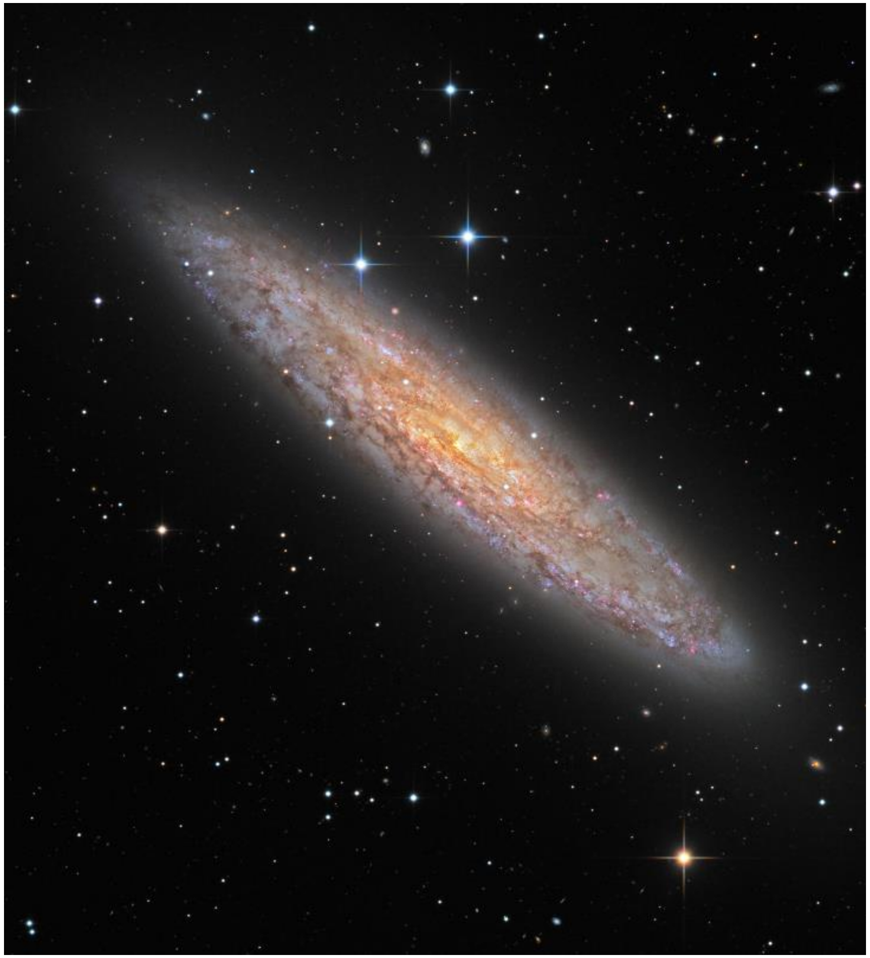
Lecture (1) Plan:

- **Hydro Turbulence and Magneto-Hydro Turbulence**
- **Non-thermal particle transport equation in magnetic turbulence**
- **The Galactic magnetic field environment**
- **Possible role of advection in non-thermal particle transport in the Galaxy**
- **The extragalactic magnetic field environment**

Thermal Emission from a Local Galaxy

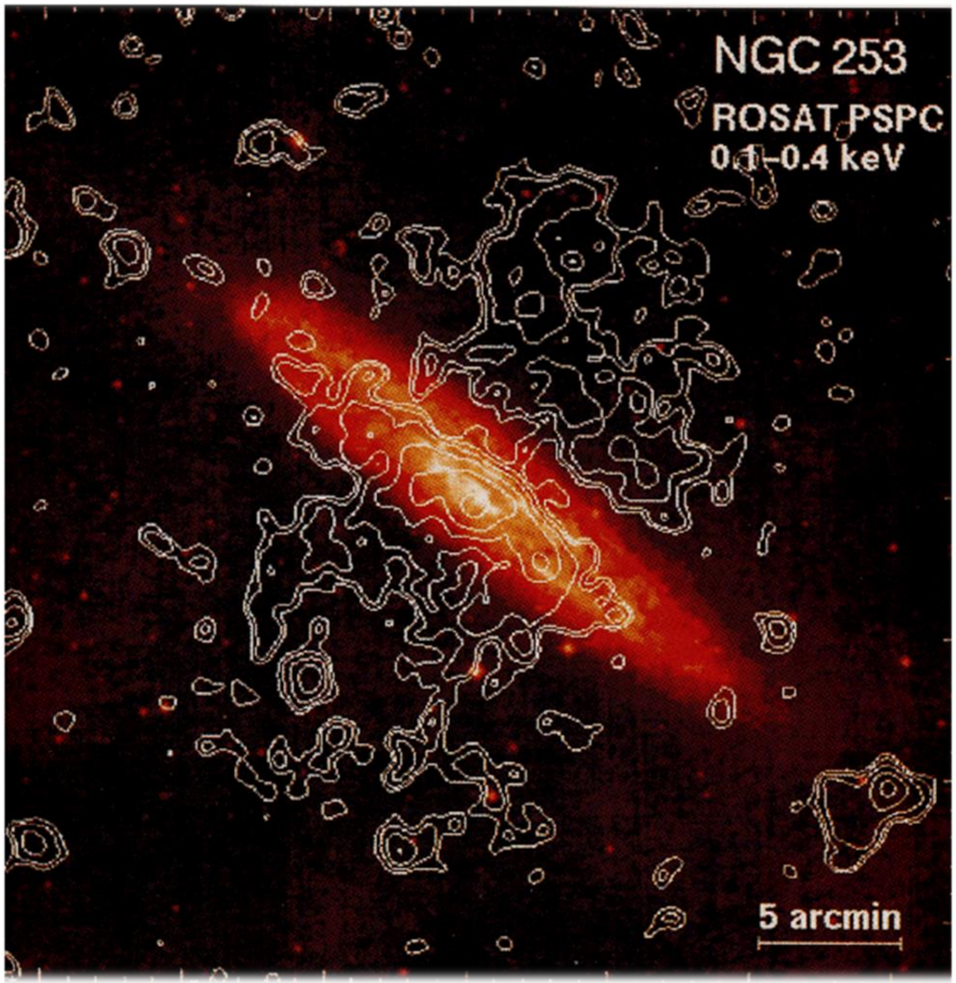
Discovered in optical by Caroline Herschel in 1783



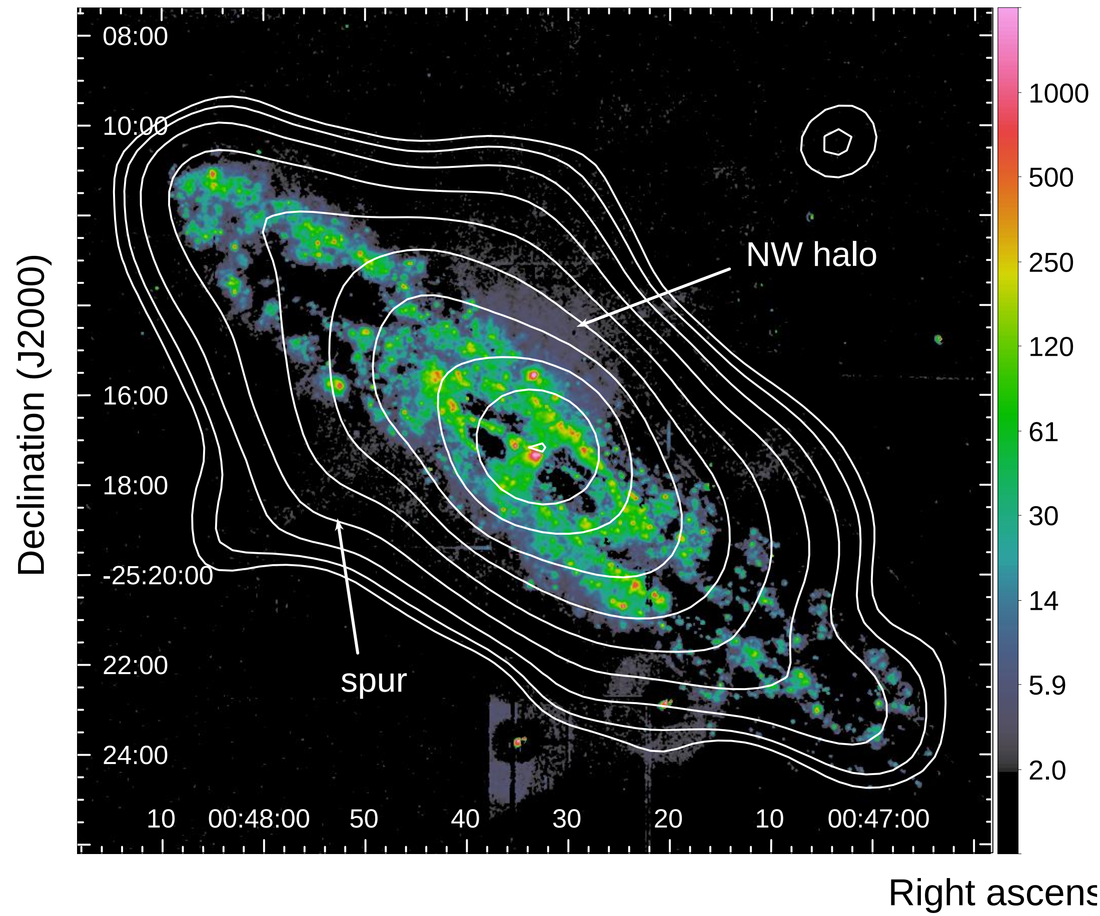
Galaxy: NGC 253

Same Galaxy Viewed in Non-Thermal Emission

1991- ROSAT



2017- GLEAM



Hydro Turbulence

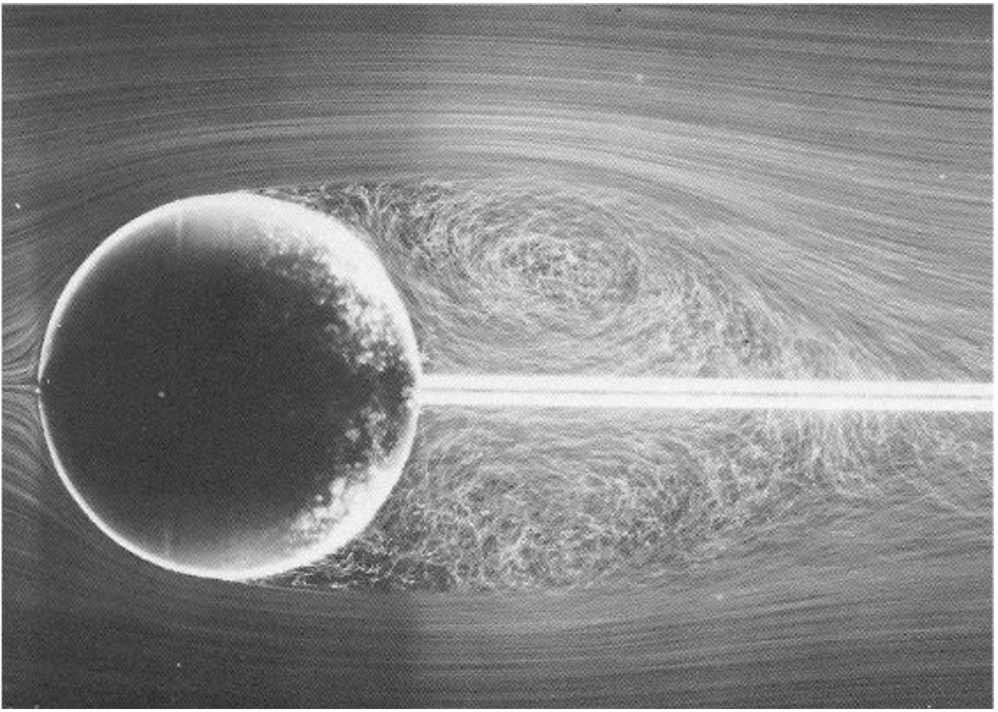
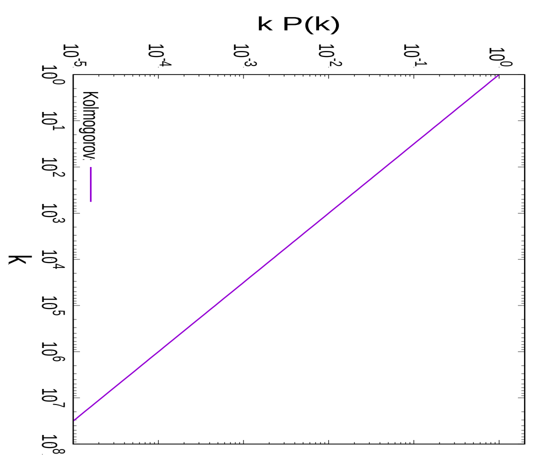
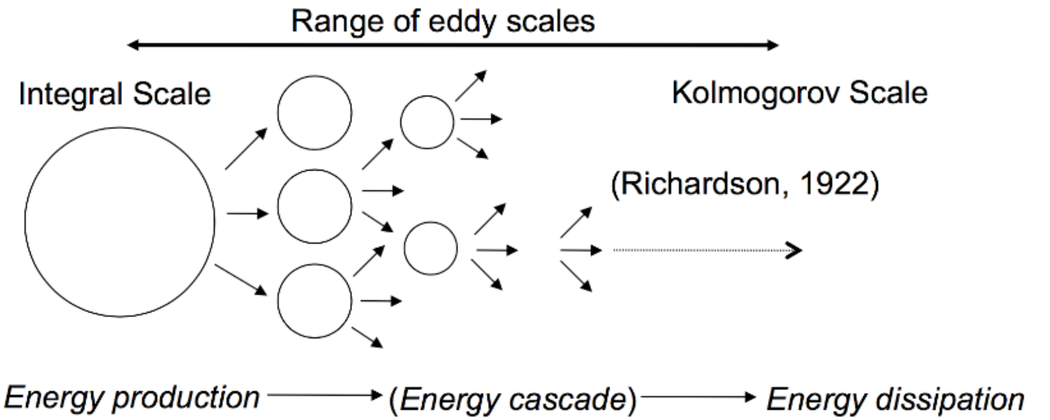


Image from University of Sydney

Richardson, 1922

> *Big whorls have little whorls*
> *That feed on their velocity;*
> *And little whorls have lesser whorls*
> *And so on to viscosity.*





Hydrodynamics

A brief comment-

$$\frac{\partial \rho \mathbf{v}}{\partial \mathbf{t}} + \nabla \cdot \mathbf{P} = \rho \mathbf{g}$$

Momentum flux conservation

$$\mathbf{P} = \mathbf{pI} + \rho \mathbf{vv}$$

Spatial part of stress energy tensor

$$\rho \frac{\partial \mathbf{v}}{\partial \mathbf{t}} + \rho(\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla \mathbf{p} + \rho \mathbf{g}$$

Magneto-Hydrodynamics

A brief comment-

$$\frac{\partial \rho \mathbf{v}}{\partial \mathbf{t}} + \nabla \cdot (\mathbf{P} - \mathbf{P_M}) = \rho \mathbf{g}$$

Momentum flux conservation

$$\mathbf{P} = \mathbf{pI} + \rho \mathbf{vv}$$

$$\mathbf{P_M} = -\frac{\mathbf{B^2}}{\mathbf{8}\pi}\mathbf{I} + \frac{\mathbf{BB}}{\mathbf{4}\pi}$$

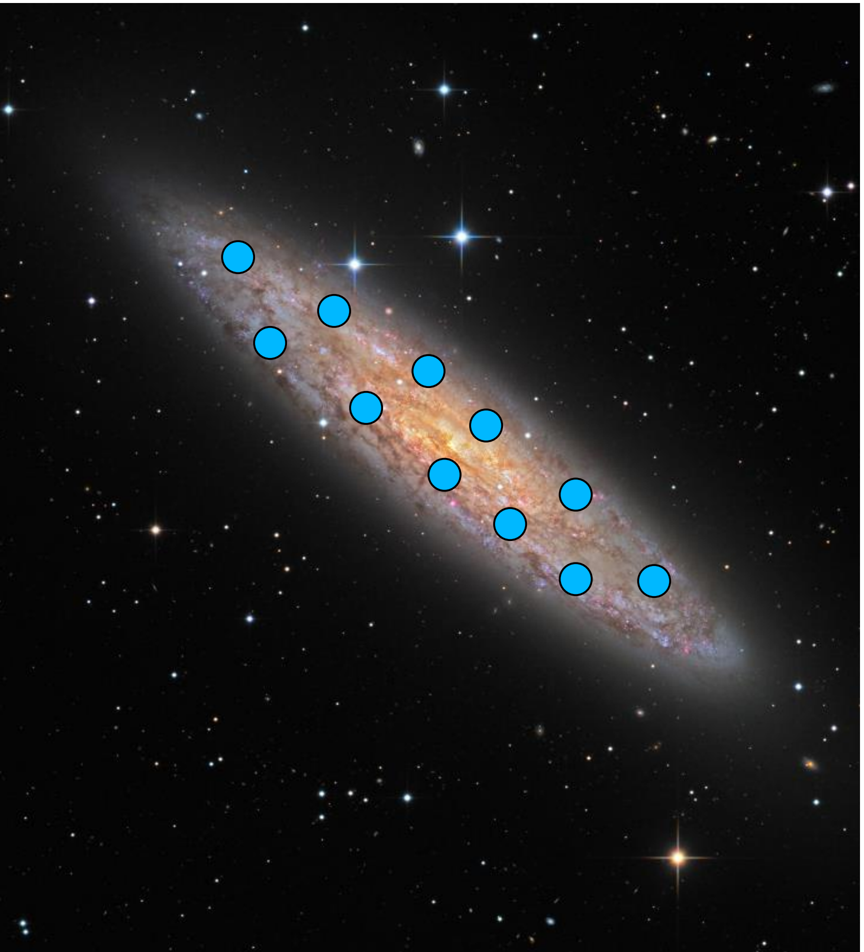
Maxwell stress tensor

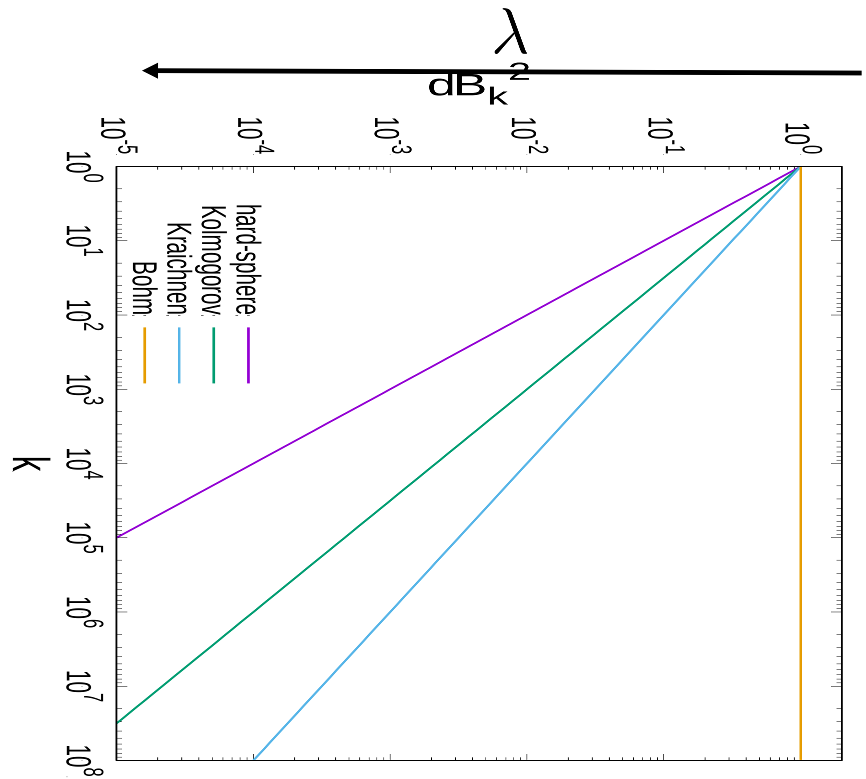
Galactic Magneto-Hydro Turbulence

One of the key drivers is thought to be Supernova explosions

$$\delta \mathbf{B}^2 = \int \frac{\mathbf{d}(\delta \mathbf{B}^2)}{\mathbf{d}\ln \mathbf{k}} \mathbf{d}\ln \mathbf{k} = \int \delta \mathbf{B}_\mathbf{k}^2 \mathbf{d}\ln \mathbf{k}$$

$$\delta \mathbf{B}_\mathbf{k}^2 = \delta \mathbf{B}_\mathbf{0}^2 \left(\frac{\mathbf{k}}{\mathbf{k_0}}\right)^{\mathbf{1}-\alpha}$$

