Recent BABAR results on measurement of exclusive hadronic cross sections

Evgeny Solodov
representing the BABAR Collaboration

BudkerINP and Novosibirsk State University
solodov@inp.nsk.su

Hadron Spectroscopy: The Next Big Step
MITP virtual workshop
New precision measurement of $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section

First measurements of $e^+e^- \rightarrow \pi^+\pi^-4\pi^0$ and $e^+e^- \rightarrow \pi^+\pi^-3\pi^0\eta$ cross sections

First measurements of $e^+e^- \rightarrow 2(\pi^+\pi^-)3\pi^0$ and $e^+e^- \rightarrow 2(\pi^+\pi^-)2\pi^0\eta$ cross sections
\[ a_{\mu} = \frac{1}{2}(g - 2)_{\mu} \] precision test of SM

SM prediction for muons:

\[ a_{\mu} = a_{\mu}^{QED} + a_{\mu}^{EW} + a_{\mu}^{hadr} \]

Absolute value dominated by \( a_{\mu}^{QED} + a_{\mu}^{EW} \)

Error dominated by \( a_{\mu}^{hadr} \): not calculable perturbatively

\( a_{\mu}^{hadr} \): LQCD or data-driven dispersive approach

For \( \sqrt{s} \lesssim 2 \text{ GeV} \) finite number of final states contribute:

\( \sigma(e^+e^- \rightarrow \text{hadrons}) \) can be obtained as sum of all exclusive cross sections
**BABAR** hadronic cross section measurements using ISR

Initial State Radiation from $e^+e^-$ allows to measure cross sections at all center-of-mass energies $\sqrt{s'}$ below the nominal $\sqrt{s}$ of the beams:

$$\frac{d\sigma(s; s'; \theta_\gamma)}{ds'd\theta_\gamma} = W(s; s'; \theta_\gamma) \cdot \sigma_X(s')$$

- **tag** photon to identify ISR events
- hadrons in fiducial detector region
- fully reconstruct the final state
- kinematic fit: energy resolution

**Boost** $\Rightarrow$ harder momentum spectrum for daughter particles

- cross sections down to threshold
- measure $\sigma$ at all $\sqrt{s}$ simultaneously
- large ”effective” luminosity

**ISR Luminosity $L^{ISR}$**

<table>
<thead>
<tr>
<th>$\sqrt{s'}$ [GeV]</th>
<th>$L^{ISR}$ (pb$^{-1}$ / 0.1 GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 GeV</td>
<td>$L_{BABAR} = 232$ fb$^{-1}$</td>
</tr>
</tbody>
</table>

E. Solodov

MITP Workshop

March 14-25, 2022

4
Light hadrons cross sections measured by BABAR
Light hadrons cross sections measured in ISR by \textit{BABAR}

Many \textbf{first measurements:} \hspace{1cm} (superseded results omitted)

\begin{align*}
\pi^+\pi^-\pi^0 \\
\pi^+\pi^-\pi^0\pi^0\pi^0\pi^0 \text{ and } \pi^+\pi^-\pi^0\pi^0\pi^0\eta \\
2(\pi^+\pi^-)\pi^0\pi^0\pi^0 \text{ and } 2(\pi^+\pi^-)\pi^0\pi^0\eta \\
\pi^+\pi^-\pi^0\pi^0\pi^0 \text{ and } \pi^+\pi^-\pi^0\pi^0\eta \\
\pi^+\pi^-\eta \\
\pi^+\pi^-\pi^0\pi^0 \\
K^0_SK^{\pm}\pi^0 \text{ and } K^0_SK^{\pm}\eta \\
K^0_SK^0\pi^0, \ K^0_SK^0\eta, \text{ and } K^0_SK^0\pi^0\pi^0 \\
K^+K^- \text{ (\gamma undetected)} \\
K^0_SK^0_{K^0_L\pi^+\pi^-}, \ K^0_SK^0_{K^0_L\pi^+\pi^-}, \text{ and } K^0_SK^0_{K^+K^-} \\
K^+K^- \\
p\bar{p} \\
p\bar{p} \ (E_{cm} : 3.0 \div 6.5 \text{ GeV}) \\
\pi^+\pi^-\pi^+\pi^- \\
K^+K^-\pi^+\pi^-, \ K^+K^-\pi^0\pi^0, \text{ and } K^+K^-K^+K^- \\
\pi^+\pi^- \\
K^+K^-\eta, \ K^+K^-\pi^0 \text{ and } K^0_SK^{\pm}\pi^0 \\
\Lambda\bar{\Lambda}, \ \Lambda\bar{\Sigma}^0, \text{ and } \Sigma^0\bar{\Sigma}^0 \\
2(\pi^+\pi^-)\pi^0, \ 2(\pi^+\pi^-)\eta, \ K^+K^-\pi^+\pi^-\pi^0 \text{ and } K^+K^-\pi^+\pi^-\eta \\
3(\pi^+\pi^-), \ 2(\pi^+\pi^-)\pi^0 \text{ and } K^+K^-2(\pi^+\pi^-) \\
\end{align*}

