Acceleration of Cosmic Rays

Diffusive Shock Acceleration

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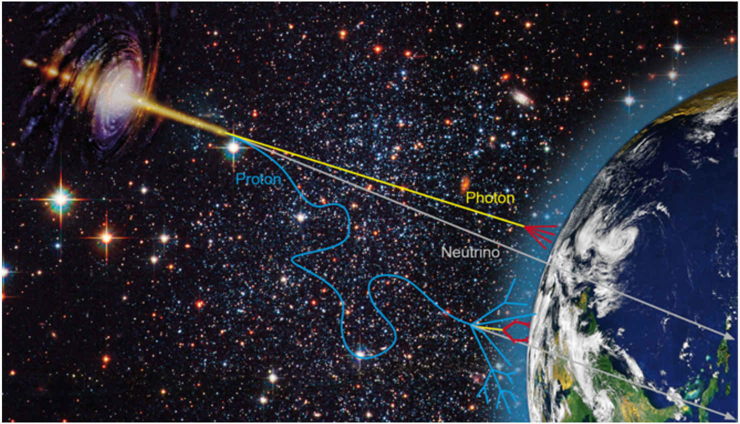
Introduction

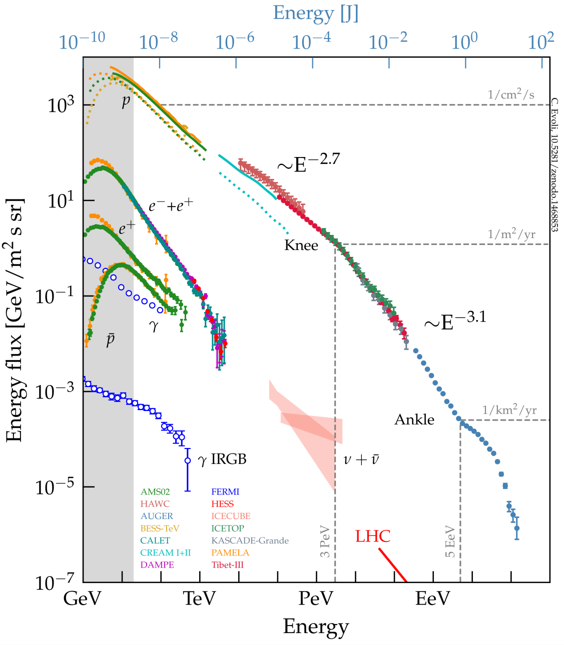
Photons • Cosmic Rays • Neutrinos • Gravitational Waves



The multi-messenger spectrum...

...of the **non-thermal Universe**

- Cosmic Rays
- Photons (γ rays)
- Neutrinos (ν)



FERMI'S ORIGINAL IDEA (1949)

On the Origin of the Cosmic Radiation

ENRICO FERMI
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magmetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

- Two magnetized clouds moving to each other
- This scenario is not very common in the Universe
- Inefficient energy gain

$$\frac{\Delta E}{E} \propto \left(\frac{V}{c}\right)^2 \ll 1$$

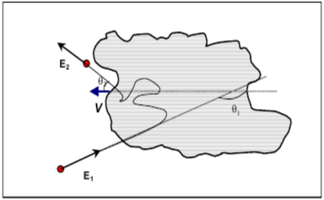


Figure 3.2: Sketch of a collision of a charged particle with moving magnetic cloud.

Including shocks (1954)

GALACTIC MAGNETIC FIELDS AND THE ORIGIN OF COSMIC RADIATION*

E. Fermi
Institute for Nuclear Studies, University of Chicago
Received September 11, 1953

Recently de Hoffmann and Teller[5] have discussed the features of magnetohydrodynamic shocks. They show, in particular, that at a shock front sudden variations in direction and intensity of the field are likely to occur. One is tempted to identify the boundaries of many clouds of the galactic diffuse matter with shock fronts. If this is correct, we have a source of magnetic discontinuities. Probably many of these discontinuities will be rather small. However, either their cumulative effect or the effect of some occasional major discontinuity will tend to convert the angle of pitch that a previous trap acceleration has reduced to a small value back to a statistical distribution corresponding to isotropy of direction. At this moment the particle is ready for a new trap acceleration.

