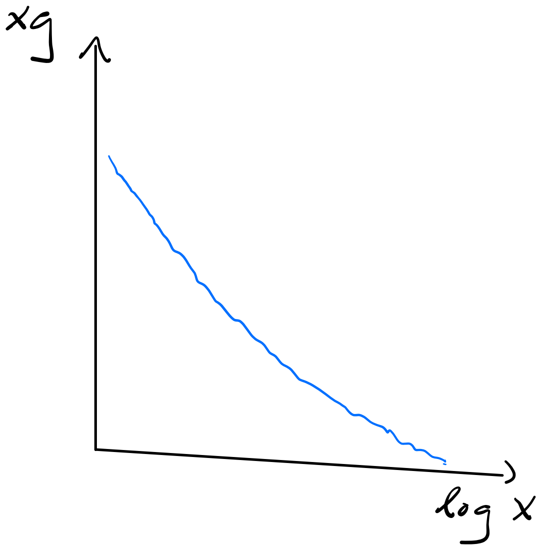
Vector mesons and diffraction in the color-glass condensate

Outline:

1. Color-glass condensate approach to gluon saturation
2. Recap of diffraction
   - Probing protons and nuclei with diffractive processes
   - Why is this interesting for gluon saturation?
3. Diffractive vector meson production
   - Why vector mesons?
4. Theoretical models for mesons
   - Heavy mesons: nonrelativistic QCD
   - Light mesons: twist expansion

Gluon saturation

Gluon density increases rapidly with decreasing x (increasing energy)



$$xg \sim (1/x)^{\delta}$$

Increase too rapid

⇒ violation of unitarity

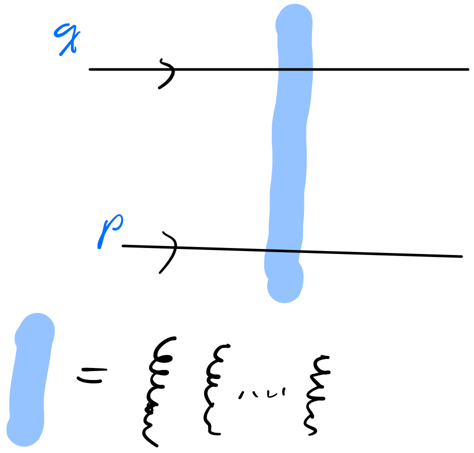
⇒ has to be tamed by nonlinear effects of QCD (gluon recombination)

⇒ gluon saturation

Naturally taken into account in the dipole picture:

- Target as a classical color field
  = color-glass condensate

- Interaction described in terms of Wilson lines

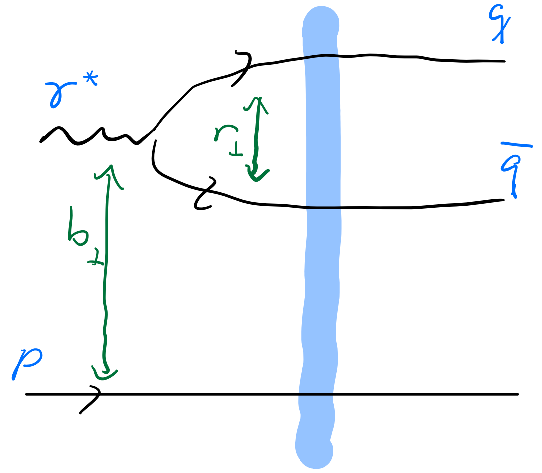


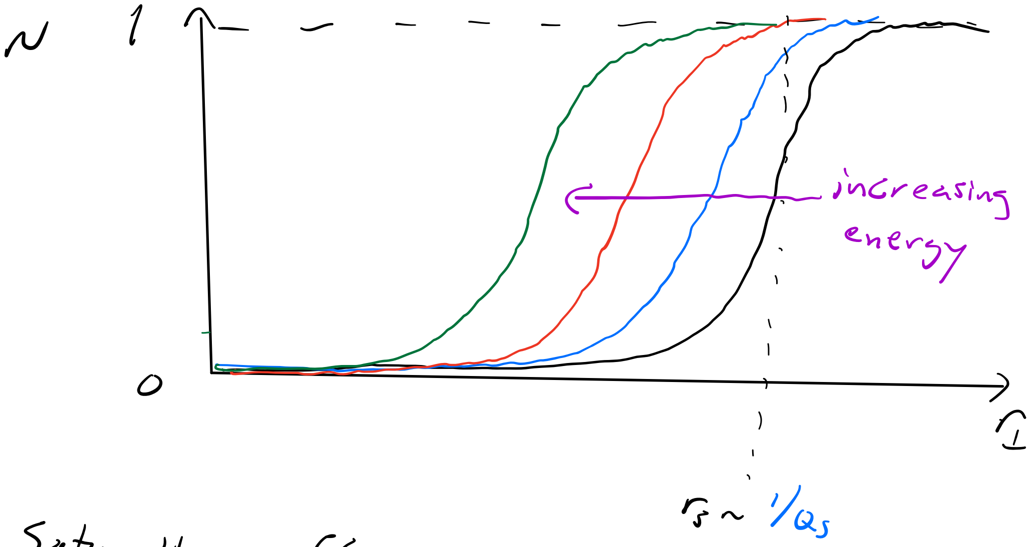
$q\bar{q}$-scattering with the target:

dipole amplitude N ($A_{q\bar{q}}$ in Tuomas's notes)

$$N(r_\perp, b_\perp, x) = 1 - \frac{1}{N_c}\left\langle \mathrm{tr}\left[V_q V_{\bar{q}}^{\dagger}\right]\right\rangle$$

average over target "color configurations"





