Chiral Transition and Axial Anomaly in QCD and Related Topics

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Zimanyi's @ 2005 Photo by S. Nagamiya

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Plan of lecture

- Notions of chiral symmetry and chiral anomaly
- Effective chiral models with anomaly included
- Effective restoration of U_A(1) symmetry at finite temperature and density
- QCD phase diagram with anomaly incorporated and possible signature of the QCD Critical Point
- Summary

$$\frac{1-\gamma_5}{2} q_i \equiv q_{iL} \to L_{ij} q_{jL}, \quad \text{(left handed)}$$

$$\frac{1+\gamma_5}{2} q_i \equiv q_{iR} \to R_{ij} q_{jR}, \quad \text{(right handed)}$$

$$L = \exp(i\boldsymbol{\theta}_L \cdot \boldsymbol{\lambda}/2) \equiv U(\boldsymbol{\theta}_L),$$

$$R = \exp(i\boldsymbol{\theta}_R \cdot \boldsymbol{\lambda}/2) = U(\boldsymbol{\theta}_R),$$

$$\boldsymbol{\theta}_{L,R} \cdot \boldsymbol{\lambda} = \sum_{a=0}^{8} \theta^a_{L,R} \lambda^a.$$

$$\gamma_5 q_L = -q_L, \quad \gamma_5 q_R = q_R.$$

For $N_f = 3$, the chiral transformation forms

Direct prod. $U_L(3) \otimes U_R(3) \simeq (U_L(1) \otimes U_R(1)) \otimes SU_L(3) \otimes SU_R(3)$

Chiral Invariance of Classical QCD Lagrangian in the chiral limit (**m=0**)

$$\bar{q}\gamma^\mu q = \bar{q}_L\gamma^\mu q_L + \bar{q}_R\gamma^\mu q_R$$

$$\rightarrow \quad \bar{q}_L L^\dagger \gamma^\mu L q_L + \bar{q}_R R^\dagger \gamma^\mu R q_R$$

$$= \quad \bar{q}_L\gamma^\mu q_L + \bar{q}_R\gamma^\mu q_R$$

$$= \bar{q}\gamma^\mu q$$ invariant!

In the chiral limit (m=0),

$\bar{q}\ \gamma^\mu D_\mu q$; Chiral invariant

$$D_\mu = \partial_\mu - igt^a A^a_\mu$$

$$\mathcal{L}_0^{cl} = \bar{q}(i\gamma^\mu D_\mu - m)q - \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}$$;Chiral invariant!

(the m is crossed out with an X)

Q^a the generators of a continuous transformation

$\partial^\mu j^a_\mu = 0$; $j^a_\mu(x)$ Noether current

eg. Chiral transformation for $SU_L(2) \otimes SU_R(2)$

$Q^a_5 = \int d\mathbf{x}\, \bar{q}\gamma^0\gamma_5\tau^a/2q$

Notice; $[iQ^a_5, \bar{q}(x)i\gamma_5\tau^b q(x)] = -\delta^{ab}\bar{q}(x)q(x)$

$\forall a$

$\exists a$

The symmetry is spontaneously broken.

Now, $\langle 0|\bar{q}q|0\rangle = \langle 0|[Q^a_5, \bar{q}\gamma_5\tau^a q]|0\rangle$

$\langle 0|\bar{q}q|0\rangle \neq 0 \rightarrow Q^a_5|0\rangle \neq 0$

Chiral symmetry is spontaneously broken!

The non-perturbative nature of QCD vacuum

Gell-Mann-Oakes-Renner

$$f_\pi^2 m_{\pi^\pm}^2 \simeq -\hat{m}\langle \bar{u}u + \bar{d}d\rangle \qquad \hat{m} = (m_u + m_d)/2$$

using

$f_\pi = 93$ MeV and $\hat{m}(1\text{GeV}) = (7 \pm 2)$ MeV,

We have

$$\langle \bar{u}u\rangle \simeq \langle \bar{d}d\rangle \simeq [-(225 \pm 25) MeV]^3 \quad \text{at} \quad \mu^2 = 1\text{GeV}$$

QCD sum rules for heavy-quark systems,

$$\langle \frac{\alpha_s}{\pi} F^a_{\mu\nu} F_a^{\mu\nu}\rangle = (350 \pm 30 \text{ MeV})^4$$

Special Chiral transformations

(i) $\boldsymbol{\theta}_L = \boldsymbol{\theta}_R \equiv \boldsymbol{\alpha}$ → $q \to U(\boldsymbol{\alpha})q, \quad \bar{q} \to \bar{q}U^\dagger(\boldsymbol{\alpha})$

gauge transformation: $U_V(N_f)$

generator; $Q^a = \int d\mathbf{x} q^\dagger(x)\lambda^a/2q(x)$

(ii) $\boldsymbol{\theta}_L = -\boldsymbol{\theta}_R \equiv -\boldsymbol{\beta}$ → $q \to U(\boldsymbol{\beta}\gamma_5)q, \quad \bar{q} \to \bar{q}U^\dagger(\boldsymbol{\beta}\gamma_5)$

Axial gauge transformation: $U_A(N_f)$

generator; $Q_5^a = \int d\mathbf{x} q^\dagger(x)\lambda^a\gamma_5 q(x)$

$SU_V(N_f)$ $$\partial_\mu(\bar{q}\gamma^\mu\lambda^a q) = i\sum_{i,j}^{N_f}\bar{q}_i(m_i - m_j)\lambda^a q_j \quad (a = 0 \sim N_f^2 - 1)$$

$i, j =$ u,d,s, ...

$SU_A(N_f)$ $$\partial_\mu(\bar{q}\gamma^\mu\gamma_5\lambda^a q) = i\sum_{i,j}^{N_f}\bar{q}_i(m_i + m_j)\gamma_5\lambda^a q_j \quad (a = 1 \sim N_f^2 - 1)$$

$U_A(1)$ $$\partial_\mu(\bar{q}\gamma^\mu\gamma_5 q) = i\sum_{i}^{N_f}\bar{q}_i 2m_i\gamma_5 q_i + 2N_f\frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a \quad (\tilde{F}^{\lambda\rho}_a = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F^a_{\mu\nu})$$

Chiral Anomaly

Quantum effects!

Dilatation $$\partial_\mu D^\mu = \Theta^\mu_\mu = (1 + \gamma_m)\sum_{i}^{N_f}\bar{q}_i m_i q_i + \frac{\beta}{2g}F^a_{\mu\nu}F^{\mu\nu}_a$$

Dilatation(scale) Anomaly

$\Theta_{\mu\nu}$; energy-momentum tensor of QCD

$$\gamma_m = 2\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{101}{12} - \frac{5}{18}N_f\right)\left(\frac{\alpha_s}{\pi}\right)^2 + \cdots \; .$$

Some symmetries existing in the classical level are broken explicitly in the quantum level. Quantum Anomaly

G= $U_L(3) \otimes U_R(3)$ 2x(8+1)=18

H= $U_V(1) \otimes SU_f(3)$ 1+8=9

# of NG-bosons= dim *G* - dim *H* = *18 – 9 = 9* (?)

Nambu-Goldstone Theorem

★ # of the lightest pseudo-scalar mesons

$\pi^{\pm}, \pi^0$(140) $K^{\pm}, K^0, \overline{K^0}$(500) η(550) << η' (958)

3 + 4 + 1 = 8 $\neq 9$!

------ $U_A(1)$ Problem

c.f. Without the anomaly,

$m_{\eta_0} \leq \sqrt{3} m_\pi$

-- S.Weinberg('75) --

$\exists$ $U_A(1)$ Anomaly $\partial_\mu(\bar{q}\gamma_\mu\gamma_5 q) = 2N_f \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$ Operator Equation!

$\neq 0$

even in the chiral limit!

+ Instantons

The resolution of U_A(1) Problem

Ref. S.Weinberg, QTF 2 (Cambridge, 1996);
K. Fujikawa and H.Suzuki, PI and QA, (Oxford, '04)

- There are `big' as well as `small' or regular gauge transformations.
- Owing to the instanton configuration which connects the gauge-non-equivalent vacuua $|n\rangle$, by a big gauge transformation, the QCD vacuum becomes a theta vacuum,

$$|\theta\rangle \equiv \sum_n e^{i\theta n} |n\rangle$$

$\langle \theta | F$ [illegible] $| \rangle$

- Thus, the Nambu-Goldstone Theorem can not apply to this channel.

Rough sketch of a proof of absence of NG boson

Anomalous Ward-Takahashi identity:

$$\partial^x_\mu \langle \theta | T^* (\overline{\psi}(x)\gamma^\mu \gamma_5 \psi(x)) \overline{\psi}(y)\gamma_5\psi(y) | \theta \rangle - 2N_f \langle \theta | T^* (\frac{g^2}{16\pi^2} \mathrm{tr} F \square \cdots)\gamma_5 \psi(y) | \theta \rangle$$

$$= 2\delta(x-y) \langle \theta | \overline{\psi}(y)\psi(y) | \theta \rangle$$

Fourier tr.

And integrate the both side (or take the 0 momentum limit:

Owing to instanton, $\int dx \langle \theta | (\mathrm{tr} F \square \cdots \psi(0) | \vartheta \rangle \neq 0$

Then, Chiral SB, $\langle \theta | \overline{\psi}(0)\psi(0) | \theta \rangle \neq 0$

can be compatible without massless pole in the 1st term in LHS.

c.f Weinding number

$$\frac{1}{16\pi^2} \int dx (\mathrm{tr} F \square \cdots \quad : \text{intejer}$$

Construction of Effective Lagrangian:

Def. $\bar{q}_j q_i + i\bar{q}_j i\gamma_5 q_i$

$$I_n = \mathrm{tr}(\Phi\Phi^\dagger)^n, \qquad (n = 1, 2, 3, \ldots) \quad \mathrm{U}_L(3)\otimes\mathrm{U}_R(3)\text{-invariant}$$

$$\Phi = \sum_{a=0}^{8} \Phi_a \lambda_a / \sqrt{2} \qquad (\because \ \mathrm{tr}\lambda_a\lambda_b = 2\delta_{ab})$$

$$\Phi_a = \mathrm{tr}\Phi\lambda_a / \sqrt{2} = \bar{q}(1-\gamma_5)\lambda_a q / \sqrt{2}$$

$$= \hat{\sigma}_a + i\hat{p}_a, \quad \text{with} \quad \hat{\sigma}_a = \bar{q}\lambda_a q / \sqrt{2} \quad \hat{p}_a = \bar{q}i\gamma_5\lambda_a q / \sqrt{2}$$

$$I_1 = \sum_{a,b=0}^{8} \Phi_a \Phi_b^\dagger \mathrm{tr}\lambda^a \lambda^b / 2 = \sum_{a=0}^{8} \Phi_a \Phi_a^\dagger,$$

$$= \sum_{a=0}^{8} [\hat{\sigma}_a^2 + \hat{p}_a^2]$$

$\exists$ $\mathrm{U}_A(1)$ Anomaly:

$\det\Phi, \quad \det\Phi^\dagger$: $\mathrm{U}_V(1) \otimes \mathrm{SU}_L(3) \otimes \mathrm{SU}_R(3)$ -inv.

but X $\mathrm{U}_A(1)$

$I_D = \det\Phi + \det\Phi^\dagger$; Hermite

Effective Model; $\mathbf{SU}_L(\mathbf{3})\otimes\mathbf{SU}_R(\mathbf{3})$-$\sigma$ model

$$\mathcal{L}_\sigma^{(0)} = 1/2\cdot(\mathrm{tr}\partial_\mu\Phi\partial^\mu\Phi) - 1/2\cdot\mu^2 I_1 - \lambda I_1^2 - \gamma I_2 + \tau I_D$$

I.Vacuum :

$$\langle\frac{\partial\mathcal{L}_0}{\partial\Phi^\dagger}\rangle = 0$$

$$\langle\Phi\rangle \to L(\vartheta_L)\langle\Phi\rangle R^\dagger(\theta_R)$$

If $\theta_L = \theta_R$, i.e., $SU_V(3)$,

$\langle\Phi\rangle$ is invariant, but otherwise not..

$$2(3\lambda+\gamma)\varphi_0^2 - \tau\varphi_0 + \mu^2/2 = 0$$

$$\therefore \quad \varphi_0 = \frac{\tau+\sqrt{\tau^2-4\mu^2(3\lambda+\gamma)}}{4(3\lambda+\gamma)} \quad \text{for } \mu^2 < 0$$

2. Meson spectra:

$$\Phi = \varphi_0 \mathbf{1} + \Phi', \quad \Phi' = \frac{1}{\sqrt{2}}(S + iP)$$

$$S = \sum_{a=0}^{8} S_a \lambda_a \qquad P = \sum_{a=0}^{8} P_a \lambda_a$$

Meson masses;

(1) ps-mesons

π, K, η_8

$$m^{(8)2}_{\mathrm{ps}} = \mu^2 + 4\varphi_0^2(3\lambda + \gamma) - 2\varphi_0\tau = 0$$

