From a viscous fluid to particles

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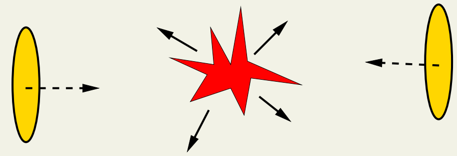
Outline

I. Hydro and identified particle observables

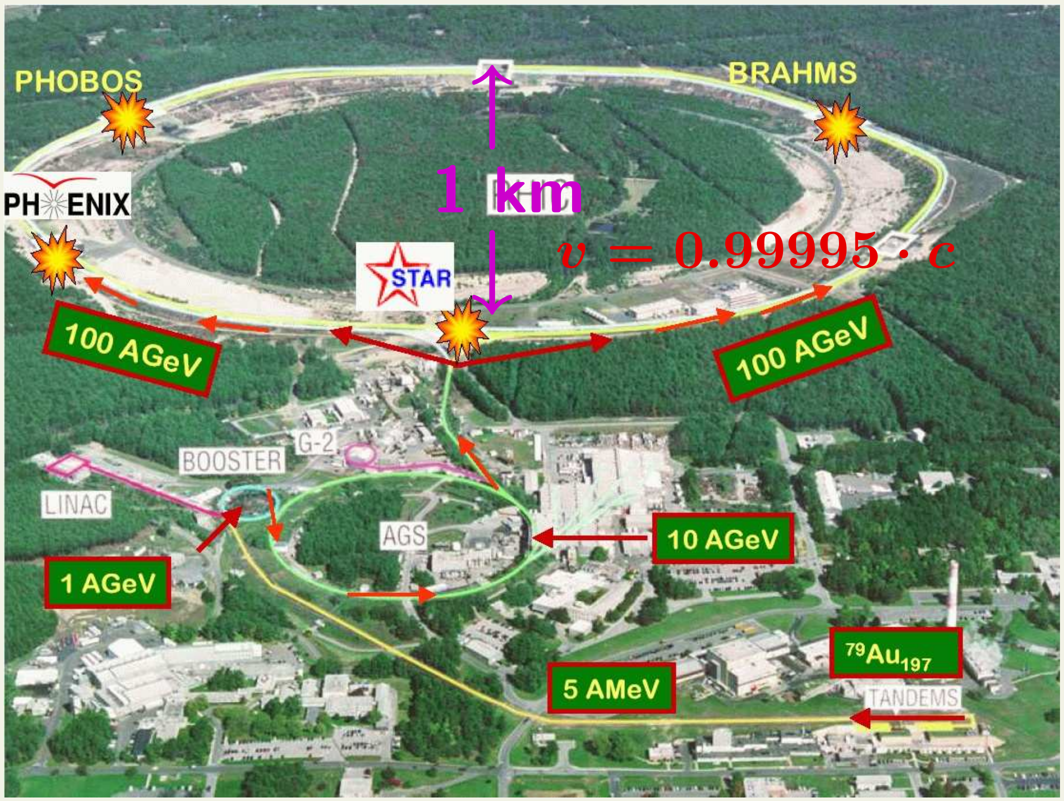
II. Converting fluid to particles

III. Summary

"Little bang":  plasma of quarks and gluons

- $T \sim 200 - 400$ MeV
- perturbative at high energies
- electric AND magnetic screening

Relativistic Heavy Ion Collider



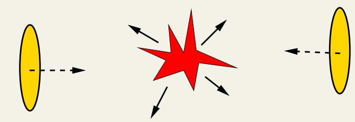
since 2000-

Au+Au: $\sqrt{s} = 200$ GeV/nucleon

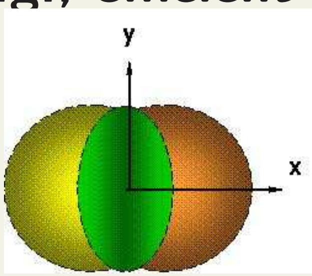
Large Hadron Collider

already running!

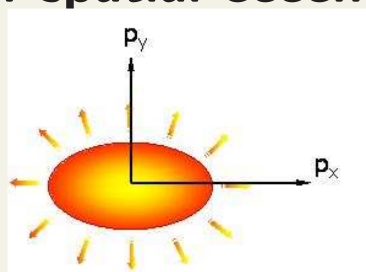
Pb+Pb: $\sqrt{s} = 2700$ GeV/nucl

RHIC collisions look largely thermalized $\rightarrow$ should be able to measure equation of state and viscosity 

e.g., efficient conversion of spatial eccentricity to momentum anisotropy



$\longrightarrow$



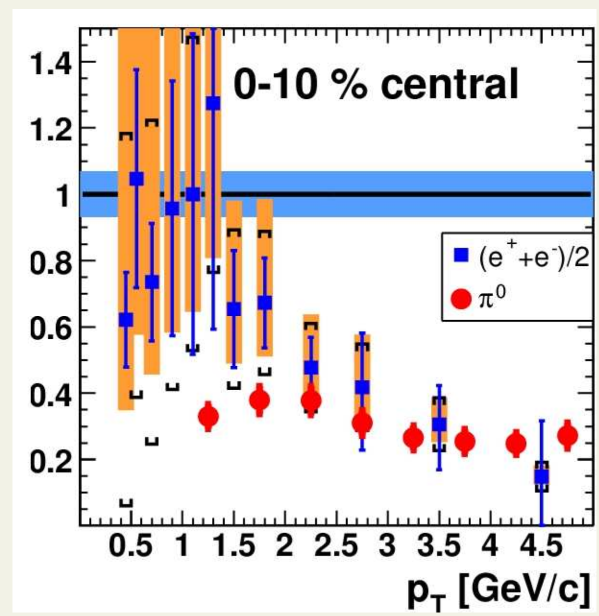
"elliptic flow"

$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$

$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \equiv \langle \cos 2\phi_p \rangle$$

large energy loss, even for heavy quarks

$$R_{AA} = \frac{measured\ yield}{expected\ yield\ for\ dilute\ system}$$



I. Hydro paradigm and particles

Identified particle observables are crucial

- help pin down initial conditions
- hold the key to equation of state

Hydrodynamics

describes systems near local equilibrium, in terms of macroscopic variables

$e(x)$, $p(x)$, $n_B(x)$ - **energy density, pressure, baryon density**

$u^\mu(x) \equiv \gamma(v)(1, \vec{v}(x))$ - **flow velocity**

conservation laws: $\partial_\mu T^{\mu\nu}(x) = 0$, $\partial_\mu N_B^\mu(x) = 0$

$T^{\mu\nu}$: **energy-momentum tensor,** N_B^μ: **baryon current**

medium given through equation of state $p(e, n_B)$

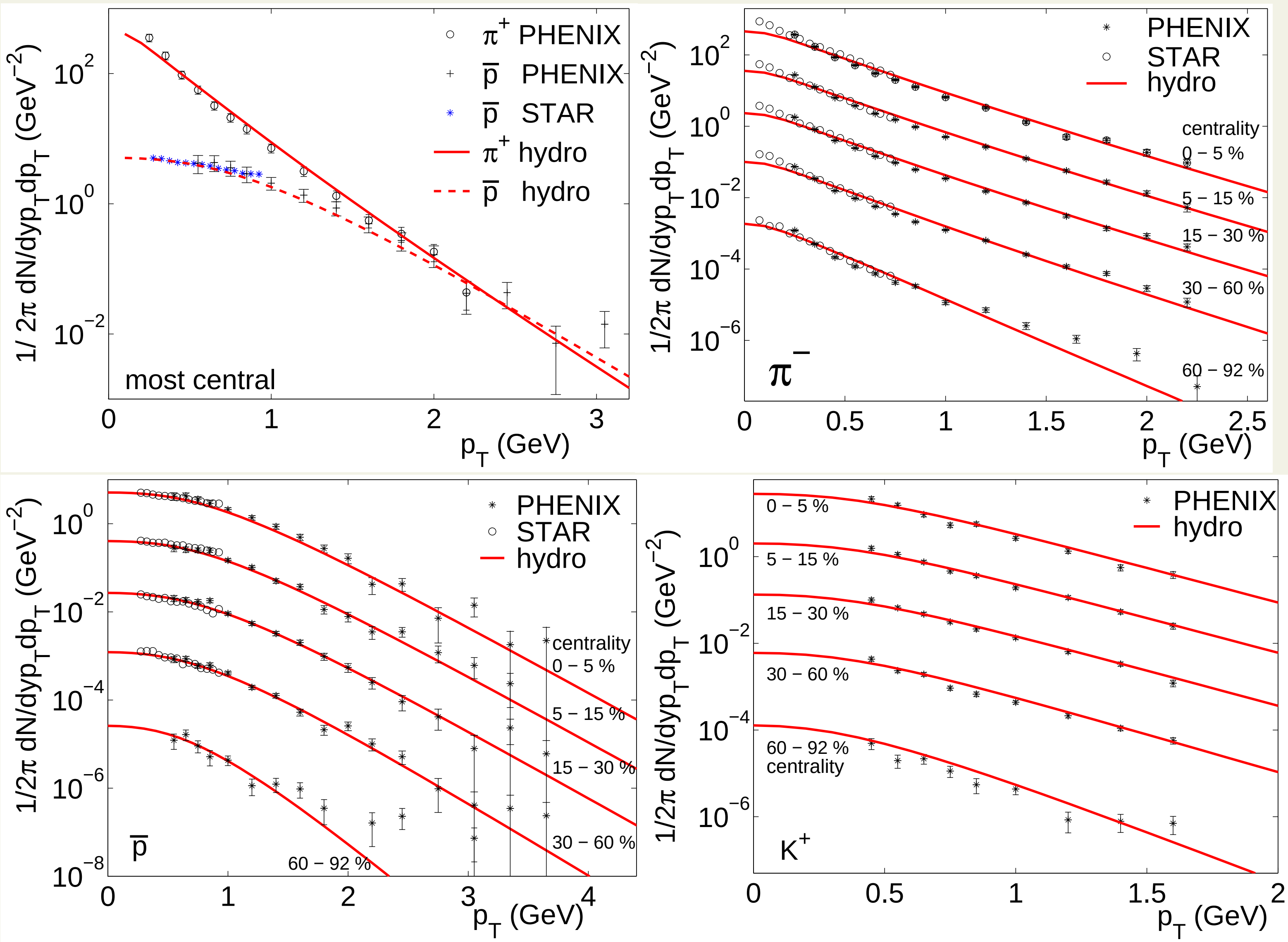
Local Rest frame (comoving frame)

ideal $\equiv$ **local equilibrium** $T_{LR}^{\mu\nu} = diag(e, p, p, p)$, $N_{B,LR}^\mu = (n_B, 0)$

nonideal $\equiv$ **small deviations from local equilibrium** $\rightarrow$ **dissipation**

∼ 2000-01: Ideal hydro

Au+Au @ RHIC: spectral shapes work quite well Kolb & Heinz, nucl-th/0305084



$\tau_0 = 0.6$ fm

$\varepsilon_0 = 25$ GeV/fm^3

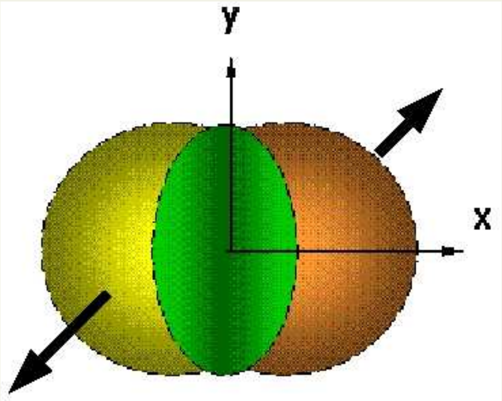
$T_0 \approx 360$ MeV

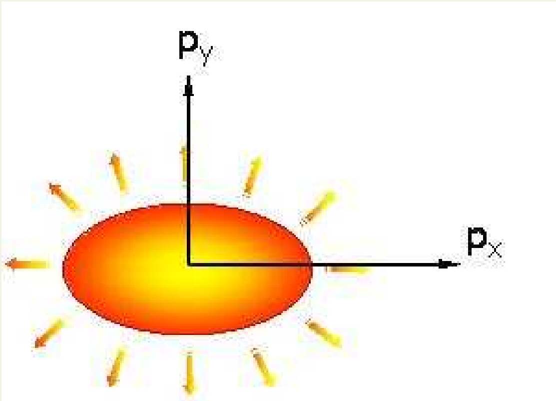
$T_{fo} \approx 130$ MeV

[75% wounded
+ 25% binary geom
bag EOS
$s_0 = 110$ / fm^3]

Elliptic flow (v_2)

spatial anisotropy $\longrightarrow$ final azimuthal momentum anisotropy

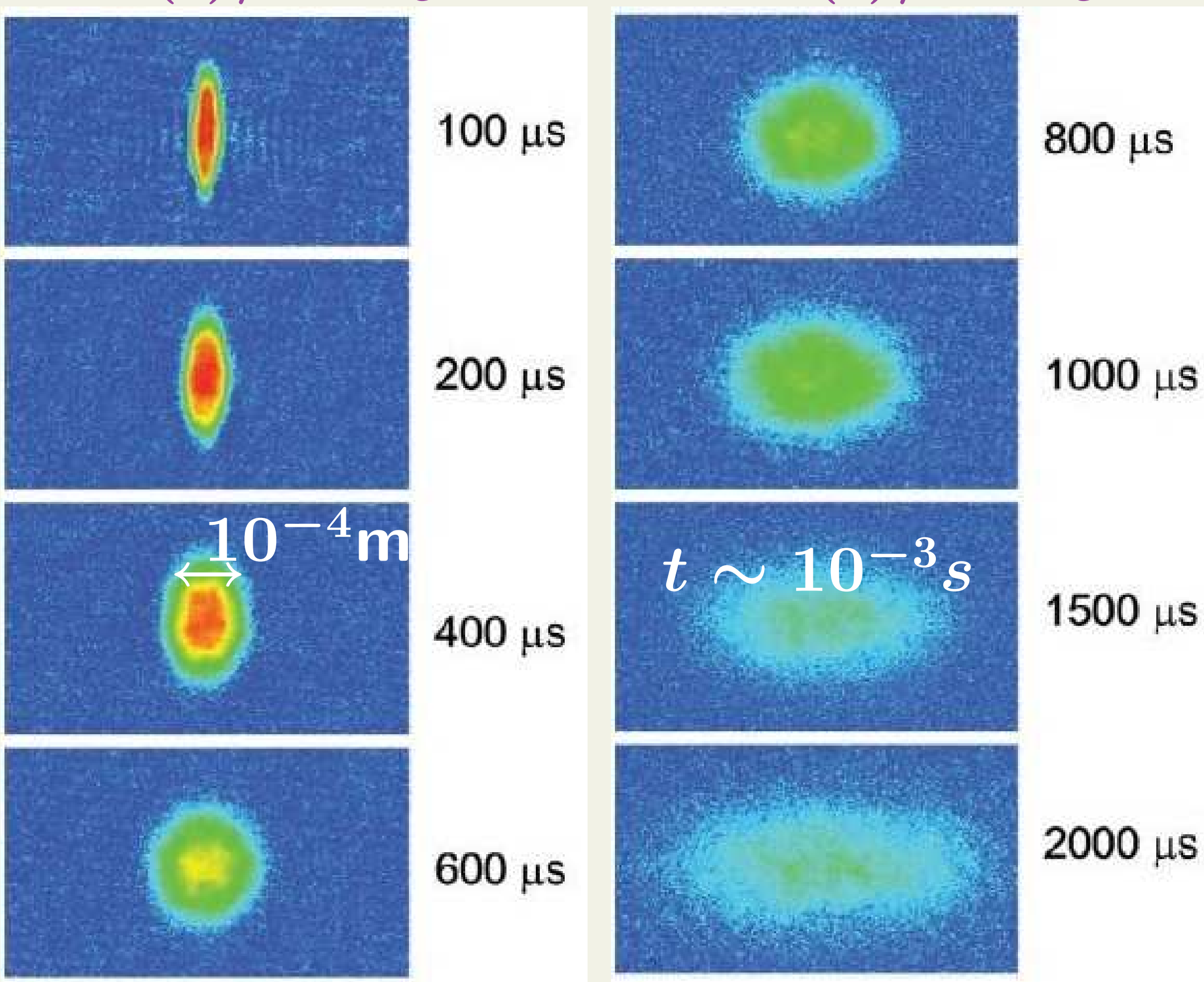
$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$ 

$$\longrightarrow \quad v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$ 

- measure of early pressure gradients
- sensitive to interaction strength (degree of thermalization)

also seen in ultra-cold atomic systems

$dN(t)/dx\,dy$ $\qquad$ $dN(t)/dx\,dy$



droplet of ^6Li

$T \sim 10^{-6}$ **K**
$n \sim 10^{19}$**/m**^{3}

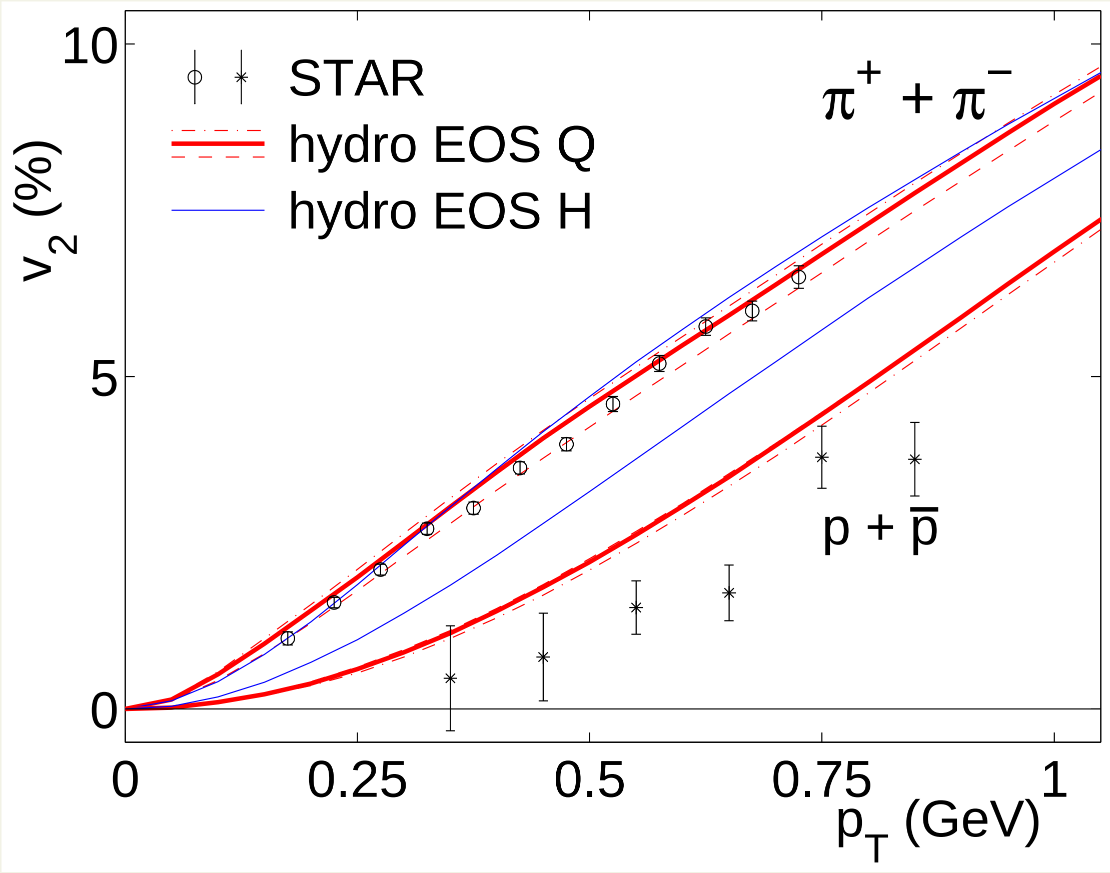
unitarity limit
(Feshbach resonance)

$$\sigma(k) = 4\pi/k^2$$

O'Hara et al, Science 298 ('03)

$\sim$ 2000-01: Ideal hydro

minimum-bias Au+Au at RHIC Kolb, Heinz, Huovinen et al ('01), nucl-th/0305084



$\tau_{therm} = 0.6$ **fm** **,** $\langle e \rangle_{init} \sim 5GeV/fm^3$**,** $T_{freezeout} = 120$ **MeV**

pion-proton splitting favors QGP phase transition over pure hadron gas

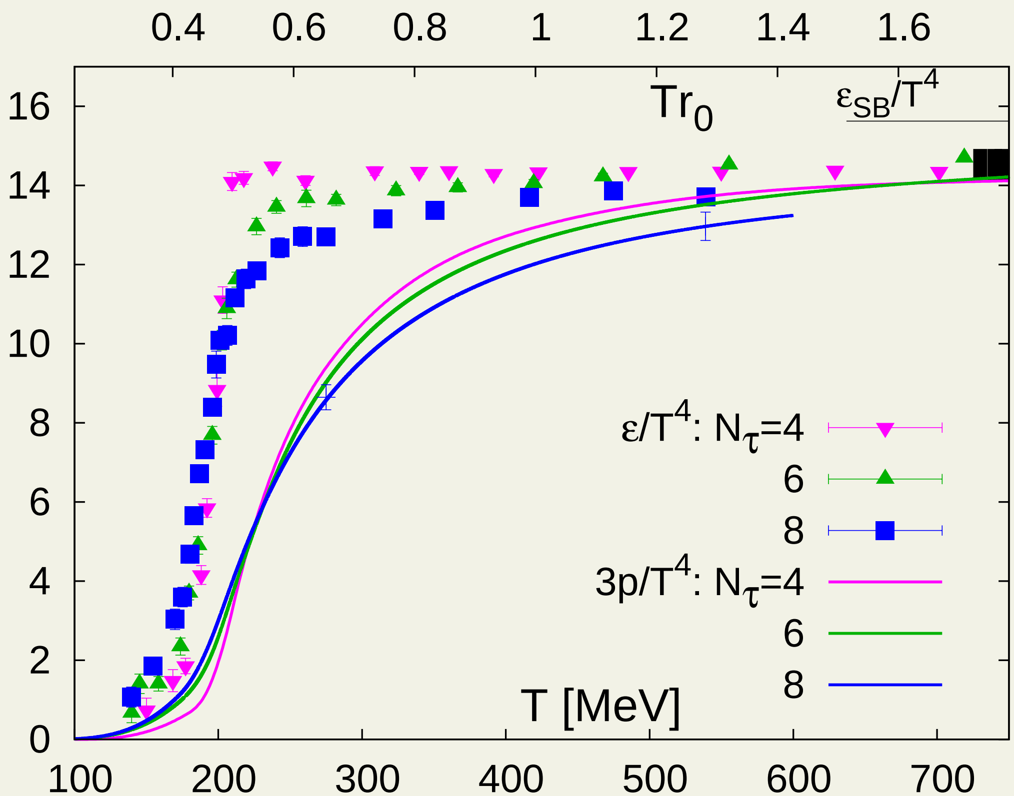
Two recent refinements:

- realistic equation of state
- QGP viscosity

QCD equation of state ($\mu_B = 0$)

QCD on a space-time lattice - $Z = \int \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, \mathcal{D}A\, e^{-S_E^{QCD}}$

A. Bazavov et al, PRD80 ('09)



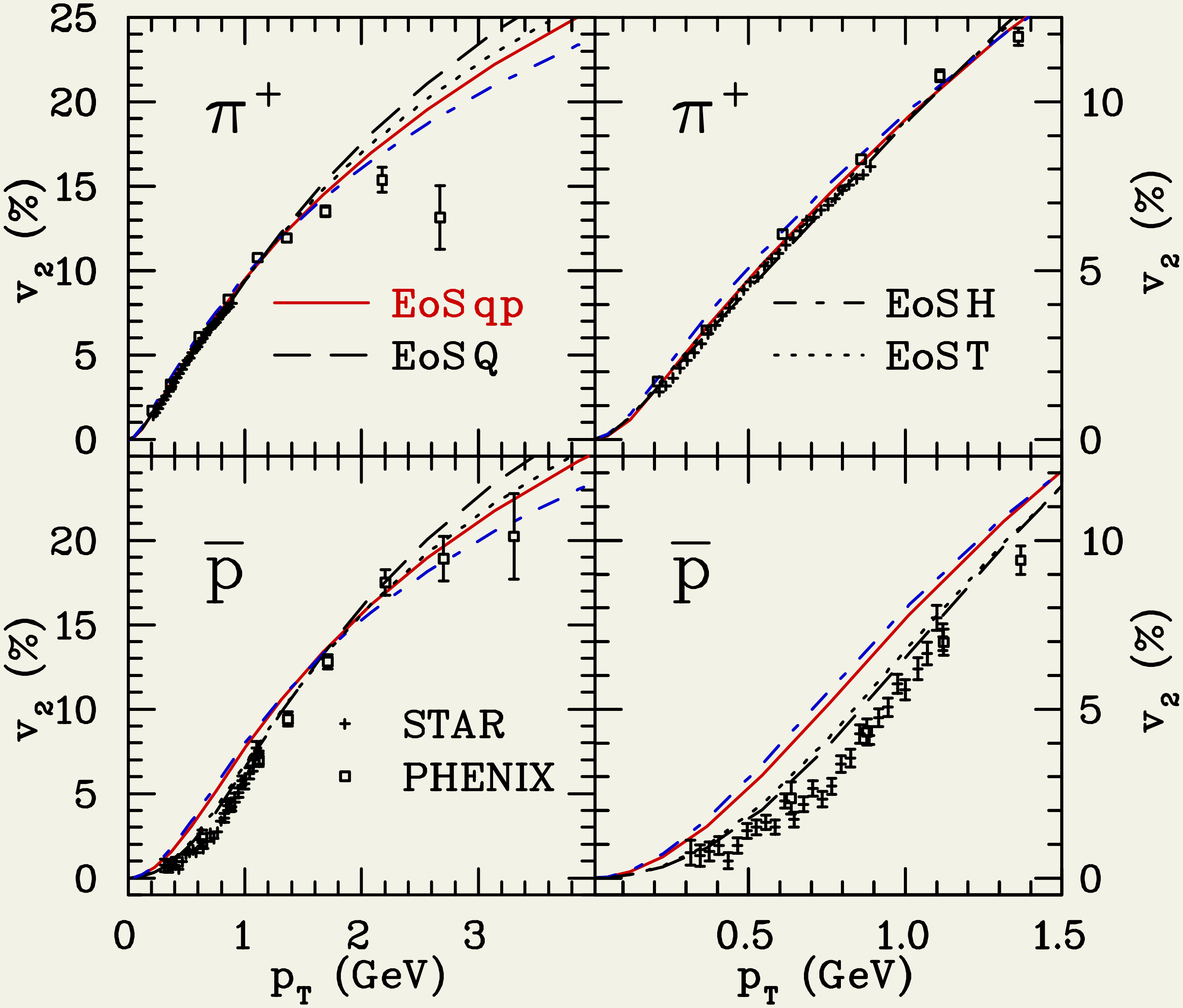
$$\frac{e_{SB}}{T^4} \approx \frac{d.o.f.}{3}$$

cross-over with a jump in effective degrees of freedom near T_c

quite robust results, though still evolving - $T_c \approx 170 - 180(-200)$ **MeV**

Ideal hydro + realistic EOS

serious eyesore for hydro paradigm: realistic EOS gives same as hadron gas?!



Huovinen, NPA761, 296 ('05)

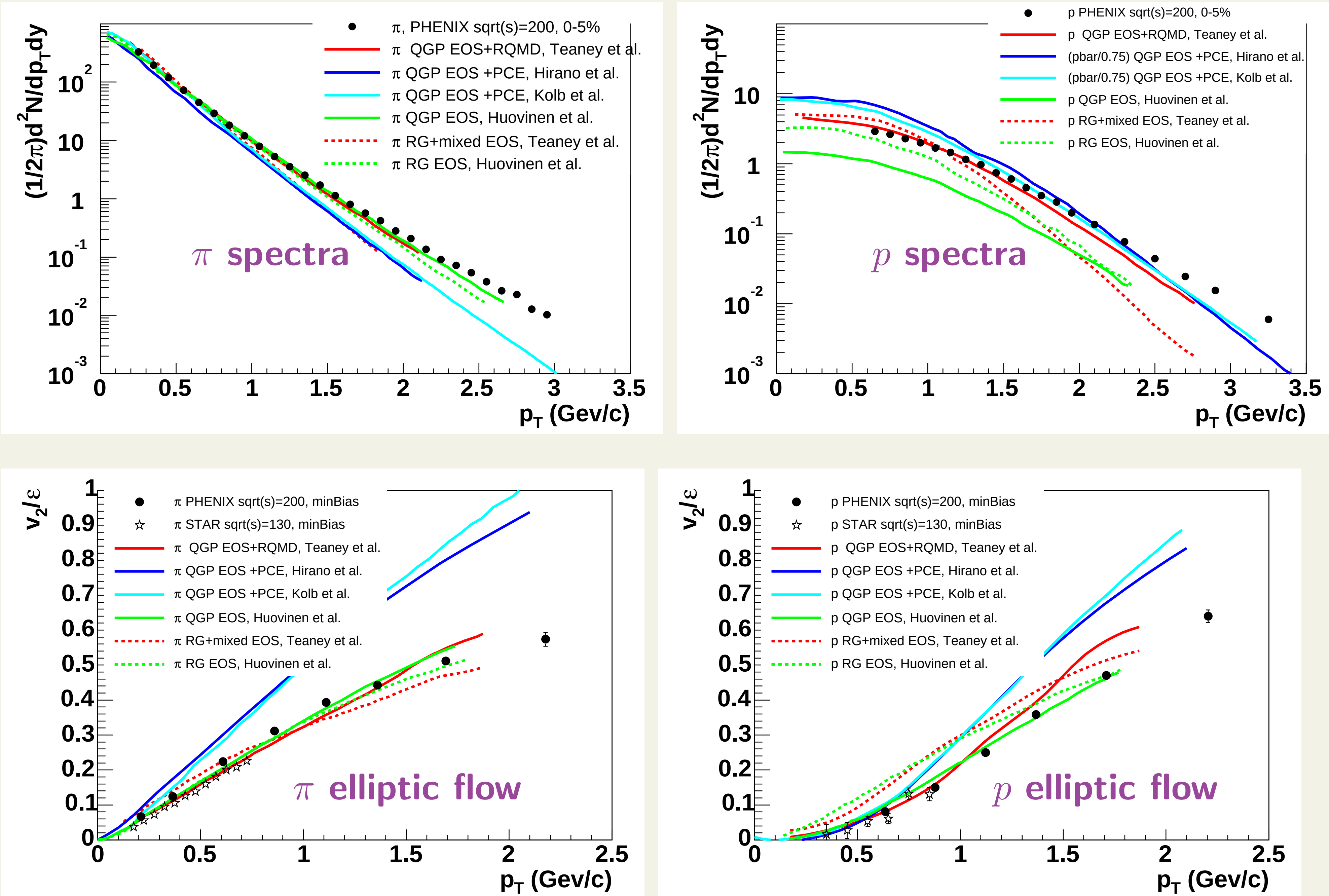
Q: bag model

qp: lattice fit ($T_c = 170$ MeV)

H: hadron gas

T: interpolated $\varepsilon(T)$ between hadron gas and $\varepsilon \propto T^4$ plasma

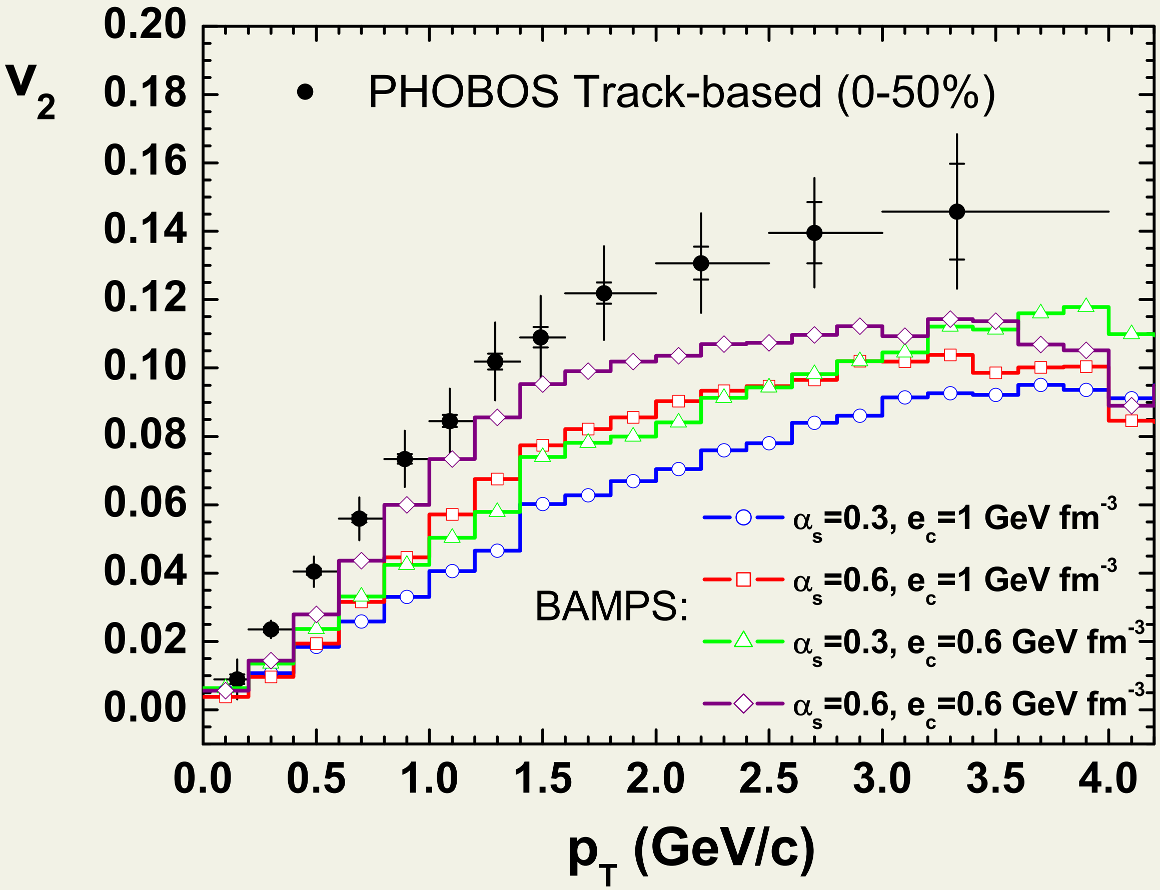
Hope: dissipation could help. 2005 PHENIX White Paper NPA757, 184 ('05)**: for best agreement, must couple hydro to late-stage hadronic transport.**



a contender paradigm (waiting for independent confirmation)

quark-gluon transport with elastic $2 \to 2$ AND radiative $3 \leftrightarrow 2$

Xu & Greiner, ('08)



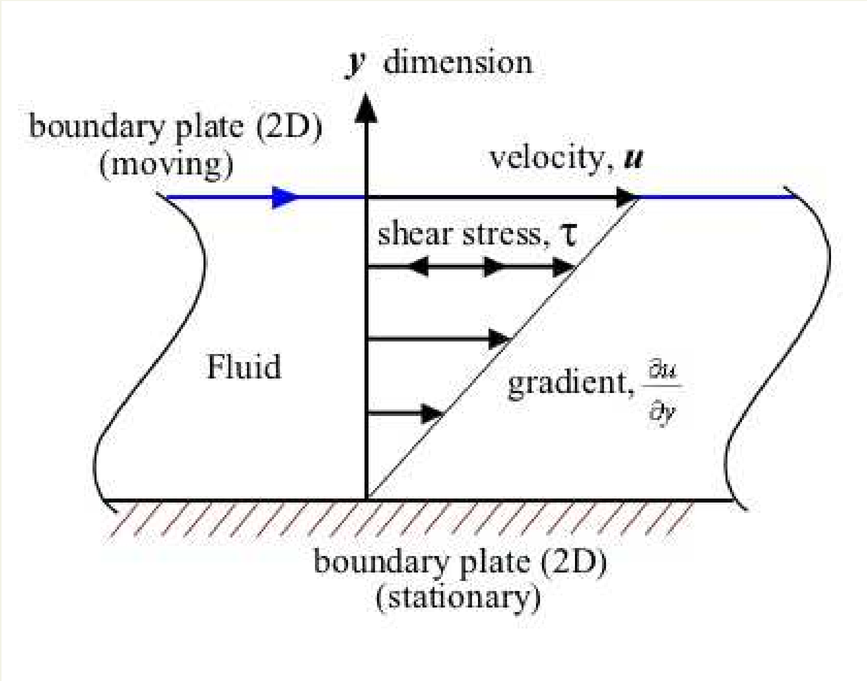
Viscous QGP(?)

(Shear) viscosity

1687 - I. Newton **(Principia)**

$$T_{xy} \equiv \frac{F_x}{A} = -\eta \frac{\partial u_x}{\partial y}$$

η: shear viscosity
reduces velocity gradients
$\Rightarrow$ dissipation



1985 - Heisenberg $\Delta E \cdot \Delta t$ + kinetic theory: $\eta/s \geq \hbar/15k_B$

Gyulassy & Danielewicz, PRD 31 ('85)

2004 - string theory AdS/CFT**:** $\eta/s \geq 1/4\pi \cdot \hbar/k_B$

Policastro, Son, Starinets, PRL87 ('02); Kovtun, Son, Starinets, PRL94 ('05)

revised to $\eta/s \geq 4\hbar/(25\pi)$ Brigante et al, PRL101 ('08)

or even lower Camanho et al, arXiv:1010.1682

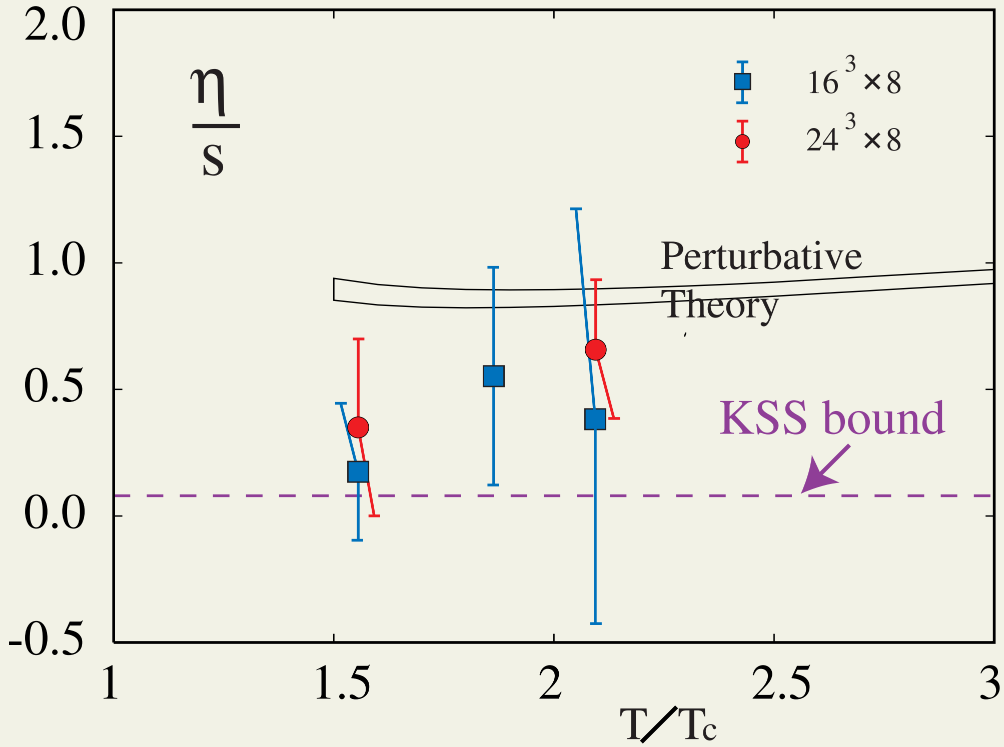
"minimal viscosity" or not - need to test it experimentally

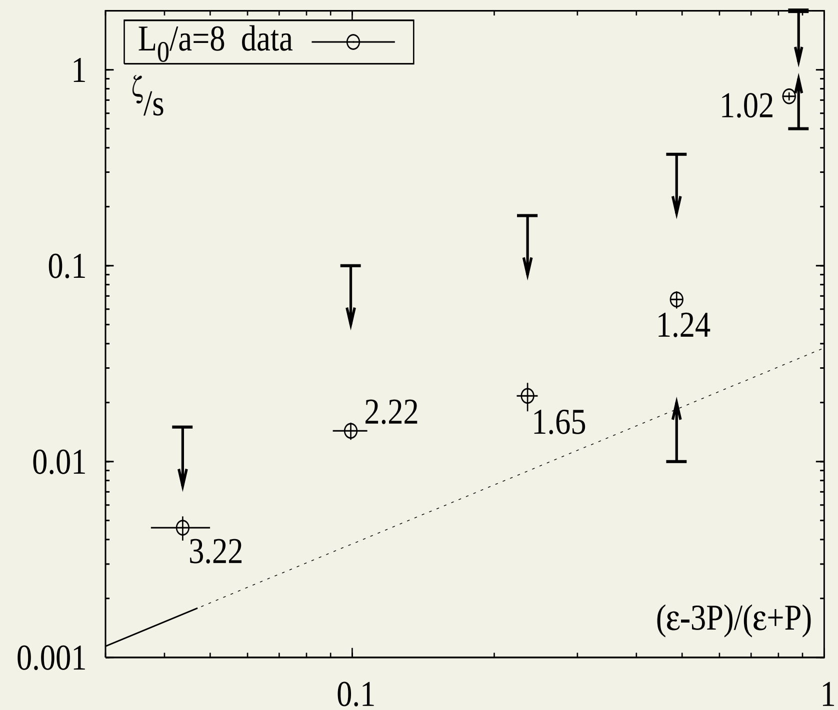
Viscosity in QCD - not known

perturbation theory ($T \gg T_c$): large $\eta/s > \mathcal{O}(1)$, small $\zeta/s \sim 0.02\alpha_s^2 \sim 0$

Arnold, Moore, Yaffe, JHEP 0305 ('03); Arnold, Dogan, Moore, PRD74 ('06)

lattice QCD estimates: shear Nakamura & Sakai, NPA774 ('06) **bulk** Meyer, PRD76 ('07)



