Cosmology

Part 1: Fundamentals

IDPASC

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Constantinos Skordis — IDPASC, 30 Aug 2022

Cosmology

Lot’s of questions

What is the particle content of the universe?

What are the fundamental interactions at play?

How did galaxies form and evolve?

What were the initial conditions of the Universe?

— Interdisciplinary field

Kinetic theory and statistical physics

Particle physics

Gravitational physics (e.g. General Relativity)

Quantum field theory

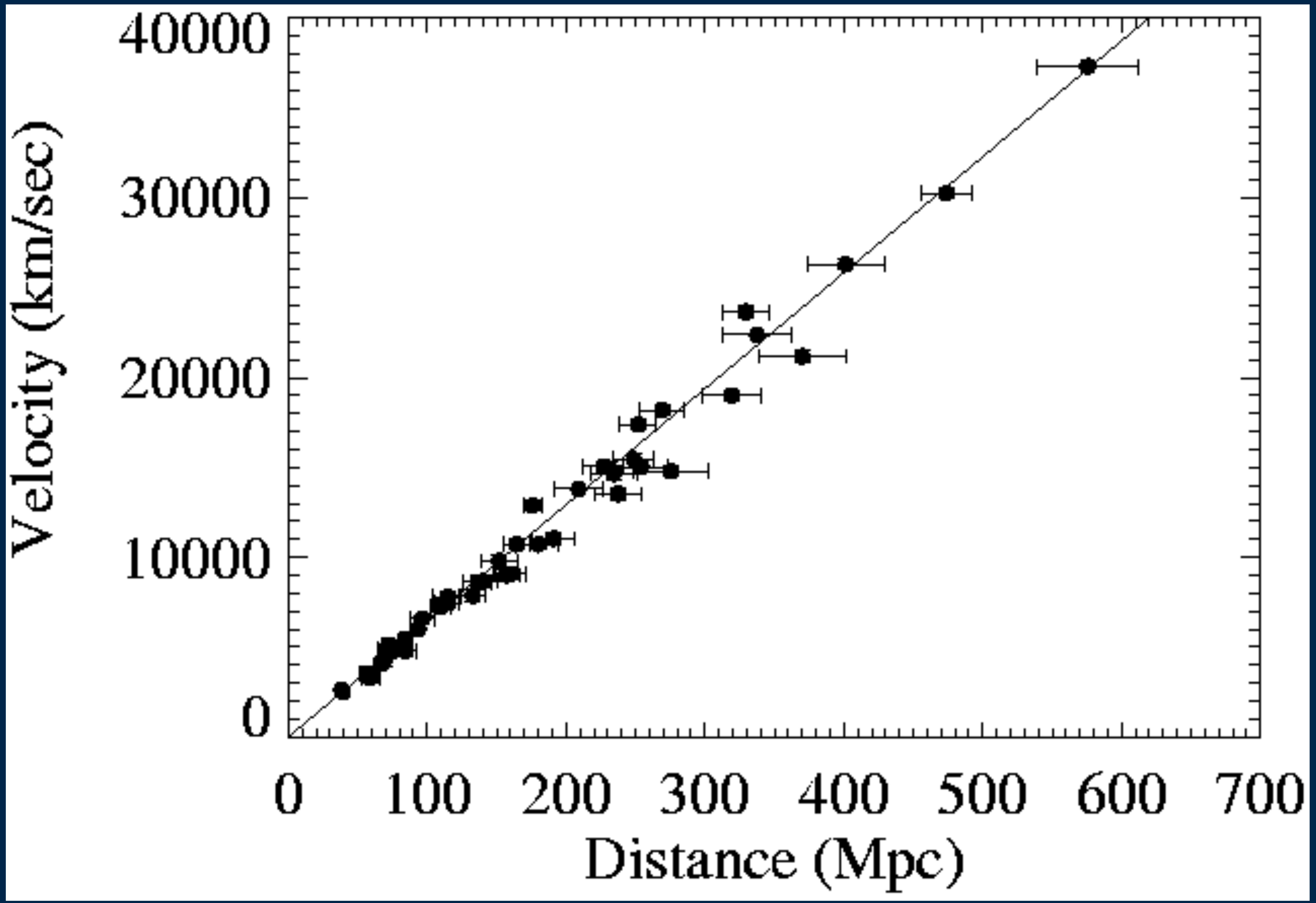
Numerical techniques (e.g. N-body simulations, ODE/PDE solving)

Astronomy

Astrophysics

Statistical techniques and data analysis

Hubble law



The Universe is expanding

Velocity

Physical distance

$$v = cz = H_0 r$$

(Small z)

Redshift

Hubble constant

$$H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$$
$$\sim \frac{h}{10^{10}\text{years}}$$
$$\sim 2 \times 10^{-42} h \;\; GeV$$

Data suggest $h \sim 0.7$

Features:

- The Universe is expanding

- The observable patch of the Universe has a radius of $\sim 3000Mpc$

$$1Mpc \simeq 3.26 \times 10^6 \text{years} \simeq 3.08 \times 10^{22} m \simeq 1.56 \times 10^{38} GeV^{-1}$$

- The Universe is statistically homogeneous and isotropic on scales larger than $\sim 100Mpc$
  with well-developed inhomogeneous structure (stars, galaxies, clusters, filaments) on smaller scales

- Gravity plays a fundamental role at all stages of the evolution of the Universe → (Assumption) General Relativity

- The Universe is full of thermal microwave radiation, $\bar{T}_{CMB} = 2.7255K$

- The Universe contains baryonic matter, roughly 1 baryon per 10^9 photons, and very little anti-matter.

- Baryons contribute ~5%   Rest is:
  - ~27% "Dark matter"
  - ~68% "Cosmological constant" (Dark energy)

Unit conventions

$$c = k_B = \hbar = 1$$

Speed of light   Boltzmann constant   Planck constant / 2π

Energy $1GeV = 1.602 \times 10^{-10} J = 1.5637 \times 10^{38} Mpc^{-1}$

Mass $1Kg = 5.6095 \times 10^{26} GeV = 8.7714 \times 10^{64} Mpc^{-1}$

Temperature $1K = 8.61698 \times 10^{-14} GeV = 1.34744 \times 10^{25} Mpc^{-1}$

Length $1m = 5.0677 \times 10^{15} GeV^{-1} = 3.2409 \times 10^{-23} Mpc$

Time $1s = 1.51925 \times 10^{24} GeV^{-1} = 9.7160 \times 10^{-15} Mpc = 2.997925 \times 10^{8} m$

