Standard Model and open problems

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Disclaimer

Many lectures already about the Standard Model (SM) an extensions.

So I will focus mostly on discussing **why the SM is the way it is.**

Only 3 hours of lectures: there will be many **gaps, that you will have to fill** yourself.

Exercises thought to bring questions that you might not have considered before. Very worth trying all them!

I will be **very happy to discuss** not only during the lectures, but also outside.

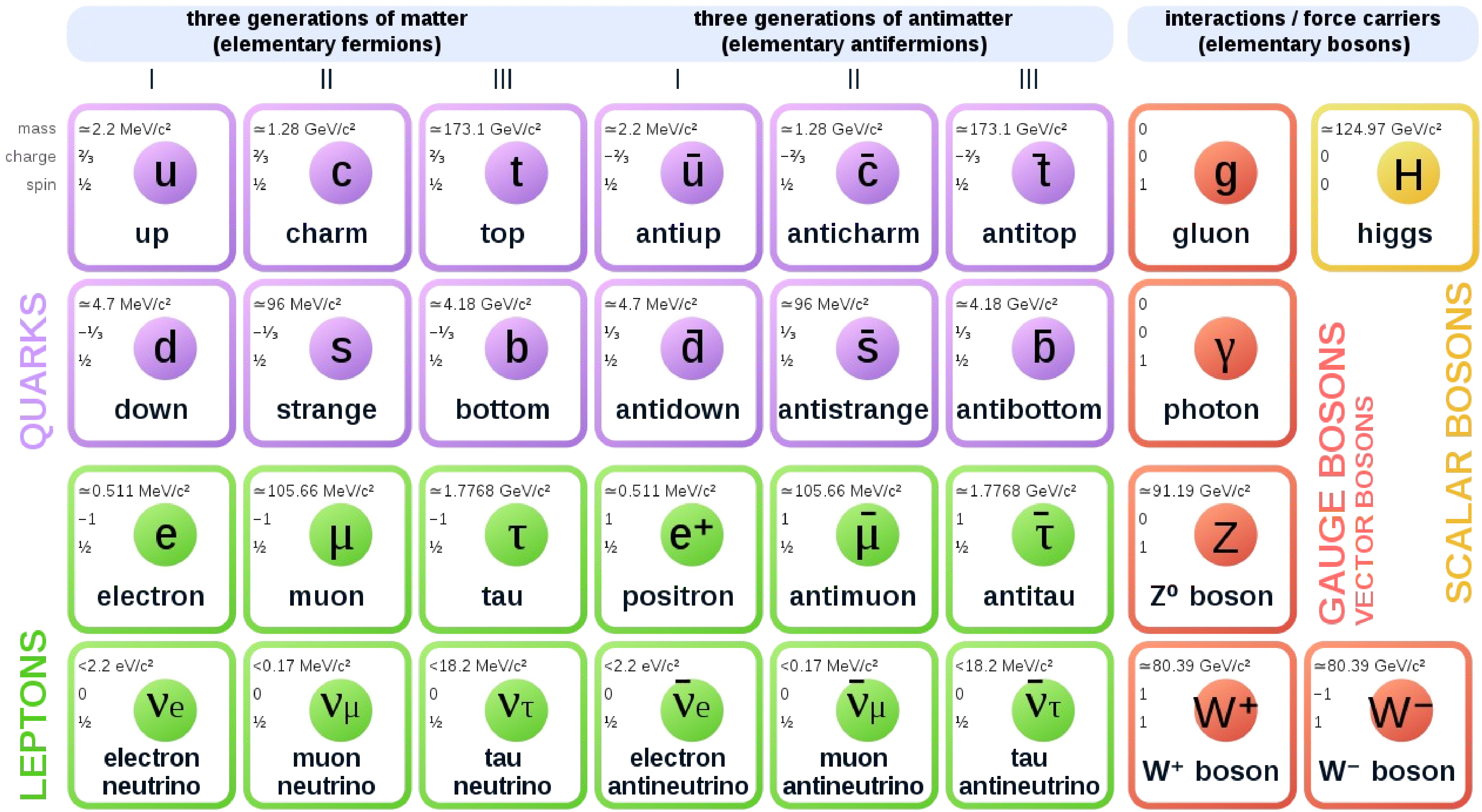
References

J. I. Illana, "*The Standard Model*", online.

M. D. Schwartz, "*Quantum field theory and the Standard Model*", Cambrigde University Press

D. M. Straub, "*Flavour Physics: Theory*", online.

Standard Model of Elementary Particles



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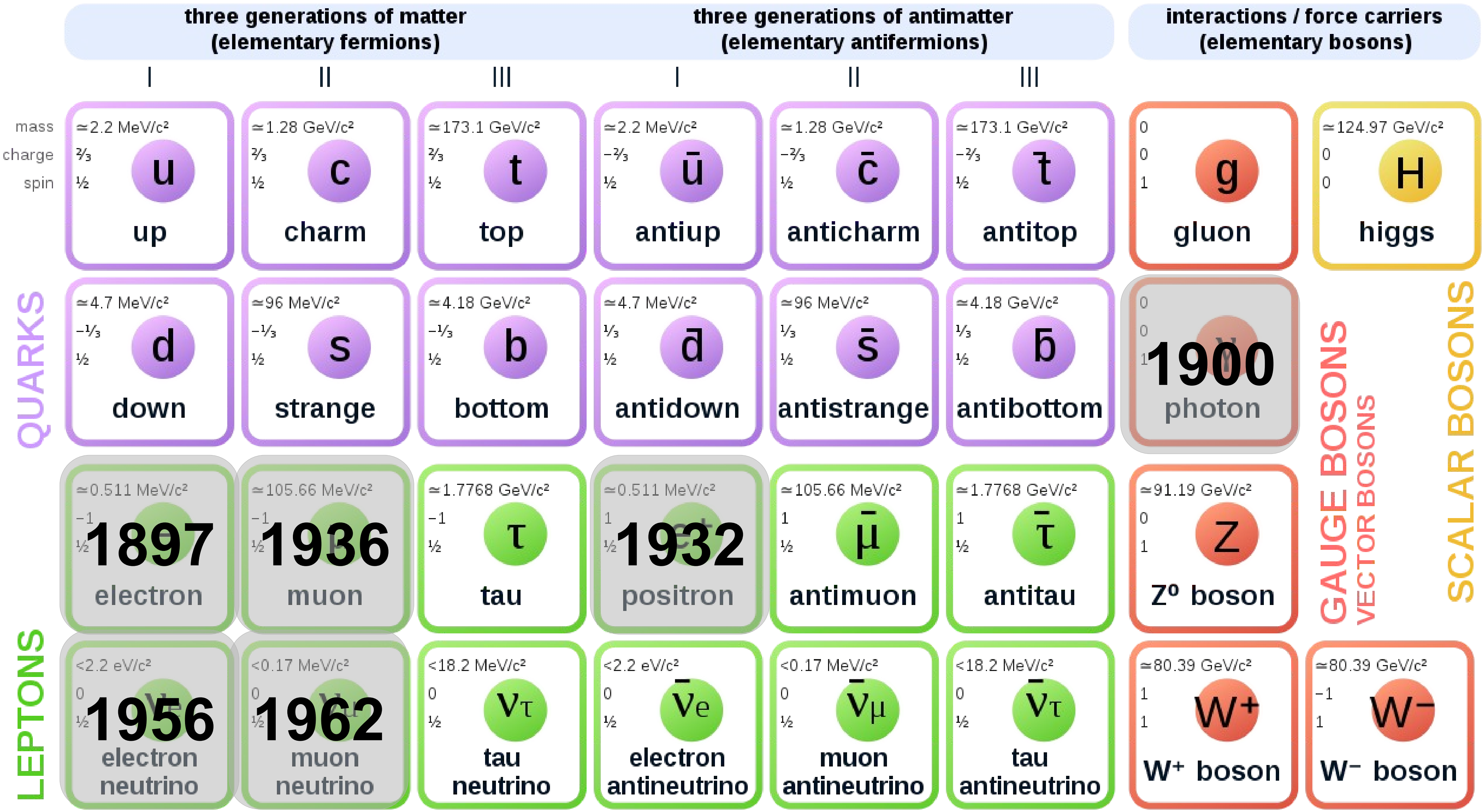

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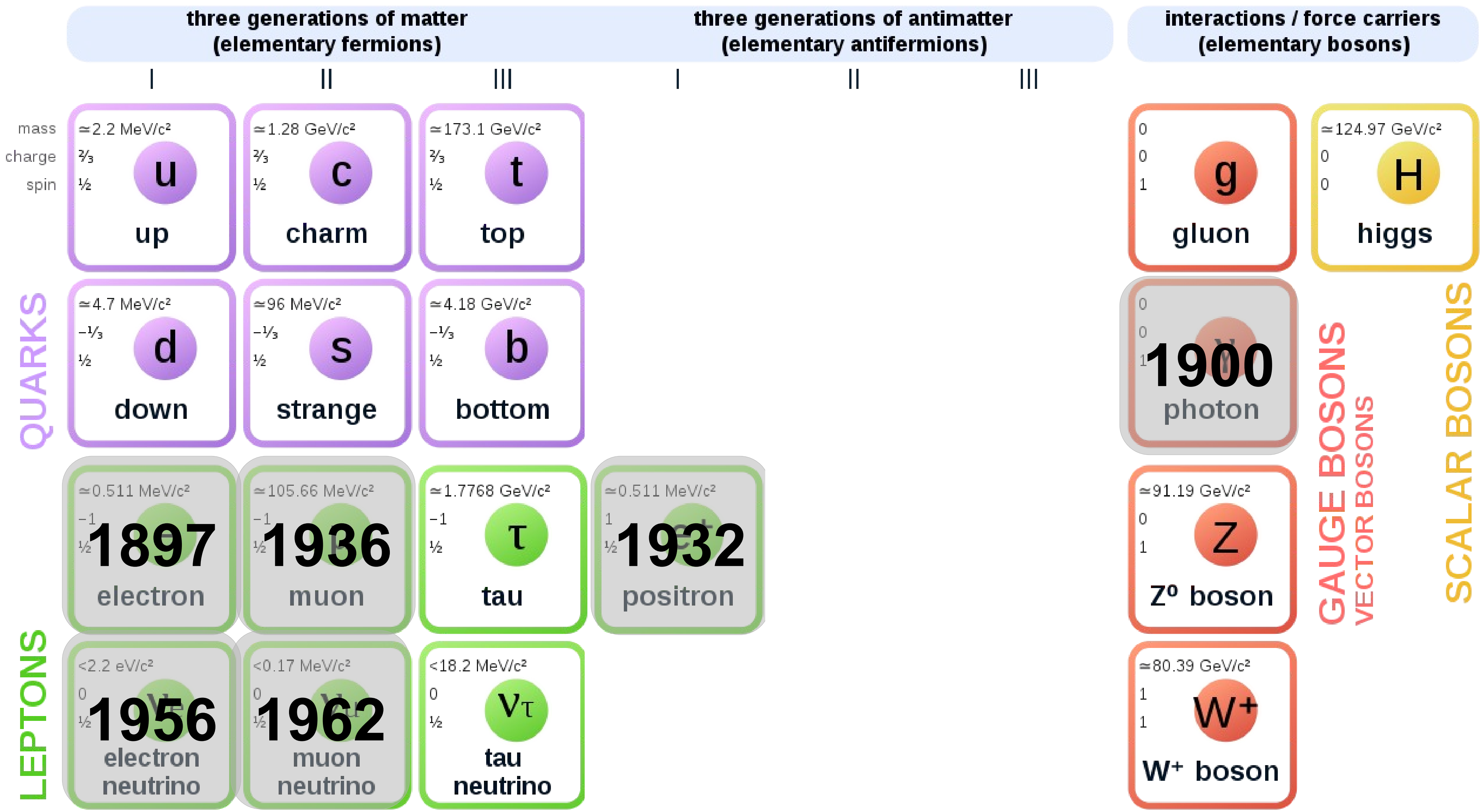

