QCD and PDFs

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ERC adG YoctoLHC

These lectures will …

- explain main theoretical and experimental results leading to development of quantum chromodynamics (QCD) and outline its concepts
- teach you how to calculate scaling violations for parton distribution functions (PDFs)
- give a taste of rich phenomenology of PDFs

Plan of lectures:

- **Lecture 1**: The quark model, deep inelastic scattering (DIS), the parton model, main concepts of quantum chromodynamics (QCD)
- **Lecture 2**: Scaling violations in QCD, DGLAP evolution equations, factorization theorem
- **Lecture 3**: Phenomenology of proton and photon PDFs

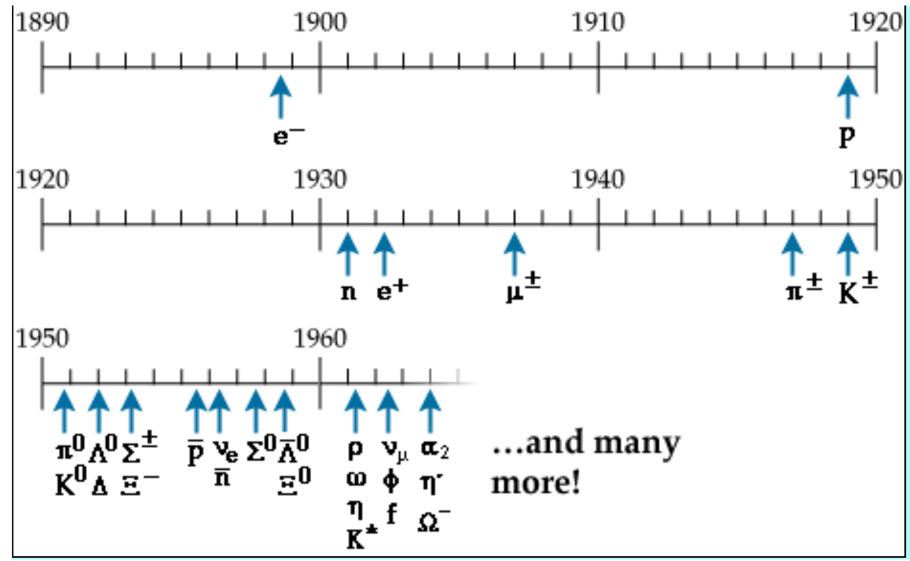
Literature:

- **Lecture 1**: Halzen, Martin, Quarks and Leptons: An Introductory Course in Modern Particle Physics (1984); Kronfeld. Quigg, “Resource Letter: Quantum Chromodynamics”, arXiv:1002.5032 [hep-ph]; Gross, Klempt et al. “50 Years of Quantum Chromodynamics”, Eur. Phys. J C (2023) 1125
- **Lecture 2**: Dokshitzer, Diakonov, Troian, “Hard Processes in Quantum Chromodynamics”, Phys. Rept. 58 (1980) 269; Sterman et al., “Handbook of perturbative QCD”, Rev. Mod. Phys. 67 (1995) 157-248
- **Lecture 3**: Aschenauer, Thorne, Yoshida, “Structure functions”, Review of Particle Physics, Particle Data Group; Nisius, Phys. Rept. 332 (2000) 165-317 [arXiv:hep-ex/9912049].

Flavor symmetry of strong interaction

• 50s–60s: Discovery of many new baryons and mesons (hadrons)

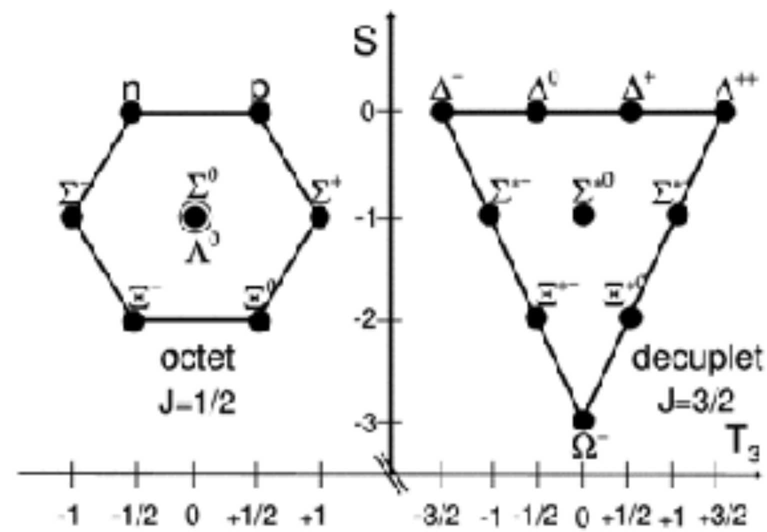
• Interaction is mediated by the strong interaction (nuclear force) acting at short distances ~ 1 fermi=10^{-15} m and short times ~ 10^{-23} s



• Various theoretical approaches (Regge poles, current algebra, Yukawa interactions) → impossible to construct quantum field theory of strong interactions.

• 1961, Gell-Mann, Ne'eman, The Eightfold Way: classification of hadrons using approximate flavor SU(3) symmetry → all hadrons grouped into multiplets

• → prediction of Ω^- confirmed in 1964
• → Gell-Mann-Okubo mass formula
• → idea of quarks



The quark model

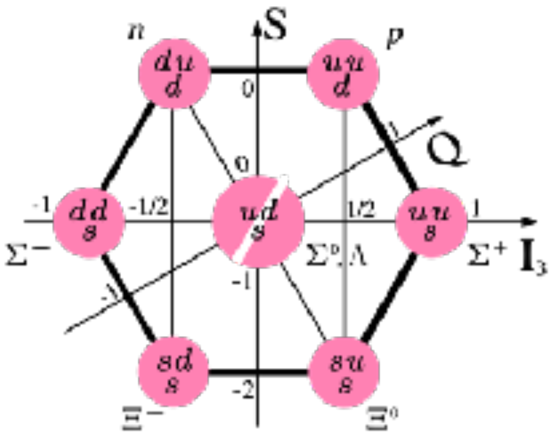
- 1964, Gell-Mann and Zweig: hadrons are made up of 3 quarks forming the fundamental representation of flavor SU(3) $\rightarrow q(u,d,s) = \mathbf{3}$ and $\bar{q}(\bar{u},\bar{d},\bar{s}) = \bar{\mathbf{3}}$.

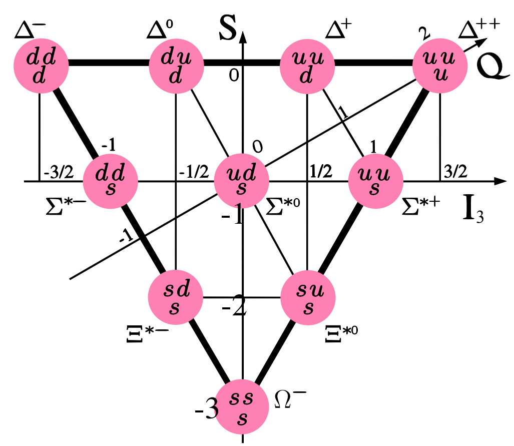
- Quarks have fractional electric charges (2/3,-1/3,-1/3), baryon number B=1/3, spin 1/2, isospin I_z=(1/2,-1/2,0), and strangeness S=(0,0,-1).

- Mesons are $q\bar{q}$ bound states: $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1} \rightarrow$ flavor octets and singlets with allowed quantum numbers $J^{PC} = (0^{-+}, 1^{--}, \ldots)$

- Baryons are qqq states: $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_{M_S} \oplus \mathbf{8}_{M_A} \oplus \mathbf{1}_A$

- Spin content of baryon multiples: $\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2} = \mathbf{4}_S \oplus \mathbf{2}_{M_S} \oplus \mathbf{2}_{M_A} \rightarrow$ S=3/2, 1/2





- Reasonable description of baryon static properties, e.g., the ratio of neutron and proton magnetic moments: $\mu_n/\mu_p = -2/3$ vs. $\mu_n/\mu_p(\text{exp.}) = -0.68497945(58)$

Quarks and color

- Initially quarks treated as fictional due to non-observation of free particles with a fractional charge.

- Second challenge: problem with Fermi statistics since decuplet ground-state wave function appears to be symmetric in space × flavor × spin: $\Delta^{++} = uuu$ or $\Omega^{-} = sss$.

- 1964/65, Greenberg; Han, Nambu; Fritzsch, Gell-Mann: quarks carry an additive quantum number color → need at least $N_c=3$ colors (red, green, blue) for the baryon wave function to be antisymmetric:

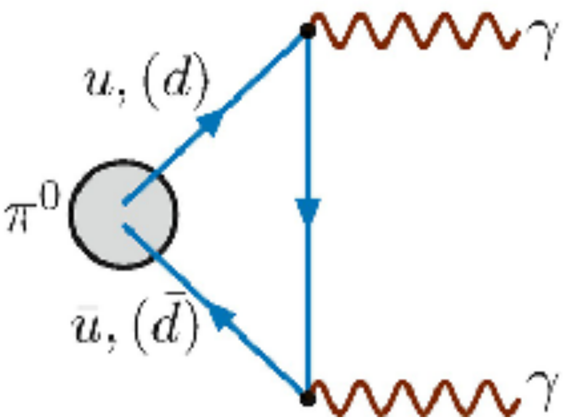
$$\Psi(q_1, q_2, q_2) = \Psi_{\text{space}}(x_1, x_2, x_3)\Psi_{\text{flavor}}(f_1, f_2, f_3)\Psi_{\text{spin}}(s_1, s_2, s_3)\Psi_{\text{color}}(c_1, c_2, c_3)$$

$$\Psi_{\text{color}}(c_1, c_2, c_3) = \frac{1}{\sqrt{6}}(\text{RGB} - \text{GRB} + \text{GBR} - \text{RGB} + \text{BRG} - \text{BGR})$$

- Experimental evidence of color-triplet quark model:

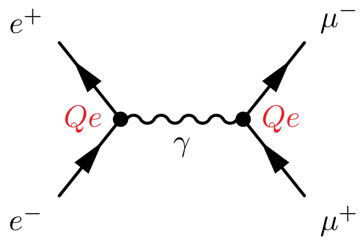
$\pi^0 \to \gamma\gamma$ decay: $\Gamma = \frac{\alpha^2}{2\pi}\frac{N_c^2}{3^3}\frac{m_\pi^3}{f_\pi^2} = 7.75 \text{ eV}$

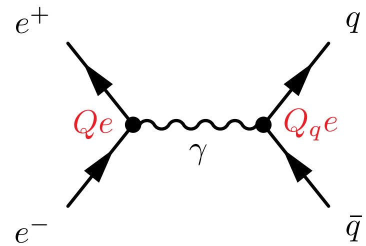
vs. $\Gamma(\text{exp.}) = (7.86 \pm 0.54) \text{ eV}$



