Eikonal scattering

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This lecture: meant as a background/introduction to make lectures on small-x, esp. Penttala, Iancu, more understandable.

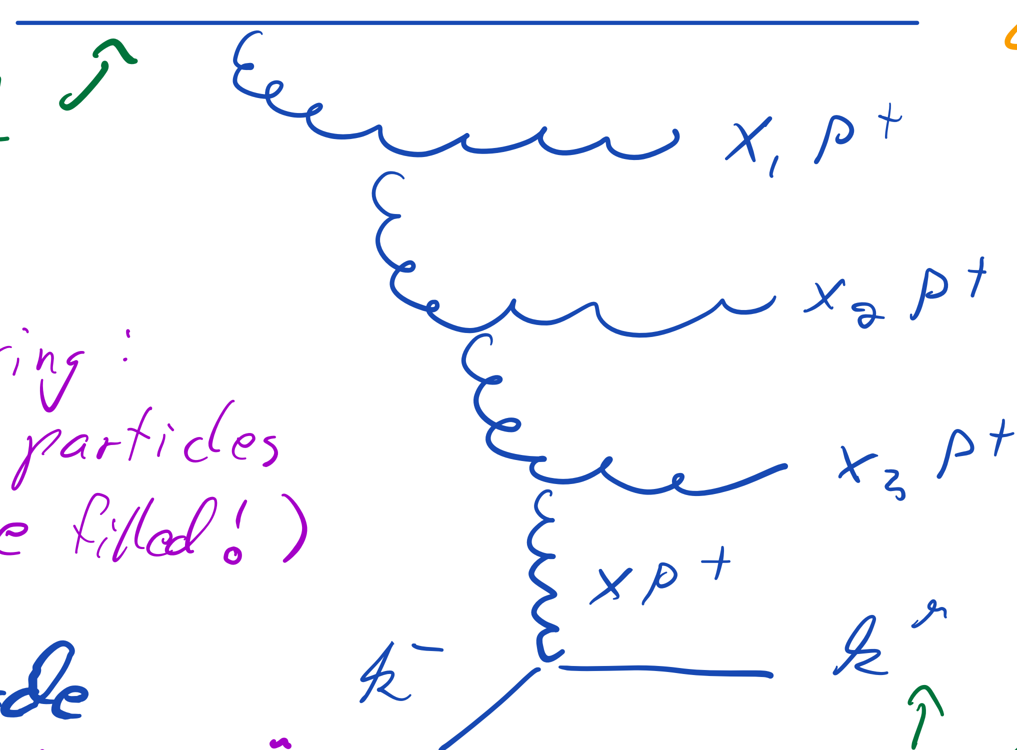
I will try to explain 3 things:

1. Why glue? We believe that high energy scattering in QCD is dominated by a large number of gluons = color field
2. Wilson line The light-like Wilson line is the eikonal (= high energy) scattering amplitude in a color field
3. Dipole picture When applied to Deep Inelastic (electron-proton) scattering, this leads to a picture where the virtual photon is a quark-antiquark dipole

High energy = small x

Light-cone variables $p^\pm = \frac{1}{\sqrt{2}}(p^0 \pm p^3)$ $\quad p^2 = 2p^+p^- - p_\perp^2$

High-energy p^+
right-moving colored
particle



Left-moving
particle

produced
particle
$\Rightarrow$ on shell

$\Leftarrow$ Picture: rapidity ordered

$1 \gg x_1 \gg x_2 \gg \ldots \gg x$

$s = 2p^+k^- \to \infty$

$k^2 = 0 = 2xp^+k^- - k_\perp^2 = 0$

$\Rightarrow \boxed{x = \frac{k_\perp^2}{s} \to 0}$

High-energy scattering:
cascade of virtual particles
(phase space likes to be filled!)

