What wee partons reveal about hadron structure at high energies and the dynamics of confinement-III

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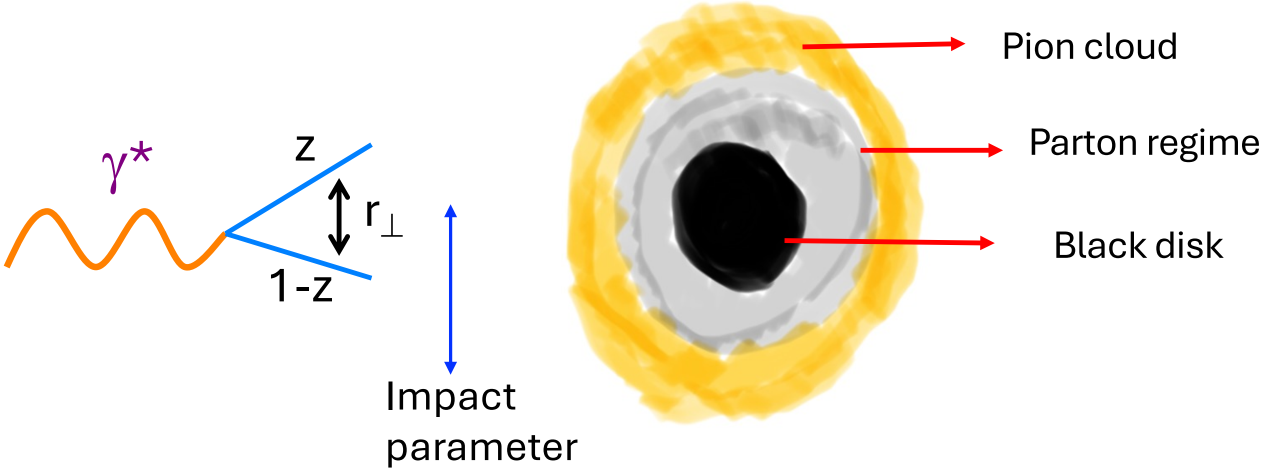
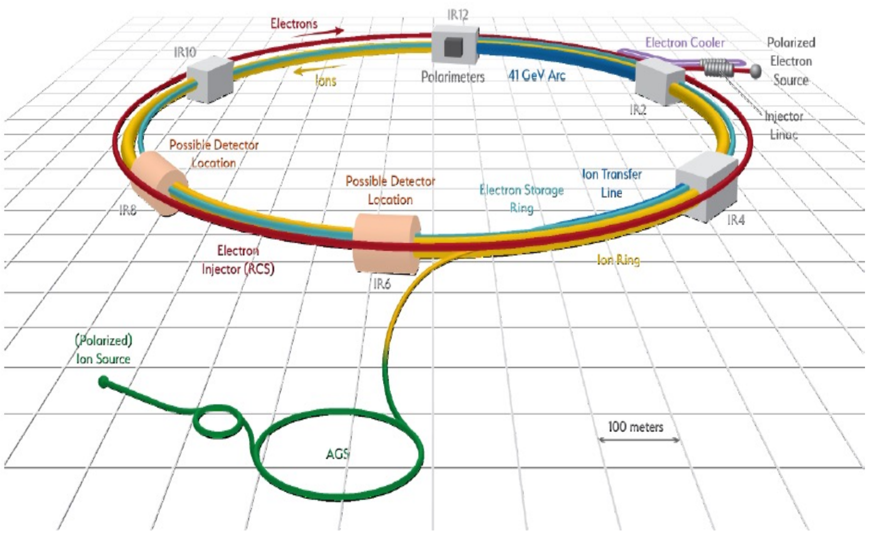
The elephant in the room



Though our increased understanding of gluon saturation helps understand some features of the proton wavefunction, we still don't understand the mystery of how

partons (quarks and gluons) → hadrons (pions, kaons,...)

A powerful femtoscope for precision studies: The Electron-Ion Collider



Polarized protons up to 275 GeV; Nuclei up to ~ Z/A*275 GeV/n

- Existing RHIC complex: Storage (Yellow), injectors (source, booster, AGS)
- Need few modifications
- RHIC beam parameters fairly close to those required for EIC@BNL

Electrons up to 18 GeV

- Storage ring, provides the range sqrt(s) = 20-140 GeV. Beam current limited by RF power of 10 MW
- Electron beam with variable spin pattern (s) accelerated in on-energy, spin transparent injector (Rapid-Cycling-Synchrotron) with 1-2 Hz cycle frequency
- Polarized e-source and a 400 MeV s-band injector LINAC in the existing tunnel

Design optimized to reach 10^{34} $cm^{-2}sec^{-1}$
1000 times HERA – EIC will be the brightest collider ever

Unprecedently clean studies of QCD dynamics (circa 2034) across proton, light (polarized0 and heavy nuclei

