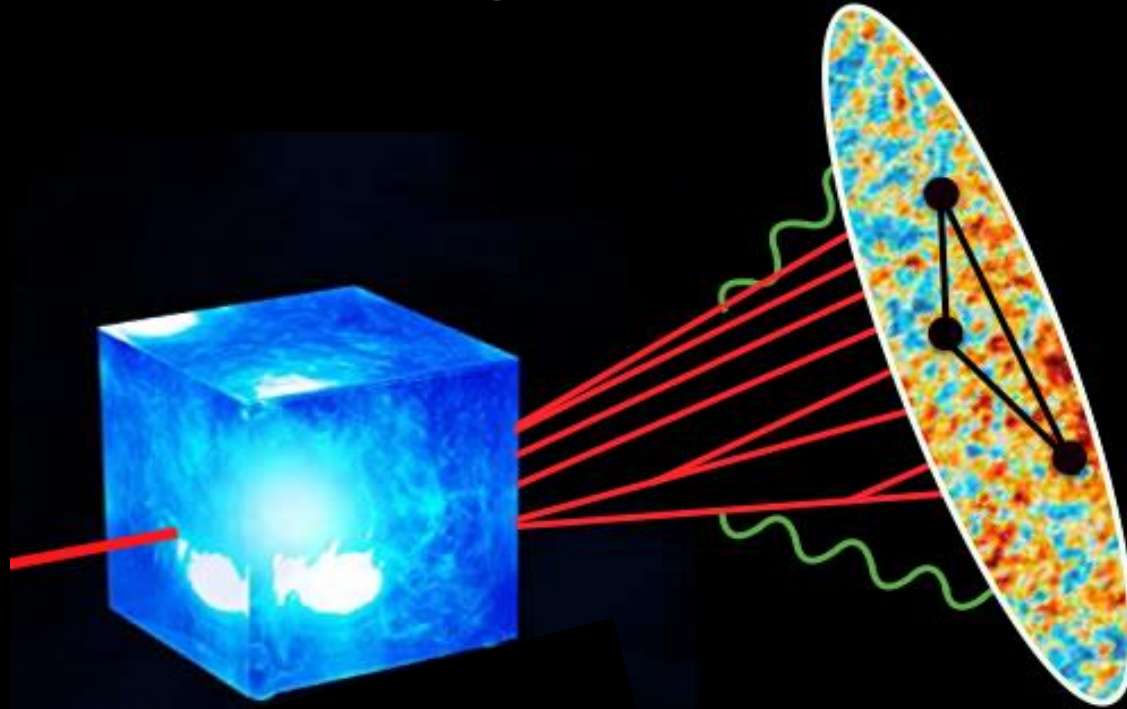


# A Coherent View of Jet Quenching from Energy Correlators



Jack Holguin

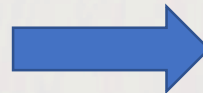
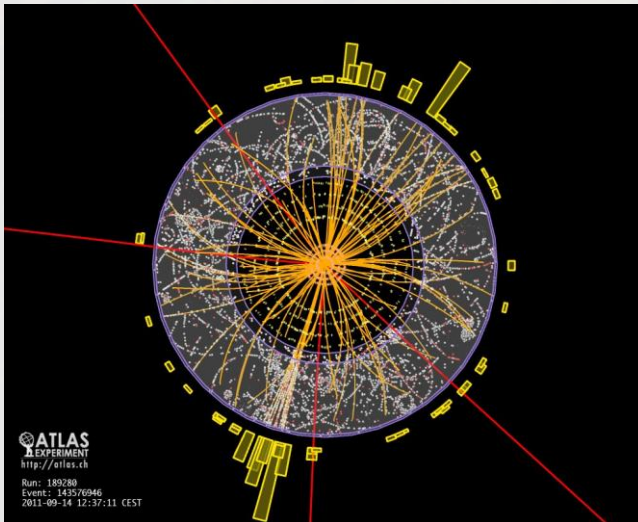
In collaboration with Carlota Andres, Fabio Dominguez, Cyrille Marquet, Ian Moulton

# Introduction

Context:

- There has been an exceptionally successful program of studying QCD in vacuum and medium with event shape observables and algorithmic substructure.

$$\frac{d\sigma}{de_1 \dots de_n} = \sum_N \int d\sigma_N \delta(f_1(N, e_1, \dots, e_n) - e_1) \dots \delta(f_n(N, e_1, \dots, e_n) - e_n)$$



$\{e_1, e_2, \dots, e_n\}$

# Introduction

## Context:

- However, there is an interesting disconnect between much of the developments in collider QCD phenomenology and formal theory.

arXiv:1309.0769v2 [hep-th] 1 May 2014

Conformal collider physics:  
Energy and charge correlations

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Princeton, NJ 08540, USA

We study observables in a conformal field theory which are used to describe hadronic events at colliders. We focus on the energies deposited on calorimeters placed at a large distance. We consider initial states produced by an operator insertion and study the properties of the energy correlation functions for conformal field theories. We find that the small angle singularities of energy correlation functions are universal and can be described by a simple formula.

Abstract

We present a new approach to computing event shape distributions or, more precisely, charge flow correlations in a generic conformal field theory (CFT). These infrared finite observables are familiar from collider physics studies and describe the angular distribution of global charges in outgoing radiation created from the vacuum by some source. The charge flow correlations can be expressed in terms of Wightman correlation functions in a certain limit. We explain how to compute these quantities starting from their Euclidean analogues by means of a non-trivial analytic continuation which, in the framework of CFT, can be performed elegantly in Mellin space.

13/02/2023

8v3 [hep-th] 26 Aug 2019

Light-ray operators in conformal field theory

Petr Kravchuk and David Simmons-Duffin

Walter Burke Institute for Theoretical Physics, Caltech, Pasadena, California 91125, USA

ABSTRACT: We argue that every CFT contains light-ray operators labeled by a continuous spin  $J$ . When  $J$  is a positive integer, light-ray operators become integrals of local operators over a null line. However for non-integer  $J$ , light-ray operators are genuinely nonlocal and give the analytic continuation of our construction. We generalize the construction to arbitrary dimensions.

CAI  
CER

May 2020

The light-ray OPE and conformal colliders

Murat Khabib, Petr Kravchuk<sup>b</sup>, David Simmons-Duffin<sup>a</sup>, and Alexander L. Fitzpatrick

CERN-PH-TH/2013-211  
IPhT-T13-210  
LAPTH-047/13

From correlation functions to event shapes

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Princeton, NJ 08544, USA

Abstract

We present a new approach to computing event shape distributions or, more precisely, charge flow correlations in a generic conformal field theory (CFT). These infrared finite observables are familiar from collider physics studies and describe the angular distribution of global charges in outgoing radiation created from the vacuum by some source. The charge flow correlations can be expressed in terms of Wightman correlation functions in a certain limit. We explain how to compute these quantities starting from their Euclidean analogues by means of a non-trivial analytic continuation which, in the framework of CFT, can be performed elegantly in Mellin space.

Xiv:1610.05308v1 [hep-th] 17 Oct 2016

Averaged Null Energy Condition from Causality

Thomas Hartman, Sandipan Kundu, and Amirhossein Tajdini

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Abstract

Unitary, Lorentz-invariant quantum field theories in flat spacetime obey microcausality: commutators vanish at spacelike separation. For interacting theories in more than two dimensions, we show that this implies that the averaged null energy,  $\int du T_{uu}$ , must be positive. This non-local operator appears in the operator product expansion of local operators in the lightcone limit, and therefore contributes to  $n$ -point functions. We derive a sum rule that isolates this contribution and is manifestly positive. The argument also applies to certain higher spin operators other than the stress tensor, generating an infinite family of new constraints of the form  $\int du X_{uv} T_{uv} \geq 0$ . These lead to new inequalities for the coupling constants of spinning operators in conformal field theory, which include as special cases (but are generally stronger than) the existing constraints from the lightcone bootstrap, deep inelastic scattering, conformal collider methods, and relative entropy. We also comment on the relation to the recent derivation of the averaged null energy condition from relative entropy, and suggest a more general formulation.

CALT-TH 2018-018

Modular Hamiltonians for Deformed Half-Spaces and the Averaged Null Energy Condition

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Abstract

We study modular Hamiltonians corresponding to the vacuum state for deformed half-spaces in relativistic quantum field theories on  $\mathbb{R}^{1,d-1}$ . We show that in addition to the averaged null energy condition, there are new constraints on the coupling constants of spinning operators in conformal field theory, which include as special cases (but are generally stronger than) the existing constraints from the lightcone bootstrap, deep inelastic scattering, conformal collider methods, and relative entropy. We also comment on the relation to the recent derivation of the averaged null energy condition from relative entropy, and suggest a more general formulation.

# Introduction

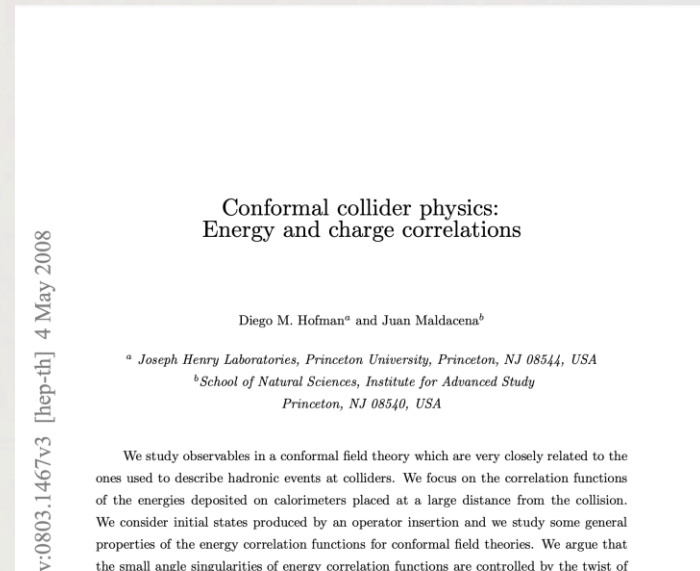
What's the middle ground?

$$\mathcal{E}(\vec{n}) = \lim_{r \rightarrow \infty} \int_0^{\infty} dt \, r^2 n^i T_{0i}(t, r\vec{n})$$

The energy flow operator is an ANEC operator, it can be computed directly in CFTs admitting many new computational techniques.

Its correlation functions are also directly collider observables – Energy Correlators.

We want to apply these to HL collisions to study jet quenching.



# Introduction

Very quickly, what is  $\mathcal{E}(\vec{n})$ ?

$\mathcal{E}(\vec{n}) =$  Idealised calorimeter output at a solid angle labelled by  $\vec{n}$ .