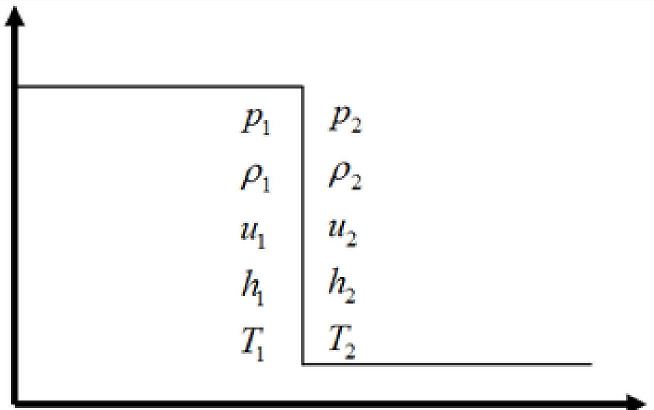
But… he didn't perform the calculations

Shocks

Collisionless shocks

Astrophysical shocks are collisionless

- Shock transition region: $\Delta x \ll$ Coulomb collisions length
- Supersonic plasma: Mach number
  $M_1 = v_1/C_s = v_1/\sqrt{\gamma_{\rm ad} P_1/\rho_1} \gg 1$



Rankine-Hugoniot relations (jump conditions)

- Density and velocity

$$\frac{\rho_2}{\rho_1} = \frac{v_1}{v_2} = \frac{(\gamma_{\rm ad} + 1)M_1^2}{(\gamma_{\rm ad} - 1)M_1^2 + 2}$$

- Pressure

$$\frac{P_2}{P_1} = \frac{2\gamma_{\rm ad}M_1^2 - (\gamma_{\rm ad} - 1)}{\gamma_{\rm ad} + 1}$$

- Temperature ($T_2 = P_2/(K_{\rm B} n_2)$)

$$\frac{T_2}{T_1} = \frac{[2\gamma_{\rm ad}M_1^2 - (\gamma_{\rm ad} - 1)][(\gamma_{\rm ad} - 1)M_1^2 + 2]}{(\gamma_{\rm ad} + 1)^2 M_1^2}$$

Strong shocks

In the limit of strong shocks ($M_0 \gg 1$) and $\gamma_{\rm ad} = 5/3$

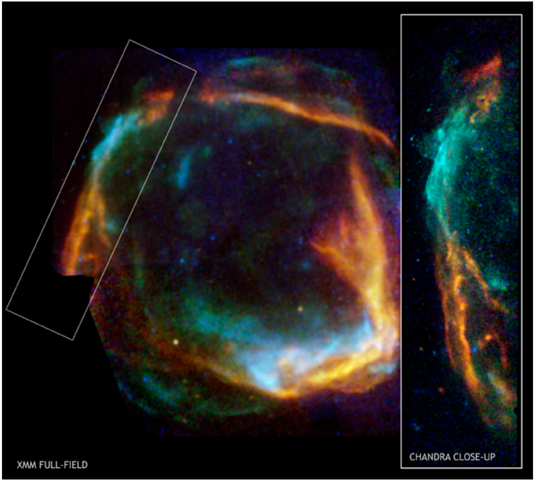
$$\lim_{M_1 \to \infty} \frac{\rho_2}{\rho_1} = \frac{\gamma_{\rm ad} + 1}{\gamma_{\rm ad} - 1} = 4$$

Therefore ...