469 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 104, 112003 (2021) \\
469 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 104, 112004 (2021) \\
469 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 103, 092001 (2021) \\
469 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 98, 112015 (2018) \\
469 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 97, 052007 (2018) \\
454 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 96, 092009 (2017) \\
454 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 95, 092005 (2017) \\
469 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 95, 052001 (2017) \\
469 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 92, 072008 (2015) \\
469 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 89, 092002 (2014) \\
232 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 88, 032013 (2013) \\
469 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 87, 092005 (2013) \\
469 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 88, 072009 (2013) \\
454 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 85, 112009 (2012) \\
454 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 86, 012008 (2012) \\
232 fb$^{-1}$ \hspace{1cm} Phys.Rev.Lett. 103, 231801 (2009) \\
232 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 77, 092002 (2008) \\
230, fb$^{-1}$ \hspace{1cm} Phys. Rev. D 76, 092006 (2007) \\
232 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 76, 092005 (2007) \\
232 fb$^{-1}$ \hspace{1cm} Phys. Rev. D 73, 052003 (2006)
\( e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \)

It gives the second largest contribution to \( a_{\mu}^{HLO} \) and its error.

Previous BABAR measurement based on 20\% of dataset available now.
Cross section had been measured in \( 1.05 \div 3 \) GeV.

New measurement using the whole dataset extends cross section below 1.05 GeV, in the region of \( \rho, \omega \) and \( \phi \) resonances.

Accuracy on \( a_{\mu}^{HLO} \) contribution due to \( e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \) currently \( \approx 3\% \), new measurement will improve accuracy to \( \approx 1.5\% \).
$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \gamma_{ISR}$

Detect all final state particles
Select events using kinematic fit (cut on $\chi^2$)
Several additional cuts reduce background by factor 2

Remaining ISR and $q\bar{q}$ background subtracted using simulation normalized to data.

Above 1.1 GeV sizeable FSR background from $e^+ e^- \rightarrow a_1 \gamma$, $a_2 \gamma$ processes.
Estimated by pQCD with 100% uncertainty.
up to 8% contribution near 1.3 GeV

$\omega$ region
0.67-0.87 GeV

Tight selection
Standard selection
Below 1.1 GeV the mass spectrum has a sharp structure unfolding required to determine true spectrum

cross section result depends on the assumed mass resolution

The $\omega$ and $\phi$ widths are well known

$\Rightarrow$ use data to correct the simulated resolution function

Tails of the resolution depend on the $\chi^2$ cut applied in selecting events:

$\Rightarrow$ try more than one cut value
Fit to the $\pi^+\pi^-\pi^0$ mass spectrum

The mass spectrum fitted with VDM model including

$$\omega(782) + \omega(1420) + \omega(1680) + \phi(1020)$$

resonances

$\omega(782)$ and $\phi$ widths fixed to PDG average

+ the rare $\rho(770) \to 3\pi$ decay

For $\chi^2 < 20$ (nominal fit) the mass spectrum is well described by introducing an additional Gaussian smearing to the MC resolution function

$$\sigma_s = 1.5 \pm 0.2 \text{ MeV}$$

$$m_\omega - m_{PDG} = 0.042 \pm 0.055 \text{ MeV}$$

$$m_\phi - m_{PDG} = 0.095 \pm 0.084 \text{ MeV}$$

For $\chi^2 < 40$ (cross check): additional Lorentzian smearing required to describe tails

