What wee partons reveal about hadron structure at high energies and the dynamics of confinement-II

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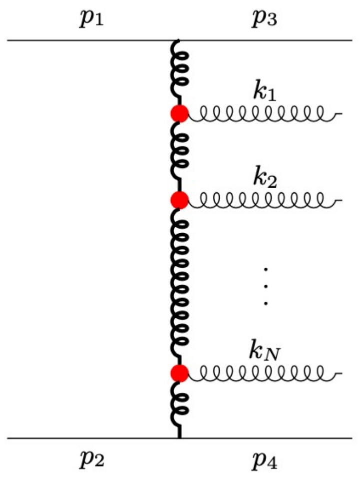
The BFKL Pomeron: 2→ N QCD amplitudes in Regge asymptotics*



Compute multiparticle in multi-Regge kinematics of QCD:

$$y_0^+ \gg y_1^+ \gg y_2^+ \gg \cdots \gg y_N^+ \gg y_{N+1}^+ \qquad \text{with} \qquad \boldsymbol{k}_i \simeq \boldsymbol{k}$$

BFKL ladder is ordered in rapidity . Produced partons are wee in longitudinal momentum(` ` slow”) but hard in transverse momentum – weak coupling Regge regime

RG description rapidity of evolution given by the BFKL Hamiltonian
Very rapid growth of the amplitude with energy

A(s,t) = $s^{\alpha(t)}$ with $\alpha(t) = \alpha_0$ + α' $|t|$ BFKL pomeron

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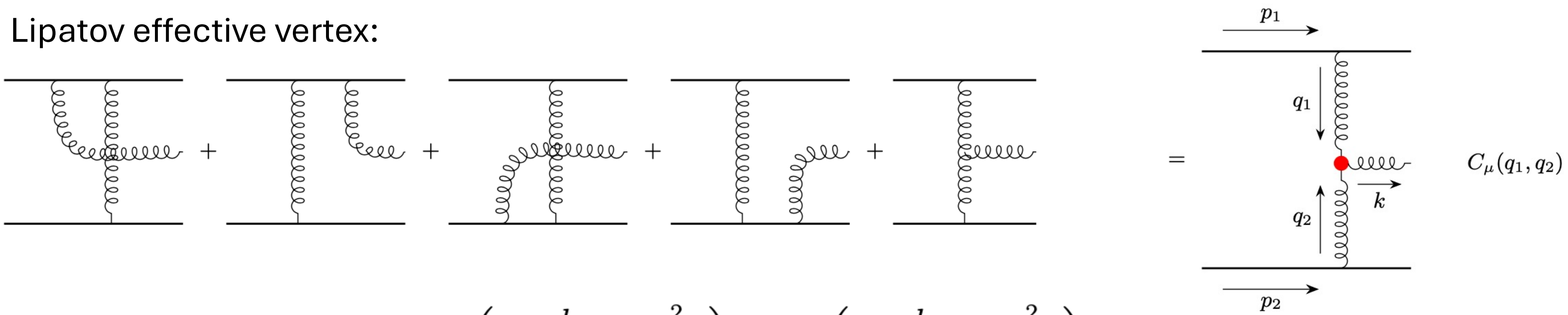
* Asymptotics is the calculus of approximations. It is used to solve hard problems that cannot be solved exactly and to provide simpler forms of complicated results

BFKL: Building blocks

Lipatov effective vertex:



$$C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) \simeq -\boldsymbol{q}_{1\mu} + \boldsymbol{q}_{2\mu} + p_{1\mu}\left(\frac{p_2 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_1^2}{p_1 \cdot k}\right) - p_{2\mu}\left(\frac{p_1 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_2^2}{p_2 \cdot k}\right)$$

Gauge covariant, satisfies $k_\mu\, C^\mu$ =0

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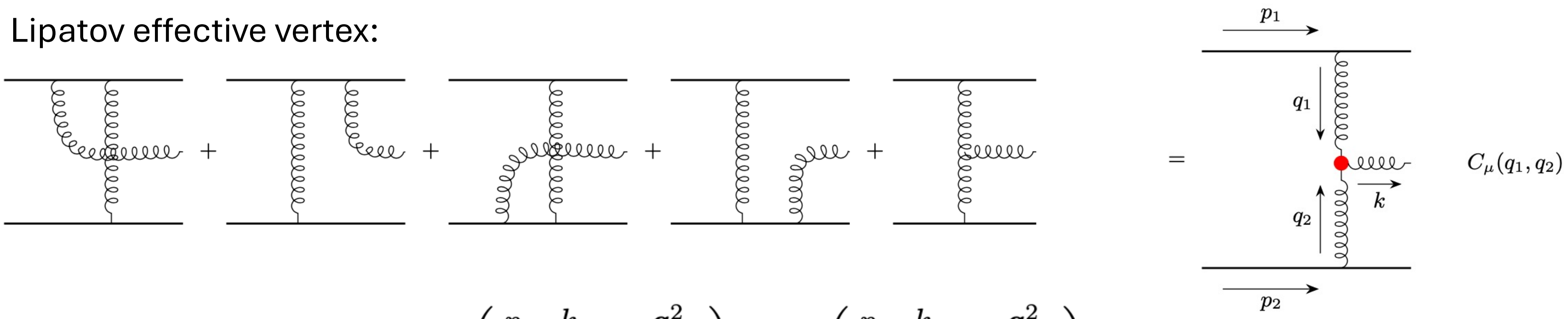

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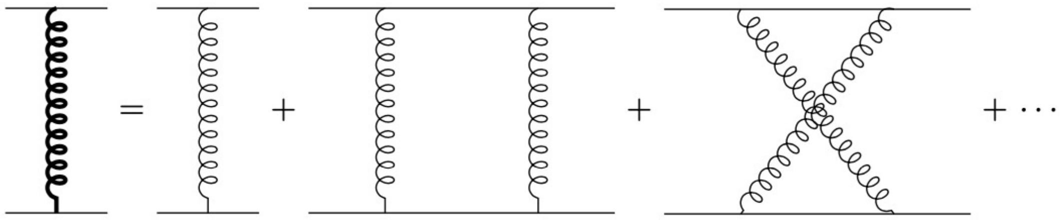
Reggeized gluon:

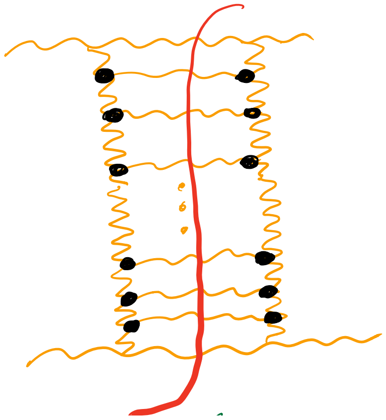


$$\frac{1}{t_i} \to \frac{1}{t_i} e^{\alpha(t_i)(y_{i-1} - y_i)}$$

$$\alpha(t) = \alpha_s N_c \, t \int \frac{d\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2 (\boldsymbol{q} - \boldsymbol{k})^2} \,, \qquad t = -\boldsymbol{q}^2$$

$2 \rightarrow N + 2$ amplitude in the Regge limit: the BFKL equation

BFKL Pomeron: compound color singlet state of two reggeized gluons



$$\mathrm{Im}\,A(s,t) \propto \sum_{n=0}^{\infty} (\alpha_S C_T)^{n+2}$$

C_T is color factor

$$\times \int \prod_{i=1}^{n} \frac{dy_i}{4\pi} \prod_{j=1}^{n+1} \frac{d^2 q_{j\perp}}{(2\pi)^2}$$

Phase space factors

$$\times\ 2is \prod_{l=1}^{n+1} \frac{1}{t_l t_{l'}}\, e^{(y_{l-1}-y_l)[\alpha(t_l)+\alpha(t_l')]}$$

Reggeized propagators on both sides of cut

$$\times \prod_{m=1}^{n} (C_\mu C^\mu)[q_m, q_{m+1}]$$

Product of Lipatov vertices

$$\sigma_{tot} = 2\,\mathrm{Im}\,A(s,t=0)$$

$$= s^{\lambda} \text{ with } \lambda = \frac{4\alpha_s N_c \ln_e 2}{\pi}$$

$$\simeq 0.5 \text{ for } \alpha_s = 0.2$$

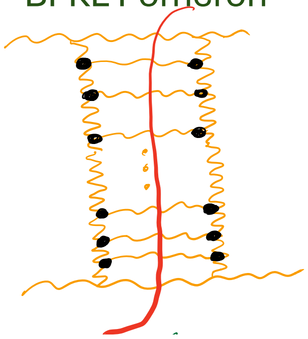
Real and virtual corrections
combine to cancel
infrared divergence !

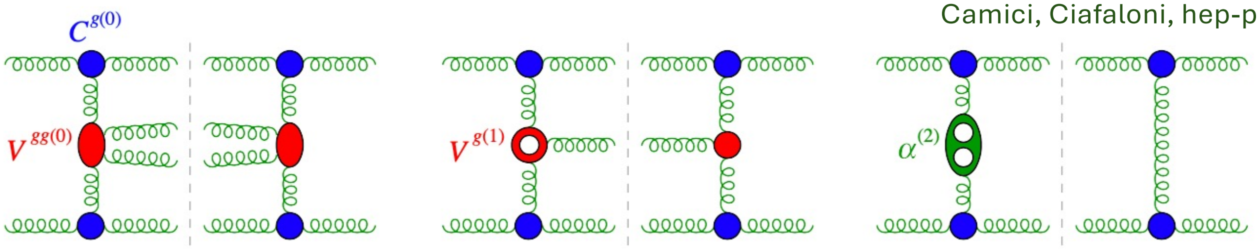
Strongly violates Froissart bound

Resummed NLO BFKL : $\lambda \approx 0.3$

$2 \rightarrow \mathrm{N} + 2$ amplitude in the Regge limit: the NLL BFKL equation

BFKL Pomeron



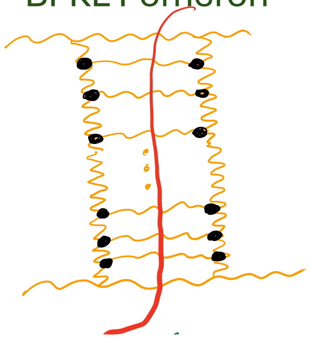


Fadin, Lipatov, hep-ph/9802290
Camici, Ciafaloni, hep-ph/8903389

Regge factorization at NLL $[\alpha_S(\alpha_S\ \mathrm{Ln(s/t)})^n]$: Includes one loop corrections to the Lipatov vertex ($V^{g(1)}$) and two loop corrections to the Regge trajectory ($\alpha^{(2)}$)