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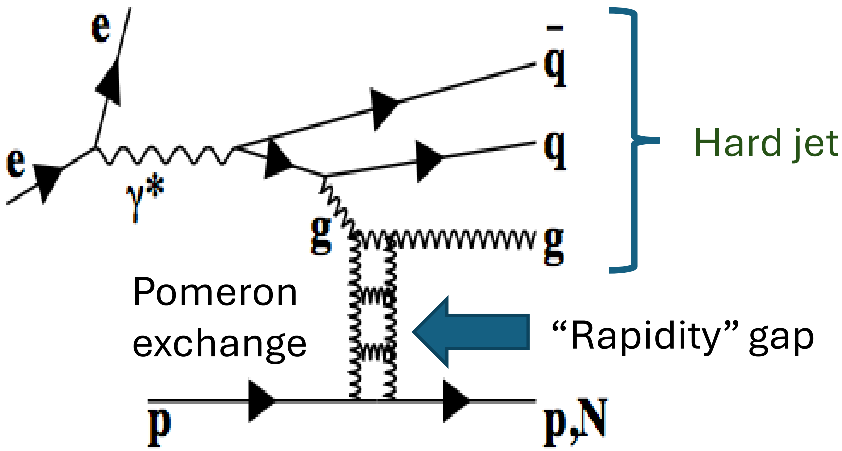
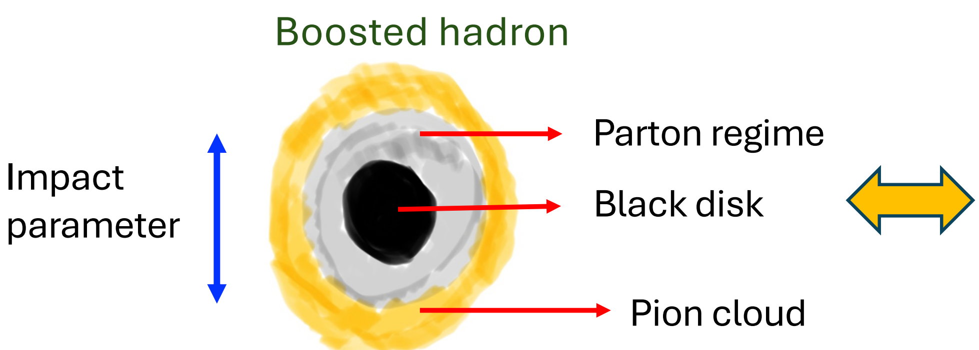
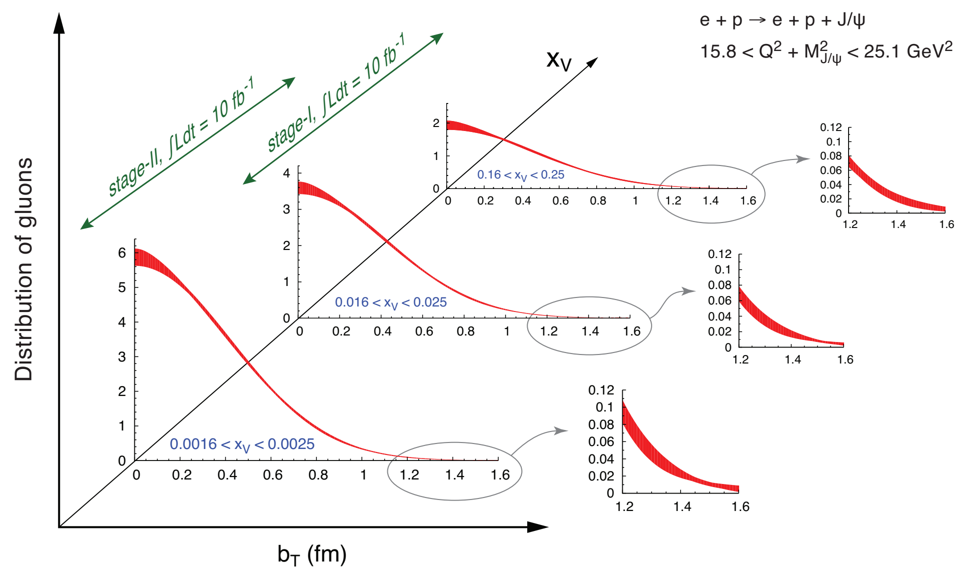


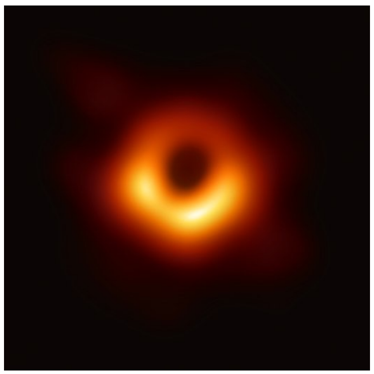
Diffractive DIS probes the structure of the pomeron in protons and nuclei



3-D quark-gluon tomography

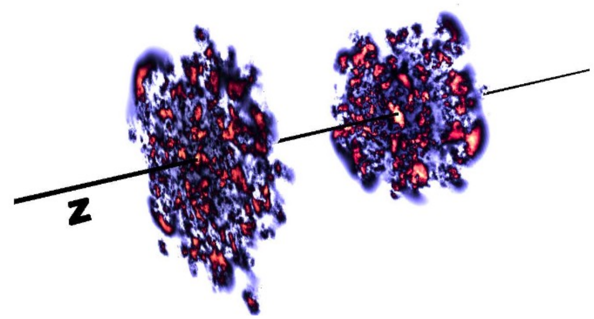


Universal features of 2→ N scattering in QCD and gravity from shockwave collisions



$M_{BH}=(6.5 \pm 0.2_{stat} \pm 0.7_{sys}) \times 10^9\, M_\odot$
at center of Messier 87
Event Horizon Telescope image of photon ring