Cosmological Principle

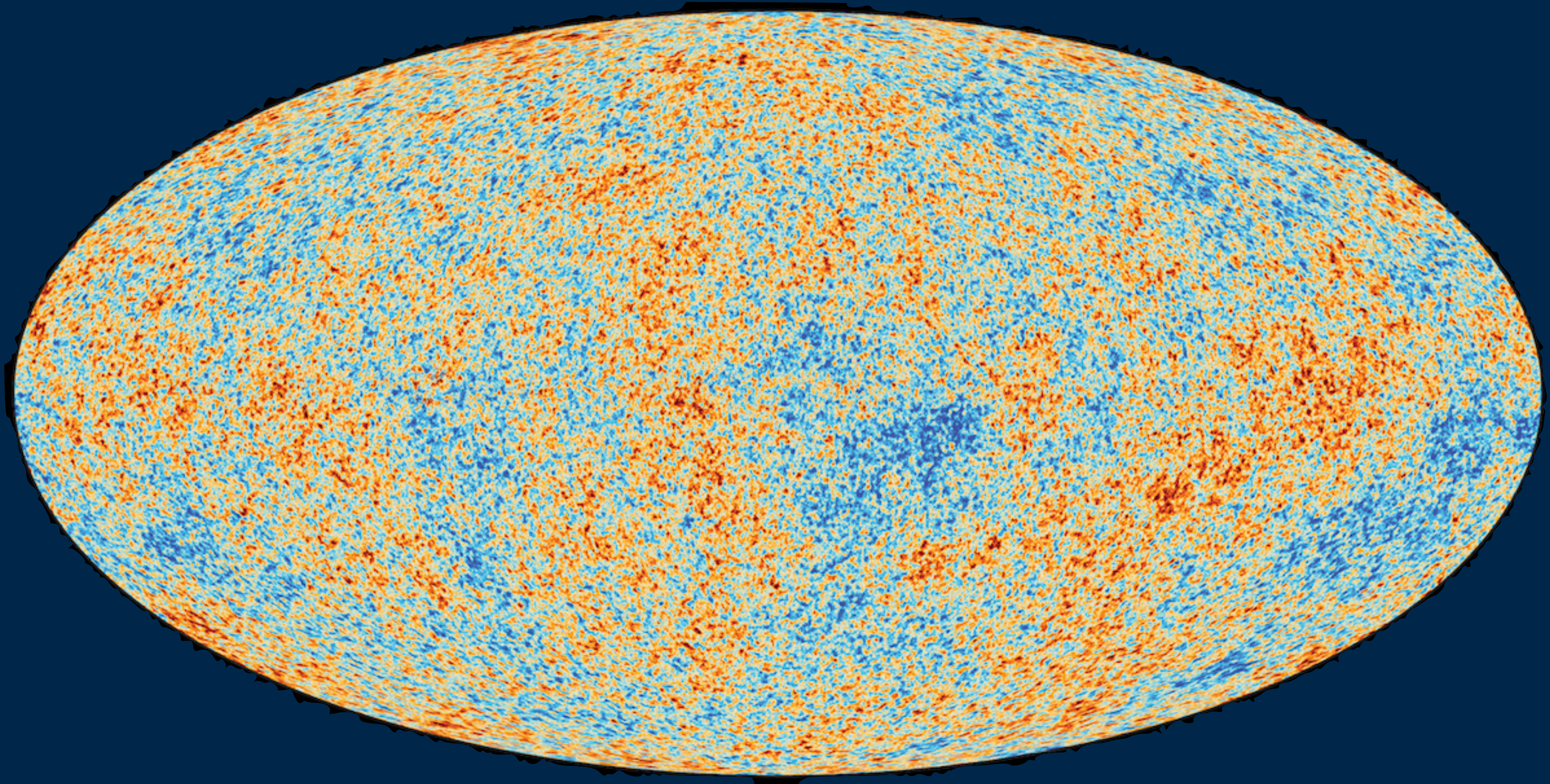
In an average sense

Universe is homogeneous and isotropic on very large scales

Looks the same along any direction

Looks the same at each point

Cosmic Microwave Background



Relic thermal radiation

$\bar{T}_{CMB} = 2.7255K$

Planck Satellite

Temperature Fluctuations $\lesssim 10^{-5}$ → Level of isotropy

Ehlers-Geren-Sachs Theorem

Ehlers, J., Genen P. & Sachs R. K., J. Math. Phys. J. 9, 1344 (1968)

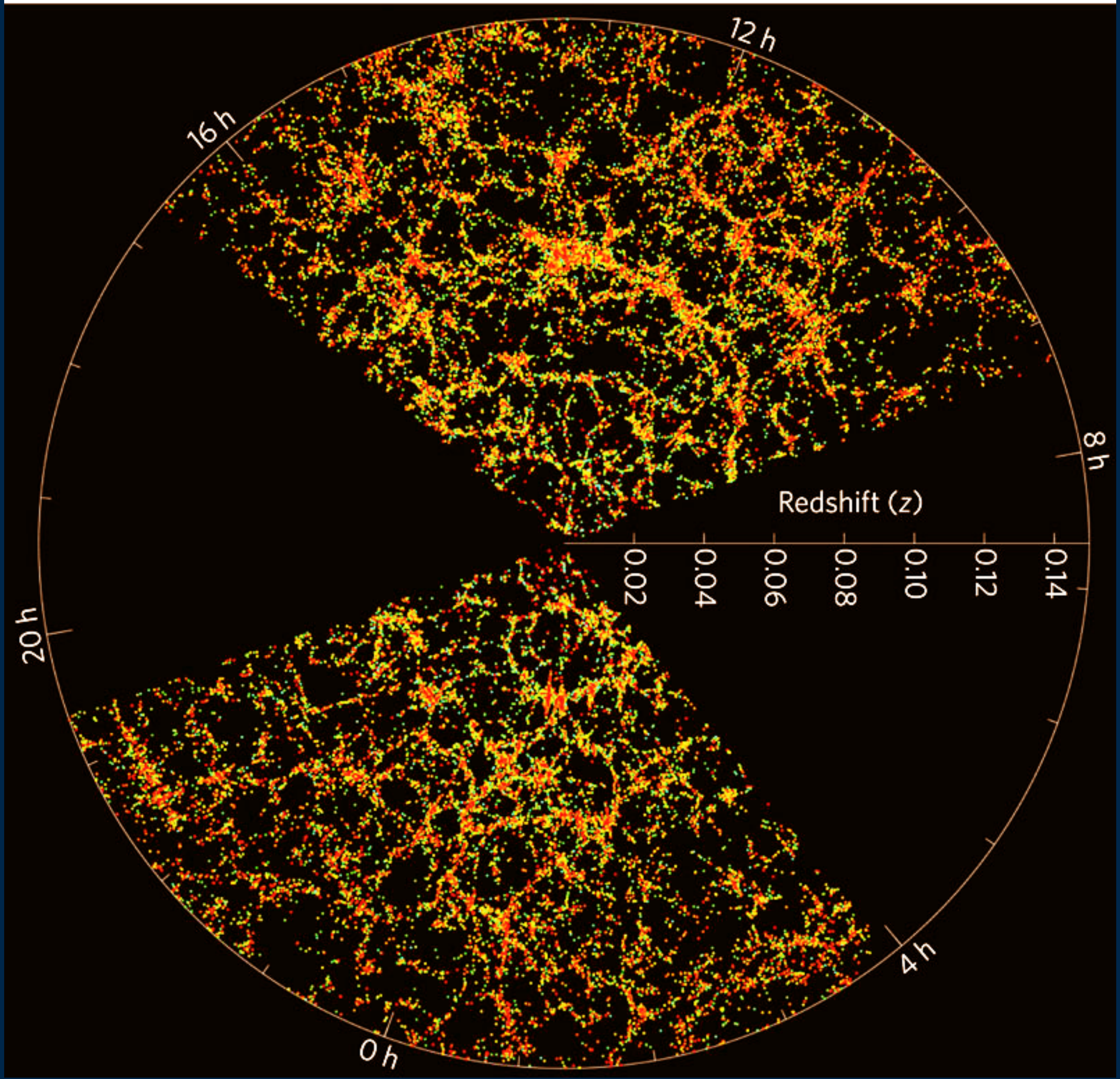
Isotropy of CMB + Copernican principle (+ GR) → Homogeneity