η_1

$$m^{(0)2}_{\mathrm{ps}} = 6\tau\varphi_0 \neq 0$$

$$\langle \frac{\partial \mathcal{L}_0}{\partial \Phi^\dagger} \rangle = 0$$

Anomaly term

(2) scalar-mesons

$$m^{(8)2}_{\mathrm{s}} = \mu^2 + 12\varphi_0^2(\lambda + \gamma) + 2\varphi_0\tau,$$

$$m^{(0)2}_{\mathrm{s}} = 2(\mu^2 - \varphi_0\tau),$$

A dynamical Ciral Lagrangian with Axial Anomaly

M. Kobayashi and T. Maskawa ('70),
G. 't Hooft ('76)

$$\mathcal{L} = \bar{q} i\gamma \cdot \partial q + \sum_{a=0}^{8} \frac{g_S}{2} [(\bar{q}\lambda_a q)^2 + (\bar{q} i\lambda_a \gamma_5 q)^2] - \bar{q} m q + g_D [\det \bar{q}_i (1-\gamma_5) q_j + \mathrm{h.c.}]$$

T.K. Soryushiron Kenkyu (1988),
T.K. and T. Hatsuda, Phys. Lett. B (1988);
Phys. Rep. 247 (1994)

A presentation of Chiral Anomaly:

$$\partial_\mu A_5^\mu = 2iN_f g_D (\det\Phi - \mathrm{h.c.}) + 2i\bar{q} m \gamma_5 q \qquad \Phi_{ij} = \bar{q}_j (1-\gamma_5) q_i$$

$$\Updownarrow$$

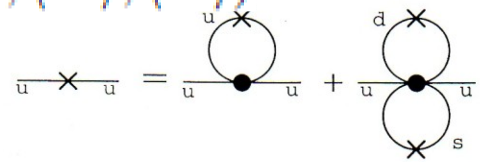
$$\partial_\mu A_5^\mu = 2N_f \frac{g^2}{32\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} + 2i\bar{q} m \gamma_5 q$$

Anomaly eq. of QCD

Note: $g_D < 0$ **consistent with the instanton-induced interaction**

1.The vacuum in MFA: $\phi = \langle \Phi \rangle_0 \equiv \mathrm{diag}(\langle \bar{u}u \rangle, \langle \bar{d}d \rangle, \langle \bar{s}s \rangle)$

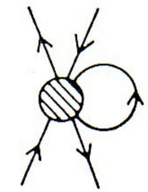
$$M_u = m_u - 2g_s\alpha - 2g_D\beta\gamma,$$
$$M_d = m_d - 2g_s\beta - 2g_D\alpha\gamma,$$
$$M_s = m_s - 2g_s\gamma - 2g_D\alpha\beta,$$



$$(\alpha, \beta, \gamma) \equiv (\langle \bar{u}u \rangle, \langle \bar{d}d \rangle, \langle \bar{s}s \rangle)$$

The residual interaction in the new vacuum:

$$\mathcal{L}_{res} = g_S : \mathrm{Tr}(\Phi^\dagger \Phi) :$$
$$+g_D : [\mathrm{Tr}(\phi\Phi^2) - \mathrm{Tr}(\phi\Phi)\mathrm{Tr}\Phi - \frac{1}{2}\mathrm{Tr}\Phi^2\mathrm{Tr}\phi + \frac{1}{2}\mathrm{Tr}\phi(\mathrm{Tr}\Phi)^2 + \mathrm{h.c.}] :$$
$$+g_D : (\det\Phi + \mathrm{h.c.}) :,$$



Flavor mixing of η and η' mesons

$$\mathcal{L}^{\eta}_{res} = \frac{1}{2}\sum_{a,b=8,0} : \eta_a G^P_{ab}\eta_b :,$$
$$\eta_a \equiv \bar{q}i\gamma_5\lambda_a q,$$

In the flavor basis:

$$(\bar{u}i\gamma_5 u, \bar{d}i\gamma_5 d, \bar{s}i\gamma_5 s)$$

$$G^P = \begin{pmatrix} g_S + \frac{1}{3}(2\alpha+2\beta-\gamma)g_D & -\frac{\sqrt{2}}{6}(2\gamma-\alpha-\beta)g_D \\ -\frac{\sqrt{2}}{6}(2\gamma-\alpha-\beta)g_D & g_S - \frac{2}{3}(\alpha+\beta+\gamma)g_D \end{pmatrix} \Longleftrightarrow G_{\eta\pi^0} = 2\begin{pmatrix} g_s & -g_D\gamma & -g_D\beta \\ -g_D\gamma & g_S & -g_D\alpha \\ -g_D\beta & -g_D\alpha & g_S \end{pmatrix}$$

An effect of the anomaly term:

$$G^P_{00} \equiv g_S - \frac{2}{3}(\alpha+\beta+\gamma)g_D < G^P_{88} \equiv g_S + \frac{1}{3}(2\alpha+2\beta-\gamma)g_D$$

less attractive in the singlet channel than in the octet

Flavor mixing by the anomaly term!

$g_D = 0 \longrightarrow$ ideal mixing.

The propagator and mixing angle

$$D(q^2) = -G_P^{-1}\left(\frac{1}{1+G_P \Pi^P(q^2)}\right) \longrightarrow T(\theta_\eta) D^{-1}(q^2) T(\theta_\eta)^{-1} = \mathrm{diag}(D_\eta^{-1}(q^2), D_{\eta'}^{-1}(q^2))$$

$$T(\theta_\eta) = \begin{pmatrix} \cos\theta_\eta & -\sin\theta_\eta \\ \sin\theta_\eta & \cos\theta_\eta \end{pmatrix}$$

diagonalize with a mixing matrix;

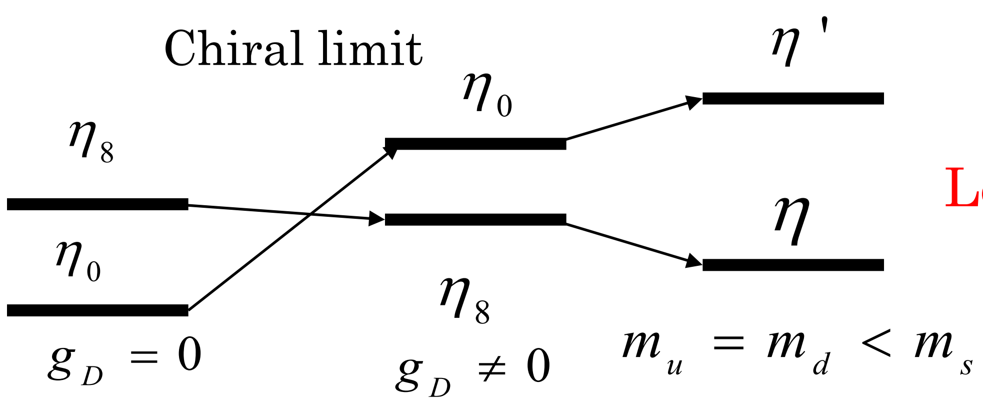
(i) $g_D = 0$ $\theta_\eta = -54.75°$; $\eta = (\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d)/\sqrt{2}$ and $\eta' = \bar{s}i\gamma_5 s$

(ii) $g_D \neq 0$ $m_u = m_d = m_s$; flavor symmetric $\longrightarrow$ $\theta_\eta = 0$

$\eta = \eta_8 = (\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d - 2\bar{s}i\gamma_5 s)/\sqrt{6}$ and $\eta' = \eta_0 = (\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d + \bar{s}i\gamma_5 s)/\sqrt{3}$.

(iii) $g_D \neq 0$ and $m_u = m_d = 5.5\,\mathrm{MeV} \neq m_s = 135.7\,\mathrm{MeV}$ **realistic case**

$\theta_\eta(m_\eta^2) = -20.9°$ $m_{\eta'} = 957.5$ MeV (fitted) $m_\eta = 486.5$ MeV



Level crossing by the anomaly!

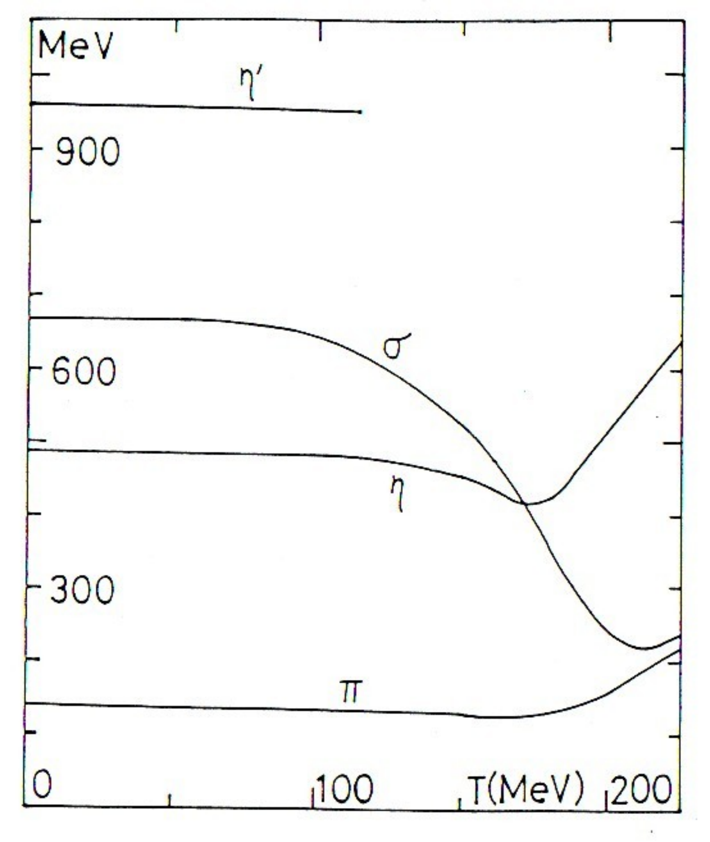
At Finite Temperature and Density

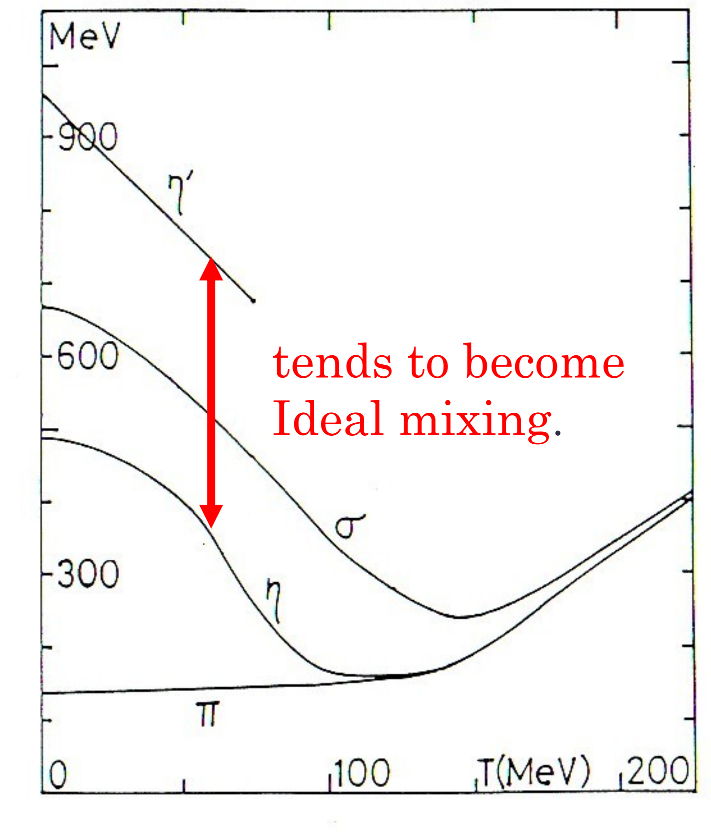
Effective restoration of axial symmetry at finite temperature

R. Pisarski and F. Wilczeck(1984)

$\eta - \eta'$

T. K. Phys. Lett. B (1989)





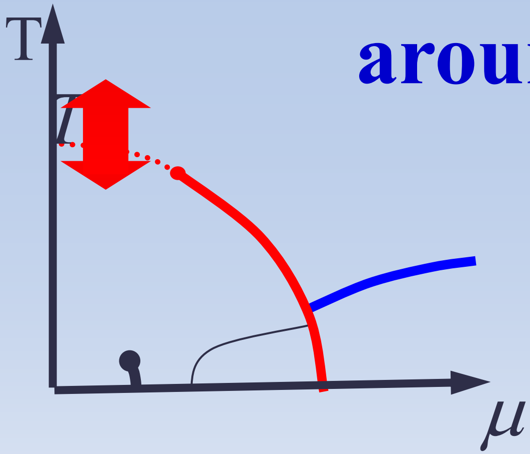
$$g_D(T) = g_D(T=0)\exp[-(T/T_0)^2].$$

$T_0 = 100$ MeV

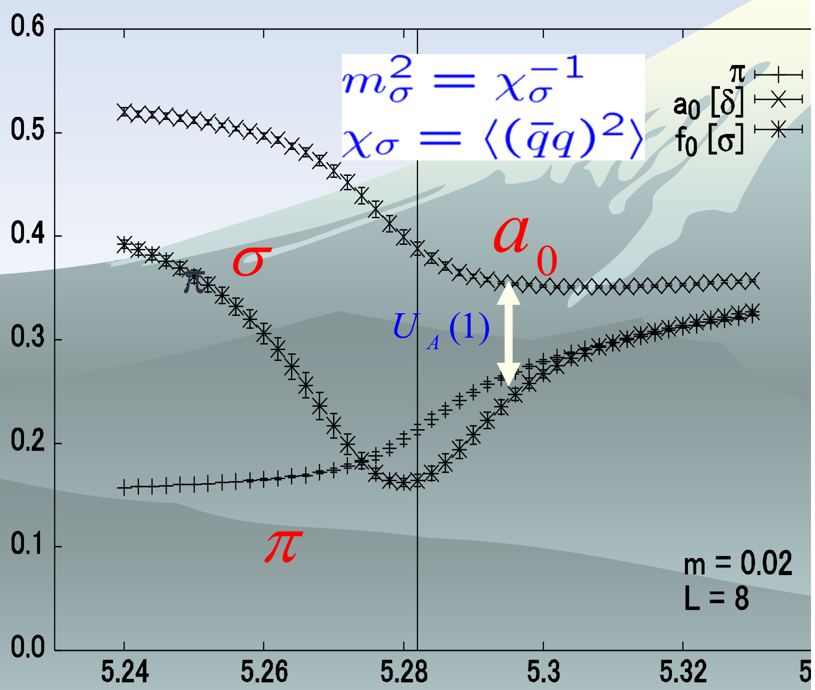
T-independent g_D

NJL model with Kobayashi-Maskawa-’t Hooft term; T.K. and T.Hatsuda (1988)