$$\rho_2 = 4\rho_1 \qquad \text{and} \qquad v_2 = \frac{v_1}{4}$$

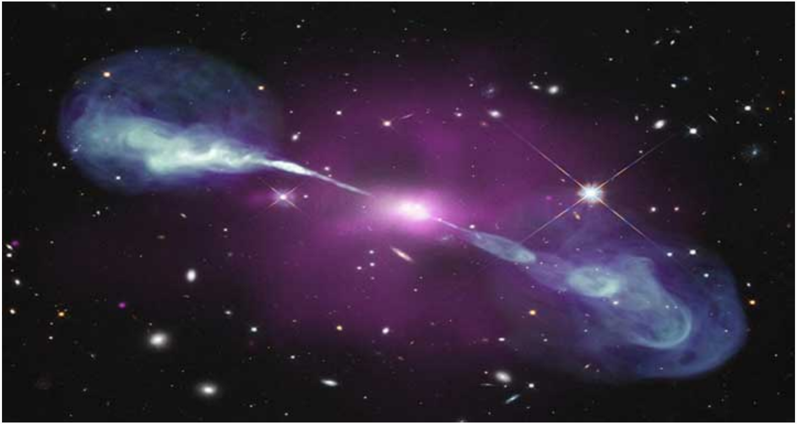
$$P_2 = \frac{3}{4}\rho_0 v_1^2 \qquad \text{and} \qquad T_2 \sim 2 \times 10^{-9} v_1^2 \, \mathrm{K}$$

Shocks are very common in the Universe!



RCW 86 (Chandra and XMM-Newton X-ray data) - J. Vink

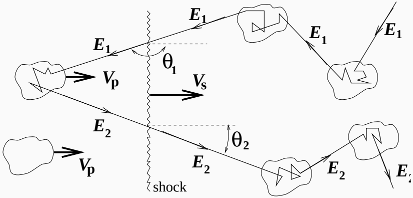
Radiogalaxies are a sub class of Active Galactic Nuclei



Diffusive Shock Acceleration

Diffusive Shock Acceleration (DSA) - Fermi I

- Cosmic rays are isotropic on either side of the shock due to small angle scattering off magnetic field fluctuations
- Isotropization allows particles to cross the shock more efficiently
- Every times the CR crosses the shock, a net energy gain is received
- The resulting spectrum of particles is independent of the diffusion regime
- The acceleration efficiency depends on the scattering efficiency



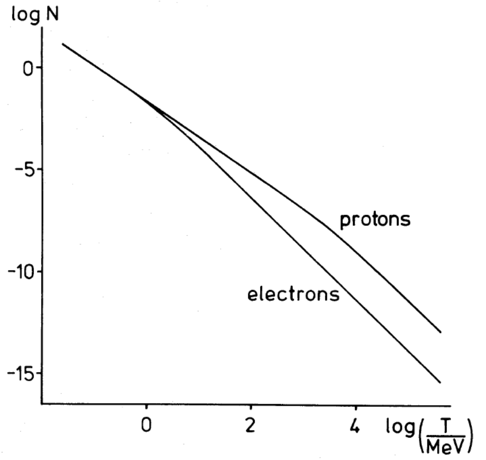
Derivation of the Universal power law (70's)

Microscopic approach

- Bell 1978a,b

Macroscopic approach through the Fokker-Planck equation

- Axford, Leer & Skadron 1977
- Krymskii 1977
- Blandford & Ostriker 1978

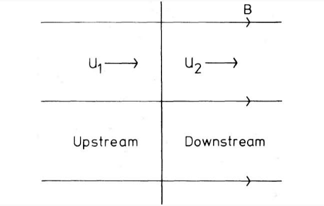


Bell 1978b

Tony Bell's approach

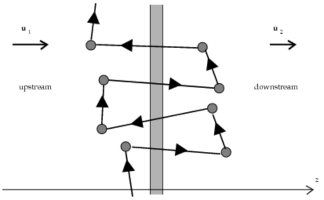
Evaluation of the number of particles located at the shock versus the number of particles that escape downstream. Only particles that do not escape are reaady for one more cycle of acceleration

- u_1 : Upstream velocity
- u_2 : Downstream velocity
- $v_{\rm sh} = u_1$: Shock velocity