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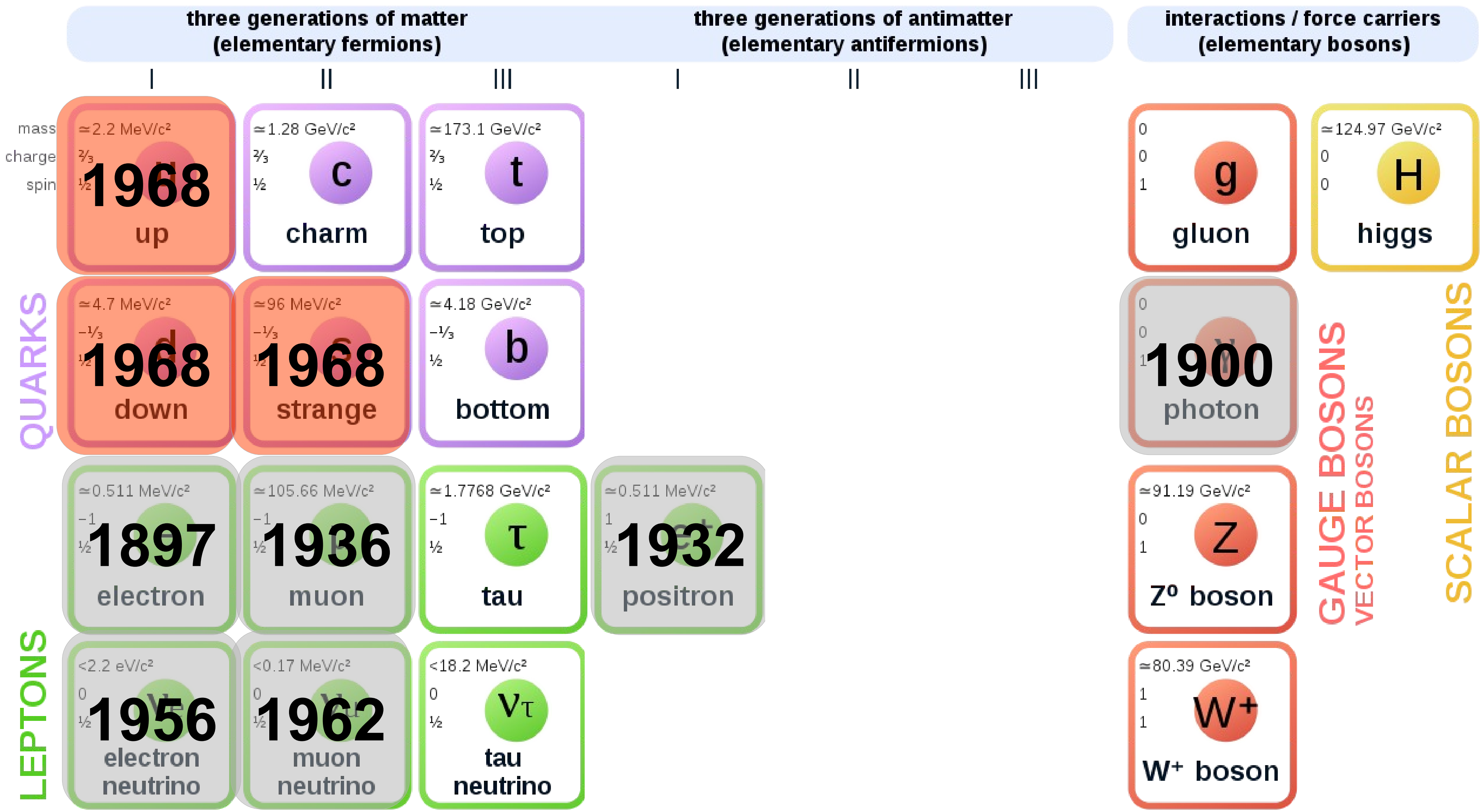

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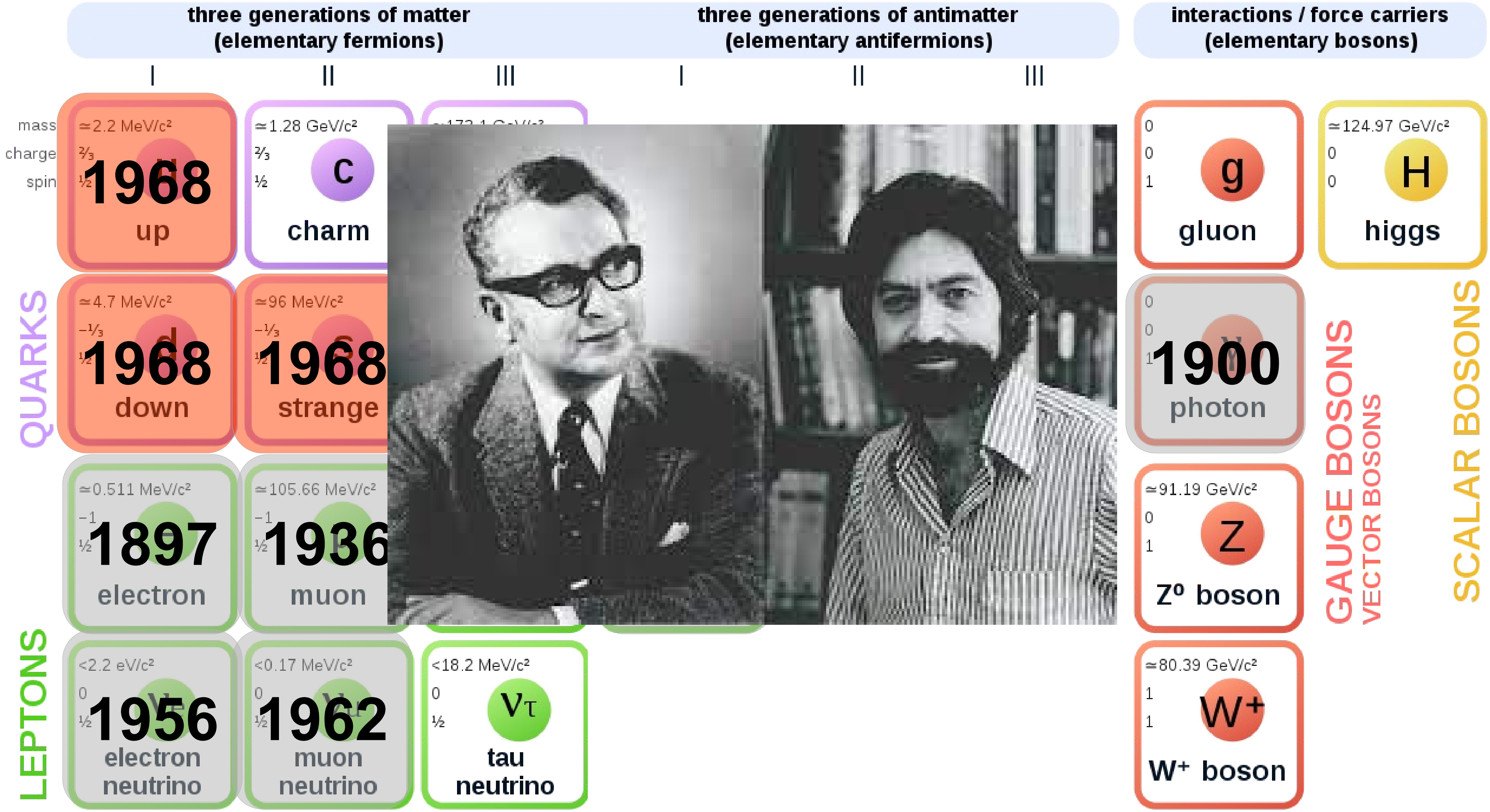

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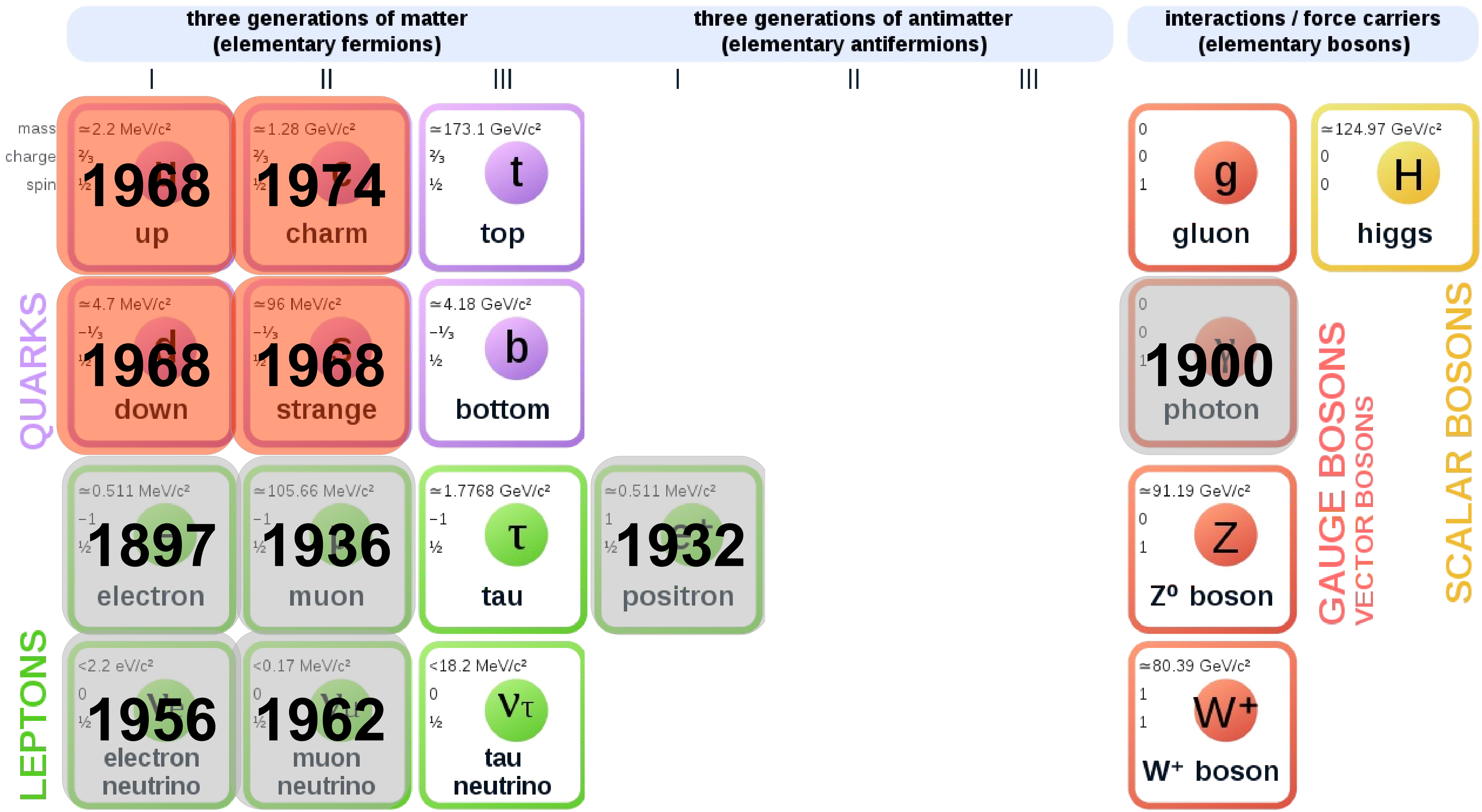