10^9 km ⟷ 10^{-19} km

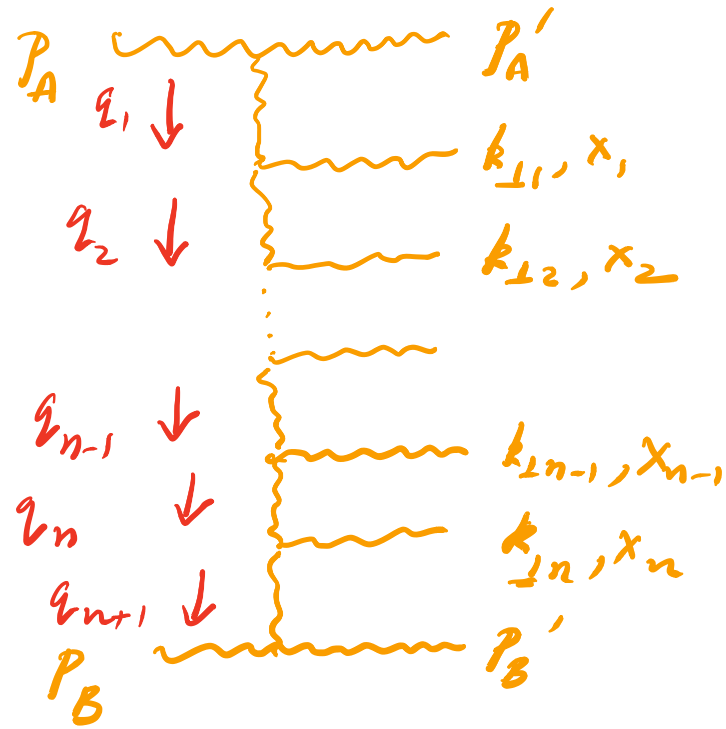


Collisions of Color Glass Condensate gluon states in nuclei, arXiv:1206.6805

Based in part on recent work (arXiv:2311.03463, 2312.03507, 2312.11652 and 2406.10483) with Himanshu Raj (Simons Confinement+ QCD Strings Collaboration Fellow at Stony Brook)

S-matrix picture of 2 → N scattering in GR at trans-Planckian energies

t'Hooft, Gross-Mende, Verlinde[2] ,…

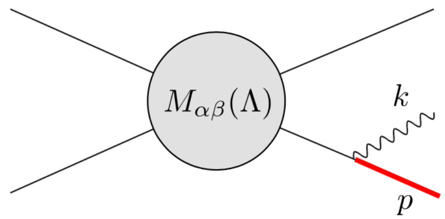


What is the role of "wee*" gravitons in trans-Planckian scattering in gravity?

Wee gravitons are not soft gravitons

Soft theorem's: exponentiation of clouds of soft photons and gravitons

In gravity, Low's theorem gives



$$\lim_{k\to 0} M^{\mu\nu}_{\alpha\beta} = \lim_{k\to 0}\sqrt{8\pi G}\,\frac{p^\mu p^\nu}{p\cdot k - i\epsilon} \qquad M_{\alpha\beta}(\Lambda)$$

Weinberg's soft graviton theorem:

$$\mathcal{S}^{(2)}_{fi} \simeq \exp\left\{-i\sum_{n,m=1}^{2} S^{nm}_{\mathrm{IR}}\right\}\mathcal{S}^{(2)}_{fi,\mathrm{UV}}$$

$$S^{nm}_{\mathrm{IR}} \simeq \frac{GM^2}{2\pi}\gamma\coth\gamma\,\frac{1+\tanh^2\gamma}{\sqrt{(1-\tanh^2\gamma}}\,\mathrm{Ln}\left(\frac{\Lambda}{\lambda}\right)$$

with $\eta' = \pm 1$

and cusp angle defined in terms of scalar product of velocities

$$\cosh\gamma^{nm} := \frac{\beta^n\cdot\beta^m}{\sqrt{(\beta^n)^2(\beta^m)^2}}$$

Wee gravitons are part of the "hard" matrix element with $k_\perp \geq \Lambda$

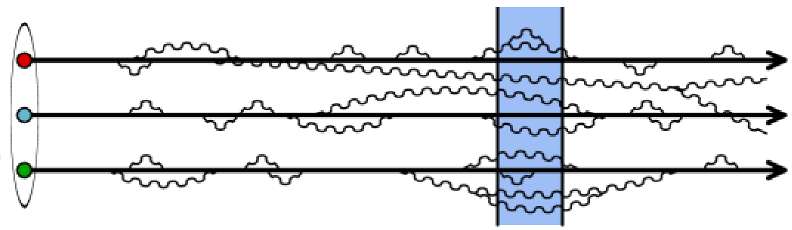
Gluon bremsstrahlung: ` ` Herculean task…might even not be possible”

After magisterially disposing of the problem of IR divergences of soft photons and gravitons, Weinberg notes…

Weinberg, Phys. Rev. 140 (1965) B516

Weinberg

But these remarks do not apply to theories involving charged massless particles. In such theories (including the Yang-Mills theory) a soft photon emitted from an external line can itself emit a pair of soft charged massless particles, which themselves emit soft photons, and so on, building up a cascade of soft massless particles each of which contributes an infrared divergence. The elimination of such complicated interlocking infrared divergences would certainly be a Herculean task, and might even not be possible.



We may be thankful that the zero charge of soft photons and the zero gravitational mass of soft gravitons saves the real world from this mess. Perhaps it would not be too much to suggest that it is the infrared divergences that prohibit the existence of Yang-Mills quanta or other charged massless particles.

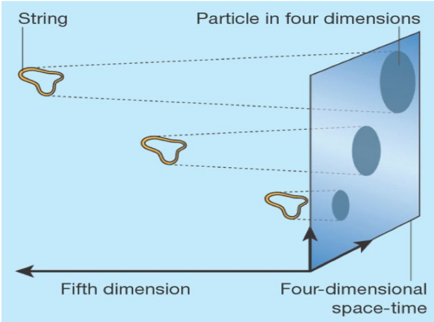
8 years prior to asymptotic freedom!