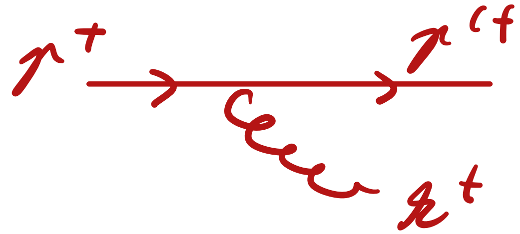
Emissions in cascade
prefer spin-1 "in t-channel"
$\Rightarrow$ gluons $\left(\sim \ln\frac{1}{x} \sim \frac{1}{\alpha_s}\right)$

QED different: no
$\gamma \to \gamma\gamma$ vertex

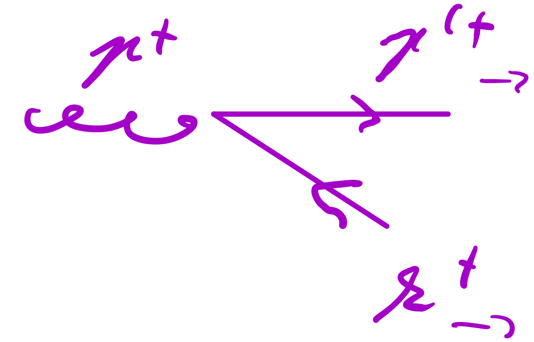
1. Many gluons $\to$ classical color field

Side note: why is emission of spin 1 favored?

Consider $q \to q g$



vs $g \to q\bar{q}$



in the soft limit $p^+ \simeq p'^+ \gg k^+$

(we want k^+ not too big because we are interested in the measured particle with $k_\perp^2 \ll s$ not carrying too much of the high energy s)

- Gluon is spin 1: needs to couple to a vector. In the limit $p^+ \gg k^+$ spin does not matter, and the only available vector is p^μ $\Rightarrow \mathcal{M}_g \sim p^+$ for $k^+ = cst$, $p^+ \to \infty$
- (Anti)quark is spin $1/2$: couples to "$\sqrt{\ }$ of vector" $\Rightarrow \mathcal{M}_{\bar{q}} \sim \sqrt{p^+}$ for $k^+ = cst$, $p^+ \to \infty$. Suppressed for small $\frac{k^+}{p^+}$.

  (actually $\mathcal{M} \sim \sqrt{k^+ p'^+}$)

Lots of gluons

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This is what the result of

$g \to gg \to ggg \to \ldots$

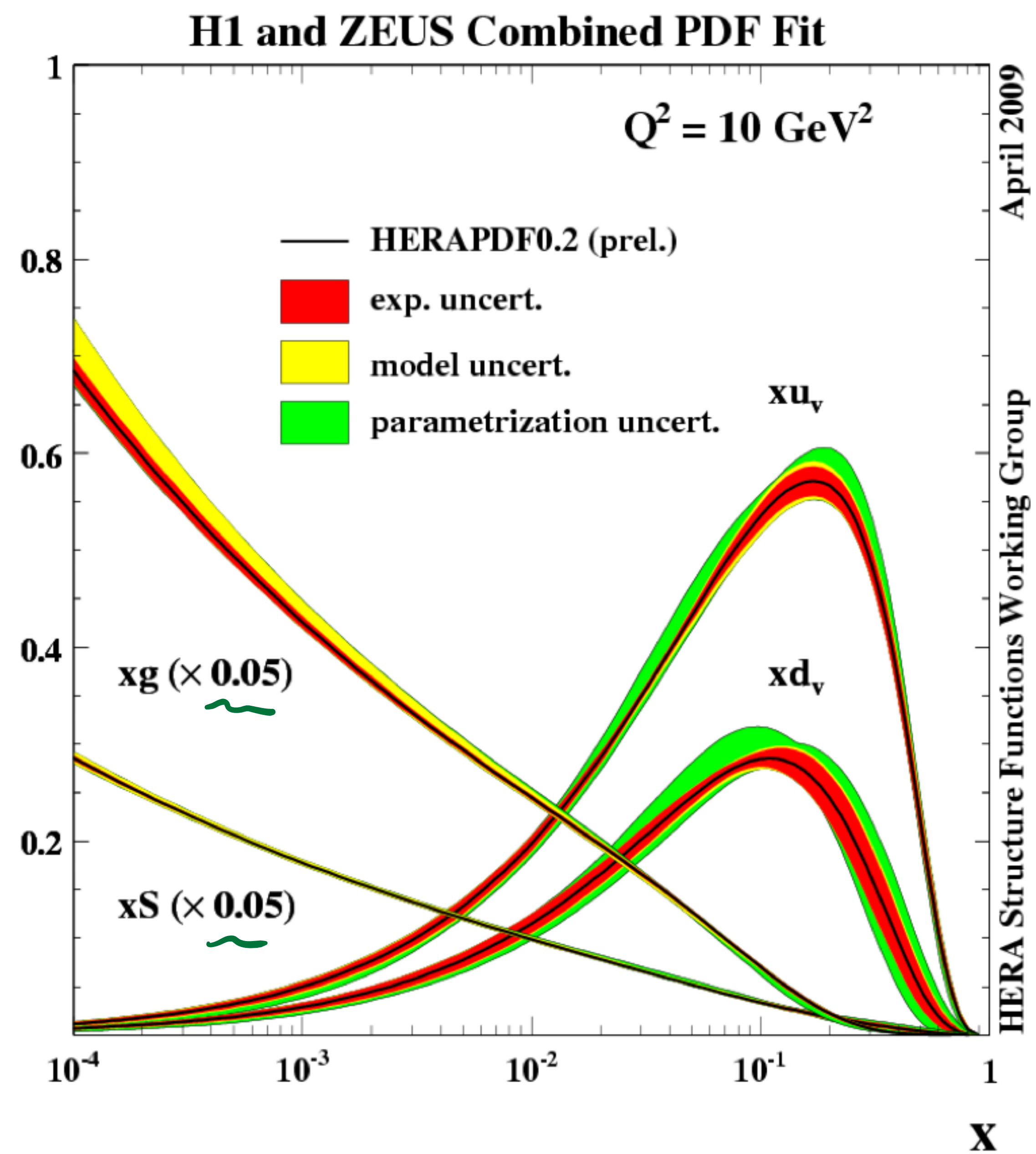
looks like.