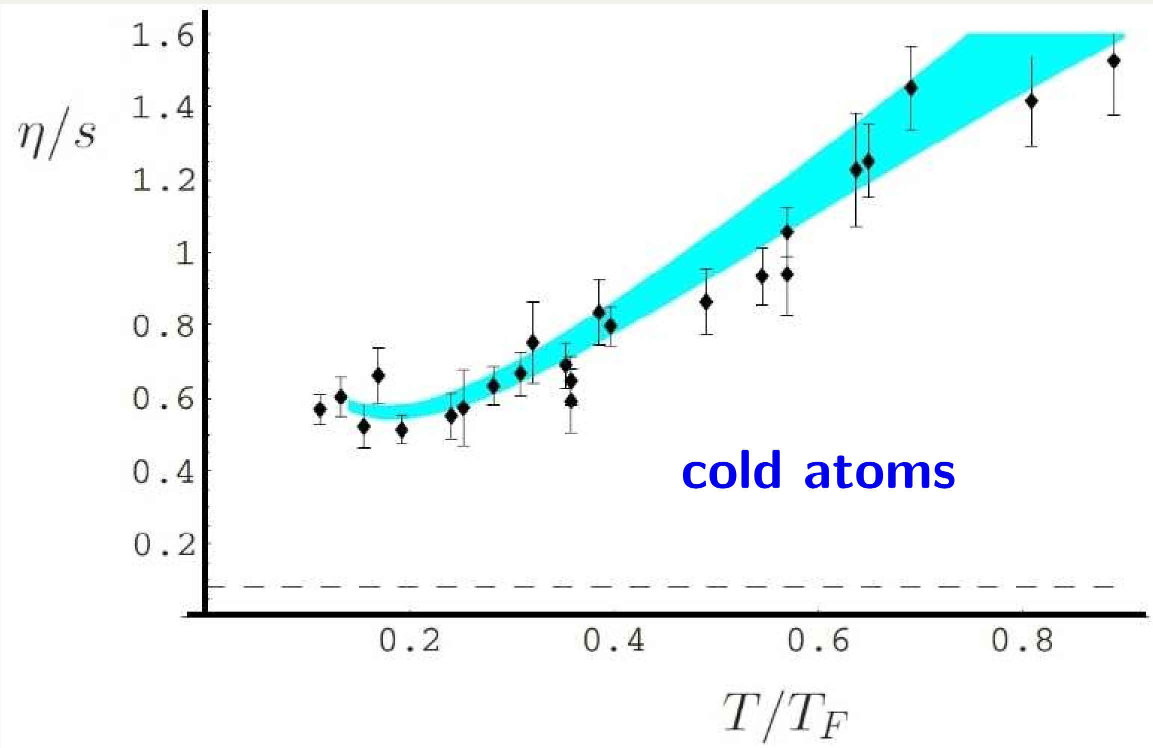


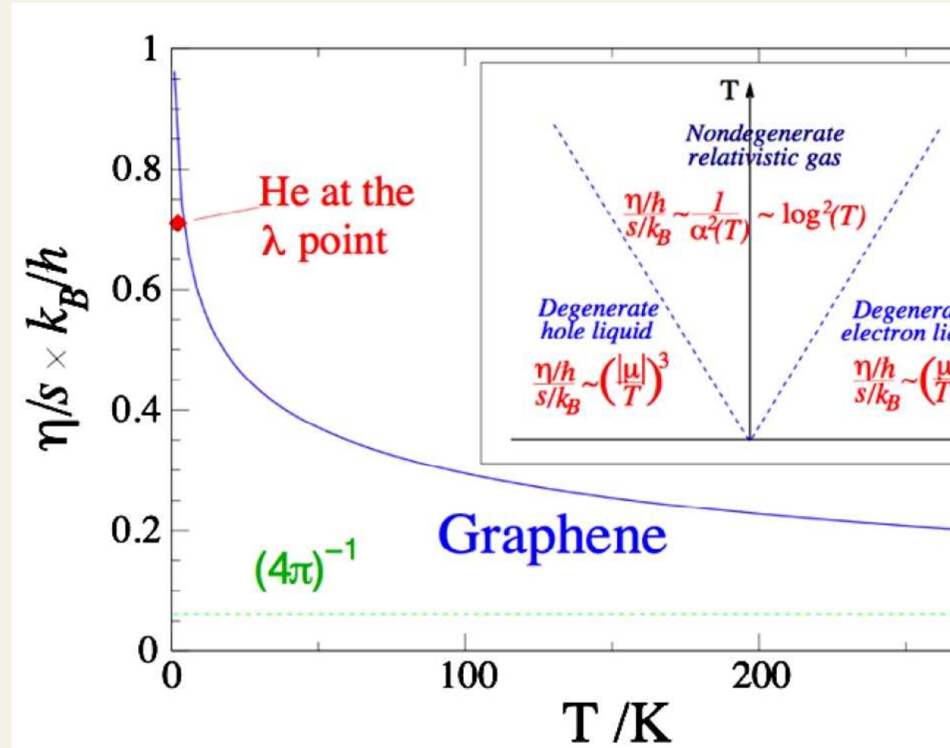
inversion problem - you compute $G(\tau) = \int_0^\infty d\omega\, \sigma(\omega) K(\omega,\tau)$ at 8-12 points τ_i, but you need $d\sigma/d\omega$ at $\omega = 0$ to obtain viscosities

viscosities at RHIC and LHC not known but could indeed be small

Schafer, PRA 76 ('07)

Mueller et al, PRL103 ('09)





$$\sim 5 \times \frac{1}{4\pi}$$

$$\sim 3 \times \frac{1}{4\pi}$$

What viscosity does

entropy production:

$$\partial_\mu S^\mu = \frac{\Pi^2}{\zeta T} - \frac{q^\mu q_\mu}{\kappa T^2} + \frac{\pi^{\mu\nu}\pi_{\mu\nu}}{2\eta T} \qquad > \quad 0$$

sound damping:

$$\omega(k) = c_s k - \frac{i}{2} k^2 \Gamma_s + \mathcal{O}(k^3) \qquad\qquad \Gamma_s \equiv \frac{\frac{4}{3}\eta + \zeta}{e + p}$$

sound attenuation length

slower cooling:

$$dE = -p dV + T dS \qquad (dS > 0)$$

anisotropic pressure: **e.g., longitudinal expansion, with shear only**

$$T^{\mu\nu} = diag(e,\ p + \pi_L/2,\ p + \pi_L/2,\ p - \pi_L)$$

Dissipative frameworks

- **causal relativistic hydrodynamics** Israel, Stewart; ... Muronga, Rischke; Teaney et al; Romatschke et al; Heinz et al, DM & Huovinen ... Niemi et al... Ván et al..

$$\partial_\mu T^{\mu\nu} = 0 \qquad\qquad (\mu_B \to 0)$$

$$T^{\mu\nu} = (e+p)u^\mu u^\nu - pg^{\mu\nu} + \pi^{\mu\nu} - \Pi\Delta^{\mu\nu}$$

$$\dot{\pi}^{\mu\nu} = F^{\mu\nu}(e,u,\pi,\Pi) \quad , \qquad \dot{\Pi} = G(e,u,\pi,\Pi)$$

e.g. Israel-Stewart theory

- **covariant transport** Israel, de Groot,... Zhang, Gyulassy, DM, Pratt, Xu, Greiner...

$$p^\mu \partial_\mu f = C_{2\to 2}[f] + C_{2\leftrightarrow 3}[f] + \cdots$$

fully causal and stable

near hydrodynamic limit, transport coefficients and relaxation times:

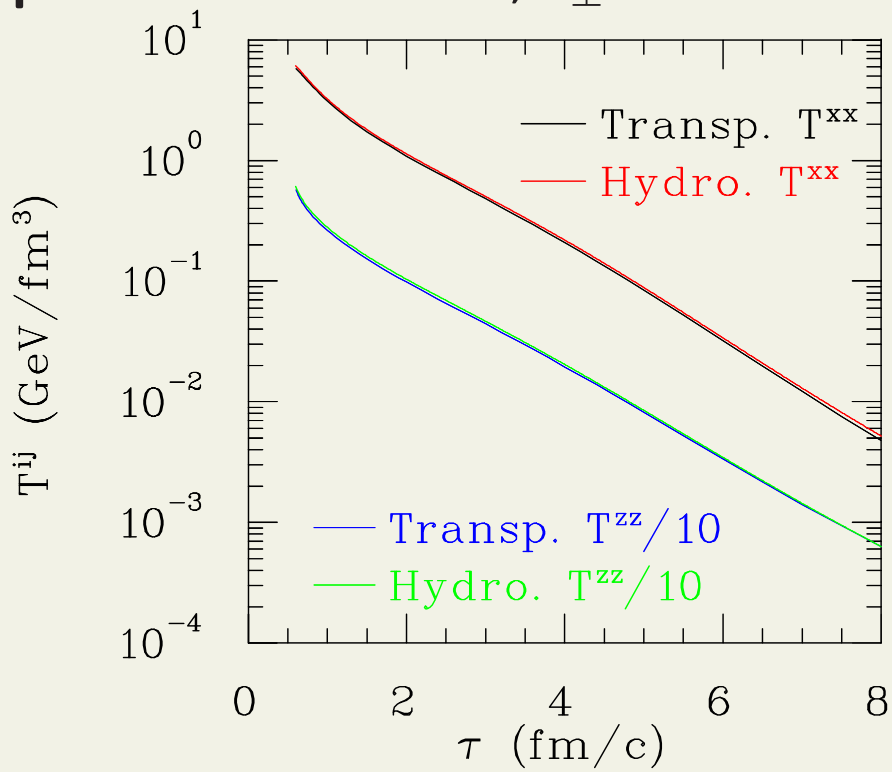
$$\eta \approx 1.2T/\sigma_{tr}, \qquad \tau_\pi \approx 1.2\lambda_{tr}$$

when viscosity is small, transport becomes viscous hydro

Au+Au at RHIC, $b = 8$ fm

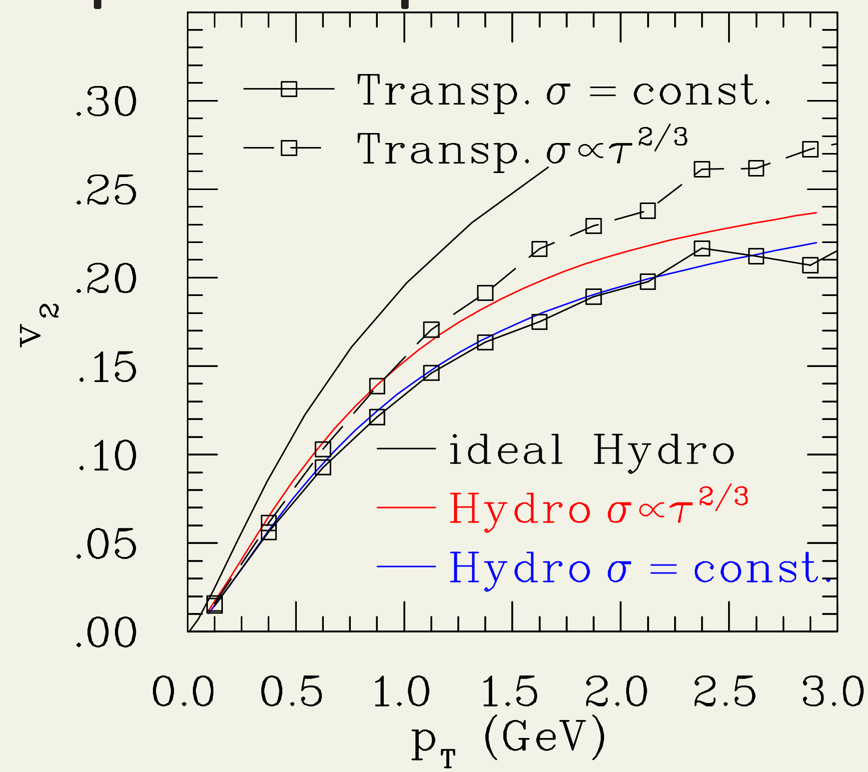
Huovinen & DM ('08)

pressure in the core, $r_\perp < 1$ fm



$\boldsymbol{\eta/s \approx 1/(4\pi)}$ $(\sigma \propto \tau^{2/3})$

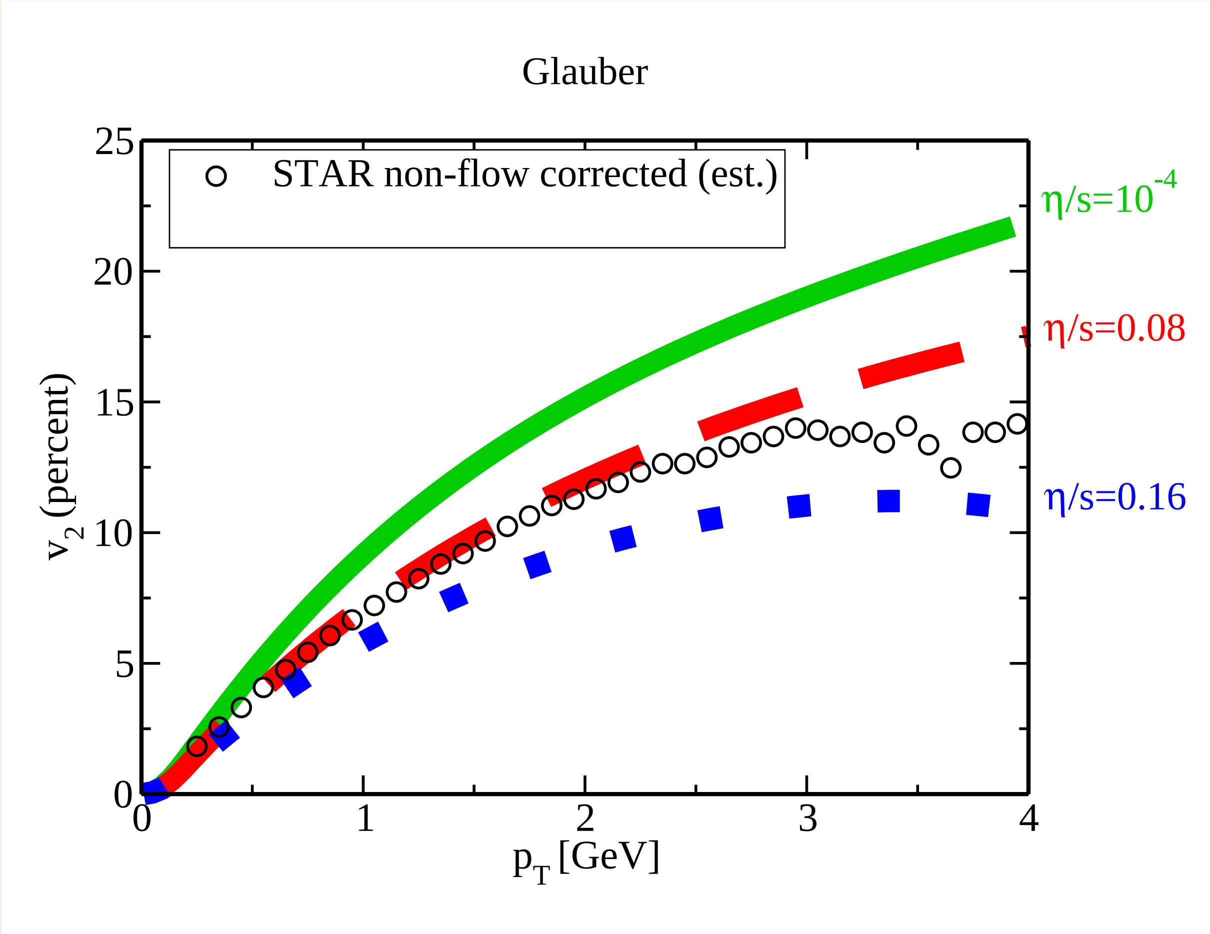
elliptic flow vs pT



- $\sigma =$ **const** $\sim 47mb$
- $\eta/s \approx 1/(4\pi)$, **i.e.,** $\sigma \propto \tau^{2/3}$

Shear viscosity from RHIC data

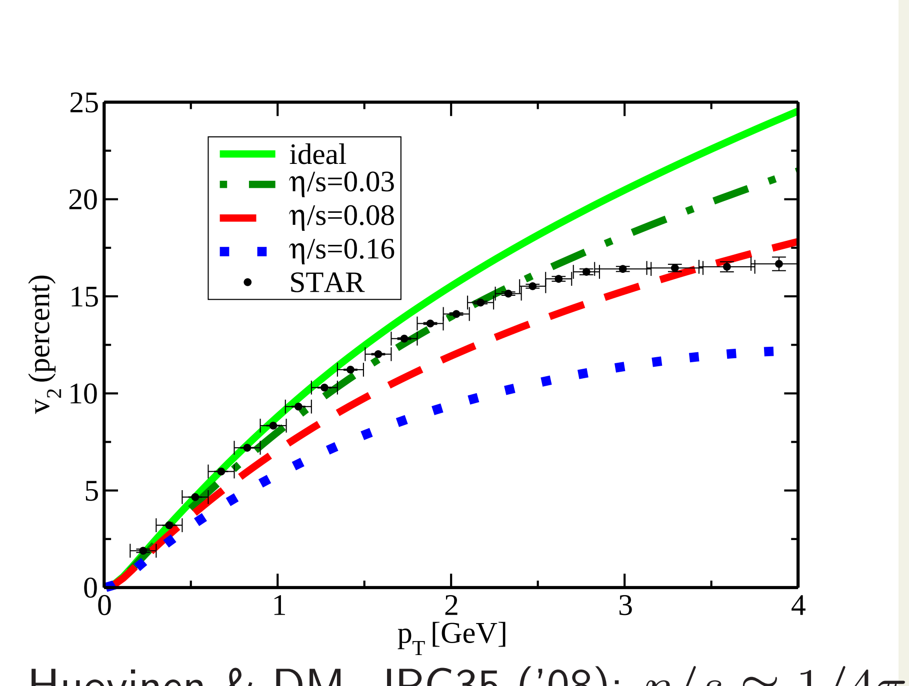
Romatschke & Luzum, PRC78 ('08): **Au+Au data vs 2+1D viscous hydrodynamics**



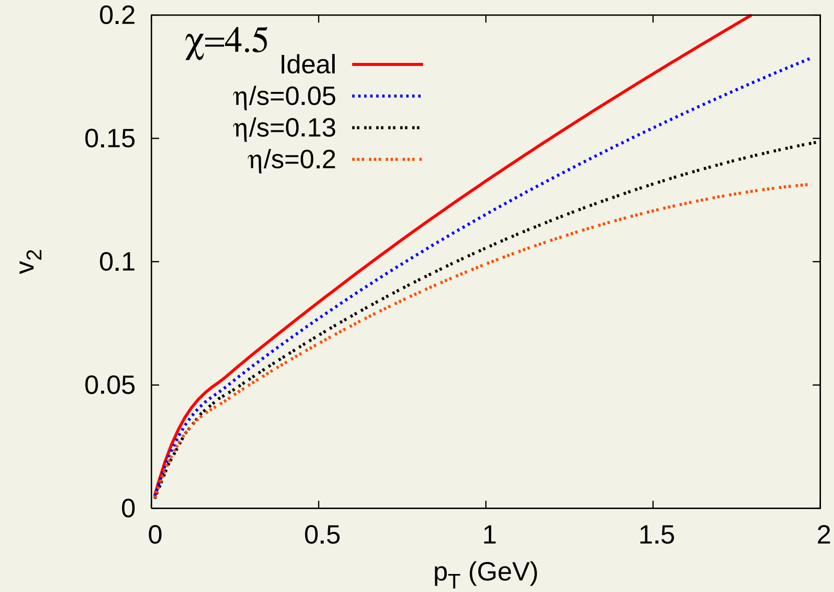
although in the end gave **conservative estimate** $\eta/s \lesssim 0.5$

many uncertainties: hydro validity, $\eta/s(T)$**, initial conditions, decoupling...**

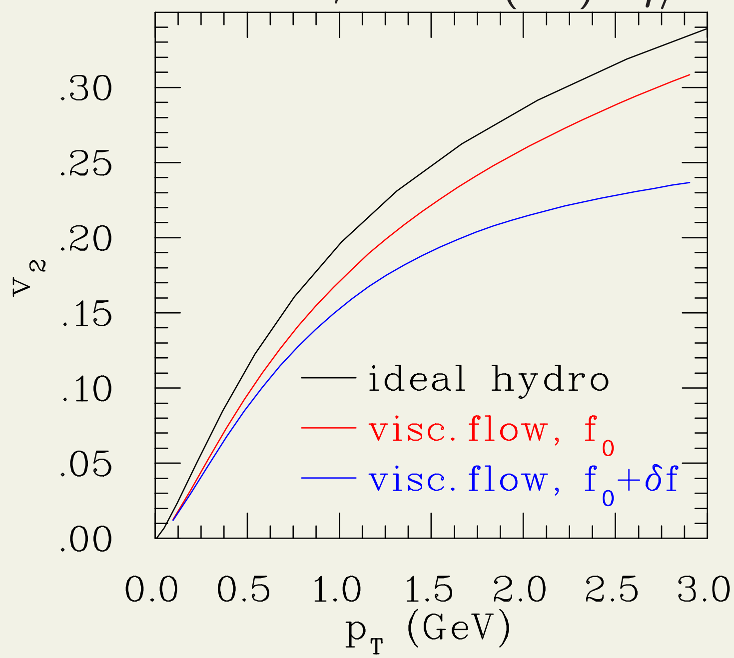
Romatschke & Romatschke, PRL99 ('07)



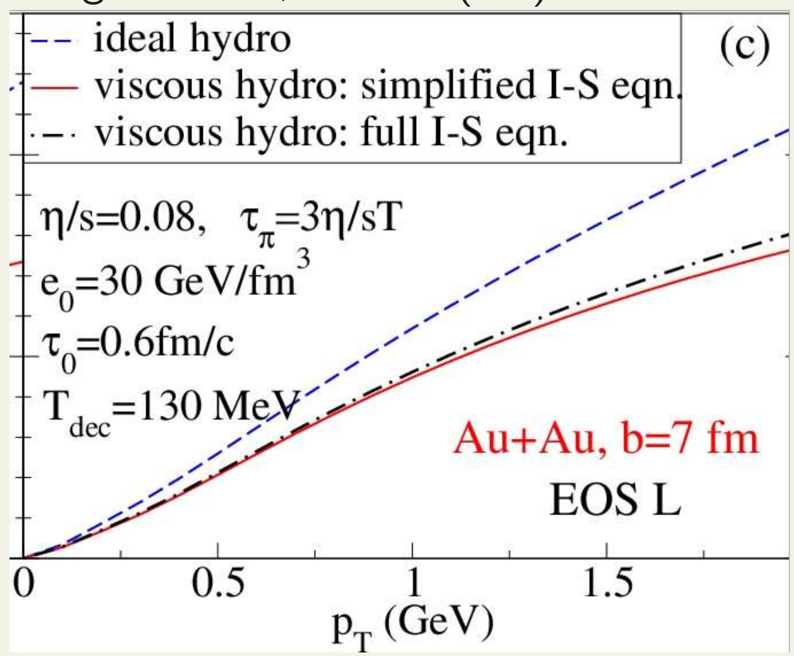
Dusling & Teaney, PRC77



Huovinen & DM, JPG35 ('08): $\eta/s \approx 1/4\pi$

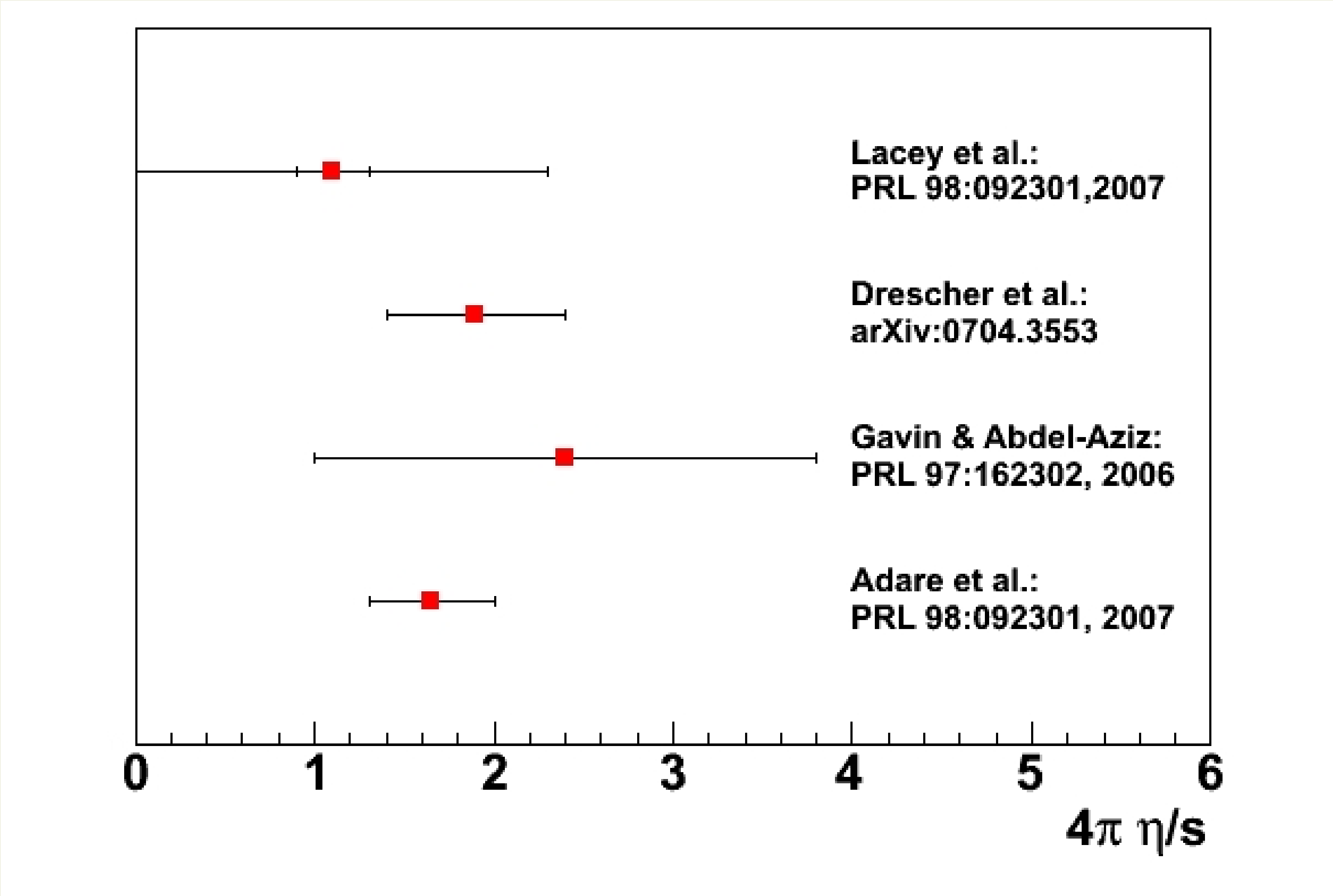


Song & Heinz, PRC78 ('08)



Consensus on $\sim 20-30$% effects for $\eta/s = 1/(4\pi)$ in Au+Au at RHIC.