Gauge-Gravity correspondence

The classic one is the AdS/CFT correspondence

Clean dictionary to derive results in a strongly coupled gauge theory some of which may be universal (a candidate being the KSS bound on $\frac{\eta}{s}$)

Our universe is de Sitter and N=4 SUSY YM is not QCD



J.Maldacena, Nature 2003

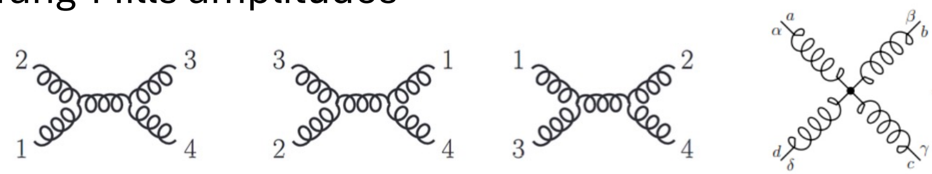
Gauge-Gravity correspondence

Double copy between QCD and Gravity amplitudes

Old idea (Kawai-Lewellyn-Tye) based on relations between closed and open string amplitudes – in "low energy" limit between Einstein & Yang-Mills amplitudes

$$M_4^{\text{tree}}(1,2,3,4) = \left(\frac{\kappa}{2}\right)^2 s\, A_4^{\text{tree}}(1,2,3,4)\, A_4^{\text{tree}}(1,2,4,3)$$

$$\kappa = 32\,\pi^2\, G_N$$



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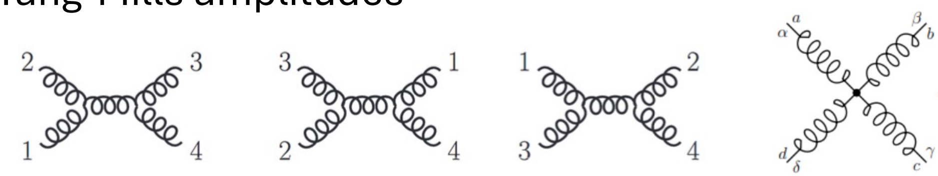

Remarkable "BCJ" color-kinematics duality

Bern, Carrasco,Johansson,arXiv:0805.3993

Tree level $gg \to gg$ amplitudes (with on shell legs) can be written as

$$i\mathcal{A}_4^{\rm tree} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

with the s channel color factor $c_s = -2 f^{a_1 a_2 b} f^{b a_3 a_4}$

kinematic factor $n_s = -\frac{1}{2}\Big\{\Big[(\epsilon_1.\epsilon_2)p_1^\mu + 2(\epsilon_1.p_2)\epsilon_2^\mu - (1\leftrightarrow 2)\Big]\Big[(\epsilon_3.\epsilon_4)p_3^\mu + 2(\epsilon_3.p_4)\epsilon_4^\mu - (3\leftrightarrow 4)\Big] + s\Big[(\epsilon_1.\epsilon_3)(\epsilon_2.\epsilon_4) - (\epsilon_1.\epsilon_4)(\epsilon_2.\epsilon_3)\Big]\Big\}$

Tree level gravity amplitude obtained by replacing color factors by kinematic factors

$$i\mathcal{A}_4^{\rm tree}|_{c_i\to n_i, g\to\kappa/2} = i\mathcal{M}_4^{\rm tree} = \left(\frac{\kappa}{2}\right)^2\left(\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}\right)$$

Significant on-going work on extension to loop amplitudes

Review: Bern et al., arXiv: 1909.01358

$2 \to N + 2$ amplitudes in trans-Planckian gravitation scattering: from wee partons to Black Holes

HIGH-ENERGY SCATTERING IN QCD AND IN QUANTUM GRAVITY AND TWO-DIMENSIONAL FIELD THEORIES

L.N. LIPATOV*

We construct effective actions describing high-energy processes in QCD and in quantum gravity with intermediate particles (gluons and gravitons) having the multi-Regge kinematics. The S-matrix for these effective scalar field models contains the results of the leading logarithmic approximation and is unitary. It can be expressed in terms of correlation functions for two field theories acting in longitudinal and transverse two-dimensional subspaces.

Effective action and all-order gravitational eikonal at planckian energies

AMATI,CIAFALONI,VENEZIANO **NPB403 (1993)707**

Building on previous work by us and by Lipatov, we present an effective action approach to the resummation of all semiclassical (i.e. $O(\hbar^{-1})$) contributions to the scattering phase arising in high-energy gravitational collisions. By using an infrared-safe expression for Lipatov's effective action, we derive an eikonal form of the scattering matrix and check that the superstring amplitude result is reproduced at first order in the expansion parameter R^2/b^2, where R, b are the gravitational radius and the impact parameter, respectively. If rescattering of produced gravitons is neglected, the longitudinal coordinate dependence can be explicitly factored out and exhibits the characteristics of a shock-wave metric while the transverse dynamics is described by a reduced two-dimensional effective action. Singular behaviours in the latter, signalling black hole formation, can be looked for.

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The World as a Hologram

LEONARD SUSSKIND

Wee partons, by contrast, are not subject to Lorentz contraction. This implies that in the Feynman Bjorken model, the halo of wee partons eternally "floats" above the horizon at a distance of order $10^{-13} cm$ as it transversley spreads. The remaining valence partons carry the various currents which contract onto the horizon as in the Einstein Lorentz case.

By contrast, both the holographic theory and string theory require all partons to be wee. No Lorentz contraction takes place and the entire structure of the string floats on the stretched horizon. I have explained in previous articles how this behavior prevents the accumulation of arbitrarily large quantities of information near the horizon of a black hole. Thus we are led full circle back to Bekenstein's principle that black holes bound the entropy of a region of space to be proportional to its area.

***J.Math.Phys*. 36 (1995) 6377; 3721 cites !**

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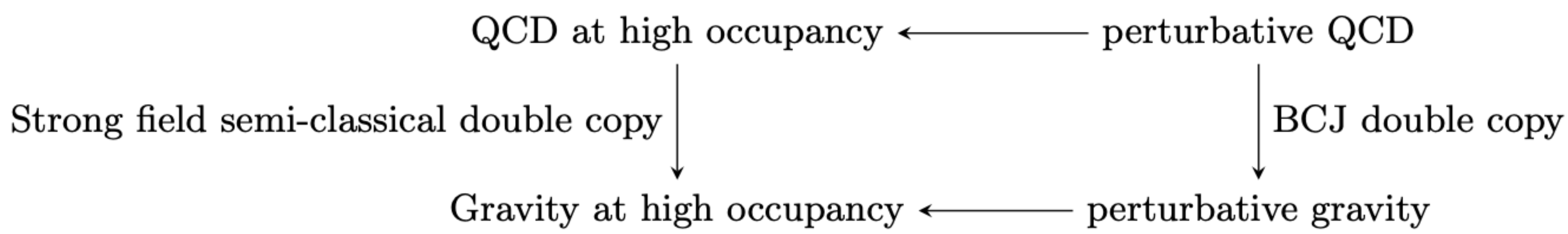
→ 30+ years of work by ACV et al. exploring gravitational shockwave collisions I
n this 2-D EFT framework

