Cosmology

Lecture 3: Large scale structure and Cosmic Microwave Background

Constantinos Skordis — IDPASC, 2 Sep 2022

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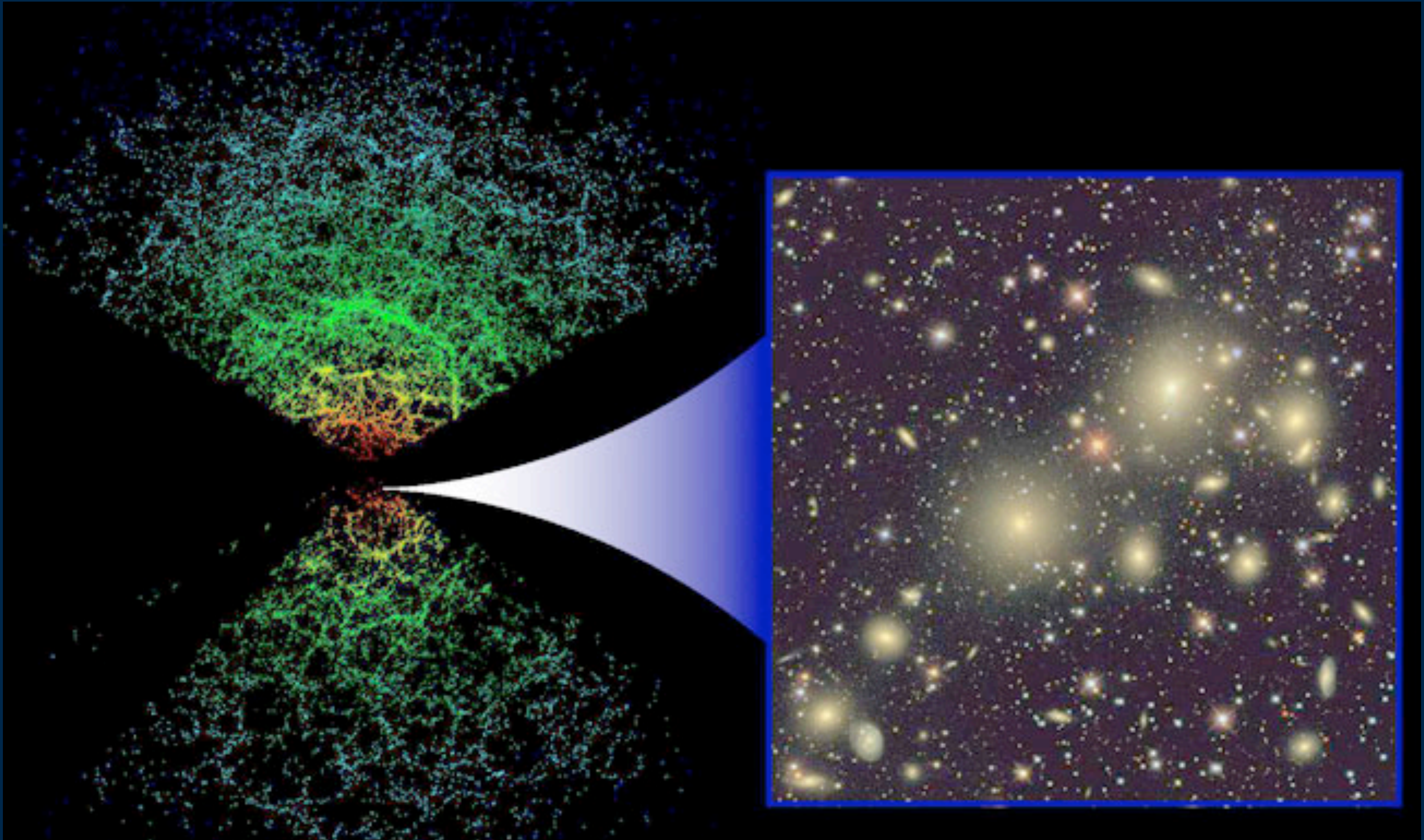
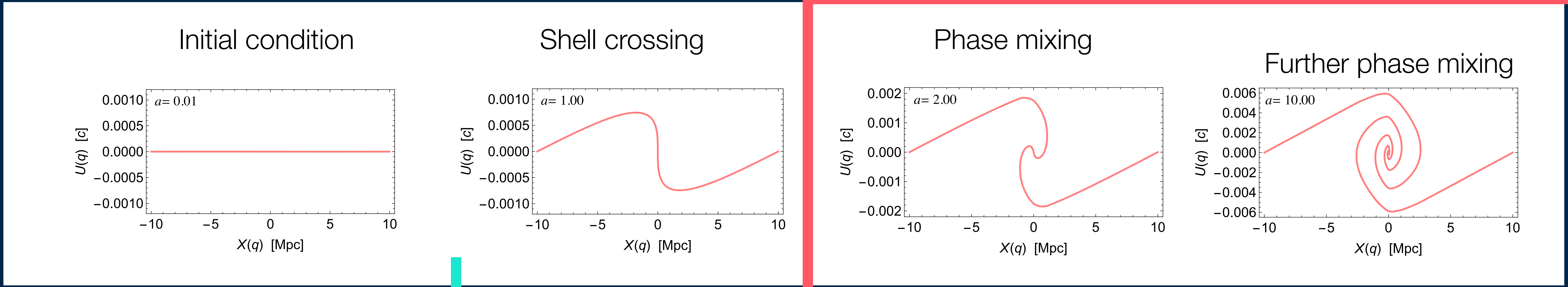


(from Sloan Digital Sky Survey)

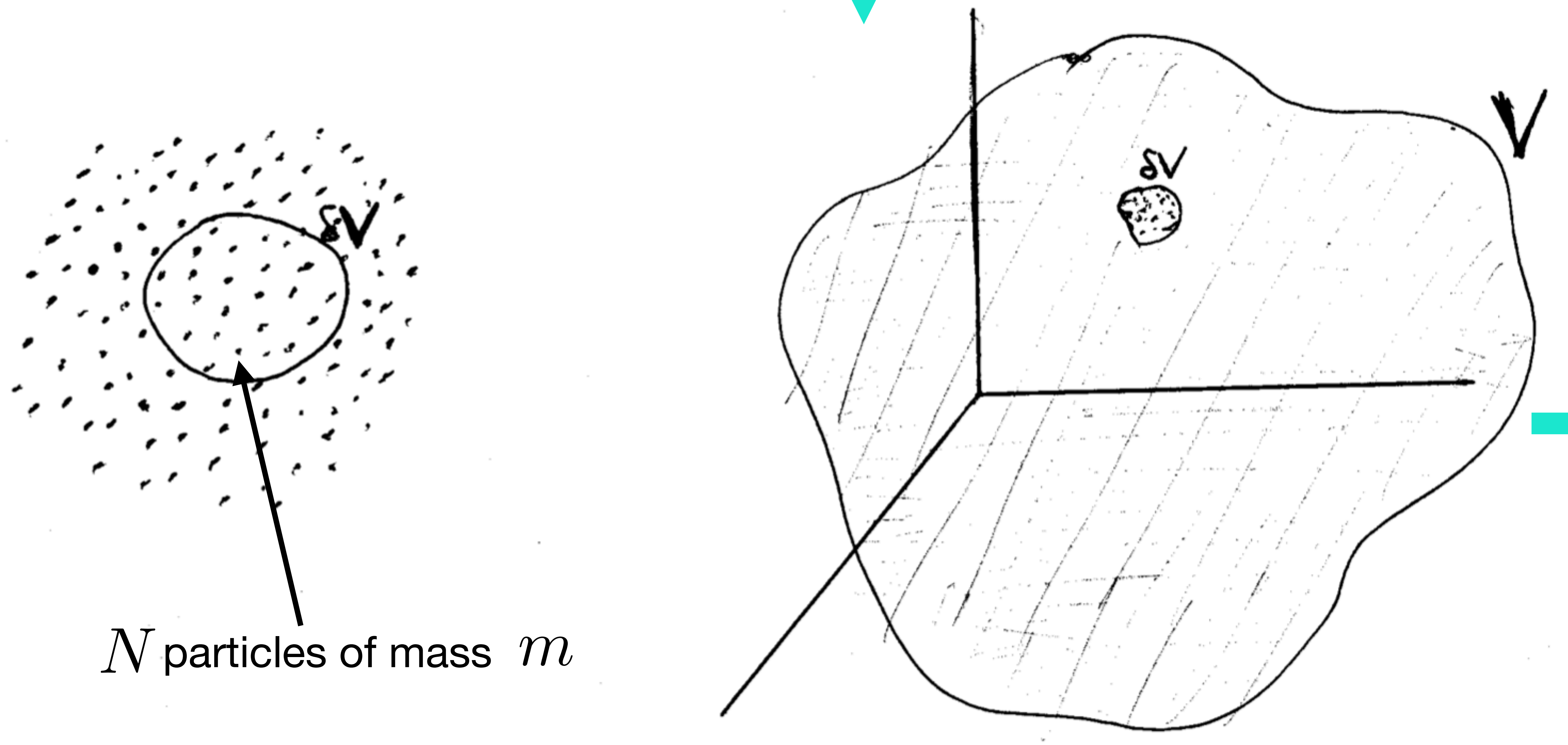


Fluid approximation:

- Particle trajectories do not cross (no collisions)
- No phase mixing
- Any small δV contains large number of particles

Use N-body simulations, Vlasov solvers, Schoedinger-Poisson Method

- Gadget: https://wwwmpa.mpa-garching.mpg.de/gadget4/
- Enzo: https://enzo-project.org/
- Arepo: https://arepo-code.org/
- RAMSES: https://www. ics.uzh.ch/~teyssier/ramses/RAMSES.html



- Energy density field $\rho(t, \vec{x}) = \lim_{\delta V \to 0} \frac{Nm}{\delta V}$
- Velocity field $\vec{v}(t, \vec{x})$

Newtonian Gravitational Collapse

$$\frac{dM}{dt} = \int dV \frac{\partial \rho}{\partial t}$$

Continuity equation
(Mass change through closed surface)

$$\frac{\partial \rho}{\partial t} + \vec{v} \cdot \vec{\nabla} \rho = -\rho \vec{\nabla} \cdot \vec{v}$$

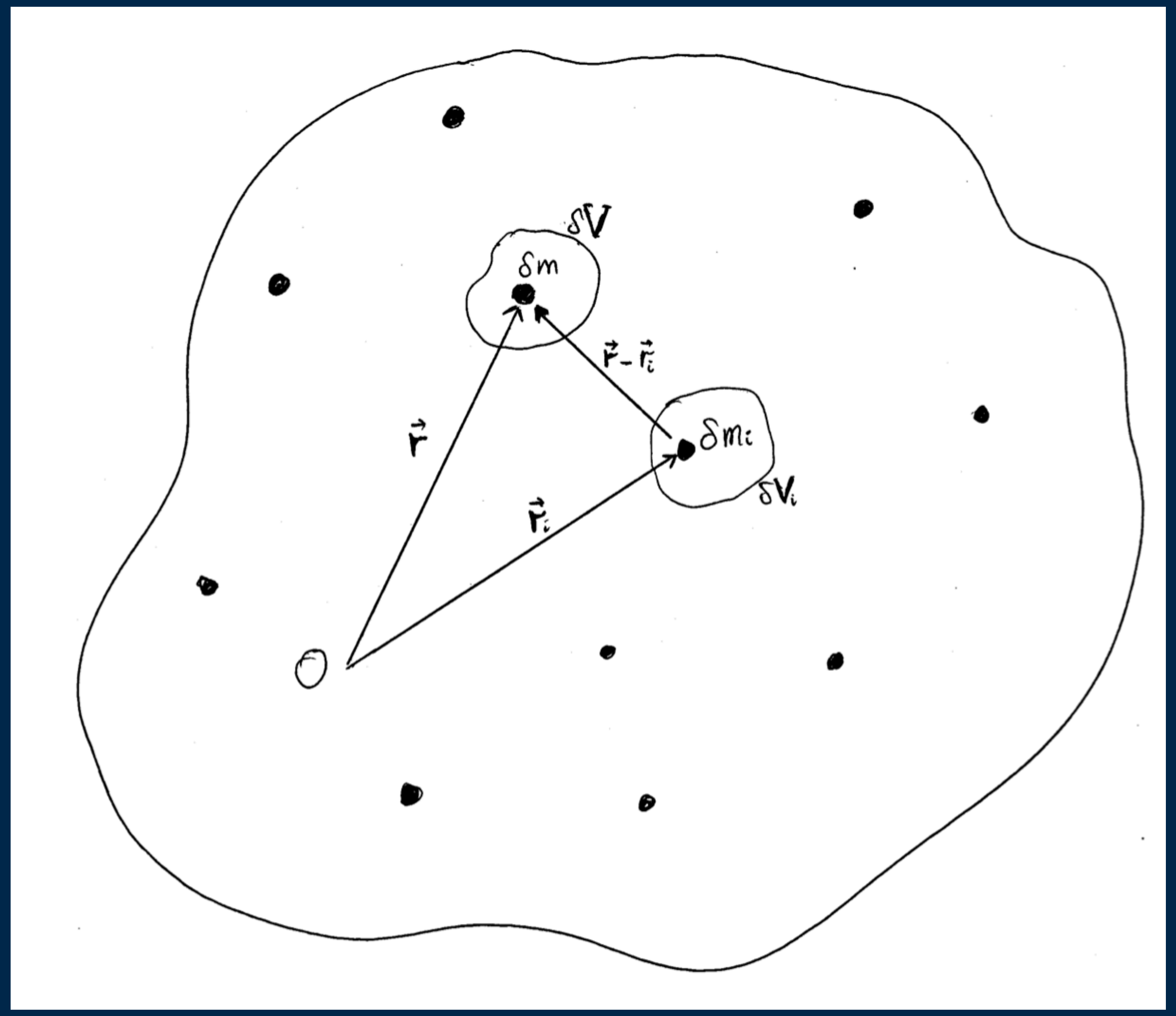
Euler eq.
(Newton-II: Acceleration = sum forces)

$$\rho \left[\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla} \right) \vec{v} \right] = -\vec{\nabla} P - \rho \vec{\nabla} \Phi$$

Poisson eq.
(Gravity)

$$\nabla^2 \Phi = 4\pi G \rho$$

$$\frac{\delta F}{\delta V}(r) \sim -G \frac{\delta M}{\delta V}(r) \sum_i \frac{\delta m_i}{|r - r_i|^2} \sim \frac{\delta \Phi}{\delta V}$$



Equation of state

$$\vec{\nabla} P = C_s^2 \vec{\nabla} \rho$$

Use a trick — Taylor expansion

$$\rho = \bar{\rho} + \epsilon \delta \rho$$
$$\vec{v} = \bar{\vec{v}} + \epsilon \delta \vec{v}$$

$$\epsilon \ll 1$$

Set $\epsilon^2 \to 0$ → Equations become linear

background is homogeneous and static:

$$\frac{\partial \bar{\rho}}{\partial t} = \vec{\nabla} \bar{\rho} = 0 \qquad \bar{\vec{v}} = 0$$

Jeans instability

Define density contrast: $\delta \equiv \frac{\delta\rho}{\rho}$

Use Fourier space $\vec{\nabla}^2 \to -k^2$

$$\ddot{\delta} + \left(k^2 C_s^2 - 4\pi G\bar{\rho}\right)\delta = 0$$

$k^2 C_s^2 > 4\pi G\bar{\rho}$ Oscillations (stable)

$k^2 C_s^2 < 4\pi G\bar{\rho}$ Exponential collapse

Dividing line: $k = \frac{\sqrt{4\pi G\bar{\rho}}}{C_s}$

Jeans length: $\lambda_J = \frac{2\pi}{k} = C_s\sqrt{\frac{\pi}{G\bar{\rho}}}$

Only wavelengths larger than the Jeans length can collapse to form bound objects

Collapse in an expanding Universe

Use Newtonian theory

suffices for sub-horizon fluctuations
for non-relativistic matter

Continuity equation $\frac{\partial \rho}{\partial t} + \vec{v} \cdot \vec{\nabla} \rho = -\rho \vec{\nabla} \cdot \vec{v}$

Euler eq. $\rho \left[\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla} \right) \vec{v} \right] = -\vec{\nabla} P - \rho \vec{\nabla} \Phi$

Poisson eq. (Gravity) $\nabla^2 \Phi = 4\pi G \rho$

Equation of state $\vec{\nabla} P = C_s^2 \vec{\nabla} \rho$

(Same eps as before)

