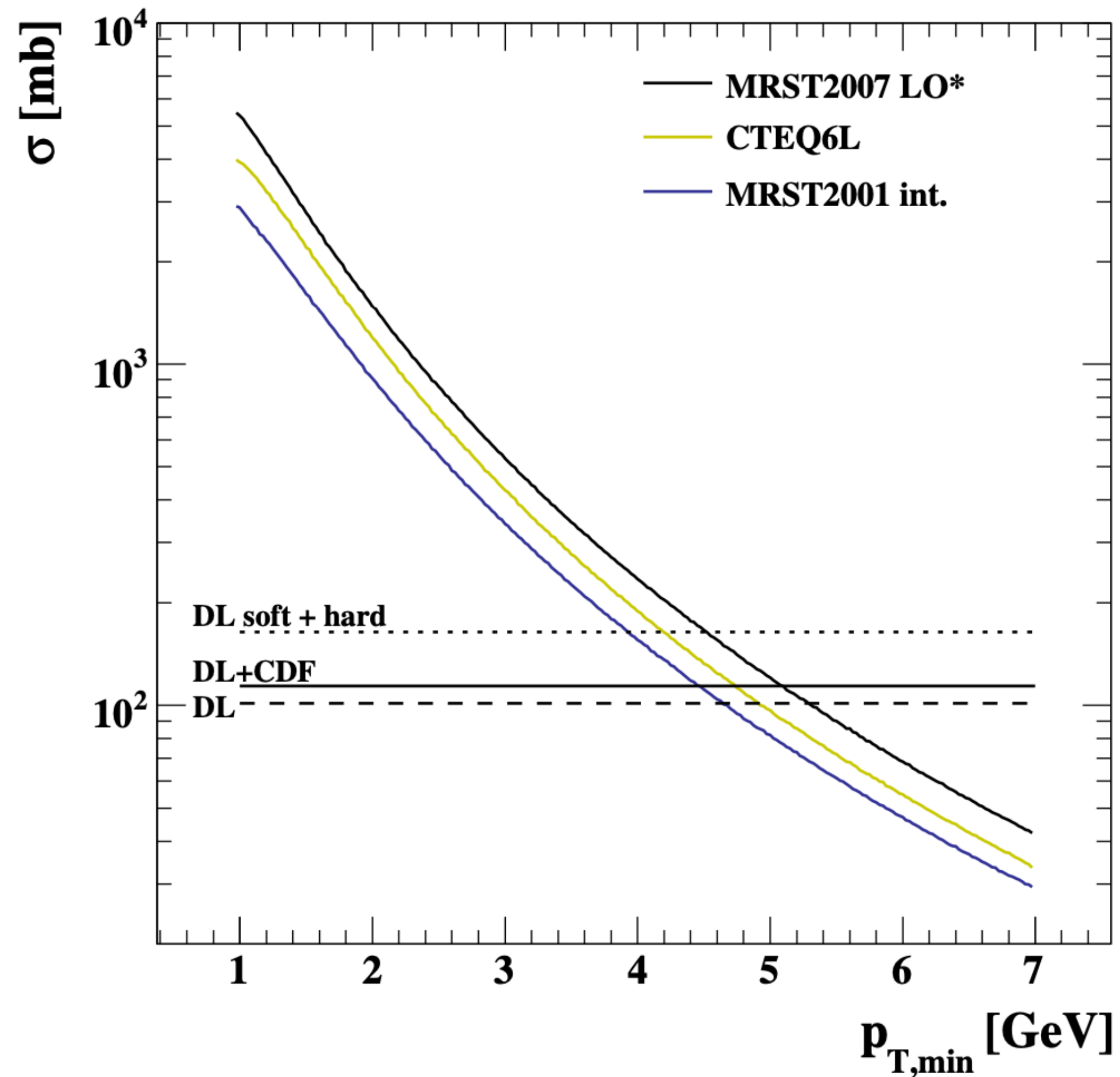
$$\sigma_{\text{total}} = \sigma_{\text{Elastic}} + \sigma_{\text{Single Diffractive}} + \sigma_{\text{Double Diffractive}} + \sigma_{\text{Non Diffractive}}$$





