



Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali del Sud



Statistical hadronization and coalescence models

Vincenzo Minissale

13/02/2023

Padova - “QCD challenges from pp to AA collisions”

Outline

Hadronization:

- Fragmentation
- Coalescence model
- Statistical Hadronization

Heavy hadrons in AA collisions:

- Λ_c , D spectra and ratio: RHIC and LHC

Heavy hadrons in small systems (pp @ 5.02 TeV):

- Λ_c/D^0
- Ξ_c/D^0 , Ω_c/D^0

Multicharm production

Heavy flavour Hadronization

Microscopic approach:

Fragmentation:

production from hard-scattering processes (PDF+pQCD).

Fragmentation functions: data parametrization, assumed “universal”

$$\sigma_{pp \rightarrow h} = PDF(x_a, Q^2) PDF(x_b, Q^2) \otimes \sigma_{ab \rightarrow q\bar{q}} \otimes D_{q \rightarrow h}(z, Q^2)$$

Parton shower: String fragmentation(Lund model – PYTHIA)

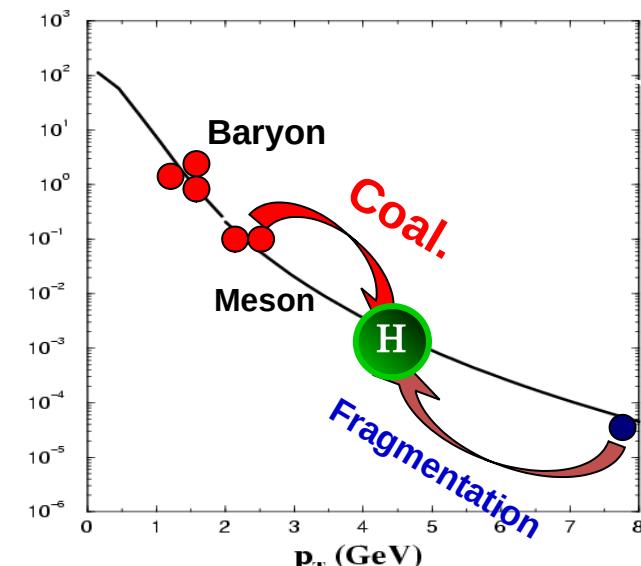
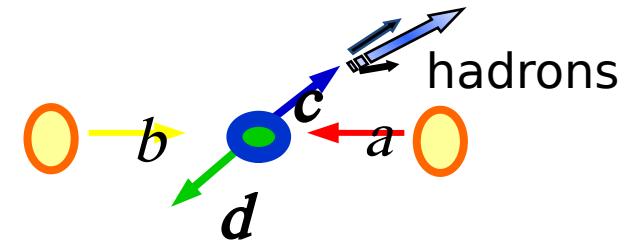
+colour reconnection(interaction from different scattering)

Cluster decay (HERWIG)

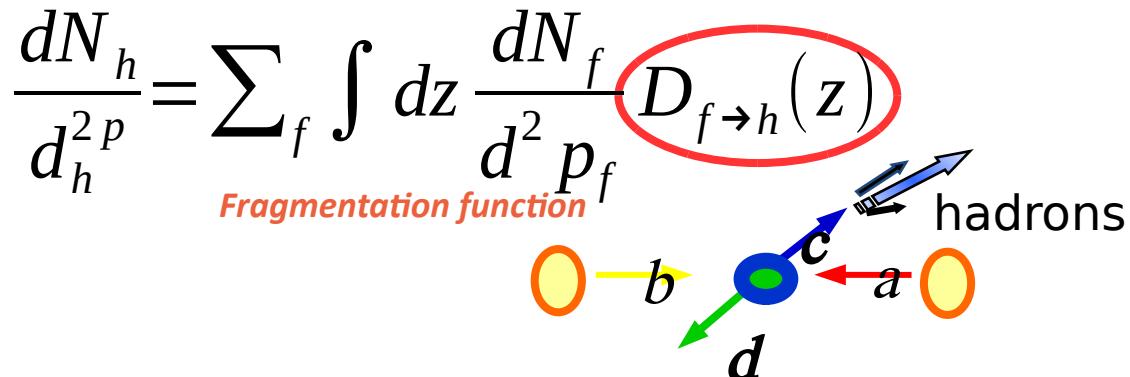
Coalescence: recombination of partons in QGP close in phase space

$$\frac{dN_{Hadron}}{d^2 p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3} f_q(x_i, p_i) f_w(x_1, \dots, x_n; p_1, \dots, p_n) \delta(p_T - \sum_i p_{iT})$$

Have described first AA observations in light sector for the enhanced baryon/meson ratio and elliptic flow splitting



Catania Model: Coalescence + Fragmentation



The distribution function is evaluated at the Fixed-Order plus Next-to-Leading-Log (FONLL)

M. Cacciari, P. Nason, R. Vogt, PRL 95 (2005) 122001

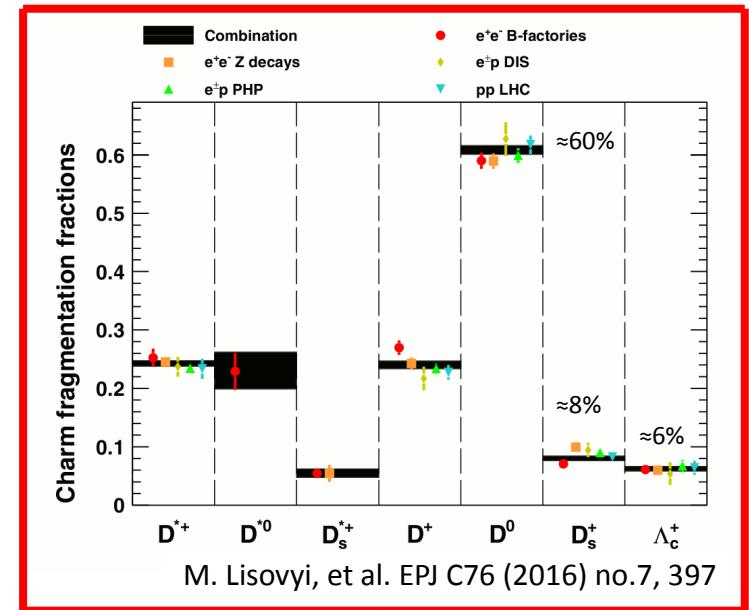
In AA: bulk+charm evolution with Relativistic Transport Boltzmann Equation

We use the Peterson fragmentation function

C. Peterson, D. Schalatter, I. Schmitt, P.M. Zerwas PRD 27 (1983) 105

$$D_{f \rightarrow h}(z) \propto \frac{1}{z \left[1 - \frac{1}{z} - \frac{\epsilon}{1-z} \right]^2}$$

Slightly modified to reproduce tail of the Λ_c/D^0



Charm Fragmentation Fraction ($c \rightarrow h$)
Measurement in $e^\pm p$, $e^+ e^-$ and old pp data

$$\left(\frac{\Lambda_c^+}{D^0} \right)_{p\bar{p}} \simeq 0.1 \quad \left(\frac{D_s^+}{D^0} \right)_{e^+ e^-} \simeq 0.13$$

Catania Model: Coalescence + Fragmentation

Statistical factor colour-spin-isospin

$$\frac{dN_{Hadron}}{d^2 p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3}$$

LIGHT

Thermal+flow for u,d,s ($p_T < 3$ GeV)

$$\frac{dN_{q,\bar{q}}}{d^2 p_T} \sim \exp\left(-\frac{\gamma_T - p_T \cdot \beta_T \mp \mu_q}{T}\right)$$

$$\beta(r) = \frac{r}{R} \beta_{max}$$

$$V = \pi R^2 \tau \cosh(y_z), R(\tau_f) = R_0(1 + \beta_{max} \tau_f)$$

$$\text{PbPb@5ATeV(0-10%)}: \tau_f = 8.4 \frac{fm}{c} \rightarrow V_{|y|<0.5} = 4500 fm^3$$

+quenched minijets for u,d,s ($p_T > 3$ GeV)

Parton Distribution function

$$f_q(x_i, p_i)$$

Hadron Wigner function

$$f_W(x_1, \dots, x_n; p_1, \dots, p_n) \delta(p_T - \sum_i p_{iT})$$

CHARM

In AA collisions charm distribution from the studies of R_{AA} and v_2 of D-meson to determine the Space Diffusion coefficient:

parton simulations solving relativistic Boltzmann transport equation

Coalescence simulation in a fireball with radial flow for light quarks \rightarrow dimension set by experimental constraints

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Wigner function – Wave function

$$\Phi_M^W(\mathbf{r}, \mathbf{q}) = \int d^3 r' e^{-i\mathbf{q}\cdot\mathbf{r}'} \phi_M(\mathbf{r} + \frac{\mathbf{r}'}{2}) \phi_M^*(\mathbf{r} - \frac{\mathbf{r}'}{2})$$

$\phi_M(\mathbf{r})$ meson wave function

Wigner function width fixed by root-mean-square charge radius from quark model

C.-W. Hwang, EPJ C23, 585 (2002)

C. Albertus et al., NPA 740, 333 (2004)

$$\langle r^2 \rangle_{ch} = \frac{3}{2} \frac{m_2^2 Q_1 + m_1^2 Q_2}{(m_1 + m_2)^2} \sigma_{r1}^2 + \frac{3}{2} \frac{m_3^2 (Q_1 + Q_2) + (m_1 + m_2)^2 Q_3}{(m_1 + m_2 + m_3)^2} \sigma_{r2}^2$$

$$\sigma_{ri} = 1/\sqrt(\mu_i \omega) \quad \mu_1 = \frac{m_1 m_2}{m_1 + m_2} \quad \mu_2 = \frac{(m_1 + m_2)m_3}{m_1 + m_2 + m_3}$$

Assuming gaussian wave function

$$f_H(\dots) = \prod_{i=1}^{N_q-1} A_w \exp\left(-\frac{x_{ri}^2}{\sigma_{ri}^2} - p_{ri}^2 \sigma_{ri}^2\right)$$

only one width coming from $\phi_M(\mathbf{r})$, constraint $\sigma_r \sigma_p = 1$

Meson	$\langle r^2 \rangle_{ch}$	σ_{p1}	σ_{p2}
$D^+ = [c\bar{d}]$	0.184	0.282	—
$D_s^+ = [\bar{s}c]$	0.083	0.404	—
Baryon	$\langle r^2 \rangle_{ch}$	σ_{p1}	σ_{p2}
$\Lambda_c^+ = [udc]$	0.15	0.251	0.424
$\Xi_c^+ = [usc]$	0.2	0.242	0.406
$\Omega_c^0 = [ssc]$	-0.12	0.337	0.53

Catania Model: Coalescence + Fragmentation

Statistical factor colour-spin-isospin

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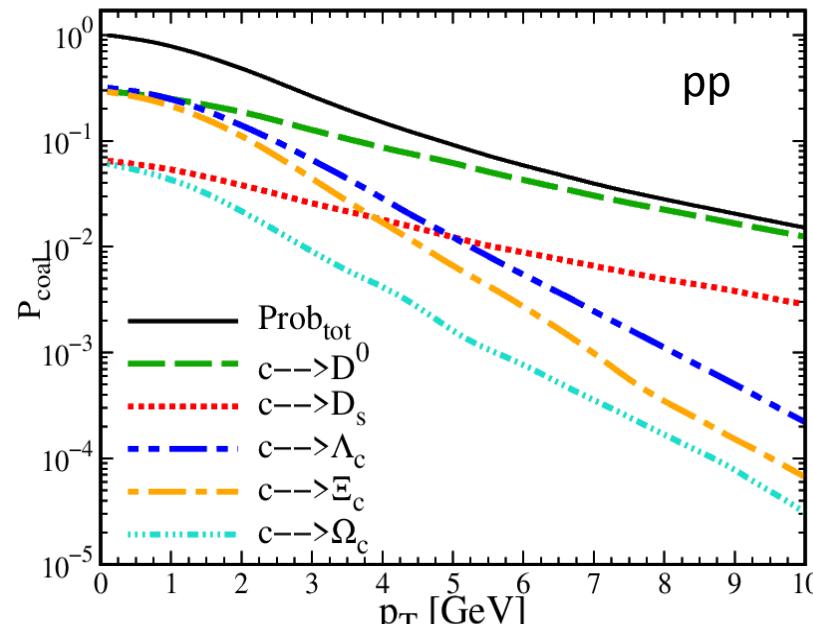
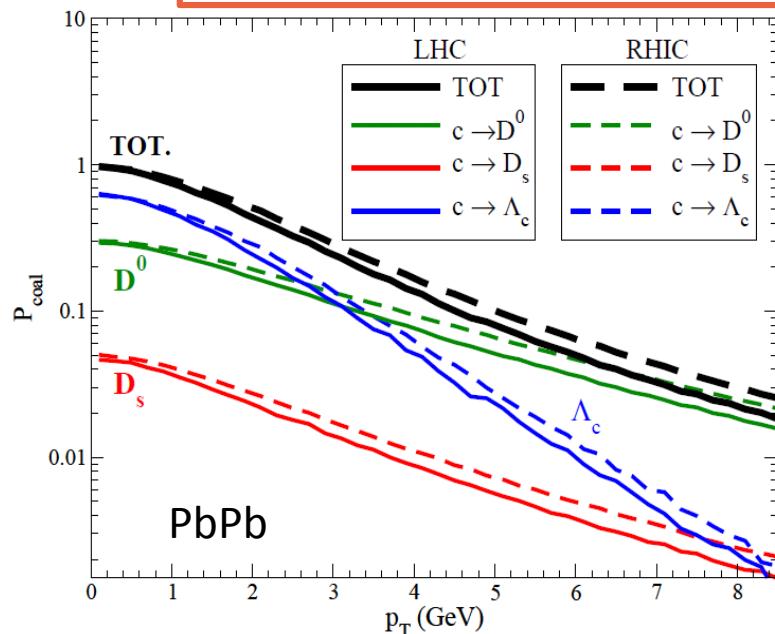
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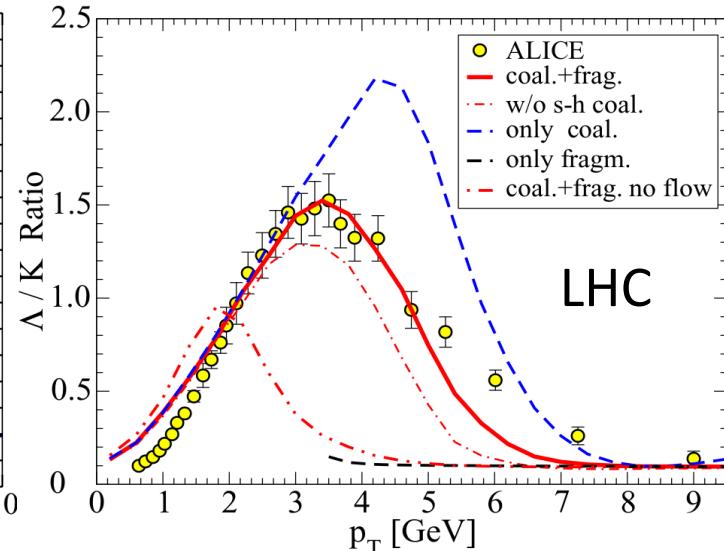
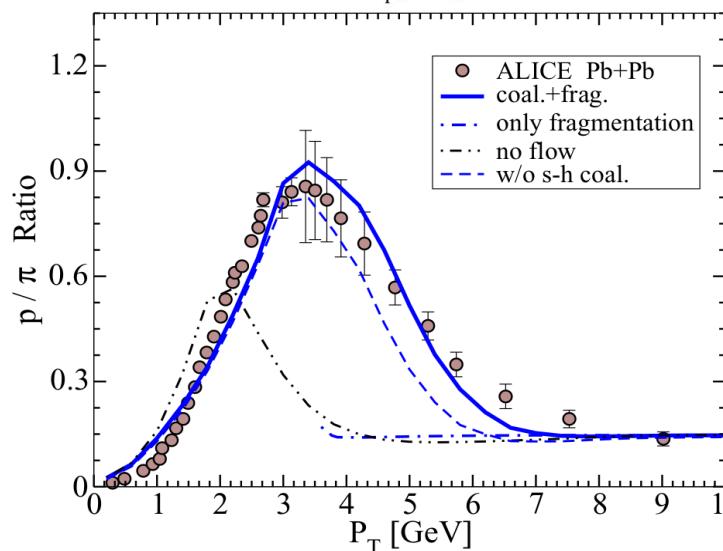
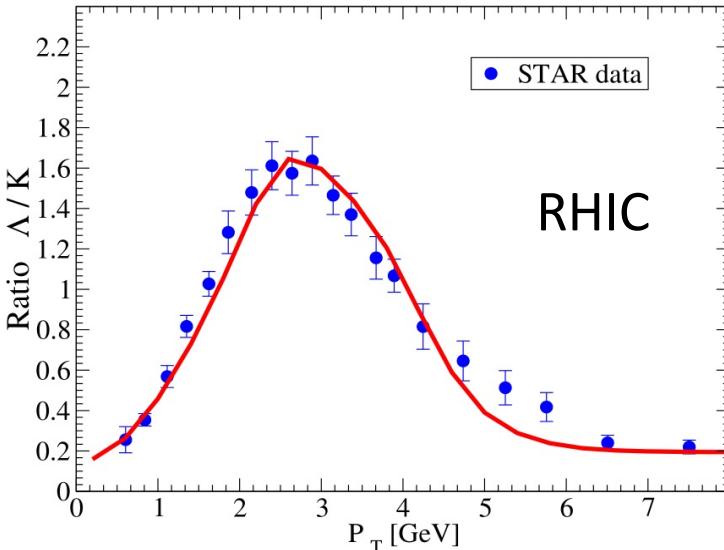
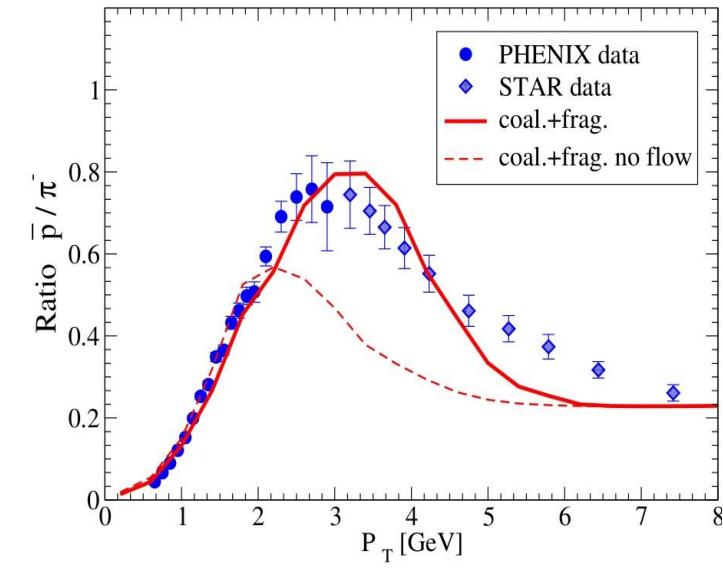
$$f_W(x_1, \dots, x_n; p_1, \dots, p_n) \delta(p_T - \sum_i p_{iT})$$

- Normalization of $f_W(\dots)$ requiring that $P_{coal}=1$ at $p=0$
- The charm that does not coalesce undergo fragmentation



Light baryon to meson ratio at RHIC & LHC

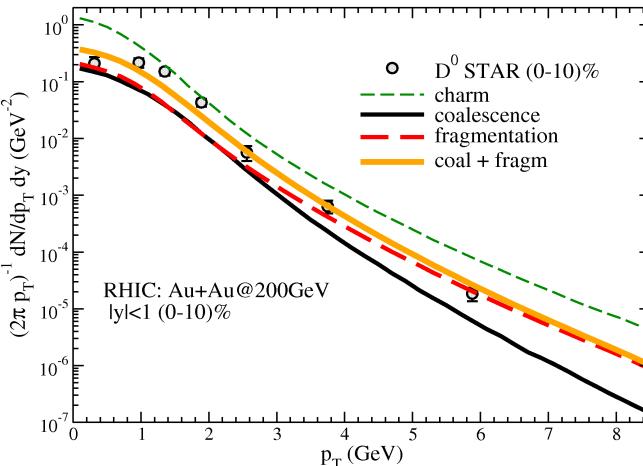
Minissale, Scardina, Greco, Phys.Rev. C 92 (2015) 5,054904



- coalescence naturally predict a baryon/meson enhancement in the region $p_T \simeq 2\text{--}4\text{GeV}$ with respect to pp collisions
- Lack of baryon yield in the region $p_T \simeq 5\text{--}7\text{GeV}$

wave function widths σ_p of baryon and mesons are the same at RHIC and LHC!