Double Copy: gluon → gravitational radiation in shockwave collisions

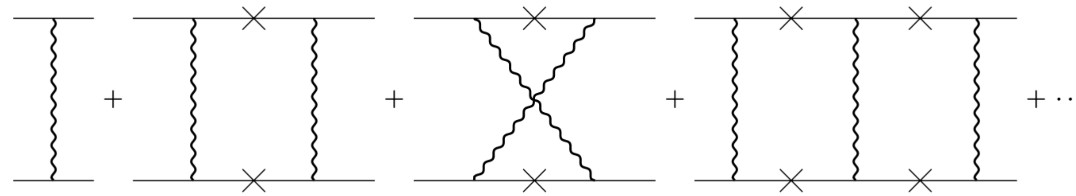


Bern, Carrasco, Johannson,
arXiv: 1004.0476

Monteiro,O'Connell,White, arXIv:1410.0239
Goldberger, Ridgeway, arXiv:1611.03493

From QCD to gravity in Regge asymptotics: reggeization

In Einstein gravity, at large impact parameters, dominant contribution is eikonal scattering

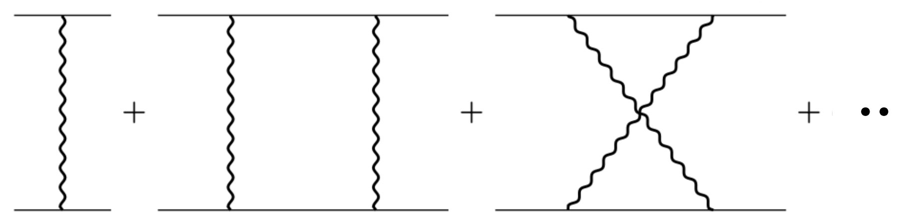


$$i\mathcal{M}_{\rm Eik} = 2s \int d^2\boldsymbol{b}\; e^{-i\mathbf{q}\cdot\mathbf{b}} \left(e^{i\chi(\boldsymbol{b},s)} - 1\right) \qquad \text{with} \quad \chi(\boldsymbol{b},s) = \frac{\kappa^2 s}{2}\int \frac{d^2\boldsymbol{k}}{(2\pi)^2}\frac{1}{\boldsymbol{k}^2} e^{i\boldsymbol{b}\cdot\boldsymbol{k}}$$

Genuine loop contributions formally suppressed by R_S^2/b^2

$$\mathcal{M}^{(1)} \sim \frac{\kappa^2}{8\pi^2}\left(-i\pi s \log\left(\frac{-t}{\Lambda^2}\right) + t\log\left(\frac{s}{-t}\right)\log\left(\frac{-t}{\Lambda^2}\right)\right)$$

Eikonal Loop

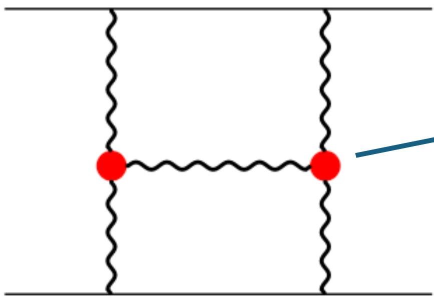


Graviton Regge trajectory $\alpha(t) = -\kappa^2 t \int \frac{d^2\boldsymbol{k}}{(2\pi)^2}\frac{1}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2}\left[(\boldsymbol{k}\cdot(\boldsymbol{q}-\boldsymbol{k}))^2\left(\frac{1}{\boldsymbol{k}^2}+\frac{1}{(\boldsymbol{q}-\boldsymbol{k})^2}\right) - \boldsymbol{q}^2\right], \qquad \boldsymbol{q}^2 = -t$

The IR virtual divergence cancels in the inclusive cross-section

Lipatov, PLB 116B (1982); JETP 82 (1982)

From QCD to gravity in Regge asymptotics: Lipatov vertex



H-diagram of Amati, Ciafaloni, Veneziano

Gravitation Lipatov vertex:

$$\Gamma_{\mu\nu}(\boldsymbol{q}_1,\boldsymbol{q}_2) \equiv \frac{1}{2}C_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2)C_\nu(\boldsymbol{q}_1,\boldsymbol{q}_2) - \frac{1}{2}N_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2)N_\nu(\boldsymbol{q}_1,\boldsymbol{q}_2)$$

Double copy of QCD Lipatov vertex

Double copy of QED Bremsstrahlung vertex

$$N_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2) = \sqrt{\boldsymbol{q}_1^2\boldsymbol{q}_2^2}\left(\frac{p_{1\mu}}{p_1\cdot k} - \frac{p_{2\mu}}{p_2\cdot k}\right)$$

S-matrix power counting a la ACV:

$$S = e^{2i(\delta_0+\delta_1+\delta_2+\cdots)} \qquad \delta_0 = Gs\log\left(\frac{L}{b}\right)\,, \qquad \delta_1 = \frac{6G^2s}{\pi b^2}\log s\,, \qquad \delta_2 = \frac{2G^3s^2}{b^2}\left[1+\frac{i}{\pi}\log s\left(\log\frac{L^2}{b^2}+2\right)\right]$$

Leading Eikonal term (real)

Sub-leading quantum gravity correction $\sim \frac{l_P^2}{b^2}$

Sub-leading loop contribution $\sim \frac{R_S^2}{b^2}$
- includes absorptive piece