$\text{fraction} = 0.7 \pm 0.2\%$; $\gamma = 63 \pm 35 \text{ GeV}$

consistent results for all other parameters

The data spectrum CANNOT be adequately described with $B(\rho \to 3\pi) \equiv 0$
Fits to the $\pi^+\pi^-\pi^0$ mass spectrum ($\chi^2 < 40$)

(solid): Lorentzian smearing and $\rho \rightarrow 3\pi$
(dash): No Lorentzian smearing and no $\rho \rightarrow 3\pi$
(dot): Lorentzian smearing and no $\rho \rightarrow 3\pi$

$\chi^2/\nu = 136/127$
$\chi^2/\nu = 201/131$
$\chi^2/\nu = 180/129$
**Fit results on resonance parameters**

For $\omega(782)$ and $\phi(1020)$ the products $\Gamma_{ee} \times B_{3\pi}$ are in reasonable agreement with world average values:

\[
\Gamma(\omega \to e^+ e^-) \cdot B(\omega \to \pi^+ \pi^- \pi^0) = (0.5698 \pm 0.0031 \pm 0.0082) \text{ keV}
\]

world average: $(0.557 \pm 0.011)$ keV

\[
\Gamma(\phi \to e^+ e^-) \cdot B(\phi \to \pi^+ \pi^- \pi^0) = (0.1841 \pm 0.0021 \pm 0.0080) \text{ keV}
\]

world average: $(0.1925 \pm 0.0043)$ keV

The rare decay $\rho \to \pi^+ \pi^- \pi^0$ is observed with significance greater than $6\sigma$ the value and the relative phase wrt to the $\omega(782)$ amplitude are in agreement with the only previous measurement by SND


\[
B(\rho \to \pi^+ \pi^- \pi^0) = (0.88 \pm 0.23 \pm 0.30) \times 10^{-4}
\]

SND: $(1.01^{+0.54}_{-0.34} \pm 0.34) \times 10^{-4}$

\[
\phi_\rho - \phi_\omega = -(99 \pm 9 \pm 15)^{\circ}
\]

SND: $-(135^{+17}_{-13} \pm 9)^{\circ}$
$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ cross section below 1.1 GeV

The parameters of the smearing function determined in the VDM fit are used to correct the simulated resolution function.

The unfolding is performed using the IDS (iterative dynamically stabilized) method (B. Malaescu, arXiv:0907.3791)

Systematic uncertainty at $\omega(782)$ and $\phi(1020)$ peak $\approx 1.3\%$
$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ cross section below 1.1 GeV:
comparison with previous measurements

**SND**:

**CMD-2**:

SND - BABAR difference $\simeq 2\%$
below syst. (3.4% SND, 1.4% BABAR)
CMD-2 (1.8% stat and 1.3% syst) is
$\simeq 7\%$ smaller than BABAR
$\simeq 2.7\sigma$ difference

SND - BABAR difference $\simeq 11\%$
syst: 5% (SND); 1.4% (BABAR)
CMD-2 - BABAR difference $\simeq 3\%$
syst: 2.5% (CMD-2); 1.4% (BABAR)
$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ cross section above 1.1 GeV

No smearing is needed above 1.1 GeV. Syst. uncertainty: $4 \div 15\%$ dominated by background subtraction

Significant localized differences around 1.25 GeV and 1.5 GeV between $\text{BABAR}$ and SND (Eur.Phys.J. C 80, 993 (2020))
Impact on $a_{\mu}^{3\pi}$

<table>
<thead>
<tr>
<th>$M_{3\pi}$ GeV/$c^2$</th>
<th>$a_{\mu}^{3\pi} \times 10^{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.62–1.10</td>
<td>$42.91 \pm 0.14 \pm 0.55 \pm 0.09$</td>
</tr>
<tr>
<td>1.10–2.00</td>
<td>$2.95 \pm 0.03 \pm 0.16$</td>
</tr>
<tr>
<td>&lt; 2.00</td>
<td>$45.86 \pm 0.14 \pm 0.58$</td>
</tr>
<tr>
<td>&lt; 1.80[A]</td>
<td>$46.21 \pm 0.40 \pm 1.40$</td>
</tr>
<tr>
<td>&lt; 1.97[B]</td>
<td>$46.74 \pm 0.94$</td>
</tr>
<tr>
<td>&lt; 2[C]</td>
<td>$44.32 \pm 1.48$</td>
</tr>
</tbody>
</table>


The value of $a_{\mu}^{3\pi}$ calculated using the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross-section is in reasonable agreement with earlier calculations and the error on this contribution is reduced by a factor $\approx 2$. 
Is anything still missing?