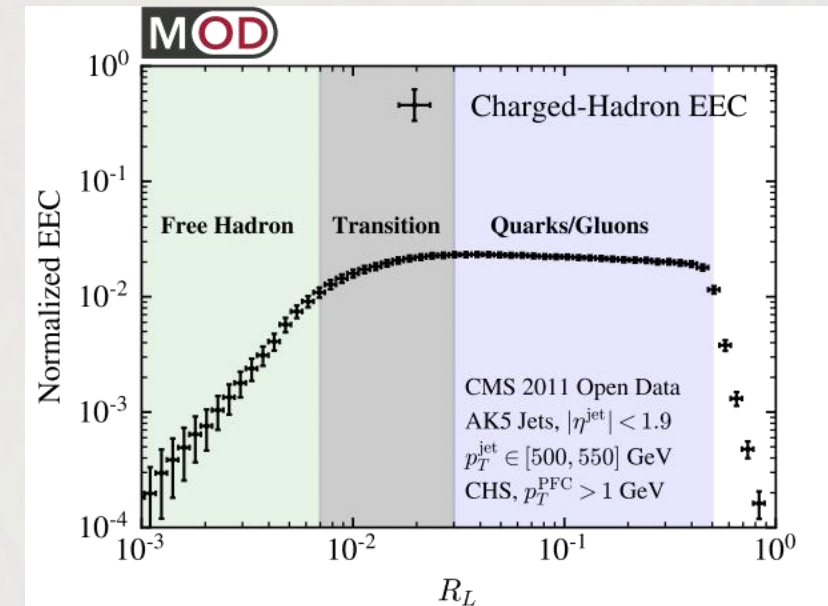
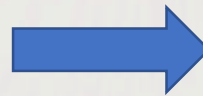
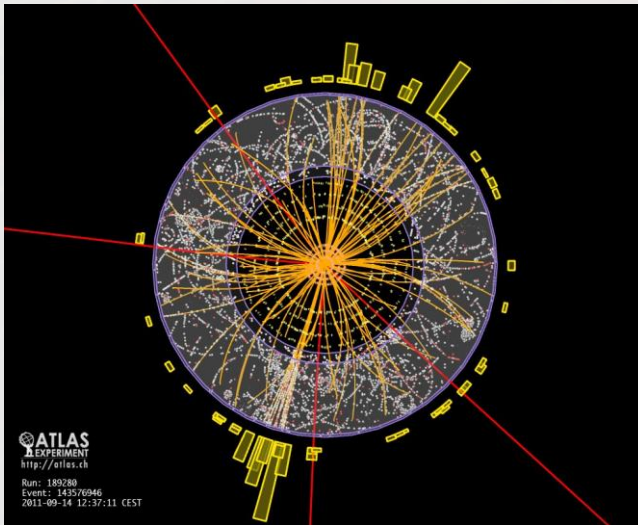
Correlation functions of  $\mathcal{E}(\vec{n})$  quantify the correlations between the average calorimeter outputs at different points across the celestial sphere from a particular process.

They are functions of the angles between the calorimeters.



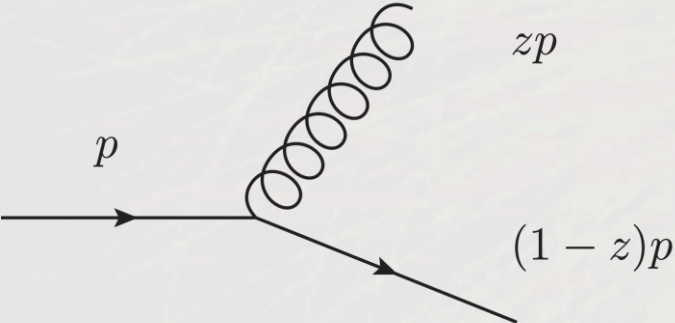
# Introduction

Important difference between correlators and more ‘typical’ observables.



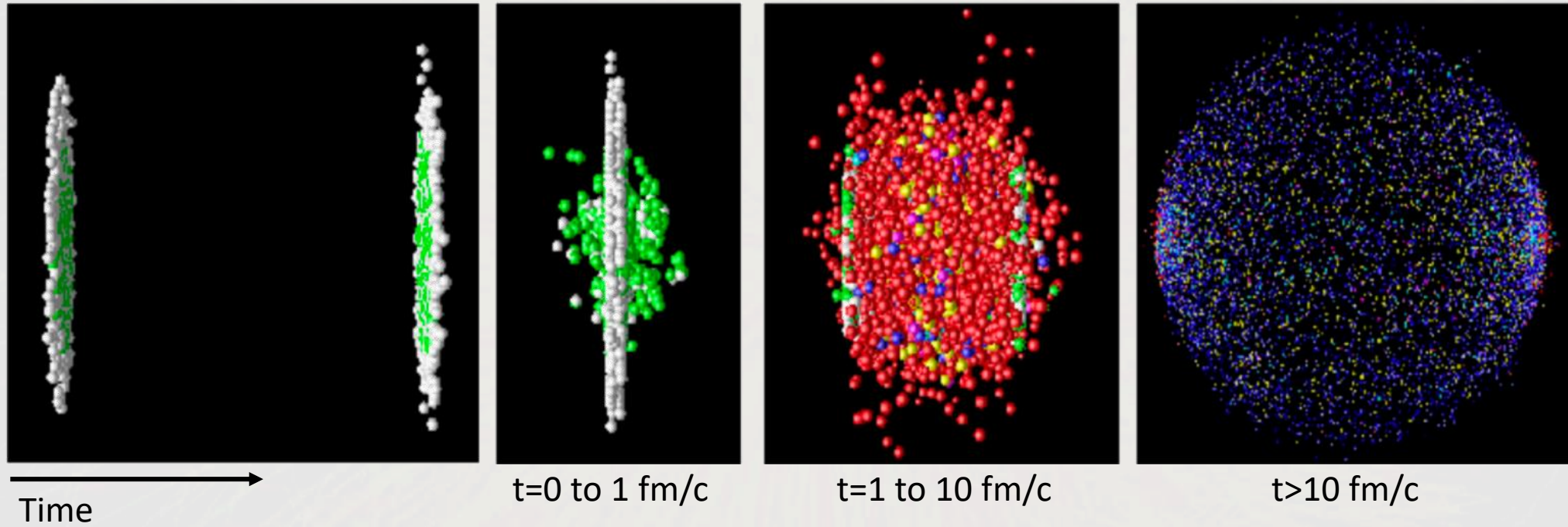
# Introduction

Additional to the computational benefits from formal theory, phenomenologically energy correlators are very good at isolating parts of multiscale dynamics.


$$t_f = \frac{2}{z(1-z)p_0\theta^2} \xrightarrow{z \approx \frac{1}{2}} \theta^2 \sim t_f^{-1} \sim p_{\text{exchanged}}^2$$

The angular size of a correlation often can be interpreted as a time parameter for the physics inducing the correlations.

# Introduction



The QGP in heavy ion collisions (HIC):

– 20 years of HIC at RHIC, 10 years of HIC at the LHC, sPHENIX coming soon.



# Computing Correlation Functions of $\mathcal{E}(\vec{n})$

## Recap

- Correlation functions in statistics:
  - $\text{Corr}_2(X, Y) = \langle XY \rangle - \langle X \rangle \langle Y \rangle$  (also just the covariance)
  - $\text{Corr}_3(X, Y, Z) = \langle XYZ \rangle - \langle X \rangle (\langle Y \rangle \langle Z \rangle - \text{Corr}_2(Y, Z))$
  - ...
- $\langle \mathcal{E}(\vec{n}_1) \dots \mathcal{E}(\vec{n}_i) \rangle \equiv \overline{\text{Corr}_i}(\mathcal{E}(\vec{n}_1), \dots, \mathcal{E}(\vec{n}_i))$  rather than a time ordered QFT correlator. Strictly they are Wightman functions.

# Computing Correlation Functions of $\mathcal{E}(\vec{n})$

Which correlation function is the one for us?

- In the earlier slides the 2-point correlator gives a sensitive probe of hadronisation.
- In [2201.08393](#) the 3-point provided a sensitive probe to the top mass.

Look to what is currently done and successful.

- $R_{AA}$  can be expressed as a function of one-point correlators + corrections:
  - $R_{AA} = \langle N_{AA} \rangle / (\langle N_{\text{Coll}} \rangle \langle N_{pp} \rangle)$ .  $\langle N \rangle$  is the one point correlator of the number operator and due to momentum conservation  $\langle N \rangle \approx \langle \mathcal{E} \rangle / \langle Q \rangle$ .
- In effect,  $R_{AA}$  gives access to the simplest but also least sensitive correlator.
- Let us systematically increase the sensitivity by looking directly at the 2-point correlator.

# Computing Correlation Functions of $\mathcal{E}(\vec{n})$

$$\frac{\langle \mathcal{E}^n(\mathbf{n}_1) \mathcal{E}^n(\mathbf{n}_2) \rangle}{Q^{2n}} = \frac{1}{\sigma} \sum_{ij} \int \frac{d\sigma_{ij}}{d\mathbf{n}_i d\mathbf{n}_j} \frac{E_i^n E_j^n}{Q^{2n}} \delta^{(2)}(\mathbf{n}_i - \mathbf{n}_1) \delta^{(2)}(\mathbf{n}_j - \mathbf{n}_2)$$

Where  $i, j$  are final state hadrons and  $\sigma_{ij}$  is the inclusive cross section to produce  $i, j$  with a hard scale  $Q$ .

We integrate out the global  $O(3)$  symmetry due to find the distribution we're interested in.

$$\frac{d\Sigma^{(n)}}{d\theta} = \int d\mathbf{n}_{1,2} \frac{\langle \mathcal{E}^n(\mathbf{n}_1) \mathcal{E}^n(\mathbf{n}_2) \rangle}{Q^{2n}} \delta(\mathbf{n}_2 \cdot \mathbf{n}_1 - \cos \theta)$$

Let me now set up the perturbative calculation we perform.

# Computing Correlation Functions of $\mathcal{E}(\vec{n})$

The average momentum exchange between the two correlator points goes as  $\sim \theta Q$ , the wide angle region where  $\theta Q \gg \Lambda_{\text{QCD}}$  is largely determined by perturbative physics. We therefore write the observable as a sum over inclusive partonic cross-sections:

$$\begin{aligned} \frac{d\Sigma^{(n)}}{d\theta} = & \frac{1}{\sigma} \int dE_{q,g} \frac{d\hat{\sigma}_{qg}}{d\theta dE_q dE_g} \frac{E_g^n E_q^n}{Q^{2n}} + \frac{1}{\sigma} \int dE_{g_1,g_2} \frac{d\hat{\sigma}_{g_1 g_2}}{d\theta dE_{g_1} dE_{g_2}} \frac{E_{g_1}^n E_{g_2}^n}{Q^{2n}} \\ & + \frac{1}{\sigma} \int dE_{q_1,q_2} \frac{d\hat{\sigma}_{q_1 q_2}}{d\theta dE_{q_1} dE_{q_2}} \frac{E_{q_1}^n E_{q_2}^n}{Q^{2n}} + (\text{perm. } q \leftrightarrow \bar{q}) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{\theta Q}\right) \end{aligned}$$

In  $pp$  collisions this is a simple application of CSS inclusive factorisation and can be convoluted with fragmentation or track functions. We must assume this also holds in  $AA$  collisions.