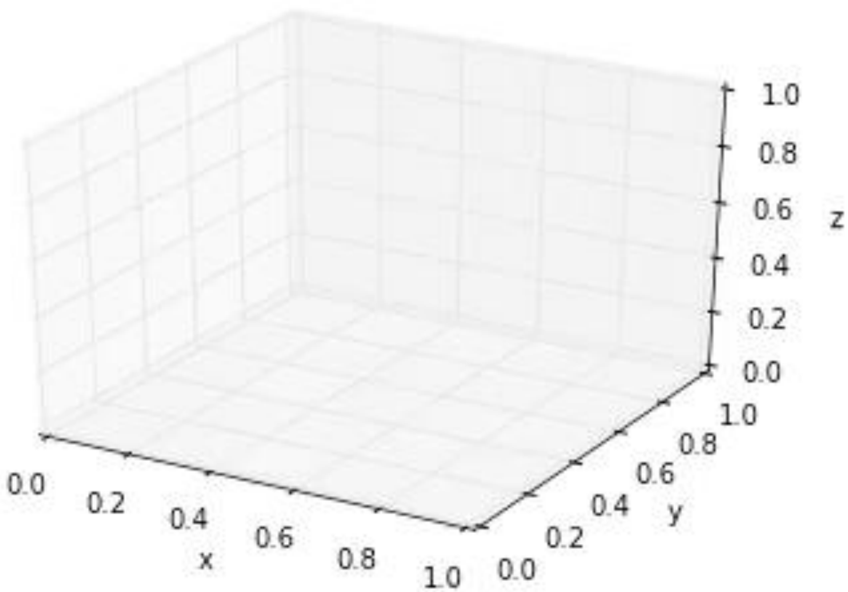


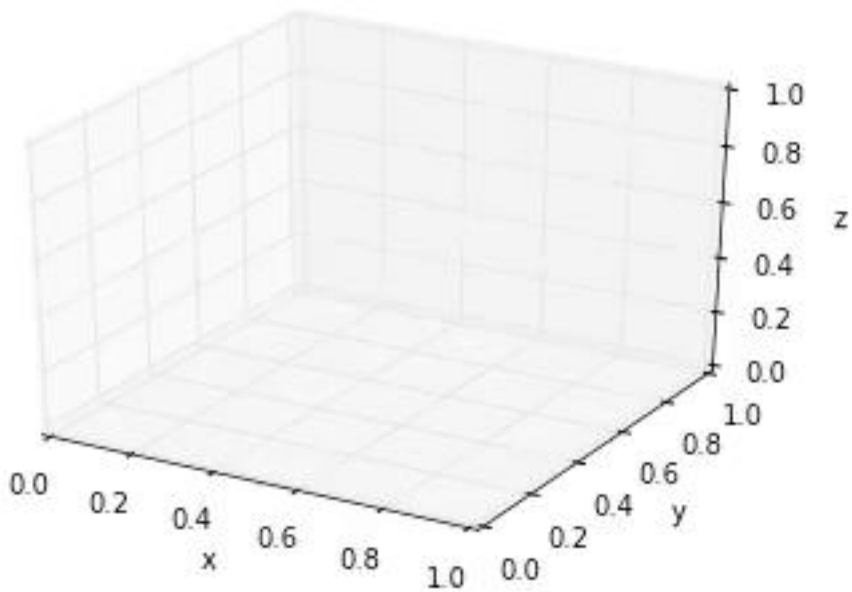


Note for MHD turbulence, the theoretically expected turbulence index is still debated

Charged Particles in Magnetic Fields

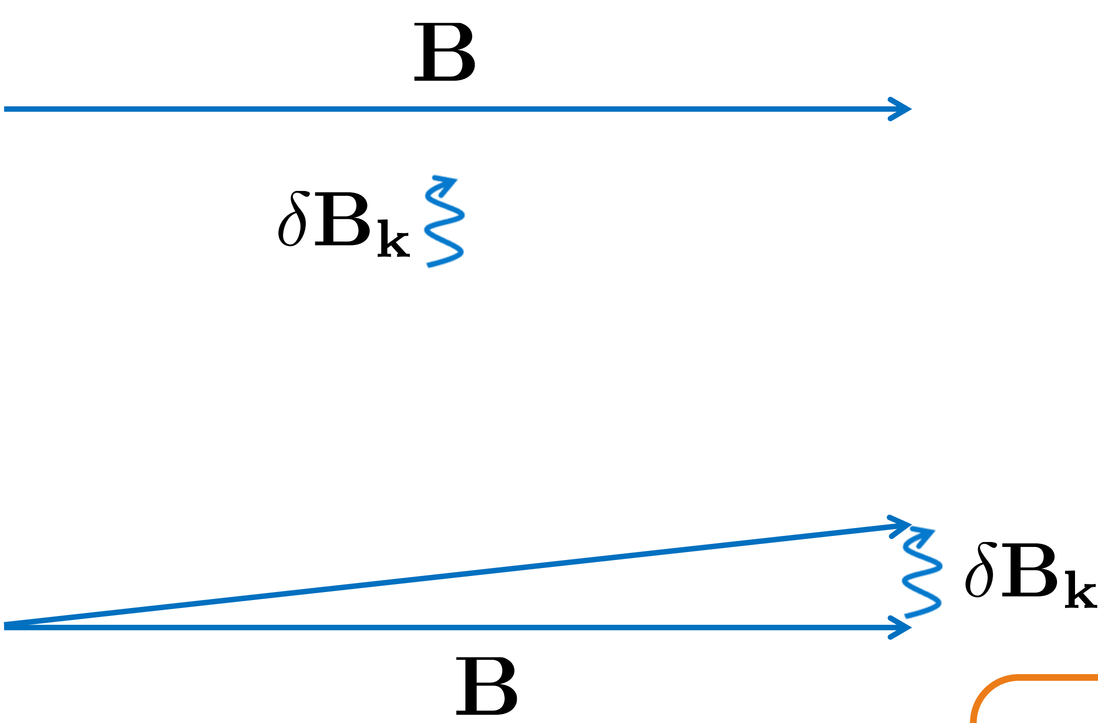
Note- a lot of what you **may have** studied about charged particle propagation in magnetic fields **likely** assumed magnetic field variation was on much longer length scales than particle Larmor radius.





Particle Diffusion in Magnetic Turbulence (Quasi-Linear Theory)?

The propagation of cosmic rays is dictated by the magnetic field landscape they live in.



$$\delta\theta = \frac{\delta\mathbf{B_k}}{\mathbf{B}}$$

$$\langle \boldsymbol{\Delta}\theta^{\mathbf{2}} \rangle = \mathbf{N}\langle \delta\theta^{\mathbf{2}} \rangle = \left(\frac{\mathbf{t}}{\mathbf{t_{lar}}}\right)\langle \delta\theta^{\mathbf{2}} \rangle$$

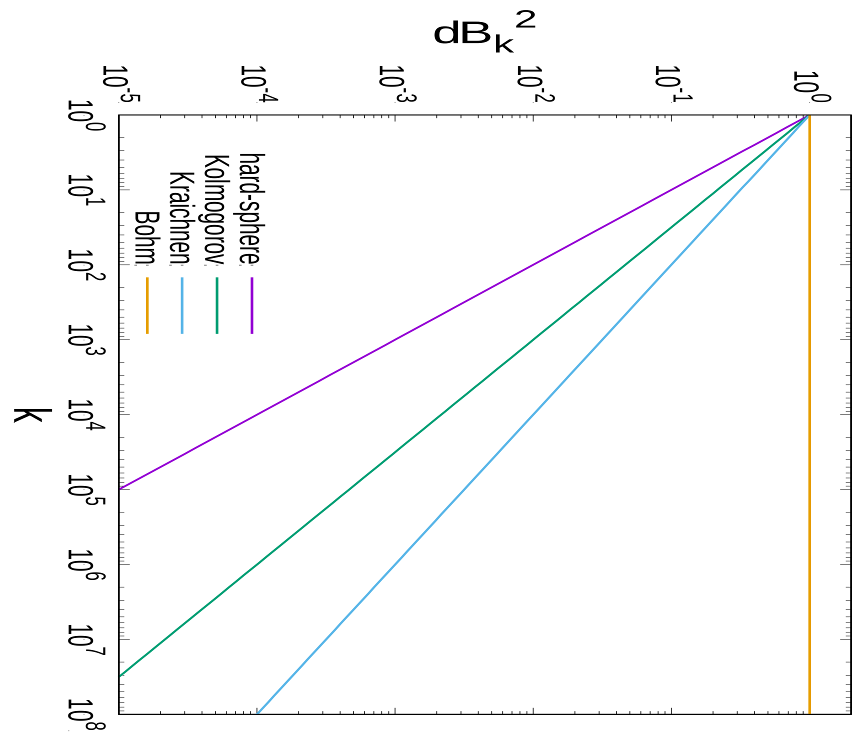
$$\mathbf{D}_{\theta\theta} = \frac{\boldsymbol{\Delta}\theta^{\mathbf{2}}}{\mathbf{t}} = \frac{\mathbf{1}}{\mathbf{t_{lar}}}\left(\frac{\delta\mathbf{B_k^2}}{\mathbf{B^2}}\right)$$

Spatial Diffusion in Magnetic Turbulence?

$$t_{scat} \approx \frac{1}{D_{\theta\theta}}$$

$$\frac{D_{xx}}{c} \approx t_{scat}$$

$$\frac{D_{xx}}{c} \approx t_{lar} \left(\frac{B^2}{\delta B_k^2} \right)$$



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Transport Equation

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \nabla_{\mathbf{x}} \cdot (\mathbf{D}_{\mathbf{xx}} \nabla_{\mathbf{x}} \mathbf{f}) + \mathbf{Q}$$

Transport (Continuity) Equation

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} + \nabla_{\mathbf{x}} \cdot \mathbf{j} = \mathbf{Q}$$

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \nabla_{\mathbf{x}} \cdot (\mathbf{D}_{\mathbf{xx}} \nabla_{\mathbf{x}} \mathbf{f}) + \mathbf{Q}$$

$$\mathbf{j} = -\mathbf{D}_{\mathbf{xx}} \nabla_{\mathbf{x}} \mathbf{f}$$

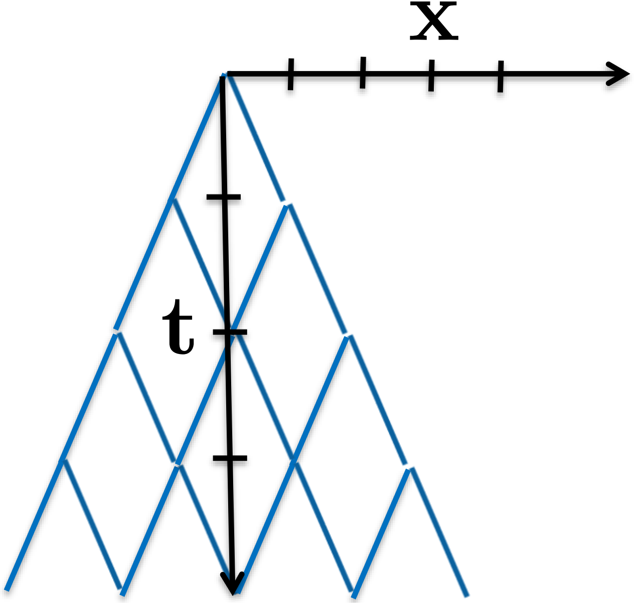
Charged Particle Motion in Turbulent Magnetic Fields

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \nabla_{\mathbf{x}} \cdot (\mathbf{D}_{\mathbf{xx}} \nabla_{\mathbf{x}} \mathbf{f}) + \mathbf{Q}$$

Diffusion

Source term

Random Walks



$$\mathbf{f(x,t) = \frac{t!}{([t-x]/2)!([x+t]/2)!(2^t)}}$$

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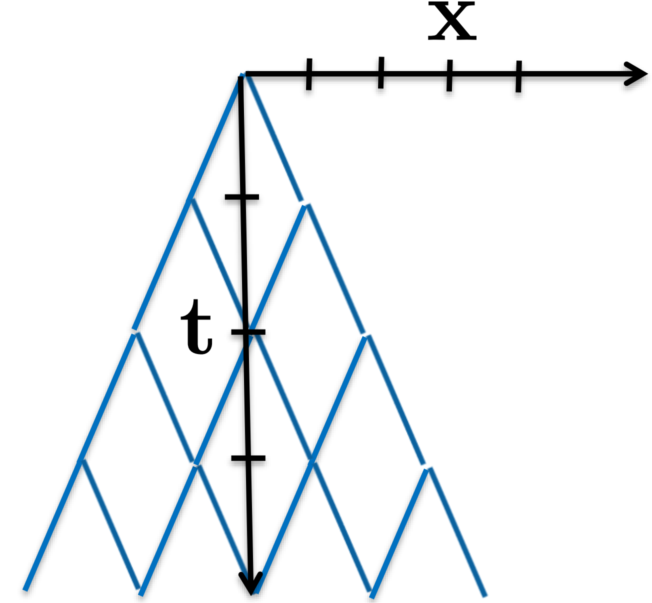
Random Walks



$$\gamma(t+1) = t!$$

$$\gamma(t+1) = \int_0^\infty x^t e^{-x} dx$$

$$f(x,t) = \frac{\gamma(t+1)}{[\gamma([t-x]/2+1)\gamma([x+t]/2+1)](2^t)}$$

$$f(x,t) \approx \frac{e^{-x^2/(2t)}}{(2\pi t)^{1/2}}$$

Suggest you all have a go at demonstrating this.

Steady State Distribution Around a Source of Diffusing Particles

cosmic rays diffuse in magnetic field turbulence

Note- expressions on previous slide in dimensionless units,

$$\mathbf{t \rightarrow 2Dt}$$

$$\mathbf{f(r,t) \approx \frac{e^{-r^2/(4Dt)}}{(4\pi Dt)^{3/2}}}$$

3D Green's function

$$\mathbf{F(r) = \int_0^\infty f(r,t)dt}$$

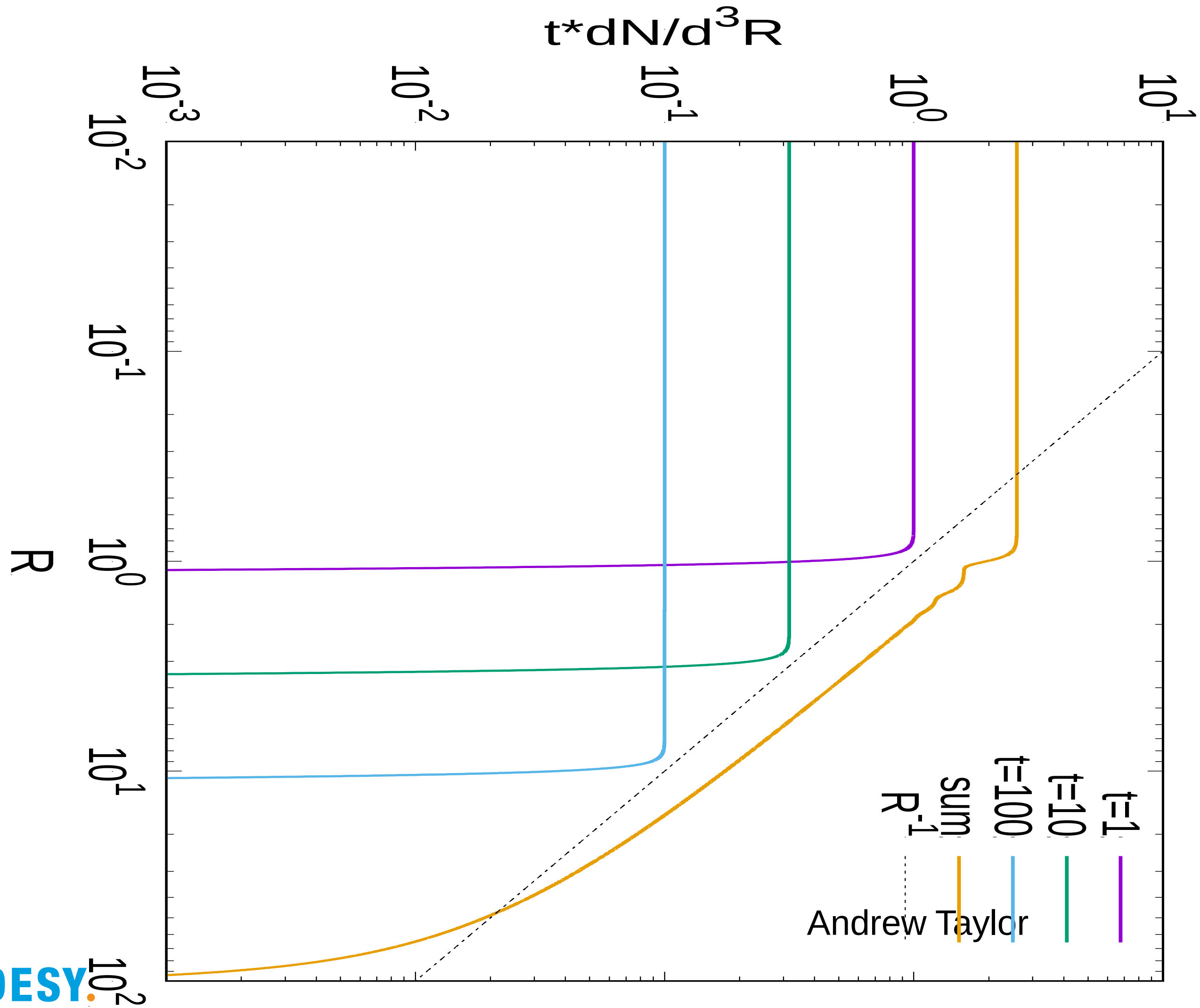
$$\mathbf{= \frac{1}{Dr}}$$

Suggest you all have a go at demonstrating this!

