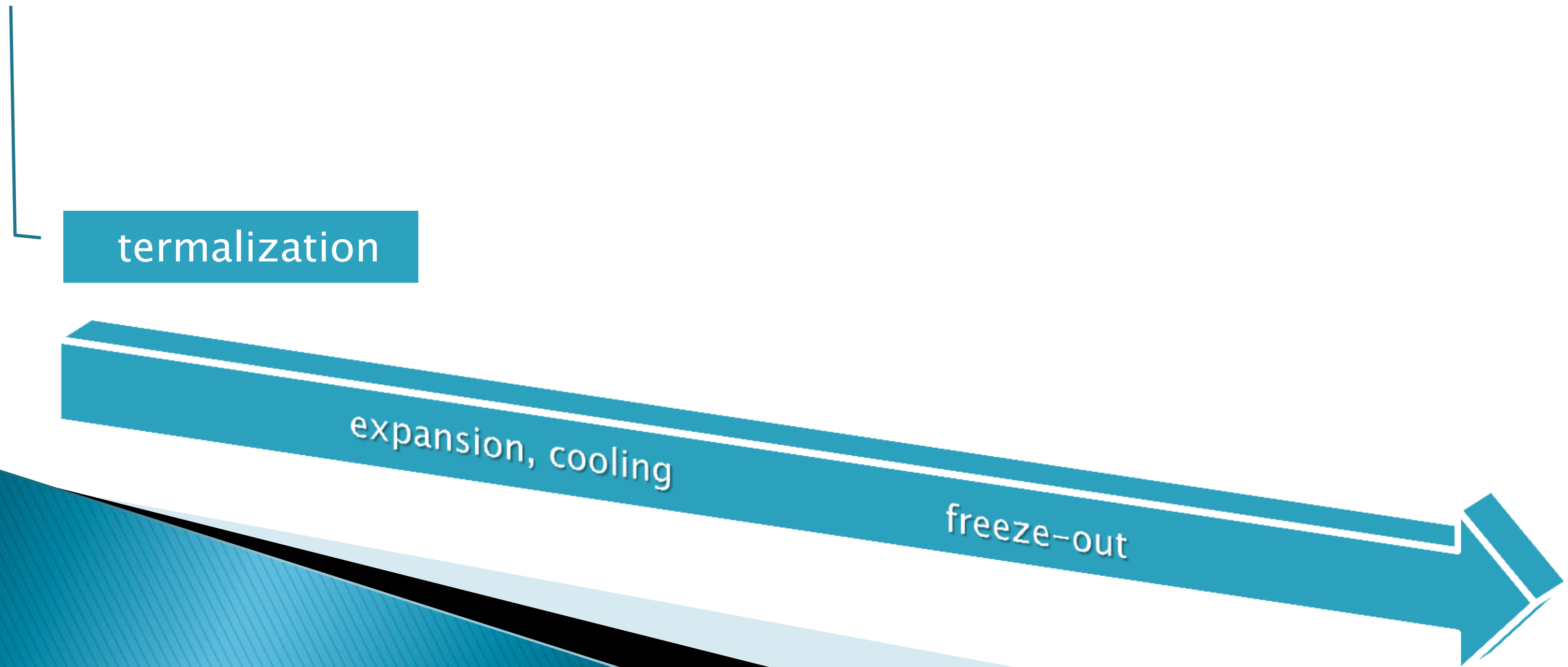
Initial temperature and EoS from photonspectra in Au+Au collisions

Presenter: **Imre Májer** – Eötvös University Physics student
Supervisor: **Máté Csanád** – Department of Atomic Physics

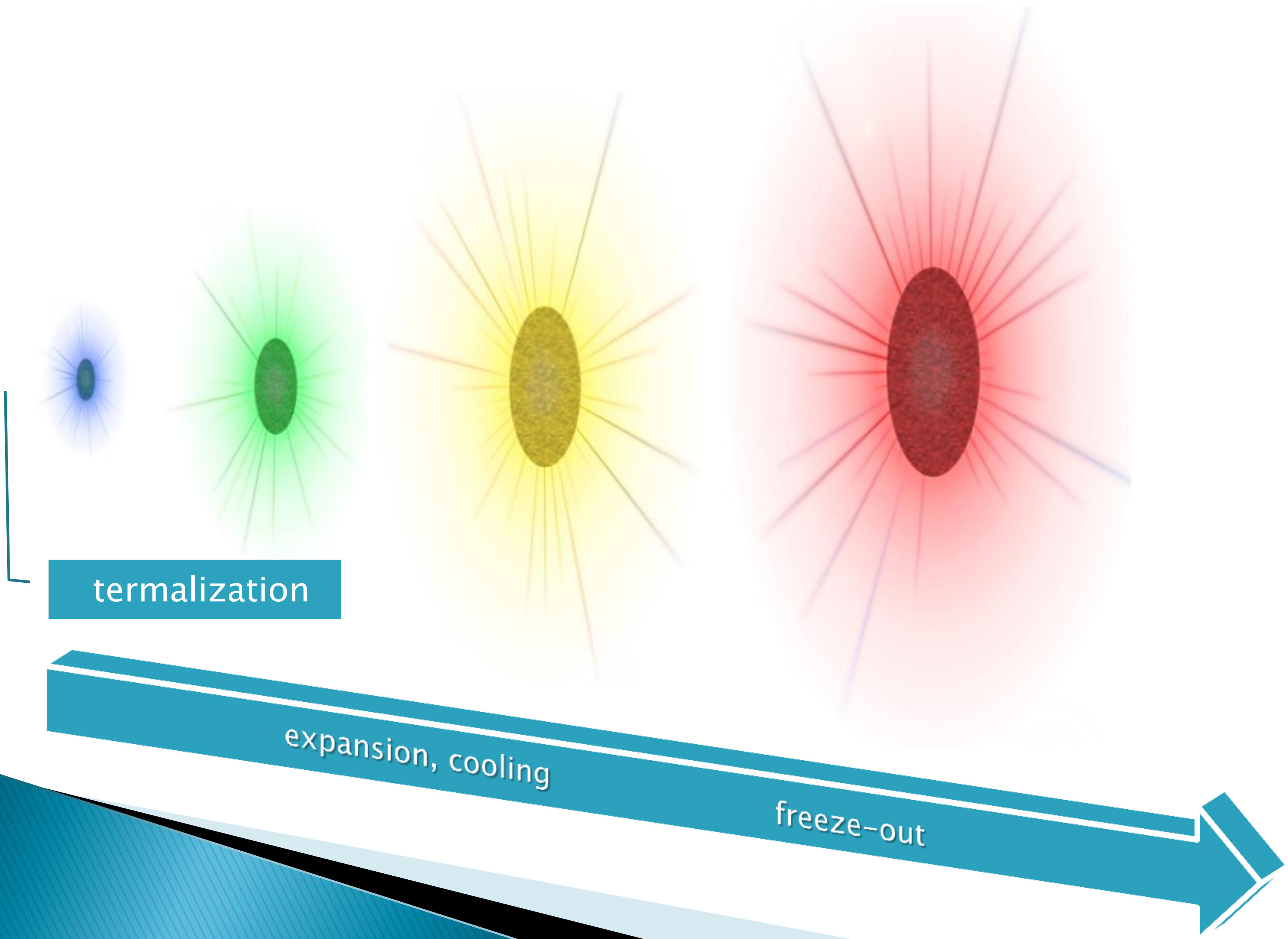
RHIC MILESTONES

- New, strongly interacting medium
  - (Nucl.Phys., A757:184-283, 2005)
- Collective behaviour
  - (Phys.Rev.Lett., 91:182301, 2003)
- Perfect fluid hydro
  - (Phys.Rev.Lett., 98:172301, 2007)
- Quark degrees of freedom
  - (Phys.Rev.Lett. 98, 162301,2007)
- Direct photon data, time evolution can be analyzed
  - (Phys.Rev.Lett., 104:132301, 2010)