Note power counting:

- $g \to gg$: $\alpha_s \ln \frac{1}{x} \sim 1$
- $g \to q\bar{q}$: α_s

Gluons drive dynamics

Sea quarks are $\sim \alpha_s$ correction



Scattering off classical field

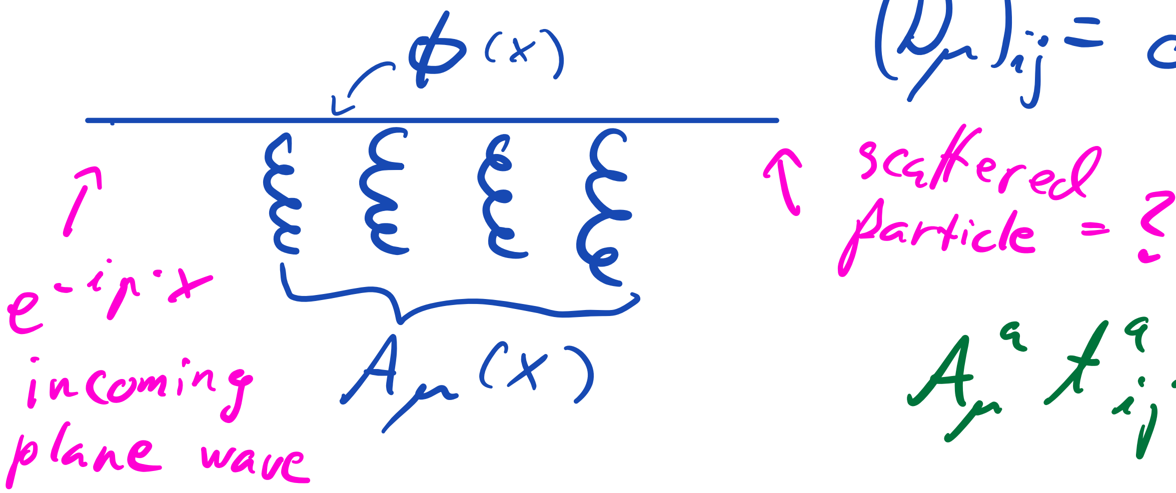
OK, we have a target of gluons. How does a colored particle (in some representation R of $SU(N_c)$) scatter off it?

E.O.M. (equation of motion) $\quad \underbrace{D_\mu}_{} D^\mu \underbrace{\phi(x)}_{\text{wave function of particle}} = 0 \quad$ (mass $=0$)

$$D_\mu = \partial_\mu - ig A_\mu$$

Full color structure $\phi_i \quad i = 1 \ldots d_R$ = dimension of representation

$$(D_\mu)_{ij} = \delta_{ij}\partial_\mu - ig A_\mu^a \underbrace{t^a_{ij}}_{\text{generator of reps. } R} \qquad a = 1 \ldots N_c^2 - 1, \quad i,j = 1 \ldots d_R$$



$$A_\mu^a t^a_{ij} = A_\mu \leftarrow d_R \times d_R \text{ matrix!}$$

Ansatz & high energy limit

Without bkg field solution is $e^{-ip^+x^-}$: plane wave.
$p^+ \gg$ any other scale $\Rightarrow$ expect $\phi(x) \sim e^{-ip^+x^-} \cdot$ [slowly varying function]