$2 \rightarrow N+2$ amplitude in the Regge limit: the NLL BFKL equation

BFKL Pomeron



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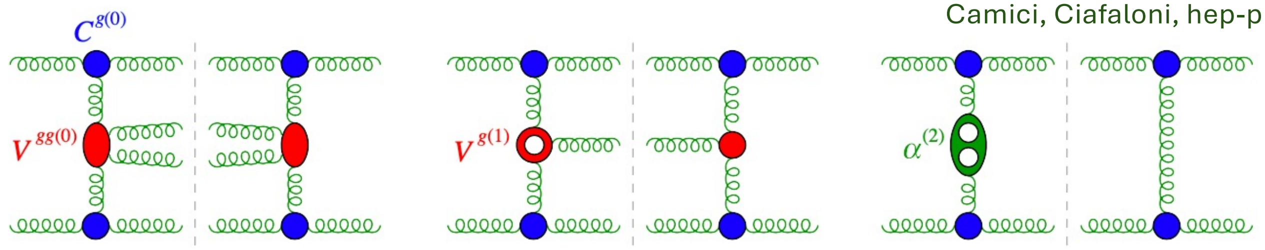

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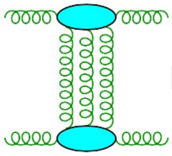
Beyond NLL:

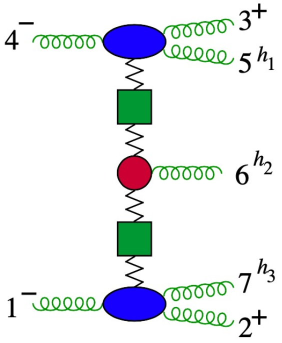


Three reggeized gluon exchange corresponds to Regge cut in angular momentum plane – this can be computed

Falcioni et al., arXiv: 2111.10664,
arXIv:2112.11098

Multi-Regge limit of planar SYM $\mathcal{N} = 4$:

Figures from excellent review of state-of-the art:
Del Duca, Dixon, arXiv:2203.13026



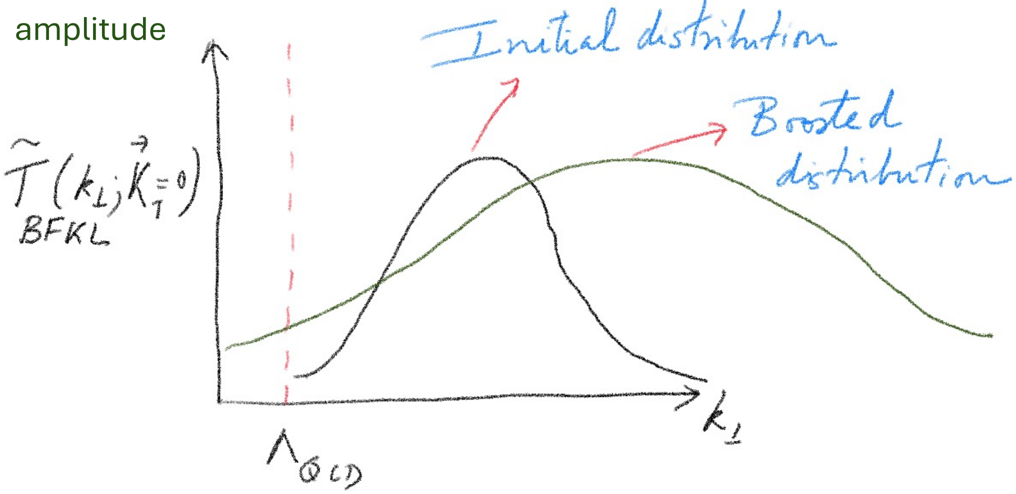
At large t'Hooft coupling, AdS/CFT duality between amplitudes and minimal area surfaces with closed light-like polygon boundaries

Dual conformal tranformations → BDS ansatz; rich mathematical structure of MHV amplitudes in MRK kinematics

BDS: Bern, Dixon, Smirnov
See for example, Dixon, Liu, Miczajka, arXiv:2110.11388

BFKL: infrared diffusion and gluon saturation

BFKL forward amplitude



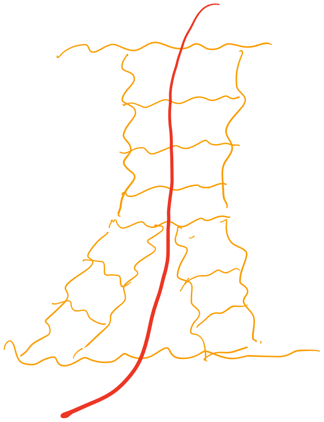
For a fixed large Q^2 there is an $x_0(Q^2)$ such that below x_0 the OPE breaks down…

significant nonperturbative corrections in the leading twist coefficient and anomalous dimension functions due to diffusion of gluons to small values of transverse momentum.

A. H. Mueller, PLB 396 (1997) 251

NLL BFKL does not cure infrared diffusion

Gluon saturation cures infrared diffusion



+ other higher twist cuts of O(1) when gluon occupancy $N \equiv \frac{xG_A(x,Q_S^2)}{2(N_c^2-1)\pi R_A^2 Q_S^2} = \frac{1}{\alpha_S(Q_S)}$

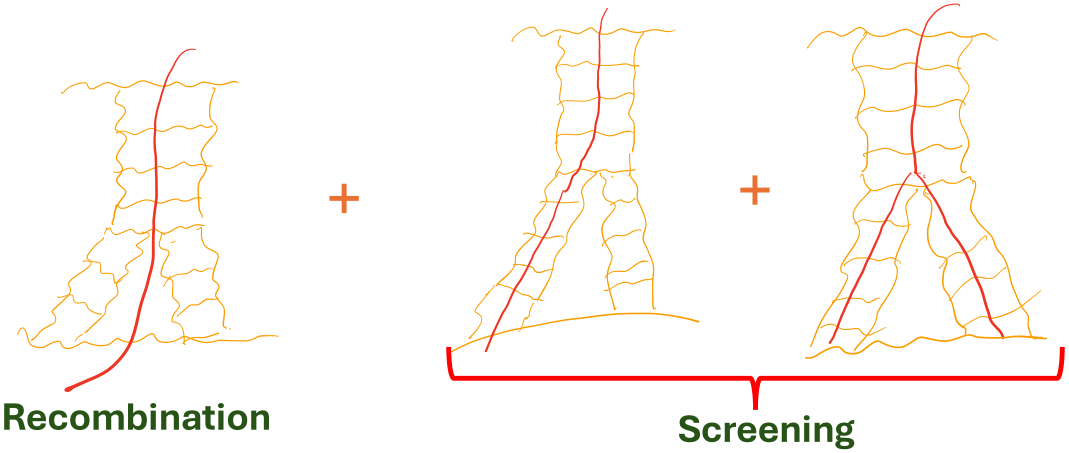
Classicalization when $\alpha_S(Q_S) \ll 1$ for saturation scale $Q_S \gg \Lambda_{QCD}$

Breakdown of OPE: Multi-Pomeron and Reggeon exchange

Gribov,Levin,Ryskin (1983)
Mueller, Qiu (1986)

Rapid BFKL growth leads to large phase-space occupancy N at high energies
→ novel many-body gluodynamics

Partons recombine and screen – many-body “shadowing”



$$= -\frac{[\alpha_s G(x,Q^2)]^2}{Q^2R^2}$$

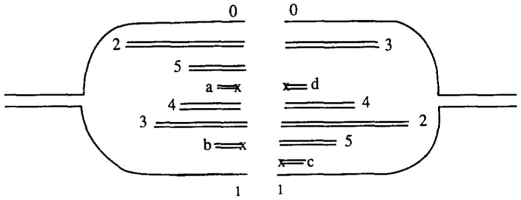
All-twist power suppressed contributions
- “death by a million cuts” equally important in the high occupancy regime $N \equiv \frac{xG_A(x,Q_S^2)}{2(N_c^2-1)\pi R_A^2 Q_S^2} = \frac{1}{\alpha_S(Q_S)}$

Classicalization when $\alpha_S(Q_S) \ll 1$ for saturation scale $Q_S \gg \Lambda_{QCD}$

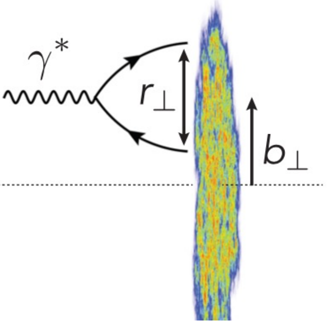
Gluon saturation: classicalization and perturbative unitarization

s-channel "dipole" scattering picture
– more convenient for multi-pomeron interactions

Mueller, NPB415 (1994) 373
Mueller, Patel, hep-ph/9403256



Boosted projectile



Boosted target

$$\sigma_{q\bar{q}P}(r_\perp, x) = \sigma_0\left[1 - \exp\left(-r_\perp^2\, Q_s^2(x)\right)\right]$$

