

Stars and nuclear/particle processes in them

*Jan Ebr
Fyzikální ústav AV ČR, v.v.i.,
5.9.2022*

Stars

- balls of gas in hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}$$

- ideal gas

$$P = \frac{\rho kT}{m_m}$$

- temperature -> pressure
- what happens without heat source?
 - cooling -> liquid/solid state
 - electron degeneration (brown dwarfs)
 - max. diameter only slightly larger than Jupiter

- collapse of gas cloud -> star; potential energy

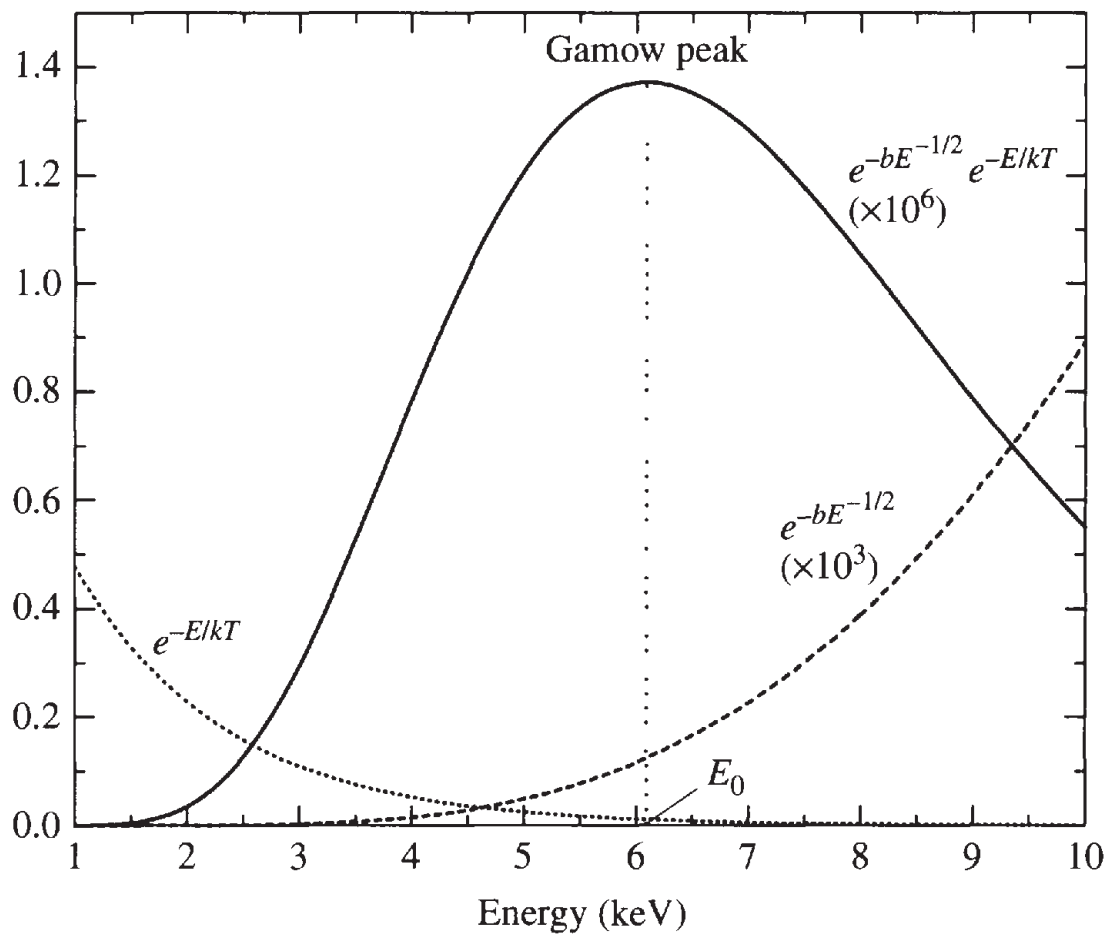
$$U \sim \frac{GM^2}{R}$$

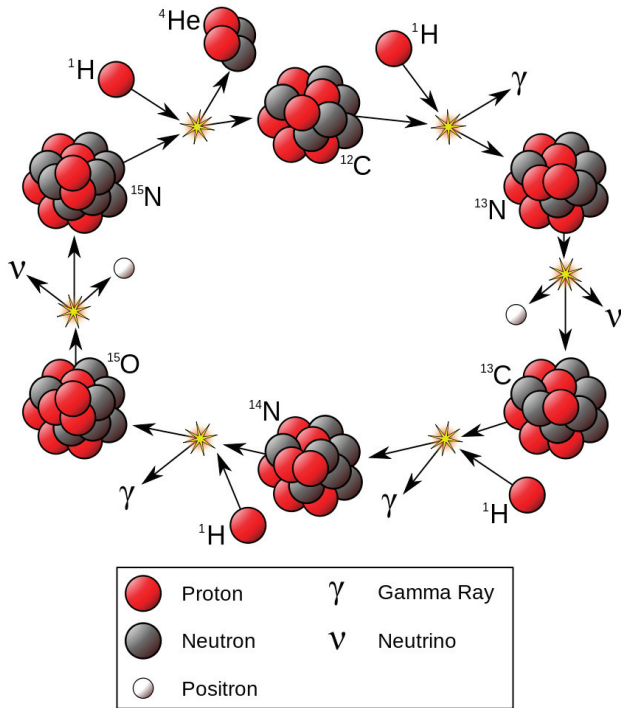
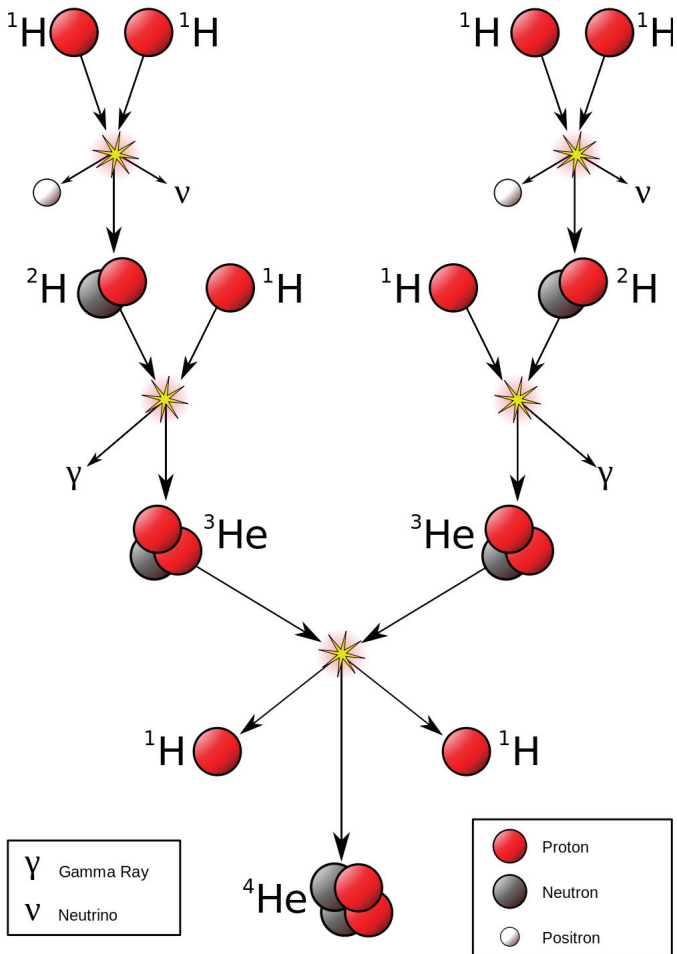
- virial theorem: half goes to kinetic energy (heat)
- Sun core: 15 mil. K -> ~ 2 keV per particle