Note the finite number of terms in the sum over partonic cross sections!

# Computing Correlation Functions of $\mathcal{E}(\vec{n})$

We now re-parameterise the medium contribution to each partonic cross section. Note this is not a factorisation, just a parameterisation.

$$\frac{d\hat{\sigma}_{ij}}{d\theta dE_i dE_j} = \left(1 + F_{\text{med}}^{(ij)}(E_i, E_j, \theta)\right) \frac{d\hat{\sigma}_{ij}^{\text{vac}}}{d\theta dE_i dE_j}$$

And using this parameterisation we can now compute terms not dependent on  $F_{\text{med}}^{(ij)}$  using the well developed frameworks from  $pp$  physics (pick your favourite between the celestial OPE, SCET, or jet calculus).

I'll show results at LO+NLL for the  $pp$ -like terms later on. NLO+NNLL is available in the literature. This part is well understood and not the focus of my talk.



# Computing Correlation Functions of $\mathcal{E}(\vec{n})$

Now we must focus on the ‘medium’ terms that contain the physics intrinsic to HI collisions. So far we’ve not approximated anything other than assuming perturbative factorisation. Let’s introduce some new helpful variables:


$$\int dE_{q,g} F_{\text{med}}^{(qg)} \frac{d\hat{\sigma}_{qg}^{\text{vac}}}{d\theta dE_q dE_g} \frac{E_q^n E_g^n}{Q^{2n}} = \int dz d\mu_s F_{\text{med}}^{(qg)} \frac{d\hat{\sigma}_{qg}^{\text{vac}}}{d\theta dz d\mu_s} z^n (1 - z - \mu_s/Q)^n$$

where  $z = E_q/Q$  and  $\mu_s = Q - E_q - E_g > 0$  is the energy scale of the radiation over which the perturbative cross sections are inclusive.

With this parameterisation and assuming we are measuring quark jets:

$$\begin{aligned} \sum_{ij \in \{g, q, \bar{q}\}} \int dE_{i,j} F_{\text{med}}^{(ij)} \frac{d\hat{\sigma}_{ij}^{\text{vac}}}{d\theta dE_i dE_j} \frac{E_i^n E_j^n}{Q^{2n}} &= \int dz F_{\text{med}}^{(qg)} \frac{d\hat{\sigma}_{qg}^{\text{vac}}}{d\theta dz} z^n (1 - z)^n \left( 1 + \mathcal{O}\left(\frac{\bar{\mu}_s}{Q}\right) + \mathcal{O}\left(\alpha_s(\theta Q) \ln \theta \frac{\bar{\mu}_s^n}{Q^n}\right) \right) \end{aligned}$$

$\bar{\mu}_s/Q \sim \sqrt{\Lambda_{\text{QCD}}/Q}$



# Computing Correlation Functions of $\mathcal{E}(\vec{n})$

Thus we will compute our observable from the master formula:

$$\frac{d\Sigma^{(n)}}{d\theta} = \frac{1}{\sigma} \int dz \left( g^{(n)}(\theta, \alpha_s) + F_{\text{med}}(z, \theta) \right) \frac{d\sigma_{qg}^{\text{vac}}}{d\theta dz} z^n (1-z)^n \left( 1 + \mathcal{O}\left(\frac{\bar{\mu}_s}{Q}\right) \right) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{\theta Q}\right)$$

As promised, the  $pp$ -like part at LO+NLL:

$$\frac{1}{\sigma} \frac{d\sigma_{qg}^{\text{vac}}}{d\theta dz} = \frac{\alpha_s(\theta Q)}{\pi} C_F \frac{1 + (1-z)^2}{z \theta} + \mathcal{O}(\alpha_s^2, \theta^0)$$

$$g^{(1)} = \left( \left[ \left( \frac{\alpha_s(Q)}{\alpha_s(\theta Q)} \right)^{\frac{\hat{\gamma}^{(3)}}{\beta_0}} \right]_{qq} + \frac{2n_f(\gamma_{qq}(2) - \gamma_{qq}(3)) + \gamma_{gg}(2) - \gamma_{gg}(3)}{\gamma_{qq}(2) - \gamma_{qq}(3) + \gamma_{gq}(2) - \gamma_{gq}(3)} \left[ \left( \frac{\alpha_s(Q)}{\alpha_s(\theta Q)} \right)^{\frac{\hat{\gamma}^{(3)}}{\beta_0}} \right]_{gq} \right) + \mathcal{O}\left(\alpha_s(Q)^n \ln(\theta)^{n-1} \Big|_{n \geq 1}\right) + \mathcal{O}(\theta), \quad (\text{A.23})$$

where  $\hat{\gamma}(J) = \begin{pmatrix} \gamma_{qq}(J), & 2n_f \gamma_{qg}(J), & 0 \\ \gamma_{qg}(J), & \gamma_{gg}(J), & 0 \\ 0, & 0, & \gamma_{g\tilde{g}}(J) \end{pmatrix}$  is the spin- $J$  twist-2 QCD anomalous dimension matrix.

# Computing $F_{\text{med}}$

This is what we're interested in. To progress further we will have to input models for the interaction between a jet and a medium. We will rely on the BDMPS-Z approach and use various approximations to solve for  $F_{\text{med}}^{(ij)}$ .

Jet modification in HIC key points which we must consider:

Energy loss through medium-induced radiation

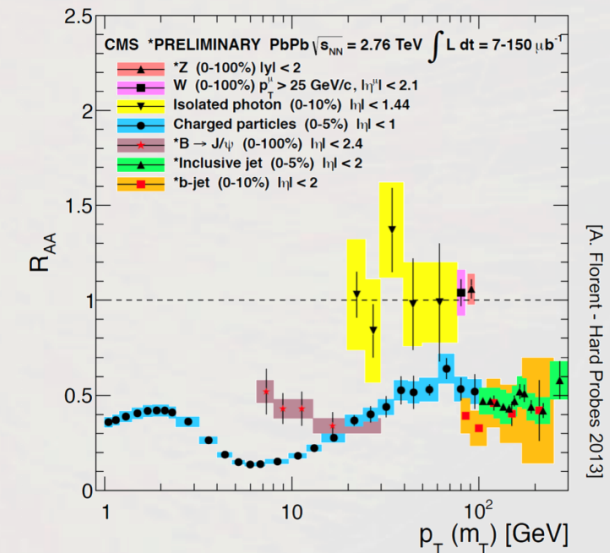
- Widely studied in terms of suppression of jets and high- $p_T$  hadrons.

Colour coherence effects – what we want to study!

- Breaking of angular ordering.
- Expected to modify jet inner structure.
- Often understood in terms of simplified calculations, but not yet unequivocally seen in observables.

[arXiv:1512.08107](https://arxiv.org/abs/1512.08107), [1710.03237](https://arxiv.org/abs/1710.03237), [1812.05111](https://arxiv.org/abs/1812.05111), [2010.00028](https://arxiv.org/abs/2010.00028), [2210.07901](https://arxiv.org/abs/2210.07901) and more

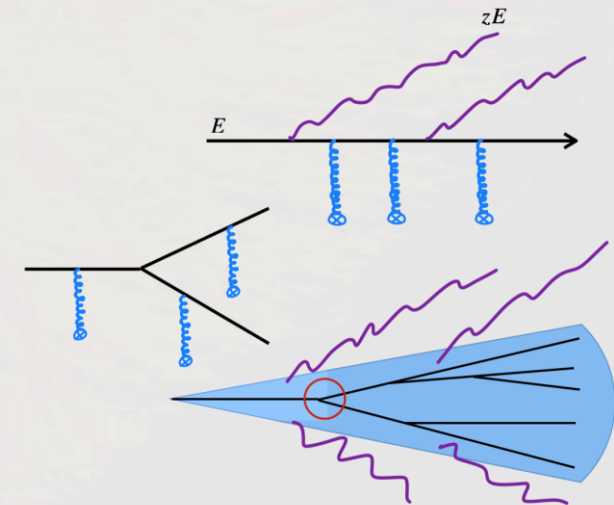
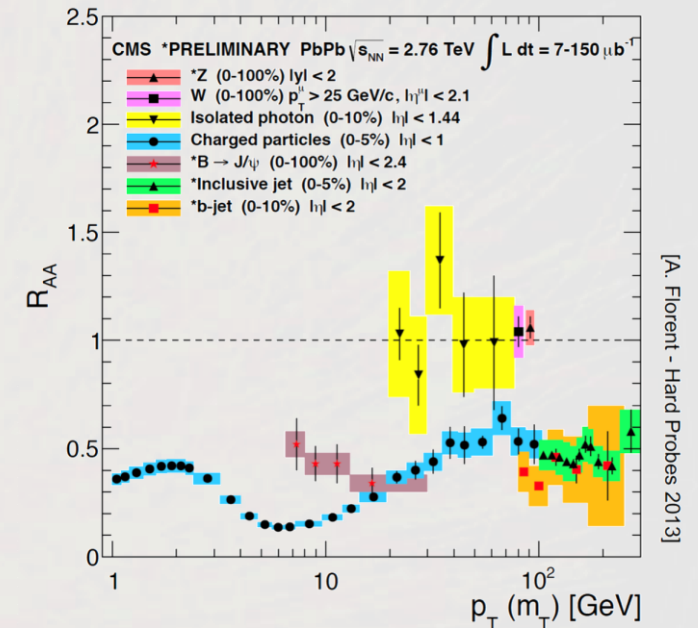
We won't yet consider medium response – this will likely need Monte Carlo.