↑

We are not a
privileged observer

Almost Isotropy of CMB + Copernican principle (+ GR) → Almost Homogeneity

Stoeger W. R., Maartens R. & Ellis G. F. R., Astrophys. J. 443, 1 (1995)



Sloan Digital Sky survey

Copernican principle can be tested

e.g. Using WiggleZ data:
I. Morag et al, MNRAS 425, 116S (2012)

Approaches homogeneity at
$\sim 80 - 100h^{-1}Mpc$

Describing a homogeneous-isotropic Universe

3D Euclidean space is homogeneous and isotropic

Pythagorean theorem measures distance (also infinitesimally): $d\ell^2 = dx^2 + dy^2 + dz^2$

Special relativity spacetime interval: $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$

Space is expanding — but is this means that space and time must be treated separately

Can't do this respecting special relativity

Try: $ds^2 = -dt^2 + a^2(t)\left[dx^2 + dy^2 + dz^2\right]$

Scale factor

3D Euclidean space remains homogeneous and isotropic

Spatial part of 4D metric is expanding if $\dot{a} > 0$

But how do we determine the scale factor?

General covariance: Physics does not depend on the coordinate system used to describe it

$$ds^2 = -dt^2 + a^2(t)\left[dx^2 + dy^2 + dz^2\right] \xrightarrow{\text{e.g.}} ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

Cartesian coordinates

Spherical coordinates

Coordinates: set of 4 scalars $x^\mu = \{t, x, y, z\} \longrightarrow x^{\mu'} = \{t, r, \theta, \phi\}$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \longrightarrow ds^2 = g_{\mu'\nu'} dx^{\mu'} dx^{\nu'}$$

$$dx^\mu = \frac{\partial x^\mu}{\partial x^{\mu'}} dx^{\mu'} \qquad g_{\mu\nu} = \frac{\partial x^{\mu'}}{\partial x^\mu}\frac{\partial x^{\nu'}}{\partial x^\nu} g_{\mu'\nu'}$$

Geometric objects transforming like this are called "Tensors"

General tensor $T^{\mu\alpha\ldots}{}_{\nu\beta\ldots}$

Contravariant components (upper indices)

Covariant components (lower indices)

Einstein summation convention: repeated indices summed over

$$\sum_{\mu=0}^{3}\sum_{\nu=0}^{3} g_{\mu\nu} dx^\mu dx^\nu \equiv g_{\mu\nu} dx^\mu dx^\nu$$

Indices $\mu, \nu, \rho, \mathrm{etc} = \{0 \ldots 3\}$

General Relativity — Gravity as geometry of spacetime

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Metric tensor $g_{\mu\nu}(t, \vec{x})$ - 4x4 symmetric matrix (10 components) determined by Einstein equations

- Generally, not equivalent to Minkowski metric $\frac{\partial x^{\mu'}}{\partial x^\mu} \frac{\partial x^{\nu'}}{\partial x^\nu} g_{\mu'\nu'} \neq \eta_{\mu\nu}$ except in (sometimes) vacuum

- Is invertible: $g^{\mu\rho} g_{\rho\nu} = \delta^\mu{}_\nu$ → $v^\mu = g^{\mu\nu} v_\nu$, $v_\mu = g_{\mu\nu} v^\nu$ Raising/lowering of tensor indices

Identity matrix

Friedman-Lemaître-Robertson-Walker (FLRW) spacetime: the geometry of a homogeneous-isotropic universe

Conventionally $a = 1$ Today

Scale factor – determined by matter content

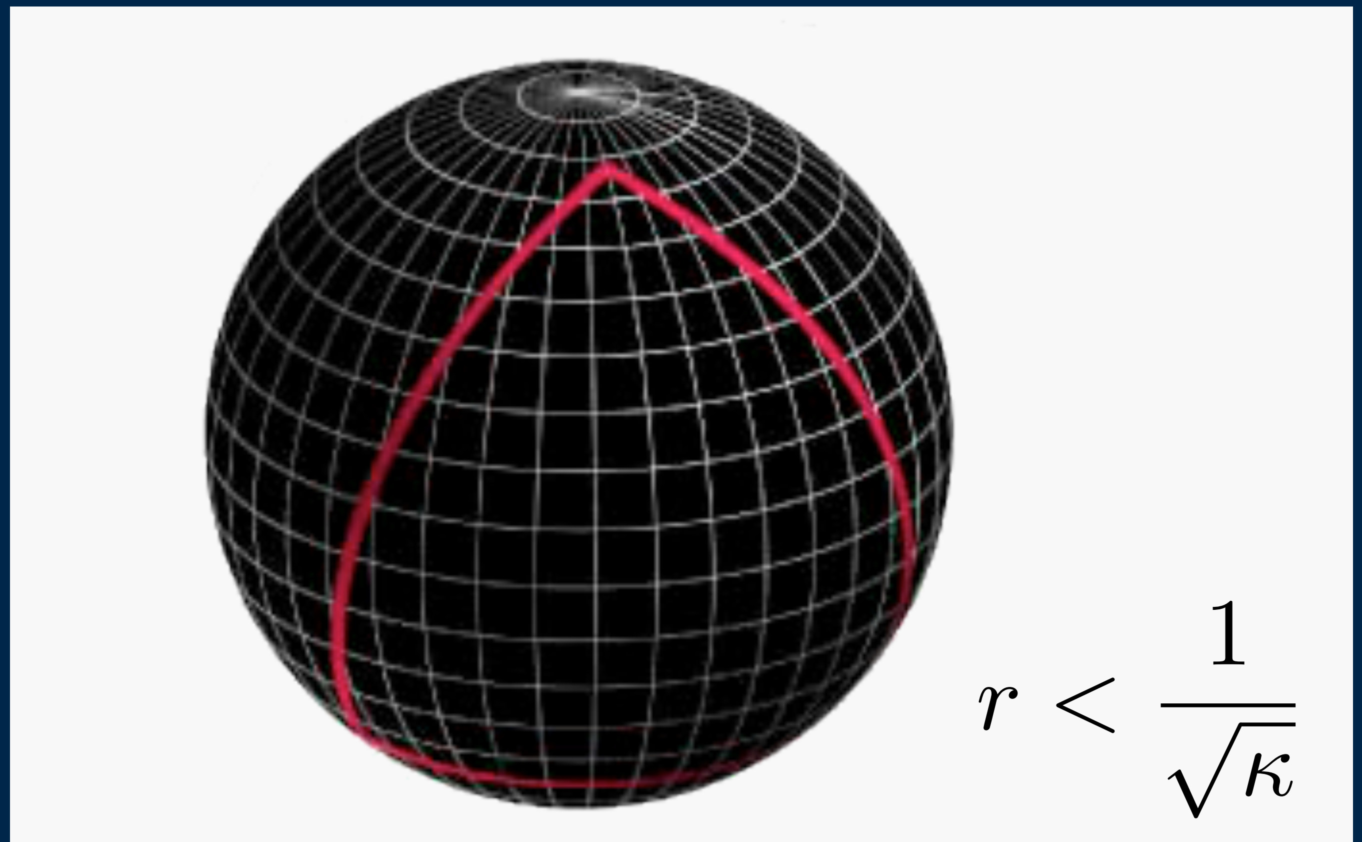
$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