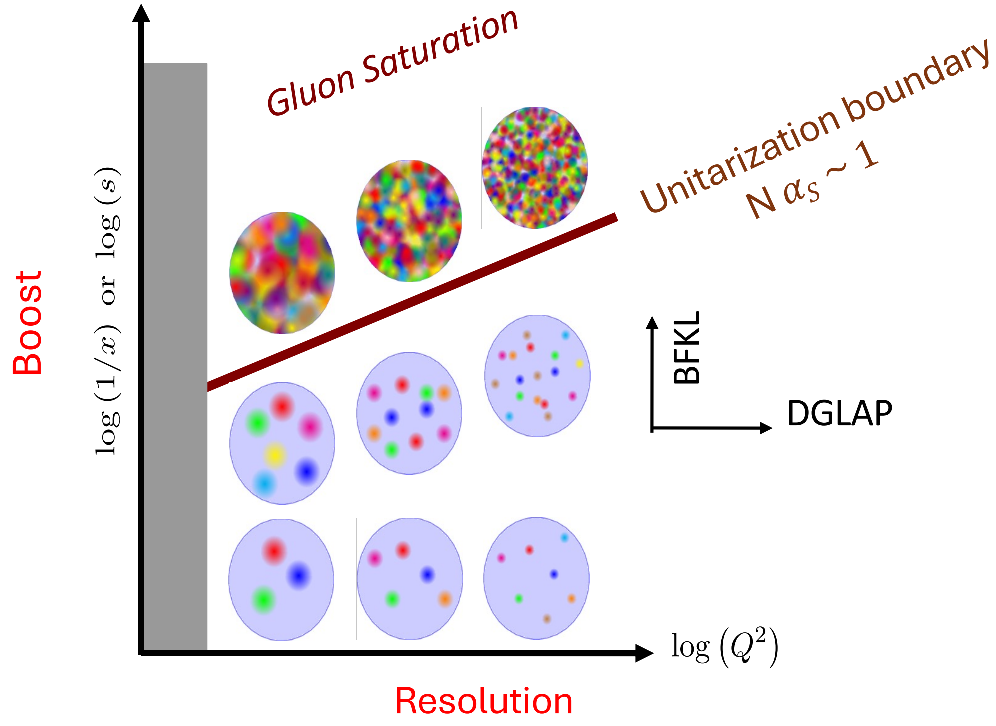
Emergent semi-hard scale $Q_s^2(x) = Q_0^2\left(\frac{x_0}{x}\right)^\lambda$ → BFKL eigenvalue

Color transparency for $r_\perp^2 Q_S^2$ << 1 ($\sigma \propto A$)

Color opacity ("black disk") for $r_\perp^2 Q_S^2$ >> 1 ($\sigma \propto A^{2/3}$)

QCD picture of observed "shadowing" at small x

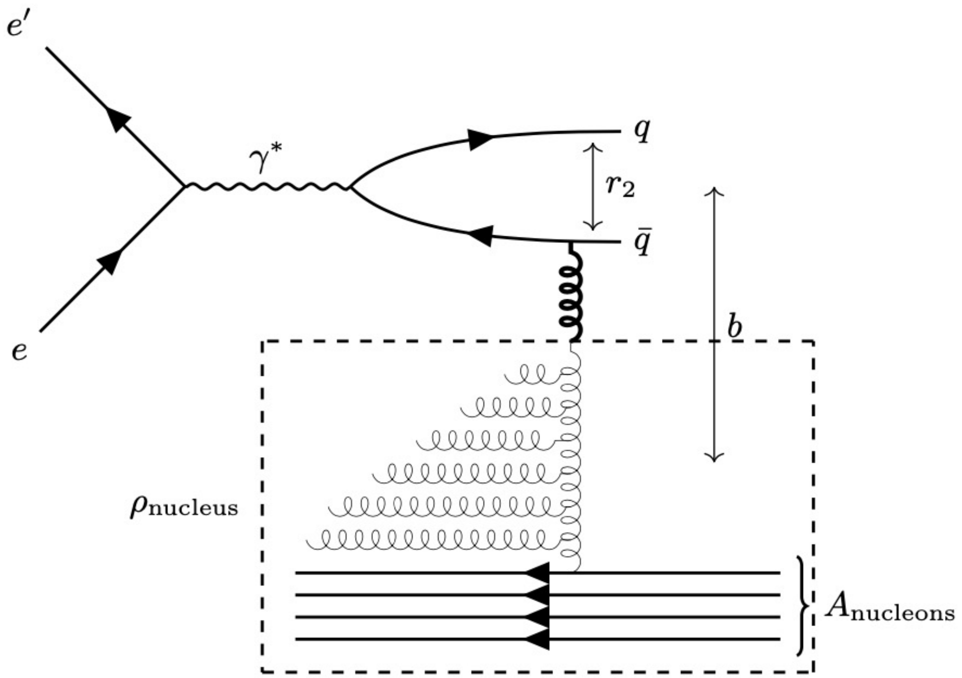
Gluon saturation: classicalization and unitarization of cross-sections



Saturation – nontrivial fixed point – defines emergent scale Q_S^2(x)≫ Λ_{QCD}^2

Color Glass EFT

McLerran, RV (1994)



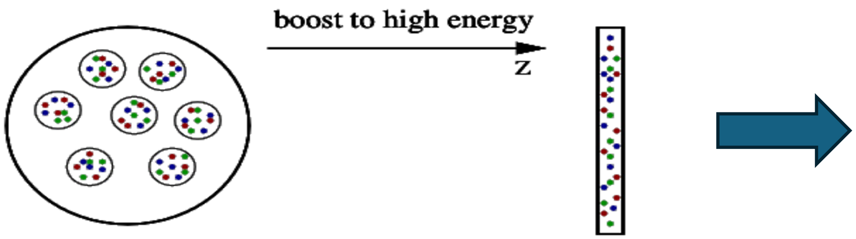
Small x

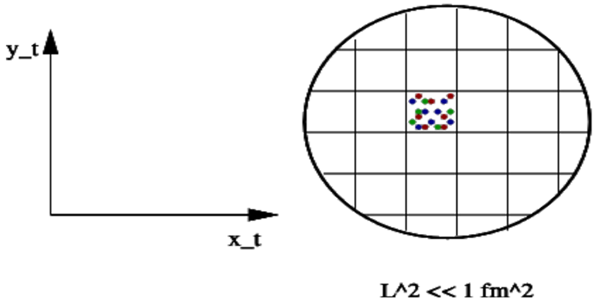
Large x

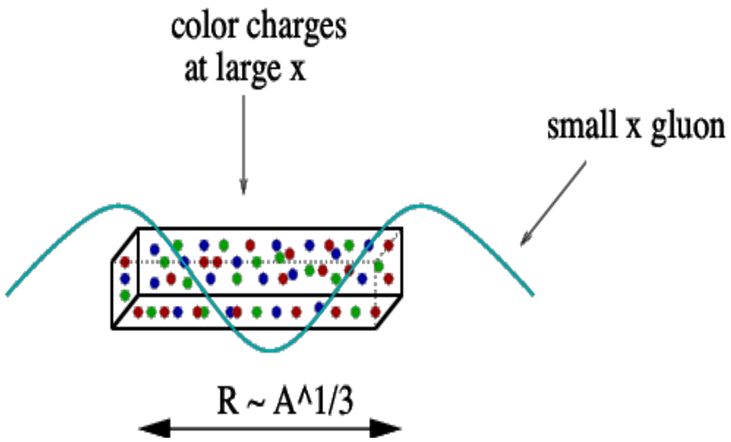
Fundamental basis for CGC EFT: large x modes are static on the light cone.
small x “wee” modes are dynamical

This “Born-Oppenheimer” separation of time scales, allows one to focus on the dynamics of the highly occupied wee modes coupled to to large x modes

Color Glass EFT-classical color charges







$$\lambda_{\rm wee} \approx \frac{1}{k^+} \equiv \frac{1}{xP^+} >> \lambda_{\rm val.} \equiv \frac{Rm_p}{P^+} => x << A^{-1/3}$$

Typical representation is a high dimensional “classical” representation: [Q,Q] ~$\frac{\hbar}{N}$ Q→ 0

So, instead of summing over discrete color charges of various possible distributions of small x modes, one can perform a classical path integral instead – “mean field” approximation

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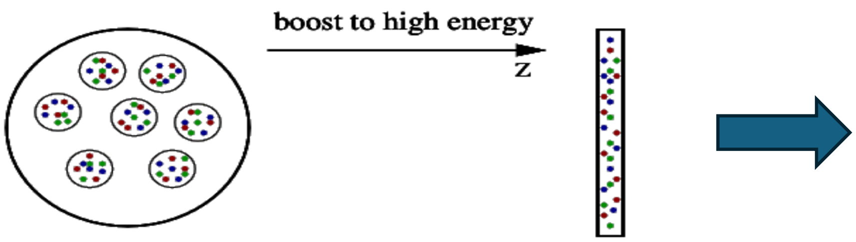

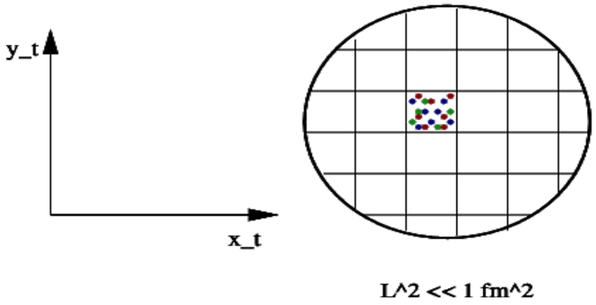

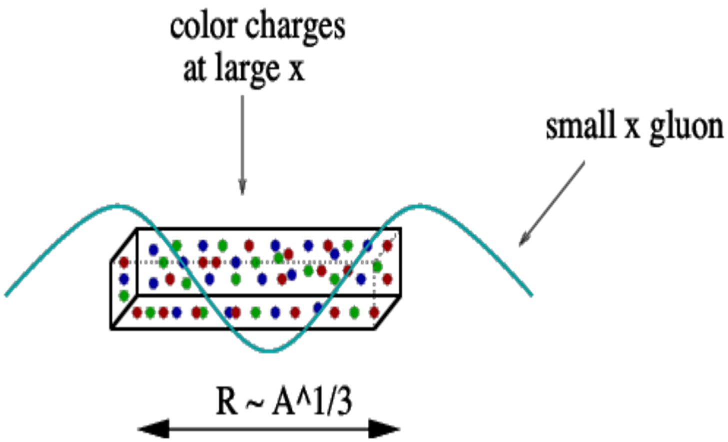

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McLerran,RV (1994)

Color Glass- path integral over replicas

$$\mathcal{Z}[j] = \int [d\rho] W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA] \delta(A^+) e^{iS_{\Lambda^+}[A,\rho] - \int j \cdot A}}{\int^{\Lambda^+} [dA] \delta(A^+) e^{iS_{\Lambda^+}[A,\rho]}} \right\}$$

For large nuclei, A >> 1,

Random walk in color space of SU(3)

$$W_{\Lambda^+} = \exp\left(- \int d^2 x_\perp \left[\frac{\rho^a \rho^a}{2\,\mu_A^2} - \frac{d_{abc}\,\rho^a \rho^b \rho^c}{\kappa_A} \right] \right)$$

$\Uparrow$ Pomeron excitations $\Uparrow$ Odderon excitations

Jeon, RV, hep-ph/0406169

$W_{\Lambda^+}[\rho]$: nonpert. gauge inv. weight functional defined at initial $x_0 = \Lambda^+ / P^+$