Ansatz $\phi(x) = e^{-ip^+x^-}\varphi(x)$, $\quad \dfrac{\partial_\mu \varphi}{\varphi} \ll p^+$

$$(\partial_\mu - igA_\mu)(\partial^\mu - igA^\mu)\left(e^{-ip^+x^-}\varphi(x)\right) = 0$$

derivative can act on either

$\partial_\mu\partial^\mu = 2\partial_+\partial_- - \nabla_\perp^2$
↳ no $(p^+)^2$ - term

$A_+ = A^-$!

Leading term: $\quad 2e^{-ip^+x^-}p^+\left[\underbrace{(\partial_+ - igA_+)\varphi}_{=0\ !}\right] + \mathcal{O}((p^+)^0) = 0$

neglect these: eikonal

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Wilson line

$\partial_+ \varphi = igA^- \varphi \Rightarrow \varphi(x) = \underbrace{\mathbb{P}\, e^{ig\int_{-\infty}^{x^+} dy^+ A^-(x^-, y^+, x_\perp)}}\ \varphi_0$

$\mathbb{P}$ = path ordering

$\boxed{\text{g. Wilson line } V(x_\perp)}$

- φ_0 is d_R - component vector : color state of incoming particle
- $V(x_\perp)$ is $d_R \times d_R$ color matrix
- Path-ordering : $\mathbb{P}(A(x^+)B(y^+)) = \begin{cases} A(x^+)B(y^+) & \text{if } x^+ > y^+ \\ B(y^+)A(x^+) & \text{if } x^+ < y^+ \end{cases}$

  but better practical definition : $\mathbb{P}$ = defined by differential equation

- Eikonal $x_\perp$ does not change

$p_\perp$ does change, but p^+ so large that $x_\perp$ does not have time to change

Wave out $\sim V(x_\perp) \times$ [wave in] $\Rightarrow$ $\boxed{V(x_\perp) - 1 = \text{scattering amplitude}}$

Wavelengths

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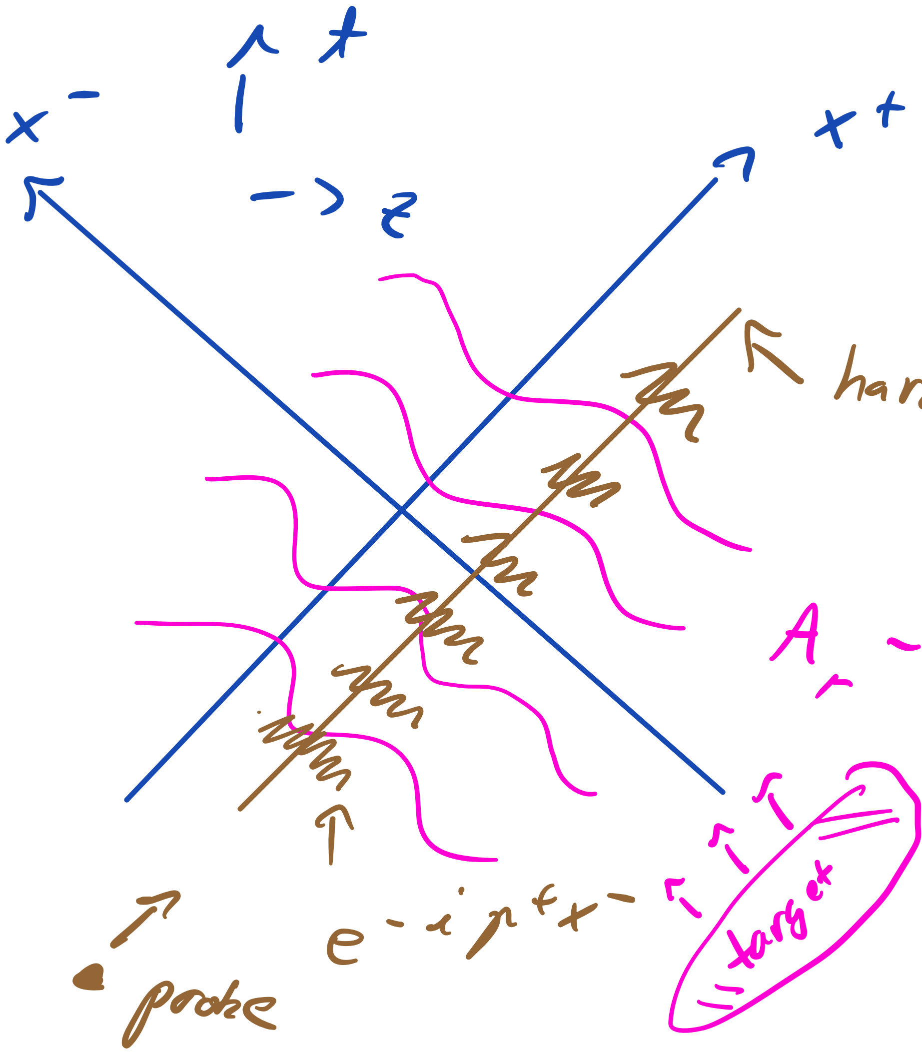
In principle $V(x_\perp, x^-)$, but hard particle $e^{-ip^+x^-}$ does not see x^--dependence