# Computing $F_{\text{med}}$

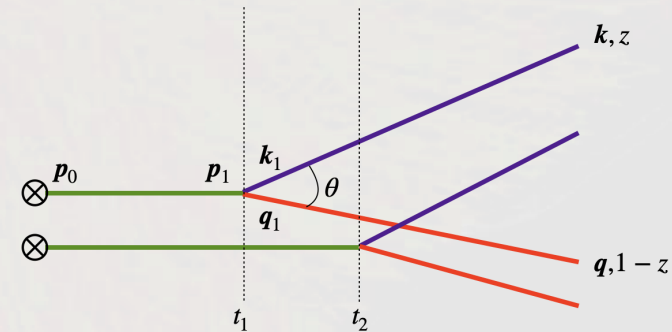
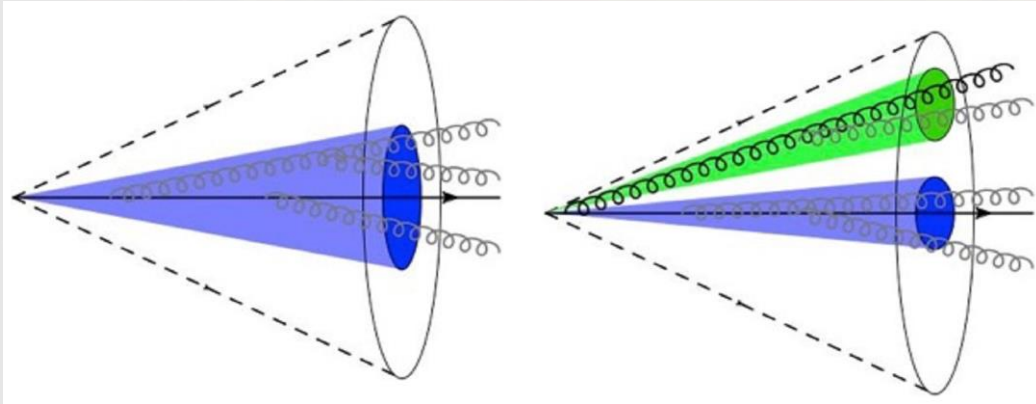
From energy loss to energy correlator jet substructure:

- For typical energy loss calculation we only need the
  - soft limit  $z \ll 1$
  - Soft divergence of the vacuum vertex
- For energy correlator jet substructure with colour coherence
  - Interplay of emissions from multiple sources or multiple scatters
  - Harder vertices – the energy weight removes the essential soft divergence. Phase space volume matters more.



# Computing $F_{\text{med}}$

Leading structure to study is a (quark) jet fragmenting into a jet and a subjet in the presence of a medium.



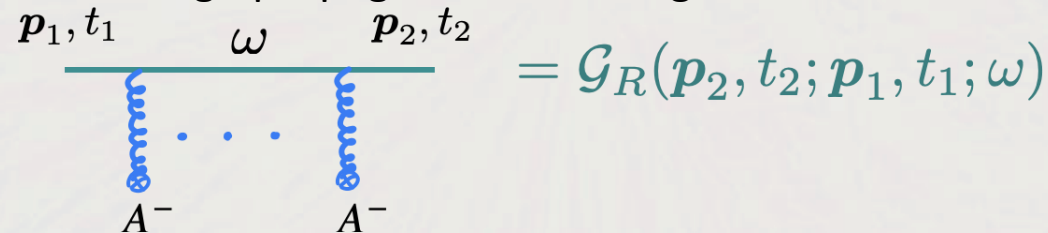
$$\mathcal{M}^{\alpha\beta} = \frac{1}{2E} \int_{\mathbf{p}_0 \mathbf{p}_1 \mathbf{k}_1 \mathbf{q}_1} \int_{t_0}^{\infty} dt_1 (2\pi)^2 \delta^{(2)}(\mathbf{p}_1 - \mathbf{k}_1 - \mathbf{q}_1) \mathcal{G}_{R_b}^{\alpha\alpha_1}(\mathbf{k}, L; \mathbf{k}_1, t_1; zE) \\ \times \mathcal{G}_{R_c}^{\beta\beta_1}(\mathbf{q}, L; \mathbf{q}_1, t_1; (1-z)E) V(\mathbf{k}_1 - z\mathbf{p}_1, z) T^{\alpha_1\beta_1\gamma_1} \mathcal{G}_{R_a}^{\gamma_1\gamma}(\mathbf{p}_1, t_1; \mathbf{p}_0, t_0; E) \mathcal{M}_0^\gamma(E, \mathbf{p}_0)$$



# Computing $F_{\text{med}}$

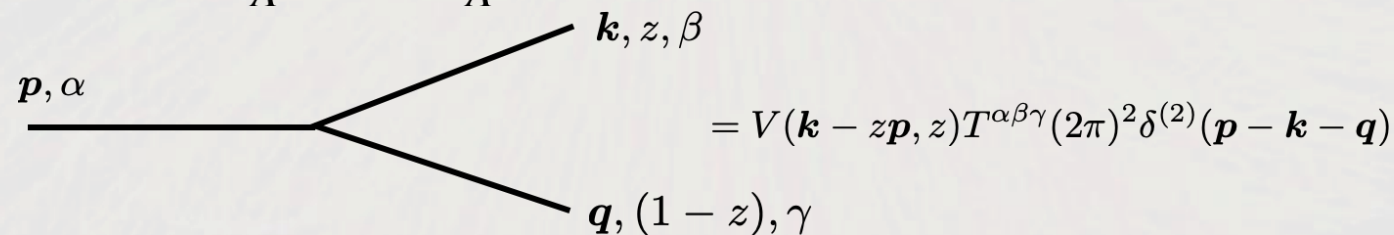
The formalism we use, based in BDMPS-Z:

- All particles have a large longitudinal momentum compared to their transverse momenta and therefore there is a decoupling between transverse and longitudinal dynamics
- We work in a mixed representation with momentum coordinates in the transverse direction and “time” (+ coordinate) in the longitudinal direction.
- Multiple scatterings resummed through propagators in a background field



$$= \mathcal{G}_R(\mathbf{p}_2, t_2; \mathbf{p}_1, t_1; \omega)$$

- Vacuum vertices



$$= V(\mathbf{k} - z\mathbf{p}, z) T^{\alpha\beta\gamma} (2\pi)^2 \delta^{(2)}(\mathbf{p} - \mathbf{k} - \mathbf{q})$$

- Background field averaged at the level of the cross section

$$\langle A^{a-}(\mathbf{q}_1, t_1) A^{b-\dagger}(\mathbf{q}_2, t_2) \rangle = \delta^{ab} \delta(t_2 - t_1) \delta^{(2)}(\mathbf{q}_1 - \mathbf{q}_2) v(\mathbf{q}_1)$$

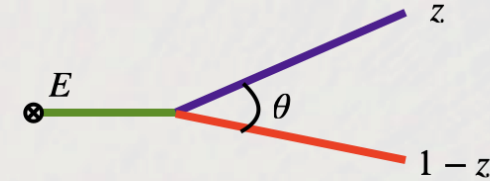
# Computing $F_{\text{med}}$

Full evaluation keeping  $z$  and  $\theta$  not yet achieved.

Two available approximations:

- Opacity expansion ( $N = 1$ ) [arXiv:1807.03799](https://arxiv.org/abs/1807.03799)
  - Unitarity problems can lead to negative cross sections.
  - Recursive formulas to generate all orders (not yet implemented numerically).
- “Tilted” Wilson lines
  - Resums multiple scatterings in the eikonal approximation. [arXiv:1907.03653](https://arxiv.org/abs/1907.03653)
  - Assumes semi-hard splittings ( $z$  not too small). [arXiv:2107.02542](https://arxiv.org/abs/2107.02542)
  - We implement this using both a Yukawa and HO potential for medium scatterings and for now using the leading colour limit.

# Computing $F_{\text{med}}$



For intuition, focus on HO for a bit.

- For a static medium of length  $L$  within the harmonic approximation one can read off the relevant scales directly from the formulas  
[arXiv:1907.03653](https://arxiv.org/abs/1907.03653)
  - (Vacuum) formation time:

$$t_f = \frac{2}{z(1-z)E\theta^2} \quad \theta_L \sim (EL)^{-1/2}$$

Below  $\theta_L$  all emissions have a formation time larger than  $L$ . This emerges as complete cancellations between dipole and quadrupole (in-out and in-in) terms in  $F_{\text{med}}^{(ij)}$  driving it rapidly to zero below  $\theta_L$ .

- Decoherence time:

$$S_{12}(\tau) = e^{-\frac{1}{12}\hat{q}(1+z^2)\theta^2\tau^3}$$

$$t_d \sim (\hat{q}\theta^2)^{-1/3} \quad \theta_c \sim (\hat{q}L^3)^{-1/2}$$

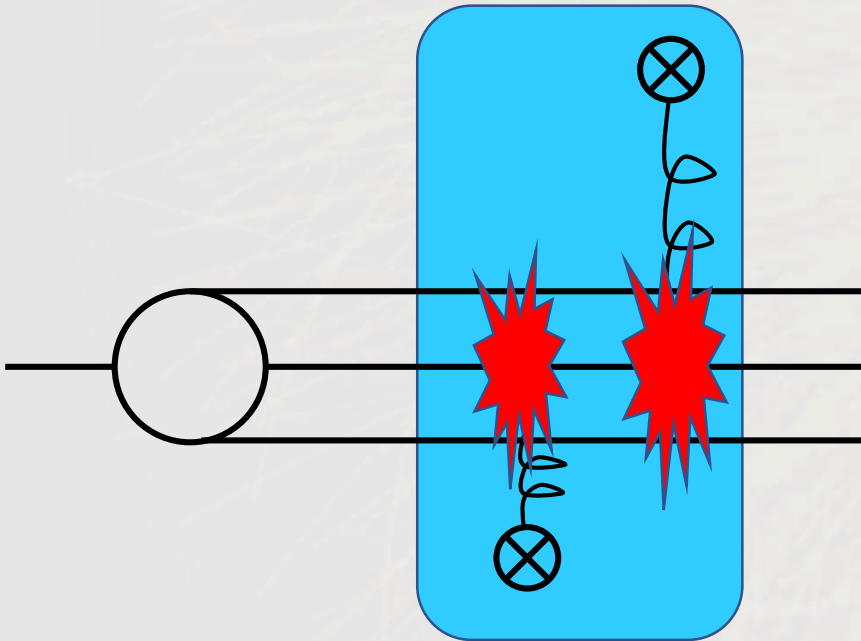
Below  $\theta_c$  emissions do not colour decohere and the medium does not independently resolve them. This emerges as an exponential suppression in the factorisable dipole terms within  $F_{\text{med}}^{(ij)}$  forcing it to become small below  $\theta_c$ .