ALSO - ballpark agreement with estimates based on kinetic theory, transverse momentum fluctuations, heavy-quark diffusion,... Zajc @ QM2009



II. From viscous fluid to particles

For viscous hydro calculations, identified particle observables are challenging

(yet unsolved problem)

Hydro $\longrightarrow$ particles

heavy-ion applications in the end must match hydrodynamics to a particle description

- **in local equilibrium - one-to-one maping**

$$T^{\mu\nu}_{LR} = diag(e,p,p,p) \qquad \Leftrightarrow \qquad f_{eq,i} = \frac{g_i}{(2\pi)^3} e^{-p_i^\mu u_\mu / T}$$

- **near local equilibrium - one-to-many**

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + \delta T^{\mu\nu} \qquad \Leftarrow \qquad f_i = f_{eq,i} + \delta f_i$$

corrections crucially affect basic observables - spectra, elliptic flow, ...

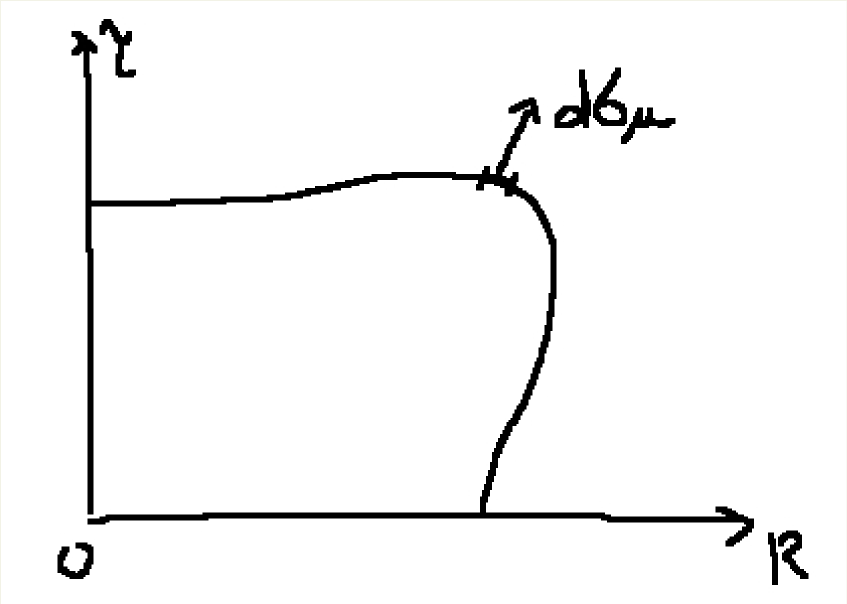
unavoidable whether we do pure hydro or hydro + transport

a separate issue: Cooper-Frye freezeout ("$t \neq const$")

Separate issue: Cooper-Frye

Cooper & Frye, PRD10 ('74)

fluid to a gas on a 3D hypersurface (e.g., $T(x) = T_{fo}$)



$$E\,\frac{dN}{d^3p} = p^\mu d\sigma_\mu(x)\, f_{gas}(T(x), \mu(x), u(x), \vec{p})$$

$d\sigma_\mu$: **hypersurface normal at** x

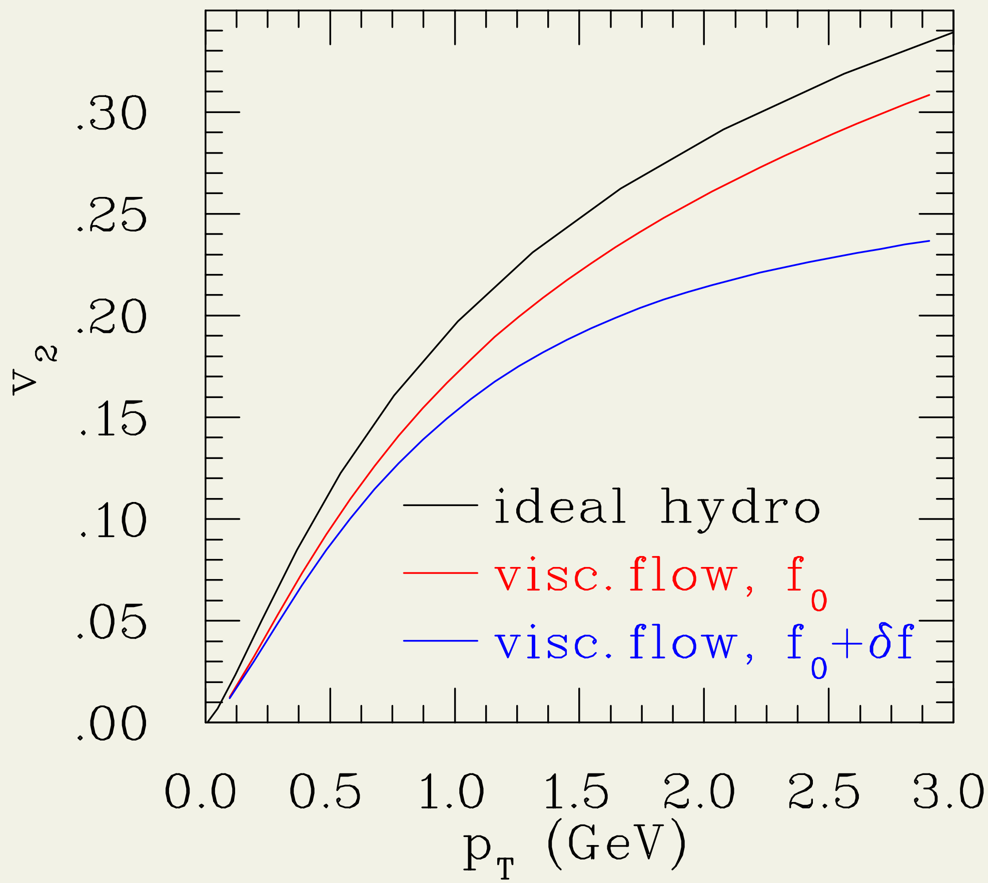
conversion at constant time is OK: $d\sigma^\mu = d^3x\,(1, \vec{0})$

$$\Rightarrow dN = f_{gas}(T(x), \mu(x), u(x), \vec{p})\, d^3x\, d^3p$$

but for arbitrary hypersurface, negative yield when $p^\mu d\sigma_\mu(x) < 0$

TWO effects: - **dissipative corrections to hydro fields** u^μ, T, n
- **dissipative corrections to thermal distributions** $f \to f_0 + \delta f$

Huovinen & DM ('08) $\boldsymbol{\eta/s \approx 1/(4\pi)}$ $(\sigma \propto \tau^{2/3})$

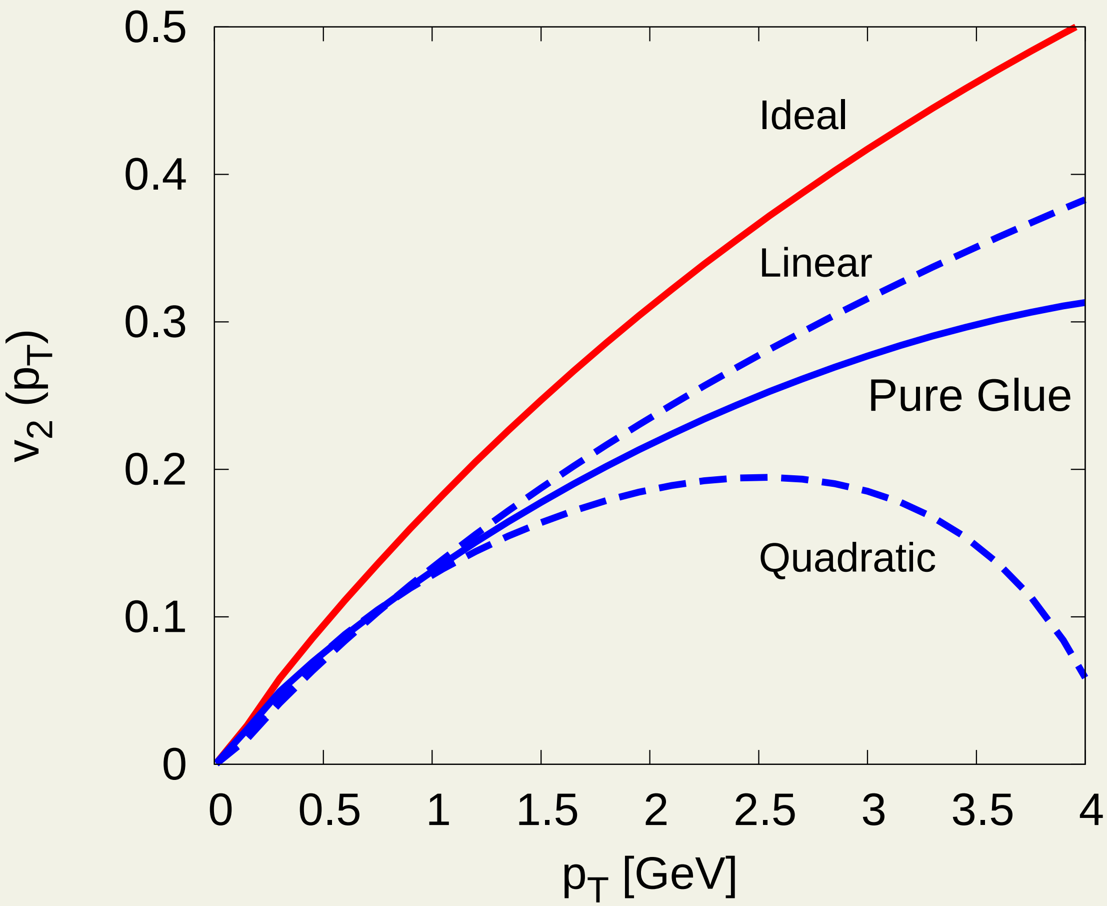


Grad:

$$\delta f = f_0 \left[1 + \frac{p^\mu p^\nu \pi_{\mu\nu}}{8 n T^6} \right]$$

most of the v_2 **reduction comes from phase space correction** δf

Teaney et al, PRC81 ('10)



energy dependence of δf affects observables at higher p_T

for one-component massless gas, with viscous shear only

$$\delta f \equiv f_{eq} \times C(\chi)\, \pi^{\mu\nu} \frac{p_\mu p_\nu}{T^2}\, \chi(\frac{p}{T})$$

from Grad's ansatz: $\chi \equiv 1$

this is a starting point in deriving IS hydro from kinetic theory

from linear response: $\chi(x) \sim x^\alpha$ **with** $-1 \lesssim \alpha \lesssim 0$ Dusling, Teaney, Moore, ('09)

but δf **blows up at large momenta** $\Rightarrow$ **approximation breaks down**

check these from nonequilibrium transport...

Test in 0+1D Bjorken $\rightarrow$ $f = f(p_T, \xi, \tau)$, where $\xi \equiv \eta - y$

i) compute f from full nonequilibrium transport

ii) from f, determine $T^{\mu\nu}$ and δf

iii) estimate δf from $T^{\mu\nu}$ alone via various ansatzes for $\chi(p/T)$ (e.g. Grad's)

$$\delta f \propto f_{eq} \frac{\pi_L}{16p} \left(\frac{p_T}{T}\right)^{\alpha+2} [\mathrm{ch}(2\xi) - 2]$$

iv) compare $\delta f^{estimated}$ with δf^{real}

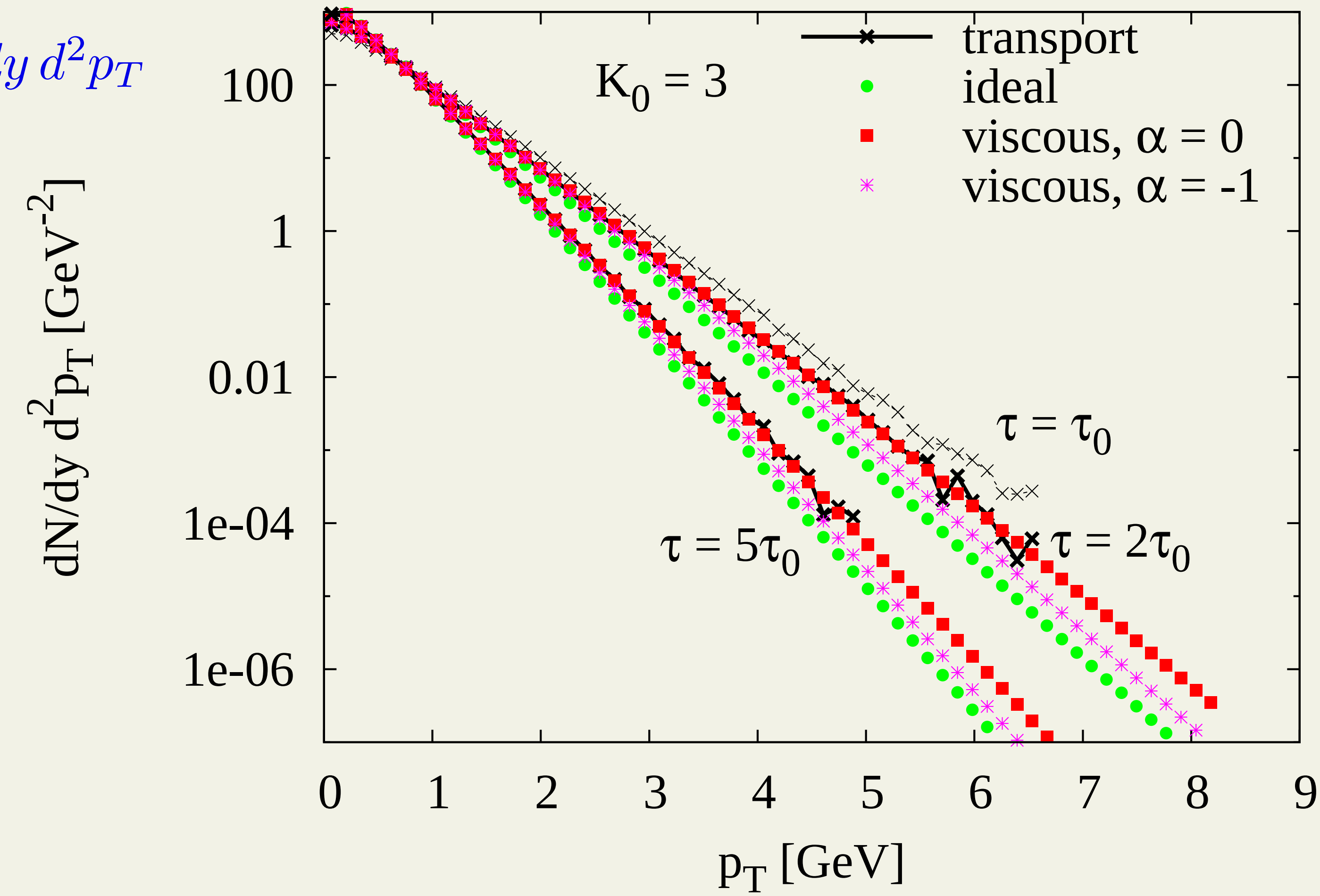
For simplicity, compare integrated quantities $dN(\tau)/dp_T^2 dy|_{y=0}$ and $dN(\tau)/d\xi$

drive calculation by inverse Knudsen number $K_0 = \tau/\lambda_{tr} \propto (\eta/s)^{-1}$

DM ('09):

spectra in 0+1D Bjorken scenario at $\tau = 2\tau_0$ and $5\tau_0$
for $\eta/s \sim 0.1$, local equilibrium initconds $\pi^{\mu\nu}(\tau_0) = 0$

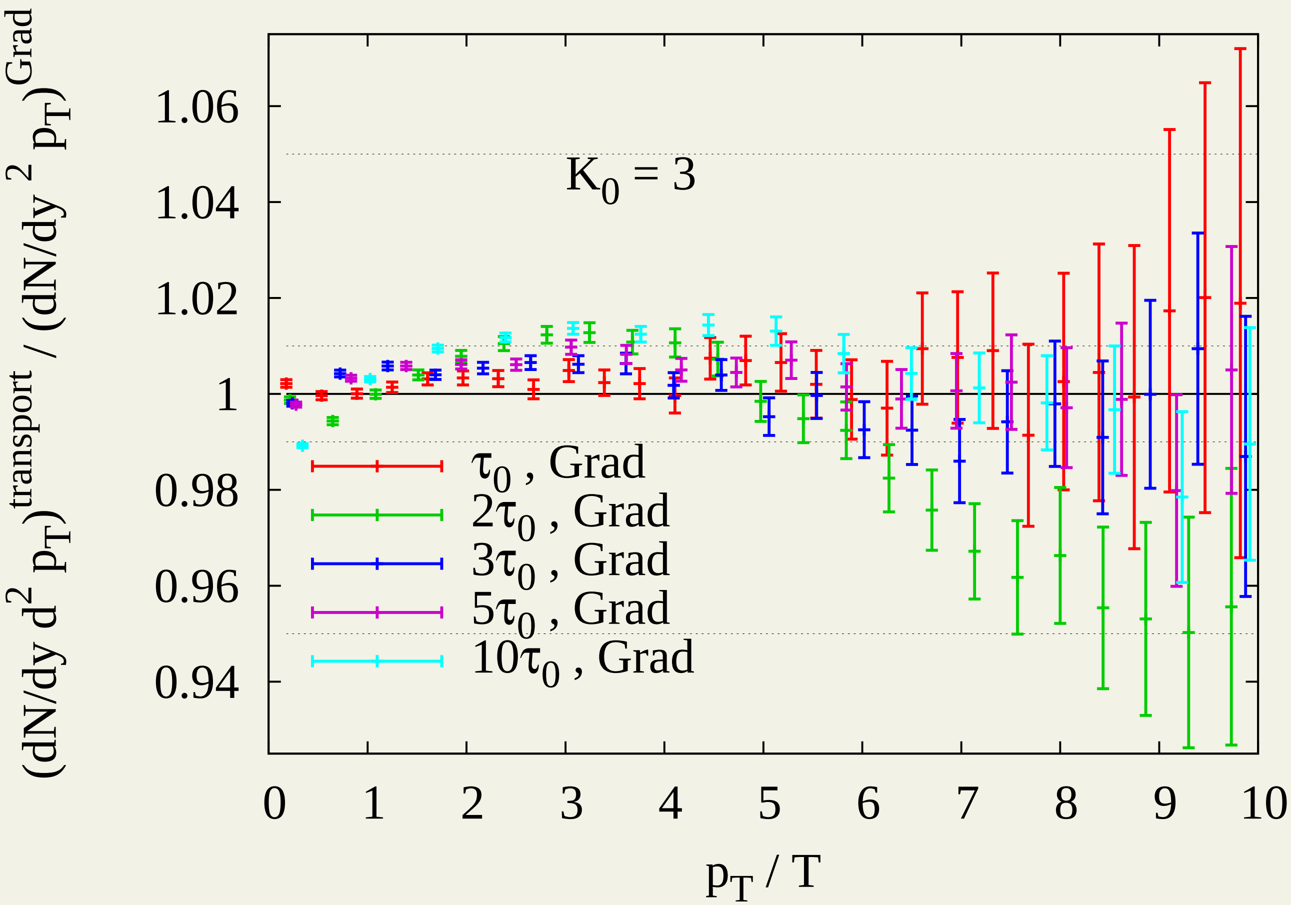
$dN/dy\, d^2p_T$



Grad ansatz ($\alpha = 0$) works surprisingly well - on a log plot at least

ratio - transport spectra / Grad approximation, $\eta/s \sim 0.1$

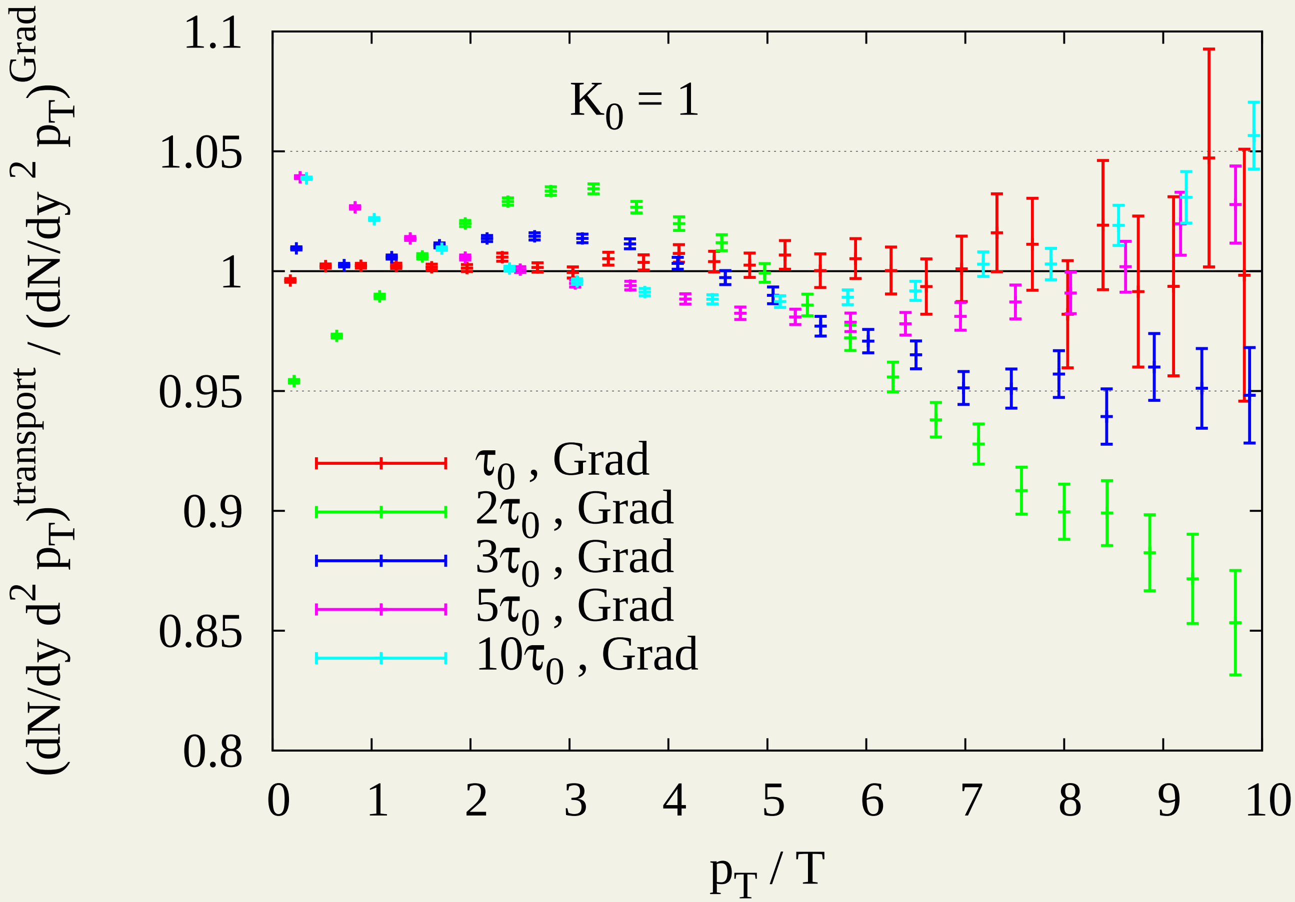
DM ('09):



