# Standard Model and open problems

**Mikael Chala**

(Universidad de Granada)

11th IDPASC School, Olomouc; September 2, 2022

# Some structural facts

## The rho parameter

$$\mathrm{H} = \begin{pmatrix} 0 \\ \frac{h+v}{\sqrt{2}} \end{pmatrix}$$

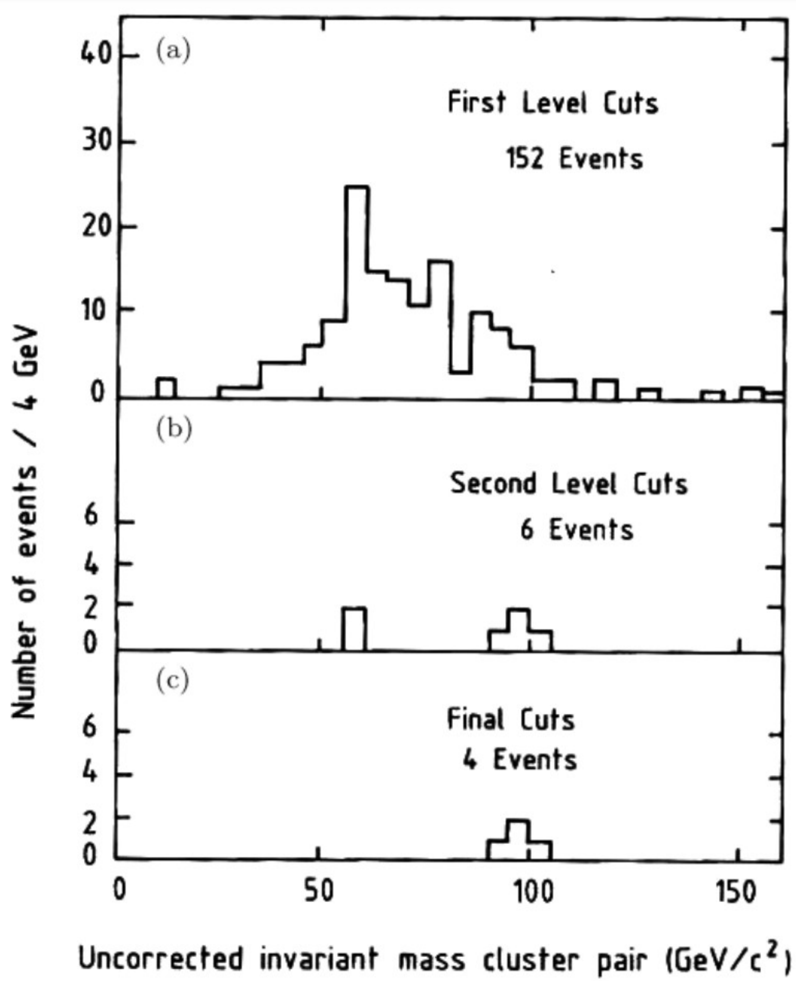
$$\rho = \frac{m_W^2}{m_Z^2 c_W^2} = 1$$

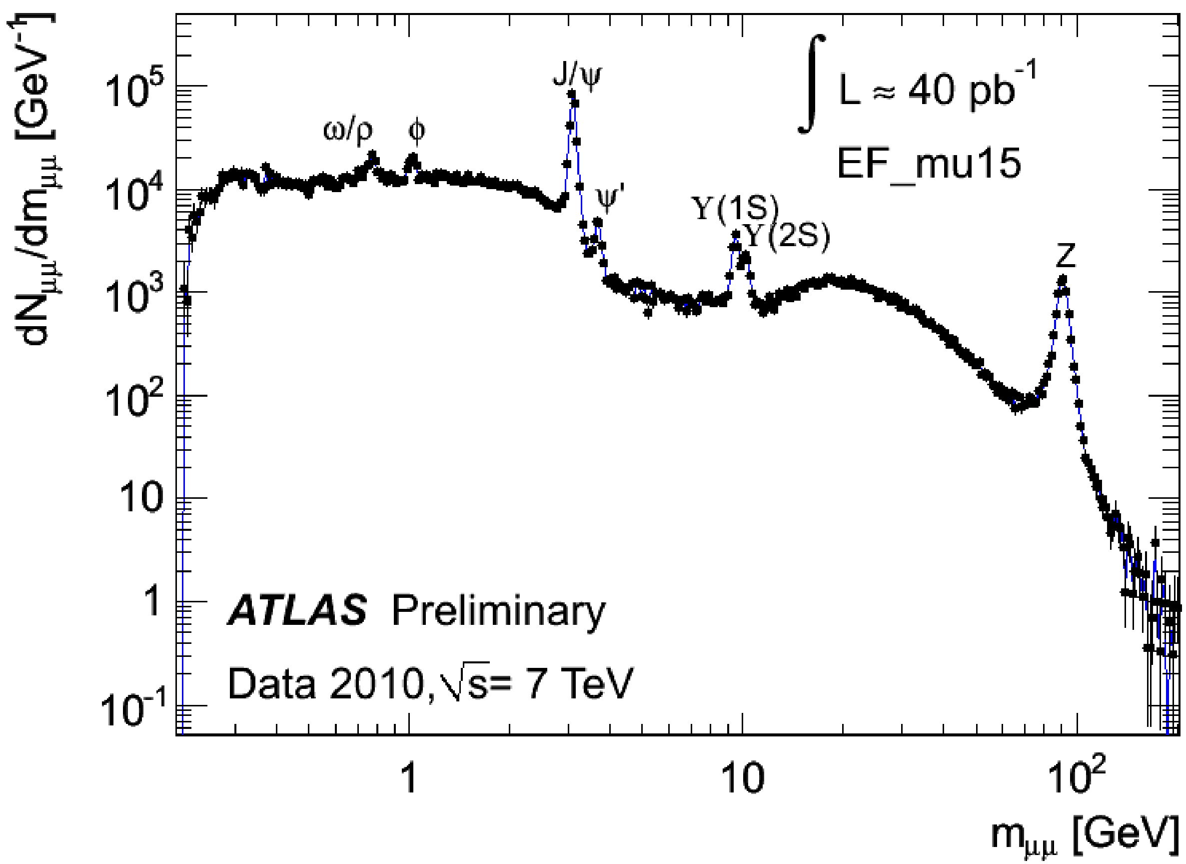
$$T^{\pm} = T_1 \pm i T_2$$

$$\begin{aligned} D_\mu \mathrm{H} &= \left(\partial_\mu + igT_I W_\mu^I + ig' Y B_\mu\right) \mathrm{H} \\ &= \left[\partial_\mu + \frac{ig}{\sqrt{2}}\left(T^+ W_\mu^+ + T^- W_\mu^-\right) + \frac{ig}{c_W}\left(T_3 - s_W^2 Q\right) Z_\mu + ieQA_\mu\right] \mathrm{H} \end{aligned}$$

$$(D_\mu \mathrm{H})^\dagger D^\mu \mathrm{H} \supset \underbrace{\frac{g^2 v^2}{4}}_{m_W^2} W_\mu^+ W^{\mu -} + \underbrace{\frac{g^2 v^2}{8 c_W^2}}_{\frac{1}{2} m_Z^2} Z_\mu Z^\mu$$

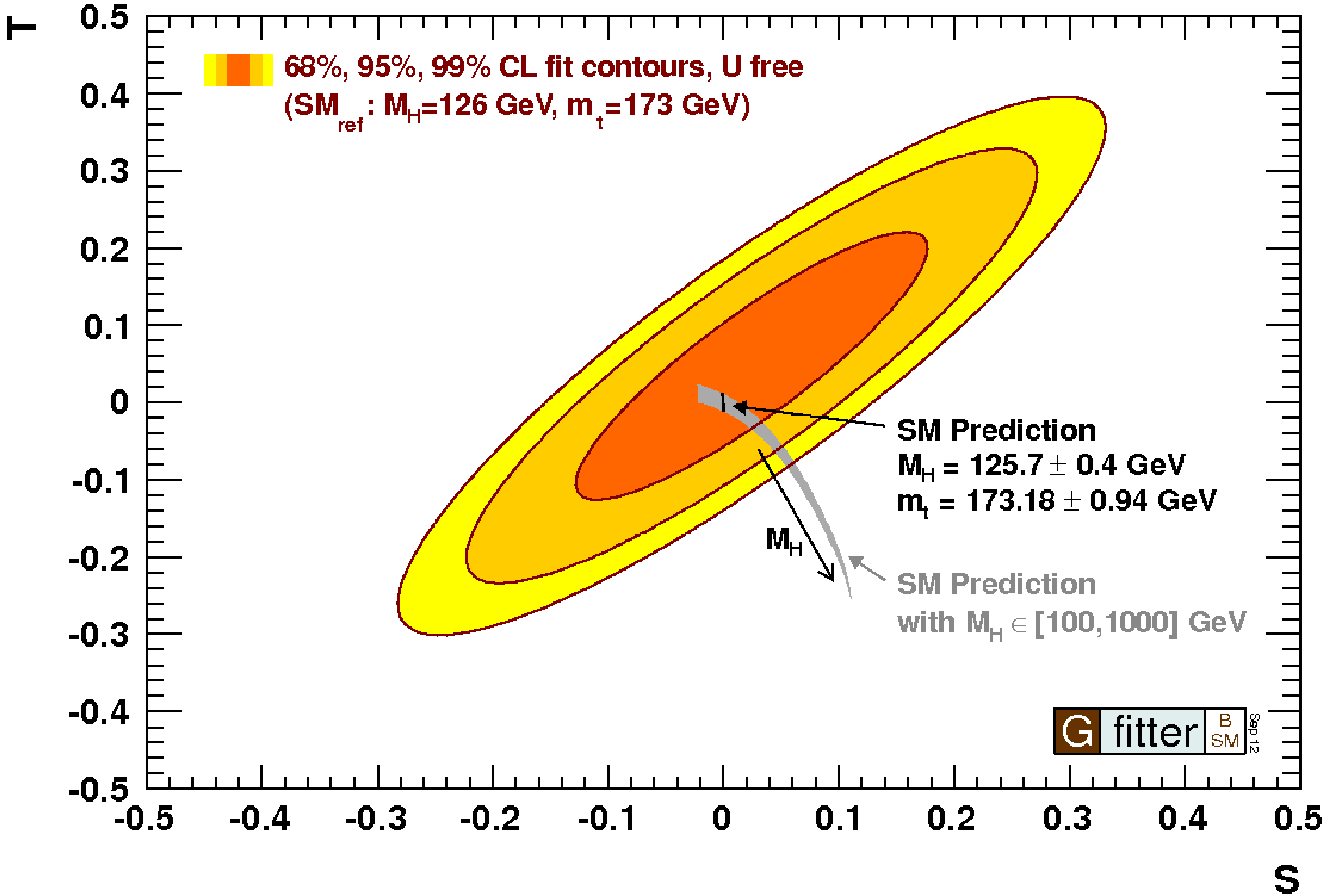
# Some structural facts



# Some structural facts

## The T (rho-1) parameter



**Proposed exercise**: Show that if the Higgs boson is a colourless SU(2) triplet with Y=1, then the rho parameter differs from 1