Return probability

- $P_{\rm esc}$ is the probability to escape downstream
- $P_{\rm ret} = 1 - P_{\rm esc}$ is the probability to cross the shock back to the upstream



Flux of particles passing from upstream to downstream: $n_0 \frac{v}{4}$

Flux of particles escaping downstream: $n_0 u_2$

$$P_{\rm esc} n_0 \frac{v}{4} = n_0 u_2 \Rightarrow P_{\rm esc} = u_2 \frac{4}{v}$$

Note that if $v \sim c$ and $u_2 \ll c$, then $P_{\rm esc} \sim 0$ and $P_{\rm ret} \sim 1$

Energy transformation

- Energy transformation

$$E' = \gamma(E + v_{\rm sh} p \cos(\theta))$$

Non relativistic shock, $\gamma \sim 1$ and $E = pc$. Therefore,

$$E' - E = v_{\rm sh} p \cos(\theta) \Rightarrow \frac{\Delta E}{E} = \frac{v_{\rm sh}}{c} \cos(\theta)$$

- Fractional energy change when the particle goes from upstream to downstream

$$\left\langle \frac{\Delta E}{E} \right\rangle_{ups \rightarrow downs} = \frac{2}{3} \frac{v_{\rm sh}}{c}$$

- Fractional energy change when the particle goes from upstream to downstream and back to the upstream (1 cycle)

$$\left\langle \frac{\Delta E}{E} \right\rangle_{\rm cycle} = 2 \left\langle \frac{\Delta E}{E} \right\rangle_{\rm ups \rightarrow downs} = \frac{4}{3} \frac{v_{\rm sh}}{c}$$

Spectrum of particles

After k cycles, the energy of particles is increased by

$$E = E_0 \left(1 + \left\langle \frac{\Delta E}{E} \right\rangle_{\text{cycle}}\right)^k$$

$$\ln\left(\frac{E}{E_0}\right) = k \ln\left(1 + \left\langle \frac{\Delta E}{E} \right\rangle_{\text{cycle}}\right)$$

where

$$k = \frac{\ln(E/E_0)}{\ln\left(1 + \frac{\Delta E}{E}\right)},$$

and

$$\left\langle \frac{\Delta E}{E} \right\rangle_{\text{cycle}} = \frac{4}{3}\frac{V_{\text{s}}}{c}$$

Spectrum of particles

$$
\begin{aligned}
J &\propto (1-P_{\rm esc})^k \\
J &= C(1-P_{\rm esc})^{\frac{\ln(E/E_0)}{\ln(1+\Delta E/E)}}; \qquad C = \rm cte \\
\ln J &= C' + \frac{\ln(E/E_0)}{\ln(1+\Delta E/E)}\ln(1-P_{\rm esc}); \qquad C' = \ln(c) = \rm cte
\end{aligned}
$$

Finally

$$J = c'' - (\Gamma - 1)\ln(E);$$

where

$$\Gamma = 1 - \frac{\ln(1-P_{\rm esc})}{\ln(1+\Delta E/E)} = 1 - \frac{\ln(1 - \frac{4}{\xi}\frac{V_{\rm sh}}{v})}{\ln(1 + \frac{4}{3}\frac{(\xi-1)}{\xi}\frac{V_{\rm sh}}{c})}.$$

Spectrum of particles

If $x \sim 0$:

$$\ln(1+x) \approx x - \frac{x^2}{2} + \ldots \qquad \text{and} \qquad \ln(1-x) \approx -x + \frac{x^2}{2} - \ldots,$$

therefore

$$\begin{aligned} \Gamma &\approx 1 - \frac{-\frac{4V_\mathrm{s}}{\xi v}}{\frac{4}{3}\frac{(\xi-1)}{\xi}\frac{V_\mathrm{s}}{c}} \\ \Gamma &\approx 1 + \frac{3}{\beta(\xi-1)} \end{aligned}$$

$$\beta \sim 1 \quad \Longrightarrow \quad \Gamma = \frac{\xi - 1 + 3}{\xi - 1}$$

$$\Gamma = \frac{\xi + 2}{\xi - 1}$$

$$J(E) \propto E^{-\Gamma}$$

Strong shock

$$\xi = 4 \quad \Longrightarrow \quad \Gamma = 2$$

Maximum energy of particles

Acceleration timescale

The spectrum of accelerated particles doesn't depend of the diffusion regime. However.... the acceleration time does