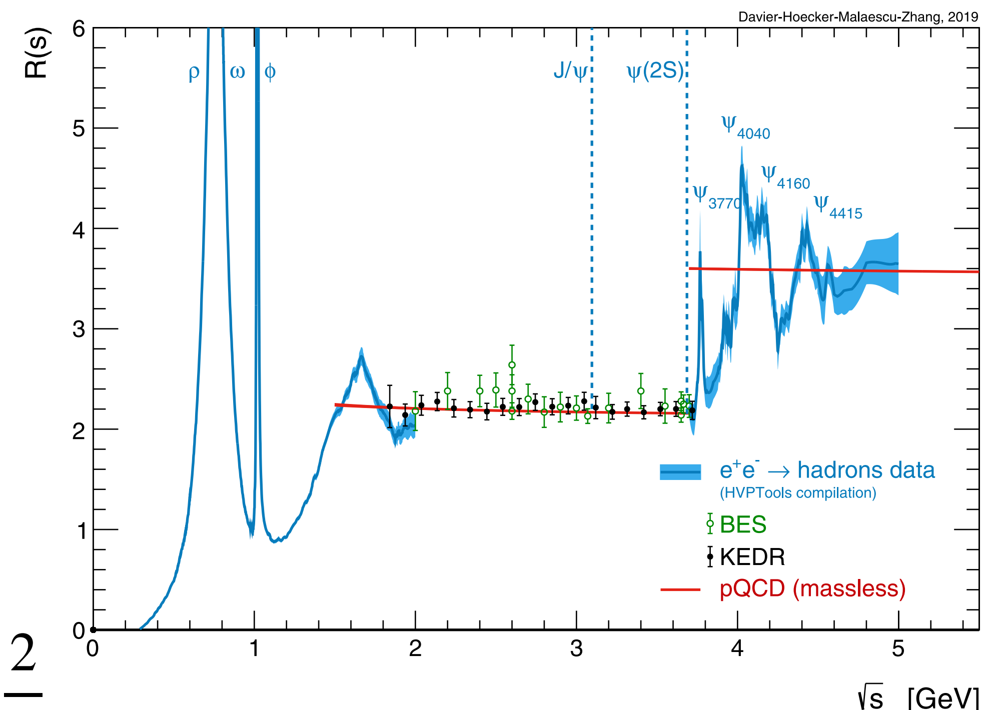
e^+e^- annihilation and R-factor

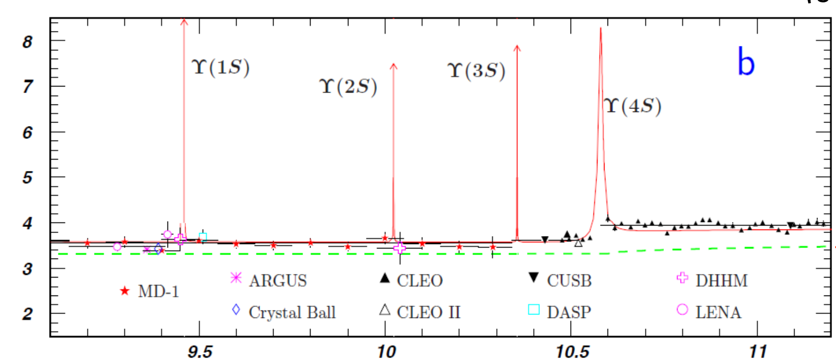
- R-factor for e^+e^- annihilation to hadrons: $R = \dfrac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_q Q_q^2$





- $s = (p_{e^+} + p_{e^-})^2$ invariant center-of mass energy squared
- $\sqrt{s} > m_s \to R = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 2$
- $\sqrt{s} > m_c \to R = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9}\right) = 3\frac{1}{3}$
- $\sqrt{s} > m_b \to R = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9}\right) = 3\frac{2}{3}$
- $\to$ discovery of charm quark in 1974 and bottomonia in 1978 (bottom quark discovered in pA at Fermilab in 1977).

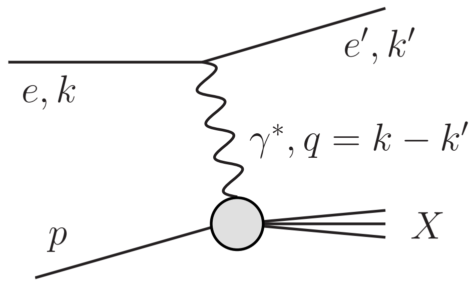


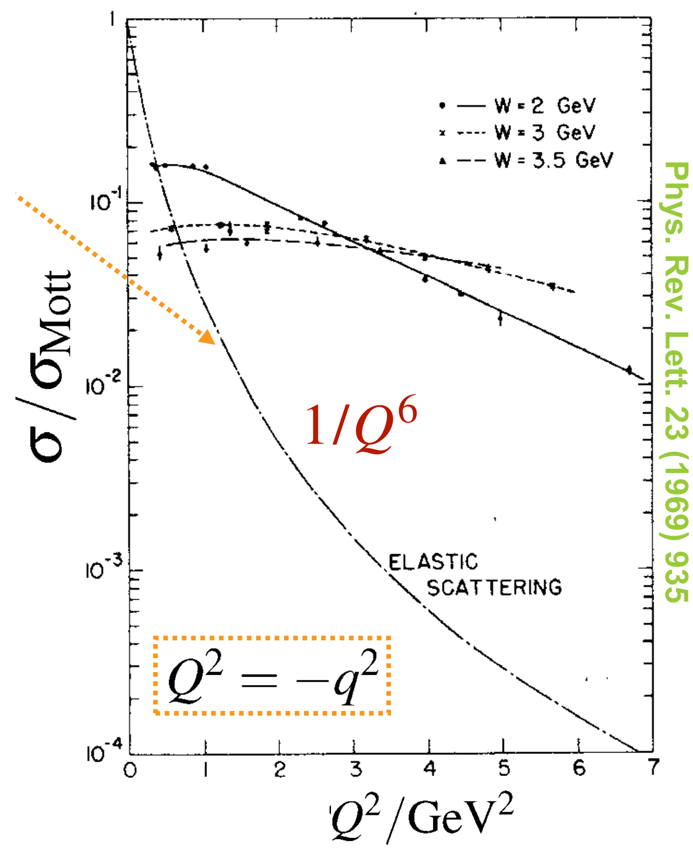


Deep inelastic scattering and Bjorken scaling

• Proton has internal structure: magnetic moment $\mu_p/\mu_N = 2.79284734463(82) \neq 2$ (Stern, Nobel Prize 1943) and elastic form factors (Hofstadter, Nobel Prize 1961).

• 1968/69, SLAC-MIT experiments on deeply inelastic electron-nucleon scattering:





• Photon virtuality $Q^2 = -q^2 = 4EE' \sin^2 \frac{\theta}{2}$

• Photon energy $\nu = E - E'$

• Invariant energy $W^2 = (q+p)^2 \approx 2\nu m_p$

• Bjorken variable $x = \frac{Q^2}{2(p \cdot q)} = \frac{Q^2}{2m_p(E - E')}$

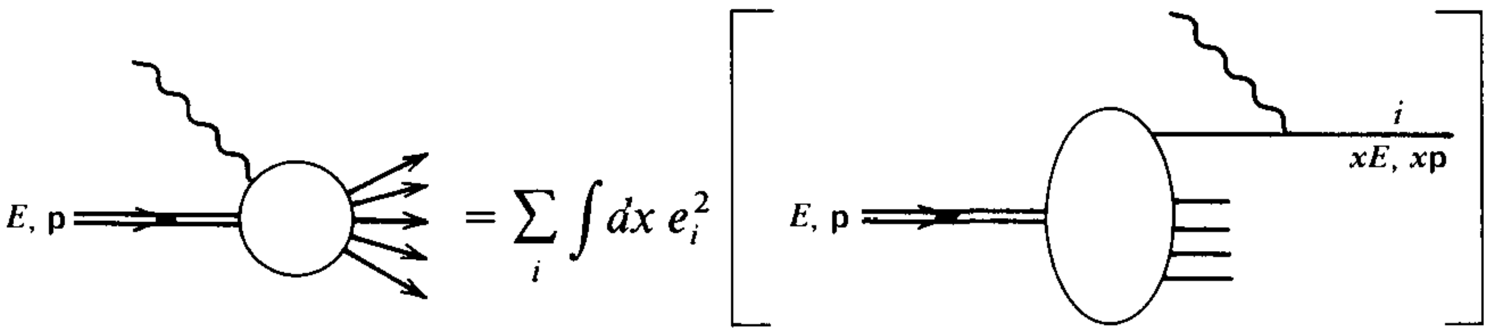
• Bjorken limit: Q^2, W are large and x is fixed.

• For large scattering angles θ (large Q^2) $\rightarrow \sigma/\sigma_{\text{Mott}}$ is much larger than given by elastic form factors and scales, i.e., depends only on x and not on (x, Q^2).

• 1968, Bjorken: Theoretical prediction of the scaling using current algebra.

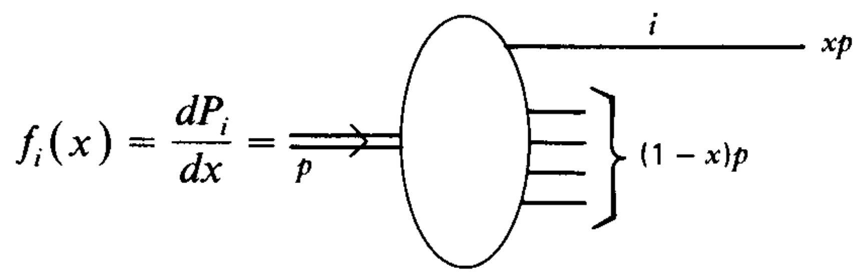
The parton model

• 1969, Feynman: Bjorken scaling of inelastic scattering off proton can be interpreted as elastic scattering off point-like constituents of the proton, partons.



• In frame, where the proton has very high energy (infinite momentum frame), proton = collection of collinear massless partons carrying momentum fraction x of the parent proton.

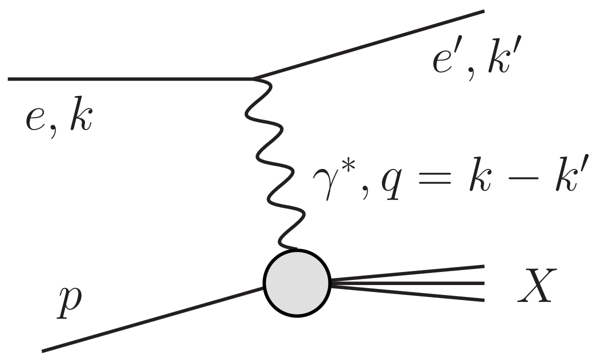
• Parton distribution function (PDF) $f_i(x)$ is the probability to find parton i that carries a momentum fraction x.



• Momentum sum rule: $\sum_i \int_0^1 dx x f_i(x) = 1$

DIS and structure functions (1/3)

- Electron-proton deep inelastic scattering (DIS)



- Photon virtuality $Q^2 = -q^2 = -(k-k')^2$
- Bjorken variable $x = \dfrac{Q^2}{2(p \cdot q)}$
- Momentum fraction carried by photon $y = \dfrac{(p \cdot q)}{(p \cdot k)}$

- Scattering amplitude for this graph: $\mathscr{M} = \dfrac{e^2}{q^2}\bar{u}(k')\gamma^\mu u(k)\langle X | J_\mu(0) | p\rangle$

- DIS cross section: $d\sigma = \dfrac{|\overline{\mathscr{M}}|^2}{4\sqrt{(k \cdot p)^2}} dLips$

- Lorentz invariant phase space:

$$dLips = (2\pi)^4 \delta^4(k + p - k' - p_X)\frac{d^3\overrightarrow{k'}}{2E_{k'}(2\pi)^3}\sum_X \frac{d^3\overrightarrow{p_X}}{2E_{p_X}(2\pi)^3}$$

- Squaring the amplitude: $d\sigma = \dfrac{16\pi^3\alpha^2 m_p}{q^4(k \cdot p)} L^{\mu\nu} W_{\mu\nu} \dfrac{d^3\overrightarrow{k'}}{2E_{k'}(2\pi)^3}$

DIS and structure functions (2/3)

- Leptonic tensor:

$$L^{\mu\nu} = \frac{1}{2}\sum_{\text{pol}} \bar{u}(k)\gamma^{\nu}u(k')\bar{u}(k')\gamma^{\mu}u(k) = \frac{1}{2}Tr(\hat{k}'\gamma^{\mu}\hat{k}\gamma^{\nu}) = 2\left(k'^{\mu}k^{\nu} + k'^{\nu}k^{\mu} - g^{\mu\nu}(k'\cdot k)\right)$$

- Hadronic tensor:

$$4\pi m_p W_{\mu\nu} = \frac{1}{2}\sum_{\text{pol}}\sum_{X}(2\pi)^4\delta^4(k+p-k'-p_X)\langle p\,|\,J_\nu(0)\,|\,X\rangle\langle X\,|\,J_\mu(0)\,|\,p\rangle\frac{d^3\overrightarrow{p_X}}{2E_{p_X}(2\pi)^3}$$

- In one-photon approximation for unpolarized DIS, 2 independent Lorentz structures respecting current conservation $q^{\mu}W_{\mu\nu} = 0$.