Fluctuations of chiral order parameter around Tc in Lattice QCD



$$\chi_m = \frac{\partial}{\partial m}\left\langle \bar{q}q \right\rangle = \left\langle (\bar{q}q)^2 \right\rangle$$



Cf. Lattice Calculation of the *generalized masses*

F. Karsch, Lect. Note Phys. **583** (2002), 20

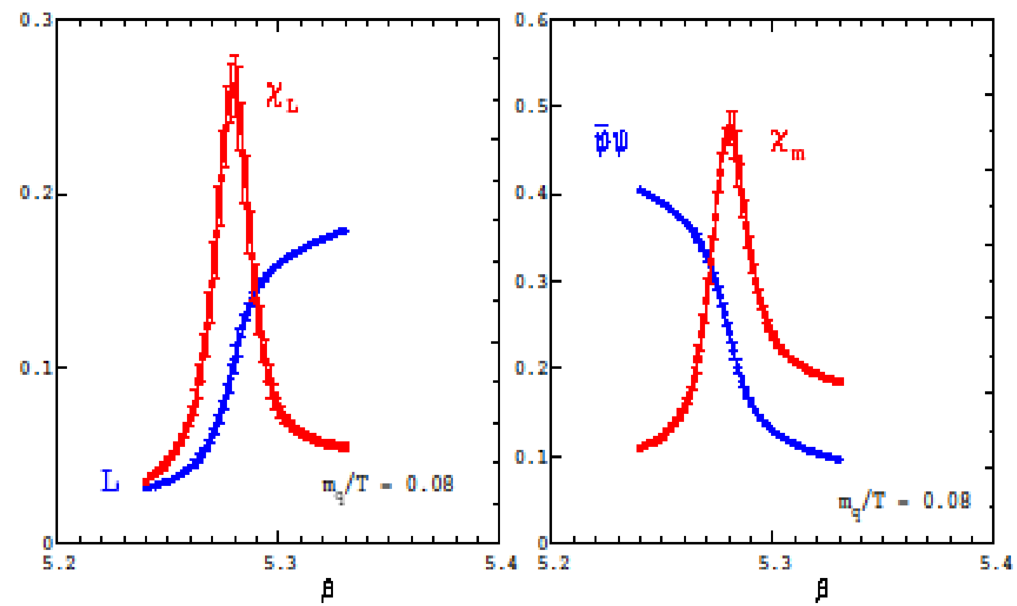


Fig. 2. Deconfinement and chiral symmetry restoration in 2-flavour QCD: Shown is $\langle L \rangle$ (left), which is the order parameter for deconfinement in the pure gauge limit ($m_q \to \infty$), and $\langle \bar{\psi}\psi \rangle$ (right), which is the order parameter for chiral symmetry breaking in the chiral limit ($m_q \to 0$). Also shown are the corresponding susceptibilities as a function of the coupling $\beta = 6/g^2$.

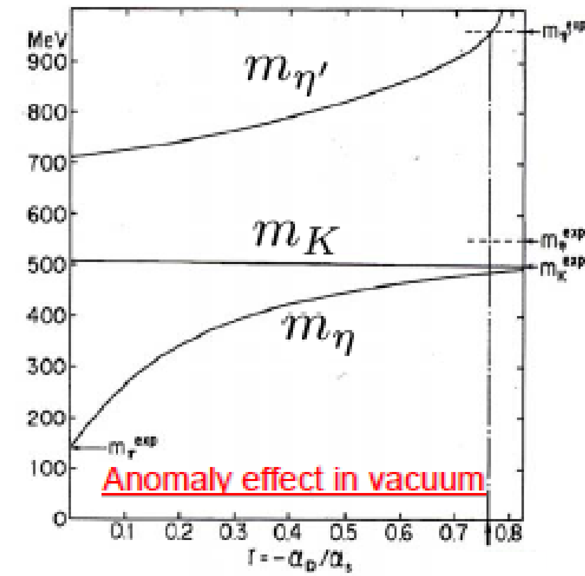
the softening of the σ with increasing T: Eff. Res. of U_A(1)

$\eta'(958)$ mesic nuclei formation

<u>H.N.</u>, S. Hirenzaki, PRL94(05)232503
<u>H. N.</u>, M. Takizawa and S. Hirenzaki, PRC74(06)045203
H. Nagahiro@NFQCD2010

- $\eta'(958)$ meson … close connection to the $U_A(1)$ anomaly
  - many theoretical works
    - in vacuum / at finite temperature / at finite density
      - R. D. Pisarski, R. Wilczek, PRD29(84)338
      - T. Kunihiro, T. Hatsuda, PLB206(88)385 / T. Kunihi
      - V. Bernard, R.L.Jaffe and U.-G.Meissner, NPB308(1
      - Y. Kohyama, K.Kubodera and M.Takizawa, PLB208
      - K. Fukushima, K.Onishi, K.Ohta, PRC63(01)045203
      - P. Costa *et al.*,PLB560(03)171, PRC70(04)025204, e
  - **poor experimental information at finite density**
- $U_A(1)$ anomaly in-medium from the viewpoint of "me
  - the η' properties, especially **mass shift**, at finite density
- **Nambu-Jona-Lasinio model** with the **KMT interaction**

Kunihiro, Hatsuda, PLB206(88)385



$$\mathcal{L} = \bar{q}(i\,\partial\!\!\!/ - m)q + \frac{g_s}{2}\sum_a\left[(\bar{q}\lambda_a q)^2 + (i\bar{q}\lambda_a\gamma_5 q)^2\right]$$
$$+ g_D\left[\det \bar{q}_i(1-\gamma_5)q_j + h.c.\right]$$

explicit breaking the $U_A(1)$ sym.

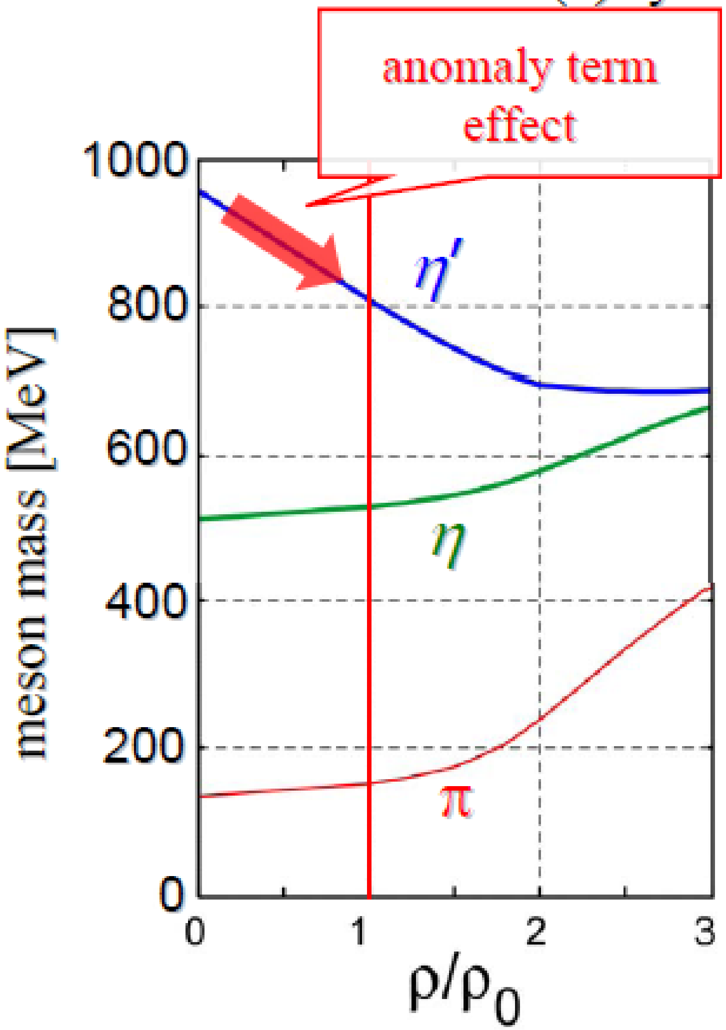
Kobayashi, Maskawa Prog.Theor.Phys.44, 1422 (70), G. 't Hooft, Phys.Rev.D14,3432 (76)

η' mass shift in medium $\rho_u = \rho_d,\ \rho_s = 0$

H.　Nagahiro@NFQCD2010

- we consider the SU(2) sym. matter as the sym. nuclear matter.

P. Costa et al.,PLB560(03)171, PRC70(04)025204, etc …



parameters (in vacuum)

P. Rehberg, et al., PRC53(96)410.

Λ = 602.3 [MeV]
$g_S \Lambda^2$ = 3.67
$g_D \Lambda^5$ = -12.36
$m_{u,d}$ = 5.5 [MeV]
m_s = 140.7 [MeV]

$M_{u,d}$ = 367.6 [MeV]
M_s = 549.5 [MeV]
$\langle \bar{u}u \rangle^{1/3}$ = -241.9 [MeV]
$\langle \bar{s}s \rangle^{1/3}$ = -257.7 [MeV]
$m_{\eta'}$ = 958 [MeV]
m_η = 514 [MeV]
m_π = 135 [MeV]

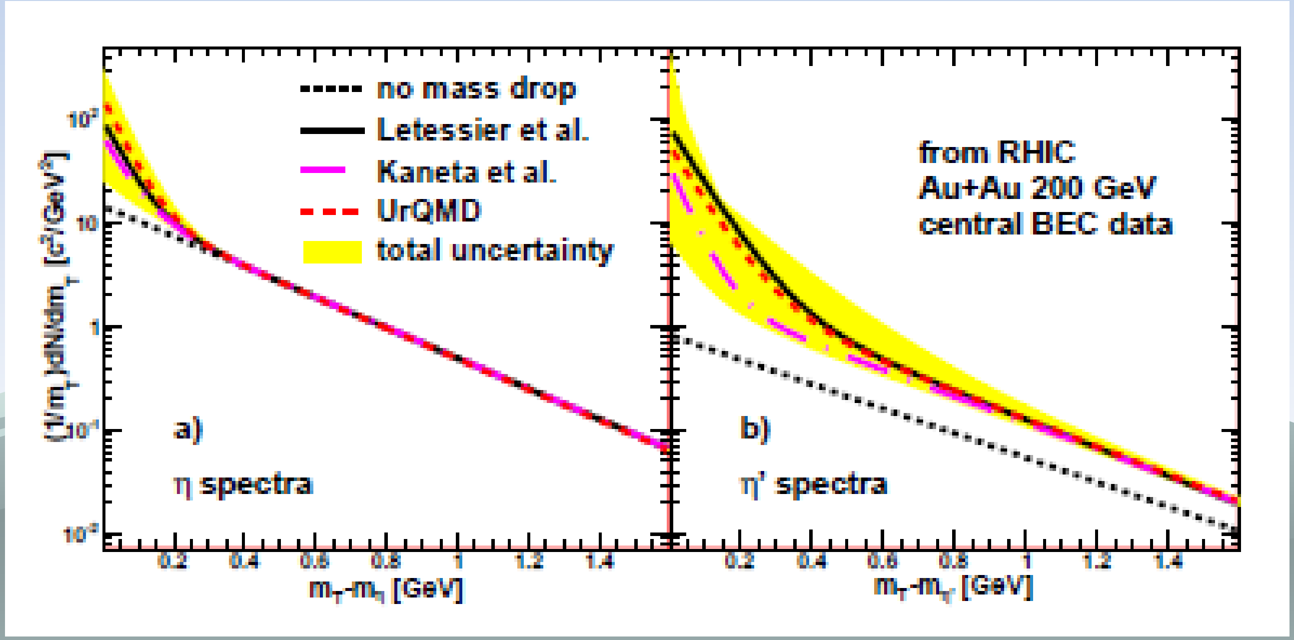
η and η' mass shifts @ ρ_0

$$\Delta m_{\eta'} \sim -150 \text{ MeV @ } \rho_0$$

$$\Delta m_{\eta} \sim +20 \text{ MeV @ } \rho_0$$

We can see the large medium effect even at normal nuclear density.