$S_{\Lambda^+}[A, \rho]$: Yang-Mills action + gauge-inv. coupling of sources to fields (Wilson line)

Saddle point of CGC action: Weizsäcker-Williams and color memory

A_i =0

$A_i = -\frac{-1}{ig}\, U\partial_i\, U^\dagger$

x^- =0

$$D_i \frac{dA^{i,a}}{dy} = g\rho^a(x_t, y)$$

$$U = P\exp\left(i\int_y^\infty dy' \,\frac{\rho(x_t, y')}{\nabla_t^2} \right)$$

$y=\ln(x^-/x_0^-)$

solution of the YM-eqns: two pure gauges (zero field strength) separated by shockwave discontinuity

U represents the *color memory* effect

- a color rotation and $p_T \sim Q_S$ kick experienced by quark-antiquark pair traversing the shock wave

Pate, Raclariu, Strominger, PRL (2017)

Ball,Pate,Raclariu,Strominger,RV,
Annals of Physics (407 2019) 15

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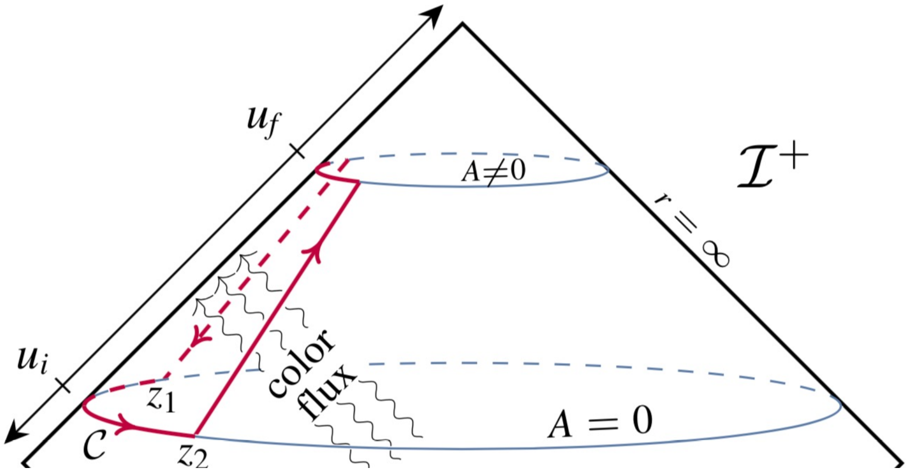
solution of the YM-eqns: two pure gauges (zero field strength) separated by shockwave discontinuity

Stereographic projection of dynamics of glue to celestial sphere at “null infinity”

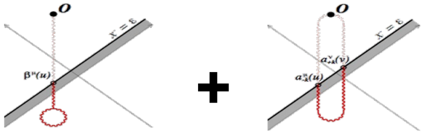
Mathematically analogous to an observable “gravitational memory effect” in GR

Deep connections to asymptotic BMS extension of Poincare group in gravity and to soft theorems



Renormalization group evolution of color charges with x

$$\mathcal{O}_{\rm NLO} = \Bigg($$



$\chi(x_\perp, y_\perp)$

$$\Bigg)\mathcal{O}_{\rm LO}$$

$$\langle \mathcal{O}_{\rm LO} + \mathcal{O}_{\rm NLO} \rangle = \int [d\tilde{\rho}] W[\tilde{\rho}] \left[\mathcal{O}_{\rm LO} + \mathcal{O}_{\rm NLO} \right] \; = \int [d\tilde{\rho}] \left\{ \left[1 + \ln\left(\frac{\Lambda^+}{p^+} \right) \mathcal{H} \right] W_{\Lambda^+} \right\} \mathcal{O}_{\rm LO}$$

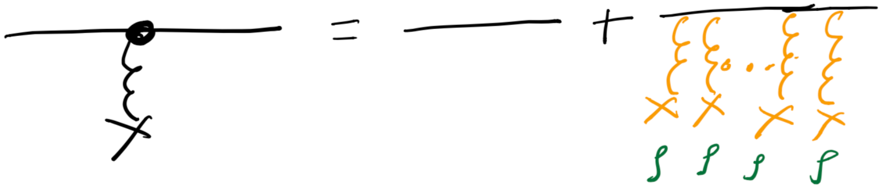
Independence of l.h.s on Λ^+ =>

$$\boxed{\frac{\partial W[\tilde{\rho}]}{\partial Y} = \mathcal{H}\, W[\tilde{\rho}]}$$

JIMWLK* Hamiltonian –see Iancu lectures

* JIMWLK:Jalilian-Marian,Iancu,McLerran,Weigert,Leonidov,Kovner

Shock wave propagators in the CGC



$$= S_0(p)\, T_q(p,q)\, S_0(q)$$

$$T_q(p,q) = (2\pi)\,\delta(p^- - q^-)\,\gamma^- \,\mathrm{sign}(p^-)$$
$$* \int d^2 z_\perp\, e^{-i(p_\perp - q_\perp) z_\perp}\, V^{\mathrm{sign}(p^-)}(z_\perp)$$

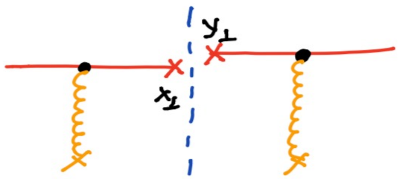
$$= G_0(p)\, T_g\, G_0(q)$$

$$T_g^{\mu\nu}(p,q) = (2\pi)\,\delta(p^- - q^-)\,(2p^-)\,\mathrm{sign}(p^-)$$
$$* \int d^2 z_\perp\, e^{-i(p_\perp - q_\perp) z_\perp}\, U^{\mathrm{sign}(p^-)}(z_\perp)$$

McLerran, RV (1994, 1998)
Balitsky (1995)
Ayala,Jalilian-Marian,McLerran,Venugopalan (1995)
Hebecker, Weigert (1997)
Balitsky, Belitsky (2001)

An example: quark scattering on dense target (forward p+A)

LO



$$\text{Amp.} \times \text{c.c.} \rightarrow 1 - \frac{1}{N_c}\left\langle \underbrace{\text{Tr}\left(V(x_\perp)\, V^\dagger(y_\perp)\right)}_{x_2}\right\rangle$$

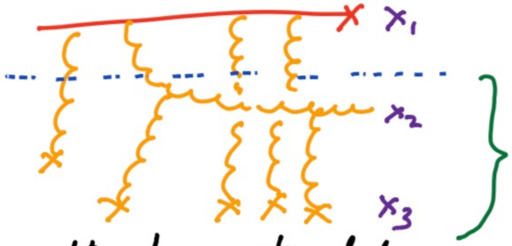
DIPOLE CORRELATOR

BUILDING BLOCK OF HIGH ENERGY QCD

Balitsky-Kovchegov equation:
RG evolution of dipole correlator
(JIMWLK for $N_c \gg 1$ and A$\gg$ 1)

if $x_1 >> x_2 >> x_3$

NLO



These soft bremsstrahlung contributions absorbed in RG evolution of DIPOLE CORRELATOR

$$S_{Y_1} \equiv \frac{1}{N_c}\left\langle \text{Tr}\left(V_{x_\perp} V^\dagger_{y_\perp}\right)\right\rangle_{Y_1}$$

$$\rightarrow \frac{1}{N_c}\left\langle \text{Tr}\left(V_{x_\perp} V^\dagger_{y_\perp}\right)\right\rangle_{Y_2}$$

$$\frac{dS}{dY} = K_{BFKL} \otimes (S - SS)$$

SOFT BREMSSTRAHLUNG
+ MULTIPLE SCATTERING ≡ SHADOWING

Classicalization and perturbative unitarization

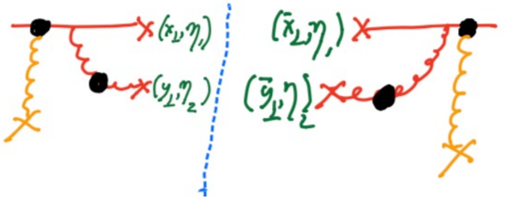
BK unitarizes cross-section when $S \to 0$: "Black disc"

This occurs when $N \sim 1/\alpha_s$ when dipole size $r_\perp = x_\perp - y_\perp \equiv \frac{1}{Q_s}$

In "dilute" regime, expanding around $S \sim 1$, to lowest order in $\rho/\nabla_\perp^2$, obtain BFKL eqn!

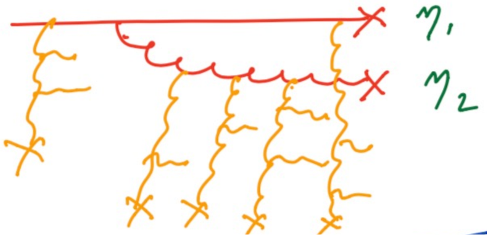
Semi-inclusive final states

$pA \to qg + X$ CROSS-SECTION



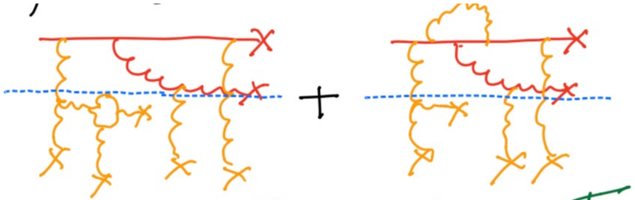
SENSITIVE TO BOTH 2-POINT DIPOLE WILSON LINE CORRELATOR & NOVEL 4-POINT QUADRUPOLE CORRELATOR

SOFT BREMSSTRAHLUNG ALSO SHADOWS SUCH CORRELATORS



JIMWLK RG equations describe bremsstrahlung of DIPOLES, QUADRUPOLES, SEXTUPOLES, ... in an infinite hierarchy