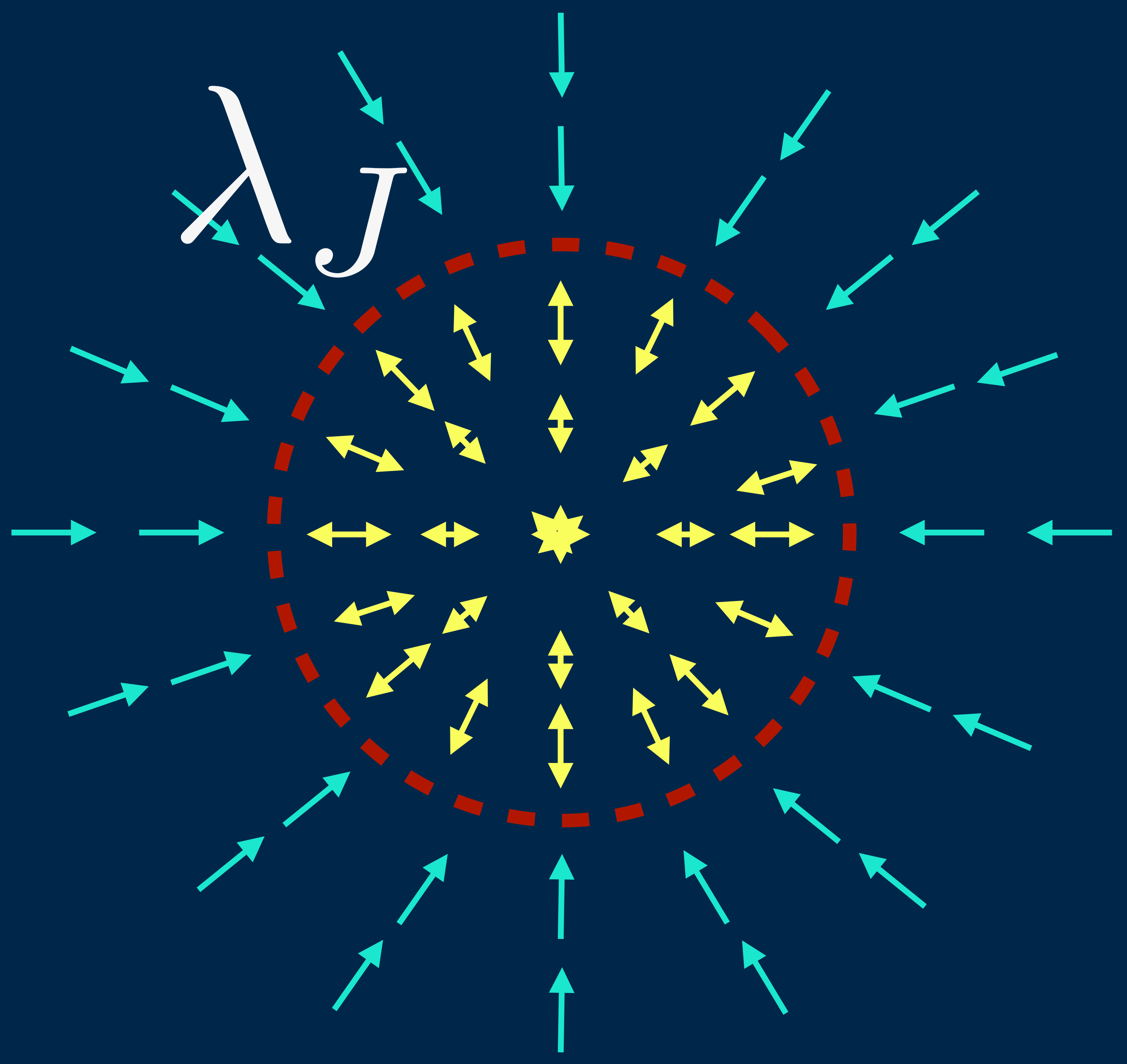
Taylor-expand $\rho = \bar{\rho} + \epsilon \delta \rho$, $\vec{v} = \bar{\vec{v}} + \epsilon \delta \vec{v}$, $\epsilon \ll 1$ — Set $\epsilon^2 \to 0$ → Equations become linear

Background is homogeneous and expanding: $\bar{\vec{v}} = H \vec{x}$

FRW energy conservation! $\dot{\bar{\rho}} + 3H\bar{\rho} = 0$

Collapse in an expanding Universe

$$\longrightarrow \quad \ddot{\delta} + 2H\dot{\delta} + \left(\frac{k^2 C_s^2}{a^2} - 4\pi G\bar{\rho}\right)\delta = 0$$



Differences arise

- Expansion introduces a damping term: $2H\dot{\delta}$
- Scales are stretched by $a(t)$
- Background density is evolving $\bar{\rho} \propto a^{-3}$
- Jeans length is time dependent $\lambda_J = \frac{C_s}{a}\sqrt{\frac{\pi}{G\bar{\rho}}}$
- A given perturbation may switch between growth and stasis

Matter fluctuations: Growth or decay

Within Jeans length: oscillations (Bessel functions with decreasing amplitude)

Outside Jeans length: $k \ll k_J$ ignore $k^2 C_s^2$

1. Matter domination: $a \sim t^{2/3}$ $8\pi G\bar{\rho} \sim \frac{1}{t^2}$

$$\rightarrow \quad \ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0 \quad \rightarrow \quad \delta = \delta_0 t^{2/3} + \frac{\delta_1}{t}$$

Power-law growth

2. Radiation domination: $a \sim \sqrt{t}$ $8\pi G\bar{\rho} = 3H^2\Omega_m \to 0$

$$\rightarrow \quad \ddot{\delta} + \frac{1}{t}\dot{\delta} = 0 \qquad \delta = \delta_0 + \delta_1 \ln t$$

Mészáros effect: During radiation era matter fluctuations grow only logarithmically at best

Can baryons form structure?

It all depends on two things:

- the Baryon Jeans mass
- the age of the Universe

Simplified assumptions:
- only species are baryons+photons
- recombination occurs at equality

Before recombination: $a \approx \Omega_{0\gamma}^{1/4}\sqrt{2H_0 t}$ and $c_s^2 = \frac{1}{3}$

Baryon Jeans length: $\lambda_J = \sqrt{\frac{\pi}{3G\rho_b}} = \frac{2\pi}{3H_0}\sqrt{\frac{2}{\Omega_{0b}}}\Omega_{0\gamma}^{3/8}(2H_0 t)^{3/4}$

Horizon diameter $\lambda_H = 4t$

$$\frac{\lambda_J}{\lambda_H} \propto \frac{1}{(H_0 t)^{1/4}}$$

As $t \to 0$ ➔ $\lambda_J \gg \lambda_H$

$t \to t_*$ ➔ $\frac{\lambda_J}{\lambda_H} \to \frac{\pi\sqrt{2}}{3} > 1$

Therefore before recombination $\lambda_J > \lambda_H$ ➔ baryons cannot collapse to form structures

After recombination $C_s^2 \to 0$ ➔ $\lambda_J \ll \lambda_H$ and baryons can collapse into structures

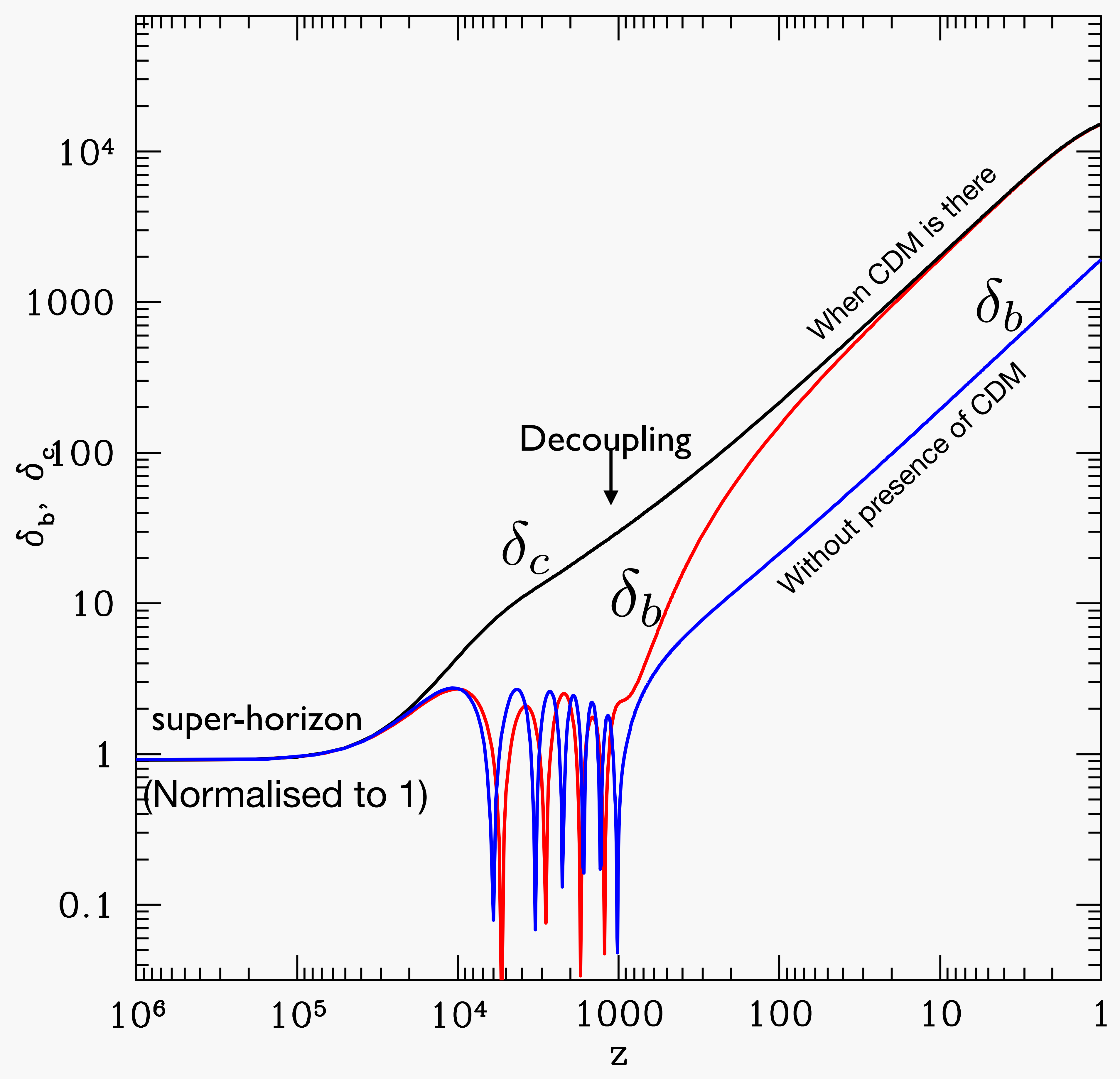
Structure formation

Sub-horizon fluctuations of uncoupled non-relativistic species

- Power-law growth during matter domination as $t^{2/3}$
- Very small logarithmic growth during radiation domination

Can baryons form structure by themselves?

YES, but… with a delay.
The universe is not old enough to produce the structure we see. A baryon-only universe would have looked very different.



One way out: Cold Dark Matter*

CDM does not couple to photons and gets a head start.

Once baryons decouple they follow the potential wells already created by CDM

$$\vec{\nabla}^2\Phi = 4\pi G a^2 \bar{\rho}_m \left(\Omega_b\delta_b + \Omega_c\delta_c\right)$$

(* another possibility is warm dark matter)

(* yet another, possibly, extending General Relativity)

Probes of Large Scale Structure

Correlation function $\xi(\vec{r}) = \int d^3r' \langle \delta(\vec{r})\delta(\vec{r}' + \vec{r}) \rangle$

Gives the correlation of fluctuations separated by $\vec{r}$

Matter power spectrum $\langle \delta_{\vec{k}} \delta_{\vec{k}'} \rangle = (2\pi)^3 P(k) \delta^{(3)} \left(\vec{k} - \vec{k}' \right)$

Correlation function is the Fourier transform of the Power spectrum $\xi(\vec{r}) = \int d^3k e^{-i\vec{k}\cdot\vec{r}} P(k)$

How do we estimate. $P(k)$?

Can use the distribution of galaxies and their velocities

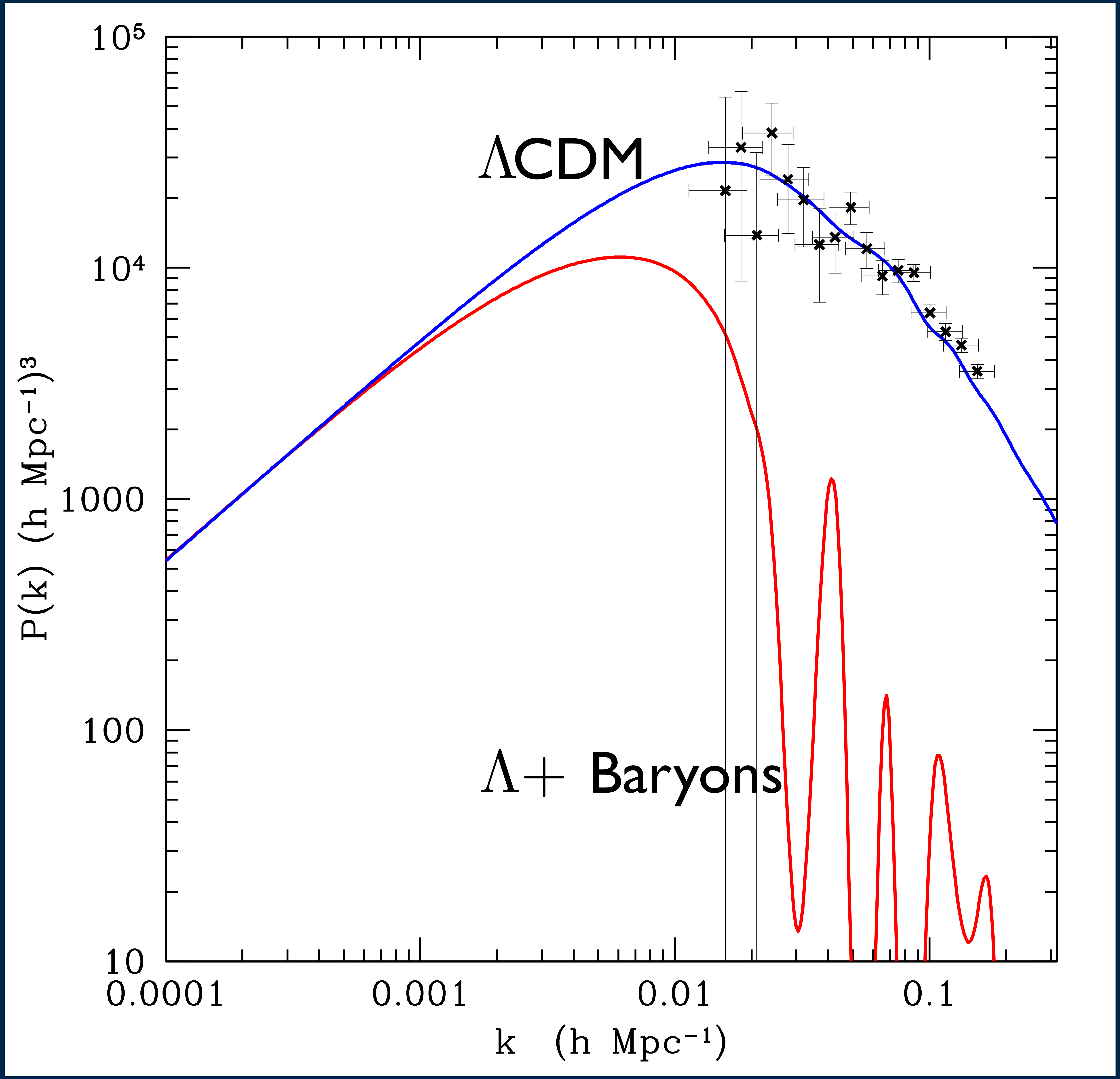
Set of Power spectra

- Galaxy-Galaxy $P_{gg}(k)$
- Galaxy-Velocity $P_{gv}(k)$
- Velocity-Velocity $P_{vv}(k)$

But galaxies may not trace the underlying matter field exactly (light does not follow mass)

Bias b s.t. $\delta_g = b\delta$ $P_{gg}(k) = b^2 P(k)$

The matter power spectrum



Neutrinos and Large Scale Structure

Collision-less particles stream out of over-dense regions and into under-dense regions (free-streaming)

