TECHNISCHE UNIVERSITÄT WIEN

The (3+1)D dilute Glasma

Based on:

A. Ipp, M. Leuthner, D. I. Müller, S. Schlichting, **K. Schmidt** and P. Singh. *Energy-momentum tensor of the dilute (3+1)D glasma*. In: *Phys. Rev. D* 109.9 (2024), p. 094040. arXiv: 2401.10320 [hep-ph]

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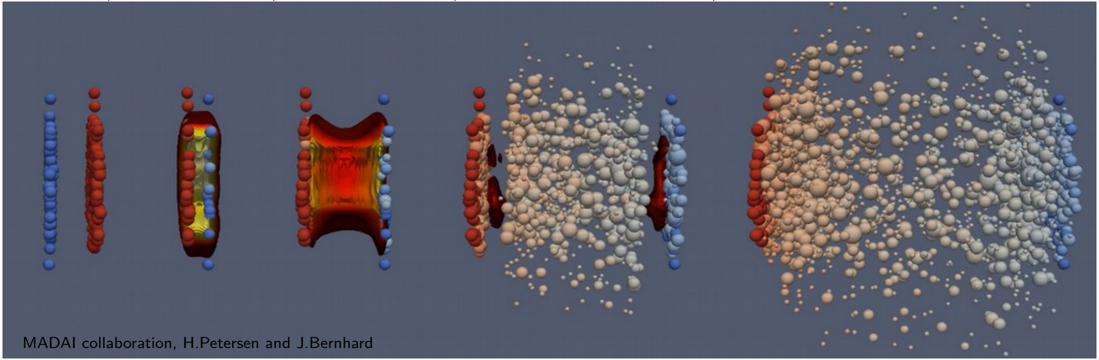
Relativistic heavy-ion collisions

Initial state	Pre-equilibrium	Local equilibrium	Hadronization		Freeze-out



Color Glass Condensate	Glasma	Quark-gluon plasma	Cooper-Frye	UrQMD	Detectors

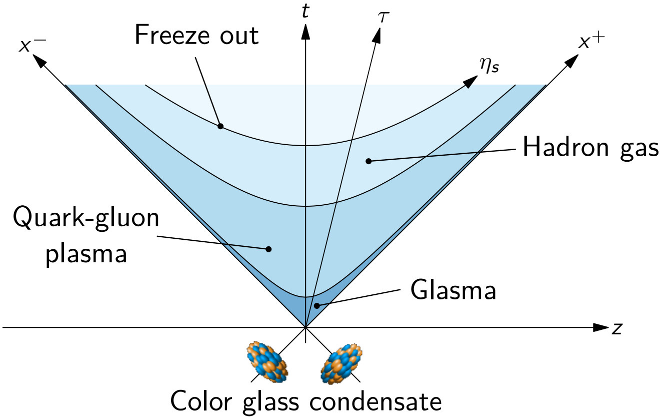
Spacetime picture

Coordinates

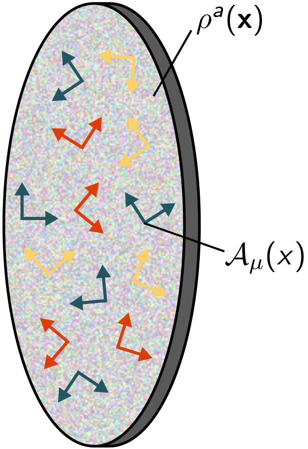
- $z \ldots$ beam axis
- $\eta_s \ldots$ rapidity
- $\tau \ldots$ proper time
- $x^{\pm} \ldots$ light cone

Milne frame

- $\tau = \sqrt{2x^+x^-} = \sqrt{t^2 - z^2}$
- $\eta_s = \frac{1}{2} \ln\left(\frac{x^+}{x^-}\right) = \mathrm{artanh}\left(\frac{z}{t}\right)$



Color glass condensate



Effective field theory for high energy QCD

- Leading order classical Yang-Mills
- Hard partons: $\mathcal{J}^\mu(x) = \delta^\mu_\mp \rho(x^\pm, \mathbf{x})$
- Soft partons: $\mathcal{A}_\mu(x)$
- YM Eqs: $\mathcal{D}_\mu \mathcal{F}^{\mu\nu} = \mathcal{J}^\mu$
- Covariant gauge $\partial_\mu \mathcal{A}^\mu = 0$ solution

  $-\Delta_\perp \mathcal{A}^\mu(x) = \mathcal{J}^\mu(x)$

Dilute approximation

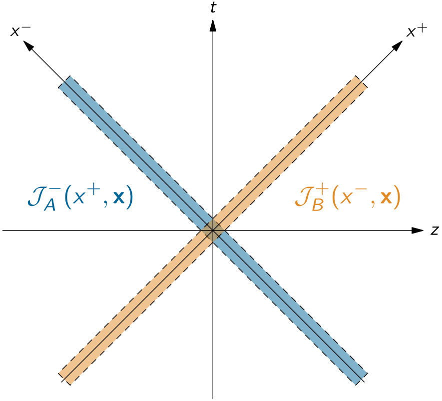
Single nuclei source terms

$$\mathcal{J}^{\mu}_{A/B}(x^{\pm}, \mathbf{x}) = \delta^{\mu}_{\mp}\, \rho_{A/B}(x^{\pm}, \mathbf{x})$$