Steady State Distribution Around a Source of Diffusing Particles

$$\int \mathbf{f(r,t)dt} = \int \mathbf{tf(r,t)dlnt}$$

Smart to look at this



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Energy Densities Around Non-Thermal Sources

$$\mathbf{U_X = \frac{L_X}{4\pi r^2 c}}$$

Object looks like a point source

$$\mathbf{U_{CR} = \frac{L_{CR}}{Dr}}$$

Dipole observed

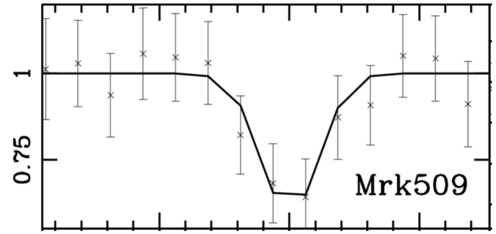
$$\frac{\mathbf{dN}}{\mathbf{d}\cos\theta} \propto \left(\mathbf{1} + \frac{\lambda_{\mathbf{scat}}}{\mathbf{r_s}}\cos\theta\right)$$

Diffusion + Advection?

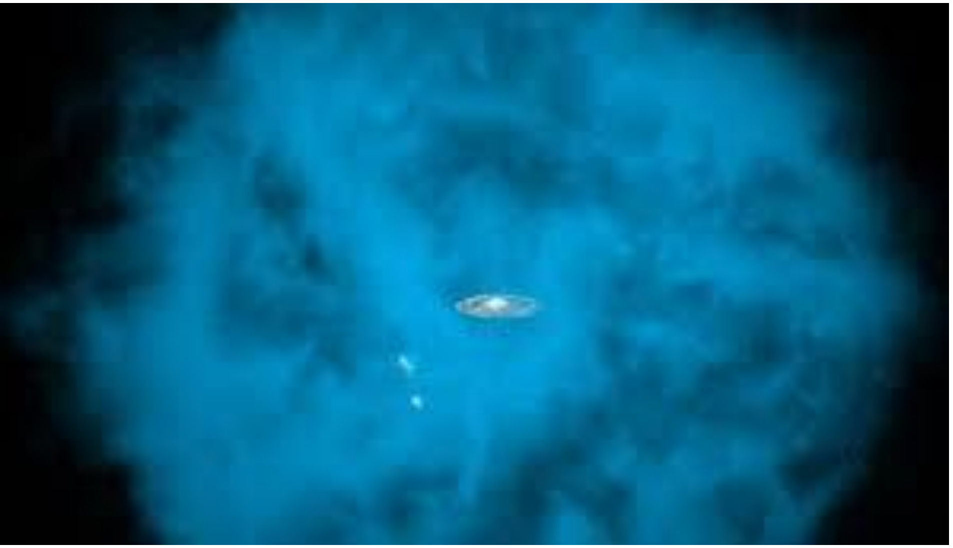
$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = -\nabla \cdot (\mathbf{vf} - \mathbf{D}\nabla \mathbf{f}) + \frac{\mathbf{1}}{\mathbf{p^2}} \frac{\partial}{\partial \mathbf{p}} \left[(\nabla \cdot \mathbf{v}) \frac{\mathbf{p^3}}{\mathbf{3}} \mathbf{f} \right] + \frac{\mathbf{Q}}{\mathbf{p^2}}$$

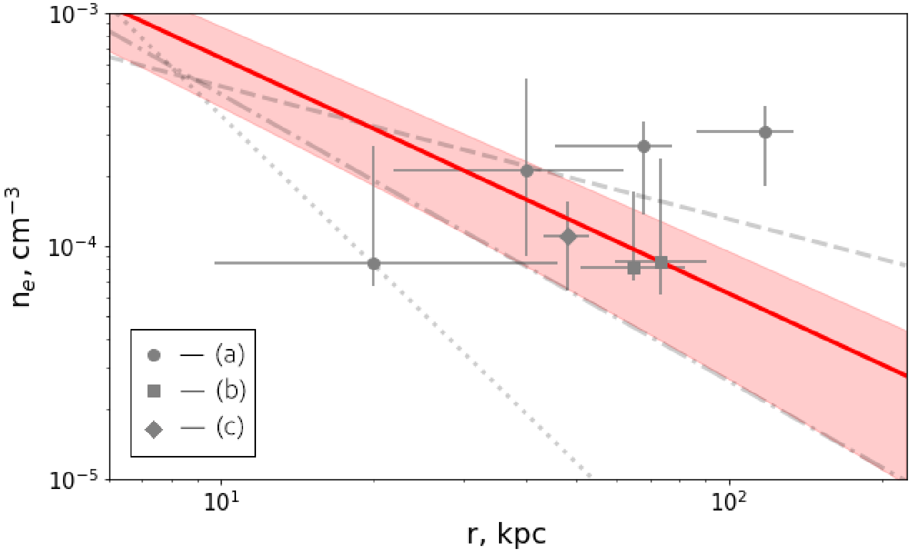
The Big Unknown in the Galactic Magnetic Field- The Halo!

Both Suzaku and Chandra X-ray observations of bright AGN (Mkr 501, PKS 2155, NGC 3783) indicate the presence of a hot local absorber.



Martynenko MNRAS, 511, (2022)





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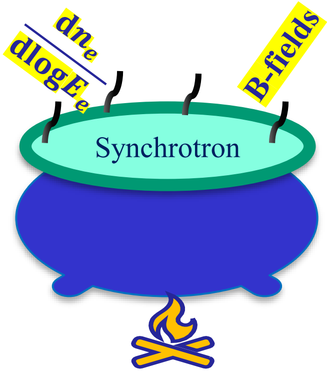
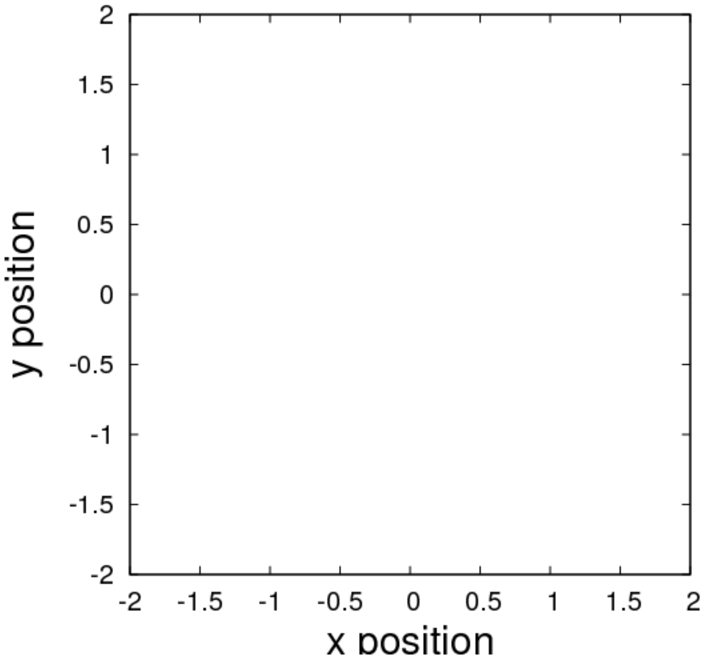
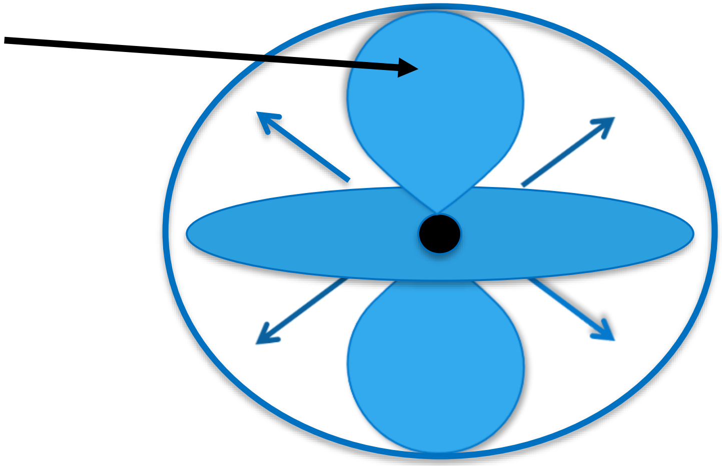
Key Astrophysical Magnetic Field Probe

Synchrotron Emission



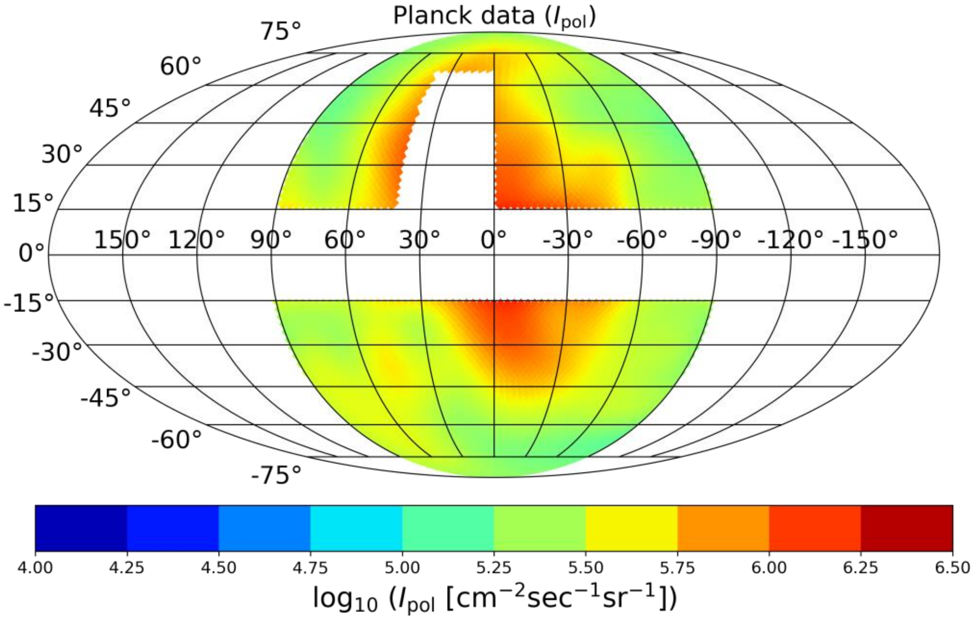


Synchrotron emission has revealed the Galactic bubble regions (whose existence was only discovered in the last 15 years!)



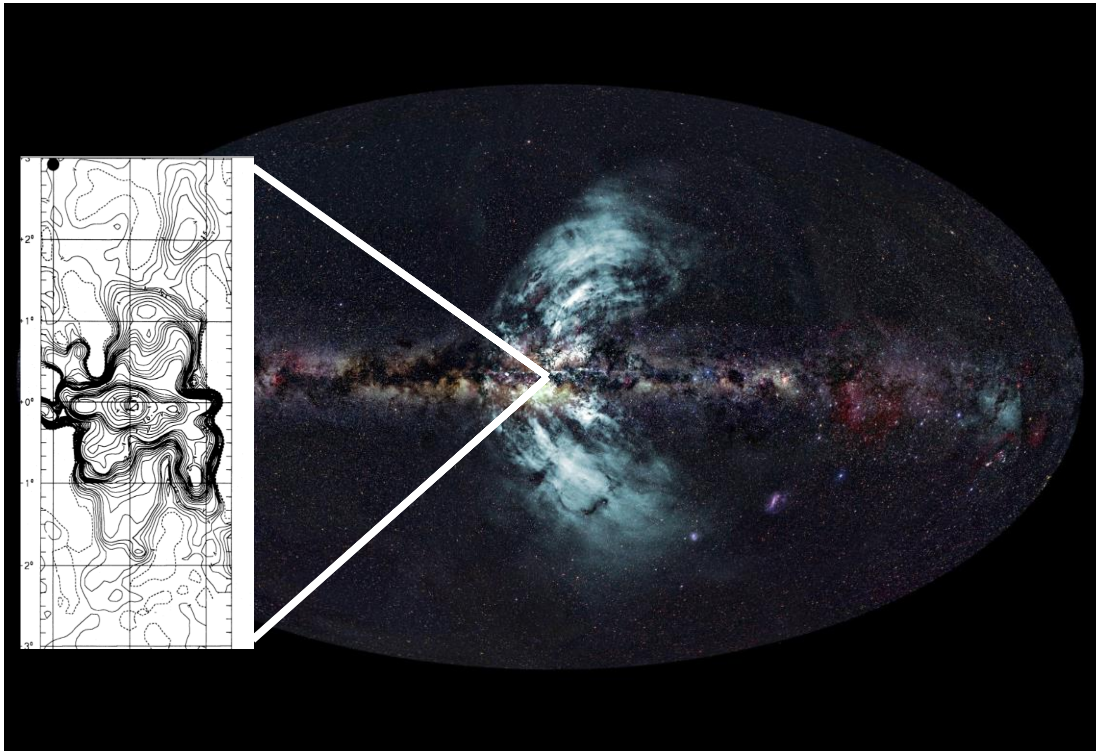
Galactic Halo Synchrotron Emission

WMAP/Planck Microwave Haze



Shaw, et al. arxiv: 2202.06780

Radio Bubbles

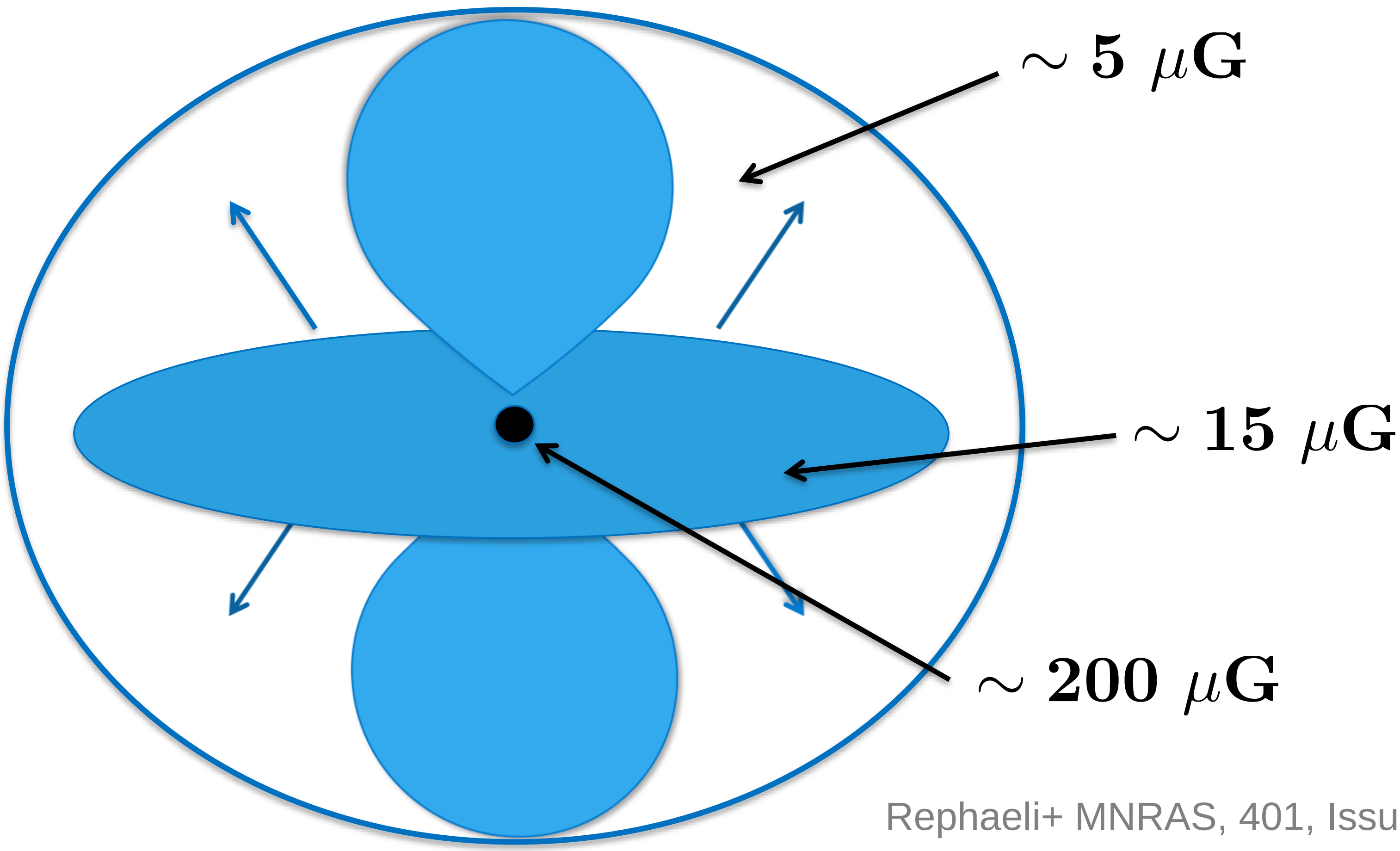


Pohl+, A&A 262 441 1992

Carretti+, *Nature* **volume 493**, 2013

Magnetic Fields in the Galaxies

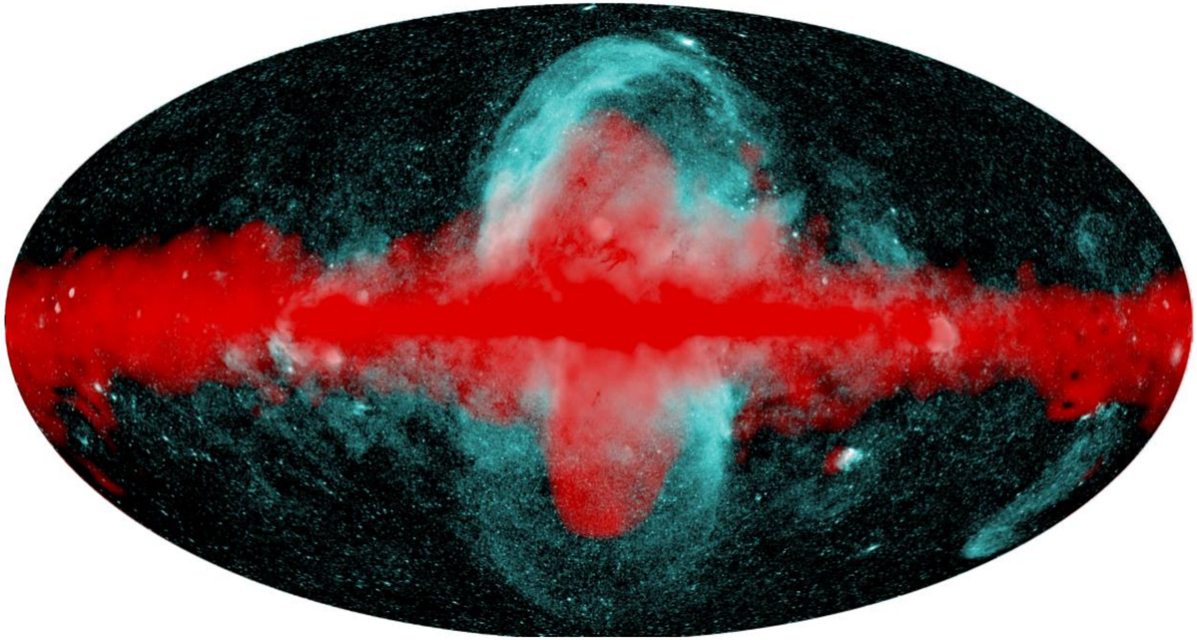
Elstner+ A&A 568, A104 (2014)