- Coulomb barrier for two protons:

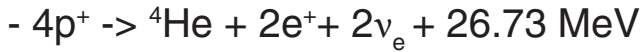
$$V = \frac{e^2}{4\pi r}$$

- just touching $r \sim 1.2$ fm -> $V \sim 0.6$ MeV
- quantum tunneling (Gamov peak)

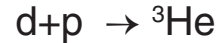




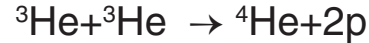
p-p chain



8×10^9 years



4 seconds



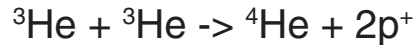
30000 years

- p-p chain dominant in the Sun

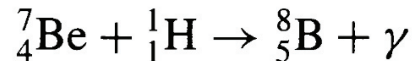
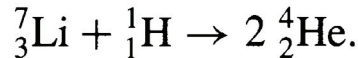
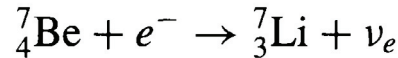
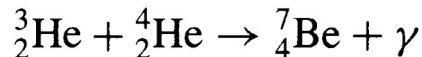
CNO-cycle in more massive stars (steep temperature dependence)

- 1. step in the Sun 0.4% contribution from "pep" process $p^+ + e^- + p^+ \rightarrow d + \nu_e$

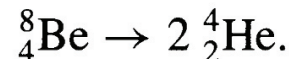
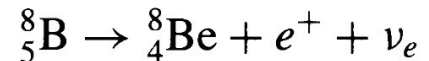
- last step 69% PPI



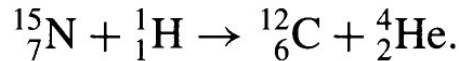
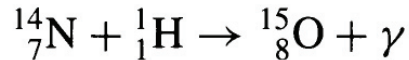
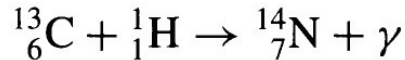
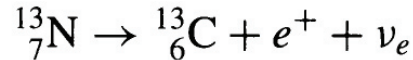
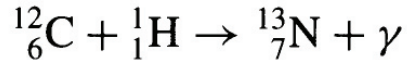
- 31% PPII



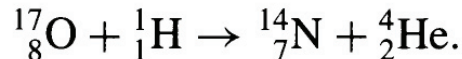
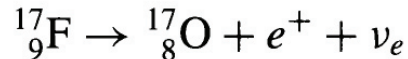
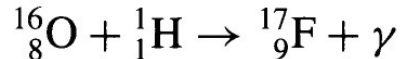
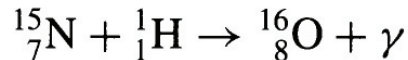
- rarely (0.3 % wrt. electron capture) PPIII