$\delta_2 \gg \delta_1$ for $R_S \gg l_P$

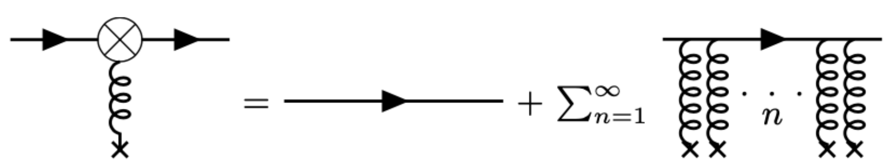
From QCD to gravity in Regge asymptotics: shock wave propagators

Gravitational shock wave propagator:

$$\tilde{G}_{\mu\nu\rho\sigma}(p,p') = \tilde{G}^0_{\mu\nu\rho\sigma}(p)(2\pi)^4\delta^{(4)}(p-p') + \tilde{G}^0_{\mu\nu\alpha\beta}(p)\mathcal{T}^{\alpha\beta\gamma\delta}(p,p')\tilde{G}^0_{\gamma\delta\rho\sigma}(p')$$

with gravitational effective vertex:

$$\mathcal{T}_{\mu\nu\rho\sigma}(p,p') = -\frac{1}{2}\underbrace{\left(\Lambda_{\mu\rho}\Lambda_{\nu\sigma} + \Lambda_{\mu\sigma}\Lambda_{\nu\rho} - \Lambda_{\mu\nu}\Lambda_{\rho\sigma}\right)}_{\text{Sum over gravitational polarization tensors}} 4\pi i (p')^-\delta(p^- - (p')^-)\int d^2\boldsymbol{z}\, e^{i(\boldsymbol{p}-\boldsymbol{p}')\cdot\boldsymbol{z}}\underbrace{\left(e^{if_1(\boldsymbol{z})p'_+} - 1\right)}_{\text{Gravitational Wilson line}}$$



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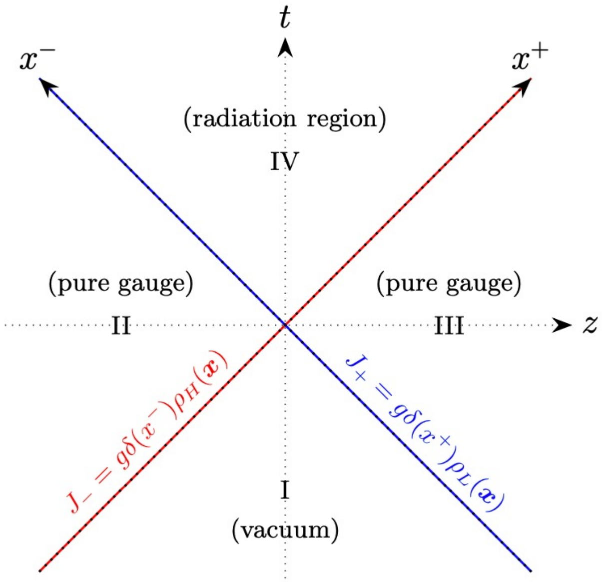
"Double-copy" of QCD Wilson line!

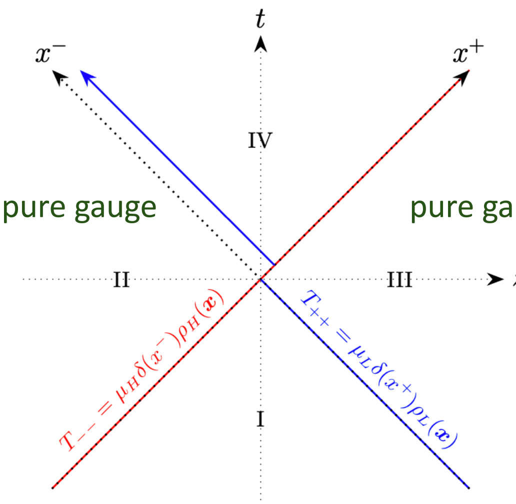
Here f_1 is the shock wave metric – equivalent of CGC shock wave field - and one replaces color matrix $T^a \to p'_+$

The gravitational polarization tensors are likewise double-copies of the QCD polarization tensors