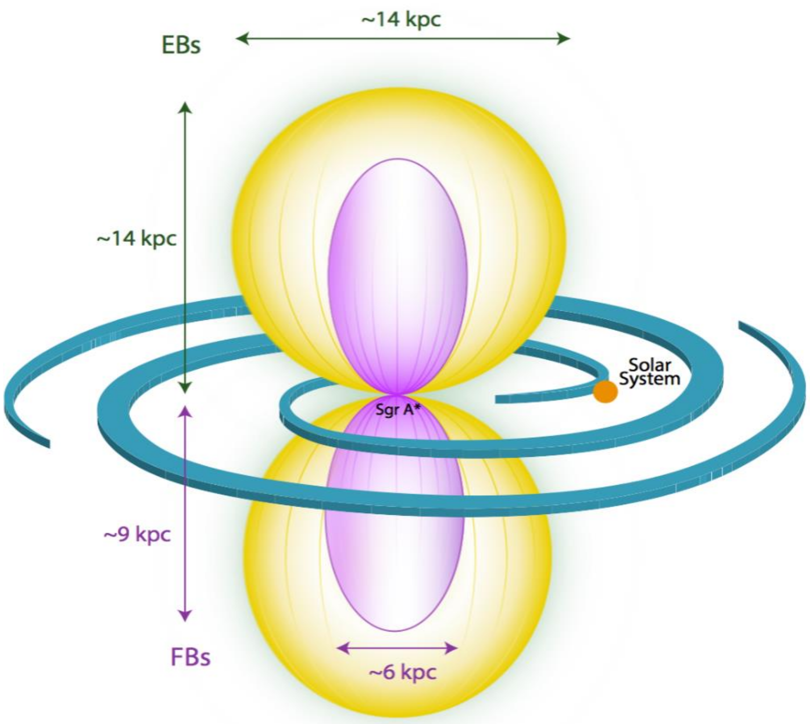


Rephaeli+ MNRAS, 401, Issue 1 (2010)

…….though note- ApJ 645:186–198 (2006)

The Fermi/eRosita Bubbles



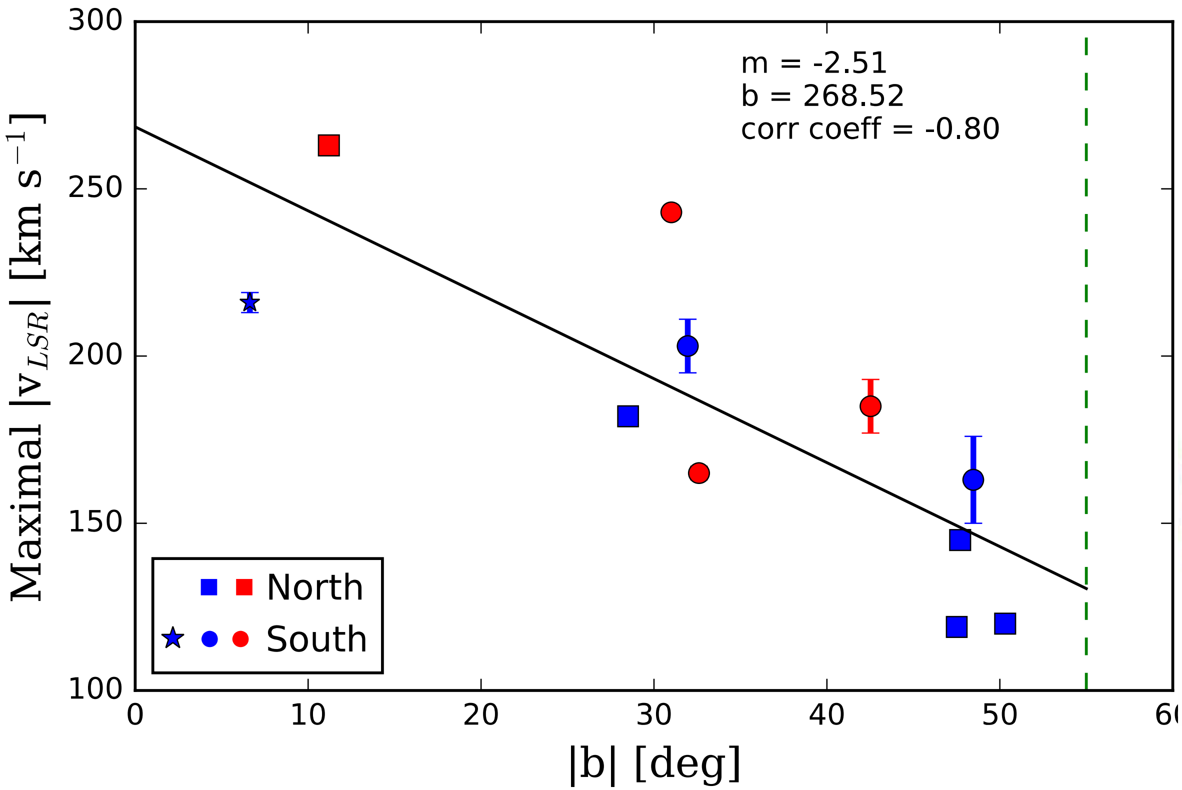


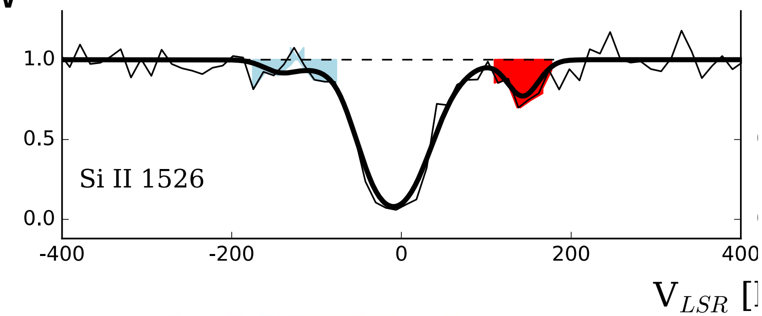
Su, M., et al. ApJ 724, 1044–1082 (2010)

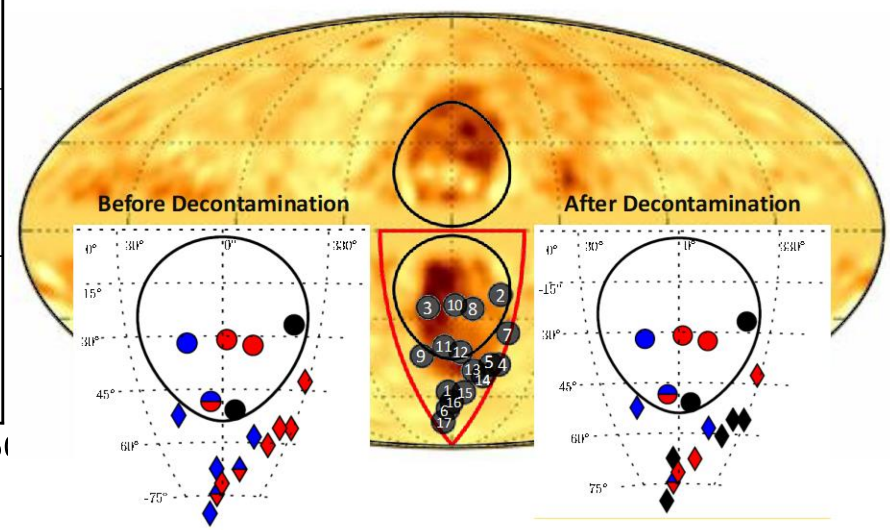
Predehl, P., et al. Nature 588, 227–231 (2020)

Advection Within the Bubbles?

The Doppler shift of absorption lines for lines-of-sight which pass through these bubbles reveals evidence of a coherent velocity flow



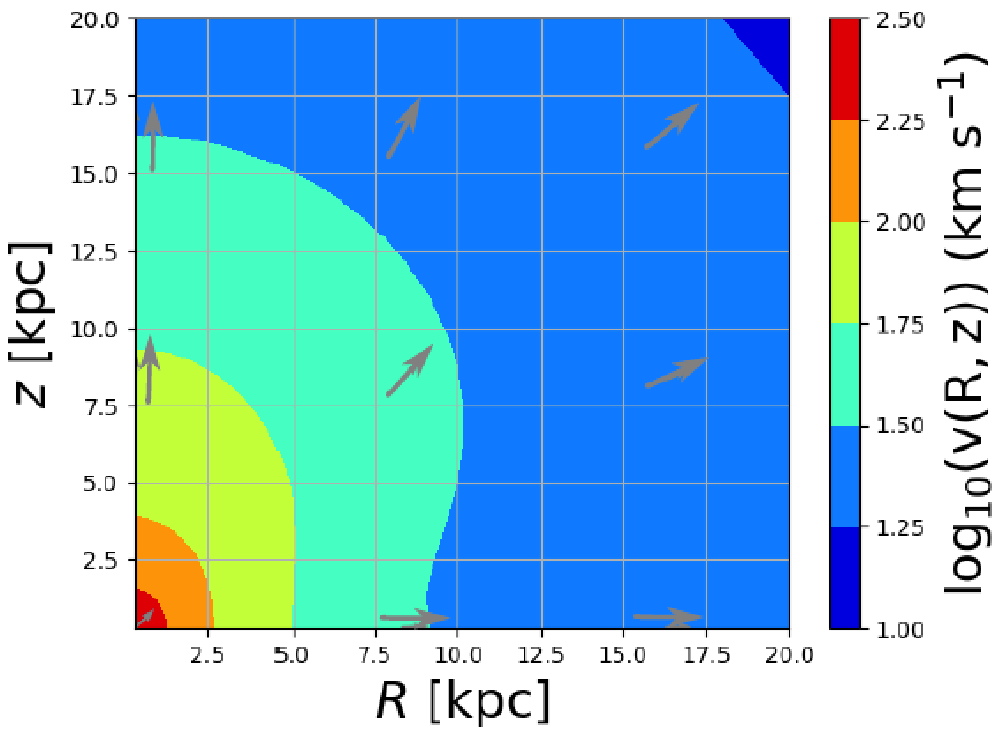




Karim+, Ap.J. 860 (2018)

Diffusion/Advection of Cosmic Rays into the Halo?

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = -\nabla \cdot (\mathbf{vf} - \mathbf{D}\nabla\mathbf{f}) + \frac{\mathbf{1}}{\mathbf{p}^2}\frac{\partial}{\partial \mathbf{p}}\left[(\nabla \cdot \mathbf{v})\frac{\mathbf{p}^3}{\mathbf{3}}\mathbf{f}\right] + \frac{\mathbf{Q}}{\mathbf{p}^2}$$



Tourmente+, arxiv:2207.09189

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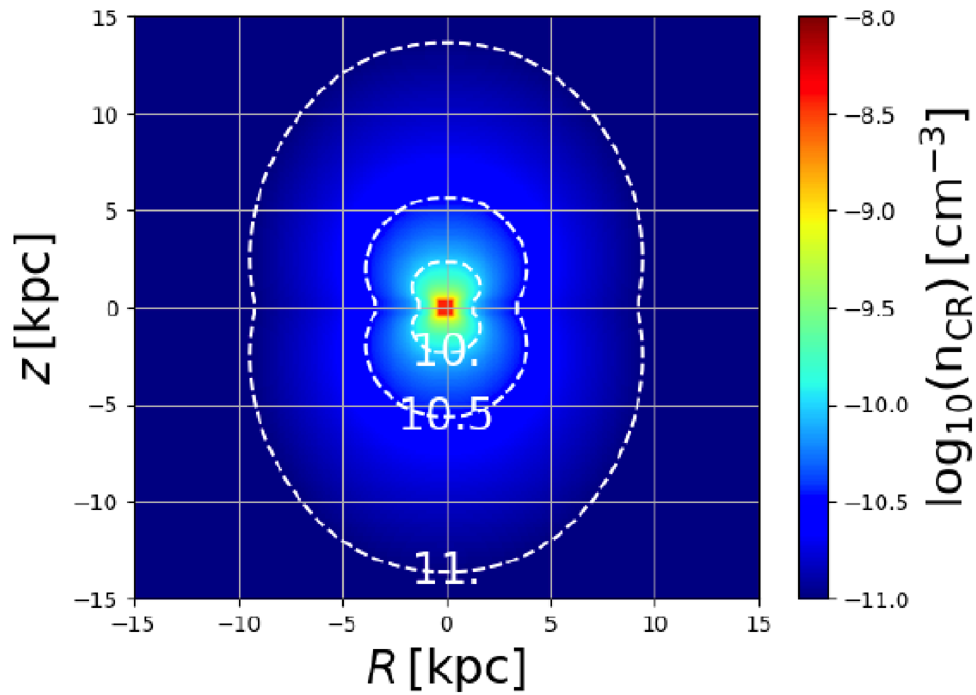

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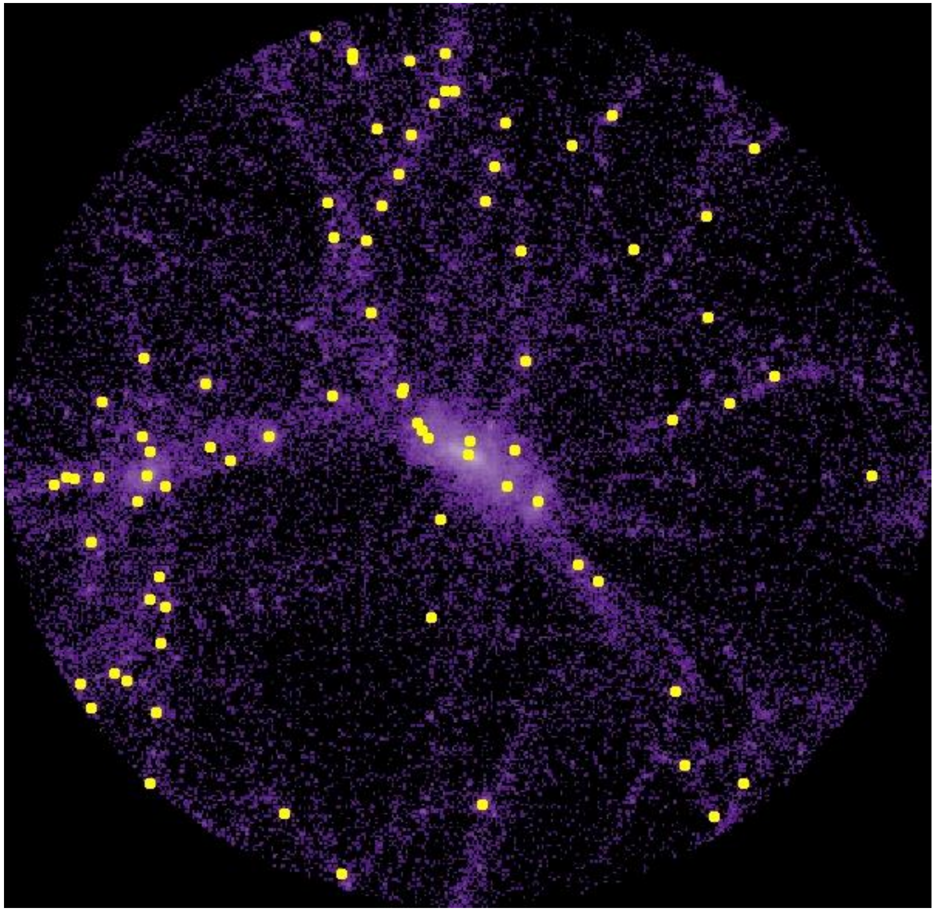
An Even Bigger Unknown! Extragalactic Magnetic Fields

The homogeneous scale for the Universe is thought to be 100 Mpc – is possible that the magnetic field in local extragalactic space is structured (the matter is structured on these scales).

What is the EGMF structure/strength in the inhomogeneous region around the Milky Way?