# Modelling the Pedestal

[Sjöstrand,Zijl] PRD36(1987)2019



Total LHC cross section is exceeded at low  $p_T$

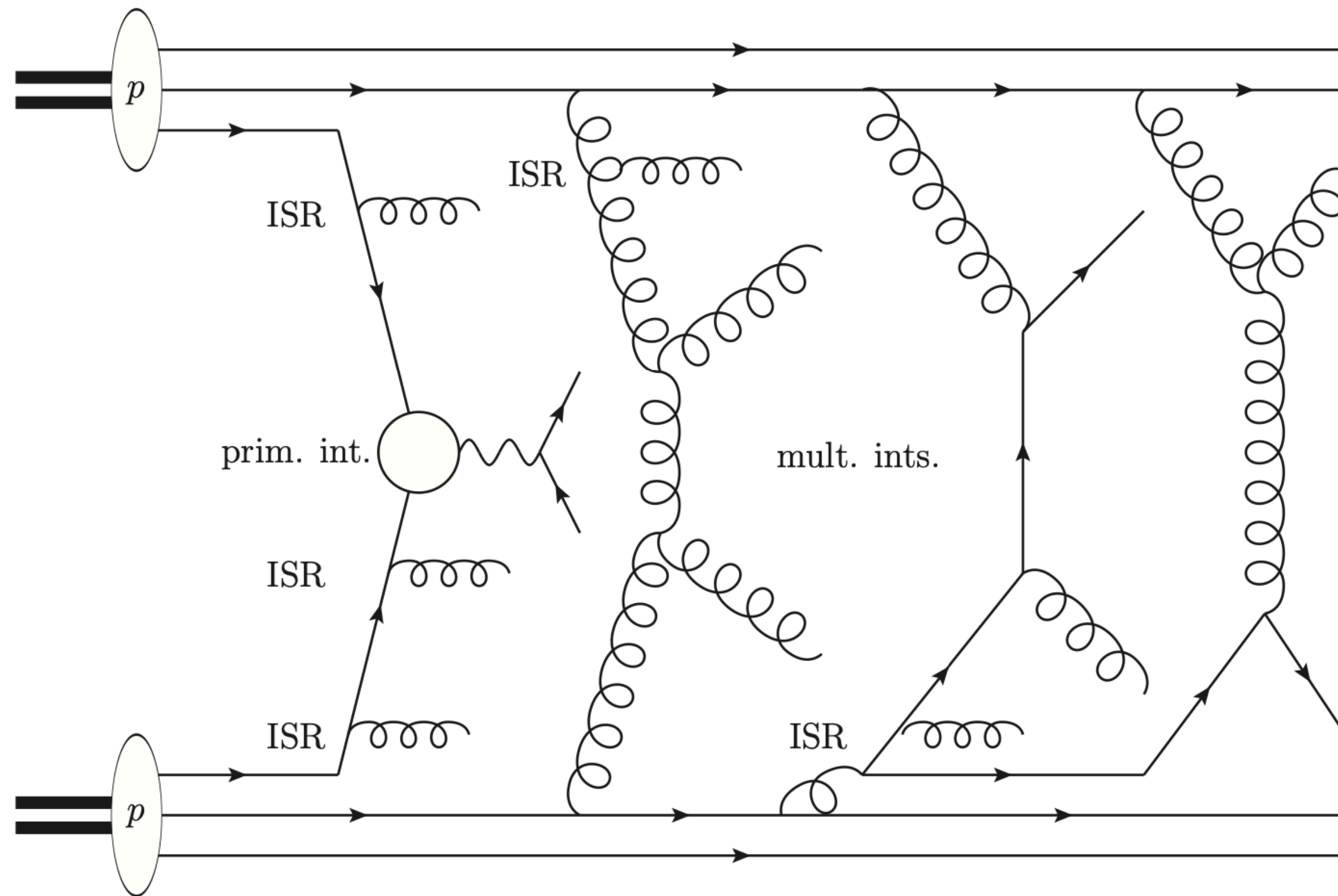
We can interpret this as additional parton-parton scattering per proton collision

$$\langle n \rangle = \frac{\sigma_{\text{Hard}}}{\sigma_{\text{Non Diffractive}}}$$

Main free parameter is  $P_{Tmin}$

# Modelling the Pedestal

[Sjöstrand,Zijl] PRD36(1987)2019

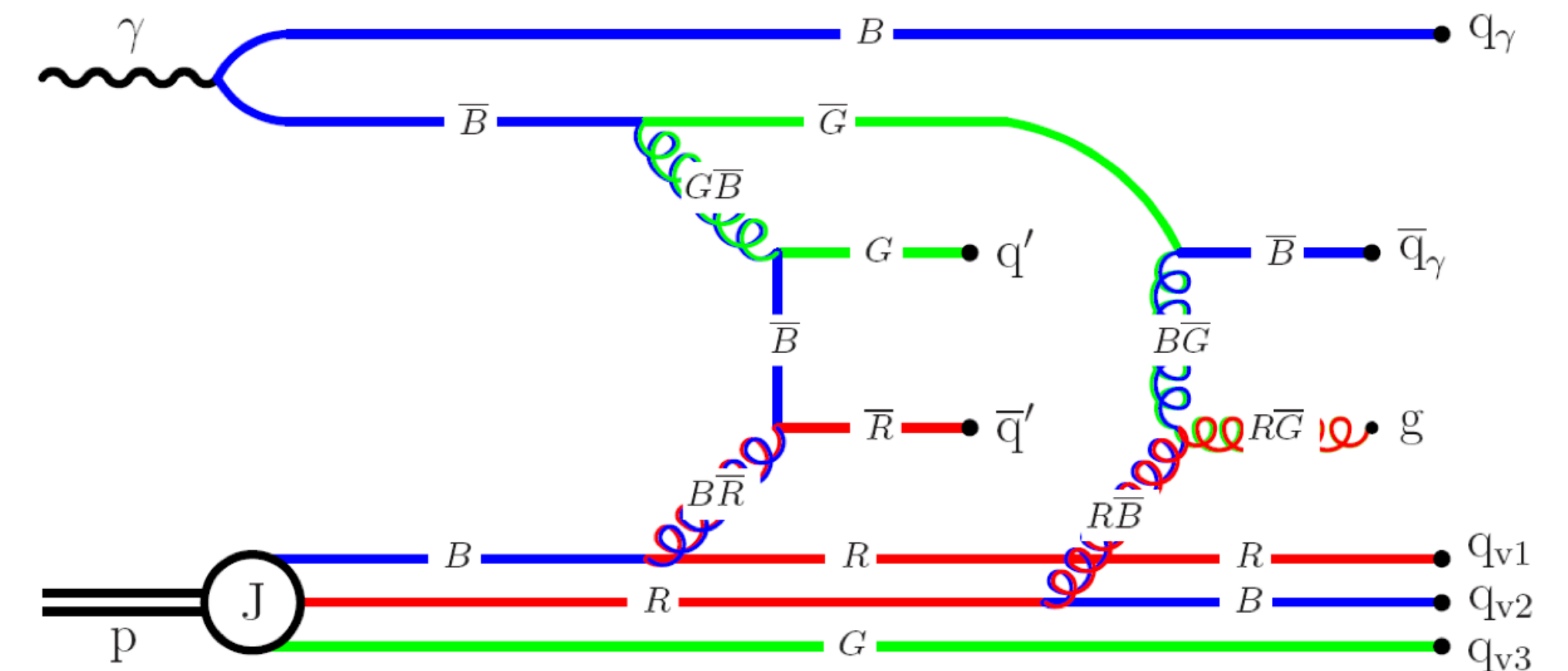


- ❖ We will have  $n$  independent secondary interactions
- ❖ Model each as a perturbative interaction with a non-perturbative form-factor correction