Saturation effects: $r_\perp \gtrsim 1/Q_s$, Q_s = saturation scale

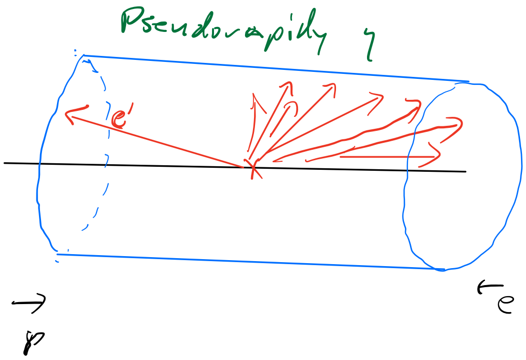
Q_s increases with energy $\Rightarrow$ saturation effects become larger

Diffraction

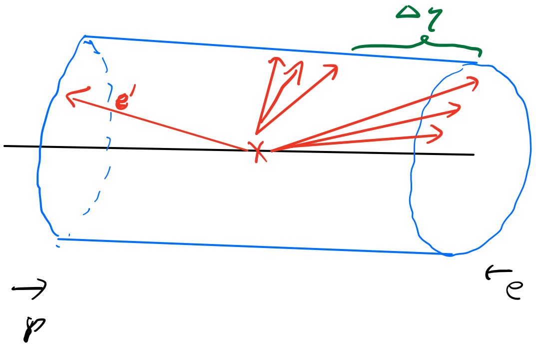
What is diffraction?

HERA: e+p-collider

~10% of events diffractive



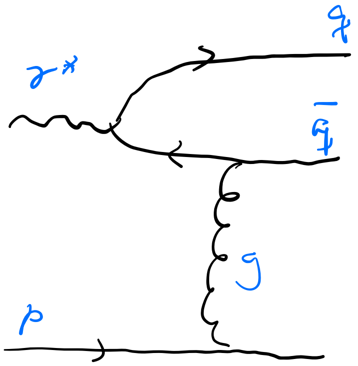
Non-diffractive:
no rapidity gap



Diffractive:
rapidity gap Δη

How can we understand the difference between diffractive and non-diffractive processes?

Compare the following:



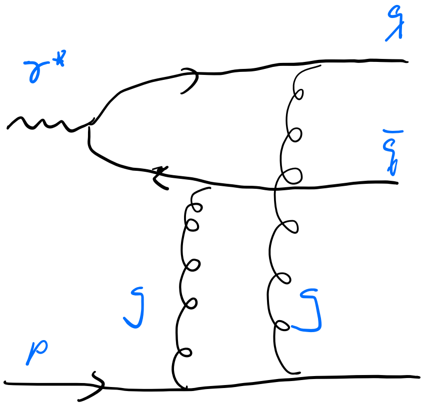
Final $q\bar{q}$ color octet

Confinement: all measured final particles have to be color singlets!

⇒ neutralization of color between $q\bar{q}$ and proton remnant in the final state

⇒ lots of soft radiation between the final-state particles

⇒ rapidity gap filled by soft particles



Consider the final $q\bar{q}$ in color-singlet state

⇒ $q\bar{q}$ and proton remnant already color neutral, no soft particle production

⇒ final particles $q\bar{q} \to X$ and $p^* \to Y$ well separated in rapidity

⇒ rapidity gap $\Delta\eta$

Diffraction: color-neutral interaction with the target (without long-distance effects)

Diffractive events need at least two gluons exchanged with the target

(c.f. non-diffractive: one gluon is enough)

⇒ more sensitive to gluon density!

⇒ more sensitive to gluon saturation

		Collinear factorization		Small x
Inclusive:	$\sigma \sim$	$xg(x,Q^2)$	$\sim$	$N(r,x)$
Diffractive:	$\sigma_D \sim$	$[xg(x,Q^2)]^2$	$\sim$	$N(r,x)^2$

xg = gluon PDF

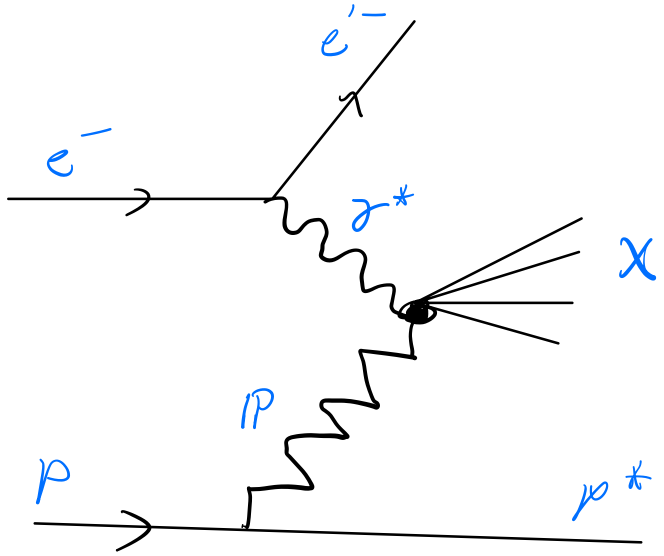
N = dipole amplitude

Comparing inclusive and diffractive events is interesting to understand saturation:

$$\frac{\sigma_D}{\sigma} \sim \frac{N^2}{2N} \sim \begin{cases} \frac{(W^\delta)^2}{2W^\delta} \sim W^\delta, & \text{dilute limit (no saturation)} \\ \frac{1^2}{2\cdot 1} \simeq \frac{1}{2} & \text{black-disk limit} \end{cases}$$

Here W is the energy $(x \sim 1/W)$

Variables



$\mathbb{P}$ = "pomeron" (color-neutral interaction, not a real particle!)