Full Yang-Mills equations in future light cone

$$D_{\mu} F^{\mu\nu} = J^{\mu}$$

$$J^{\mu}(x) = \mathcal{J}^{\mu}_{A}(x) + \mathcal{J}^{\mu}_{B}(x) + j^{\mu}(x)$$
$$A^{\mu}(x) = \mathcal{A}^{\mu}_{A}(x) + \mathcal{A}^{\mu}_{B}(x) + a^{\mu}(x)$$



Dilute approximation

Single nuclei source terms

$$\mathcal{J}^{\mu}_{A/B}(x^{\pm}, \mathbf{x}) = \delta^{\mu}_{\mp}\ \rho_{A/B}(x^{\pm}, \mathbf{x})$$

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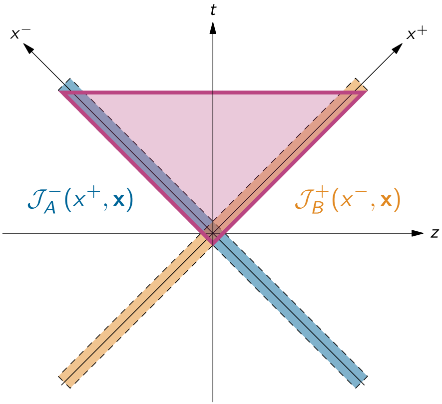
$a^{\mu}(x) \ldots$ Glasma field



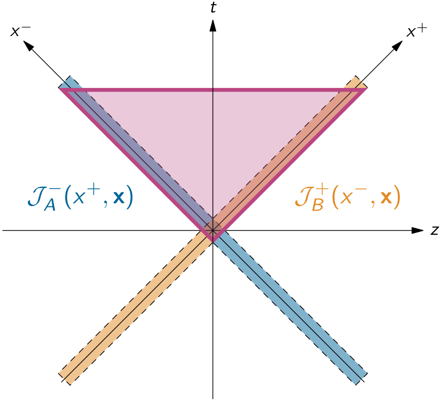
Dilute approximation

Full Yang-Mills equations in future light cone

$$D_\mu F^{\mu\nu} = J^\mu$$

$$J^\mu(x) = \mathcal{J}^\mu_A(x) + \mathcal{J}^\mu_B(x) + j^\mu(x)$$
$$A^\mu(x) = \mathcal{A}^\mu_A(x) + \mathcal{A}^\mu_B(x) + a^\mu(x)$$

$a^\mu(x)$ . . . Glasma field



Expansion in weak source terms

- a^μ and j^μ capture $O(\rho^n_A \rho^m_B)$ with $n, m \geq 1$
- Dilute limit: only keep order $\rho_A \rho_B$

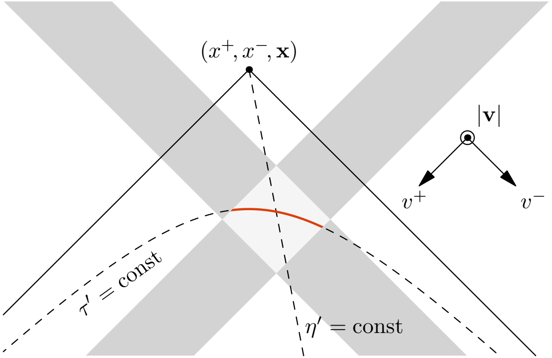
Glasma field strength tensor

$$v = \left(\frac{|\mathbf{v}|}{\sqrt{2}}\mathrm{e}^{+\eta'}, \frac{|\mathbf{v}|}{\sqrt{2}}\mathrm{e}^{-\eta'}, \mathbf{v}\right), \;\; v^2 = 0$$

$$f^{+-} = -\frac{g}{2\pi}\int_{\mathbf{v}}\int_{-\infty}^{\infty} d\eta' \, V$$

$$f^{\pm i} = \frac{g}{2\pi}\int_{\mathbf{v}}\int_{-\infty}^{\infty} d\eta' \, \left(V^{ij} \mp \delta^{ij} V\right) w^j \frac{e^{\pm\eta'}}{\sqrt{2}}$$

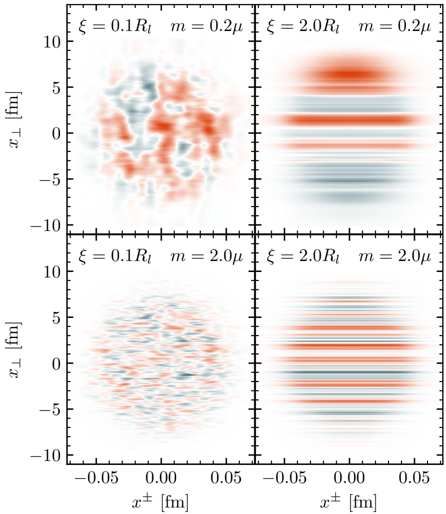
$$f^{ij} = -\frac{g}{2\pi}\int_{\mathbf{v}}\int_{-\infty}^{\infty} d\eta' \, V^{ij}$$



$$V := f_{abc} t^c \partial^i \mathcal{A}_A^{-a}\left(x^+ - \frac{|\mathbf{v}|}{\sqrt{2}}e^{+\eta'}, \mathbf{x} - \mathbf{v}\right)\partial^i \mathcal{A}_B^{+b}\left(x^- - \frac{|\mathbf{v}|}{\sqrt{2}}e^{-\eta'}, \mathbf{x} - \mathbf{v}\right)$$

$$V^{ij} := f_{abc} t^c \left(\partial^i \mathcal{A}_A^{-a}(\dots)\partial^j \mathcal{A}_B^{+b}(\dots) - \partial^j \mathcal{A}_A^{-a}(\dots)\partial^i \mathcal{A}_B^{+b}(\dots)\right)$$