Gluon and graviton production in shock wave collisions



Collision of gluon shock waves



Collision of gravitational shock waves

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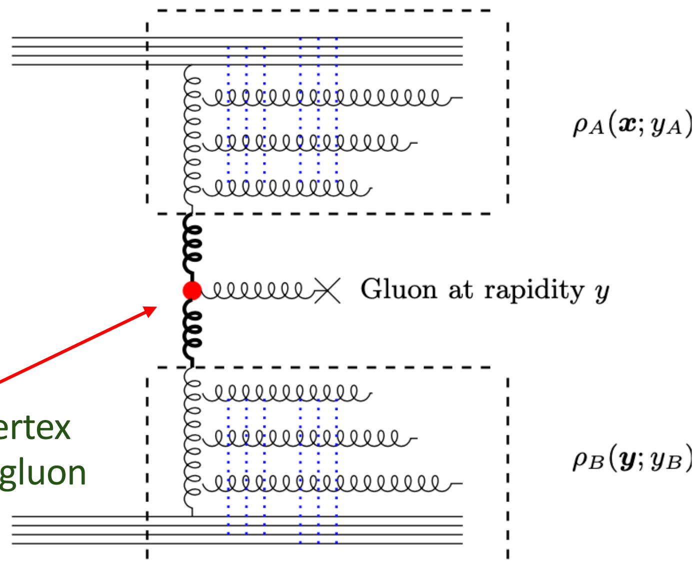
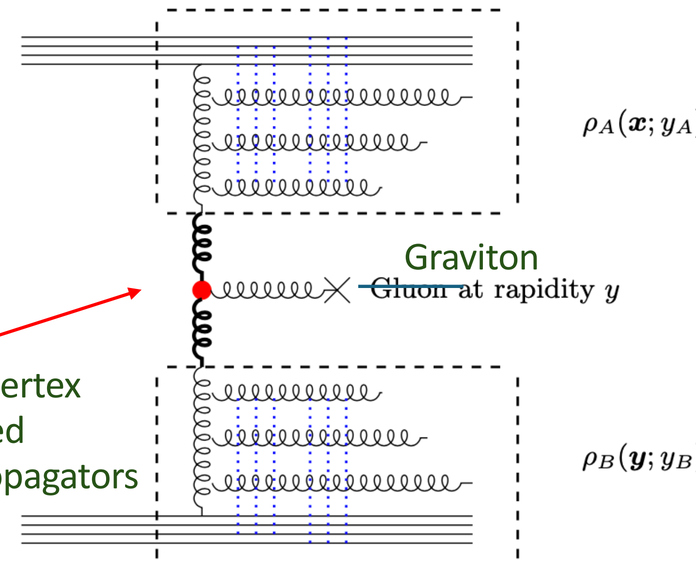
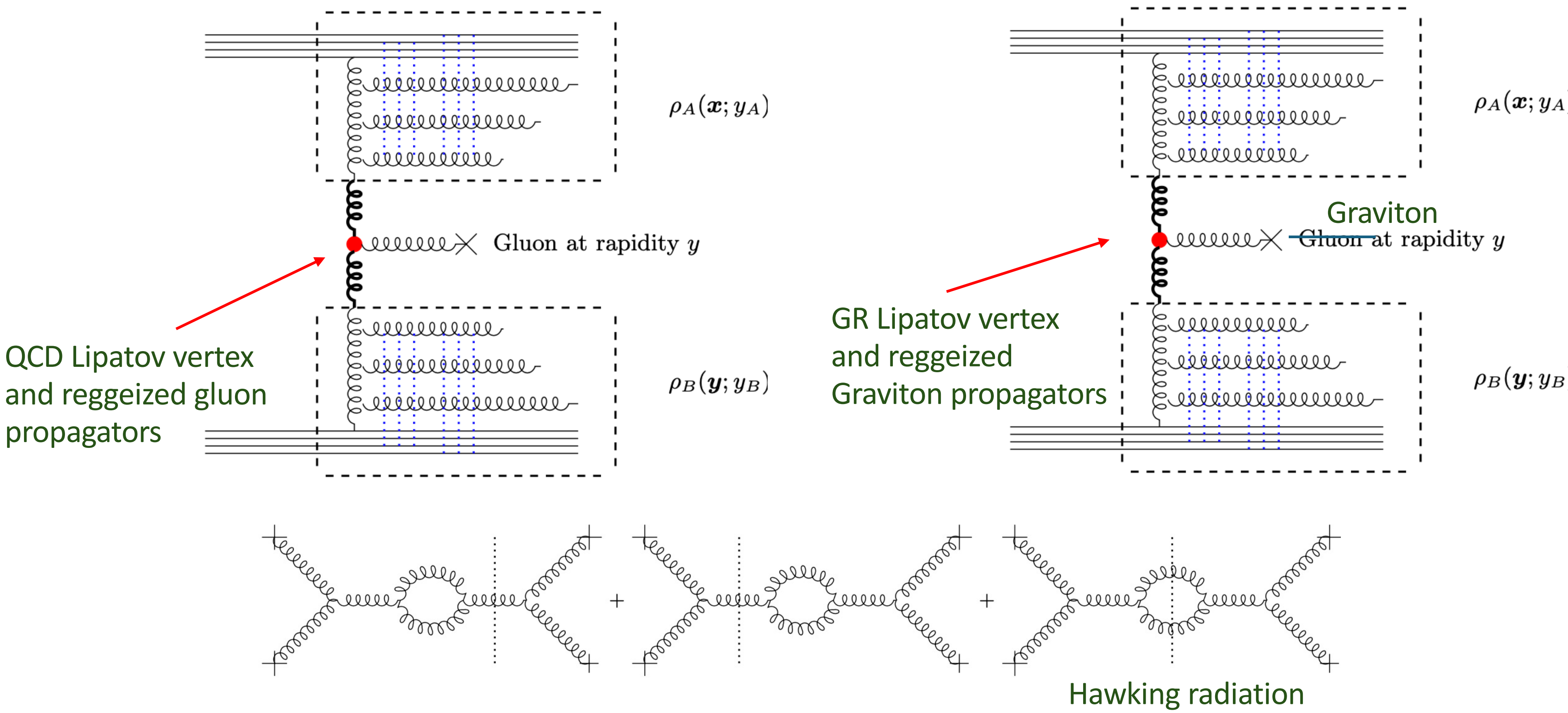


QCD Lipatov vertex
and reggeized gluon
propagators



GR Lipatov vertex
and reggeized
Graviton propagators

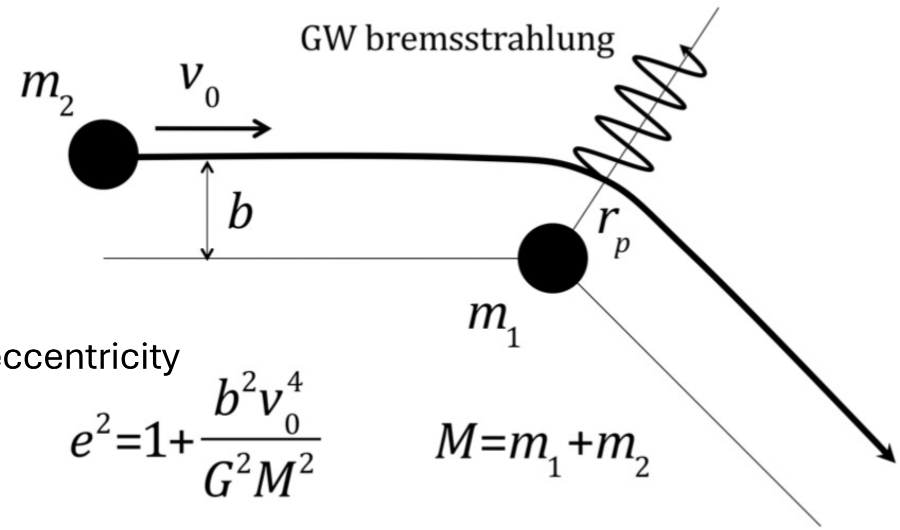
RG for graviton production in shock wave collisions: Gravitational BFKL