Sadly, LLx resummation not good enough for precision studies



NLLx corrections to RG equations UNIVERSAL

NLO corrections to observable PROCESS DEPENDENT

* PARTS OF THIS ARE COMPUTED

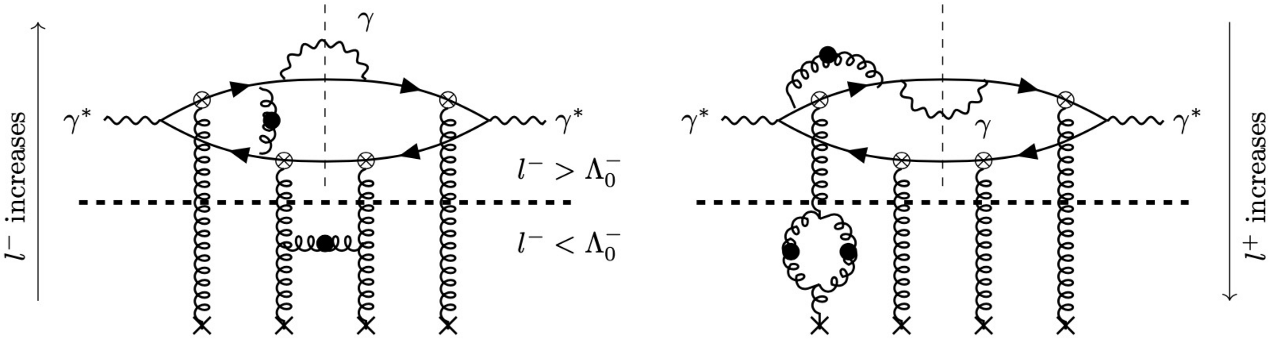
→ NLO JIMWLK "available"

NLO BK more accessible

* PARTS IMPOSED: Eg; RUNNING COUPLING

CGC EFT state-of-the art: NLO+NLLx accuracy

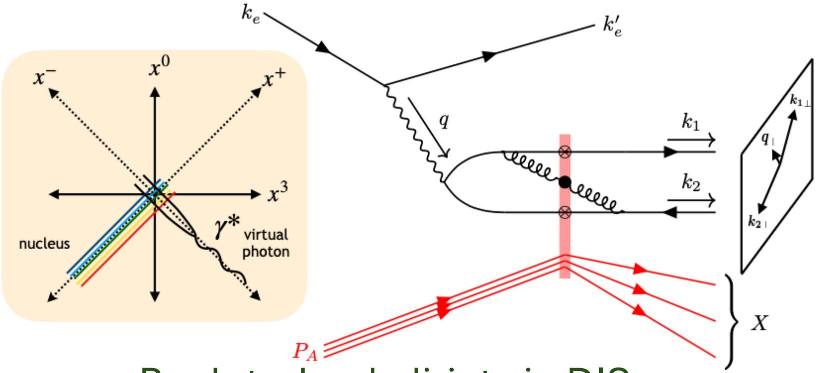
Examples:I) photon+dijets in DIS



Roy, RV, arXiv:1911.04530

A large number of NLO computations by multiple groups using different methods

Gluon Weizsäcker-Williams distribution: complete NLO results

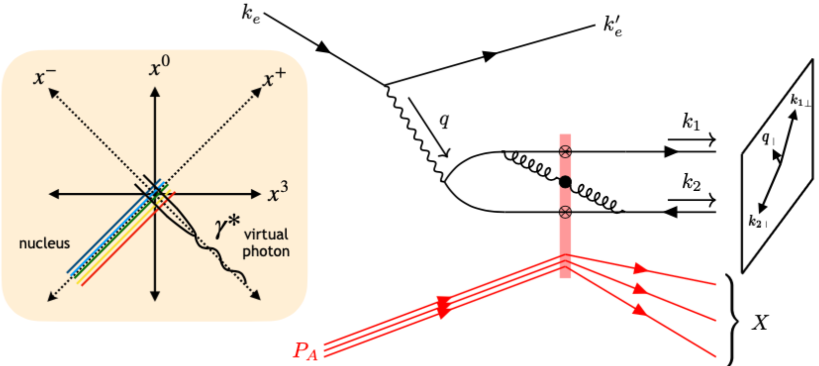


Back-to-back di-jets in DIS

Factorization of small-x TMDs to NLO accuracy

Caucal,Salazar,Schenke,Stebel,RV, arXiv:2308.00022, (PRL 2024)

Gluon Weizsäcker-Williams distribution: complete NLO results



Back-to-back di-jets in DIS

Caucal,Salazar,Schenke,Stebel,RV, arXiv:2308.00022, (PRL 2024)

Factorization of small-x TMDs to NLO accuracy

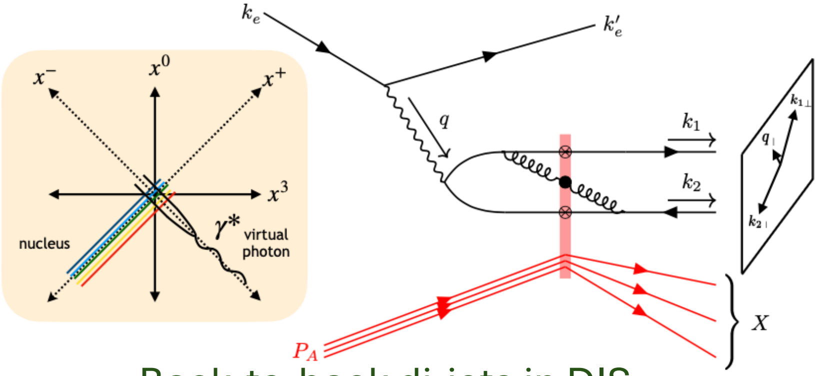
$$\begin{aligned} \mathrm{d}\sigma^{(0),\lambda=\mathrm{T}} &= \mathcal{H}_{\mathrm{LO}}^{0,\lambda=\mathrm{T}} \int \frac{\mathrm{d}^2\boldsymbol{B}_\perp}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \boxed{\hat{G}^0_{\eta_c}(\boldsymbol{r}_{bb'},\mu_0)\mathcal{S}(\boldsymbol{P}_\perp^2,\mu_0^2)} \\ &\times \left\{1 + \frac{\alpha_s(\mu_R)N_c}{2\pi} f_1^{\lambda=\mathrm{T}}(\chi,z_1,R) + \frac{\alpha_s(\mu_R)}{2\pi N_c} f_2^{\lambda=\mathrm{T}}(\chi,z_1,R) + \alpha_s(\mu_R)\beta_0 \ln\left(\frac{\mu_R^2}{P_\perp^2}\right)\right\} \\ &+ \mathcal{H}_{\mathrm{LO}}^{0,\lambda=\mathrm{T}} \int \frac{\mathrm{d}^2\boldsymbol{B}_\perp}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \boxed{\hat{h}^0_{\eta_c}(\boldsymbol{r}_{bb'},\mu_0)\mathcal{S}(\boldsymbol{P}_\perp^2,\mu_0^2)} \\ &\times \frac{-2\chi^2}{1+\chi^4}\left\{\frac{\alpha_s(\mu_R)N_c}{2\pi}\left[1+\ln(R^2)\right] + \frac{\alpha_s(\mu_R)}{2\pi N_c}\left[-\ln(z_1z_2R^2)\right]\right\} + \mathcal{O}\left(\frac{q_\perp}{P_\perp},\frac{Q_s}{P_\perp},\alpha_s R^2,\alpha_s^2\right) \end{aligned}$$

$\hat{G}^0$ and $\hat{h}^0$ respectively are unpolarized and linearly polarized WW distributions,

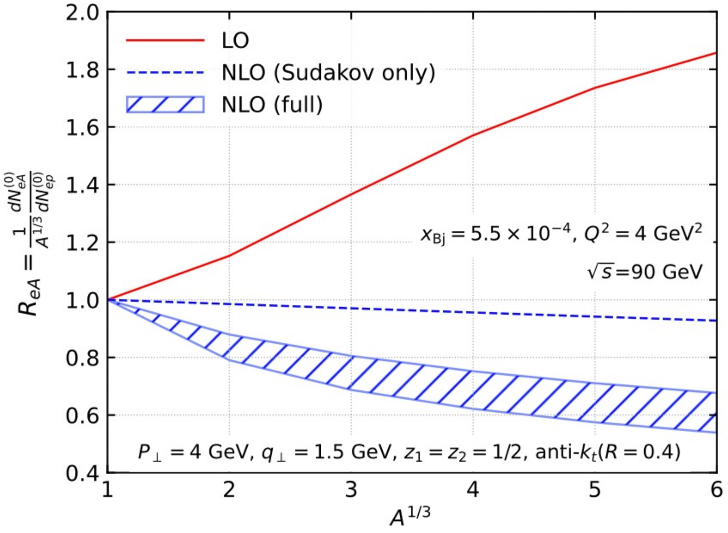
$\mathcal{S}$ the Sudakov soft factor resumming double+single logs in P_T/q_T

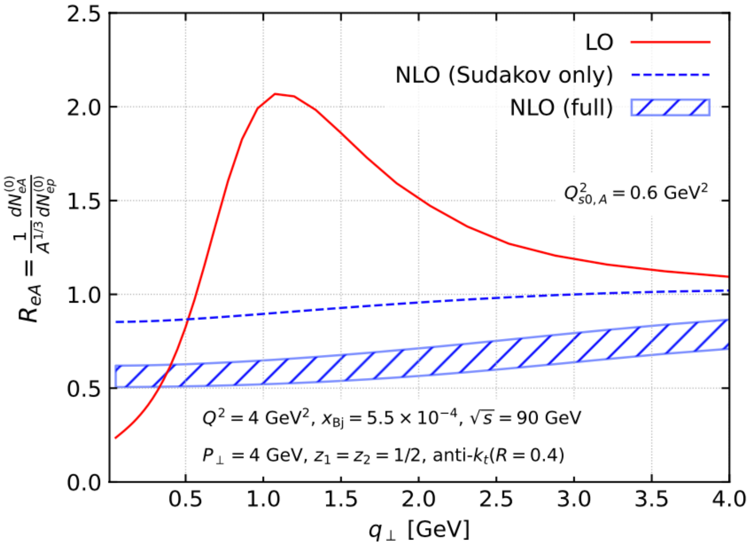
f_1 and f_2 are finite pure $O(\alpha_S)$ contributions

Gluon Weizsäcker-Williams distribution: complete NLO results



Back-to-back di-jets in DIS



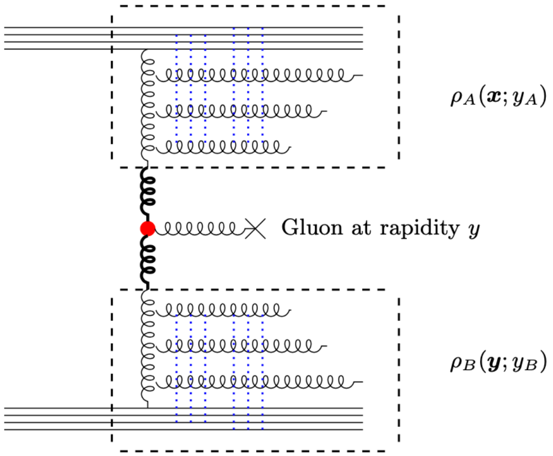


Global analyses to extract "universal" TMDs from p+A collisions at the LHC and e+A collisions from the EIC