- Particles moving in a turbulent magnetic field diffuse on a times-scale $t_{\rm diff} = R^2/D$, where R is the diffusion length
- The diffusion coefficient is $D = \lambda c/3$, where λ is the mean free path
- D is a big unknown in CR physics. We assume that $D \propto E^\delta$
- Bohm diffusion regime[1]: $D_{\rm Bohm} = r_{\rm g} c/3$

[1] $r_{\rm g} = E/qB$ is the Larmor radius of a relativistic particle

Cycle timescale

Balance between away from the shock upstream and advection downstream creates a CR precursor located at $L = \frac{D_\mathrm{u}}{u}$ upstream of the shock

- Flux of particles passing from upstream to downstream: $n_0 \frac{c}{4}$
- Number of CR in the precursor per unit area: $n_0 L$
- Average time a particle spend upstream: $t_\mathrm{u} \sim \frac{n_0 L}{n_0 c/4} = 4 \frac{D_\mathrm{u}}{uc}$

Similarly...

- Average time a particle spend downstream: $t_\mathrm{d} \sim 4 \frac{D_\mathrm{d}}{u_\mathrm{d} c}$

Cycle time between upstream and downstream:
$t_\mathrm{cycle} = t_\mathrm{u} + t_\mathrm{d} = 4 \left(\frac{D_\mathrm{u}}{U} + \frac{D_\mathrm{d}}{u_\mathrm{d}} \right) \frac{1}{c}$

Acceleration timescale

- Energy gain per cycle:

$$\left\langle \frac{\Delta E}{E} \right\rangle_{\mathrm{cycle}} = \frac{4}{3}\frac{v}{c} \sim \frac{v}{c}$$

- Acceleration timescale

$$t_{\mathrm{acc}} = t_{\mathrm{cycle}} \left\langle \frac{\Delta E}{E} \right\rangle_{\mathrm{cycle}} \sim 4\frac{D_{\mathrm{u}} + 4D_{\mathrm{d}}}{u^2}$$

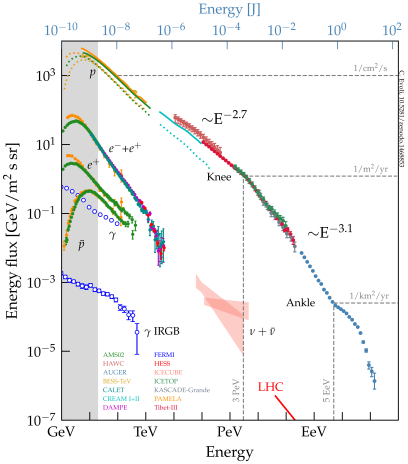
by assuming Bohm diffusion: $D_{\mathrm{u}} = D_{\mathrm{Bohm}} = r_{\mathrm{g}}c/3 = (E/qB)c/3$, and therefore $D_{\mathrm{d}} \sim D_{\mathrm{u}}/4$ (B-compression at the shock)

$$t_{\mathrm{acc}} = \frac{8}{3}\frac{E}{Bv_{\mathrm{sh}}^2}$$

Maximum energy

we balance t_{acc} with cooling and dynamical timescales

- Lifetime of the source (mostly fro protos and heavy ions)
- Radiative cooling (mostly for electrons)