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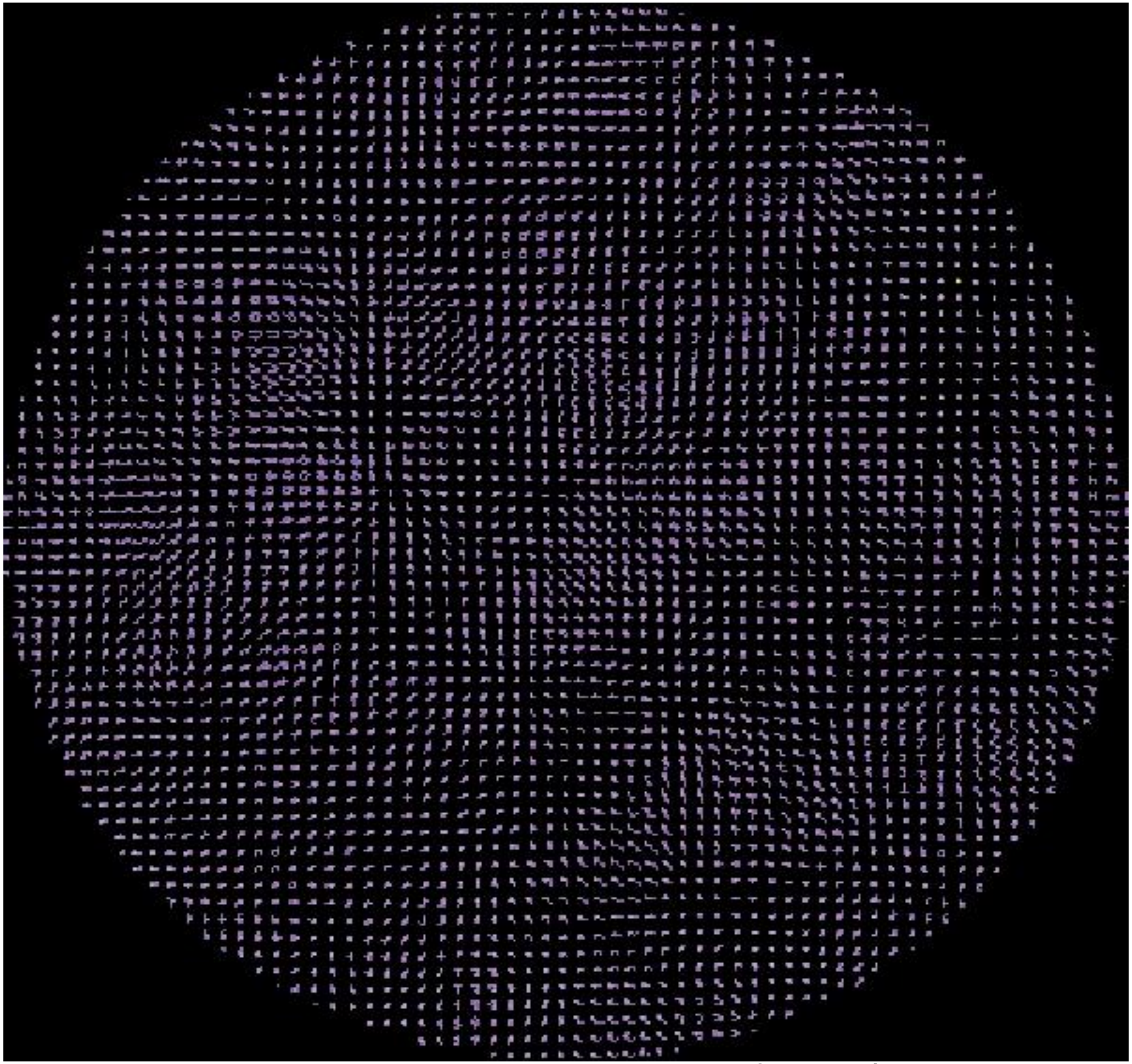
Extragalactic Magnetic Field Origin?



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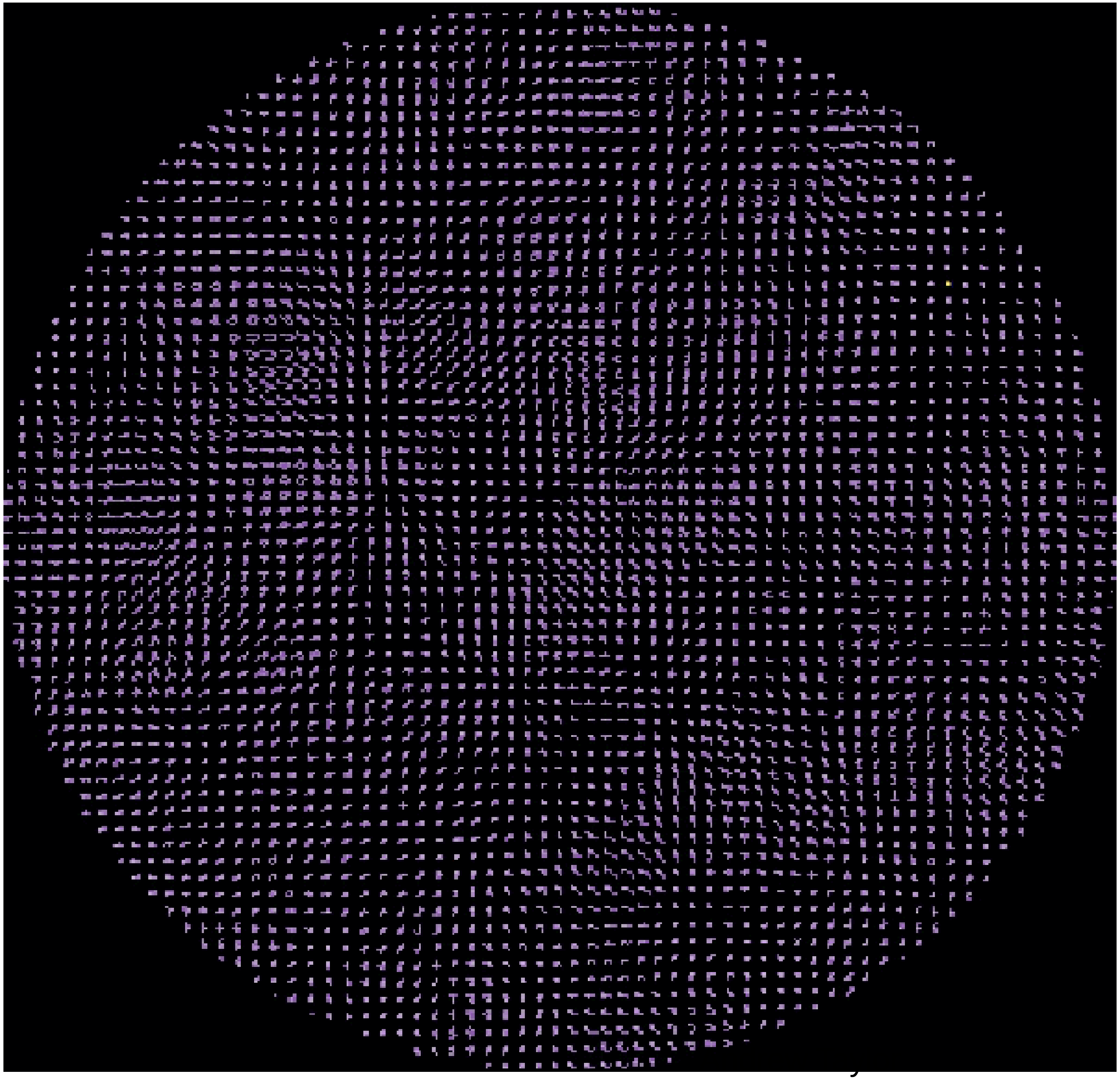
$z = 40$

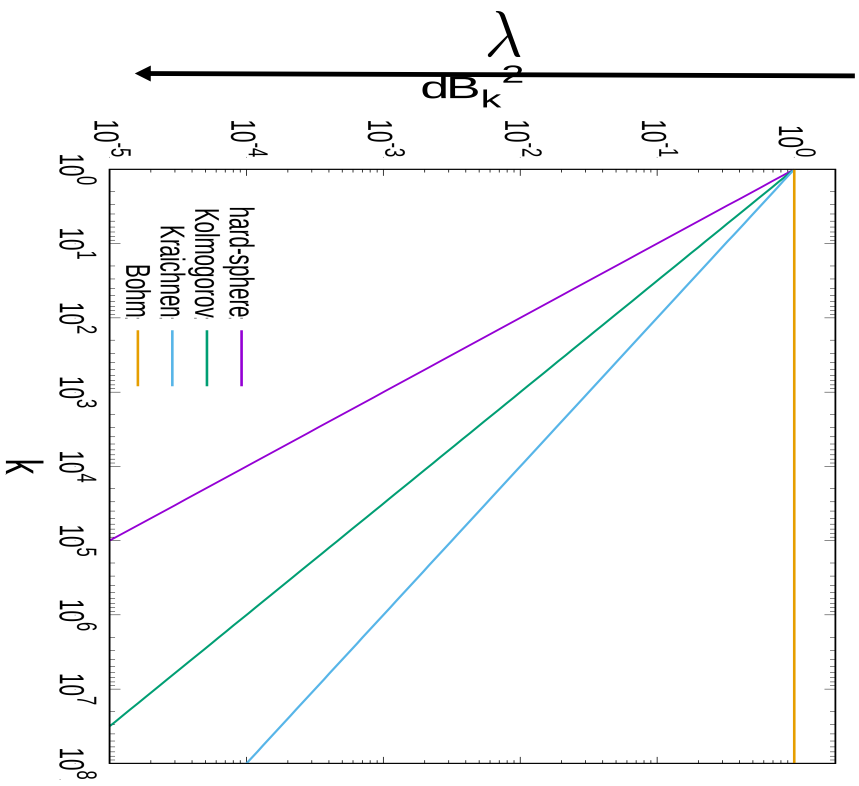
Seed B-field strength?

Extragalactic Magnetic Field Origin?

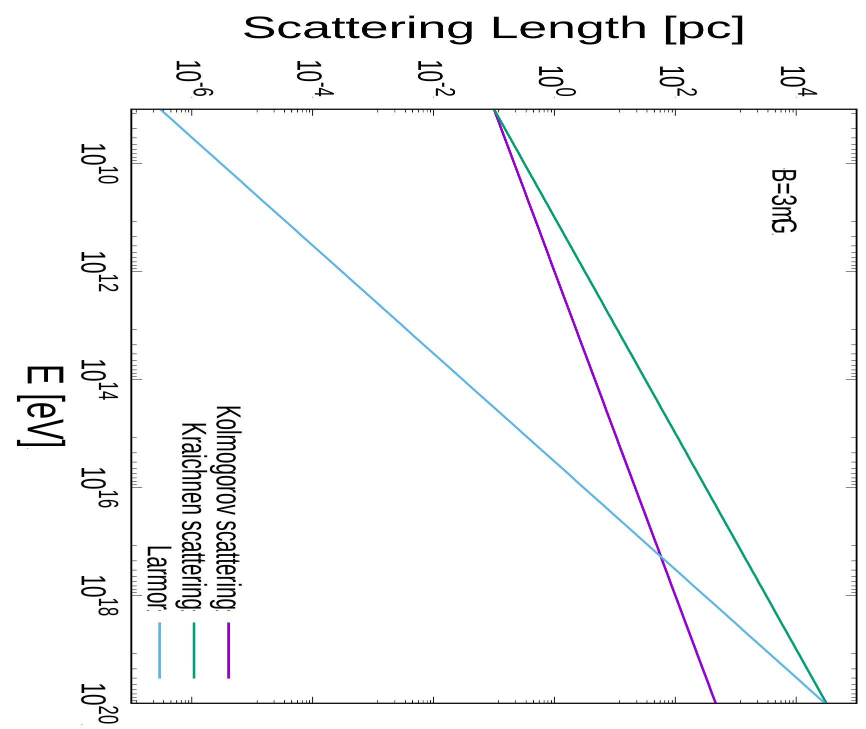


...compression and dynamo action lead to ~μG B-field strength growth on galactic scales

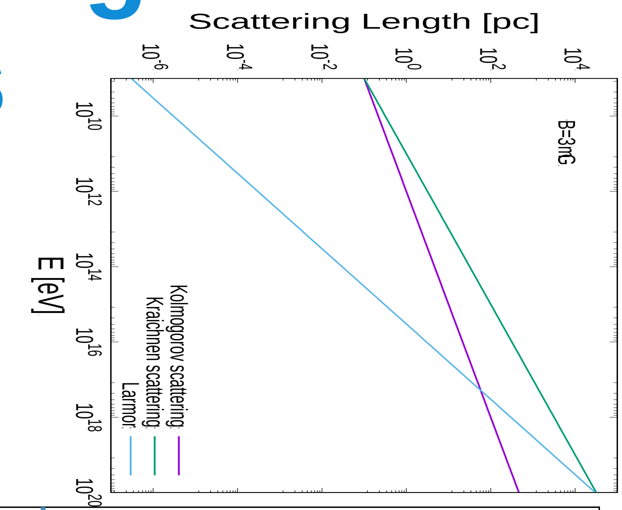
Propagation through Extragalactic Magnetic Fields



$$\frac{D_{xx}}{c} \approx t_{lar} \left(\frac{B^2}{\delta B_k^2} \right)$$



Propagation through Extragalactic Magnetic Fields



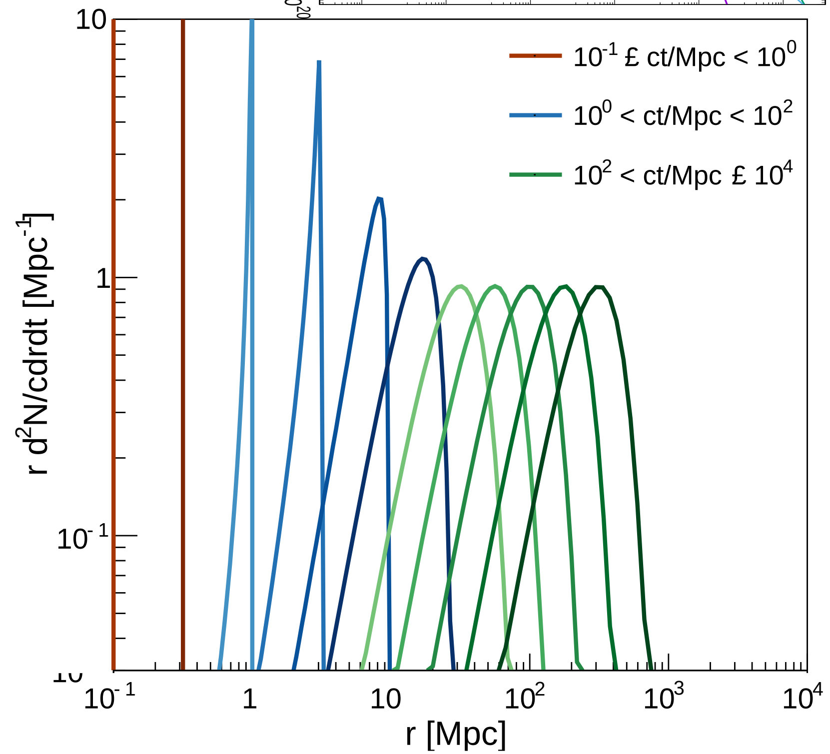
3 Phases of Propagation:

1. Ballistic
2. Ballistic/Gaussian
3. Gaussian

] Juttner

$$\frac{\mathbf{dN}}{\mathbf{dr}} = \frac{\mathbf{r^2}\alpha\mathbf{e}^{(-\alpha/\sqrt{[\mathbf{1}-(\mathbf{r}/\mathbf{ct})^{\mathbf{2}}]})}}{(\mathbf{ct})^{\mathbf{3}}\mathbf{K_1}(\alpha)[\mathbf{1}-(\mathbf{r}/\mathbf{ct})^{\mathbf{2}}]^{\mathbf{2}}}$$

$$\alpha = \mathbf{tc^2}/\mathbf{2D}$$



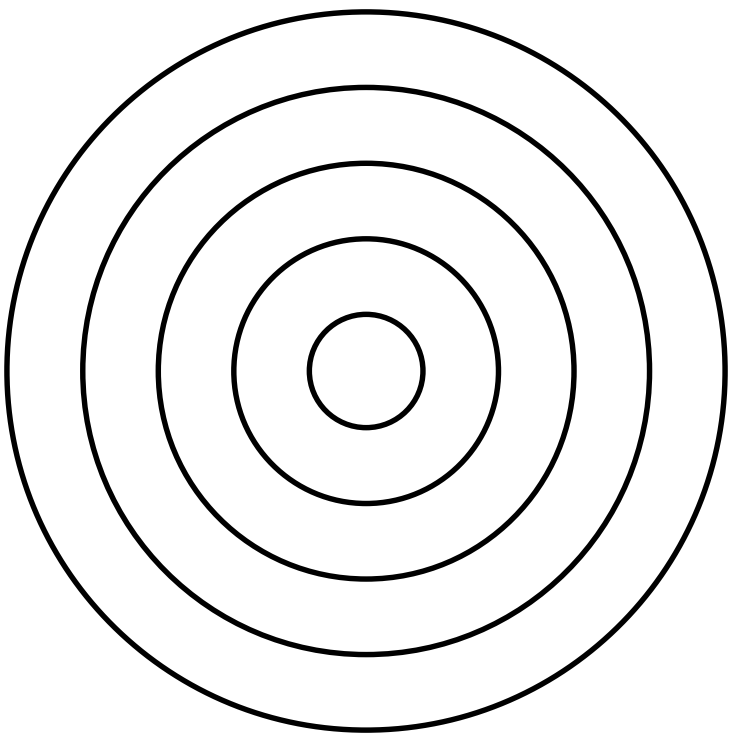
Aloisio, R. +, ApJ 693 2009,

Lang, R. +, PRD 102 (2020)

Extragalactic Magnetic Field Effects

Olbers Paradox for extragalactic cosmic rays:

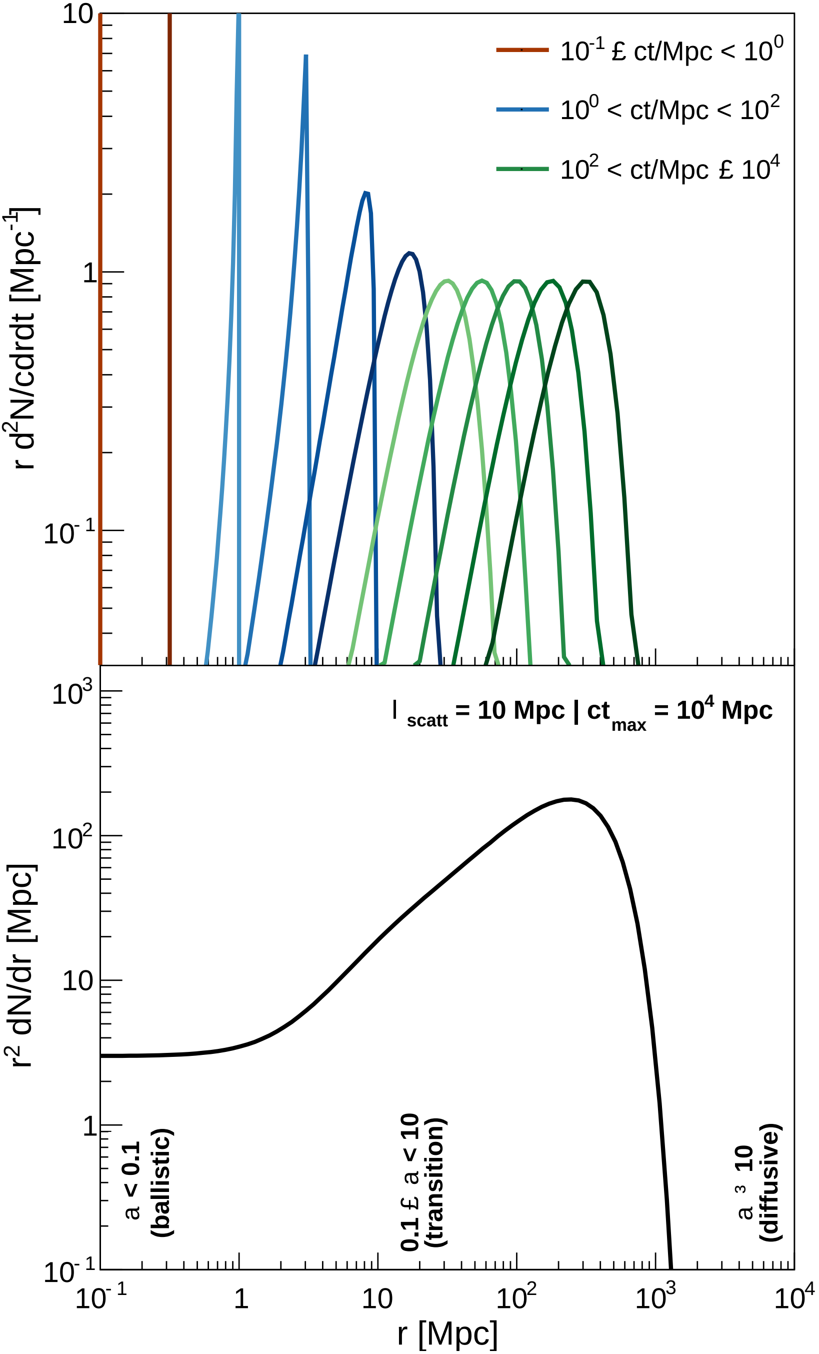
1) Without extragalactic magnetic fields (ie. ballistic propagation)
2) With extragalactic magnetic fields (ie. diffusive propagation)



$$\mathbf{dF = \frac{1}{r^2} ndV}$$

$$\mathbf{F_t = \int_0^{r_{max}} \frac{dF}{dr} dr}$$

Magnetic Field Horizon



$$F_t = \int_0^{r_{max}} \frac{dF}{dr} dr$$

Constant for ballistic propagation

If cosmic ray sources were continuously distributed in space, magnetic fields wouldn’t alter the total cosmic ray spectrum at Earth.

How does the discreet nature of cosmic ray sources alter this statement?

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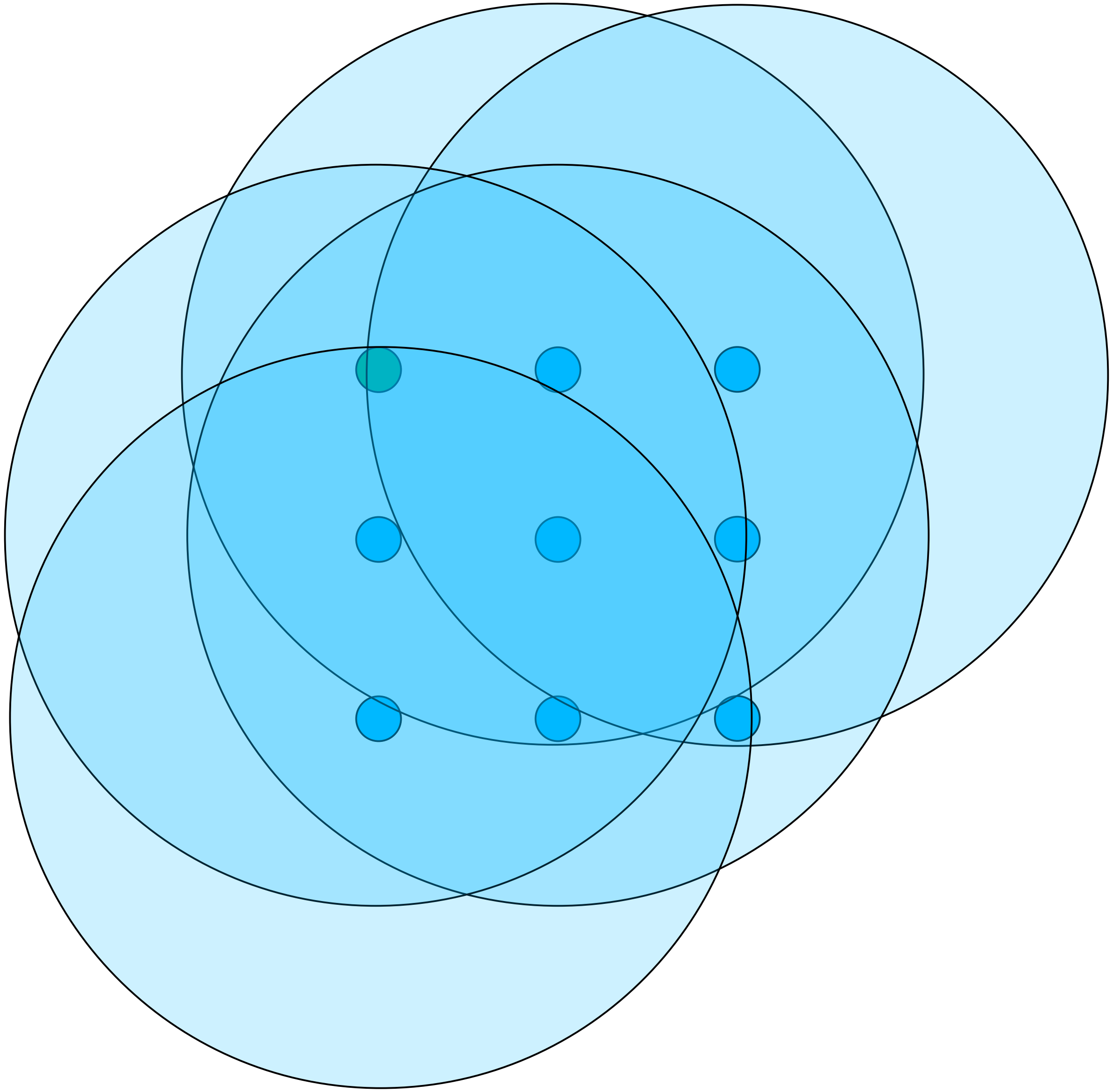
Magnetic Field Horizon



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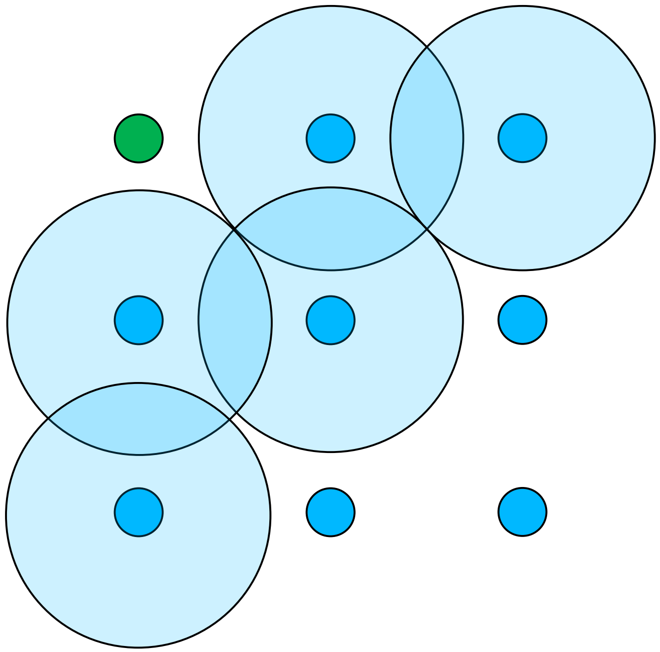
Magnetic Field Horizon

$$\mathbf{l_{MH} = (D_{xx} c t_H)^{1/2}}$$



Once l_{MH} becomes smaller than r_s cosmic rays from the nearest sources become suppressed

Energy Dependent Magnetic Horizon

$$\mathbf{l_{MH}} = (\mathbf{D_{xx} t_H})^{\mathbf{1/2}} = \mathbf{60} \left(\frac{\mathbf{D_{xx}}}{\mathbf{1\ Mpc}} \right)^{\mathbf{1/2}} \left(\frac{\mathbf{t_H}}{\mathbf{4000\ Mpc}} \right)^{\mathbf{1/2}} \mathbf{Mpc}$$

If the diffusion coefficient, D_{xx}, is energy dependent, the magnetic horizon is also energy dependent.

Extragalactic cosmic rays cannot arrive to the Milky Way at low energies!

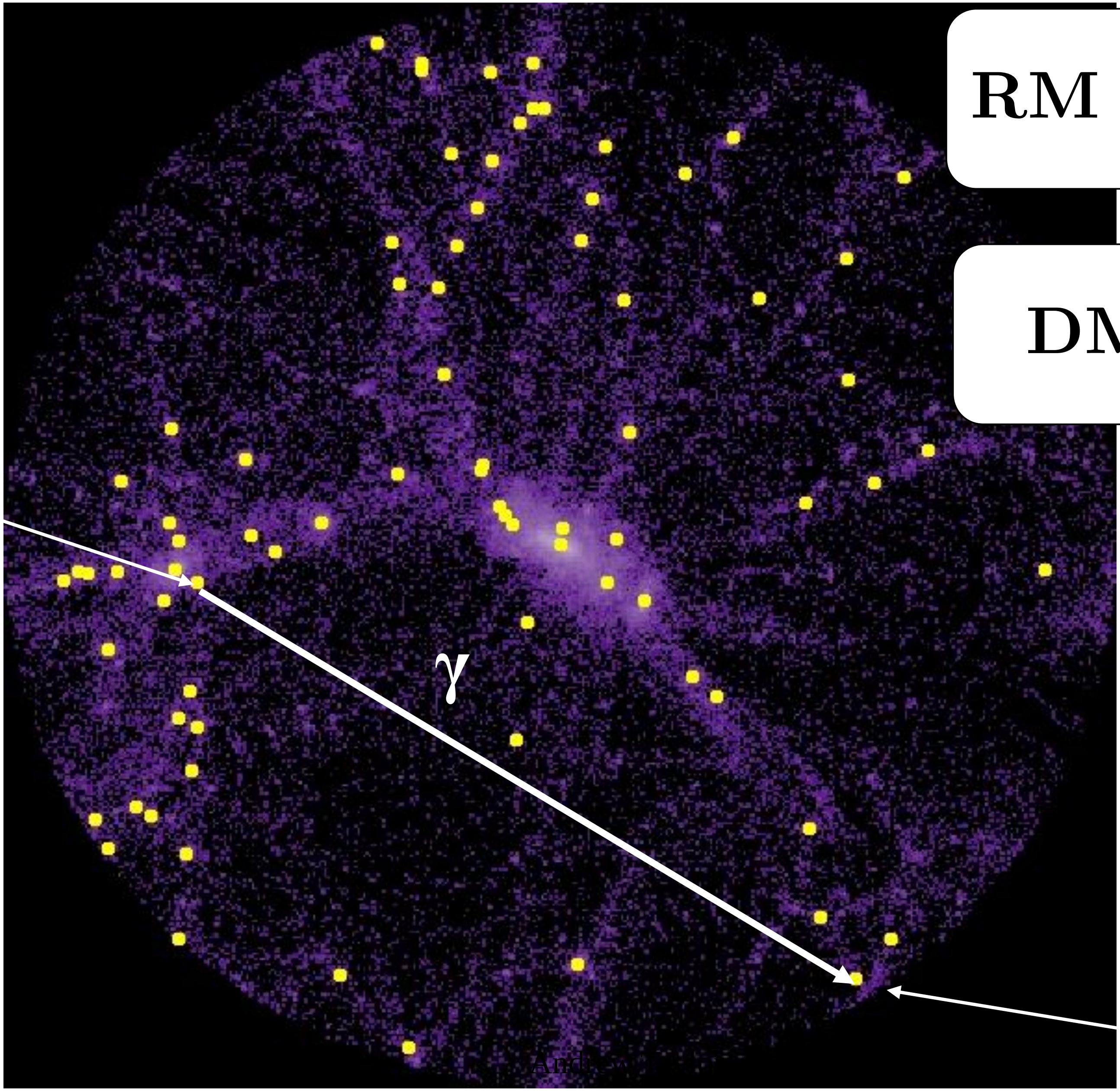
Conclusion

- Cascades in hydrodynamics and magneto-hydrodynamics lead to the formation of turbulence
- Charged particle propagation is dictated by magnetic structure, and in particular magnetic turbulence
- Our knowledge of the magnetic structure of the Milky Way (+ other galaxies) is particularly poor in the Galactic halo region
- Advection may also be playing a role in ”low” energy cosmic ray transport in the Galaxy
- The magnetic structure in our local inhomogeneous patch of the Universe is even more poorly probed
- Extragalactic magnetic fields prevent the arrival of ”low” energy cosmic rays from even the most local sources (the magnetic horizon)