Grad spectra are ≈ 1% accurate even at $p_T/T = 6$(!)

ratio - transport spectra / Grad approximation, $\eta/s \sim 0.3$

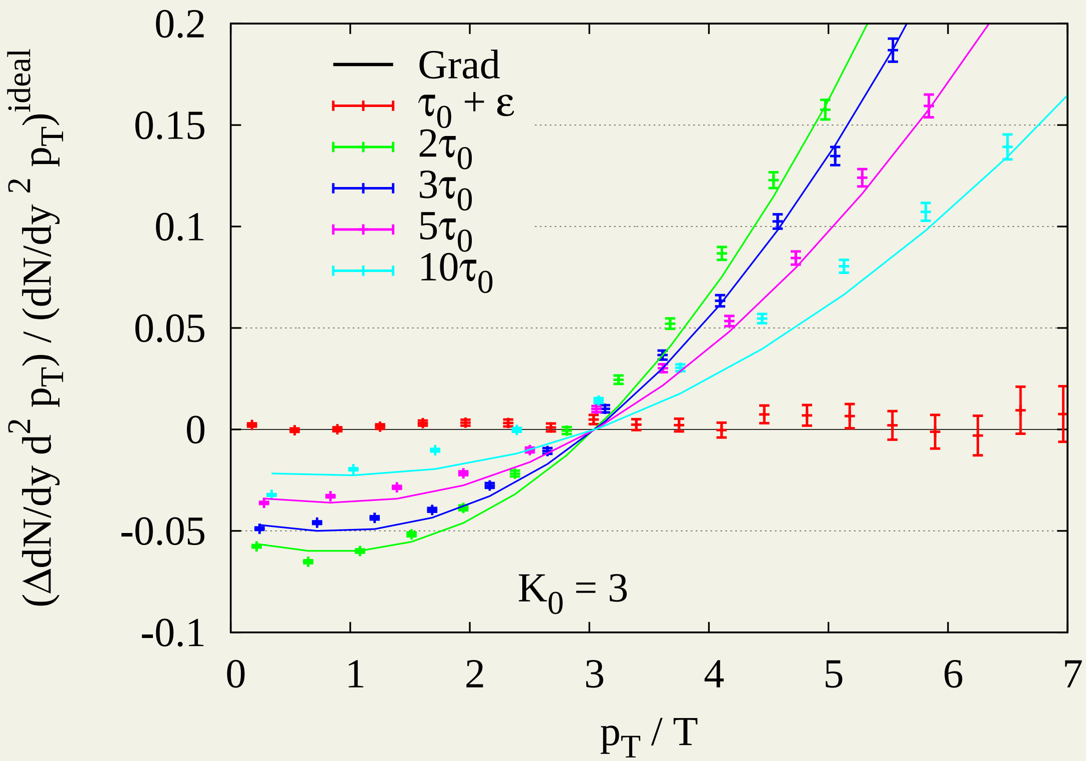
DM ('09):



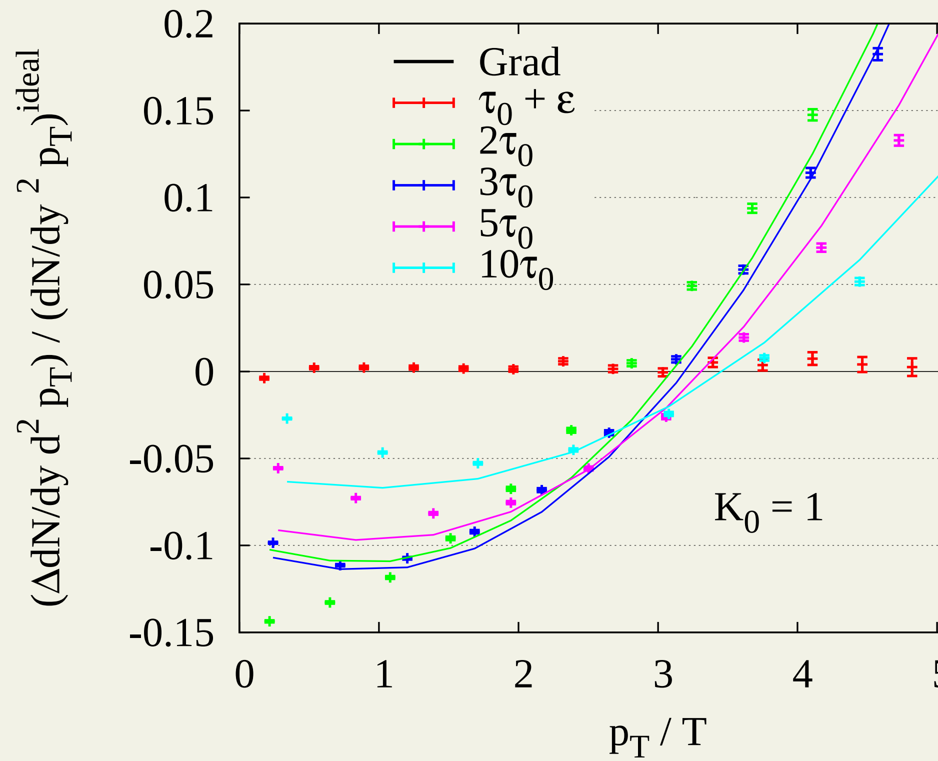
for higher viscosity, accuracy worsens - should affect other observables also

highlight viscous correction: spectra / ideal spectra

$\eta/s \approx 0.1$



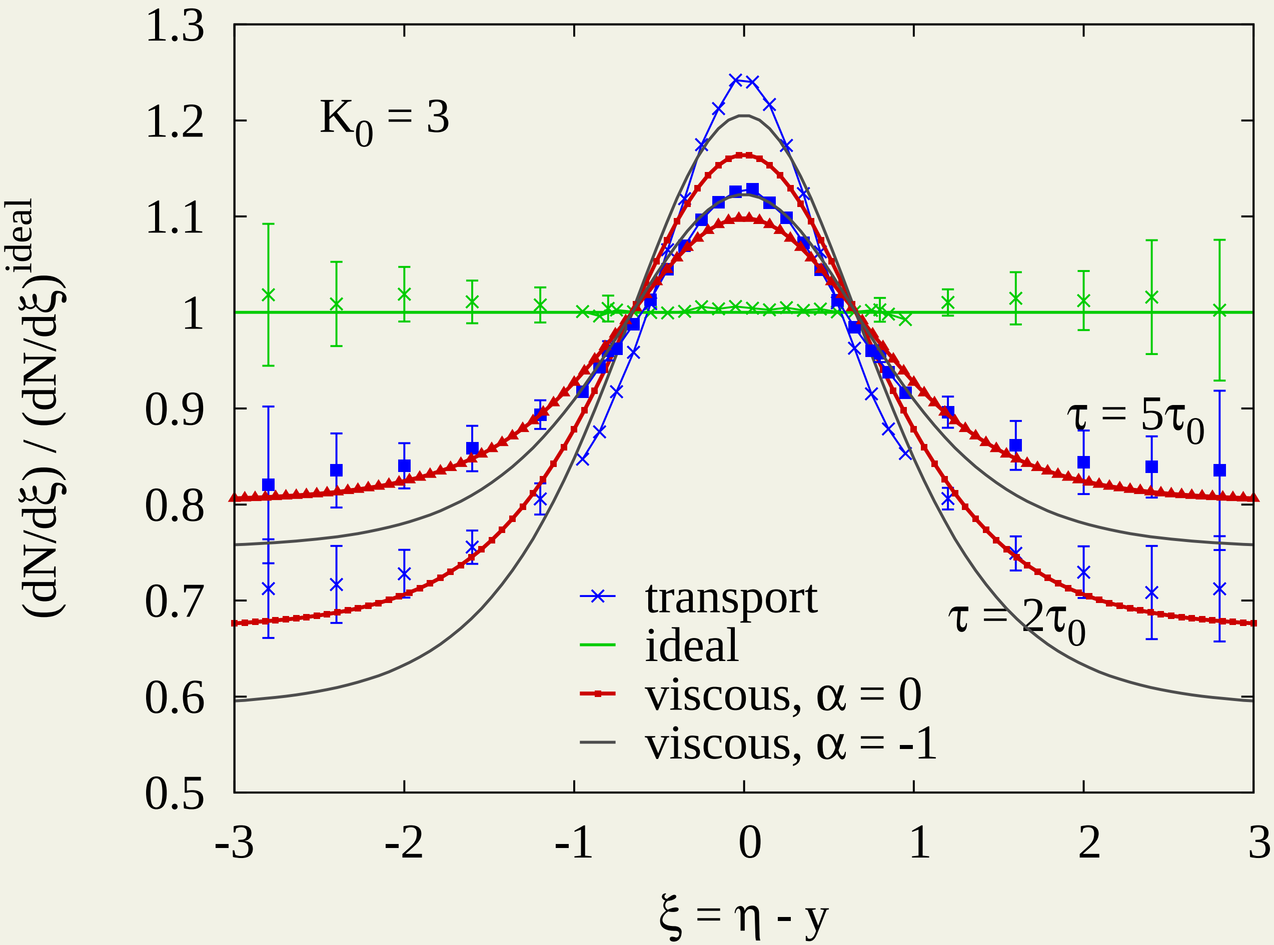
$\eta/s \approx 0.3$



for higher viscosity, $20-40$**% error in viscous correction at low** p_T

Grad ansatz not as good for rapidity $\xi \equiv \eta - y$ correlation

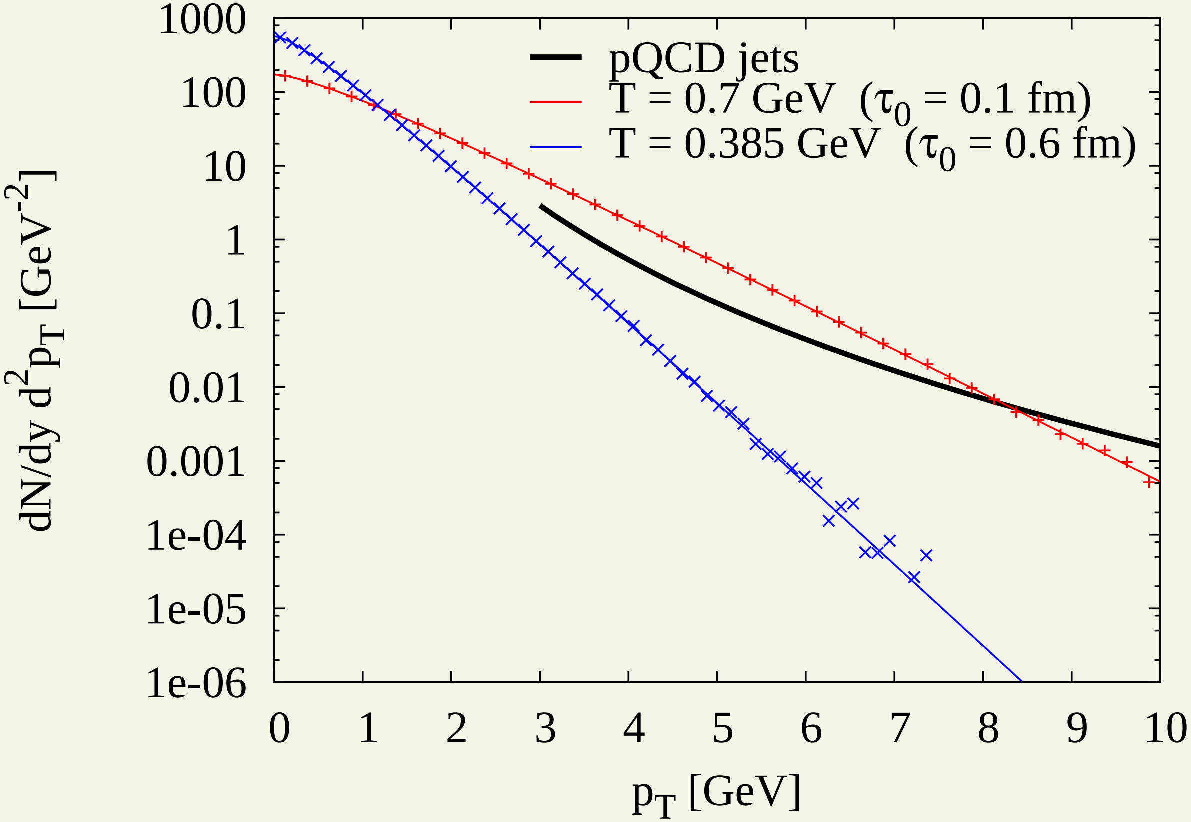
$\frac{dN/d\xi}{dN^{ideal}/d\xi}$ **distributions relative to IDEAL hydro,** $\eta/s \sim 0.1$ DM ('09-'10)



Interplay with pQCD jets

hydro accuracy is further limited by pQCD power-law tails

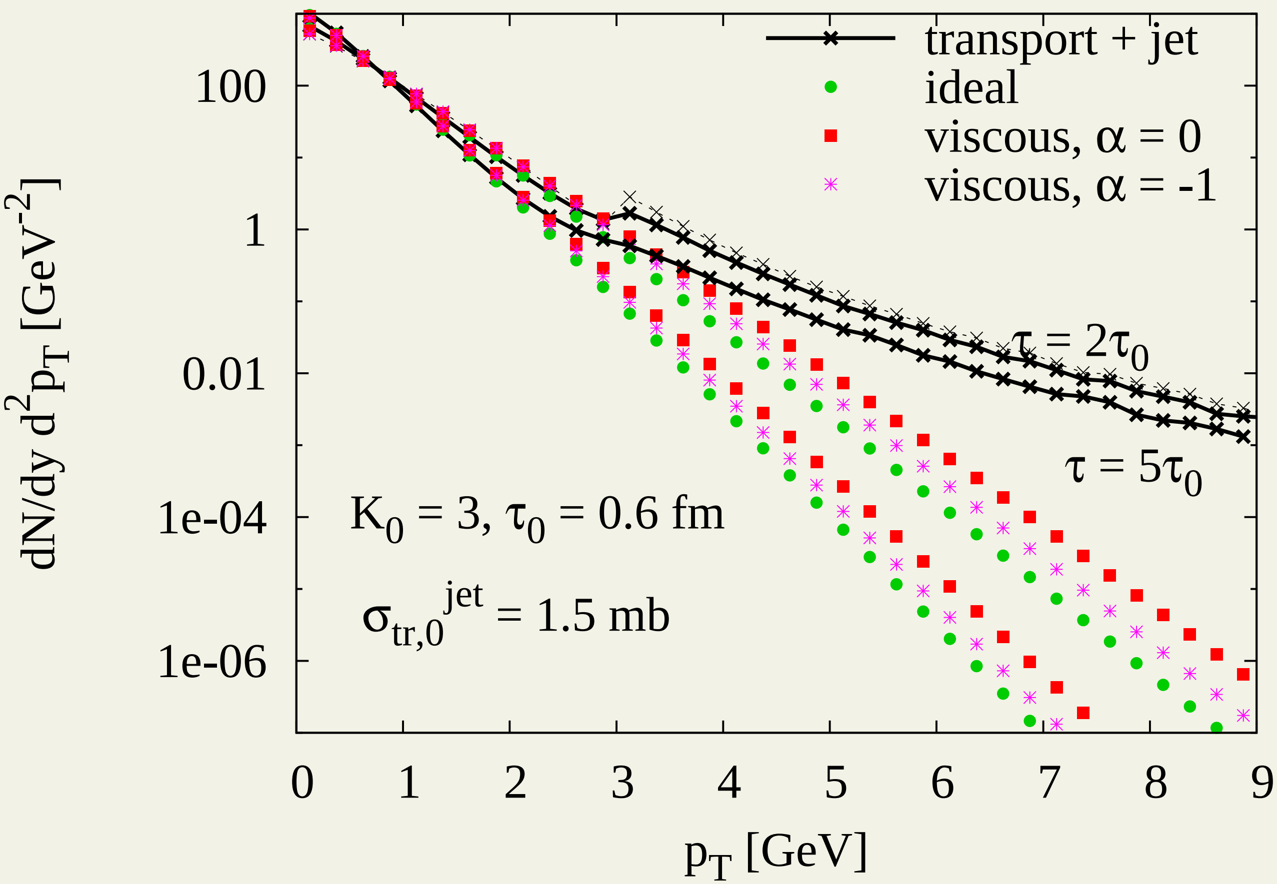
parton $dN/dp_T^2\,dy$ - central $Au + Au$ @ RHIC



study this in a two-component jet + bulk model

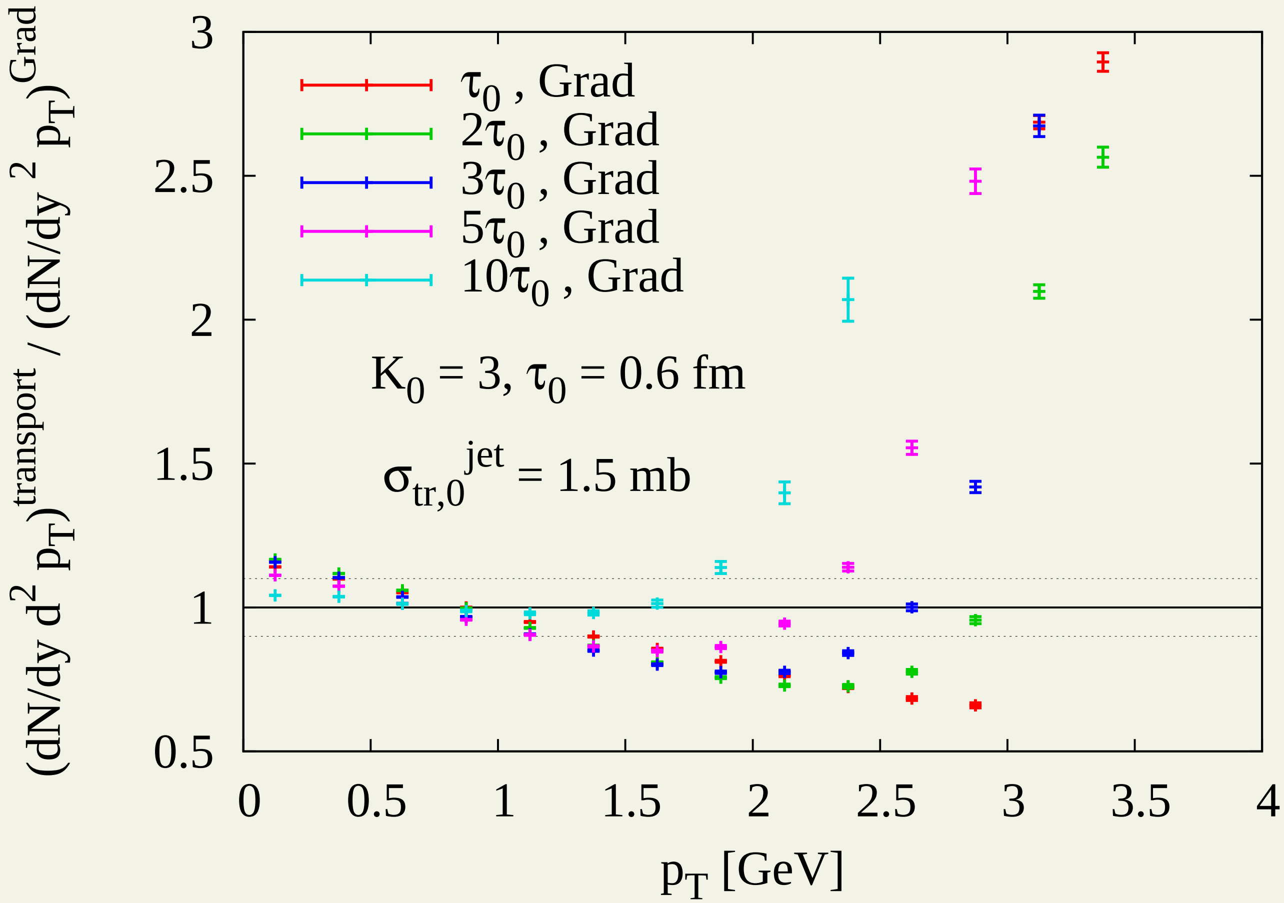
for jets: pQCD cross sections, for bulk: strong interactions ($\eta/s \approx 0.1$)