# Some structural facts

Lepton and baryon numbers (accidentally) conserved:
Neutrinos are massless; proton is stable

$$\mathcal{O}^{(5)} = (\overline{L_L^c}\tilde{\phi}^*)(\tilde{\phi}^\dagger L_L)$$

$$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$$

$$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$$

$$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$$

$$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$$

# Some structural facts

Lepton and baryon numbers are broken non-perturbatively:

$$\Delta L = \Delta B = 3$$

1601.03654

$$qq \rightarrow \overline{lll}\,\overline{q}\,\overline{q}\,\overline{q}\,\overline{q}\,\overline{q}\,\overline{q}\,\overline{q}$$

$$\Gamma \sim e^{-\frac{(4\pi)^2}{g^2}}$$

$$\Gamma \sim T^4$$

At high temperature

# Some structural facts

Suppressed flavour-changing neutral currents

$$L_Y \to -\frac{v}{\sqrt{2}}\left(1+\frac{h}{v}\right)\left(y^u_{ij}\overline{u^i_L}u^j_R + y^d_{ij}\overline{d^i_L}d^j_R + y^l_{ij}\overline{l^i_L}l^j_R + \text{h.c.}\right)$$

$$(\mathcal{U}^u_L)^\dagger y^u \mathcal{U}^u_R = \text{diag}(y_u, y_c, y_t)$$
$$(\mathcal{U}^d_L)^\dagger y^d \mathcal{U}^d_R = \text{diag}(y_d, y_s, y_b)$$

$$c^V_Z \overline{\mathbf{u_L}}\gamma_\mu \mathbf{u_L} Z_\mu \to c^V_Z \overline{\mathbf{u_L}}(\mathcal{U}^u_L)^\dagger \gamma_\mu \mathcal{U}^u_L \mathbf{u_L} Z^\mu = c^V_Z \overline{\mathbf{u_L}}\gamma_\mu \mathbf{u_L} Z_\mu$$



FCNC amplitudes can
not arise at tree level

# Some structural facts

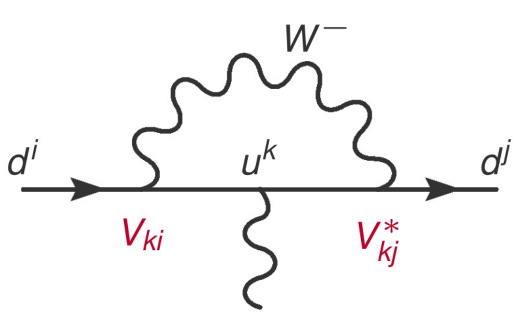
Suppressed flavour-changing neutral currents

$$-\frac{g}{2\sqrt{2}}\left\{\left[\overline{u^i}\gamma^\mu(1-\gamma^5)V_{ij}d^j+\overline{\nu^i}\gamma^\mu(1-\gamma^5)l^j\right]W^+_\mu+\text{h.c.}\right\}$$

$$V = (\mathcal{U}^u_L)^\dagger \mathcal{U}^d_L$$

CKM matrix, it's unitary

FCNC amplitudes can arise at one loop, but suppressed:




$$\sum V_{ki}V^*_{kj}F(m_{u^k}) \sim \delta_{ij}F(m)$$

For equal mass (GIM mechanism!)

# Some structural facts

Suppressed flavour-changing neutral currents

**Proposed exercise**: figure out an extension of the Standard Model that gives tree-level FCNCs.

# Some structural facts

CKM matrix for three generations:

$$V = (\mathcal{U}_L^u)^\dagger \mathcal{U}_L^d$$

CP violation!

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

# Some structural facts

All Shakarov conditions for baryogenesis are present within the SM:

- Baryon number violation

- C violation

- CP violation

Why the three are needed for generating $\Delta B$ ?

# Some structural facts

Let's see what happens in we have baryon number violation but C is conserved:

$$\Gamma(p^+ \rightarrow e^+ \gamma)$$

$$||$$

$$\Gamma(p^+_L \rightarrow e^+_R \gamma_L) + \Gamma(p^+_R \rightarrow e^+_L \gamma_R)$$

|| because C is conserved ||

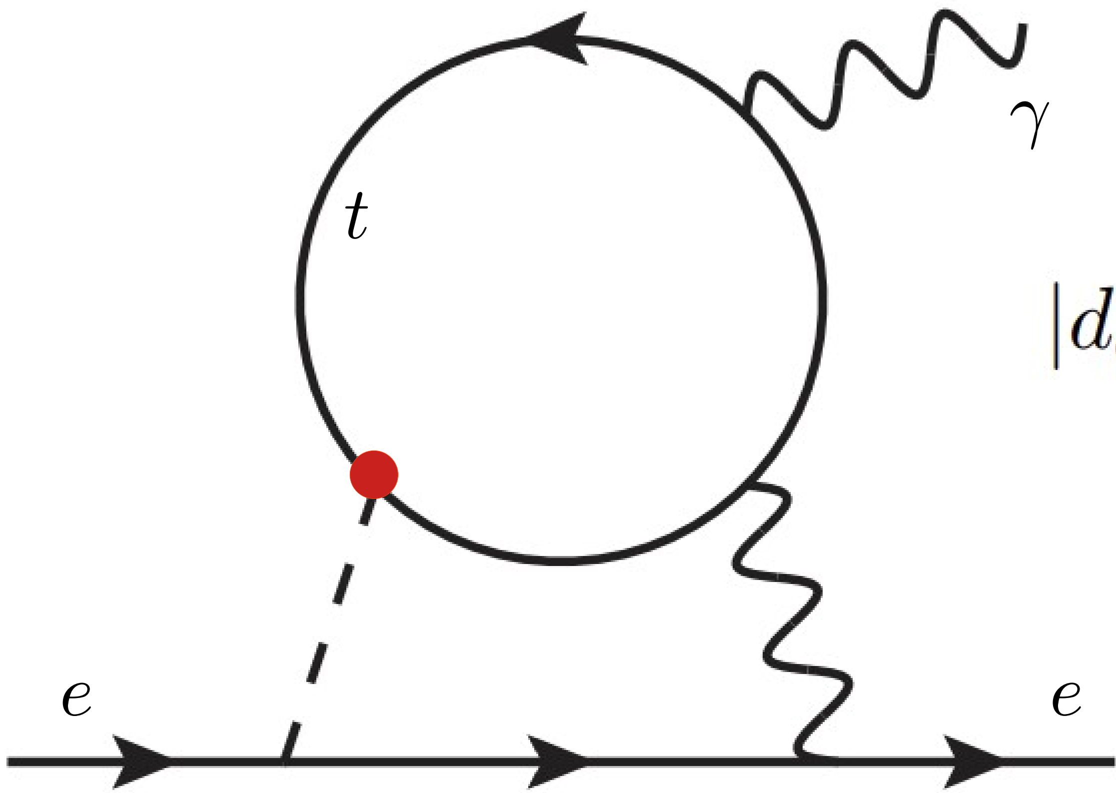
$$\Gamma(p^-_L \rightarrow e^-_R \gamma_L) \qquad \Gamma(p^-_R \rightarrow e^-_L \gamma_R)$$

# Some structural facts

**Proposed exercise:** Show that CP is also needed or else there exist processes that wash out the baryon asymmetry

However, it is well known that the amount of CPV within the SM it is not enough for baryogenesis

One can add new sources of CPV, but this (seemingly) conflicts with experiment; e.g.:



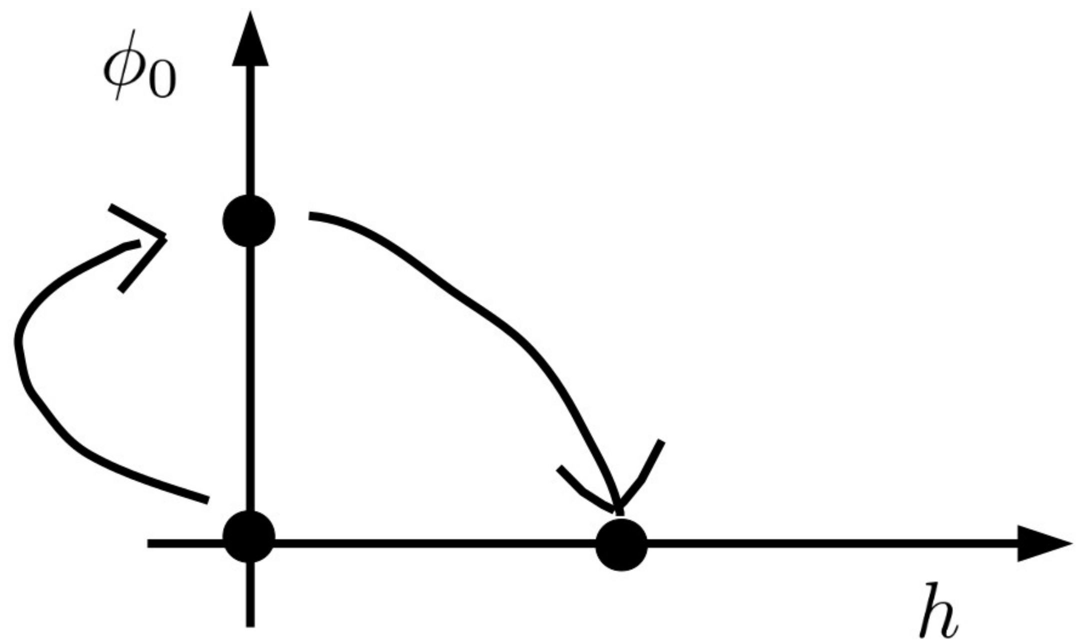

$$-\frac{i}{2}d_e(\mu)\,\bar{e}\sigma_{\mu\nu}\gamma_5 e F^{\mu\nu}$$

$$|d_e| < 1.1 \cdot 10^{-29}\,\mathrm{e}\cdot\mathrm{cm}$$

ACME experiment

A way out: spontaneous CPV.

CPV must occur in the early Univerise, namely at high T (it can be negligible at current experiments!)




$$m_t \sim y_t \left[ \langle h \rangle + i \frac{c}{f} \langle \phi_0 \rangle \right] = |m_t| e^{i\Theta}$$

# Some structural facts

**Anomalies**: classical symmetries broken at the quanutm level.
Global anomalies are OK, but gauge anomalies are forbidden!

Among other conditions, for the fields of the SM we get:

$$Q_\nu + Q_e + N_c(Q_u + Q_d) = -1 + \frac{1}{3}N_c = 0 \quad \Rightarrow \quad N_c = 3 \quad (!!)$$

Leptons and quarks needed in every generation!