- Structure functions W_1 and W_2 parametrize composite structure of proton:

$$W_{\mu\nu} = -W_1\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \frac{W_2}{m_p^2}\left(p_\mu - \frac{(p\cdot q)}{q^2}q_\mu\right)\left(p_\nu - \frac{(p\cdot q)}{q^2}q_\nu\right)$$

- More common notation using structure functions $F_1 = m_p W_1$ and $F_2 = \nu W_2$:

$$W_{\mu\nu} = -\frac{F_1}{m_p}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \frac{F_2}{\nu m_p^2}\left(p_\mu - \frac{(p\cdot q)}{q^2}q_\mu\right)\left(p_\nu - \frac{(p\cdot q)}{q^2}q_\nu\right)$$

- $\longrightarrow F_1 = \left(-\frac{1}{2}g^{\mu\nu} + \frac{2x^2}{Q^2}p^{\mu}p^{\nu}\right)m_p W_{\mu\nu}$ and $\frac{F_2}{x} = \left(-g^{\mu\nu} + \frac{12x^2}{Q^2}p^{\mu}p^{\nu}\right)m_p W_{\mu\nu}$

DIS and structure functions (3/3)

- After some algebra: $L^{\mu\nu}W_{\mu\nu} = \frac{2Q^2}{m_p}F_1 + \frac{2Q^2}{m_p}F_2\frac{1}{xy^2}\left(1 - y - x^2y^2\frac{m_p^2}{Q^2}\right)$

- Phase space of scattered electron: $\frac{d^3\overrightarrow{k'}}{2E_{k'}(2\pi)^3} = \frac{1}{2(2\pi)^2}\frac{m_pE}{Q^2}y^2dxdQ^2$ using $(x, Q^2) \to (E', \cos\theta)$ transformation with Jakobian: $dxdQ^2 = \frac{Q^2}{m_p}\frac{1-y}{y^2}dE'd\cos\theta$

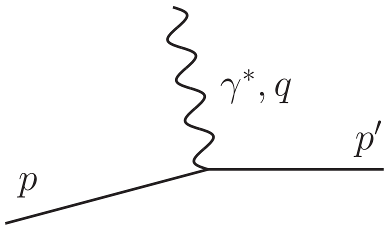
- Putting all factors together: $\frac{d\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\frac{1}{x}\left(xy^2F_1 + F_2\left(1 - y - \frac{x^2y^2m_p^2}{Q^2}\right)\right)$

- Reduced cross section: $d\sigma_r = \frac{xQ^4}{2\pi\alpha^2Y_+}\frac{d\sigma}{dxdQ^2} = F_2 - \frac{y^2}{Y_+}F_L$, where $Y_+ = 1 + (1-y)^2$

- Longitudinal structure function $F_L = F_2 - 2xF_1$ proportional to the cross section of longitudinally polarized photons.

DIS in quark parton model (1/2)

- In the parton model, DIS cross section is convolution of the cross section for scattering off a parton with its momentum distribution $d\sigma = \sum_q \int_0^1 d\xi \hat{\sigma}_0^q(\xi p) f_q(\xi)$
- At the level of hadronic tensors: $W_{\mu\nu} = \sum_q \int_0^1 \frac{d\xi}{\xi} \hat{W}^q_{\mu\nu} f_q(\xi)$
- Direct calculation of hadronic tensor for partons = spin-1/2 fermions of charge e_q:



Now p, p' refer to quark momenta

$$4\pi m_p \hat{W}^q_{\mu\nu} = \frac{e_q^2}{2} \sum_{\text{pol}} (2\pi)^4 \delta^4(p + q - p') \frac{d^3p'}{2E_{p'}(2\pi)^3} \bar{u}(p)\gamma_\nu u(p')\bar{u}(p')\gamma_\mu u(p)$$

- Sum over polarizations: $4\pi m_p \hat{W}^q_{\mu\nu} = \frac{e_q^2}{2}(2\pi)^4 \delta^4(p + q - p') \frac{d^3p'}{2E_{p'}(2\pi)^3} Tr(\hat{p}'\gamma_\mu \hat{p}\gamma_\nu)$
- Trick to handle phase space integrals: $\frac{d^3p'}{2E_{p'}} = \delta(p'^2)\Theta(E_{p'})d^4p'$
- $\rightarrow 4\pi m_p \hat{W}^q_{\mu\nu} = \frac{e_q^2}{2}(2\pi)\delta((p+q)^2)\Theta(E_{p'}) Tr(\hat{p}'\gamma_\mu \hat{p}\gamma_\nu)$

DIS in quark parton model (2/2)

- Since $(p+q)^2 = 2(p\cdot q) - Q^2 = (Q^2/x)(\xi - x) \rightarrow m_p \hat{W}^q_{\mu\nu} = \frac{e_q^2}{4}\frac{x}{Q^2}\delta(\xi - x) Tr(\hat{p}'\gamma_\mu \hat{p}\gamma_\nu)$

- Projecting out Lorentz structures: $-g^{\mu\nu} m_p \hat{W}^q_{\mu\nu} = e_q^2 x \delta(\xi - x)$ and $p^\mu p^\nu \hat{W}^q_{\mu\nu} = 0$.

- Recalling similar projection for the proton hadronic tensor and the connection between $W_{\mu\nu}$ and $\hat{W}^q_{\mu\nu} \rightarrow F_1(x) = \frac{1}{2}\sum_q e_q^2 f_q(x)$ and $F_2(x) = \sum_q e_q^2 x f_q(x)$

- The Callan-Gross relation $F_2(x) = 2xF_1(x)$ is a consequence of spin 1/2 of partons $\rightarrow$ the longitudinal structure function $F_L = 0 \rightarrow$ not true in full QCD.

- Partonic Born cross section: $\frac{d\hat{\sigma}_0}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left(\frac{y^2}{2} + \left(1 - y - \frac{x^2y^2m_p^2}{Q^2}\right)\right)$

- DIS cross section in quark parton model: $\frac{d\sigma}{dxdQ^2} = \frac{d\hat{\sigma}_0}{dxdQ^2}\sum_q e_q^2 f_q(x)$

- The parton model explains the Bjorken scaling of $d\sigma/d\hat{\sigma}$ since the structure functions $F_{1,2}(x)$ and PDFs $f_q(x)$ depend only on one variable $x \rightarrow$ not true in full QCD, see Lecture 2.

Quark PDFs of the proton (1/3)

- It is natural to identify partons with quarks.

*Note that massless current quarks ≠ massive quarks of the naive quark model.

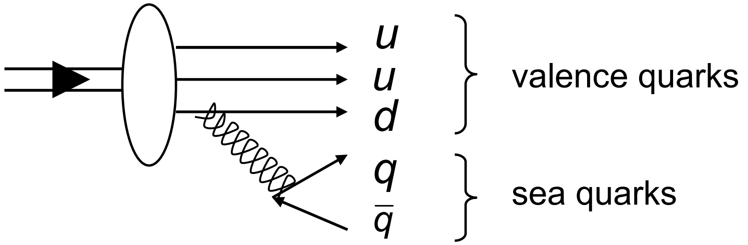
- The proton structure function:

$$\frac{F_2^{ep}(x)}{x} = \left(\frac{2}{3}\right)^2 \left(u(x) + \bar{u}(x)\right) + \left(\frac{1}{3}\right)^2 \left(d(x) + \bar{d}(x)\right) + \left(\frac{1}{3}\right)^2 \left(s(x) + \bar{s}(x)\right)$$

- The neutron structure function using isospin symmetry:

$$\frac{F_2^{en}(x)}{x} = \left(\frac{2}{3}\right)^2 \left(d(x) + \bar{d}(x)\right) + \left(\frac{1}{3}\right)^2 \left(u(x) + \bar{u}(x)\right) + \left(\frac{1}{3}\right)^2 \left(s(x) + \bar{s}(x)\right)$$

- It is customary to split quark distributions into the valence and sea parts: $u(x) = u_{val}(x) + u_s(x)$ and $\bar{u}(x) = u_s(x)$.



- Sum rules for valence quarks:

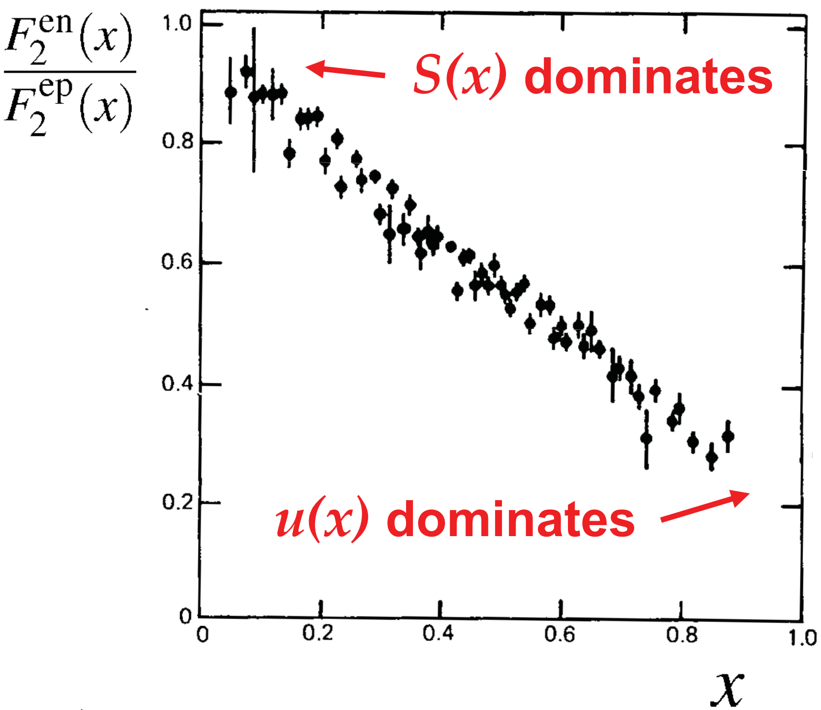
$$\int_0^1 dx[u(x) - \bar{u}(x)] = \int_0^1 dx u_{val}(x) = 2$$

$$\int_0^1 dx[d(x) - \bar{d}(x)] = \int_0^1 dx d_{val}(x) = 1, \int_0^1 dx[s(x) - \bar{s}(x)] = 0$$

Valence quarks carry proton quantum numbers, sea quarks are radiated in pairs.