A long ways to go – since such NLO (NNLO in usual pQCD counting) analyses in p+A at the LHC are not available

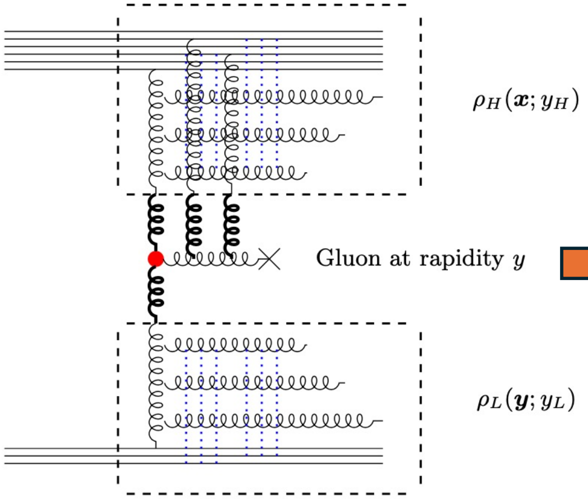
Caucal,Salazar,Schenke,Stebel,RV, arXiv:2308.00022, (PRL 2024)

Gluon shockwave collisions: Lipatov vertex and reggeization



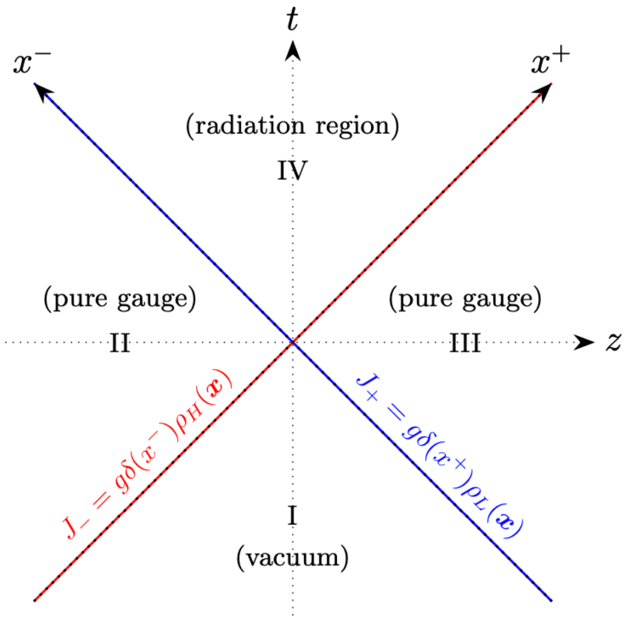
“dilute-dilute” collisions

($\frac{\rho_A}{\Delta_\perp^2}$, $\frac{\rho_B}{\Delta_\perp^2} \ll 1$)

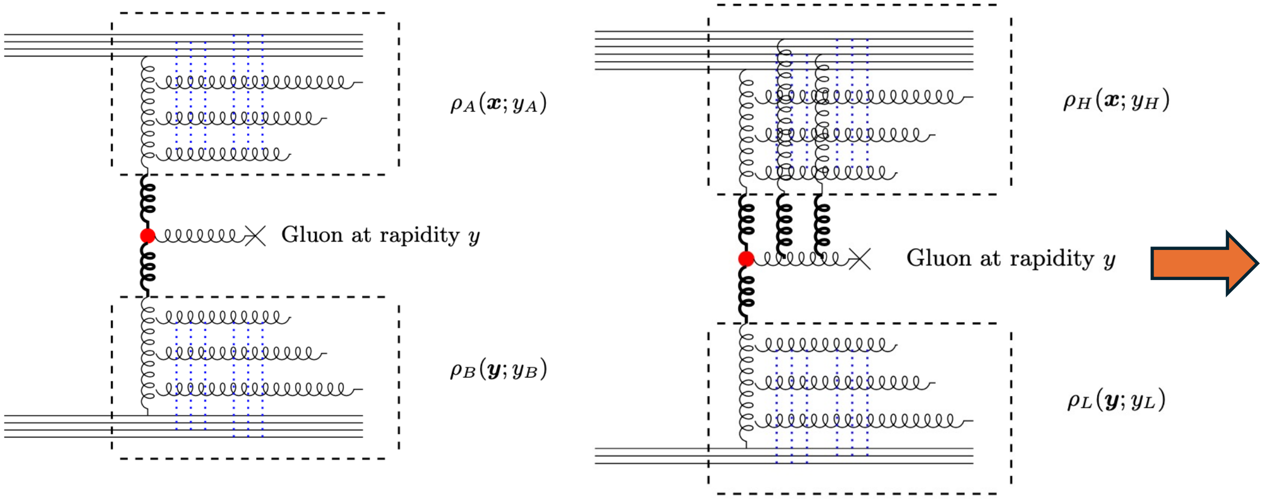


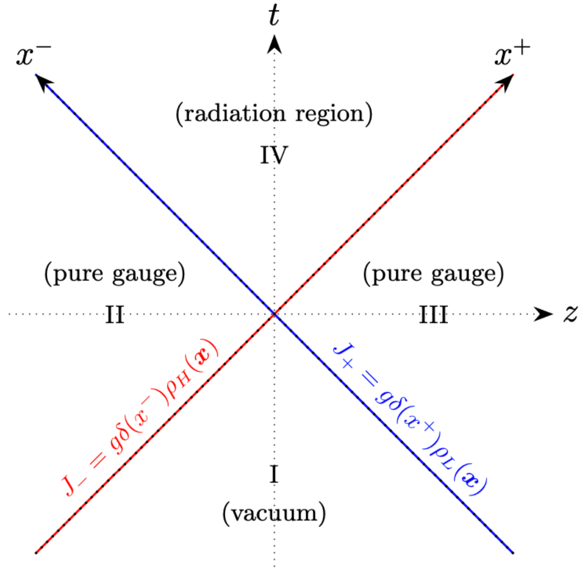
“dilute-dense” collisions

($\frac{\rho_L}{\Delta_\perp^2}$ << 1 , $\frac{\rho_H}{\Delta_\perp^2} \sim O(1)$)



Gluon shockwave collisions: Lipatov vertex and reggeization





Weizs$\ddot{a}$cker-Williams gluon radiation field in light cone gauge

$$a_i(k) = -\frac{2ig}{k^2+i\epsilon}\int\frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\left(q_{2i}-k_i\frac{\boldsymbol{q}_2^2}{\boldsymbol{k}^2}\right)\underbrace{\frac{\rho_L(\boldsymbol{q}_2)}{\boldsymbol{q}_2^2}\left(U(\boldsymbol{k}+\boldsymbol{q}_2)-(2\pi)^2\delta^2(\boldsymbol{k}+\boldsymbol{q}_2)\right)}$$

$$U(x^-,\boldsymbol{x})\delta\left(x^+\right) = \exp\left(ig\int_{-\infty}^{x^-}dz^-\bar{A}_-(z^-,\boldsymbol{x})\cdot T\right)$$

$$\bar{A}_\mu(x^-,\boldsymbol{x}) = -g\delta_{\mu-}\delta\left(x^-\right)\frac{\rho_H\left(\boldsymbol{x}\right)}{\square_\perp}$$

Lipatov vertex
in A^- = 0 gauge

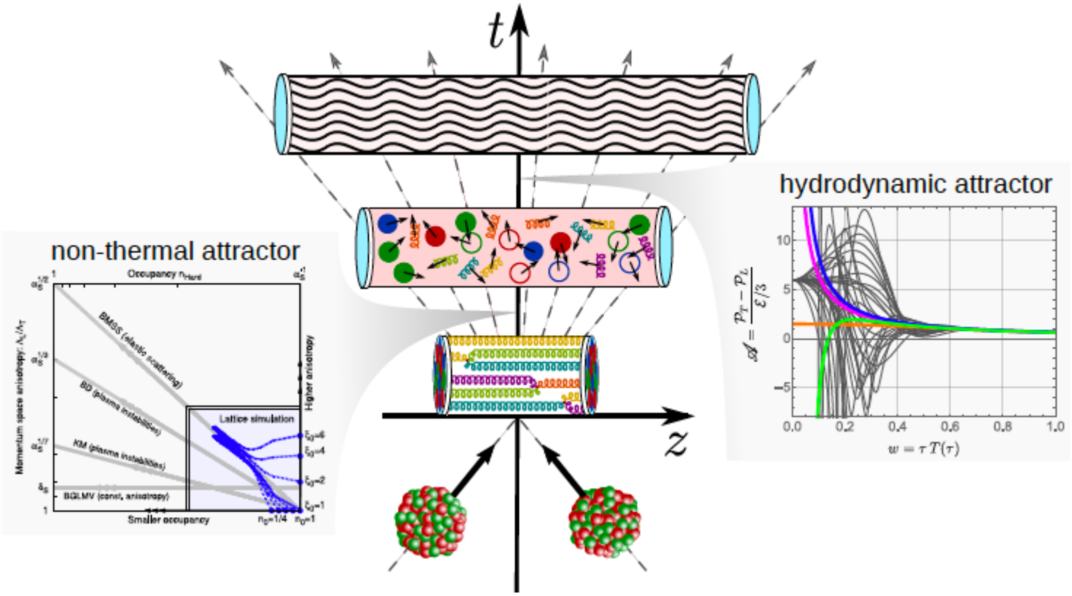
reggeized gluons from
semi-classical source dists.