# Color connections and beam remnants

[Sjöstrand,Skands] hep-ph/0402078

- ❖ Secondary scatterings need to be color-connected to something
- ❖ Simplest model would decouple them from proton remnants
- ❖ Newer models embed scattering into color topology



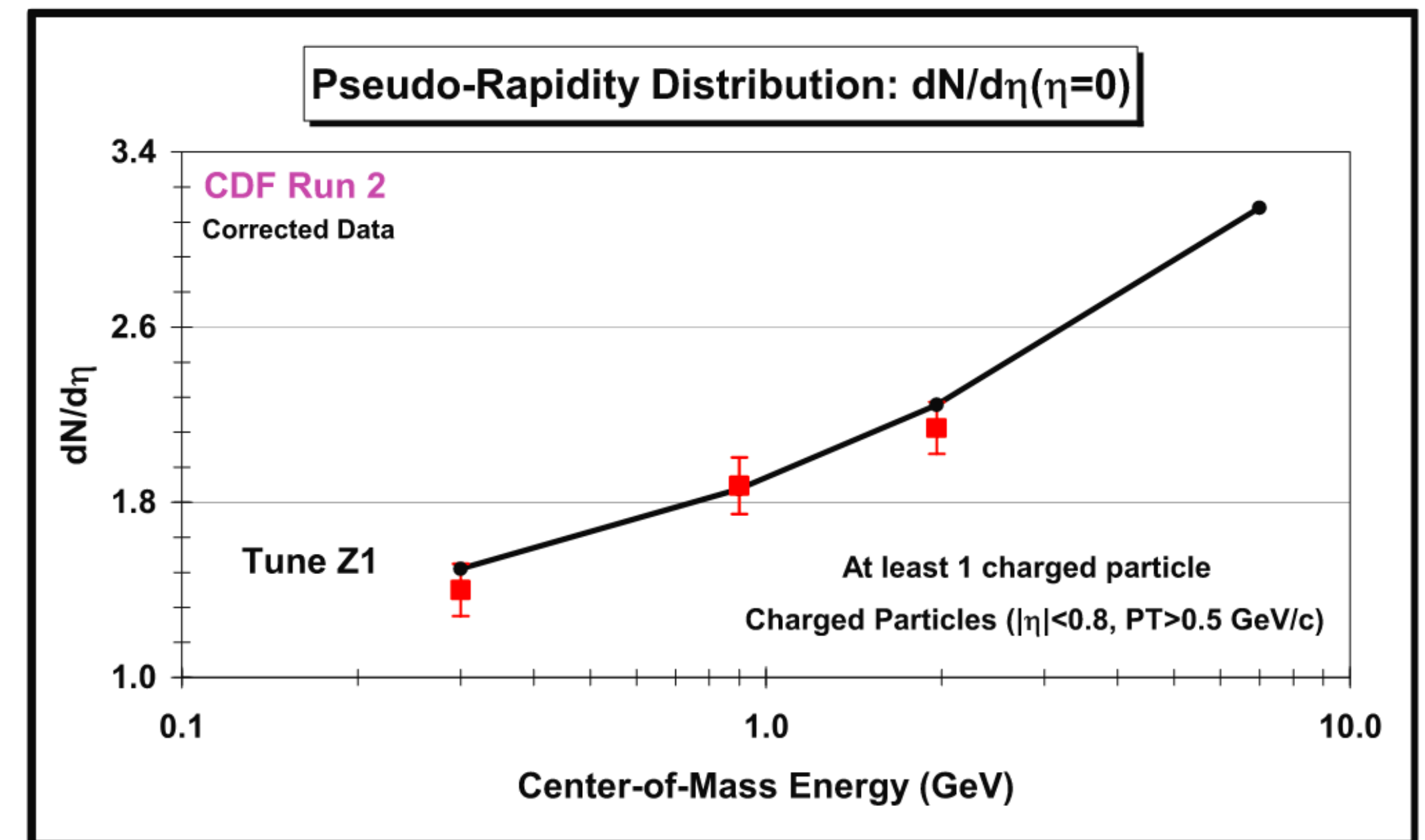
# A model for minimum bias

[Butterworth,Forshaw,Seymour] hep-ph/9601371

- ❖ Peripheral collision of particles in a collision
- ❖ Take the parton distribution in a beam as