Quark PDFs of the proton (2/3)

- Ratio of proton and neutron structure functions: $\frac{1}{4} \leq \frac{F_2^{en}(x)}{F_2^{ep}(x)} \leq 4$



- At small x, see quarks dominate: $\frac{F_2^{en}(x)}{F_2^{ep}(x)} \to 1$
- At large x, $u_v(x) \gg d_v(x) \gg q_s(x)$: $\frac{F_2^{en}(x)}{F_2^{ep}(x)} \to \frac{1}{4}$
- Many more inequalities and approximate sum rules for electron-nucleon and neutrino-nucleon scattering. Some examples:

- Gottfried sum rule: $\int_0^1 \frac{dx}{x}(F_2^{ep}(x) - F_2^{en}(x)) = \frac{1}{3}$
- Adler sum rule: $\int_0^1 \frac{dx}{x}(F_2^{\nu n}(x) - F_2^{\nu p}(x)) = 2$
- Gross-Llewellyn-Smith sum rule: $\int_0^1 \frac{dx}{x}(F_3^{\nu p}(x) + F_3^{\nu n}(x)) = 3$

Quark PDFs of the proton (3/3)

- Fraction of proton momentum carried by quarks:

$$\int_0^1 dx\, x[u(x) + \bar{u}(x) + d(x) + \bar{d}(x) + s(x) + \bar{s}(x)] \approx \int_0^1 dx\, x[u(x) + \bar{u}(x) + d(x) + \bar{d}(x)] = \epsilon_u + \epsilon_d$$

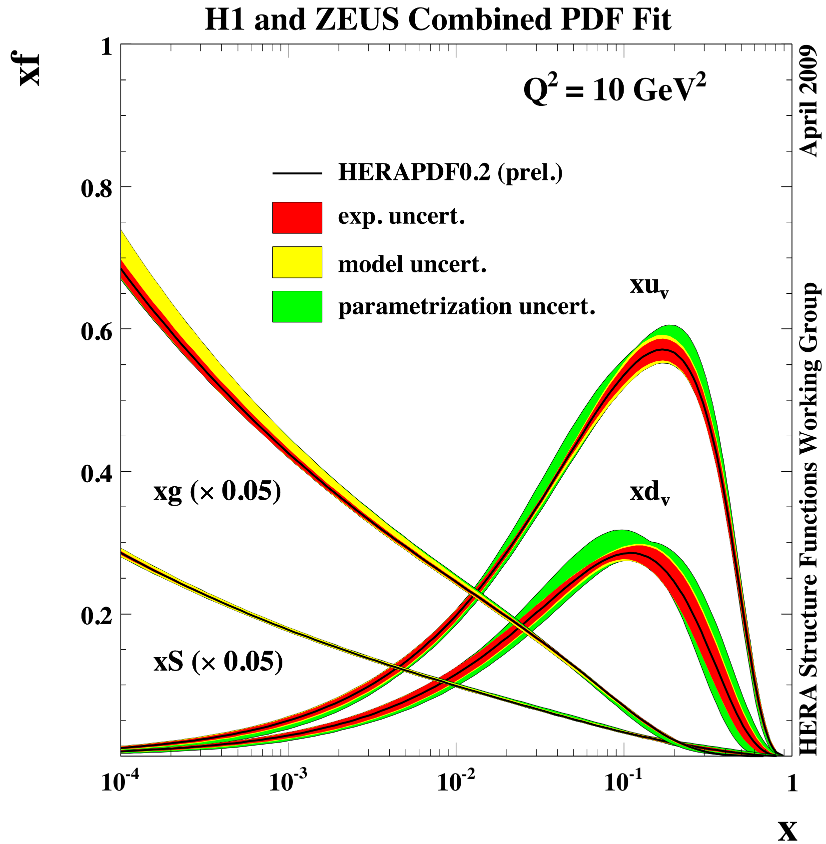
- From experimentally measured structure functions:

$$\int_0^1 dx F_2^{ep}(x) = \frac{4}{9}\epsilon_u + \frac{1}{9}\epsilon_d \approx 0.18, \quad \int_0^1 dx F_2^{en}(x) = \frac{1}{9}\epsilon_u + \frac{4}{9}\epsilon_d \approx 0.12$$

- $\rightarrow \epsilon_u = 0.36$, $\epsilon_d = 0.18$ $\rightarrow$ quarks carry about 50% of the proton momentum.

- The rest of the proton momentum is carried by neutral partons, which we identify with gluons.

- Modern picture of the valence, sea quark and gluon PDFs on the proton.



Color symmetry as a gauge group

- 1972, Fritzsch, Gell-Mann: non-observation of colored quarks (confinement) → promote color symmetry of hadron wave function to $SU(3)_c$ gauge symmetry of the strong interactions.

- Degrees of freedom: quarks (flavor=u,d,s,c,t,b) in fundamental representation and gauge fields (gluons) in adjoint representation of $SU(3)_c$.

- Local gauge transformation (rotations in color space):

$\psi'(x) = U(x)\psi(x) = e^{i\omega^a(x)T^a}\psi(x)$ and $A'_\mu(x) = U(x)(A_\mu(x) - \frac{i}{g}\partial_\mu)U^\dagger(x)$, where $A_\mu = A^a_\mu T^a$

- Copying quantum electrodynamics, QED (Yang, Mills, 1954), the classical Lagrangian of QCD: $\mathscr{L}_{\rm QCD} = \sum_f \bar\psi_f(i\partial_\mu + gT^aA^a_\mu - m_f)\psi_f - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$

- Gluon field tensor: $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$, $a = N_c^2 - 1 = 1,\ldots,8$

- Generators T^a form the Lie algebra: $[T^a, T^a] = if^{abc}T^c$ with antisymmetric structure constants f^{abc}.

- Color algebra: $Tr(T^aT^b) = T_F\delta^{ab}, \sum_a T^aT^a = C_F\hat{I}, \ \sum_{b,c} f^{abc}f^{dbc} = C_A\delta^{ab}$, where

$T_F = \frac{1}{2}, C_F = \frac{4}{3}, C_A = 3 \rightarrow$ determine color factors in perturbative QCD.

Running QCD coupling

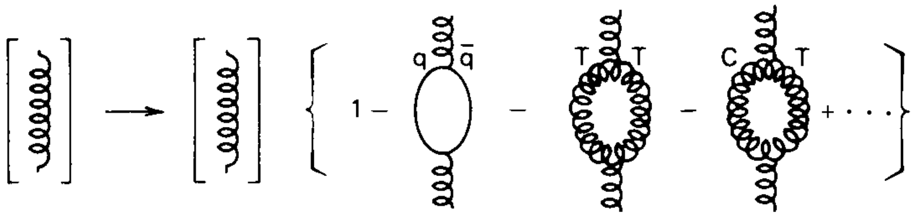
• Essential feature of QCD is self-interaction of gluons:

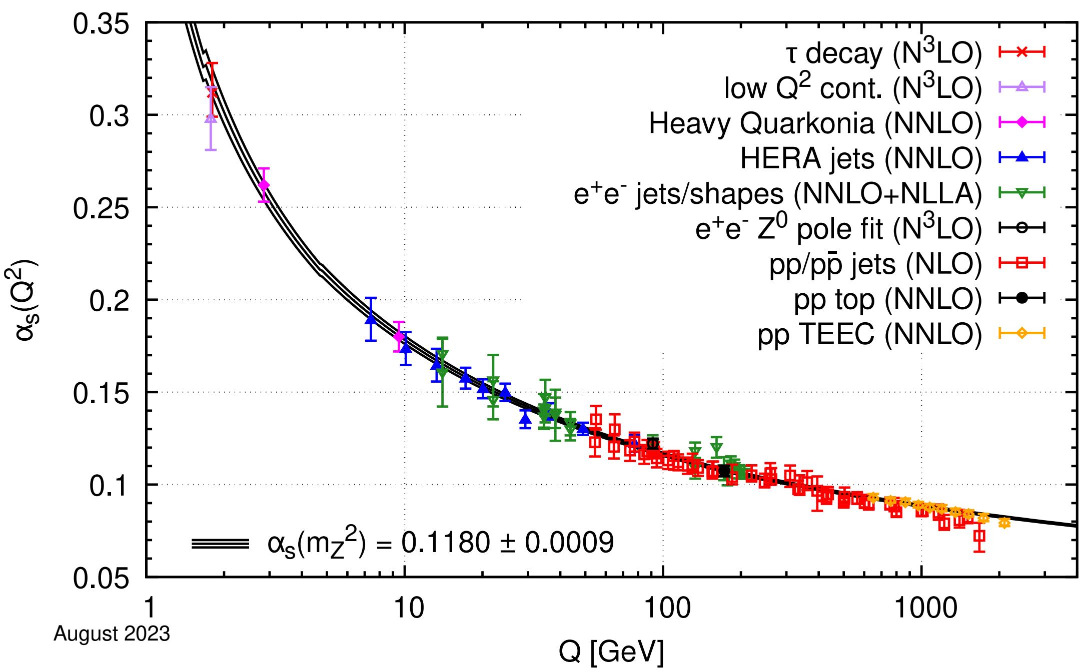
$$\mathscr{L}_{\text{QCD}} = \mathscr{L}_0 + gA^a_\mu \sum_f \bar{\psi}_f T^a A^a_\mu \psi_f - gf^{abc}(\partial_\mu A^a_\nu)A^{b\mu}A^{c\nu} - g^2 f^{eab} f^{ecd} A^a_\mu A^b_\nu A^{c\mu} A^{d\nu}$$

• 1973, Gross, Wilczek, Politzer, asymptotic freedom: the QCD coupling constant $\alpha_s(Q^2) = g^2/(4\pi)$ decreases at large values of Q^2 or short distances,

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2n_f)\log(Q^2/\mu^2)}$$

• Vacuum polarization loop in gluon propagator: screening due to quarks overcomed by anti-screening due to gluons:





• → explains Bjorken scaling for large Q^2 and predicts its violation.

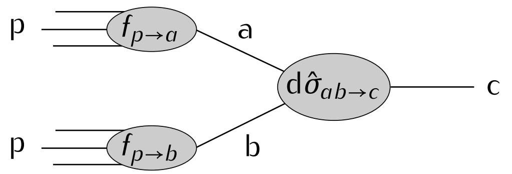
• In the opposite limit of small Q^2 and large distances, $\alpha_s(Q^2)$ becomes large → "infrared slavery" and confinement of quarks into color-singlet hadrons.

Summary: Foundations of QCD

- The quark model for hadron spectroscopy, Gell-Mann, Nobel Prize 1969
- Bjorken scaling in DIS, Friedman, Kendall, Taylor, Nobel Prize 1990
- Asymptotic freedom of QCD, Gross, Politzer, Wilczek, Nobel Prize 2004
- Discovery of J/ψ, Richter, Ting, Nobel Prize 1976
- Renormalization of Yang-Mills gauge theory, t' Hooft, Veltman, Nobel Prize 1999
- Spontaneous broken symmetry, Nambu, Kobayashi, Maskawa, Nobel prize 2008

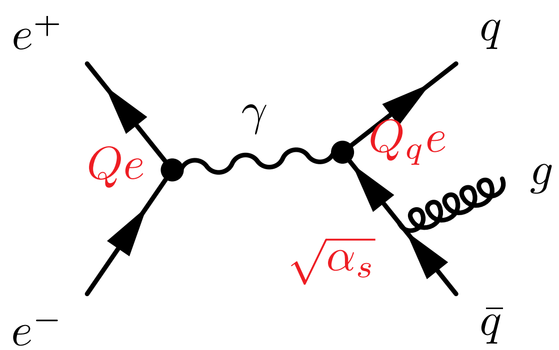
In QCD as in a quantum field theory, the ability to describe high-energy scattering rests on 2 concepts:

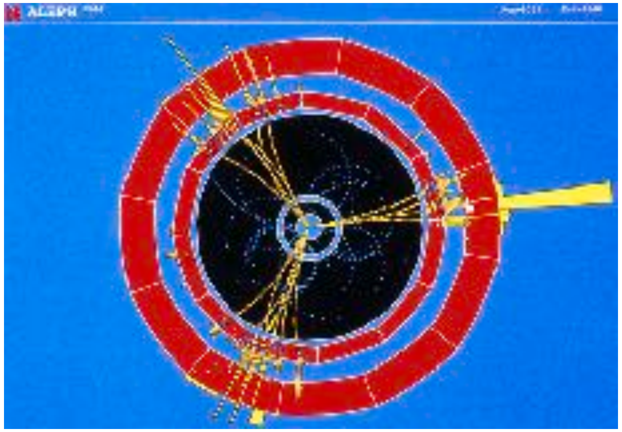
- Renormalization: handles infinities in loop integrals and allows to sum all orders of perturbation theory, Tomonaga, Schwinger, Feynman, Nobel Prize 1965; t' Hooft, Veltman, Nobel Prize 1999; Wilson, Nobel Prize 1982
- Factorization: separation of short-distance matrix elements described by perturbative QCD from long-distance PDFs describing hadron structure, Collins, Soper, Sterman, 1987/89



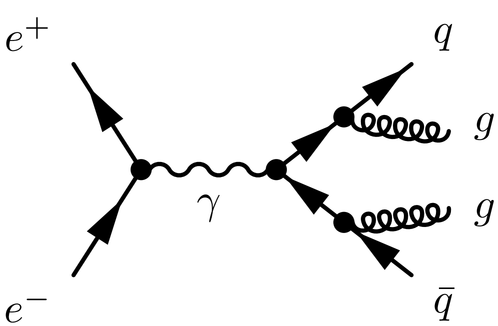
Experimental confirmation of QCD (1/2)

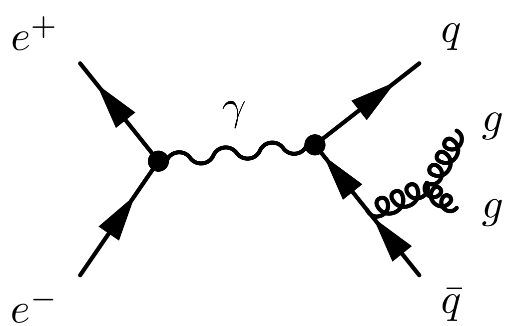
- Scaling violations of the structure functions in ep deep inelastic scattering
- 1978, PETRA, Observation of 3-jet events in e^+e^- annihilation: evidence for gluons with the angular distribution consistent with spin-1 vector gluons.