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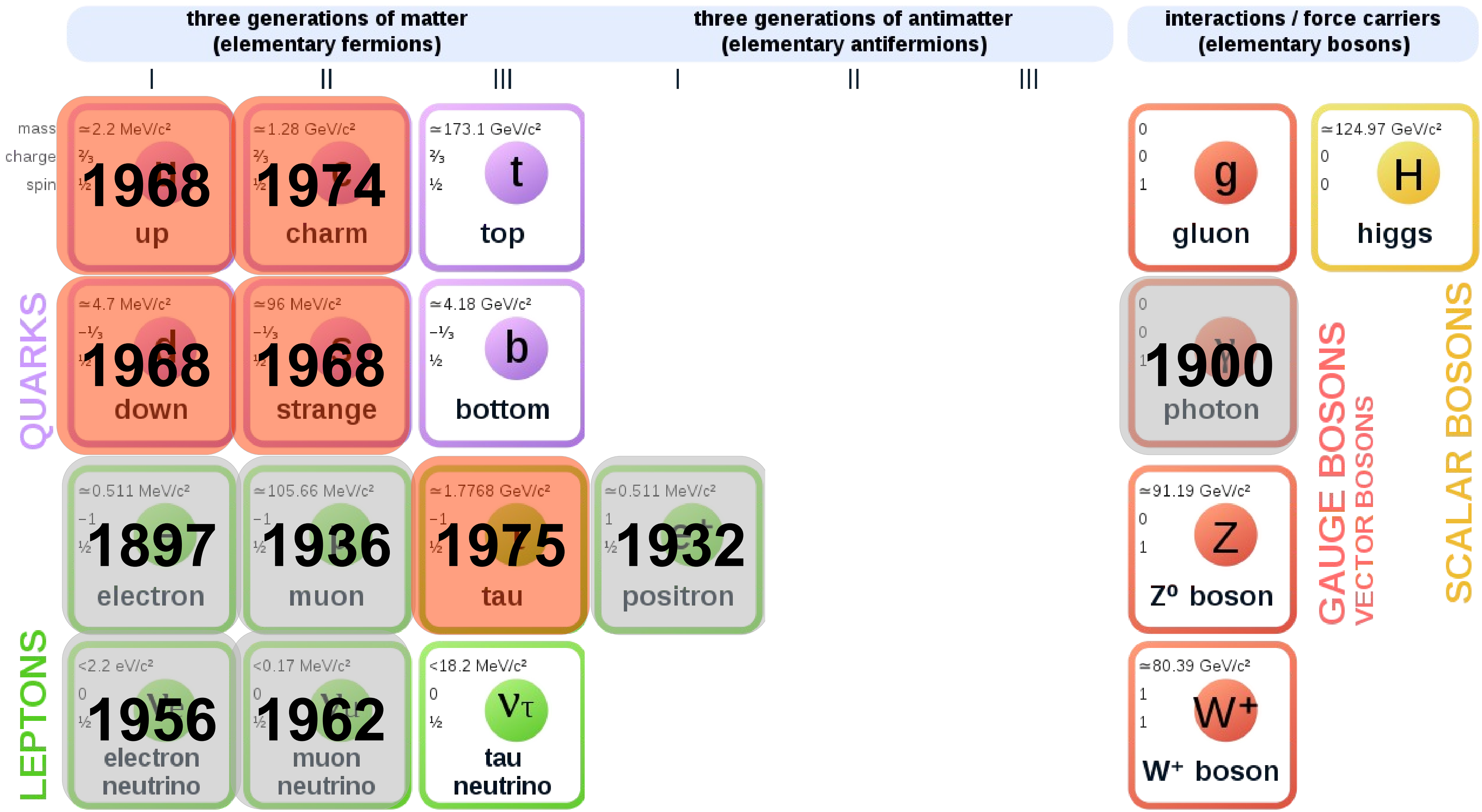

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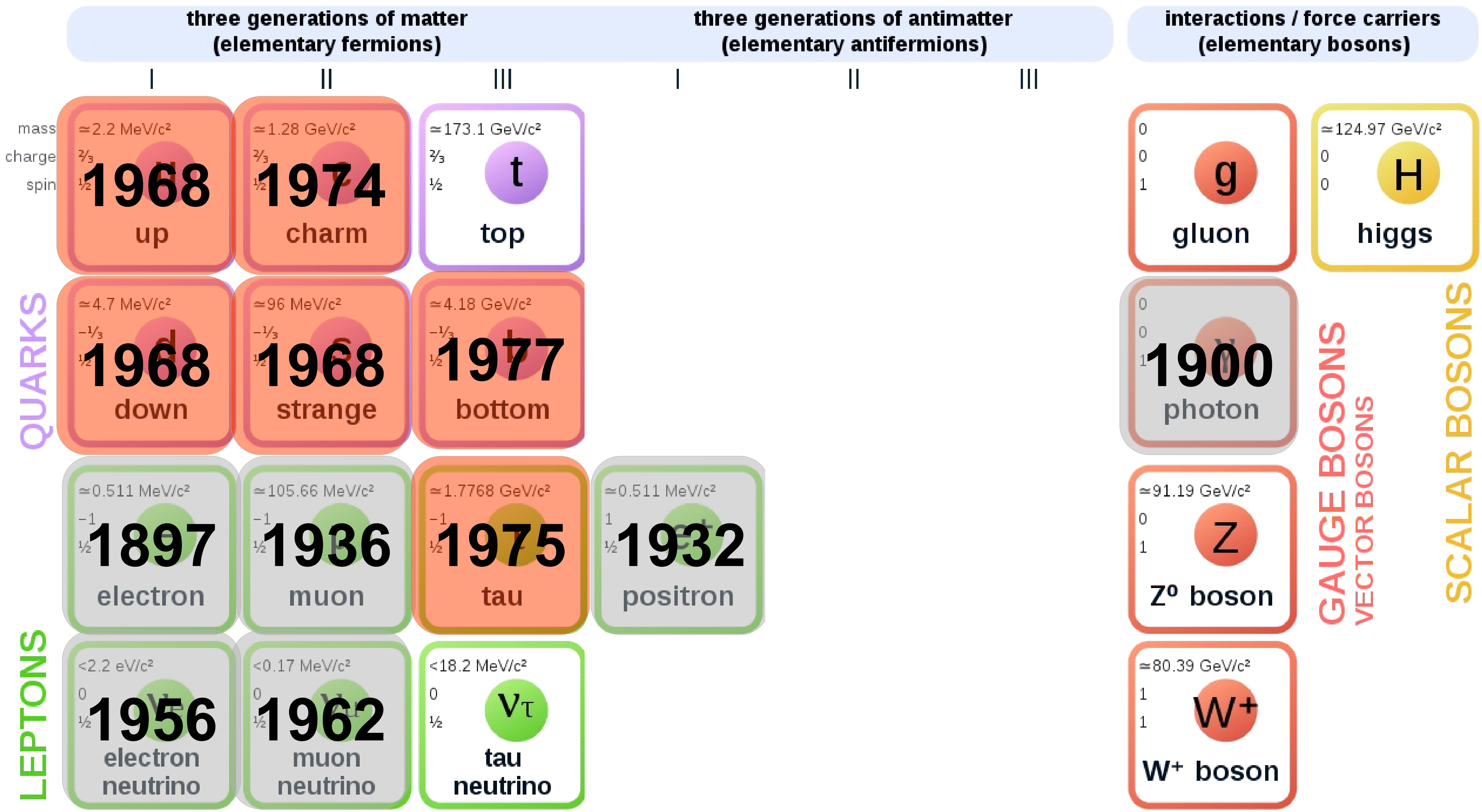

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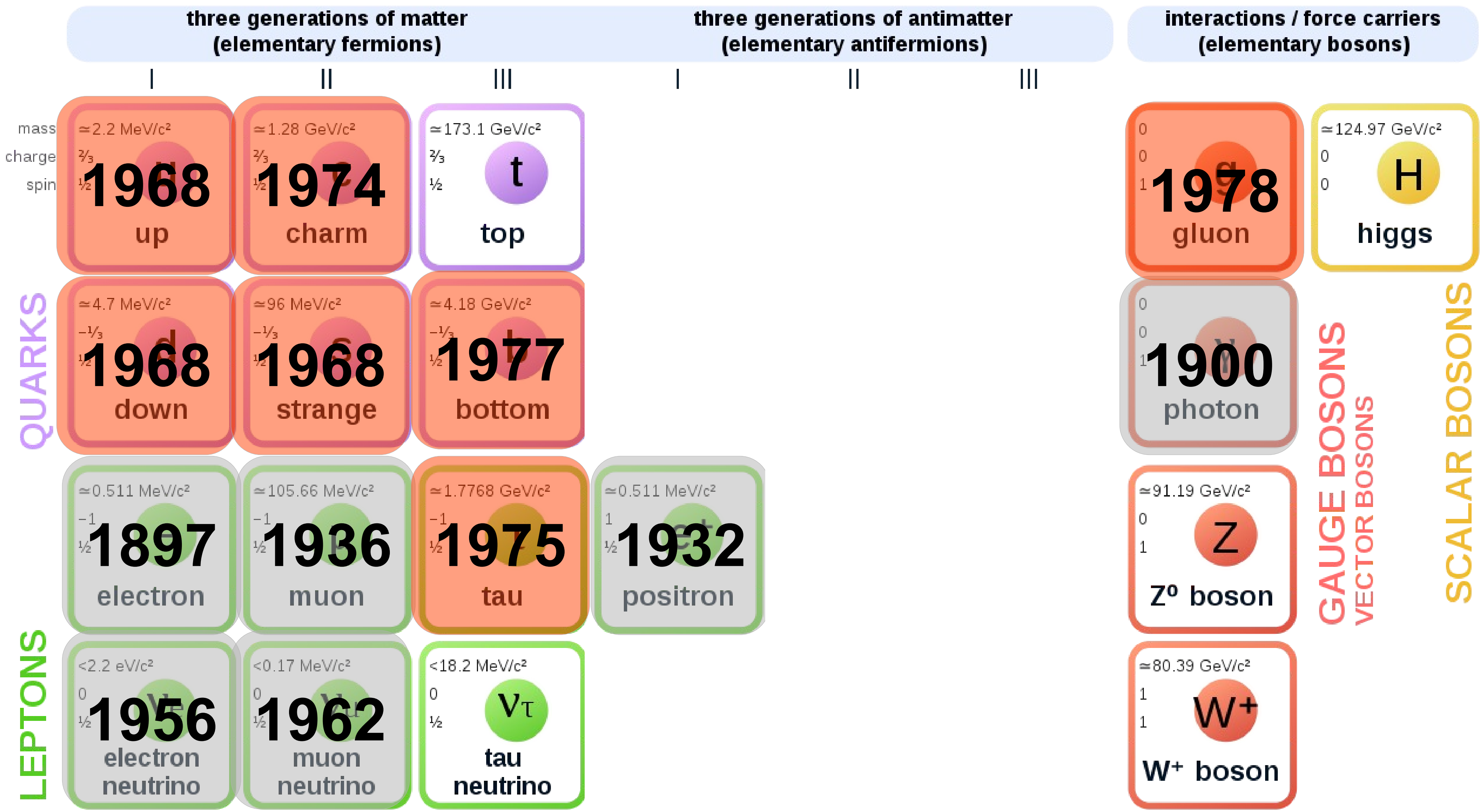