spectra vs hydro approximation DM ('09)-('10)



$$[\text{bulk: } \eta/s \sim 0.1, \quad \text{jets: } \sigma_{tr} = T_0^2/T^2 \cdot 1.5 \text{ mb}, \quad T_0 = 385GeV]$$

ratio - transport spectra / Grad approximation



$$[\text{bulk: } \eta/s \sim 0.1, \quad \text{jets: } \sigma_{tr} = T_0^2/T^2 \cdot 1.5 \text{ mb}, \quad T_0 = 385GeV]$$

jets spoil accuracy of Grad ansatz for $p_T \gtrsim 1.5$ GeV

Hydro $\to$ gas mixture

must be tackled to address IDENTIFIED particle data

(!) from ONE set of viscous fields we need to obtain δf_i for EVERY species

commonly used "democratic" prescription:

$$\delta f_i \equiv f_i^{eq} \times \frac{\pi^{\mu\nu}}{2(e+p)} \frac{p_{\mu,i} p_{\nu,i}}{T^2} \qquad (i = \pi, K, p, \Lambda, ...)$$

ignores equilibration dynamics

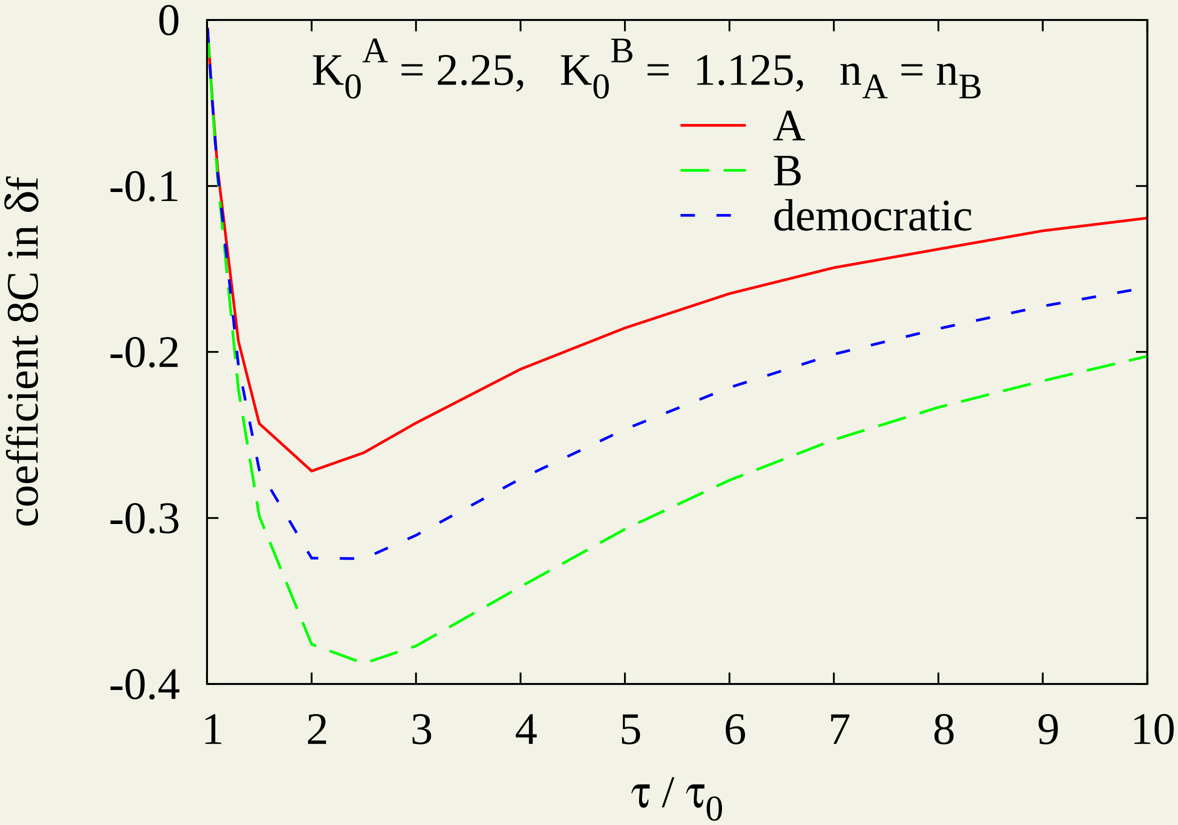
$$K_i \sim \frac{\tau}{\lambda_i} \sim \tau \sum_j n_j \sigma_{ij}$$

key drivers: <u>relative</u> Knudsen numbers between species K_j / K_i

Democratic vs $2 \to 2$ transport

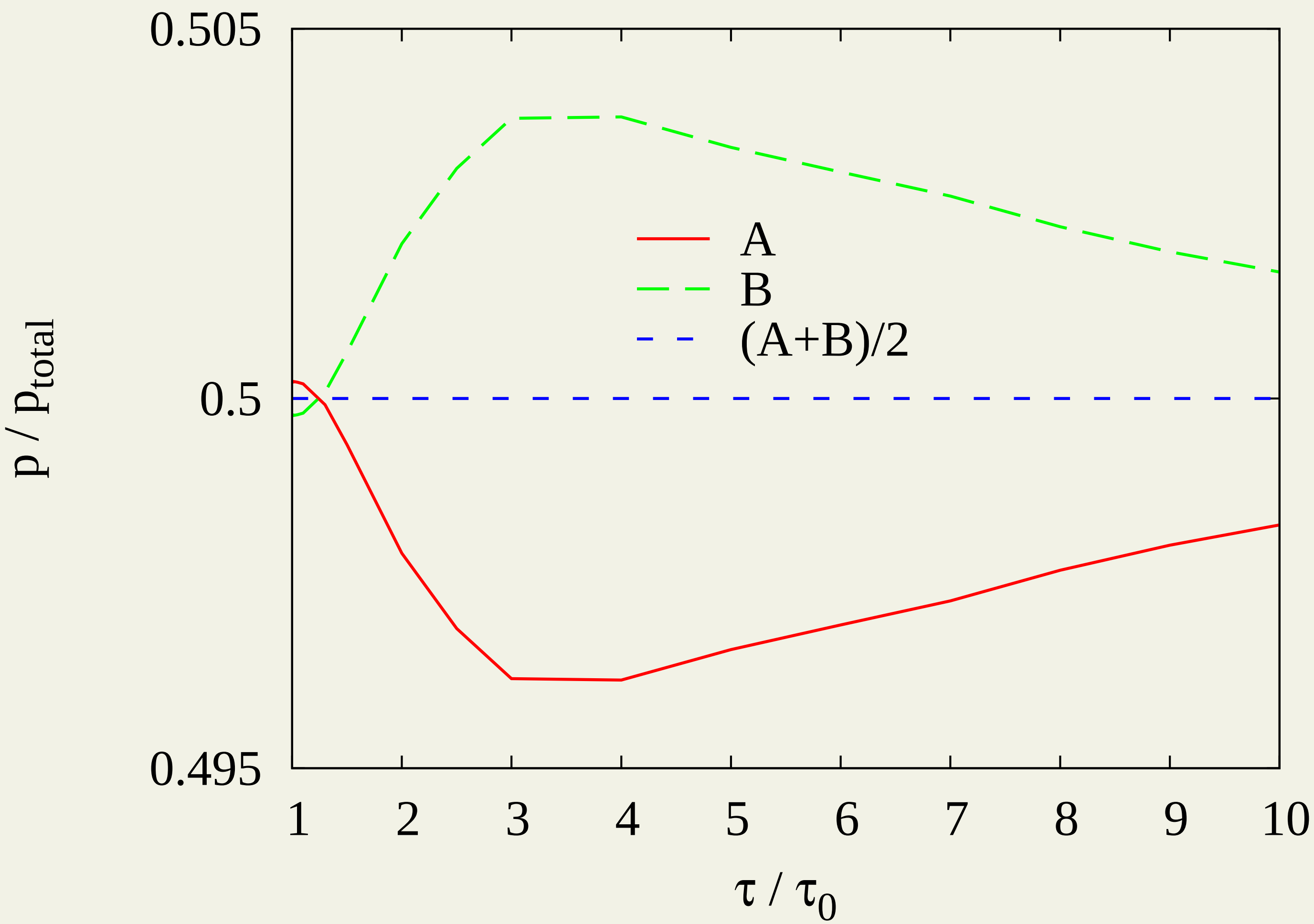
2-component 0+1D Bjorken test DM ('10) - A **equilibrates twice as fast as** B

$$\delta f_i = C_i \, (p_T/T)^2 (\mathrm{sh}^2 y - 1/2) f_i^{eq} \qquad \pi_{L,i}/p_i = 8C_i$$



"democratic" ansatz misses viscous effects by $\sim\ 20 - 25$**%**

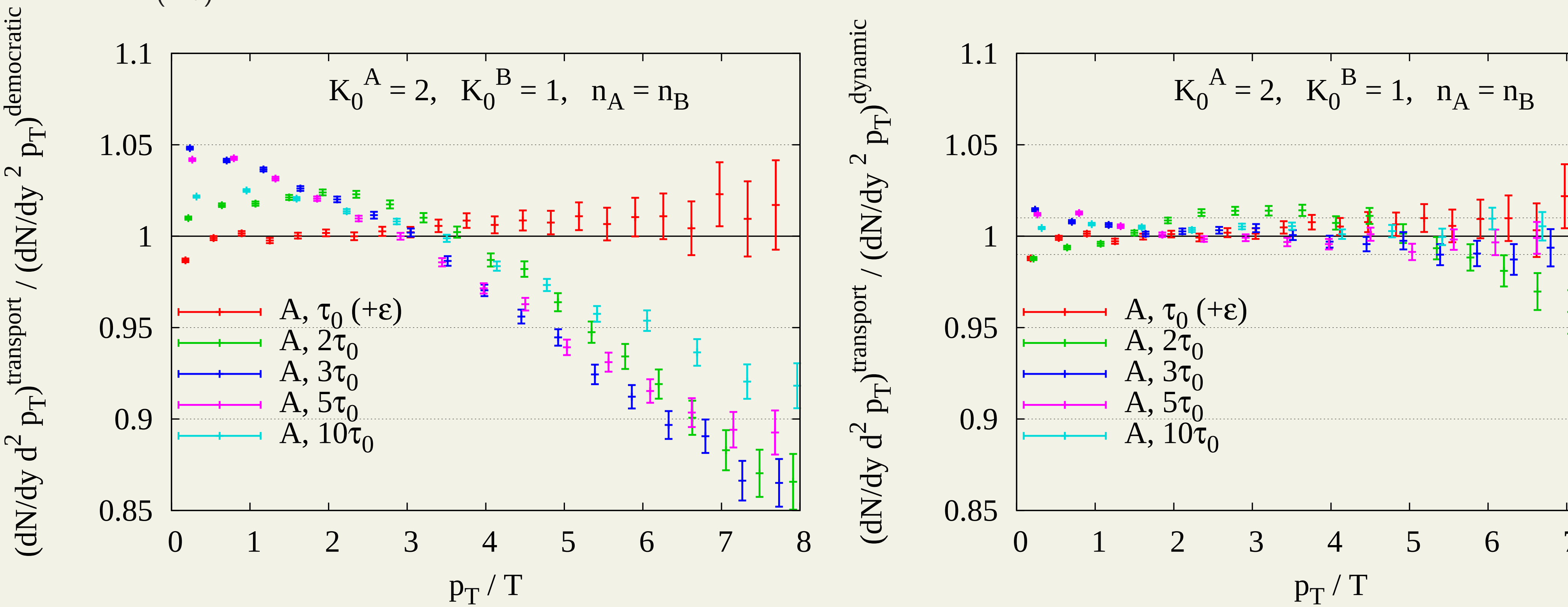
pressure evolution



VERY slightly more pressure work for species "A"

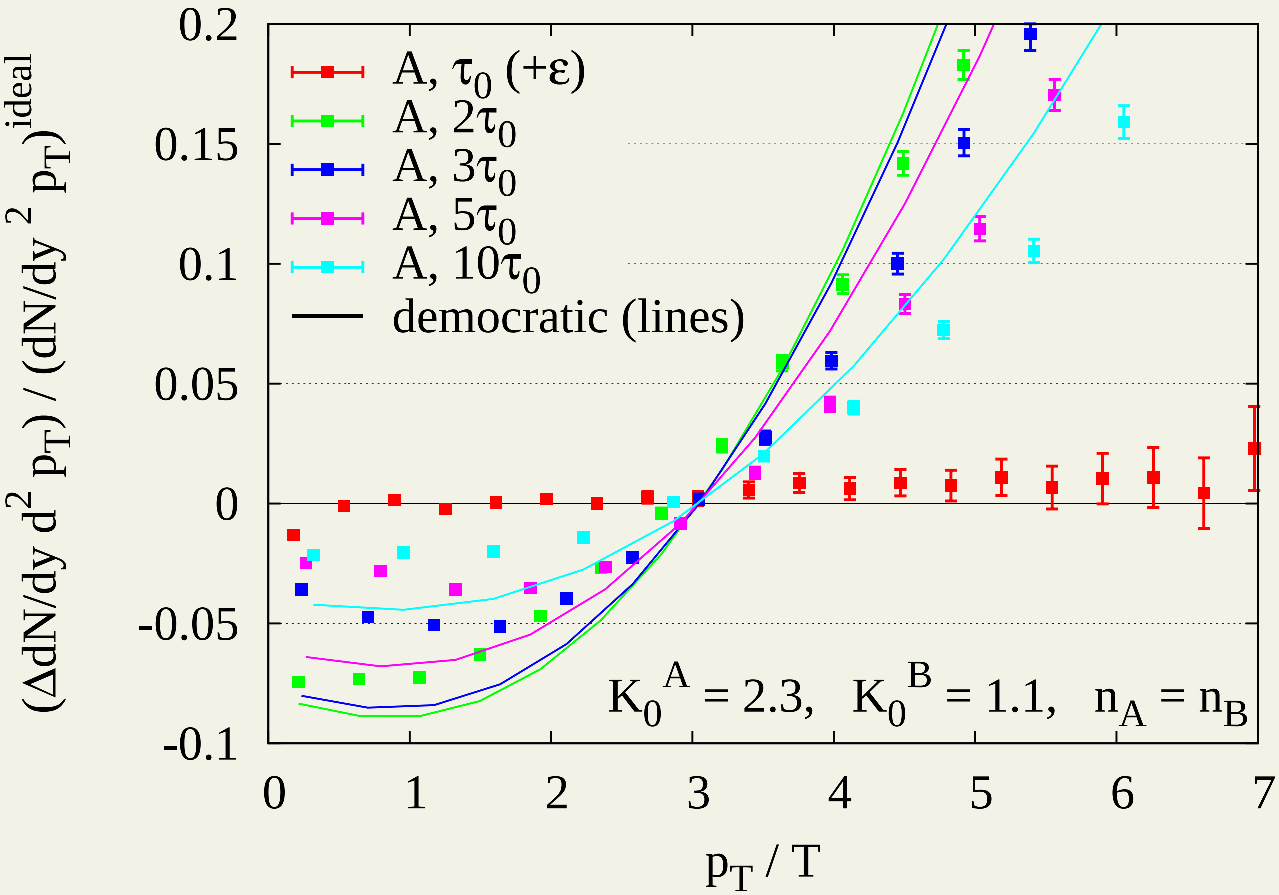
transport spectra / "democratic" Grad vs transport / dynamical Grad

DM ('10)



"democratic" Grad prescription not accurate

highlight dissipative correction: spectra for A / ideal ansatz



20-100% error in dissipative correction, depending on p_T and time

So... we must obtain dynamically determined partial shear stresses $\pi_i^{\mu\nu}$ - BEFORE we can convert to particles

$\Rightarrow$ **will a one-component viscous hydro be sufficient?**

usual derivation of hydro from kinetic theory based on Grad ansatz gives coupled set of equations between $\pi_i^{\mu\nu}$

Closed Equations – Bulk and Shear

$$\frac{d\Pi_{(i)}}{d\tau} + \frac{\Pi_{(i)}}{\tau_{\Pi(i)}} + \sum_{j\neq i} \frac{\Pi_{(j)}}{\tau_{\Pi(i)(j)}} = -\left(\beta_{\zeta(i)} + \zeta_{\Pi\Pi(i)}\Pi_{(i)}\right)\theta + \zeta_{\Pi\pi(i)}\pi^{\mu\nu}_{(i)}\sigma_{\mu\nu}$$
$$-\zeta_{\Pi n(i)}\partial_\mu n^\mu - \alpha_{\Pi n(i)} n^\mu_{(i)}\dot{u}_\mu - \beta_{\Pi n(i)} n^\mu_{(i)}\nabla_\mu \alpha_{0(i)}$$
$$-\zeta_{\Pi q(i)}\partial_\mu q^\mu - \alpha_{\Pi q(i)} q^\mu_{(i)}\dot{u}_\mu - \beta_{\Pi q(i)} q^\mu_{(i)}\nabla_\mu \alpha_{0(i)}$$

$$\frac{d\pi^{\langle\mu\nu\rangle}_{(i)}}{d\tau} + \frac{\pi^{\mu\nu}_{(i)}}{\tau_{\pi(i)}} + \sum_{j\neq i}\frac{\pi^{\mu\nu}_{(j)}}{\tau_{\pi(i)(j)}} = 2\left(\beta_{\eta(i)} + \eta_{\pi\Pi(i)}\Pi_{(i)}\right)\sigma^{\mu\nu} - 2\eta_{\pi\pi(i)}\pi^{\langle\mu}_{\alpha(i)}\sigma^{\nu\rangle\alpha} + 2\pi^{\langle\mu}_{\alpha(i)}\omega^{\nu\rangle\alpha} - \left(\frac{1}{2} + \frac{7}{6}\eta_{\pi\pi(i)}\right)\pi^{\mu\nu}_{(i)}\theta$$
$$+2\eta_{\pi n(2)(i)}\nabla^{\langle\mu} n^{\nu\rangle}_{(i)} + 2\beta_{\pi n(i)} n^{\langle\mu}_{(i)}\nabla^{\nu\rangle}\alpha_{0(i)} - 2\alpha_{\pi n(i)} n^{\langle\mu}_{(i)}\dot{u}^{\nu\rangle}$$
$$+2\eta_{\pi q(2)(i)}\nabla^{\langle\mu} q^{\nu\rangle}_{(i)} + 2\beta_{\pi q(i)} q^{\langle\mu}_{(i)}\nabla^{\nu\rangle}\alpha_{0(i)} - 2\alpha_{\pi q(i)} q^{\langle\mu}_{(i)}\dot{u}^{\nu\rangle}$$

Eqs. will depend on

$\Pi_{(i)}, q^\mu_{(i)}, \pi^{\mu\nu}_{(i)}, \nu^\mu_{(i)}$ **MORE variables!**

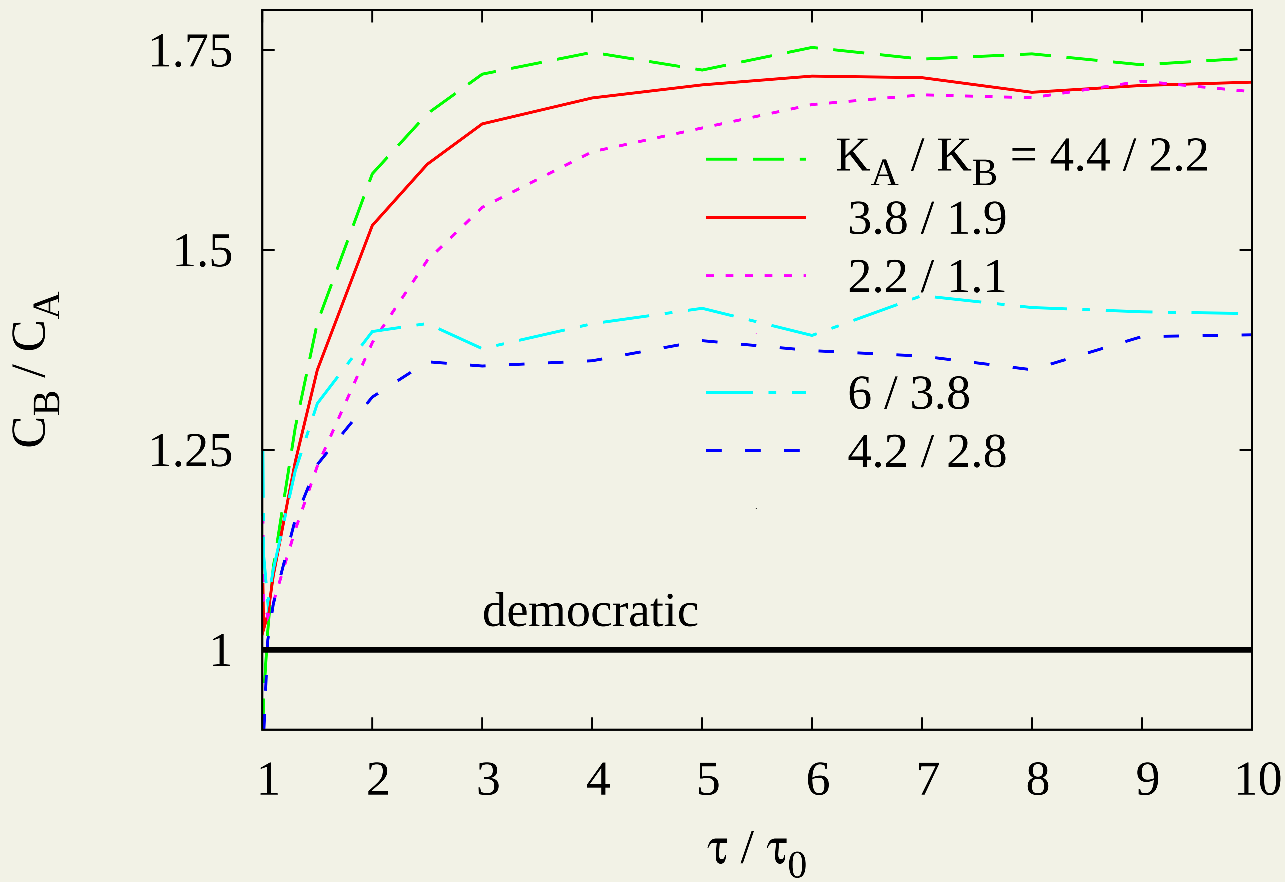
$\Pi, q^\mu, \pi^{\mu\nu}, \nu^\mu$ (possible?)