End of Lecture

A Radio Probe

AGN

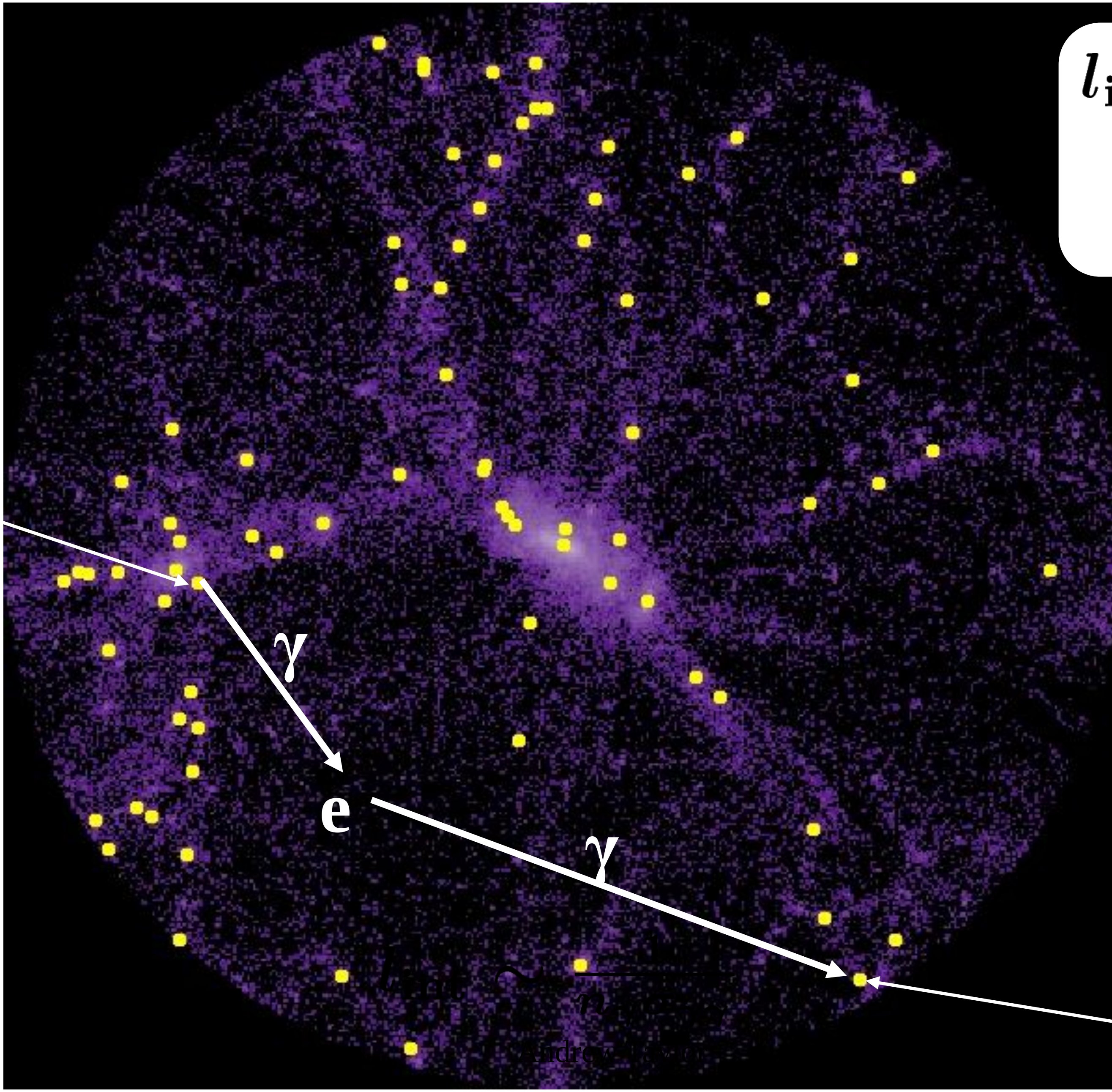


$$\mathbf{RM} = \int \mathbf{n_e B_{\parallel} dl}$$

$$\mathbf{DM} = \int \mathbf{n_e dl}$$

Milky Way

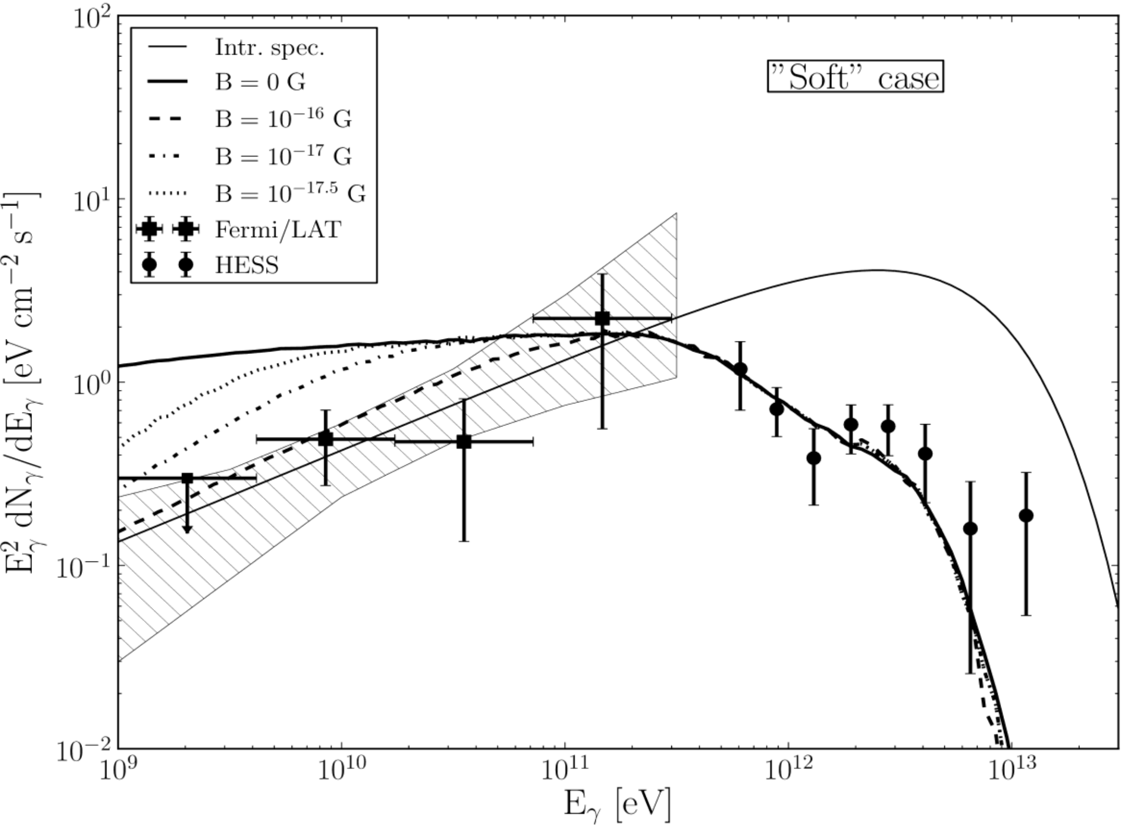
A Gamma-Ray Probe

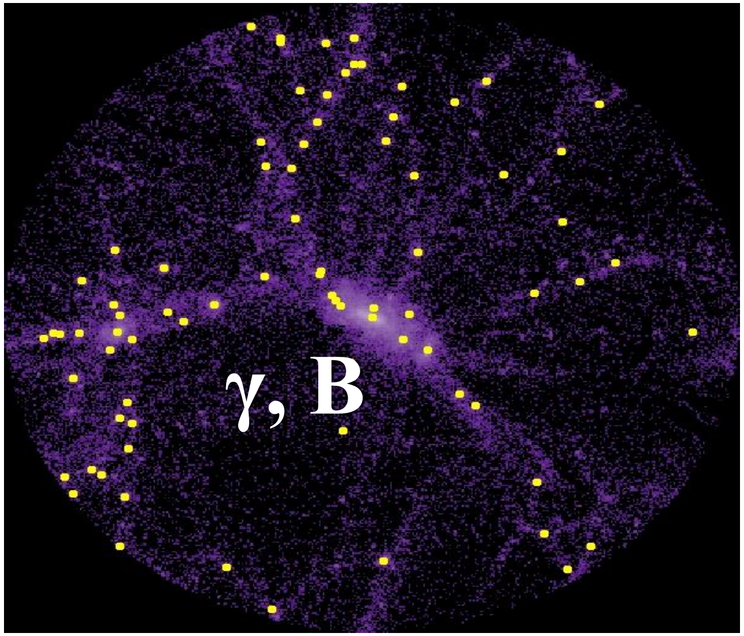


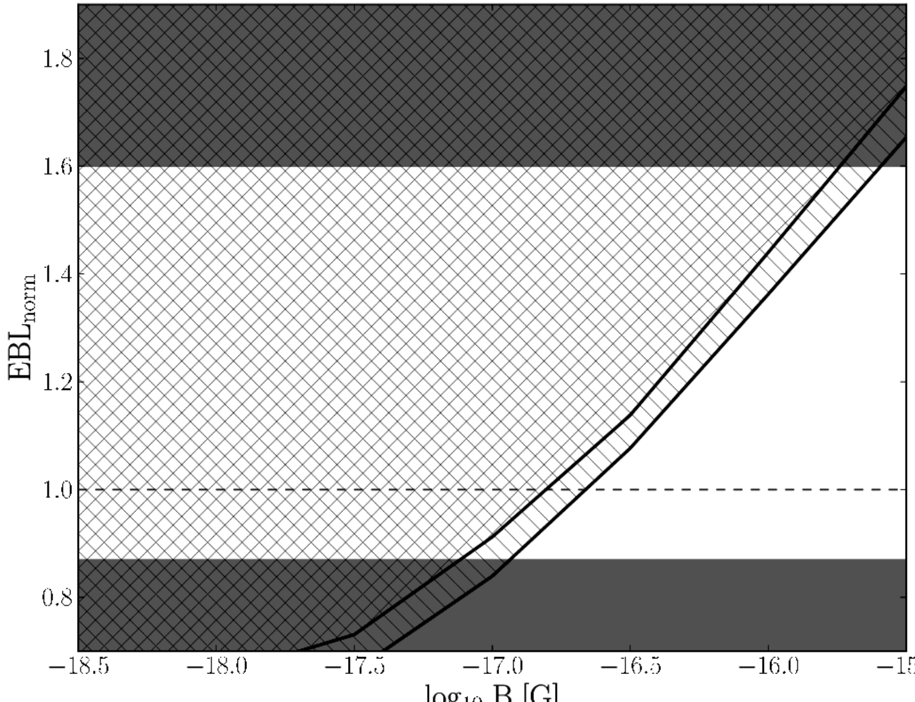
$$l_{\text{int}} \sim \frac{1}{n_\gamma \sigma_{\gamma\gamma}}$$

$$\sim 30 \text{ Mpc}$$

Probing Extragalactic Radiation + Magnetic Fields?







astro-ph/1101.0932 Taylor et al.

astro-ph/1112.2534 Vovk et al.

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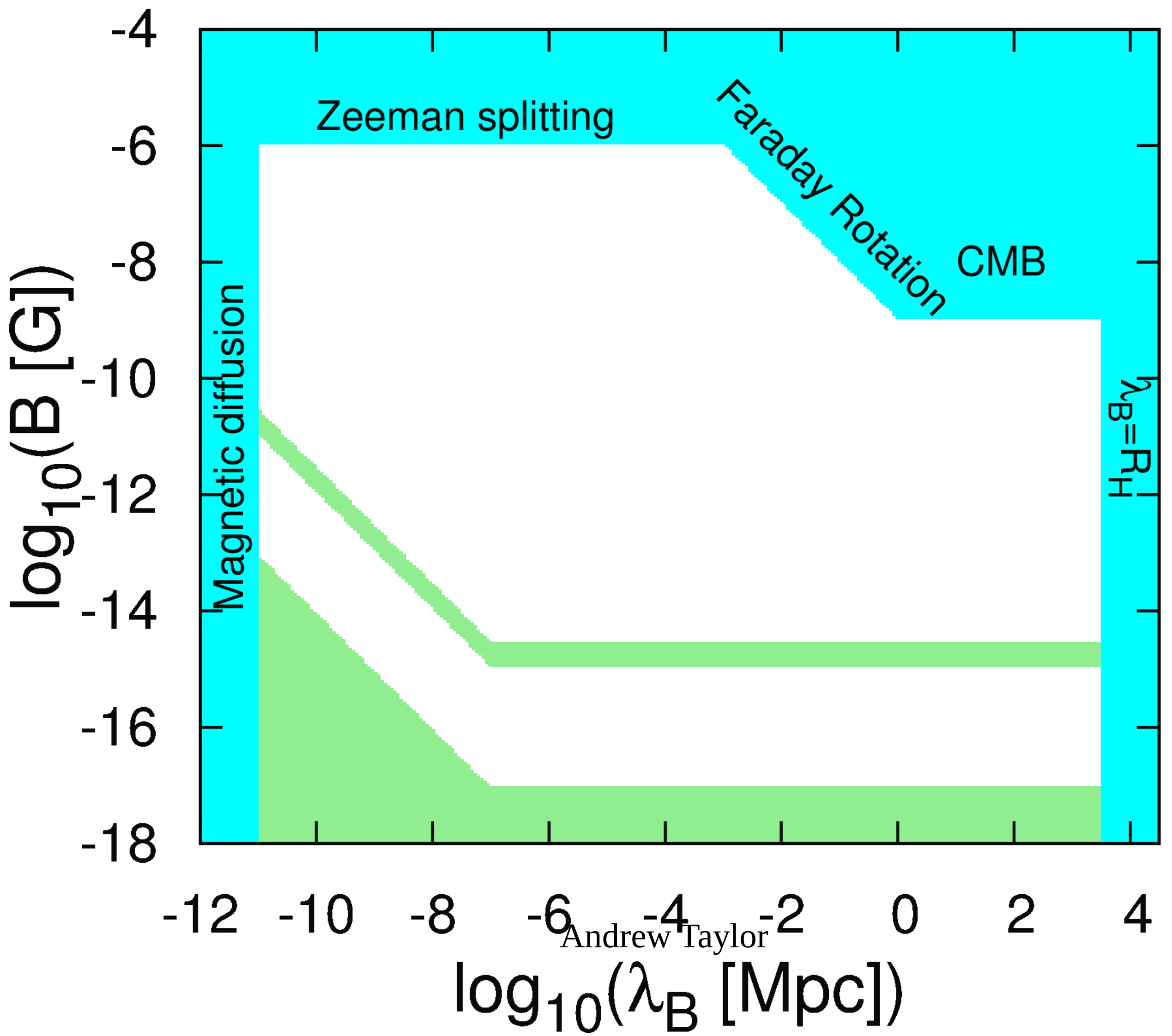
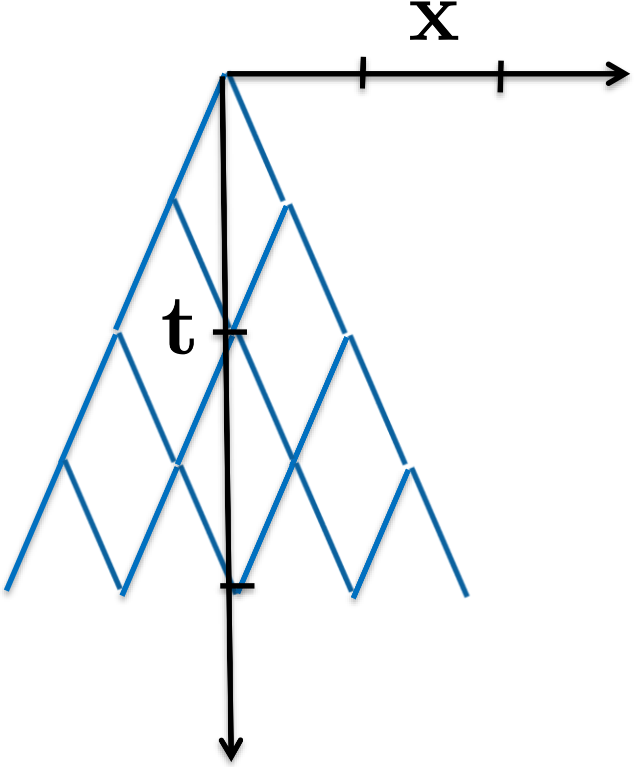
Extragalactic Magnetic Field is Hugely Uncertain



Extra Slides

Random Walks



$$\gamma(t+1) = t!$$

$$\gamma(t+1) = \int_0^\infty x^t e^{-x} dx$$

$$f(x,t) = \frac{\gamma(t+1)}{[\gamma([t-x]/2+1)\gamma([x+t]/2+1)](2^t)}$$

$$f(x,t) \approx \frac{e^{-x^2/(2t)}}{(2\pi t)^{1/2}}$$

Random Walks

Stirling's approximation

$$\gamma(\mathbf{x}+\mathbf{1}) \approx (\mathbf{2}\pi\mathbf{x})^{\mathbf{1/2}}(\mathbf{x}/\mathbf{e})^{\mathbf{x}}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) \approx \frac{\mathbf{2}^{-\mathbf{t}}}{(\mathbf{2}\pi)^{\mathbf{1/2}}} \frac{\mathbf{t}^{\mathbf{1/2}}\mathbf{t}^{\mathbf{t}}}{[(\mathbf{t}^{\mathbf{2}}-\mathbf{x}^{\mathbf{2}})/\mathbf{4}]^{\mathbf{t/2}}[(\mathbf{t}^{\mathbf{2}}-\mathbf{x}^{\mathbf{2}})/\mathbf{4}]^{\mathbf{1/2}}} \left(\frac{\mathbf{t}-\mathbf{x}}{\mathbf{t}+\mathbf{x}}\right)^{\mathbf{x/2}}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) \approx \frac{\mathbf{2}}{(\mathbf{2}\pi\mathbf{t})^{\mathbf{1/2}}} \left[\mathbf{1}-\frac{\mathbf{x}^{\mathbf{2}}}{\mathbf{t}^{\mathbf{2}}}\right]^{-\mathbf{t/2}} \left[\mathbf{1}-\frac{\mathbf{x}^{\mathbf{2}}}{\mathbf{t}^{\mathbf{2}}}\right]^{-\mathbf{1/2}} \left(\frac{\mathbf{1}+\mathbf{x}/\mathbf{t}}{\mathbf{1}-\mathbf{x}/\mathbf{t}}\right)^{-\mathbf{x/2}}$$

Random Walks

Consider log of this expression

$$\log\left[1-\frac{\mathbf{x}^2}{\mathbf{t}^2}\right]^{-\mathbf{t}/2} \approx \frac{\mathbf{x}^2}{2\mathbf{t}}$$

$$\log\left[1-\frac{\mathbf{x}^2}{\mathbf{t}^2}\right]^{-1/2} \approx \frac{\mathbf{x}^2}{2\mathbf{t}^2}$$

$$\log\left(\frac{1+\mathbf{x}/\mathbf{t}}{1-\mathbf{x}/\mathbf{t}}\right)^{-\mathbf{x}/2} \approx \log\left(1+\frac{2\mathbf{x}}{\mathbf{t}}\right)^{-\mathbf{x}/2} \approx -\frac{\mathbf{x}^2}{\mathbf{t}}$$

Random Walks

Gathering, throwing away the second term, and re-exponentiating

$$\mathbf{f(x,t) \propto e^{-x^2/(2t)}}$$

$$\int_{-\infty}^{\infty} \mathbf{f(x,t)dx = 1} \quad \Longrightarrow \quad \mathbf{f(x,t) = \frac{e^{-x^2/2t}}{(2\pi t)^{1/2}}}$$

How would this calculation change for 2D and 3D random walks?

Random Walks

The distribution function shapes stay the same, only their normalization changes.

$$\mathbf{f(R,t) \propto e^{-R^2/2t}}$$

For 2D

$$\int_{-\infty}^{\infty} \mathbf{f(R,t)dA = 1} \quad \Rightarrow \quad \mathbf{f(R,t) = \frac{e^{-R^2/2t}}{2\pi t}}$$

Random Walks

The distribution function shapes stay the same, only their normalization changes.

$$\mathbf{f(r,t) \propto e^{-r^2/2t}}$$

For 3D

$$\int_{-\infty}^{\infty} \mathbf{f(r,t)dV = 1} \quad \Rightarrow \quad \mathbf{f(r,t) = \frac{e^{-r^2/2t}}{(2\pi t)^{3/2}}}$$

Saturation of Steady-State Integral

3D system-

$$\mathbf{f(r,t)} = \frac{\mathbf{e^{-r^2/(4Dt)}}}{\mathbf{(4\pi Dt)^{3/2}}}$$

$$\mathbf{F(r)} = \int_0^\infty \mathbf{f(r,t)dt}$$

Change of variable-

$$\mathbf{x} = \frac{\mathbf{r^2}}{\mathbf{4Dt}}$$

$$\int_0^{\mathbf{t_H}} \mathbf{f(r,t)dt} = \frac{\mathbf{1}}{\mathbf{Dr}} \int_{\mathbf{r^2/Dt_H}}^\infty \mathbf{x^{-1/2}e^{-x}dx}$$

Saturation of Steady-State Integral

$$\gamma(t+1) = \int_0^\infty x^t e^{-x} dx$$

$$F(r) = \int_0^\infty f(r,t)dt$$

$$= \frac{1}{Dr} \int_{r^2/Dt_H}^\infty x^{-1/2} e^{-x} dx$$

$$= \frac{1}{Dr} [1.0 - \Gamma(1/2, r^2/Dt_H)]$$

Incomplete gamma function

Repeat this for 1D and 2D systems

Saturation of Steady-State Integral

2D system-

$$\gamma(t+1) = \int_0^\infty x^t e^{-x} dx$$

$$f(R,t) = \frac{e^{-R^2/(4Dt)}}{(4\pi Dt)}$$

$$F(R) = \frac{1}{DR}\int_{R^2/Dt_H}^\infty x^{-1} e^{-x} dx$$

$$= \frac{1}{DR}[1.0 - \Gamma(0, R^2/Dt_H)]$$

Saturation of Steady-State Integral

1D system-

$$\gamma(\mathbf{t}+\mathbf{1}) = \int_{\mathbf{0}}^{\infty} \mathbf{x}^{\mathbf{t}} \mathbf{e}^{-\mathbf{x}} \mathbf{dx}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) = \frac{\mathbf{e}^{-\mathbf{x}^2/(4\mathbf{Dt})}}{(4\pi\mathbf{Dt})^{1/2}}$$

$$\mathbf{F}(\mathbf{x}) = \frac{\mathbf{1}}{\mathbf{Dx}} \int_{\mathbf{x}^2/\mathbf{Dt}_{\mathrm{H}}}^{\infty} \mathbf{y}^{-3/2} \mathbf{e}^{-\mathbf{y}} \mathbf{dy}$$

$$= \frac{\mathbf{1}}{\mathbf{Dx}} [\mathbf{1.0} - \Gamma(-\mathbf{1}/\mathbf{2}, \mathbf{x}^2/\mathbf{Dt}_{\mathrm{H}})]$$

