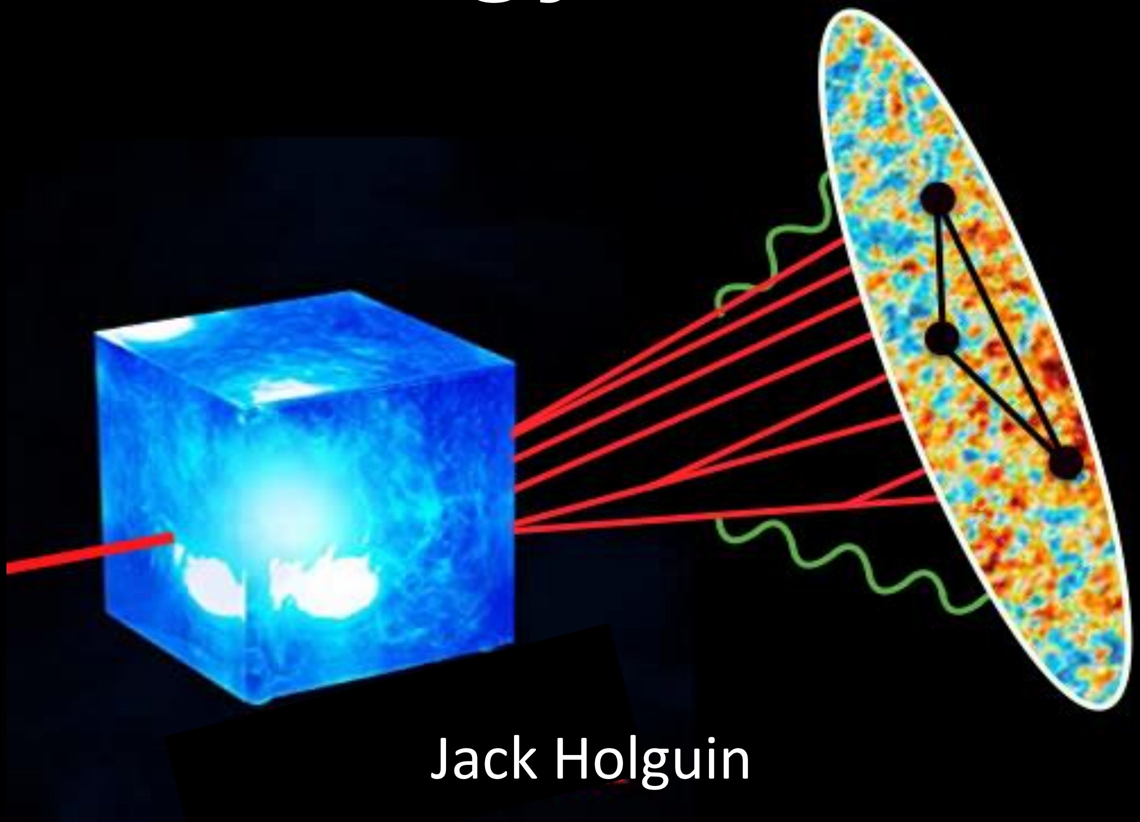
A Coherent View of Jet Quenching from Energy Correlators



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In collaboration with Carlota Andres, Fabio Dominguez, Cyrille Marquet, Ian Moult

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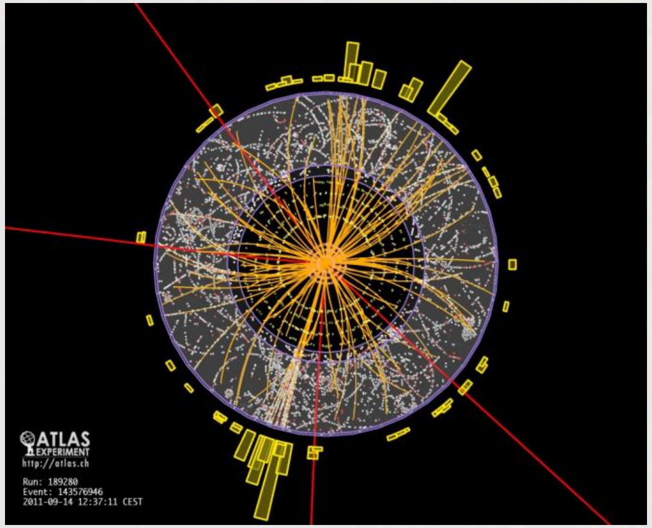
Introduction

Context:

- There has been an exceptionally successful program of studying QCD in vacuum and medium with event shape observables and algorithmic substructure.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}e_1 \dots \mathrm{d}e_n} = \sum_N \int \mathrm{d}\,\sigma_N\, \delta(f_1(N, e_1, \dots, e_n) - e_1) \dots \delta(f_n(N, e_1, \dots, e_n) - e_n)$$



$$\{e_1, e_2, \dots, e_n\}$$

Introduction

Context:

- However, there is an interesting disconnect between much of the developments in collider QCD phenomenology and formal theory.

Introduction

What's the middle ground?

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\ r^2 n^i T_{0i}(t, r\vec{n})$$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Conformal collider physics:
Energy and charge correlations

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We study observables in a conformal field theory which are very closely related to the ones used to describe hadronic events at colliders. We focus on the correlation functions of the energies deposited on calorimeters placed at a large distance from the collision. We consider initial states produced by an operator insertion and we study some general properties of the energy correlation functions for conformal field theories. We argue that the small angle singularities of energy correlation functions are controlled by the twist of

The energy flow operator is an ANEC operator, it can be computed directly in CFTs admitting many new computational techniques.

Its correlation functions are also directly collider observables – Energy Correlators.

We want to apply these to HI collisions to study jet quenching.

Introduction

Very quickly, what is $\mathcal{E}(\vec{n})$?

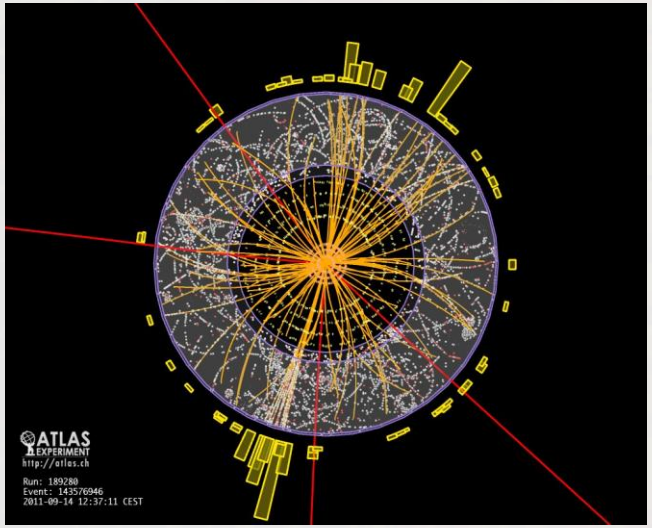
$\mathcal{E}(\vec{n}) =$ Idealised calorimeter output at a solid angle labelled by $\vec{n}$.

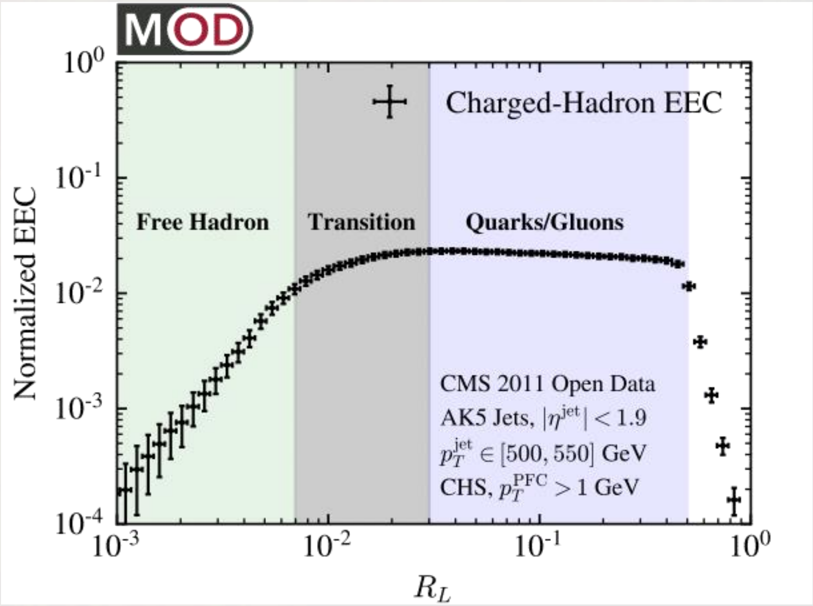
Correlation functions of $\mathcal{E}(\vec{n})$ quantify the correlations between the average calorimeter outputs at different points across the celestial sphere from a particular process.

They are functions of the angles between the calorimeters.

Introduction

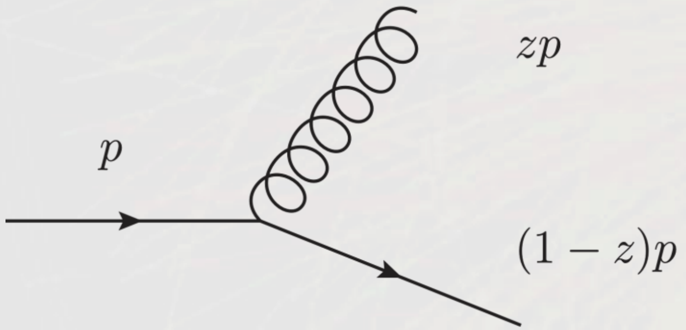
Important difference between correlators and more ‘typical’ observables.