# Some numerical accidents?

## Gauge coupling evolution

$$\beta_g = 16\pi^2 \mu \frac{dg}{d\mu} \Rightarrow g(\mu)$$

# Some numerical accidents?

## Renormalisation in a nutshell

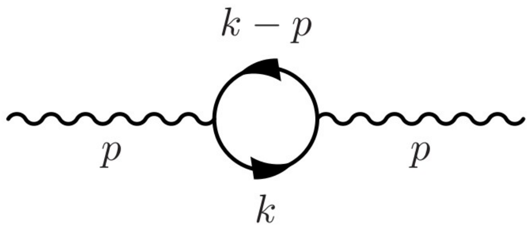
All divergences are local, namely they can be cast in Lagrangian form

$$L_{QED}^{one-loop} = Z_\psi\, i\bar{\psi}\gamma^\mu D_\mu \psi, \quad Z_\psi = 1 + \mathcal{O}(1/\epsilon), \quad D = 4 - 2\epsilon$$

$$= (Z_\psi \bar{\psi}\gamma^\mu \partial_\mu \psi + \sqrt{Z_A} Z_e Z_\psi e \bar{\psi}\gamma^\mu \psi A_\mu)$$

=1

$$Z_e = Z_A^{-1/2} \Rightarrow$$



# Some numerical accidents?

Gauge coupling running. Renormalisation in a nutshell

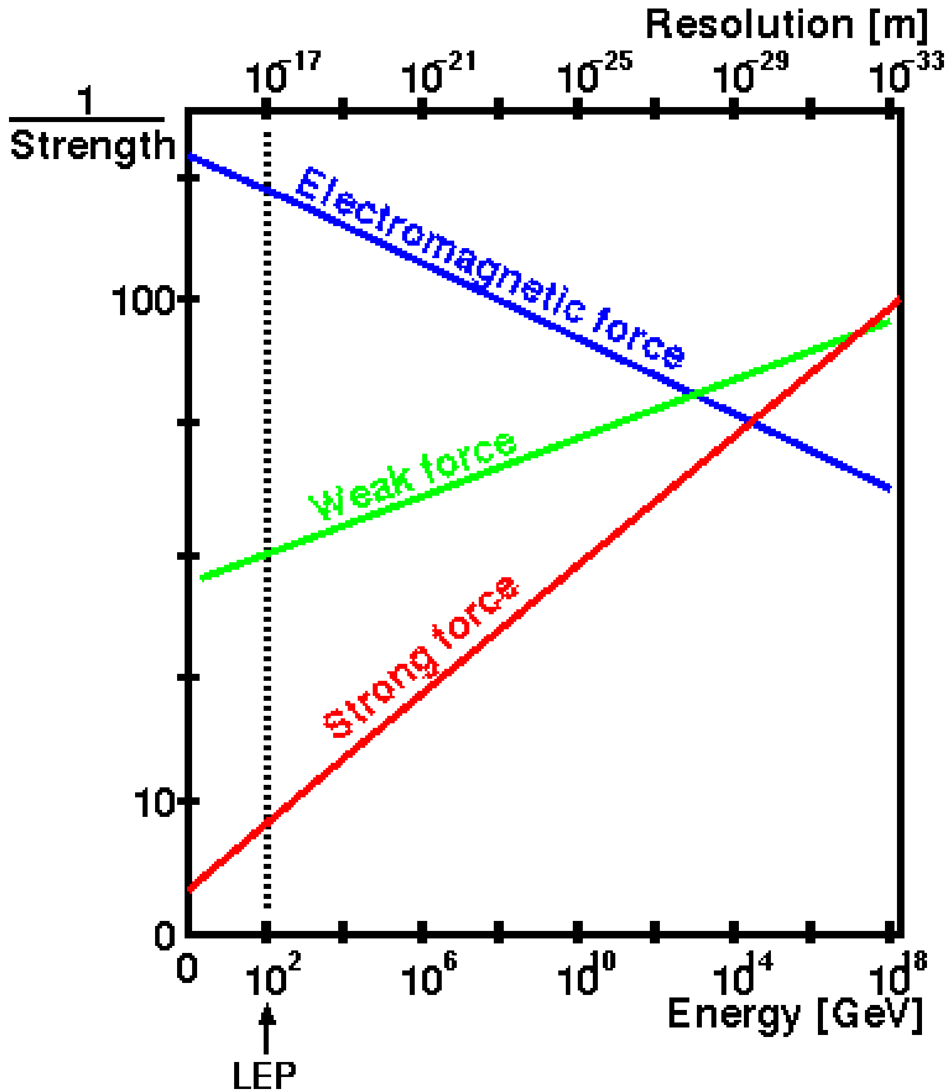
**Anecdotal comment**: This does not work for Yang-Mills out of the box, because quantization breaks gauge invariance (gauge fixing)

$\rightarrow$ Use the so-called background field method instead for QCD and SU(2)!

**Proposed exercise**: Compute the beta function of QED and demonstrate that it grows at high energies

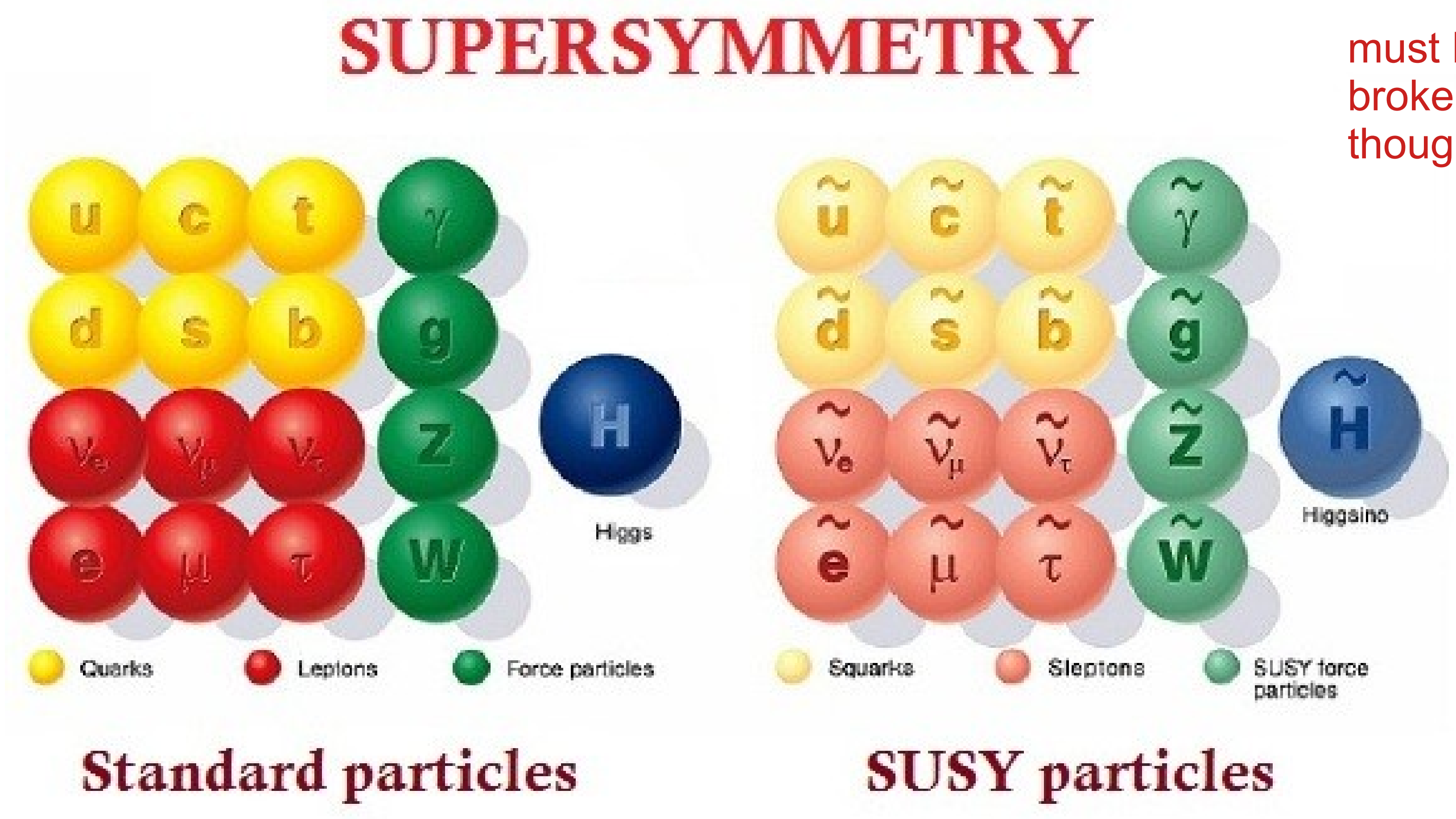
# Some numerical accidents?

Gauge coupling running



# Some numerical accidents?

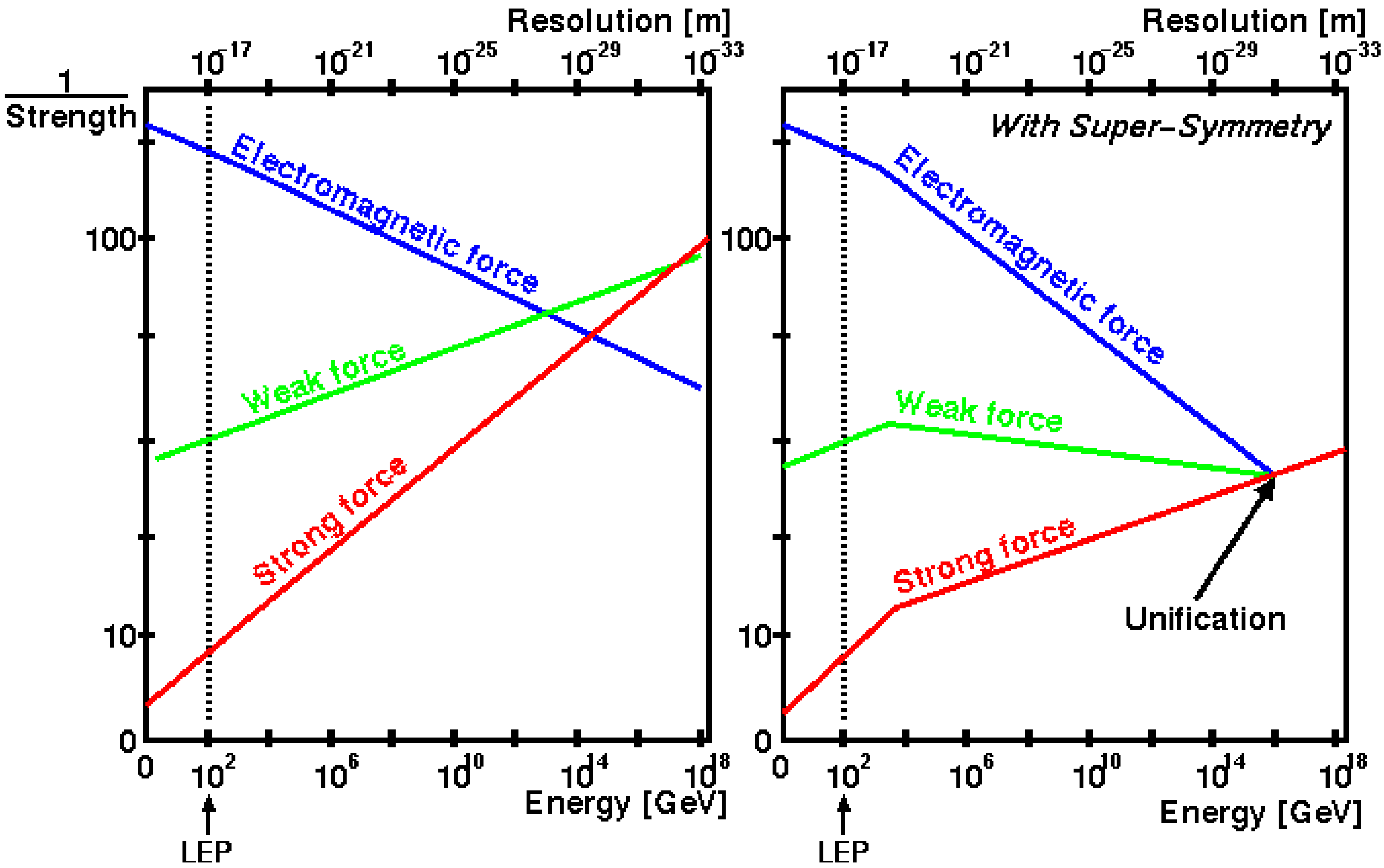
Gauge coupling running: Let's talk about SUSY




must be broken though!