If  $\theta_L > \theta_c$  then  $\theta_c$  becomes irrelevant

# Interpretation for $F_{\text{med}}$

$$\theta_c \gg \theta_L$$

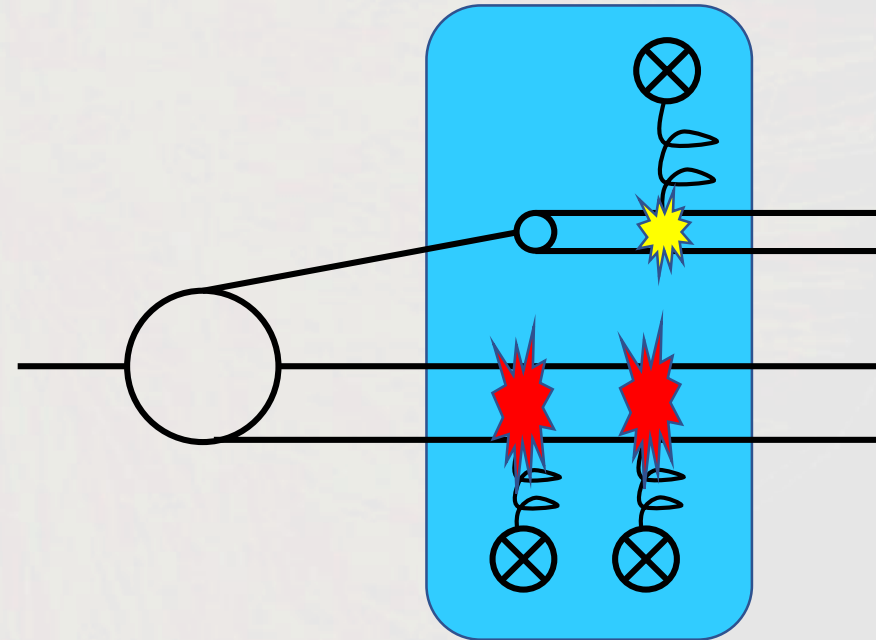
$$E \gg \hat{q}L^2$$



For angles  $\theta_c \gg \theta \gg \theta_L$ , the quark jet undergoes some minimal energy loss but the substructure is not resolved.

$$\theta_c \ll \theta_L$$

$$E \ll \hat{q}L^2$$



Initial splitting can be resolved by the medium when  $\theta \gg \theta_L$ . Broadening and energy loss occur.

# Numerical evaluation of $F_{\text{med}}$

Numerical integration is needed

$$F_{\text{med}} = 2 \int_0^L \frac{dt_1}{t_f} \left[ \int_{t_1}^L \frac{dt_2}{t_f} \cos \left( \frac{t_2 - t_1}{t_f} \right) \mathcal{C}^{(4)}(L, t_2) \mathcal{C}^{(3)}(t_2, t_1) - \sin \left( \frac{L - t_1}{t_f} \right) \mathcal{C}^{(3)}(L, t_1) \right]$$

$$\mathcal{C}_{gq}^{(3)}(t_2, t_1) = \frac{1}{N_c^2 - 1} \left\langle \text{tr}[V_2^\dagger V_1] \text{tr}[V_0^\dagger V_2] - \frac{1}{N_c} \text{tr}[V_0^\dagger V_1] \right\rangle.$$

$$\begin{aligned} \mathcal{C}_{gq}^{(3)}(t_2, t_1) &= e^{-\frac{1}{2} \int_{t_1}^{t_2} ds n(s) [N_c(\sigma_{02} + \sigma_{12}) - \frac{1}{N_c} \sigma_{01}]} \\ &= e^{-\frac{1}{12} \hat{q}(t_2 - t_1)^3 \theta^2 \left( 1 + z^2 + \frac{2z}{N_c^2 - 1} \right)}. \end{aligned}$$

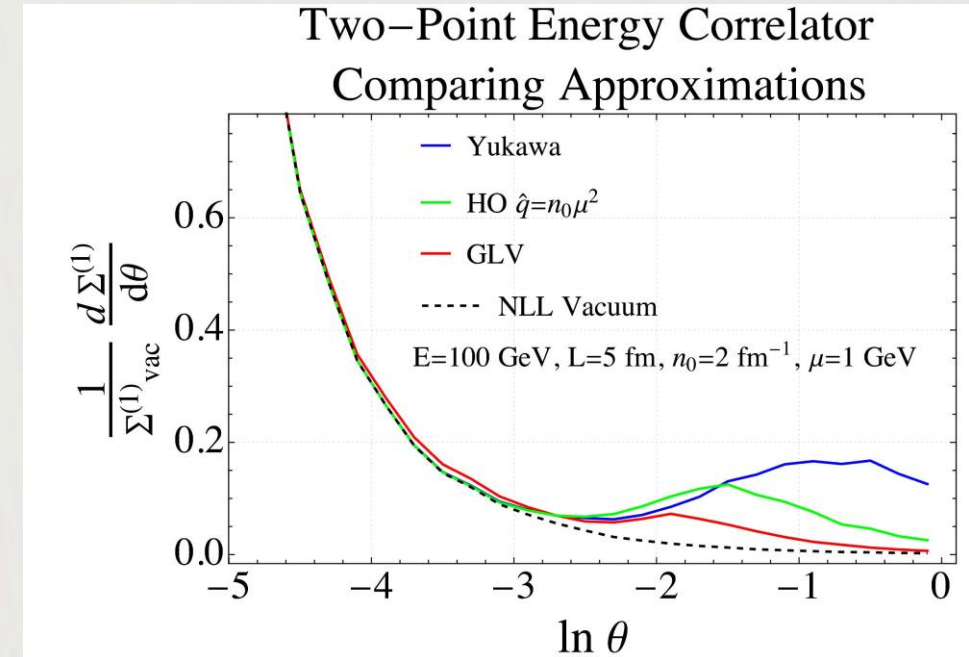
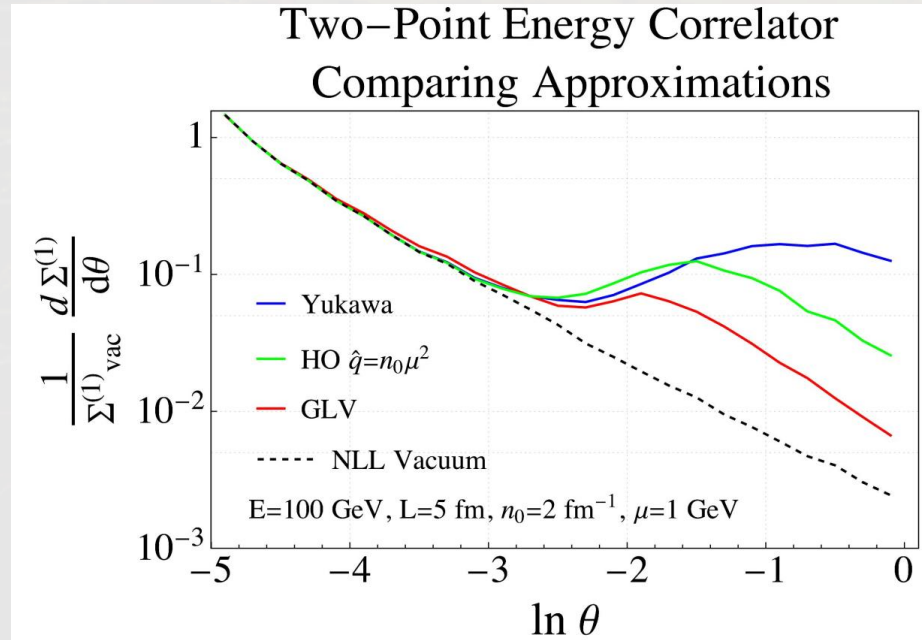
$$\mathcal{C}_{gq}^{(4)}(L, t_2) = \frac{1}{N_c^2 - 1} \left\langle \text{tr}[V_1^\dagger V_1 V_2^\dagger V_2] \text{tr}[V_2^\dagger V_2] - \frac{1}{N_c} \text{tr}[V_1^\dagger V_1] \right\rangle,$$

$$\begin{aligned} \frac{1}{N_c^2} \langle \text{tr}[V_1 V_2^\dagger V_2 V_1^\dagger] \text{tr}[V_2 V_2^\dagger] \rangle &\simeq e^{-\frac{1}{4} \hat{q} \theta^2 (t - t_2)(t_2 - t_1)^2 (1 - 2z + 3z^2)} \\ &\times \left( 1 - \frac{1}{2} \hat{q} \theta^2 z(1 - z)(t_2 - t_1)^2 \int_{t_2}^t ds e^{-\frac{1}{12} \hat{q} \theta^2 [(s - t_2)^2 (2s - 3t_1 + t_2) + 6z(1 - z)(s - t_2)(t_2 - t_1)^2]} \right) \end{aligned}$$





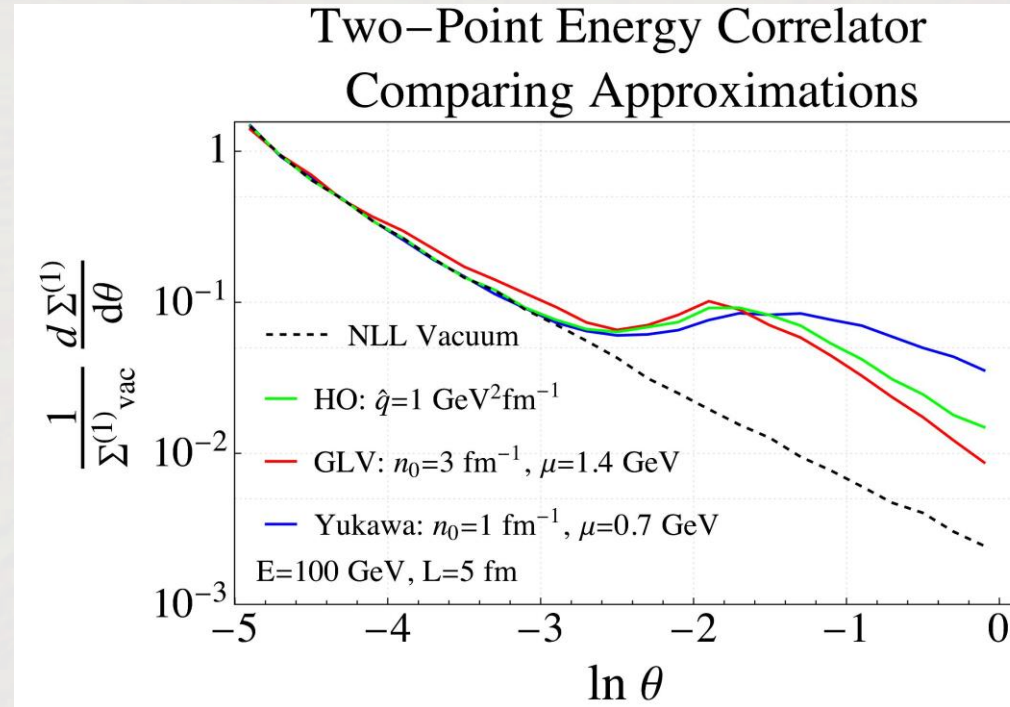
# Numerical evaluation of $F_{\text{med}}$



General features: an enhancement which begins above  $\theta_L$ , at  $\theta \gg \theta_L$  the enhancement peaks and then settles into a new medium dependent scaling law.