Massive neutrinos undergo collision-less damping

$$\lambda_{FS} \approx 20Mpc \times \frac{30eV}{m_\nu}$$

↓

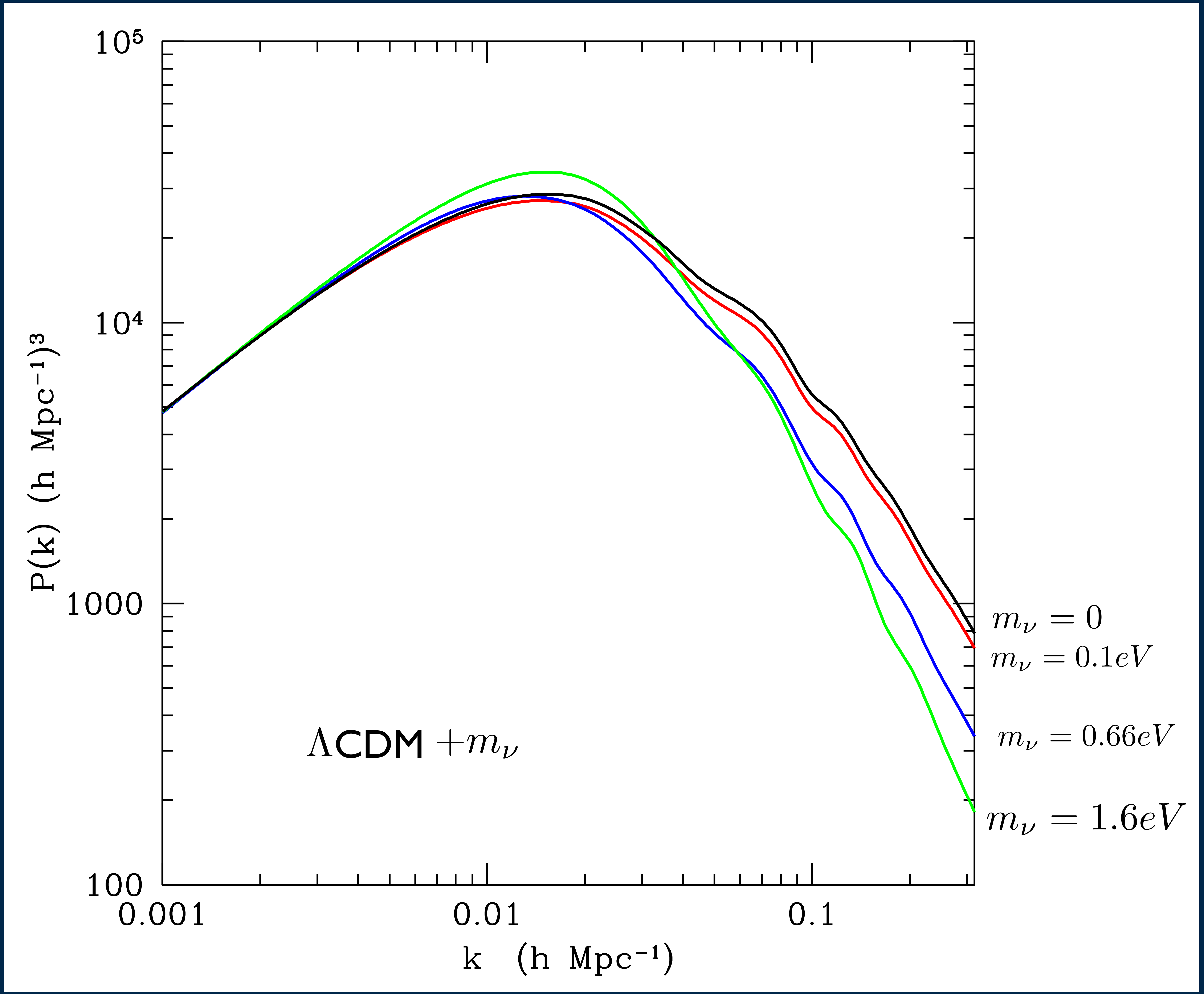
$$k_{FS} \approx 0.28 \frac{m_\nu}{1eV} Mpc^{-1}$$

Structure formation is suppressed

Current limits:

$\sum m_\nu < 0.24eV$ (Planck 2018)

$\sum m_\nu < 0.12eV$ CMB + BAO (Planck 2018)

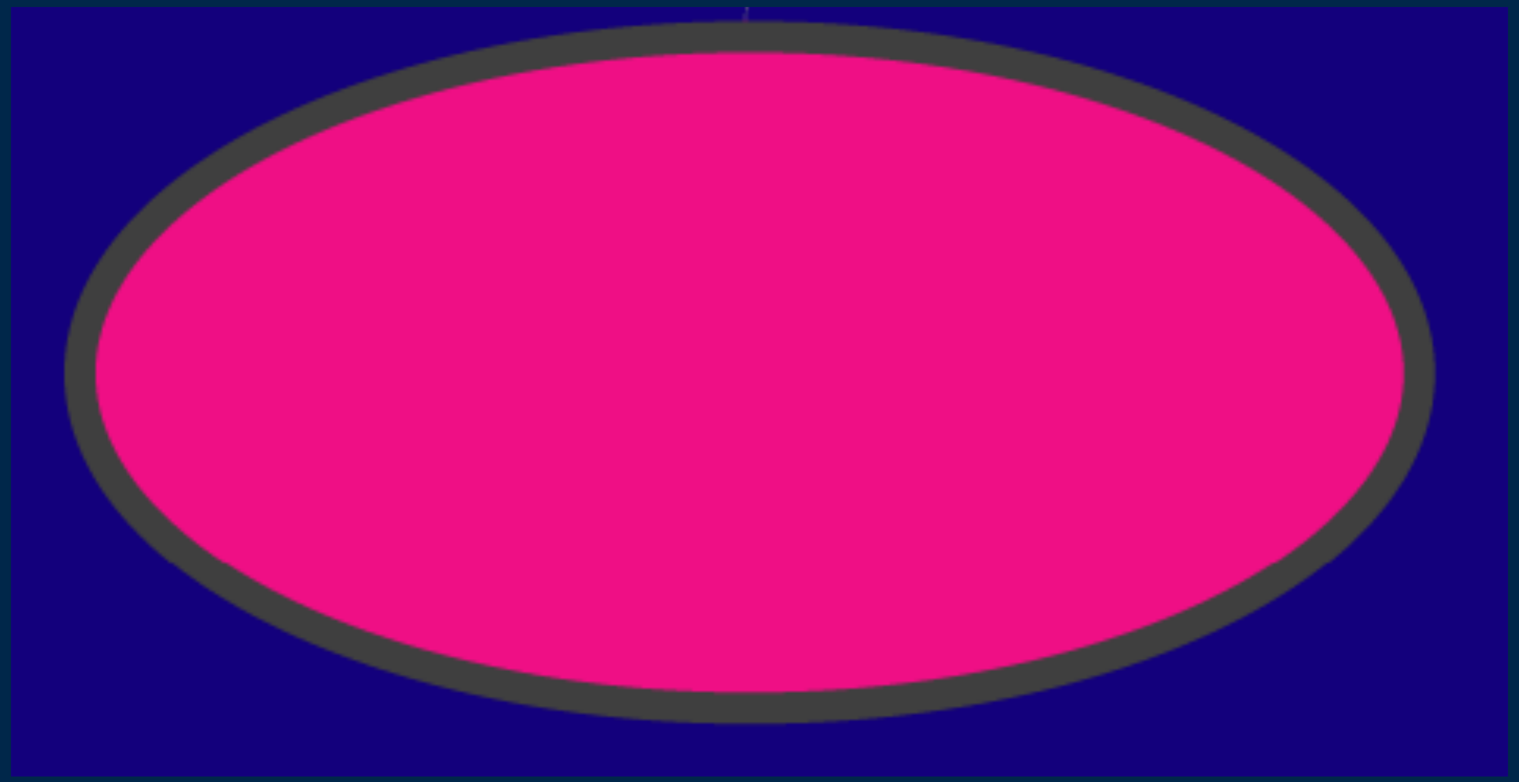


CMB anisotropies

CMB anisotropies (boring parts)

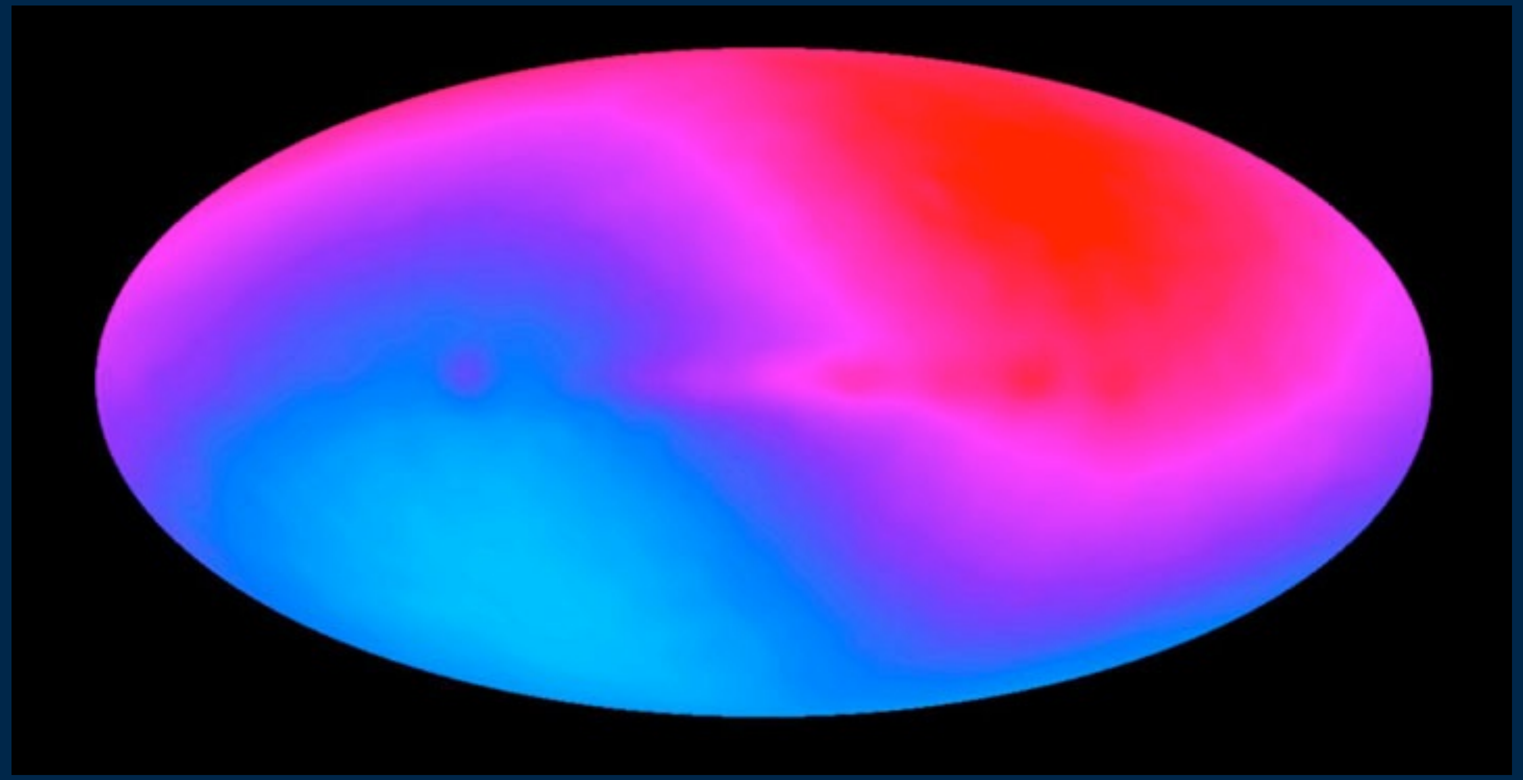
Monopole

$\bar{T}_{CMB} = 2.7255K$



Isotropic part,
same everywhere

Dipole

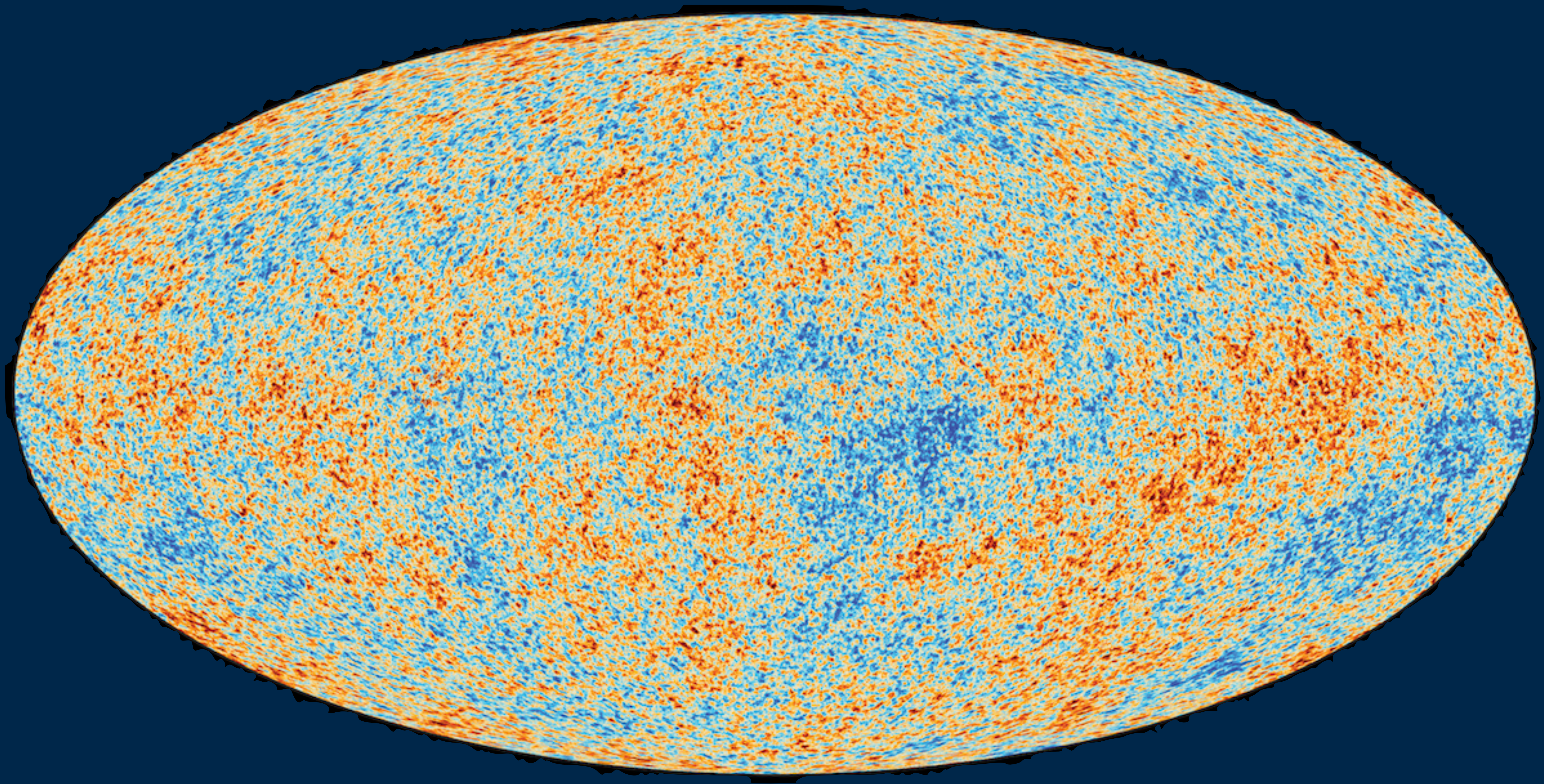


- 1000 times smaller than the monopole
- Dopper effect due to our motion wrt to rest frame of CMB
- Corresponding to a velocity of $v = (627 \pm 22)\, km/s$
- In the direction of galactic longitude $l = (276 \pm 3)^\circ$ $b = (30 \pm 3)^\circ$

CMB anisotropies (monopole + dipole subtracted)

Temperature slightly different on different patches on the sky.