(Spherical coordinates)

Spatial curvature

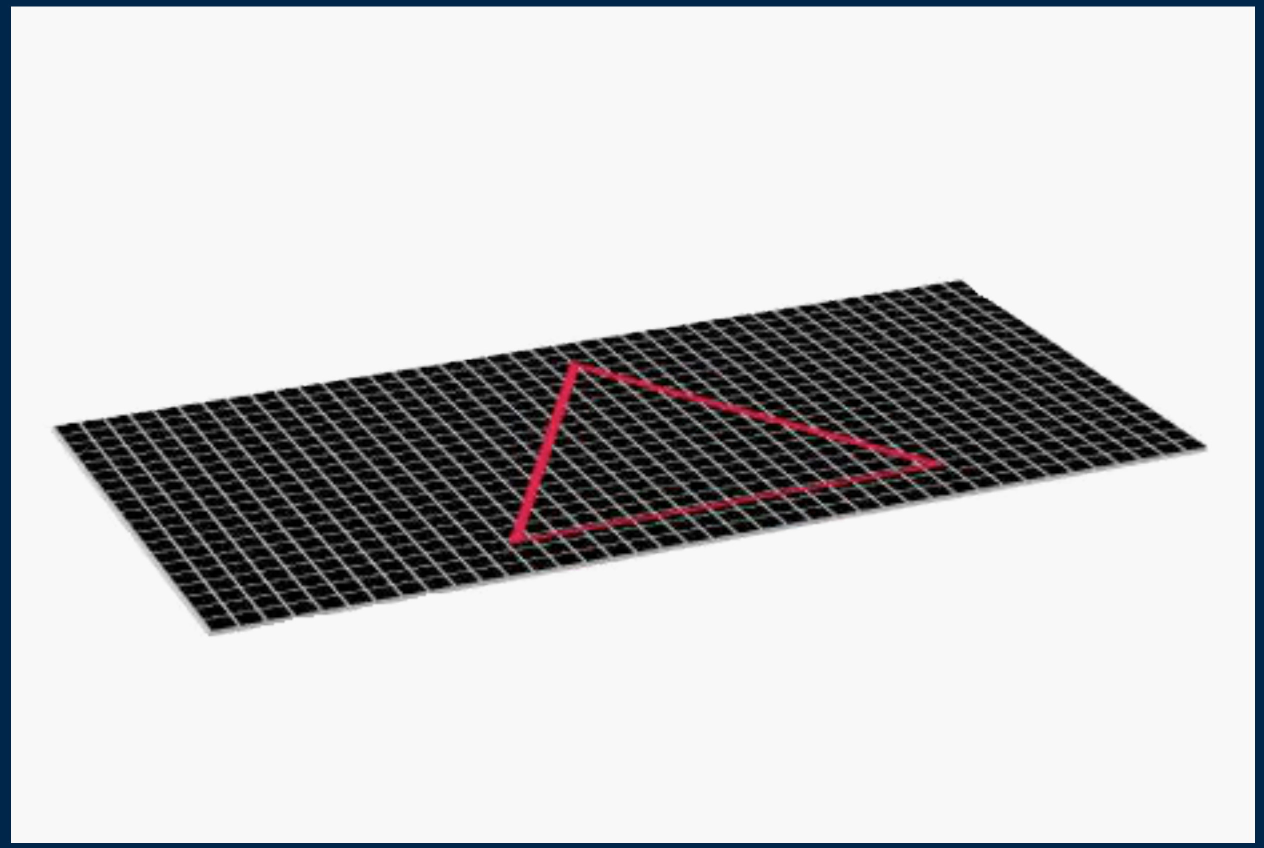
$\kappa > 0$

Positively curved (e.g. 3-sphere)



$\kappa = 0$

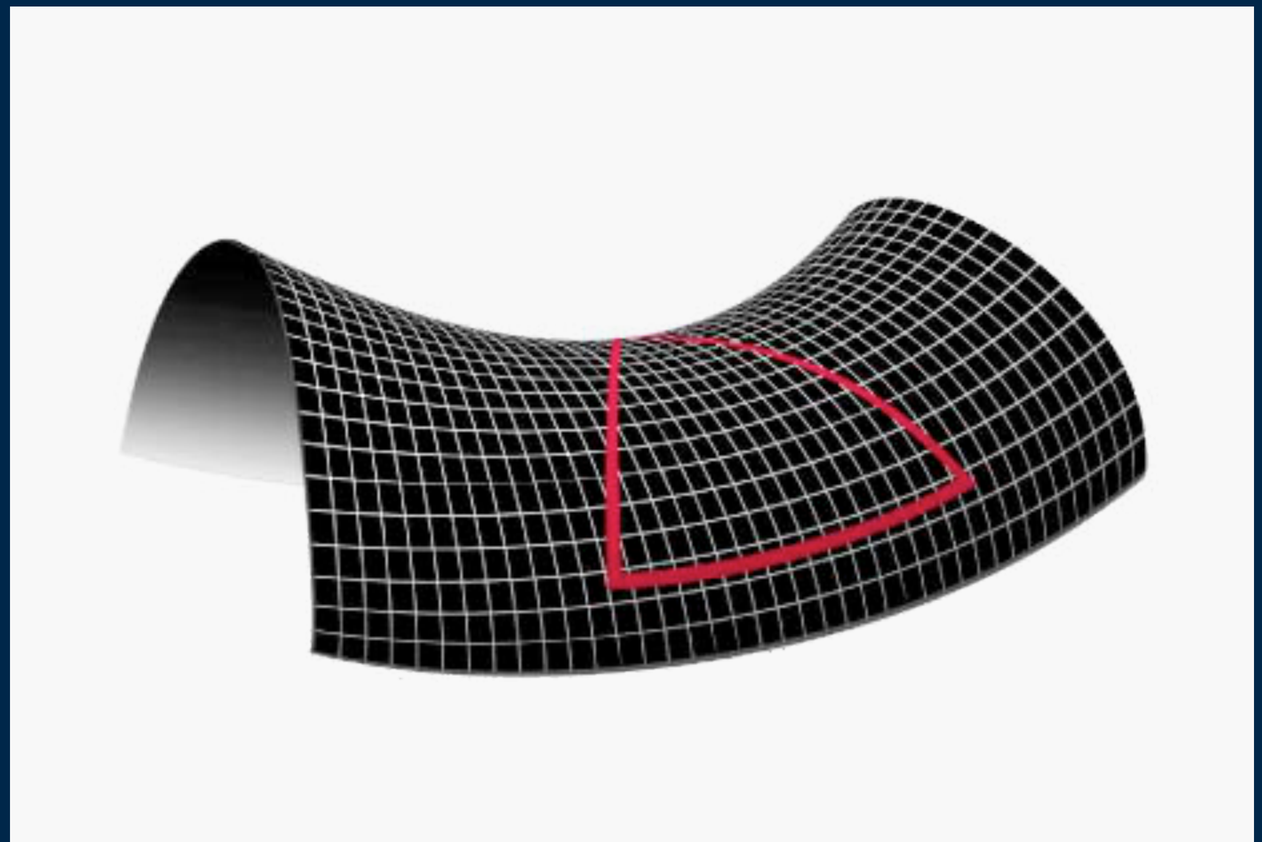
Spatially flat (Euclidean)