Because of large, highly segmented calorimeter BaBar has advantage for study of the multi-photon reactions over detectors at VEPP2000 (SND and CMD-3) even effective integrated luminosity is already lower. It was demonstrated for the study of the reactions \( e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0, \pi^+ \pi^- \pi^+ \pi^- \pi^0 \pi^0, K_S K^\pm \pi^\mp \pi^0 \pi^0, K^- K^+ \pi^0 \pi^0 \pi^0, \ldots \)\(^{1}\) may help to understand if this deviation is real.

Currently, the sum of exclusive cross sections near 2.0 GeV shows a systematic deviation from the QCD predictions. BABAR, SND and CMD-3 measurements of previously unmeasured processes, e.g. \( e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0 \pi^0, \pi^+ \pi^- \pi^+ \pi^- \pi^0 \pi^0 \pi^0, K_S K^\pm \pi^\mp \pi^0 \pi^0, K^- K^+ \pi^0 \pi^0 \pi^0, \ldots \)\(^{1}\)

\[ e^+ e^- \rightarrow 2(\pi^+ \pi^-)3\pi^0 \text{ and } 2(\pi^+ \pi^-)2\pi^0\eta \] (first measurement)

All candidate events with 4 oppositely charged tracks, one \( \gamma_{ISR} \) photon candidate, 2 photon pairs with \( m_{\gamma \gamma} \) compatible with \( \pi^0 \) and a third photon pair + kinematic fit. Signal events selected based on \( \chi^2 < 50 \); background from \( \chi^2 \) sidebands. + some additional cuts to reduce background.

Fit the third photon pair mass distribution in bins of \( m(2(\pi^+ \pi^-)2\pi^0 \gamma \gamma) \) to determine the \( 2(\pi^+ \pi^-)3\pi^0 \) and \( 2(\pi^+ \pi^-)2\pi^0\eta \) yields to determine cross sections.
Intermediate states in $e^+e^- \rightarrow 2(\pi^+\pi^-)3\pi^0$

Good agreement with previously measured cross sections in different decay modes of $\omega$, $\eta$

$$e^+e^- \rightarrow \omega \pi^+\pi^-\pi^0\pi^0$$


New: No other measurements

BR($\omega \rightarrow 2\pi^0$) is taken into account

No significant evidence for other intermediate states

(not shown: $\rho^\pm \pi^\mp \pi^+\pi^- 2\pi^0$)
$e^+ e^- \rightarrow 2(\pi^+ \pi^-)3\pi^0$ and $2(\pi^+ \pi^-)2\pi^0\eta$: charmonium