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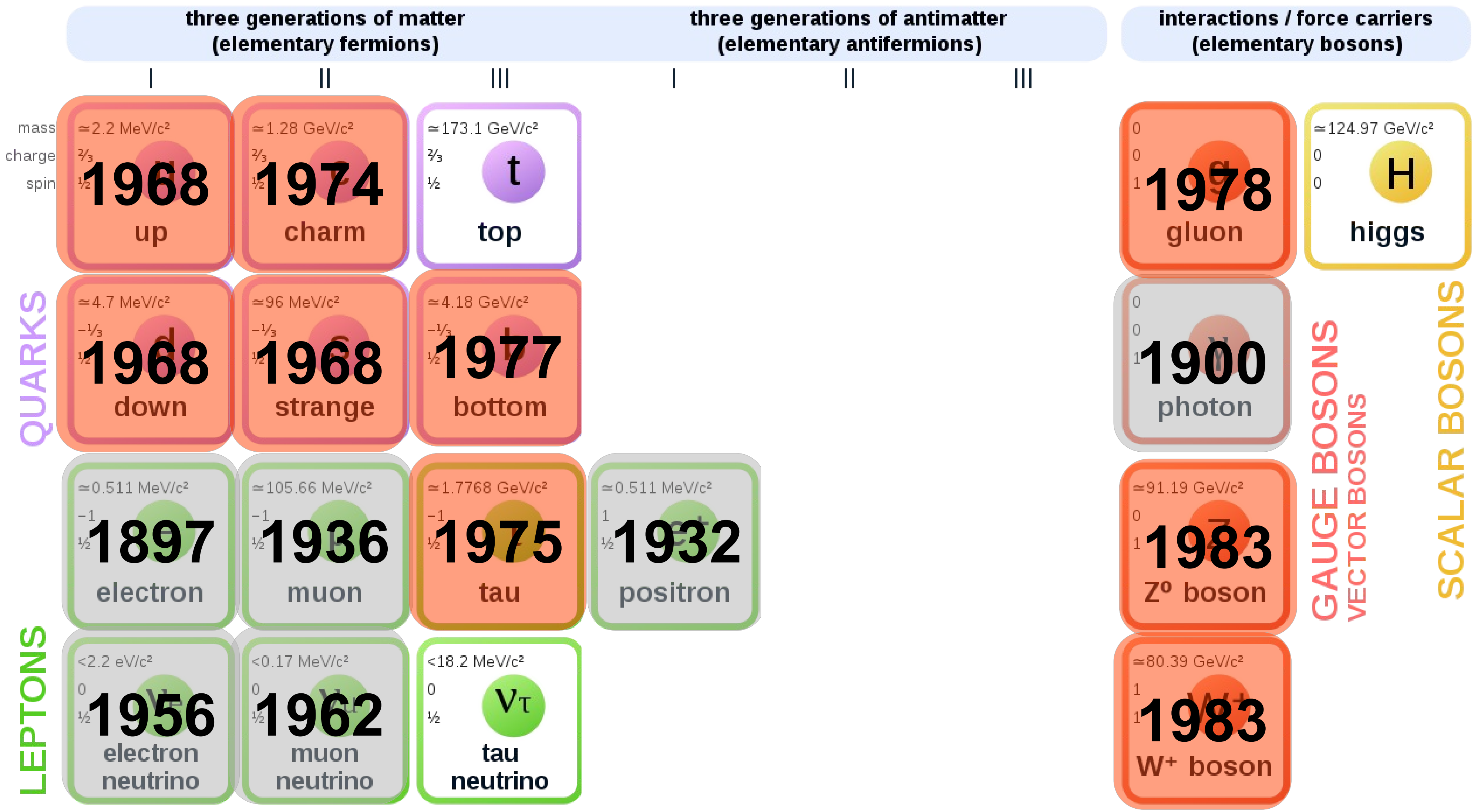

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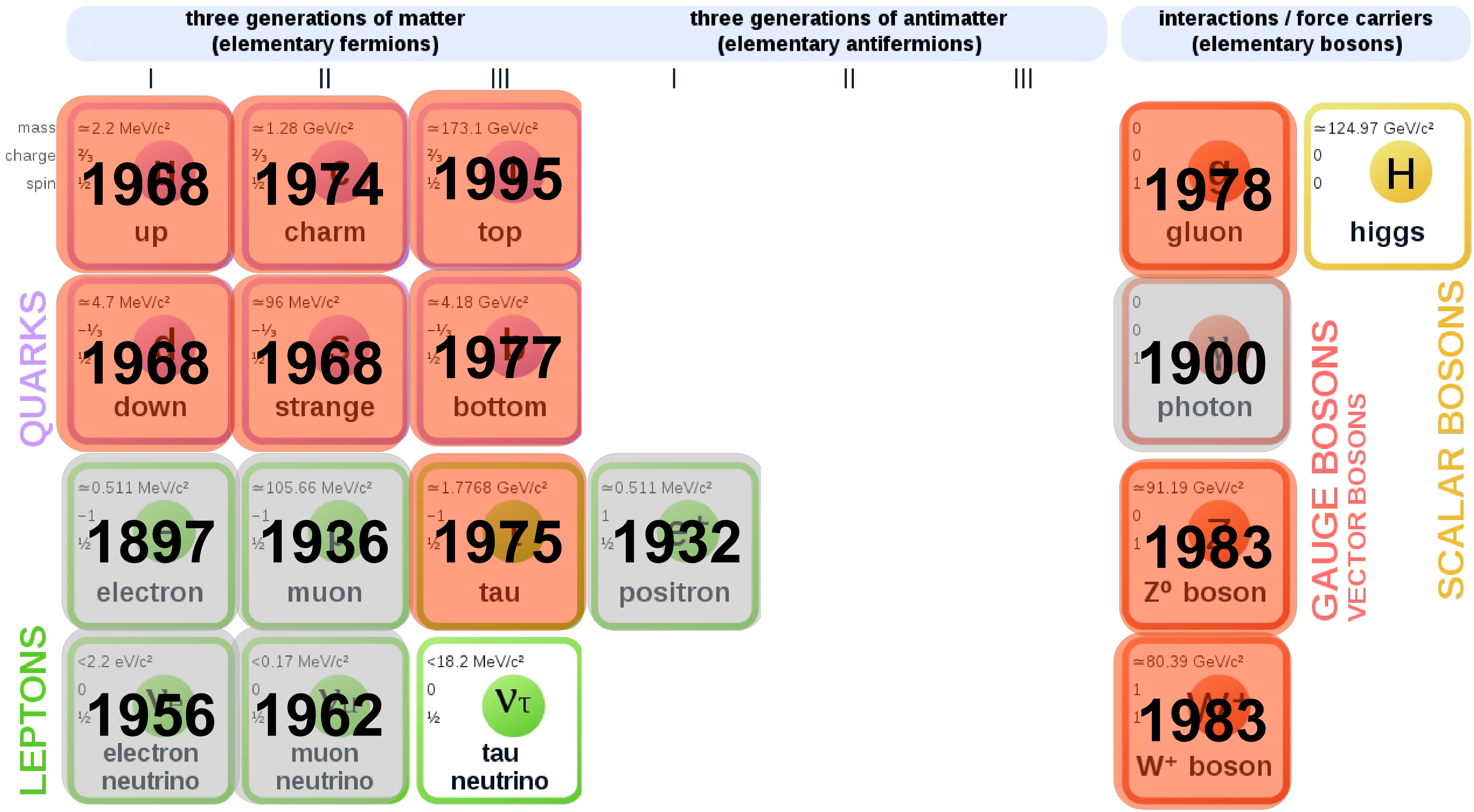

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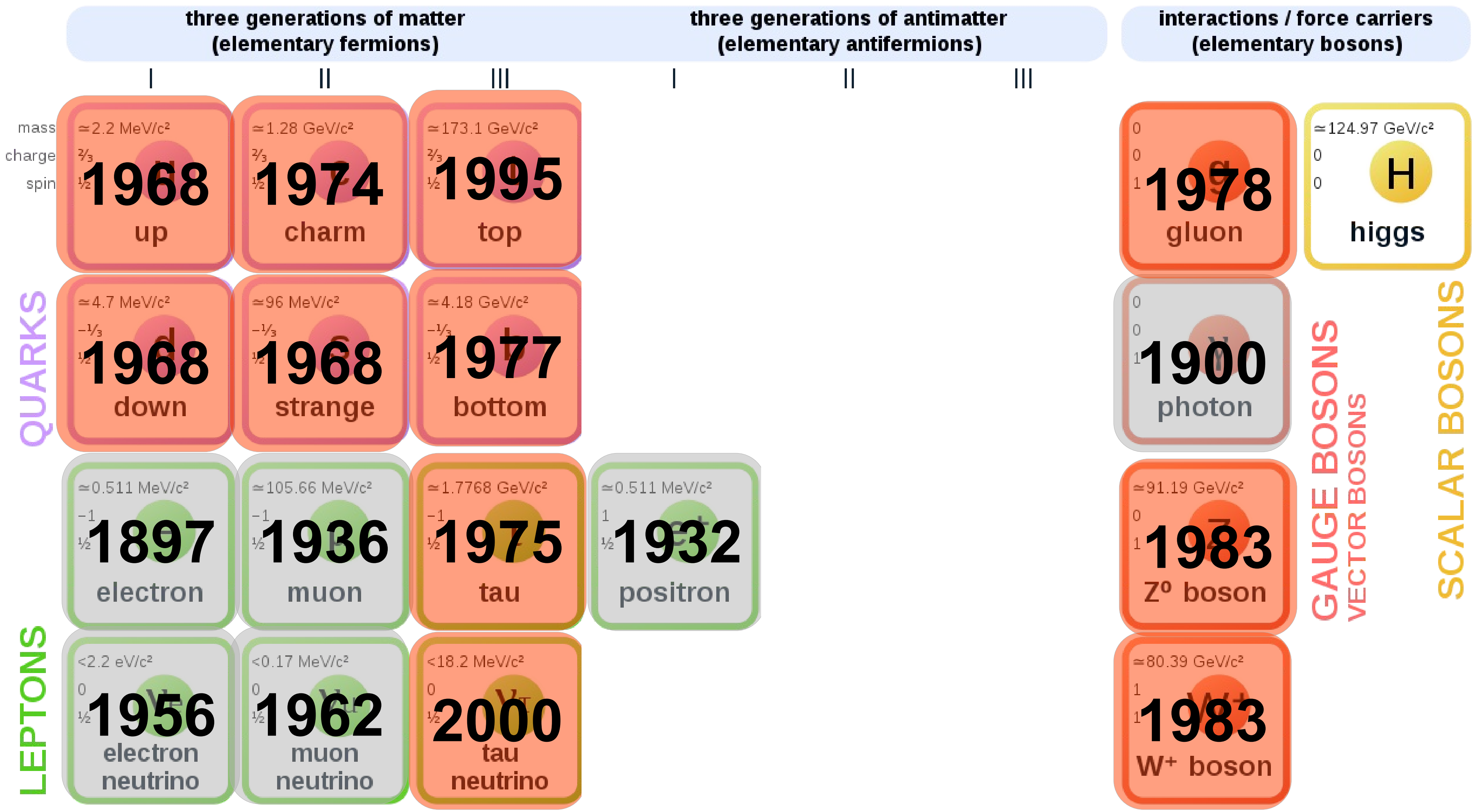