1 part in 100 000

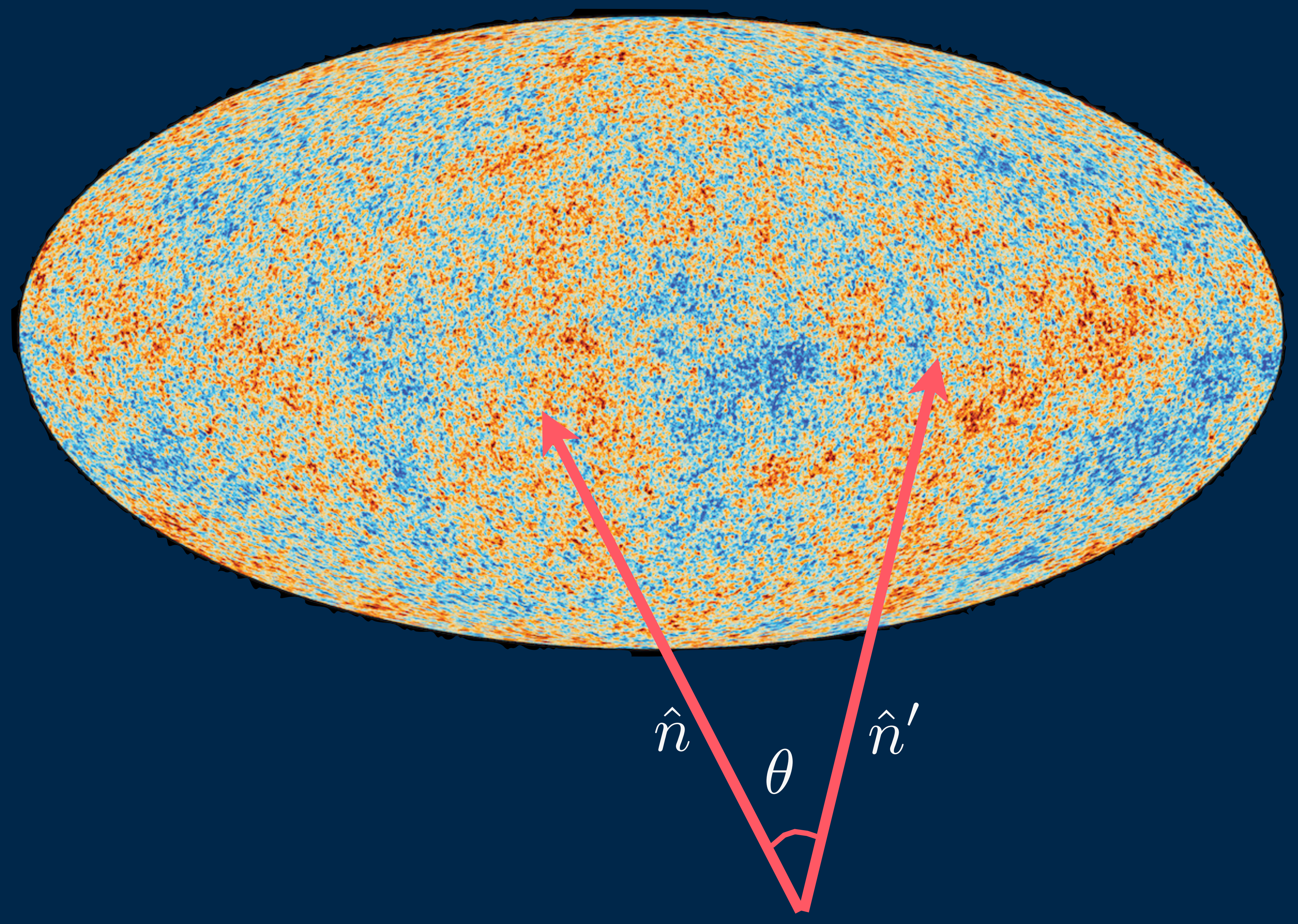


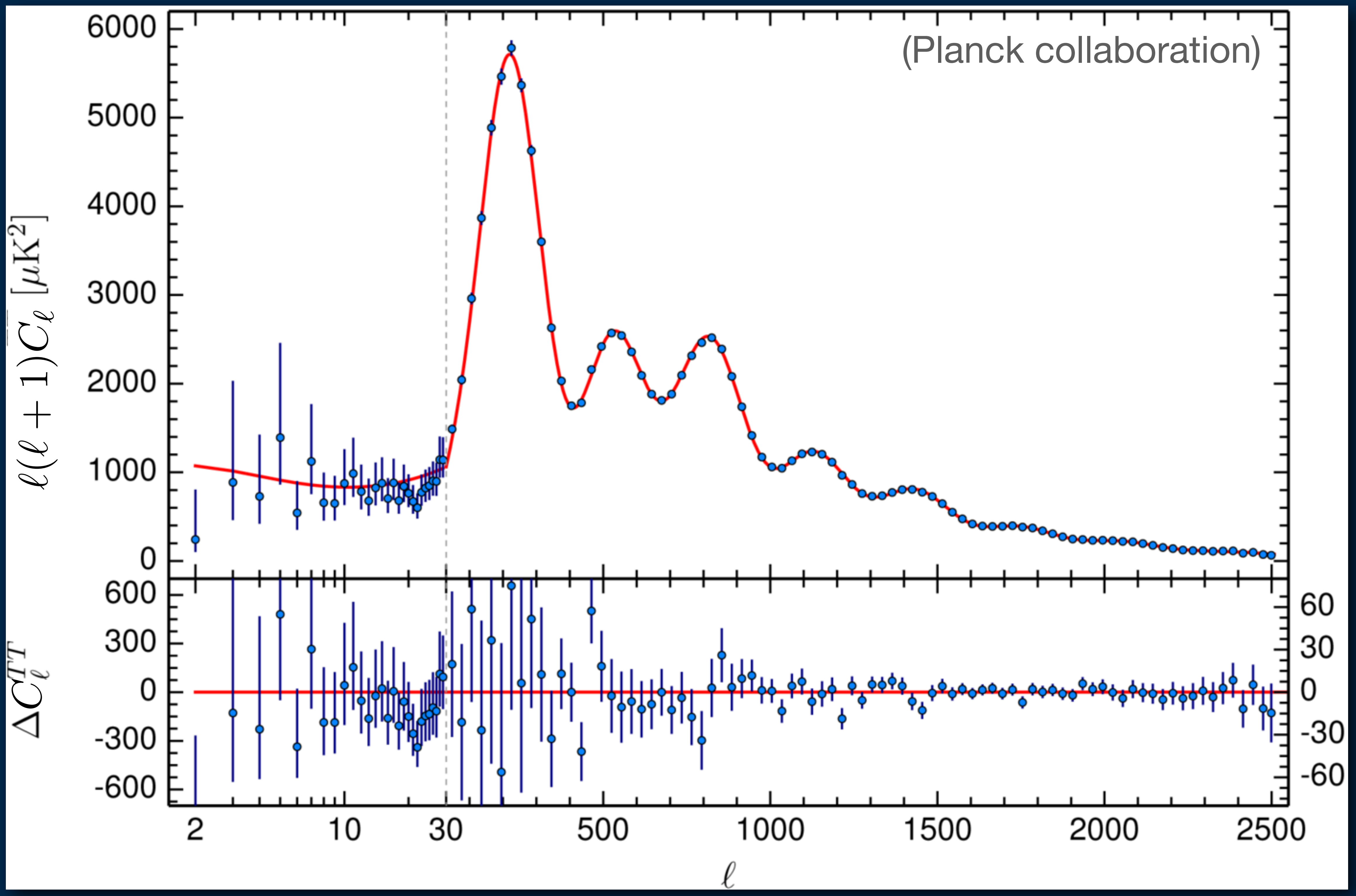
Planck Satellite

Temperature anisotropy in direction $\hat{n}$

$$\Theta(\hat{n}) = \frac{T(\hat{n}) - \bar{T}_{CMB}}{\bar{T}_{CMB}} \lesssim 10^{-5}$$

CMB angular power spectrum





$$\langle \Theta(\hat{n})\,\Theta(\hat{n}')\rangle = \sum_\ell (2\ell+1)\,C_\ell P_\ell(\cos\theta)$$

$P_\ell(\mu)$: Legendre polynomials

$\mu = [-1, 1]$

C_ℓ : Angular power spectrum

C_0 Monopole

C_1 Dipole

C_2 Quadrupole

$\cdots$

Functions defined in $[-1, 1]$

Can be expanded in terms of $P_\ell(\mu)$

$P_0 = 1$ $\quad P_1 = \mu$ $\quad P_2 = \dfrac{3\mu^2 - 1}{2}$ $\quad \cdots$

Fluctuations in the Universe

CMB anisotropies: need to go beyond the homogeneous and isotropic Universe

Simplified assumption: *ALMOST* homogeneous and isotropic

Split all variables into background (FLRW) + small fluctuation

e.g. metric $$ds^2 = -(1+2\Psi)dt^2 + a^2(1-2\Phi)\delta_{ij}dx^i dx^j$$

Two gravitational potentials: $\Psi(t,\vec{x})$ $\Phi(t,\vec{x})$

Photons: $$f(t,\vec{x},\vec{p}) = \frac{1}{e^{p/T(t,\vec{x},\vec{p})} - 1}$$

Space-dependent temperature

$$\Delta(t,\vec{x},\vec{p}) = \frac{T(t,\vec{x},\vec{p}) - \bar{T}_\gamma(t)}{\bar{T}_\gamma(t)} = \sum_\ell (2\ell+1)\Delta_\ell(t,k)P_\ell(\hat{x}\cdot\hat{p})$$

$$C_\ell = \frac{2}{\pi}\int dk k^2 P_0(k)\Delta_\ell(t_0,k)|^2$$

Initial power spectum (inflation)

Do the same for baryons, CDM and neutrinos (density fluctuation, velocity, …)

Fourier expansion of every fluctuation variable

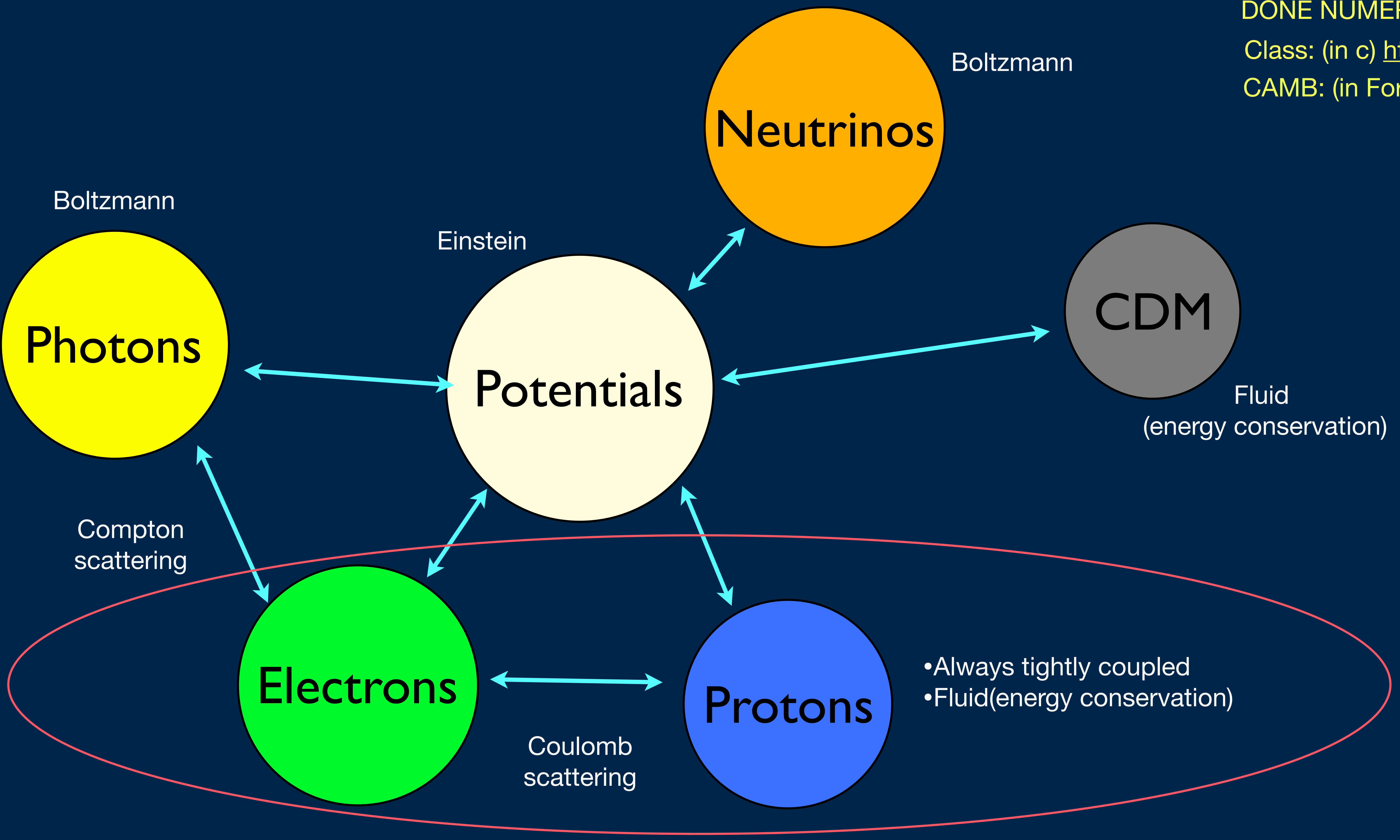
Einstein-Fluid-Boltzmann system

The evolution of the metric potentials, baryon and dark matter fluid variables, and photon and neutrino temperature fluctuations

DONE NUMERICALLY with codes:

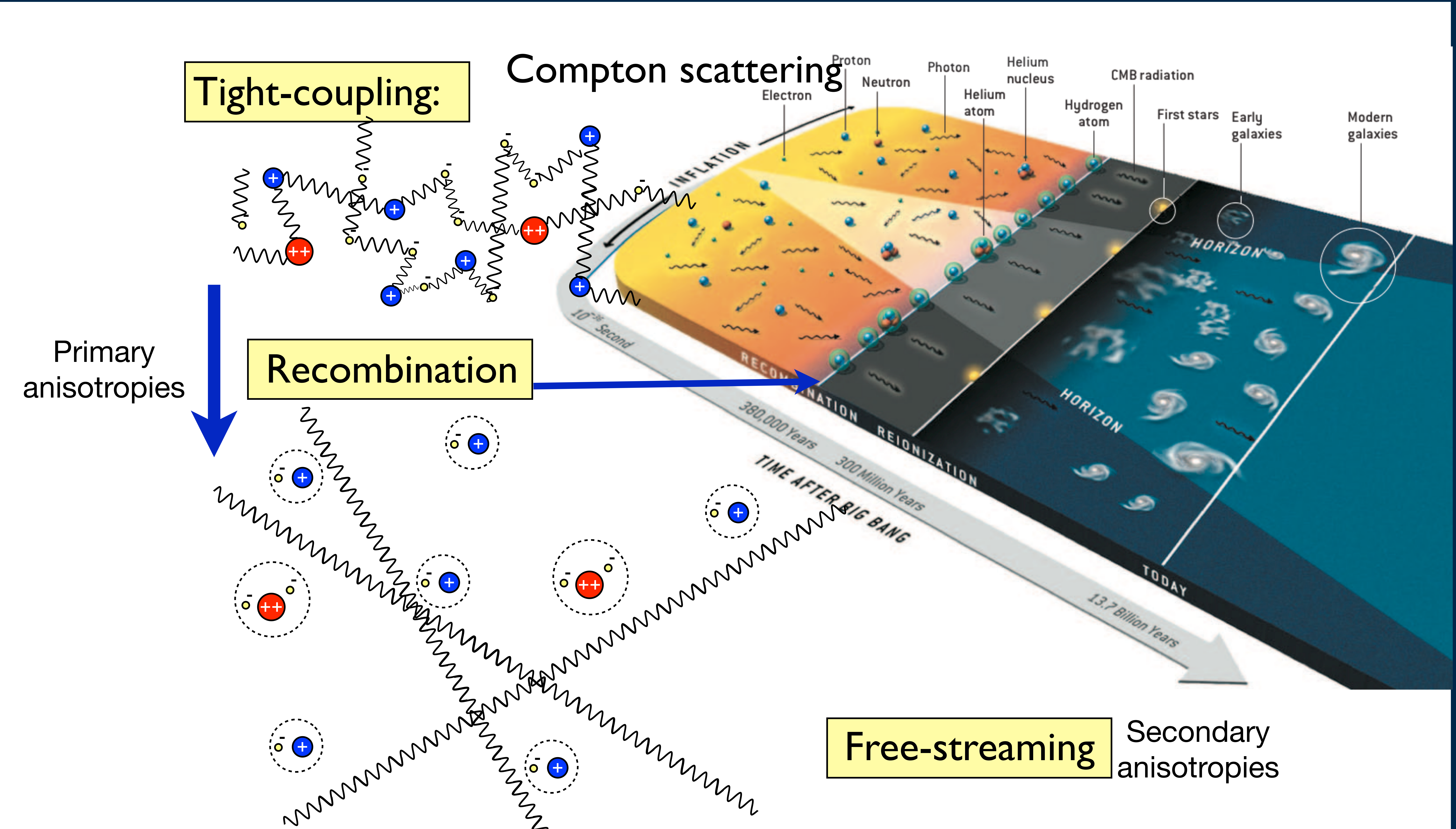
Class: (in c) https://lesgourg.github.io

CAMB: (in Fortran & Python) https://camb.readthedocs.io/

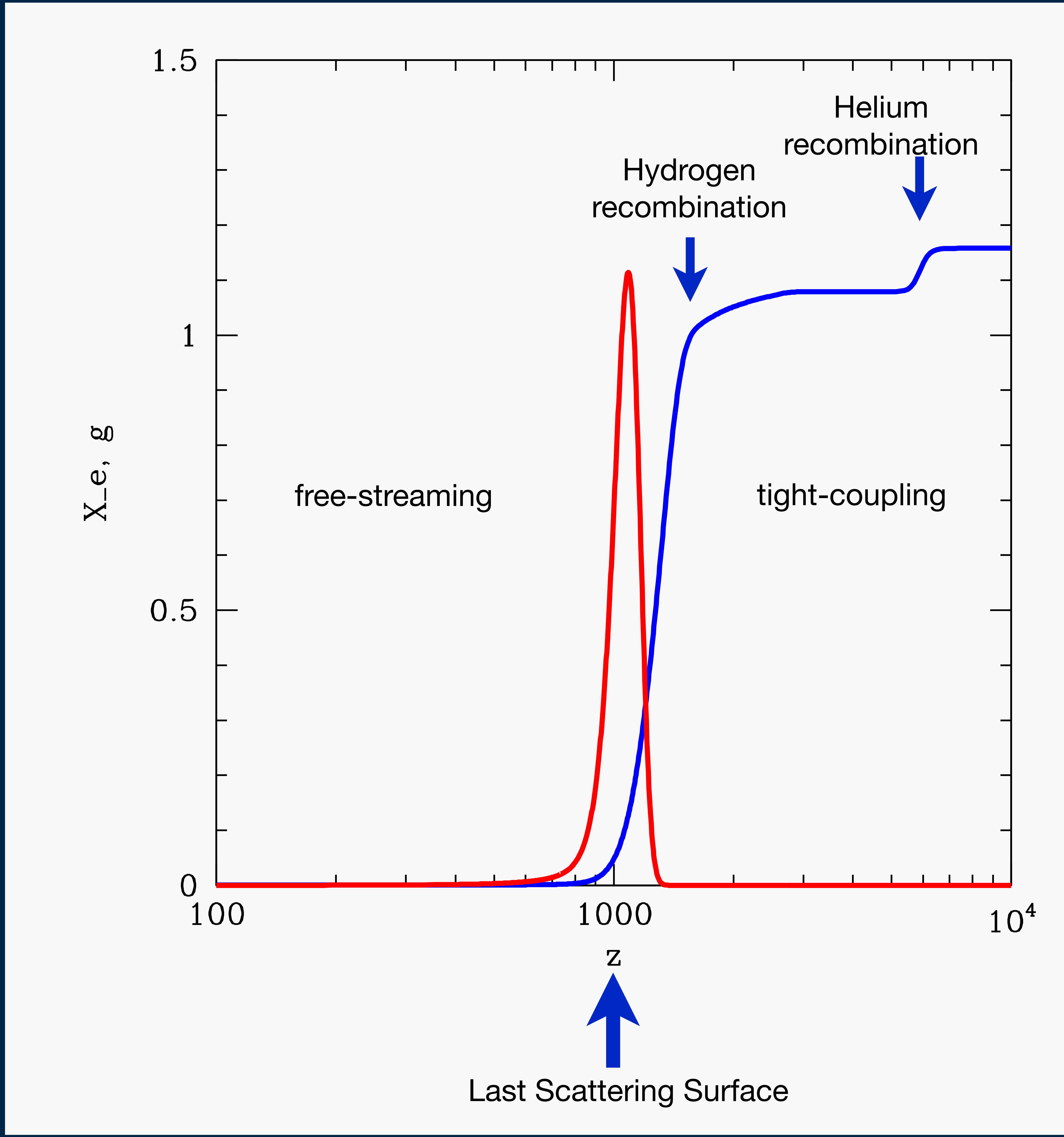


(from S.Dodelson)