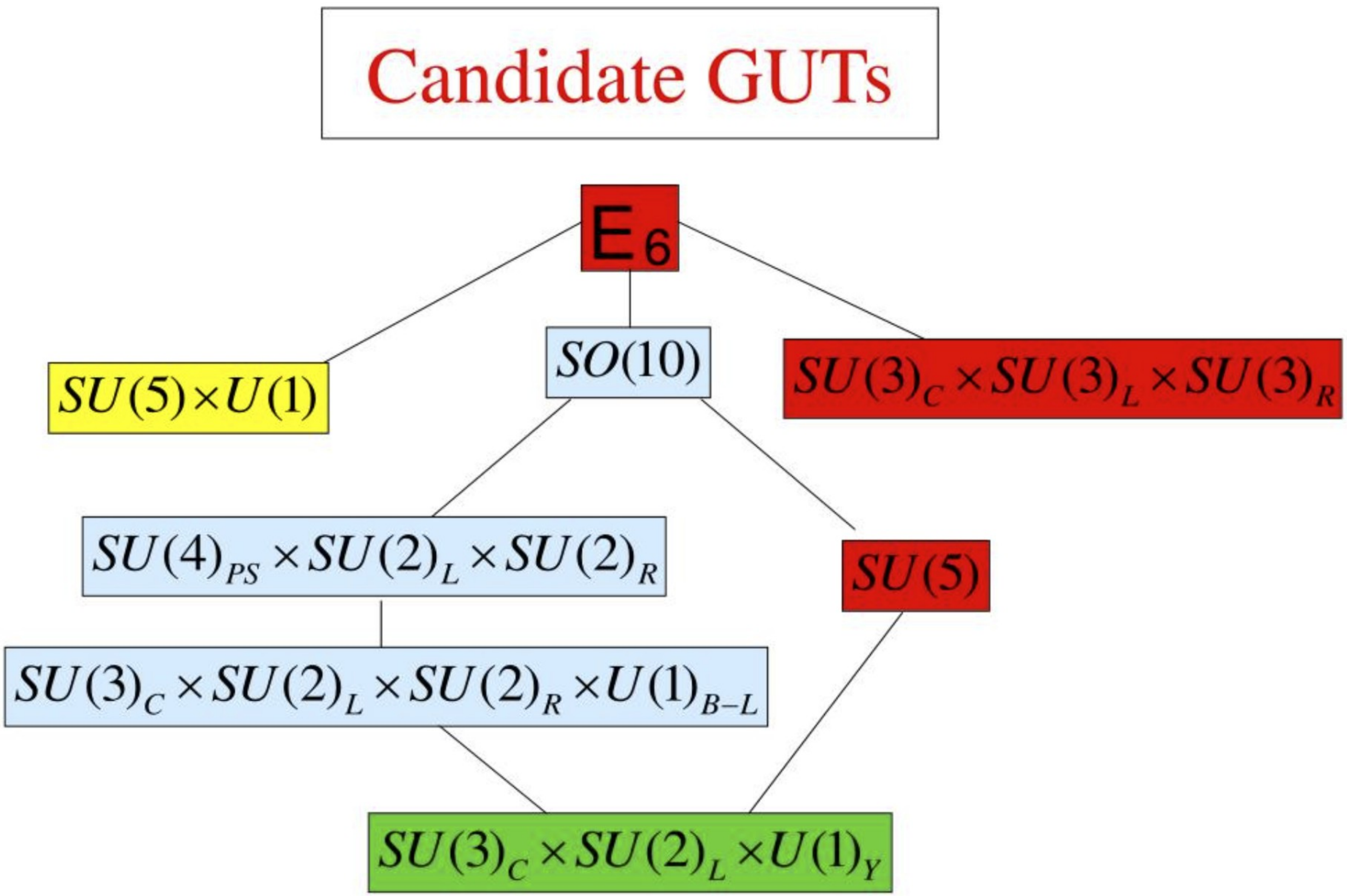
# Some numerical accidents?

Gauge coupling running: Let's talk about SUSY



# Some numerical accidents?

So, at very high energies we could have a single group...



# Some numerical accidents?

SU(5)

$$\bar{\mathbf{5}}_F = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix} \qquad \mathbf{10}_F = \left(\begin{array}{ccc|cc} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ \hline -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{array}\right)$$

SO(10)

$$16_F = (Q, u^c, d^c, L, \nu^c, e^c)$$

Still, 3 families like in the SM

# Some numerical accidents?

Georgi–Jarlskog mass relation

$$m_b = m_\tau, \quad m_\mu = 3m_s, \quad m_d = 3m_e$$

Relations that hold at the GUT scale

# Some numerical accidents?

One group to unify them all...

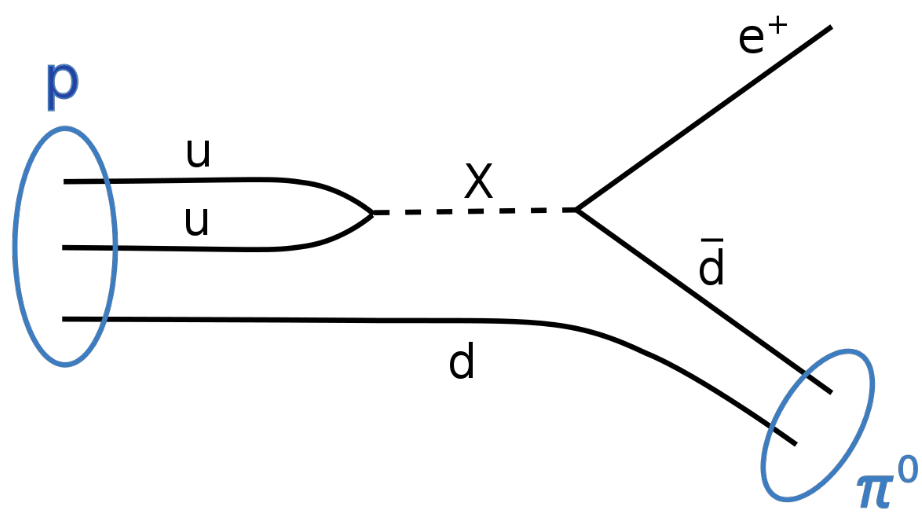
$$\underbrace{\mathbf{171}}_{\mathrm{SU}(19)} \to \underbrace{4\mathrm{Q} + 4\mathrm{u}^c + 5\mathrm{d}^c + 5\mathrm{L} + 4\mathrm{e}^c + \mathrm{Q}^c + \mathrm{u} + 2\mathrm{d} + 2\mathrm{L}^c + \mathrm{e} + (\text{more vector fermions})}_{\mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}}$$

Ekstedt, Fonseca and
Malinsky '20

# Some numerical accidents?

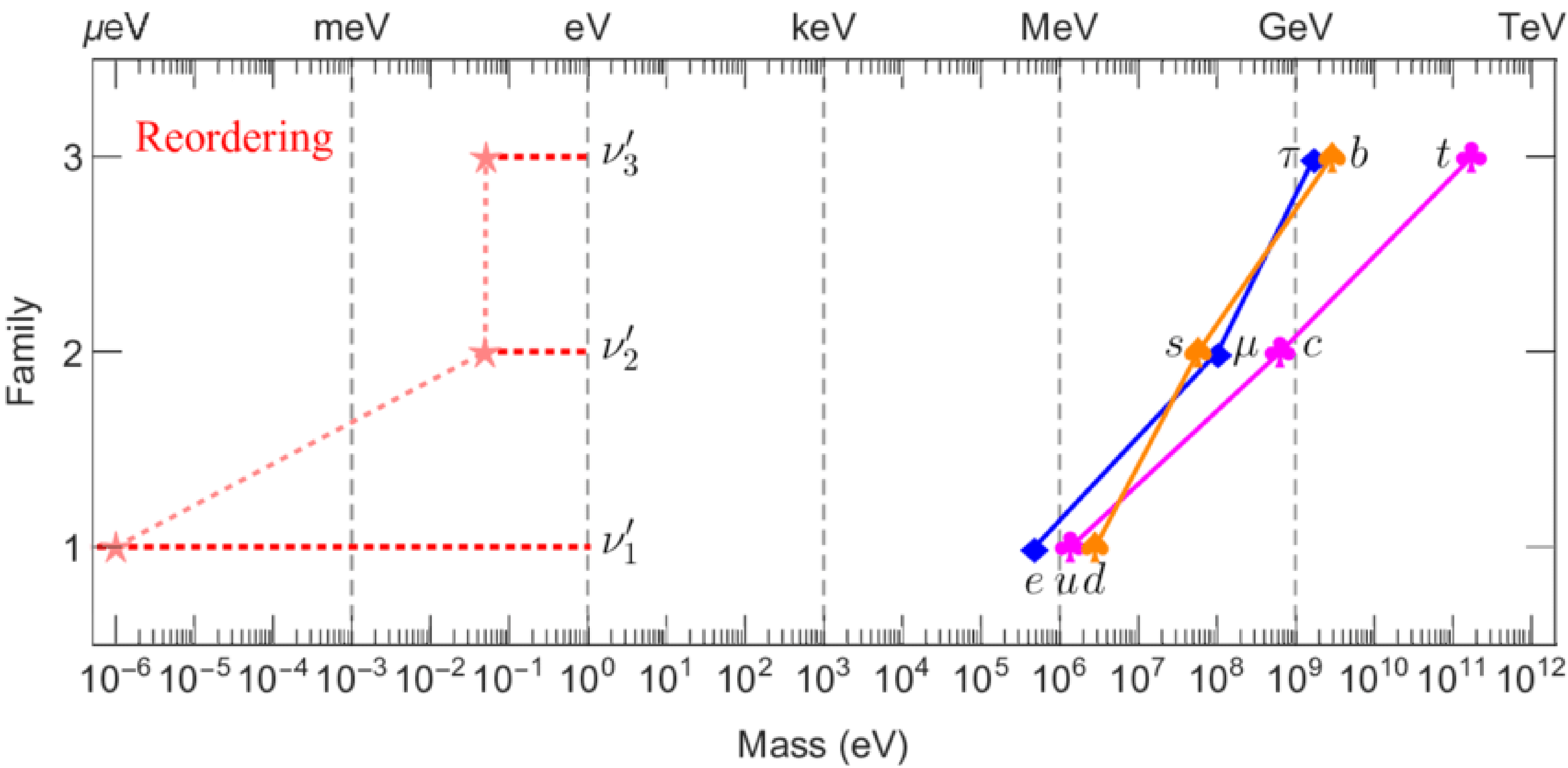
Problems:

* hard computations
* gravity effects not always under control
* decoupling all scalars but the Higgs
* **proton decay!**



# Some numerical accidents?

Mass hierarchy




Zhi-zhong Xing and Jing-yu Zhu
2017 Chinese Phys. C 41 123103

# Some numerical accidents?

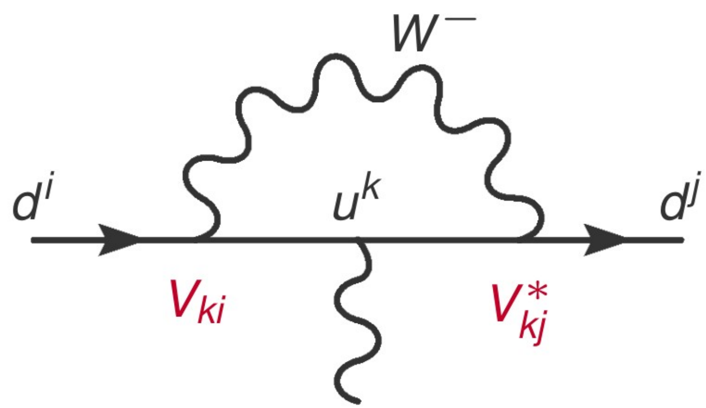
Hierarchical CKM

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97370 \pm 0.00014 & 0.2245 \pm 0.0008 & 0.00382 \pm 0.00024 \\ 0.221 \pm 0.004 & 0.987 \pm 0.011 & 0.0410 \pm 0.0014 \\ 0.0080 \pm 0.0003 & 0.0388 \pm 0.0011 & 1.013 \pm 0.030 \end{bmatrix}$$

$$V_{\mathrm{CKM}} \sim 1$$

# Some numerical accidents?

Hierarchical CKM




Complementary to the GIM mechanism!

$$\sum V_{ki}V^*_{kj}F(m_{u^k}) = V_{ui}V^*_{uj}F(m_u) + V_{ci}V^*_{cj}F(m_c) + V_{ti}V^*_{tj}F(m_t)$$

$$\sim (V_{ui}V^*_{uj} + V_{ci}V^*_{cj})F(0) + V_{ti}V^*_{tj}F(m_t)$$

$$\sim V_{ti}V^*_{tj}[F(m_t) - F(0)]$$

Some (many?) people claim this hierarchy of masses **cries** for an explanation

One possibility: GUTs, but hard to make them work properly

Other possibility: Frogatt-Nielsen-like models:

$$L_Y = y_t \overline{q_L^3} H t_R + y_u \overline{q_L^1} H u_R + y_u \overline{q_L^1} H \frac{S}{f} u_R$$

[e.g.: Q(qL1) = Q(uR) = -1, Q(S) = 2, Q(rest) = 0]

$$y_t \sim y_u \Rightarrow m_t \gg m_u$$

You predict the hierarchy?