CNO cycle



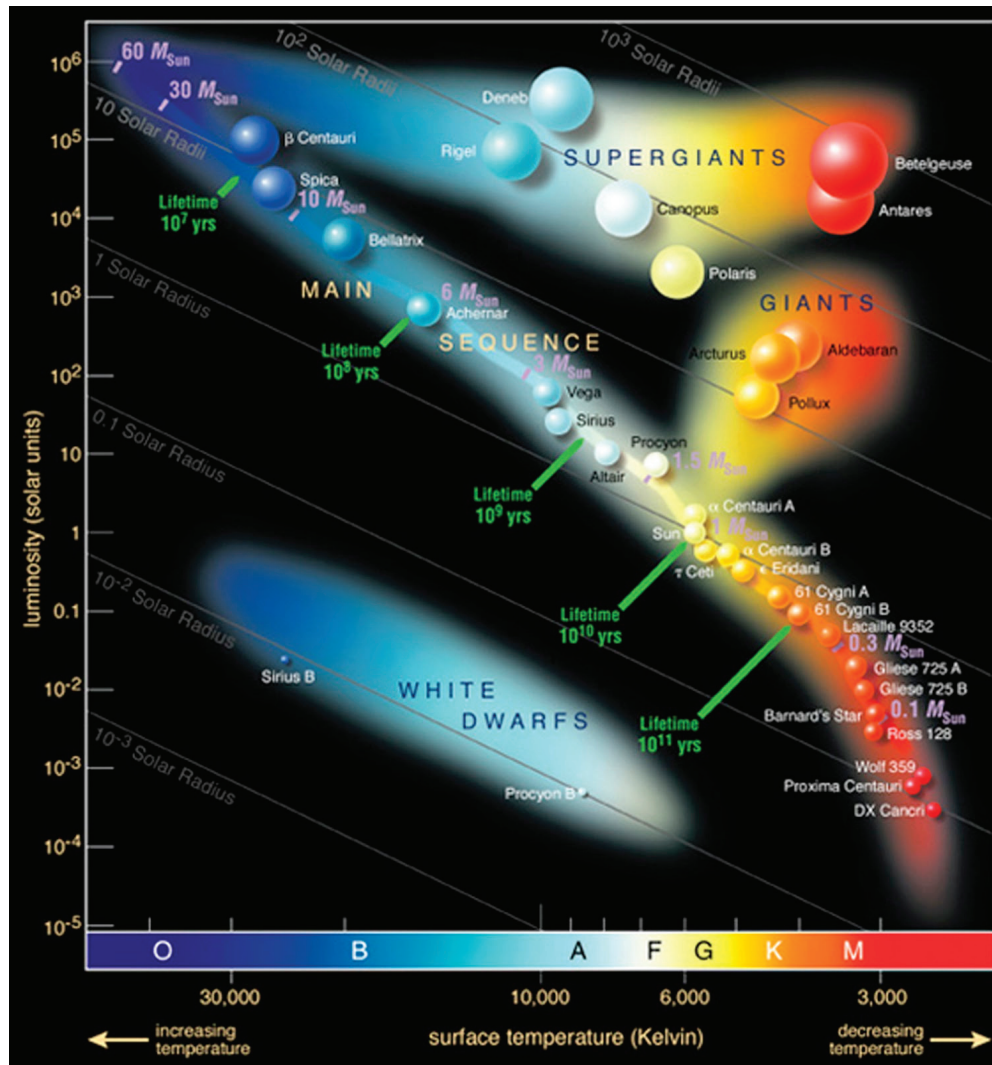
- last reaction in 0.04 % cases:



Hertzsprung-Russell diagram

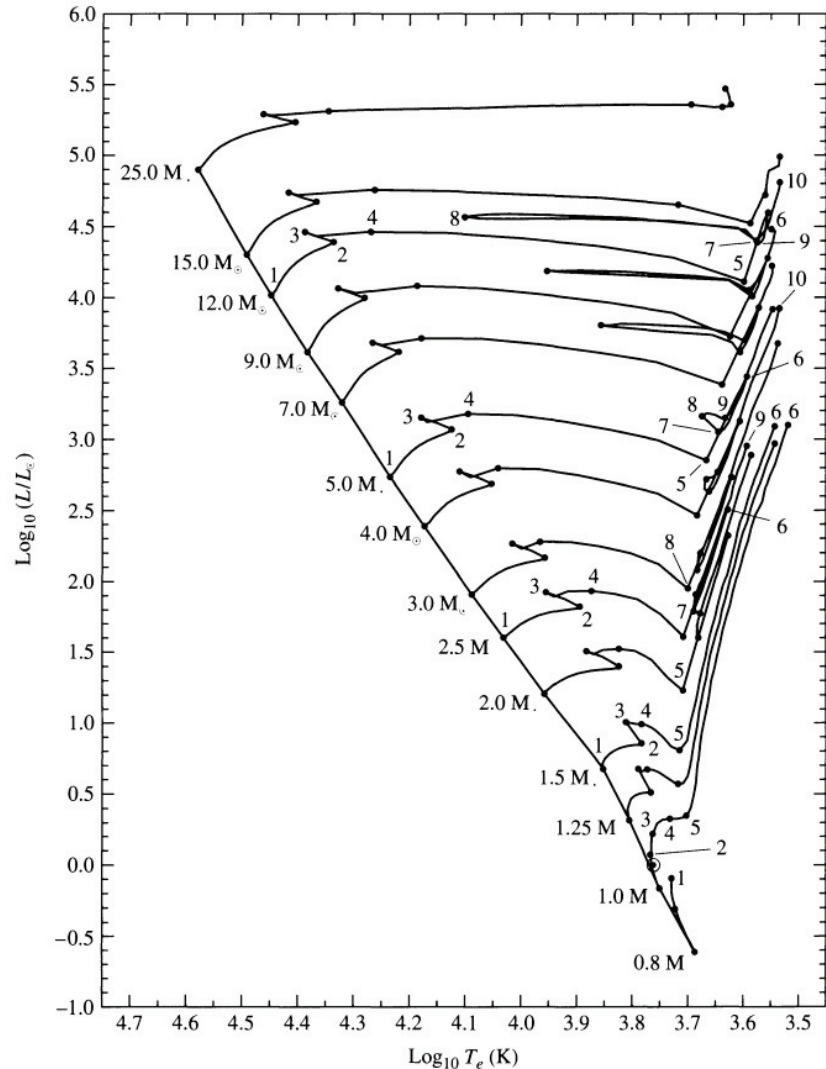
- main sequence =
hydrogen burning
in core - stable
equilibrium between
production and loss
of energy, depends
on mass

- Sun: 10 billion years
- $85 M_{\text{Sun}}$: 2.8 mil. yr.