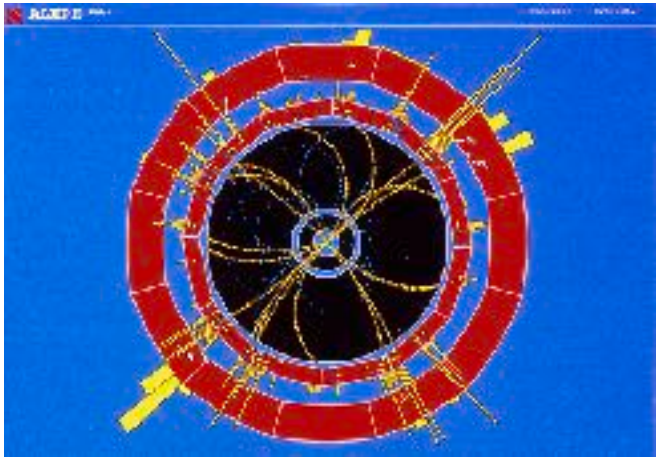




- 1981, PETRA, Observation of 4-jet events: evidence for gluon self-interaction, i.e. non-abelian nature of QCD.

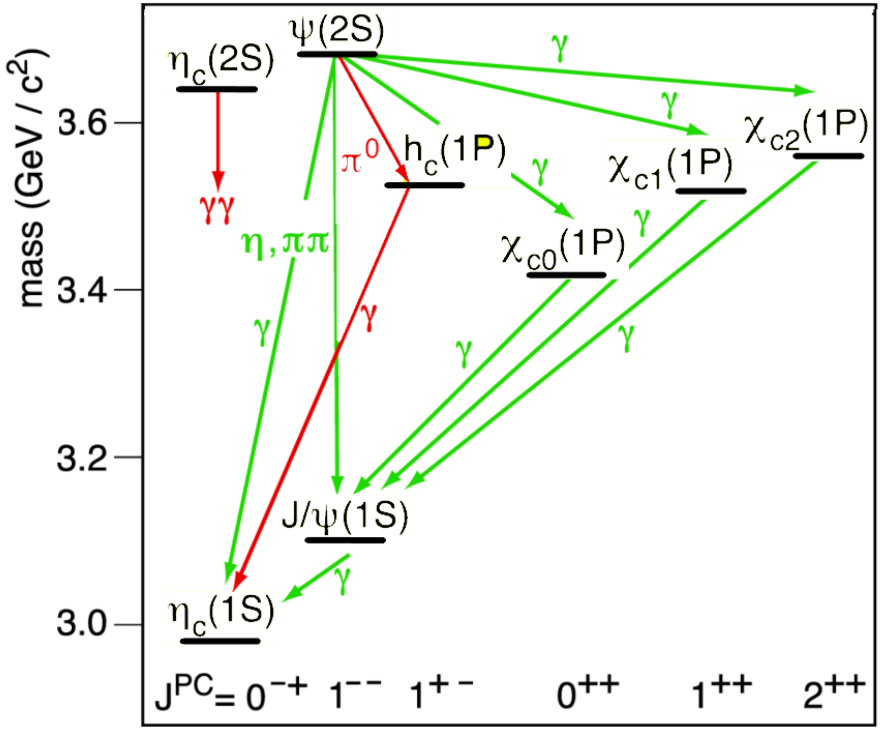






Experimental confirmation of QCD (2/2)

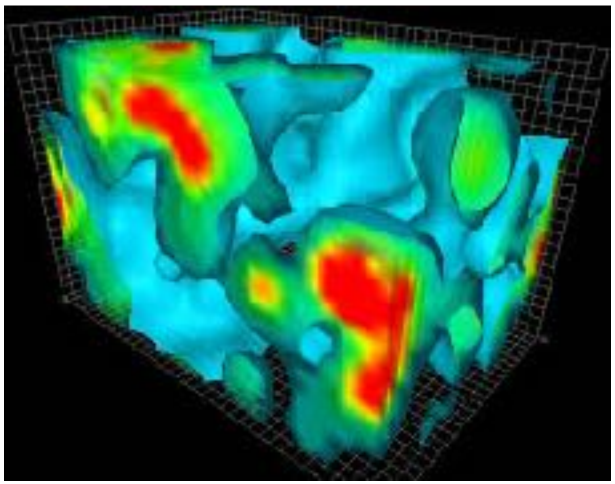
- November 1974, “November revolution”: discovery of J/ψ meson and its exited states suggesting a new charm quark → charmonium spectrum described using QCD potential models → evidence of color charge interaction and acceptance of Standard Model.



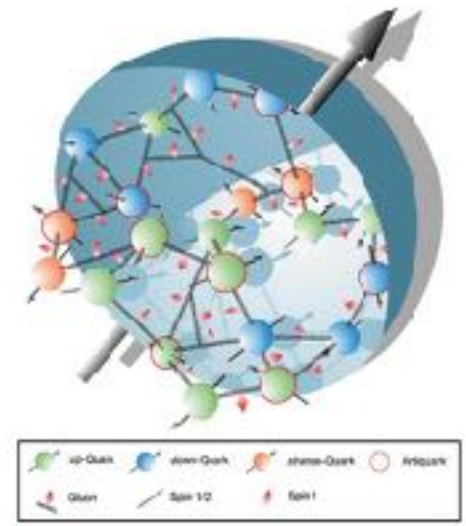
- QCD predicts existence of exotic hadrons, which are not allowed in the quark model: XYZ tetraquarks (Belle, 2003; BES, 2013), pentaquarks (LHCb, 2015), glueball and hybrid mesons (candidates available, need confirmation).

QCD: a broad picture

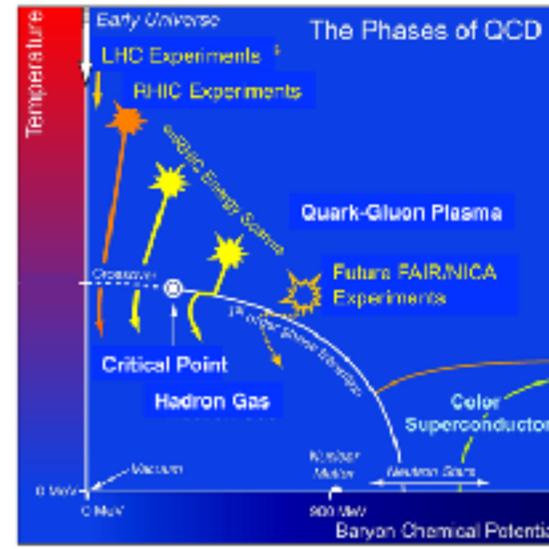
- Simple QCD Lagrangian leads to a vast array of successes in explaining and predicting phenomena in low-energy and high-energy nuclear physics.
- It is active field of research with many open questions:



Structure of QCD vacuum, confinement, and origin of mass



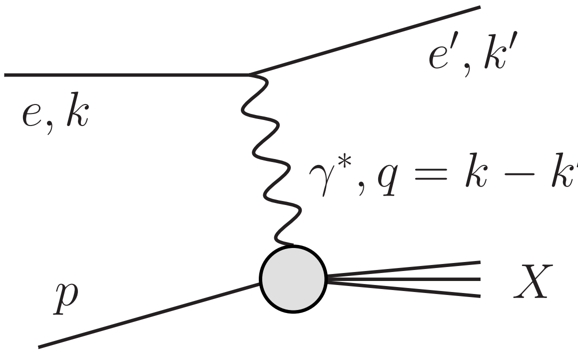
Quark-gluon structure of proton and nuclei including spin and k_T



QCD phase transitions, new regimes of strong interactions, quark-gluon plasma

- Focus of these lectures is collinear PDFs of the proton, nuclei and photon:
  - fundamental structure probed in high-energy scattering processes
  - initial condition for heavy-ion scattering
  - Standard Model precision studies and background for searches of new physics

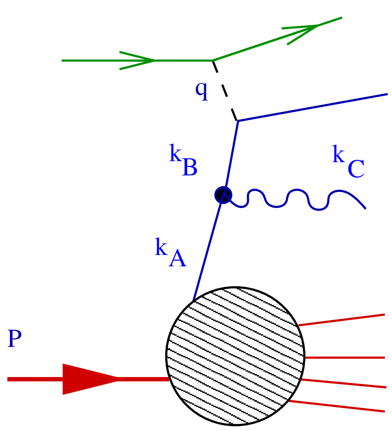
Origin of scaling violations in QCD



- The parton model predicts exact Bjorken scaling of the DIS structure functions $F_{1,2}(x)$.

- In full QCD, the scaling is only approximate with logathmic dependence on the photon virtuality $q^2 = -Q^2$.

- Fundamental reason:
  - parton model: transverse momenta of partons are limited, $k_t \leq \mu_0 \sim 1$ GeV
  - QCD: quark and gluon k_t allowed to be large, $k_t \leq \sqrt{Q^2}$



- Probability w to produce extra partons is large despite small coupling constant $\alpha_s(Q^2)$: $w \propto \frac{\alpha_s}{2\pi} \int^{Q^2} \frac{dk_\perp^2}{k_\perp^2} = \frac{\alpha_s}{2\pi} \ln Q^2 \sim 1$

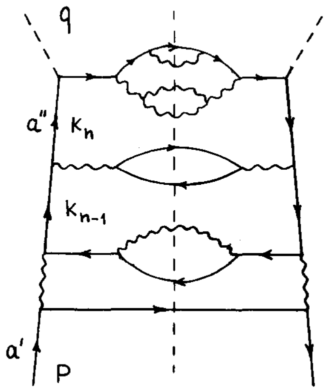
- Leading Logarithmic Approximation (LLA):

$$F_{1,2}(x, Q^2) = \sum_{n=0}^{\infty} f_n(x) \left(\frac{\alpha_s}{2\pi} \ln Q^2 \right)^n$$

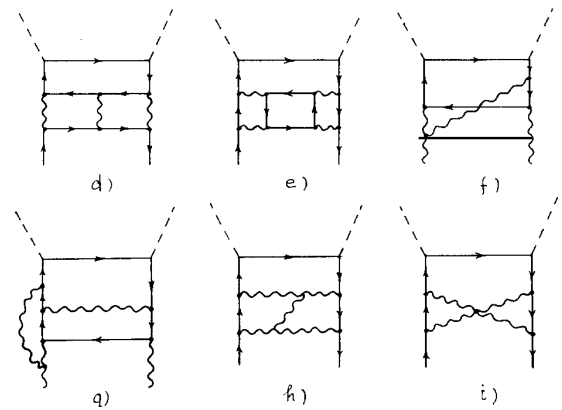
- Such logarithmic behavior is a general feature of QFT with dimensionless coupling constant → the "virtual coat" of partons depends on the resolution Q^2.