Nuclear model



Color charges from Gaussian probability

$$\langle \rho^a(x^\pm, \mathbf{x}) \rangle = 0$$

$$\langle \rho^a(x^\pm, \mathbf{x}) \rho^b(y^\pm, \mathbf{y}) \rangle = g^2 \mu^2 \delta^{ab} \times \sqrt{T(x^\pm, \mathbf{x})} \sqrt{T(y^\pm, \mathbf{y})} \times U_\xi(x^\pm - y^\pm) \delta^{(2)}(\mathbf{x} - \mathbf{y})$$

Observables

Energy-momentum tensor

$$T^{\mu\nu} = 2\,\mathrm{Tr}\left[f^{\mu\rho} f_{\rho}^{\ \nu} + \frac{1}{4} g^{\mu\nu} f^{\rho\sigma} f_{\rho\sigma}\right]$$

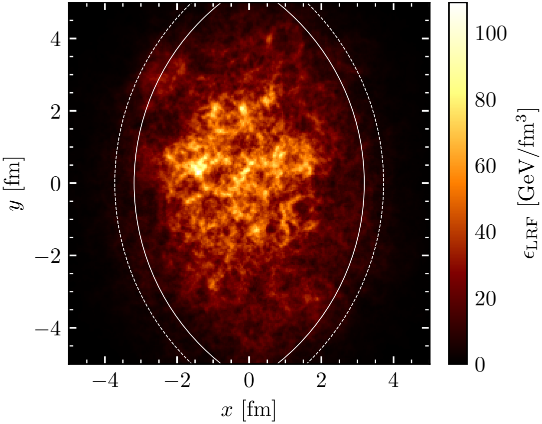
Local rest frame energy density ϵ_{LRF}

$$T^{\mu}_{\ \nu} u^{\nu} = \epsilon_{\mathrm{LRF}} u^{\mu}$$

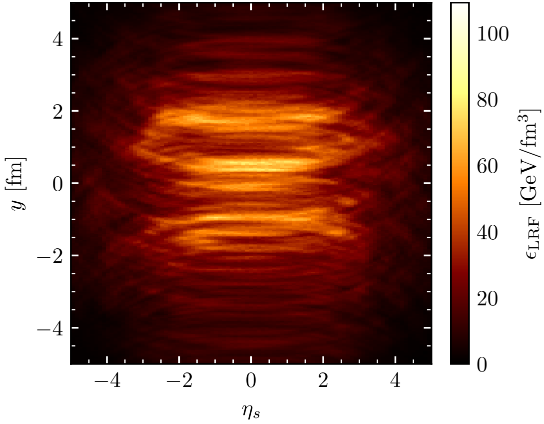
(u^{μ} is the only timelike eigenvector of $T^{\mu}_{\ \nu}$. $u^{\mu} u_{\mu} = 1$)

3D structure of $T^{\mu\nu}$

$\sqrt{s_{\mathrm{NN}}} = 200$ GeV Au+Au at $\tau = 0.4$ fm/c



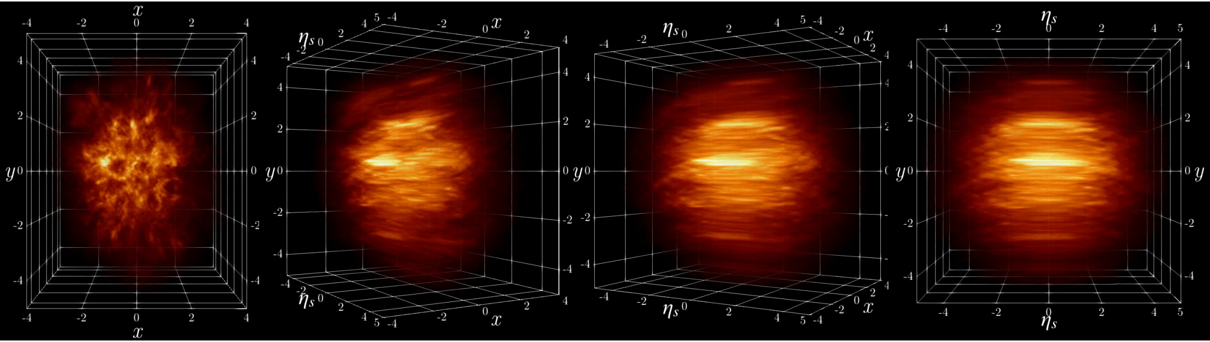
Almond shape



"Flux tube" structure

3D structure of $T^{\mu\nu}$

$\sqrt{s_{\mathrm{NN}}} = 200$ GeV Au+Au at $\tau = 0.4$ fm/c

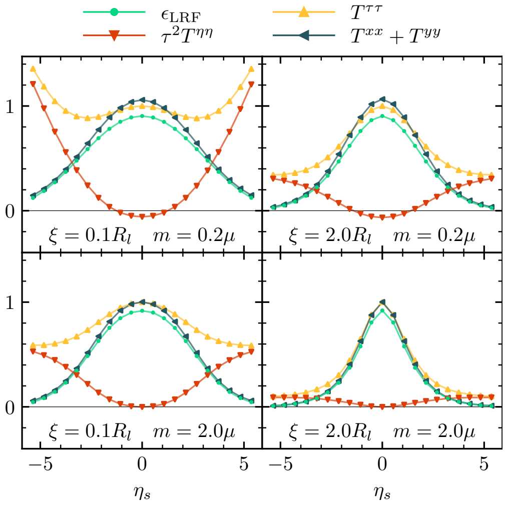


Full video available online: PhysRevD.109.094040 or [2401.10320]