Data from ALICE Coll. JHEP 09 (2012) 112



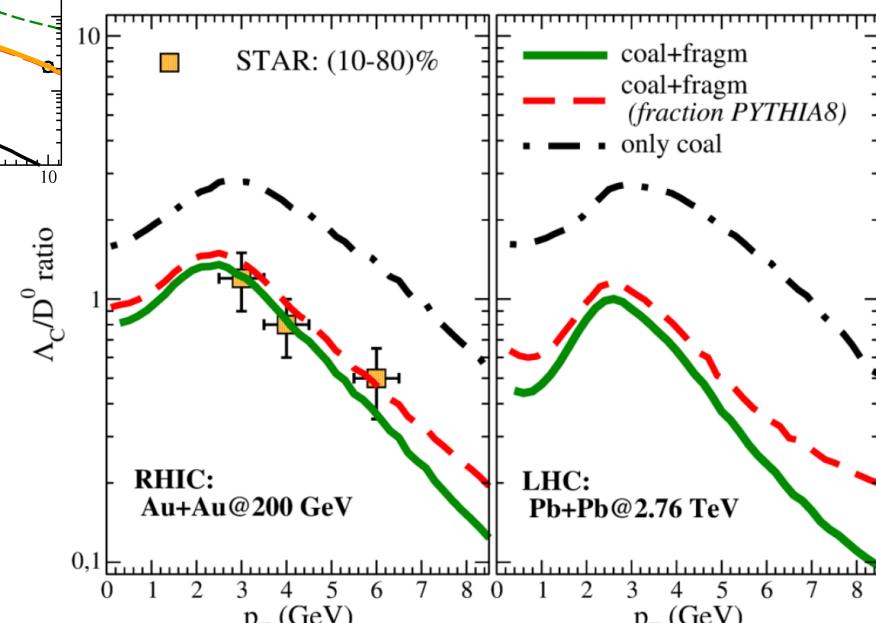
Only Coalescence ratio is similar at both energies.

Fragmentation ~ 0.1 at both energies.

the **combined ratio is different** because the coalescence over fragmentation ratio at LHC is smaller than at RHIC

Therefore at LHC the larger contribution in particle production from fragmentation leads to a final ratio that is smaller than at RHIC.

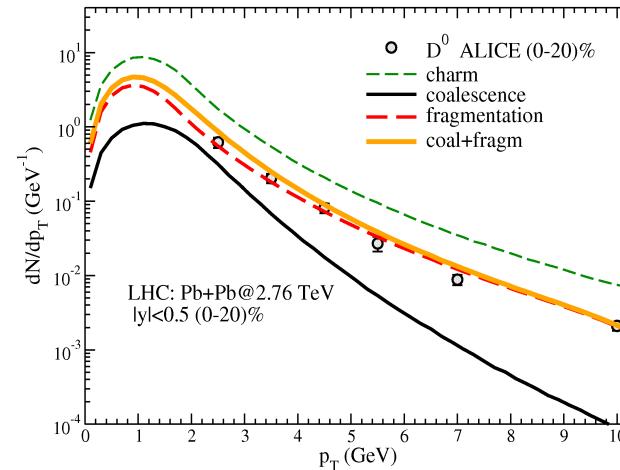
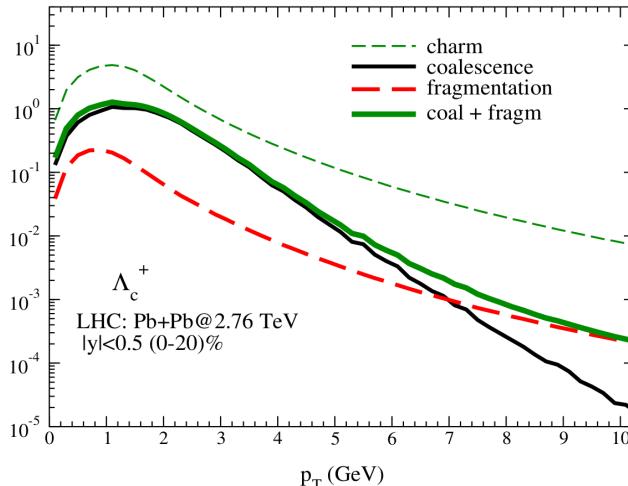
Coalescence lower at LHC than at RHIC



STAR Coll., Phys.Rev.Lett. 124 (2020) 17, 172301

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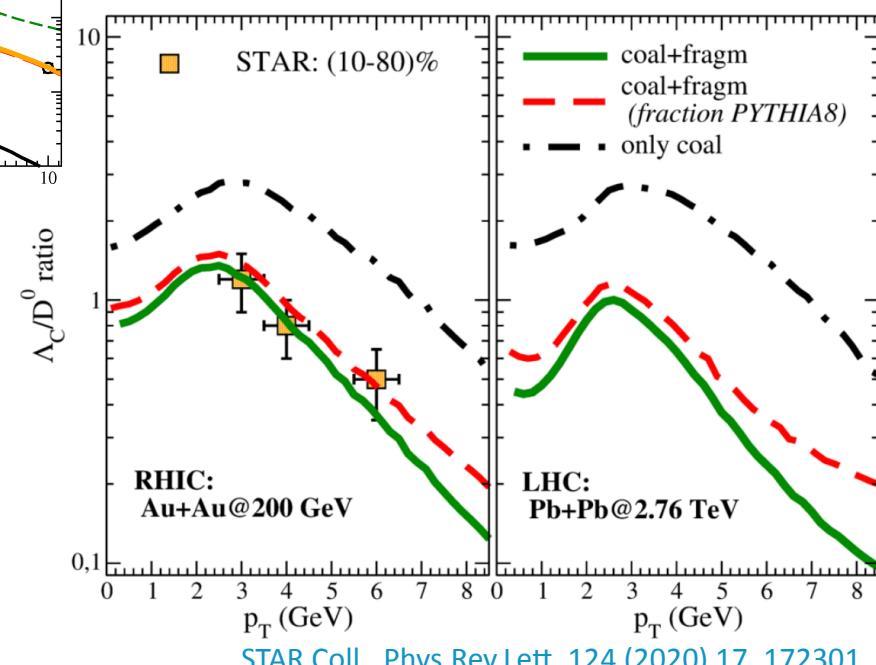
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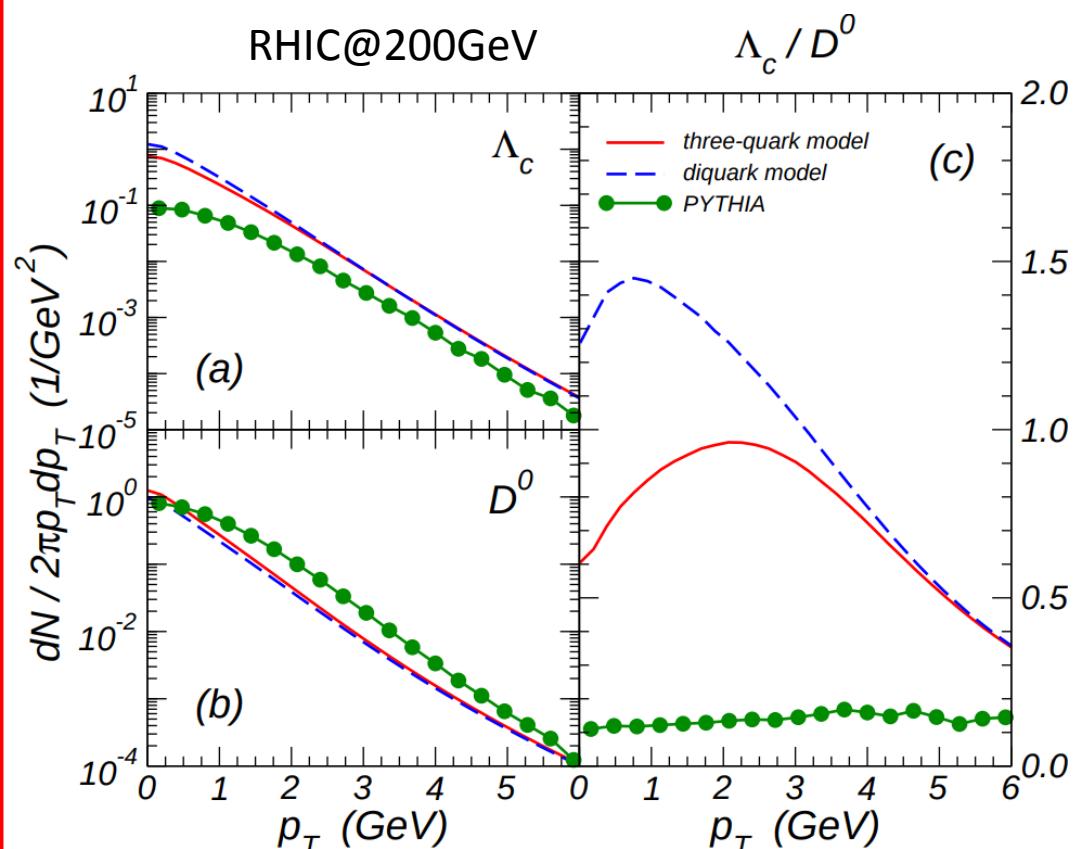
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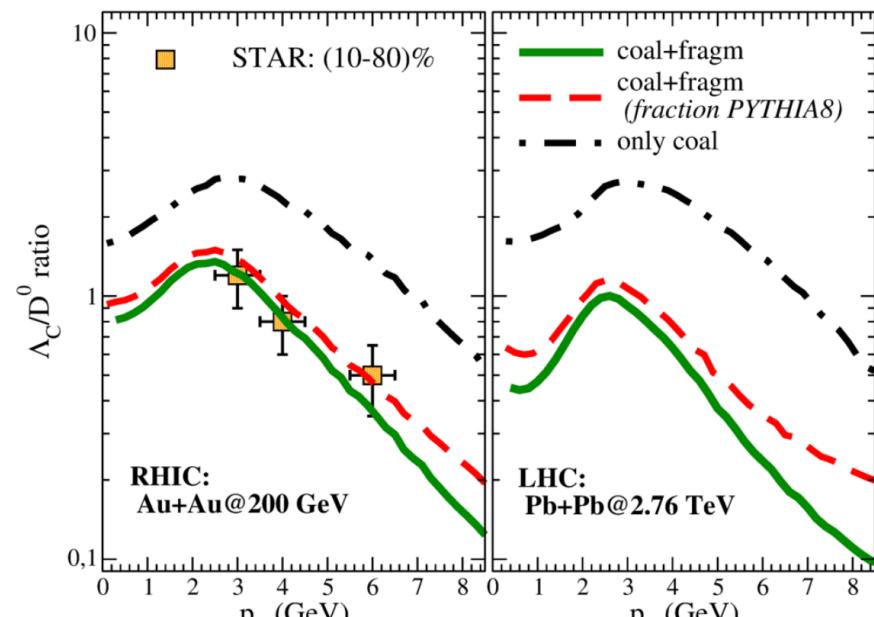
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STAR Coll., Phys.Rev.Lett. 124 (2020) 17, 172301



First prediction about baryon over meson ratio in charm sector by Oh,Ko,Lee,Yasui Phys.Rev.C 79 (2009) 044905



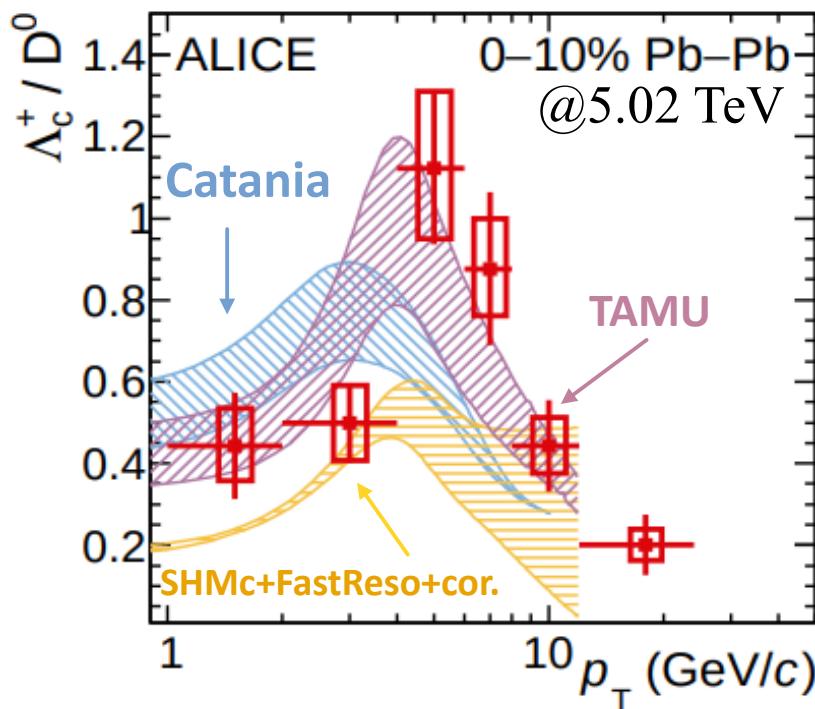
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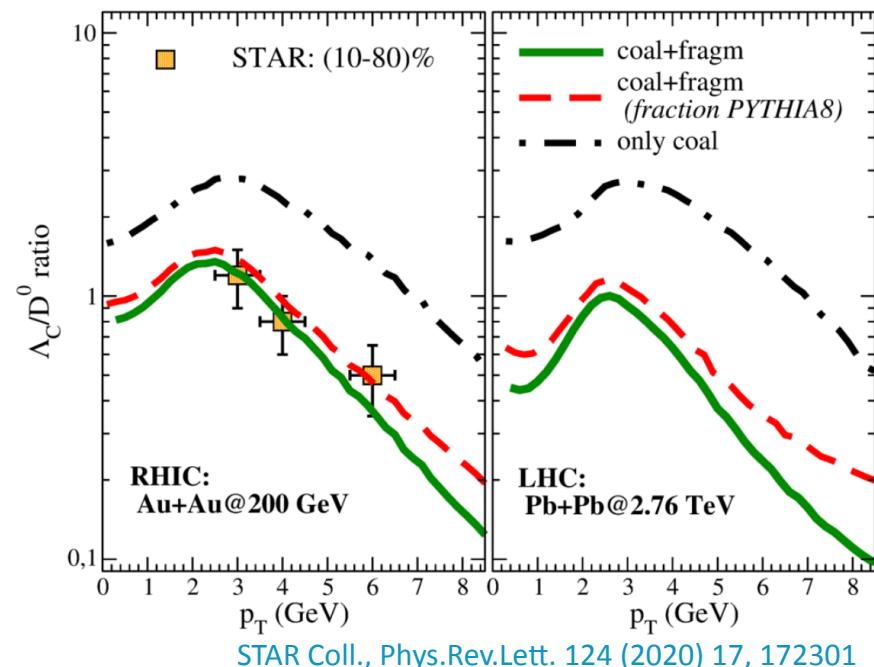
Results for 0-10% in PbPb @5.02TeV:

Consistent with the trend shown at RHIC and LHC @2.76TeV

Available data at low p_T → differences recombination vs SHM



ALICE Coll. arXiv:2112.08156v1



STAR Coll., Phys.Rev.Lett. 124 (2020) 17, 172301

S. Plumari, V. Minissale et al., Eur. Phys. J. C78 no. 4, (2018) 348

Baryons in Resonance Recombination Model (RRM)

The 3-body hadronization process in RRM are conducted in 2 steps

STEP 1

quark-1 and quark-2 recombine into a diquark,

$$q_1(p_1) + q_2(p_2) \rightarrow dq(p_{12})$$

The diquark spectrum in analogy to meson formation

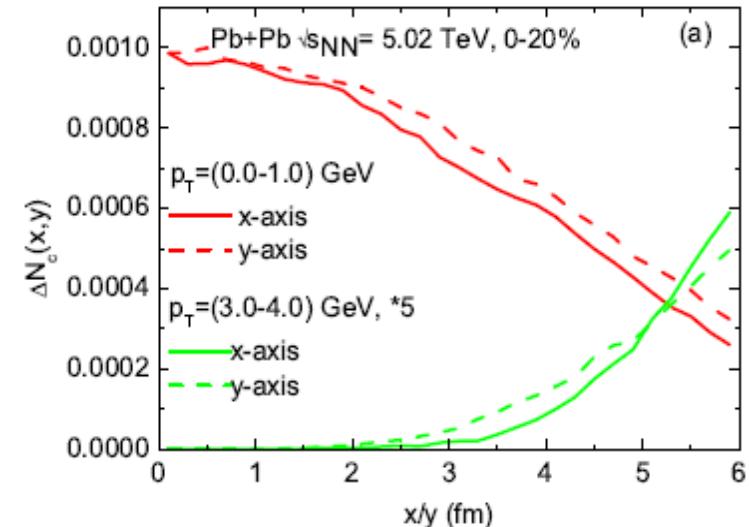
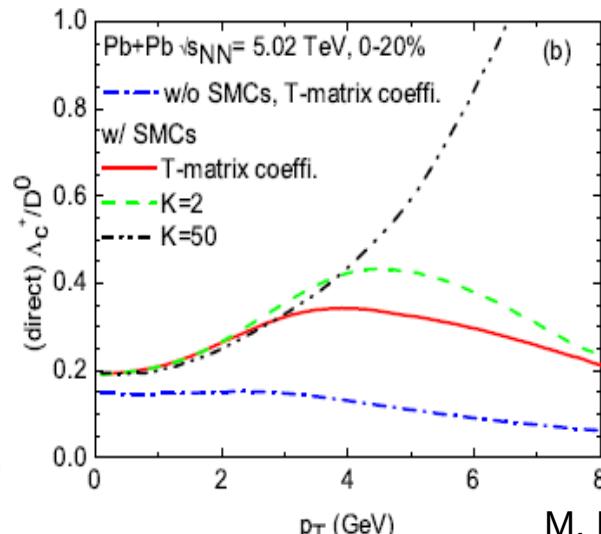
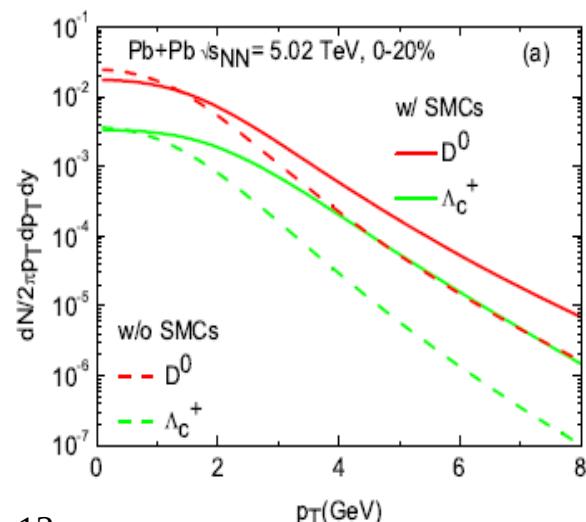
STEP 2

the diquark recombines with quark-3 into a baryon

$$dq_1(p_{12}) + q_3(p_3) \rightarrow B$$

The baryon spectrum in analogy to meson formation

$$f_B(\vec{x}, \vec{p}) = \frac{\gamma_B}{\Gamma_B} \int \frac{d^3 \vec{p}_1 d^3 \vec{p}_2 d^3 \vec{p}_3}{(2\pi)^6} \frac{\gamma_{dq}}{\Gamma_{dq}} f_1(\vec{x}, \vec{p}_1) f_2(\vec{x}, \vec{p}_2) \\ \times f_3(\vec{x}, \vec{p}_3) \sigma_{dq}(s_{12}) v_{\text{rel}}^{12} \sigma_B(s) v_{\text{rel}}^{dq3} \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$



Space-momentum correlation

p_T=0-1GeV: c quarks preferentially populate the inner regions of the fireball

p_T=3-4GeV: c quarks populate the outer regions of the fireball

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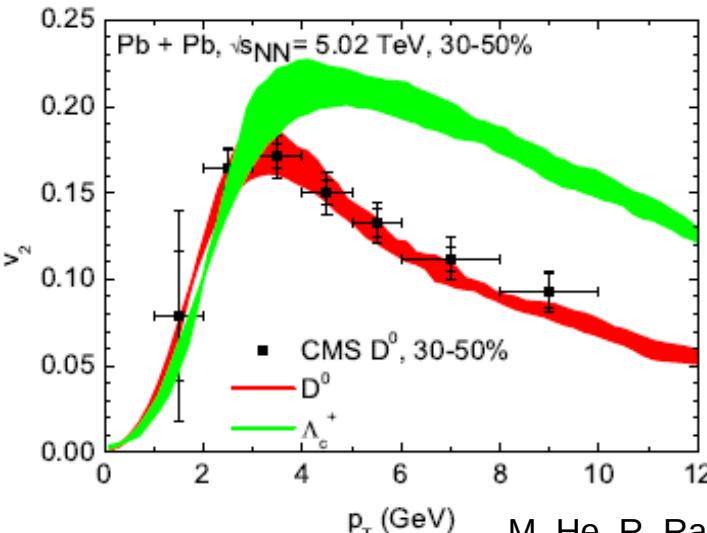
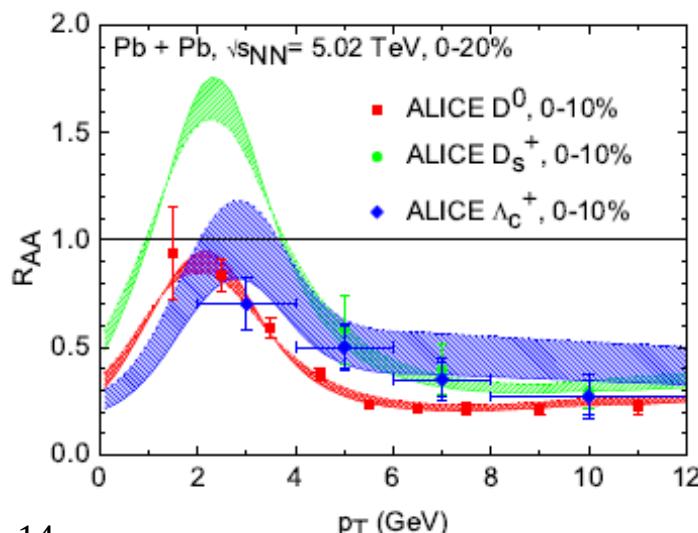
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HF hadro-chemistry improved by employing a large set of “missing” HF baryon states not listed by PDG, but predicted by the relativistic-quark model