Introduction

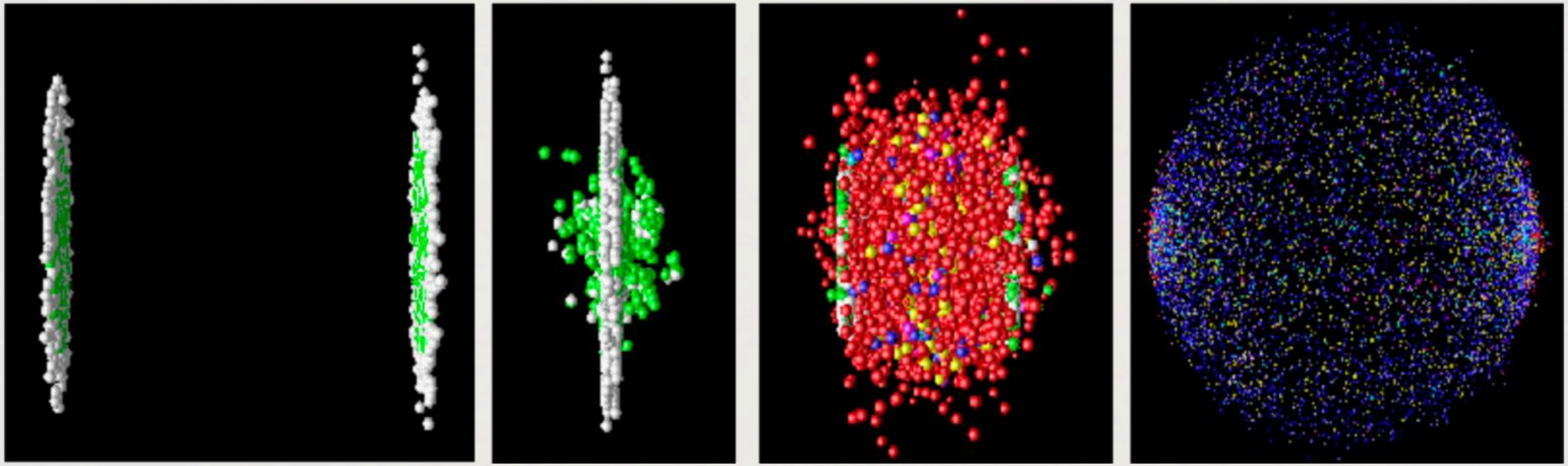
Additional to the computational benefits from formal theory, phenomenologically energy correlators are very good at isolating parts of multiscale dynamics.



$$t_f = \frac{2}{z(1-z)p_0\theta^2} \xrightarrow{z \approx \frac{1}{2}} \theta^2 \sim t_f^{-1} \sim p_{\mathrm{exchanged}}^2$$

The angular size of a correlation often can be interpreted as a time parameter for the physics inducing the correlations.

Introduction



Time

t=0 to 1 fm/c

t=1 to 10 fm/c

t>10 fm/c

The QGP in heavy ion collisions (HIC):

– 20 years of HIC at RHIC, 10 years of HIC at the LHC, sPHENIX coming soon.

Computing Correlation Functions of $\mathcal{E}(\vec{n})$

Recap

- Correlation functions in statistics:
  - $\mathrm{Corr}_2(X,Y) = \langle XY \rangle - \langle X \rangle \langle Y \rangle$ (also just the covariance)
  - $\mathrm{Corr}_3(X,Y,Z) = \langle XYZ \rangle - \langle X \rangle \big(\langle Y \rangle \langle Z \rangle - \mathrm{Corr}_2(Y,Z) \big)$
  - …

- $\langle \mathcal{E}(\vec{n}_1) \ldots \mathcal{E}(\vec{n}_i) \rangle \equiv \overline{\mathrm{Corr}}_i(\mathcal{E}(\vec{n}_1), \ldots, \mathcal{E}(\vec{n}_i))$ rather than a time ordered QFT correlator. Strictly they are Wightman functions.

Computing Correlation Functions of $\mathcal{E}(\vec{n})$

Which correlation function is the one for us?

- In the ealier slides the 2-point correlator gives a sensitive probe of hadronisation.
- In 2201.08393 the 3-point provided a sensitive probe to the top mass.

Look to what is currently done and successful.

- R_{AA} can be expressed as a function of one-point correlators + corrections:
  - $R_{AA} = \langle N_{AA}\rangle / (\langle N_{\text{Coll}}\rangle \langle N_{pp}\rangle)$. $\langle N\rangle$ is the one point correlator of the number opertator and due to momentum conservation $\langle N\rangle \approx \langle \mathcal{E}\rangle / \langle Q\rangle$.
- In effect, R_{AA} gives access to the simplest but also least sensitive correlator.
- Let us systematically increase the sentivity by looking directly at the 2-point correlator.

Computing Correlation Functions of $\mathcal{E}(\vec{n})$

$$\frac{\langle \mathcal{E}^n(\boldsymbol{n}_1)\mathcal{E}^n(\boldsymbol{n}_2)\rangle}{Q^{2n}} = \frac{1}{\sigma}\sum_{ij}\int \frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}\boldsymbol{n}_i \mathrm{d}\boldsymbol{n}_j}\frac{E_i^n E_j^n}{Q^{2n}}\delta^{(2)}(\boldsymbol{n}_i - \boldsymbol{n}_1)\delta^{(2)}(\boldsymbol{n}_j - \boldsymbol{n}_2)$$

Where i, j are final state hadrons and σ_{ij} is the inclusive cross section to produce i, j with a hard scale Q.

We integrate out the global $O(3)$ symmetry due to find the distribution we're interested in.

$$\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} = \int \mathrm{d}\boldsymbol{n}_{1,2}\frac{\langle \mathcal{E}^n(\boldsymbol{n}_1)\mathcal{E}^n(\boldsymbol{n}_2)\rangle}{Q^{2n}}\delta(\boldsymbol{n}_2 \cdot \boldsymbol{n}_1 - \cos\theta)$$

Let me now set up the perturbative calculation we perform.

Computing Correlation Functions of $\mathcal{E}(\vec{n})$

The average momentum exchange between the two correlator points goes as $\sim \theta Q$, the wide angle region where $\theta Q \gg \Lambda_{\mathrm{QCD}}$ is largely determined by perturbative physics. We therefore write the observable as a sum over inclusive partonic cross-sections:

$$\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} = \frac{1}{\sigma}\int \mathrm{d}E_{q,g}\,\frac{\mathrm{d}\hat{\sigma}_{qg}}{\mathrm{d}\theta\mathrm{d}E_q\mathrm{d}E_g}\frac{E_g^n E_q^n}{Q^{2n}} + \frac{1}{\sigma}\int \mathrm{d}E_{g_1,g_2}\,\frac{\mathrm{d}\hat{\sigma}_{g_1 g_2}}{\mathrm{d}\theta\mathrm{d}E_{g_1}\mathrm{d}E_{g_2}}\frac{E_{g_1}^n E_{g_2}^n}{Q^{2n}}$$
$$+ \frac{1}{\sigma}\int \mathrm{d}E_{q_1,q_2}\,\frac{\mathrm{d}\hat{\sigma}_{q_1 q_2}}{\mathrm{d}\theta\mathrm{d}E_{q_1}\mathrm{d}E_{q_2}}\frac{E_{q_1}^n E_{q_2}^n}{Q^{2n}} + (\text{perm. } q \leftrightarrow \bar{q}) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\theta\, Q}\right)$$

In pp collisions this is a simple application of CSS inclusive factorisation and can be convoluted with fragmentation or track functions. We must assume this also holds in AA collisions.

Note the finite number of terms in the sum over partonic cross sections!

Computing Correlation Functions of $\mathcal{E}(\vec{n})$

We now re-parameterise the medium contribution to each partonic cross section. Note this is not a factorisation, just a parameterisation.

$$\frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}\theta \mathrm{d}E_i \mathrm{d}E_j} = \left(1 + F^{(ij)}_{\mathrm{med}}(E_i, E_j, \theta)\right) \frac{\mathrm{d}\hat{\sigma}^{\mathrm{vac}}_{ij}}{\mathrm{d}\theta \mathrm{d}E_i \mathrm{d}E_j}$$

And using this parameterisation we can now compute terms not dependent on $F^{(ij)}_{med}$ using the well developed frameworks from pp physics (pick your favourite between the celestial OPE, SCET, or jet calculus).

I'll show results at LO+NLL for the pp-like terms later on. NLO+NNLL is available in the literature. This part is well understood and not the focus of my talk.

Computing Correlation Functions of $\mathcal{E}(\vec{n})$

Now we must focus on the 'medium' terms that contain the physics intrinsic to HI collisions. So far we've not approximated anything other than assuming perturbative factorisation. Let's introduce some new helpful variables:

$$\int \mathrm{d}E_{q,g}\, F_{\mathrm{med}}^{(qg)} \frac{\mathrm{d}\hat{\sigma}_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta \mathrm{d}E_q \mathrm{d}E_g} \frac{E_q^n E_g^n}{Q^{2n}} = \int \mathrm{d}z\, \mathrm{d}\mu_{\mathrm{s}}\, F_{\mathrm{med}}^{(qg)} \frac{\mathrm{d}\hat{\sigma}_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta \mathrm{d}z \mathrm{d}\mu_{\mathrm{s}}} z^n (1 - z - \mu_{\mathrm{s}}/Q)^n$$

where $z = E_q/Q$ and $\mu_s = Q - E_q - E_g > 0$ is the energy scale of the radiation over which the perturbative cross sections are inclusive.

With this parameterisation and assuming we are measuring quark jets:

$$\bar{\mu}_{\mathrm{s}}/Q \sim \sqrt{\Lambda_{\mathrm{QCD}}/Q}$$

$$\sum_{ij\in\{g,q,\bar{q}\}} \int \mathrm{d}E_{i,j}\, F_{\mathrm{med}}^{(ij)} \frac{\mathrm{d}\hat{\sigma}_{ij}^{\mathrm{vac}}}{\mathrm{d}\theta \mathrm{d}E_i \mathrm{d}E_j} \frac{E_i^n E_j^n}{Q^{2n}}$$
$$= \int \mathrm{d}z\, F_{\mathrm{med}}^{(qg)} \frac{\mathrm{d}\hat{\sigma}_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta \mathrm{d}z} z^n (1-z)^n \left(1 + \mathcal{O}\left(\frac{\bar{\mu}_{\mathrm{s}}}{Q}\right) + \mathcal{O}\left(\alpha_{\mathrm{s}}(\theta Q) \ln \theta \frac{\bar{\mu}_{\mathrm{s}}^n}{Q^n}\right)\right)$$

Computing Correlation Functions of $\mathcal{E}(\vec{n})$

Thus we will compute our observable from the master formula:

$$\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} = \frac{1}{\sigma}\int \mathrm{d}z\left(g^{(n)}(\theta,\alpha_\mathrm{s}) + F_\mathrm{med}(z,\theta)\right)\frac{\mathrm{d}\sigma^\mathrm{vac}_{qg}}{\mathrm{d}\theta\mathrm{d}z} z^n(1-z)^n\left(1+\mathcal{O}\left(\frac{\bar{\mu}_\mathrm{s}}{Q}\right)\right) + \mathcal{O}\left(\frac{\Lambda_\mathrm{QCD}}{\theta\, Q}\right)$$