Neglect $\begin{cases} x^- = \text{target LC time dependence} \\ k^+\text{-transfer from target} \end{cases}$



hard particle path $\int dx^+$

A^- -> low k^+ gluons, small wavelength in x^-

Color (obvious)

Glass

Condensate

$gA \sim 1$

=> $V(x_\perp)$ everywhere allowed by $SU(N_c)$)

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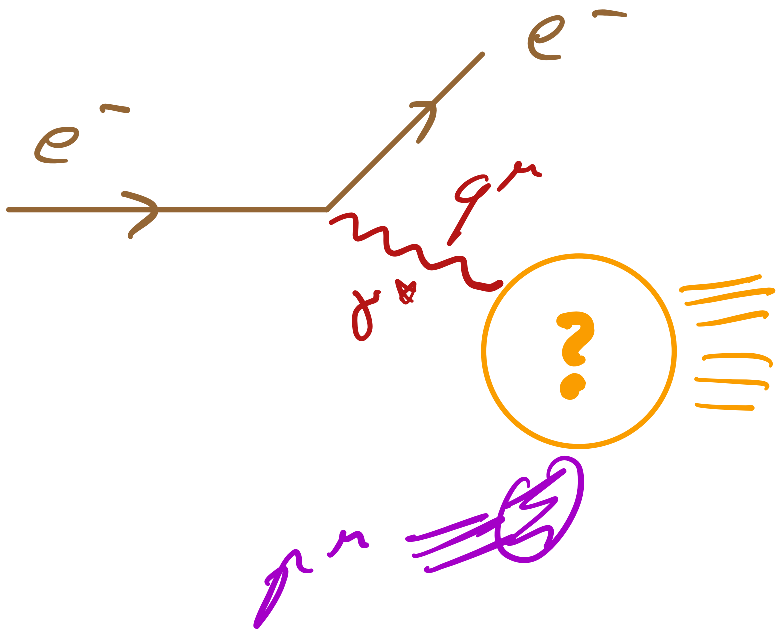
Deep Inelastic Scattering off glue



γ^* does not interact with g, only with $q/\bar{q}$

- Collinear / IMF / parton picture: g fluctuates to $q\bar{q} \to \gamma^*$ scatter

- 3. "Dipole picture": $\gamma^* \to q\bar{q}$, then $q\bar{q}$ scatters off glue

Cross-section is Lorentz-invariant, but physical picture not
proton $\ni$ partons, hit by γ^* <–> $\gamma^* \ni$ partons, hit by gluon
shockwave