Possible experimental evidence of the reduction of eta' meson mass

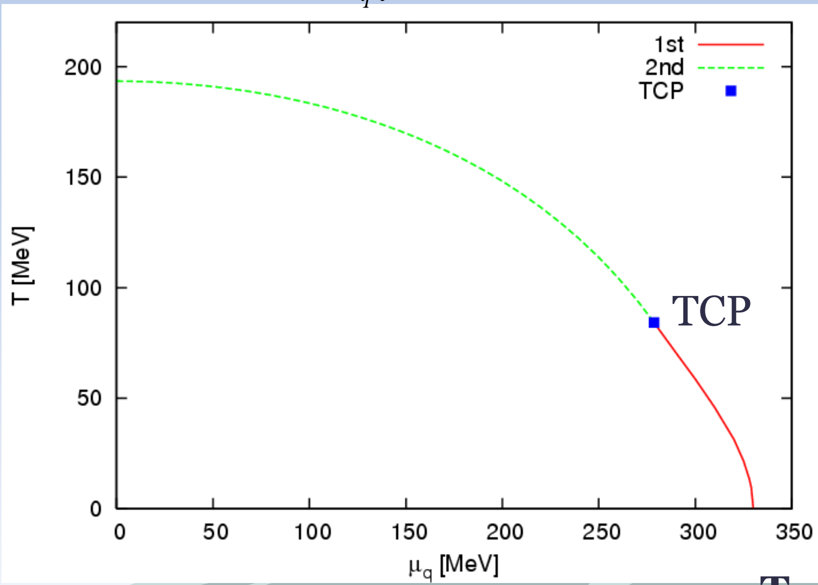


R. Vertesi, T. Csorgo and J. Sziklai, PRL, 105, 182301 (2010); arXiv.0912.5526[nucl-ex]

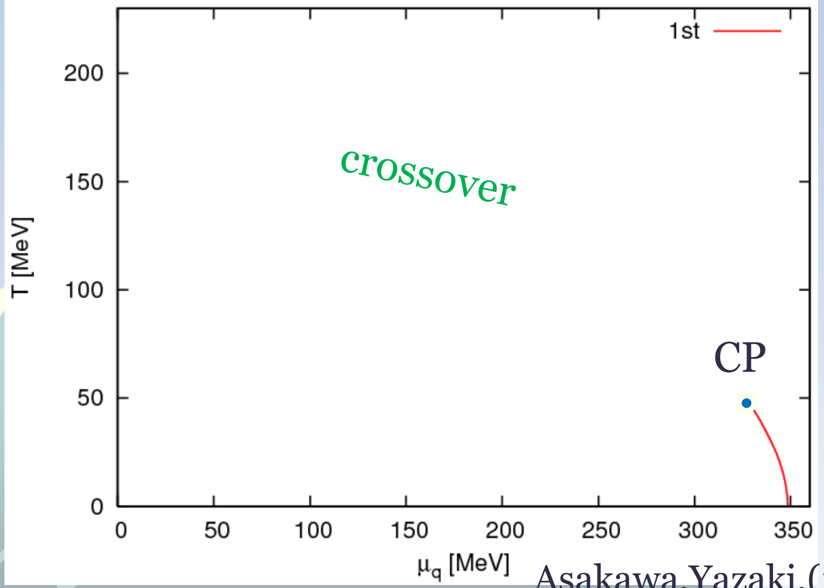
Effects of Vector Interaction and Anomaly on QCD Phase Diagram

Phase diagram in NJL model

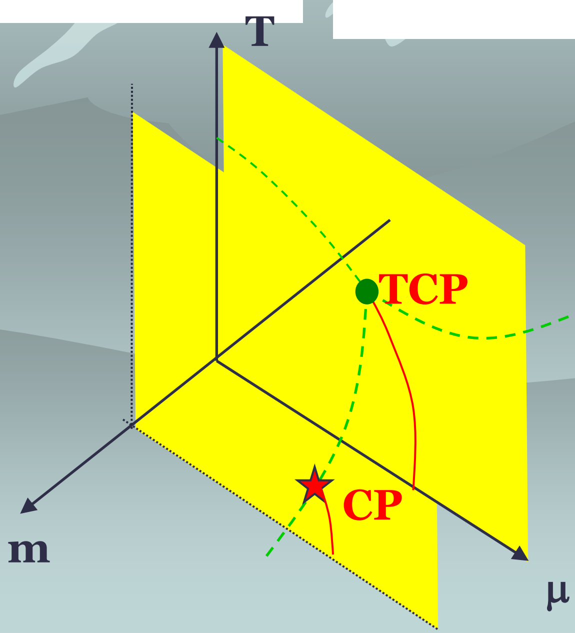
$m_{q0} = 0$



$m_{q0} = 5.5$ MeV

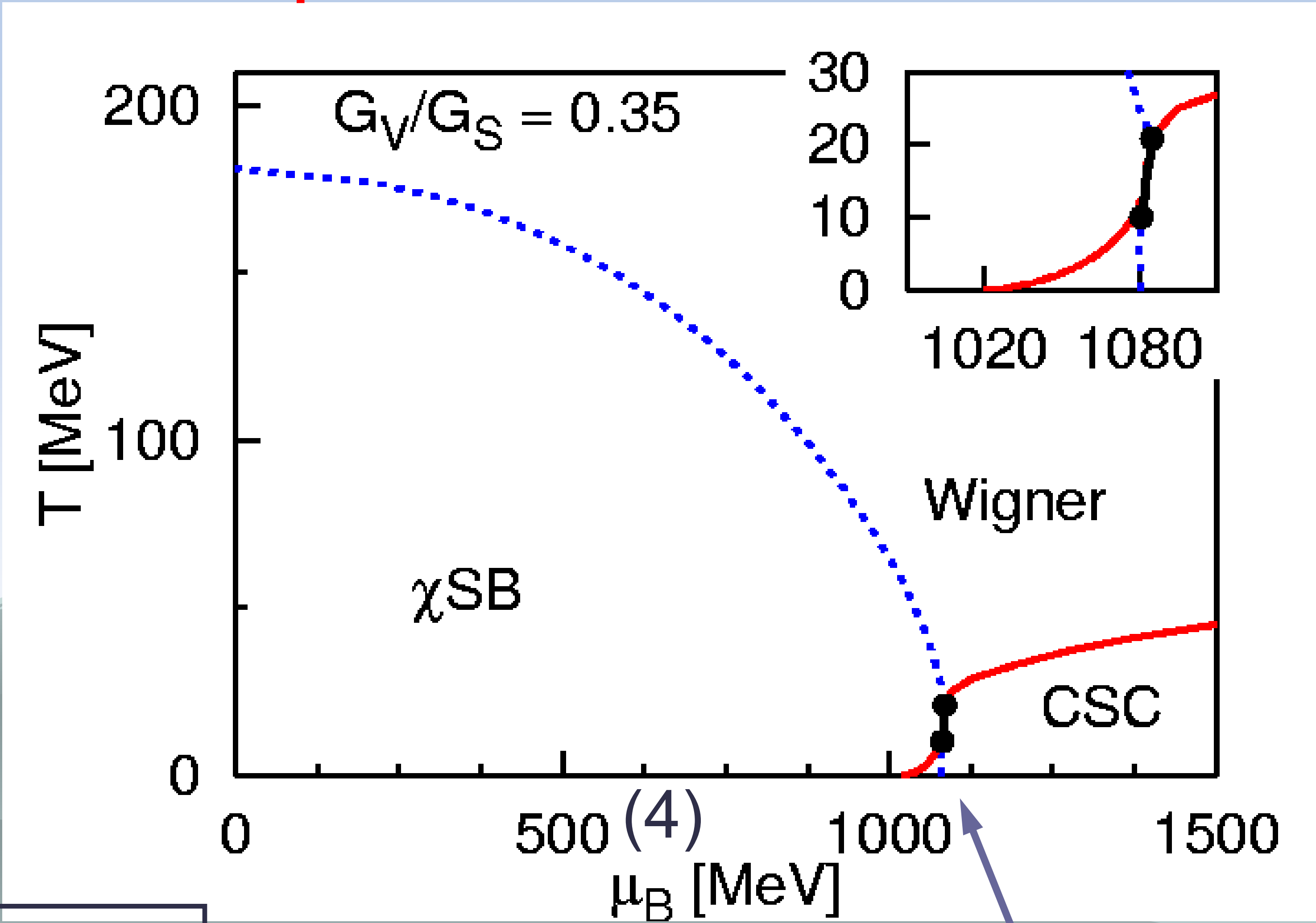


Asakawa,Yazaki,(1989)



With color superconductivity transition incorporated: Two critical end point!

M. Kitazawa, T. Koide, Y. Nemoto and T.K., PTP ('02)



(4)

$G_V / G_S = 0.35$

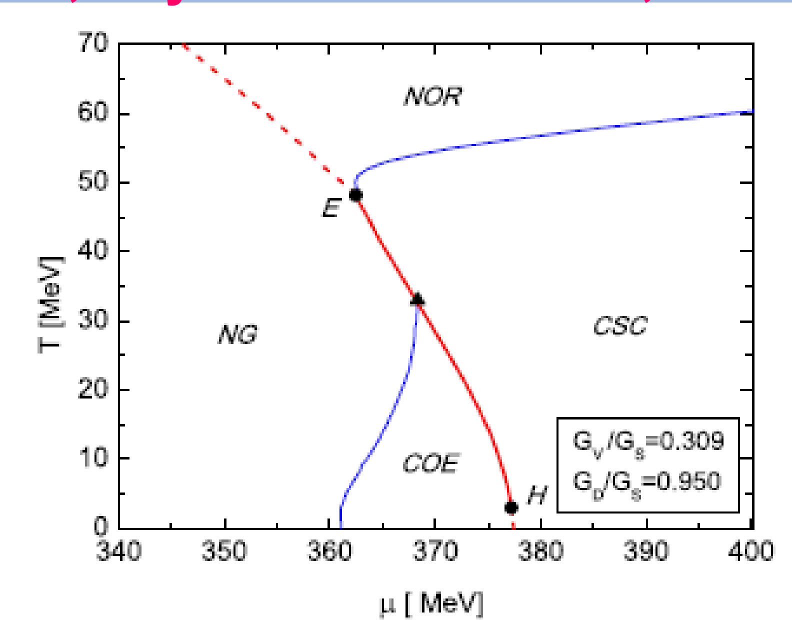
Another end point appears from lower temperature, and hence **there can exist <u>two end points</u>** in some range of G_V!

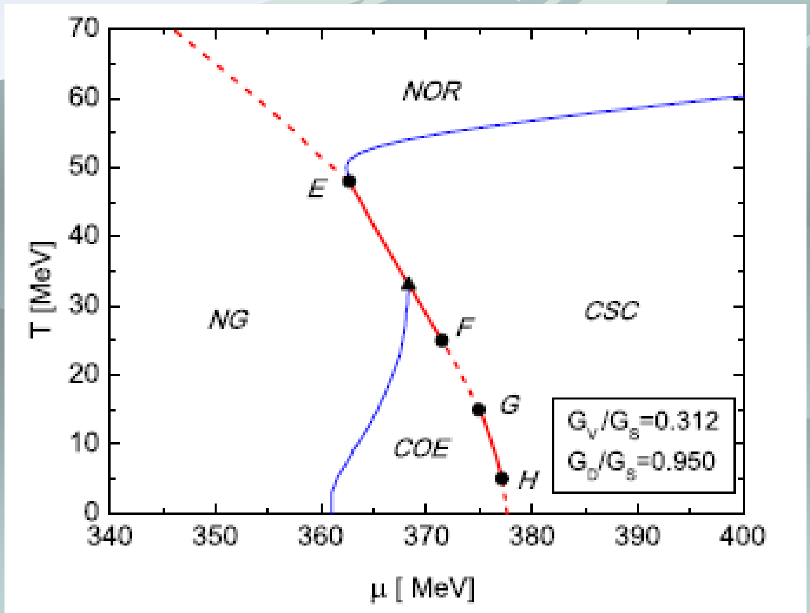
Z. Zhang and T. K., Phys.Rev.D80:014015,2009.;

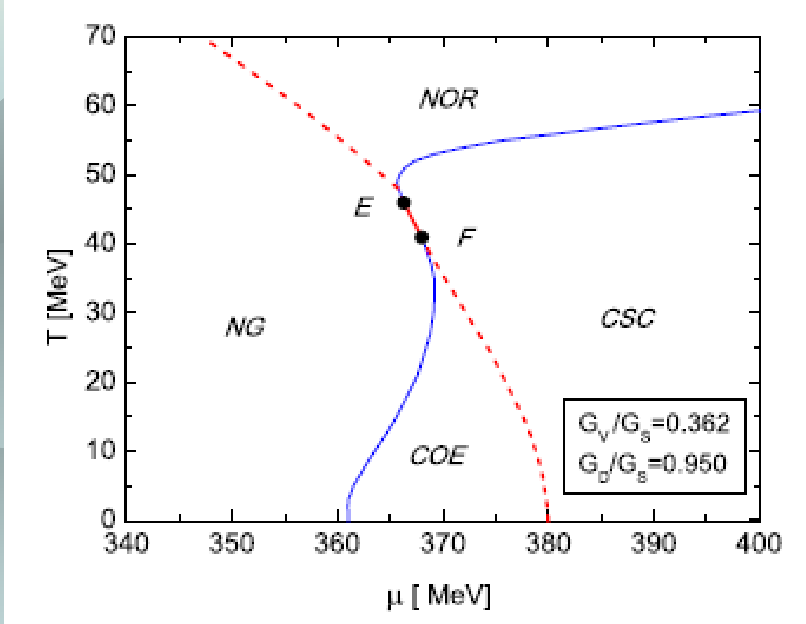
2+1 flavor case

$m_{u,d} = 5.5\text{MeV}$ $m_s = 140\text{MeV}$

Similar to the two-flavor case, with multiple critical points.