As promised, the pp-like part at LO+NLL:

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma^\mathrm{vac}_{qg}}{\mathrm{d}\theta\mathrm{d}z} = \frac{\alpha_\mathrm{s}(\theta Q)}{\pi}\; C_\mathrm{F}\frac{1+(1-z)^2}{z\;\theta} + \mathcal{O}(\alpha_\mathrm{s}^2,\theta^0)$$

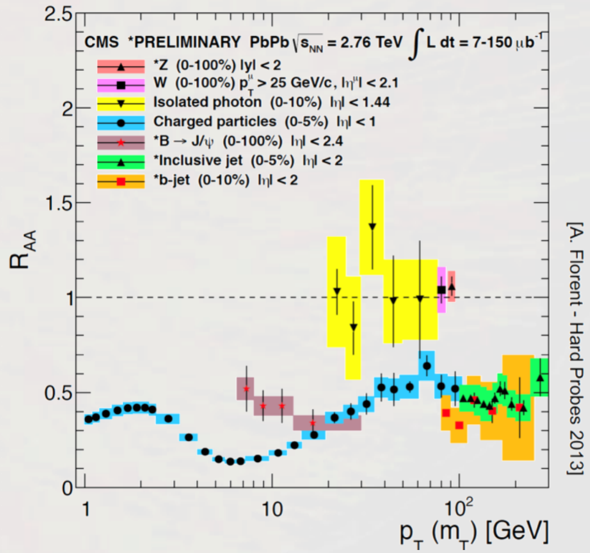
$$\begin{aligned} g^{(1)} = &\left(\left[\left(\frac{\alpha_\mathrm{s}(Q)}{\alpha_\mathrm{s}(\theta Q)}\right)^{\frac{\hat{\gamma}(3)}{\beta_0}}\right]_{qq} + \frac{2n_f(\gamma_{qg}(2)-\gamma_{qg}(3))+\gamma_{gg}(2)-\gamma_{gg}(3)}{\gamma_{qq}(2)-\gamma_{qq}(3)+\gamma_{gq}(2)-\gamma_{gq}(3)}\left[\left(\frac{\alpha_\mathrm{s}(Q)}{\alpha_\mathrm{s}(\theta Q)}\right)^{\frac{\hat{\gamma}(3)}{\beta_0}}\right]_{gq}\right) \\ &+ \mathcal{O}\left(\alpha_\mathrm{s}(Q)^n \ln(\theta)^{n-1}\big|_{n\geq 1}\right) + \mathcal{O}(\theta), \end{aligned} \tag{A.23}$$

where $\hat{\gamma}(J) = \begin{pmatrix} \gamma_{qq}(J), & 2n_f\gamma_{qg}(J), & 0 \\ \gamma_{gq}(J), & \gamma_{gg}(J), & 0 \\ 0, & 0, & \gamma_{g\tilde{g}}(J) \end{pmatrix}$ is the spin-J twist-2 QCD anomalous dimension matrix.

Computing F_{med}

This is what we're interested in. To progress further we will have to input models for the interaction between a jet and a medium. We will rely on the BDMPS-Z approach and use various approximations to solve for $\mathrm{F}_{\mathrm{med}}^{(ij)}$.

Jet modification in HIC key points which we must consider:



Energy loss through medium-induced radiation

- Widely studied in terms of suppression of jets and high-p_T hadrons.

Colour coherence effects – what we want to study!

- Breaking of angular ordering.
- Expected to modify jet inner structure.
- Often understood in terms of simplified calculations, but not yet unequivocally seen in observables.

arXiv:1512.08107, 1710.03237, 1812.05111, 2010.00028, 2210.07901 and more

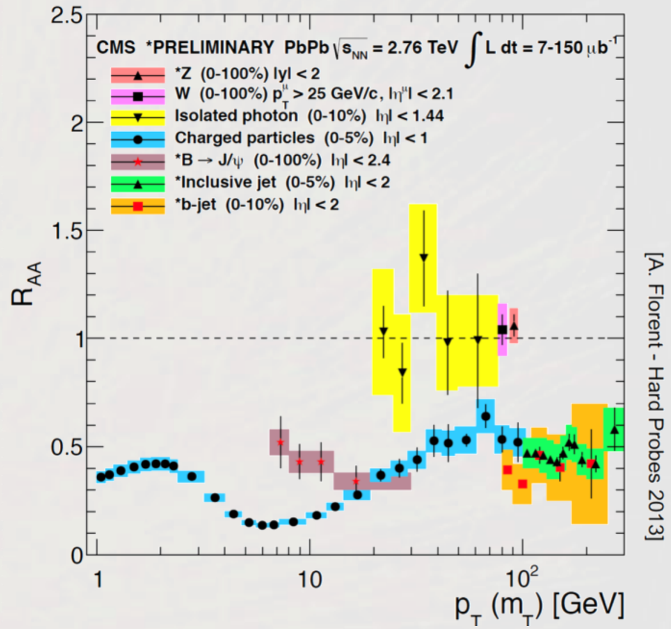
We wont yet consider medium response – this will likely need Monte Carlo.

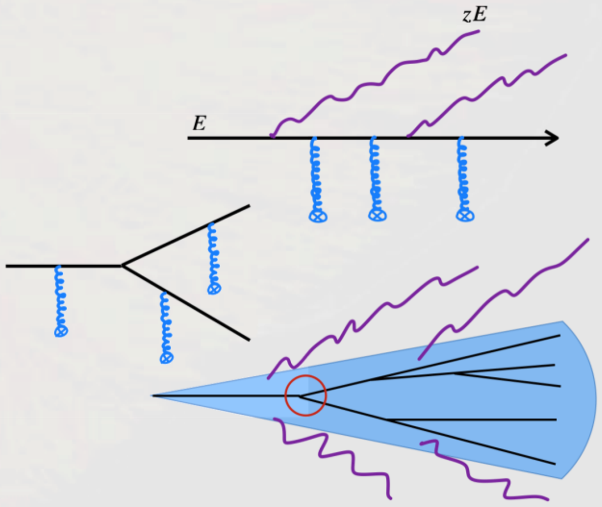
Computing F_{med}

From energy loss to energy correlator jet substructure:

- For typical energy loss calculation we only need the
  - soft limit $z \ll 1$
  - Soft divergence of the vacuum vertex

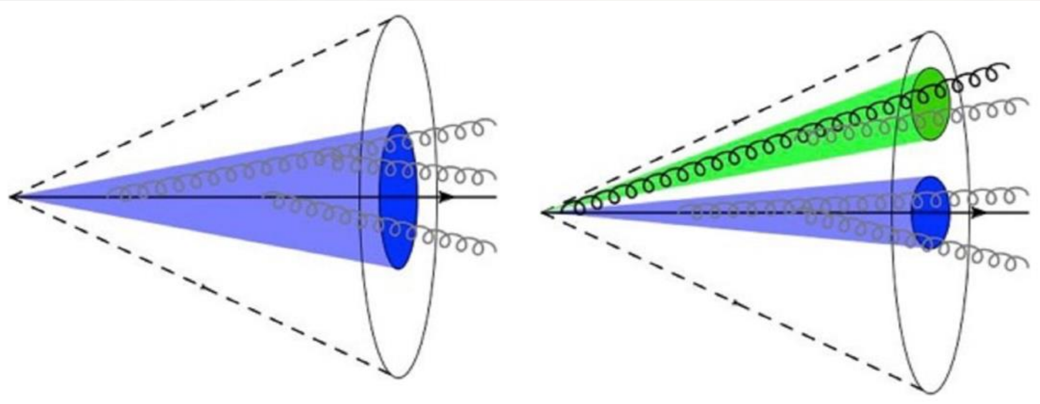
- For energy correlator jet substructure with colour coherence
  - Interplay of emissions from multiple sources or multiple scatters
  - Harder vertices – the energy weight removes the essential soft divergence. Phase space volume matters more.

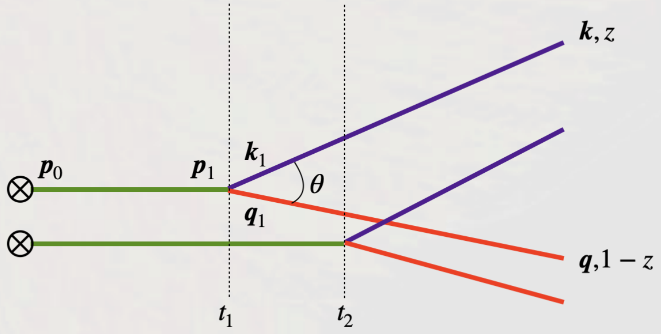




Computing F_{med}

Leading structure to study is a (quark) jet fragmenting into a jet and a subjet in the presence of a medium.