When the hydrogen in the core runs out

- burning moves to a shell around the core - leaving main sequence
- hot, but isothermal core stable up to some size (Schönberg–Chandrasekhar limit, 10–15% of the star's mass)
- breaking the limit -> contraction -> temperature increase -> more reactions
- complication: electron degeneracy



Degenerate Fermi gas

- complete degeneracy = all possible states filled ($T > 0 \rightarrow$ partial)

- number of states, Fermi momentum:
$$N = g_e \int \frac{d^3x d^3p}{h^3} = 2V \frac{4\pi p_F^3}{3h^3}$$

$$p_F = h \left(\frac{3n}{8\pi} \right)^{1/3}$$

- non-relativistic:

$$P = \frac{1}{3} m n \langle v^2 \rangle = \frac{8\pi m_e}{3} \int_0^{p_F} p^2 \left(\frac{p}{m_e} \right)^2 \frac{dp}{h^3} = \frac{8\pi p_F^5}{15 m_e h^3} = \left(\frac{3}{8\pi} \right)^{2/3} \frac{h^2 n^{5/3}}{5 m_e}$$

- relativistic case ($p_F \gg m_e c$) $\sim n^{4/3}$

- white dwarf: stable due to degeneracy pressure

- $\sim < 0.5 M_{\text{sun}}$: He (takes longer than the age of the Universe)
- most C-O (from stars below $8 M_{\text{sun}}$ initial mass)
- small part O-Ne-Mg ($8-10 M_{\text{sun}}$ initial mass)

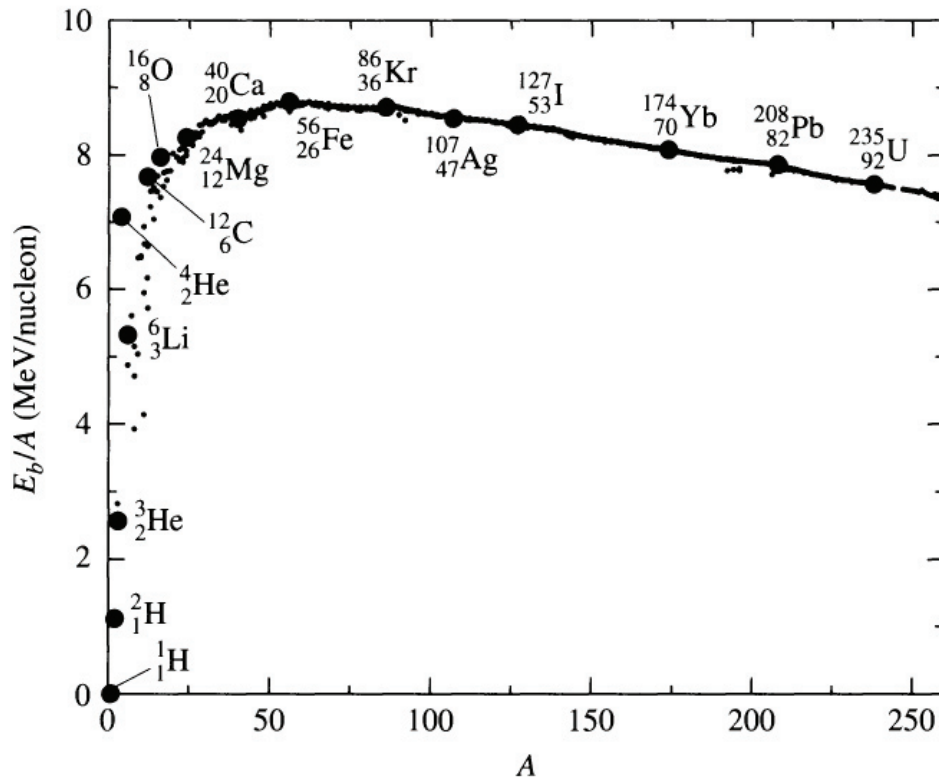
- Chandrasekhar limit: $1.44 M_{\text{sun}}$

Helium -> carbon

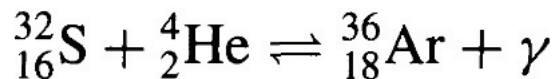
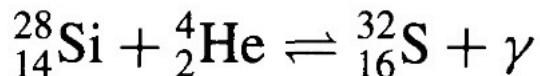
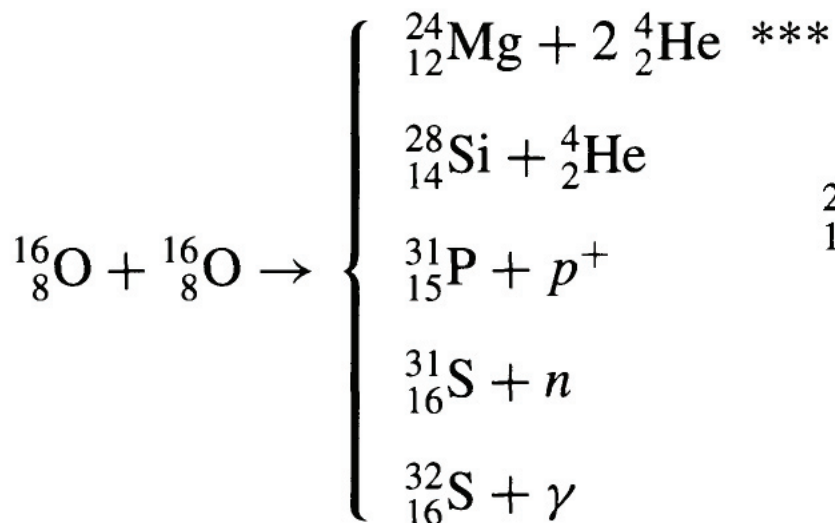
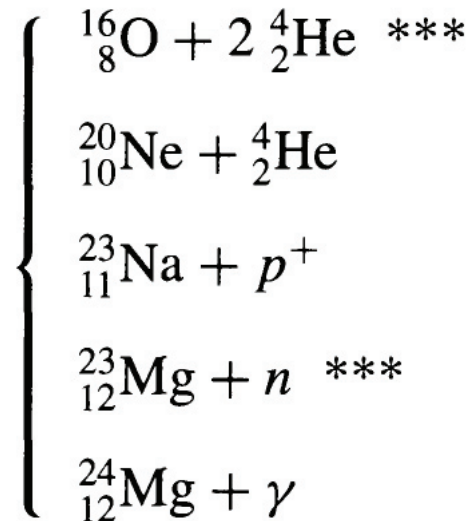
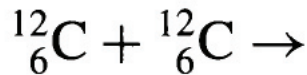
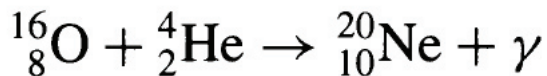
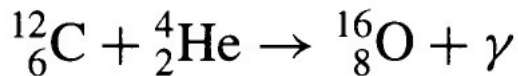
- in a star mostly ^1H and ^4He ; other nuclei burned quickly
- no stable nucleus $A=5$ or 8
- the only way is 3-alpha: $^4\text{He} + ^4\text{He} \rightarrow ^8\text{Be}$ $T = 8 \times 10^{-17} \text{ s}$
- requires 92 keV extra (at least 10^8 K)
- $^4\text{He} = 3728.40 \text{ MeV}$, $^8\text{Be} = 7456.89 \text{ MeV}$, total 11185.29 MeV
- $^{12}\text{C} = 11177.93 \text{ MeV}$, but $^{12}\text{C}^* = 11185.63 \text{ MeV}$ exists (only $+7.7 \text{ MeV}$)
- additional 300 keV needed, but the cross-section is small without the resonance and Be decays quickly - resonance predicted by this!
- $M < 1.8 M_{\text{Sun}}$: He core degenerate (allows S-C limit violation) -> sudden ignition of He->C in the core = helium flash = sever seconds of power output $10^{11} \times \text{Sun}$ (not seen from the outside of the star)
 - degeneracy pressure independent of temperature, $3\alpha \sim T^{40}$
- heavier stars ignite the reaction before reaching degeneracy in the core