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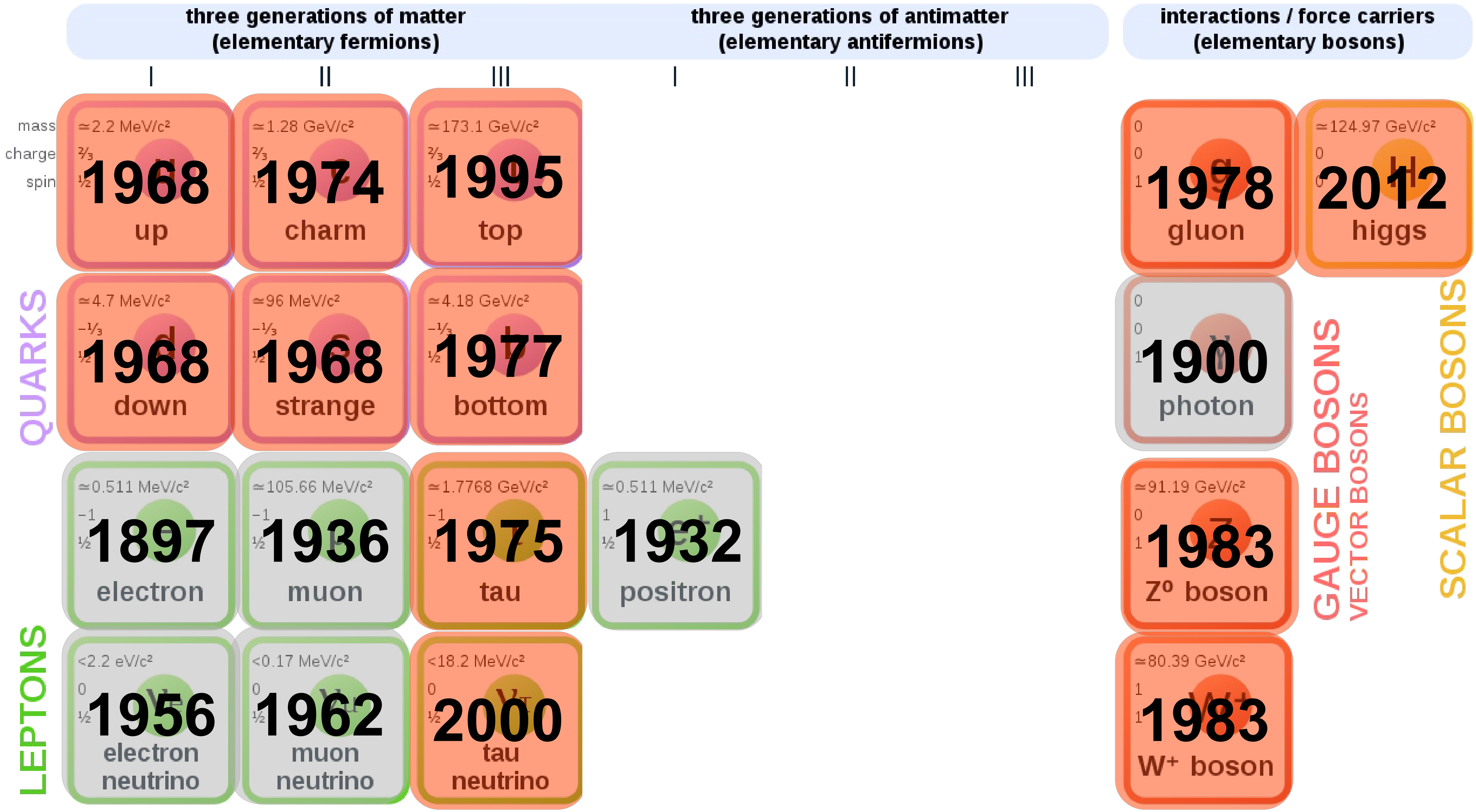

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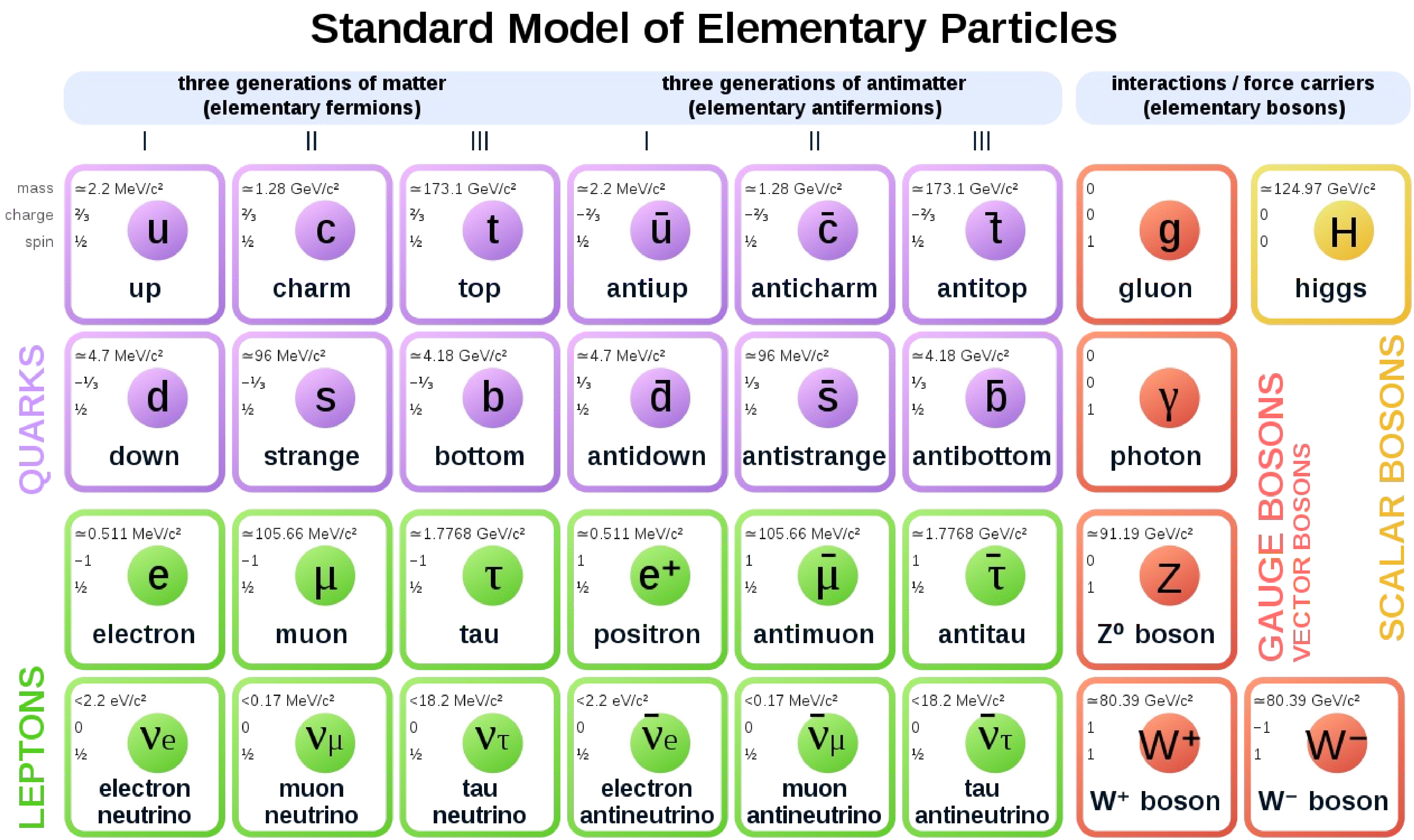
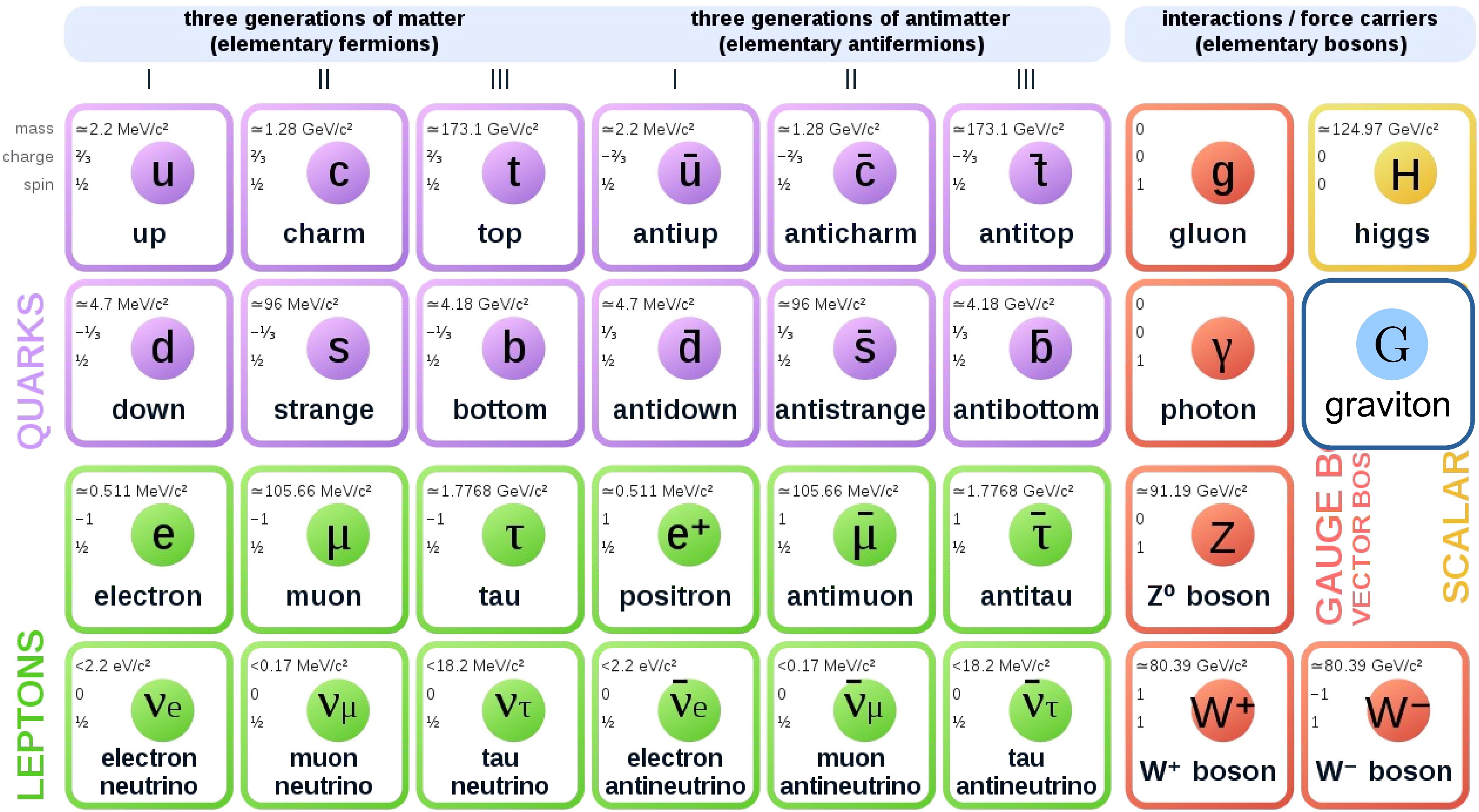