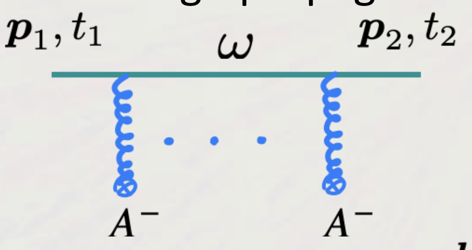


$$\mathcal{M}^{\alpha\beta} = \frac{1}{2E}\int_{\boldsymbol{p}_0\boldsymbol{p}_1\boldsymbol{k}_1\boldsymbol{q}_1}\int_{t_0}^{\infty} dt_1\,(2\pi)^2\delta^{(2)}(\boldsymbol{p}_1-\boldsymbol{k}_1-\boldsymbol{q}_1)\mathcal{G}_{R_b}^{\alpha\alpha_1}(\boldsymbol{k},L;\boldsymbol{k}_1,t_1;zE)$$
$$\times\,\mathcal{G}_{R_c}^{\beta\beta_1}(\boldsymbol{q},L;\boldsymbol{q}_1,t_1;(1-z)E)\,V(\boldsymbol{k}_1-z\boldsymbol{p}_1,z)T^{\alpha_1\beta_1\gamma_1}\,\mathcal{G}_{R_a}^{\gamma_1\gamma}(\boldsymbol{p}_1,t_1;\boldsymbol{p}_0,t_0;E)\,\mathcal{M}_0^{\gamma}(E,\boldsymbol{p}_0)$$

Computing F_{med}

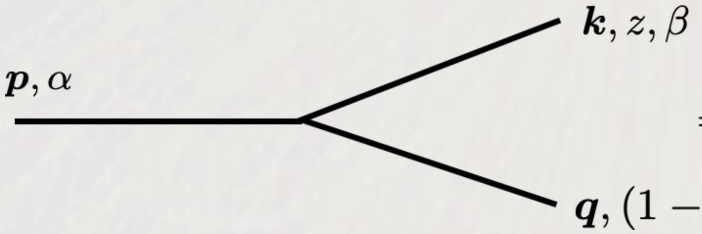
The formalism we use, based in BDMPS-Z:

- All particles have a large longitudinal momentum compared to their transverse momenta and therefore there is a decoupling between transverse and longitudinal dynamics
- We work in a mixed representation with momentum coordinates in the transverse direction and “time” (+ coordinate) in the longitudinal direction.
- Multiple scatterings resumed through propagators in a background field



$$= \mathcal{G}_R(\boldsymbol{p}_2, t_2; \boldsymbol{p}_1, t_1; \omega)$$

- Vacuum vertices



$$= V(\boldsymbol{k} - z\boldsymbol{p}, z) T^{\alpha\beta\gamma} (2\pi)^2 \delta^{(2)}(\boldsymbol{p} - \boldsymbol{k} - \boldsymbol{q})$$

- Background field averaged at the level of the cross section

$$\left\langle A^{a-}(\boldsymbol{q}_1, t_1) A^{b-\dagger}(\boldsymbol{q}_2, t_2) \right\rangle = \delta^{ab} \delta(t_2 - t_1) \delta^{(2)}(\boldsymbol{q}_1 - \boldsymbol{q}_2) v(\boldsymbol{q}_1)$$

Computing F_{med}

Full evaluation keeping z and θ not yet achieved.

Two available approximations:

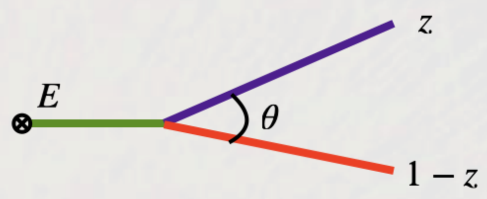
- Opacity expansion ($N = 1$) arXiv:1807.03799
  - Unitarity problems can lead to negative cross sections.
  - Recursive formulas to generate all orders (not yet implemented numerically).

- “Tilted” Wilson lines
  - Resums multiple scatterings in the eikonal approximation.
  - Assumes semi-hard splittings (z not too small).
  - We implement this using both a Yukawa and HO potential for medium scatterings and for now using the leading colour limit.

arXiv:1907.03653

arXiv:2107.02542

Computing F_{med}



For intuition, focus on HO for a bit.

- For a static medium of length within the harmonic approximation one can read off the relevant scales directly from the formulas
  arXiv:1907.03653
  - (Vacuum) formation time:

$$t_f = \frac{2}{z(1-z)E\theta^2} \qquad \theta_L \sim (EL)^{-1/2}$$

  - Decoherence time:

$$S_{12}(\tau) = e^{-\frac{1}{12}\hat{q}(1+z^2)\theta^2\tau^3}$$

$$t_d \sim (\hat{q}\theta^2)^{-1/3} \qquad \theta_c \sim (\hat{q}L^3)^{-1/2}$$

If $\theta_L > \theta_c$ then θ_c becomes irrelevant

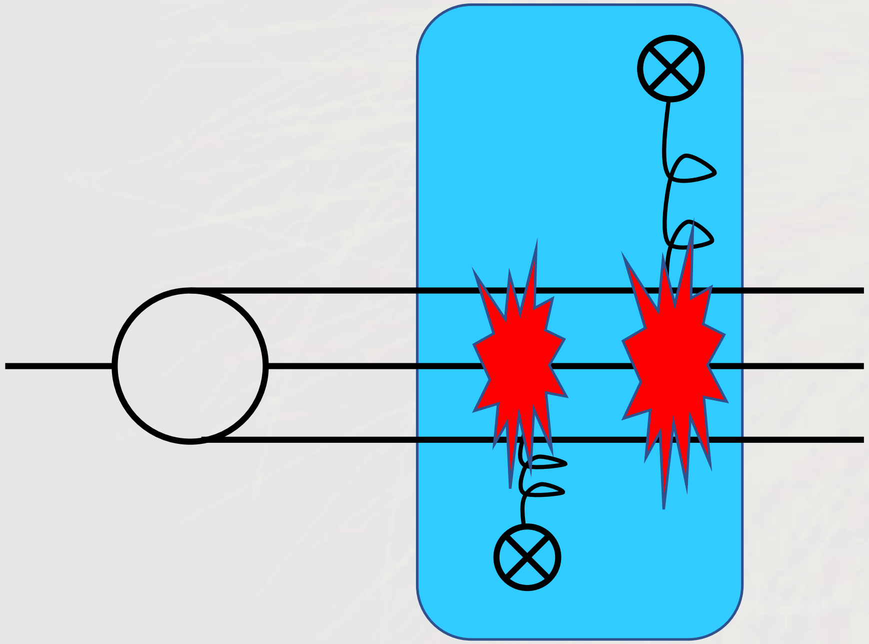
Below θ_L all emissions have a formation time larger than L. This emerges as complete cancellations between dipole and quadrapole (in-out and in-in) terms in $\mathrm{F}_{\mathrm{med}}^{(ij)}$ driving it rapidly to zero below θ_L.

Below θ_c emissions do not colour decohere and the medium does not independently resolve them. This emerges as an exponential suppression in the factorisable dipole terms within $\mathrm{F}_{\mathrm{med}}^{(ij)}$ forcing it to become small below θ_c.

Interpretation for F_{med}

$\theta_{\mathrm{c}} \gg \theta_{\mathrm{L}}$

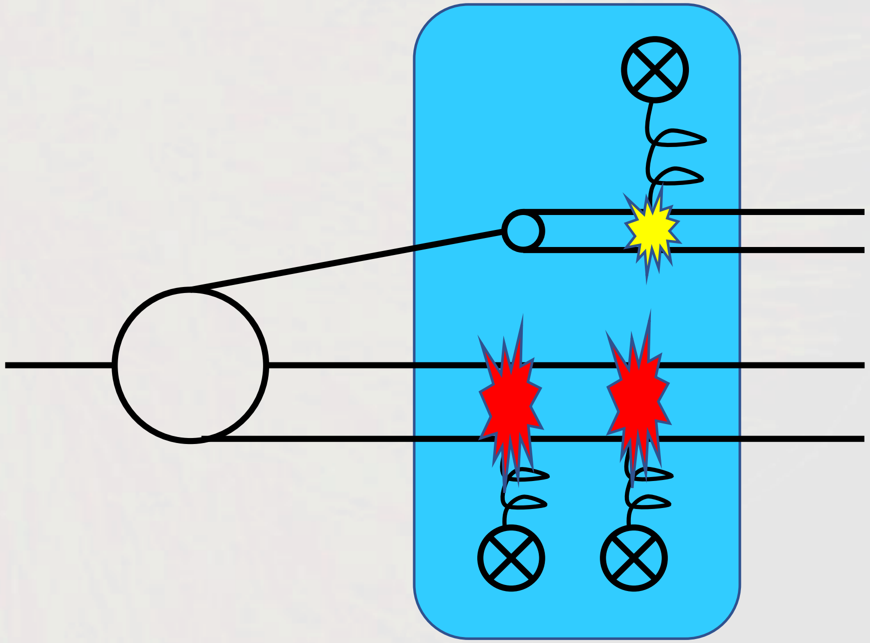
$E \gg \hat{q}L^2$



For angles $\theta_{\mathrm{c}} \gg \theta \gg \theta_{\mathrm{L}}$, the quark jet undergoes some minimal energy loss but the substructure is not resolved.

$\theta_{\mathrm{c}} \ll \theta_{\mathrm{L}}$

$E \ll \hat{q}L^2$



Initial splitting can be resolved by the medium when $\theta \gg \theta_{\mathrm{L}}$. Broadening and energy loss occur.

Numerical evaluation of F_{med}

Numerical integration is needed

$$F_{\text{med}} = 2\int_0^L \frac{\mathrm{d}t_1}{t_\mathrm{f}} \left[\int_{t_1}^L \frac{\mathrm{d}t_2}{t_\mathrm{f}} \cos\left(\frac{t_2 - t_1}{t_\mathrm{f}}\right) \mathcal{C}^{(4)}(L, t_2)\mathcal{C}^{(3)}(t_2, t_1) - \sin\left(\frac{L - t_1}{t_\mathrm{f}}\right) \mathcal{C}^{(3)}(L, t_1) \right]$$

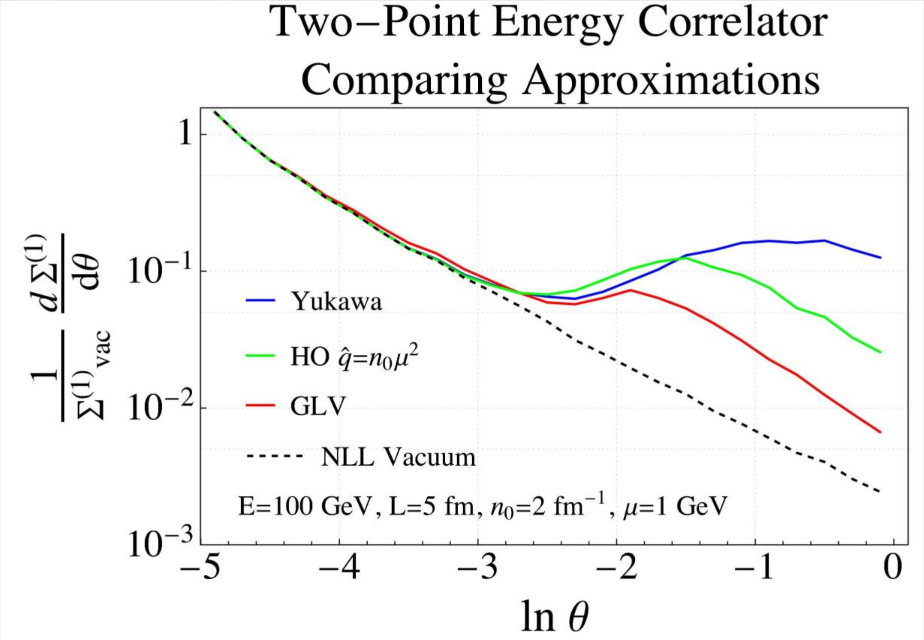
$$\mathcal{C}^{(3)}_{gq}(t_2, t_1) = \frac{1}{N_c^2 - 1} \left\langle \mathrm{tr}[V_2^\dagger V_1]\, \mathrm{tr}[V_0^\dagger V_2] - \frac{1}{N_c} \mathrm{tr}[V_0^\dagger V_1] \right\rangle .$$