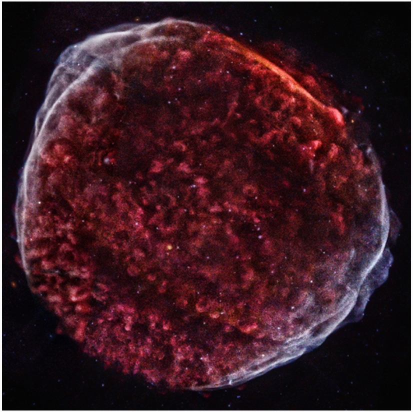


Maximum energy of electrons

Electrons cool down very efficiently via synchrotron emission

$t_{\rm acc} = t_{\rm synchr} \Rightarrow E_{\rm max} \propto v_{\rm sh} B^{-0.5}$

Synchrotron cooling length:
$\frac{l_{\rm synchr}(\nu)}{\rm arcsec} \sim \left(\frac{\nu}{10^{18}{\rm Hz}}\right)^{-\frac{1}{2}} \left(\frac{B}{100}\right)^{-\frac{3}{2}} \left(\frac{v_{\rm sh}}{10000{\rm kms}^{-1}}\right).$



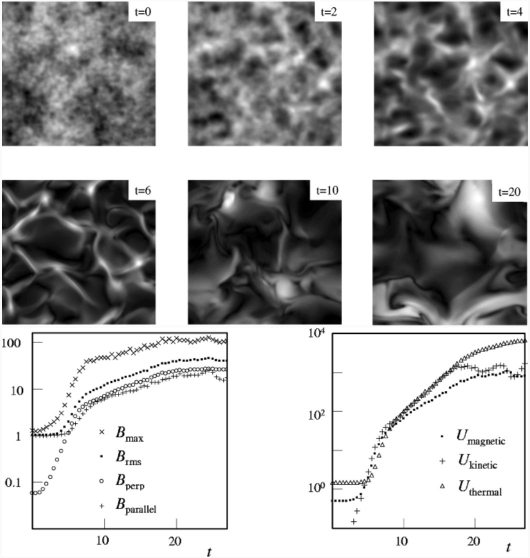
Magnetic field amplification by Bell instabilities

Dispersion relation

$$\omega^2 - k^2 v_\mathrm{A}^2 - k\zeta \frac{v_\mathrm{sh}^2}{r_\mathrm{gm}} = 0$$

- Alfvén (resonant):
  $k^2 v_\mathrm{A}^2 > k\zeta \frac{v_\mathrm{sh}^2}{r_\mathrm{gm}}$
- Bell (non resonant):
  $k^2 v_\mathrm{A}^2 < k\zeta \frac{v_\mathrm{sh}^2}{r_\mathrm{gm}}$

  Magnetic field amplification!

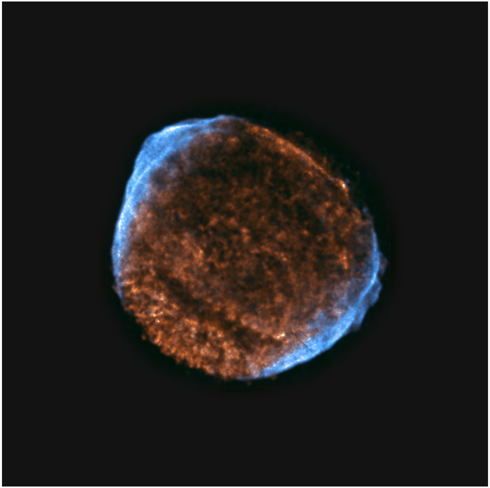


Bell (2004, 2005)

Bell instabilities

Strong magnetic fields ($\gg 1\,\mu$G) are required to explain the thin (synchrotron) X-ray filaments in supernova remnants

- Non-resonant hybrid (Bell) instabilities can amplify the ISM magnetic field up to $\sim 100\,\mu$G