Dipole picture

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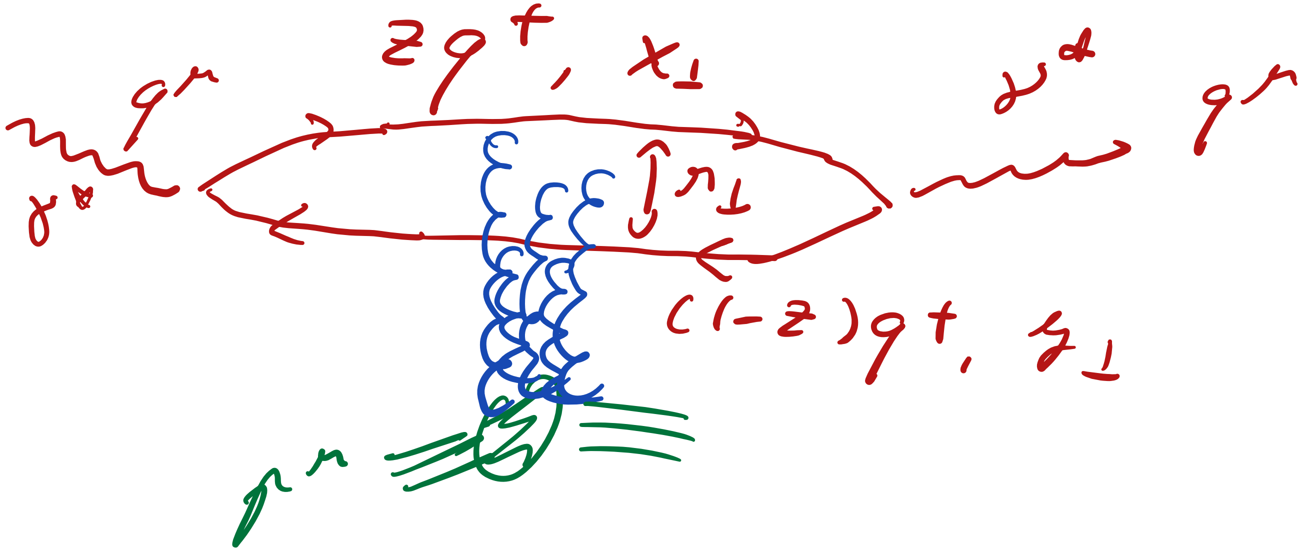


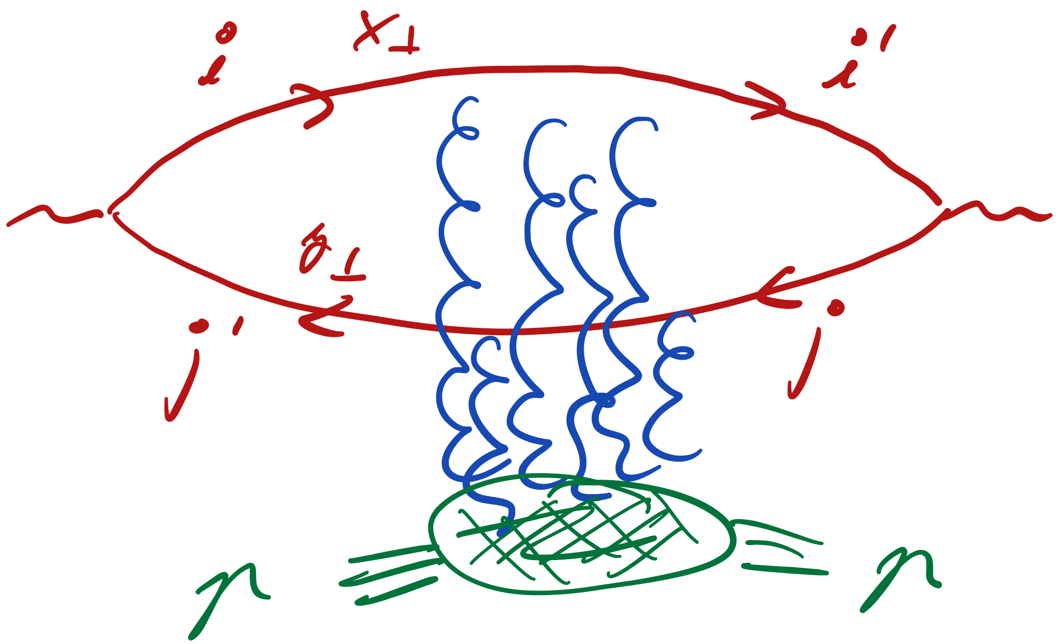
Optical thm:
$\sigma_{tot} \sim 2 \times \mathrm{Im}\, A_{el}$

$$\sigma_{tot}^{\gamma^* + p/A} = \int_0^1 dz \int d^2 r_\perp \underbrace{\left| \psi^{\gamma^* \to q\bar{q}}(z, r_\perp) \right|^2}\ \underbrace{2 A_{q\bar{q}}(r_\perp)}$$