Standard Model of Elementary Particles
three generations of matter (elementary fermions)
three generations of antimatter (elementary antifermions)
interactions / force carriers (elementary bosons)
I
II
III
I
II
III
mass
charge
spin
QUARKS
LEPTONS
≃2.2 MeV/c² ⅔ ½ u up
≃1.28 GeV/c² ⅔ ½ c charm
≃173.1 GeV/c² ⅔ ½ t top
≃2.2 MeV/c² −⅔ ½ ū antiup
≃1.28 GeV/c² −⅔ ½ c̄ anticharm
≃173.1 GeV/c² −⅔ ½ t̄ antitop
0 0 1 g gluon
≃124.97 GeV/c² 0 0 H higgs
≃4.7 MeV/c² −⅓ ½ d down
≃96 MeV/c² −⅓ ½ s strange
≃4.18 GeV/c² −⅓ ½ b bottom
≃4.7 MeV/c² ⅓ ½ d̄ antidown
≃96 MeV/c² ⅓ ½ s̄ antistrange
≃4.18 GeV/c² ⅓ ½ b̄ antibottom
0 0 1 γ photon
≃0.511 MeV/c² −1 ½ e electron
≃105.66 MeV/c² −1 ½ μ muon
≃1.7768 GeV/c² −1 ½ τ tau
≃0.511 MeV/c² 1 ½ e⁺ positron
≃105.66 MeV/c² 1 ½ μ̄ antimuon
≃1.7768 GeV/c² 1 ½ τ̄ antitau
≃91.19 GeV/c² 0 1 Z Z⁰ boson
<2.2 eV/c² 0 ½ νe electron neutrino
<0.17 MeV/c² 0 ½ νμ muon neutrino
<18.2 MeV/c² 0 ½ ντ tau neutrino
<2.2 eV/c² 0 ½ ν̄e electron antineutrino
<0.17 MeV/c² 0 ½ ν̄μ muon antineutrino
<18.2 MeV/c² 0 ½ ν̄τ tau antineutrino
≃80.39 GeV/c² 1 1 W⁺ W⁺ boson
≃80.39 GeV/c² −1 1 W⁻ W⁻ boson
GAUGE BOSONS
VECTOR BOSONS
SCALAR BOSONS

wikipedia

Standard Model of Elementary Particles



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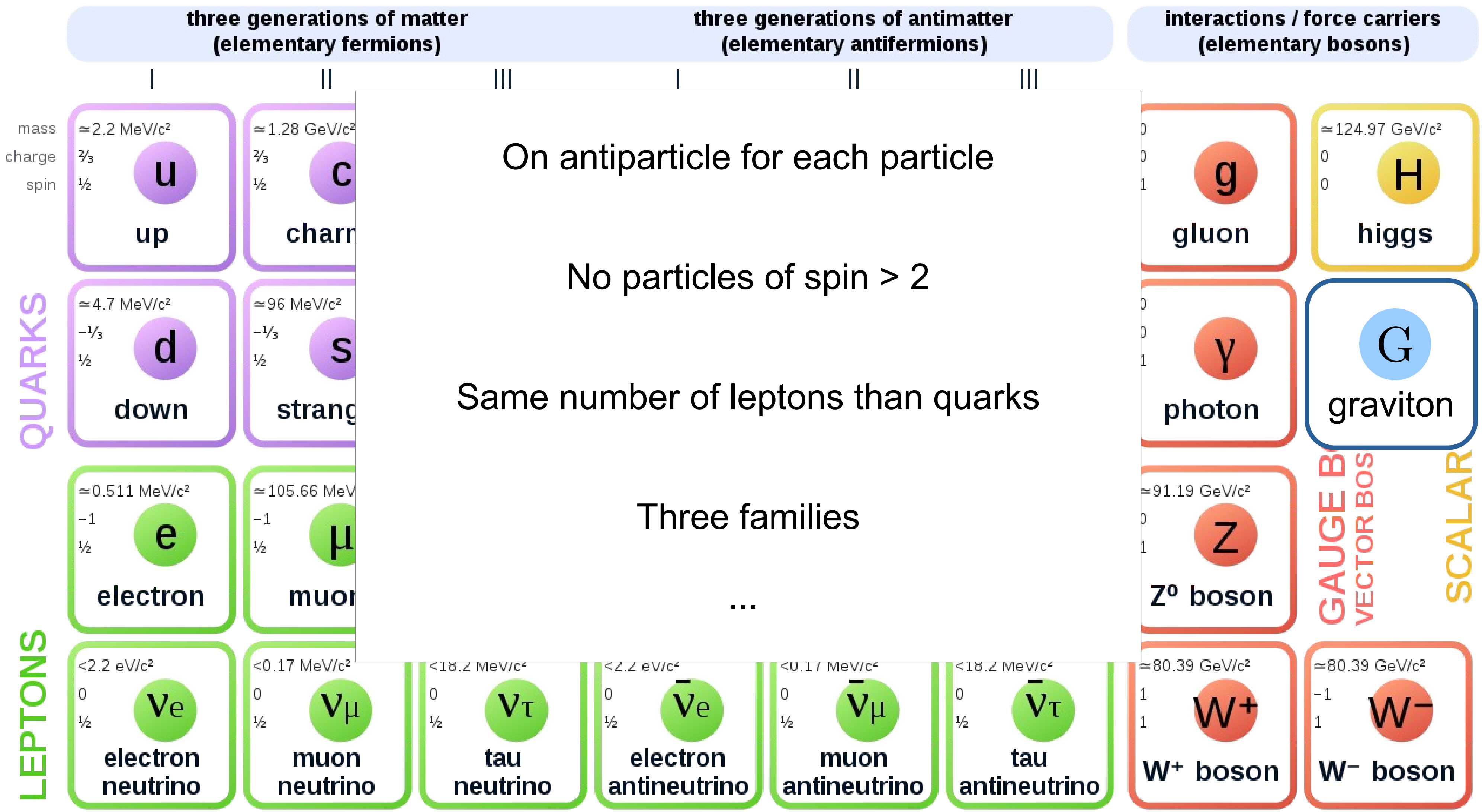


On antiparticle for each particle

No particles of spin > 2

Same number of leptons than quarks

Three families

...

wikipedia

Multiplets	$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	I	II	III	$Q = T_3 + Y$
Quarks	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	$\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$ $-\frac{1}{3} = -\frac{1}{2} + \frac{1}{6}$
	$(\mathbf{3}, \mathbf{1}, \frac{2}{3})$	u_R	c_R	t_R	$\frac{2}{3} = 0 + \frac{2}{3}$
	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$	d_R	s_R	b_R	$-\frac{1}{3} = 0 - \frac{1}{3}$
Leptons	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$	$0 = \frac{1}{2} - \frac{1}{2}$ $-1 = -\frac{1}{2} - \frac{1}{2}$
	$(\mathbf{1}, \mathbf{1}, -1)$	e_R	μ_R	τ_R	$-1 = 0 - 1$
	$(\mathbf{1}, \mathbf{1}, 0)$	ν_{e_R}	ν_{μ_R}	ν_{τ_R}	$0 = 0 + 0$
Higgs	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	(3 families of quarks & leptons)			

$$\begin{aligned}\mathcal{L}_{SM} &= -\frac{1}{4}G^A_{\mu\nu}G^{\mu\nu}_A - \frac{1}{4}W^a_{\mu\nu}W^{\mu\nu}_a - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ \overline{q^\alpha_L} i \not{D} q^\alpha_L + \overline{l^\alpha_L} i \not{D} l^\alpha_L + \overline{u^\alpha_R} i \not{D} u^\alpha_R + \overline{d^\alpha_R} i \not{D} d^\alpha_R + \overline{e^\alpha_R} i \not{D} e^\alpha_R \\ &+ (D_\mu \phi)^\dagger (D^\mu \phi) - \mu^2 |\phi|^2 - \lambda |\phi|^4 - \left(y^u_{\alpha\beta} \overline{q^\alpha_L} \tilde{\phi} u^\beta_R + y^d_{\alpha\beta} \overline{q^\alpha_L} \phi d^\beta_R + y^e_{\alpha\beta} \overline{l^\alpha_L} \phi e^\beta_R + \text{h.c.} \right)\end{aligned}$$

$\phi^T \phi$?

What about operators of the following form?

$$\theta G^{A}_{\mu\nu} \tilde{G}^{\mu\nu A} \qquad H^\dagger D^2 H + \text{h.c.}$$

$$(H^\dagger \sigma_a H)(H^\dagger \sigma_a H)$$

$$(2 \times 2) \times (2 \times 2) = (1 + 3) \times (1 + 3) = 1 + 3 + 3 + 1 + 3 + 5$$

$$(H^\dagger H)^2$$

Proposed exercise: Demonstrate that the Standard Model Higgs Lagrangian is the most general renormalisable Lagrangian for the Higgs

$$D_\mu = \partial_\mu - ieQA_\mu$$

$$\psi(x) \rightarrow e^{-iQ\theta(x)}\psi(x)$$

$$A_\mu \rightarrow A_\mu + \frac{1}{e}\partial_\mu\theta(x)$$

$$\overline{\psi}\gamma^\mu D_\mu\psi$$

remains invariant!

Explanation in terms of R+QM

Unitary Poincare representations are infinite-dimensional

$$\mathcal{H} = \{\epsilon_+^\mu(p), \epsilon_-^\mu(p),\ \forall p\}$$

Most Poincare transformations act as $\Lambda\epsilon(p) \to \epsilon(p')$. There is though a special subgroup of transformations for each p; denoted as the little group, that leave p invariant.