PDG: $5\Lambda_c, 3\Sigma_c, 8\Xi_c, 2\Omega_c$

RQM: $18\Lambda_c, 42\Sigma_c, 62\Xi_c, 34\Omega_c$

Coalescence : LBT

S. Cao, K. Sun, S. Li, S. Liu, W. Xing, G. Qin, and C. Ko, PLB 807 (2020) 135561.

F. Liu, W. Xing, X. Wu, G. Qin, S. Cao, and X. Wang, EPJC 82 (2022) 4, 350.

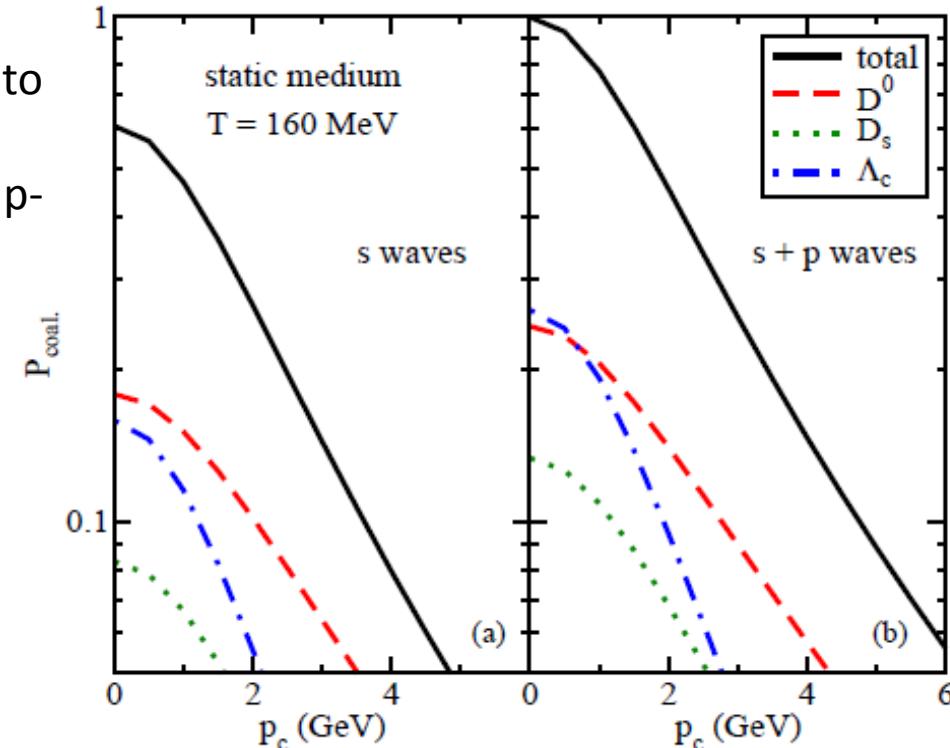
$$f_h(p'_h) = \int \left[\prod_i dp_i f_i(p_i) \right] W(\{p_i\}) \delta(p'_h - \sum_i p_i)$$

- The quark wave functions in the meson is assumed to be those of a harmonic oscillator potential
- The Wigner functions for mesons are in the s and p-wave states

$$W_s = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 k^2},$$

$$W_p = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} \frac{2}{3} \sigma^2 k^2 e^{-\sigma^2 k^2}$$

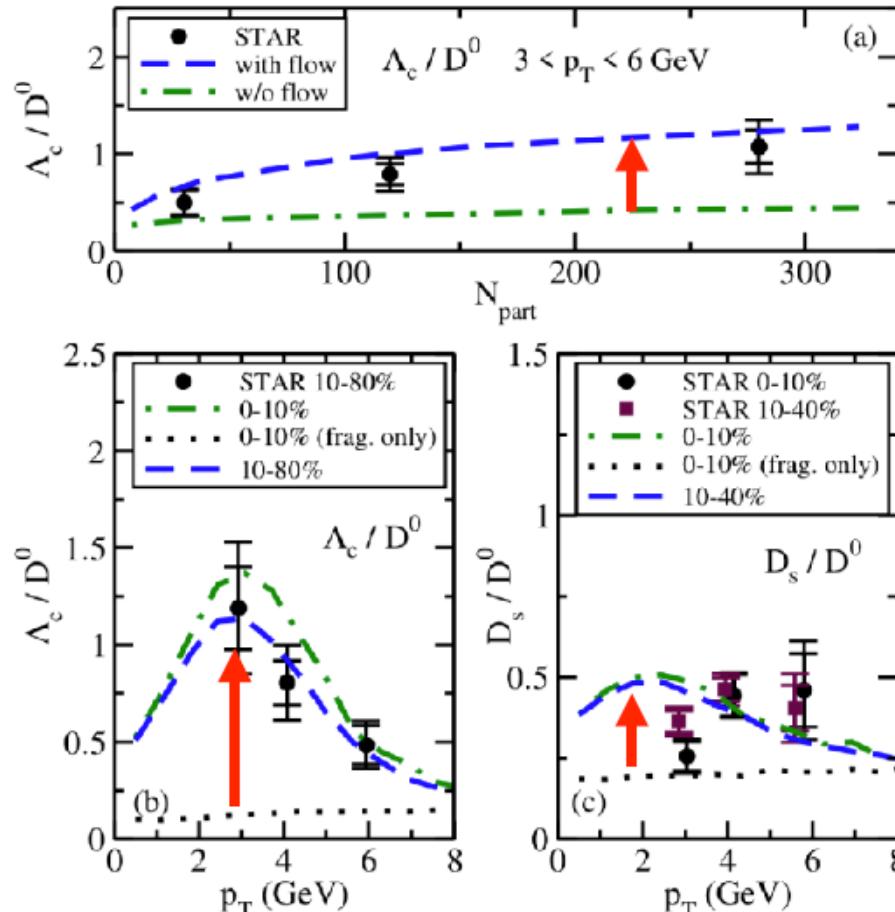
The oscillator frequency is fixed to impose that the total coalescence probability for zero-momentum charm quark is equal to 1 when s and p states are included.



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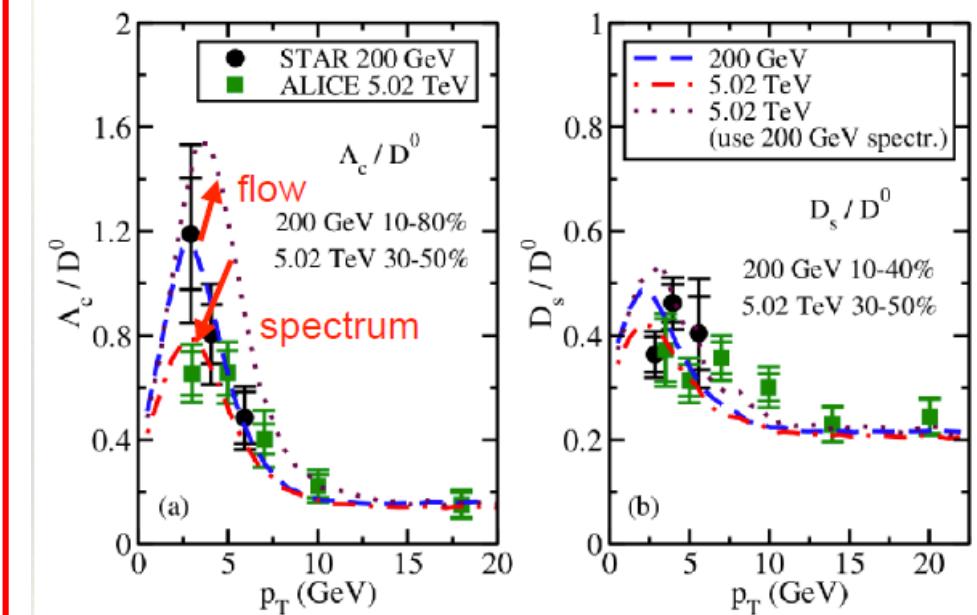
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Stronger QGP flow boost on heavier hadrons
 \Rightarrow increasing Λ_c/D^0 ratio with N_{part}

harder initial charm spectra at LHC reduces the Λ_c/D^0 ratio



Statistical Thermal Model (SHM) + charm(SHMc)

grand canonical partition function

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln [1 \pm \exp(-(E_i - \mu_i)/T)]$$

chemical potential \leftrightarrow
conservation quantum numbers
(N_B , N_s , N_c)

Equilibrium + hadron-resonance gas + freeze-out temperature.

Production depends on hadron masses and degeneracy, and on system properties.

Charm hadrons according to thermal weights

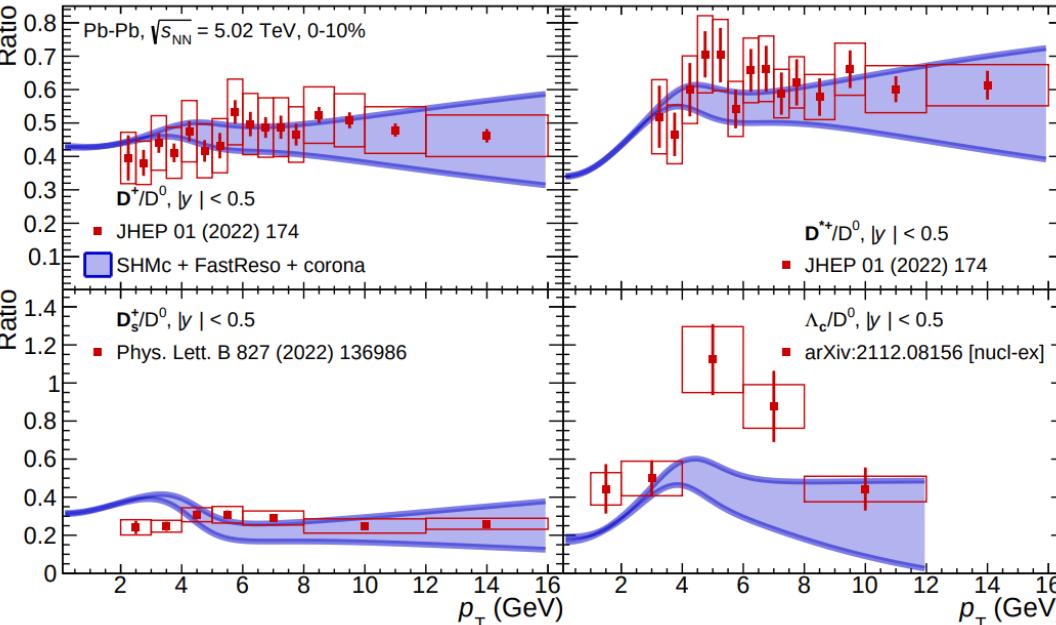
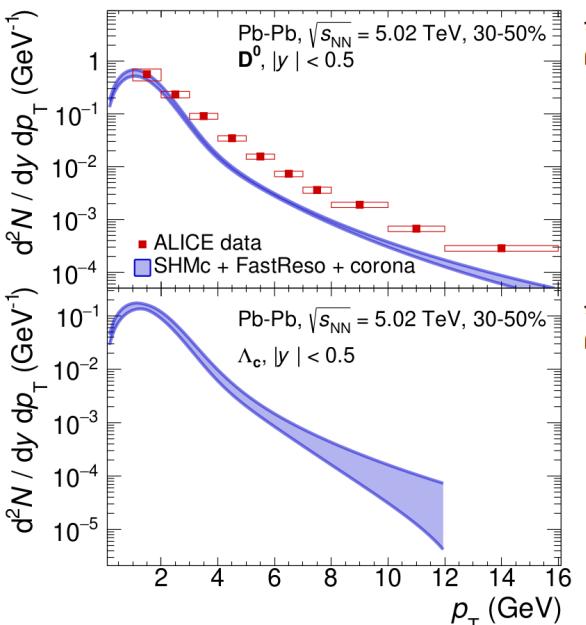
the total charm content of the fireball is fixed by the measured open charm cross section.

$$N_{c\bar{c}}^{dir} = \frac{1}{2} g_c V \left(\sum_i n_{D_i}^{th} + n_{\Lambda_{ci}}^{th} \right) + g_c^2 V \left(\sum_i n_{\psi_i}^{th} + n_{\chi_i}^{th} \right)$$

pQCD production $N_{c, \text{anti-}c} = 9.6 \rightarrow g_c = 30.1$ (charm fugacity)

Andronic et al.,
JHEP 07 (2021) 035

SHMc yields+blast wave
 $\rightarrow p_T$ spectra



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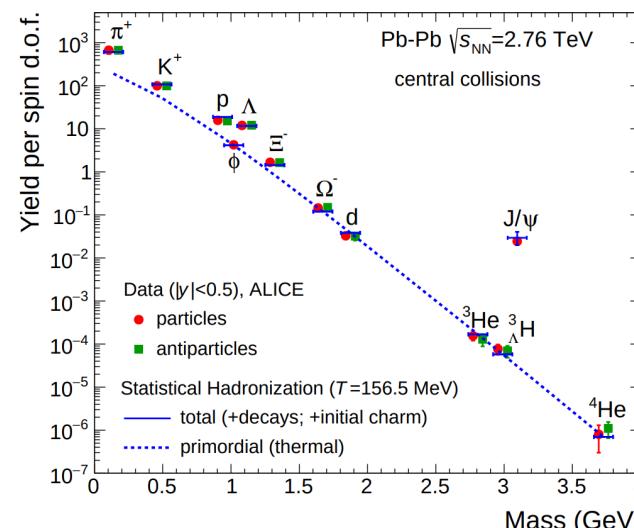
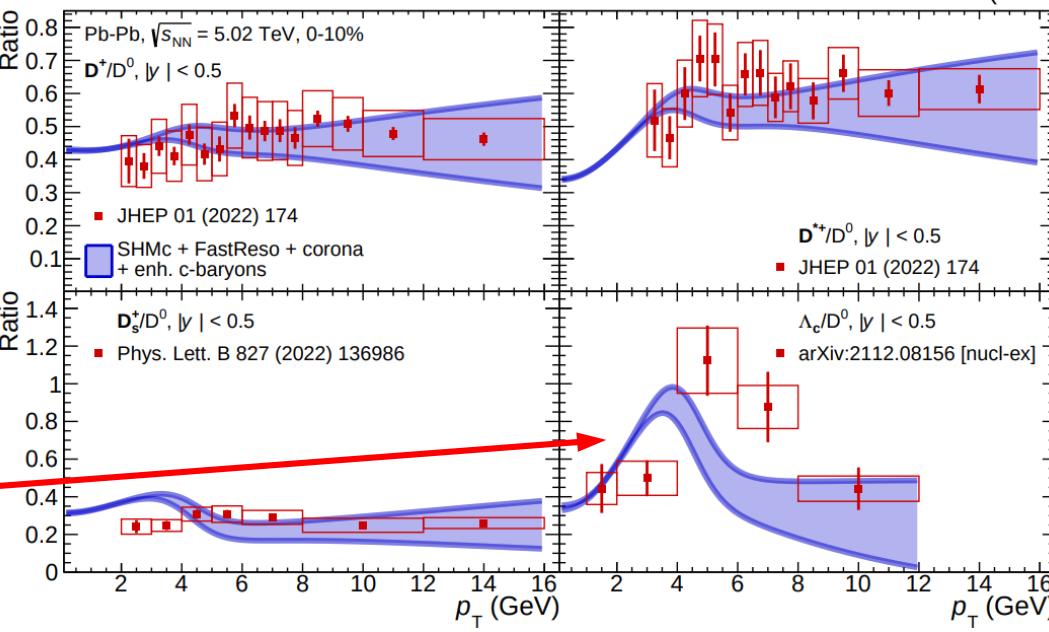
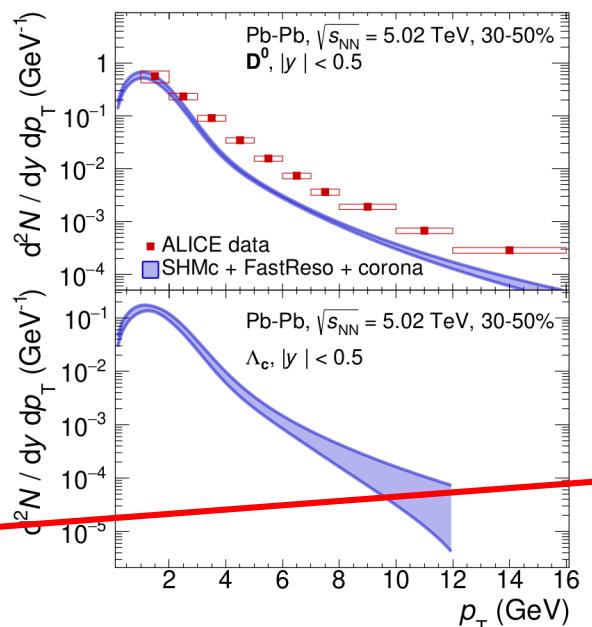
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With enhanced set
of charmed baryons



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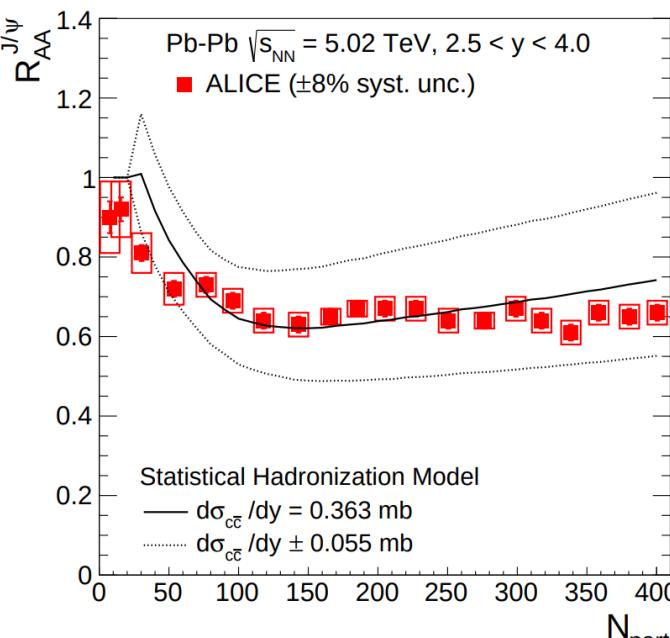
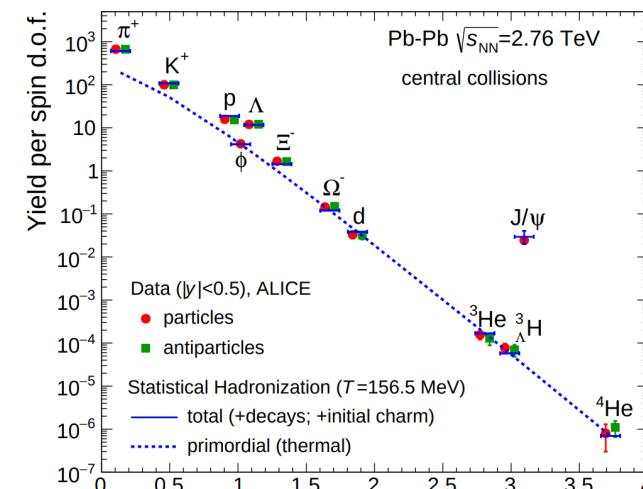
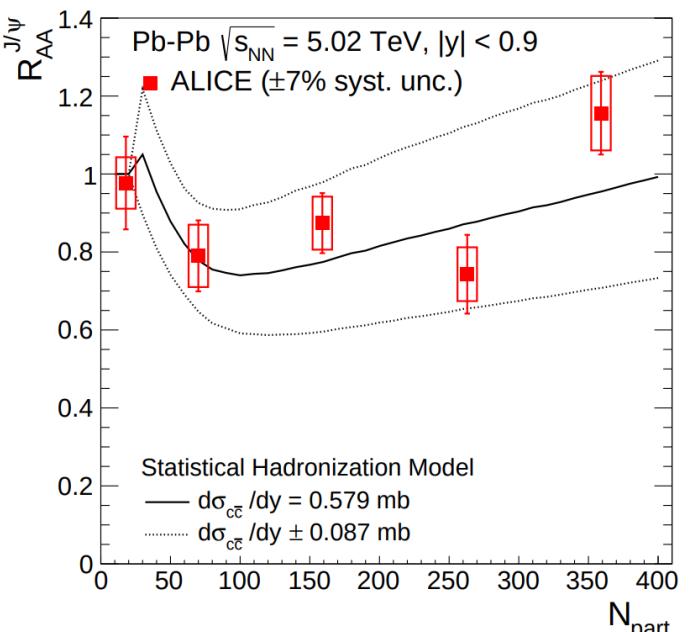
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Charmonium:

Regeneration mechanism in
agreement with data vs N_{part}

uncertainties arise from charm
cross sections, nPDF, corona
thickness



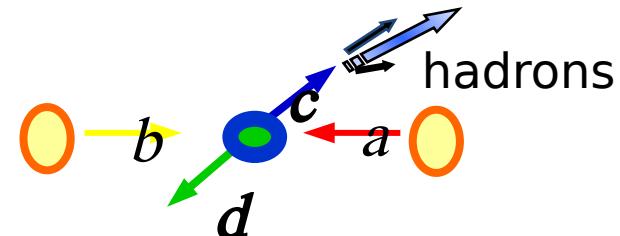
Heavy flavour Hadronization

Fragmentation: production from hard-scattering processes

(PDF+pQCD).