$$\frac{dn_a(x, \mathbf{b})}{d^2\mathbf{b}dx} = f_a(x) G(\mathbf{b})$$

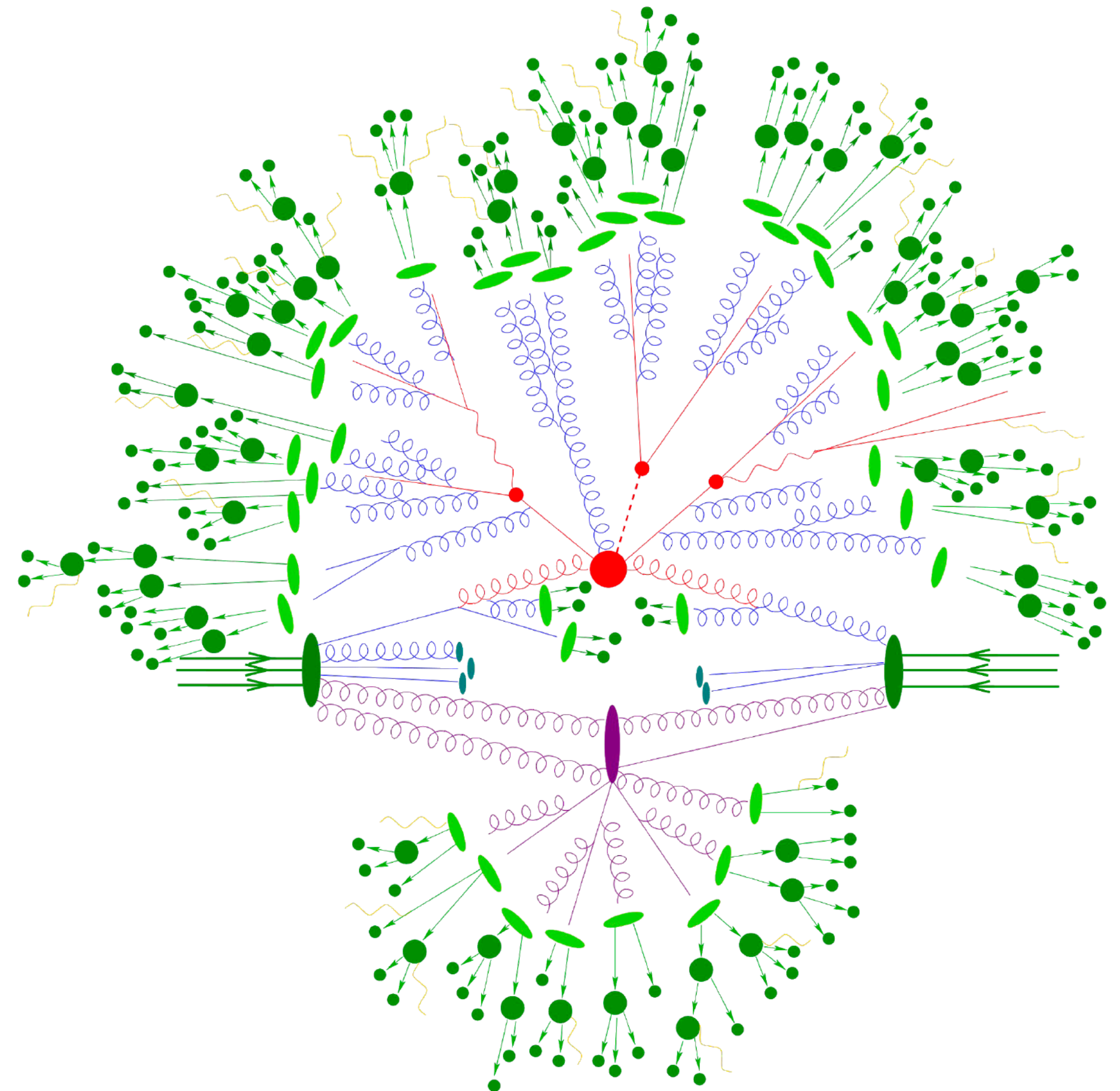
$$G(\mathbf{b}) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\exp(\mathbf{k} \cdot \mathbf{b})}{(1 + \mathbf{k}^2/\mu^2)^2}$$





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# Hadron Decays

- Most hadrons are unstable and will **decay**
- In practice there are  $\approx \mathcal{O}(1000)$  decay channels
- PDG contains tables of branching ratios

## Issues

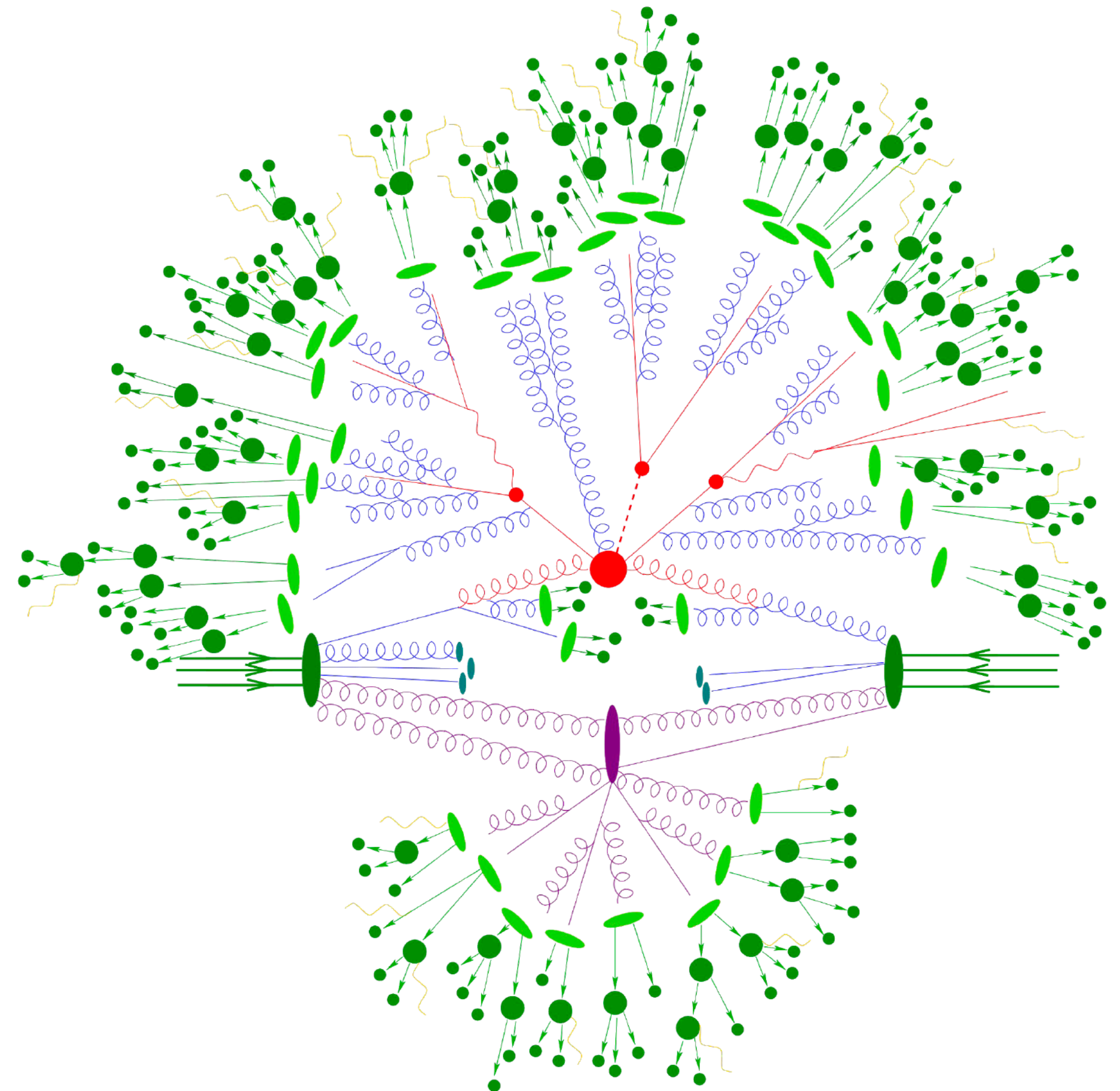
- Not all resonances have been measured
- BR rarely add up to 100% and some have large uncertainties
- Can lead to non-trivial effects in the hadronization, event shapes, etc..