X = diffractive final state

p^* = proton remnant (can stay as a proton or dissociate into other particles)

Relevant variables:

Q^2 = photon virtuality

M_X^2 = invariant mass of the final state

$t = (p_\gamma - p_X)^2$ = momentum exchange

W^2 = center-of-mass energy for $\gamma^* p$ system

$$x_{I\!P} = \frac{Q^2+M_x^2-t}{W^2+Q^2+m_p^2}$$ = longitudinal momentum fraction carried by the pomeron (analogous to Bjorken x)

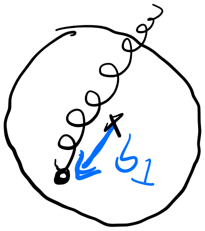
A lot of variables to study the target!

A comparison:

- Inclusive DIS: described in terms of structure functions $F(x, Q^2)$
- Diffractive DIS: in terms of diffractive structure functions $F(x_{I\!P}, Q^2, t, M_x^2)$

t-dependence especially interesting:

$$t \simeq -\Delta_\perp^2, \quad iM \sim \int d^2b_\perp \, e^{-i\Delta_\perp \cdot b_\perp} N(r_\perp, b_\perp)$$



transverse momentum transfer $\Delta_\perp$ $\xleftrightarrow{\text{Fourier transform}}$ impact parameter $b_\perp$

By measuring $\Delta_\perp$ we gain information about the impact-parameter dependent gluon distribution!

Coherent and incoherent diffraction

Coherent

Final proton p^* stays intact

Incoherent

Final proton p^* dissociates

How can we describe this theoretically?

Good-Walker picture

Total diffractive production:

$$\sigma \sim \sum_X \langle p|\mathcal{M}|X\rangle\langle X|\mathcal{M}^*|p\rangle = \langle p|\,|\mathcal{M}|^2|p\rangle$$

Coherent production:

$$\sigma_{coh} \sim \langle p|\mathcal{M}|p\rangle\langle p|\mathcal{M}^*|p\rangle = |\langle p|\mathcal{M}|p\rangle|^2$$

Incoherent production:

$$\sigma_{incoh} = \sigma - \sigma_{coh} \sim \langle p|\,|\mathcal{M}|^2|p\rangle - |\langle p|\mathcal{M}|p\rangle|^2$$

Small x: elements $\langle p | \mathcal{O} | p \rangle$ can be written as averages over proton's color configurations

$$\langle p | \mathcal{O} | p \rangle \equiv \langle \mathcal{O} \rangle$$

$$\Rightarrow \begin{cases} \sigma \sim \langle |\mathcal{M}|^2 \rangle \\ \sigma_{coh} \sim |\langle \mathcal{M} \rangle|^2 \\ \sigma_{incoh} \sim \langle |\mathcal{M}|^2 \rangle - |\langle \mathcal{M} \rangle|^2 \end{cases}$$