In general it mixes polarizations with p. In fact, polarizations differing on a multiple of p must be considered equivalent!

$$\epsilon_\mu \to \epsilon_\mu + \alpha p_\mu$$

$$A_\mu \to A_\mu + \partial_\mu \alpha$$

Explanation in terms of R+QM

$$\epsilon_\mu \to \epsilon_\mu + \alpha p_\mu \qquad\qquad \left(A_\mu \to A_\mu + \partial_\mu \alpha \right)$$

This is a consequence of using **fields** for describing **particles**

Can we do particle physics without fields? YES

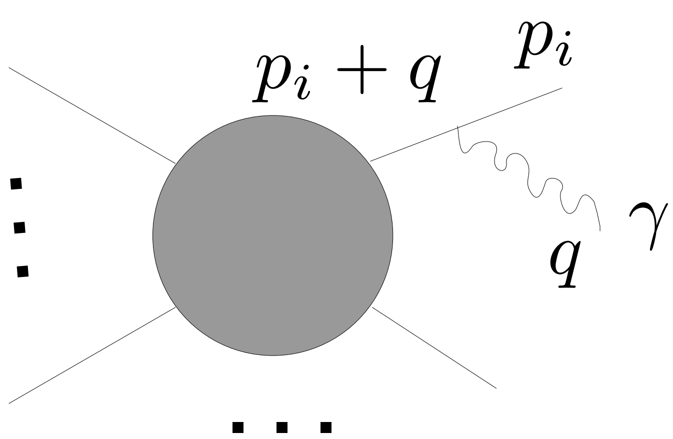
In that case then, gauge redundancy is not present? YES

Do we actually do computations this way? NOT ALWAYS

Not always clear how to enforce locality and unitarity; not a well define perturbation theory

Implications

Amplitudes in the soft limit



$$\mathcal{M} = \sum_i \mathcal{M}_0 \times Q_i \frac{\{p_i \cdot \epsilon, \cancel{q \cdot \epsilon}, \ldots\}}{(q + p_i)^2}$$

$$= \mathcal{M}_0 \times \sum_i Q_i \frac{p_i \cdot \epsilon}{p_i \cdot q}$$

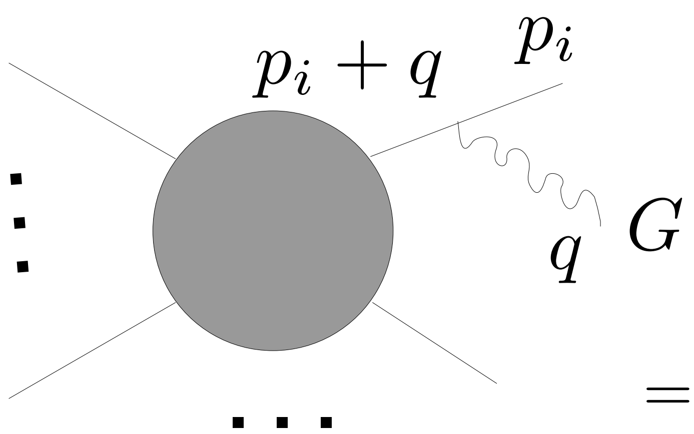
$$= \mathcal{M}_0 \sum_i Q_i \frac{p_i \cdot (\epsilon + \alpha q)}{p_i \cdot q}$$

$$\Rightarrow \alpha \sum_i Q_i \frac{p_i \cdot q}{p_i \cdot q} = 0 \rightarrow \sum_i Q_i = 0$$

charge conservation!

Implications

Amplitudes in the soft limit



$$\mathcal{M} = \mathcal{M}_0 \times \sum_i \kappa_i \frac{p_i^\mu p_i^\nu \epsilon_{\mu\nu}}{p_i \cdot q}$$

$$= \mathcal{M}_0 \sum_i \kappa_i \frac{p_i^\mu p_i^\nu (\epsilon_{\mu\nu} + \alpha_\mu q_\nu + \alpha_\nu q_\mu + \alpha q_\mu q_\nu)}{p_i \cdot q}$$

$$\Rightarrow \alpha_\mu \sum_i \kappa_i \frac{p_i^\mu p_i \cdot q}{p_i \cdot q} = 0 \Rightarrow \sum_i \kappa_i p_i^\mu = 0$$

+ mom cons: $\sum_i p_i^\mu = 0$ $\qquad \Rightarrow \kappa_i = \kappa$ Equivalence principle!

Implications

Amplitudes in the soft limit

Proposed exercise: do the same for s=3, and convince yourself that high spins must be non interacting

$$D_\mu = \partial_\mu - igT_A F_\mu^A$$

$$F_{\mu\nu}^A = \partial_\mu F_\nu^A - \partial_\nu F_\mu^A + g f_{ABC} F_\mu^B F_\nu^C$$

This combination is also gauge invariant.

For Abeian groups, f vanishes, so no self interactions of bosons

This, again, can be better understood on the basis of R+QM

The Higgs mechanism

Boson and fermion masses are not allowed by gauge invariance

$$m_\psi \overline{\psi_L} \psi_R$$

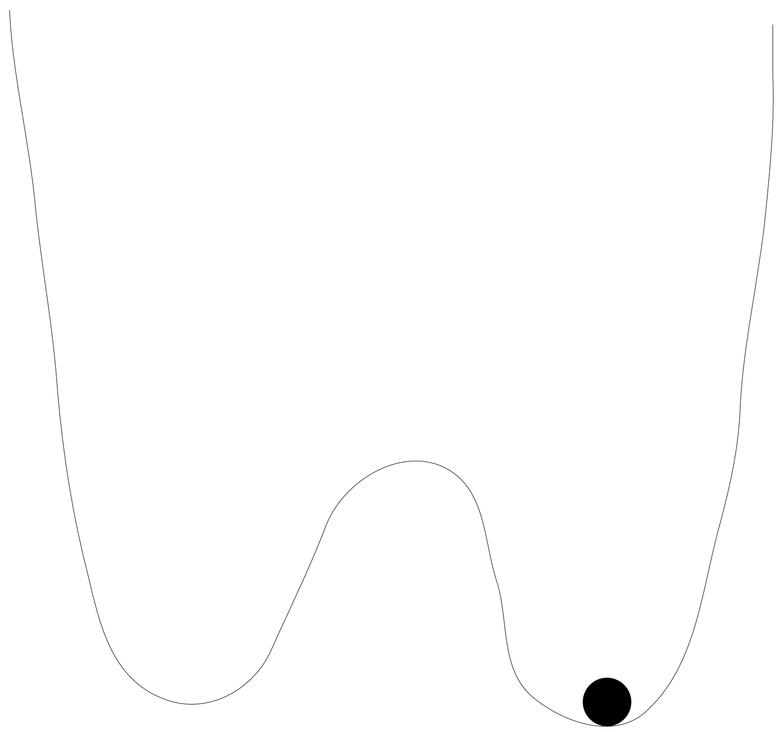
If the RH exists, then neutrino masses are possible

The Higgs mechanism

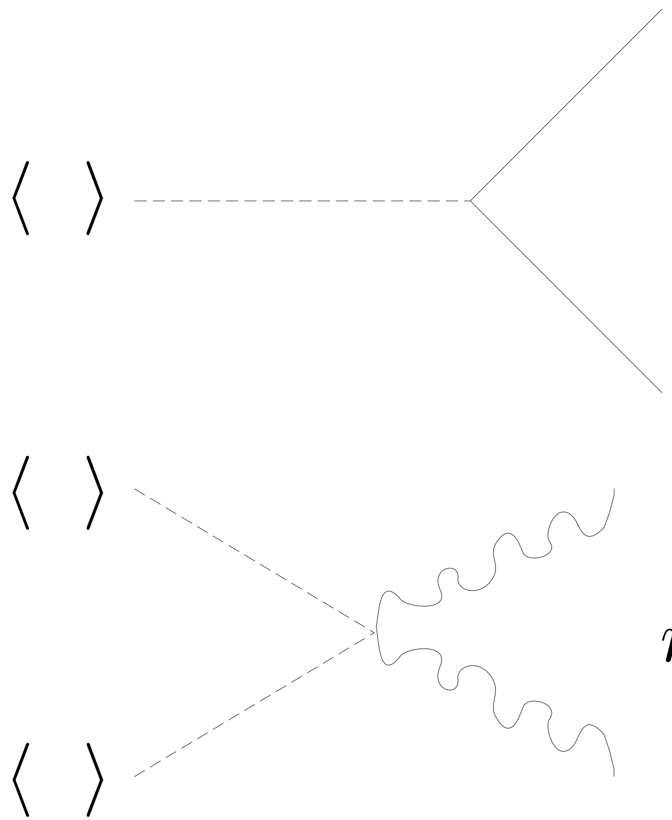
The electroweak symmetry is broken in the vacuum:

$$\langle \mathrm{H} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$(T_3 + Y)\langle \mathrm{H} \rangle = 0$$



$T = 0$



$$m_\psi \propto y_\psi v$$

$$m_V^2 \propto g^2 v^2$$

The Higgs mechanism

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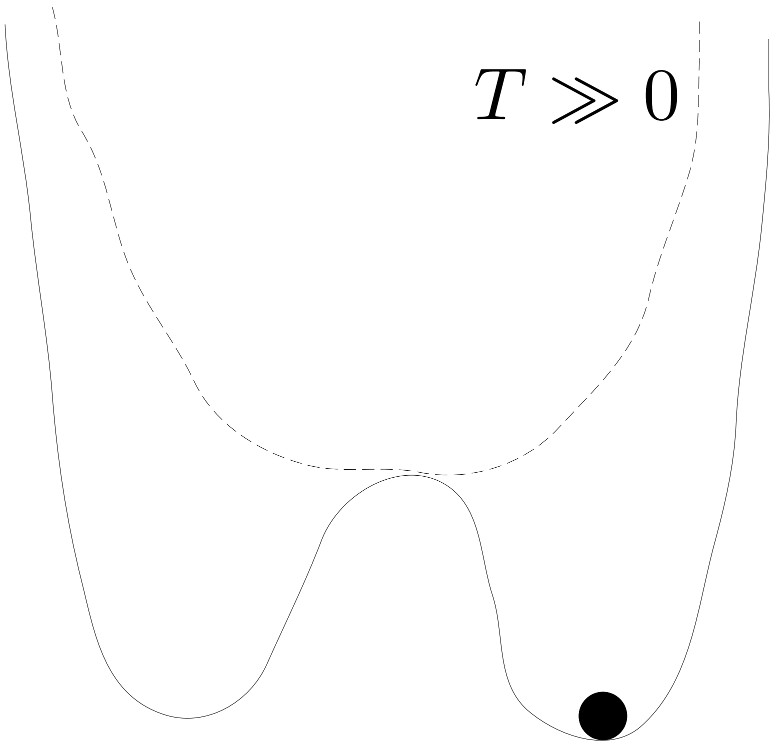

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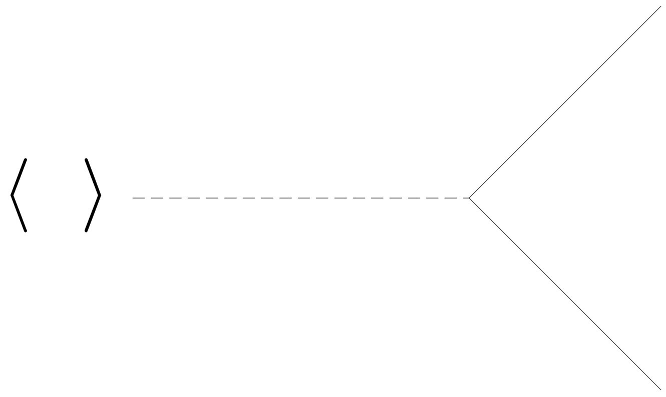