Rapidity profiles

- Transverse integrals
  $\tau \int_{\mathbf{x}} T^{\mu\nu}$
  $\tau \int_{\mathbf{x}} \epsilon_{\mathrm{LRF}}$
- Normalized to $T^{\tau\tau}(\eta_s = 0)$
- τ and η tensor components are problematic
- Tracelessness $T^{\mu}{}_{\mu} = 0$

10 central events
at $\tau = 0.4\mathrm{fm/c}$
at $\sqrt{s_{\mathrm{NN}}} = 200$ GeV

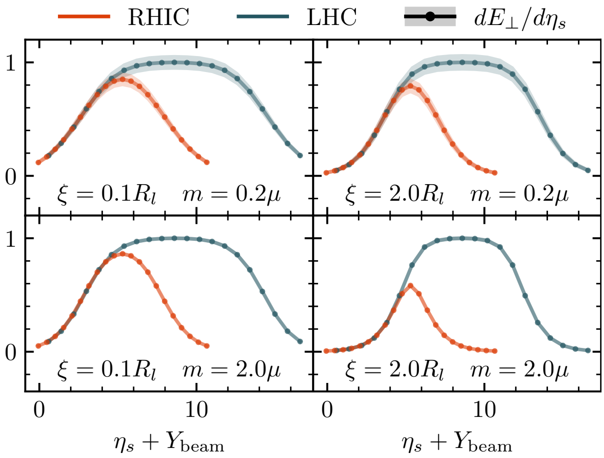


Limiting fragmentation

- Shapes at large η_s overlap
- Normalized to LHC $\eta_s = 0$
- Differential transverse energy

$$\frac{dE_\perp}{d\eta_s} = \tau \int_{\mathbf{x}} \left(T^{xx}(\tau, \eta_s, \mathbf{x}) + T^{yy}(\tau, \eta_s, \mathbf{x}) \right)$$

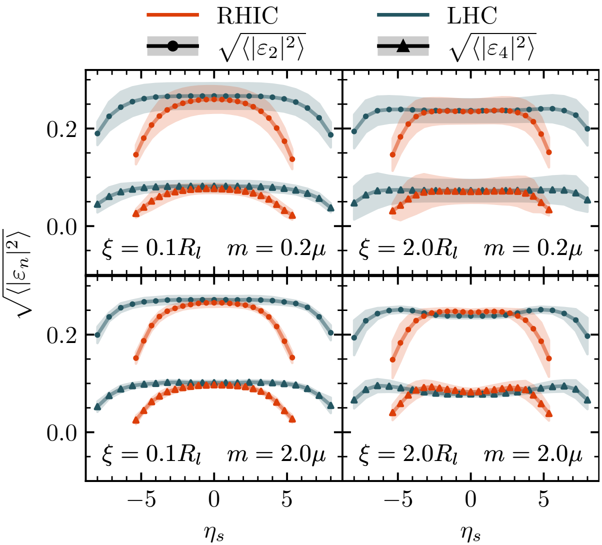
10 central events
at $\tau = 0.4$ fm/c
RHIC: $\sqrt{s_{\mathrm{NN}}} = 200$ GeV
LHC: $\sqrt{s_{\mathrm{NN}}} = 2700$ GeV



Eccentricity

- Transverse structure

$$\varepsilon_n(\tau,\eta_s) = \frac{\int_{\mathbf{x}} \gamma \epsilon_{\mathrm{LRF}}\, \bar{r}^n \exp\left(in\bar{\phi}\right)}{\int_{\mathbf{x}} \gamma \epsilon_{\mathrm{LRF}}\, \bar{r}^n}$$



10 events with impact parameter
at $\tau = 0.4$ fm/c
RHIC: $\sqrt{s_{\mathrm{NN}}} = 200$ GeV
LHC: $\sqrt{s_{\mathrm{NN}}} = 2700$ GeV

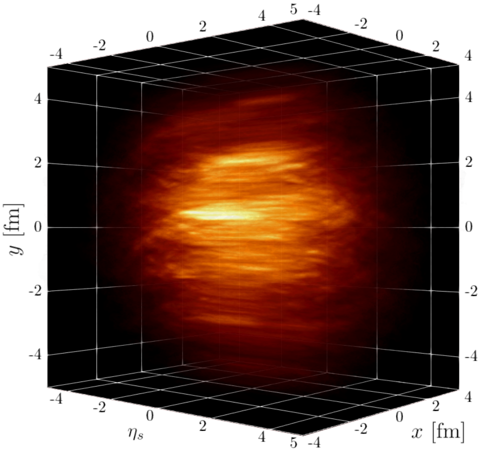
Summary

Our model

- Yang-Mills equations expanded up to $\rho_A \rho_B$
- 3D nuclear model with longitudinal correlations