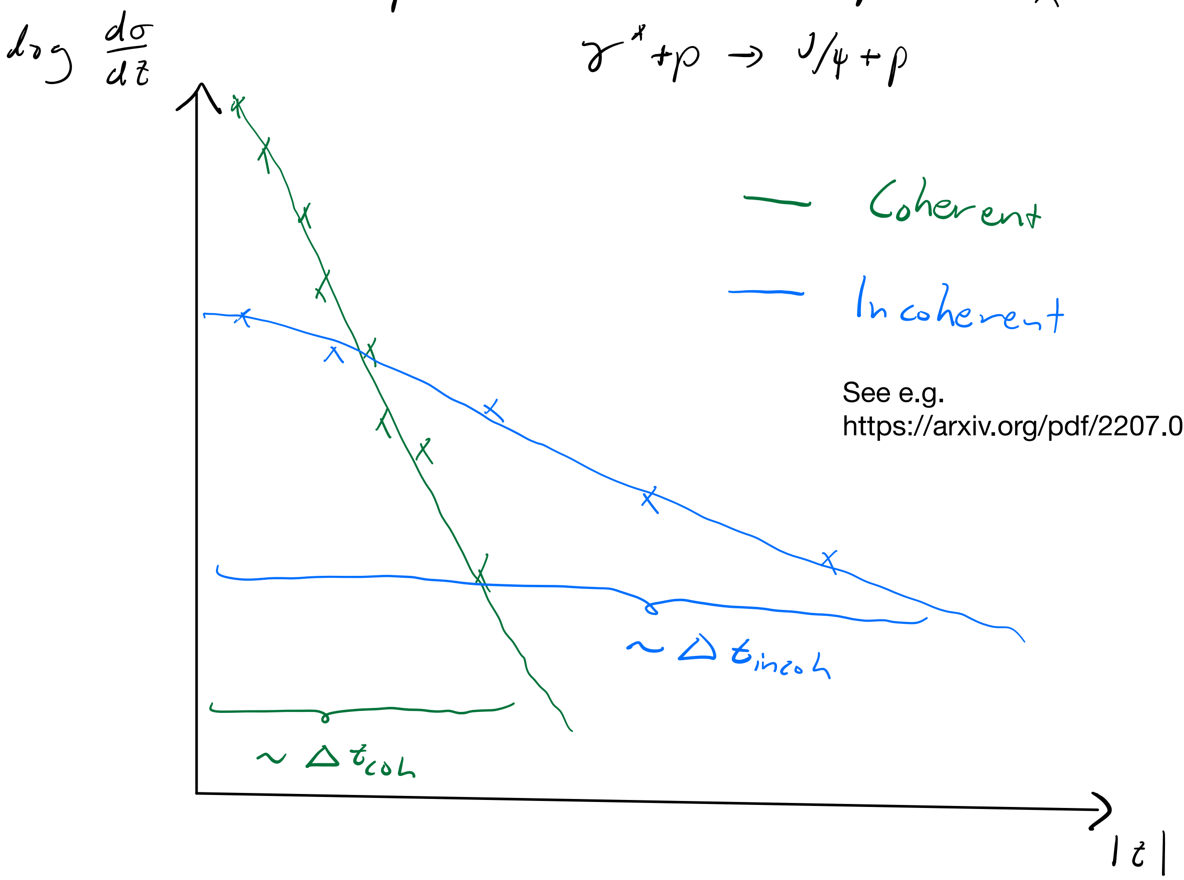
We get the following interpretation:

σ_{coh}: Probes average interaction with the target

σ_{incoh}: Probes fluctuations of the interaction with the target ("variance" of the interaction)

Example: Exclusive J/ψ production

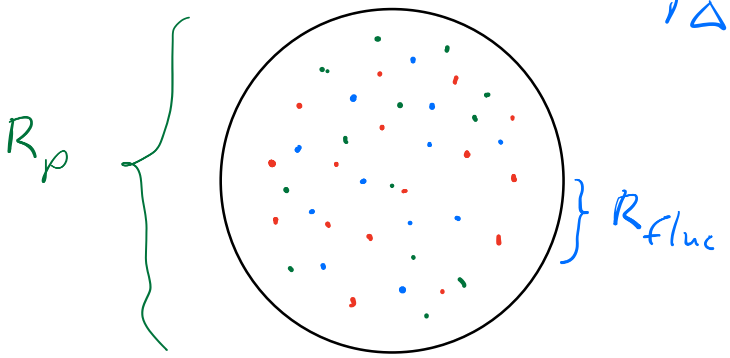
$$\gamma^* + p \to J/\psi + p$$



See e.g.
https://arxiv.org/pdf/2207.03712

Coherent: target size $R_p \sim \frac{1}{\sqrt{\Delta t_{coh}}}$

Incoherent: fluctuation length scale $R_{fluc} \sim \frac{1}{\sqrt{\Delta t_{incoh}}}$

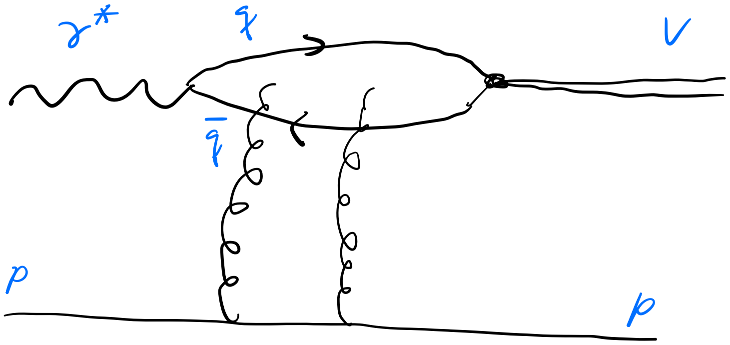


Diffractive vector meson production in DIS

Consider exclusive single-particle production:

- Single particle → diffractive!

$$\gamma^* + p \rightarrow V + p$$



Photons are vector particles: $J^{PC} = 1^{--}$

⇒ dominant exclusive particle production: vector mesons

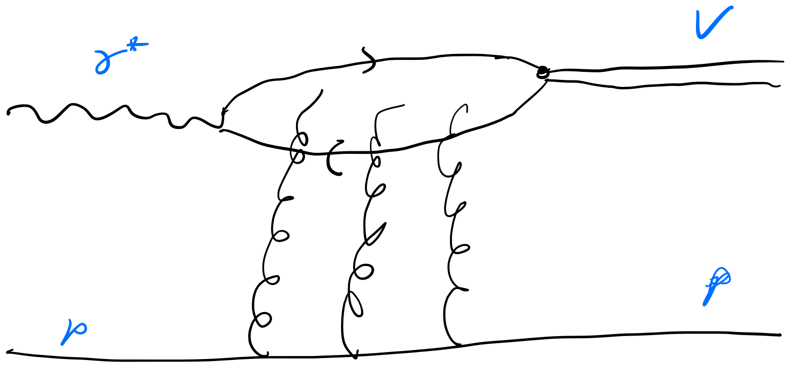
- No exchange of quantum numbers (color, spin) with the target (also known as pomeron exchange)
- Production of other particles would require an exchange of orbital angular momentum

⇒ suppression (requires non-zero momentum transfer t)

Example: **E**xclusive production of a pseudoscalar particle $(J^{PC}=0^{-+})$

C-parity is flipped: $\overset{\gamma^*}{1^{--}} \Rightarrow \overset{V}{0^{-+}}$

$\Rightarrow$ Requires an exchange of at least 3 gluons:



C-parity flip: "odderon" exchange

$\Rightarrow$ Suppressed compared to vector mesons

Also: in DIS dominant contribution from vector mesons with $q\bar{q}$ of the same flavor: e.g.