Fragmentation functions: data parametrization, assumed “universal”

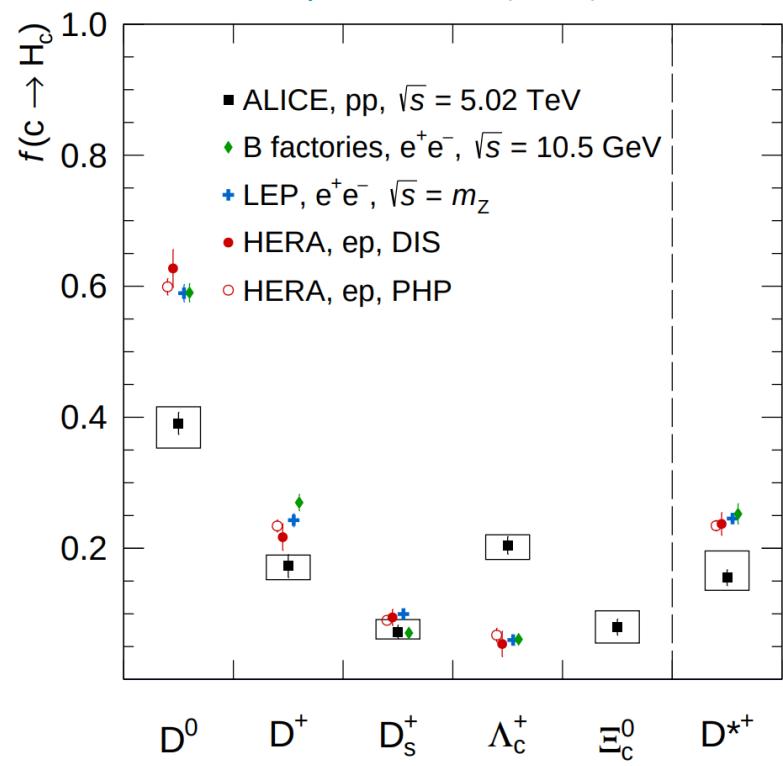
$$\sigma_{pp \rightarrow h} = PDF(x_a, Q^2) PDF(x_b, Q^2) \otimes \sigma_{aa \rightarrow q\bar{q}} \otimes D_{q \rightarrow h}(z, Q^2)$$



ALICE, Phys. Rev. D 105 (2022) 1, L011103

Things get more complicated after experimental evidence with ALICE in pp@5TeV:

Fragmentation fractions ($c \rightarrow h$) depends on collision system



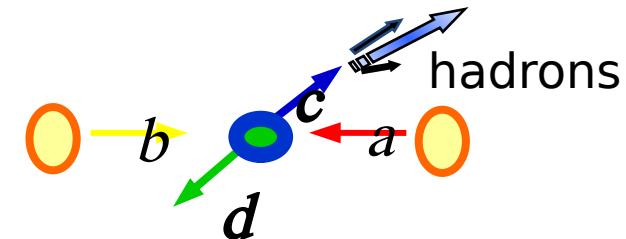
Heavy flavour Hadronization

Fragmentation: production from hard-scattering processes

(PDF+pQCD).

Fragmentation functions: data parametrization, assumed “universal”

$$\sigma_{pp \rightarrow h} = PDF(x_a, Q^2) PDF(x_b, Q^2) \otimes \sigma_{aa \rightarrow q\bar{q}} \otimes D_{q \rightarrow h}(z, Q^2)$$



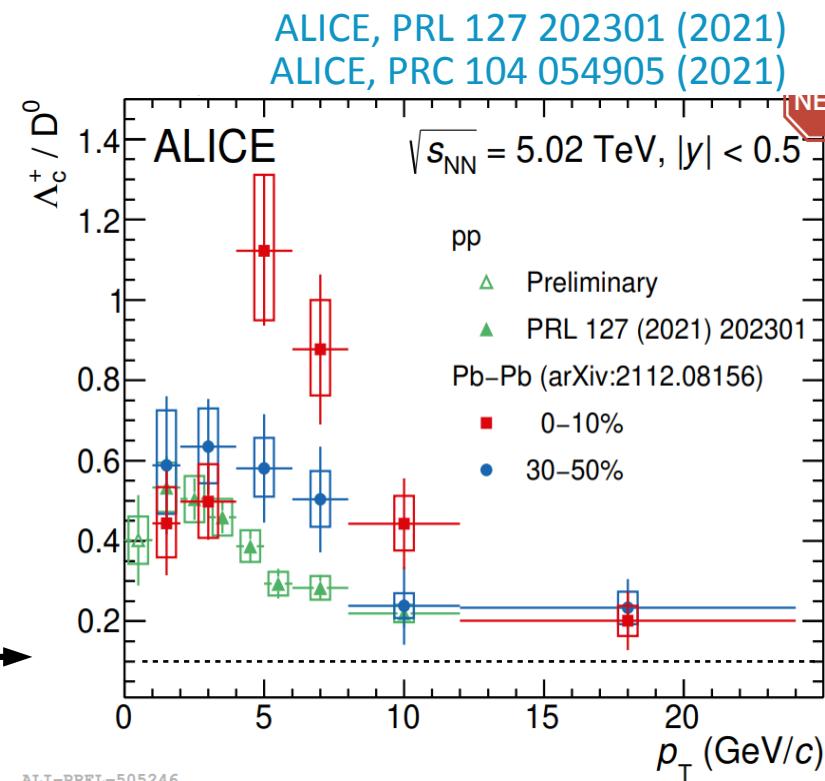
Things get more complicated after experimental evidence with ALICE in pp@5TeV:

Fragmentation fractions ($c \rightarrow h$) depends on collision system

Baryon/meson ratio is underestimated.

Peculiar p_T dependence, rise and fall

$$\left(\frac{\Lambda_c^+}{D^0} \right)_{e^+ e^-} \simeq 0.1$$

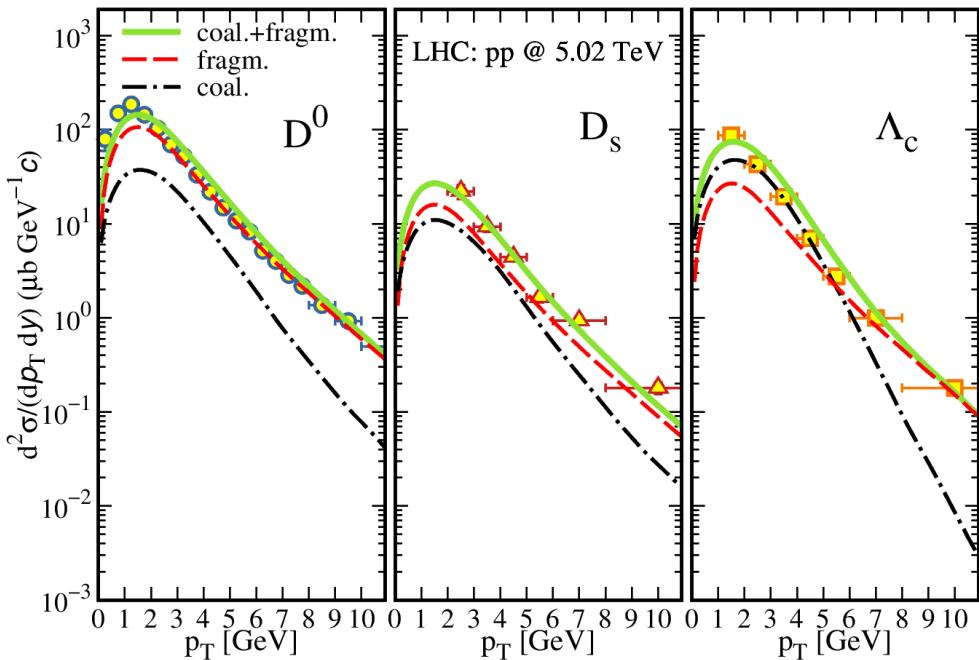


Small systems: Coalescence in pp?

What if:

- Assuming QGP formation also in pp?
- What coalescence+fragmentation predicts in this case?

V. Minissale, S. Plumari, V. Greco, Physics Letters B 821 (2021) 136622



Data from:

S. Acharya et al. (ALICE), Eur. Phys. J. C 79, 388 (2019)

ALICE Coll., Phys. Rev. Lett. 127 (2021) 20, 202301 - Phys. Rev. C 104 (2021) 5, 054905

If we assume in $p+p$ @ 5 TeV a medium similar to the one simulated in hydro:

p+p @ 5 TeV

- $\tau_{pp}=2 \text{ fm}/c$
- $\beta_0=0.4$
- $R=2.5 \text{ fm}$
- $V \sim 30 \text{ fm}^3$

LIGHT

■ Thermal Distribution ($p_T < 2 \text{ GeV}$)

$$\frac{dN_q}{d^2 r_T d^2 p_T} = \frac{g_g \tau m_T}{(2\pi)^3} \exp\left(-\frac{\gamma_T(m_T - p_T \cdot \beta_T)}{T}\right)$$

■ Minijet Distribution ($p_T > 2 \text{ GeV}$)
NO QUENCHING

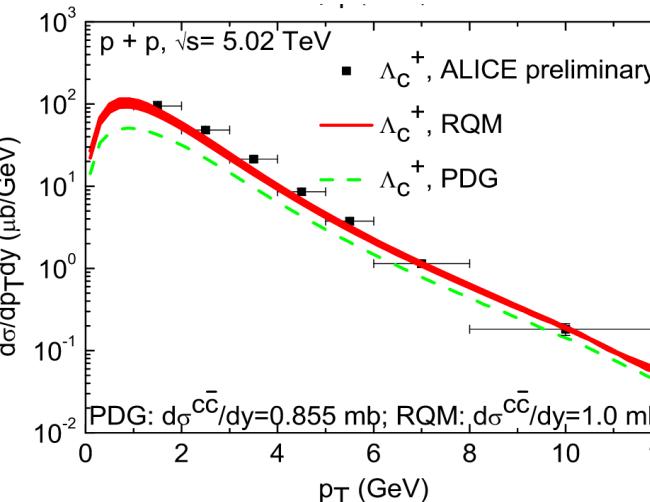
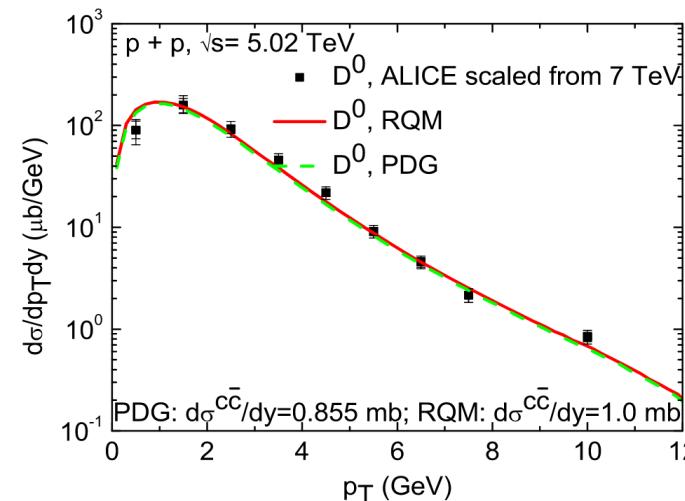
CHARM

FONLL Distribution

wave function widths σ_p of baryon and mesons kept the same from AA to pp

Small systems: Coalescence in pp?

He-Rapp, Phys.Lett.B 795 (2019) 117-121



Thermal yields to compute the charmed hadron-chemistry

Transverse-momentum spectra calculated with fragmentation of c-quark spectrum from FONLL

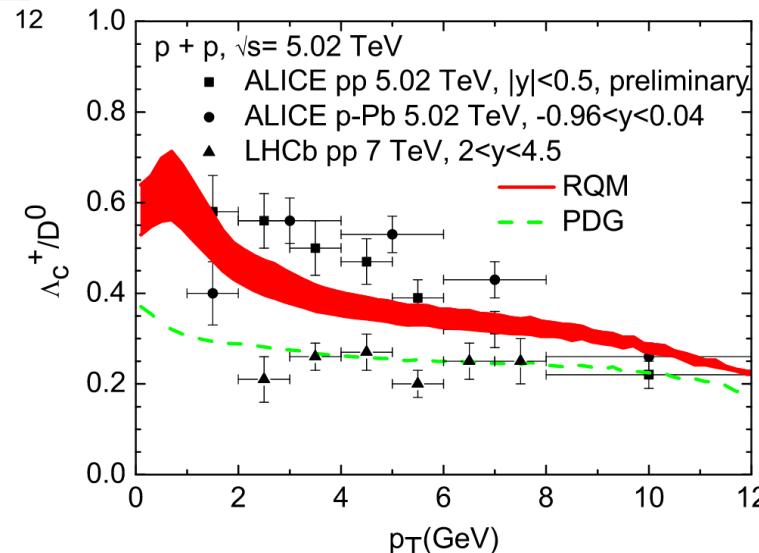
Statistical hadronization for charm hadrons:
- chemical equilibrium with different charm-hadron species

$$n_i = \frac{d_i}{2\pi^2} m_i^2 T_H K_2 \left(\frac{m_i}{T_H} \right)$$

-Increased set of baryons for the Λ_c production:

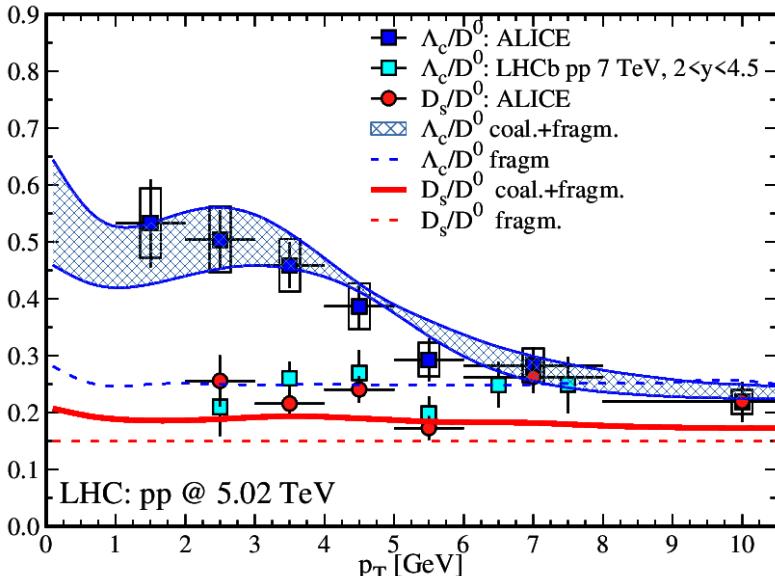
PDG: 5 Λ_c , 3 Σ_c , 8 Ξ_c , 2 Ω_c

RQM: 18 Λ_c , 42 Σ_c , 62 Ξ_c , 34 Ω_c



Small systems: Coalescence in pp?

V. Minissale, S. Plumari, V. Greco, Physics Letters B 821 (2021) 136622



Error band correspond to $\langle r^2 \rangle$ uncertainty in quark model

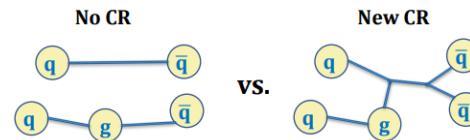
Other models:

He-Rapp, Phys.Lett.B 795 (2019) 117-121: Increase ≈ 2 to

Λ_c production: SHM with resonance not present in PDG

PYTHIA8 + color reconnection

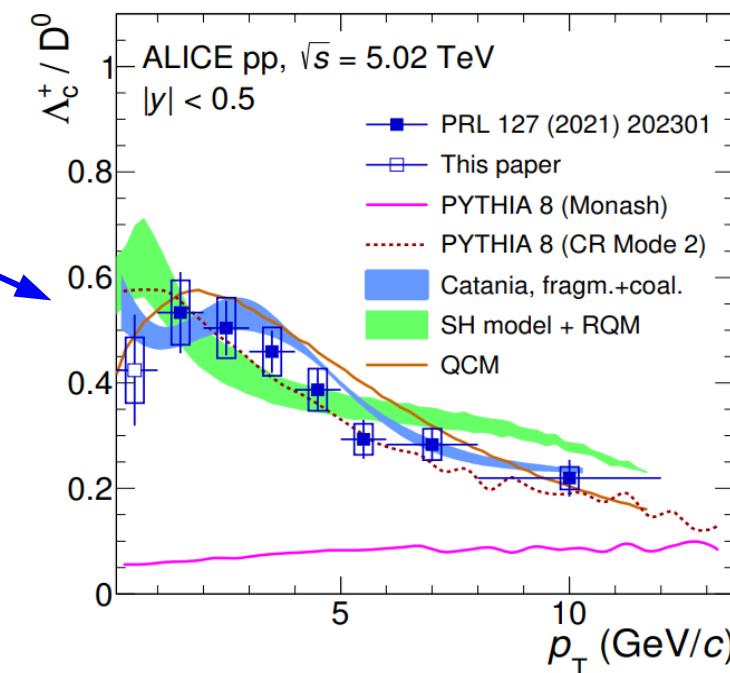
CR with SU(3) weights and string length minimization



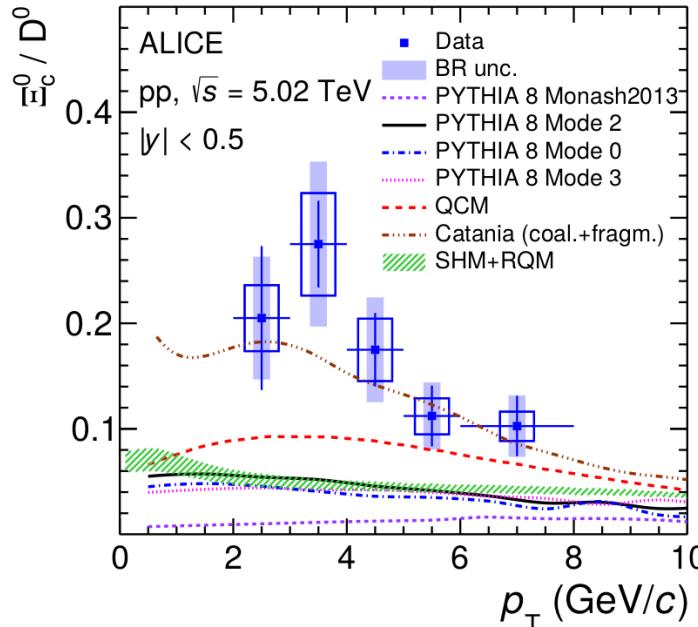
Reduction of rise-and-fall behaviour in Λ_c / D^0 ratio:

- Confronting with AA: Coal. contribution smaller w.r.t. Fragm.
- FONLL distribution flatter w/o evolution through QGP
- Volume size effect

ALICE, Phys.Rev.Lett. 127 (2021) 20, 202301
ALICE,CERN-EP-2022-261, arXiv:2211.14032 (sub. to PRC)



Small systems: Coalescence in pp?