<table>
<thead>
<tr>
<th>Measured quantity</th>
<th>Measured value (eV)</th>
<th>$J/\psi$ or $\psi(2S)$ branching fraction ($10^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{ce} \cdot B_{J/\psi \rightarrow \pi^+ \pi^- \eta} \cdot B_{\omega \rightarrow \pi^+ \pi^- \eta}$</td>
<td>$345.0 \pm 10.0 \pm 50.0$</td>
<td>$62.0 \pm 2.0 \pm 9.0$</td>
</tr>
<tr>
<td>$\Gamma_{ce} \cdot B_{J/\psi \rightarrow \omega \pi^+ \pi^- \pi^0 \eta} \cdot B_{\omega \rightarrow \pi^+ \pi^- \pi^0 \eta}$</td>
<td>$165.0 \pm 9.0 \pm 25.0$</td>
<td>$33.0 \pm 2.0 \pm 5.0$</td>
</tr>
<tr>
<td>$\Gamma_{ce} \cdot B_{J/\psi \rightarrow \pi^+ \pi^- \eta \eta} \cdot B_{\eta \rightarrow \pi^+ \pi^- \eta}$</td>
<td>$6.0 \pm 4.0 \pm 1.0$</td>
<td>$4.8 \pm 3.2 \pm 0.8$</td>
</tr>
<tr>
<td>$\Gamma_{ce} \cdot B_{J/\psi \rightarrow \omega \pi^+ \pi^- \pi^0 \eta} \cdot B_{\omega \rightarrow \pi^+ \pi^- \pi^0 \eta}$</td>
<td>$5.6 \pm 2.6 \pm 0.8$</td>
<td>$2.6 \pm 1.2 \pm 0.5$</td>
</tr>
<tr>
<td>$\Gamma_{ce} \cdot B_{J/\psi \rightarrow \pi^+ \pi^- \eta \eta} \cdot B_{\eta \rightarrow \pi^+ \pi^- \eta}$</td>
<td>$155.0 \pm 26.0 \pm 36.0$</td>
<td>$28.0 \pm 4.7 \pm 6.6$</td>
</tr>
<tr>
<td>$\Gamma_{ce} \cdot B_{J/\psi \rightarrow \rho^+ \pi^- \pi^0 \pi^0 \eta}$</td>
<td>$32.0 \pm 13.0 \pm 15.0$</td>
<td>$5.7 \pm 2.4 \pm 2.7$</td>
</tr>
<tr>
<td>$\Gamma_{ce} \cdot B_{J/\psi \rightarrow \rho^+ \pi^- \pi^0}$</td>
<td>$9.1 \pm 2.6 \pm 1.4$</td>
<td>$4.2 \pm 1.2 \pm 0.6$</td>
</tr>
<tr>
<td>$\Gamma_{ce} \cdot B_{J/\psi \rightarrow \pi^+ \pi^- \eta \eta} \cdot B_{\eta \rightarrow \pi^+ \pi^- \eta}$</td>
<td>$33.0 \pm 5.0 \pm 5.0$</td>
<td>$14.0 \pm 2.0 \pm 2.0$</td>
</tr>
<tr>
<td>$\Gamma_{ce} \cdot B_{\psi(2S) \rightarrow \pi^+ \pi^- \pi^0 \eta \eta} \cdot B_{\eta \rightarrow \pi^+ \pi^- \eta}$</td>
<td>$14.8 \pm 2.6 \pm 2.2$</td>
<td>$34.7 \pm 6.1 \pm 5.2$</td>
</tr>
<tr>
<td>$\Gamma_{ce} \cdot B_{\psi(2S) \rightarrow \omega \pi^+ \pi^- \pi^0 \eta \eta} \cdot B_{\omega \rightarrow \pi^+ \pi^- \pi^0}$</td>
<td>$19.2 \pm 4.5 \pm 3.2$</td>
<td>$23.8 \pm 5.6 \pm 3.6$</td>
</tr>
<tr>
<td>$\Gamma_{ce} \cdot B_{\psi(2S) \rightarrow \rho^+ \pi^- \pi^0 \pi^0 \eta}$</td>
<td>$18.0 \pm 4.0 \pm 3.0$</td>
<td>$8.7 \pm 1.9 \pm 1.5$</td>
</tr>
<tr>
<td>$\Gamma_{ce} \cdot B_{\psi(2S) \rightarrow \rho^+ \pi^- \pi^0}$</td>
<td>$&lt; 1.9$ at 90% C.L.</td>
<td>$&lt; 2.0$ at 90% C.L.</td>
</tr>
<tr>
<td>$\Gamma_{ce} \cdot B_{\psi(2S) \rightarrow \omega \pi^+ \pi^- \pi^0 \eta \eta} \cdot B_{\eta \rightarrow \pi^+ \pi^- \eta}$</td>
<td>$&lt; 2.3$ at 90% C.L.</td>
<td>$&lt; 2.4$ at 90% C.L.</td>
</tr>
<tr>
<td>$\Gamma_{ce} \cdot B_{\psi(2S) \rightarrow \pi^+ \pi^- \eta \eta} \cdot B_{\eta \rightarrow \pi^+ \pi^- \eta}$</td>
<td>$1.2 \pm 0.6$</td>
<td>No entry</td>
</tr>
</tbody>
</table>
$e^+ e^- \rightarrow \pi^+ \pi^- 4\pi^0$ and $\pi^+ \pi^- 3\pi^0 \eta$ (first measurement)

Events with 2 oppositely charged tracks, one $\gamma_{ISR}$ photon candidate, 3 photon pairs with $m_{\gamma\gamma}$ compatible with $\pi^0$ and a fourth photon pair

Signal events selected based on $\chi^2 < 70$; background from $\chi^2$ sidebands some additional cuts to reduce background