$\omega = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$, $\rho^0 = \frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$, $\phi = s\bar{s}$, $J/\psi = c\bar{c}$, $\Upsilon = b\bar{b}$

(also higher-energy states)

We will focus on these from now on.

Vector meson decay

Decays of heavy vector mesons have some interesting properties.

1. Decay width can be very narrow
   (e.g. compared to pseudoscalars)

$c\bar{c}$: $\Gamma(\eta_c) \approx 30\ \text{MeV}$ $\quad$ $\Gamma(J/\psi) \approx 90\ \text{keV}$

$b\bar{b}$: $\Gamma(\eta_b) \approx 10\ \text{MeV}$ $\quad$ $\Gamma(\Upsilon(1S)) \approx 50\ \text{keV}$

Such particles also have relatively common electromagnetic decays.

2. Higher-order states:
   Decay width suddenly becomes large

$c\bar{c}$: $\Gamma(\psi(2S)) \approx 300\ \text{keV}$ $\quad$ $\Gamma(\psi(3770)) \approx \underline{30\ \text{MeV}}$

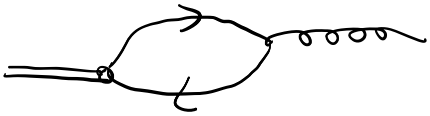
$b\bar{b}$: $\Gamma(\Upsilon(2S)) \approx 30\ \text{keV}$ $\quad$ $\Gamma(\Upsilon(3S)) \approx 20\ \text{keV}$

$\Gamma(\Upsilon(4S)) \approx \underline{20\ \text{MeV}}$

How can we understand these properties?

Let's consider the lowest-order diagrams for the decay.

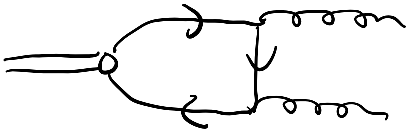
$V \to g^*$



Color-singlet to color-octet:
Color not conserved
⇒ prohibited

✗

$V \to gg$

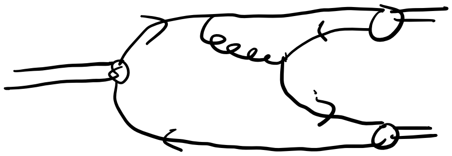


Prohibited by spin-parity conservation

✗

Note: pseudoscalar could decay in this way
⇒ larger decay width for pseudoscalars

$V \to h\bar{h}$



Possible only if $M_V \gtrsim 2M_h$

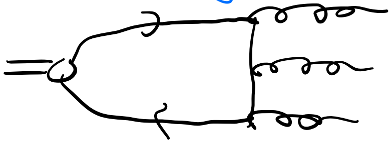
$c\bar{c}$: $2M_D = 3740$ MeV

$b\bar{b}$: $2M_B = 10\,560$ MeV

✓/✗

⇒ light vector mesons,
$\psi(3770)$, $\Upsilon(4S)$, ...

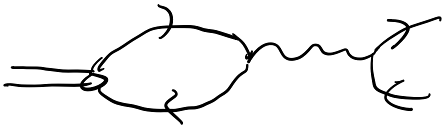
$V \to ggg$



Not prohibited
⇒ Occurs in nature

✓

$V \to \gamma^* \to l^+ l^-$



Not prohibited
(same quantum numbers)

✓

Ratio (neglecting $V \to h\bar{h}$)

$$\frac{V \to \ell^+\ell^-}{\text{QCD decays}} \sim \frac{\Gamma(V \to \ell^+\ell^-)}{\Gamma(V \to ggg)} \sim \frac{\left(\frac{\alpha_{em}}{2\pi}\right)^2}{\left(\frac{\alpha_s}{2\pi}\right)^3} \sim 4\ \%$$

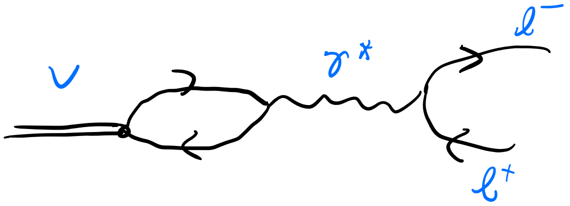
$\alpha_s = 0{,}2$

Experiment:

$J/\psi : \sim 6\%$ $\quad\quad$ $\Upsilon(1S) : \sim 2{,}5\%$

Most common QED decay:

$$V \to \gamma^* \to \ell^+\ell^-$$



Extremely clean: only a dilepton pair in the detectors!