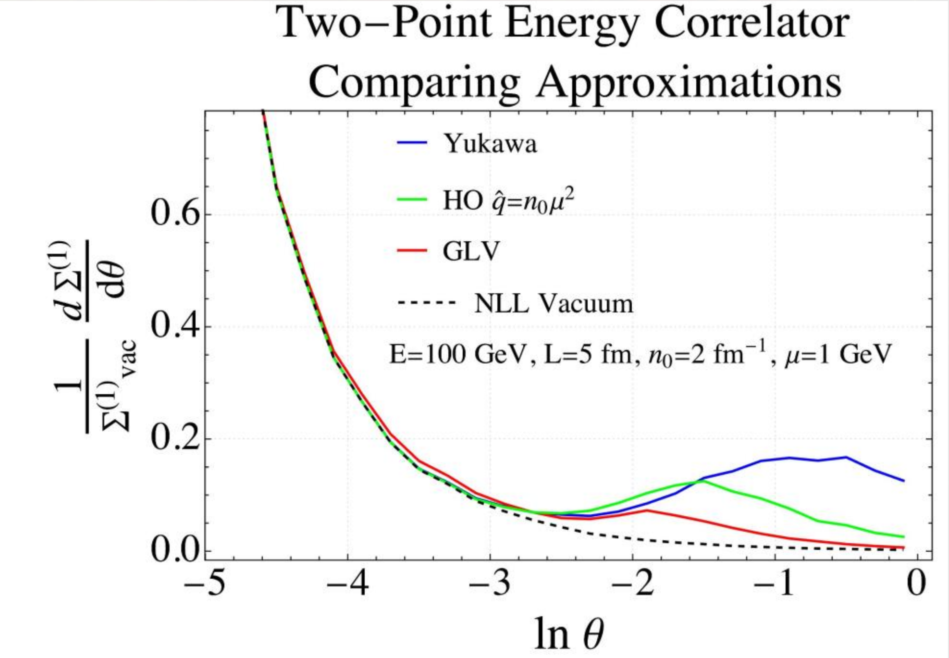
$$\begin{aligned}\mathcal{C}^{(3)}_{gq}(t_2, t_1) &= \mathrm{e}^{-\frac{1}{2}\int_{t_1}^{t_2} \mathrm{d}s\, n(s)[N_c(\sigma_{02}+\sigma_{12}) - \frac{1}{N_c}\sigma_{01}]} \\ &= \mathrm{e}^{-\frac{1}{12}\hat{q}(t_2-t_1)^3\theta^2\left(1+z^2+\frac{2z}{N_c^2-1}\right)} .\end{aligned}$$

$$\mathcal{C}^{(4)}_{gq}(L, t_2) = \frac{1}{N_c^2 - 1} \left\langle \mathrm{tr}[V_{\bar{1}}^\dagger V_1 V_2^\dagger V_{\bar{2}}]\, \mathrm{tr}[V_{\bar{2}}^\dagger V_2] - \frac{1}{N_c} \mathrm{tr}[V_{\bar{1}}^\dagger V_1] \right\rangle ,$$

$$\begin{aligned}&\frac{1}{N_c^2}\langle \mathrm{tr}[V_1 V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger]\, \mathrm{tr}[V_2 V_{\bar{2}}^\dagger]\rangle \simeq \mathrm{e}^{-\frac{1}{4}\hat{q}\theta^2(t-t_2)(t_2-t_1)^2(1-2z+3z^2)} \\ &\times \left(1 - \frac{1}{2}\hat{q}\theta^2 z(1-z)(t_2-t_1)^2 \int_{t_2}^t \mathrm{d}s\, \mathrm{e}^{-\frac{1}{12}\hat{q}\theta^2\left[(s-t_2)^2(2s-3t_1+t_2)+6z(1-z)(s-t_2)(t_2-t_1)^2\right]}\right)\end{aligned}$$

Numerical evaluation of F_{med}

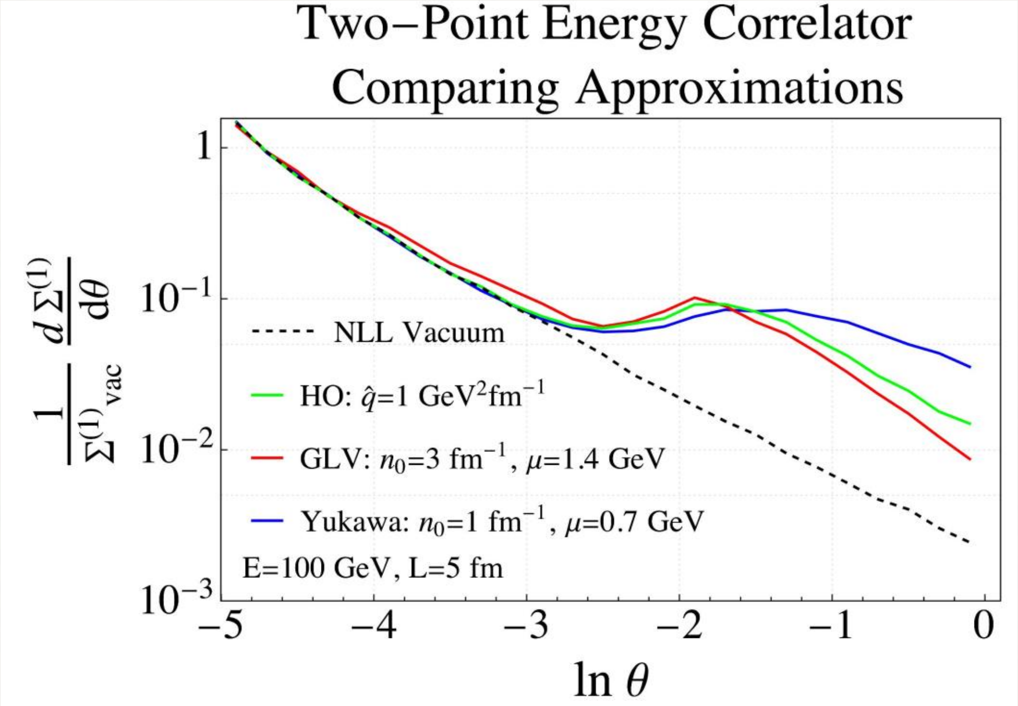




General features: an enhancement which begins above θ_L, at $\theta \gg \theta_L$ the enhancement peaks and then settles into a new medium dependent scaling law.

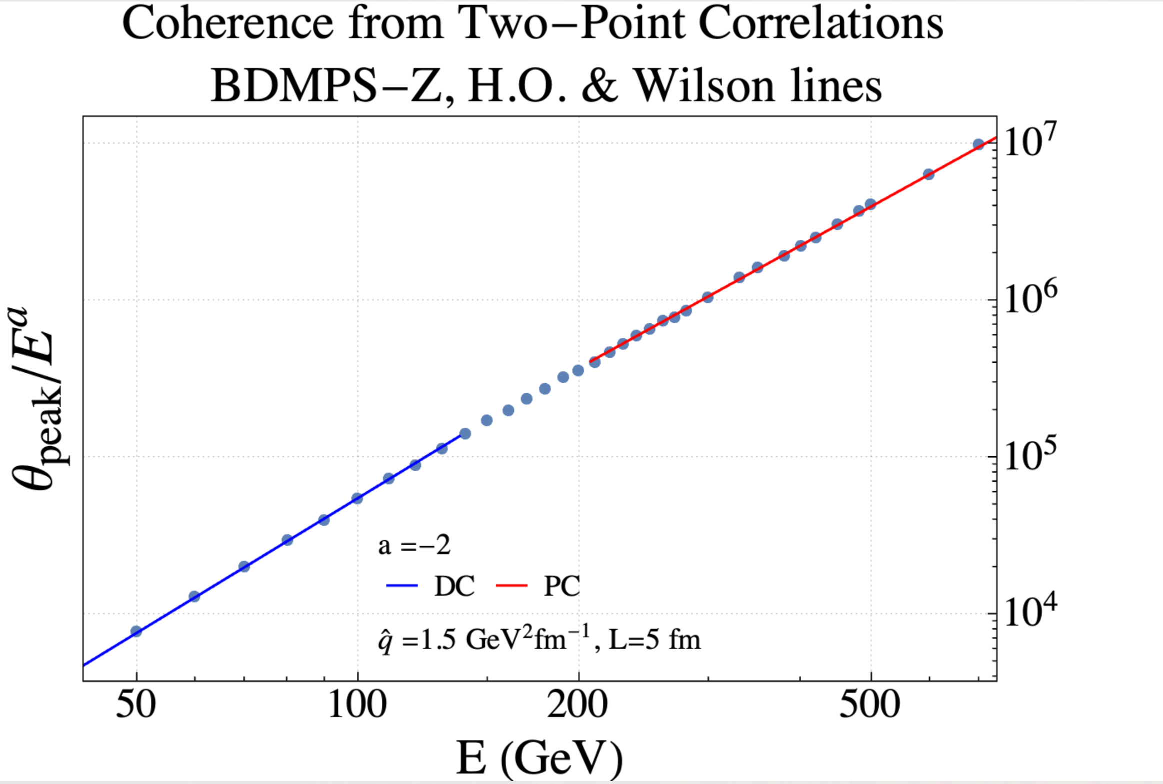
Amplitudes appear model dependent.