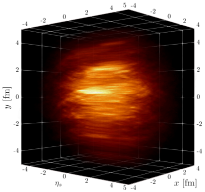
Our results

- Energy-momentum tensor of the Glasma
- Longitudinal and transverse structure



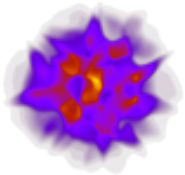
ϵ_{LRF} for $\sqrt{s_{\mathrm{NN}}} = 200$ GeV Au+Au at $\tau = 0.4$ fm/c

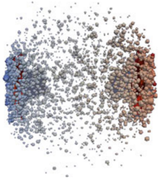
Outlook — Modeling all stages



$$
\begin{aligned}
\mathcal{T}^{\mu\nu}_{\text{hydro}} = \mathcal{T}^{\mu\nu}_{\text{ideal}} &\\
+ \pi^{\mu\nu} &\\
- (g^{\mu\nu} - u^\mu u^\nu)\Pi &
\end{aligned}
$$

$$\mathcal{T}^{\mu\nu}_{\text{ideal}} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu}$$



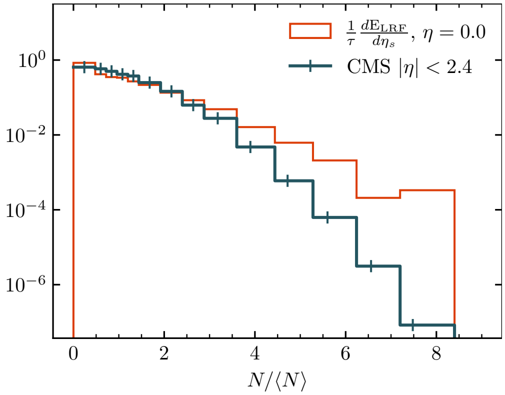


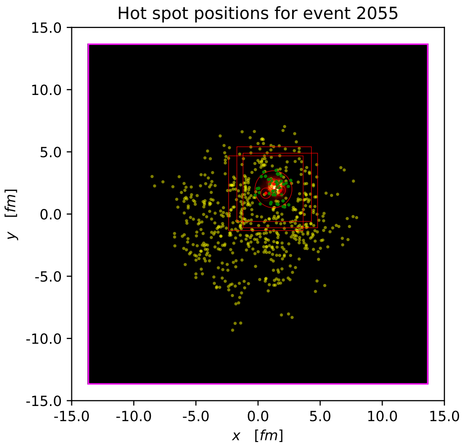
Dilute Glasma →	Landau matching →	Equilibrium QGP →	Hadron gas
$\tau \lesssim 0.2\,\frac{\text{fm}}{c}$	$\tau = \tau_{\text{switch}}$	$1\,\frac{\text{fm}}{c} \lesssim \tau \lesssim 10\,\frac{\text{fm}}{c}$	$10\,\frac{\text{fm}}{c} \lesssim \tau$
Classical Yang-Mills	$T^\mu{}_\nu u^\nu = \epsilon u^\mu$	Rel. viscous hydrodynamics	Kinteic theory
Our code	$\pi^{\mu\nu} = T^{\mu\nu}_{\text{glasma}} - T^{\mu\nu}_{\text{ideal}}$ $\Pi = \frac{\epsilon}{3} - P_{\text{EOS}}(\epsilon)$	MUSIC	UrQMD

Image credits: Schenke et al. [1009.3244], Becattini et al. (2021).

Outlook — p+Pb $\sqrt{s_{\mathrm{NN}}} = 5.02$ TeV

5000 events histogram





CMS data from Chatrchyan et al. [1305.0609]

References

[1] A. Ipp, M. Leuthner, D. I. Müller, S. Schlichting, **K. Schmidt** and P. Singh. *Energy-momentum tensor of the dilute (3+1)D glasma*. In: *Phys. Rev. D* 109.9 (2024), p. 094040. arXiv: 2401.10320 [hep-ph].

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[5] A. Ipp, D. I. Müller, S. Schlichting and P. Singh. *Spacetime structure of (3+1)D color fields in high energy nuclear collisions*. In: *Phys. Rev. D* 104.11 (2021), p. 114040. arXiv: 2109.05028 [hep-ph].

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Acknowledgements

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FWF Österreichischer Wissenschaftsfonds

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Backup Slides

Coordinate systems

- $\mathbf{x} = (x, y) \ldots$ transverse plane
- $z \ldots$ beam axis
- $\phi \in [0, 2\pi) \ldots$ azimuthal angle
- $\theta \in [0, \pi) \ldots$ polar angle
- $\eta \ldots$ pseudorapidity
  $\eta = -\ln\left[\arctan(\theta/2)\right]$
- $\mathrm{y} \ldots$ rapidity
  $\mathrm{y} = \frac{1}{2}\ln\frac{E+p_z}{E-p_z} \approx \eta$ for $p \gg m$