For higher quarkonia ($\psi(2S)$, $\Upsilon(2S)$...) decays like

$$\psi(2S) \to J/\psi + \text{anything}$$

also possible → experimental signal more complicated

Meson wave function

Formation (or decay) of the vector meson is nonperturbative.

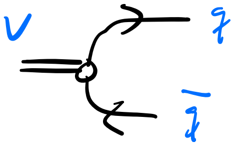
To describe it, expand the state in terms of partonic Fock states:

$$|V\rangle = \int d[q\bar{q}]\ \psi^{q\bar{q}} |q\bar{q}\rangle + \int d[q\bar{q}g]\, \psi^{q\bar{q}g} |q\bar{q}g\rangle + \ldots$$

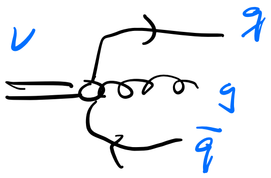
$d[n]$ = phase space for Fock state n

ψ^{n} = nonperturbative wave function for Fock state n

$\psi^{q\bar{q}} \sim$



$\psi^{q\bar{q}g} \sim$



How do we deal with ψ^{n}?

Heavy (vector) mesons

Focus on quarkonia states

$c\bar{c}$: η_c, J/ψ, χ_c, h_c, $\eta_c(2S)$, $\psi(2S)$, ...

$b\bar{b}$: η_b, Υ, χ_b, h_b, $\eta_b(2S)$, $\Upsilon(2S)$, ...

Quark mass large, $M_V \approx 2m_q$

velocity of the quark/antiquark

$\Rightarrow$ kinetic energy small $\Rightarrow$ nonrelativistic system! $(v \ll 1)$

We can use nonrelativistic QCD (NRQCD) to describe the systems.

NRQCD:

- an effective field theory
- Idea: start from the heavy quark Lagrangian

$$\mathcal{L}_{heavy} = \bar{\Psi}(i\gamma^\mu D_\mu - m_q)\Psi \text{ and expand in velocity}$$

$$\longrightarrow \psi^+\left(iD_t + \frac{1}{2m_q}\bar{D}^2 + \dots\right)\psi \leftarrow \text{quark term}$$

$$+ \chi^+\left(iD_t - \frac{1}{2m_q}\bar{D}^2 + \dots\right)\chi \leftarrow \text{antiquark term}$$

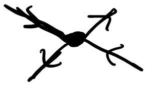
$$\Psi \sim \begin{pmatrix} \psi \\ \chi \end{pmatrix}$$

- This Lagrangian conserves heavy quarks and antiquarks

$\Rightarrow$ Add mixing terms like $\mathcal{O} = \psi^+\chi\chi^+\psi$ to describe production and decay

$$\delta\mathcal{L} = \sum_i \frac{f_i}{m_q^2}\mathcal{O}_i$$

$\mathcal{O} \sim$



- expansion in the (relative) velocity v and $\alpha_s \sim v$
- write particles in terms of different spin and color states

Example:

spectroscopic notation for the spin and orbital angular momentum of the $q\bar{q}$ pair ↓

$$|J/\psi\rangle = \mathcal{O}(v^0)\,|^3S_1^{[1]}\rangle + \mathcal{O}(v)\,|^3P_1^{[8]}g\rangle + \mathcal{O}(v^{3/2})\,|^1S_0^{[8]}g\rangle$$

$$+\,\mathcal{O}(v^2)\,|^3D_1^{[1]}\rangle + \dots$$

color state of the $q\bar{q}$ system

suppression in velocity

- describe particles in terms of universal long-distance matrix elements (LDMEs)

Example: For $J/\psi \to q\bar{q}$ wave function

$$|\psi(\vec{r}=0)|^2 = \frac{1}{N_c}\langle \mathcal{O}_1(^3S_1)\rangle_{J/\psi} \qquad \mathcal{O}_1(^3S_1) = \psi^\dagger\sigma^i\chi\,\chi^\dagger\sigma^i\psi$$

Leading order in v and α_s:

only one universal nonperturbative constant: extremely simple!

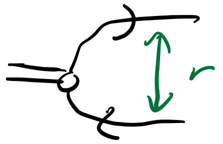
- Can be extracted from e.g. the decay width

$V \to \ell^+\ell^-$ (for vector mesons)

Light (vector) mesons

Focus on LO wave function $\psi^{q\bar{q}}(r,z)$.

r = transverse separation between $q\bar{q}$



$z = p^+_q / p^+_V$

= longitudinal momentum fraction carried by the quark

Consider a process with a hard scale $Q^2 \gg M_V^2$.