Assuming additional PDG resonances with

$J=3/2$ and decay to Ω_c^0 additional to $\Omega_c^0(2770)$

$\Omega_c^0(3000), \Omega_c^0(3005), \Omega_c^0(3065), \Omega_c^0(3090), \Omega_c^0(3120)$

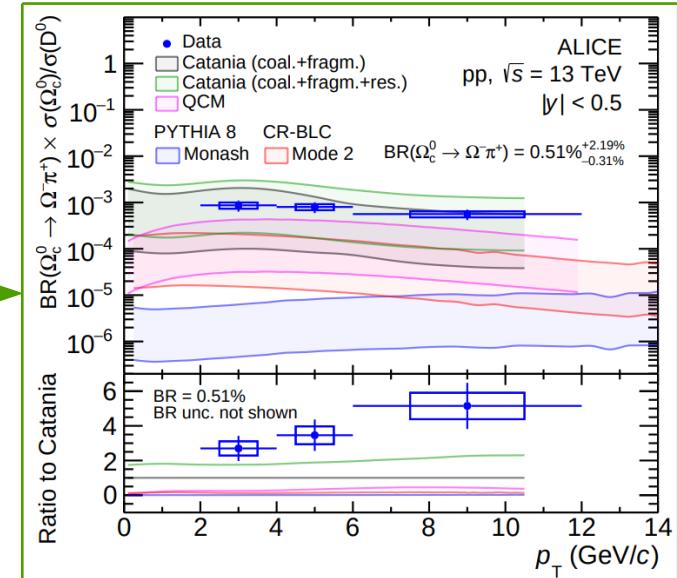
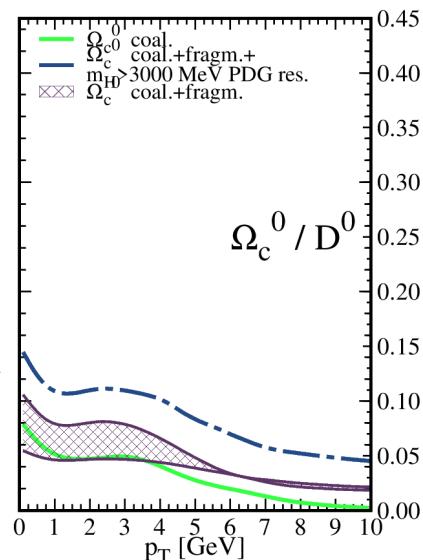
supply an idea of how these states may affect
the ratio

Error band correspond to $\langle r^2 \rangle$ uncertainty in
quark model

New measurements of heavy hadrons at ALICE:

- Ξ_c^0 / D^0 ratio, same order of Λ_c^+ / D^0 : coalescence gives enhancement
- very large Ω_c^0 / D^0 ratio, our model does not get the big enhancement

Uncertainties bands
coming from the
Branching Ratio error



ALICE Coll. JHEP 10 (2021) 159
ALICE Coll. arXiv:2205.13993

V. Minissale, S. Plumari, V. Greco, Physics Letters B 821 (2021) 136622

Statistical Thermal Model (SHM) + charm(SHMc)

grand canonical partition function

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln [1 \pm \exp(-(E_i - \mu_i)/T)]$$

chemical potential \leftrightarrow conservation quantum numbers (N_B , N_s , N_c)

Equilibrium + hadron-resonance gas + freeze-out temperature.

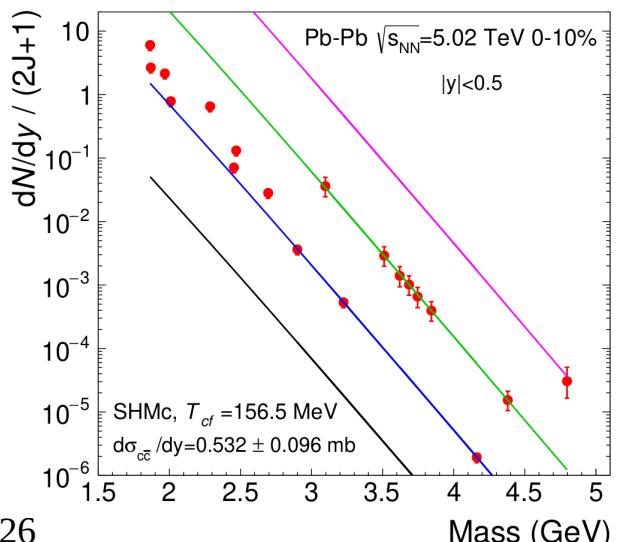
Production depends on hadron masses and degeneracy, and on system properties.

charm hadrons according to thermal weights

pQCD production $N_{c,\text{anti-}c} = 9.6 \rightarrow g_c = 30.1$ (charm fugacity)

the total charm content of the fireball is fixed by the measured open charm cross section.

$$N_{c\bar{c}}^{\text{dir}} = \frac{1}{2} g_c V \left(\sum_i n_{D_i}^{\text{th}} + n_{\Lambda_{ci}}^{\text{th}} \right) + g_c^2 V \left(\sum_i n_{\psi_i}^{\text{th}} + n_{\chi_i}^{\text{th}} \right)$$



MULTICCHARMED HADRONS

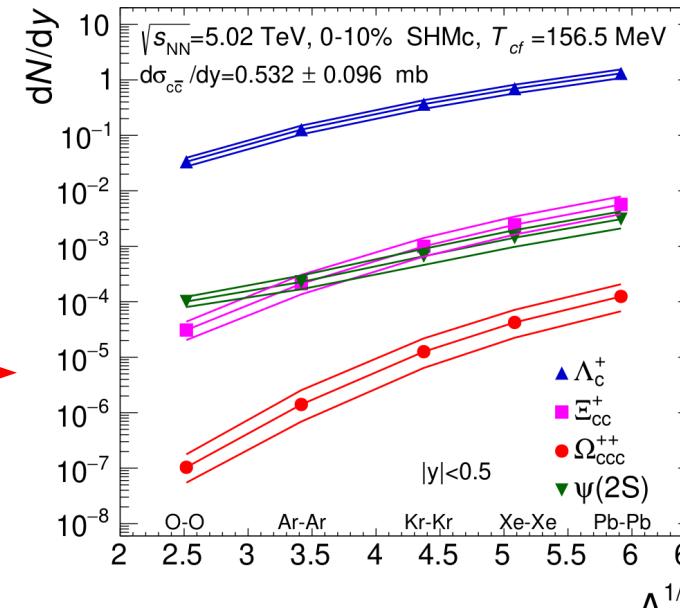
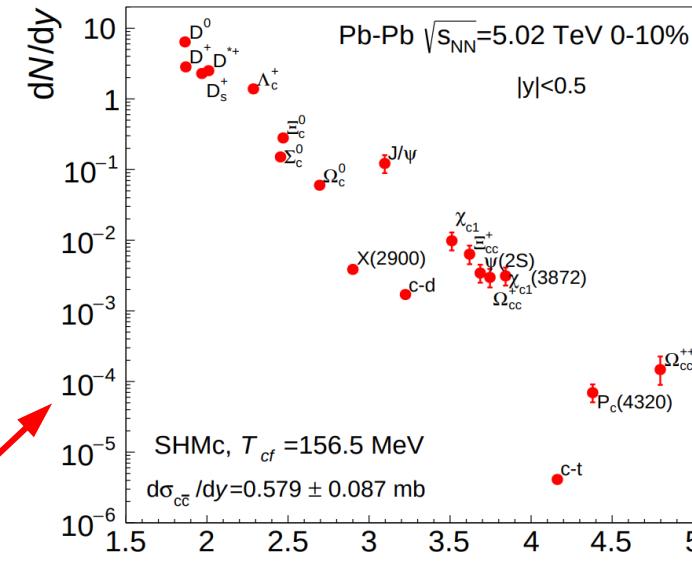
degeneracy-normalized particle yields are straight line for fixed charm quark number

$$dN/dy \propto M^{3/2} \exp(-M/T_{cf})$$

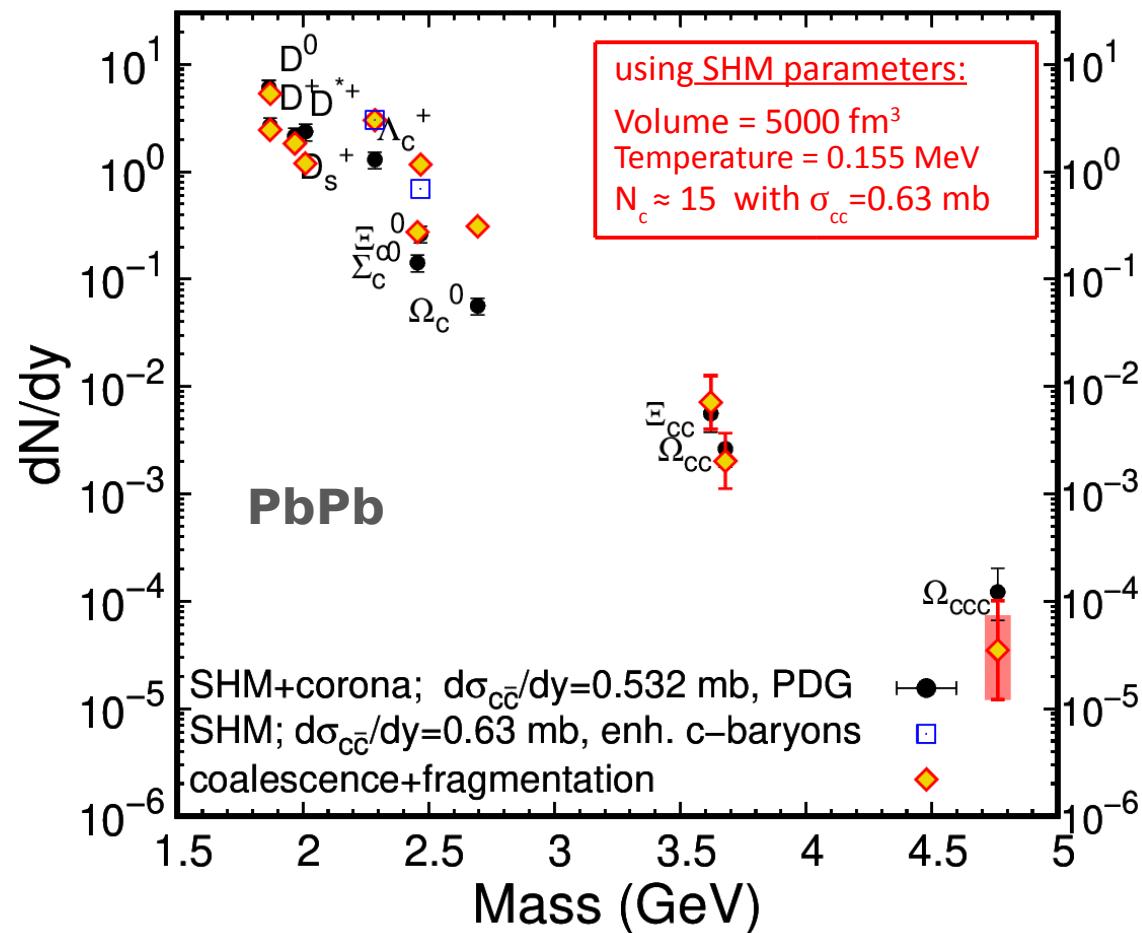
Evolution with AA systems

$$\frac{dN}{dy}(h_i) = \frac{dN^{PbPb}}{dy}(h_i) \left(\frac{A}{208} \right)^{(\alpha+3)/3} \frac{f_{can}(\alpha, A)}{f_{can}(\alpha, Pb)}$$

α number of constituent charm quark



Yields in PbPb from coalescence vs SHM

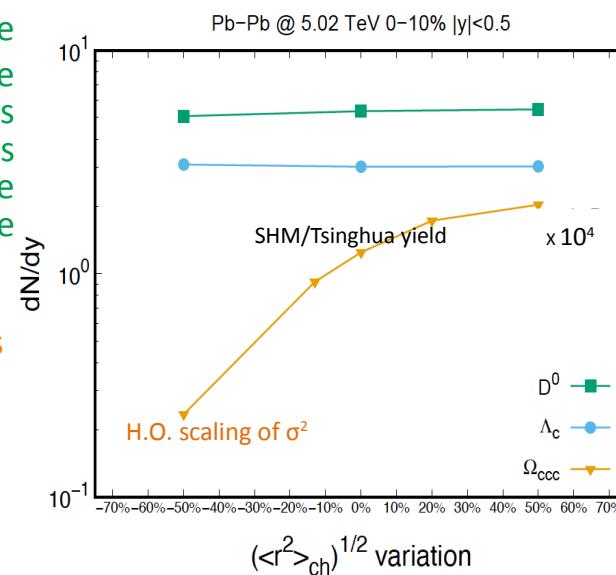


$\Sigma_c^0, \Xi_c^0, \Omega_c^0$, widths from quark model
 Ξ_{cc}, Ω_{cc} widths obtained rescaling with harm. oscillator
 $\sigma_{ri} = \frac{1}{\sqrt{\mu_i \omega}}$ $\mu_1 = \frac{m_1 m_2}{m_1 + m_2}; \mu_2 = \frac{(m_1 + m_2)m_3}{m_1 + m_2 + m_3}$

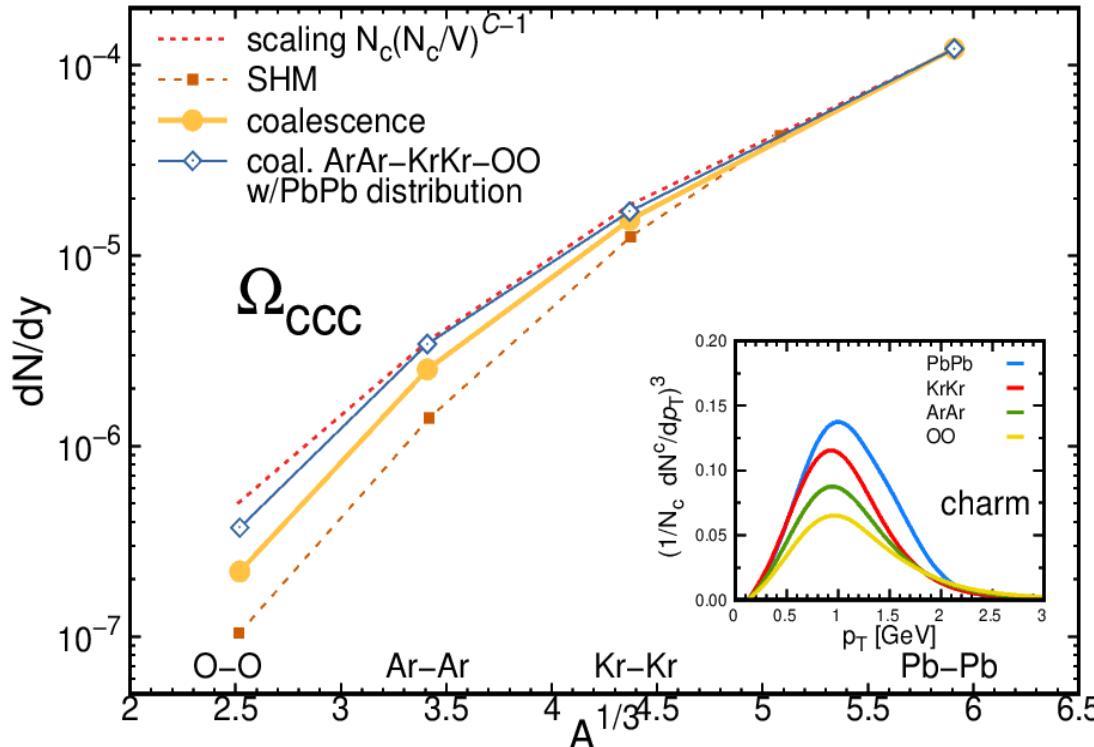
- upper limit: charm thermal distribution Ω_{ccc}
- lower limit: PbPb distribution with widths rescaled as standard Harm. Oscill. (ω from Ω_c^0)
- box upper limit: $\sigma_r \cdot \sigma_p \approx 1.5 + \langle r \rangle = 0.5$ fm
 as in Tsinghua PLB746 (2015) [Solution of Schrödinger eq. under $V(r)$]

D^0 and Λ_c determine the majority of the yield, the radius variation is compensated by the constraint on the charm hadronization

A ± 50% in the radius of Ω_{ccc} induces a change in the yield by about 1 order of magnitude



Yields scaling with A



Scaling of SHM (for $A>40$)

$$\frac{dN^{AA}}{dy}(h^i) = \frac{dN^{PbPb}}{dy}(h^i) \left(\frac{A}{208}\right)^{(\alpha+3)/3} \frac{f_{can}(\alpha, A)}{f_{can}(\alpha, Pb)}$$

For coalescence, in an homogeneous density background in equilibrium at fixed T , discarding flow and wave functions effects the expected scaling is:

$$V \left(\frac{N_c}{V}\right)^c = N_c \left(\frac{N_c}{V}\right)^{C-1}$$

with $N_c \propto A^{4/3}$ and $V \propto A$
 \rightarrow the scaling corresponds to $\frac{dN}{dy} \propto A^{\frac{C+3}{3}}$

like in SHM w/o canonical suppression

- If the p_T -distribution does not change we obtain the scaling expected
- There is an effect due to different charm distributions. In Ar-Ar it reduces Ω_{ccc} by ≈ 1.3 factor, in O-O it is ≈ 1.7
- the cube of the distribution gives an idea of this difference, but Wigner function mitigate the effect
- A larger production of coalescence w.r.t. SHM for small systems:

- Lack of canonical suppression!

- e-b-e fluctuations can enhance production? $\langle N^3 \rangle > \langle N \rangle^3$

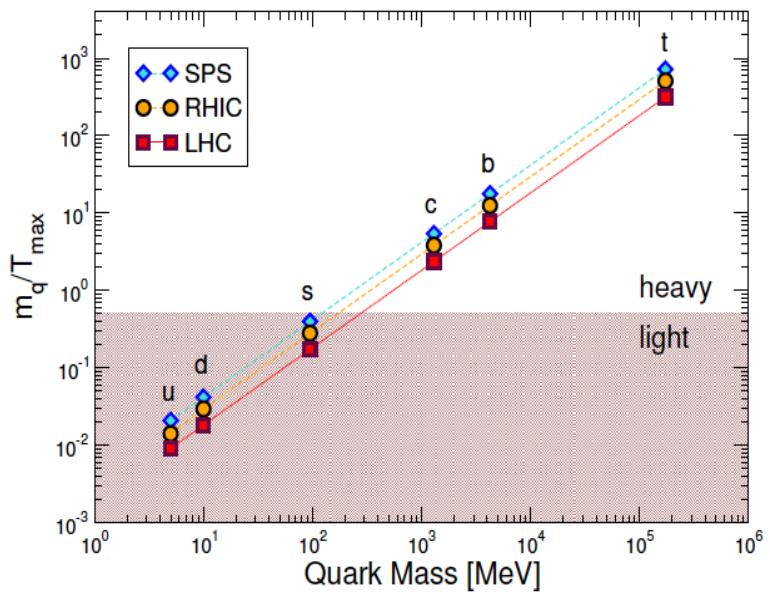
Thank you

Backup Slides

Specific of Heavy Quarks

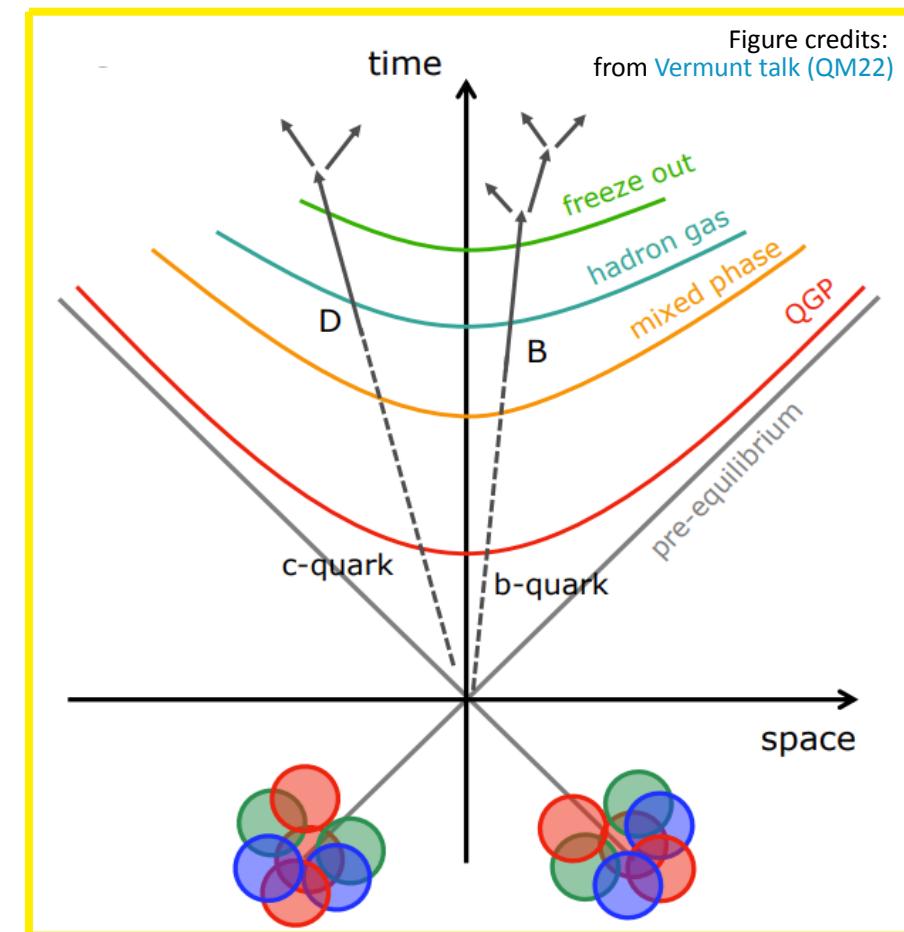
- $m_{c,b} \gg \Lambda_{\text{QCD}}$
produced by pQCD process (out of equilibrium)
- $m_{c,b} \gg T_0$
negligible thermal production
- $\tau_0 \ll \tau_{\text{QGP}}$
- $\tau_{\text{therm.}} \approx \tau_{\text{QGP}} \gg \tau_{g,q}$