$\gamma^* \to q\bar{q}$ light cone wave function $\Rightarrow$ Enforces $r_\perp \sim \frac{1}{Q}$

Dipole amplitude: $q\bar{q}$ interacts with color field

Dipole amplitude



$S = 1 - A \iff A = 1 - S$

- γ^* color neutral: $\overbrace{\frac{1}{\sqrt{N_c}}\delta_{ij'}}^{\text{normalized}}\ \overbrace{\frac{1}{\sqrt{N_c}}\delta_{i'j}}^{\text{projector}}$
- q S-matrix: Wilson line $V_{i'i}(x_\perp)$
- $\bar{q}$ S-matrix: conjugate $V^*_{j'j}(y_\perp) = V^\dagger_{jj'}(y_\perp)$

Dipole amplitude $A_{q\bar q} = 1 - S_{q\bar q} = 1 - \left\langle \frac{1}{N_c}\delta_{ij'}\delta_{i'j}\, V_{i'i}\, V^\dagger_{j'j} \right\rangle$

$$\boxed{= 1 - \frac{1}{N_c}\left\langle \mathrm{Tr}\, V(x_\perp)\, V^\dagger(y_\perp) \right\rangle}$$

$\langle\ \rangle$ = average over target state

Saturation

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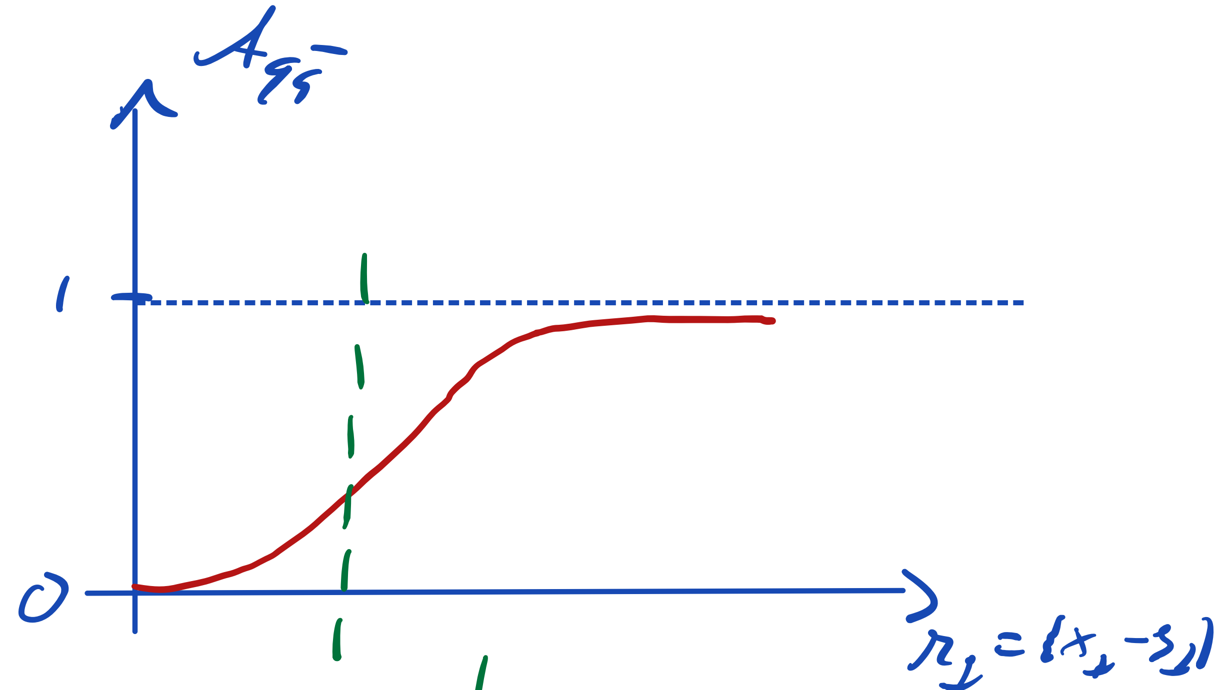
$$A_{q\bar{q}}(x_\perp - y_\perp) = 1 - \frac{1}{N_c}\left\langle \mathrm{Tr}\, V(x_\perp) V^\dagger(y_\perp) \right\rangle$$

- $x_\perp = y_\perp \Rightarrow VV^\dagger = \mathbb{1} \Rightarrow A_{q\bar{q}} = 0$