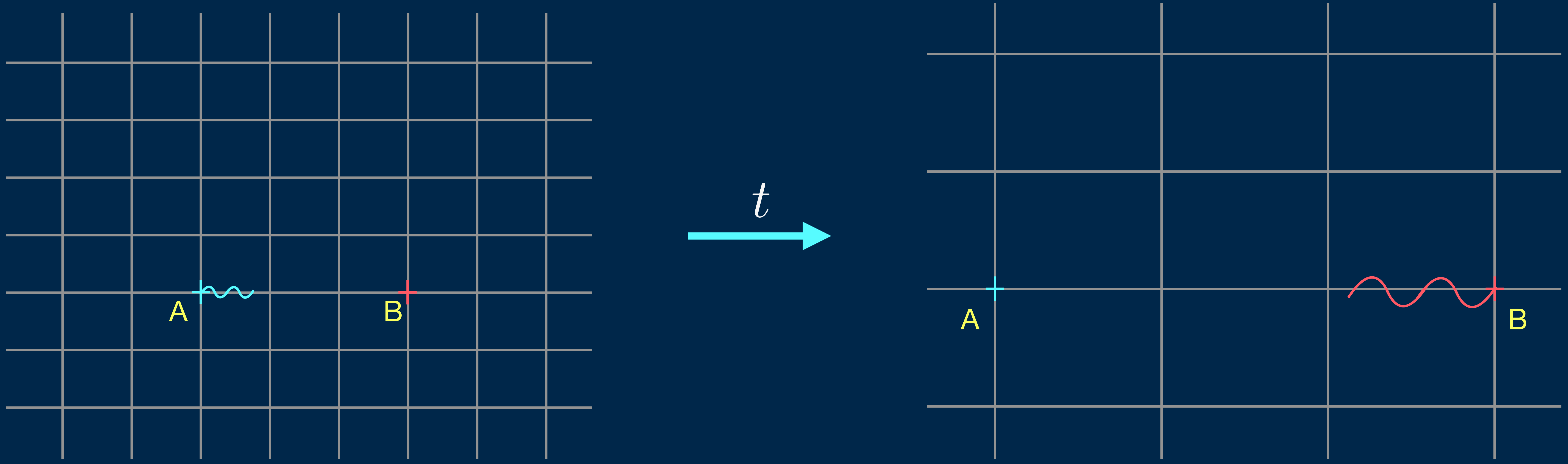
Preferred by the data

$\kappa < 0$

Negatively curved (e.g. hyperbolic 3-space)



Physical vs co-moving distance — Back to Hubble law



$$r_{phys} = a\, r$$

$$v = \dot{r}_{phys} = \dot{a}\, r = H(t) r_{phys}$$

Generalized Hubble law

$$H = \frac{\dot{a}}{a}$$

Hubble parameter

$$H_0 = H(t_0)$$

Today

In the absence of forces, particle trajectories are geodesics

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0$$

$x^\mu(\lambda)$ Solution gives geodesic

Affine parameter

Christoffel symbols $\Gamma^\alpha_{\mu\nu} \equiv \frac{1}{2}g^{\alpha\beta}\left(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}\right)$

Subject to constraint: $g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = L$

- $L = -1$ Timelike geodesics: massive particles $\lambda \to s$
- $L = 0$ Null geodesics: massless particles
- $L = 1$ Spacelike geodesics: no physical particles $\lambda \to s$

Connection - covariant derivative $\nabla_\lambda T^{\mu\ldots}{}_{\nu\ldots} \equiv \partial_\lambda T^{\mu\ldots}{}_{\nu\ldots} + \Gamma^\mu_{\lambda\rho}T^{\rho\ldots}{}_{\nu\ldots} - \Gamma^\rho_{\lambda\nu}T^{\mu\ldots}{}_{\rho\ldots} + (\ldots)$

Energy of massless particles

Use definition of 4-momentum $\frac{dx^\mu}{d\lambda} = P^\mu = \left(E, P^i\right)$ → $\frac{dP^\mu}{d\lambda} = -\Gamma^\mu_{\alpha\beta} P^\alpha P^\beta$

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0$$

Take $\mu = 0$ component

$$\frac{dE}{d\lambda} = -\Gamma^0_{ij} P^i P^j = -a\dot{a}P^2$$

$$= \frac{dt}{d\lambda}\frac{dE}{dt}$$

$$\Rightarrow E\frac{dE}{dt} = -a\dot{a}P^2$$

Massless particles: $g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = L = 0$ → $P = \frac{E}{a}$

$$\Rightarrow \frac{dE}{dt} = -\frac{\dot{a}}{a}E$$

Energy today (at $a = 1$)

Solution: $E = \frac{E_0}{a(t)}$

Particle's energy decays
(That's why the redshift)