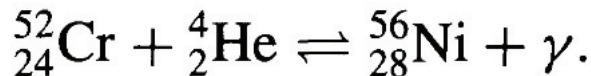
Further thermonuclear reactions

- energetically beneficial until iron/nickel
- only in very massive stars (otherwise a stable white dwarf)



Further thermonuclear reaction



$$\vdots$$


Further reactions

- near the "iron peak" low energy yield

- neutrino losses increase with temperature (from thermal electron-positron pairs)

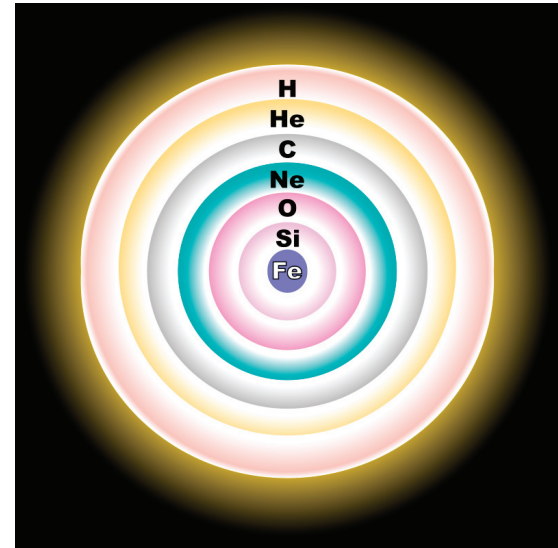
=> contraction, temperature increase -> power output increase

- a degenerate Fe-core without an energy source grows inside

- nb.: Chandrasekhar limit $1.44 M_{\text{Sun}}$

-

Fusion of:	Time to complete	Core temperature	Core density (kg m^{-3})
H	$7 \times 10^6 \text{ yr}$	$6 \times 10^7 \text{ K}$	5×10^4
He	$5 \times 10^5 \text{ yr}$	$2 \times 10^8 \text{ K}$	7×10^5
C	600 yr	$9 \times 10^8 \text{ K}$	2×10^8
Ne	1 yr	$1.7 \times 10^9 \text{ K}$	4×10^9
O	0.5 yr	$2.3 \times 10^9 \text{ K}$	1×10^{10}
Si	1 day	$4.1 \times 10^9 \text{ K}$	3×10^{10}