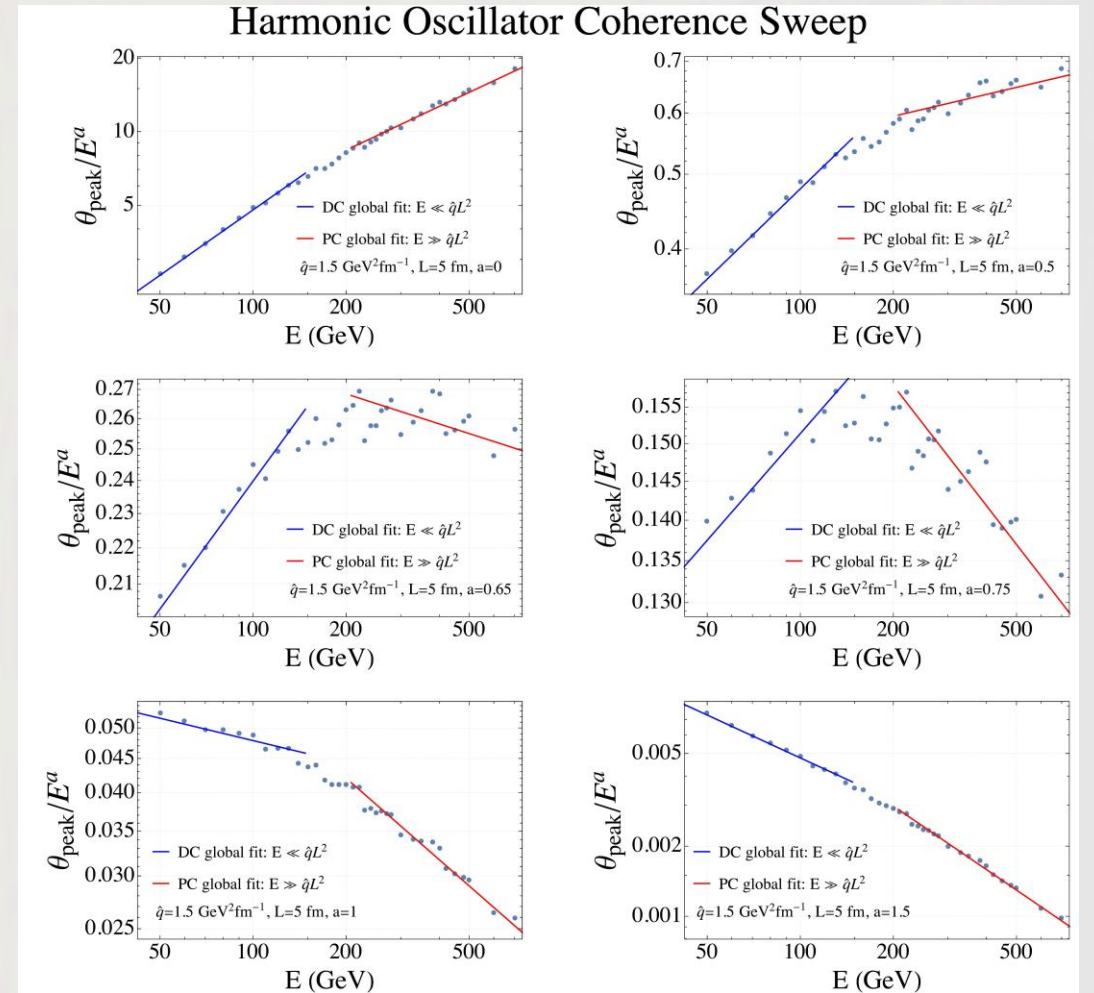
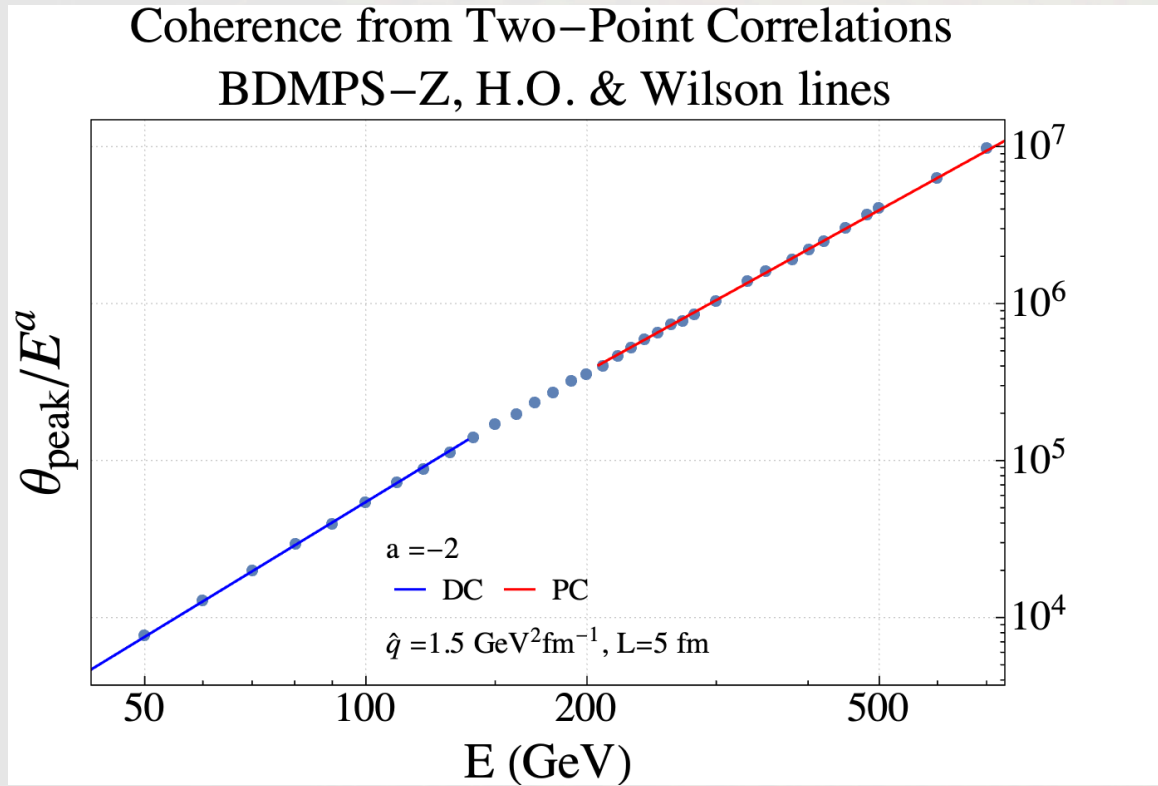
Amplitudes appear model dependent.

# Numerical evaluation of $F_{\text{med}}$



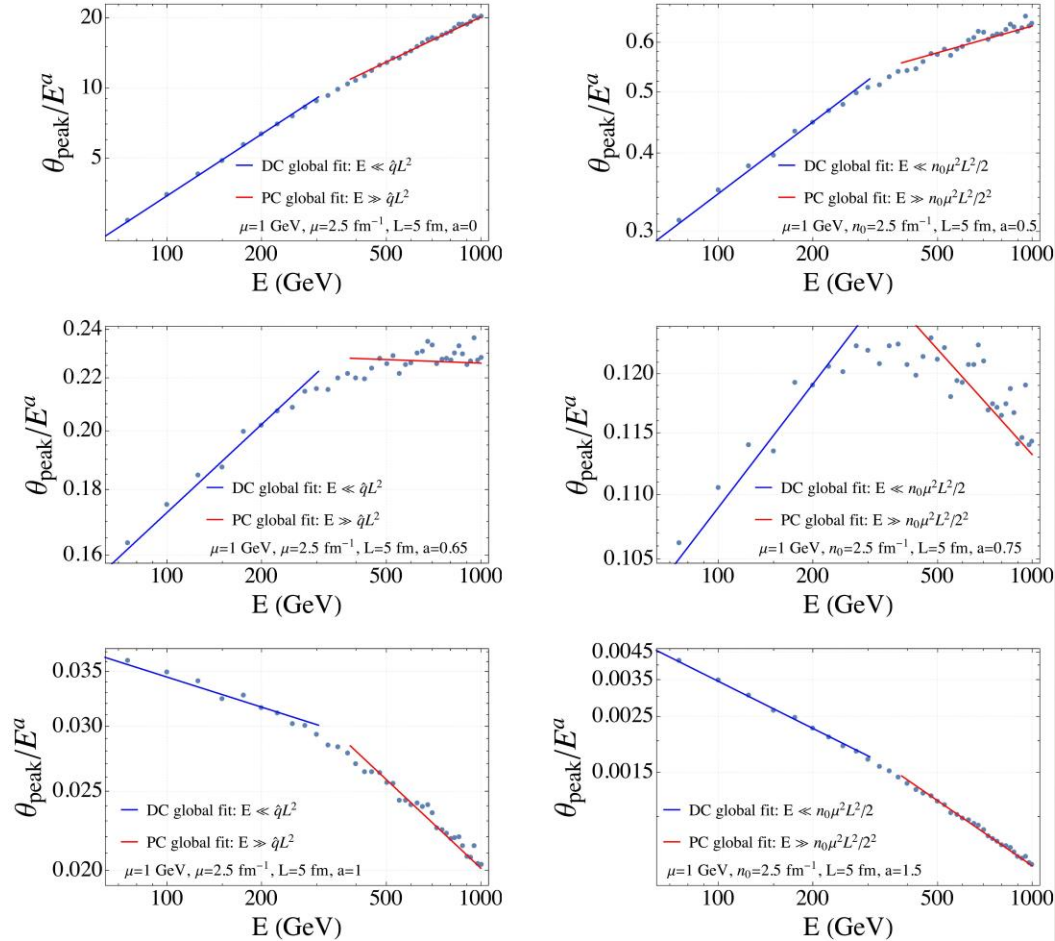
Whilst amplitudes are very model dependent, the differences can be fairly well absorbed into variation of the model parameters (not so much the wide angle though).