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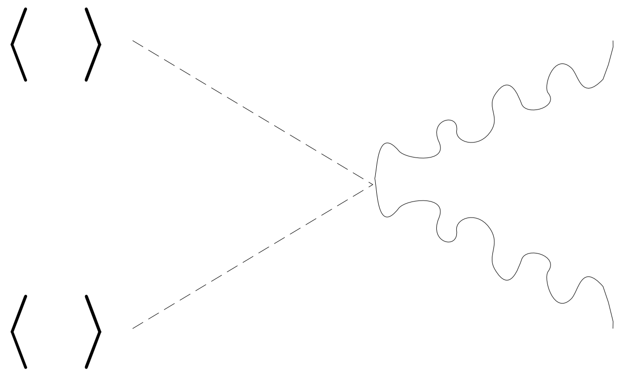
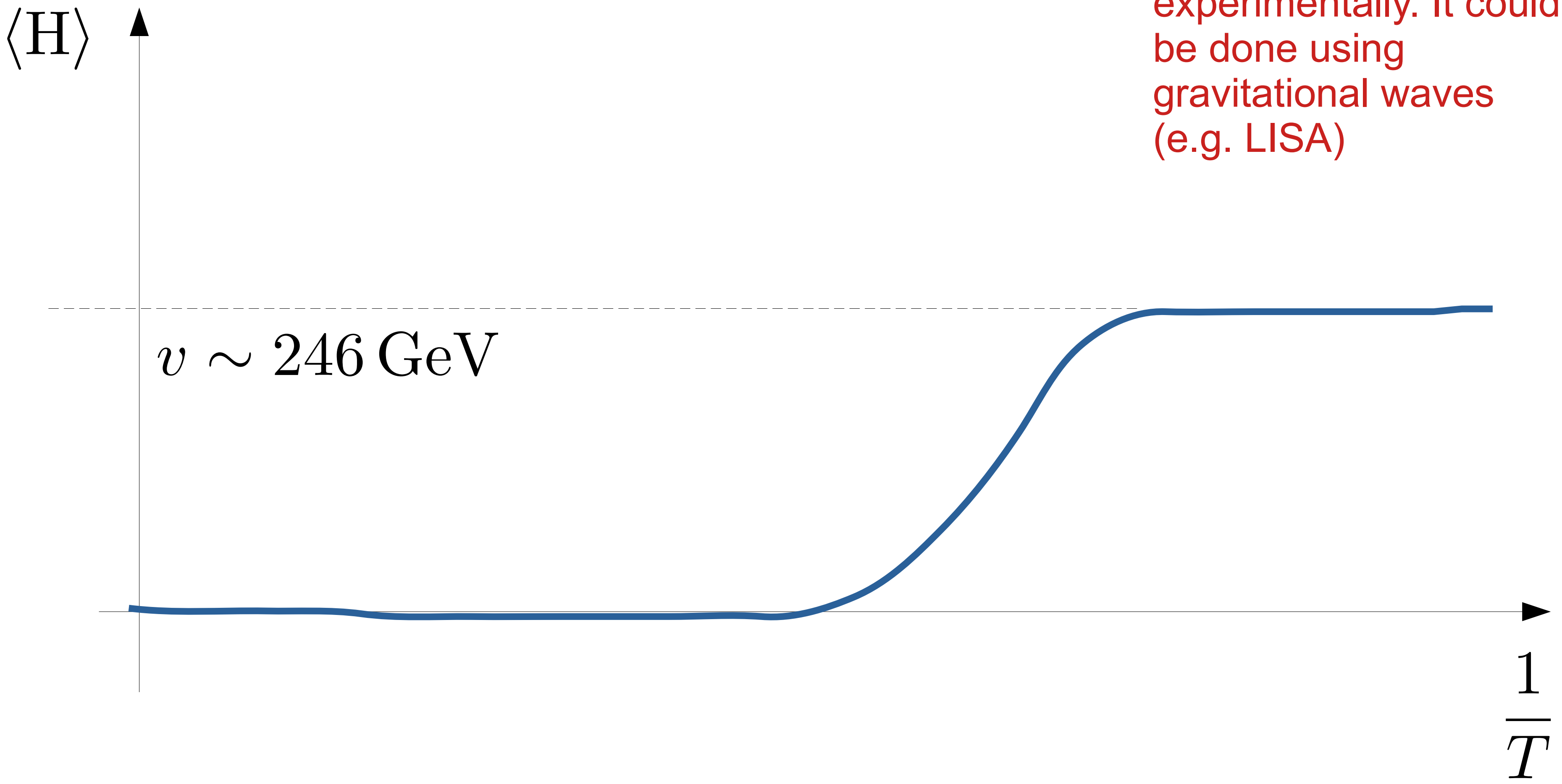


$m_V^2 \propto g^2 v^2$

The Higgs mechanism

Anecdotal content: the electroweak phase transition within the SM



It has not been tested experimentally. It could be done using gravitational waves (e.g. LISA)

The Higgs mechanism

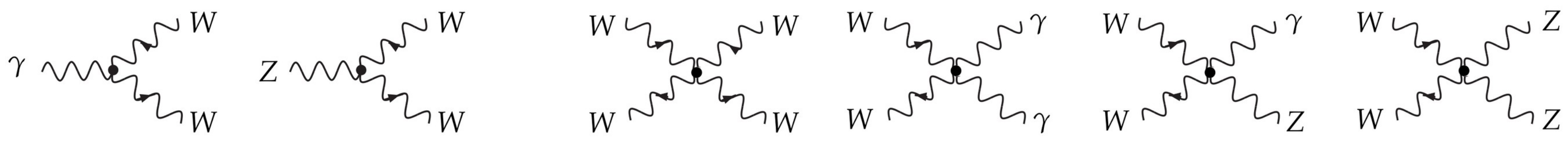
The B and the third component of W mix after EWSB:

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

$s_W \equiv \sin\theta_W \,, \quad c_W \equiv \cos\theta_W$

$\theta_W =$ weak mixing angle

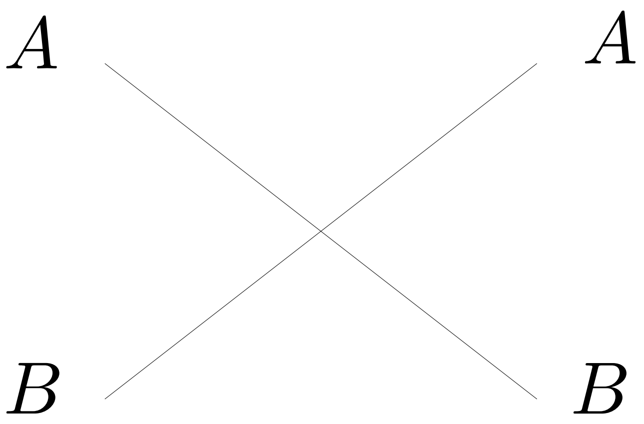
(1) $\boxed{e = g s_W = g' c_W}$ (2) $\boxed{Q = T_3 + Y}$



Proposed exercise: Show that there are no vertices with only Z and photons within the SM.
What if renormalisability is abandoned?

The Higgs mechanism

Importantly, the Higgs is also needed to maintain the unitarity of the SM at high energies:



$$\sigma_{\text{tot}}(AB \to AB) = \frac{1}{32\pi E_{\text{CM}}^2} \int d\cos\theta |\mathcal{M}(\theta)|^2$$

$$\mathcal{M}(\theta) = 16\pi \sum_{j=0}^{\infty} a_j (2j+1) P_j(\cos\theta)$$

Legendre polynomials

The Higgs mechanism

Importantly, the Higgs is also needed to maintain the unitarity of the SM at high energies:

Optical theorem
(i.e. unitarity)

$$\Longrightarrow \quad |a_j| \leq 1$$

The Higgs mechanism

Importantly, the Higgs is also needed to maintain the unitarity of the SM at high energies:

$$\sigma_{\text{tot}} = \frac{16\pi}{E_{\text{CM}}^2} \sum_{j=0}^{\infty} (2j+1) \left| a_j \right|^2$$

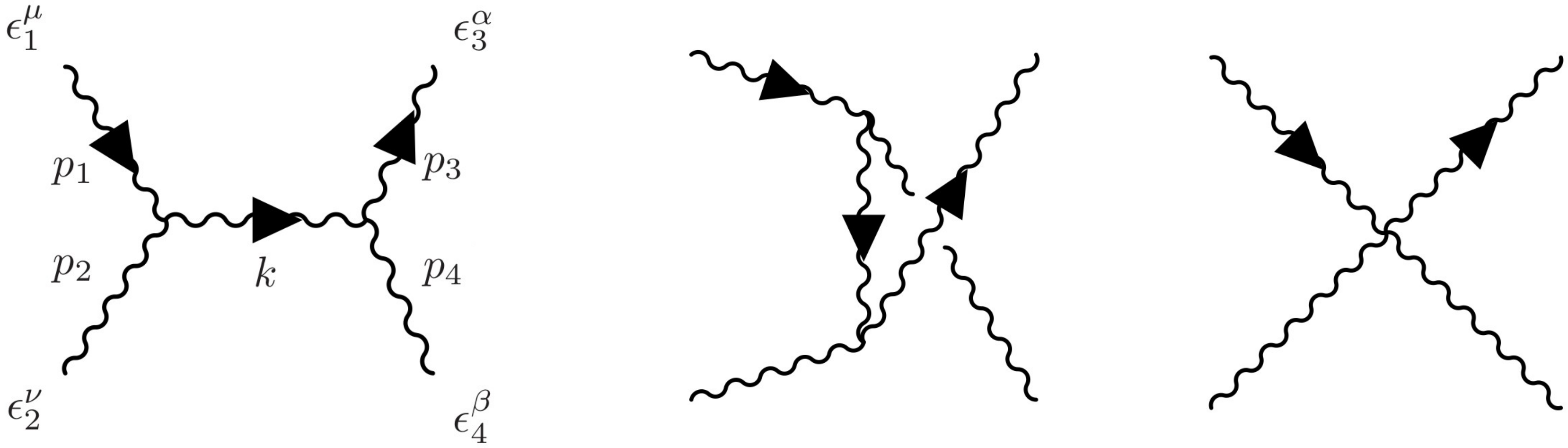
Optical theorem

$$\text{Im}\mathcal{M}(AB \to AB \text{ at } \theta = 0) = 2E_{\text{CM}} |\vec{p}_i| \sum_X \sigma_{\text{tot}}(AB \to X)$$

$$\geq 2E_{\text{CM}} |\vec{p}_i| \sigma_{\text{tot}}(AB \to AB),$$

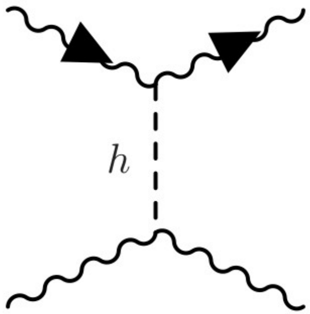
$$\sum_{j=0}^{\infty} (2j+1) \text{Im}(a_j) \geq \frac{2\,|\vec{p}_i|}{E_{\text{CM}}} \sum_{j=0}^{\infty} (2j+1) \left| a_j \right|^2$$

$$\sigma\left(W_L^+(p_1)\,Z_L(p_2) \rightarrow W_L^+(p_3)\,Z_L(p_4)\right)$$



$$\mathcal{M} \sim \frac{t}{m^2} + \mathcal{O}(1)$$

Unitarity violation!

$\mathcal{M}_h =$  $= \quad -\frac{t}{m^2} + \mathcal{O}(1)$

Proposed exercise: Using the previous results, demonstrate that the SM without the Higgs is not a valid theory for E > few TeV