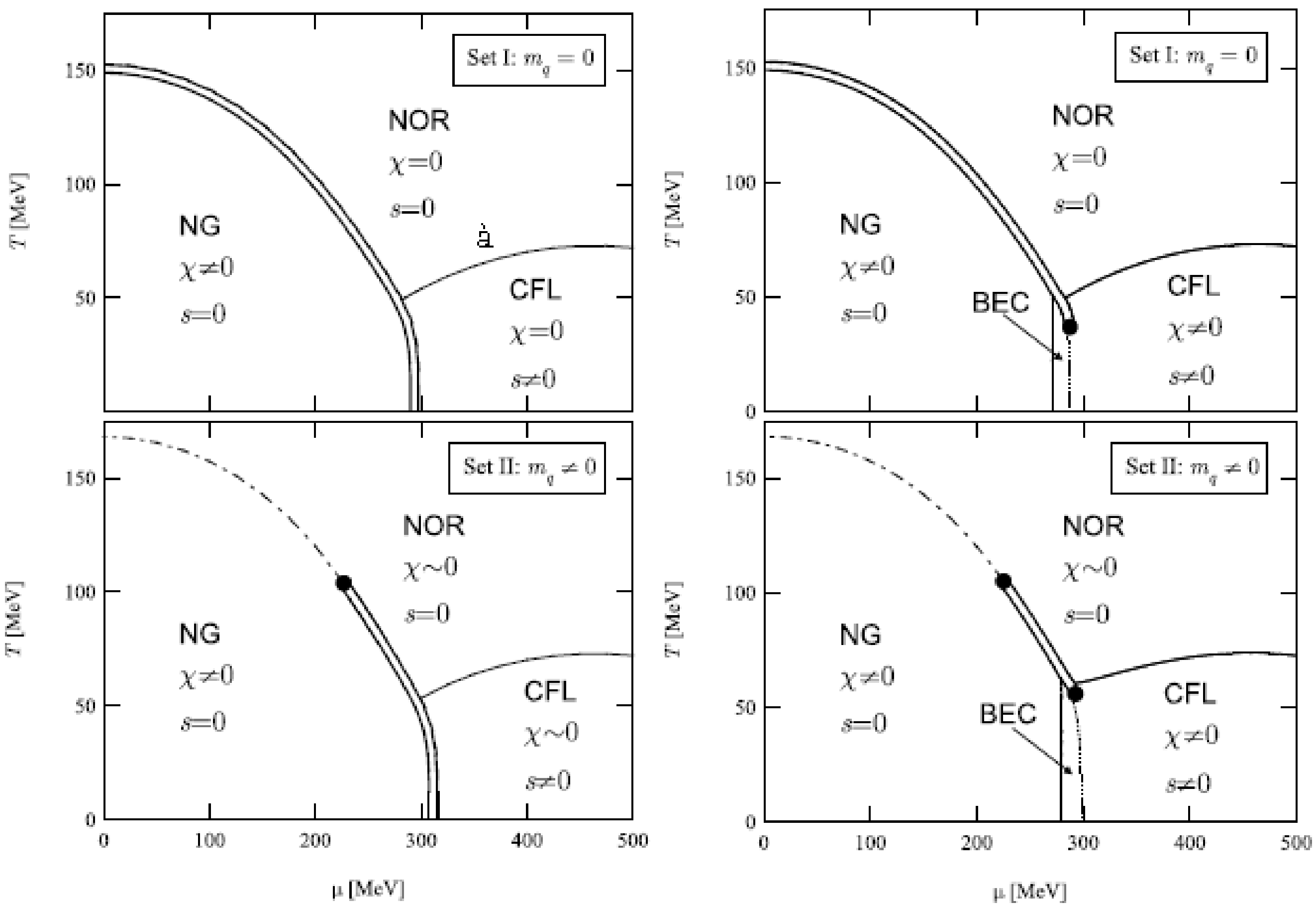
How about the anomaly-induced new critical point?

a la Hatsuda-Tachibana-Yamamoto-Baym (2006)

$$\mathscr{L}^{(6)}_{\chi d} = K' \left(\mathrm{tr}[(d_R^\dagger d_L)\phi] + \mathrm{h.c.} \right)$$

Fiertz-tr. KMT term with diquark-diquark-chiral int.

(A) In flavor-symmetric limit; Abuki et al, PRD**81** (2010)



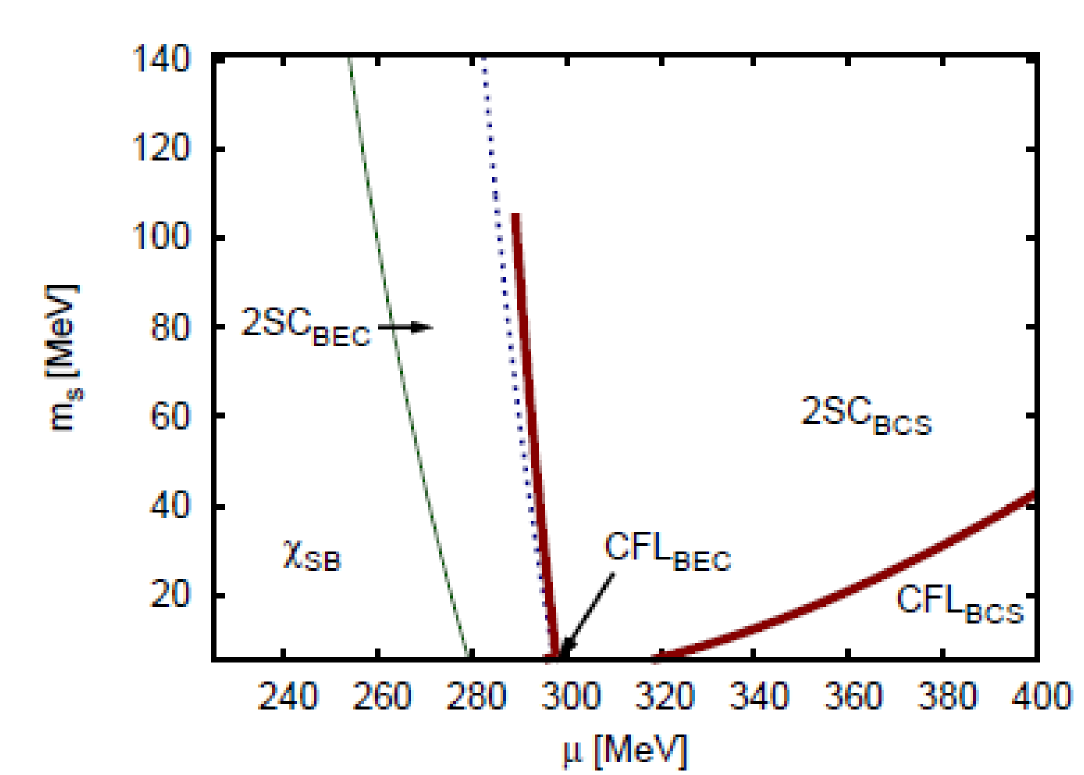
(a): $K' = 0$

(b): $K' \neq 0$

(B) Realistic case with massive strange quark;

$m_u = m_d = 5.5$ MeV << $m_s = 140.7$ MeV

H. Basler and M. Buballa, arXiv:1007.5198

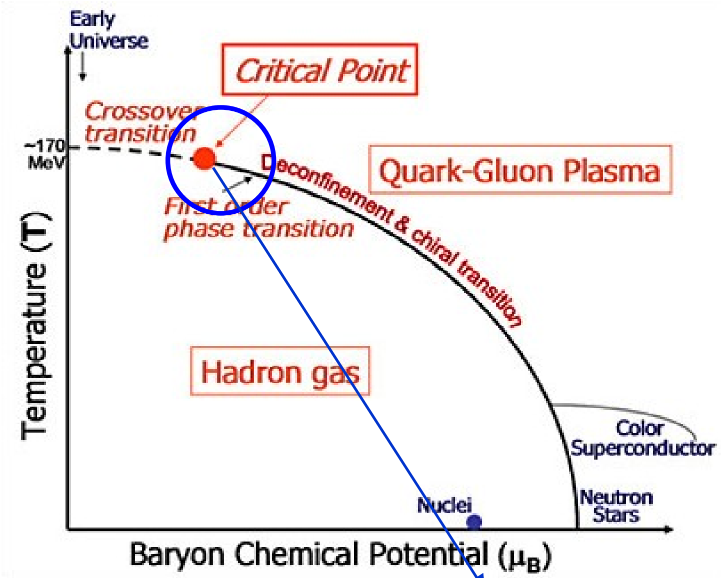


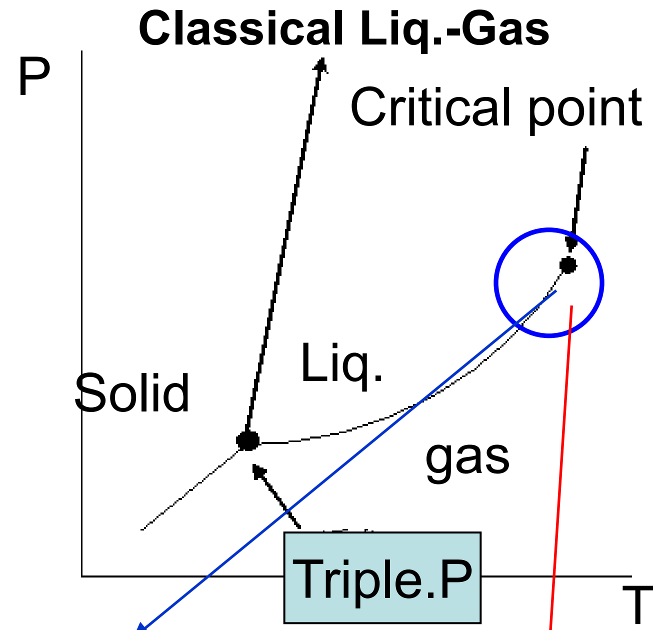
Notice!
Without charge neutrality nor vector interaction.

phases and new endpoints. On the other hand, the low-temperature critical endpoint, which was found earlier in the same model without 2SC pairing, is almost removed from the phase diagram and cannot be reached from the low-density chirally broken phase without crossing a preceding first-order phase boundary. For physical quark masses no additional critical endpoint is found.

Possible Signal of the existence of the QCD Critical Point

Plausible QCD phase diagram:





The same universality class; Z2

H. Fujii, PRD 67 (03) 094018;H. Fujii and M.Ohtani,Phys.Rev.D70(2004)
Dam. T. Son and M. A. Stephanov, PRD70 ('04) 056001

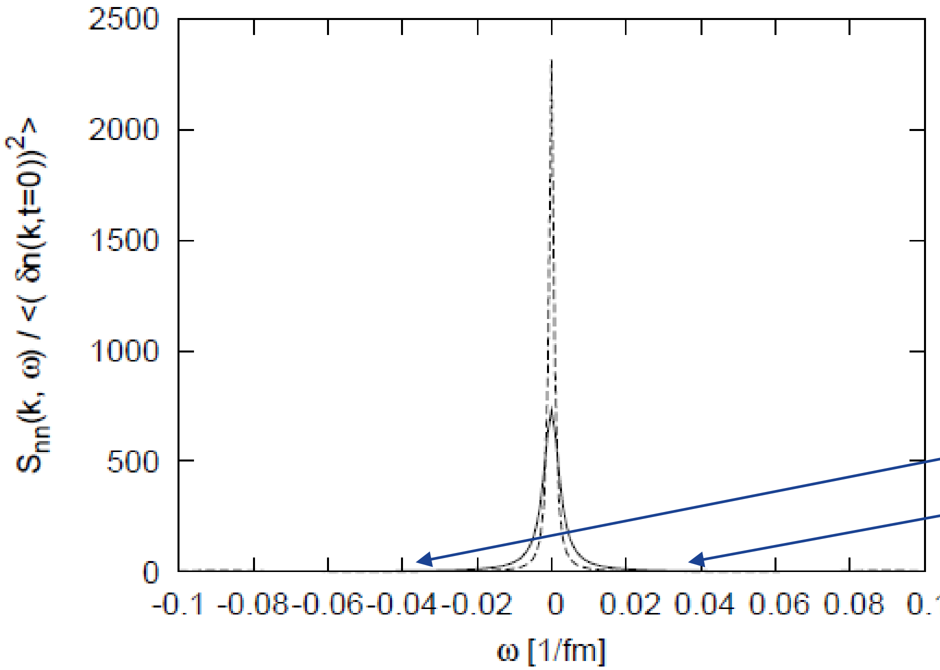
Fluctuations of conserved quantities such as the number and energy are the soft mode of QCD critical point!
The sigma mode is a slaving mode of the density.