Typical dipole size $r^2 \sim 1/Q^2$

$\Rightarrow$ Perform a Taylor expansion for $\psi^{q\bar{q}}$ in terms of r^2.

$$\psi^{q\bar{q}}(r,z) = \underbrace{\psi^{q\bar{q}}(0,z)}_{\mathcal{O}(1)} + \underbrace{r^i \partial_i \psi^{q\bar{q}}(0,z)}_{\mathcal{O}(M_V/Q)} + \frac{1}{2}\underbrace{r^i r^j \partial_i \partial_j \psi^{q\bar{q}}(0,z)}_{\mathcal{O}(M_V^2/Q^2)} + \ldots$$

Depends on the meson polarization:

- Longitudinal: $\psi^{q\bar{q}}_L \sim f_V\, \phi(z) + \mathcal{O}(r^2)$
- Transverse: $\psi^{q\bar{q}}_\lambda \sim f_V\, \varepsilon^i_\lambda r^i \phi'(z) + \mathcal{O}(r^3)$

$\phi(z)$ = distribution amplitudes $\qquad f_V$ = decay constant

A more formal definition for $\phi(z)$ with twist expansion

field operators

Wilson line

$$\psi^{q\bar{q}} \sim \langle 0 | \bar{\Psi}(r_q)\, \Gamma\, W[r_q, r_{\bar{q}}]\, \Psi(r_{\bar{q}}) | V \rangle = \underset{\mathcal{O}(1)}{\text{twist-2}} + \underset{\mathcal{O}(M_V/Q)}{\text{twist-3}} + \ldots$$

Dirac matrix

Leading twist (twist 2):

only need longitudinal polarization with $\phi(z)$

In practice the distribution amplitudes also depend on Q^2: $\phi(z) \to \phi(z,Q^2)$

Scale dependence due to renormalization of the quark fields.

- Evolution in Q^2 given by the ERBL equation (analogous to DGLAP):

$$\frac{\partial}{\partial \log Q^2} \phi(z,Q^2) = \frac{\alpha_s C_F}{2\pi} \int_0^1 dz' \, K(z',z) \, \phi(z,Q^2)$$

Perturbative Q^2-dependence, only need an initial condition

Intuitive interpretation: we integrate over the momenta of the $q\bar{q}$ pair until Q^2:

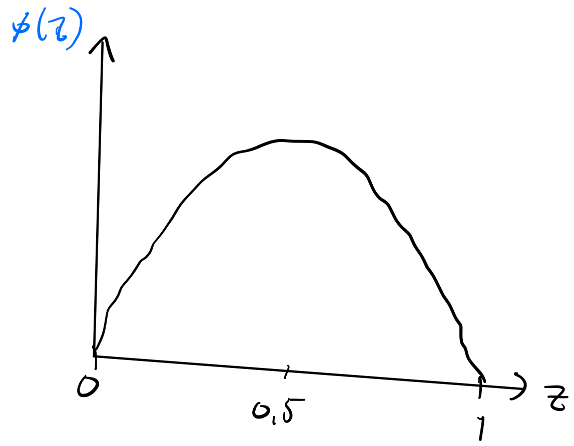
Fourier transform of $\psi^{q\bar{q}}(z,r_\perp)$

$$\phi(z,Q^2) \sim \int^{k_\perp^2 < Q^2} \frac{d^2 k_\perp}{(2\pi)^2} \, \tilde{\psi}^{q\bar{q}}(z,k_\perp)$$

- Asymptotic form:

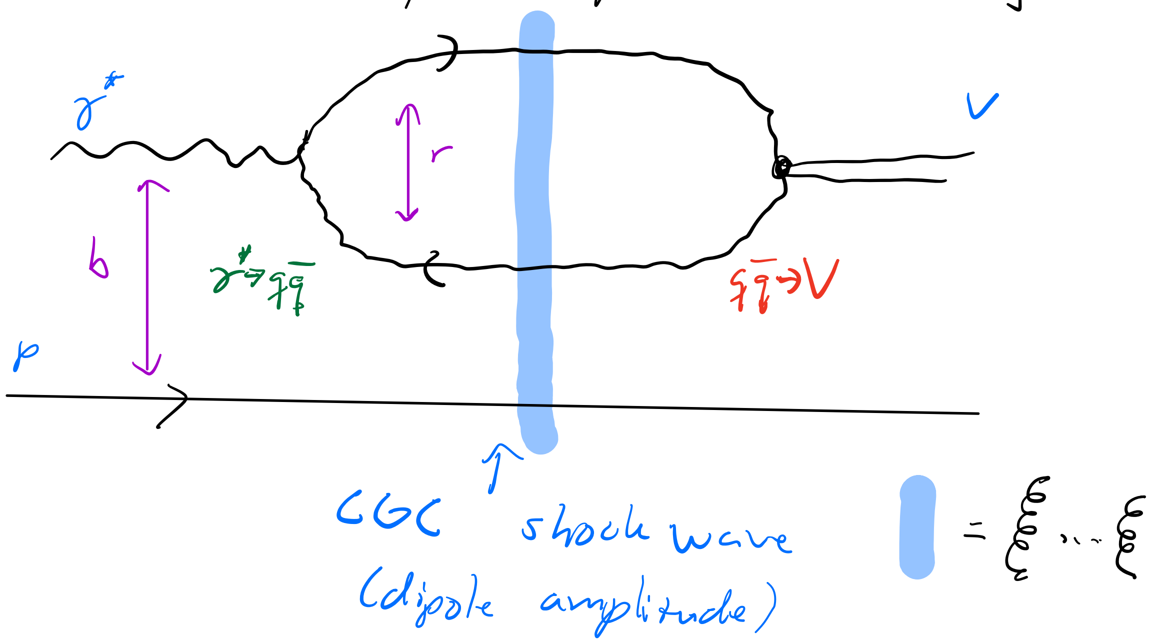
$$Q^2 \to \infty: \quad \phi(z,Q^2) \to 6z(1-z)$$

In this limit only nonperturbative constant is f_V! (can be determined from the decay width $V \to l^+ l^-$)