Parton ladder in LLA

- The axial gauge $A^+(x) = A^0(x) + A^3(x) = 0$ → only 2 physical gluon polarizations contribute → no need to introduce ghosts+only ladder-type diagrams contribute:



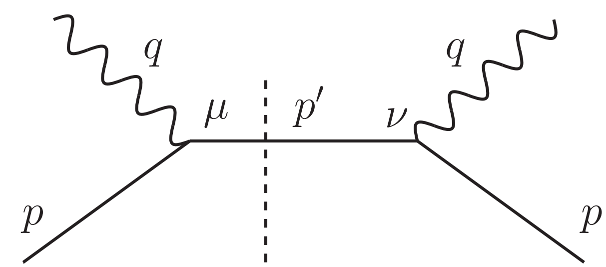
Typical diagram for $\gamma^* p$ DIS



Examples of "non-parton" diagrams that do not contribute to LLA

- Leading order (LO) in α_s (Born term) →

- We are interested in the structure functions ~ $|\mathscr{M}|^2$ → use cut diagram notation → dashed lines show cut propagators on mass shell → $2\pi\delta_+(p'^2)$



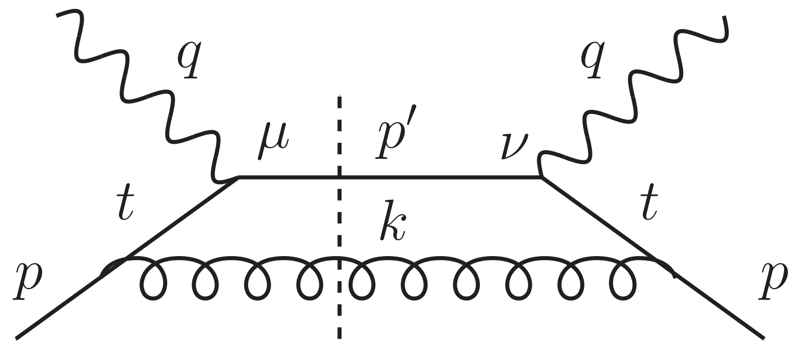
- Direct calculation:

$$4\pi m_p \hat{W}_{\mu\nu}(\gamma^* q \to q) = \frac{N_c}{N_c}\frac{e_q^2}{2}(2\pi)\delta_+((p+q)^2)Tr(\hat{p}'\gamma_\mu \hat{p}\gamma_\nu) = \pi e_q^2 \frac{x}{Q^2}\delta(\xi - x)Tr(\hat{p}'\gamma_\mu \hat{p}\gamma_\nu)$$

→ same as in the parton model.

Parton ladder: real gluon emission (1/2)

- Let us start adding corrections using a perturbation series in α_s, but keeping only $\log Q^2$-enhanced terms → LLA.
- Real gluon emission off initial quark:



$$4\pi m_p \hat{W}_{\mu\nu}(\gamma^* q \to qg) = \int \frac{d^4k}{(2\pi)^4} 2\pi\delta_+(p'^2) 2\pi\delta_+(k^2) C_F \frac{g_s^2 e_q^2}{2} \sum_{\text{pol}} \bar{u}(p)\hat{\epsilon}\frac{\hat{t}}{t^2}\gamma_\nu \hat{p}'\gamma_\mu \frac{\hat{t}}{t^2}\hat{\epsilon}^* u(p)$$

- Color factor for this diagram is $(1/N_c)Tr(T^aT^a) = C_F = 4/3$
- Sudakov decomposition of four-vectors in terms of light-cone vectors p and n: $l_\mu = \alpha p_\mu + \beta n_\mu + l_{\perp\mu}$, where $p^2 = n^2 = (l_\perp \cdot p) = (l_\perp \cdot n) = 0$
- Parton momenta: $p = p$, $q = -(x/\xi)p + n$, $k = (1-z)p + \beta n + \mathbf{k}_\perp$
- Trace calculation: $\sum_{\text{pol}} \ldots = Tr(\hat{p}'\gamma_\mu \hat{t}\hat{\epsilon}^* \hat{p}\hat{\epsilon}\hat{t}\gamma_\nu) \approx 2\mathbf{k}_\perp^2 \frac{1}{1-z}\frac{1+z^2}{1-z} Tr(\hat{p}'\gamma_\mu \hat{p}\gamma_\nu)$
- Delta functions: $\delta_+(p'^2) = \delta((Q^2/x)(z\xi - x)) = \frac{x}{zQ^2}\delta(\xi - x/z)$,

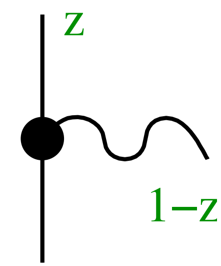
Parton ladder: real gluon emission (2/2)

- $\delta_+(k^2) = \delta(2(p\cdot n)(1-z)\beta - \mathbf{k}_\perp^2) = \dfrac{1}{2(p\cdot n)(1-z)}\delta(\beta - \mathbf{k}_\perp^2/[2(p\cdot n)(1-z)])$
- Loop integration: $d^4k = (p\cdot n)dzd\beta d^2\mathbf{k}_\perp = \pi(p\cdot n)dzd\beta d\mathbf{k}_\perp^2$
- Putting everything together:

$$4\pi m_p \hat{W}_{\mu\nu}(\gamma^* q \to qg) = \int_x^1 \frac{dz}{z}\int_{m^2}^{Q^2}\frac{d\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2}\delta(\xi - x/z)\frac{\alpha_s}{2\pi}P_{qq}(z)\frac{x}{Q^2}\pi e_q^2 Tr(\hat{p}'\gamma_\mu\hat{p}\gamma_\nu)$$

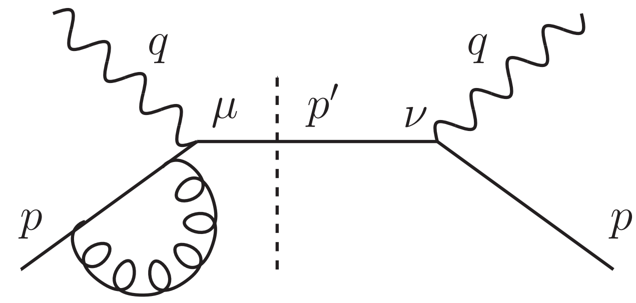
- Integration over $\mathbf{k}_\perp^2$ gives: $\displaystyle\int_{m^2}^{Q^2}\frac{d\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2} = \log(Q^2/m^2)$, where m^2 regulates small-$\mathbf{k}_\perp$ divergence corresponding to **collinear** gluon emission.

- $P_{qq}(z) = C_F\dfrac{1+z^2}{1-z}$ is the quark-quark splitting function ~ probability for a quark to emit quark a with momentum fraction z and a gluon with momentum fraction $1-z$.



- Diverges at $z\to 1$, which is unphysical → regulated by virtual correction to the quark propagator.

Parton ladder: gluon loop



• In addition to real gluon emission, ladder-type graphs include virtual corrections → lead to ultraviolet (UV) divergences → handled by renormalization of quark propagator.

• Instead of an explicit calculation, notice that they are concentrated at $z = 1$ → of the form $\delta(1-z)$.

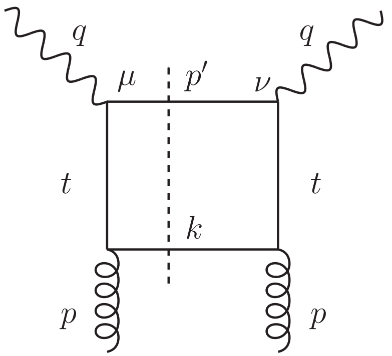
• Total probability (Born+α_s correction) of finding a quark inside a quark is 1 → $\int_0^1 dz P_{qq}(z) = 0$.

• Virtual corrections regularize $P_{qq}(z)$ at $z = 1$ → so-called "+ prescription"

$$\int_0^1 dz \frac{f(x)}{(1-z)_+} = \int_0^1 dz \frac{f(z) - f(1)}{1-z}$$

• These conditions fix the numerical coefficient in front of $\delta(1-z)$. Final result for quark-quark splitting function: $P_{qq}(z) = C_F \frac{1+z^2}{(1-z)_+} + 2\delta(1-z)$.

Gluon-initiated parton ladder



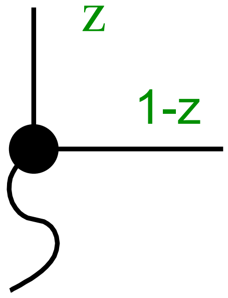
- A parton ladder can also be initiated by gluons.
- Contribution to the hadronic tensor at parton level:

$$4\pi m_p \hat{W}_{\mu\nu}(\gamma^* g \to q q) = \int \frac{d^4k}{(2\pi)^4} 2\pi\delta_+(p'^2) 2\pi\delta_+(k^2) T_R g_s^2 e_q^2 \frac{1}{2} \sum_{\text{pol}} \frac{1}{t^4} Tr(\hat{p}'\gamma_\mu \hat{t}\hat{\epsilon}\hat{k}\hat{\epsilon}^* \hat{t}\gamma_\nu)$$

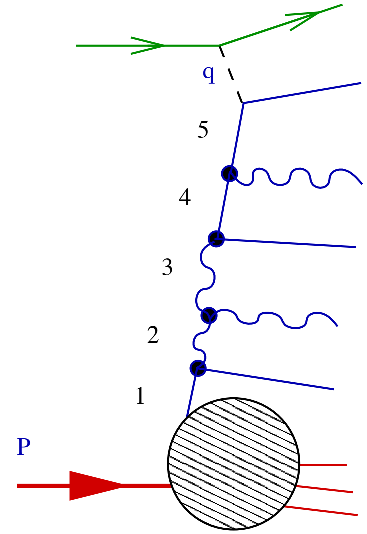
- Color factor for this diagram is $1/(N_c^2-1)Tr(T^aT^a) = 1/2 = T_R$
- Using Sudakov decomposition, after some algebra:

$$4\pi m_p \hat{W}_{\mu\nu}(\gamma^* g \to q q) = \int_x^1 \frac{dz}{z} \int_{m^2}^{Q^2} \frac{d\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2} \delta(\xi - x/z) \frac{\alpha_s}{2\pi} P_{qg}(z) \frac{x}{Q^2} \pi e_q^2 Tr(\hat{p}'\gamma_\mu \hat{p}\gamma_\nu)$$

- $P_{qg}(z) = T_R(z^2 + (1-z)^2)$ is the gluon-quark splitting function ~ probability for a gluon to emit a quark a with momentum fraction z and a quark with momentum fraction $1-z$.