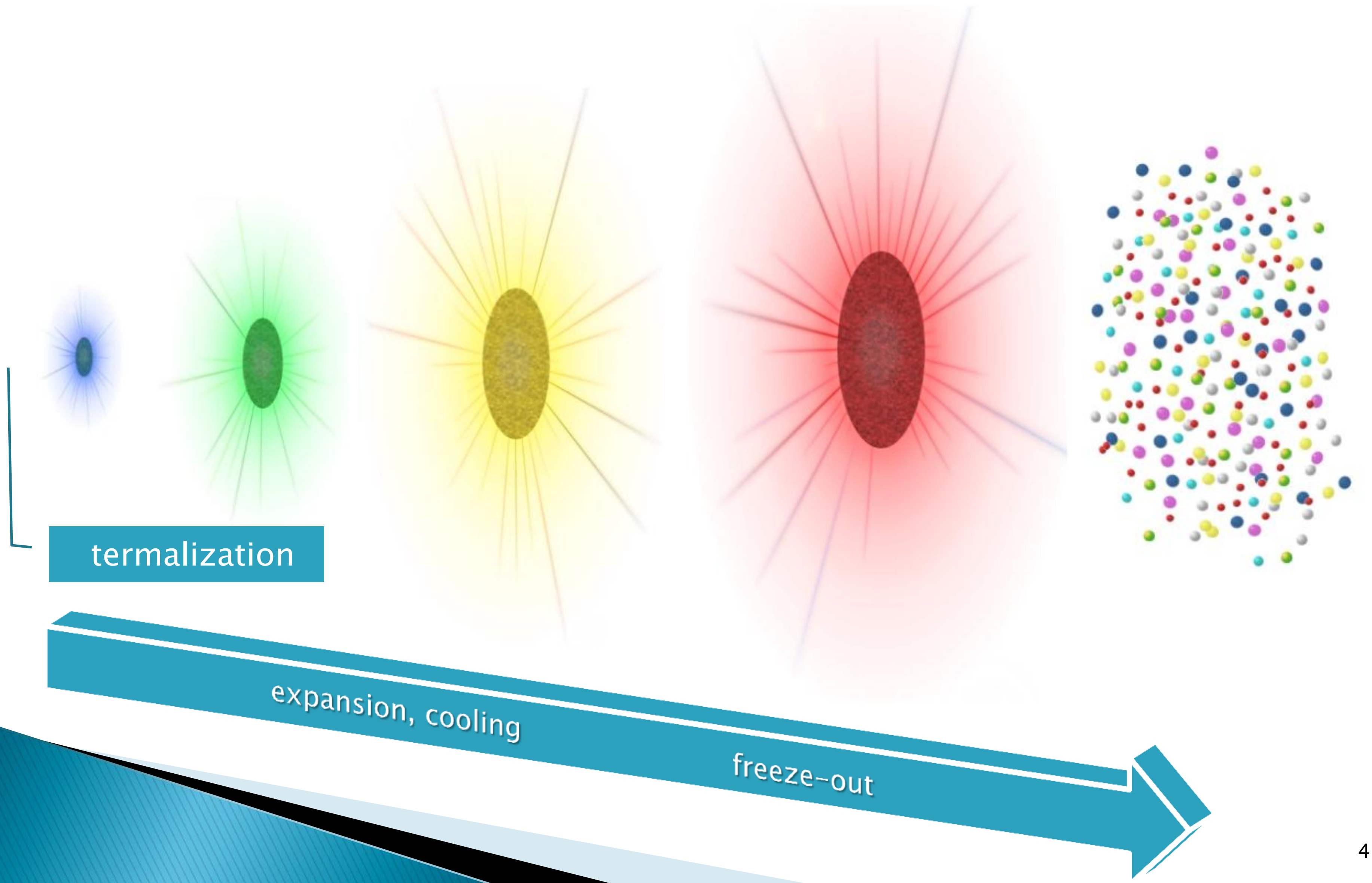
Time development – Spectra



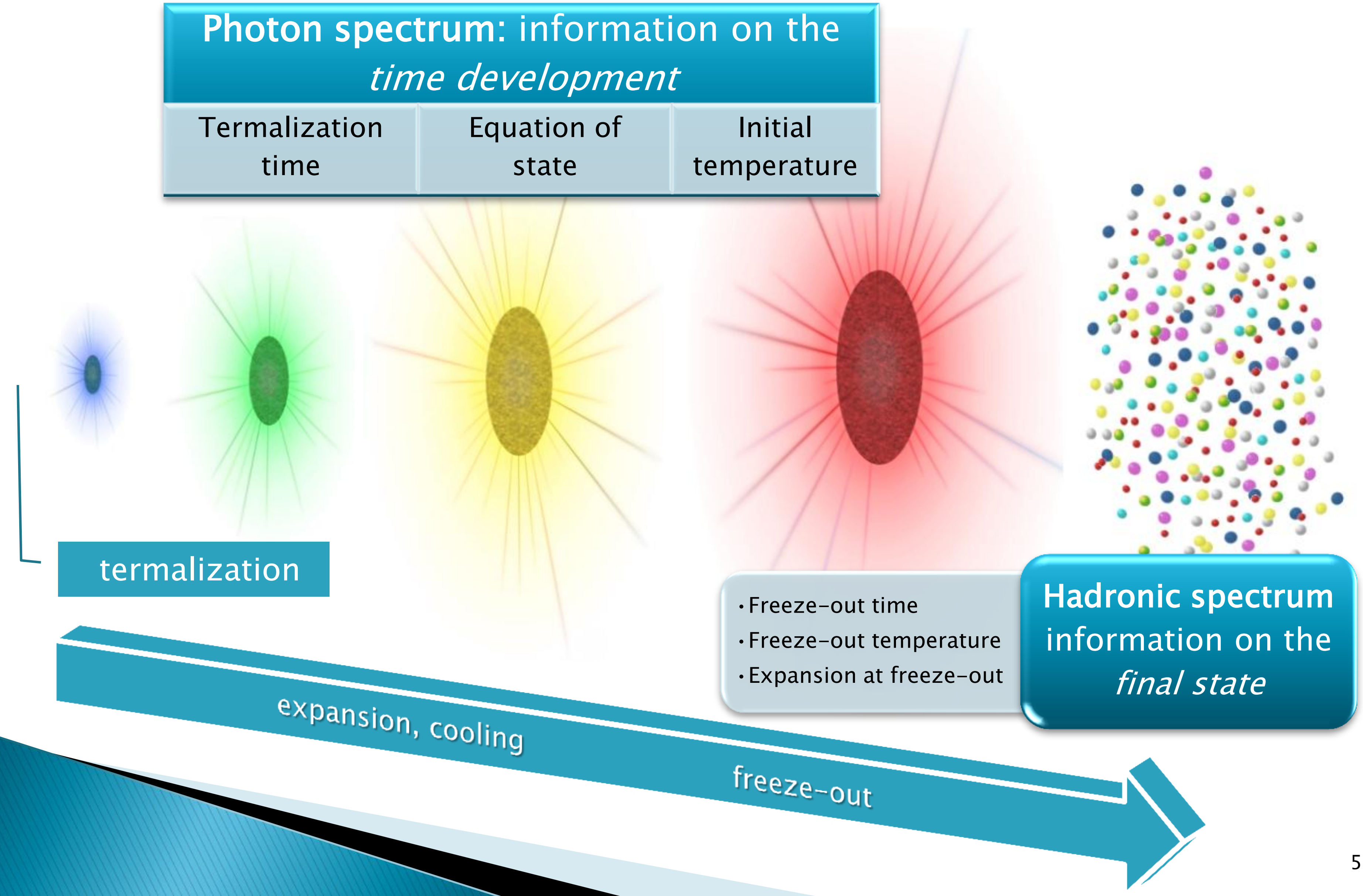
TIME DEVELOPMENT – SPECTRA



Time development – Spectra



Time development – Spectra



Spectra information

Physical quantities we can get from different spectra:

- Hadronic spectra
  - Freeze-out temperature (T_0)
  - Freeze-out time (τ_0)
  - Freeze-out expansion velocities ($\dot{X}$, $\dot{Y}$, $\dot{Z}$ ⟺ ϵ, u_t, $\dot{Z}$)
- Photon spectra
  - Equation of state (κ)
  - Termalization time ($t_{initial}$)
  - Initial temperature ($T_{initial}$)

Physical Quantities

How do we get the physical quantities describing the system?

1.
- Source function for a particle type $S(x^{\mu}, \boldsymbol{p})$
  - Unknown functions (u^{μ}, n, T, p)

2.
- Hydrodynamic model
  - We get the unknown functions (u^{μ}, n, T, p)

3.
- Observables: spectra $N_1(\boldsymbol{p})$
  - Free parameters are the physical quantities (eg. κ, T_0, τ_0, …)

4.
- Comparing to data
  - Fitting the spectra → free parameters

- The probability of particle creation at a given place and time with a given momentum: $S(x^\mu, \boldsymbol{p})$
- Photons: bosons ⟶ Bose–Einstein distribution
- $$S(x^\mu, \boldsymbol{p})d^4x = \frac{dt\, p_\mu d^3\Sigma^\mu}{e^{p_\mu u^\mu/T} - 1} = \frac{p_\mu u^\mu}{e^{p_\mu u^\mu/T} - 1} d^3x\, dt$$
  - $p_\mu d^3\Sigma^\mu$ – Cooper–Frye factor gives the integration measure
- Integrate the source function: invariant momentum distribution

$$N_1(\boldsymbol{p}) = \int S(x^\mu, \boldsymbol{p})\, d^4x$$

One of the most important observable!

- Relativistic hydro for perfect fluids (Landau):

$$\partial_\mu (n u^\mu) = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

- Equation of state ($\epsilon = \kappa p = \kappa n T$)
  - Assuming an avarage constant κ
- Ideal solution:
  - Relativistic
  - 1+3 dimensional
  - Realistic (eg. ellipsoidal) geometry
  - Anisotropic Hubble-flow ($\dot{r}(t)/r(t)$ = const.) after some time
  - Accelerating at the start of the time development

Some known solutions:

Solutions	Basic properties	Equation of state
Landau-Khalatnyikov	1+1D, accelerating, implicit	$\epsilon = \kappa n T$
Hwa-Björken	1+1D, non-accelerating, estimate of the initial energy density	$\epsilon = \kappa n T$
Csörgő, Nagy, Csanád Phys.Rev.C77:024908, 2008	1+1D / 1+3D spherical symmetry, accelerating	$\epsilon - B = \kappa(p + B)$
Bialas et al. Phys. Rev. C76, 054901 (2007).	1+1D, Between Hwa-Björken and Landau-Khalatnyikov	$\epsilon = \kappa n T$
Csörgő, Csernai, Hama, Kodama Heavy Ion Phys., A21:73-84, 2004	1+3D, **ellipsoidal** symmetry, non-accelerating	$\epsilon = \kappa n T$

The examined solution (Csörgő et al., 2004):

- *Relativistic, 1+3 dimensional*

$$n(\boldsymbol{x},\tau) = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \nu(s)$$

$$T(\boldsymbol{x},\tau) = T_0 \left(\frac{\tau_0}{\tau}\right)^{3/\kappa} \frac{1}{\nu(s)}$$

$$p(\tau) = p_0 \left(\frac{\tau_0}{\tau}\right)^{3(\kappa+1)/\kappa}$$

- Scaling parameter – *ellipsoidal symmetry*: $s = \frac{r_x^2}{X(t)^2} + \frac{r_y^2}{Y(t)^2} + \frac{r_z^2}{Z(t)^2}$
- Arbitrary function of the scaling parameter: $\nu(s) = e^{-bs/2}$
  - $b < 0$ is the good temperature gradient
- *Hubble-flow:* $u^\mu = \gamma \left(1, \frac{\dot{X}(t)}{X(t)} r_x,\ \frac{\dot{Y}(t)}{Y(t)} r_y,\ \frac{\dot{Z}(t)}{Z(t)} r_z\right)$
- *Non-accelerating:* $\dot{X}(t) = \dot{X}_0 t$ $\quad \dot{Y}(t) = \dot{Y}_0 t$ $\quad \dot{Z}(t) = \dot{Z}_0 t$

3. Observables

- Observable: invariant one-particle spectrum

3D: $$N_1(\boldsymbol{p}) = E\frac{d^3N}{d^3\boldsymbol{p}} = \frac{d^3N}{p_t\,dp_t\,d\varphi\,dy}$$

y – rapidity
$(E\,dy = dp_z)$

2D: $$N_1(p_t,\varphi) = \frac{dN}{p_t\,dp_t}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos(n\varphi)\right) = N_1(p_t)\left(1 + 2\sum_{n=1}^{\infty} v_n \cos(n\varphi)\right)$$

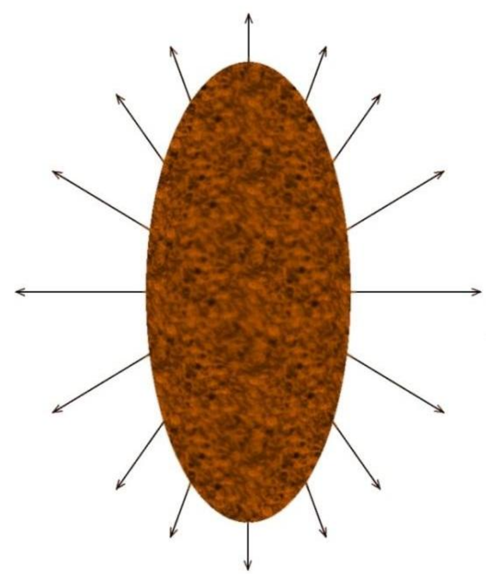
Fourier series of the azimuthal distribution function

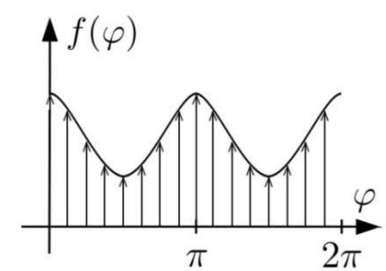
- Inverse Fourier-transform:

1D: $$N_1(p_t) = \frac{1}{2\pi}\int_0^{2\pi} N_1(p_t,\varphi)\,d\varphi$$

1D: $$v_2(p_t) = \frac{\int_0^{2\pi} N_1(p_t,\varphi)\cos(2\varphi)\,d\varphi}{N_1(p_t)}$$

- v_2 is the most important (elliptic flow)





The exact analitic result of the second order Gaussian approximation:

$$N_1(p_t) = \sum_{n=0}^{\infty} (2\pi)^{\frac{3}{2}} \sqrt{\rho_x \rho_y \rho_z} \tau_0^4 T_0 \left(\frac{p_t}{T_0}\right)^{-\frac{4\kappa}{3}+1} \frac{\kappa}{3} \frac{B^n}{A^{n+\frac{4\kappa}{3}-\frac{3}{2}}} \left\{ \left[\frac{(\rho_x-1)^2 + \left(\rho_y-1\right)^2 + 2}{4} a_{0n}\right] + \left[\frac{(\rho_x-1)^2 + \left(\rho_y-1\right)^2 - 2}{4} a_{1n}\right] \Gamma\left(n+\frac{4\kappa}{3}-\frac{3}{2}, A\frac{p_t}{T_0}\xi^{\frac{3}{\kappa}}\right)\Bigg|_1^i + \left[\frac{\rho_x^2+\rho_y^2+\rho_z^2}{2} a_{0n}\right] A\,\Gamma\left(n+\frac{4\kappa}{3}-\frac{5}{2}, A\frac{p_t}{T_0}\xi^{\frac{3}{\kappa}}\right)\Bigg|_1^i \right\}$$

$$v_2(p_t) = \sum_{n=0}^{\infty} (2\pi)^{\frac{3}{2}} \sqrt{\rho_x \rho_y \rho_z} \tau_0^4 T_0 \left(\frac{p_t}{T_0}\right)^{-\frac{4\kappa}{3}+1} \frac{\kappa}{3} \frac{B^n}{A^{n+\frac{4\kappa}{3}-\frac{3}{2}}} \left\{ \left[\frac{(\rho_x-1)^2 + \left(\rho_y-1\right)^2 + 2}{4} a_{1n}\right] + \left[\frac{(\rho_x-1)^2 + \left(\rho_y-1\right)^2 - 2}{4} \left(\frac{a_{0n}+a_{2n}}{2}\right)\right] \Gamma\left(n+\frac{4\kappa}{3}-\frac{3}{2}, A\frac{p_t}{T_0}\xi^{\frac{3}{\kappa}}\right)\Bigg|_1^i + \left[\frac{\rho_x^2+\rho_y^2+\rho_z^2}{2} a_{1n}\right] A\,\Gamma\left(n+\frac{4\kappa}{3}-\frac{5}{2}, A\frac{p_t}{T_0}\xi^{\frac{3}{\kappa}}\right)\Bigg|_1^i \right\} \frac{1}{N_1(p_t)}$$

$$\rho_x = \frac{\kappa}{\kappa-3-\kappa\frac{b}{\dot{X}_0^2}} \qquad \rho_y = \frac{\kappa}{\kappa-3-\kappa\frac{b}{\dot{Y}_0^2}} \qquad \rho_z = \frac{\kappa}{\kappa-3-\kappa\frac{b}{\dot{Z}_0^2}} \qquad A = 1-\frac{\rho_x+\rho_y}{4} \qquad B = \frac{\rho_x-\rho_y}{4}$$

a_{in} are the coefficients of the first kind modified Bessel-functions

- Second-order Gaussian approximation
  - A bit complicated formula
- An approximation, when $p_t \approx 2-3$ GeV:
  - $N_1(p_t) \sim \left(\frac{p_t}{T_0}\right)^{-\frac{4\kappa}{3}+1}$
  - $v_2(p_t) = C + De^{-\frac{Ap_t}{T_0}}\left(\frac{p_t}{T_0}\right)^{-\frac{4\kappa}{3}-\frac{5}{2}} - Ee^{-\frac{Ap_t}{T_0}}\left(\frac{p_t}{T_0}\right)^{-\frac{4\kappa}{3}-\frac{3}{2}}$
    - A, C, D, E are constants

1. 2. 3. 4. Comparing to data

Parameter	Symbol	Value	Type
Freeze-out temperature	T_0	204 MeV	fixed by hadron spectrum
Freeze-out time	τ_0	7.7 fm/c	fixed by hadron spectrum
Excentricity	ϵ	0,34	fixed by hadron spectrum
Transverse expansion	u_t^2/b	$-0{,}34$	fixed by hadron spectrum
Longitudinal expansion	$\dot{Z}_0^2/b$	-1.6	fixed by hadron spectrum
Compressibility	κ	7.7 ± 0.7	fitted
Initial time	t_i	$0 - 0.7$ fm/c	acceptable interval

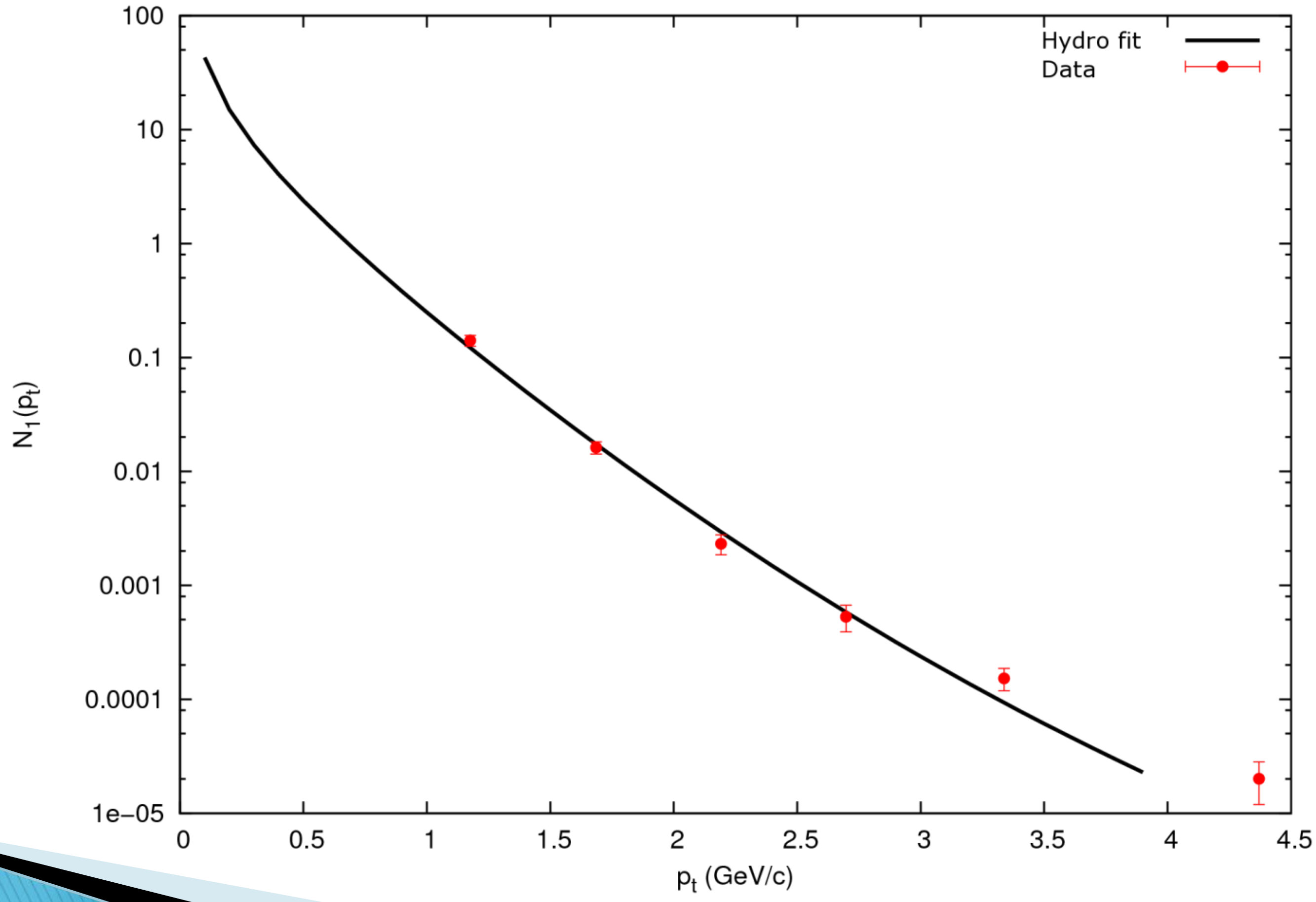
Fixed by hadron spectrum fit: (M. Csanád – M. Vargyas: Eur.Phys.J., A44:473-478, 2010.)