Exclusive vector meson production in color-glass condensate

We are now ready to compute this at leading order!



$$\frac{d}{dt}\sigma^{\gamma^*+p\to V+p} = \frac{1}{4\pi}|\mathcal{M}|^2$$

where

$$i\mathcal{M} = \int_0^1 \frac{dz}{4\pi z(1-z)}\int \underset{\text{dipole size}}{d^2\bar{r}}\,\underset{\text{impact parameter}}{d^2\bar{b}}\, e^{-i\bar{b}\cdot\overset{\text{momentum transfer}}{\bar{\Delta}}}\, \Psi^{\gamma^*\to q\bar{q}}(\bar{r},z)\, N(\bar{r},\bar{b})\, \left[\Psi^{V\to q\bar{q}}(\bar{r},z)\right]^*$$

light-cone wave functions

$\psi^{\gamma^*\to q\bar q}$: perturbative, can be calculated

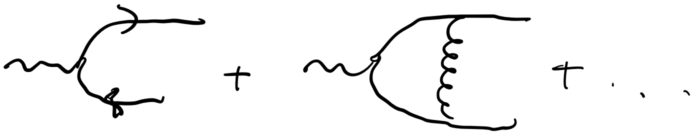
N: can be modelled using CGC

$\psi^{V\to q\bar q}$: 1. Light vector mesons with large photon virtuality Q^2: $\psi^{V\to q\bar q} \to \phi(z)$

2. Heavy vector mesons in the nonrelativistic limit: $\psi^{V\to q\bar q} \to \sqrt{\frac{1}{2N_c}\langle \mathcal{O}_1({}^3S_1)\rangle_{J/\psi}}$

We can also include higher-order corrections in QCD:

1. Corrections to the wave functions

$\psi^{\gamma\to q\bar q} =$ 

2. New Fock states

$$\psi^{\gamma\to q\bar q} N^{q\bar q} [\psi^{V\to q\bar q}]^* \qquad \underline{\psi^{\gamma\to q\bar q g}}\, \underline{N^{q\bar q g}} [\underline{\psi^{V\to q\bar q g}}]^*$$

New wave functions and Wilson line correlators appear!

State-of-the-art: NLO with approximations for the vector meson wave function

Saturation effects in exclusive J/ψ production

Measuring saturation effects: need momentum scale $\mu^2 \lesssim Q_s^2$.

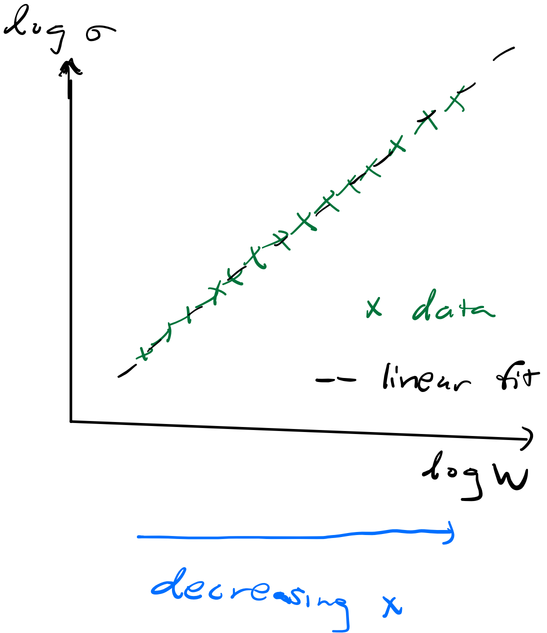
If $\mu^2 \sim \Lambda_{QCD}^2$: nonperturbative...

J/ψ mass kind of a sweet spot:

- Large enough to be perturbative
- Small enough for (hopefully) seeing saturation

$\gamma^* + p \to J/\psi + p$

https://arxiv.org/pdf/2207.03712

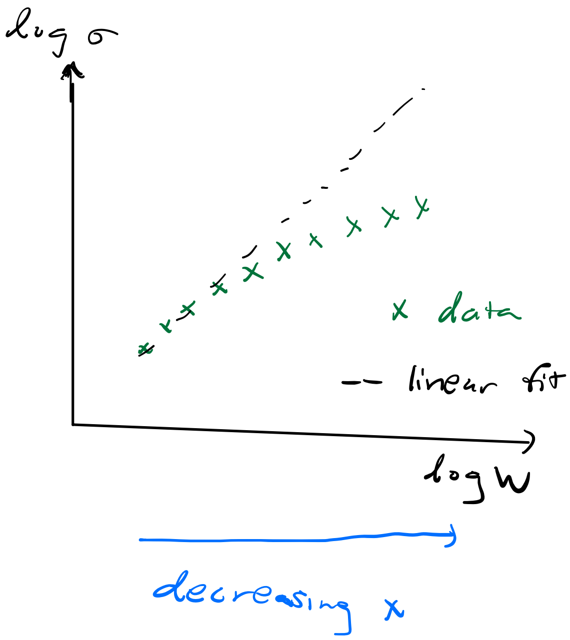


Proton targets:
Dependence completely linear

⇒ no saturation

$\gamma^* + Pb \to J/\psi + Pb$

https://arxiv.org/pdf/2312.04194



Pb targets:
Deviations from linear

⇒ saturation?

Hot topic: Is this saturation or something else?