Core-Collapse Supernova

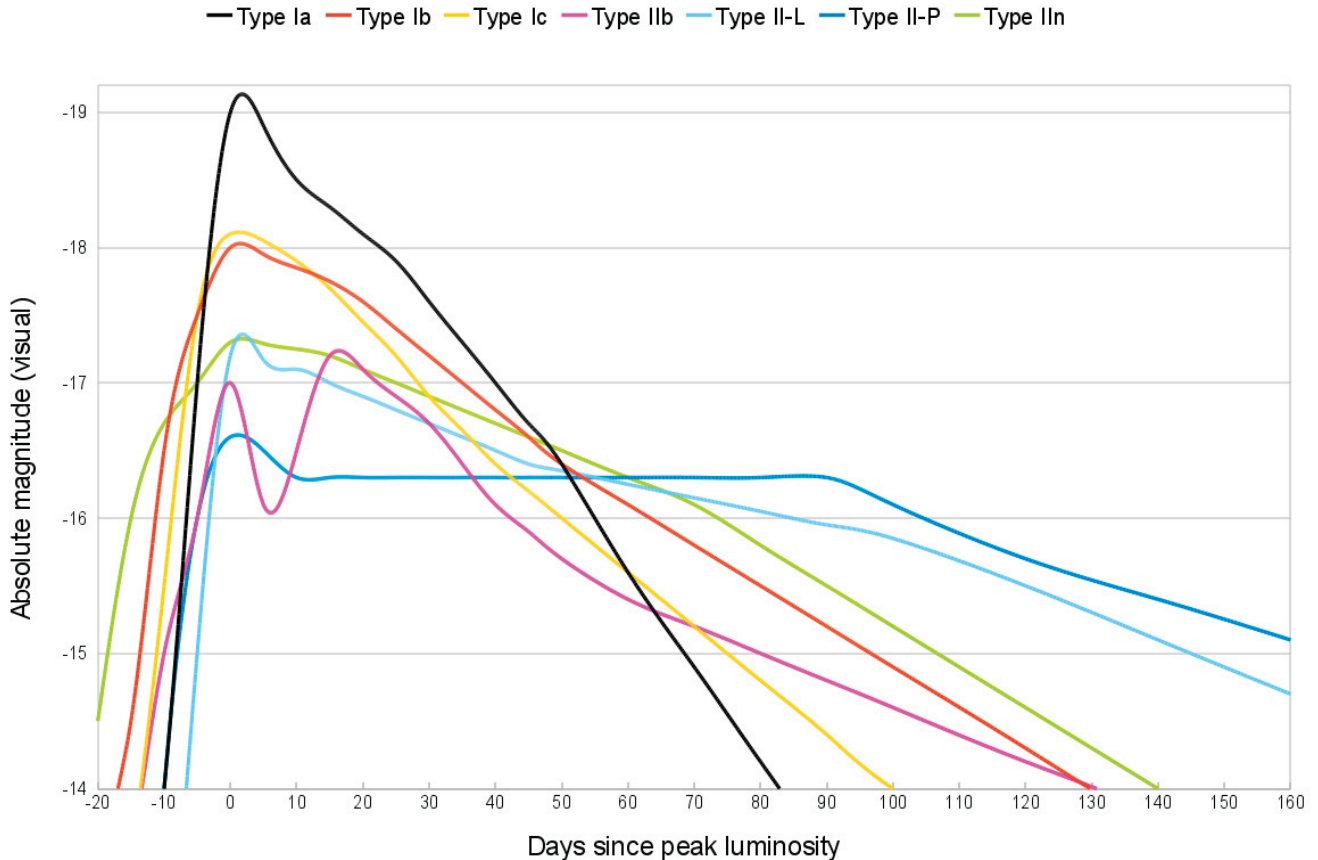
- collapse -> temperature up -> endothermal process = loss of pressure
- iron photodissociation by photons over 8 MeV ($^{56}\text{Fe} \rightarrow 13 \times ^4\text{He} + 4n$), consumes 145 MeV in total
- electron capture by proton ($e^- + p^+ \rightarrow n + \nu_e$), consumes 0.8 MeV
 - the Fermi energy of electrons is sufficient during collapse
- removes electrons from degenerate gas -> further pressure loss -> free fall collapse within 0.1 s, velocity up to 70000 km/s
- neutron degeneracy -> stops contraction -> shock wave reflected -> further photodissociation -> loss of energy, shock wave stalled
- why do we see a supernova??

Core-Collapse Supernova

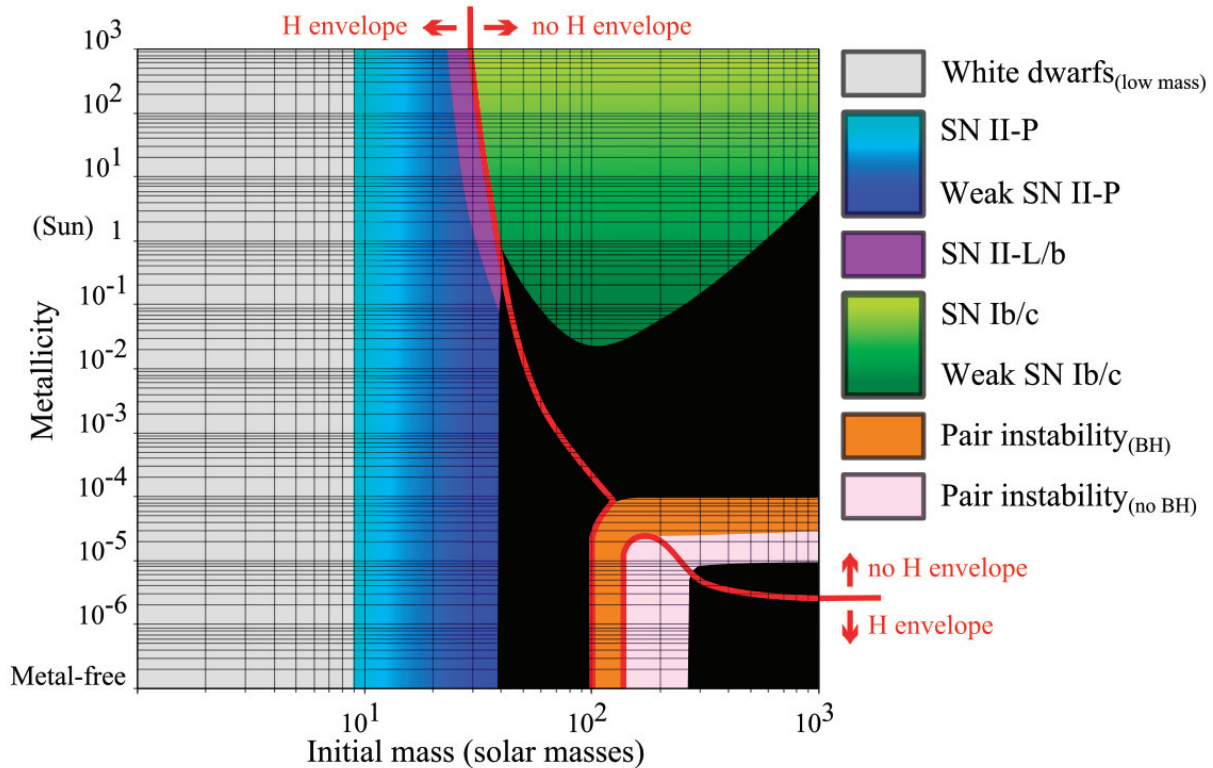
- huge amount of neutrinos created during the collapse
- even more thermal neutrinos from the emerging neutron star
 - nuclear density $\rightarrow R = r_0 A^{1/3}$ $r_0 = 1.2 \text{ fm}$
 - for $M = 1.5 M_{\text{Sun}}$ $A = 1.9 \times 10^{57}$ $\rightarrow R = 15 \text{ km}$
 - gravitational energy $3 \times 10^{46} \text{ J} = 1.8 \times 10^{59} \text{ MeV} \sim 100 \text{ MeV/nucleon}$
 - compare : 8 MeV/nucleon nuclear binding energy in iron
- neutron matter opaque even for neutrinos \rightarrow 2–10 m below surface
"neutrinosphere", emits most neutrinos within 0.1–1 s (random walk)
- 99% energy in neutrinos, only 1% captures in the shell \rightarrow "restart" of the explosion
 - still energy output as the entire galaxy