Gravitational radiation from primordial BH collisions

H. Raj, RV, in preparation

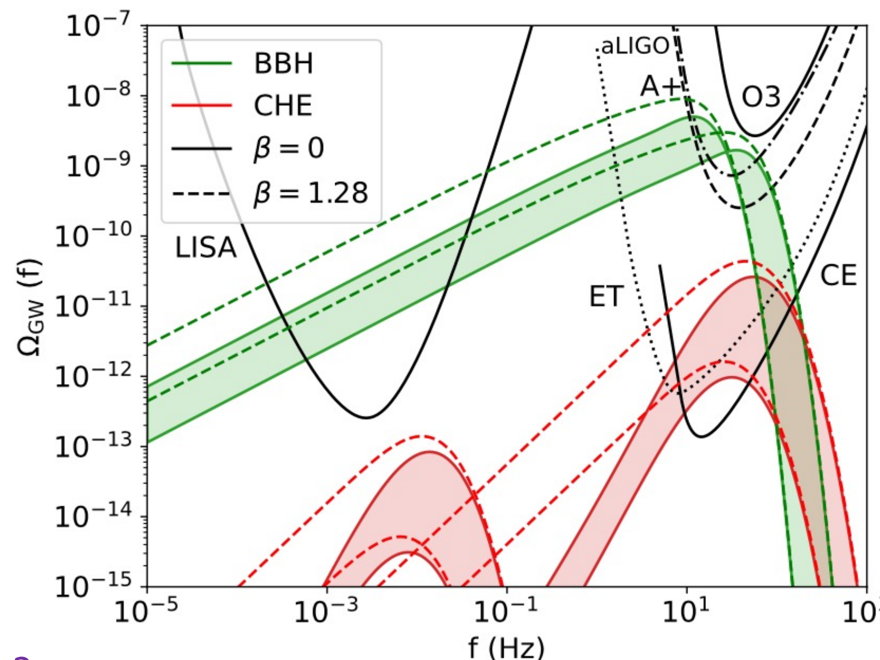
Close hyperbolic encounters (CHE) of BHs



eccentricity

$$e^2 = 1 + \frac{b^2 v_0^4}{G^2 M^2} \qquad M = m_1 + m_2$$

Stochastic GW spectrum accessible at next gen. GWOs



Interesting question: what is the capture radius
of GW radiation in ultrarelativistic shockwave collisions in GR ?

Pretorius, Khurana, arXiv:gr-qc/0702084,
Sperhake,Berti,Cardoso,Pretorius, arXiv:1211.6114
Page, arXiv:2212.03890

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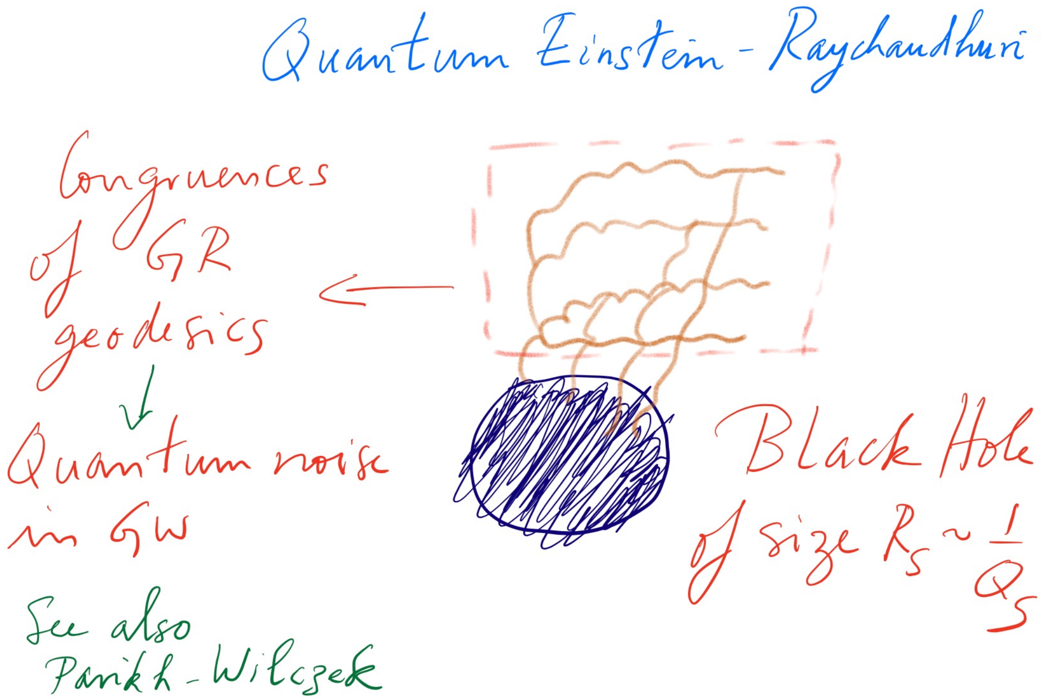
Outstanding question

Can we understand black hole formation in GR as the fixed point of a non-linear RG equation in rapidity?

Will next-generation GWO be sensitive to these quantum features of gravity?



Dvali, RV, PRD (2022)