Tight-coupling and Free-streaming



Visibility function



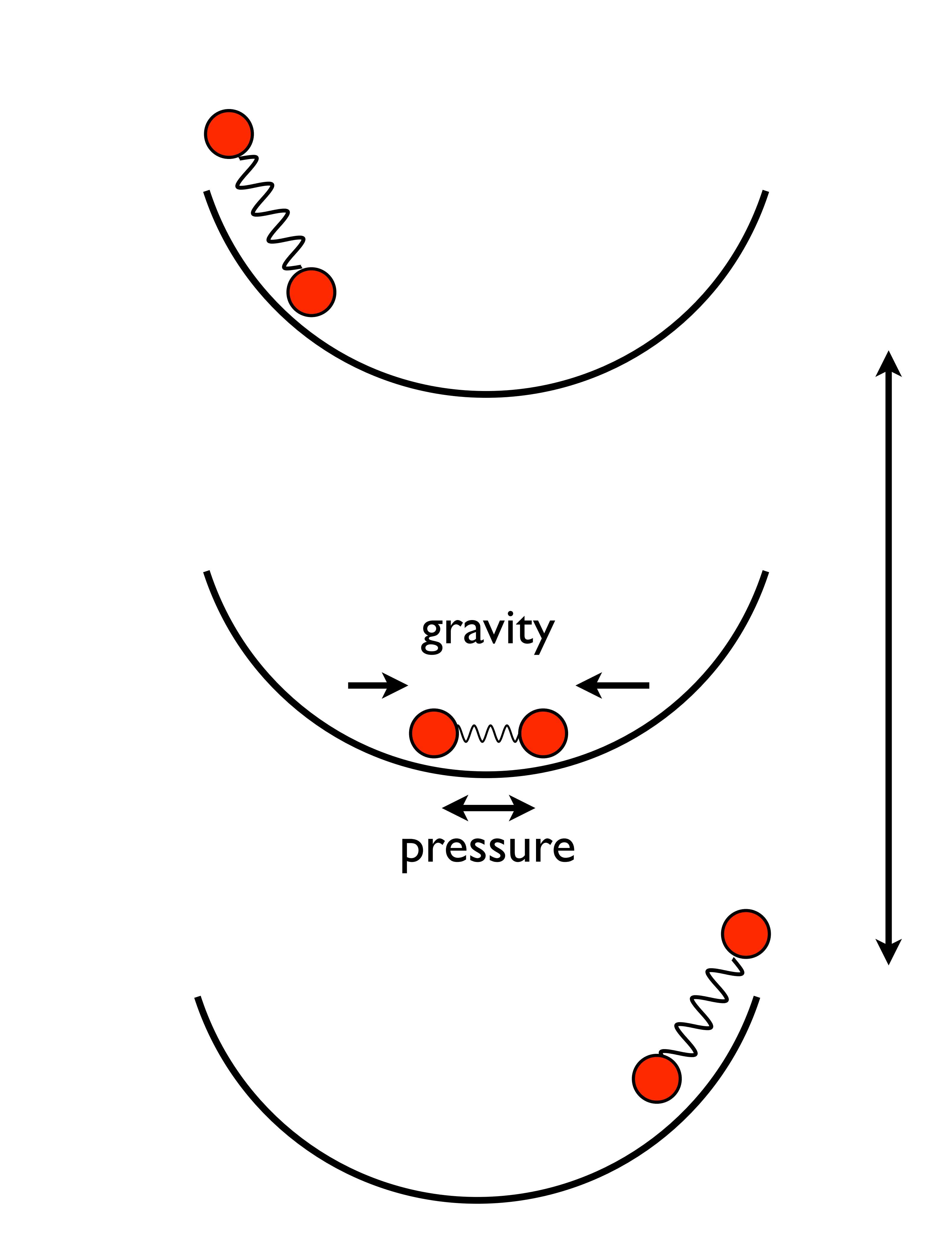
$X_e(z)$ Electron ionization fraction

number of free electrons

$g(z)$ Visibility function

Proportional to the probability that an observed photon today has last scattered at redshift z

Acoustic oscillations



Tight-coupling very efficient — destroys all $\Delta_\ell(t,k) \quad \ell \geq 2$

Full system of equations reduces to

$$\ddot{\Delta}_0 + \frac{\dot{R}}{1+R}\dot{\Delta}_0 + \frac{k^2}{3(1+R)}\Delta_0 = -\frac{k^2}{3}\Psi - \frac{1}{1+R}\frac{d}{dt}\left[(1+R)\dot{\Phi}\right]$$

Baryon-photon ratio $R = \frac{3\bar{\rho}_b}{4\bar{\rho}_\gamma}$

Gravity

Gravity

$$C_{\gamma b}^2 = \frac{1}{3(1+R)}$$

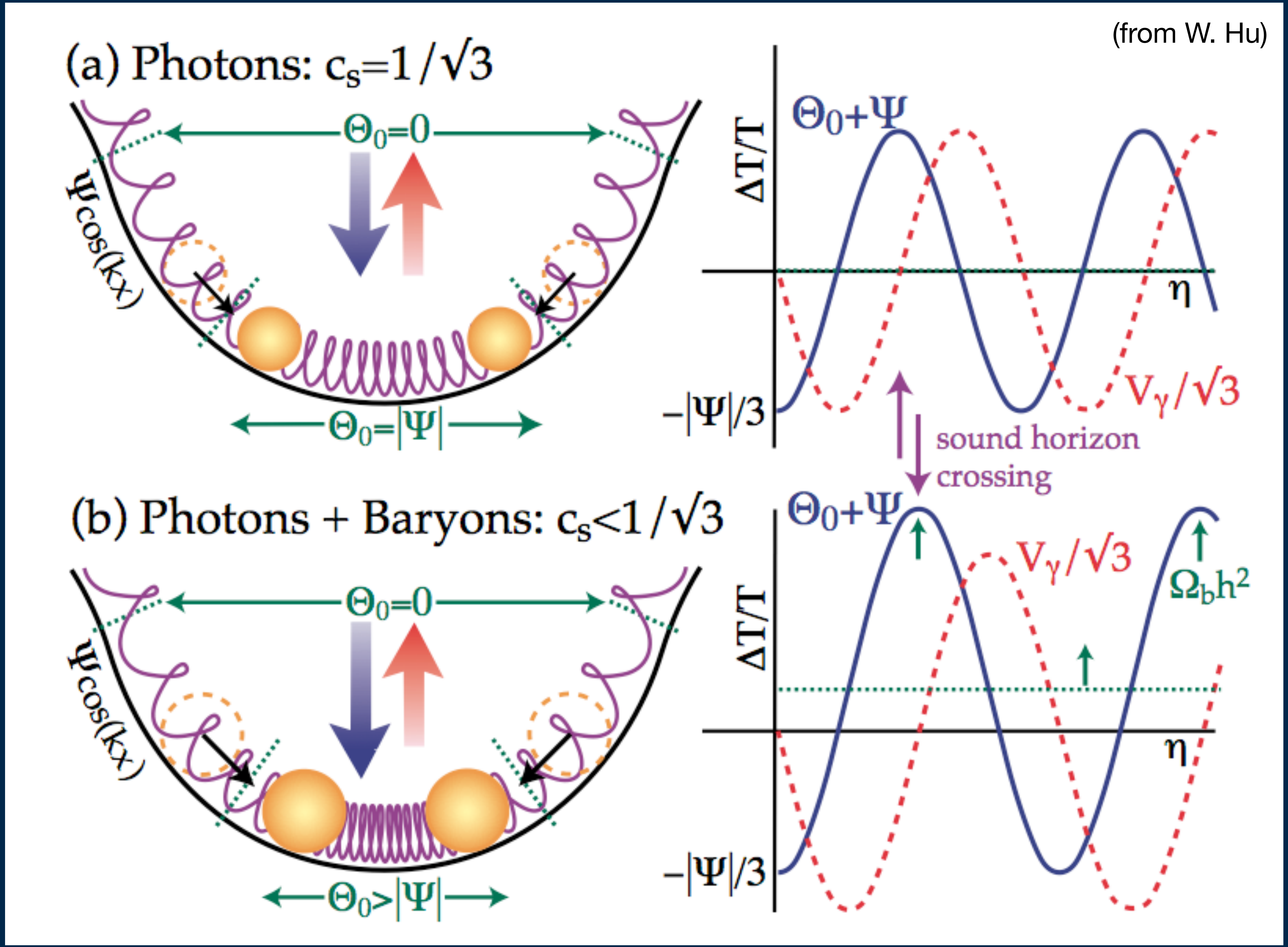
Sound horizon: $r_s = \int_0^t C_{\gamma b} dt$

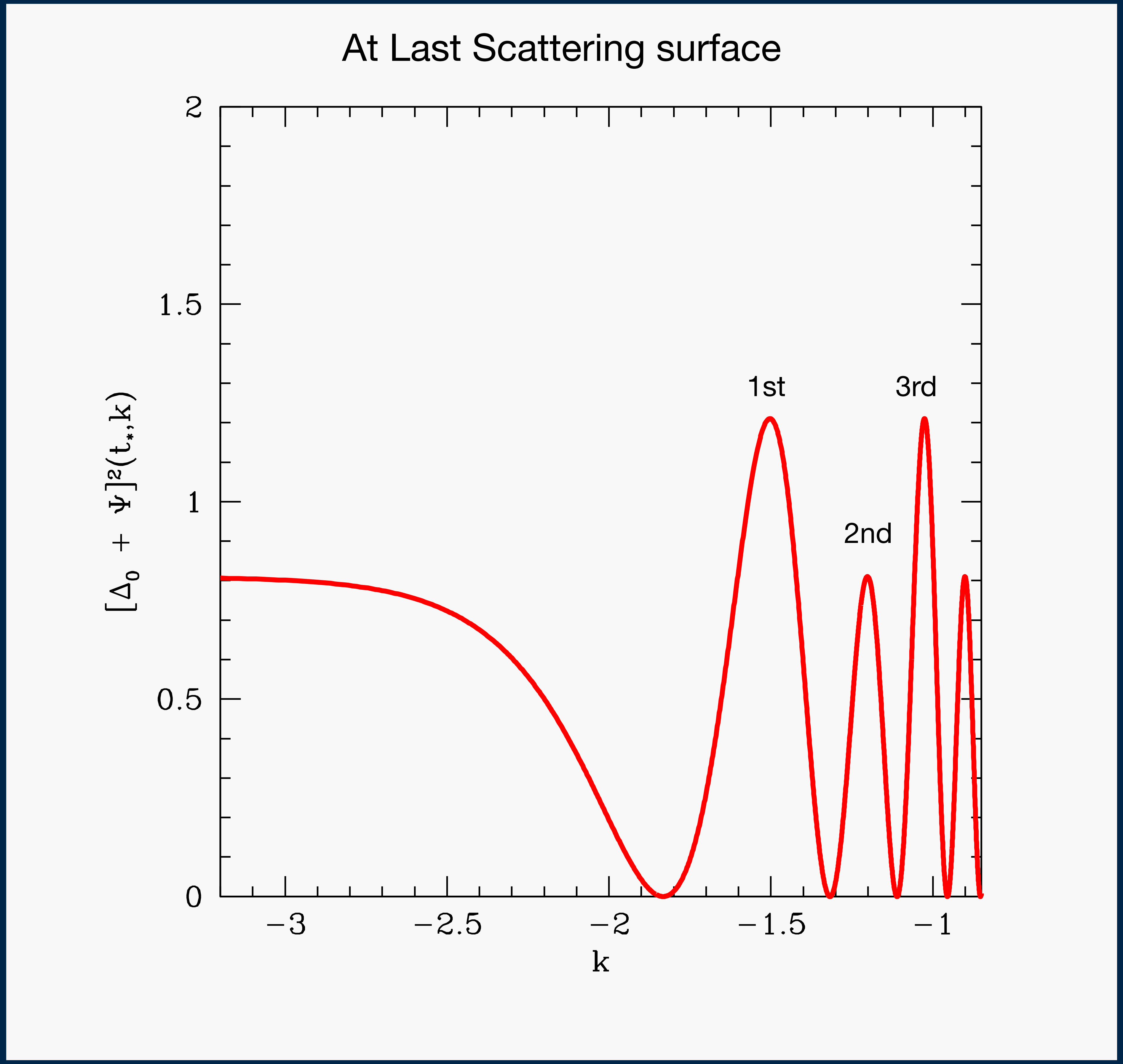
Assume slowly varying potentials and sound horizon

$$\ddot{\Delta}_0 + k^2 C_s^2 \Delta_0 \approx -\frac{k^2}{3}\Psi$$

$$(\Delta_0 + \Psi)(t_*) = A\cos(k r_s^* t_*) - R\Psi(t_*)$$

Oscillations displaced by gravity due to baryons
Frequency set by sound horizon (baryons)

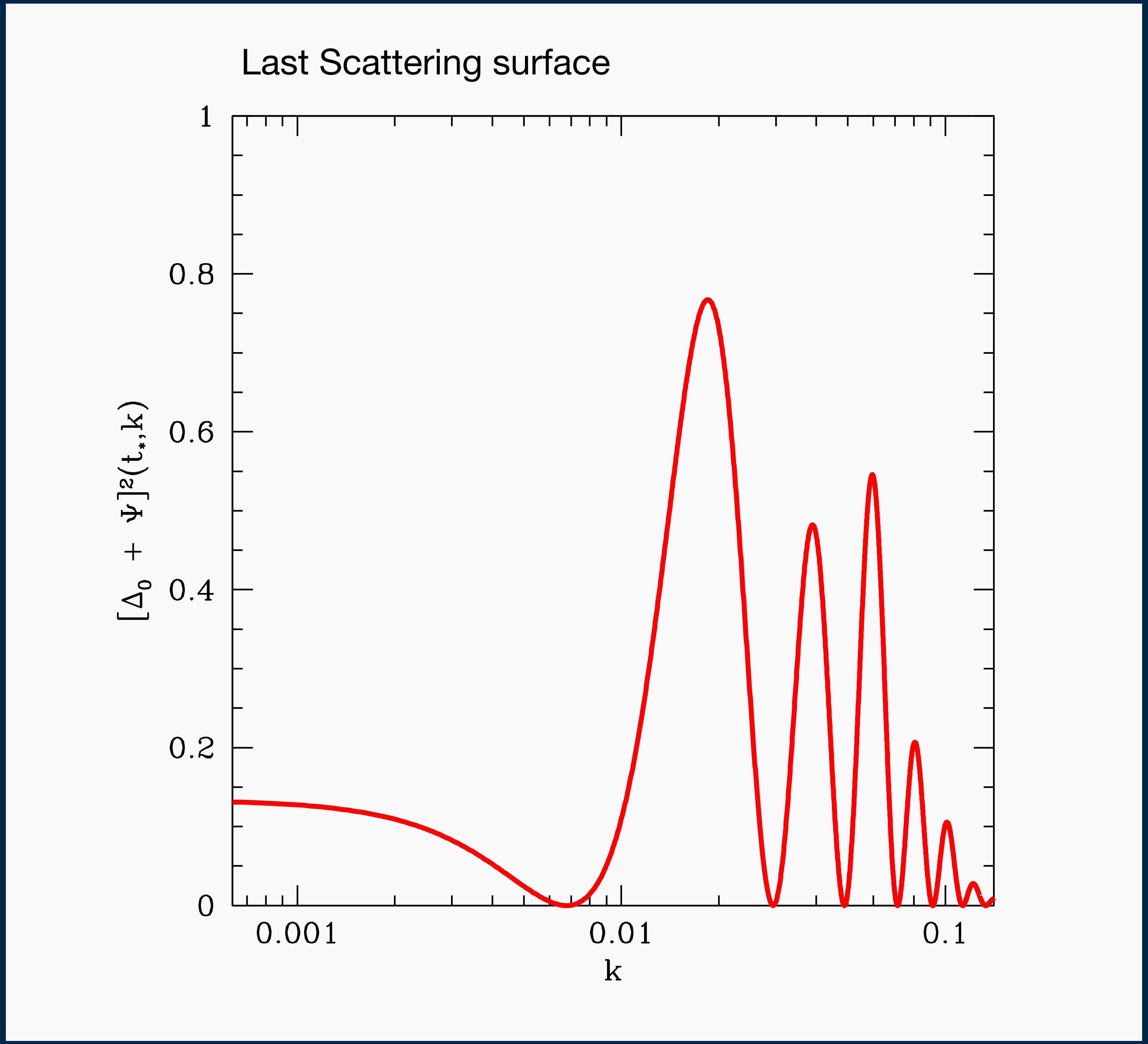

(from W. Hu)
(a) Photons: $c_s=1/\sqrt{3}$
$\Theta_0=0$
$\Psi\cos(kx)$
$\Theta_0=|\Psi|$
$\Delta T/T$
$\Theta_0+\Psi$
η
$-|\Psi|/3$
$V_\gamma/\sqrt{3}$
sound horizon
crossing
(b) Photons + Baryons: $c_s<1/\sqrt{3}$
$\Theta_0=0$
$\Psi\cos(kx)$
$\Theta_0>|\Psi|$
$\Delta T/T$
$\Theta_0+\Psi$
$V_\gamma/\sqrt{3}$
$\Omega_b h^2$
η
$-|\Psi|/3$