Ln (U) → *reggeized gluon*

Blaizot,Gelis,RV (2004)
Gelis-Mehtar-Tani (2005)

Jalilian-Marian,Jeon,RV (2000); Caron-Huot (2013)

Dense-dense shockwave collisions: heavy-ion collisions

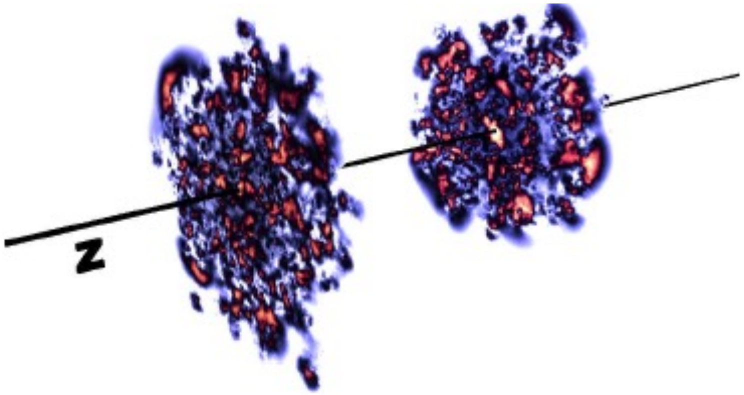


Quark-Gluon Plasma undergoing
hydrodynamic expansion

Collision of overoccupied Color Glass Condensate shockwaves

QCD thermalization: Ab initio approaches and interdisciplinary connections
Jürgen Berges, Michal P. Heller, Aleksas Mazeliauskas, and RV
Rev. Mod. Phys. **93**, 035003 (2021)

“Dense-dense” semi-classical shockwave collisions of lumpy glue



Collisions of “lumpy” gluon shock waves with $1/Q_s$ –wide ` ` fuzz” of wee partons

Important point: the width of each shock wave is not R/γ but $1/Q_S$ - this description is frame invariant

One can “prove” that quantum fluctuations about each shockwave are responsible for energy evolution in each shock wave (BK/JIMWLK)

Can be factorized from quantum fluctuations after the collision

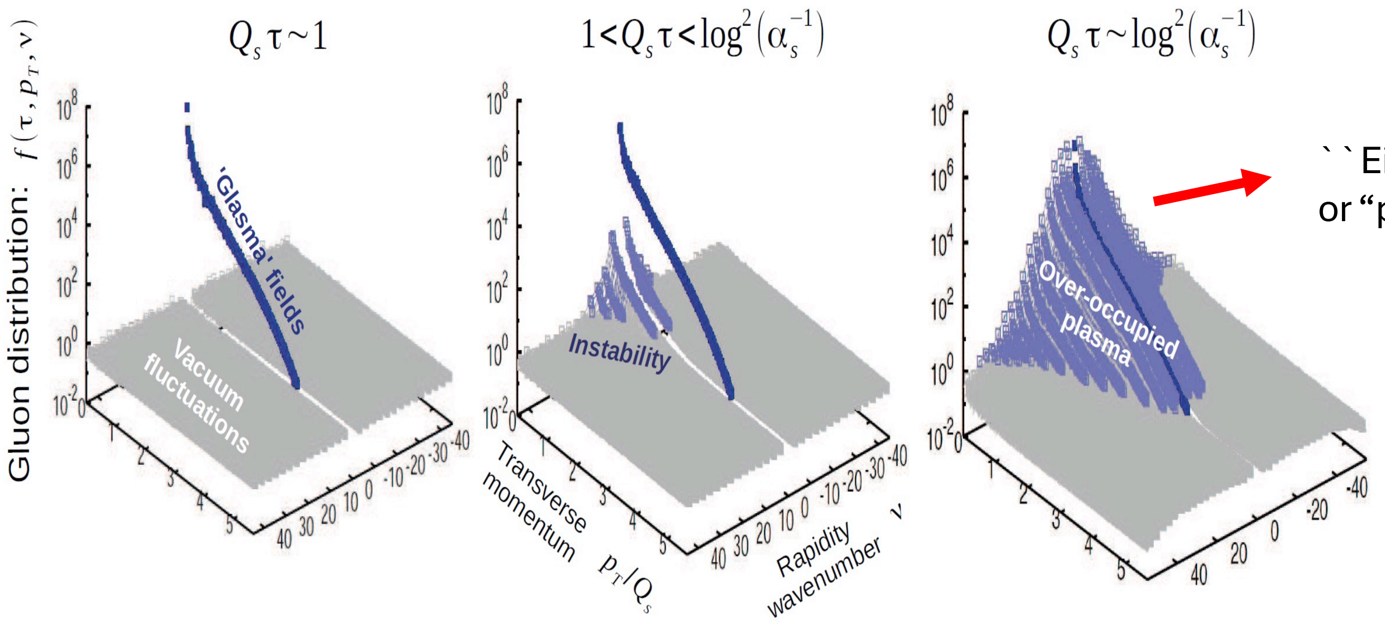
Gelis,Lappi,RV arXiv:0804.2630

Decoherence from explosive amplification of quantum fluctuations

Longitudinally expanding ``Glasma" fields are unstable to quantum fluctuations…
leading to an explosive "Weibel"-like instability.

Romatschke,RV, hep-ph/0605045

Rapid decoherence and overpopulation of all momentum modes



` ` Eigenstate thermalization"
or "pre-thermalization"

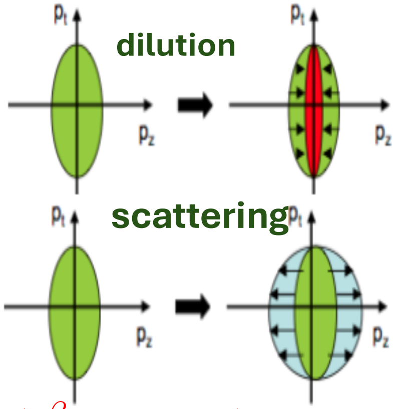
Classical-statistical real-time lattice simulations of 3+1-D gluon fields exploding into the vacuum

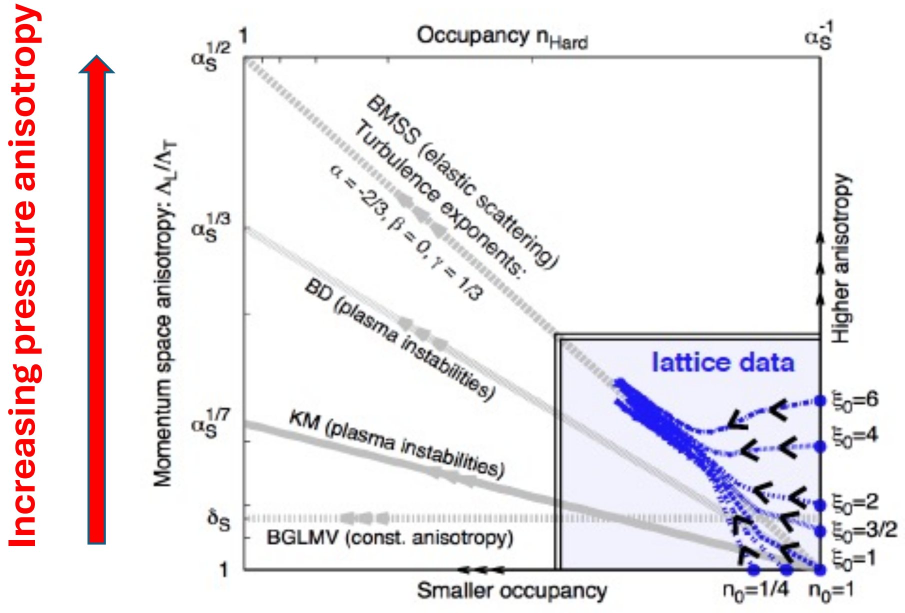
Berges,Schenke,Schlichting,RV, NPA 931 (2014) 348

Classical-statistical simulations: A turbulent attractor

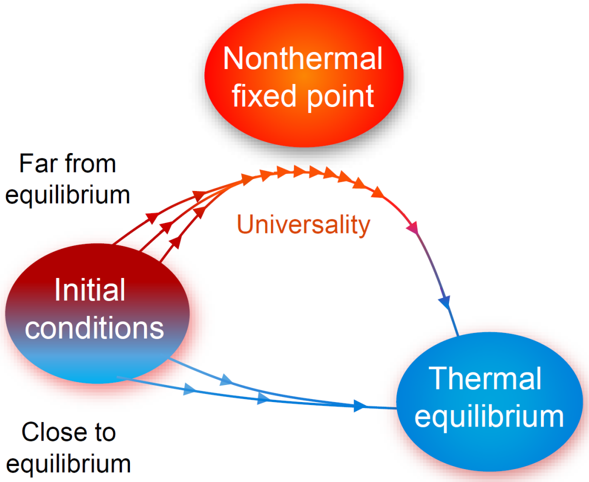
After rapid scrambling of information by quantum fluctuations, competition between dilution due to expansion and isotropization due to scattering

“Single particle” distributions become self-similar in time characterized by universal exponents – helps identify “right” kinetic theory



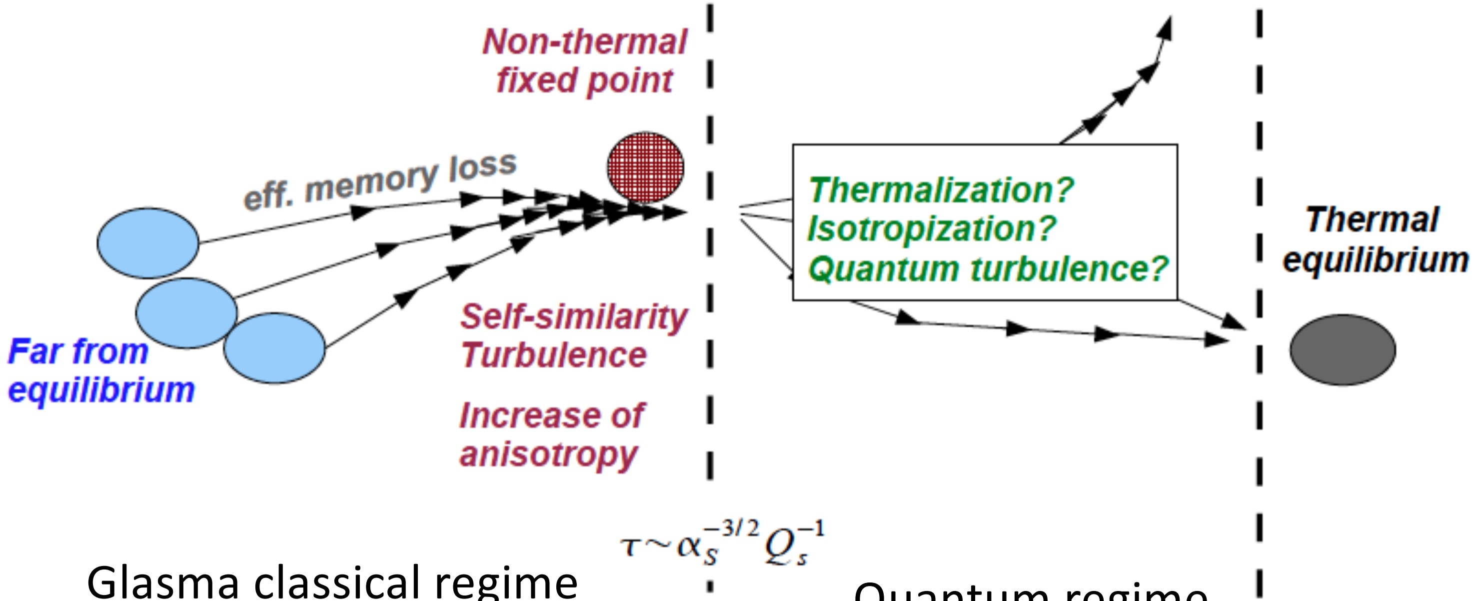


$$f(p_\perp, p_z, t) = t^\alpha \; f_S(t^\beta p_T, t^\gamma p_z)$$



Berges,Boguslavski,Schlichting,RV (2014)

Bottom-up thermalization: from nuts to soup



Non-thermal fixed point

eff. memory loss

Far from equilibrium

Self-similarity Turbulence

Increase of anisotropy

Thermalization?
Isotropization?
Quantum turbulence?