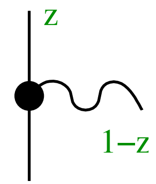


Parton ladder: more rungs and splittings

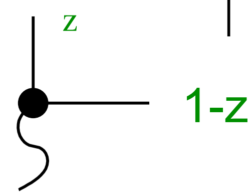


- One can add more rungs to the parton ladder and combine different types of parton-parton splittings.
- Nested integrals $\rightarrow$ to build up $\log Q^2$ contribution, transverse momenta should be strongly ordered: $-k_{1\perp}^2 \ll -k_{2\perp}^2 \ll \cdots \ll -k_{n\perp}^2 \ll Q^2$
- <u>All parton splitting functions to one-loop accuracy</u>:

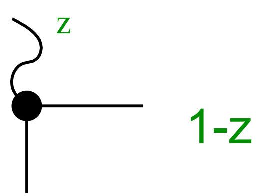
- $$P_{qq}(z) = C_F\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right)$$



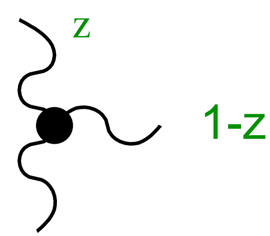
- $$P_{qg}(z) = T_R\left(z^2 + (1-z)^2\right)$$



- $$P_{gq}(z) = C_F\frac{1+(1-z)^2}{z}$$

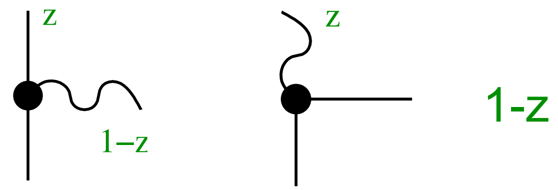


- $$P_{gg}(z) = 6\left(\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right) + \left(\frac{11}{2} - \frac{n_f}{3}\right)\delta(1-z)$$



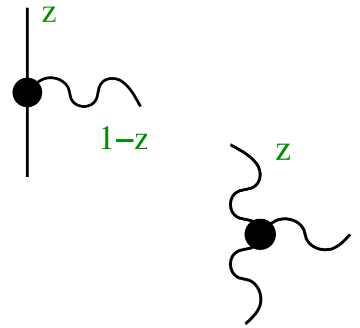
Symmetries of splitting functions

- Exchange decay products $z \to 1 - z$ (for $z < 1$):



- $P_{qq}(z) = P_{gq}(1 - z)$
- $P_{gq}(z) = P_{gq}(1 - z)$
- $P_{gg}(z) = P_{gg}(1 - z)$

- Exchange the parent and the offspring $z \to 1/z$ (for $z < 1$):



- $P_{qq}(1/z) = -\frac{1}{z}P_{qq}(z)$
- $P_{gg}(1/z) = -\frac{1}{z}P_{gg}(z)$
- $P_{gq}(1/z)/C_F = \frac{1}{z}P_{gq}(z)/T_R$

- Quark-gluon symmetry (super-symmetry relation):

- $P_{qq}(z)/C_F + P_{gq}(z)/C_F = P_{qg}(z)/T_R + P_{gg}(z)/N_c \to$ it sufficient to know quantum electrodynamics to restore the structure of the gluon self-interaction!

Q^2-dependent PDFs (1/2)

- We calculated $\mathcal{O}(\alpha_s^0)$ and $\mathcal{O}(\alpha_s^1)$ contributions to parton hadronic tensor $\hat{W}_{\mu\nu}$ keeping only terms proportional to $\log Q^2$ (LLA).

- Recalling the connection between the parton and proton hadronic tensors

$W_{\mu\nu} = \sum_{i=q,g} \int_0^1 \frac{d\xi}{\xi} \hat{W}_{\mu\nu} f_i(\xi) \rightarrow$ DIS cross section on proton in LLA:

$$\frac{d\sigma}{dx dQ^2} = \frac{d\hat{\sigma}_0}{dx dQ^2} \sum_q e_q^2 \left(f_q(x) + \frac{\alpha_s}{2\pi} \log(Q^2/m^2) \int_x^1 \frac{dz}{z} \left[P_{qq}(z) f_q(x/z) + P_{qg}(z) f_g(x/z) \right] \right) + \ldots$$

- Absorb $\log(Q^2/m^2)$ into definition of PDFs:

$$f_q(x, Q^2) = f_q(x) + \frac{\alpha_s}{2\pi} \log(Q^2/m^2) \int_x^1 \frac{dz}{z} \left[P_{qq}(z) f_q(x/z) + P_{qg}(z) f_g(x/z) \right] \ldots$$

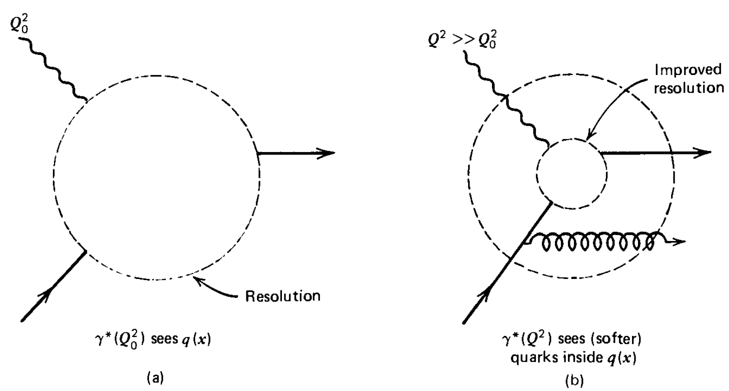
- PDFs depend on Q^2 and obey the integro-differential evolution equations, Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations, 1972-1977 $\rightarrow$ renormalization group equations for PDFs:

$$Q^2 \frac{\partial}{\partial Q^2} \begin{pmatrix} f_q(x, Q^2) \\ f_g(x, Q^2) \end{pmatrix} = \frac{\alpha_s}{2\pi} \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} f_q(Q^2) \\ f_g(Q^2) \end{pmatrix} \qquad P_{ij} \otimes f_j(Q^2) \equiv \int_x^1 \frac{dz}{z} P_{ij}(z) f_j\left(x/z, Q^2\right)$$

Q^2-dependent PDFs (2/2)

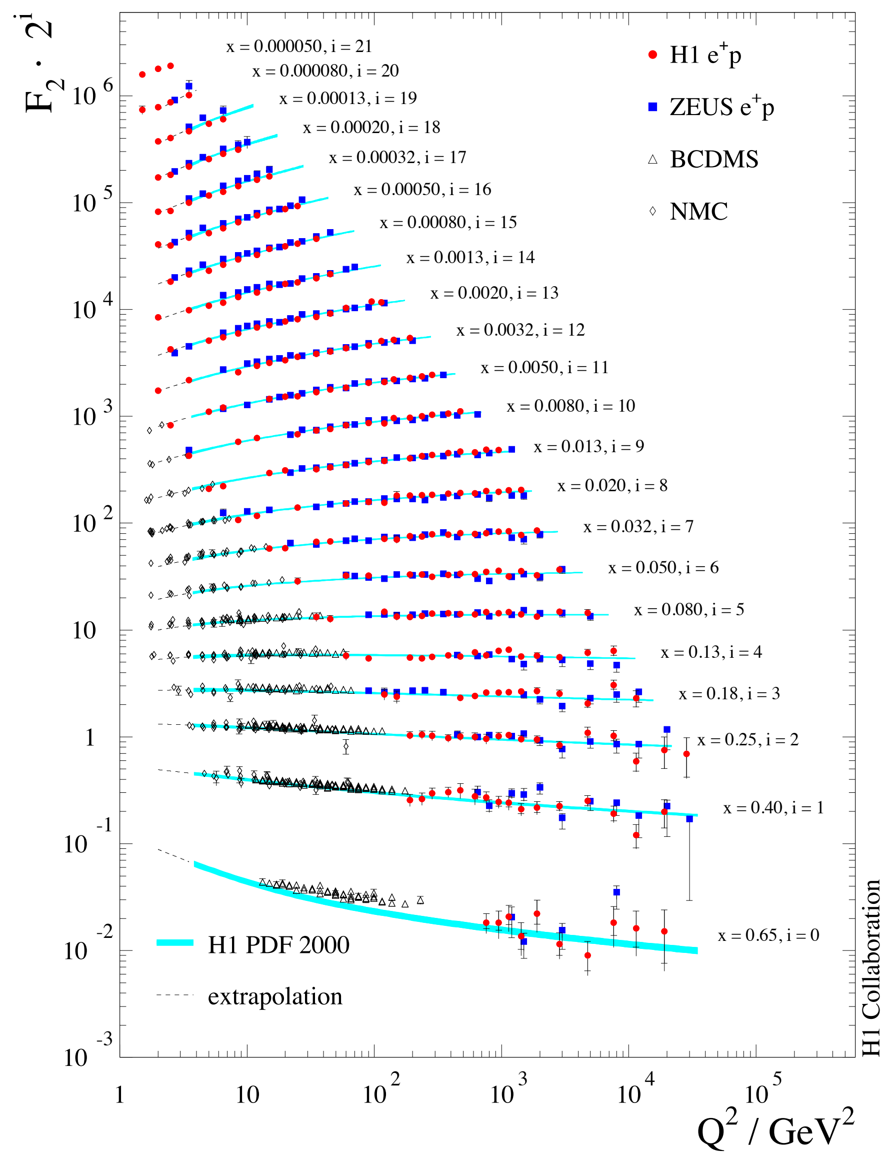
• In QCD due to parton emission, the partonic content of hadrons (proton, pion, nucleus, real photon,…) depends on the photon virtuality Q^2.

• The virtuality Q^2 determines the spacial resolution $\lambda \sim 1/\sqrt{Q^2}$: higher Q^2 give more detailed partonic picture:



• Parton emission leads to scaling violations of the DIS structure functions $F_{1,2}(x, Q^2)$.

• Large-x partons emit partons with smaller $x \to Q^2$ evolution decreases $F_2(x, Q^2)$ at large x and increases at small x.



Factorization theorem (1/4)

- We derived that in LLA $\frac{d\sigma}{dxdQ^2} = \frac{d\hat{\sigma}_0}{dxdQ^2} \sum_q e_q^2 f_q(x, Q^2)$ → the cross section of DIS on proton is a product of the LO $\gamma^* q \to q$ cross section and the quark PDF $f_q(Q^2)$ absorbing all collinear logarithmic divergences.

- It is a particular example the QCD factorization theorem, which is valid beyond LLA and order-by-order in perturbative QCD:

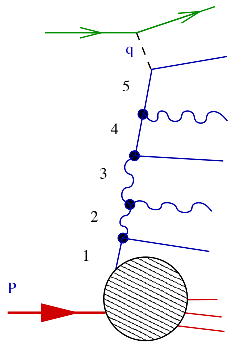
$$F_2(x, Q^2) = \sum_{i=q,\bar{q},g} \int_x^1 d\xi C_i \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) f_i(\xi, \mu^2)$$

- Coefficient functions C_i are process-specific, but do not depend on the target → calculated in perturbation series taking quarks and gluon as a target.

- PDFs $f_i(\xi, \mu^2)$ are process-independent (universal), but depend on the target.

- Factorization scale μ separates (factorizes) the perturbative and non-perturbative effects.

- $F_2(x, Q^2)$ does not depend on μ at given order of pQCD → guaranteed by the DGLAP equations.



Factorization theorem (2/4)

- To explain factorization concepts, consider DIS on a quark target.

- Expand C_i and f_i as a series in powers of α_s:
- $f_i = f_i^{(0)} + f_i^{(1)} + \ldots$
- $C_i = C_i^{(0)} + C_i^{(1)} + \ldots$
- $\rightarrow F_{2,i} = C_i^{(0)} \otimes f_i^{(0)} + C_i^{(0)} \otimes f_i^{(1)} + C_i^{(1)} \otimes f_i^{(0)} \ldots$

- At leading order, $f_{i/i}^{(0)}(\xi) = \delta(1-\xi)$, $F_{2,i}(x) = e_q^2 \delta(1-x) \rightarrow C_q^{(0)}(x) = e_q^2 \delta(1-x)$

- To one-loop accuracy and taking into account the real gluon emission, virtual corrections and non-logarithmic terms:

$$\frac{F_{2,i}^{(1)}}{x} = e_q^2 \frac{\alpha_s}{2\pi} C_F \left[\int \frac{dk_\perp^2}{k_\perp^2} \left(\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right) + \left(\frac{1+x^2}{1-x} \left(\ln \frac{1-x}{x} - \frac{3}{2} \right) + \frac{1}{4}(9+5x) \right)_+ \right]$$

- From factorization formula: $F_{2,i}^{(1)} = e_q^2 x f_{i/i}^{(1)}(x) + C_i^{(1)}(x)$
- $\rightarrow f_{i/i}^{(1)}(x, Q^2) = \dfrac{\alpha_s(Q^2)}{2\pi} \displaystyle\int_{m^2}^{Q^2} \dfrac{dk_\perp^2}{k_\perp^2} P_{qq}(x) \rightarrow$ gives the P_{qq} splitting function and the NLO quark coefficient function $C_q(x) = C_F \left(\dfrac{1+x^2}{1-x} \left(\ln \dfrac{1-x}{x} - \dfrac{3}{2} \right) + \dfrac{1}{4}(9+5x) \right)_+$

Factorization theorem (3/4)

- Expressions for the splitting functions P_{ij} at NLO in 1980s, NNLO in 2000s, and N3LO is still work in progress (approx. solutions and special cases).