Supernova light curves

- dominant decays of ^{56}Ni a ^{56}Co , but dependent on possible hydrogen shell



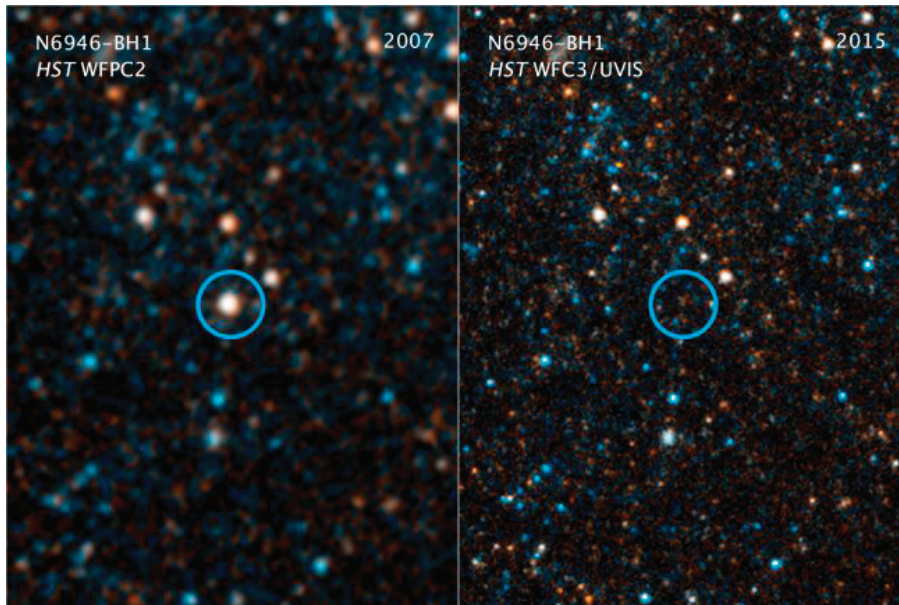
Ultimate destiny of massive stars



- special case ($130\text{--}250 M_{\text{Sun}}$) Pair Instability Supernova: catastrophic production of e^+e^- pairs in C/O cores, often with no remnant

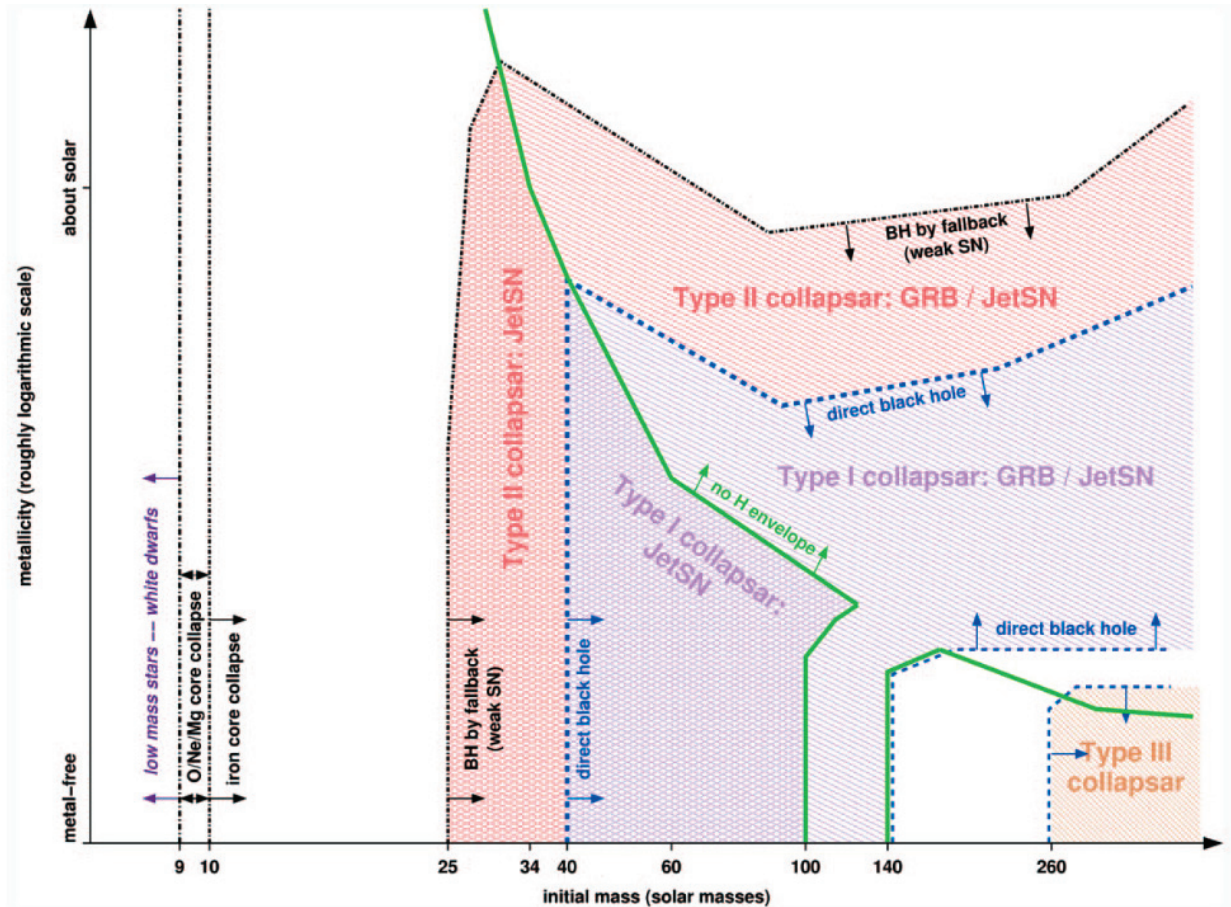
Black holes

- neutron degeneracy only up to TOV (Tolman–Oppenheimer–Volkoff) limit $\sim 2 M_{\text{Sun}}$ -> heavier remnants collapse into black holes
- "fallback": after the formation of neutron star, further infalling mass exceeds TOV limit
- direct black hole - star collapses without an explosion



Rotation change everything?