G. Denicol

Tech-qm

some hope: transport suggests universal sharing, at later times DM ('10)

$$\delta f_i = C_i \, (p_T/T)^2 (\mathrm{sh}^2 y - 1/2) f_i^{eq} \qquad \pi_{L,i}/p_i = 8C_i$$



III. Summary

- Theory should strive to address identified particle data. In the hydro paradigm, this is key in order to constrain the equation of state and initial conditions.

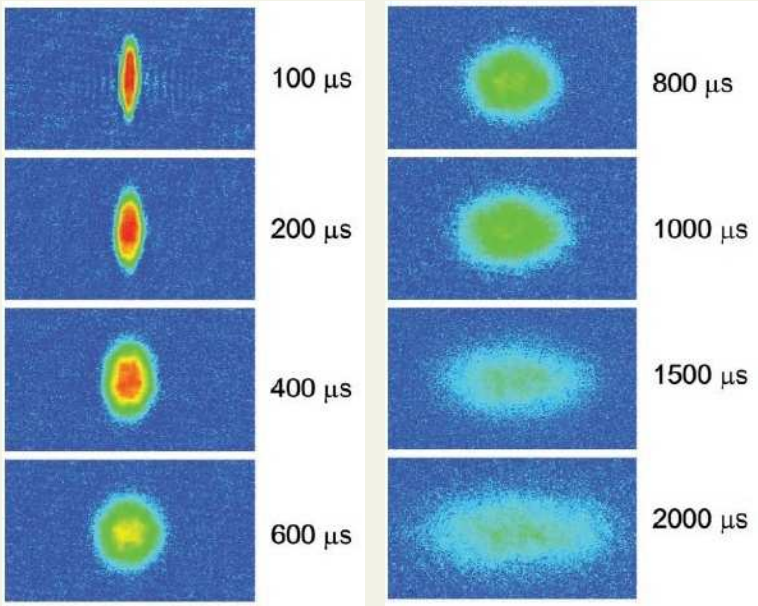
- Converting a non-ideal fluid to particles is nontrivial, independently of Cooper-Frye freezeout assumptions.

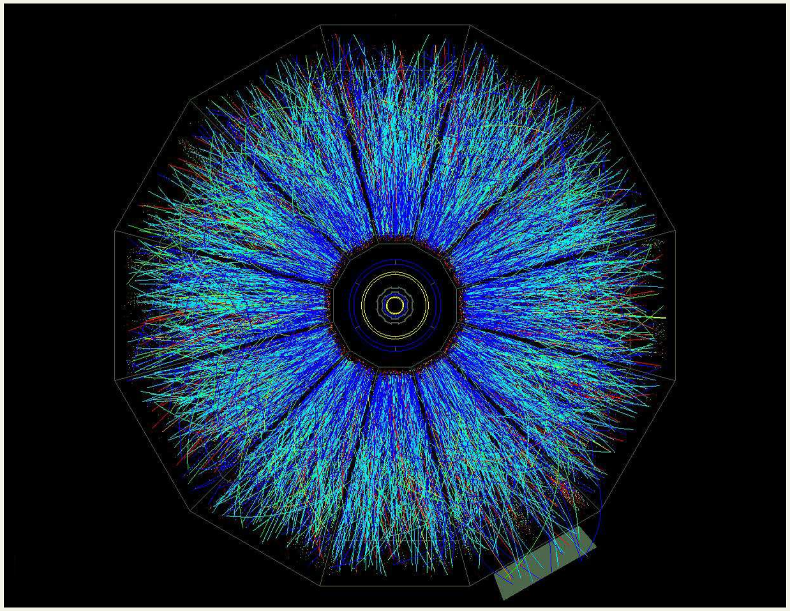
  Comparison with kinetic theory indicates that Grad's quadratic ansatz is remarkably accurate (1-2%) up to $p_T/T \sim 6$, at least with $2 \to 2$ interactions and for small shear viscosities $\eta/s \approx 0.1$.

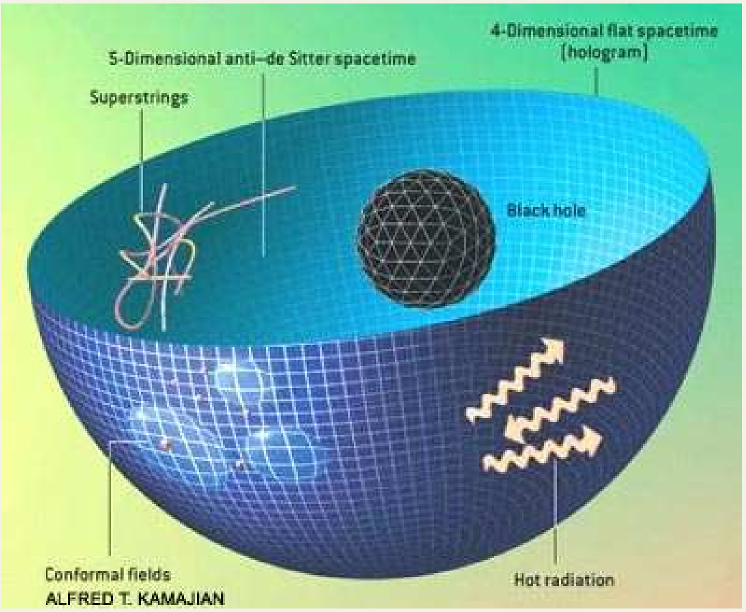
  In the multicomponent case, kinetic theory indicates that per-species dissipative corrections are driven by the relative opacities. The commonly assumed "democratic" sharing in viscous hydro calculations is unrealistic.

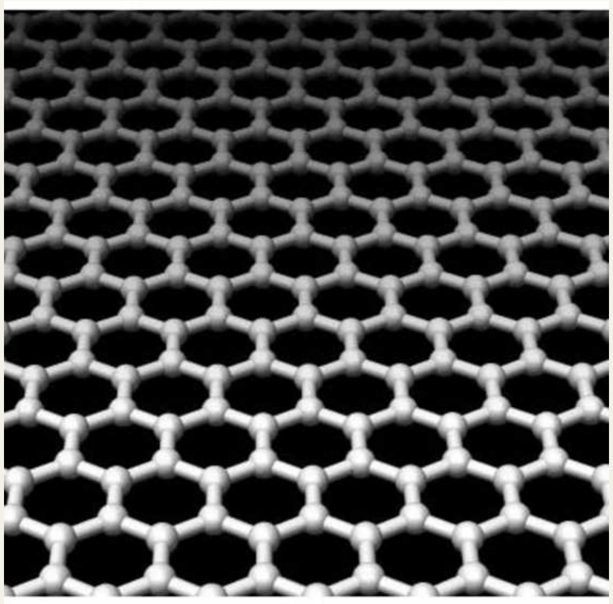
- Some open questions:

  - whether we need to extend hydrodynamics with per-species viscous fields
  - accuracy of linear response
  - any simple shortcut to the end result







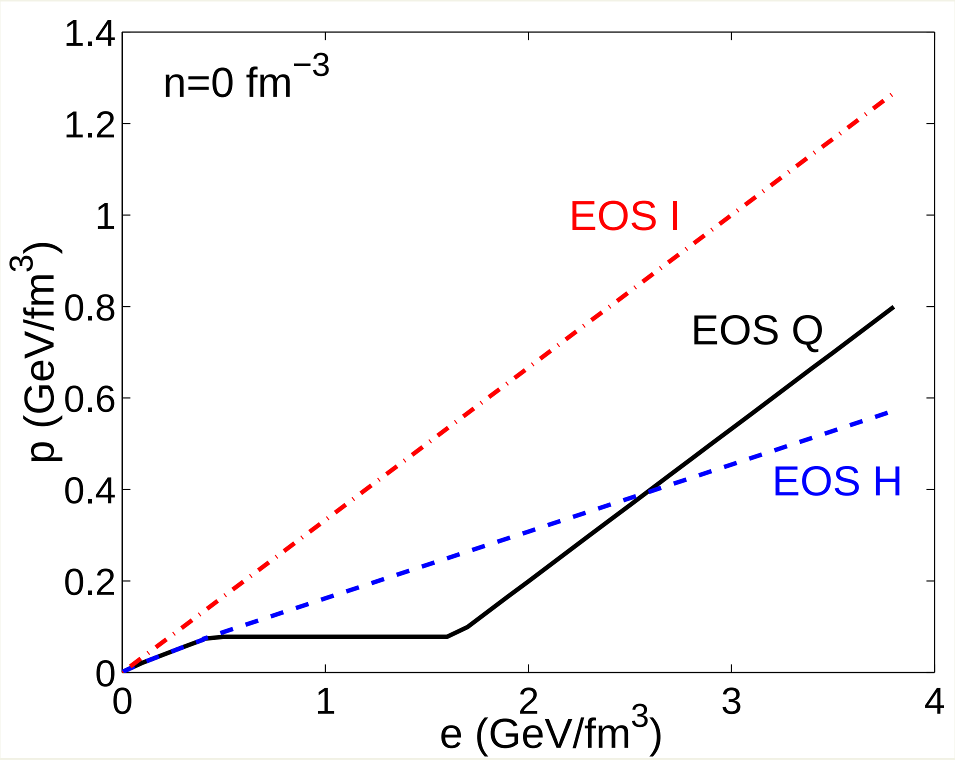


unique connections between different fields

Backup slides

“Bag” equation of state

EOS Q: simplified 1st-order phase transition parameterization



combines hadron resonance gas (EOS H) & plasma (EOS 1)

Viscous hydrodynamics

Navier-Stokes: corrections linear in gradients [Landau]

$$T^{\mu\nu}_{NS} = T^{\mu\nu}_{ideal} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\partial^\alpha u_\alpha) + \zeta\Delta^{\mu\nu}\partial^\alpha u_\alpha$$

$$N^\nu_{NS} = N^\nu_{ideal} + \kappa\left(\frac{n}{\varepsilon + p}\right)^2 \nabla^\nu\left(\frac{\mu}{T}\right)$$

$[\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu, \quad \nabla^\nu \equiv \Delta^{\mu\nu}\partial_\mu]$

η, ζ: **shear and bulk viscosity;** κ: **heat conductivity**

unfortunately NS hydro is unstable and acausal

Müller ('76), Israel & Stewart ('79), Hiscock & Lindblom, PRD31 ('85) ...

causal 2nd-order hydro: dynamical corrections

$$T^{\mu\nu} \equiv T^{\mu\nu}_{ideal} + \pi^{\mu\nu} - \Pi\Delta^{\mu\nu}\,, \qquad N^\mu \equiv N^\mu_{ideal} - \frac{n}{e+p}q^\mu$$

relaxation eqns for bulk pressure Π**, shear stress** $\pi^{\mu\nu}$**, heatflow** q^μ

e.g. Israel-Stewart theory, Öttinger-Grmela, conformal hydro, ...

Israel & Stewart, Ann.Phys 110&118; Öttinger & Grmela, PRE 56&57; Baier et al, JHEP04 ('08)

Israel-Stewart theory - complete set of equations of motion

$$
\begin{aligned}
D\Pi &= -\frac{1}{\tau_\Pi}(\Pi + \zeta\nabla_\mu u^\mu) \\
&\quad -\frac{1}{2}\Pi\left(\nabla_\mu u^\mu + D\ln\frac{\beta_0}{T}\right) \\
&\quad +\frac{\alpha_0}{\beta_0}\partial_\mu q^\mu - \frac{a_0'}{\beta_0}q^\mu D u_\mu && (1)\\
Dq^\mu &= -\frac{1}{\tau_q}\left[q^\mu + \kappa_q\frac{T^2 n}{\varepsilon + p}\nabla^\mu\left(\frac{\mu}{T}\right)\right] - u^\mu q_\nu D u^\nu && (2)\\
&\quad -\frac{1}{2}q^\mu\left(\nabla_\lambda u^\lambda + D\ln\frac{\beta_1}{T}\right) - \omega^{\mu\lambda}q_\lambda \\
&\quad -\frac{\alpha_0}{\beta_1}\nabla^\mu\Pi + \frac{\alpha_1}{\beta_1}(\partial_\lambda\pi^{\lambda\mu} + u^\mu\pi^{\lambda\nu}\partial_\lambda u_\nu) + \frac{a_0}{\beta_1}\Pi D u^\mu - \frac{a_1}{\beta_1}\pi^{\lambda\mu}D u_\lambda \\
D\pi^{\mu\nu} &= -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) - (\pi^{\lambda\mu}u^\nu + \pi^{\lambda\nu}u^\mu)D u_\lambda && (3)\\
&\quad -\frac{1}{2}\pi^{\mu\nu}\left(\nabla_\lambda u^\lambda + D\ln\frac{\beta_2}{T}\right) - 2\pi_\lambda{}^{\langle\mu}\omega^{\nu\rangle\lambda} \\
&\quad -\frac{\alpha_1}{\beta_2}\nabla^{\langle\mu}q^{\nu\rangle} + \frac{a_1'}{\beta_2}q^{\langle\mu}D u^{\nu\rangle}\ .
\end{aligned}
$$

where $A^{\langle\mu\nu\rangle} \equiv \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(A_{\alpha\beta} + A_{\beta\alpha}) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}A^{\alpha\beta}$, $\quad \omega^{\mu\nu} \equiv \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(\partial_\beta u_\alpha - \partial_\alpha u_\beta)$

Why does ideal hydrodynamics work in everyday life??

Ideal hydro is applicable when relative viscous corrections are small $\delta T^{\mu\nu}_{viscous}/T^{\mu\nu}_{ideal} \ll 1$

I.e., based on Navier-Stokes we need (with shear only)

$$\frac{\eta \nabla^i u^j}{e+p} \sim \frac{\eta \nabla^i u^j}{T\, s} \sim \frac{\eta}{s} \times \frac{v}{LT} \ll 1$$

where L **is the shortest length scale. In heavy ion physics,**

$$\frac{v}{LT} \sim \frac{1}{\tau T} \sim 1 \qquad \rightarrow \qquad \text{need } \frac{\eta}{\text{s}} \ll 1$$

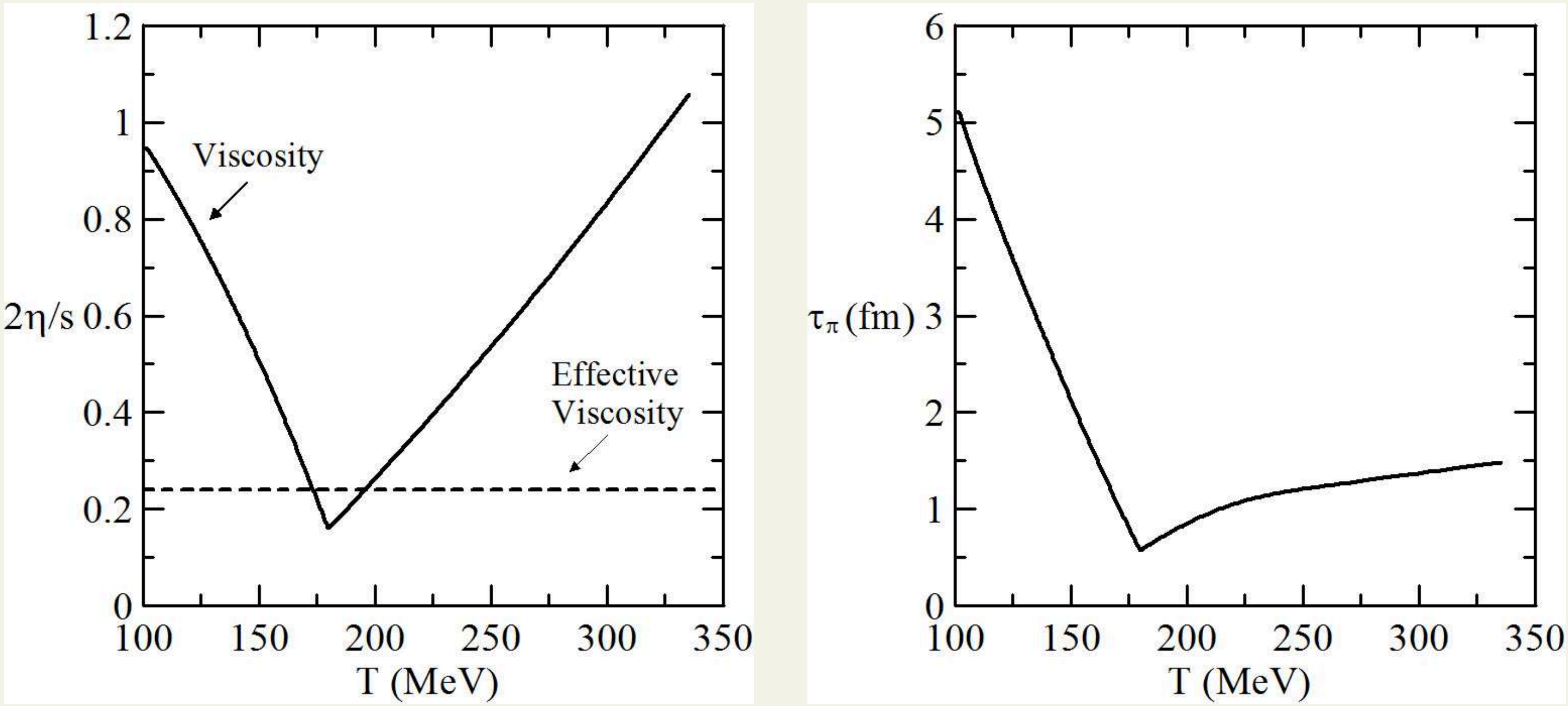
In everyday problems $v/(LT) \gg 1$**, compensating a large** η/s**.**

What we are after is not ideal hydro behavior but systems where the viscosity is near its quantum (uncertainty) limit.

Realistically, $\eta/s \neq const$

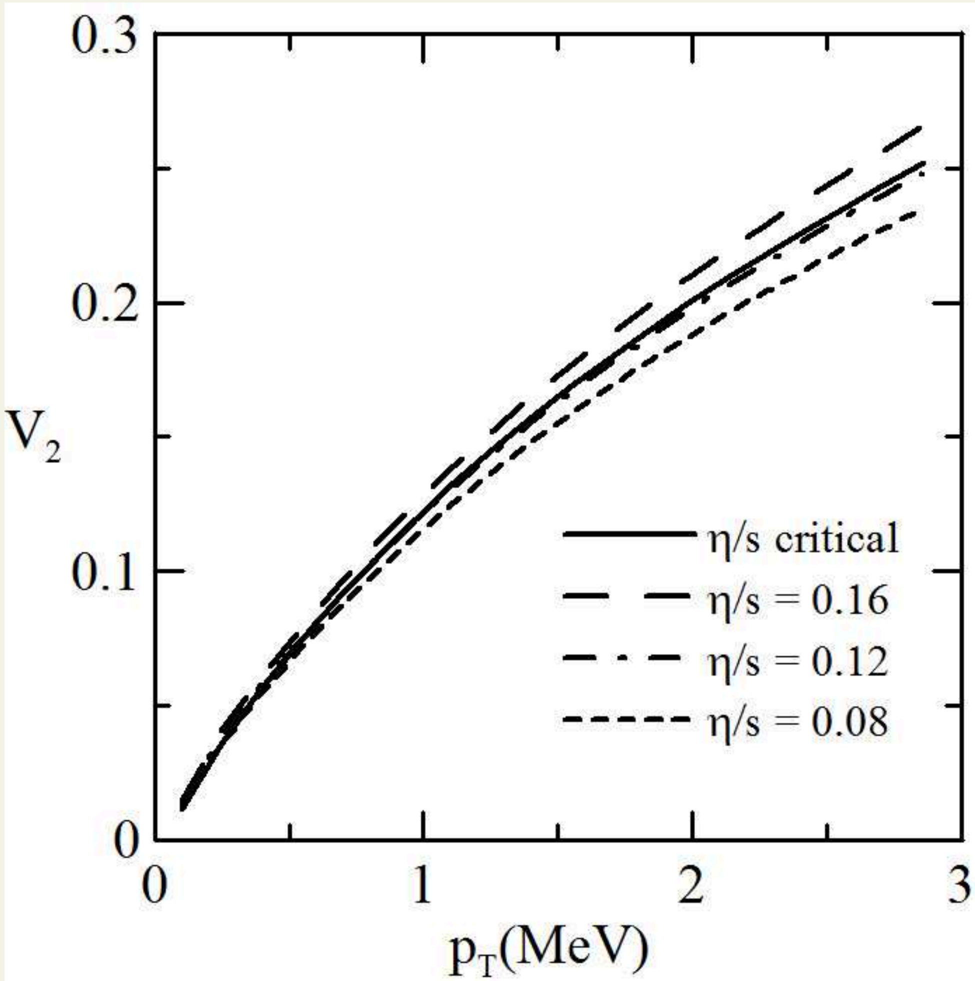
interpolate between wQGP, sQGP, hadron gas $+$ **use** $\tau_\pi \approx \eta/p$

Denicol et al, JPG37 ('10)



same elliptic flow as for low $\eta/s \approx 0.12$(!)