HQs experience the full QGP evolution
Carry informations about initial stages, more than light quarks



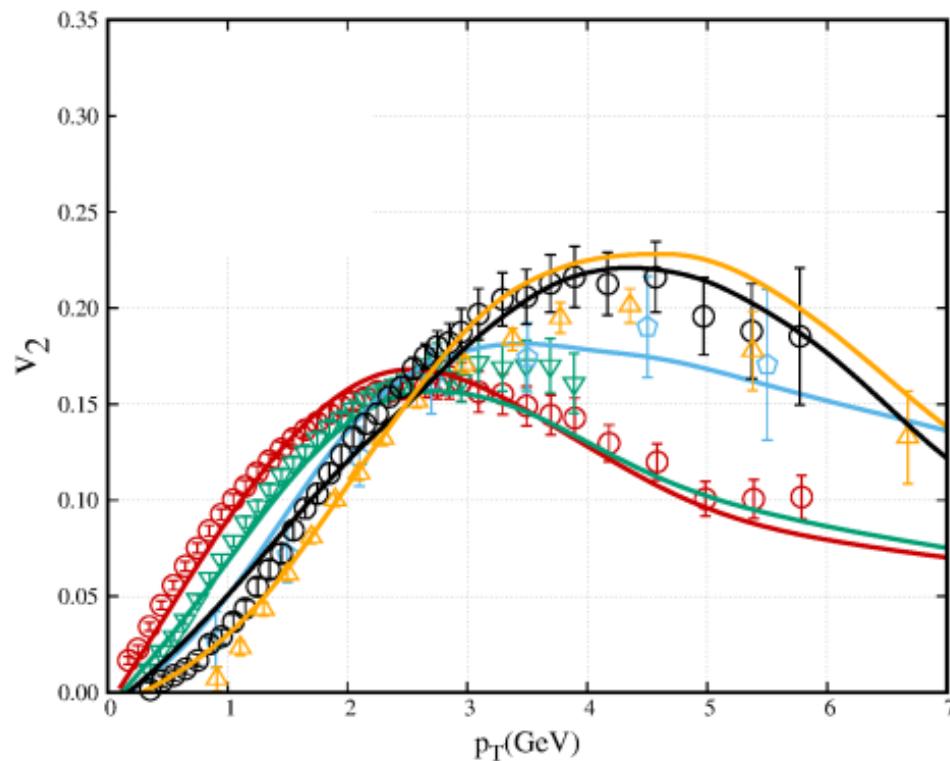
Recent reviews:

- 1) X.Dong, V. Greco Prog. Part. Nucl. Phys. 104 (2019)
- 2) A.Andronic Eur.Phys.J.C 76 (2016) 3, 107
- 3) F.Prino, R.Rapp, J.Phys.G 43 (2016) 9, 093002

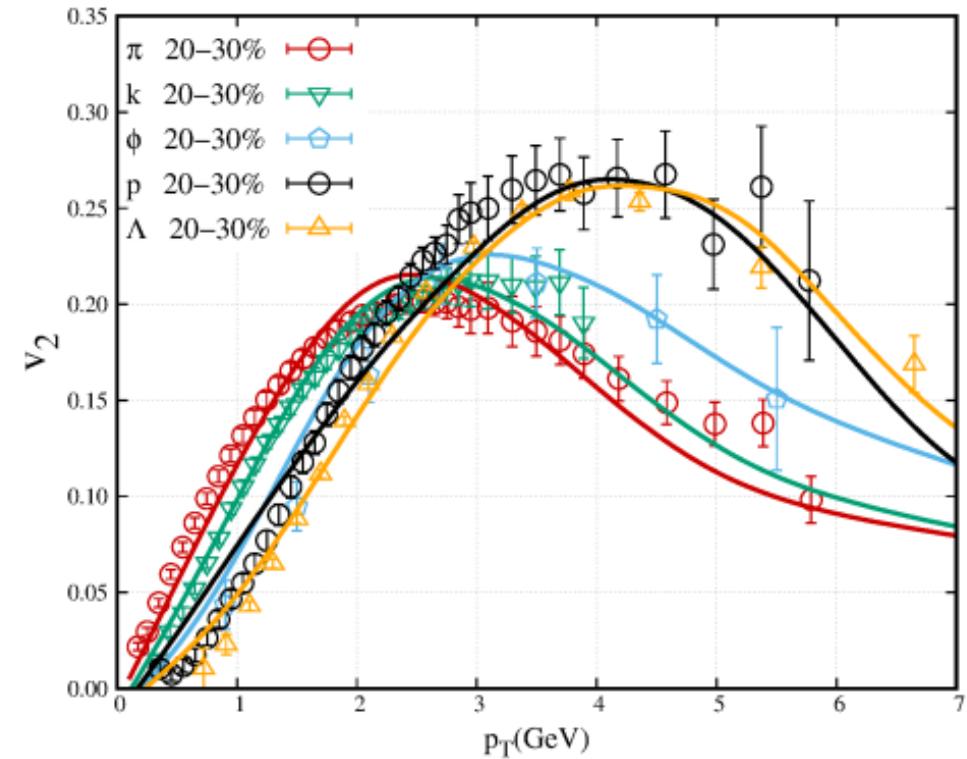


Elliptic flow of light particles LHC @2.76TeV

ALICE 10–20% 2.76TeV



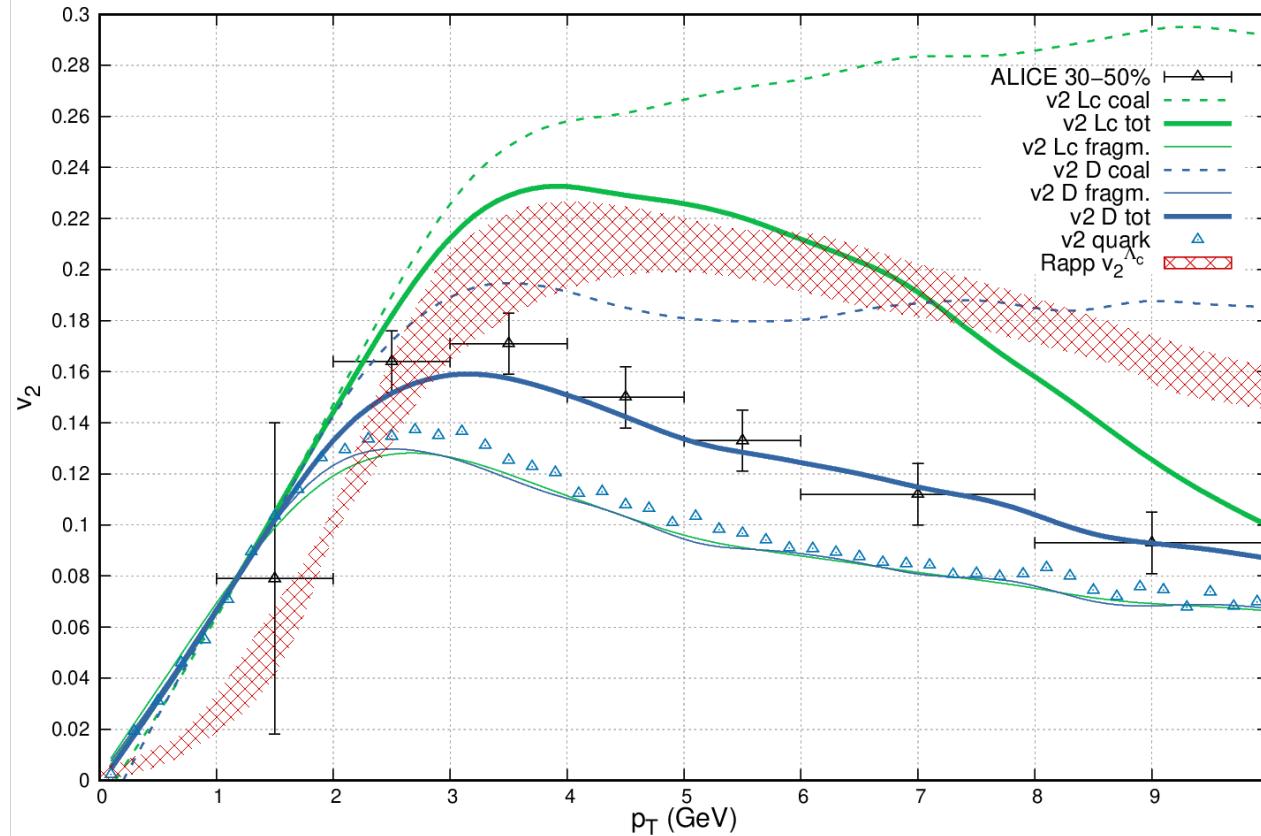
ALICE 20–30% 2.76TeV



φ meson: intermediate behaviour between meson and baryon

Elliptic flow of heavy particles

v_2 30–50% PbPb 5 TeV



splitting as in light sector: Λ_c coalescence dominant vs fragmentation

Heavy flavour: Resonance decay

Meson	Mass(MeV)	I (J)	Decay modes	B.R.
$D^+ = \bar{d}c$	1869	$\frac{1}{2}(0)$		
$D^0 = \bar{u}c$	1865	$\frac{1}{2}(0)$		
$D_s^+ = \bar{s}c$	2011	0(0)		
Resonances				
D^{*+}	2010	$\frac{1}{2}(1)$	$D^0\pi^+; D^+X$	68%,32%
D^{*0}	2007	$\frac{1}{2}(1)$	$D^0\pi^0; D^0\gamma$	62%,38%
D_s^{*+}	2112	0(1)	D_s^+X	100%
Baryon				
$\Lambda_c^+ = udc$	2286	$0(\frac{1}{2})$		
$\Xi_c^+ = usc$	2467	$\frac{1}{2}(\frac{1}{2})$		
$\Xi_c^0 = dsc$	2470	$\frac{1}{2}(\frac{1}{2})$		
$\Omega_c^0 = ssc$	2695	$0(\frac{1}{2})$		
Resonances				
Λ_c^+	2595	$0(\frac{1}{2})$	$\Lambda_c^+\pi^+\pi^-$	100%
Λ_c^+	2625	$0(\frac{3}{2})$	$\Lambda_c^+\pi^+\pi^-$	100%
Σ_c^+	2455	$1(\frac{1}{2})$	$\Lambda_c^+\pi$	100%
Σ_c^+	2520	$1(\frac{3}{2})$	$\Lambda_c^+\pi$	100%
$\Xi_c'^{+,0}$	2578	$\frac{1}{2}(\frac{1}{2})$	$\Xi_c^{+,0}\gamma$	100%
Ξ_c^+	2645	$\frac{1}{2}(\frac{3}{2})$	$\Xi_c^+\pi^-$,	100%
Ξ_c^+	2790	$\frac{1}{2}(\frac{1}{2})$	$\Xi_c'\pi$,	100%
Ξ_c^+	2815	$\frac{1}{2}(\frac{3}{2})$	$\Xi_c'\pi$,	100%
Ω_c^0	2770	$0(\frac{3}{2})$	$\Omega_c^0\gamma$,	100%

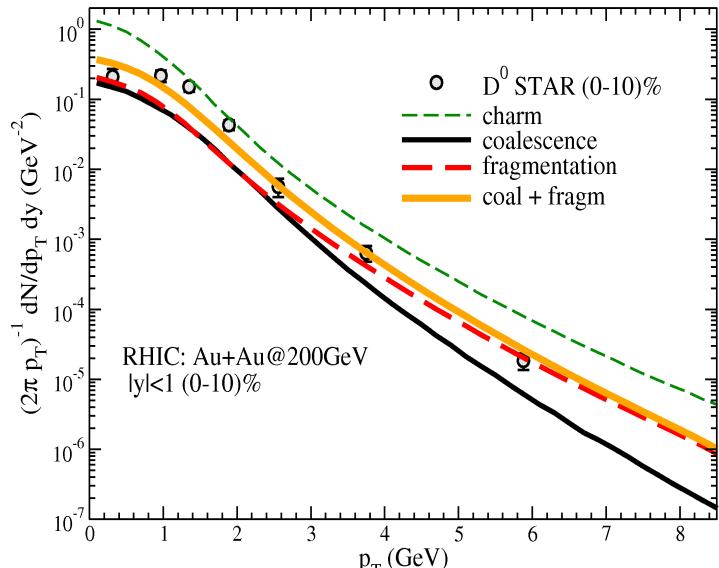
In our calculations we take into account hadronic channels including the ground states + first excited states

Statistical factor suppression for resonances

$$\frac{[(2J+1)(2I+1)]_{H^*}}{[(2J+1)(2I+1)]_H} \left(\frac{m_{H^*}}{m_H} \right)^{3/2} e^{-(m_{H^*} - m_H)/T}$$

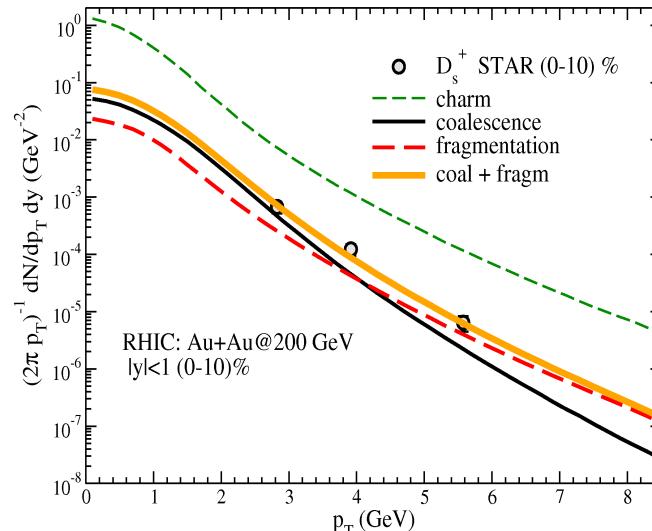
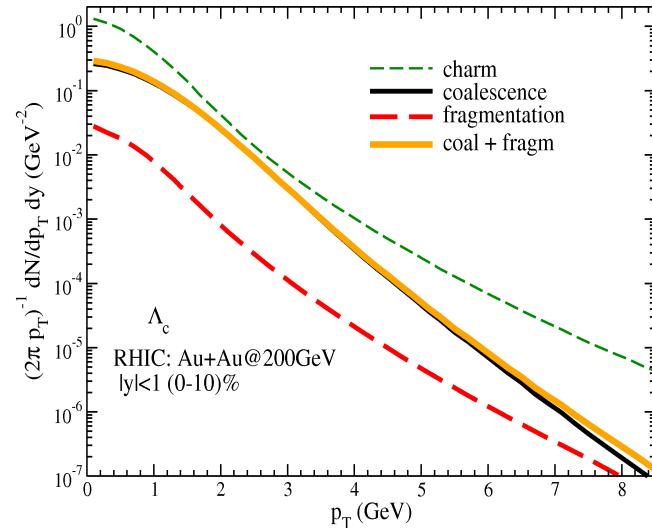
RHIC: results

S. Plumari, V. Minissale et al., Eur. Phys. J. C78 no. 4, (2018) 348



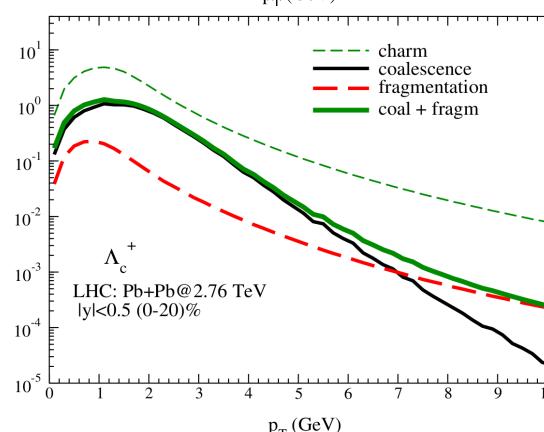
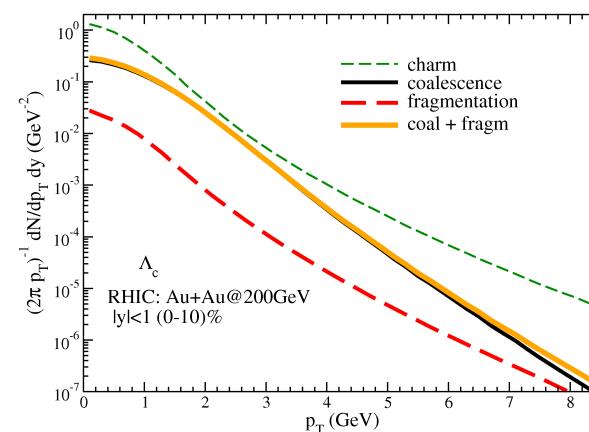
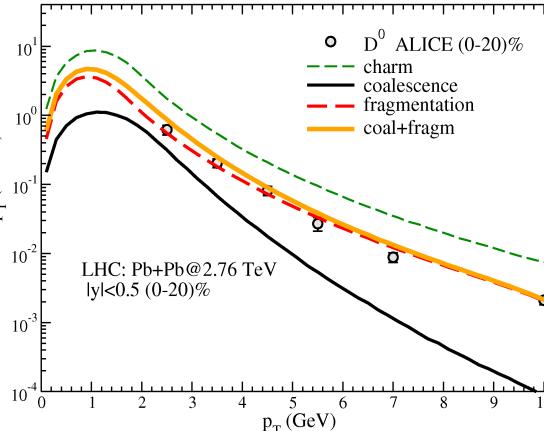
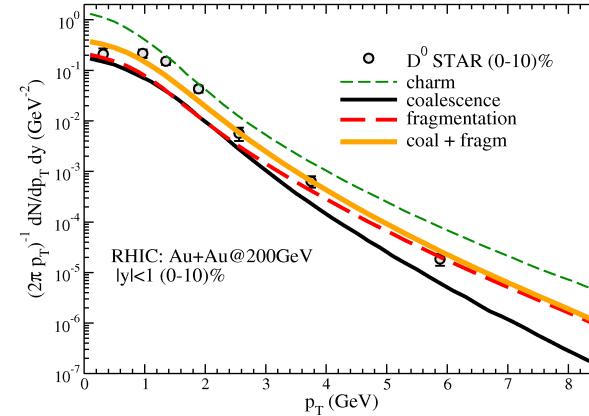
Data from STAR Coll. PRL 113 (2014) no.14, 142301

- For D^0 coalescence and fragmentation comparable at 2 GeV
- fragmentation fraction for D_s^+ are small and less than about 8% of produced total heavy hadrons
- Λ_c^+ fragmentation is even more smaller, coalescence gives the dominant contribution



wave function widths σ_p of baryon and mesons are the same at RHIC and LHC!