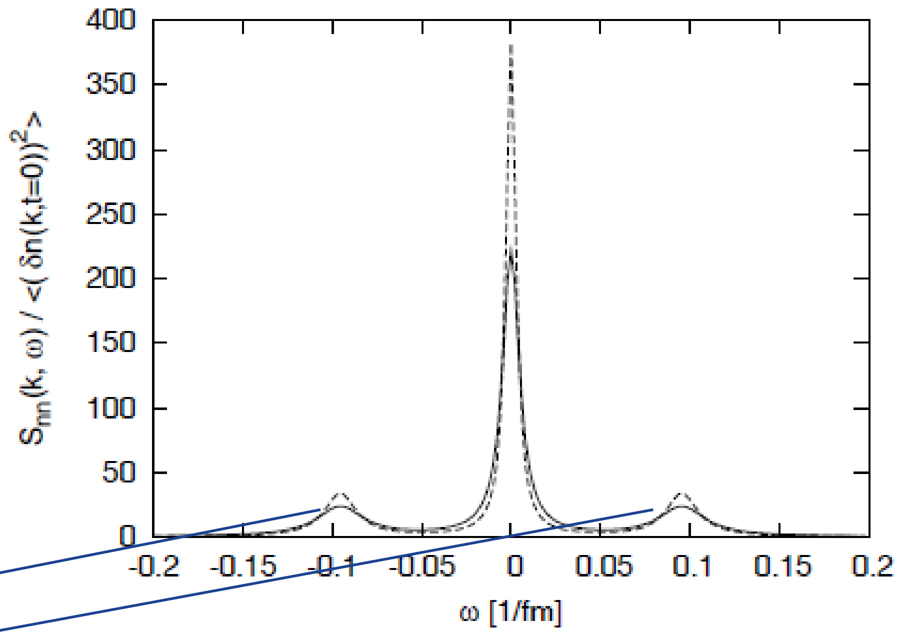
Spectral function of density fluctuation at CP

Y. Minami and T.K., (2009)

$t \equiv (T - T_c)/T_c = 0.4$

$t = 0.1$





The sound mode (Brillouin) disappears
Only an enhanced thermal mode remains.
Furthermore, the Rayleigh peak is enhanced, meaning the large energy dissipation.

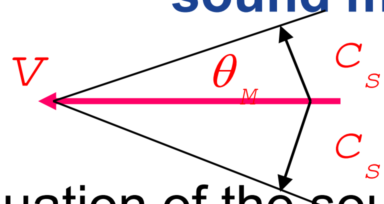
Spectral function at CP

The soft mode around QCD CP is thermally induced density fluctuations, but not the usual sound mode.

Suggesting interesting critical phenomena related to sound mode.

Possible disappearance or strong suppression of Mach cone at the QCD critical point

Mach cone ← **developed from** **sound mode as the density fluctuation**



$$\sin\theta_M = \frac{c_s}{v}$$

However,

Around the CP;

(i) Attenuation of the sound mode; the dynamical density fluctuation is hardly developed.

(ii) The enhancement of the Rayleigh peak suggests that the energy dissipation is so large that the possible density fluctuation gets dissipated rapidly.

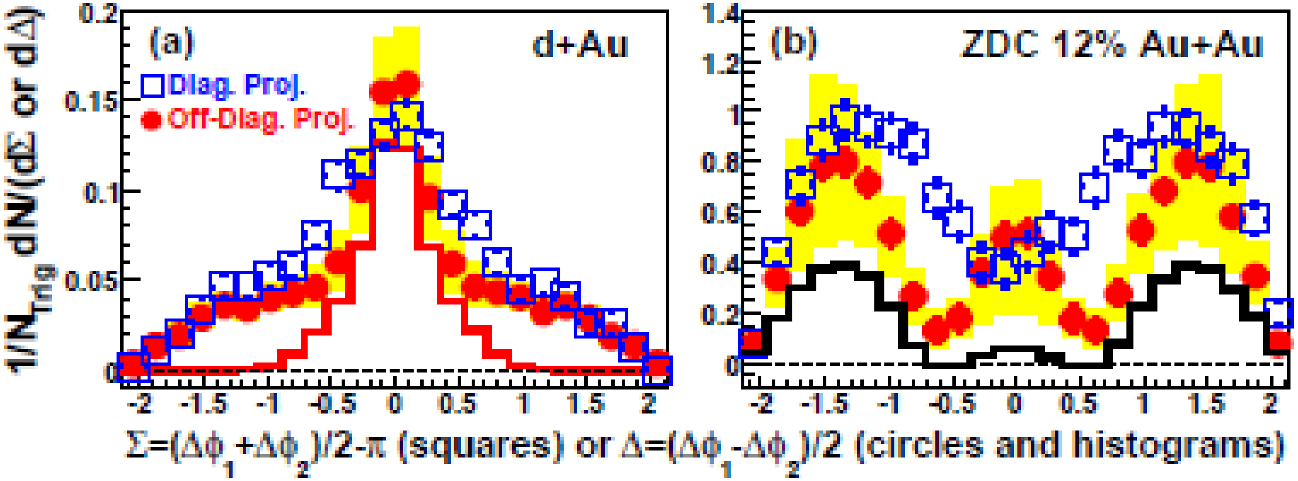
Possible disappearance or strong suppression of Mach cone at the QCD critical point!

Thus, if the identification of the Mach cone in the RHIC experiment is confirmed, possible disappearance or suppression along with the variation of the incident energy can bea signal of the existence of the critical point belonging to the same universality class as liq.-gas transition.

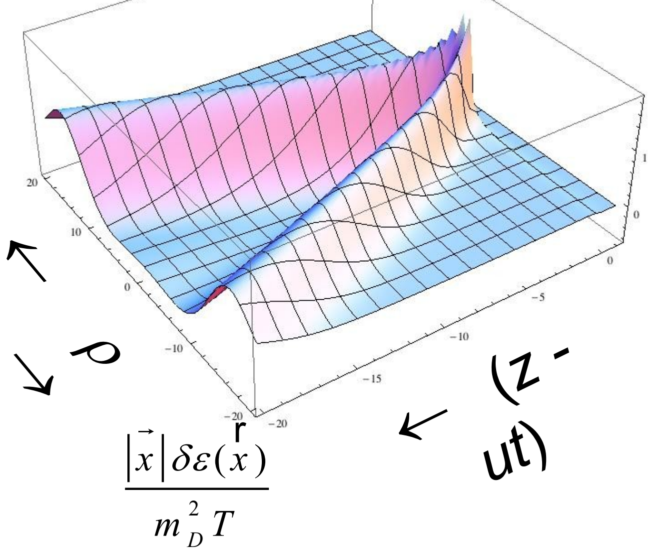
Cf. **STAR, arXiv:0805/0622.**

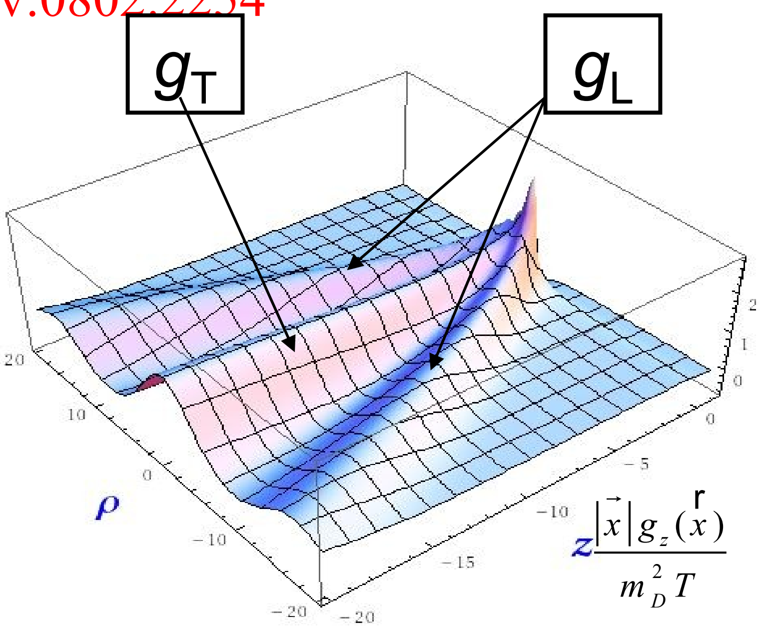
3-body correlations

Cf. the idea of Mach cone: E. Stoecker , E. Shuryak and many Others.



R. B. Neufeld, B. Muller, and J. Ruppert,arXiv:0802.2254





Summary

- Axial anomaly as well as chiral symmetry and its spontaneous breaking in QCD has been briefly reviewed: η' is massive because of the chiral anomaly and the theta vacuum of QCD.
- At finite temperature/density, instanton density gets smaller, which may lead to an effective restoration of U_A(1) symmetry.
- The soft modes of the QCD critical point (CP) are analyzed using rel. hydro. dynamics; the mechanical sound is attenuated near QCD CP, suggesting that possible Mach cone formation is suppressed around the CP, which might be an experimental signal of the existence of QCD CP.

Back UPS

$$[Q_a, Q_b] = if_{abc} Q_c$$

$$Q^a \left|0\right\rangle \neq 0 \quad \Longrightarrow \quad Q^a = X^a \qquad a = 1, 2, , , n$$

$$Q^a \left|0\right\rangle = 0 \quad \Longrightarrow \quad Q^a = Y^a \qquad a = n+1, n+2, , , N$$

$$[Y^a, Y^b] = ig_{abc} Y^c$$

$$[X^a, X^b] = ig_{abc} Y^c$$

$$[X^a, Y^b] = ig_{abc} X^c$$

Chiral-invariant operators:

$N_f = 2$ $SU_L(2) \otimes SU_R(2)$ transformation:

$L = \exp(i\boldsymbol{\theta}_L \cdot \boldsymbol{\tau}), \quad R = \exp(i\boldsymbol{\theta}_R \cdot \boldsymbol{\tau})$ $\boldsymbol{\theta} \cdot \boldsymbol{\tau} \equiv \sum_{a=1}^{3} \theta_a \tau^a$

Any function $V(\sigma^2 + \boldsymbol{\pi}^2)$ of $\sigma^2 + \boldsymbol{\pi}^2$; Invariant!

eg.1 $\mathcal{L} = 1/2 \cdot [(\partial_\mu \sigma)^2 + (\partial_\mu \boldsymbol{\pi})^2] - V(\sigma^2 + \boldsymbol{\pi}^2)$

(Linear sigma model)

eg.2 $\mathcal{L} = \bar{q} i \gamma \cdot \partial q + g/2[(\bar{q}q)^2 + (\bar{q} i \gamma_5 \boldsymbol{\tau} q)^2]$

(Nambu-Jona-Lasinio model)

$$\mathcal{L}_\sigma^{(0)} = 1/2\cdot(\mathrm{tr}\partial_\mu\Phi\partial^\mu\Phi) - 1/2\cdot\mu^2 I_1 - \lambda I_1^2 - \gamma I_2 + \tau I_D$$

I. Determination of vacuum:

$$\langle\frac{\partial\mathcal{L}_0}{\partial\Phi^\dagger}\rangle = 0$$

$$\langle\Phi\rangle \rightarrow L(\vartheta_L)\langle\Phi\rangle R^\dagger(\theta_R)$$

If $\theta_L = \theta_R$, i.e., $SU_V(3)$,

$\langle\Phi\rangle$ is invariant, but otherwise not..

$$2(3\lambda+\gamma)\varphi_0^2 - \tau\varphi_0 + \mu^2/2 = 0$$

$$\therefore \quad \varphi_0 = \frac{\tau + \sqrt{\tau^2 - 4\mu^2(3\lambda+\gamma)}}{4(3\lambda+\gamma)}$$

for $\mu^2 < 0$

2. Meson spectra:

$$\Phi = \varphi_0 \mathbf{1} + \Phi', \quad \Phi' = \frac{1}{\sqrt{2}}(S + iP)$$

$$S = \sum_{a=0}^{8} S_a \lambda_a \qquad P = \sum_{a=0}^{8} P_a \lambda_a$$

Meson masses;

(1) ps-mesons

π, K, η_8

$$m^{(8)2}_{\mathrm{ps}} = \mu^2 + 4\varphi_0^2(3\lambda + \gamma) - 2\varphi_0\tau = 0$$

$$\langle \frac{\partial \mathcal{L}_0}{\partial \Phi^\dagger} \rangle = 0$$

η_1

$$m^{(0)2}_{\mathrm{ps}} = 6\tau\varphi_0 \neq 0$$

Anomaly term

(2) scalar-mesons

$$m^{(8)2}_{\mathrm{s}} = \mu^2 + 12\varphi_0^2(\lambda + \gamma) + 2\varphi_0\tau,$$
$$m^{(0)2}_{\mathrm{s}} = 2(\mu^2 - \varphi_0\tau),$$

$$\mathcal{L}_{\rm SB} = -{\rm tr}(\epsilon_0\lambda_0 + \epsilon_8\lambda_8)(\Phi + h.c.)/2\sqrt{2}$$

Def. $c = (\sqrt{2}\epsilon_0 + \epsilon_8)/2\sqrt{6}, \quad d = -\frac{1}{2}\sqrt{\frac{3}{2}}\ \varepsilon_8$

The vacuum: $\langle\Phi\rangle = {\rm diag}(\varphi_0, \varphi_0, \varphi_0 + \varphi_1)$

The masses of NG bosons

$$m_\pi^2 = -\frac{2c}{\varphi_0}, \quad m_K^2 = -\frac{2(2c+d)}{2\varphi_0 + \varphi_1}$$

Finite owing to the SB term!