The maximum growth rate

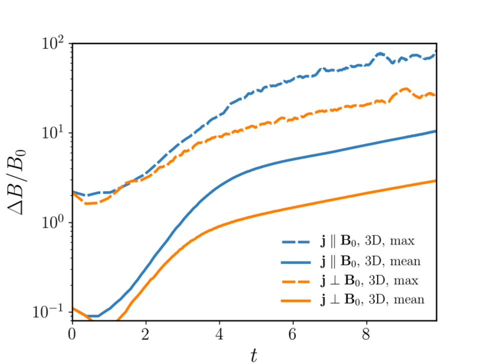
The maximum growth rate ($\Gamma_{\rm max}$) of the Bell instability is similar in both parallel and perpendicular shocks (Riquelme & Spitkovsky 2011, Matthews et al. 2017)

Parallel shocks

- $N_{100} \sim 5 - 10$
- $t_{\parallel} \sim R/v_{\rm sh}$

Perpendicular shocks

- $N_{100} \sim 25$
- $t_{\perp} \sim r_{\rm g0}/c$



Matthews et al. (2017)

SNR as the sources of Galactic cosmic rays

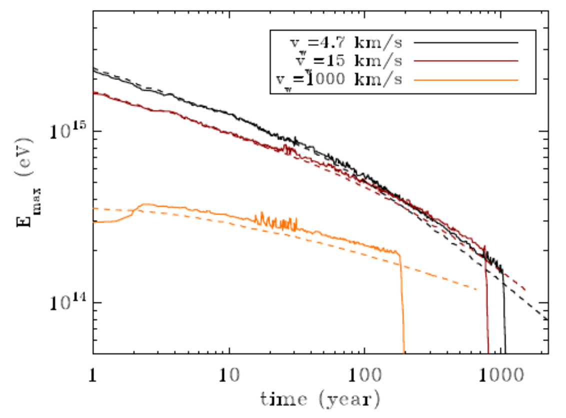
The condition $\Gamma_{\rm max} t > N_{100}$ leads to an estimation of the maximum energy of protons

$$\Gamma_{\rm max} = \frac{j}{c}\sqrt{\frac{\pi}{\rho}}$$

$$E_{\rm max} \propto v_{\rm sh}^2 R_{\rm sh}\sqrt{\rho}$$

For supernova remnants, $R_{\rm sh} \sim v_{\rm sh} t$

$$E_{\rm max} \propto v_{\rm sh}^3 t\sqrt{\rho}$$



Schure et al. (2013)

The Hillas limit (and acceleration of UHECRs)

The Hillas energy

The Hillas limit is valid for any acceleration mechanism.

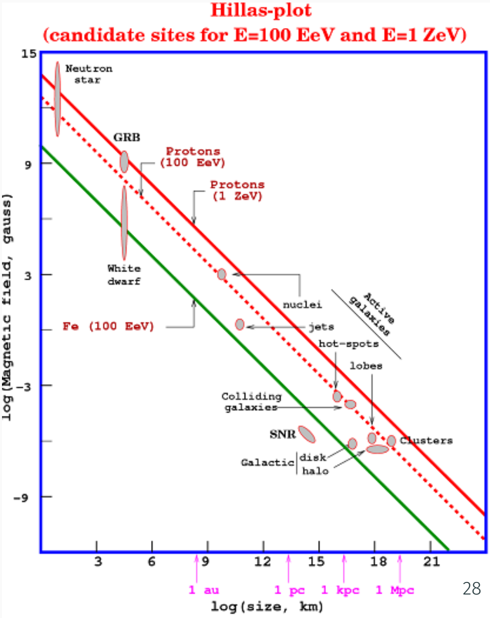
$$E = \int \mathcal{E} d\mathbf{l}$$

The electric field is $\mathcal{E} = \frac{1}{c}\mathbf{u} \times \mathbf{B}$. Therefore...

$$E = \int \mathcal{E} d\mathbf{l} \sim uBL/c$$

Hillas upper-limit on the maximum energy:

$$\left(\frac{E_{\mathrm{H}}}{100\,\mathrm{EeV}}\right) = Z\left(\frac{v}{c}\right)\left(\frac{B}{100\mu\mathrm{G}}\right)\left(\frac{L}{\mathrm{kpc}}\right)$$



The Hillas energy (alternative interpretation)

The system size R must be large enough to contain c/v times the Larmor radius r_{g} of a CR with energy E_{H}.