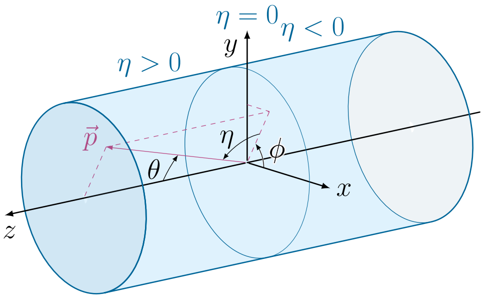


Image credits: Izaak Neutelings

Dilute approximation examples

Covariant conservation of current at order $\rho_A \rho_B$:

$$\begin{aligned} D_\mu J^\mu &= \partial_\mu J^\mu - ig[A_\mu, J^\mu] \\ &= \mathcal{D}_{A,\mu} \mathcal{J}_A^\mu + \mathcal{D}_{B,\mu} \mathcal{J}_B^\mu + \partial_\mu j^\mu - ig[\mathcal{A}_{A,\mu}, \mathcal{J}_B^\mu] - ig[\mathcal{A}_{B,\mu}, \mathcal{J}_A^\mu] = 0 \end{aligned}$$

$$\begin{aligned} \partial_- j^- &= ig\left[\mathcal{A}_B^+, \mathcal{J}_A^-\right] \\ \partial_+ j^+ &= ig\left[\mathcal{A}_A^-, \mathcal{J}_B^+\right] \end{aligned}$$

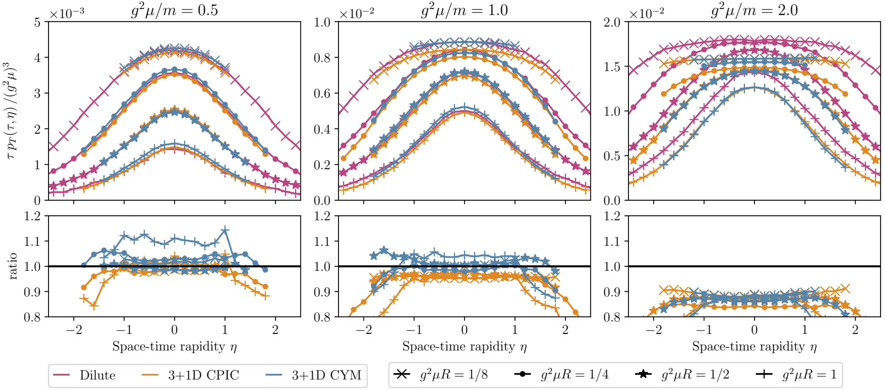
Dilute approximation examples

Yang-Mills equations at order $\rho_A \rho_B$:

$$
\begin{aligned}
[D_\mu, F^{\mu\nu}] = & \left[\mathcal{D}_{A,\mu}, \mathcal{F}_A^{\mu\nu}\right] + \left[\mathcal{D}_{B,\mu}, \mathcal{F}_B^{\mu\nu}\right] \\
& + \partial_\mu\left(\partial^\mu a^\nu - \partial^\nu a^\mu - ig\left[\mathcal{A}_A^\mu, \mathcal{A}_B^\nu\right] - ig\left[\mathcal{A}_B^\mu, \mathcal{A}_A^\nu\right]\right) \\
& - ig\left[\mathcal{A}_{A,\mu}, \partial^\mu \mathcal{A}_B^\nu - \partial^\nu \mathcal{A}_B^\mu\right] - ig\left[\mathcal{A}_{B,\mu}, \partial^\mu \mathcal{A}_A^\nu - \partial^\nu \mathcal{A}_A^\mu\right] \\
= & \mathcal{J}_A^\nu + \mathcal{J}_B^\nu + j^\nu .
\end{aligned}
$$

$$
\begin{aligned}
\partial_\mu \partial^\mu a^+ &= ig\left[\mathcal{A}_A^-, \partial_- \mathcal{A}_B^+\right] + j^+ \\
\partial_\mu \partial^\mu a^- &= ig\left[\mathcal{A}_B^+, \partial_+ \mathcal{A}_A^-\right] + j^- \\
\partial_\mu \partial^\mu a^i &= ig\left[\mathcal{A}_A^-, \partial_i \mathcal{A}_B^+\right] + ig\left[\mathcal{A}_B^+, \partial_i \mathcal{A}_A^-\right]
\end{aligned}
$$

Comparison to lattice simulations



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Simulation parameters

Param.	Name	Value(s)	Unit
N_c	No. of colors	3	-
γ	Lorentz factor	100 (R), 2700 (L)	-
$\sqrt{s_{NN}}$	c.m. energy	200 (R), 5400 (L)	GeV
R	WS radius	6.38 (R), 6.62 (L)	fm
d	WS skin depth	0.535 (R), 0.546 (L)	fm
g	YM coupling	1	-
μ	MV scale	1	GeV
m	IR cutoff	0.2, 2.0	GeV
Λ_{UV}	UV cutoff	10	GeV
ξ	correlation length	0.1, 0.5, 2.0	R_l
b	impact parameter	0, 1	R
τ	proper time	0.2, 0.4, 0.6, 0.8, 1.0	fm/c

McLerran-Venugopalan nuclear model

Gaussian distribution set by expectation values

$$\langle \rho(x^\pm, \mathbf{x}) \rangle = 0$$
$$\langle \rho^a(x^\pm, \mathbf{x})\ \rho^b(y^\pm, \mathbf{y}) \rangle = \delta^{ab} g^2 \mu^2 (x^\pm) \delta(x^\pm - y^\pm) \delta^{(2)}(\mathbf{x} - \mathbf{y})$$