Properties of the fit:

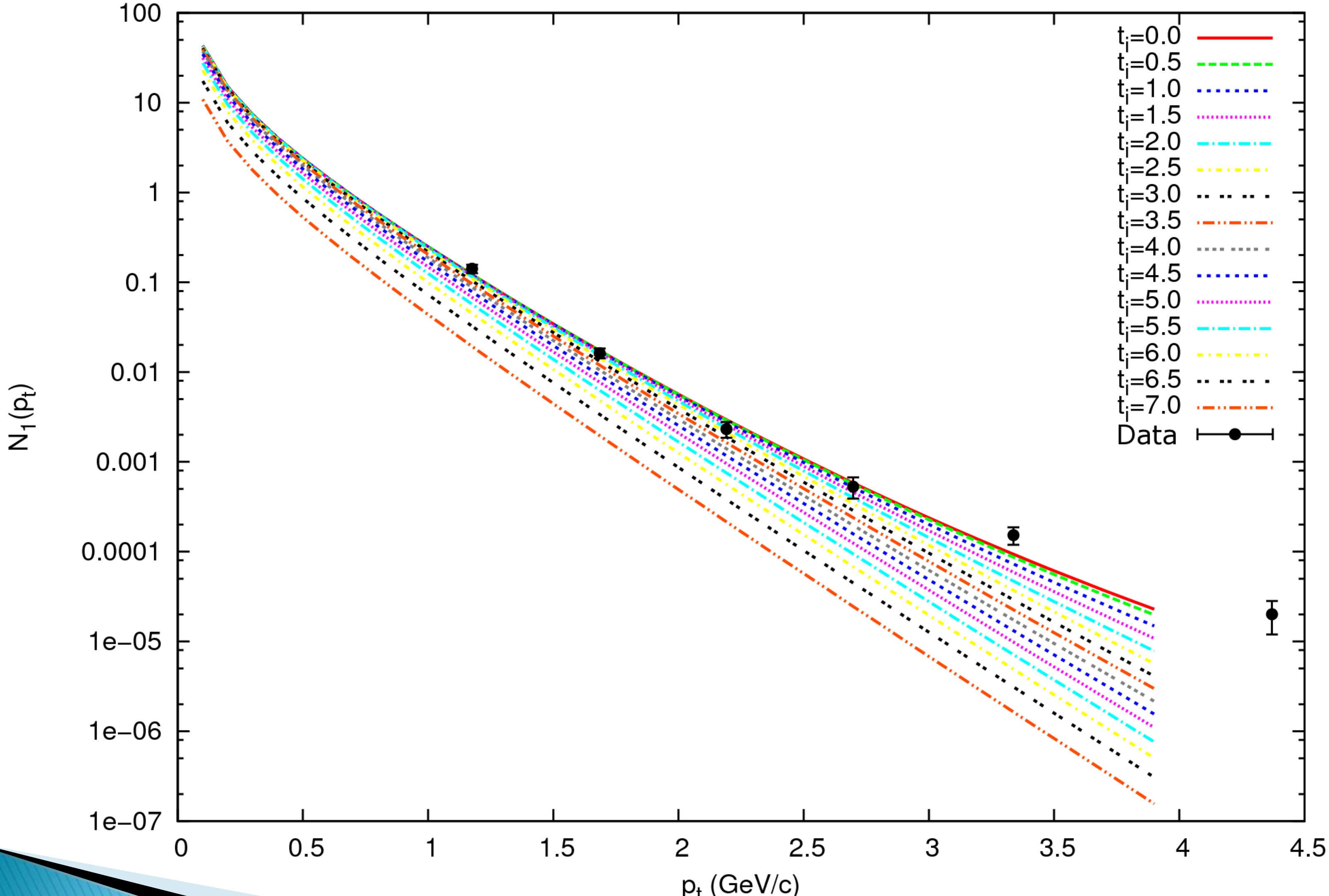
Number of datapoints		5
Fitted parameters		2
Degrees of freedom	NDF	$5 - 2 = 3$
Chi square	χ^2	7.0
Confidence rate		7.2%

1. 2. 3. 4. COMPARING TO DATA

Fit to 0–92% centrality PHENIX data:

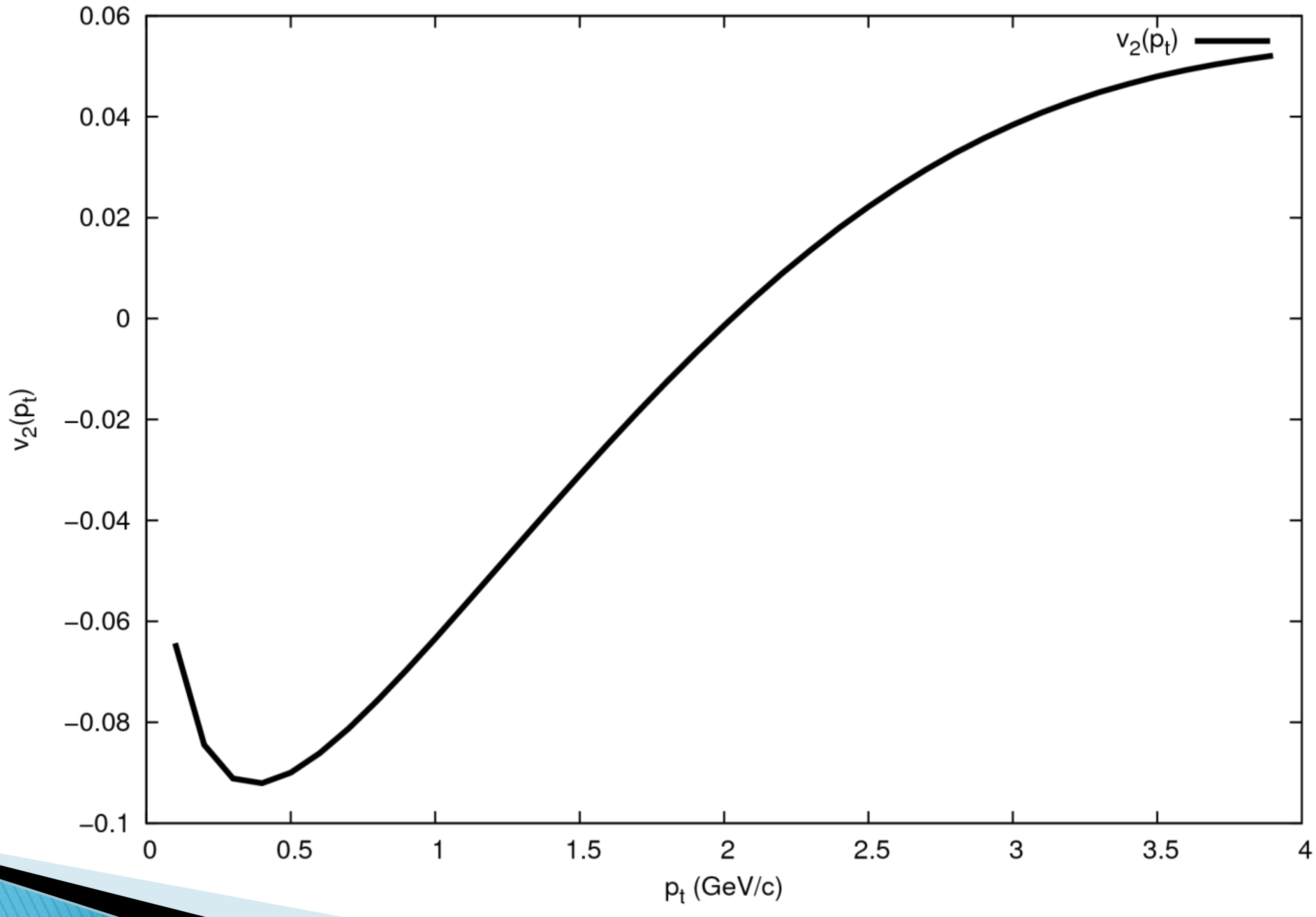


Small dependence on initial time



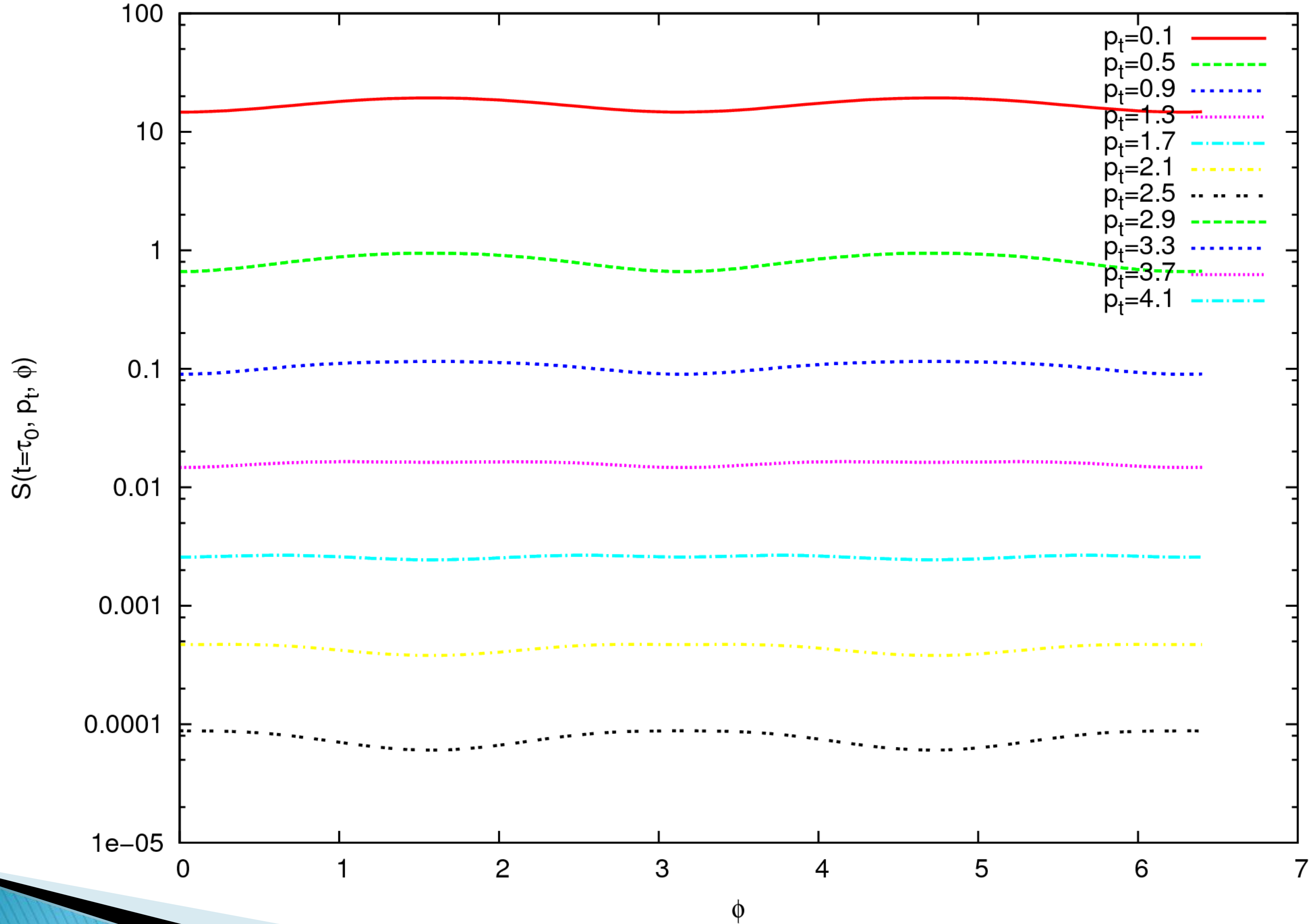
1. 2. 3. 4. Comparing to data

No measurement available for direct photon v_2



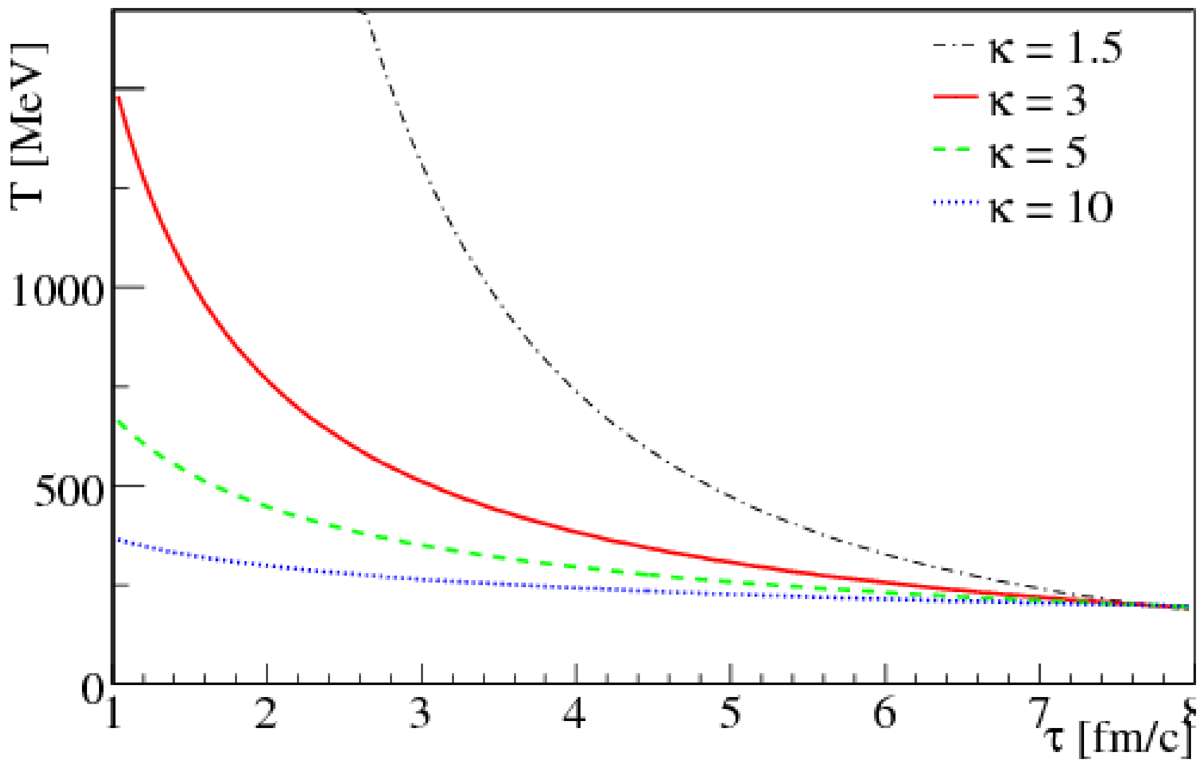
v_2 is negative at low p_t−s

$$S(t, p_t, \varphi) = \int S(x^\mu, p_t, \varphi)\, d^3x$$



1. 2. 3. 4. Comparing to data

From hadronic spectra:



From photon spectra: $\kappa = 7.7 \pm 0.7$