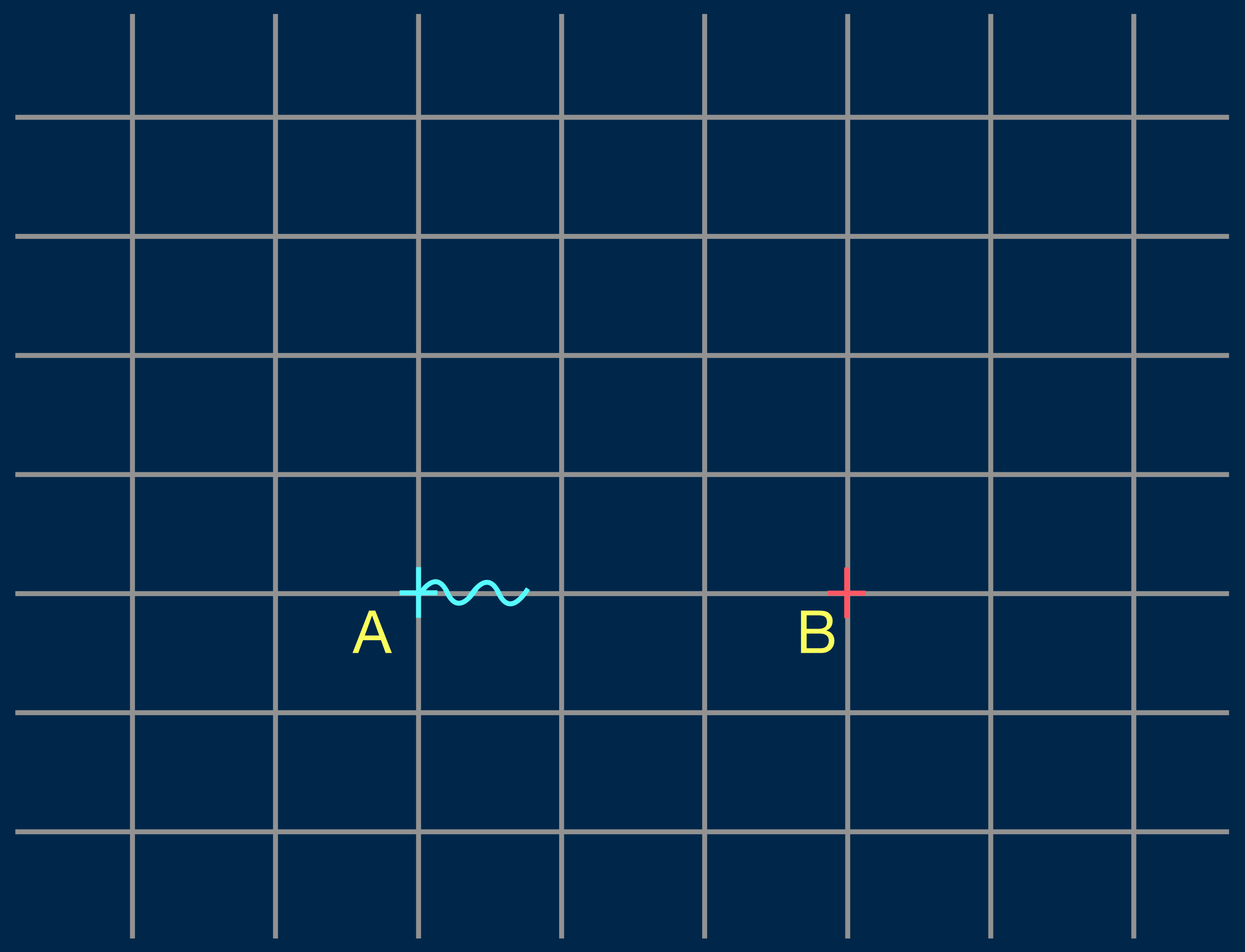
c.f. also temperature $T = \frac{T_0}{a(t)}$

Redshift and scale factor

Photon energy E_{em}

Photon wavelength $\lambda_{em} = \frac{1}{E_{em}}$

Emission $a(t_{em})$



t_{em}

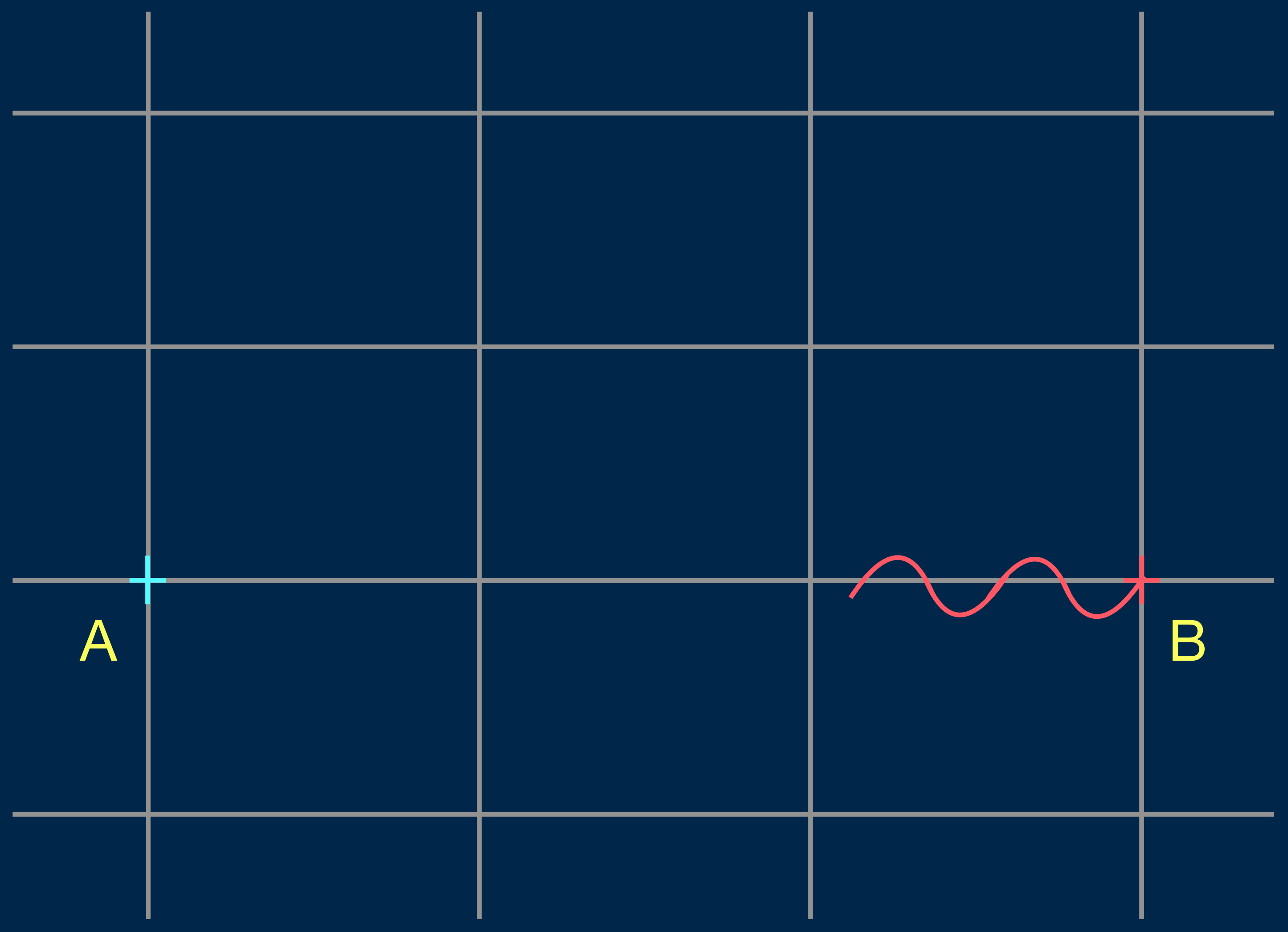
t

Today

Photon energy $E_0 = a(t)E_{em}$

Photon wavelength $\lambda_0 = \frac{1}{E_0} = \frac{\lambda_{em}}{a(t)}$

$a = 1$



t_0 (Today)