Data from: STAR Coll. PRL 113, 142301 (2014), ALICE Coll. JHEP 09 (2012) 112



RHIC

LHC



Coalescence lower at LHC than at RHIC

main contribution:
Fragmentation

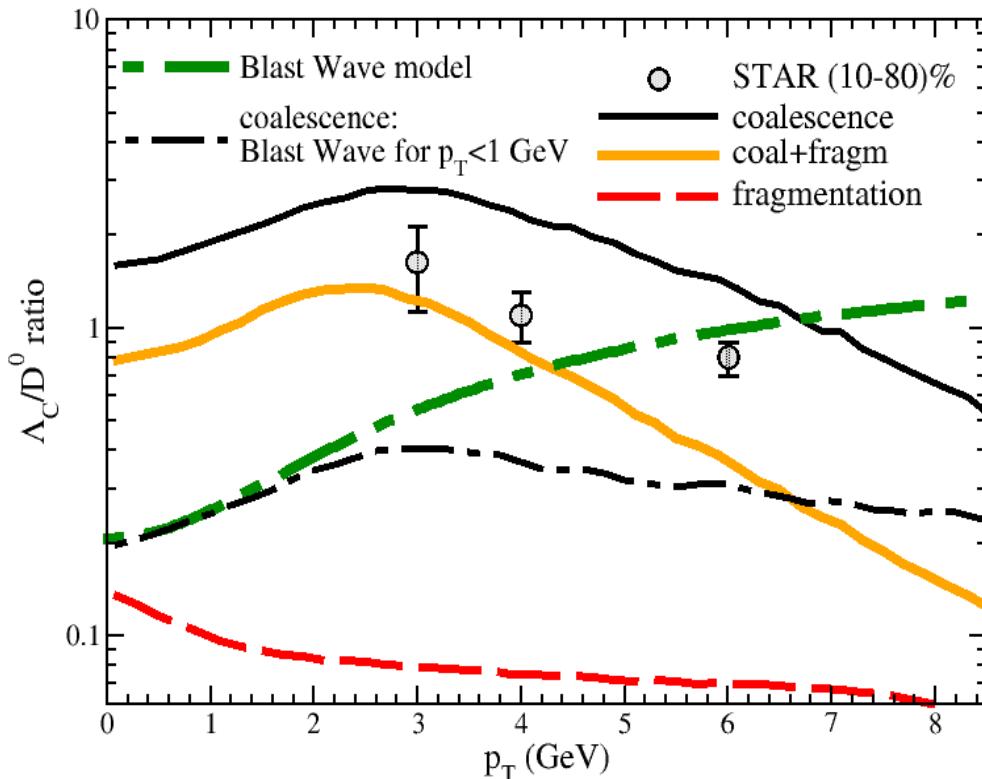


Coalescence lower at LHC than at RHIC

main contribution:
Coalescence

RHIC: Baryon/meson

STAR, Phys.Rev.Lett. 124 (2020) 17, 172301



Compared to light baryon/meson ratio
the Λ_c/D^0 ratio has a larger width
(flatter)

More flatter \rightarrow should coalescence
extend to higher p_T ? Indication also in
light sector

V. Minissale, F. Scardina, V. Greco PRC **92**, 054904 (2015)
Cho, Sun, Ko et al., PRC **101** (2020) 2, 024909

Needed data at low p_T

Coalescence : Duke

Y. Xu, S. Cao, M. Nahrgang, W. Ke, G. Qin, J. Auvinen, and S. Bass, Nucl.Part.Phys.Proc. 276 (2016) 225.
 S. Cao, G. Qin, and S. Bass, PRC92, 024907 (2015).

Instantaneous coalescence model

Hadron Wigner functions are averaged over the position space

Mesons

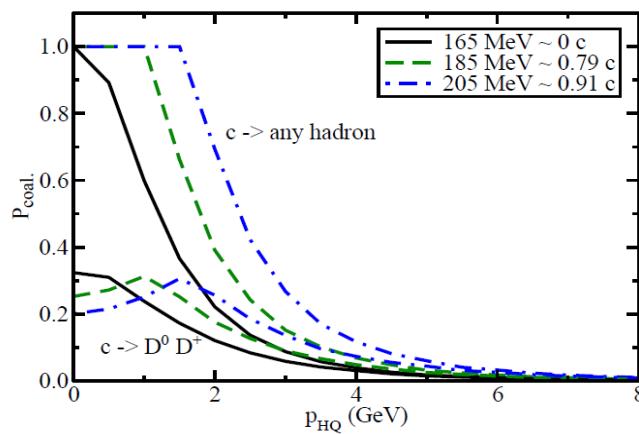
$$\frac{dn_M}{d^3 p_M} = \int d^3 p_1 d^3 p_2 \frac{dN_1}{d^3 p_1} \frac{dN_2}{d^3 p_2} f_M^W(\vec{p}_1, \vec{p}_2) \delta(\vec{p}_M - \vec{p}_1 - \vec{p}_2)$$

$$f_M^W(q^2) = g_M \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-q^2\sigma^2} \quad \vec{q} \equiv \frac{E_2^{\text{cm}} \vec{p}_1^{\text{cm}} - E_1^{\text{cm}} \vec{p}_2^{\text{cm}}}{E_1^{\text{cm}} + E_2^{\text{cm}}}$$

Baryons

$$\frac{dN_B}{d^3 p_B} = \int d^3 p_1 d^3 p_2 d^3 p_3 \frac{dN_1}{d^3 p_1} \frac{dN_2}{d^3 p_2} \frac{dN_3}{d^3 p_3} f_B^W(\vec{p}_1, \vec{p}_2, \vec{p}_3) \\ \times \delta(\vec{p}_M - \vec{p}_1 - \vec{p}_2 - \vec{p}_3).$$

$$f_B^W(q_1^2, q_2^2) = g_B \frac{(2\sqrt{\pi})^6 (\sigma_1 \sigma_2)^3}{V^2} e^{-q_1^2 \sigma_1^2 - q_2^2 \sigma_2^2} \\ \vec{q}_1 \equiv \frac{E_2^{\text{cm}} \vec{p}_1^{\text{cm}} - E_1^{\text{cm}} \vec{p}_2^{\text{cm}}}{E_1^{\text{cm}} + E_2^{\text{cm}}}, \\ \vec{q}_2 \equiv \frac{E_3^{\text{cm}} (\vec{p}_1^{\text{cm}} + \vec{p}_2^{\text{cm}}) - (E_1^{\text{cm}} + E_2^{\text{cm}}) \vec{p}_3^{\text{cm}}}{E_1^{\text{cm}} + E_2^{\text{cm}} + E_3^{\text{cm}}}$$



$$\sigma_{ri} = 1/\sqrt{\mu_i \omega} \quad \text{Harmonic oscillator relation} \\ \mu_1 = \frac{m_1 m_2}{m_1 + m_2}, \quad \mu_2 = \frac{(m_1 + m_2) m_3}{m_1 + m_2 + m_3}.$$

These two parameters are obtained by requiring the coalescence probability through all possible hadronization channels to be unity for a zero momentum heavy quark.

Coalescence : Duke

Y. Xu, S. Cao, M. Nahrgang, W. Ke, G. Qin, J. Auvinen, and S. Bass:
 S. Cao, G. Qin, and S. Bass, PRC92, 024907 (2015).

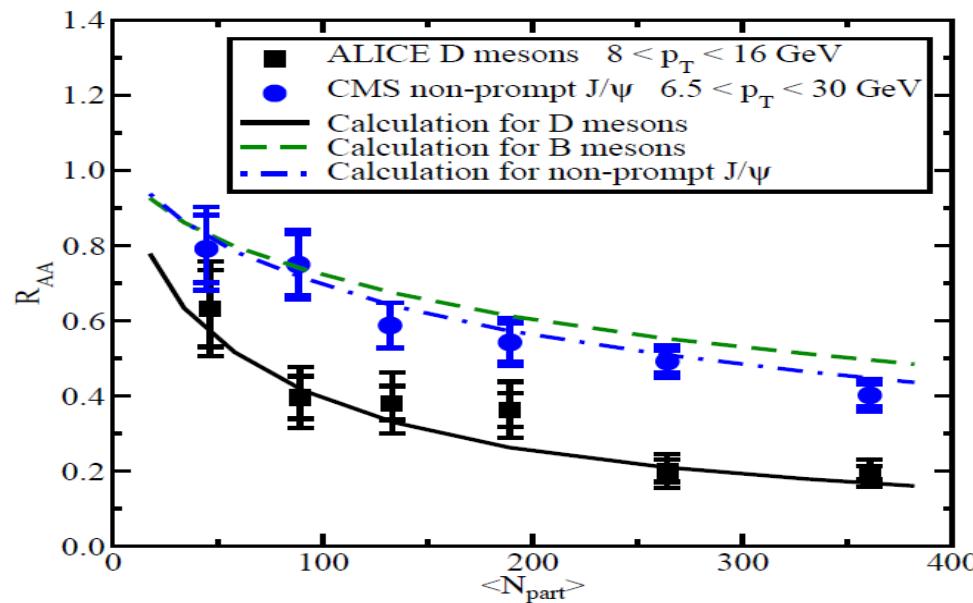
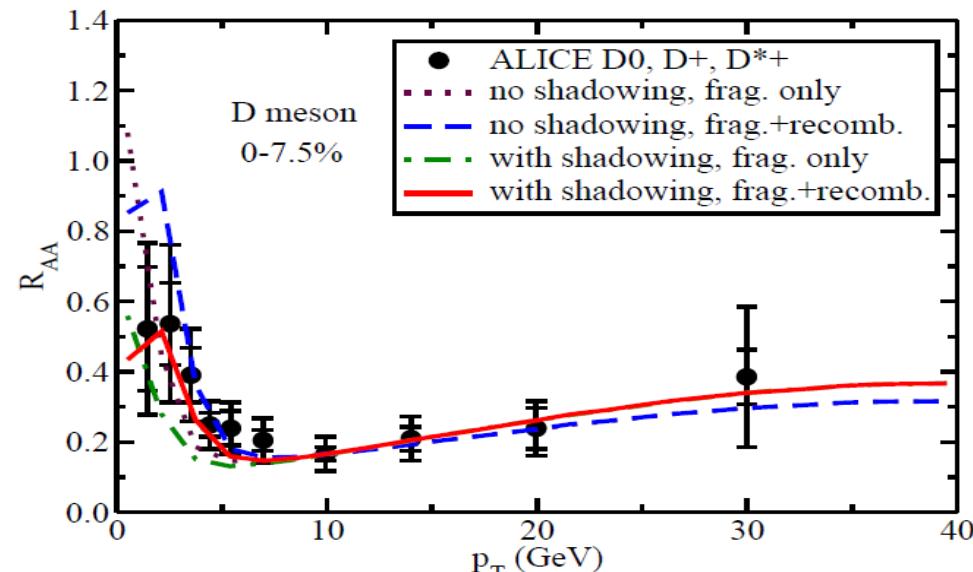
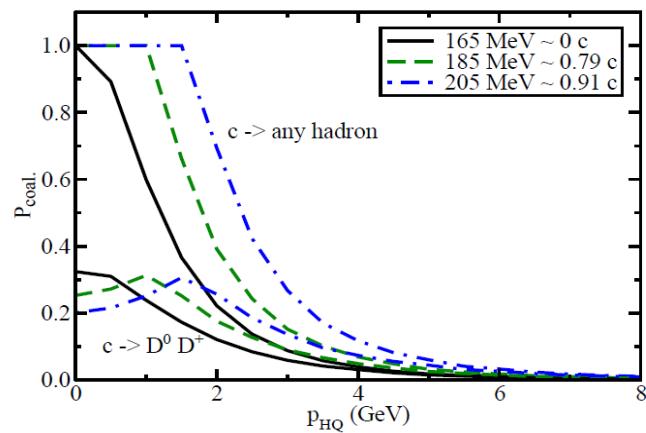
Instantaneous coalescence model

Mesons

$$\frac{dn_M}{d^3 p_M} = \int d^3 p_1 d^3 p_2 \frac{dN_1}{d^3 p_1} \frac{dN_2}{d^3 p_2} f_M^W(\vec{p}_1, \vec{p}_2) \delta(\vec{p}_M - \vec{p}_1 - \vec{p}_2)$$

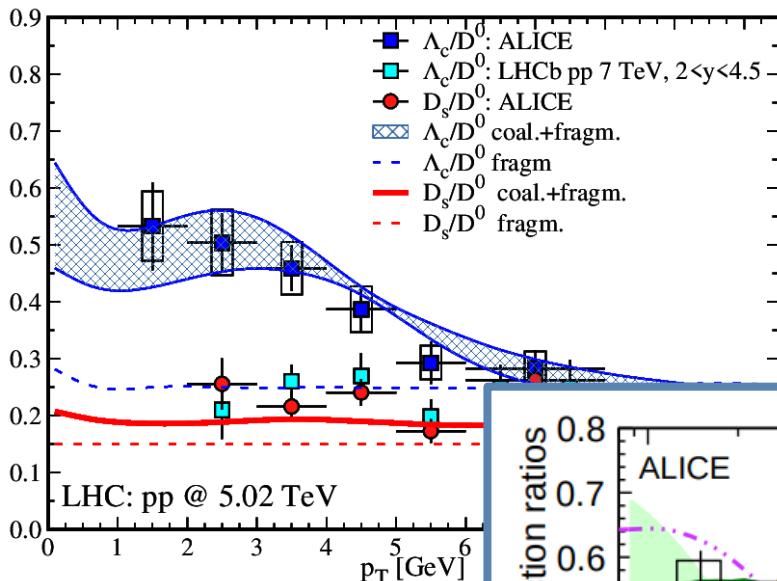
Baryons

$$\frac{dN_B}{d^3 p_B} = \int d^3 p_1 d^3 p_2 d^3 p_3 \frac{dN_1}{d^3 p_1} \frac{dN_2}{d^3 p_2} \frac{dN_3}{d^3 p_3} f_B^W(\vec{p}_1, \vec{p}_2, \vec{p}_3) \\ \times \delta(\vec{p}_M - \vec{p}_1 - \vec{p}_2 - \vec{p}_3).$$



Small systems: Coalescence in pp?

V. Minissale, S. Plumari, V. Greco, Physics Letters B 821 (2021) 136622

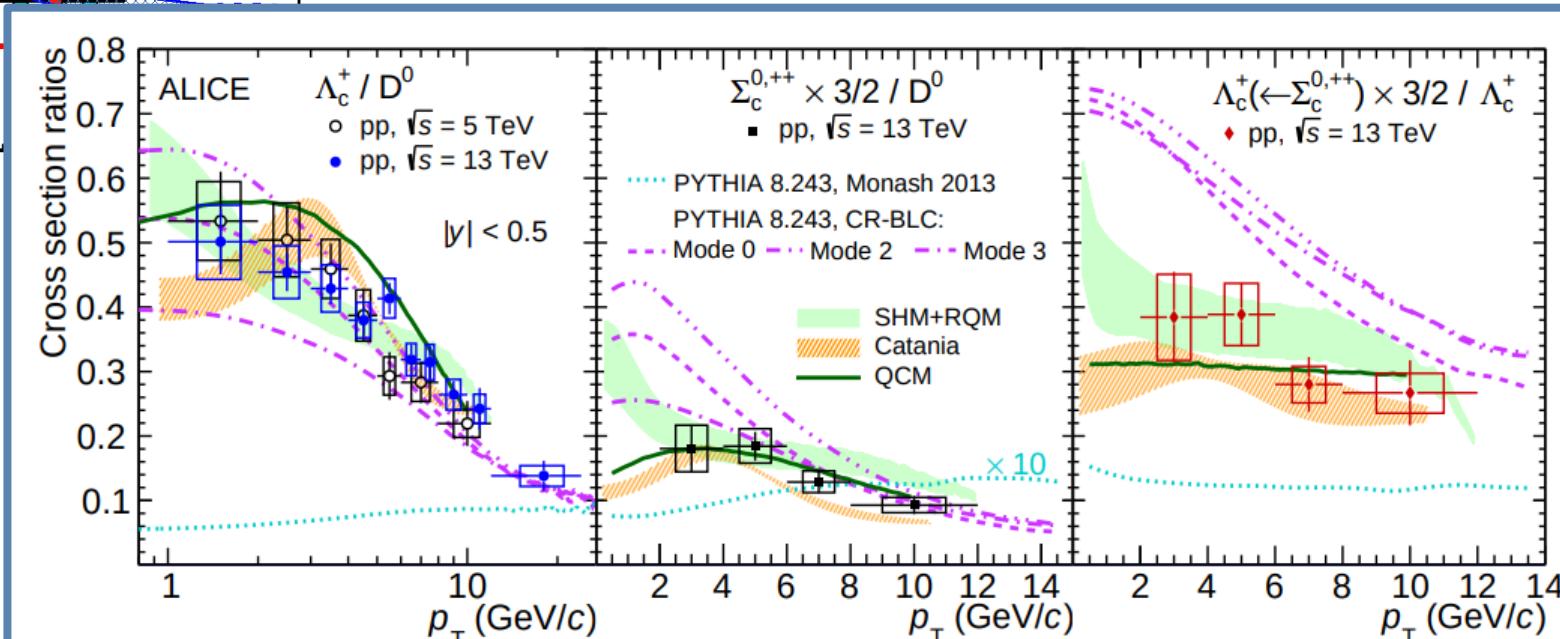


Error band correspond to

Reduction of rise-and-fall behaviour in Λ_c^+ / D^0 ratio:

- Confronting with AA: Coal. contribution smaller w.r.t. Fragm.
- FONLL distribution flatter w/o evolution trough QGP
- Volume size effect

The increase of Λ_c^+ production in pp have effect on R_{AA} of Λ_c^+



Elliptic Flow – Quark Number Scaling

Fourier expansion of the azimuthal distribution

$$f(\varphi, p_T) = 1 + 2 \sum_{n=1}^{\infty} v_n(p_T) \cos n\varphi$$

n=2 Elliptic flow

momentum anisotropy in the transverse plane

coalescence brings to

$$\begin{aligned} v_{2,M}(p_T) &\approx 2v_{2,q}(p_T/2) \\ v_{2,B}(p_T) &\approx 3v_{2,q}(p_T/3) \end{aligned}$$

Partonic
elliptic flow

Hadronic
elliptic flow

Assumption

- one dimensional
- Dirac delta for Wigner function
- isotropic radial flow
- not including resonance effect

Transport approaches

Fokker-Planck ($T \ll m_b$ soft scattering)

$$\frac{\partial}{\partial t} f_Q = \gamma \frac{\partial}{\partial p_i} [p_i f_Q] + D_p \nabla_p^2 [f_Q]$$

Drag coeff. Momentum diffusion coeff.

Background: Hydro/transport expanding bulk

-Fluctuation dissipation theorem

$$D_p = E T \gamma$$

$$D_s = \frac{T}{M_\gamma} = \frac{T^2}{D_p} = \frac{T}{M} \tau_{th}$$

-Spatial diffusion coefficient
a measure of thermalization time

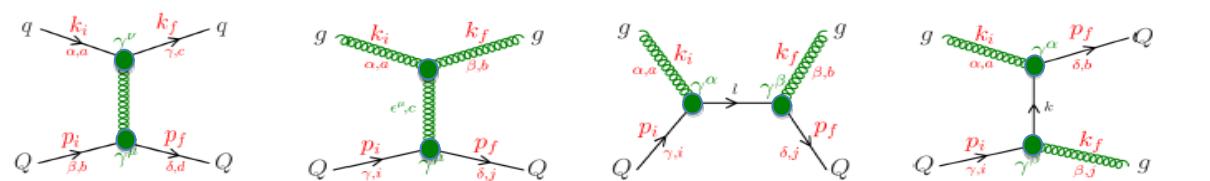
$$\langle x^2 \rangle - \langle x \rangle = 6 D_s t$$

Boltzmann kinetic transport

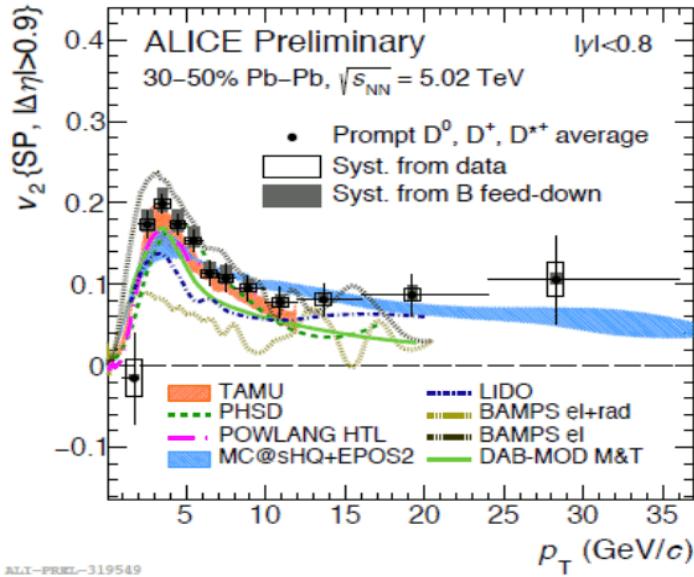
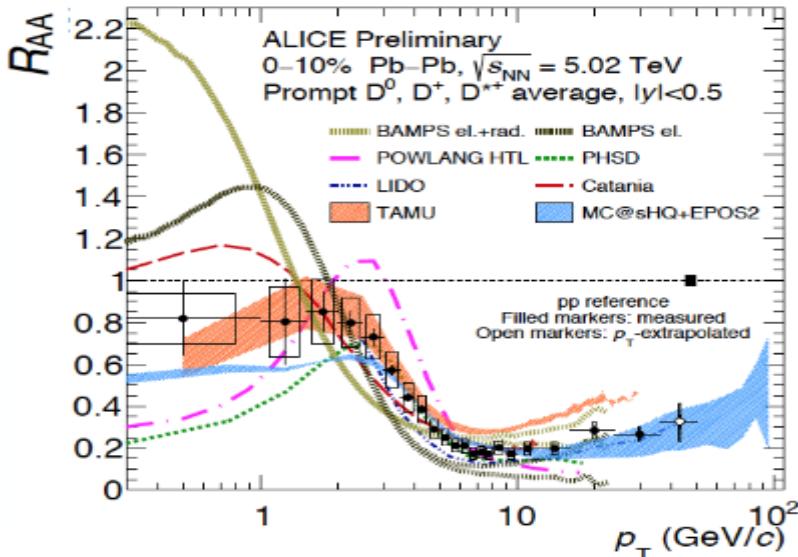
$$p^\mu \partial_\mu f_Q(x, p) = C[f_q, f_g, f_Q]$$