What's the argument?

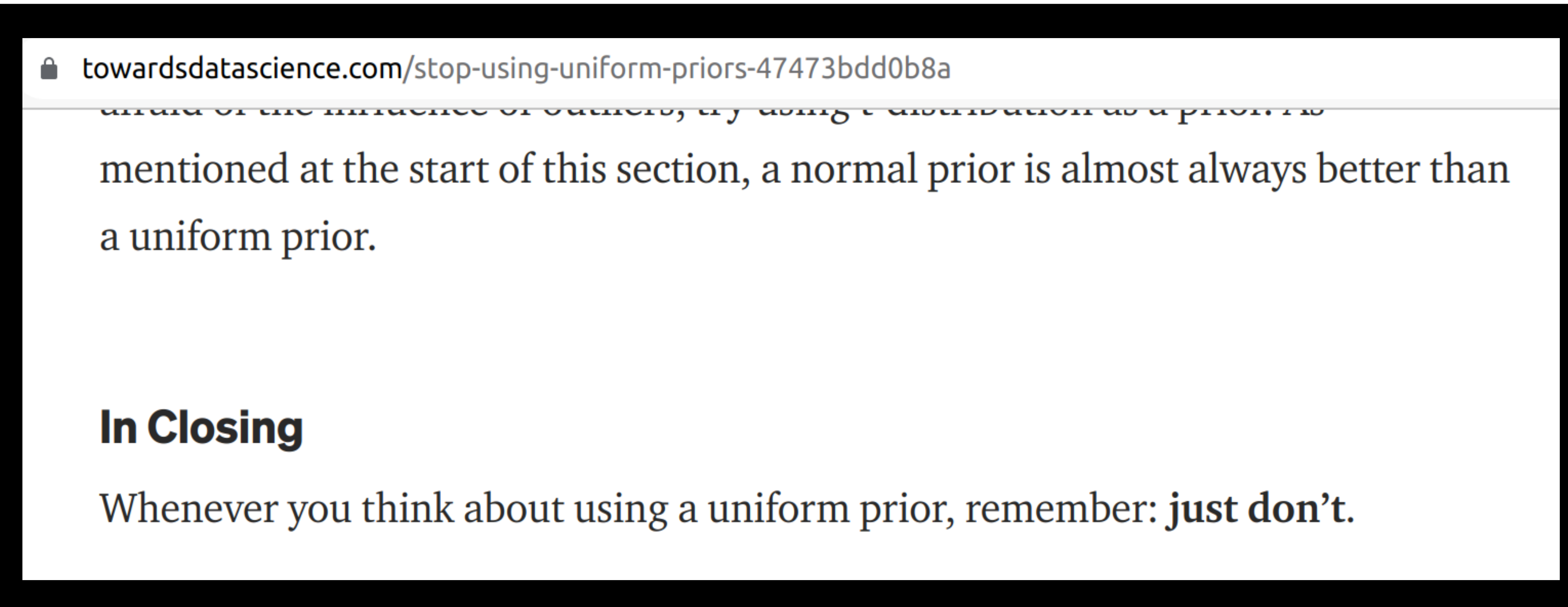
If $yt$ and $yu$ are taken randomly from a **flat distribution**, the probability of $yt/yu$ being O(1) is large, while $yt/yu$ being O(10000) is small

Why assuming a flat distribution on the first place? Because in the absence of other knowledge, flat distribution = maximal ignorance

OK, do the same reasoning but for $\log(yt)/\log(yu)$ instead. What's the result in this case?

# What's the argument?

If *yt* and *yu* are taken randomly from a **flat**


towardsdatascience.com/stop-using-uniform-priors-47473bdd0b8a

mentioned at the start of this section, a normal prior is almost always better than a uniform prior.

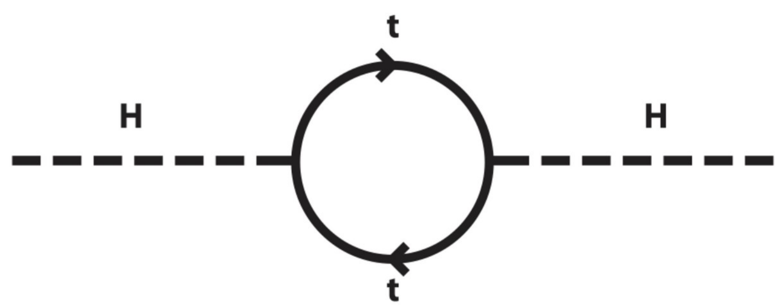
## In Closing

Whenever you think about using a uniform prior, remember: **just don't**.

instead. What's the result in this case?

# Some numerical accidents?

Higgs mass: the wrong calculation




$$\int_0^{\Lambda} \cdots \Rightarrow \ \delta m_H^2 = \left[\frac{1}{4}(9g^2 + 2g'^2) - 6y_t^2 + 6\lambda\right]\frac{\Lambda^2}{32\pi^2}$$

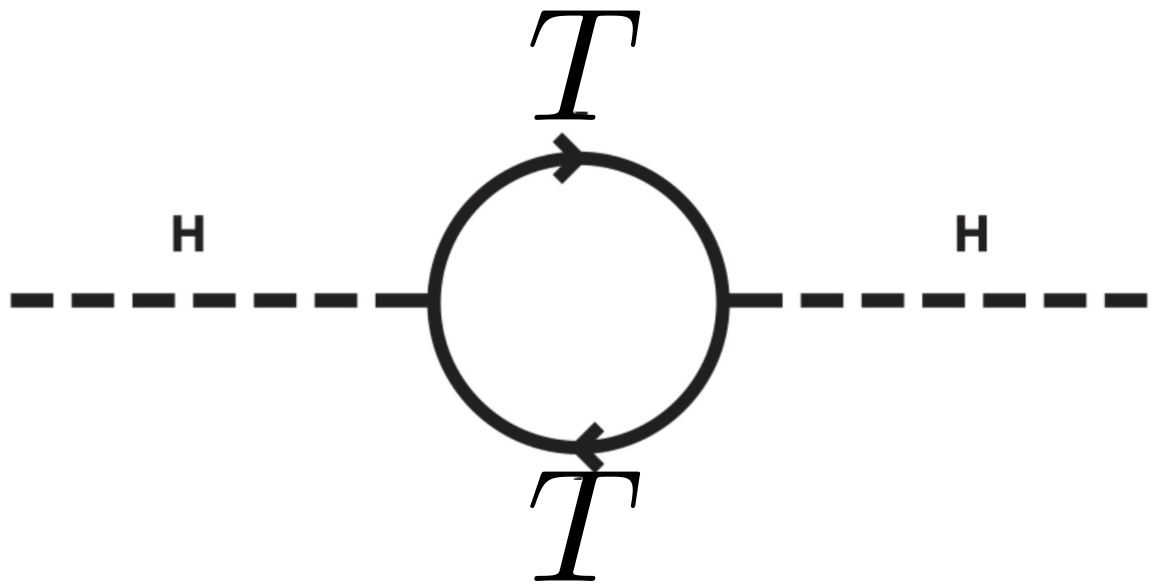
# Stability under quantum corrections

Masses and mixing parameters (including the Higgs mass!) are stable under **Standard Model** quantum corrections

$$\beta_{\mu^2} = \left[2\mathrm{Tr}(y^e y^{e\dagger}) + 6\mathrm{Tr}(y^u y^{u\dagger}) + 6\mathrm{Tr}(y^d y^{d\dagger}) - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 - 12\lambda\right]\mu^2$$

$$\beta_{y^u} = \left\{\frac{3}{2}y^u y^{u\dagger} - \frac{3}{2}y^d y^{d\dagger} + 3\left[\mathrm{Tr}(y^u y^{u\dagger}) + \mathrm{Tr}(y^d y^{d\dagger})\right] + \mathrm{Tr}(y^e y^{e\dagger}) - \frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right\} y^u$$