η_8 and η_0 are mixed to form η and η' .

$$\partial^\mu j^\pi_{5\mu} = -2\sqrt{2}c\phi_\pi \equiv m_\pi^2 f_\pi \phi_\pi,$$
$$\partial^\mu j^K_{5\mu} = -2\sqrt{2}(c + d/2)\phi_K \equiv m_K^2 f_K \phi_K$$

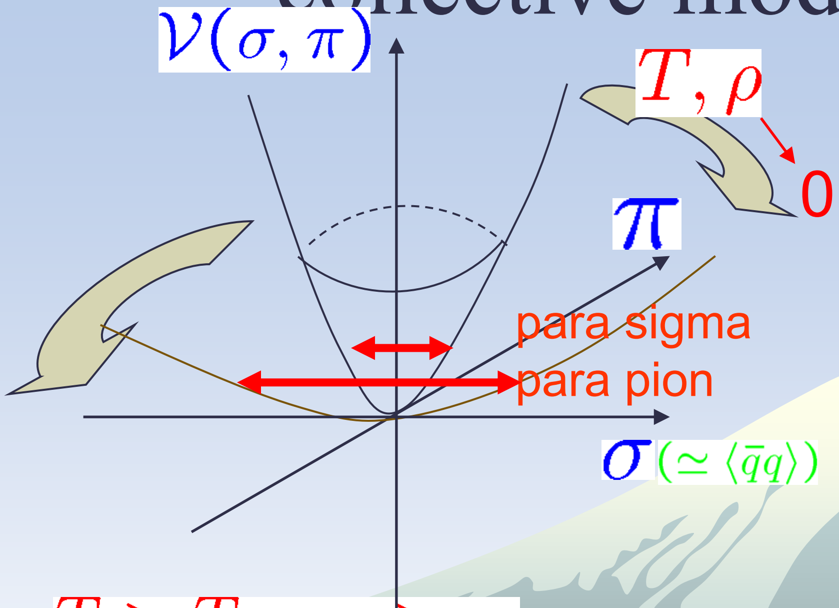
with

$$f_\pi = \sqrt{2}\varphi_0, \quad f_K = \sqrt{2}(\varphi_0 + \frac{1}{2}\varphi_1)$$

$$f_K/f_\pi = 1 + \frac{\varphi_1}{2\varphi_0} > 1$$

(= 1.25 (empirical))

Chiral Transition and the collective modes



$T > T_c \quad \rho > \rho_c$

(pairing vib.)

(pairing rotation)

$T < T_c \quad \rho < \rho_c$

The low mass sigma in vacuum is now established:

pi-pi scattering; Colangero, Gasser, Leutwyler('06) and many others

Full lattice QCD ; SCALAR collaboration ('03)

q-qbar, tetra quark, glue balls, or their mixed st's?

c.f. The sigma as the Higgs particle in QCD

$$\sigma = \sigma_0 + \tilde{\sigma}$$

ϕ ; Higgs field $\Rightarrow$ $\phi = \langle\phi\rangle + \tilde{\phi}$

Higgs particle

η'(958) mesic nuclei by (π,N) reaction

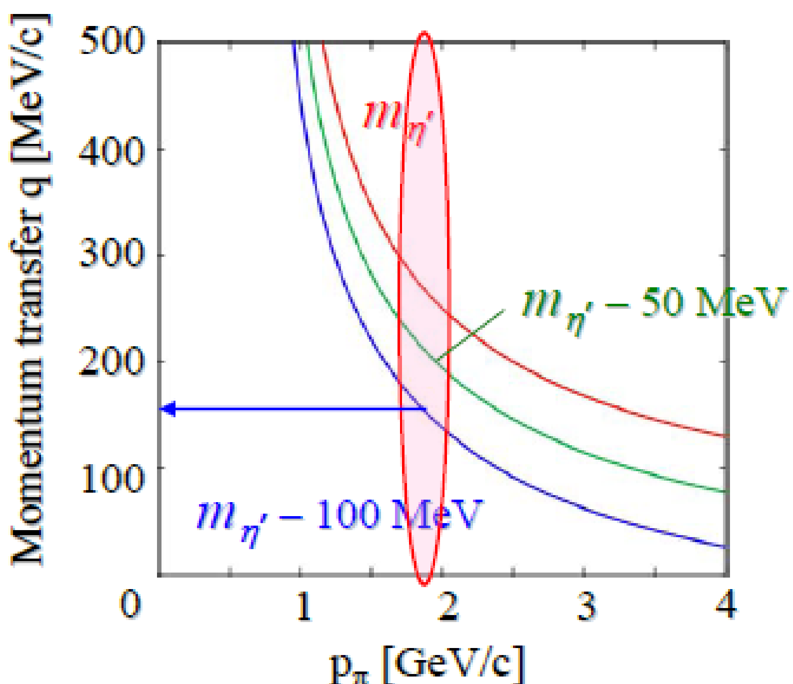
- Potential description [Energy independent]

Real Part V_0 … evaluated by possible η' mass shift at ρ_0

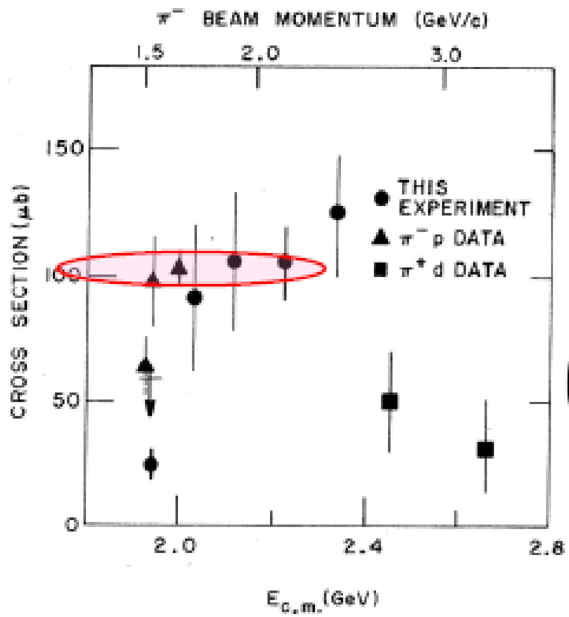
$$\Delta m(\rho) \to V(\rho(r)) = V_0 \frac{\rho(r)}{\rho_0}$$

Imaginary part W_0 … *unknown* → 20 MeV, *for example*

momentum transfer



elementary cross section $\pi^+ n \to \eta' p$

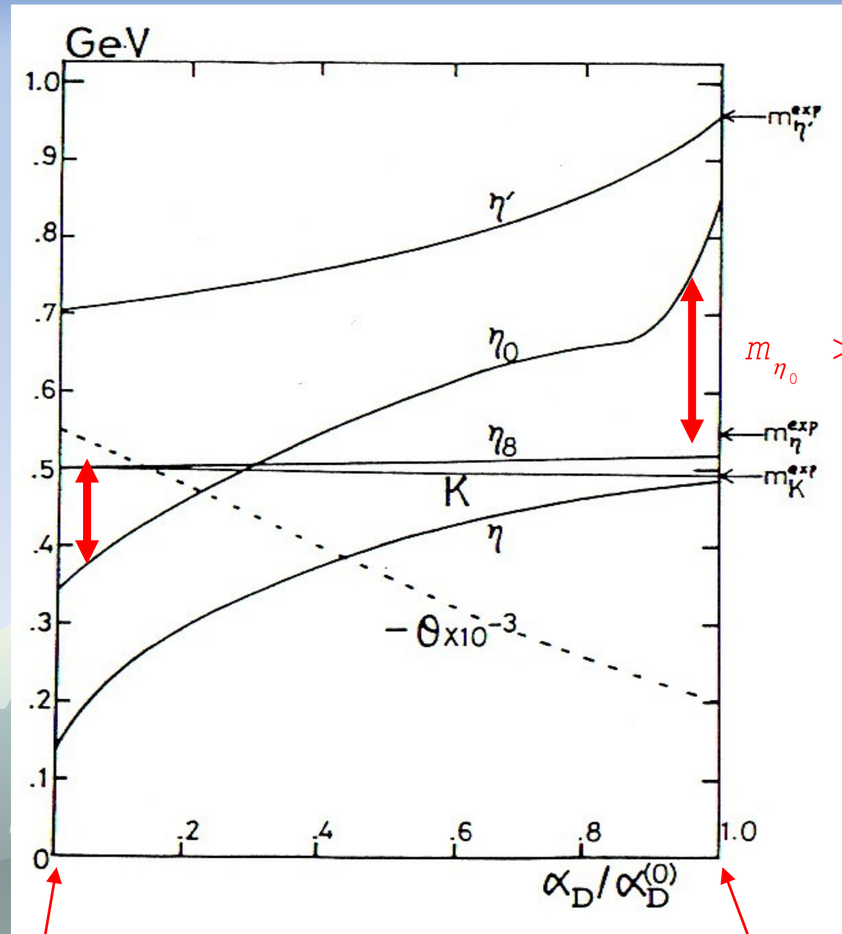


R.K.Rader *et al.*, PRD6(72)3059

$\sigma(\pi^+ n \to \eta' p) \sim 100$ μb

$$\left(\frac{d\sigma}{d\Omega}\right)^{Lab.} = 10 \sim 100\mu\text{b/sr}$$

$m_{\eta_0} < m_{\eta_8}$



$m_{\eta_0} > m_{\eta_8}$

without anomaly

with anomaly

$$\alpha_D = \frac{2 g_D N_c^2 \Lambda^5}{\pi^4}$$

Determination of the parameters:

$\Lambda = 631.4\mathrm{MeV}, \quad g_S \Lambda^2 = 3.666, \quad g_D \Lambda^5 = -9.288, \quad m_s = 135.7\,\mathrm{MeV}$

$m_u = m_d = 5.5$ MeV

$m_\pi = 138\mathrm{MeV}$, $f_\pi = 93\mathrm{MeV}$, $m_K = 495.7\mathrm{MeV}$, and $m_{\eta'} = 957.5\mathrm{MeV}$

Dynamical Chiral Symmetry Breaking and the sigma meson

Y. Nambu, **117** (1960), 648; Gauge invariance in Superconductivity → Appearance of a collective mode in the broken phase coupling to the longitudinal part of the current.

Gauge invariance

(Bogoliubov-Anderson)

Y. Nambu, PRL **4** (1960), 380;

Axial gauge (chiral) symm.

Y. Nambu and G. Jona-Lasinio, **122** (1960), 345;
Dynamical model of elementary particles based on an analogy with superconductivity.

The pion ; a (massless) collective mode associated with the dynamical breaking of chiral symmetry.
A scalar meson with the mass $2m_f$ appears as another collective mode than the pion. The sigma is a Higgs in QCD.

$$m_f \approx 300\mathrm{MeV} \rightarrow m_\sigma = 2m_f \approx 600\mathrm{MeV}$$

The feature essentially does not change with U_A(1) anomaly term incorporated;
T. Hatsuda and T.K.. Phys.Lett.B206 (1988), Z. Phy. C51 (1991)

Possible Chiral Restoration Phenomena in Other Channels

- Vector-axial vector degeneracy

$$\langle S(x)S(y)\rangle \to \langle P^a(x)P^a(y)\rangle, \quad \langle A^a_\mu(x)A^b_\nu(y)\rangle \to \langle V^a_\mu(x)V^b_\nu(y)\rangle$$

As chiral symmetry is restored,

- hadron realization of chiral symmetry; vector manifestation
- Chiral symmetry in Baryon sector;

parity doubling? What is the nature of N*(1535)?

