

Temperature dependent EoS in hydro

Máté Csanád

Eötvös University, Budapest

November 29, 2010

Equations of hydrodynamics

- Basic eqs: continuity and energy-mom. conservation

$$\partial_{\mu}(nu^{\mu}) = 0, \quad (1)$$

$$\partial_{\nu}T^{\mu\nu} = 0. \quad (2)$$

- n is some kind of number density (nonzero chemical potential)
- Energy-momentum tensor $T^{\mu\nu}$ in a perfect fluid:

$$T^{\mu\nu} = (\epsilon + p)u^{\mu}u^{\nu} - pg^{\mu\nu}, \quad (3)$$

- Equation of State additionally (κ may depend on time):

$$\epsilon = \kappa p = \kappa(T)nT \quad (4)$$

Separation of energy and momentum conservation

- Projection with u^μ : energy conservation

$$(\epsilon + p)\partial_\nu u^\nu + u^\nu \partial_\nu \epsilon = 0. \quad (5)$$

- Multiply with u^ν and subtract from the original: Euler equation

$$(\epsilon + p)u^\nu \partial_\nu u^\mu = (g^{\mu\nu} - u^\mu u^\nu)\partial_\nu p, \quad (6)$$

- Substitute $\epsilon = \kappa(T)p$ and $p = nT$.

Ellipsoidal scaling

- Introduce an ellipsoidal scaling variable:

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2} \quad (7)$$

- $s = 1$ is an ellipsoid with principal axes X , Y and Z .
- X , Y and Z are time dependent \rightarrow expansion!
- Look for u^μ such as

$$u^\mu \partial_\mu s = 0 \quad (8)$$

- Then for any function $f(s)$ $u^\mu \partial_\mu f(s) = 0$ also

Continuity

- We choose a 3D Hubble flow

$$u^\mu = \gamma \left(1, \frac{\dot{X}}{X} r_x, \frac{\dot{Y}}{Y} r_y, \frac{\dot{Z}}{Z} r_z \right) \quad (9)$$

- Fulfills $u^\mu \partial_\mu s = 0$
- Acceleration possible if X, Y, Z are not constant
- Let the density be:

$$n = n_0 \frac{V_0}{V} \nu(s) \quad (10)$$

- $\nu(s)$ arbitrary, V has to fulfill:

$$u^\mu \partial_\mu V = V \partial_\mu u^\mu \quad (11)$$

- Example: $V = \tau^3$ or $V = XYZ$

Energy conservation and temperature

- Continuity + energy conservation + $\kappa(T)$:

$$\frac{d\kappa T}{dT} u^\mu \partial_\mu T + T \partial_\mu u^\mu = 0. \quad (12)$$

- Generally true (if κ depends only on temperature)!
- Means $d(\kappa T)/dT \neq 0$ ($\partial_\mu u^\mu$ not realistic)
- Also, this is ϵ/n , expected to be monotonic in T
- Can be solved implicitly:

$$\int_{T_0}^T \left(\frac{d\kappa(T') T'}{dT'} \frac{1}{T'} \right) dT' = \ln \left(\nu(s) \frac{V_0}{V} \right) \quad (13)$$

- with arbitrary $\mu(s)$ (cancels anyway)
- If $\kappa = \text{constant}$:

$$T = T_0 \left(\frac{V_0}{V} \right)^{1/\kappa} \mu(s)^{1/\kappa} \quad (14)$$

Euler-equation

- Putting everything together:

$$\begin{aligned}(\kappa + 1)u^\nu \partial_\nu u^\mu - \frac{k + 1}{k} u^\mu \partial_\nu u^\nu &= \\ &= \frac{k + 1}{k} \partial^\mu \ln \frac{V_0}{V} + \left(\frac{\nu'(s)}{\nu(s)} + \frac{\mu'(s)}{k\mu(s)} \right) \partial_\mu s\end{aligned}\quad (15)$$

- with $k = d\kappa T/dT$
- If $\kappa = \text{constant}$, then $\kappa = k$, thus:

$$\kappa u^\nu \partial_\nu u^\mu - u^\mu \partial_\nu u^\nu = \partial^\mu \ln \left(\frac{V_0}{V} \nu(s) \mu(s)^{1/\kappa} \right) \quad (16)$$

- Easier to solve if $\nu(s) = \mu(s)^{-1/\kappa}$.