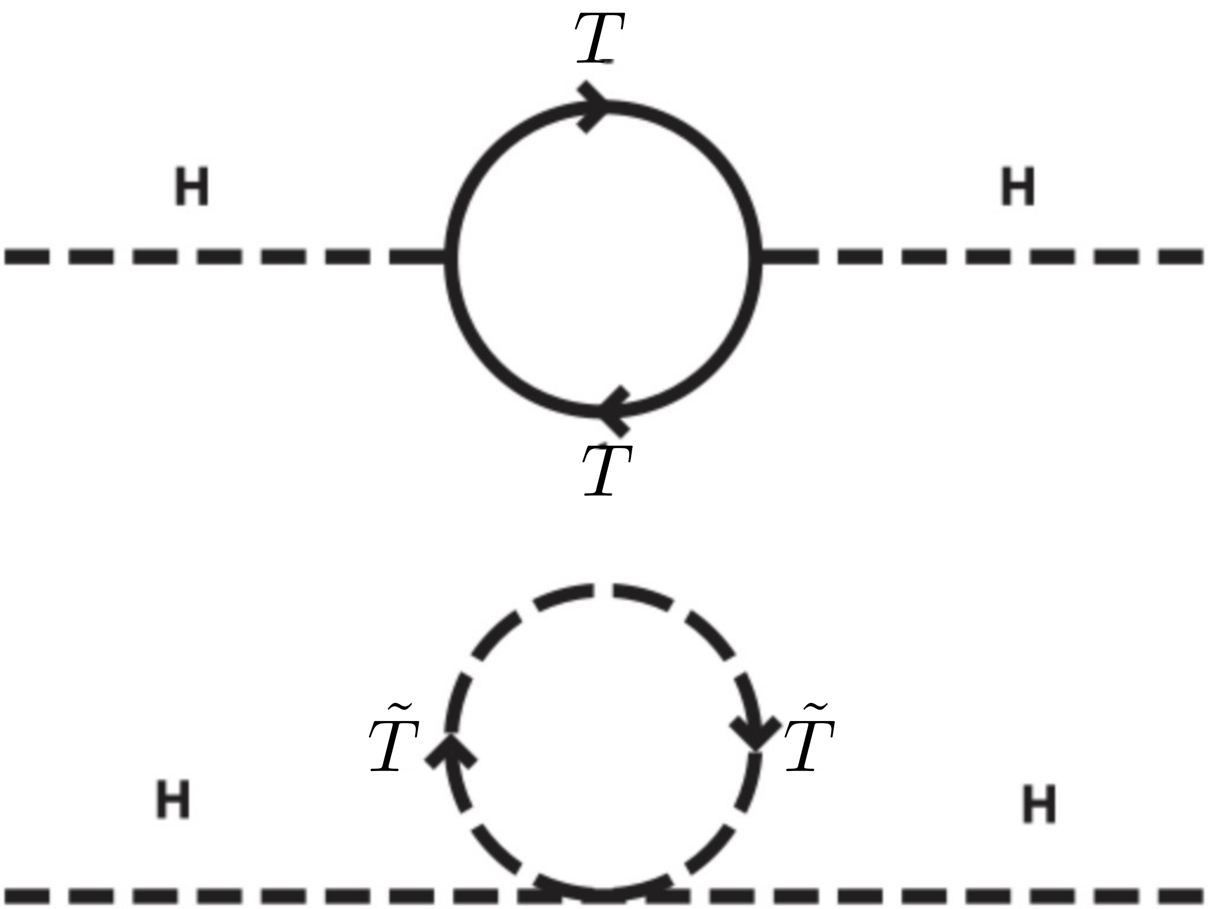
The actual hierarchy problem




$$m_H^2 \sim m_{H^0}^2 - m_T^2$$

Fine tuning subjective (like for CKM).
However, prediction of charm quark indication that naturalness might work...

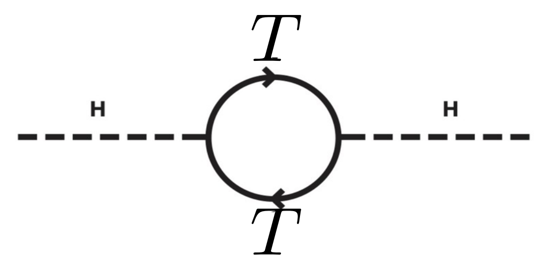
# Most clear solution: SUSY again




Difficulty: susy must be broken

# One more solution: composite Higgs models

Cannot resolve the loop to high energies because I see the Higgs structure...




The constituents of the Higgs make also other composite particles: most importantly **vector-like quarks (VLQs)**

VLQs should be much heavier than the Higgs, otherwise we would have seen them already

Solution? Copy the structure of QCD

**QCD** $$SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$$

3 Goldstones, the pions, much lighter than other hadrons

**CHMs** $$G \rightarrow H$$

4 Goldstones, the Higgs degrees of freedom, much lighter than VLQs

Who can be $G$ and $H$?

# Solution? Copy the structure of QCD

**QCD**

$$SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$$

3 Goldstones, the pions, much lighter than other hadrons

**CHMs**

$$G \to H$$

4 Goldstones, the Higgs degrees of freedom, much lighter than VLQs

Who can be $G$ and $H$?

$$SO(5) \to SO(4)$$

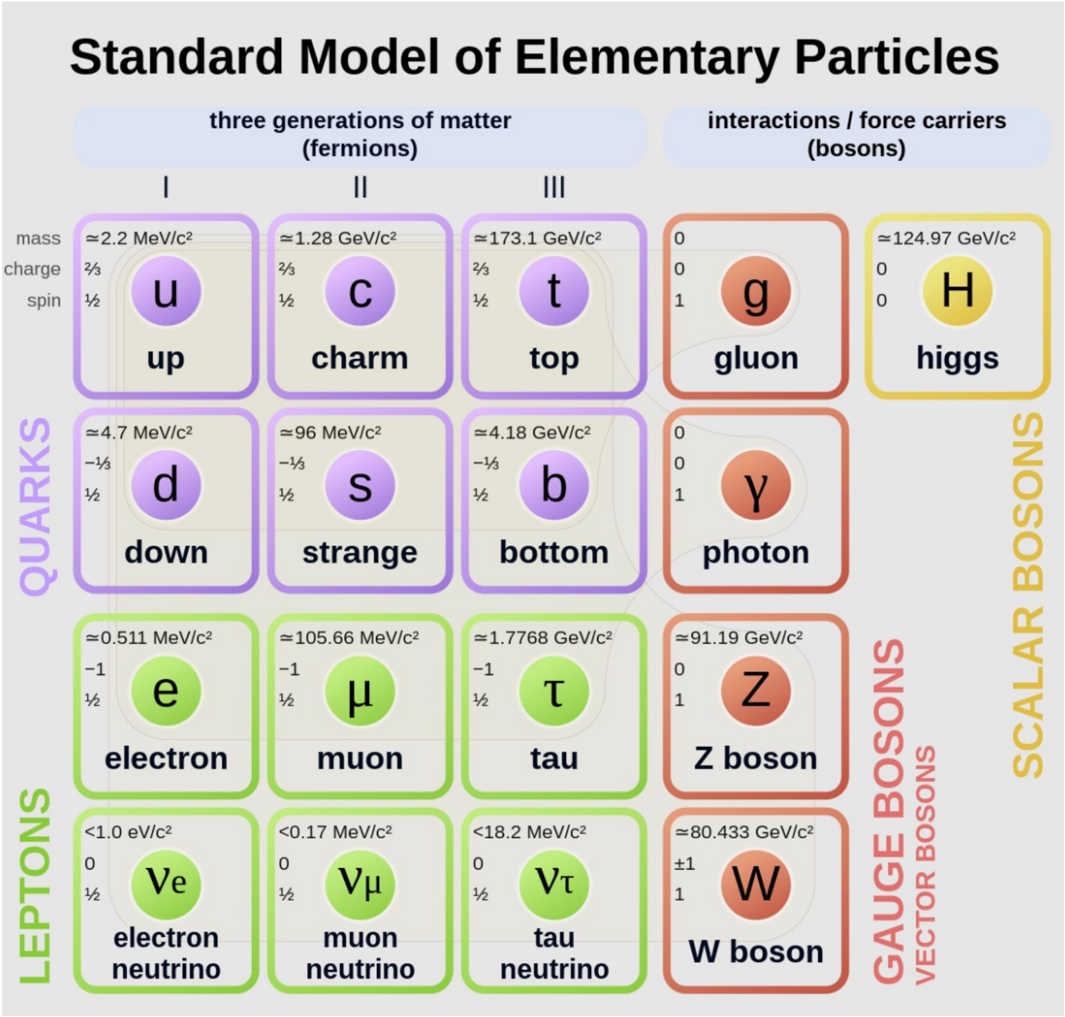
$$SO(6) \to SO(5)$$

$$\cdots$$

Difficulty: *G* must be broken

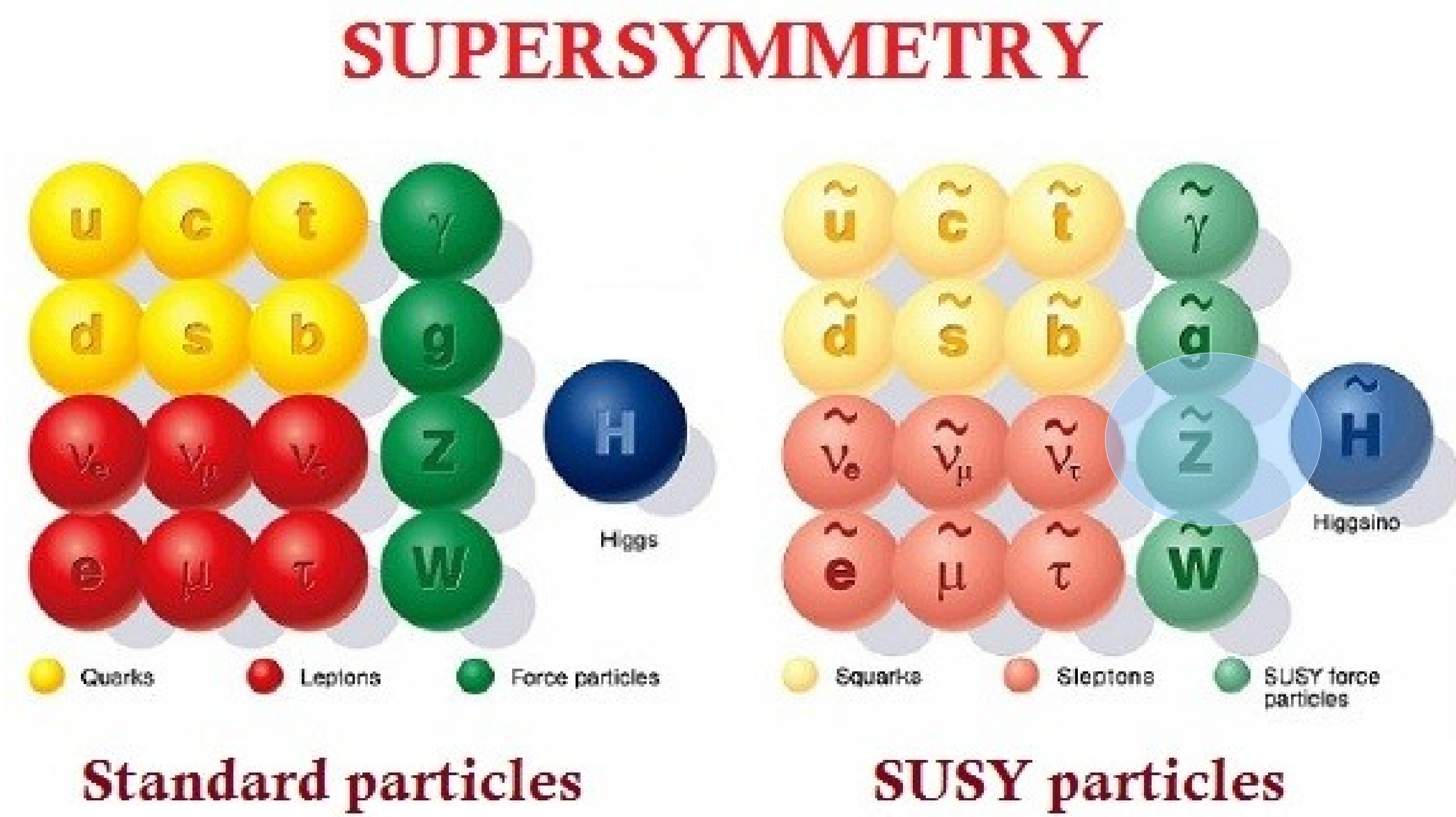
Some more fundamental problem: dark matter (DM)

One interesting hypothesis: DM is formed by **neutral weakly-interacting non-relativistic** particles



Some more fundamental problem: dark matter (DM)

One interesting hypothesis: DM is formed by **neutral weakly-interacting non-relativistic** particles



Composite Higgs models also work! e.g. SO(6)→ SO(5)