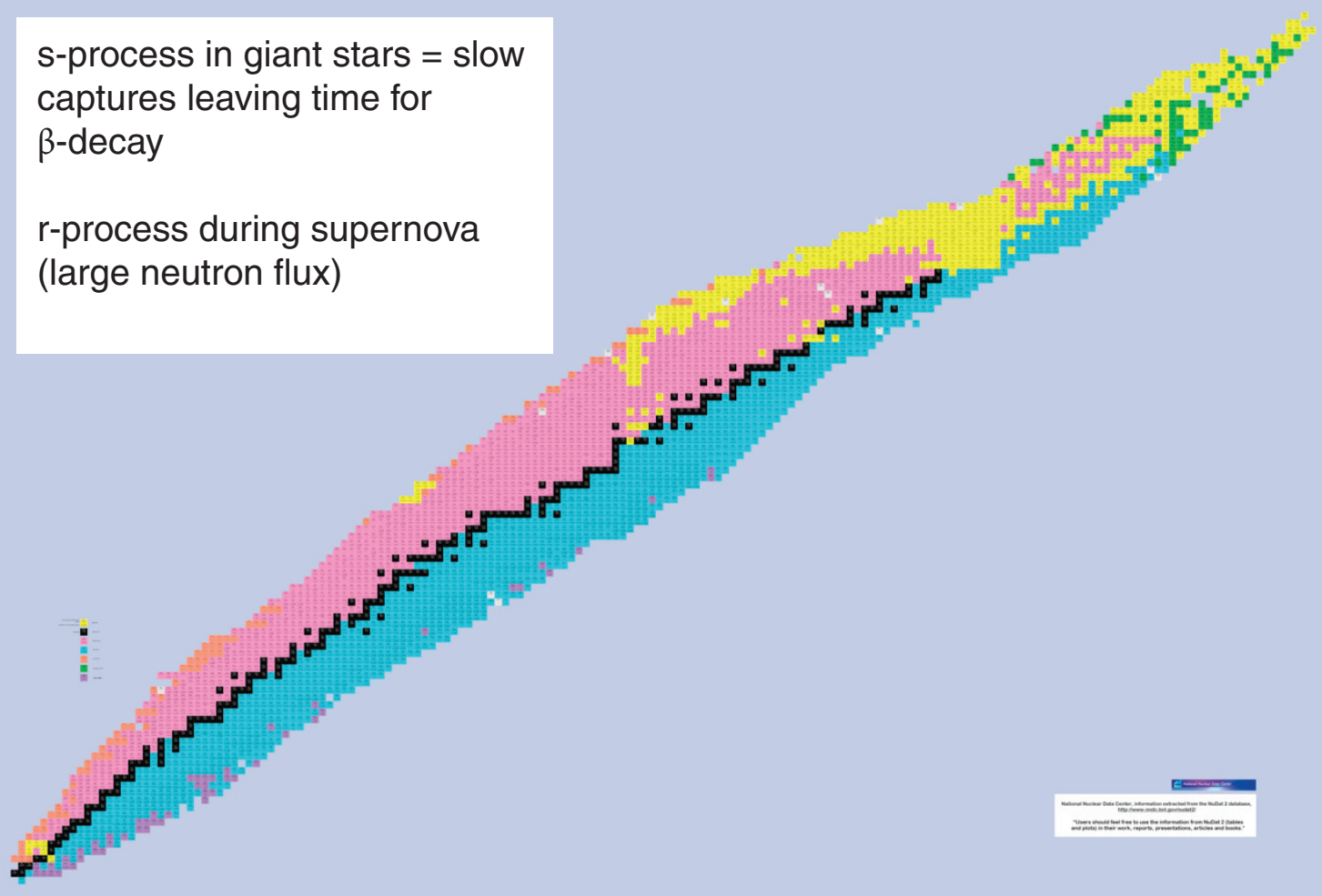
- rotating "collapsar" forming BH with accretion disc = GRB model:



Production of heavy nuclei

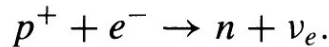
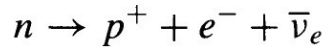
s-process in giant stars = slow captures leaving time for β -decay

r-process during supernova (large neutron flux)



Neutron stars

- after supernova $T \sim 10^{11}$ K, neutrino cooling (Urca process \sim day):



- after 1000 years $T \sim 10^6$ K on surface, 1 Sun output, but X-rays
- degeneracy increases with cooling
- typical p_F for neutrons ~ 300 MeV, for electrons ~ 50 MeV
- $n \sim p_F^3$ -> a small electron fraction sufficient
- angular momentum conservation $CMR^2\omega = \text{const.}$ -> $P_f/P_i = (R_f/R_i)^2$
 - how fast did the core spin? angular momentum exchange with shell?
 - typically \sim hours -> NS \sim ms
- "frozen" magnetic field in a conducting fluid $B \times S = \text{const.}$ -> $B \sim 10^8$ T
 - radiation along mag. field axis - when not parallel to rotation = pulsar
 - radiation losses slow down the rotation $P \sim \omega^4$
- surface smooth ~ 5 mm; "glitch" - reconfiguration due to spin changes