Fit the fourth photon pair invariant mass distribution in bins of $m(\pi^+ \pi^- 3\pi^0 \gamma\gamma)$ to determine the $\pi^+ \pi^- 4\pi^0$ and $\pi^+ \pi^- 3\pi^0 \eta$ yields to determine cross sections
Intermediate states in $e^+e^- \rightarrow \pi^+\pi^- 4\pi^0$

Intermediate states:

- $\pi^+\pi^-\pi^0\eta$
- $\omega\eta$
- $\omega 3\pi^0$
- $(\rho\pi) 3\pi^0$

and possibly $\rho^+\rho^- 2\pi^0$

above 2.9 GeV

Sum of intermediate states seem to saturate the observed cross section

Below 2 GeV agreement with SND and CMD-2 measurements of $\pi^+\pi^-\pi^0\eta$ and $\omega\eta$:  

No other measurements
\( e^+ e^- \rightarrow \pi^+ \pi^- 4\pi^0 \) and \( \pi^+ \pi^- 3\pi^0 \eta \): charmonium

\[
N_{J/\psi} = 340\pm 42 \\
N_{\psi(2S)} = 28\pm 19
\]

\[
\text{Measured} \quad \text{Quantity} \quad \text{Measured Value (eV)} \quad \text{\textit{J}/\psi or \psi(2S) Branching Fraction (10^{-3})}} \\
\begin{array}{llll}
\Gamma_{ee}^{J/\psi} \cdot B_{J/\psi \rightarrow \pi^+ \pi^- \pi^0 \pi^0 \eta \pi^0} & 35.8\pm 4.4\pm 5.4 & 6.5\pm 0.8\pm 1.0 & \text{no entry} \\
\Gamma_{ee}^{J/\psi} \cdot B_{J/\psi \rightarrow \eta \pi^+ \pi^- \pi^0 \pi^0} & 21.1\pm 1.7\pm 3.2 & 11.9\pm 0.9\pm 2.3 & \text{no entry} \\
\Gamma_{ee}^{J/\psi} \cdot B_{J/\psi \rightarrow \omega \eta \pi^+ \pi^- \pi^0 \pi^0} & 4.9\pm 2.1\pm 0.7 & 3.0\pm 1.3\pm 0.5 & 1.74\pm 0.20 \\
\Gamma_{ee}^{J/\psi} \cdot B_{J/\psi \rightarrow \omega \pi^+ \pi^- \pi^0 \pi^0} & 9.4\pm 2.3\pm 1.5 & 1.9\pm 0.5\pm 0.3 & \text{no entry} \\
\Gamma_{ee}^{J/\psi} \cdot B_{J/\psi \rightarrow \pi^+ \pi^- \pi^0 \pi^0 \pi^0} & 10.6\pm 1.6\pm 1.6 & 4.9\pm 0.8\pm 0.8 & \text{no entry} \\
\Gamma_{ee}^{\psi(2S)} \cdot B_{\psi(2S) \rightarrow \pi^+ \pi^- \pi^0 \pi^0 \pi^0} & 3.3\pm 2.3\pm 0.5 & 1.4\pm 1.0\pm 0.2 & \text{no entry} \\
\Gamma_{ee}^{\psi(2S)} \cdot B_{\psi(2S) \rightarrow \eta \pi^+ \pi^- \pi^0 \pi^0 \pi^0} & <3.0 \text{ at 90\% C.L.} & <3.5 \text{ at 90\% C.L.} & \text{no entry} \\
\Gamma_{ee}^{\psi(2S)} \cdot B_{\psi(2S) \rightarrow \omega \pi^+ \pi^- \pi^0 \pi^0 \pi^0} & <1.1 \text{ at 90\% C.L.} & <1.4 \text{ at 90\% C.L.} & <0.11 \text{ at 90\% C.L.} \\
\Gamma_{ee}^{\psi(2S)} \cdot B_{\psi(2S) \rightarrow \omega \pi^+ \pi^- \pi^0 \pi^0 \pi^0} & <1.6 \text{ at 90\% C.L.} & <0.8 \text{ at 90\% C.L.} & \text{no entry} \\
\Gamma_{ee}^{\psi(2S)} \cdot B_{\psi(2S) \rightarrow \pi^+ \pi^- \pi^0 \pi^0 \pi^0 \eta \pi^0 \pi^0} & <1.9 \text{ at 90\% C.L.} & <2.0 \text{ at 90\% C.L.} & \text{no entry}
\end{array}
\]
Conclusions