A known solution

- A known solution

$$s = \frac{r_x^2}{\dot{X}^2 t^2} + \frac{r_y^2}{\dot{Y}^2 t^2} + \frac{r_z^2}{\dot{Z}^2 t^2}, \quad (17)$$

$$u^\mu = \frac{x^\mu}{\tau}, \quad (18)$$

$$n = n_0 \frac{V_0}{V} \nu(s), \text{ with } \nu(s) = \text{arbitrary}, \quad (19)$$

$$V = \tau^3, \quad (20)$$

$$T = T_0 \left(\frac{V_0}{V} \right)^{1/\kappa} \frac{1}{\nu(s)}, \text{ with } \kappa = \text{constant} \quad (21)$$

- T. Csörgő, L. P. Csernai, Y. Hama *et al.*, Heavy Ion Phys. **A21** (2004) 73-84.
- No acceleration: $\dot{X}, \dot{Y}, \dot{Z}$ constant.
- Here $\kappa = \text{constant}$

A new solution

- A partly implicit non-acceleration new solution:

$$n = n_0 \frac{V_0}{V}, \quad (22)$$

$$u^\mu = \frac{x^\mu}{\tau}, \quad (23)$$

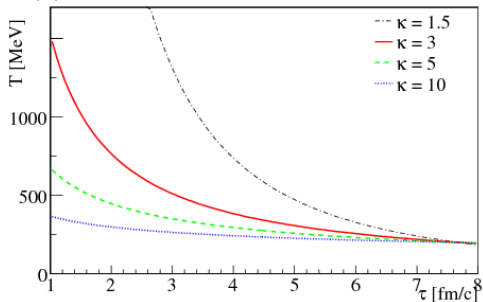
$$V = \tau^3, \quad (24)$$

$$T \text{ from } \int_{T_0}^T \left(\frac{d\kappa(T') T'}{dT'} \frac{1}{T'} \right) dT' = \ln \frac{V_0}{V} \quad (25)$$

- No acceleration, but $\kappa(T)$ possible
- Time dependence of the temperature to calculate more exactly

An example, $\kappa = \text{constant}$

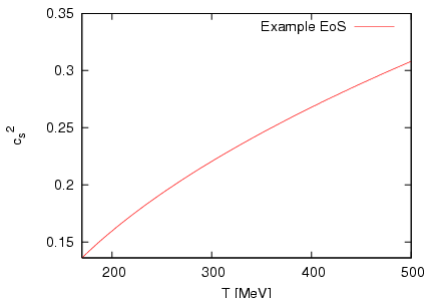
- $T(t)$ can be calculated directly:



- How about $\kappa(T)$?

An example $\kappa(T)$

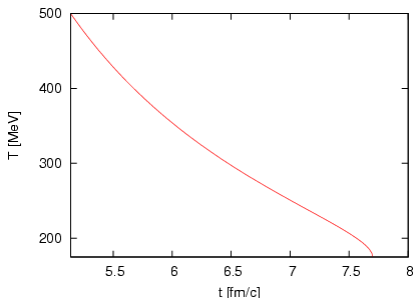
- Take an example $\kappa(T)$, displayed as $c_s^2(T)$.



- In the right regime, just took an example dependence
- Not physical, just an example!
- This can be plugged into the solution

Temperature evolution

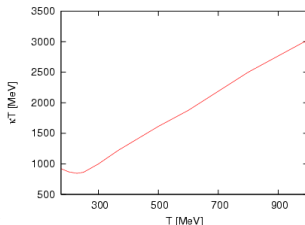
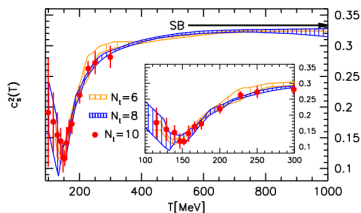
- $T(t)$ can be calculated from the implicit equation for the temperature:



- How about realistic $c_s^2(T)$?

A realistic EoS

- Take EoS from lattice QCD:



- S. Borsányi, G. Endrődi, Z. Fodor *et al.* arXiv:1007.2580
- Problem: $\frac{d\kappa T}{dT} = 0$ at $T = 200 - 250$ MeV
- Recall: $\frac{d\kappa T}{dT} u^\mu \partial_\mu T + T \partial_\mu u^\mu = 0$
- Thus $\partial_\mu u^\mu = 0$ here: unrealistic!

Summary

- A general ellipsoidally symmetric class of solutions
- A new solution with arbitrary $\kappa(T)$
- Lattice QCD EoS versus $\frac{d\kappa T}{dT} u^\mu \partial_\mu T + T \partial_\mu u^\mu$?