# Numerical evaluation of $F_{\text{med}}$

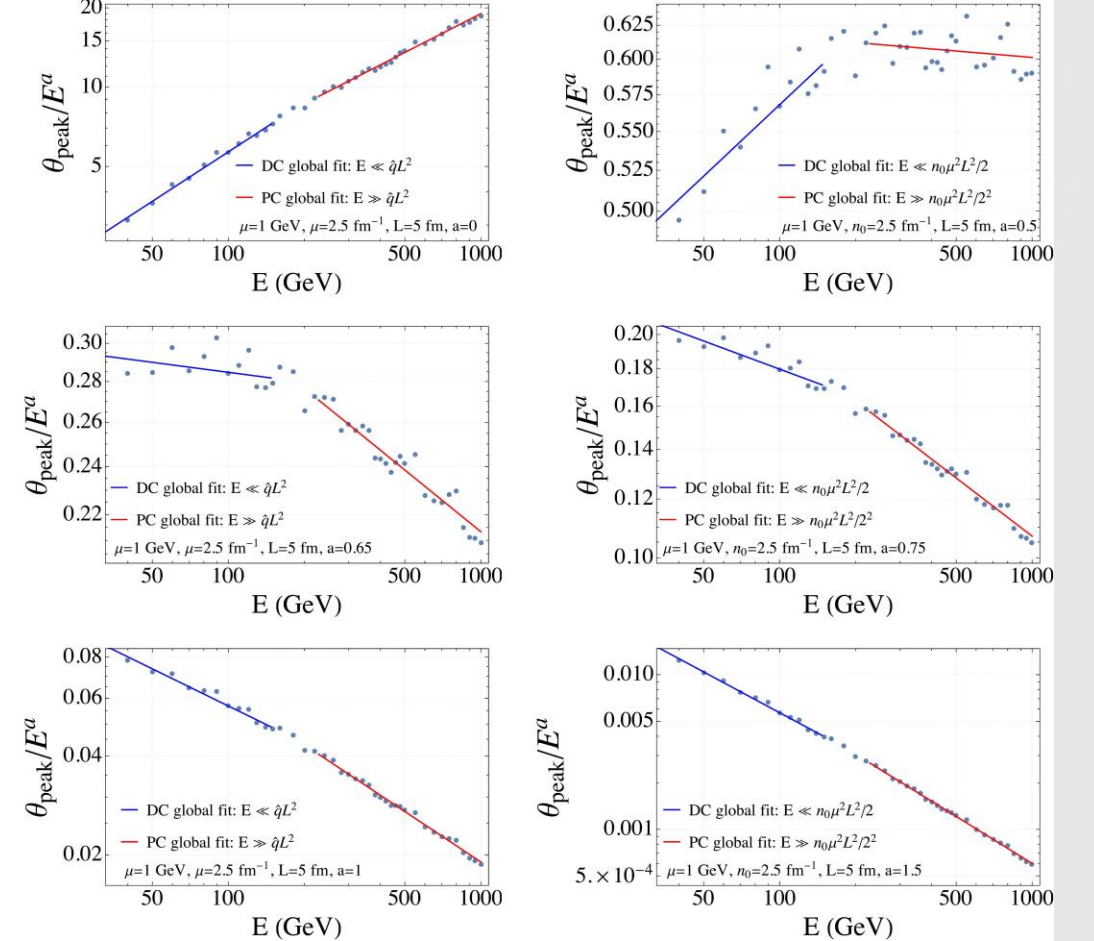


# Numerical evaluation of $F_{\text{med}}$

Yukawa Potential Coherence Sweep

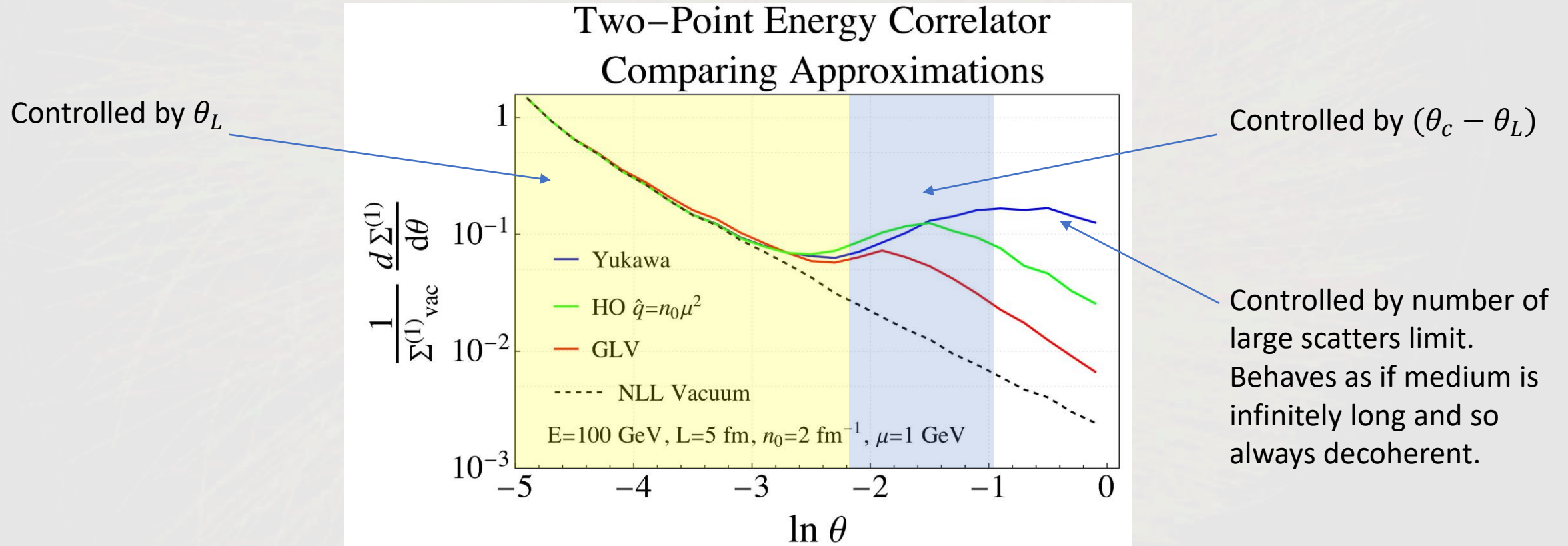


GLV Approximation Coherence Sweep





# Numerical evaluation of $F_{\text{med}}$



Provided  $E \gg E_c \sim \hat{q}L^2$



# Questions and observations

Coherence is a feature independent of the approach we take to evaluating the BDMPS-Z framework.

It doesn't seem clearly associated with a particular crisp angle or energy. Analytically, harsh boundaries on the phase-space of radiation emerge from approximations of emission kernels and medium interactions. In more complete numerical evaluations these boundaries are fuzzy.

Energy correlators provide a nice window into coherence and general features of jet quenching. Other models can be readily input – if you can compute  $F_{\text{med}}$ , you can compute this observable.

How does the observable change with a more realistic medium?

What other information can we extract? Can we access the 3-point correlator?

# Conclusions

Energy Correlators are cool and fun!



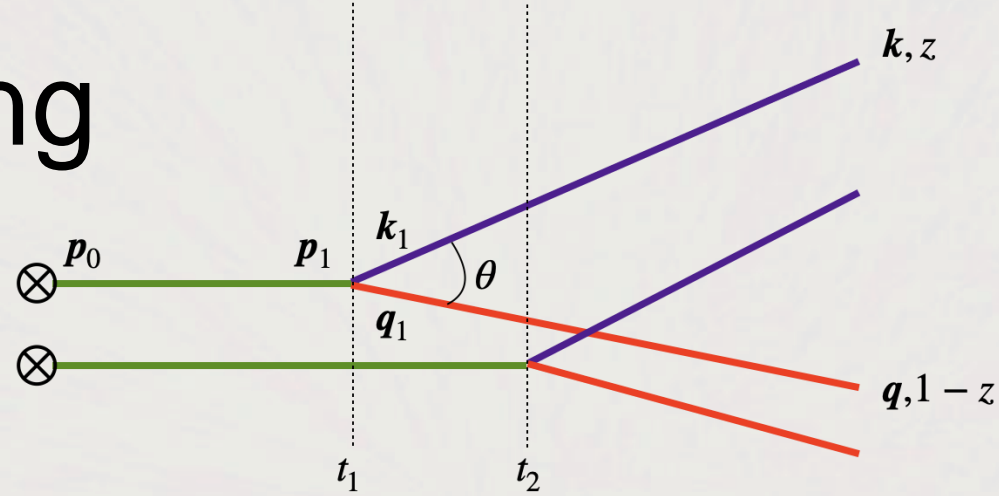
The simple and inclusive nature of energy correlators perhaps provides a window into degrees of theoretical control than can be very hard to otherwise achieve in jet substructure within H1 physics.

We've shown that (within the BDMPS-Z) framework, coherence can be accessed in a model independent way through energy correlator spectra.

There are several steps that needed to be taken to produce less idealised predictions that can be compared with experiment. Also, the novelty of these observables possibly introduces several experimental challenges. However, they show great promise and so I think it is worth the effort!

# Part N/A: Supplemental Material

# Jet Quenching



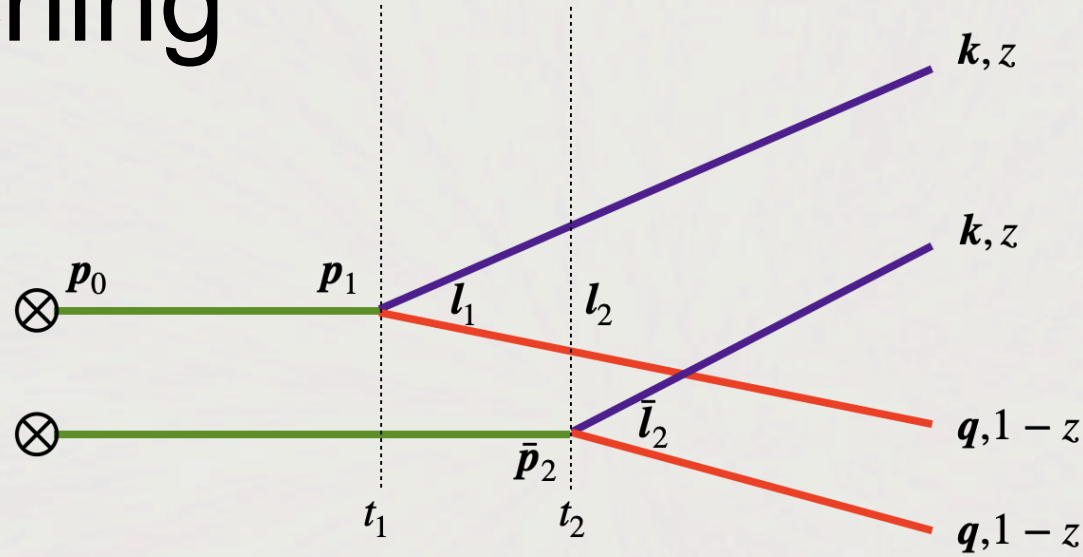
$$\mathcal{M}^{\alpha\beta} = \frac{1}{2E} \int_{\mathbf{p}_0 \mathbf{p}_1 \mathbf{k}_1 \mathbf{q}_1} \int_{t_0}^{\infty} dt_1 (2\pi)^2 \delta^{(2)}(\mathbf{p}_1 - \mathbf{k}_1 - \mathbf{q}_1) \mathcal{G}_{R_b}^{\alpha\alpha_1}(\mathbf{k}, L; \mathbf{k}_1, t_1; zE) \\ \times \mathcal{G}_{R_c}^{\beta\beta_1}(\mathbf{q}, L; \mathbf{q}_1, t_1; (1-z)E) V(\mathbf{k}_1 - z\mathbf{p}_1, z) T^{\alpha_1\beta_1\gamma_1} \mathcal{G}_{R_a}^{\gamma_1\gamma}(\mathbf{p}_1, t_1; \mathbf{p}_0, t_0; E) \mathcal{M}_0^\gamma(E, \mathbf{p}_0)$$