At Last Scattering surface
$[\Delta_0+\Psi]^2(t_*,k)$
k
1st
2nd
3rd
0
0.5
1
1.5
2
−3
−2.5
−2
−1.5
−1

Projection

After recombination → Free-streaming: photons follow null geodesics

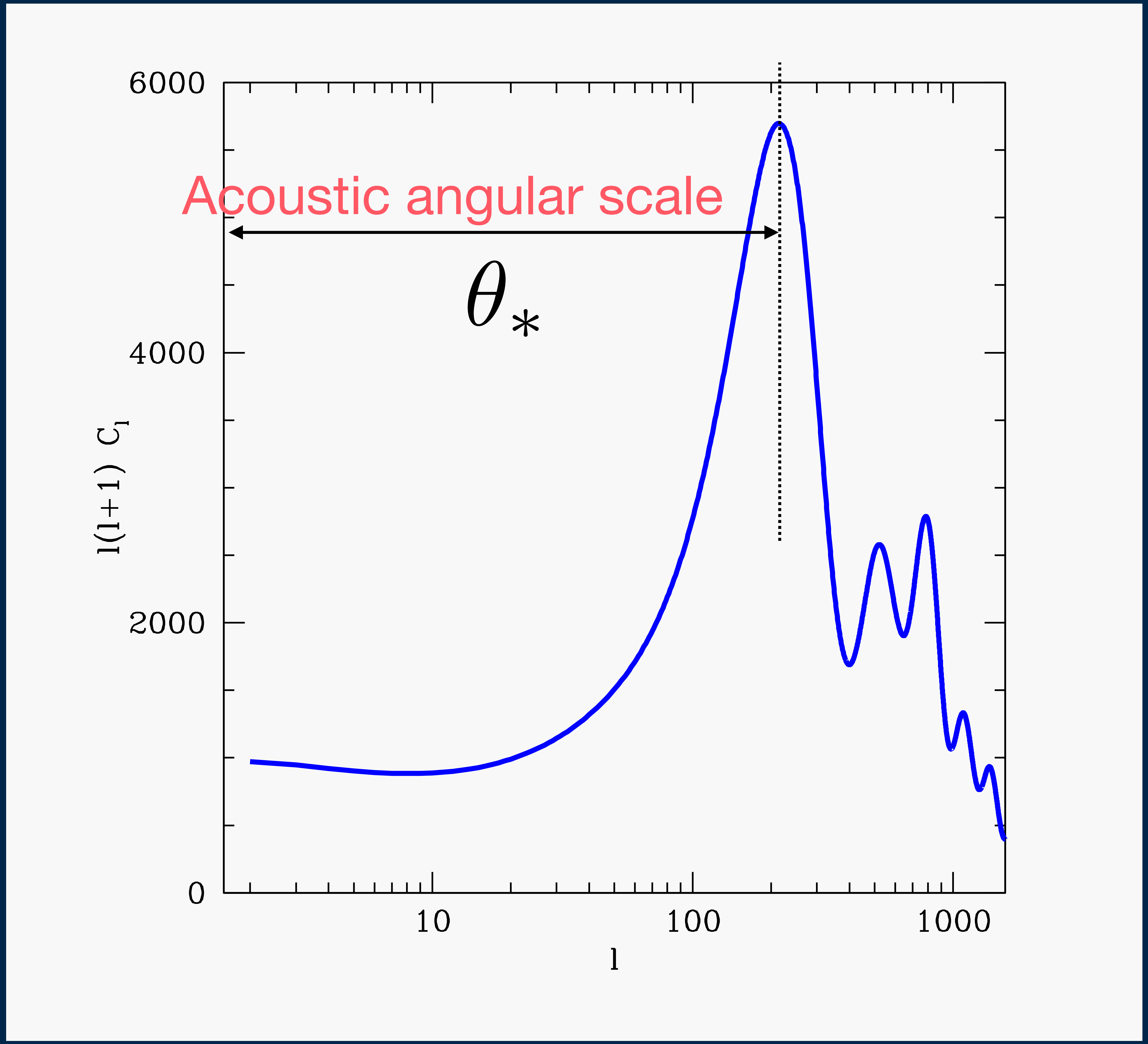
$$\Delta_\ell(t_0, k) \approx (\Delta_0 + \Psi)(t_*) j_\ell [k(t_0 - t_*)]$$



nth Peak location

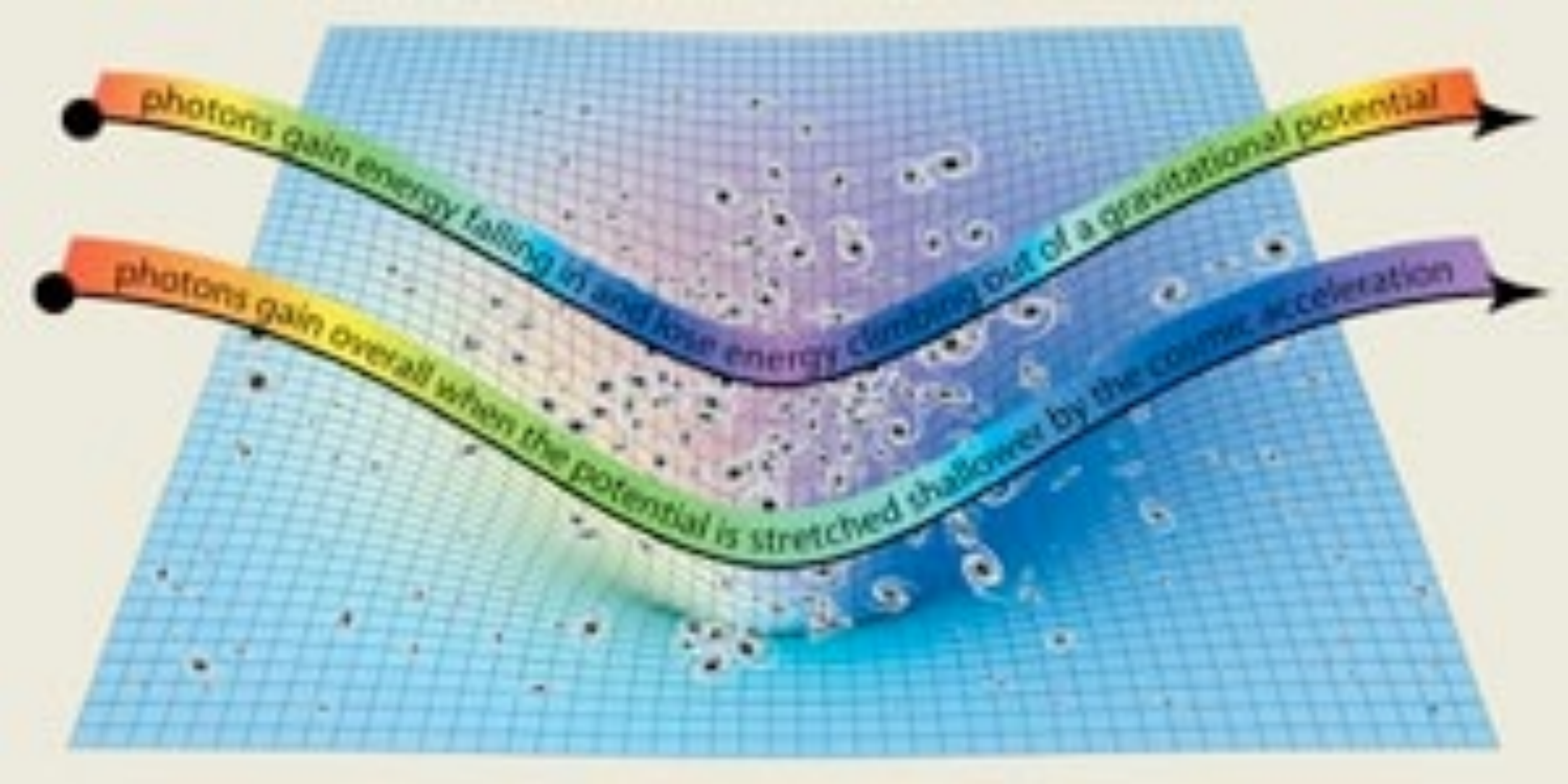
$$\ell_n = \frac{n\pi (t_0 - t_*)}{r_s}$$

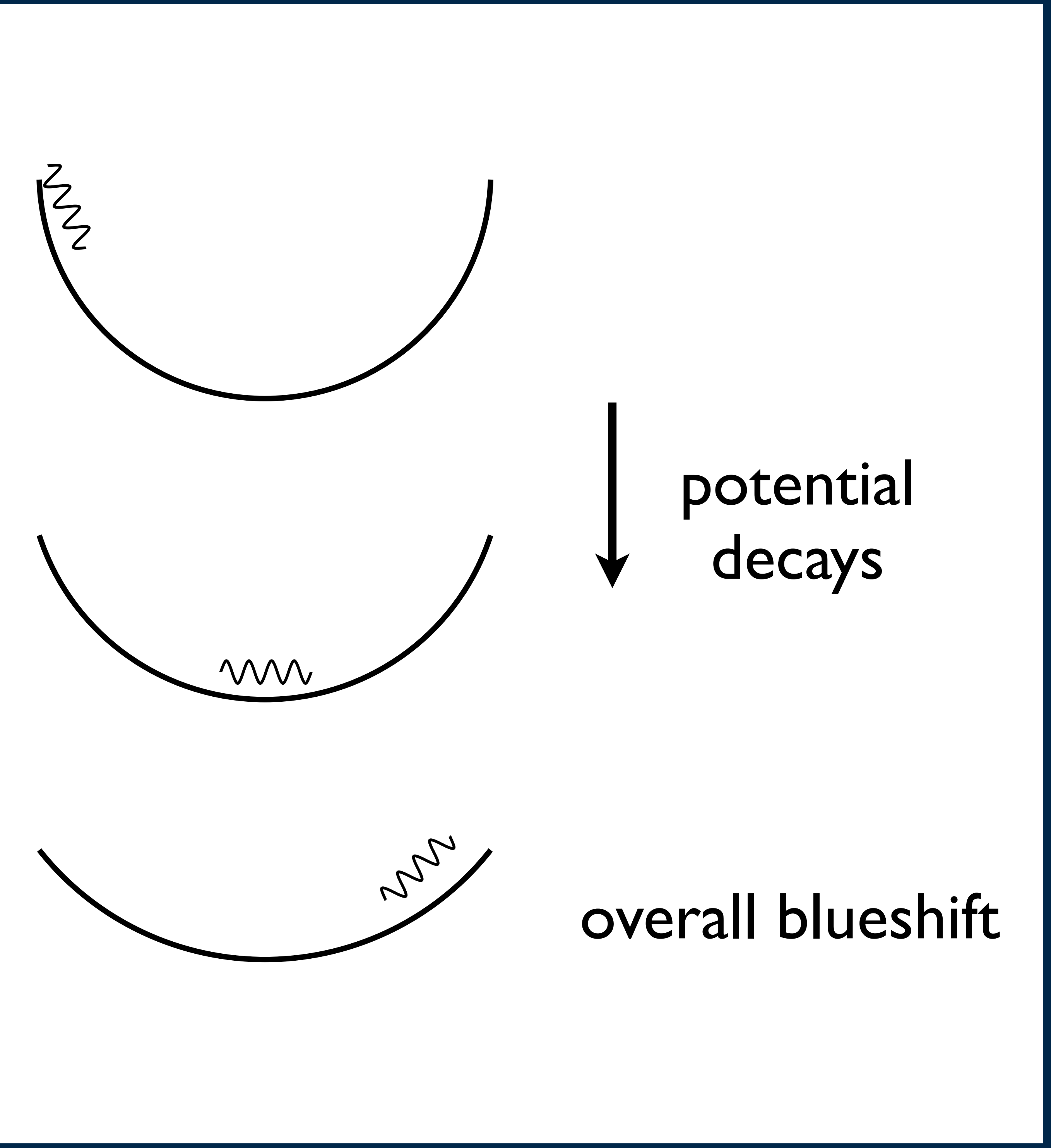
Strong dependence on: curvature, Λ, baryons



Integrated Sachs-Wolfe effect

Gravitational redshift of photons due to a potential well



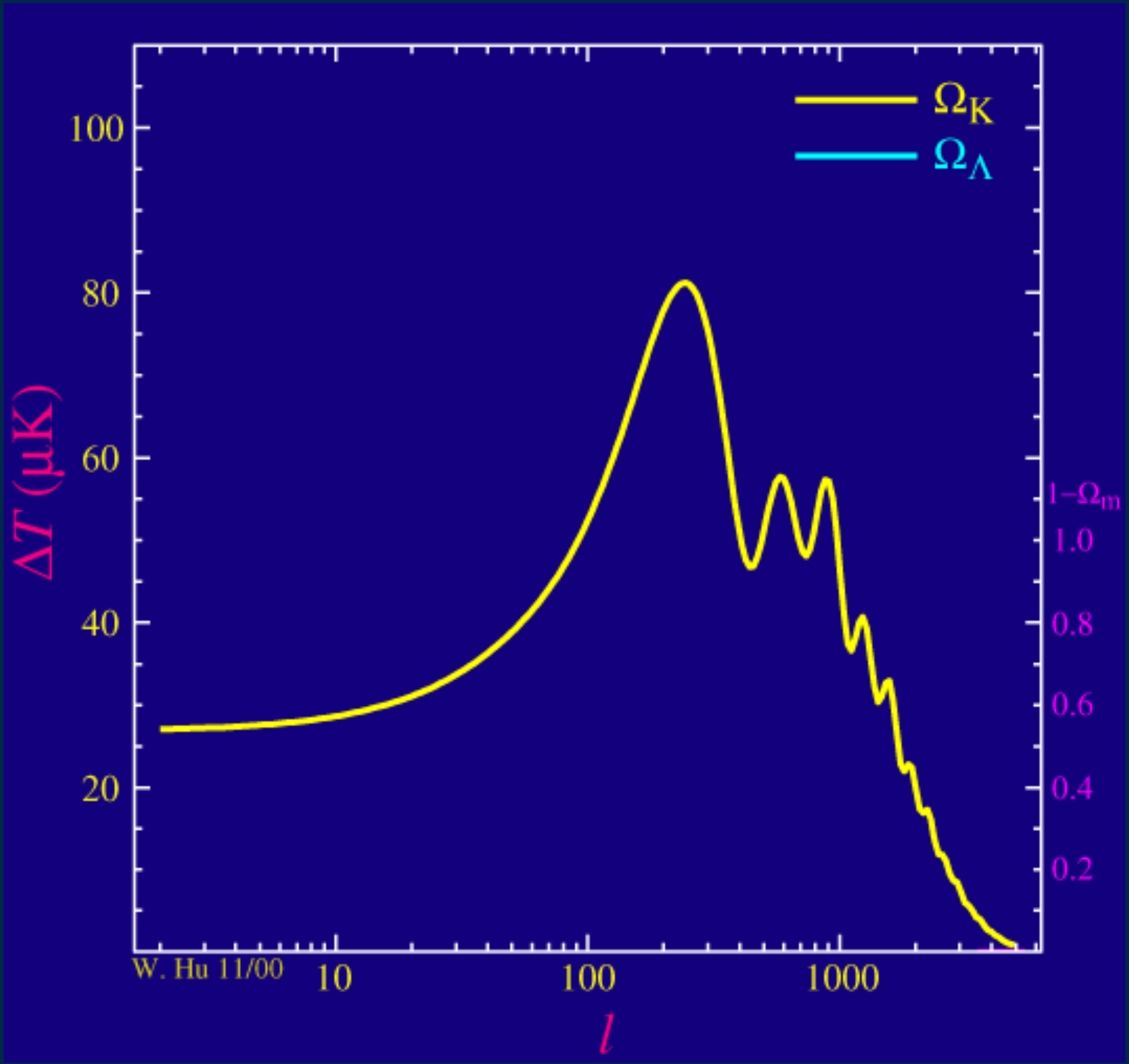


$$\dot{\Phi} \neq 0$$

potentials constant during Matter domination → No ISW

Radiation-to-matter transition: early ISW effect → small scales (Extra radiation or less matter)