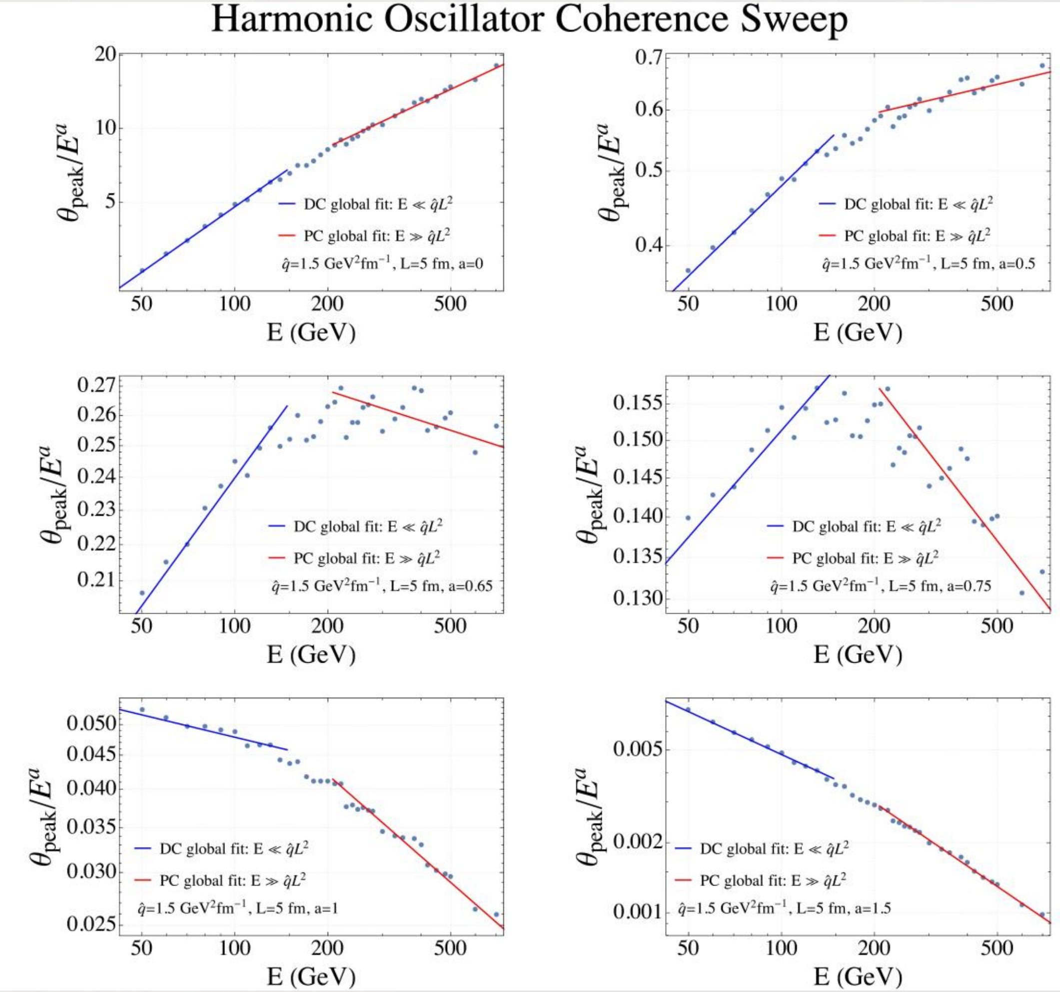
Numerical evaluation of F_{med}



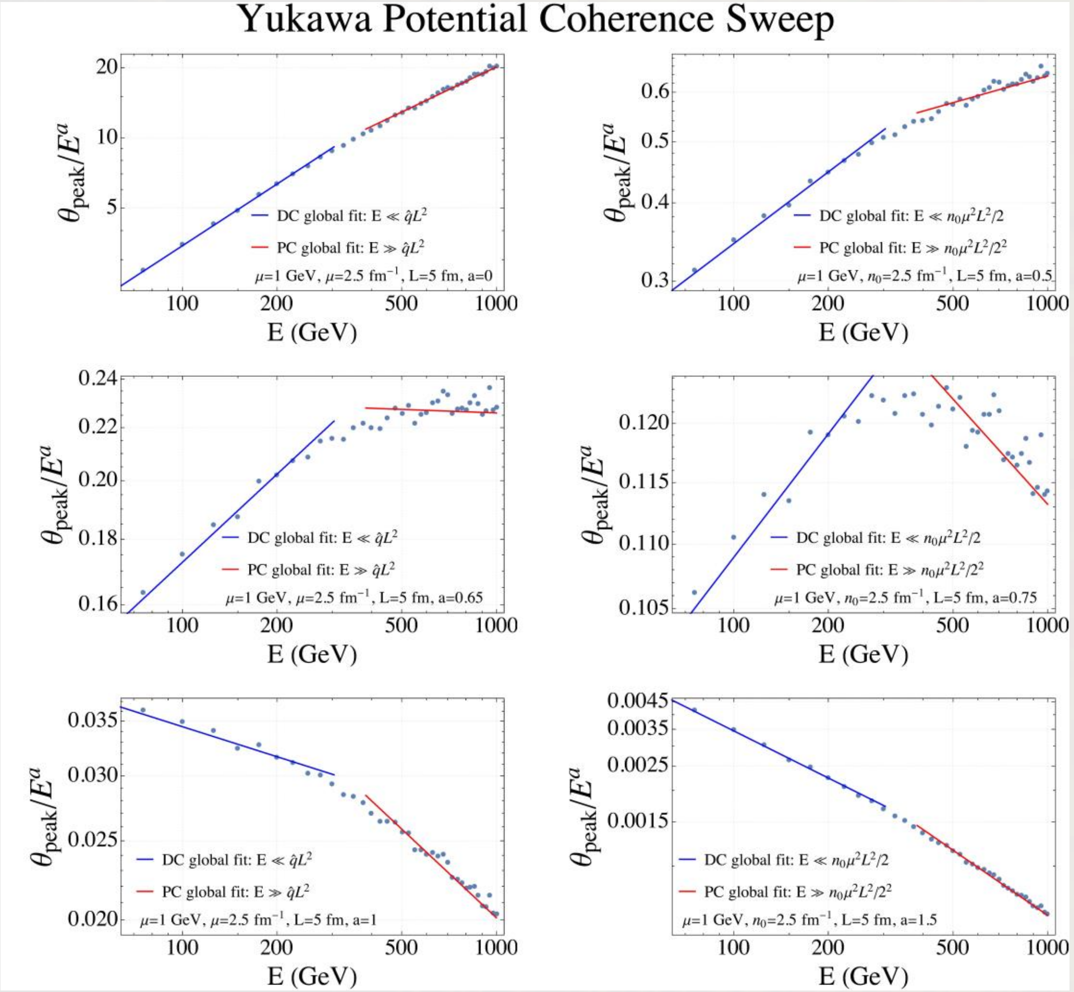
Whilst amplitudes are very model dependent, the differences can be fairly well absorbed into variation of the model parameters (not so much the wide angle though).