  Size 0 color neutral object does not interact



$r_s \sim \frac{1}{Q_s} \leftarrow$ saturation scale

= transition from pert → nonpert

~ 2-gluon → multigluon x-change

- $A_{q\bar{q}}(r_\perp) \sim \alpha_s \times G(x, Q^2) r_\perp^2$

  Color transparency

  = the perturbative behavior,

  2-gluon exchange

- $|x_\perp - y_\perp| \to \infty \quad A_{q\bar{q}} \to 1$ Saturation

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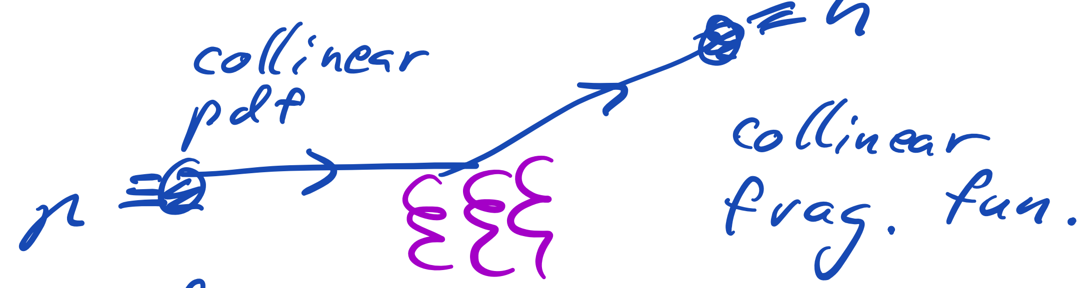
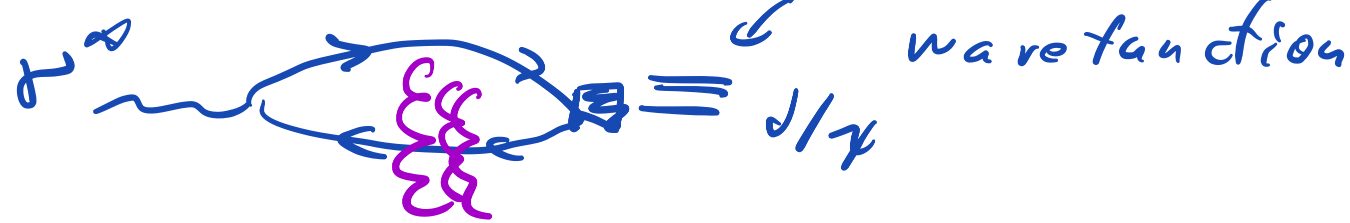
Ways forward

- Fwd rapidity pA:

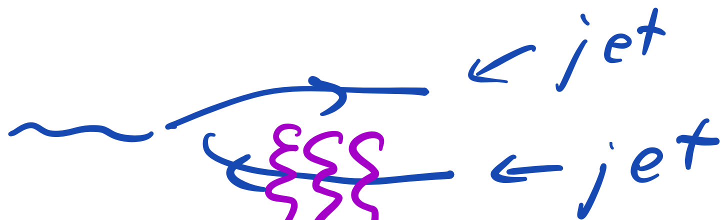


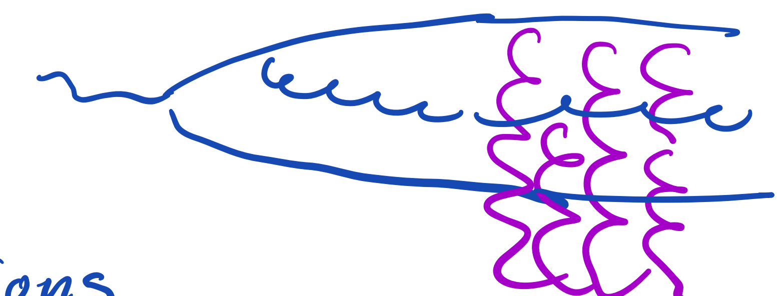
  "dipole": quark in amplitude
  + antiquark in conjugate

- Exclusive scattering
  (Penttala)



- Fwd dijets
  (Iancu)





- Add $q\bar{q} + q\bar{q}g$ → NLO cross sections
  (Venugopalan, Penttala, Iancu)
  ↓
  BK/JIMWK/BFKL evolution
  (Venugopalan, Chachamis)