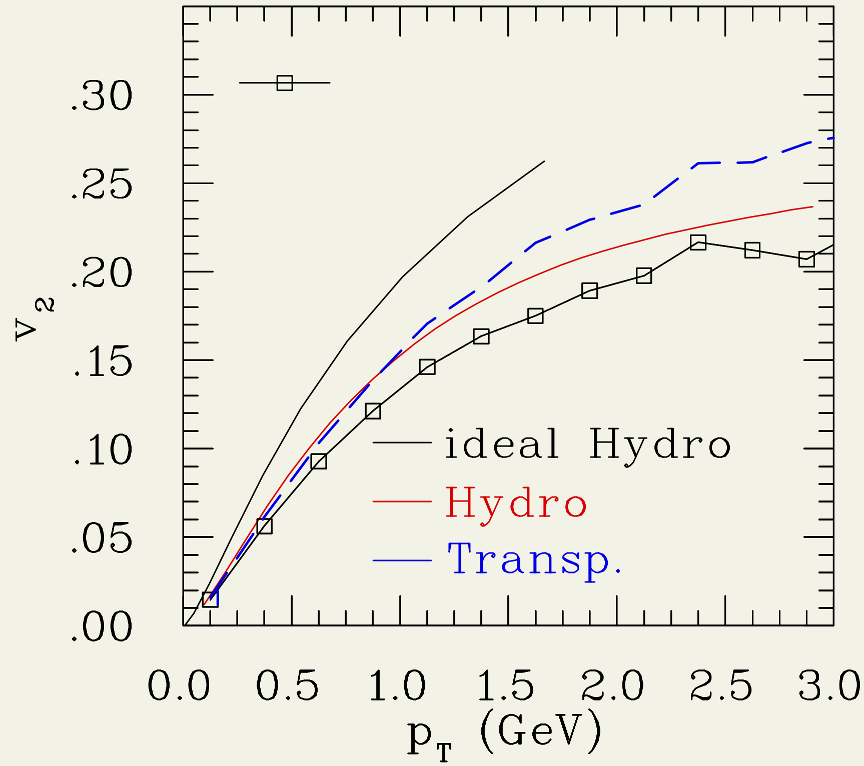
Denicol et al, JPG37 ('10)



$\Rightarrow$ **disentangling η and τ_π will likely need good theory input**

Validity of hydrodynamics

test against transport: IS hydro accurate for $\eta/s \approx 1/4\pi$ DM & Huovinen, JPG35('08)



relevant condition: high-enough inverse Knudsen number Huovinen & DM, PRC79 ('08)

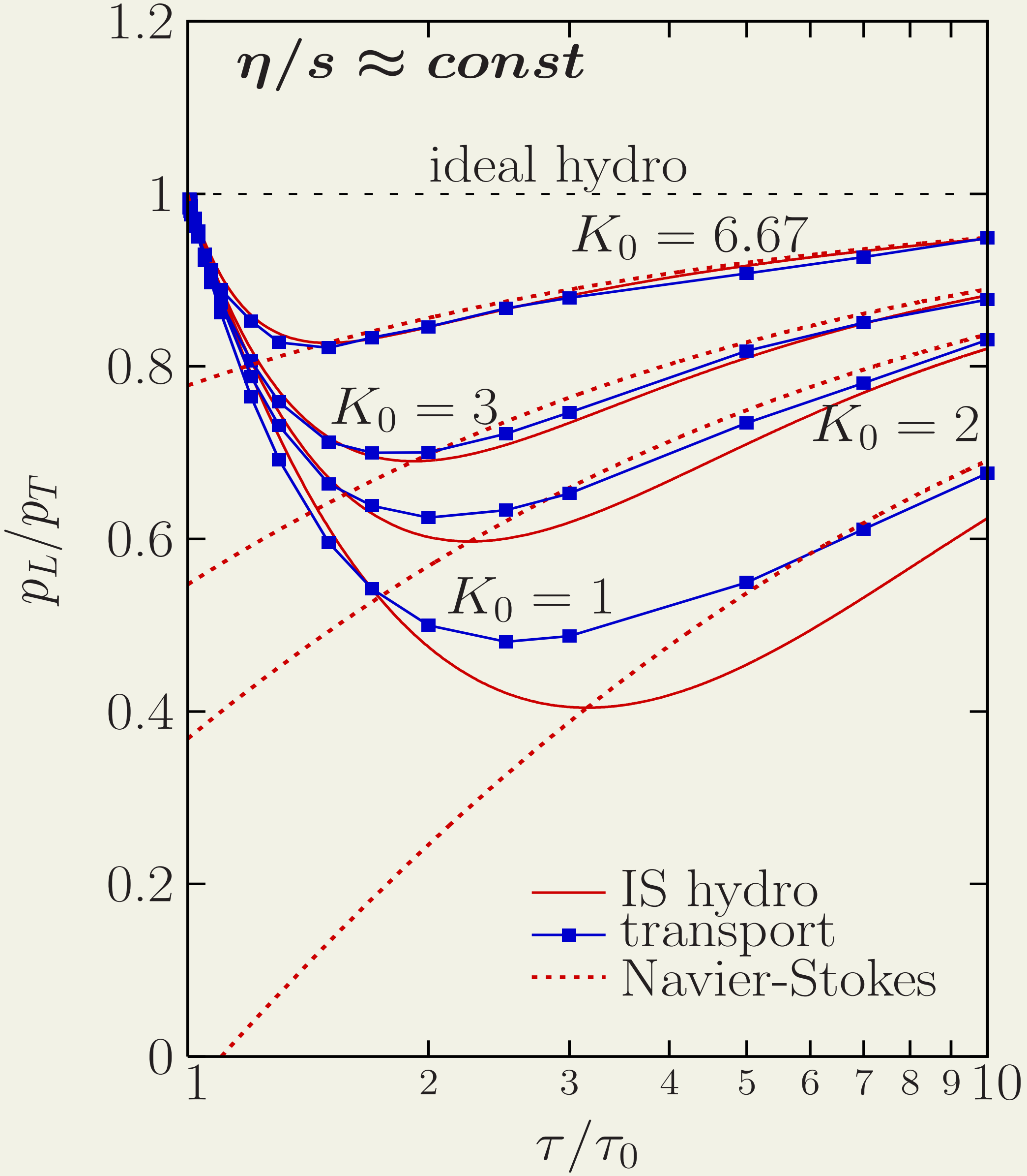
$$K_0 \equiv \frac{\tau}{\lambda_{tr}} = \frac{\tau_{exp}}{\tau_{scatt}} \approx \frac{6\tau_{exp}}{5\tau_\pi} \quad > \quad \sim 2-3$$

in terms of shear viscosity

$$\frac{\eta}{s} \sim \frac{2.6}{4\pi K_0} \lesssim 2 \times \frac{1}{4\pi}$$

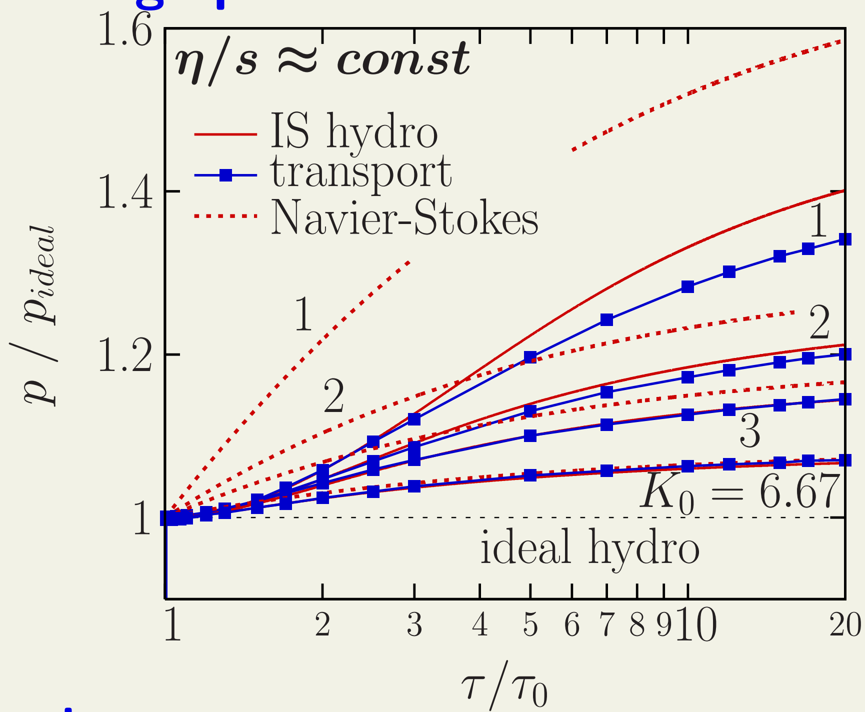
Validity of Israel-Stewart hydro (0+1D Bjorken)

pressure anisotropy T_{zz}/T_{xx}

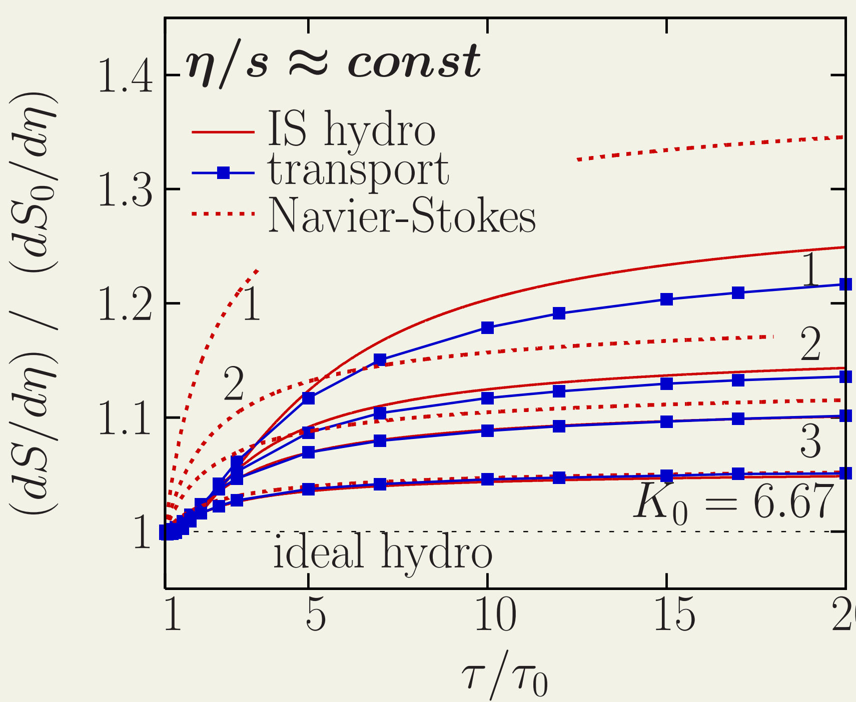


average pressure

Huovinen & DM, PRC79



entropy



IS is 10% accurate when $K_0 \equiv \tau_0/\lambda_{tr,0} \gtrsim 2$

Connection to viscosity

$$K_0 \approx \frac{T_0\tau_0}{5}\frac{s_0}{\eta_{s,0}} \approx 12.8 \times \left(\frac{T_0}{1\text{ GeV}}\right)\left(\frac{\tau_0}{1\text{ fm}}\right)\left(\frac{1/(4\pi)}{\eta_0/s_0}\right)$$

For typical RHIC hydro initconds $T_0\tau_0 \sim 1$, therefore

$$K_0 \gtrsim 2-3 \qquad \Rightarrow \qquad \frac{\eta}{s} \lesssim \frac{1-2}{4\pi} \tag{4}$$

I.e., shear viscosity cannot be many times more than the conjectured bound, for IS hydro to be applicable.

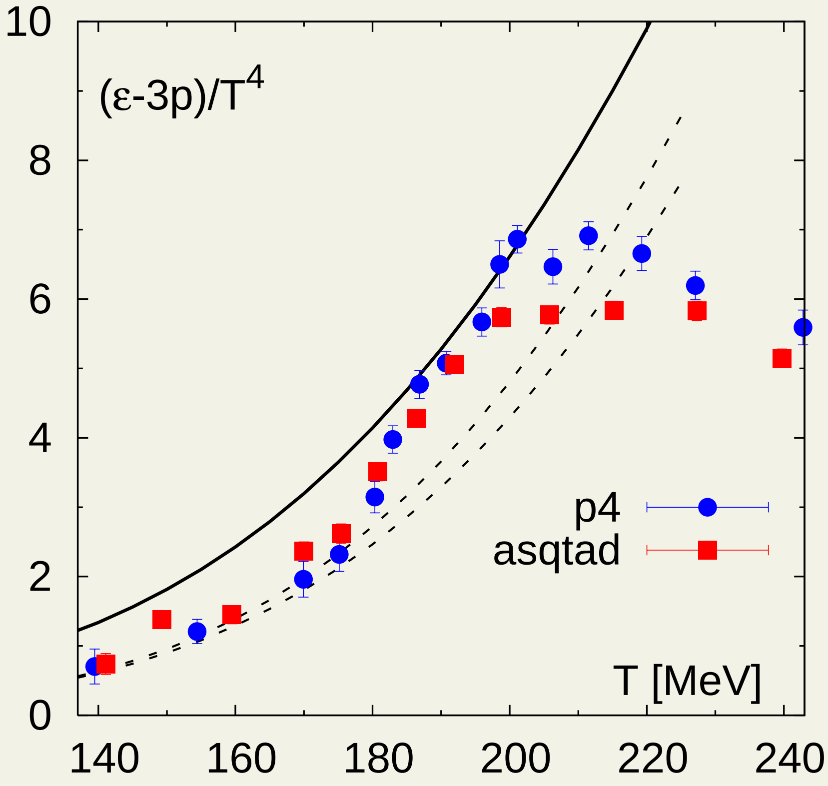
When IS hydro is accurate, dissipative corrections to pressure and entropy do not exceed 20% significantly (a necessary condition). This holds for a wide range $[0.476, 1.697]$ of initial pressure anisotropies.

(!) close to a noninteracting "resonance gas" at $T \lesssim 160 - 180$ MeV

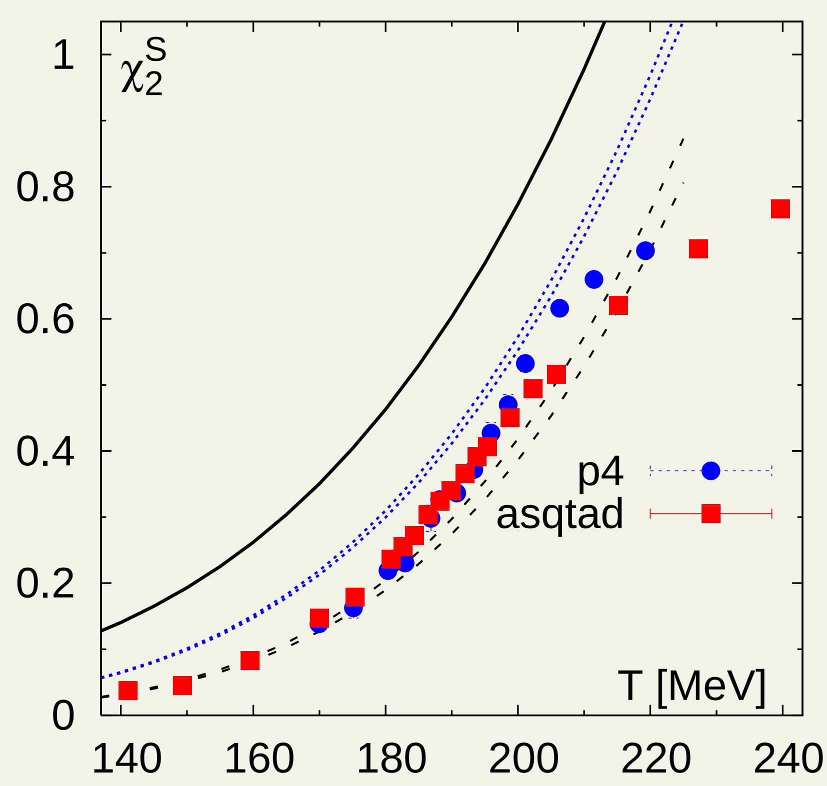
$$f_i(p) = \frac{g_i}{(2\pi)^3} \frac{1}{\exp[E_i(p) - \mu_i]/T \pm 1}$$

Huovinen & Petreczky, NPA837 ('10)

equation of state



strangeness fluctuations

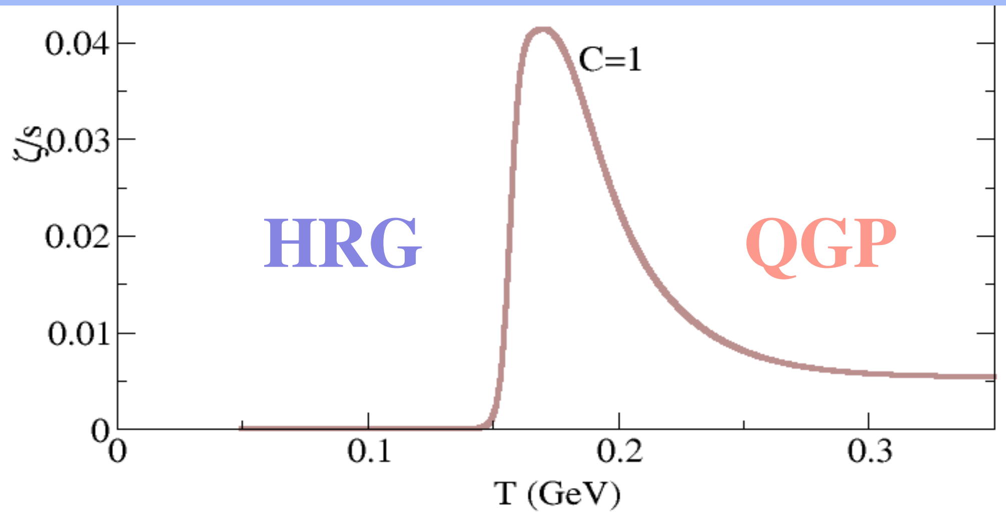


important for late-stage dynamics (hadron transport)

Song & Heinz ('09)

bulk viscosity and relaxation time

Bulk viscosity:



Relaxation times: $\tau_\Pi \sim \zeta$ also peaks near T_c,

this plays an important role for bulk viscous dynamics

N-S initialization: $\Pi_0 = -\zeta(\partial \cdot u)$

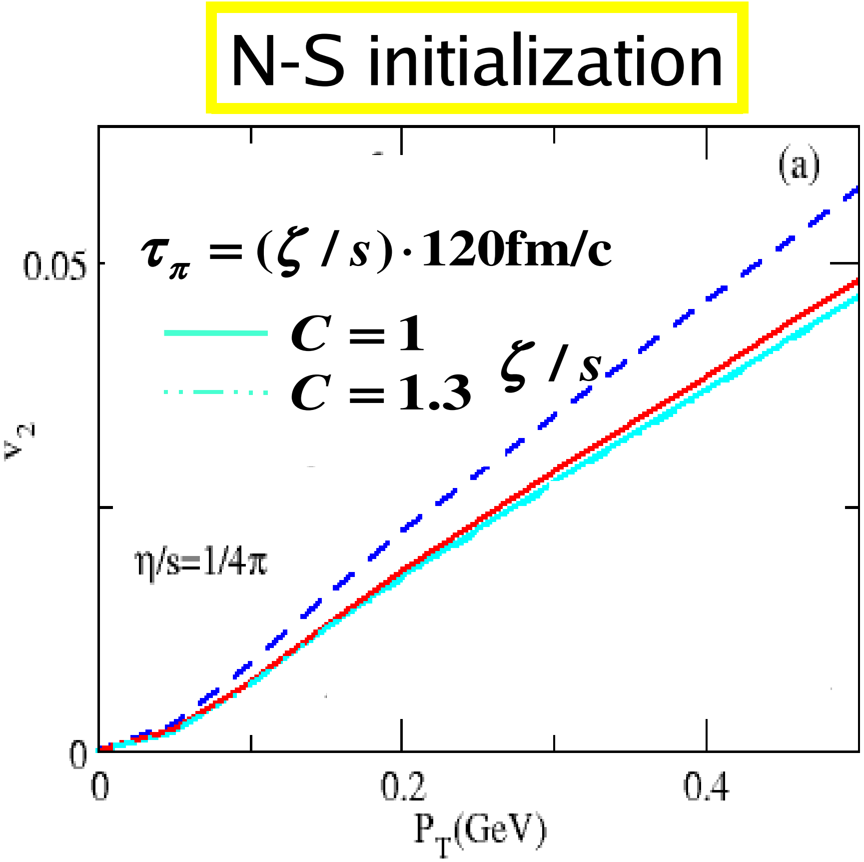
large τ_Π near T_c ⟶ **keeps large negative value of Π** in phase transition region

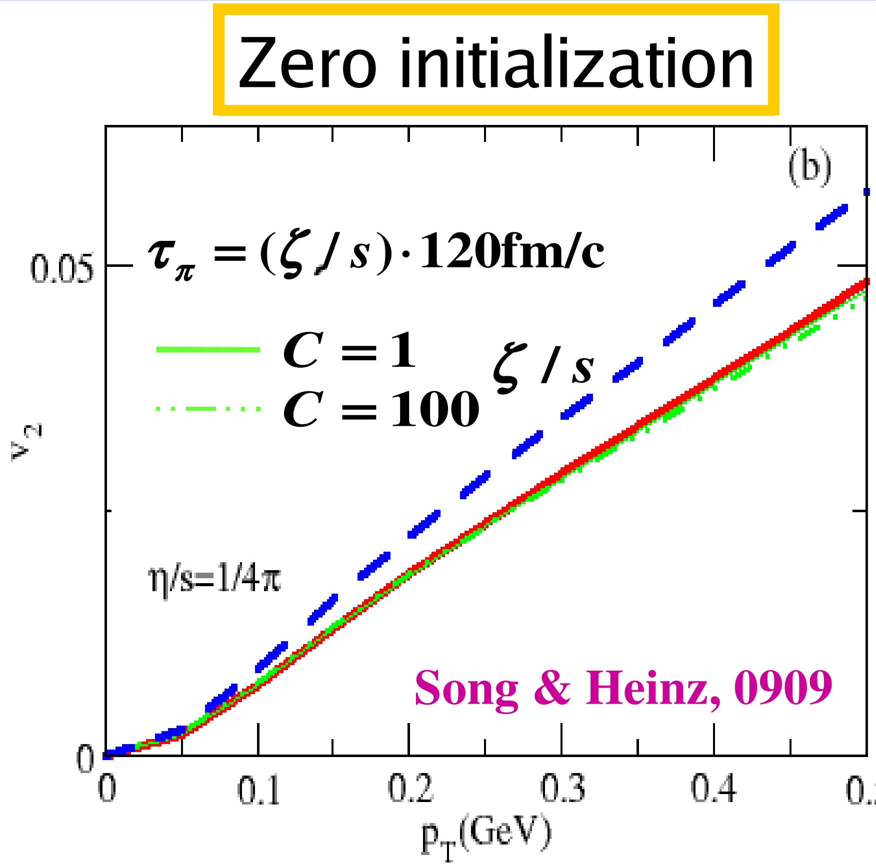
⟶ **viscous hydro breaks down** ($p + \Pi < 0$) for larger ζ/s

viscous hydro is only valid with small ζ/s ⟶ **small bulk viscous effects on v_2**

Song Heinz ('09)

Uncertainties from **bulk viscosity**





-with a critical slowing down τ_Π, effects from bulk viscosity effects are much smaller than from shear viscosity

bulk viscosity influences v_2 ~5% (N-S initial.) <4% (zero initial.)

⟺ uncertainties to η / s ~20% (N-S initial.) <15% (zero initial.)