$T_{initial} = 520$ MeV at $\tau = 0.7$ fm$/c$

SUMMARY

- Important new parameters:
  - The unknown parameter of the **EoS**:

$$\boldsymbol{\kappa = 7.7 \pm 0.7}$$

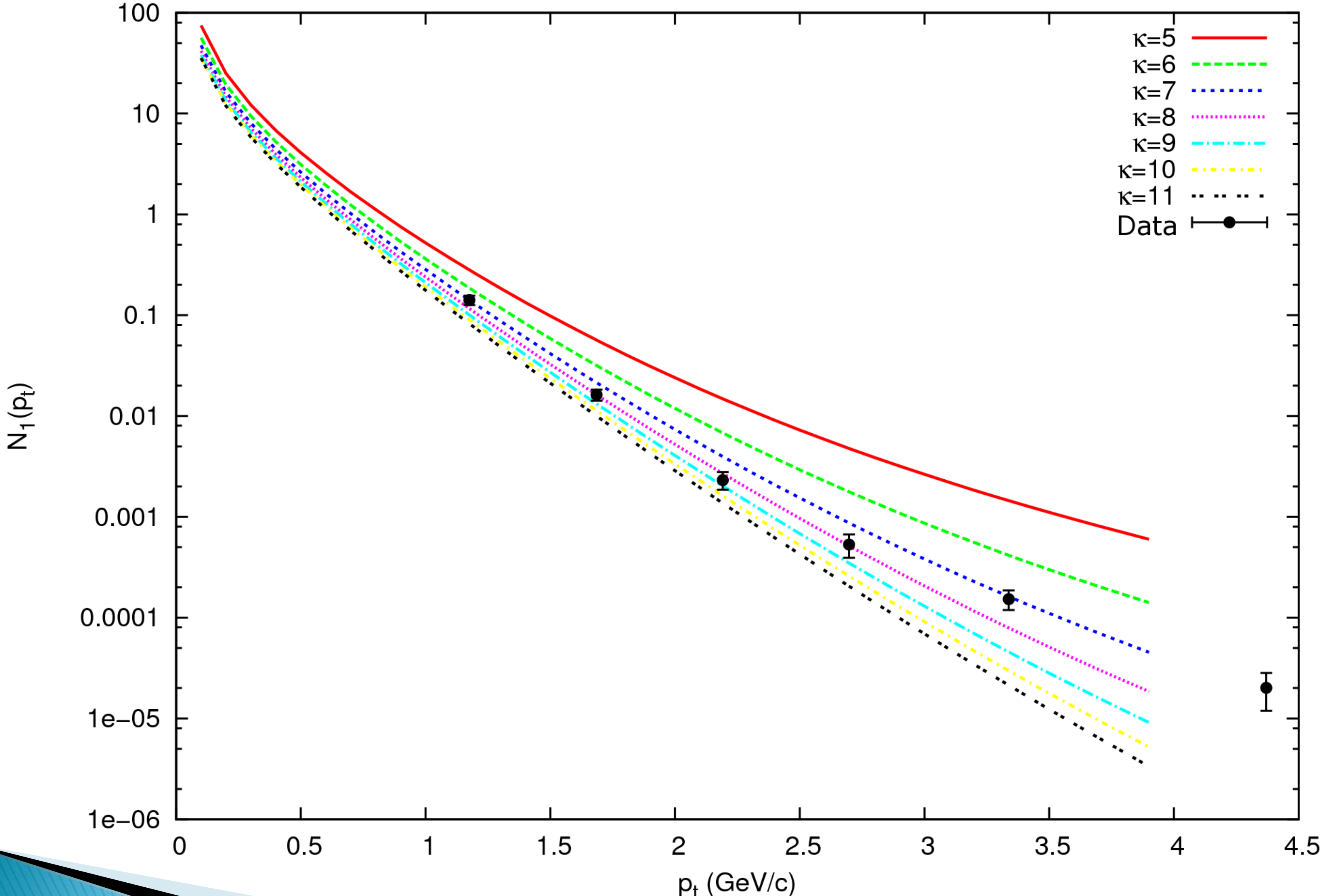
  - Corresponds to other theories (R. A. Lacey, A. Taranenko., PoS, CFRNC2006:021, 2006: $\kappa \approx 8.2$)
  - Acceptibility interval for the initial time
  - Lower bound for the initial **temperature**:

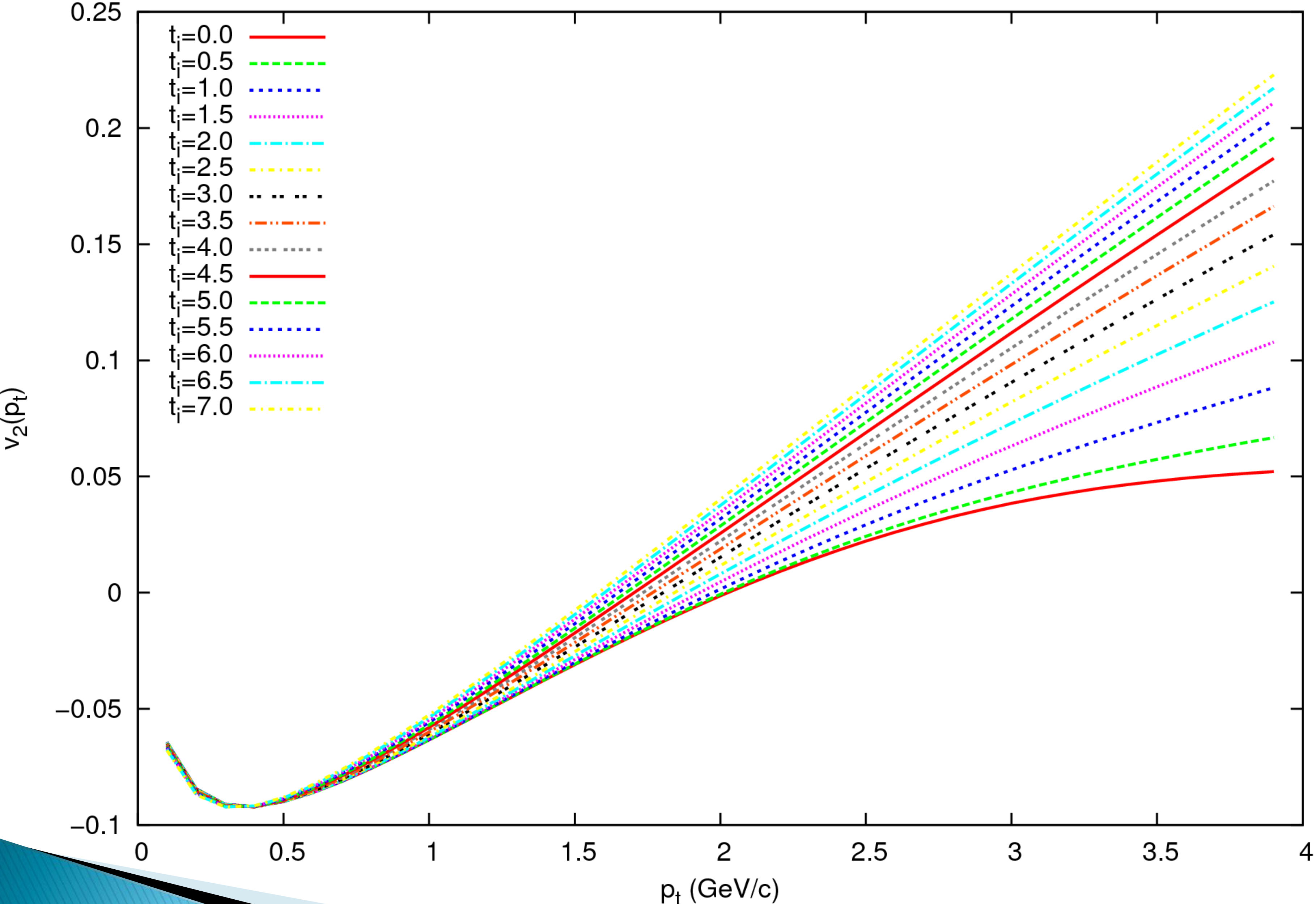
$$\boldsymbol{T_{initial} > 520\ \mathrm{MeV}}$$

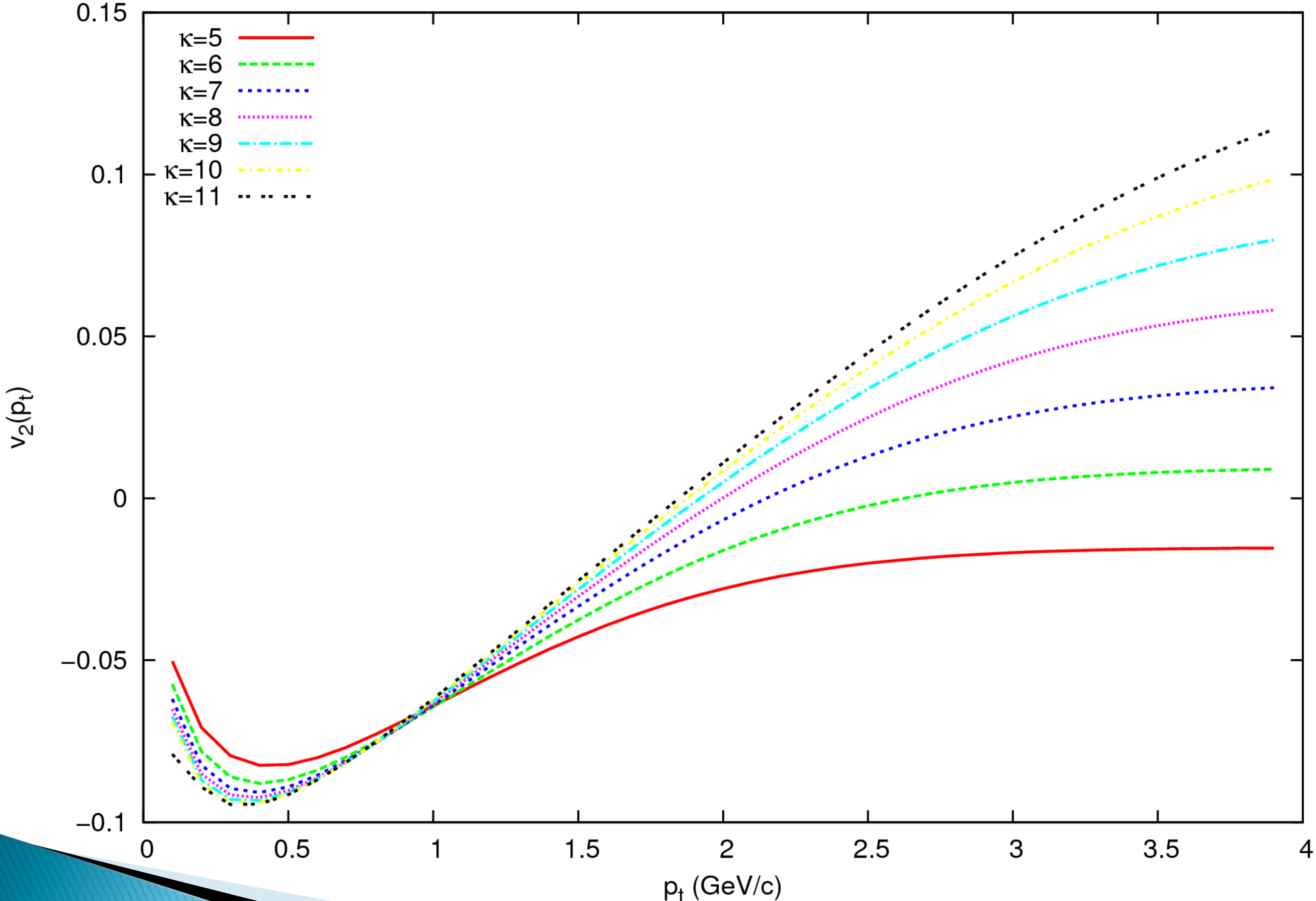
  - Other theories predict $300 - 600$ MeV (A. Adare et al., Phys.Rev.Lett., 104:132301, 2010)

Thank you for your attention!

Sensitive to κ




t_i=0.0
t_i=0.5
t_i=1.0
t_i=1.5
t_i=2.0
t_i=2.5
t_i=3.0
t_i=3.5
t_i=4.0
t_i=4.5
t_i=5.0
t_i=5.5
t_i=6.0
t_i=6.5
t_i=7.0
$v_2(p_t)$
p_t (GeV/c)
0.25
0.2
0.15
0.1
0.05
0
−0.05
−0.1
0
0.5
1
1.5
2
2.5
3
3.5
4


κ=5
κ=6
κ=7
κ=8
κ=9
κ=10
κ=11
0.15
0.1
0.05
0
−0.05
−0.1
0
0.5
1
1.5
2
2.5
3
3.5
4
$v_2(p_t)$
p_t (GeV/c)

Approximation