Redshift

$$z \equiv \frac{\lambda_0 - \lambda_{em}}{\lambda_{em}}$$

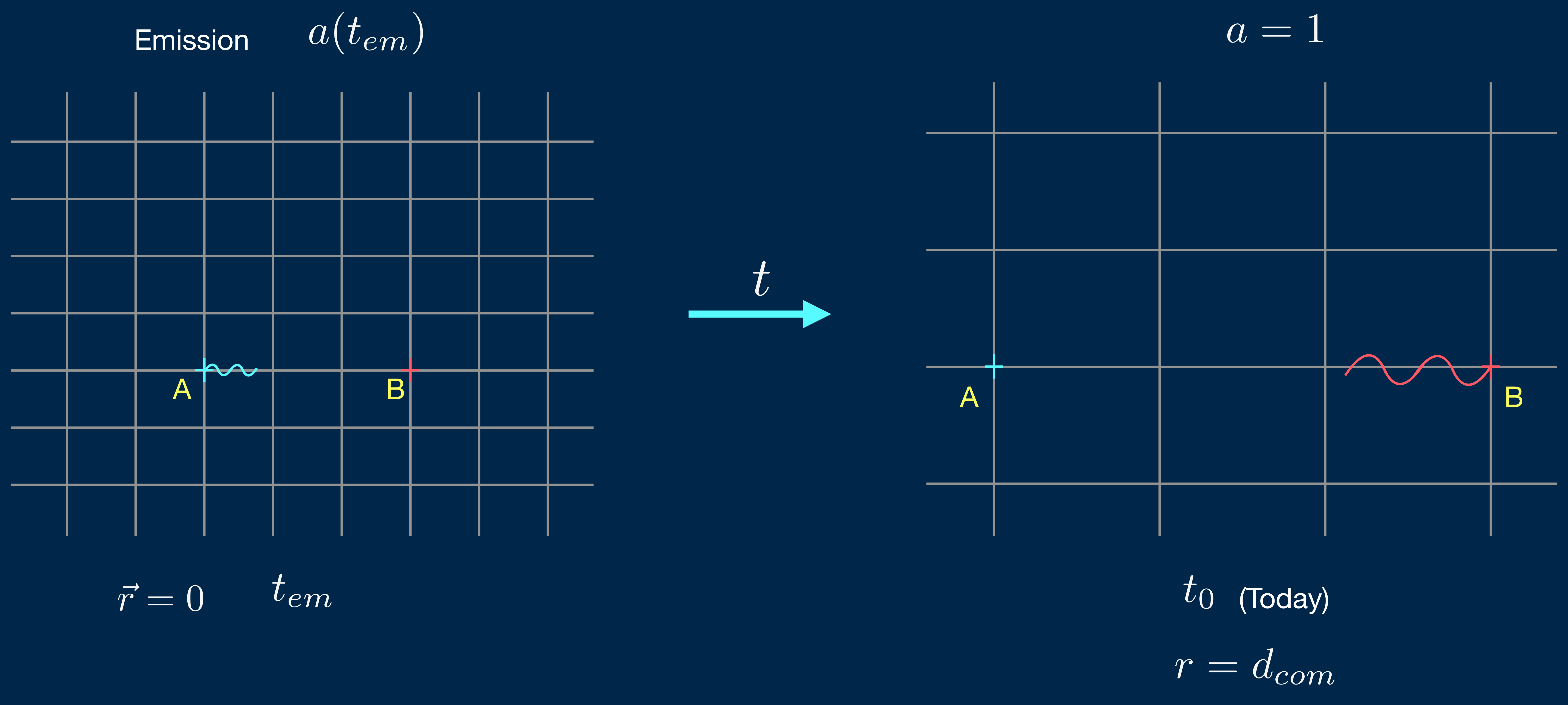
$$1 + z = \frac{1}{a}$$

Measuring distance: use light

$$\frac{dt}{a} = \frac{dr}{\sqrt{1-\kappa r^2}}$$

Horizon distance

$$\chi = \int_t^{t_0} \frac{dt'}{a(t')} = \int_a^1 \frac{da'}{a'^2 H(a')} = \int_0^z \frac{dz'}{H(z')}$$