Where each of the in-medium propagators is of the form:

$$\mathcal{G}_R(t_2, \mathbf{x}_2; t_1, \mathbf{x}_1; \omega) = \int_{\mathbf{x}_1}^{\mathbf{x}_2} \mathcal{D}\mathbf{r} \exp \left\{ \frac{i\omega}{2} \int_{t_1}^{t_2} d\xi \dot{\mathbf{r}}^2(\xi) \right\} \underbrace{\text{P exp} \left\{ ig \int_{t_1}^{t_2} d\xi A_R^-(\xi, \mathbf{r}(\xi)) \right\}}_{V_R(t_2, t_1; [\mathbf{r}] )}$$

$$\langle |\mathcal{M}|^2 \rangle \propto \left\langle \mathcal{G}_{R_b}^{\alpha\alpha_1}(\mathbf{k}, L; \mathbf{k}_1, t_1; zE) \mathcal{G}_{R_c}^{\beta\beta_1}(\mathbf{q}, L; \mathbf{q}_1, t_1; (1-z)E) \mathcal{G}_{R_b}^{\dagger\bar{\alpha}_2\alpha}(\bar{\mathbf{k}}_2, t_2; \mathbf{k}, L; zE) \right. \\ \left. \times \mathcal{G}_{R_c}^{\dagger\bar{\beta}_2\beta}(\bar{\mathbf{q}}_2, t_2; \mathbf{q}, L; (1-z)E) \mathcal{G}_{R_a}^{\gamma_1\gamma}(\mathbf{p}_1, t_1; \mathbf{p}_0, t_0; E) \mathcal{G}_{R_a}^{\dagger\bar{\gamma}\bar{\gamma}_2}(\bar{\mathbf{p}}_0, t_0; \bar{\mathbf{p}}_2, t_2; E) \right\rangle$$

# Jet Quenching

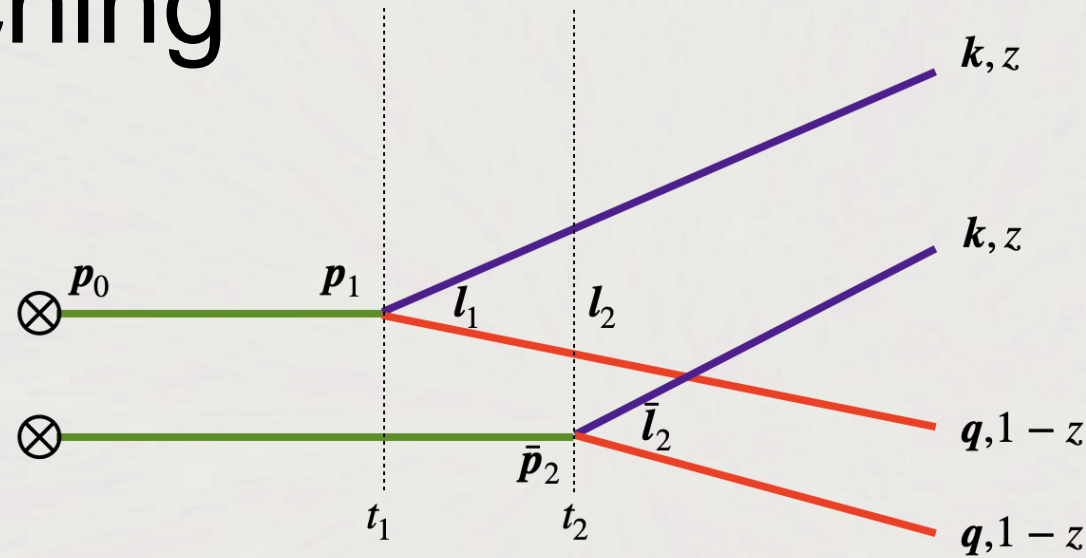


$$\begin{aligned} \frac{d\sigma}{d\Omega_k d\Omega_q} &= \frac{g^2}{z(1-z)E^2} P_{a \rightarrow bc}(z) 2\text{Re} \int_{p_0 p_1 \bar{p}_2 l_1 l_2 \bar{l}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (l_1 \cdot \bar{l}_2) \\ &\times \mathcal{S}^{(4)}((1-z)\mathbf{k} - z\mathbf{q}, L; l_2, \bar{l}_2, t_2; \mathbf{k} + \mathbf{q} - \bar{\mathbf{p}}_2, z) \\ &\times \mathcal{K}^{(3)}(l_2, t_2; l_1, t_1; \bar{\mathbf{p}}_2 - \mathbf{p}_1, z) \mathcal{P}_{R_a}(\mathbf{p}_1 - \mathbf{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}} \end{aligned}$$



# Jet Quenching

BDMPS-Z



$$\begin{aligned}
 \frac{d\sigma}{d\Omega_k d\Omega_q} &= \frac{g^2}{z(1-z)E^2} P_{a \rightarrow bc}(z) 2\text{Re} \int_{p_0 p_1 \bar{p}_2 l_1 l_2 \bar{l}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (l_1 \cdot \bar{l}_2) \\
 &\times \mathcal{S}^{(4)}((1-z)k - zq, L; l_2, \bar{l}_2, t_2; k + q - \bar{p}_2, z) \\
 \langle \mathcal{G} \mathcal{G} \mathcal{G}^\dagger \mathcal{G}^\dagger \rangle &\times \mathcal{K}^{(3)}(l_2, t_2; l_1, t_1; \bar{p}_2 - p_1, z) \mathcal{P}_{R_a}(p_1 - p_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}} \\
 &\langle \mathcal{G} \mathcal{G} \mathcal{G}^\dagger \rangle \quad \langle \mathcal{G} \mathcal{G}^\dagger \rangle
 \end{aligned}$$

# Part N/A: Supplemental Material

$$\mathcal{M}_{\gamma \rightarrow q\bar{q}} = \frac{e}{E} e^{i\frac{\mathbf{p}_1^2}{2zE}L + i\frac{\mathbf{p}_2^2}{2(1-z)E}L} \int_0^\infty dt \int_{\mathbf{k}_1, \mathbf{k}_2} [\mathcal{G}(\mathbf{p}_1, L; \mathbf{k}_1, t|zE) \bar{\mathcal{G}}(\mathbf{p}_2, L; \mathbf{k}_2, t|(1-z)E)]_{ij} \\ \times \gamma_{\lambda, s, s'}(z) \mathbf{k} \cdot \boldsymbol{\epsilon}_\lambda^* \mathcal{G}_0(\mathbf{k}_1 + \mathbf{k}_2, t|E)$$

$$\mathcal{G}(\mathbf{p}_1, t_1; \mathbf{p}_0, t_0) = \int_{\mathbf{x}_1, \mathbf{x}_2} e^{-i\mathbf{p}_1 \cdot \mathbf{x}_1 + i\mathbf{p}_0 \cdot \mathbf{x}_0} \mathcal{G}(\vec{x}_1, \vec{x}_0) \\ \mathcal{G}(\vec{x}_1, \vec{x}_0) = \int_{\mathbf{r}(t_0)=\mathbf{x}_0}^{\mathbf{r}(t_1)=\mathbf{x}_1} \mathcal{D}\mathbf{r} \exp \left[ i\frac{E}{2} \int_{t_0}^{t_1} ds \dot{\mathbf{r}}^2 \right] V(t_1, t_0; [\mathbf{r}]) \\ V(t_1, t_0; [\mathbf{r}]) = \mathcal{P} \exp \left[ ig \int_{t_0}^{t_1} dt \mathbf{t}^a A^{-,a}(t, \mathbf{r}(t)) \right]$$

$$\frac{dN^{\text{med}}}{dz d\mathbf{p}^2} = \frac{1}{4(2\pi)^2 z(1-z)} \langle |\mathcal{M}_{\gamma \rightarrow q\bar{q}}|^2 \rangle = \frac{1}{4(2\pi)^2 z(1-z)} \langle |\mathcal{M}_{\gamma \rightarrow q\bar{q}}^{\text{in}} + \mathcal{M}_{\gamma \rightarrow q\bar{q}}^{\text{out}}|^2 \rangle$$

# Part N/A: Supplemental Material

$$\frac{d\sigma_{qg}}{d\theta dz} = \frac{d\sigma_{qg}^{\text{vac}}}{d\theta dz} (1 + F_{\text{med}}(z, \theta, \hat{q}, L))$$

$$F_{\text{med}} = 2 \int_0^L \frac{dt_1}{t_f} \left[ \int_{t_1}^L \frac{dt_2}{t_f} \cos\left(\frac{t_2 - t_1}{t_f}\right) \mathcal{C}^{(4)}(L, t_2) \mathcal{C}^{(3)}(t_2, t_1) - \sin\left(\frac{L - t_1}{t_f}\right) \mathcal{C}^{(3)}(L, t_1) \right]$$

$$\mathcal{C}_{gq}^{(3)}(t_2, t_1) = \frac{1}{N_c^2 - 1} \left\langle \text{tr}[V_2^\dagger V_1] \text{tr}[V_0^\dagger V_2] - \frac{1}{N_c} \text{tr}[V_0^\dagger V_1] \right\rangle . \quad \mathcal{C}_{gq}^{(3)}(t_2, t_1) = e^{-\frac{1}{2} \int_{t_1}^{t_2} ds n(s) [N_c(\sigma_{02} + \sigma_{12}) - \frac{1}{N_c} \sigma_{01}]} \\ = e^{-\frac{1}{12} \hat{q}(t_2 - t_1)^3 \theta^2 \left(1 + z^2 + \frac{2z}{N_c^2 - 1}\right)} .$$

$$\mathcal{C}_{gq}^{(4)}(L, t_2) = \frac{1}{N_c^2 - 1} \left\langle \text{tr}[V_1^\dagger V_1 V_2^\dagger V_2] \text{tr}[V_2^\dagger V_2] - \frac{1}{N_c} \text{tr}[V_1^\dagger V_1] \right\rangle ,$$

$$\frac{1}{N_c^2} \langle \text{tr}[V_1 V_2^\dagger V_2 V_1^\dagger] \text{tr}[V_2 V_2^\dagger] \rangle \simeq e^{-\frac{1}{4} \hat{q} \theta^2 (t - t_2)(t_2 - t_1)^2 (1 - 2z + 3z^2)} \\ \times \left( 1 - \frac{1}{2} \hat{q} \theta^2 z(1 - z)(t_2 - t_1)^2 \int_{t_2}^t ds e^{-\frac{1}{12} \hat{q} \theta^2 [(s - t_2)^2 (2s - 3t_1 + t_2) + 6z(1 - z)(s - t_2)(t_2 - t_1)^2]} \right)$$