Larmor radius ($r_{\mathrm{g}} = \frac{E}{ZqB}$) = size of the source ($L$)

$$\left(\frac{B}{100\mu\mathrm{G}}\right) = \frac{1}{Z}\left(\frac{E}{100\,\mathrm{EeV}}\right)\left(\frac{L}{\mathrm{kpc}}\right)^{-1}$$

Hillas upper-limit on the maximum energy:

$$\left(\frac{E_{\mathrm{H}}}{100\,\mathrm{EeV}}\right) = Z\left(\frac{v}{c}\right)\left(\frac{B}{100\mu\mathrm{G}}\right)\left(\frac{L}{\mathrm{kpc}}\right)$$

The Lagage & Cesarsky limit

There are two assumptions behind the Hillas limit

1. particles diffuse in the Bohm regime, i.e. the mean-free path is $\sim r_{\rm g}$
2. the magnetic field B persists over distances $\sim R$ downstream of the shock

Accelerated particles interacting with magnetic turbulence of random scale size s ($\ll r_{\rm g}$) are deflected by an angle $\theta \sim s/r_{\rm g}$

Mean-free path: $\lambda = \frac{r_{\rm g}^2}{s}$

Diffusion coefficient $D = \frac{\lambda c}{3}$

Bohm diffusion coefficient $D_{\rm Bohm} = \frac{r_{\rm g} c}{3}$

$$t_{\rm acc} \sim t_{\rm life} \Rightarrow \boxed{E_{\rm LC} \sim \frac{s}{R_g} E_{\rm H} = \sqrt{\frac{S}{L}} E_{\rm H}}$$

The magnetic field is crucial

The magnetic field B in the expression for E_{Hillas} is generally a turbulent magnetic field amplified by the CR

- The magnetic energy density cannot be larger than the energy density of the CR amplifying the field.

$$\frac{B_E^2}{8\pi} = \eta(E) = \eta_0 \left(\frac{E}{E_{\mathrm{inj}}}\right)^{-(\beta+2)/2}$$

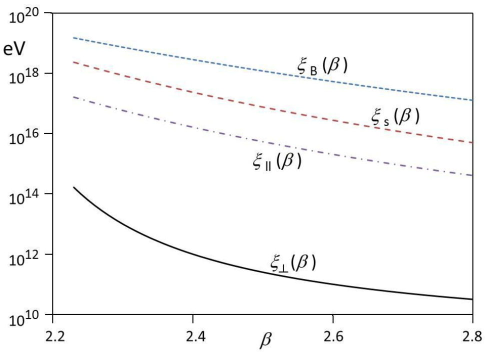
- The scale-size s of the turbulent magnetic field cannot exceed the Larmor radius of the CR driving the turbulence. If the CR proton current exists in a layer of thickness L about the shock, then $t \sim L/c$ giving

$$s \sim \eta(E) r_{\mathrm{g}} \left(\frac{L}{r_{\mathrm{g}}}\right)^2$$

Maximum energies of accelerated particles

Main effects that limit the maximum energy to which particles can be accelerated (Bell et al. 2018)[2]:

1. Steep CR spectrum (β)
2. Small-scale turbulence
3. Quasi-perpendicular shocks



[2]See also Kirk & Reville (2010), Lemoine & Pelletier (2010), Sironi et al. (2013)

Summary and conclusions

- Diffussive shock acceleration is the most efficient mechanism for accelerating particles in astrophysical sources
- Supernova remnants are the most efficient accelators in our Galaxy. They can accelarate particles up to the knee of the CR spectrum
- Acceleration of UHECRs is still under debate.
- Relativistic shocks are not good accelerators of UHECRs
- Radiogalaxies and starburst galaxies are coincident with the hotspots detected by PA, but only radiogalaxies have the power to accelerate particles up to energies larger than 10^{18} eV.

Questions?