Extension to non-trivial longitudinal structure

$$\langle \rho^a(x^\pm, \mathbf{x})\ \rho^b(y^\pm, \mathbf{y}) \rangle =$$
$$\delta^{ab}\ \underbrace{g^2 \mu^2}_{\text{strength of color charges}}\ \underbrace{T_R\left(\frac{x^\pm + y^\pm}{2}\right)}_{\text{longitudinal profile of width } R}\ \underbrace{U_\xi\left(x^\pm - y^\pm\right)}_{\text{correlations of width } \xi}\ \underbrace{T_S(\mathbf{x} - \mathbf{y})}_{\text{transverse profile of width } S}\ \underbrace{\delta^{(2)}(\mathbf{x} - \mathbf{y})}_{\text{uncorrelated}}$$

McLerran-Venugopalan nuclear model

$$\langle \rho^{a,-}(x^\pm, \mathbf{x}) \rangle = 0$$

$$\langle \rho^{a,-}(x^\pm, \mathbf{x})\ \rho^{b,-}(y^\pm, \mathbf{y}) \rangle = \delta^{ab}\, g^2 \mu^2\ T_R\left(\frac{x^\pm + y^\pm}{2}\right) U_\xi\left(x^\pm - y^\pm\right) T_S\left(\mathbf{x} - \mathbf{y}\right) \delta^{(2)}\left(\mathbf{x} - \mathbf{y}\right)$$

Single nuclei separation ansatz for gaussian T_R, T_S:

$$\langle \rho^{a,-}(x^\pm, \mathbf{x})\ \rho^{b,-}(y^\pm, \mathbf{y}) \rangle = \delta^{ab}\, g^2 \mu^2\ \sqrt{T_R\left(x^\pm\right)}\sqrt{T_S\left(\mathbf{x}\right)}\sqrt{T_R\left(y^\pm\right)}\sqrt{T_S\left(\mathbf{y}\right)} \times U_\xi^{\mathrm{mod}}\left(x^\pm - y^\pm\right) \delta^{(2)}\left(\mathbf{x} - \mathbf{y}\right)$$

$$U_\xi^{\mathrm{mod}}\left(x^\pm - y^\pm\right) = \frac{1}{\sqrt{2\pi\xi^2}} \exp\left[-(x^\pm - y^\pm)^2\left(\frac{1}{2\xi^2} - \frac{1}{8R^2}\right)\right]$$

Nuclear model details

$$\langle \rho^a(x^\pm, \mathbf{x}) \rangle = 0$$

$$\langle \rho^a(x^\pm, \mathbf{x}) \rho^b(y^\pm, \mathbf{y}) \rangle = g^2 \mu^2 \delta^{ab} \sqrt{T(x^\pm, \mathbf{x})} \sqrt{T(y^\pm, \mathbf{y})} U_\xi(x^\pm - y^\pm) \delta^{(2)}(\mathbf{x} - \mathbf{y})$$

$$T(x^\pm, \mathbf{x}) = \frac{c}{1 + \exp\left(\frac{\sqrt{2(\gamma x^\pm)^2 + \mathbf{x}^2} - R}{d}\right)}$$

$$U_\xi(x^\pm - y^\pm) = \frac{1}{\sqrt{2\pi\xi^2}} \mathrm{e}^{\frac{(x^\pm - y^\pm)^2}{8R_l^2}} \mathrm{e}^{-\frac{(x^\pm - y^\pm)^2}{2\xi^2}}$$

$$\mathcal{A}^{\mp a}_{A/B}(x^\pm, \mathbf{x}) = \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \frac{\tilde{\rho}^a_{A/B}(x^\pm, \mathbf{k})}{\mathbf{k}^2 + m^2} \mathrm{e}^{-\mathbf{k}^2/(2\Lambda^2_{\mathrm{UV}})} \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}},$$

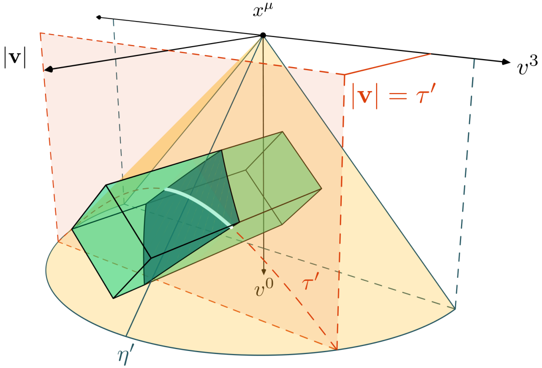
Glasma field strength tensor

$$v = \left(\frac{|\mathbf{v}|}{\sqrt{2}}\mathrm{e}^{+\eta'}, \frac{|\mathbf{v}|}{\sqrt{2}}\mathrm{e}^{-\eta'}, \mathbf{v}\right), \;\; v^2 = 0$$

$$f^{+-} = -\frac{g}{2\pi}\int\limits_{\mathbf{v}}\int\limits_{-\infty}^{\infty} d\eta' \, V$$

$$f^{\pm i} = \frac{g}{2\pi}\int\limits_{\mathbf{v}}\int\limits_{-\infty}^{\infty} d\eta' \, \left(V^{ij} \mp \delta^{ij} V\right) w^j \frac{e^{\pm\eta'}}{\sqrt{2}}$$

$$f^{ij} = -\frac{g}{2\pi}\int\limits_{\mathbf{v}}\int\limits_{-\infty}^{\infty} d\eta' \, V^{ij}$$



$$V := f_{abc} t^c \partial^i \mathcal{A}_A^{-a}\left(x^+ - \frac{|\mathbf{v}|}{\sqrt{2}} e^{+\eta'}, \mathbf{x} - \mathbf{v}\right) \partial^i \mathcal{A}_B^{+b}\left(x^- - \frac{|\mathbf{v}|}{\sqrt{2}} e^{-\eta'}, \mathbf{x} - \mathbf{v}\right)$$