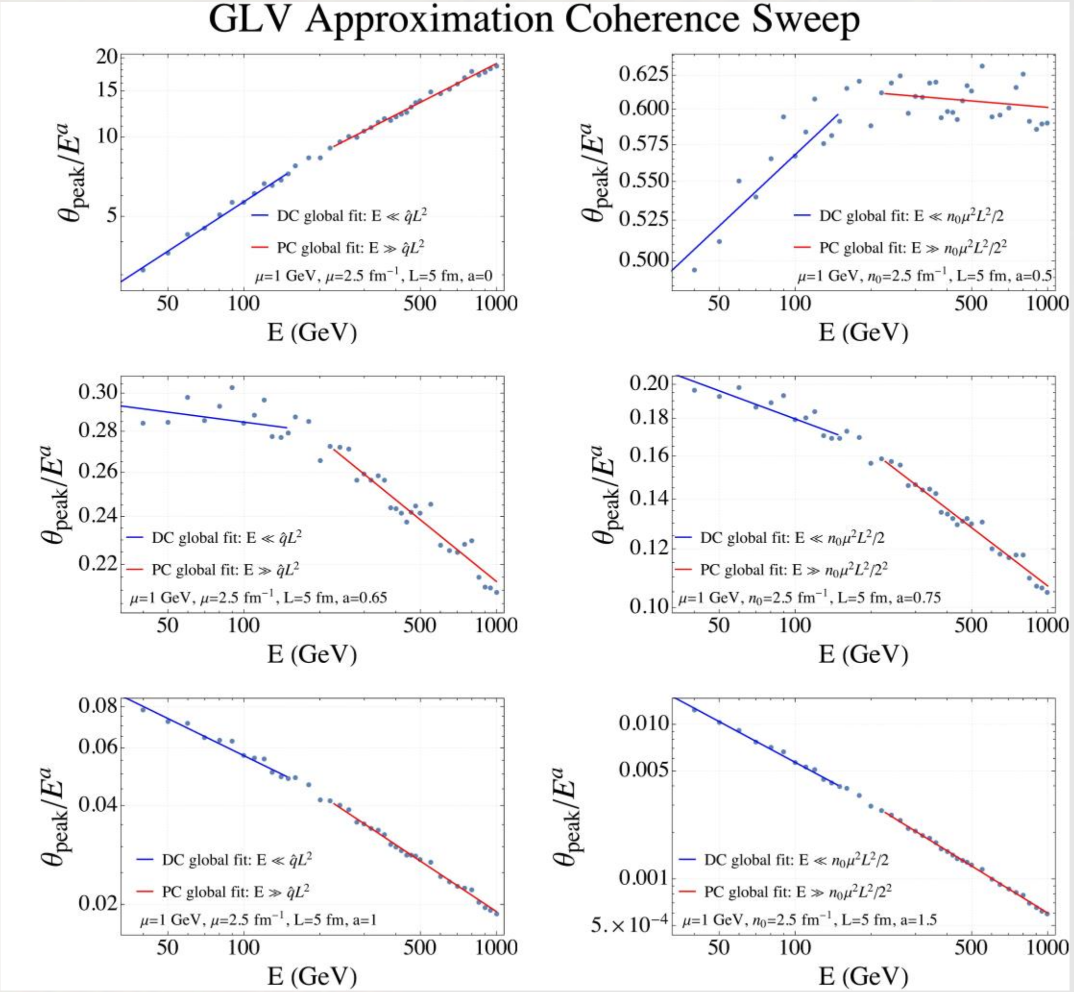
Numerical evaluation of F_{med}





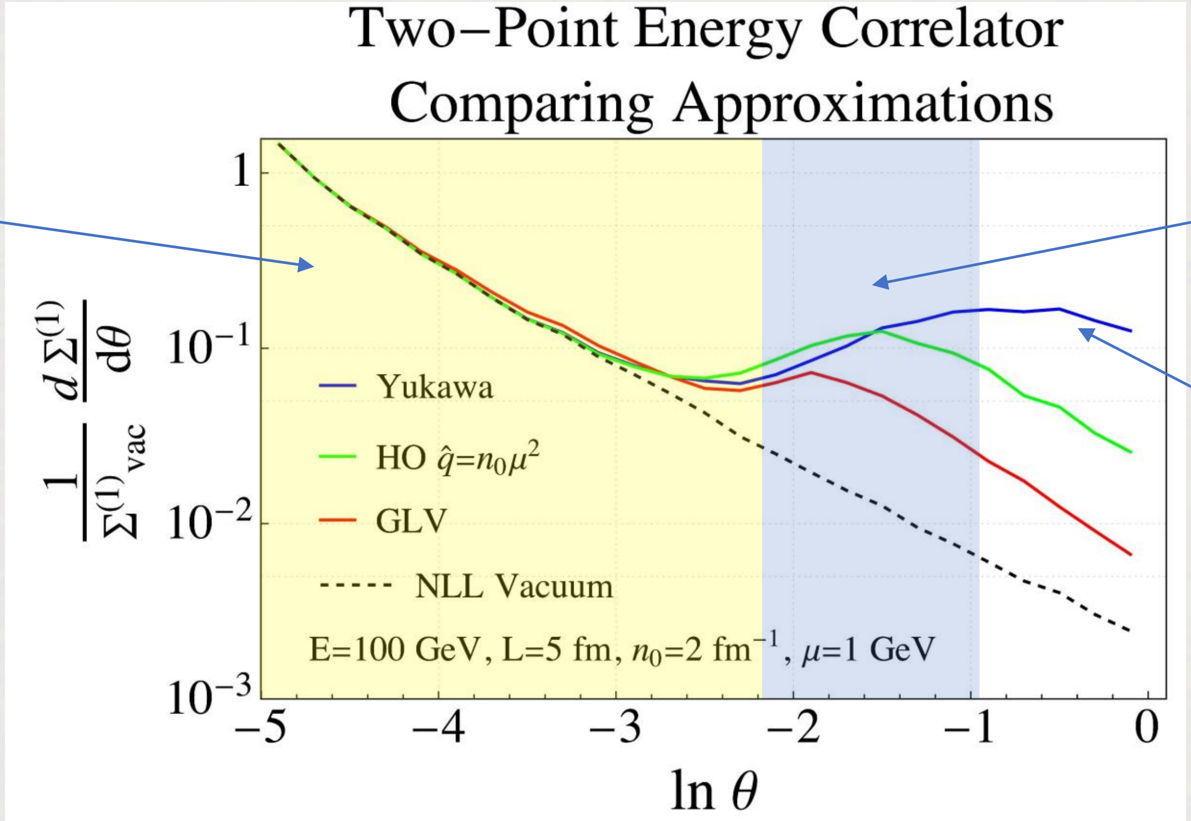
Numerical evaluation of F_{med}





Numerical evaluation of F_{med}

Controlled by θ_L



Controlled by $(\theta_c - \theta_L)$

Controlled by number of large scatters limit. Behaves as if medium is infinitely long and so always decoherent.

Provided $E \gg E_c \sim \hat{q}L^2$

Questions and observations

Coherence is a feature independent of the approach we take to evaluating the BDMPS-Z framework.

It doesn’t seem clearly associated with a particular crisp angle or energy. Analytically, harsh boundaries on the phase-space of radiation emerge from approximations of emission kernels and medium interactions. In more complete numerical evaluations these boundaries are fuzzy.

Energy correlators provide a nice window into coherence and general features of jet quenching. Other models can be readily input – if you can compute F_{med}, you can compute this observable.

How does the observable change with a more realistic medium?

What other information can we extract? Can we access the 3-point correlator?

Conclusions

Energy Correlators are cool and fun! 😁

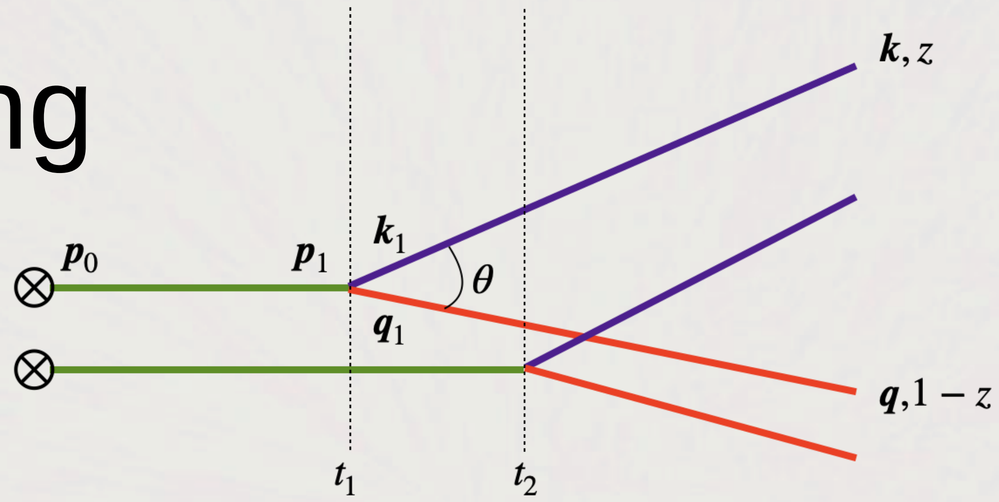
The simple and inclusive nature of energy correlators perhaps provides a window into degrees of theoretical control than can be very hard to otherwise achieve in jet substructure within HI physics.

We've shown that (within the BDMPS-Z) framework, coherence can be accessed in a model independent way through energy correlator spectra.

There are several steps that needed to be taken to produce less idealised predictions that can be compared with experiment. Also, the novelty of these observables possibly introduces several experimental challenges. However, they show great promise and so I think it is worth the effort!

Part N/A: Supplemental Material

Jet Quenching



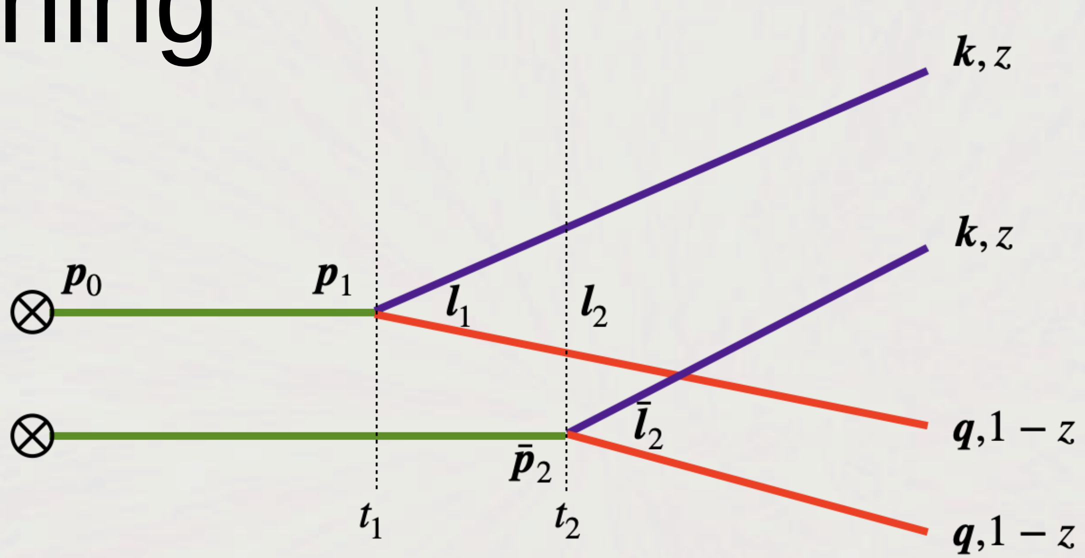
$$\mathcal{M}^{\alpha\beta} = \frac{1}{2E}\int_{\boldsymbol{p}_0\boldsymbol{p}_1\boldsymbol{k}_1\boldsymbol{q}_1}\int_{t_0}^{\infty} dt_1\,(2\pi)^2\delta^{(2)}(\boldsymbol{p}_1-\boldsymbol{k}_1-\boldsymbol{q}_1)\mathcal{G}_{R_b}^{\alpha\alpha_1}(\boldsymbol{k},L;\boldsymbol{k}_1,t_1;zE)$$
$$\times\,\mathcal{G}_{R_c}^{\beta\beta_1}(\boldsymbol{q},L;\boldsymbol{q}_1,t_1;(1-z)E)\,V(\boldsymbol{k}_1-z\boldsymbol{p}_1,z)T^{\alpha_1\beta_1\gamma_1}\,\mathcal{G}_{R_a}^{\gamma_1\gamma}(\boldsymbol{p}_1,t_1;\boldsymbol{p}_0,t_0;E)\,\mathcal{M}_0^{\gamma}(E,\boldsymbol{p}_0)$$

Where each of the in-medium propagators is of the form:

$$\mathcal{G}_R(t_2,\boldsymbol{x}_2;t_1,\boldsymbol{x}_1;\omega) = \int_{\boldsymbol{x}_1}^{\boldsymbol{x}_2}\mathcal{D}\boldsymbol{r}\,\exp\left\{\frac{i\omega}{2}\int_{t_1}^{t_2}d\xi\;\dot{\boldsymbol{r}}^2(\xi)\right\}\underbrace{\mathrm{P}\exp\left\{ig\int_{t_1}^{t_2}d\xi\;A_R^-(\xi,\boldsymbol{r}(\xi))\right\}}_{V_R(t_2,t_1;[\boldsymbol{r}])}$$

$$\left\langle|\mathcal{M}|^2\right\rangle \propto \Big\langle\mathcal{G}_{R_b}^{\alpha\alpha_1}(\boldsymbol{k},L;\boldsymbol{k}_1,t_1;zE)\mathcal{G}_{R_c}^{\beta\beta_1}(\boldsymbol{q},L;\boldsymbol{q}_1,t_1;(1-z)E)\mathcal{G}_{R_b}^{\dagger\bar{\alpha}_2\alpha}(\bar{\boldsymbol{k}}_2,t_2;\boldsymbol{k},L;zE)$$
$$\times\mathcal{G}_{R_c}^{\dagger\bar{\beta}_2\beta}(\bar{\boldsymbol{q}}_2,t_2;\boldsymbol{q},L;(1-z)E)\mathcal{G}_{R_a}^{\gamma_1\gamma}(\boldsymbol{p}_1,t_1;\boldsymbol{p}_0,t_0;E)\mathcal{G}_{R_a}^{\dagger\bar{\gamma}\bar{\gamma}_2}(\bar{\boldsymbol{p}}_0,t_0;\bar{\boldsymbol{p}}_2,t_2;E)\Big\rangle$$