$\eta$ DECAY MODES	Fraction ( $\Gamma_i/\Gamma$ )	Scale factor/ Confidence level	$p$ (MeV/c)
<b>Neutral modes</b>			
neutral modes	(71.96±0.30) %	S=1.3	–
2 $\gamma$	(39.36±0.18) %	S=1.1	274
3 $\pi^0$	(32.57±0.21) %	S=1.2	179
$\pi^0 2\gamma$	( 2.55±0.22) $\times 10^{-4}$		257
2 $\pi^0 2\gamma$	< 1.2 $\times 10^{-3}$	CL=90%	238
4 $\gamma$	< 2.8 $\times 10^{-4}$	CL=90%	274
invisible	< 1.0 $\times 10^{-4}$	CL=90%	–
<b>Charged modes</b>			
charged modes	(28.04±0.30) %	S=1.3	–
$\pi^+ \pi^- \pi^0$	(23.02±0.25) %	S=1.2	174
$\pi^+ \pi^- \gamma$	( 4.28±0.07) %	S=1.1	236
$e^+ e^- \gamma$	( 6.9 ±0.4 ) $\times 10^{-3}$	S=1.2	274
$\mu^+ \mu^- \gamma$	( 3.1 ±0.4 ) $\times 10^{-4}$		253
$e^+ e^-$	< 7 $\times 10^{-7}$	CL=90%	274
$\mu^+ \mu^-$	( 5.8 ±0.8 ) $\times 10^{-6}$		253
2 $e^+ 2e^-$	( 2.40±0.22) $\times 10^{-5}$		274
$\pi^+ \pi^- e^+ e^- (\gamma)$	( 2.68±0.11) $\times 10^{-4}$		235
$e^+ e^- \mu^+ \mu^-$	< 1.6 $\times 10^{-4}$	CL=90%	253
2 $\mu^+ 2\mu^-$	( 5.0 ±1.3 ) $\times 10^{-9}$		161
$\mu^+ \mu^- \pi^+ \pi^-$	< 3.6 $\times 10^{-4}$	CL=90%	113
$\pi^+ e^- \bar{\nu}_e + \text{c.c.}$	< 1.7 $\times 10^{-4}$	CL=90%	256
$\pi^+ \pi^- 2\gamma$	< 2.1 $\times 10^{-3}$		236
$\pi^+ \pi^- \pi^0 \gamma$	< 6 $\times 10^{-4}$	CL=90%	174
$\pi^0 \mu^+ \mu^- \gamma$	< 3 $\times 10^{-6}$	CL=90%	210

[S. Navas et al. \(Particle Data Group\), to be published in Phys. Rev. D 110, 030001 \(2024\)](#)

# Structure of LHC Events: Conclusion

- ❖ **Hard Interaction**
- ❖ **Radiative Corrections**
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- ❖ **Hadron Decays**
- ❖ **Underlying Event**



# Sherpa 3.0.0 “Erebus”

- ❖ New major release of Sherpa happened this week
- ❖ Culmination of many years of work
- ❖ [Issue/feature request on gitlab page](#)



<https://sherpa-team.gitlab.io>



# Sherpa 3.0.0 “Erebus”

## Physics Improvements and New Capabilities

- NLO EW corrections for fixed-order predictions
- Polarized cross sections for massive vector bosons [!619](#)
- EW Sudakov logarithms ([!831](#), [!464](#))
- Improved scale setting for VBF processes [!723](#)
- Add RecoLa interface [!463](#), including but not limited to on-/off-shell  $W+W+jj$  production for both t-channel (VBS) and s-channel ( $W+W+W^-$ ) production [!849](#)
- Add model for instanton production ([!813](#), [!476](#), [!447](#), [!366](#))
- Photoproduction at MC@NLO accuracy [!448](#)
- New MPI and MinBias modelling including photon MPIs [!687](#)
- Photon splittings in YFS resummation ([!641](#), [!660](#), [!663](#))
- YFS for initial state radiation in lepton-lepton collisions [!830](#)
- Add simple color reconnection model ([!361](#), [!201](#), [!173](#))
- Upgrade embedded Lund hadronisation implementation from Pythia 6 to Pythia 8 [!774](#)
- Implement UFO 2.0 support for handling form factors [!834](#)
- New on-the-fly variations for the merging cut (QCUT) parameter and for  $\text{AlphS(MZ)}$ . The latter can now be used instead of relying on implicit variations via PDF variations, e.g. to vary the strong coupling for lepton collider setups [!295](#)
- Additional new ME interfaces for external generators [!419](#)
  - Support for the MCFM BLHA interface
  - Support for MadLoop
  - Comix/Amegic programmatic interface to external generators
  - Support for resummation and full / leading color ME corrections
- First version of BLHA2 support (tested with GoSam for  $ee \rightarrow bb$ ) [!390](#)