Collision integral

$$C[f_q, f_g, f_Q] = \frac{1}{2E_1} \int \frac{d^3 p_2}{2E_2(2\pi)^3} \int \frac{d^3 p_1'}{2E_1'(2\pi)^3} [f_Q(p_1') f_{q,g}(p_2') - f_Q(p_1) f_{q,g}(p_2)] \times |M_{(q,g) \rightarrow Q}(p_1 p_2 \rightarrow p_1' p_2')| (2\pi)^4 \delta^4(p_1 + p_2 - p_1' - p_2')$$



Transport approaches



Models not really tested at $p \rightarrow 0$

The new data \rightarrow determine $D_s(T)$ more properly,
i.e. $p \rightarrow 0$ where it is defined and computed in IQCD

	Catania	Duke	Frankfurt(PHSD)	LBL	Nantes	TAMU
Initial HQ (p)	FONLL	FONLL	pQCD	pQCD	FONLL	
Initial HQ (x)	binary coll.	binary coll.	binary coll.	binary coll.	EPOS	binary coll.
Initial QGP	Glauber	Trento	Lund	Vishnu	EPOS	2d ideal hydro
QGP	Boltzm.	Vishnu	Boltzm.	Vishnu	m=0	m=0
partons	mass	m=0	m(T)	m=0	m=0	m=0
formation time QGP	0.3 fm/c	0.6 fm/c	0.6 fm/c (early coll.)	0.6 fm/c	0.3 fm/c	0.4 fm/c
interactions in between	HQ-glasma	no	HQ-preformed plasma	no		no

2018-2019

Several Collab. in joint activities:

- EMMI-RRTF:

R. Rapp et al., Nucl. Phys. A 979 (2018)

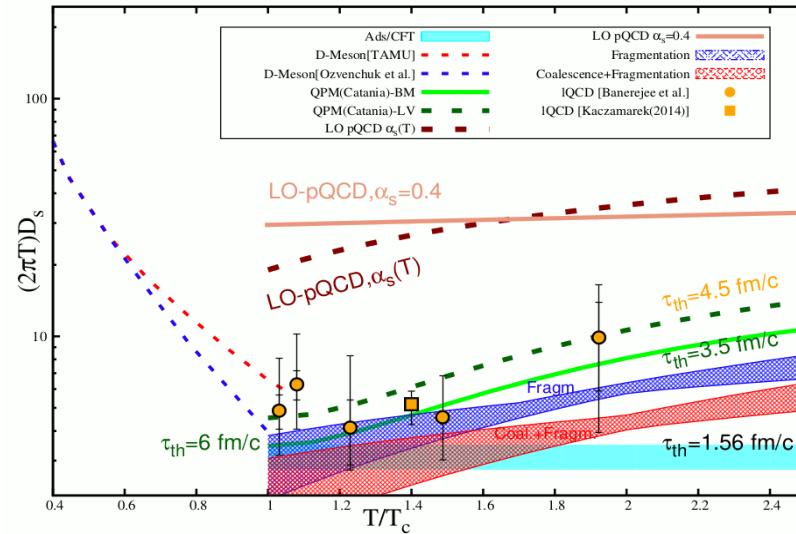
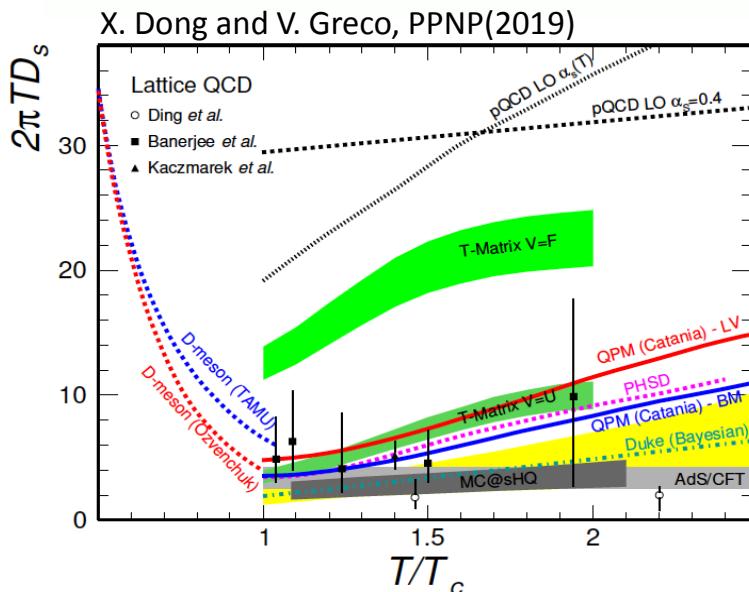
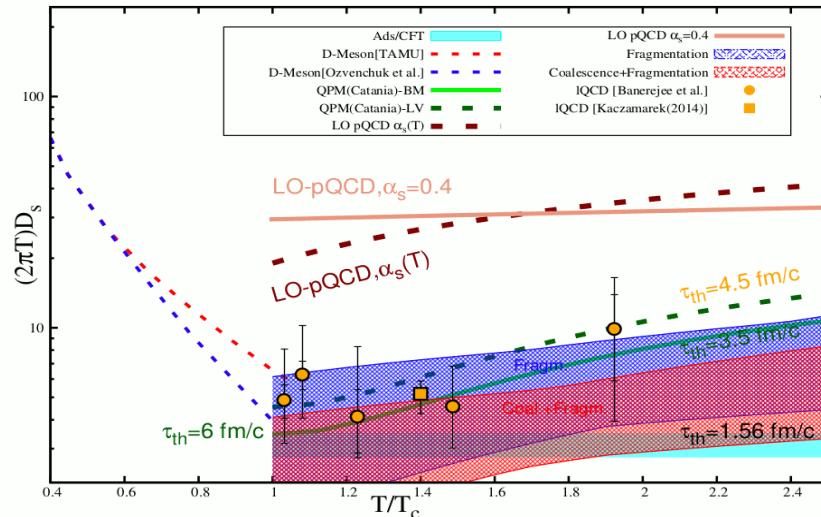
- HQ-JETS:

S. Cao et al., Phys. Rev. C 99 (2019)

- Y. Xu et al., Phys. Rev. C 99 (2019)

Transport coefficient

Z. Citron et al., CERN Yellow Rep. Monogr. 7 (2019) 1159



Different hadronization models can affect the extraction of the charm quark diffusion coefficient

2018-2019

Several Collab. in joint activities:

- EMMI-RRTF: R. Rapp et al., Nucl. Phys. A 979 (2018)
- HQ-JETS: S. Cao et al., Phys. Rev. C 99 (2019)
- Y. Xu et al., Phys. Rev. C 99 (2019)

Multicharm production Pb-Pb, Kr-Kr, Ar-Ar, O-O

Baryon				
$\Xi_{cc}^{+,++} = dec, ucc$	3621	$\frac{1}{2} (\frac{1}{2})$		
$\Omega_{scc}^+ = scc$	3679	$0 (\frac{1}{2})$		
$\Omega_{ccc}^{++} = ccc$	4761	$0 (\frac{3}{2})$		
Resonances				
Ξ_{cc}^*	3648	$\frac{1}{2} (\frac{3}{2})$	$1.71 \times g.s$	
Ω_{scc}^*	3765	$0 (\frac{3}{2})$	$1.23 \times g.s$	

like S.Cho and S.H. Lee, PRC101 (2020)
from R.A. Briceno et al., PRD 86(2012)

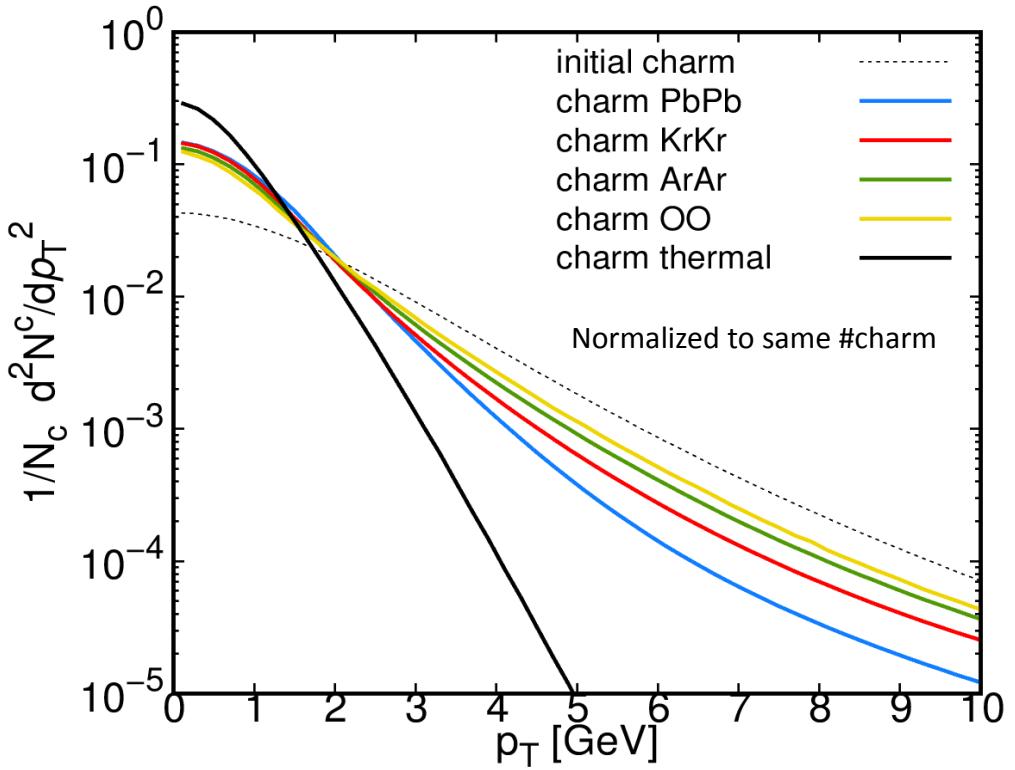
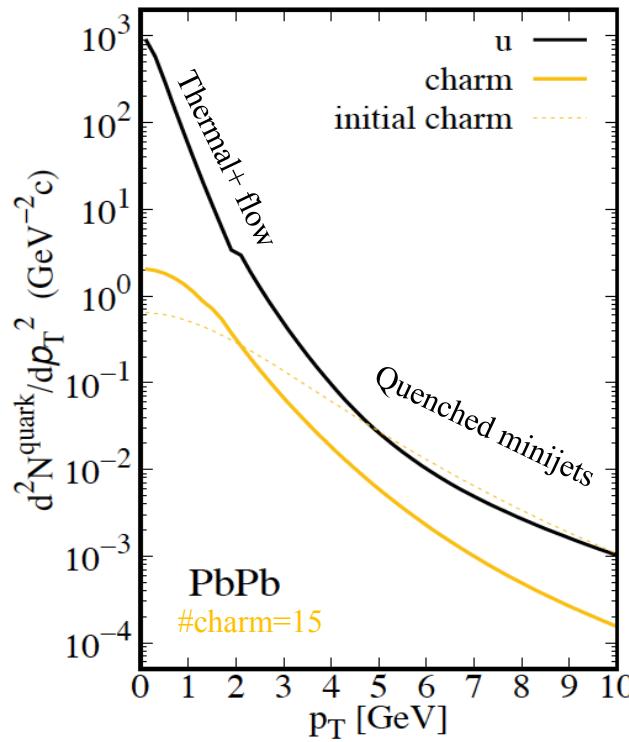
Strengths of the approach:

- Does not rely on distribution in equilibrium for charm
→ useful for small AA down to pp collisions and at $p_T > 3\text{-}4 \text{ GeV}$
- Provide a p_T dependence of spectra and their ratios vs p_T

Widths from harmonic oscillator rescaling and from $\langle r \rangle$ of Tsingua approach

	$\sigma_{p_1}(\text{GeV})$	$\sigma_{p_2}(\text{GeV})$	$\sigma_{r_1}(fm)$	$\sigma_{r_2}(fm)$
Ξ_c	0.262	0.438	0.751	0.450
Ω_c	0.345	0.557	0.572	0.354
Ξ_{cc}^ω	0.317	0.573	0.622	0.344
$\Omega_{ccc}^{\sigma_r \sigma_p = 3/2}$	0.522	0.522	0.566	0.566

Charm distribution in PbPb-KrKr-ArAr-OO from transport approach

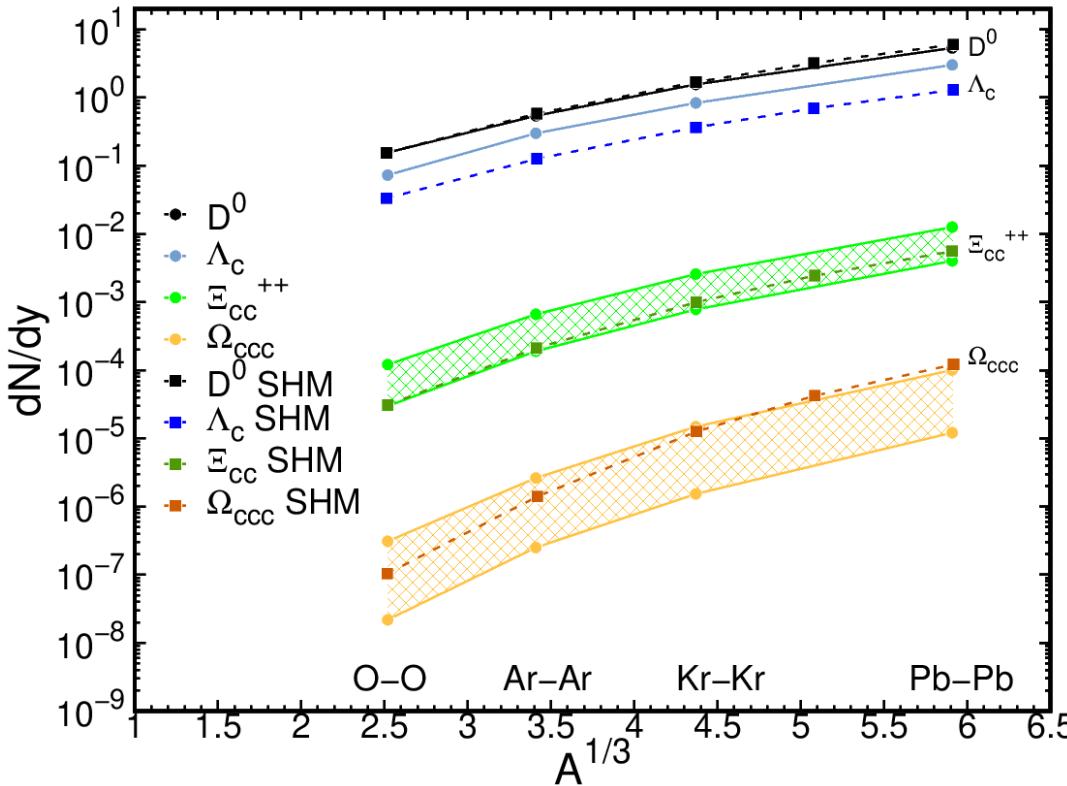


	OO	ArAr	KrKr	PbPb
$R_0(f m)$	2.76	3.75	4.9	6.5
$R_{\max}(f m)$	5.2	7.65	10.1	14.1
$\tau(f m)$	4	5	6.2	8
β_{\max}	0.55	0.6	0.64	0.7
$V_{ y <0.5}(f m^3)$	345	920	2000	5000

Volume scales with A , now we employ the same value of SHM
 A. Andronic et al., JHEP (2021) 035

Shadowing on charm included as a $K = 0.65$ factor [no p_T dependence]
 #charm= 15 (PbPb), 4.35 (KrKr), 1.5(ArAr), 0.4(00)

Multi-charm production vs A-A: Yields



Ξ_{cc}	Ω_{ccc}
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- upper limit: thermal distribution
- lower limit: PbPb distribution with widths scaled as standard Harm. Oscill. (ω from Ξ_c, Ω_c^0)

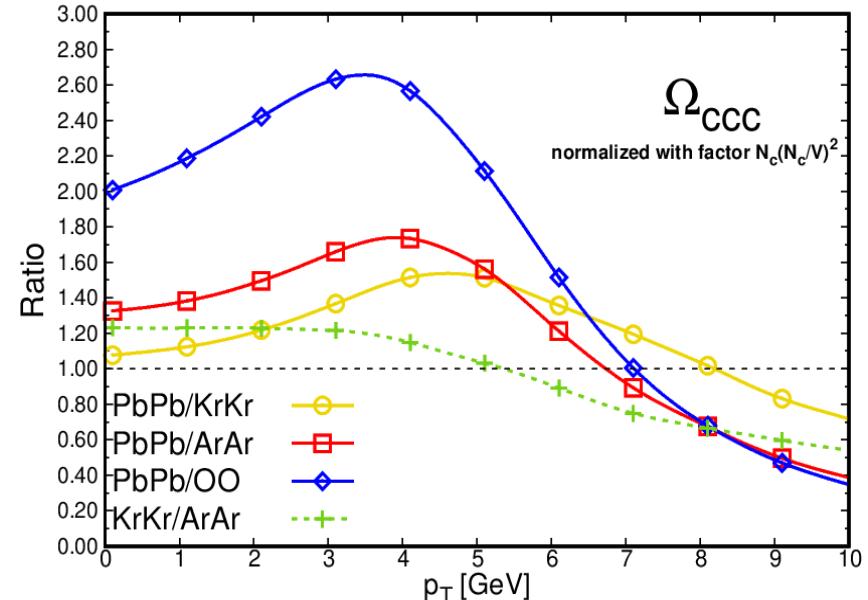
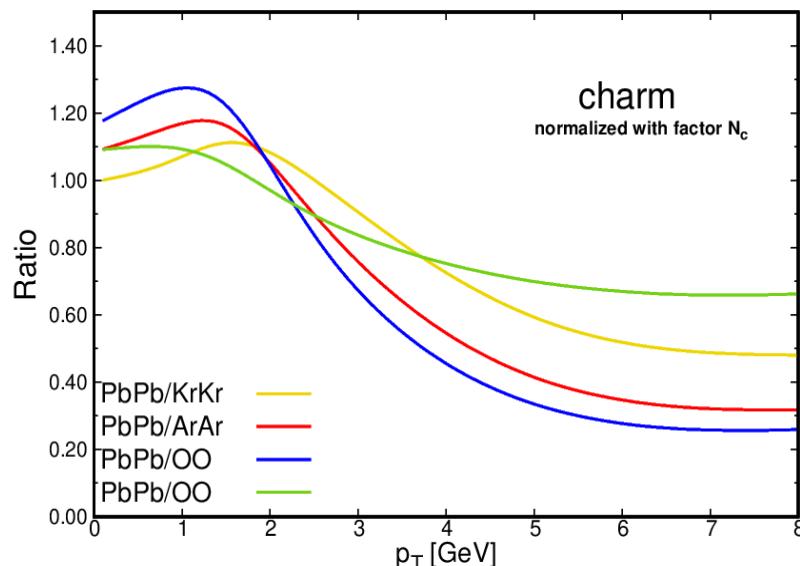
- Compatible yields within the two scenarios w.r.t. SHM
- Different trends with $A^{1/3}$ increasing the number of constituent charm quarks
Lack of canonical suppression but...

	$O - O$	$Ar - Ar$	$Kr - Kr$	$Pb - Pb$
D^0	0.156	0.543	1.564	5.343
Λ_c	0.0732	0.301	0.835	3.0123
$\Xi_{cc}^{+,++}$	$3 - 12.1 \times 10^{-5}$	$1.9 - 6.6 \times 10^{-4}$	$0.78 - 2.6 \times 10^{-3}$	$4 - 12.5 \times 10^{-3}$
Ω_{ccc}	$2.2 - 29.2 \times 10^{-8}$	$2.5 - 26.3 \times 10^{-7}$	$1.5 - 14.9 \times 10^{-6}$	$0.12 - 1.01 \times 10^{-4}$

Ratios of p_T distribution of Ω_{ccc} in PbPb/KrKr/ArAr

caveat: in O-O no N_c and V scaling with fixed distribution (multipl. factor)

The Ω_{ccc} p_T distribution, with only coalescence, in the intermediate region decreases faster in larger systems.



- It can be a meter of non-equilibrium. Translation of feature of charm spectra at low p_T into higher momentum region.
- More sensitive for multicharm respect to D mesons and Λ_c . Both effects of light quarks and fragmentation