Thermal equilibrium

$\tau \sim \alpha_S^{-3/2} Q_s^{-1}$

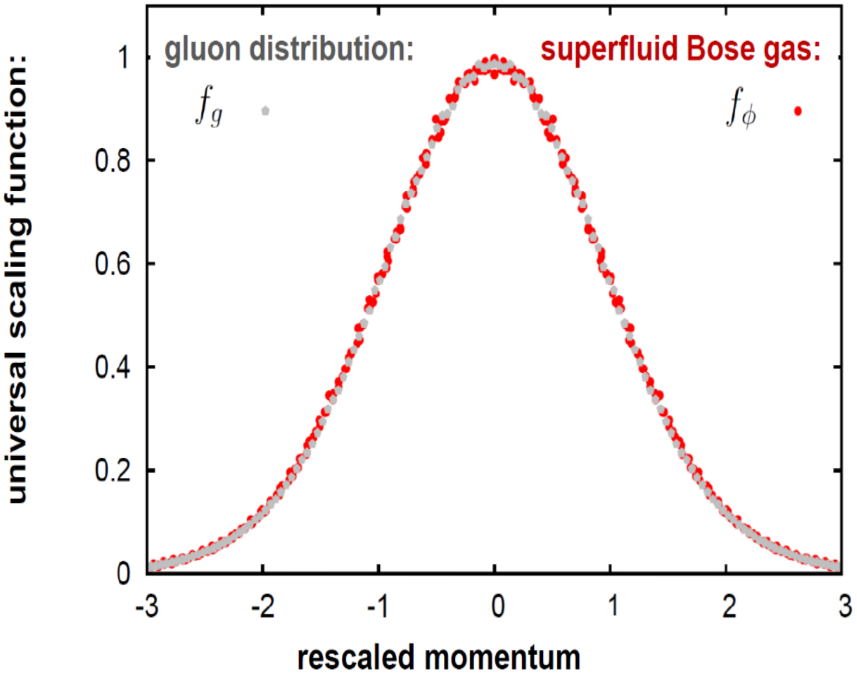
Glasma classical regime

Quantum regime of kinetic theory

Baier,Mueller,Schiff,Son, hep-ph/0009237

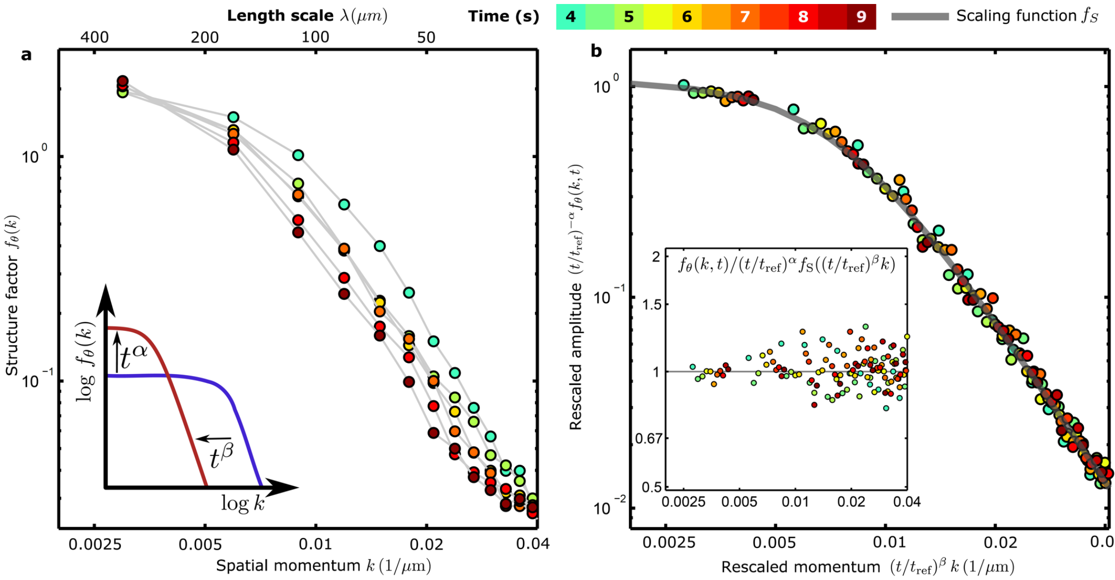
Spacetime evolution of saturated glue and overoccupied ultracold atoms

Compare evolution of saturated gluon in heavy-ion collisions to dynamics of cold atomic gases:
remarkable *universality of longitudinally expanding world's hottest and coolest fluids*



Berges,Boguslavski,Schlichting,RV, PRL (2015) Editor's suggestion

^{87}Rb BEC in a quasi 1D optical trap



Oberthaler BEC Labs, Prüfer et al, arXiv:1805.11881, *Nature* (2018)

Scalable cold-atom quantum simulator for overoccupied features of gauge theories?

R. Ott et al., arXiv:2012.10432

Bottom-up thermalization

Thermalized soft bath of gluons for $\tau > \frac{1}{\alpha_S^{5/2}} \frac{1}{Q_S}$

Thermalization temperature of $T_i = \alpha_S^{2/5} Q_S$

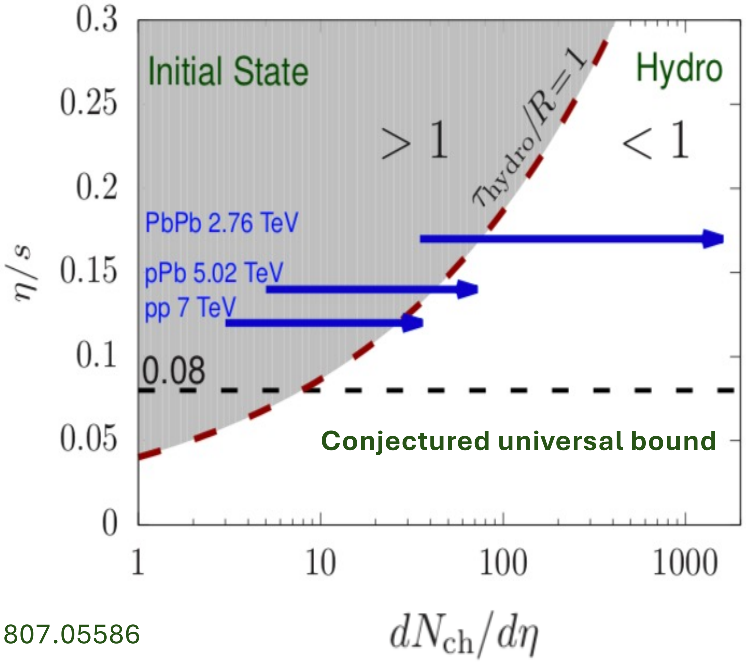
Since $\alpha_S \propto \frac{1}{\log Q_S}$

then $\tau_{\text{therm}} \propto \frac{(\log Q_S)^{5/2}}{Q_S} \rightarrow 0$ as $QS \rightarrow \infty$

Matter thermalizes almost instantaneously at asymptotic energies

CGC naturally leads to hydro if $\tau_{\text{therm}} << R$

Remarkably, in this picture, dominant feature of the entire dynamics of thermalization (across systems) is one semi-hard scale …



Mazeliauskas, arXiv:1807.05586
Kurkela,Wiedemann,Wu, arXiv:1905.05139

Bottom-up thermalization: from nuts to soup

$\tau \lesssim 1/Q_S$: quantum "crossing time" of wavefunctions with "fuzz" of wee partons of width $1/Q_S$
- lumpy "hot spot" classical configurations in transverse plane

$\frac{1}{Q_S} \leq \tau \leq \frac{1}{Q_S} \mathrm{Ln}^2\left(\frac{1}{\alpha_S^2}\right)$: Rapid scrambling of overoccupied gauge fields by exponentially growing quantum fluctuations (Weibel instabilities) generates isotropic "single particle" distributions

$\frac{1}{Q_S} \mathrm{Ln}^2\left(\frac{1}{\alpha_S^2}\right) \leq \tau \leq \frac{1}{Q_S} \frac{1}{\alpha_S^{3/2}}$: System flows to turbulent non-thermal attractor. Subsequent classical/quantum evolution until a "quantum breaking time" when occupancies are of order unity.

$\frac{1}{Q_S} \frac{1}{\alpha_S^{3/2}} \leq \tau \leq \frac{1}{Q_S} \frac{1}{\alpha_S^{5/2}}$: $2 \rightarrow 3$ kinetic processes begin to dominate. Soft radiated gluons thermalize

– but hard gluons $k_T \approx Q_S$ still far from equilibrium

$\frac{1}{Q_S} \frac{1}{\alpha_S^{5/2}} \leq \tau \leq \frac{1}{Q_S} \frac{1}{\alpha_S^{13/5}}$: Hard gluons thermalize through a turbulent quantum process

– which also describes "jet quenching"

Blaizot, Dominguez, Iancu, Mehtar-Tani (2013-2016),
Eg., Blaizot, Mehtar-Tani, 1503.05958