# YAML Input

```
(run){
  # general settings
  EVENTS 25000;

  # collider setup
  BEAM_1 2212; BEAM_ENERGY_1 6500;
  BEAM_2 2212; BEAM_ENERGY_2 6500;

  # generator parameters
  WIDTH[6] 0;

  # scale choice
  CORE_SCALE VAR{sqr(175)};
  HARD_DECAYS=1
  WIDTH[6] 0;

  # settings to accelerate event generation
  MI_HANDLER None;
  FRAGMENTATION Off;
}(run);

(processes){
  Process 93 93 -> 6 -6 93{1};
  CKKW sqr(20/E_CMS);
  Order (*,0);
  End process;
}(processes);
```

```
# collider setup
BEAMS: 2212
BEAM_ENERGIES: 6500

# scales
EXCLUSIVE_CLUSTER_MODE: 1
MEPS:
  CORE_SCALE: TTBar

# me generator settings
ME_GENERATORS: [Comix, Amegid]

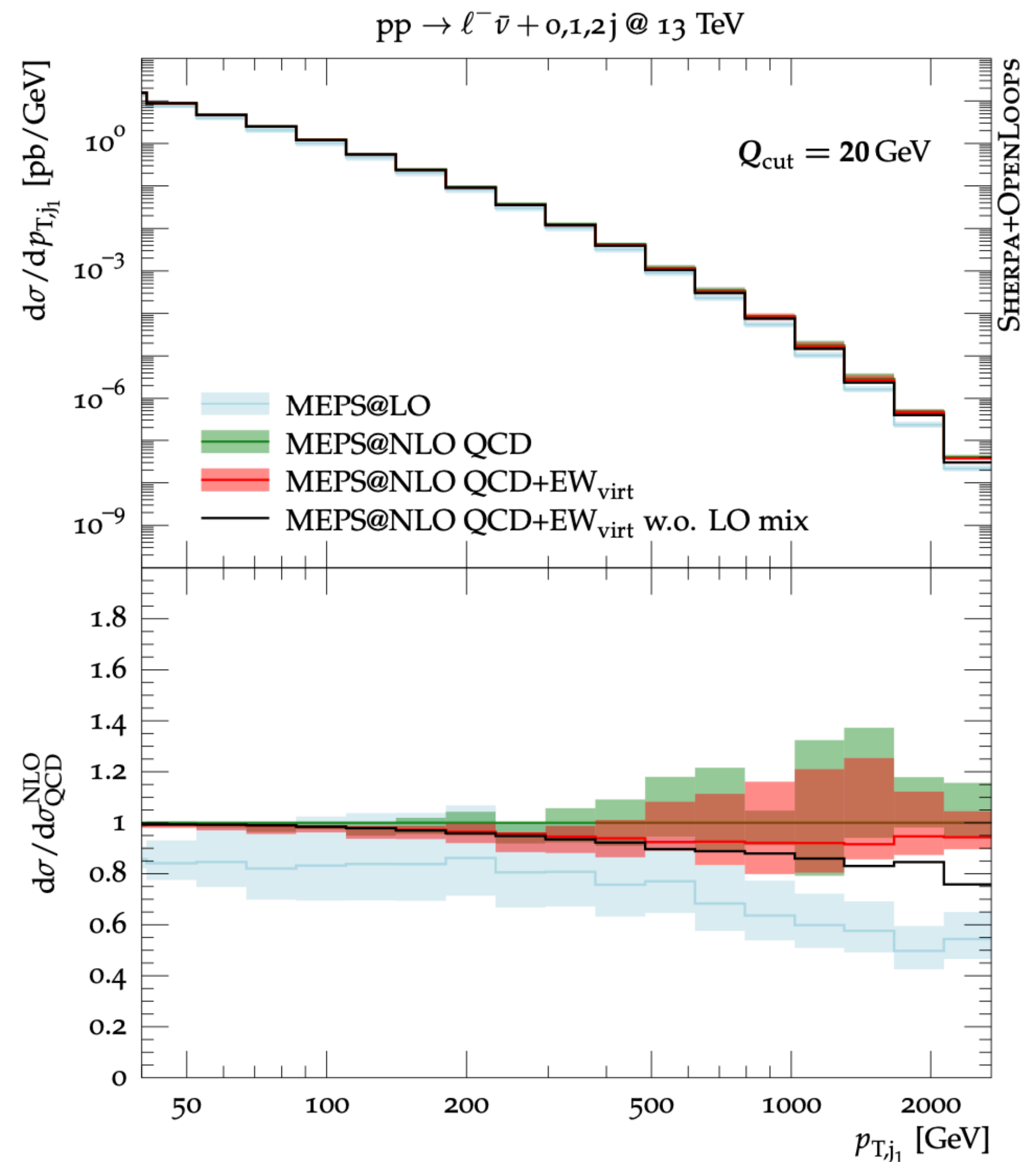
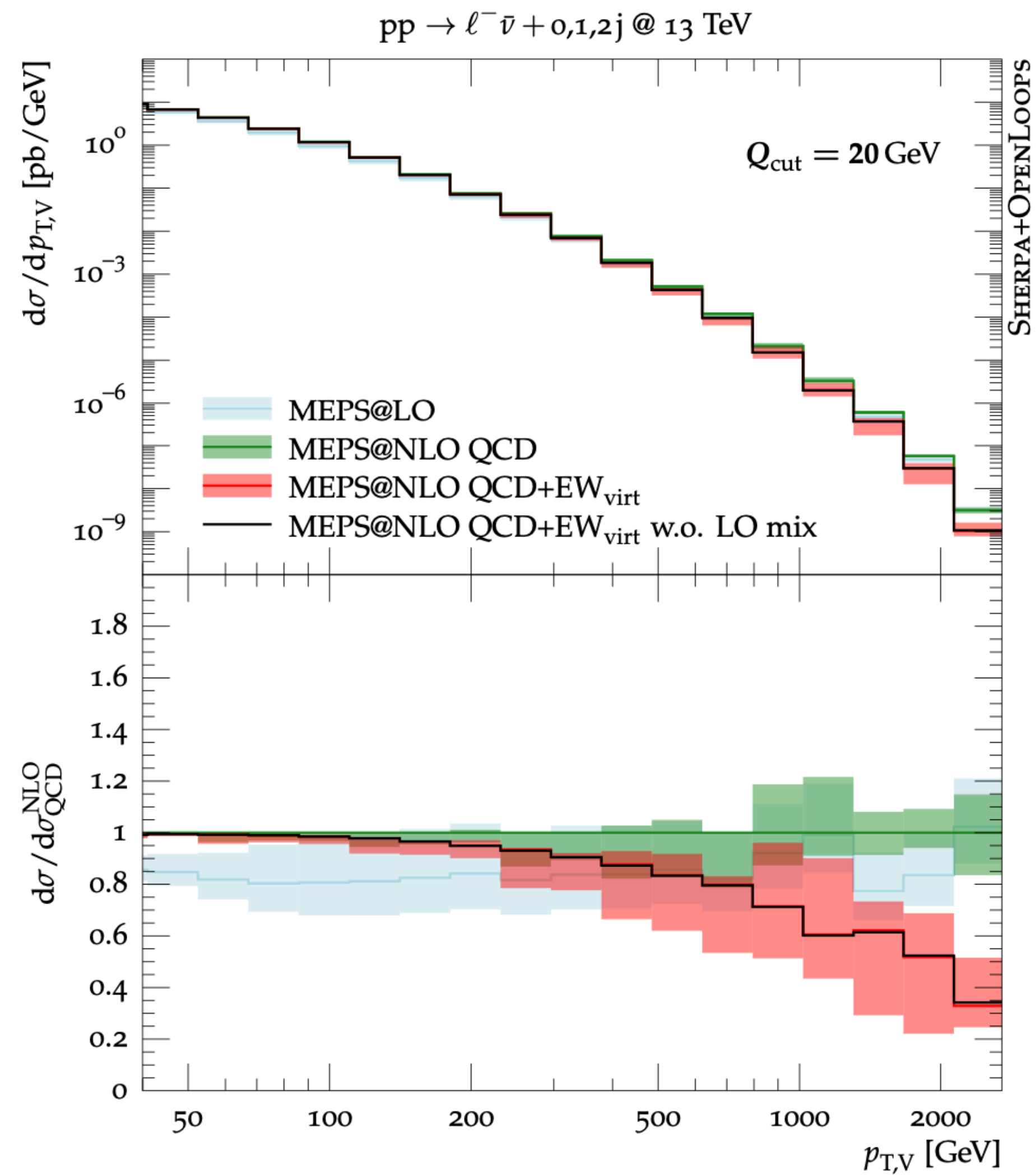
# decays
HARD_DECAYS:
  Enabled: true
  Channels:
    24,2,-1: {Status: 0}
    24,4,-3: {Status: 0}
    -24,-2,1: {Status: 0}
    -24,-4,3: {Status: 0}

PARTICLE_DATA:
  6: {Width: 0}

# on-the-fly variations
SCALE_VARIATIONS: 4.0* # 7-point scale variations

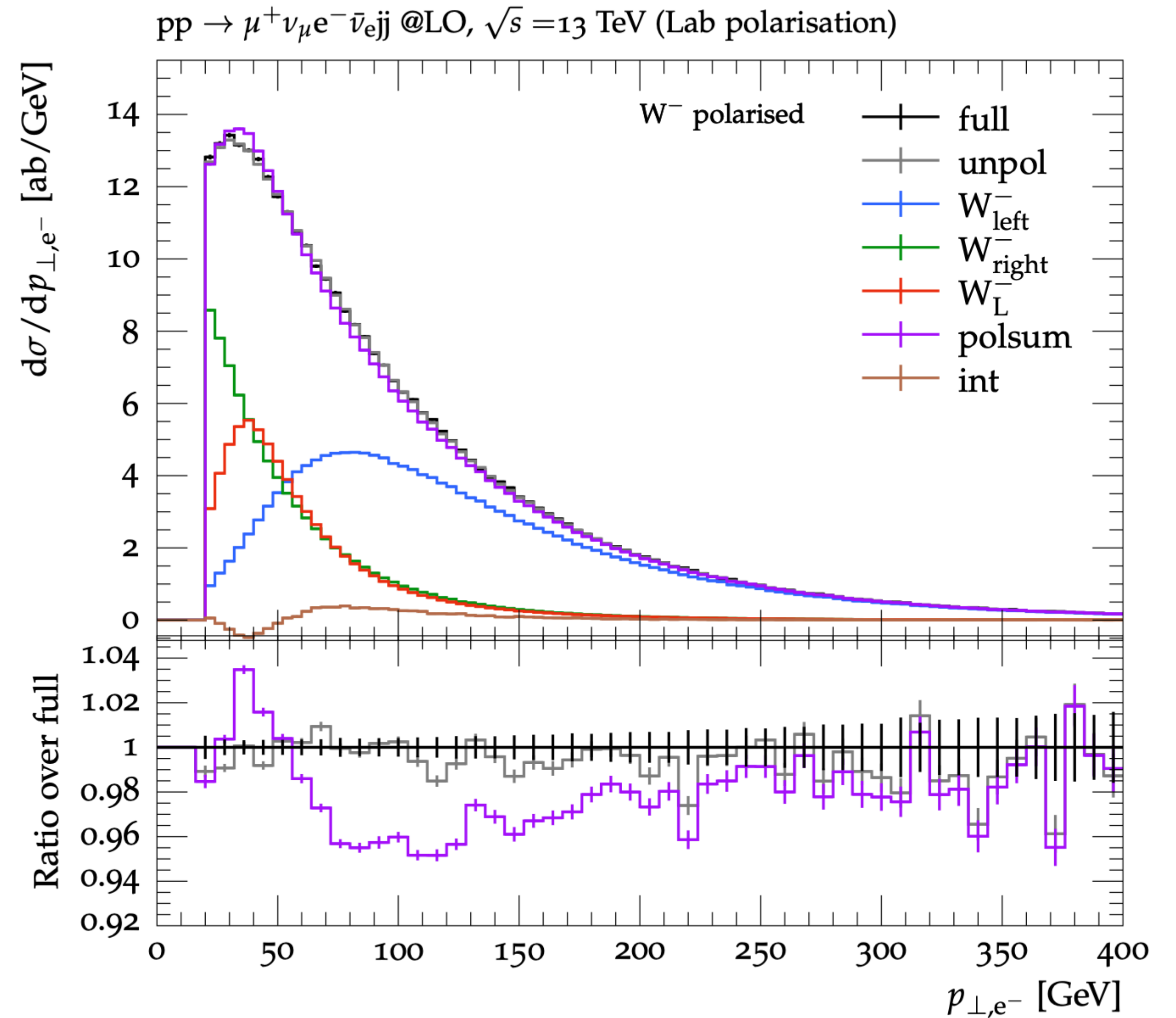
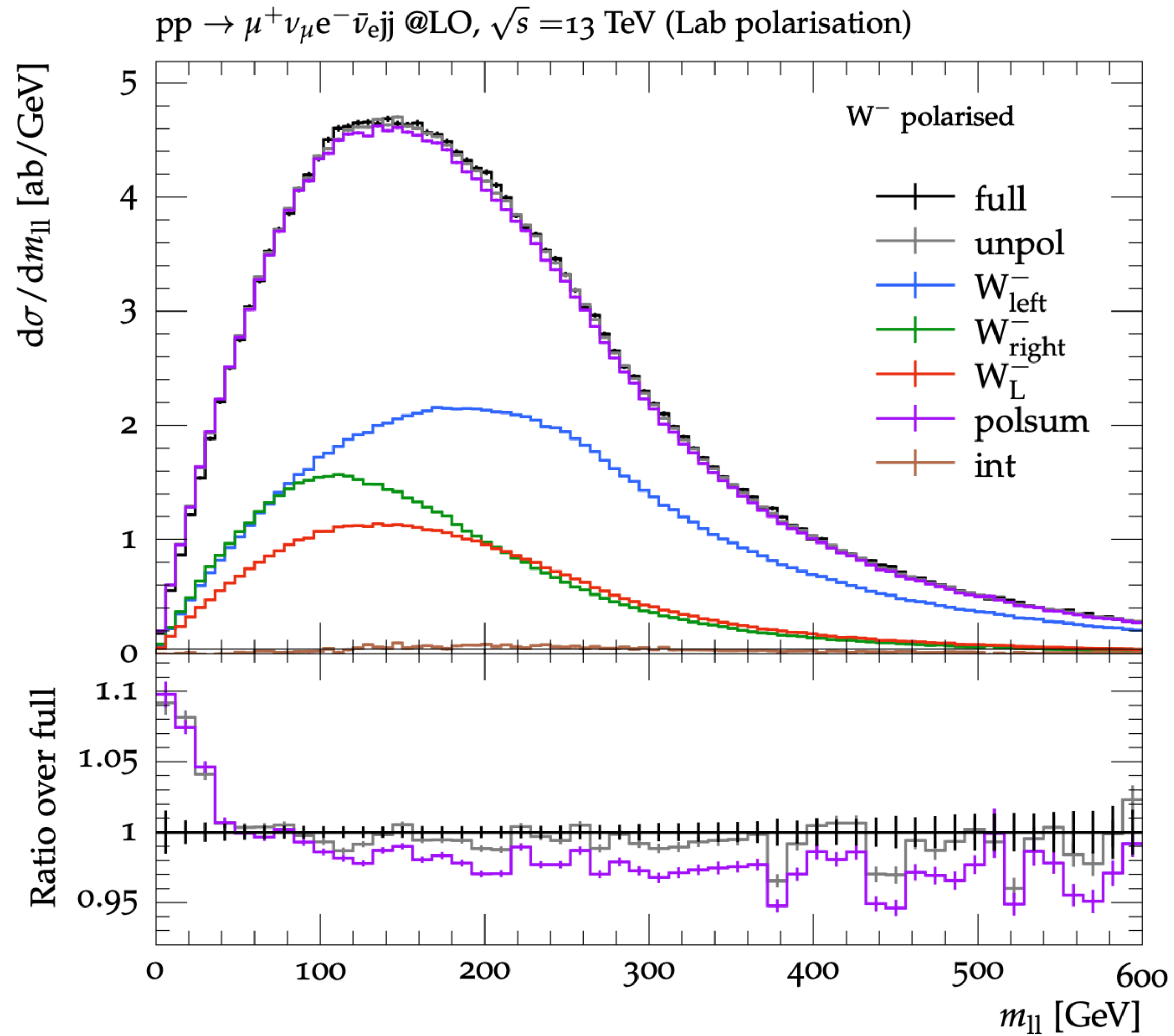
PROCESSES:
- 93 93 -> 6 -6 93{1}:
  Order: {QCD: 2, EW: 0}
  CKKW: 20
```

# EW Corrections



# Polarised Cross-Sections

Mareen Hoppe, Marek Schönherr, Frank Siegert



<https://arxiv.org/pdf/2310.14803>

- EW Sudakov approximation for parton showered event generation