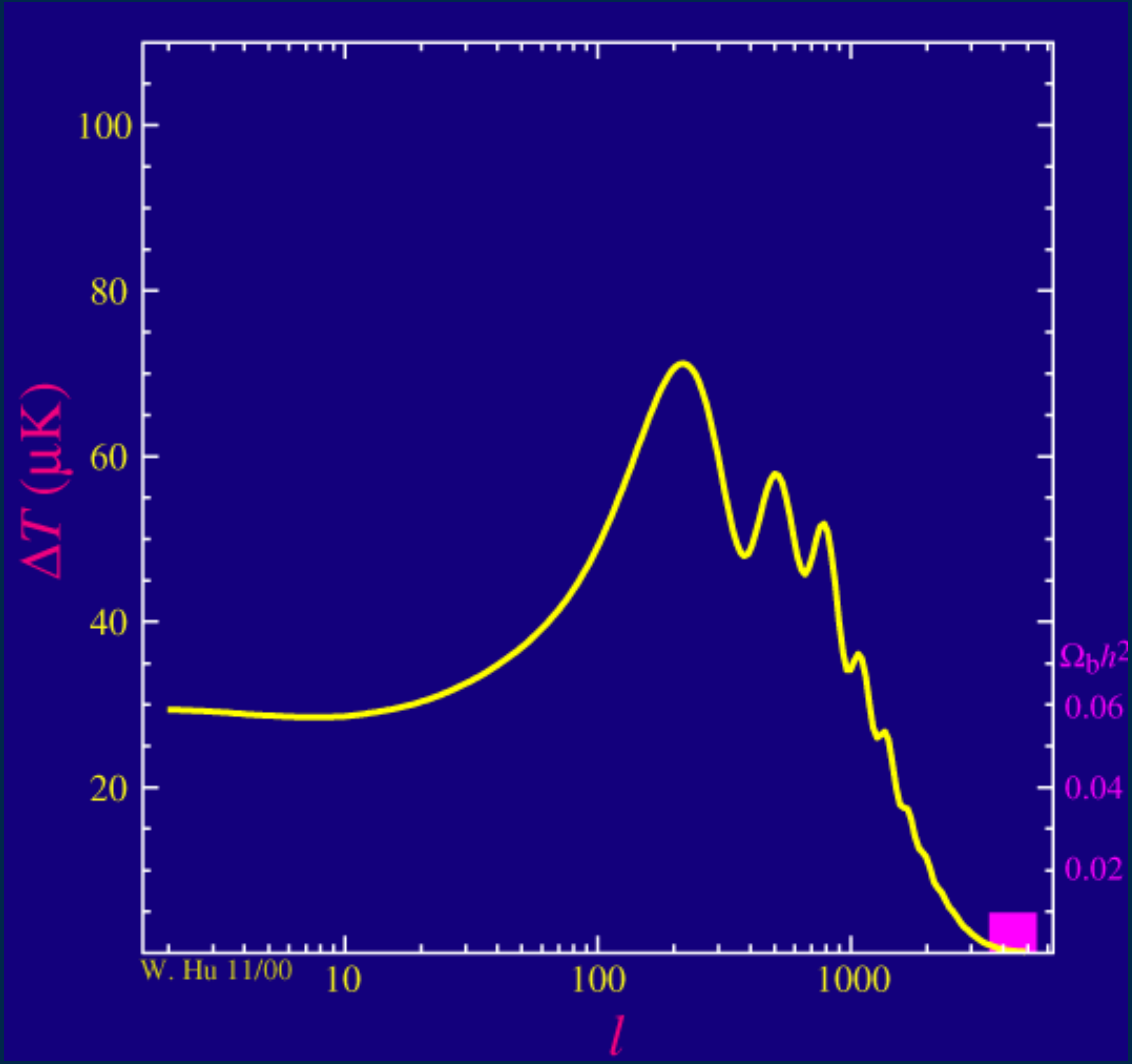
Λ Era: late ISW effect → large scales



ANIMATION:

From Wayne Hu's webpage

http://background.uchicago.edu/~whu



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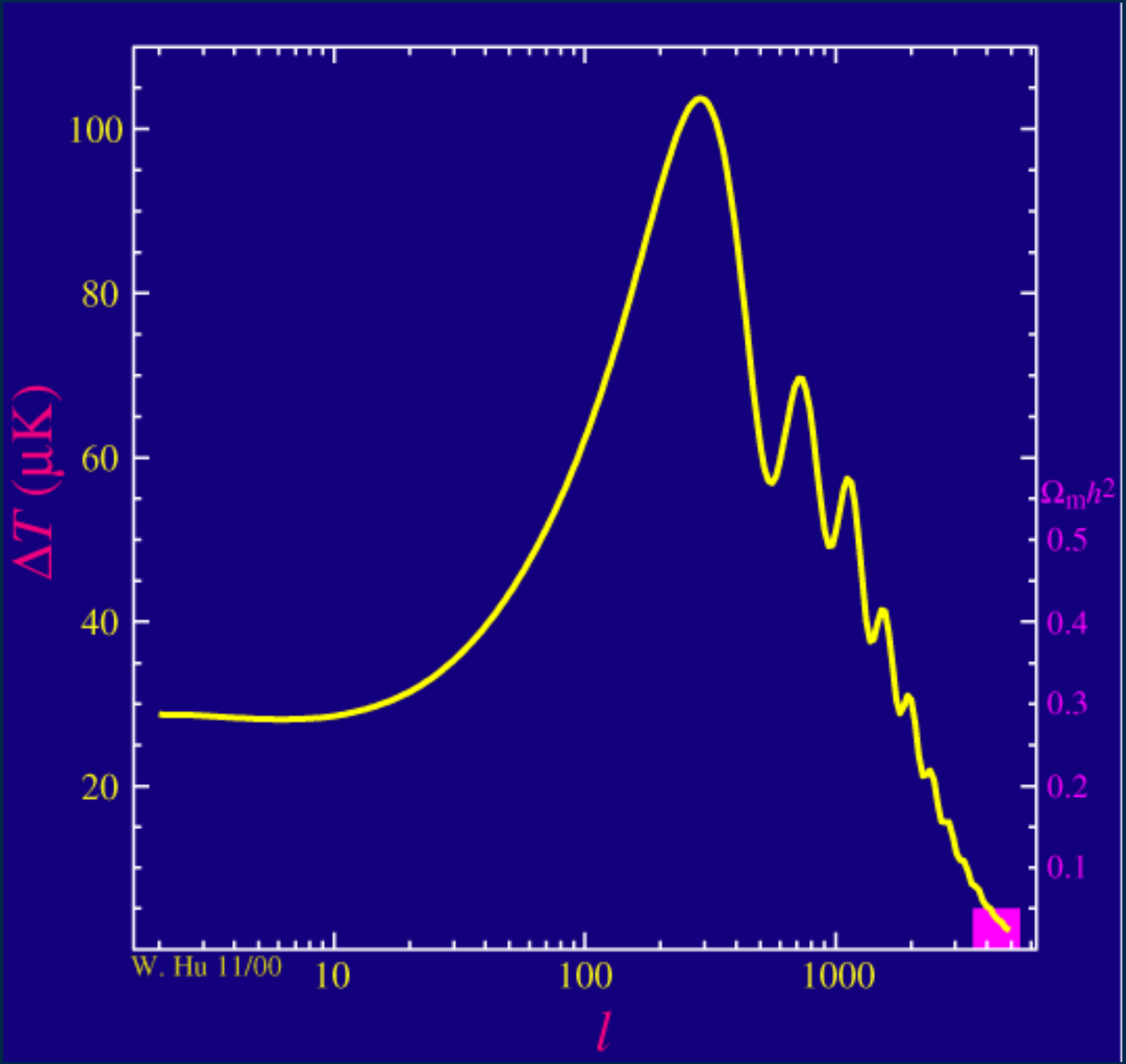

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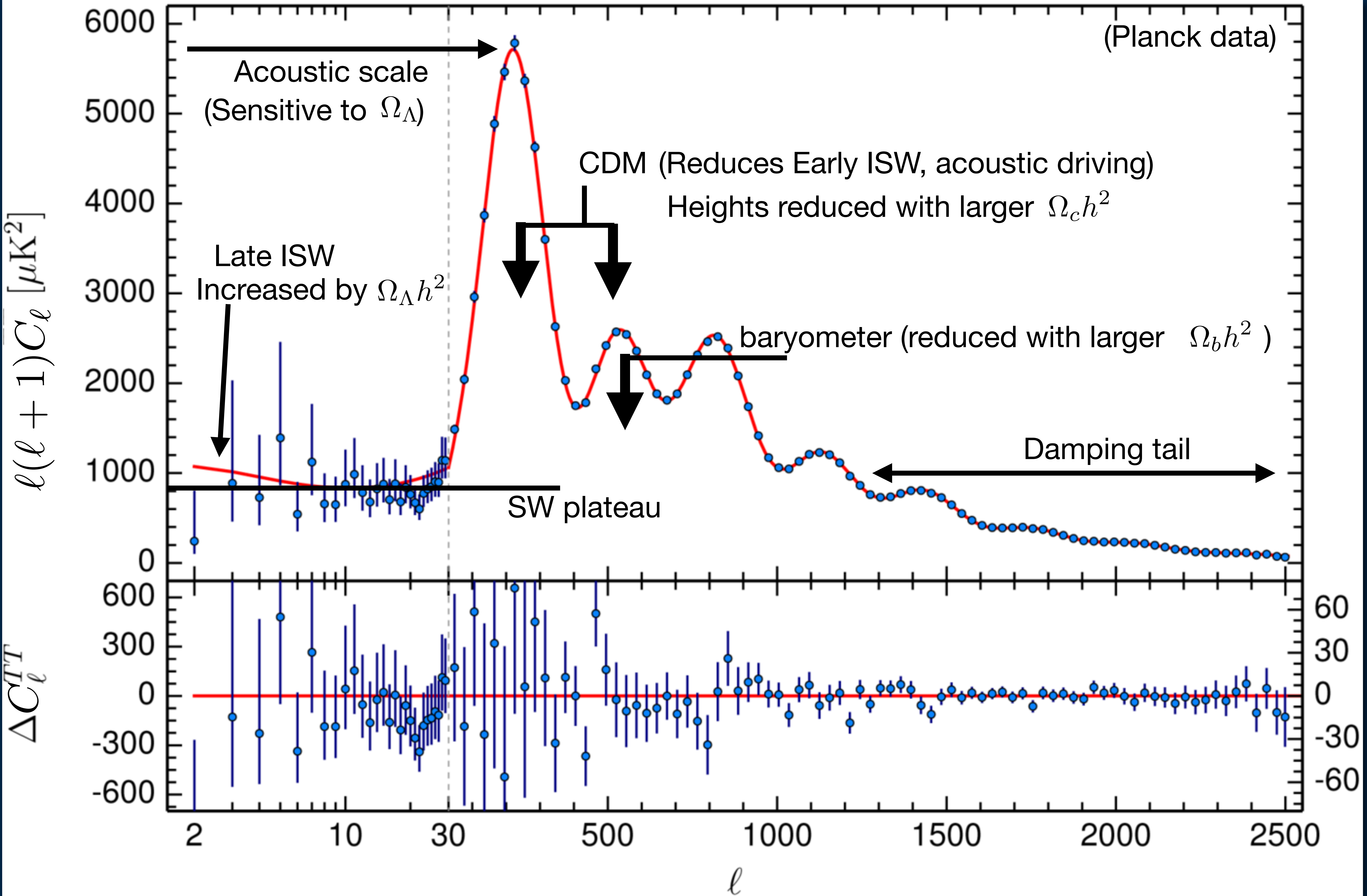
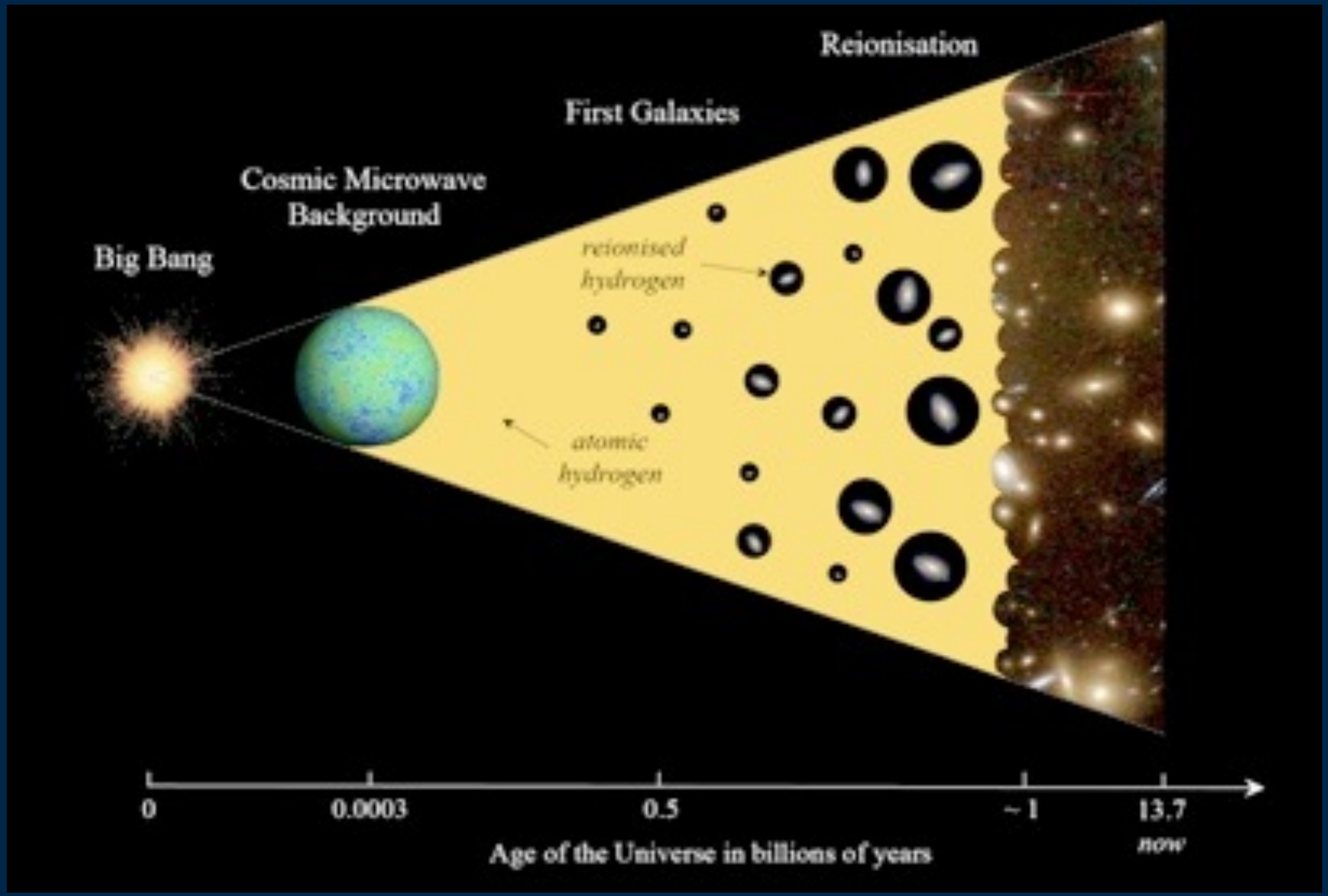