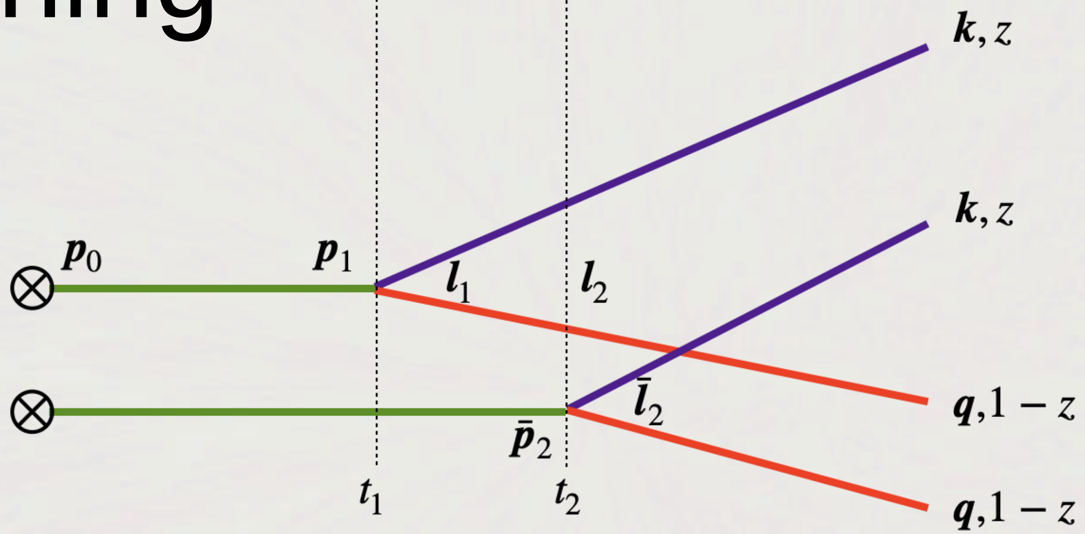
Jet Quenching



$$\frac{d\sigma}{d\Omega_k d\Omega_q} = \frac{g^2}{z(1-z)E^2} P_{a\to bc}(z)\, 2\mathrm{Re} \int_{\boldsymbol{p}_0\boldsymbol{p}_1\bar{\boldsymbol{p}}_2\boldsymbol{l}_1\boldsymbol{l}_2\bar{\boldsymbol{l}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\boldsymbol{l}_1 \cdot \bar{\boldsymbol{l}}_2)$$
$$\times\, \mathcal{S}^{(4)}((1-z)\boldsymbol{k} - z\boldsymbol{q}, L; \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2, t_2; \boldsymbol{k}+\boldsymbol{q}-\bar{\boldsymbol{p}}_2, z)$$
$$\times\, \mathcal{K}^{(3)}(\boldsymbol{l}_2, t_2; \boldsymbol{l}_1, t_1; \bar{\boldsymbol{p}}_2 - \boldsymbol{p}_1, z)\, \mathcal{P}_{R_a}(\boldsymbol{p}_1 - \boldsymbol{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}}$$

Jet Quenching

BDMPS-Z



$$\frac{d\sigma}{d\Omega_k d\Omega_q} = \frac{g^2}{z(1-z)E^2} P_{a\to bc}(z)\, 2\mathrm{Re}\int_{\boldsymbol{p}_0\boldsymbol{p}_1\bar{\boldsymbol{p}}_2\boldsymbol{l}_1\boldsymbol{l}_2\bar{\boldsymbol{l}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\boldsymbol{l}_1\cdot\bar{\boldsymbol{l}}_2)$$

$$\times\, \mathcal{S}^{(4)}((1-z)\boldsymbol{k} - z\boldsymbol{q}, L; \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2, t_2; \boldsymbol{k}+\boldsymbol{q}-\bar{\boldsymbol{p}}_2, z)$$

$$\times\, \mathcal{K}^{(3)}(\boldsymbol{l}_2, t_2; \boldsymbol{l}_1, t_1; \bar{\boldsymbol{p}}_2 - \boldsymbol{p}_1, z)\, \mathcal{P}_{R_a}(\boldsymbol{p}_1 - \boldsymbol{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}}$$

$\langle \mathcal{G}\mathcal{G}\mathcal{G}^\dagger\mathcal{G}^\dagger \rangle$

$\langle \mathcal{G}\mathcal{G}\mathcal{G}^\dagger \rangle$

$\langle \mathcal{G}\mathcal{G}^\dagger \rangle$

Part N/A: Supplemental Material

$$\mathcal{M}_{\gamma\to q\bar{q}} = \frac{e}{E}\mathrm{e}^{i\frac{\boldsymbol{p}_1^2}{2zE}L+i\frac{\boldsymbol{p}_2^2}{2(1-z)E}L}\int_0^\infty \mathrm{d}t \int_{\boldsymbol{k}_1,\boldsymbol{k}_2}\left[\mathcal{G}(\boldsymbol{p}_1,L;\boldsymbol{k}_1,t|zE)\,\bar{\mathcal{G}}(\boldsymbol{p}_2,L;\boldsymbol{k}_2,t|(1-z)E)\right]_{ij}$$
$$\times\,\gamma_{\lambda,s,s'}(z)\boldsymbol{k}\cdot\boldsymbol{\epsilon}_\lambda^*\,\mathcal{G}_0(\boldsymbol{k}_1+\boldsymbol{k}_2,t|E)$$

$$\mathcal{G}(\boldsymbol{p}_1,t_1;\boldsymbol{p}_0,t_0) = \int_{\boldsymbol{x}_1,\boldsymbol{x}_2}\mathrm{e}^{-i\boldsymbol{p}_1\cdot\boldsymbol{x}_1+i\boldsymbol{p}_0\cdot\boldsymbol{x}_0}\mathcal{G}(\vec{x}_1,\vec{x}_0)$$

$$\mathcal{G}(\vec{x}_1,\vec{x}_0) = \int_{\boldsymbol{r}(t_0)=\boldsymbol{x}_0}^{\boldsymbol{r}(t_1)=\boldsymbol{x}_1}\mathcal{D}\boldsymbol{r}\,\exp\left[i\frac{E}{2}\int_{t_0}^{t_1}\mathrm{d}s\,\dot{\boldsymbol{r}}^2\right]V(t_1,t_0;[\boldsymbol{r}])$$

$$V(t_1,t_0;[\boldsymbol{r}]) = \mathcal{P}\exp\left[ig\int_{t_0}^{t_1}\mathrm{d}t\,\mathbf{t}^a A^{-,a}(t,\boldsymbol{r}(t))\right]$$

$$\frac{\mathrm{d}N^{\mathrm{med}}}{\mathrm{d}z\mathrm{d}\boldsymbol{p}^2} = \frac{1}{4(2\pi)^2\,z(1-z)}\langle|\mathcal{M}_{\gamma\to q\bar{q}}|^2\rangle = \frac{1}{4(2\pi)^2\,z(1-z)}\langle\left|\mathcal{M}^{\mathrm{in}}_{\gamma\to q\bar{q}}+\mathcal{M}^{\mathrm{out}}_{\gamma\to q\bar{q}}\right|^2\rangle$$

Part N/A: Supplemental Material

$$\frac{\mathrm{d}\sigma_{qg}}{\mathrm{d}\theta\mathrm{d}z} = \frac{\mathrm{d}\sigma_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta\mathrm{d}z}(1 + F_{\mathrm{med}}(z,\theta,\hat{q},L))$$

$$F_{\mathrm{med}} = 2\int_0^L \frac{\mathrm{d}t_1}{t_\mathrm{f}}\left[\int_{t_1}^L \frac{\mathrm{d}t_2}{t_\mathrm{f}}\cos\left(\frac{t_2-t_1}{t_\mathrm{f}}\right)\mathcal{C}^{(4)}(L,t_2)\mathcal{C}^{(3)}(t_2,t_1) - \sin\left(\frac{L-t_1}{t_\mathrm{f}}\right)\mathcal{C}^{(3)}(L,t_1)\right]$$

$$\mathcal{C}_{gq}^{(3)}(t_2,t_1) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}[V_2^\dagger V_1]\,\mathrm{tr}[V_0^\dagger V_2] - \frac{1}{N_c}\,\mathrm{tr}[V_0^\dagger V_1]\right\rangle .$$

$$\begin{aligned}\mathcal{C}_{gq}^{(3)}(t_2,t_1) &= \mathrm{e}^{-\frac{1}{2}\int_{t_1}^{t_2}\mathrm{d}s\,n(s)[N_c(\sigma_{02}+\sigma_{12})-\frac{1}{N_c}\sigma_{01}]} \\ &= \mathrm{e}^{-\frac{1}{12}\hat{q}(t_2-t_1)^3\theta^2\left(1+z^2+\frac{2z}{N_c^2-1}\right)} .\end{aligned}$$

$$\mathcal{C}_{gq}^{(4)}(L,t_2) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}[V_{\bar{1}}^\dagger V_1 V_2^\dagger V_{\bar{2}}]\,\mathrm{tr}[V_{\bar{2}}^\dagger V_2] - \frac{1}{N_c}\,\mathrm{tr}[V_{\bar{1}}^\dagger V_1]\right\rangle ,$$

$$\begin{aligned}&\frac{1}{N_c^2}\langle \mathrm{tr}[V_1 V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger]\,\mathrm{tr}[V_2 V_{\bar{2}}^\dagger]\rangle \simeq \mathrm{e}^{-\frac{1}{4}\hat{q}\theta^2(t-t_2)(t_2-t_1)^2(1-2z+3z^2)} \\ &\times\left(1 - \frac{1}{2}\hat{q}\theta^2 z(1-z)(t_2-t_1)^2\int_{t_2}^{t}\mathrm{d}s\,\mathrm{e}^{-\frac{1}{12}\hat{q}\theta^2\left[(s-t_2)^2(2s-3t_1+t_2)+6z(1-z)(s-t_2)(t_2-t_1)^2\right]}\right)\end{aligned}$$