New measurement of the cross section $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ based on the entire Babar dataset measured in the range 0.62 ÷ 3.5 GeV. 1.3% systematic uncertainty near the maxima of $\omega(782)$ and $\phi(1020)$. The error on the leading order contribution to muon magnetic anomaly from $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0 \eta$ ($E < 2$ GeV) reduced by a factor $\approx 2$

First measurements of $e^+ e^- \rightarrow \pi^+ \pi^- 4\pi^0$ and $e^+ e^- \rightarrow \pi^+ \pi^- 3\pi^0 \eta$ cross sections

The $e^+ e^- \rightarrow \pi^+ \pi^- 4\pi^0$ cross section seems to be saturated by intermediate states: $\pi^+ \pi^- 3\pi^0 \eta$, $\omega 3\pi^0$, $(\rho \pi) 3\pi^0$ and possibly $\rho^+ \rho^- 2\pi^0$ intermediate states. All possible combinations for the $e^+ e^- \rightarrow 6\pi$ cross section have been measured by Babar iso-spin relations are not needed for the HVP calculation. New $J/\psi$ and $\psi(2S)$ decay modes

First measurements of $e^+ e^- \rightarrow 2(\pi^+ \pi^-)3\pi^0$ and $e^+ e^- \rightarrow 2(\pi^+ \pi^-)2\pi^0 \eta$ cross sections

The $e^+ e^- \rightarrow 2(\pi^+ \pi^-)3\pi^0$ cross section seems to be saturated by intermediate states: $2(\pi^+ \pi^-)\eta$, $\omega \pi^0 \eta$, $\rho^\pm \pi^\mp \pi^+ \pi^- 2\pi^0$, $\eta \pi^+ \pi^- 2\pi^0$, $\omega \pi^+ \pi^- 2\pi^0$. New $J/\psi$ and $\psi(2S)$ decay modes
BACKUP
\[ a_\mu = \frac{1}{2} (g - 2)_\mu \] precision test of SM

SM prediction for muons:
\[ a_\mu = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{hadr} \]

absolute value dominated by \( a_\mu^{QED} + a_\mu^{EW} \)

ERROR dominated by \( a_\mu^{hadr} \): not calculable perturbatively

\[ a_\mu^{hadr} : \text{LQCD or data-driven dispersive approach} \]

4.2 \( \sigma \) (WP/SM) or 1.5 \( \sigma \) (LQCD/SM)

relies on hadronic cross section measurements

Largest contribution to the integral from hadronic cross section at low energies

For \( \sqrt{s} \lesssim 2 \text{ GeV} \) finite number of final states contribute:

\[ \sigma(e^+e^- \rightarrow \text{hadrons}) \text{ can be obtained as sum of all exclusive cross sections} \]
The **BABAR** experiment

PEP-II asymmetric $e^+e^-$ collider operating at center of mass energies near the $\Upsilon(4S)$ (for most of the time)

$$\sqrt{s} = 10.58 \text{ GeV}/c^2$$

**General-purpose detector**

Asymmetric detector:

$$-0.9 < \cos \theta^* < 0.85$$

towards electron beam

excellent performance:

- vertexing
- tracking
- PID
- calorimeter

**DIRC** (PID)
- 144 quartz bars
- 11000 PMs

**1.5T solenoid**

**EMC**
- 6580 CsI(Tl) crystals

**Drift Chamber**
- 40 stereo layers

**Silicon Vertex Tracker**
- 5 layers, double sided strips

**Instrumented Flux Return**
- iron / RPCs / LSTs (muon / neutral hadrons)
Data samples

- Delivered: 531 fb⁻¹
- \( \Upsilon(4S) \): 432 fb⁻¹
- \( \Upsilon(3S) \): 30.2 fb⁻¹
- \( \Upsilon(2S) \): 14.4 fb⁻¹
- Off Peak & scans: 48 fb⁻¹

As of 2008/04/11 00:00
Light hadrons cross sections measured by \textbf{BABAR}
Substructures in $e^+ e^- \rightarrow \pi^+ \pi^- 4\pi^0$