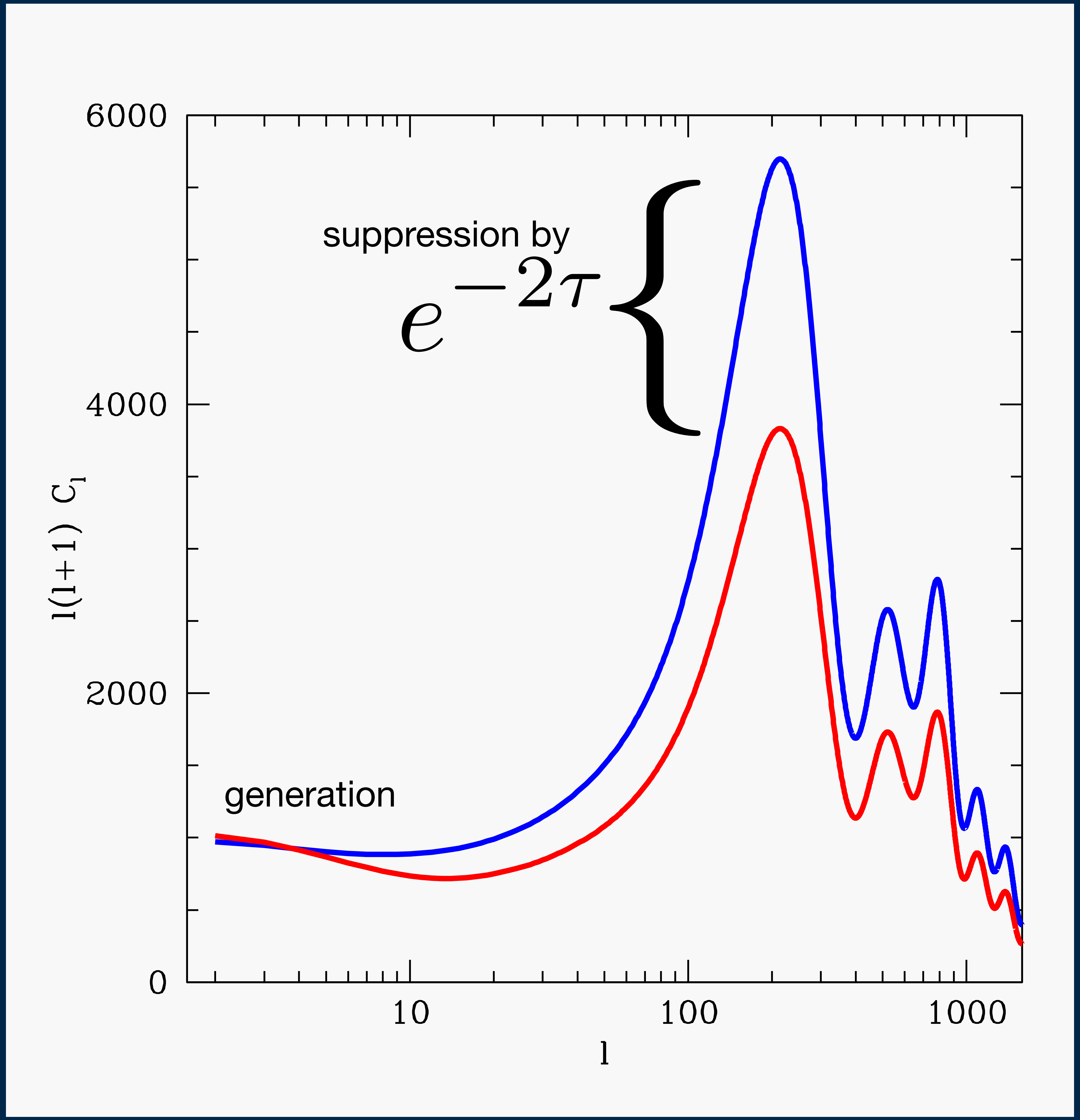
(Planck data)
Acoustic scale
(Sensitive to Ω_Λ)
CDM (Reduces Early ISW, acoustic driving)
Heights reduced with larger $\Omega_c h^2$
Late ISW
Increased by $\Omega_\Lambda h^2$
baryometer (reduced with larger $\Omega_b h^2$)
Damping tail
SW plateau
$\ell(\ell+1)C_\ell\ [\mu K^2]$
ΔC_ℓ^{TT}
ℓ
6000
5000
4000
3000
2000
1000
0
600
300
0
-300
-600
60
30
0
-30
-60
2
10
30
500
1000
1500
2000
2500

Reionization

It's tight-coupling again!

optical depth to reionization $\tau = \sigma_T \int_{t_0}^{t_r} \frac{X_e(t)}{a^2} dt$



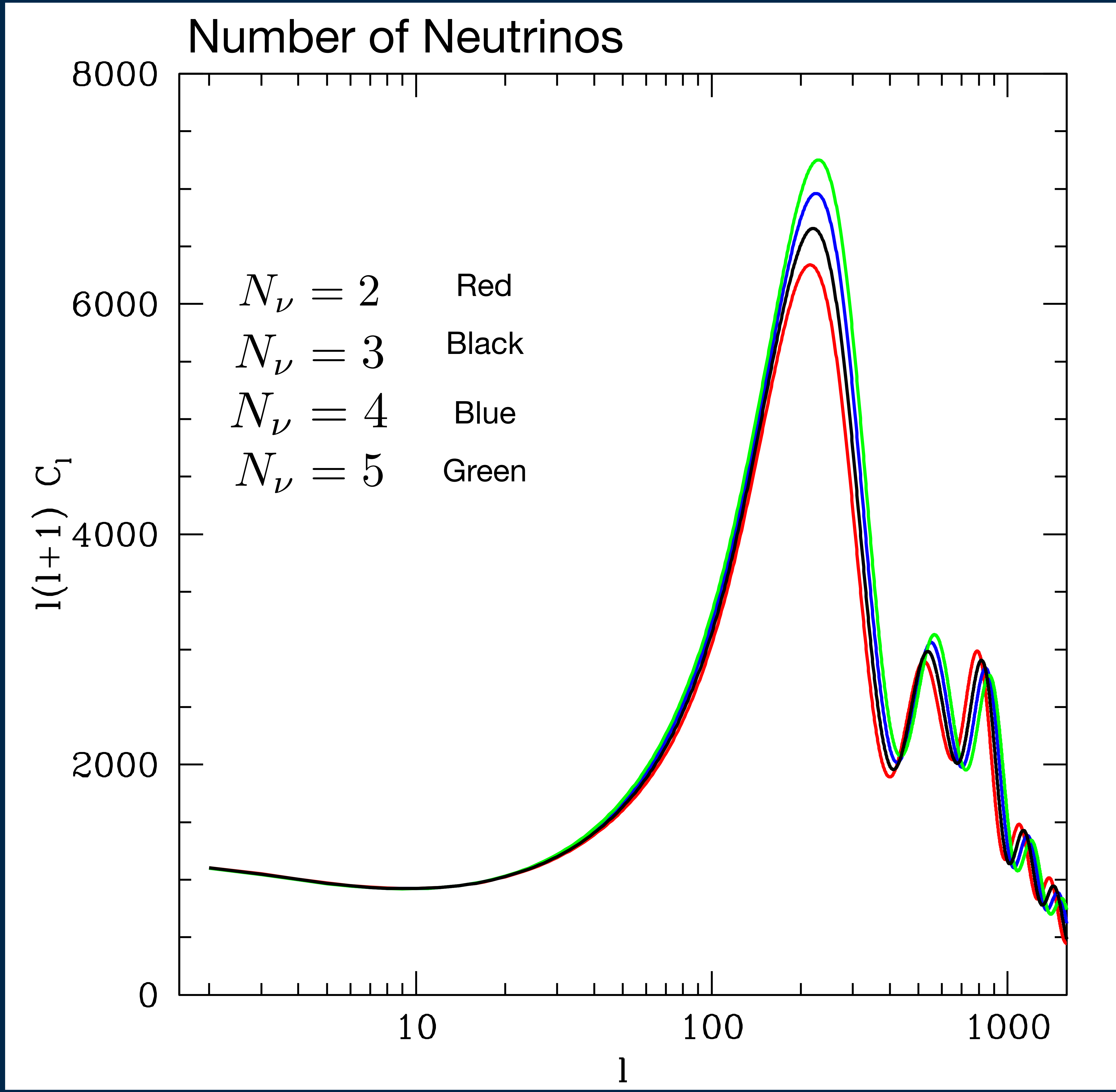


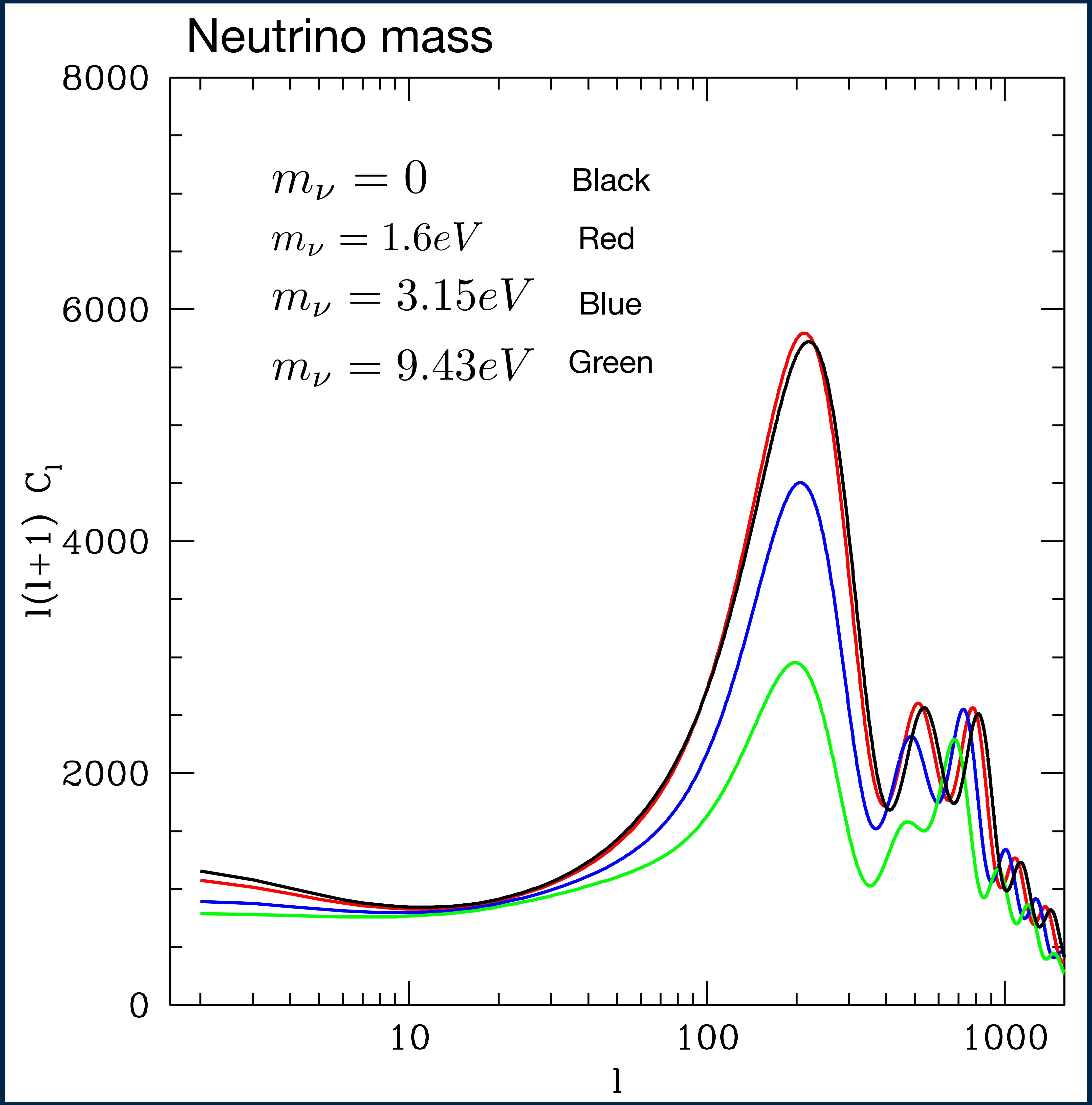
Large scales: generation of anisotropies

Small scales: suppression of anisotropies
(remember diffusion damping)

Neutrinos and CMB

For CMB primary anisotropies neutrinos are still relativistic





More neutrinos shift matter-radiation equality later

- Shifts peaks to smaller scales
- Increases power around 1st peak

Largely insensitive to small masses

Large masses: neutrinos contribute like CDM

Planck Collaboration (2018)

Parameter	TT+lowE 68% limits	TE+lowE 68% limits	EE+lowE 68% limits	TT,TE,EE+lowE 68% limits	TT,TE,EE+lowE+lensing 68% limits	TT,TE,EE+lowE+lensing+ 68% limits
$\Omega_b h^2$	0.02212 ± 0.00022	0.02249 ± 0.00025	0.0240 ± 0.0012	0.02236 ± 0.00015	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1206 ± 0.0021	0.1177 ± 0.0020	0.1158 ± 0.0046	0.1202 ± 0.0014	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04077 ± 0.00047	1.04139 ± 0.00049	1.03999 ± 0.00089	1.04090 ± 0.00031	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0522 ± 0.0080	0.0496 ± 0.0085	0.0527 ± 0.0090	$0.0544^{+0.0070}_{-0.0081}$	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10} A_s)$	3.040 ± 0.016	$3.018^{+0.020}_{-0.018}$	3.052 ± 0.022	3.045 ± 0.016	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9626 ± 0.0057	0.967 ± 0.011	0.980 ± 0.015	0.9649 ± 0.0044	0.9649 ± 0.0042	0.9665 ± 0.0038
H_0 [km s^{-1} Mpc^{-1}]	66.88 ± 0.92	68.44 ± 0.91	69.9 ± 2.7	67.27 ± 0.60	67.36 ± 0.54	67.66 ± 0.42
Ω_Λ	0.679 ± 0.013	0.699 ± 0.012	$0.711^{+0.033}_{-0.026}$	0.6834 ± 0.0084	0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m	0.321 ± 0.013	0.301 ± 0.012	$0.289^{+0.026}_{-0.033}$	0.3166 ± 0.0084	0.3153 ± 0.0073	0.3111 ± 0.0056
$\Omega_m h^2$	0.1434 ± 0.0020	0.1408 ± 0.0019	$0.1404^{+0.0034}_{-0.0039}$	0.1432 ± 0.0013	0.1430 ± 0.0011	0.14240 ± 0.00087
$\Omega_m h^3$	0.09589 ± 0.00046	0.09635 ± 0.00051	$0.0981^{+0.0016}_{-0.0018}$	0.09633 ± 0.00029	0.09633 ± 0.00030	0.09635 ± 0.00030
σ_8	0.8118 ± 0.0089	0.793 ± 0.011	0.796 ± 0.018	0.8120 ± 0.0073	0.8111 ± 0.0060	0.8102 ± 0.0060
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$	0.840 ± 0.024	0.794 ± 0.024	$0.781^{+0.052}_{-0.060}$	0.834 ± 0.016	0.832 ± 0.013	0.825 ± 0.011
$\sigma_8 \Omega_m^{0.25}$	0.611 ± 0.012	0.587 ± 0.012	0.583 ± 0.027	0.6090 ± 0.0081	0.6078 ± 0.0064	0.6051 ± 0.0058
z_{re}	7.50 ± 0.82	$7.11^{+0.91}_{-0.75}$	$7.10^{+0.87}_{-0.73}$	7.68 ± 0.79	7.67 ± 0.73	7.82 ± 0.71
$10^9 A_s$	2.092 ± 0.034	2.045 ± 0.041	2.116 ± 0.047	$2.101^{+0.031}_{-0.034}$	2.100 ± 0.030	2.105 ± 0.030
$10^9 A_s e^{-2\tau}$	1.884 ± 0.014	1.851 ± 0.018	1.904 ± 0.024	1.884 ± 0.012	1.883 ± 0.011	1.881 ± 0.010
Age [Gyr]	13.830 ± 0.037	13.761 ± 0.038	$13.64^{+0.16}_{-0.14}$	13.800 ± 0.024	13.797 ± 0.023	13.787 ± 0.020
z_*	1090.30 ± 0.41	1089.57 ± 0.42	$1087.8^{+1.6}_{-1.7}$	1089.95 ± 0.27	1089.92 ± 0.25	1089.80 ± 0.21
r_* [Mpc]	144.46 ± 0.48	144.95 ± 0.48	144.29 ± 0.64	144.39 ± 0.30	144.43 ± 0.26	144.57 ± 0.22
$100\theta_*$	1.04097 ± 0.00046	1.04156 ± 0.00049	1.04001 ± 0.00086	1.04109 ± 0.00030	1.04110 ± 0.00031	1.04119 ± 0.00029
z_{drag}	1059.39 ± 0.46	1060.03 ± 0.54	1063.2 ± 2.4	1059.93 ± 0.30	1059.94 ± 0.30	1060.01 ± 0.29
r_{drag} [Mpc]	147.21 ± 0.48	147.59 ± 0.49	146.46 ± 0.70	147.05 ± 0.30	147.09 ± 0.26	147.21 ± 0.23
k_D [Mpc^{-1}]	0.14054 ± 0.00052	0.14043 ± 0.00057	0.1426 ± 0.0012	0.14090 ± 0.00032	0.14087 ± 0.00030	0.14078 ± 0.00028
z_{eq}	3411 ± 48	3349 ± 46	3340^{+81}_{-92}	3407 ± 31	3402 ± 26	3387 ± 21
k_{eq} [Mpc^{-1}]	0.01041 ± 0.00014	0.01022 ± 0.00014	$0.01019^{+0.00025}_{-0.00028}$	0.010398 ± 0.000094	0.010384 ± 0.000081	0.010339 ± 0.000063
$100\theta_{s,eq}$	0.4483 ± 0.0046	0.4547 ± 0.0045	0.4562 ± 0.0092	0.4490 ± 0.0030	0.4494 ± 0.0026	0.4509 ± 0.0020

The End