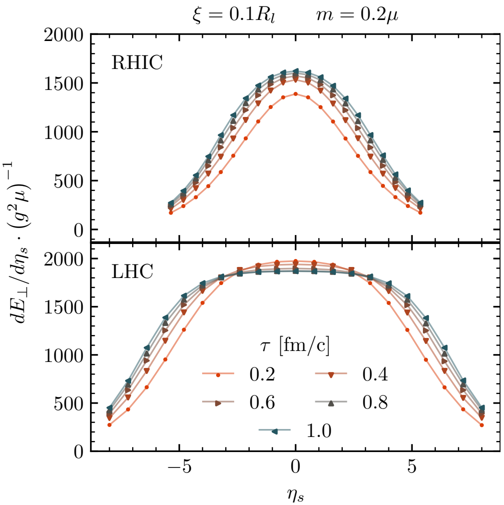
$$V^{ij} := f_{abc} t^c \left(\partial^i \mathcal{A}_A^{-a}(\dots)\partial^j \mathcal{A}_B^{+b}(\dots) - \partial^j \mathcal{A}_A^{-a}(\dots)\partial^i \mathcal{A}_B^{+b}(\dots)\right)$$

Time evolution

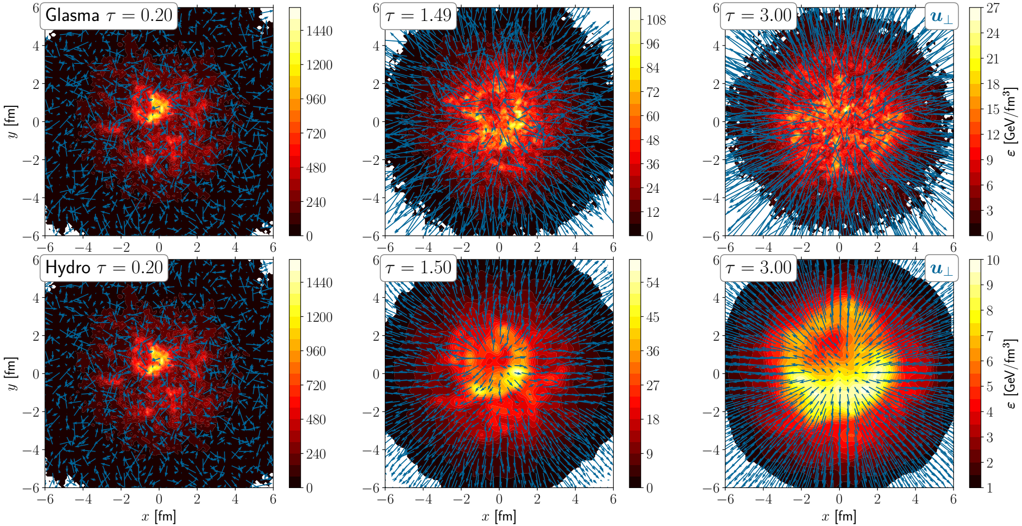
- Profiles stabilize
- Differential transverse energy

$$\frac{dE_\perp}{d\eta_s} = \tau \int_{\mathbf{x}} \left(T^{xx}(\tau, \eta_s, \mathbf{x}) + T^{yy}(\tau, \eta_s, \mathbf{x}) \right)$$

10 central events
RHIC: $\sqrt{s_{\mathrm{NN}}} = 200$ GeV
LHC: $\sqrt{s_{\mathrm{NN}}} = 2700$ GeV



Proper time τ evolution at $\eta_s = 0$



QCD phase diagram

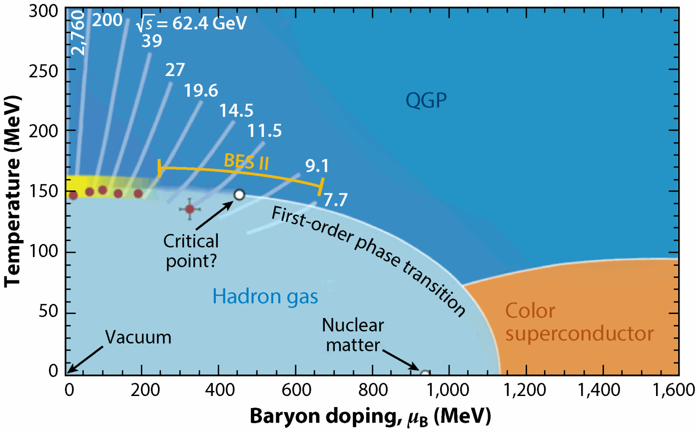


Image credits: Busza, Rajagopal and Schee [1802.04801].