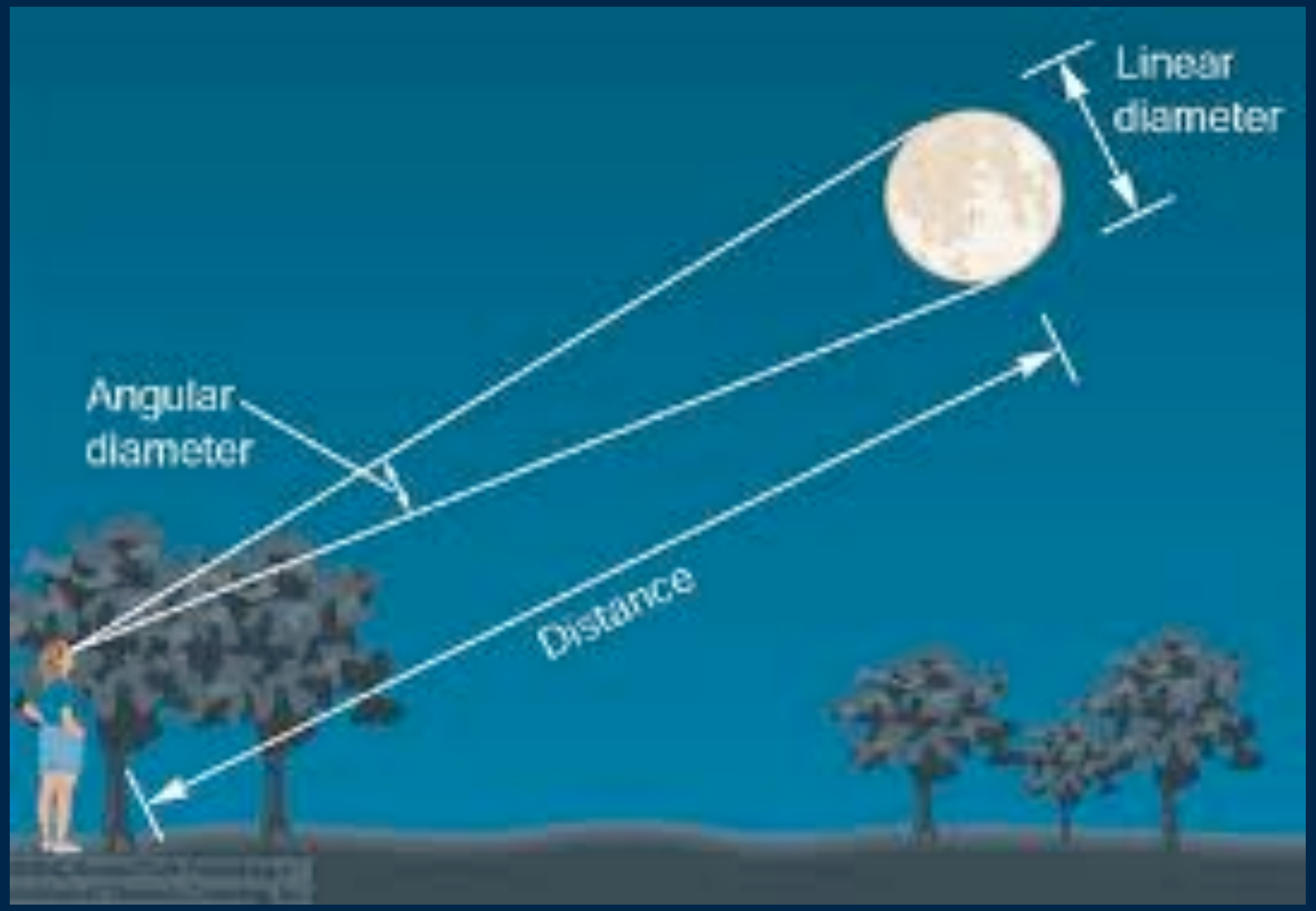
Co-moving distance

$$d_{com} = \frac{1}{\sqrt{\kappa}} \sin\left(\sqrt{\kappa}\chi\right)$$

Measuring distance

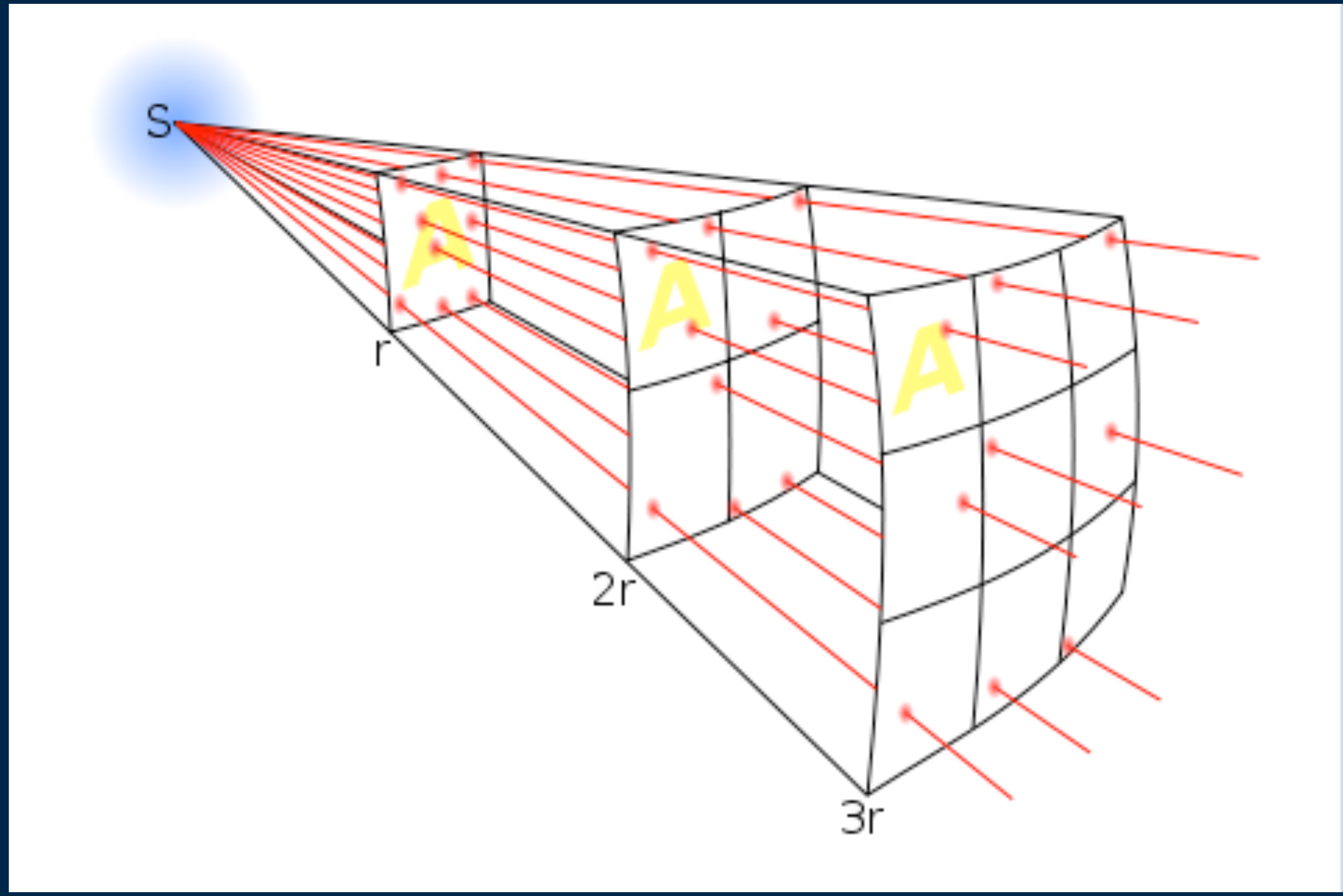
Angular diameter distance = $\frac{\text{Diameter (actual)}}{\text{Angle subtended}}$

$$d_A = a d_{com}$$



Luminosity distance

$$d_L^2 = \frac{L_s}{4\pi\mathcal{F}} = \frac{\text{Absolute luminosity of source}}{4\pi \text{ Observed flux}} = \frac{d_{com}^2}{a^2}$$



Etherington's theorem (1933): $d_L = (1+z)^2 d_A$ irrespective of the underlying theory of gravity.

Theorem fails when photon number not conserved. E.g. the photon conversion into axions.

Summary so far: Expanding, homogeneous-isotropic universe

Friedman-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

Particles follow geodesics $$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0$$

$$\Gamma^i_{0j} = H\delta^i{}_j \qquad \Gamma^0_{ij} = Hg_{ij} \qquad H(t) \equiv \frac{\dot{a}}{a}$$

$$g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = \begin{cases} -1 & \text{Massive} \\ 0 & \text{Massless} \end{cases}$$

Photon energy, wavelength, temperature $$E = \frac{E_0}{a} \qquad \lambda = a\lambda_0 \qquad T = \frac{T_0}{a}$$

Scale factor - redshift relation: $1 + z = \dfrac{1}{a}$

Distance measures $$d_{com} = \frac{1}{\sqrt{\kappa}}\sin\left(\sqrt{\kappa}\chi\right) \qquad \chi = \int_0^z \frac{dz'}{H(z')} \qquad d_A = ad_{com} = a^2 d_L$$