Extragalactic Deflections

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Random Walks

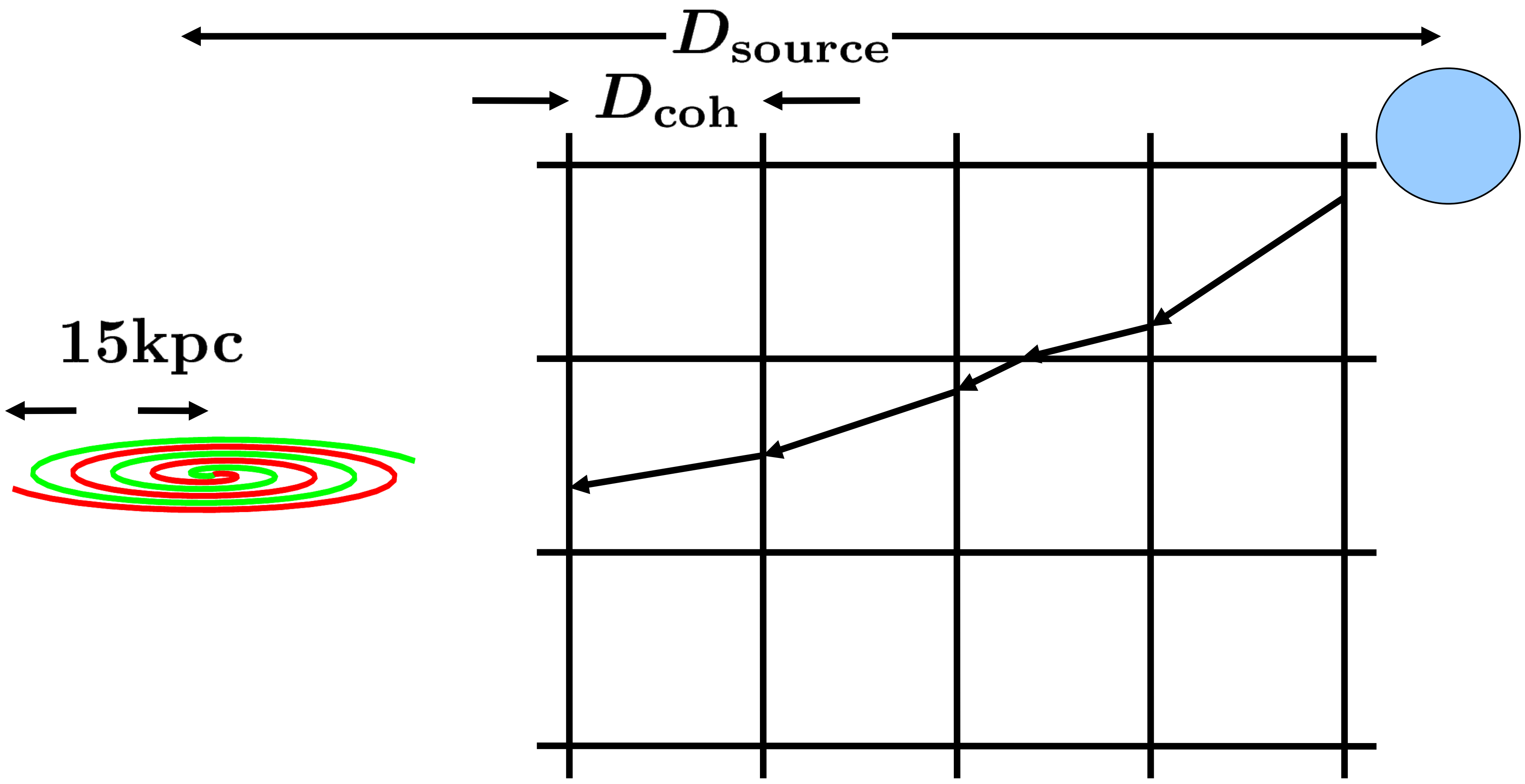
Consider log of this expression

$$\log\left[\mathbf{1}-\frac{\mathbf{x^2}}{\mathbf{t^2}}\right]^{-\mathbf{t}} \approx \frac{\mathbf{x^2}}{\mathbf{t}}$$

$$\log\left[\mathbf{1}-\frac{\mathbf{x^2}}{\mathbf{t^2}}\right]^{-\mathbf{1/2}} \approx \frac{\mathbf{x^2}}{\mathbf{2t^2}}$$

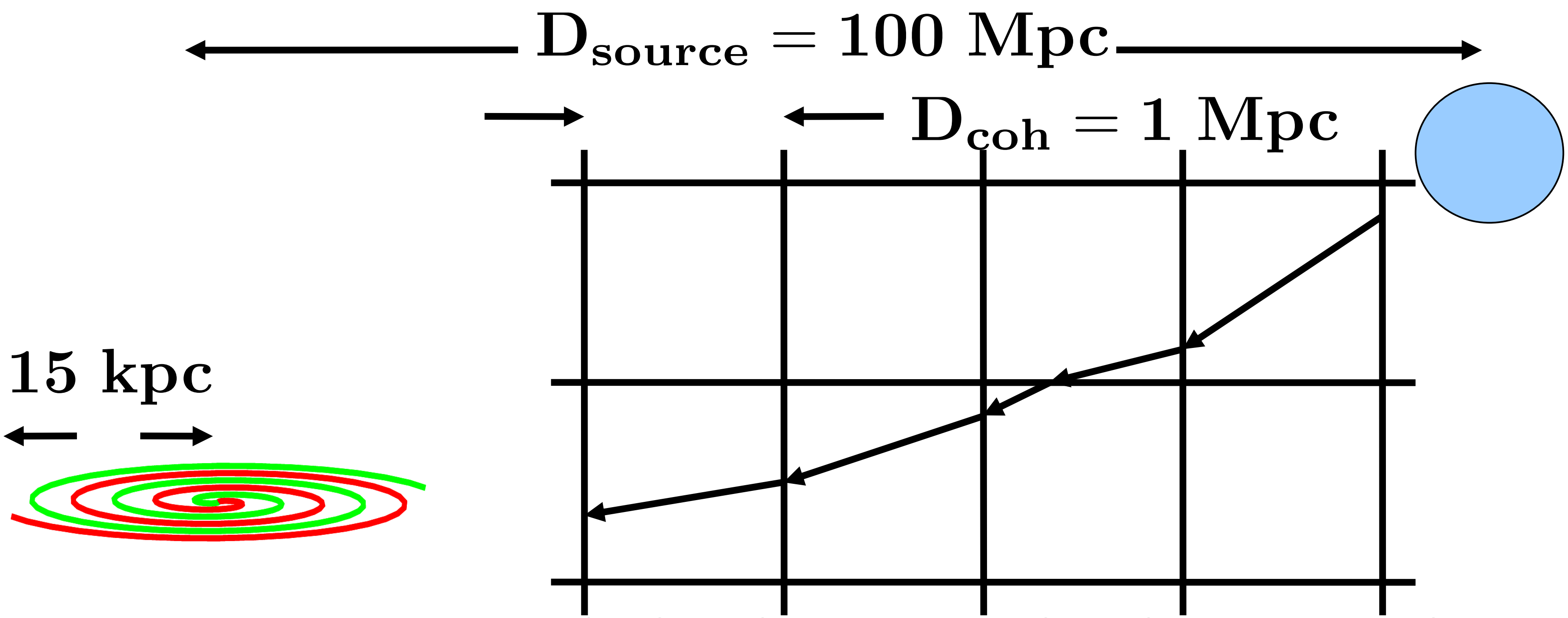
$$\log\left(\frac{\mathbf{1}+\mathbf{x/t}}{\mathbf{1}-\mathbf{x/t}}\right)^{-\mathbf{x}} \approx \log\left(\mathbf{1}+\frac{\mathbf{2x}}{\mathbf{t}}\right)^{-\mathbf{x/2}} \approx -\frac{\mathbf{2x^2}}{\mathbf{t}}$$

Those that Leave are Replaced by those that Arrive



$$N_{\rm cells} = \frac{D_{\rm source}}{D_{\rm coh}}, \; \Delta\theta_{\rm cell} = \frac{D_{\rm coh}}{R_{\rm Larmor}}, \quad \Delta\theta_{\rm tot} \approx N_{\rm cells}^{1/2}\Delta\theta_{\rm cell}$$

Those that Leave are Replaced by those that Arrive



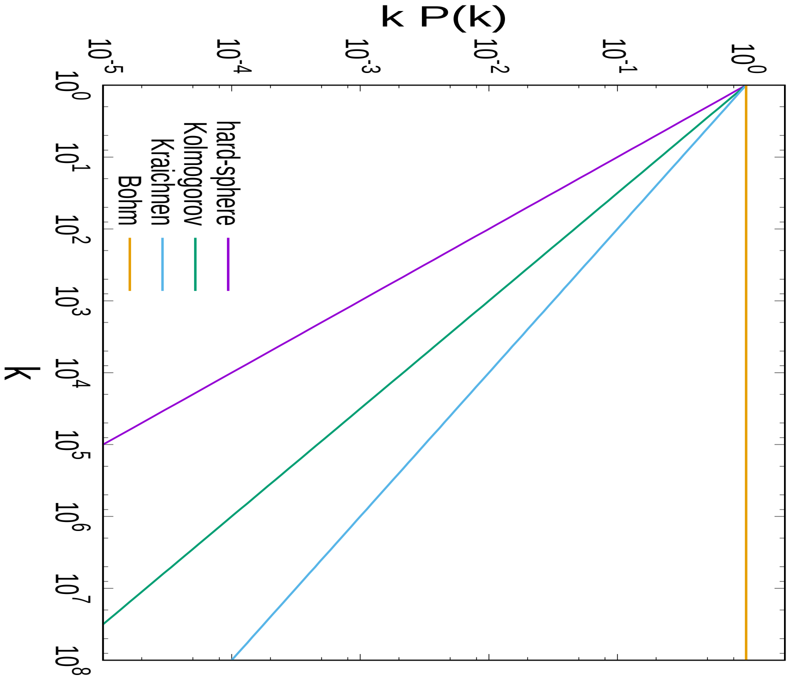
$$R_{Larmor} = 1\ \text{Gpc} \left(\frac{1}{Z}\right)\left(\frac{E}{10^{20}\ \text{eV}}\right)\left(\frac{0.01\ \text{nG}}{B}\right)$$

$$\Delta\theta_{tot} \approx N_{cells}^{1/2}\Delta\theta_{cell} \qquad [N_{cells} = 100]$$

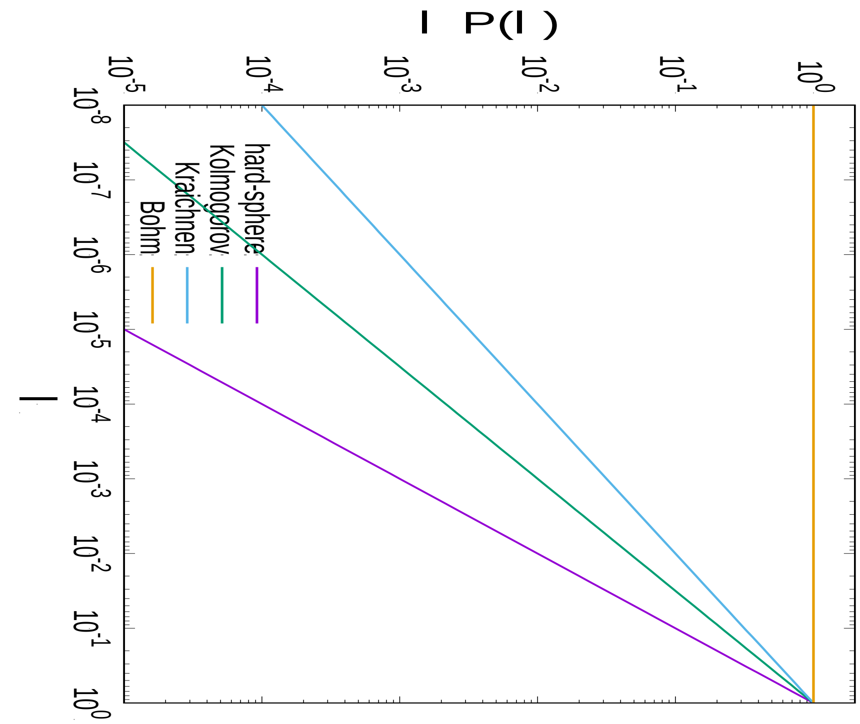
For 10^{20} eV protons: $\Delta\theta_{tot} = 0.01$ rad. (ie. 0.6°)

For 10^{20} eV iron: $\Delta\theta_{tot} = 0.26$ rad. (ie. 15°)

Supernovae as Drivers of Galactic Turbulence



$$\mathbf{P(k)} = \frac{\mathbf{dP}}{\mathbf{dk}} = \mathbf{P_0}\left(\frac{\mathbf{k}}{\mathbf{k_0}}\right)^{-\alpha}$$



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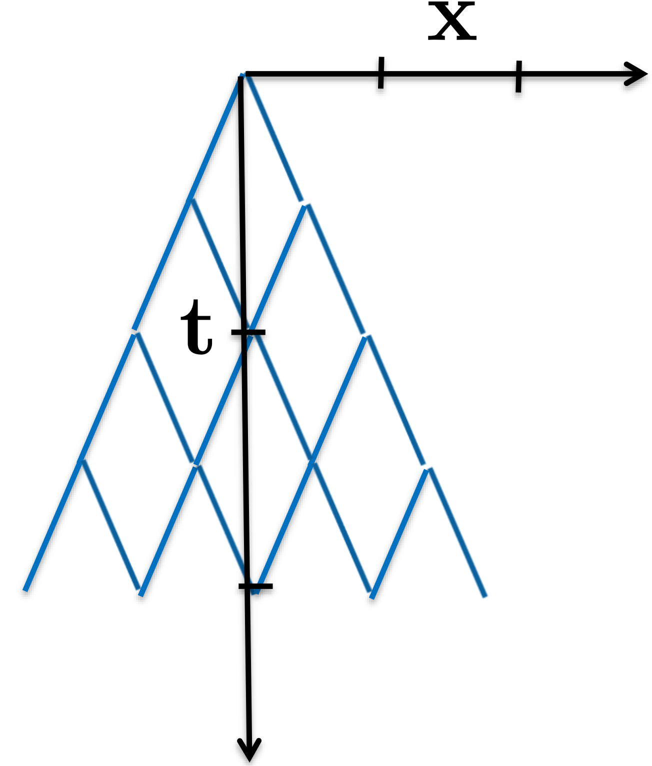
Random Walks



$$\gamma(\mathbf{t}+\mathbf{1}) = \mathbf{t}!$$

$$\gamma(\mathbf{t}+\mathbf{1}) = \int_{\mathbf{0}}^{\infty} \mathbf{x}^{\mathbf{t}}\mathbf{e}^{-\mathbf{x}}\mathbf{dx}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) = \frac{\gamma(\mathbf{2t}+\mathbf{1})}{[\gamma([\mathbf{t}-\mathbf{x}]+\mathbf{1})\gamma([\mathbf{x}+\mathbf{t}]+\mathbf{1})](\mathbf{2}^{\mathbf{2t}})}$$

Random Walks

Stirling's approximation

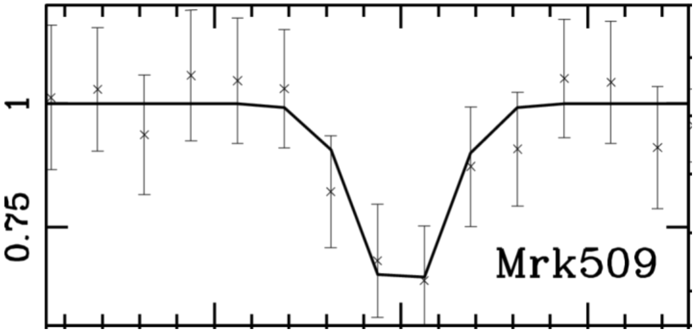
$$\gamma(\mathbf{x}+\mathbf{1}) \approx (\mathbf{2}\pi\mathbf{x})^{\mathbf{1/2}}(\mathbf{x/e})^{\mathbf{x}}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) = \frac{\gamma(\mathbf{2t}+\mathbf{1})}{[\gamma([\mathbf{t}-\mathbf{x}]+\mathbf{1})\gamma([\mathbf{x}+\mathbf{t}]+\mathbf{1})](\mathbf{2^{2t}})}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) \approx \frac{\mathbf{2^{-2t}}}{(\mathbf{2}\pi)^{\mathbf{1/2}}} \frac{(\mathbf{2t})^{\mathbf{1/2}}(\mathbf{2t})^{\mathbf{2t}}}{(\mathbf{t^2}-\mathbf{x^2})^{\mathbf{t}}(\mathbf{t^2}-\mathbf{x^2})^{\mathbf{1/2}}}\left(\frac{\mathbf{t}-\mathbf{x}}{\mathbf{t}+\mathbf{x}}\right)^{\mathbf{x}}$$

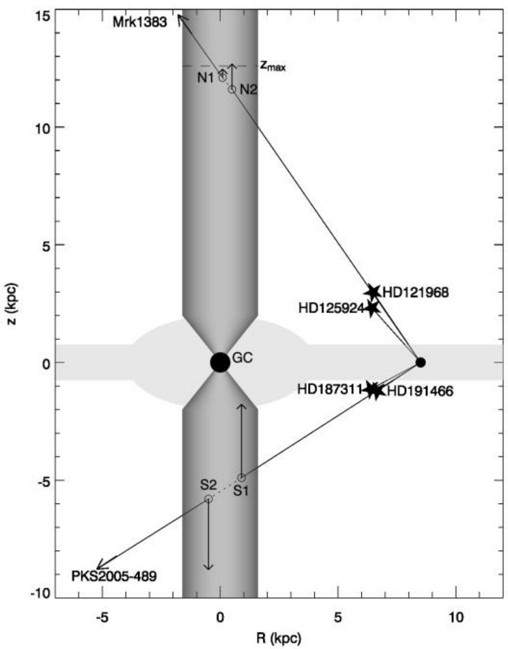
$$\mathbf{f}(\mathbf{x},\mathbf{t}) \approx \frac{\mathbf{2}}{(\mathbf{2}\pi\mathbf{t})^{\mathbf{1/2}}}\left[\mathbf{1}-\frac{\mathbf{x^2}}{\mathbf{t^2}}\right]^{-\mathbf{t}}\left[\mathbf{1}-\frac{\mathbf{x^2}}{\mathbf{t^2}}\right]^{-\mathbf{1/2}}\left(\frac{\mathbf{1}+\mathbf{x/t}}{\mathbf{1}-\mathbf{x/t}}\right)^{-\mathbf{x}}$$

Advection With the Bubbles?



Suzaku and Chandra X-ray observations of bright AGN (Mkr 501, PKS 2155, NGC 3783) indicated the presence of a hot local absorber surrounding the Milky Way

The Doppler shift of absorption lines for lines-of-sight which pass through these bubbles reveals evidence of a coherent velocity flow



Keeney et al. 2006