Standard Model and open problems

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1 Short questions (to make you think)

1. Why are the electron and the proton stable within the Standard Model?
2. What are the numbers of degrees of freedom of a massless and of a massive scalar? What about spin-1/2 fermions and spin-1 bosons?
3. How many degrees of freedom does a graviton have, and how many has the corresponding $h_{\mu\nu}$ field?
4. Why macroscopic photon systems show quantum behaviour (for example optical interference) without (fast) decoherence?
5. Which effects, not yet observed experimentally, can be expected if $U(1)_Q$ is broken?
6. Which experimental observations can not be explained if $SU(2)_L$ is unbroken?
7. Why is the Higgs field a scalar (it does not have spin)?
8. How would you measure the different free parameters of QED?
9. Figure out one extension of the Standard Model that could induce tree-level flavour-changing neutral currents.
10. Draw Feynman diagrams for Higgs production at electron-positron, proton-antiproton and proton-proton colliders.
11. How could you measure the Higgs quartic coupling?
12. Why both C and CP must be broken to account for the baryon assymmetry?
13. Why all known elementary particles have spin $s \leq 2$?

2 Detailed computations

1. Show that
$$V_H = (D^\mu H^\dagger)(D_\mu H) + \mu_H^2 H^\dagger H - \lambda (H^\dagger H)^2$$
is indeed the most general renormalisable Higgs Lagrangian.

2. Compute the decay width for $Z \to e^+ e^-$.

3. Show, by explicit calculation, that there are no vertices with only γ and Z within the Standard Model. Can you find a counterexample to this observation if renormalisability is abandoned?

4. Show that if the Higgs boson is a colourless $SU(2)_L$ triplet with $Y = 1$, then we have $\rho \equiv \dfrac{m_W^2}{c_W^2 m_Z^2} \neq 1$.

5. Compute the β function of QED and demonstrate that it grows at high energies.

6. The amplitude for an elastic 2-to-2 process can be decomposed in partial waves as $\mathcal{M}(\theta) = 16\pi \sum_{j=0}^{\infty} a_j (2j+1) P_j(\cos\theta)$, with P_j being the Legendre polynomials and where angle θ is related to the Mandesltan variable $t = (p_1 - p_3)^2$ via $t = -1/2 E_{\text{cm}}^2 (1 - \cos\theta)$. Perturbative unitarity implies that $|a_j| \leq 1$. Using this result for $W_L Z_L \to W_L Z_L$, with cross section
$$\mathcal{M} = -\frac{m_Z^2 c_W^2 e^2}{4 m_W^4 s_W^2}(s+u)\,, \tag{1}$$
demonstrate that the Standard Model without the Higgs boson is not a valid theory at energies $E \gtrsim$ few TeV. (Computing Eq. (1) is highly recommend, but it is a very long exercise.)

7. Consider an extension of the Standard Model with two $SU(2)_L$ singlets, h and κ, with hypercharges $Y = 1$ and $Y = 2$, respectively. Show that lepton number can be violated within this model. Draw one diagram inducing neutrino masses.