- Different choices to group terms into the coefficient functions and PDFs are called factorization schemes.

- Modified minimal subtraction scheme, $\overline{\mathrm{MS}}$ scheme: absorb only the collinear-divergent term (+ universal constant from dim. reg.) in PDFs.

- Another frequently option is the DIS-scheme → absorb everything in PDFs:

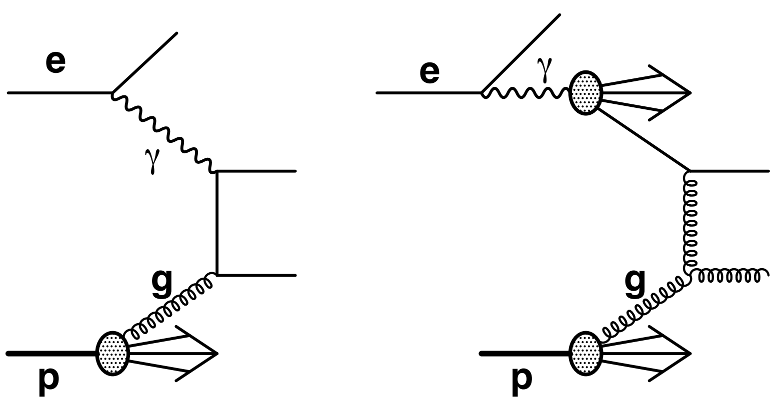
- $xf^{(1)}_{i/i}(x, Q^2)_{\mathrm{DIS}} = F^{(1)}_{2,i}/e_q^2 \rightarrow$
- $C_{q,\bar{q}}(x)_{\mathrm{DIS}} = e_q^2 \delta(1-x)$
- $C_g(x)_{\mathrm{DIS}} = 0$
- → simple parton model form of the DIS cross section and structure functions.

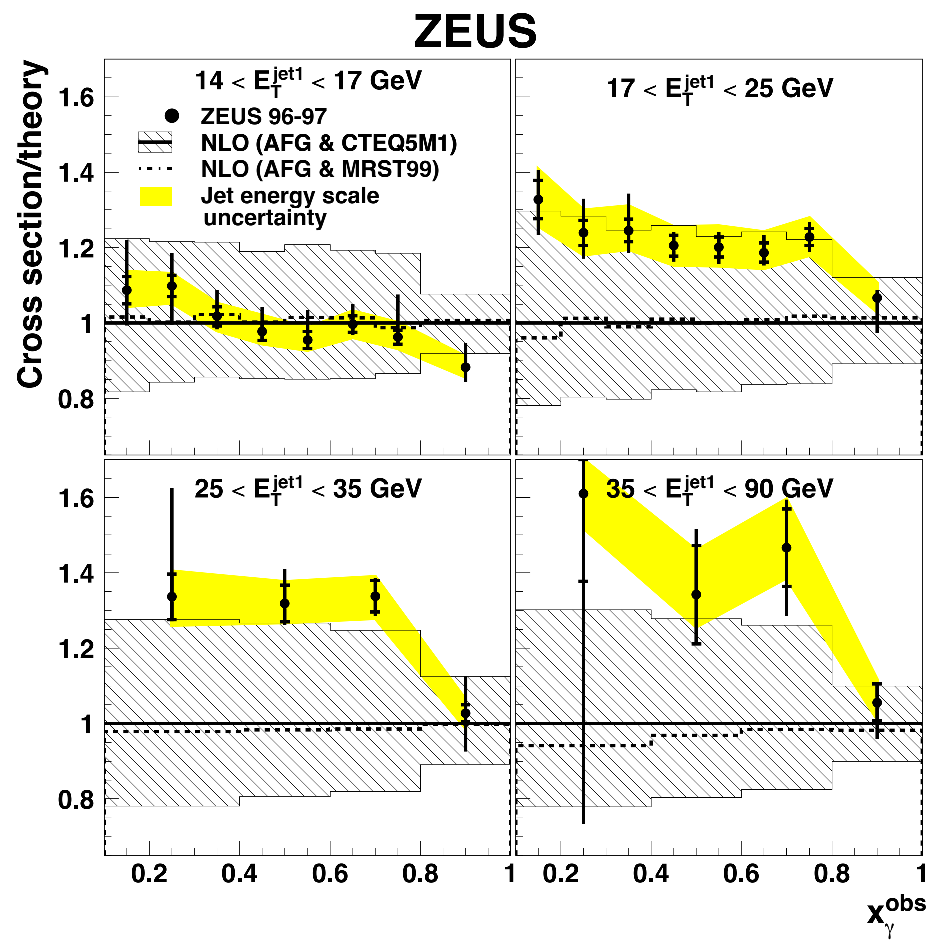
Factorization theorem (4/4)

- One usually associates the factorization scale μ with hard scale of the process: $\mu = c\sqrt{Q^2} = (0.5-2)\sqrt{Q^2}$ ($c=1$ in our DIS calculations).

- Small variations of c result in effects suppressed by additional power of α_s: $\frac{d}{d\log\mu^2}\sum_{n=0}^{N}\alpha_s(\mu^2)^n\sum_i C_i^{(n)}\otimes f_i^{(n)}(\mu^2)\sim\mathcal{O}(\alpha_s^N)$ → used to estimate missing higher order corrections (MHOU).

- Example: dijet photoproduction at HERA





Operator definition of PDFs (1/3)

- One can define PDFs as matrix elements of QCD operators separated by light-cone distances.
- Below we consider the LO (Born contribution) of one quark flavor.
- Hadronic tensor: $m_p W_{\mu\nu} = \frac{1}{4\pi} \sum_X (2\pi)^4 \delta^4(p + q - p_X) \langle p | J_\mu(0) | X \rangle \langle X | J_\nu(0) | p \rangle$
- Use $(2\pi)^4 \delta^4(p + q - p_X) = \int d^4x e^{ix(p+q-p_X)}$, translation $\langle p | J_\mu(0) | X \rangle = e^{-ix(p-p_X)} \langle p | J_\mu(x) | X \rangle$

and completeness of states X $\rightarrow$ $m_p W_{\mu\nu} = \frac{1}{4\pi} \int dx^{iqx} \langle p | J_\mu(x) J_\nu(0) | p \rangle$

- Forward Compton scattering amplitude: $T_{\mu\nu} = i \int dx^{iqx} \langle p | T\{J_\mu(x) J_\nu(0)\} | p \rangle$
- Optical theorem $W_{\mu\nu} = 2\Im m T_{\mu\nu}$
- Time-ordered product of quark e.m. currents:

$$T\{J_\mu(x) J_\nu(0)\} = e_q^2 \bar{\psi}(x) \gamma_\mu S_F(x) \gamma_\nu \psi(0) + \ldots = \int \frac{d^4p'}{(2\pi)^4} e^{-p'x} e_q^2 \bar{\psi}(x) \gamma_\mu S_F(p') \gamma_\nu \psi(0) + \ldots$$

- $S_F(p') = \frac{\hat{p}'}{p'^2 + i\epsilon}$ is the massless quark propagator.

Operator definition of PDFs (2/3)

- Changing variables $p' = q + k$ and taking the imaginary part:

$$m_p W_{\mu\nu} = \frac{e_q^2}{2} \int d^4k \int \frac{d^4x}{(2\pi)^4} e^{-ixk} \langle p \,|\, \bar{\psi}(x) \gamma_\mu (\hat{k} + \hat{q}) \gamma_\nu \psi(0) \,|\, p \rangle \delta((k+q)^2)$$

- Introduce the quark correlator $\hat{\Phi}^q_{\alpha\beta}(k) = \int \frac{d^4x}{(2\pi)^4} e^{-ixk} \langle p \,|\, \bar{\psi}_\beta(x) \psi_\alpha(0) \,|\, p \rangle$

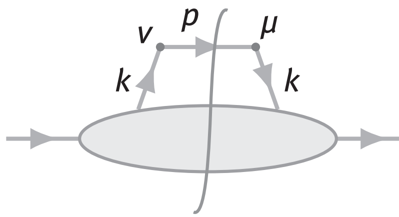
- $\rightarrow m_p W_{\mu\nu} = \frac{e_q^2}{2} \int d^4k \delta((k+q)^2) Tr\{\gamma_\mu (\hat{k} + \hat{q}) \gamma_\nu \hat{\Phi}^q(k)\}$

- We showed before that $\delta((k+q)^2) = x/Q^2 \delta(\xi - x)$. Expressing vectors in Sudakov basis: $q = -xp + n$ and $k = \xi p \rightarrow q + k = n + \ldots$

- $\rightarrow$ in the expansion of $\hat{\Phi}^q_{\alpha\beta}(k)$ in terms of independent Lorentz vectors, one can keep only $\hat{\Phi}^q_{\alpha\beta}(k) \propto \hat{p} \rightarrow \hat{\Phi}^q_{\alpha\beta}(k) = \frac{\hat{p}}{4(p \cdot n)} \int \frac{d^4x}{(2\pi)^4} e^{-ikx} \langle p \,|\, \bar{\psi}(x) \hat{n} \psi(o) \,|\, p \rangle$

- In d^4k integration, the component along p is fixed by the delta-function, while $\int dk^- d^2\mathbf{k}_\perp e^{-ixk} = (2\pi)^3 \delta(x^+) \delta^2(\mathbf{x}_\perp)$

Operator definition of PDFs (3/3)



- Putting all factors together and using $\int dx^+ d^2\mathbf{x}_\perp \delta(x^+)\delta^2(\mathbf{x}_\perp) = 1$:

$$m_p W_{\mu\nu} = \frac{e_q^2}{4}\left(x/Q^2\right) Tr\{\gamma_\mu \hat{n} \gamma_\nu \hat{p}\} \frac{p^+}{2(p\cdot n)} \int \frac{dx^-}{2\pi} e^{-ixx^-p^+} \langle p | \bar{\psi}(x) \hat{n} \psi(0) | p \rangle_{|x^+=\mathbf{x}_\perp=0}$$

- Recalling that $F_2(x,Q^2)/x = -g^{\mu\nu} m_p W_{\mu\nu} + \ldots$ and summing over quarks:

$$F_2(x,Q^2)/x = \sum_q e_q^2 \frac{p^+}{2(p\cdot n)} \int \frac{dz^-}{2\pi} e^{-ixz^-p^+} \langle p | \bar{\psi}(x) \hat{n} \psi(0) | p \rangle_{z^+=\mathbf{z}_\perp=0}$$

- Comparing with the LO expression $F_2(x,Q^2)/x = \sum_q e_q^2 f_q(x,Q^2)$
- $\rightarrow f_q(x,Q^2) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{-ixz^-p^+} \langle p | \bar{\psi}(x) \gamma^+ \psi(0) | p \rangle_{z^+=\mathbf{z}_\perp=0}$

- The quark distribution is given by a forward matrix element of two quark fields separated by the light-cone distance z^-.

- Note*: No need to introduce gauge link in $A^+(x) = 0$ gauge.