C. DeTar and T.K. (1989)

g_A of N*; small

c.f. Lattice cal.
T.T. Takahashi and T.K.,
PRD78 (2008), 011503

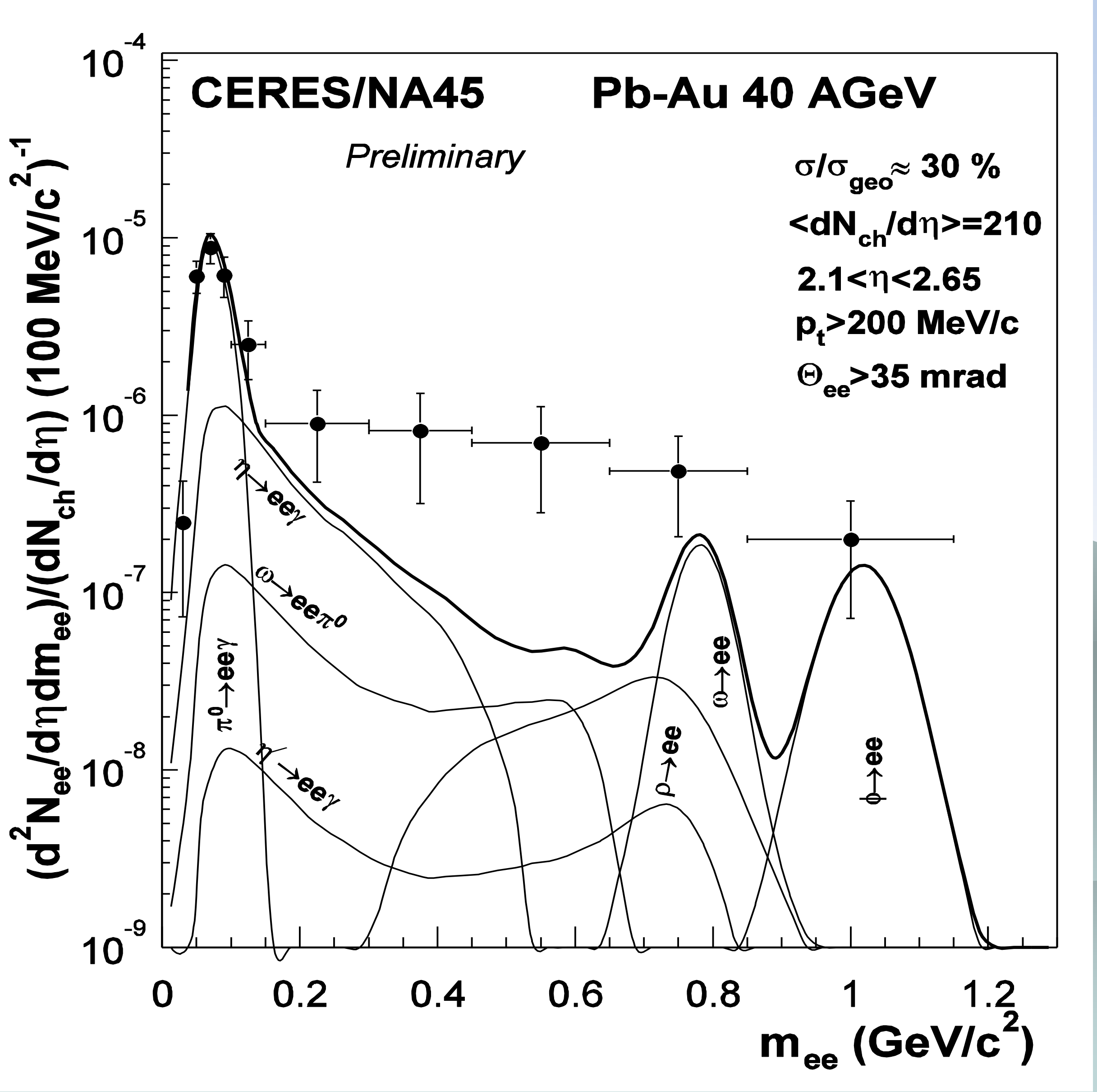
Softening of the spectral function in the Vector channel *or Scalar channel?*

$\mu \neq 0$

↓

scalar-vector mixing

T.K. (1991)



Combined effect of Vector Interaction and Charge Neutrality constraint

Z. Zhang and T. K., **Phys.Rev.D80:014015,2009**.;

$$\mathcal{L} = \bar{\psi}\,(i\partial\!\!\!/ - \hat{m}\,)\psi + G_S\left[(\bar{q}(x)q(x))^2 + (\bar{q}(x)i\gamma_5\vec{\tau}q(x))^2\right]$$

chiral

$$+\; G_D \sum_A [\bar{q}(x)\gamma_5\tau_2\lambda_A q_C(x)]\,[\bar{q}_C(x)\gamma_5\tau_2\lambda_A q(x)]$$

di-quark

$$-\, G_V \sum_{i=0}^{3}\left[(\bar{q}(x)\gamma^\mu\tau_i q(x))^2 + (\bar{q}(x)i\gamma^\mu\gamma_5\tau_i q(x))^2\right]$$

vector

$$-\, K\left\{\det_f\left[\bar{\psi}\,(1+\gamma_5)\,\psi\right] + \det_f\left[\bar{\psi}\,(1-\gamma_5)\,\psi\right]\right\}$$

anomaly

diquark-chiral density coupl.

Fierts tr.

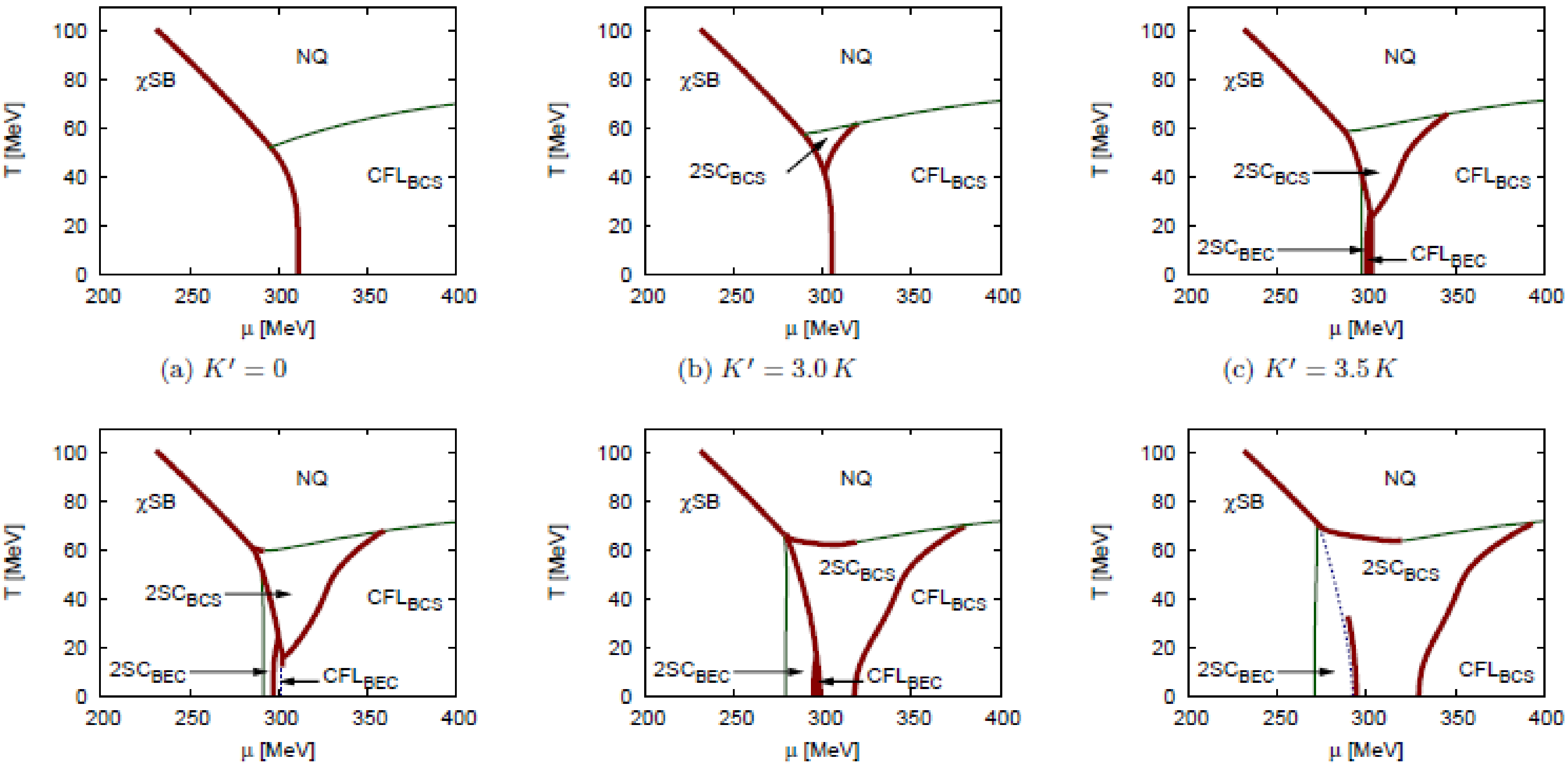
for 2+1 flavors

Kobayashi-Maskawa('70); 't Hooft ('76)

(A') Role of 2SC in 3-flavor quark matter

H. Basler and M. Buballa, arXiv:1007.5198

with equal bare quark masses of ($m_u = m_d = m_s = 5.5$ MeV).



(a) $K' = 0$ (b) $K' = 3.0\,K$ (c) $K' = 3.5\,K$ (d) $K' = 3.744\,K$ (e) $K' = 4.2\,K$ (f) $K' = 4.5\,K$

What is the soft mode at CP?

Sigma meson has still a non-zero mass at CP.
This is because the chiral symmetry is explicitly broken.

What is the soft mode at CP?

At finite density, scalar-vector mixing is present.

Phonon mode in the space-like region softens at CP.

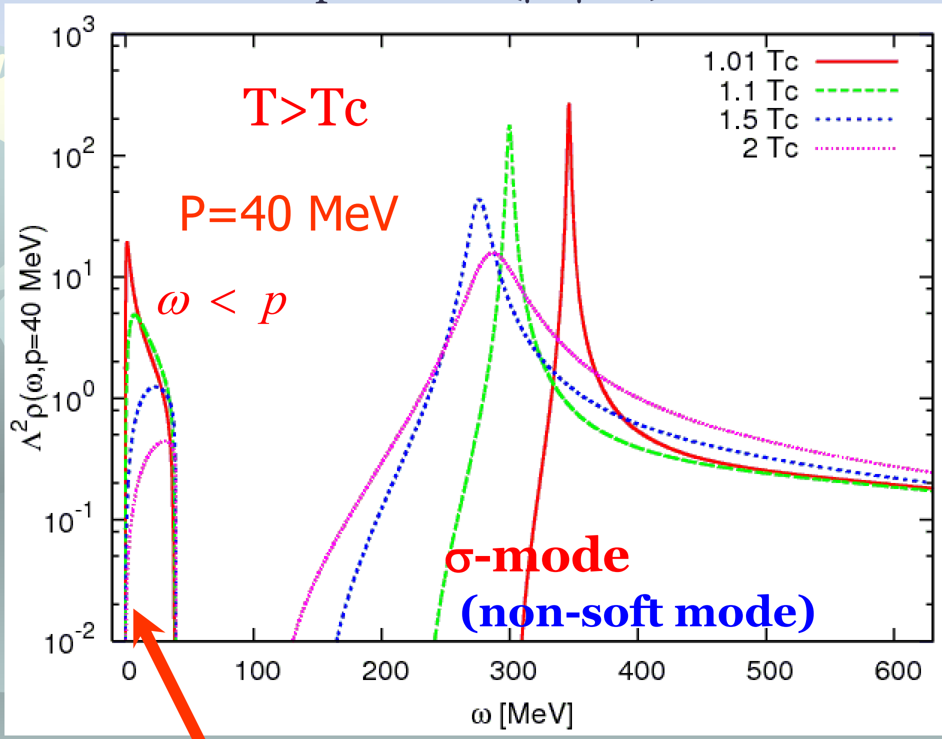
H. Fujii (2003)
H. Fujii and M.Ohtani(2004)

See also, D. T. Son and M. Stephanov (2004)

does not affect particle creation in the time-like region.

It couples to hydrodynamical modes,
leading to interesting dynamical critical phenomena.

Spectral function of the chiral condensate
T-dependence ($\mu=\mu_{CP}$)



Space-like region $\omega < p$

(the soft modes)

Spectral function of density fluctuations

Y. Minami and T.K., Prog. Theor. Phys.122, (2009),881

In the long-wave length limit, k→0

thermal mode — sound modes

$$\frac{S_{nn}(\vec{k},\omega)}{<(\delta n(\vec{k},t=0))^2>} = (1-\frac{1}{\gamma})\frac{2\Gamma_R k^2}{\omega^2+\Gamma_R^2 k^4} + \frac{1}{\gamma}\left(\frac{\Gamma_B k^2}{(\omega+c_s k)^2+\Gamma_B^2 k^4} + \frac{\Gamma_B k^2}{(\omega-c_s k)^2+\Gamma_B^2 k^4}\right)$$

Rel. effects appear only in the width of the peaks.

Rel. effects appear only in the sound mode.

$$\Gamma_R = \chi \qquad \Gamma_B = \Gamma + \frac{1}{2}c_s^2 T_0(\kappa/w_0 - 2\chi\alpha_P)$$

$$\Gamma = \frac{1}{2}[\chi(\gamma-1)+\nu_l]$$

α_P rate of isothermal exp.

thermal expansion rate: $\chi = \frac{\kappa}{n_0 C_P}$

c_s : sound velocity γ : specific heat ratio Long. Dynamical : $\nu_l = (\zeta + \frac{4}{3}\eta)/w_0$

enthalpy

Notice: $\gamma = c_p / c_n = t^{-\gamma+\alpha} \rightarrow \infty$

As approaching the critical point, the ratio of specific heats diverges!

The strength of the sound modes vanishes out at the critical point.