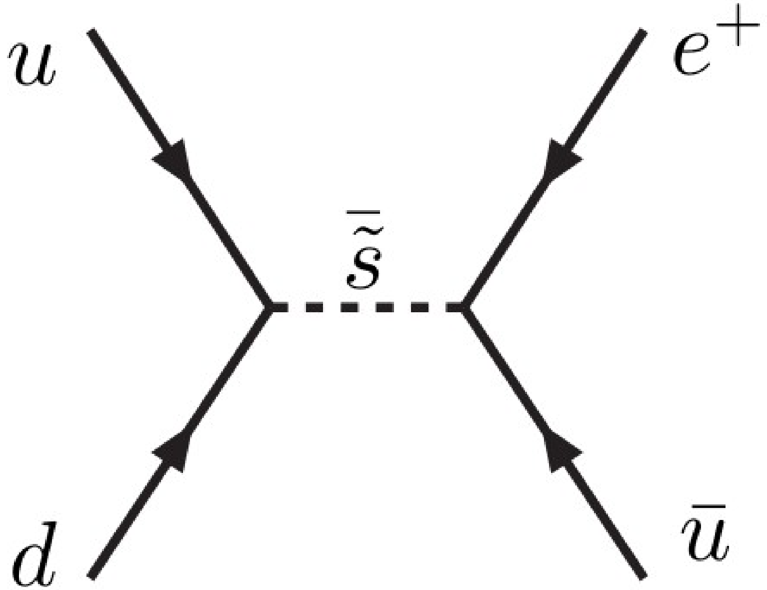
$$
\begin{aligned}
L = &\frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu S)^2 \\
&- \left\{ \frac{1}{2}\mu^2 h^2 + \frac{1}{4}\lambda_h h^4 + \frac{1}{2}\mu_S^2 S^2 + \frac{1}{3}\kappa S^3 \right. \\
&\left. + \frac{1}{4}\lambda_S S^4 + \frac{1}{3}\kappa_{hS} h^2 S + \frac{1}{4}\lambda_{hS} h^2 S^2 \right\}
\end{aligned}
$$

# A comparison with R-parity in SUSY

(motivation relies *also* on dark matter)

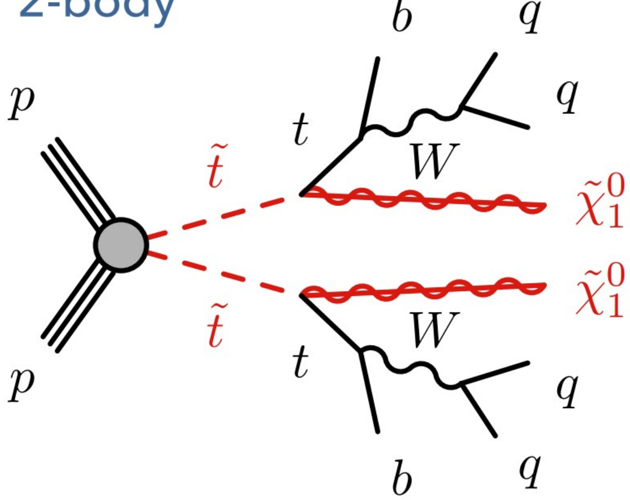
**Avoiding proton decay**

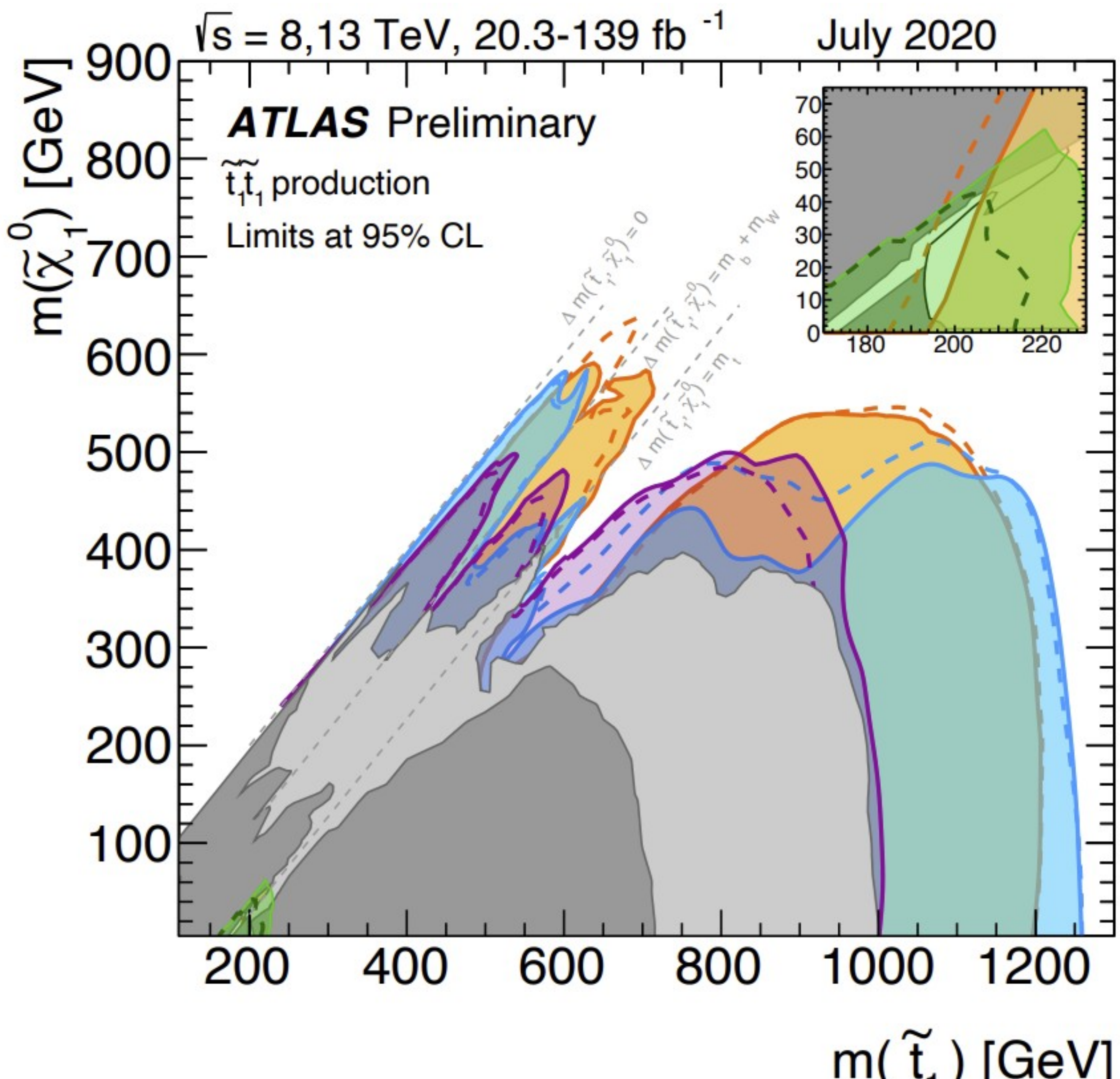
- Baryon parity
- Lepton parity
- Many others, see *e.g.* Smirnov and Visani, *9601387*. None prefered by GUT



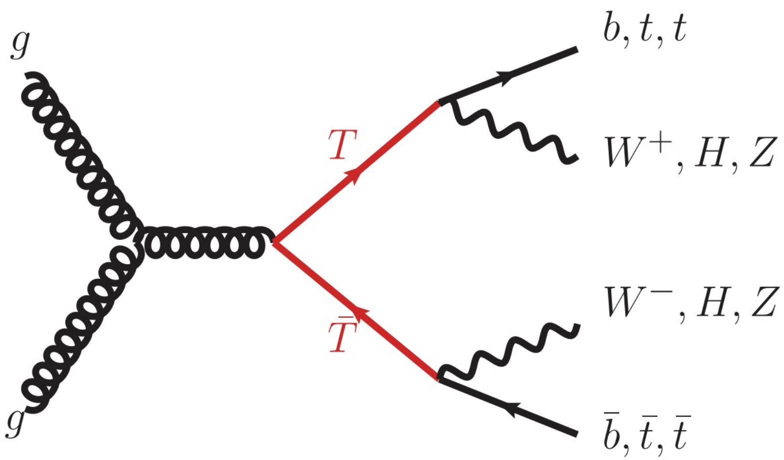
# Signals and status of SUSY and CHMs:

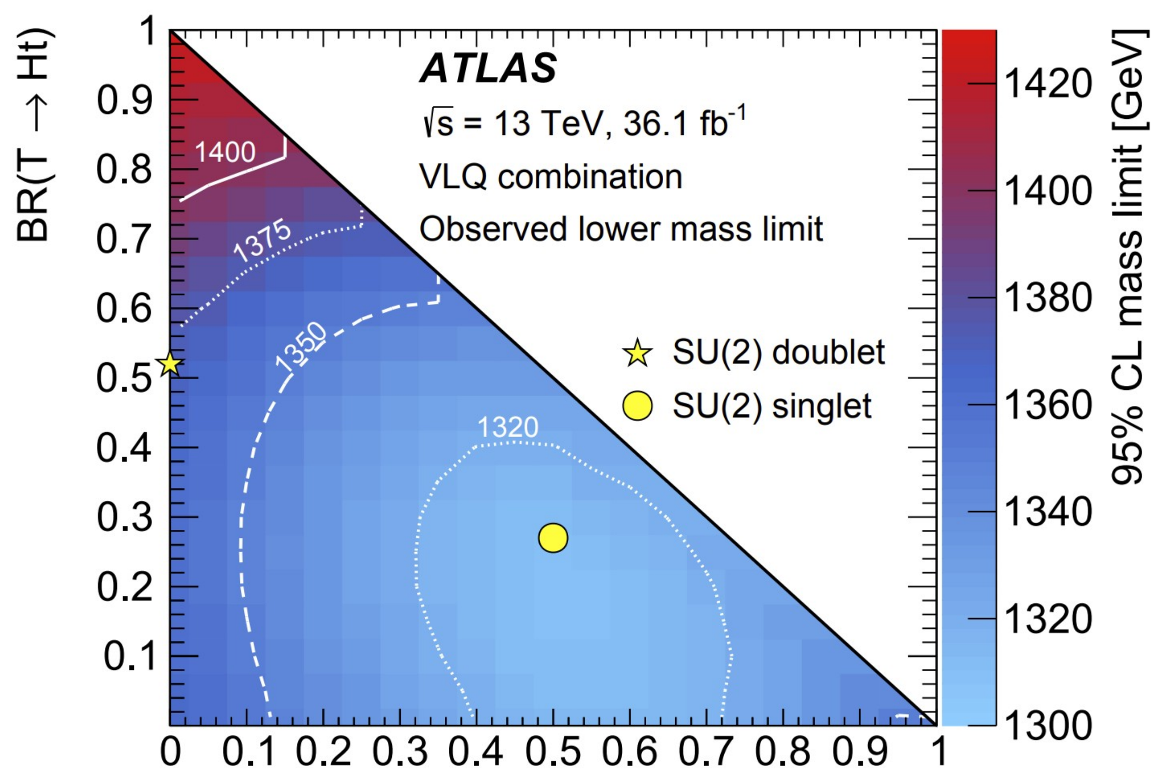
2-body



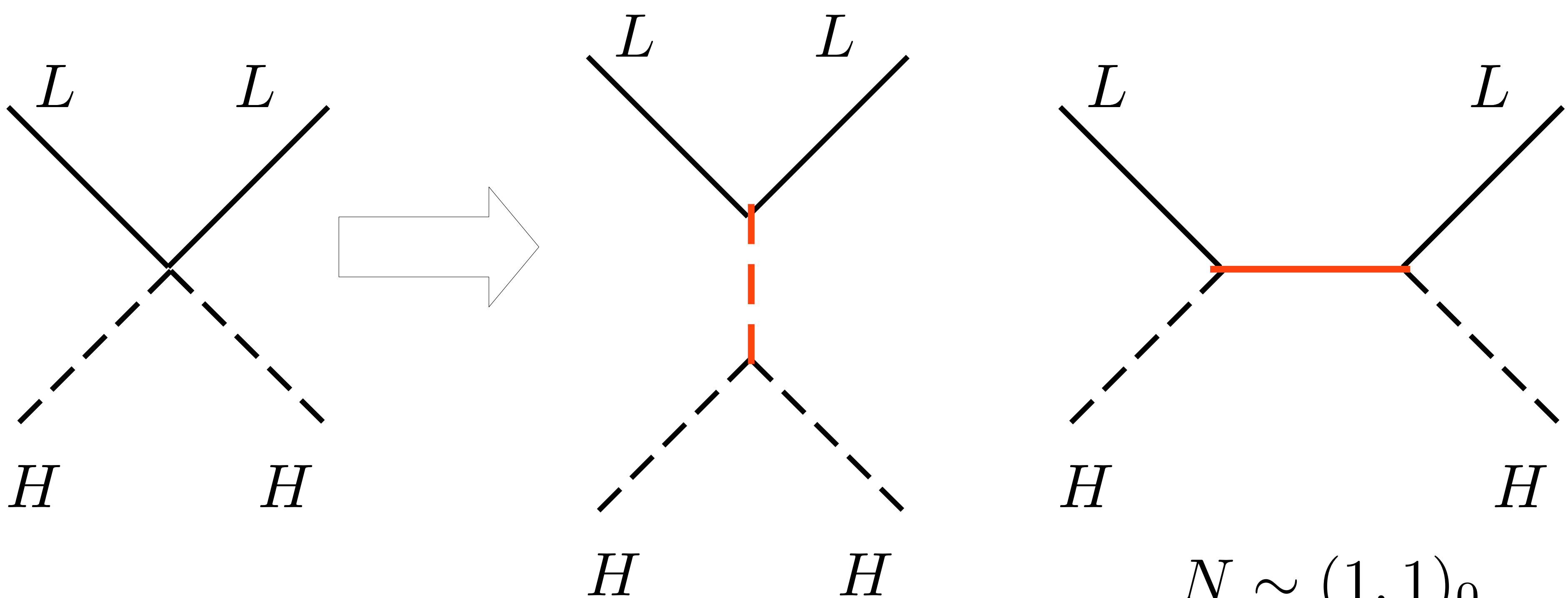
# Signals and status of SUSY and CHMs:



# Neutrino masses

They can arise at dimension-5. How can this be completed in the UV? $\mathcal{O}^{(5)} = (\overline{L_L^c}\tilde{\phi}^*)(\tilde{\phi}^\dagger L_L)$



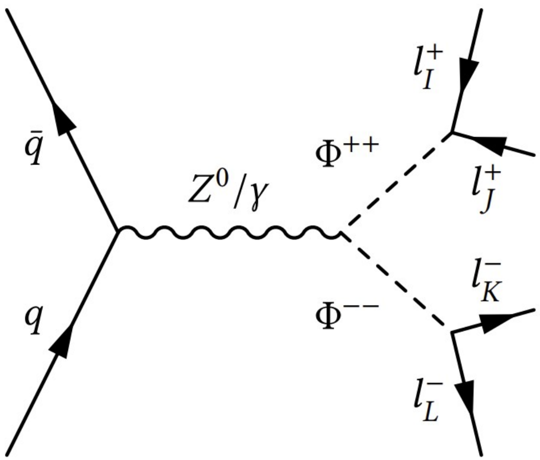

$\Delta \sim (1,3)_1$

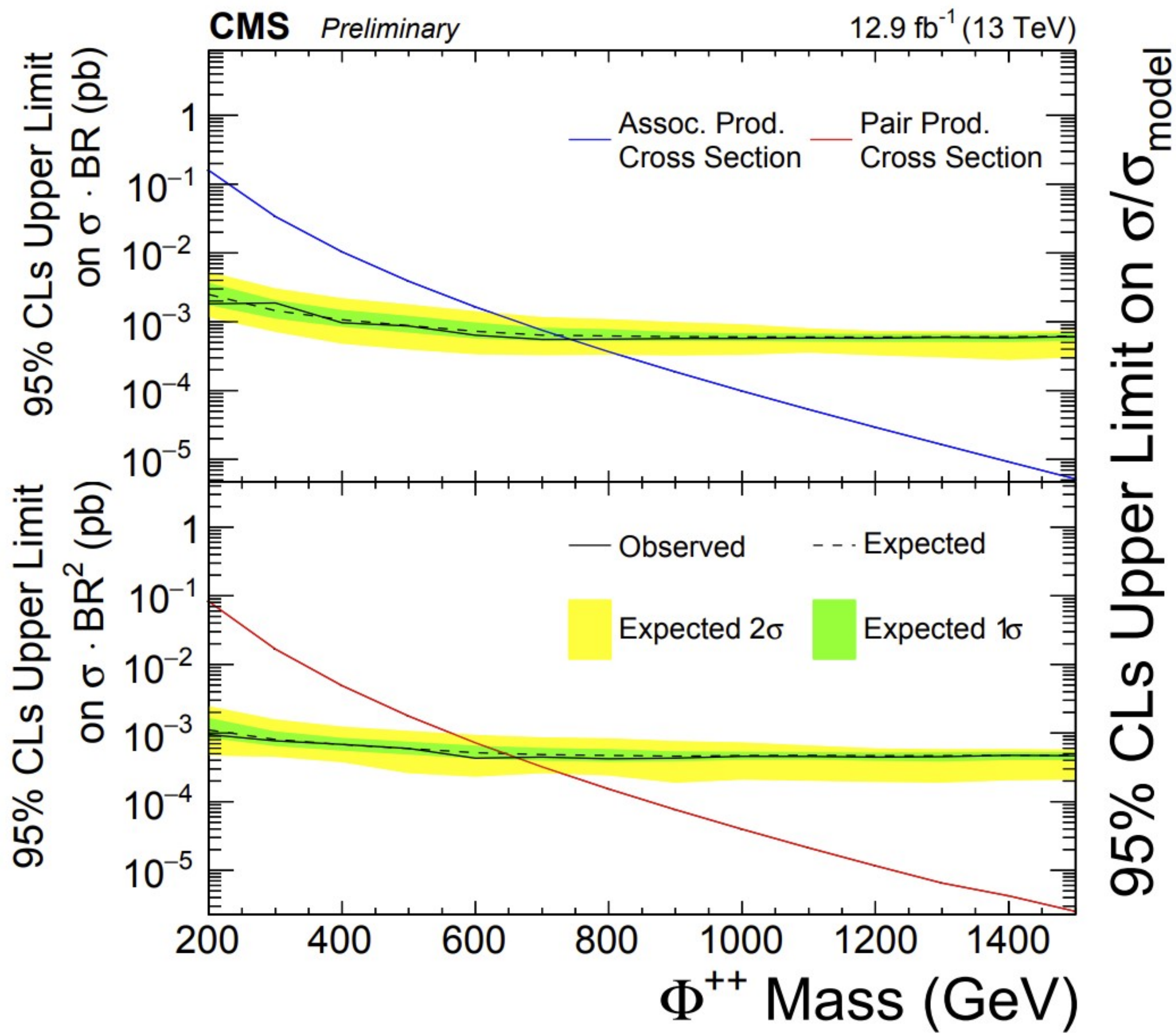
$N \sim (1,1)_0$

$E \sim (1,3)_0$

# Neutrino masses

Searches at the LHC

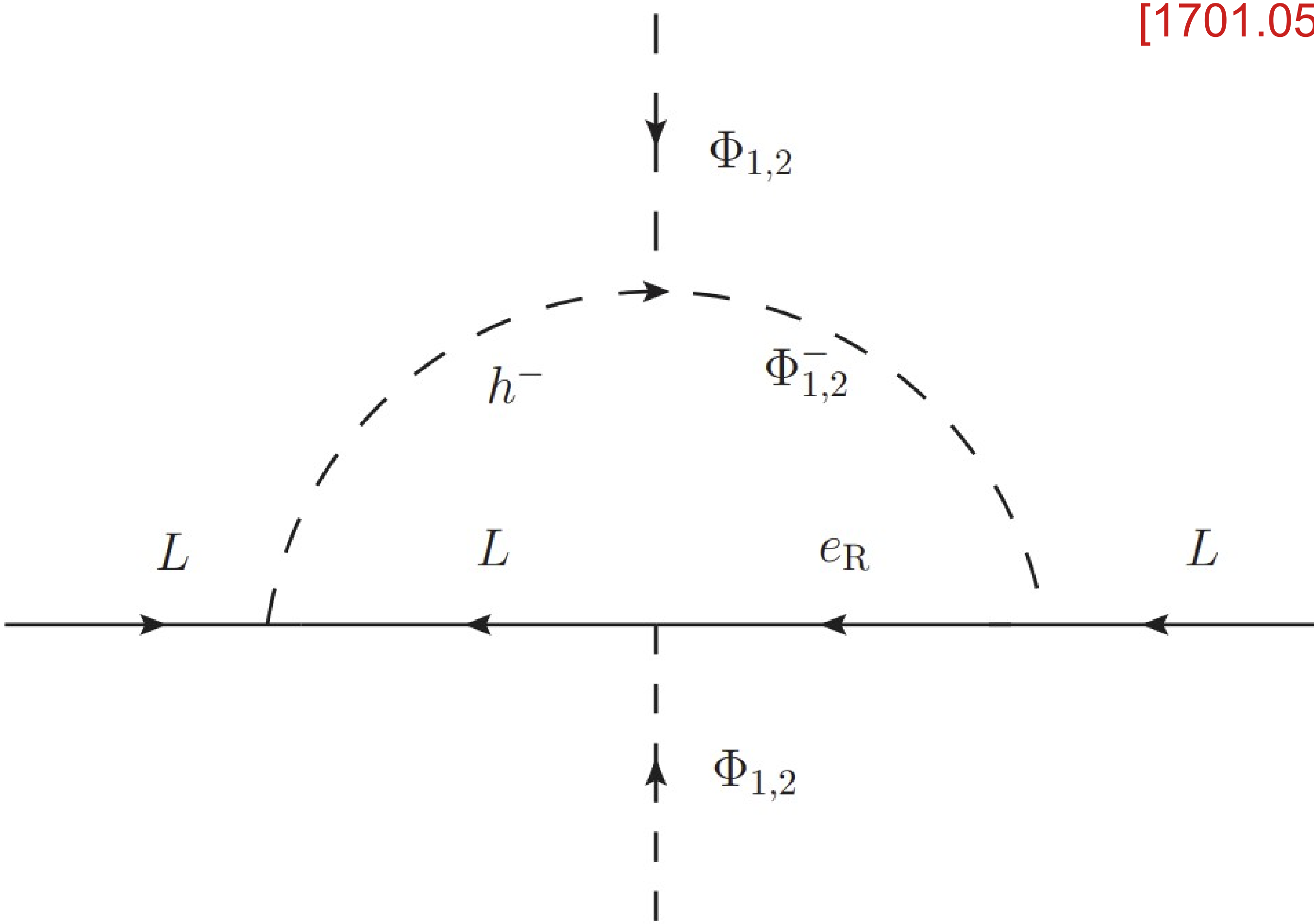


# Neutrino masses

Low-mass models?

Zee model
[picture from
[1701.05345]



- Other (potential) problems of the SM:

  * g-2, flavour anomalies, ...

  * Why three generations?
  * Why charge is quantised?

  * How to make computations without using fields? What do we learn from that?