Lecture (2) Plan:

- **Extragalactic radiation fields**
- **Cosmic ray proton interaction rates with extragalactic radiation fields**
- **Cosmic ray nuclei interaction rates with extragalactic radiation fields**
- **High Energy electron and photon interaction rates with radiation fields**
- **Cosmic ray composition and gamma-rays as a probe of the source distribution**

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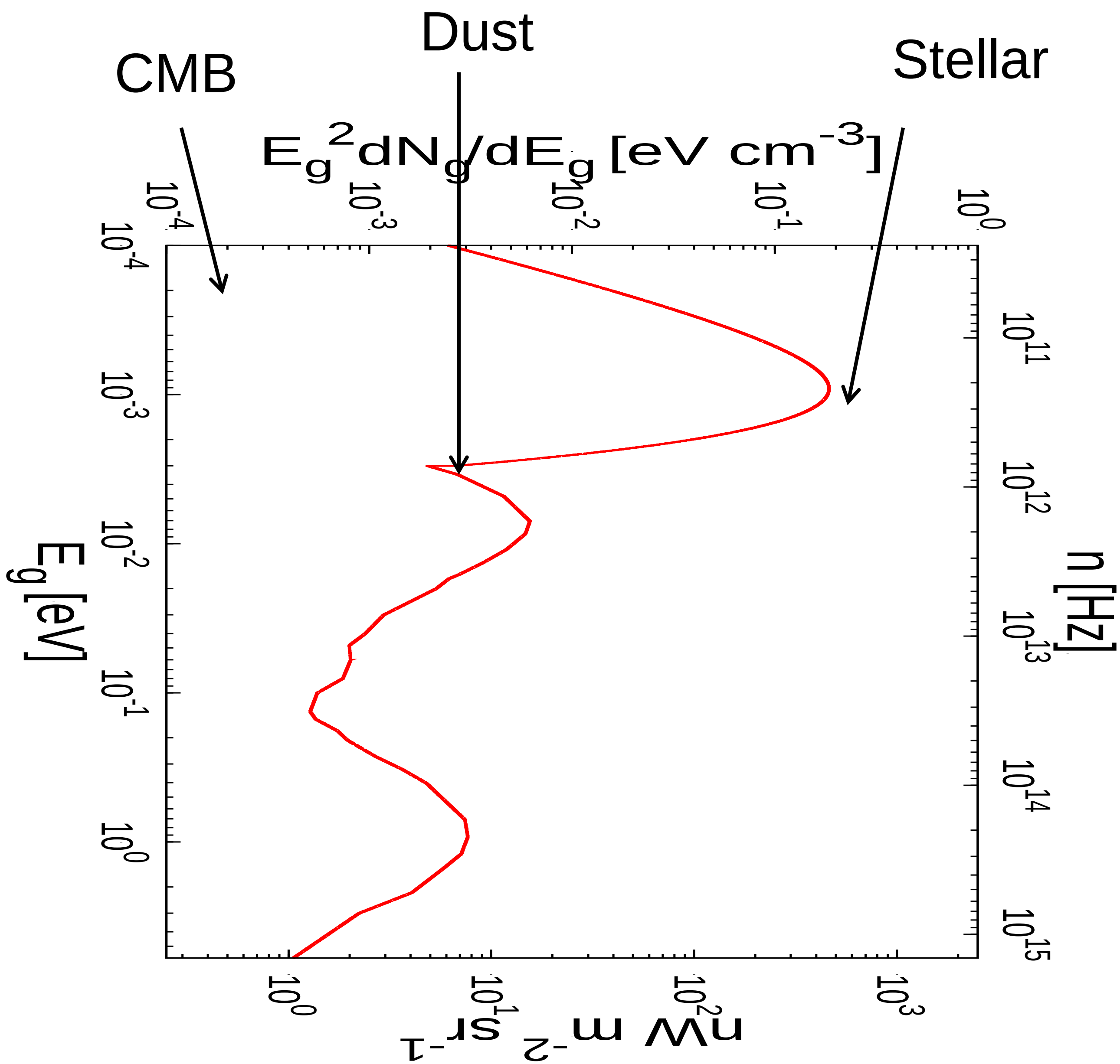
Cosmic Radiation Fields- Energy Density

$$\mathbf{U}_\gamma = \int_{\mathbf{0}}^{\infty} \mathbf{E}_\gamma \frac{\mathbf{dn}}{\mathbf{dE}_\gamma} \mathbf{dE}_\gamma$$

$$= \int_{\mathbf{0}}^{\infty} \mathbf{E}_\gamma^{\mathbf{2}} \frac{\mathbf{dn}}{\mathbf{dE}_\gamma} \mathbf{dlnE}_\gamma$$

Note- this amounts to a visual inspection version of Laplace’s integral method

Cosmic Radiation Fields- Energy Density



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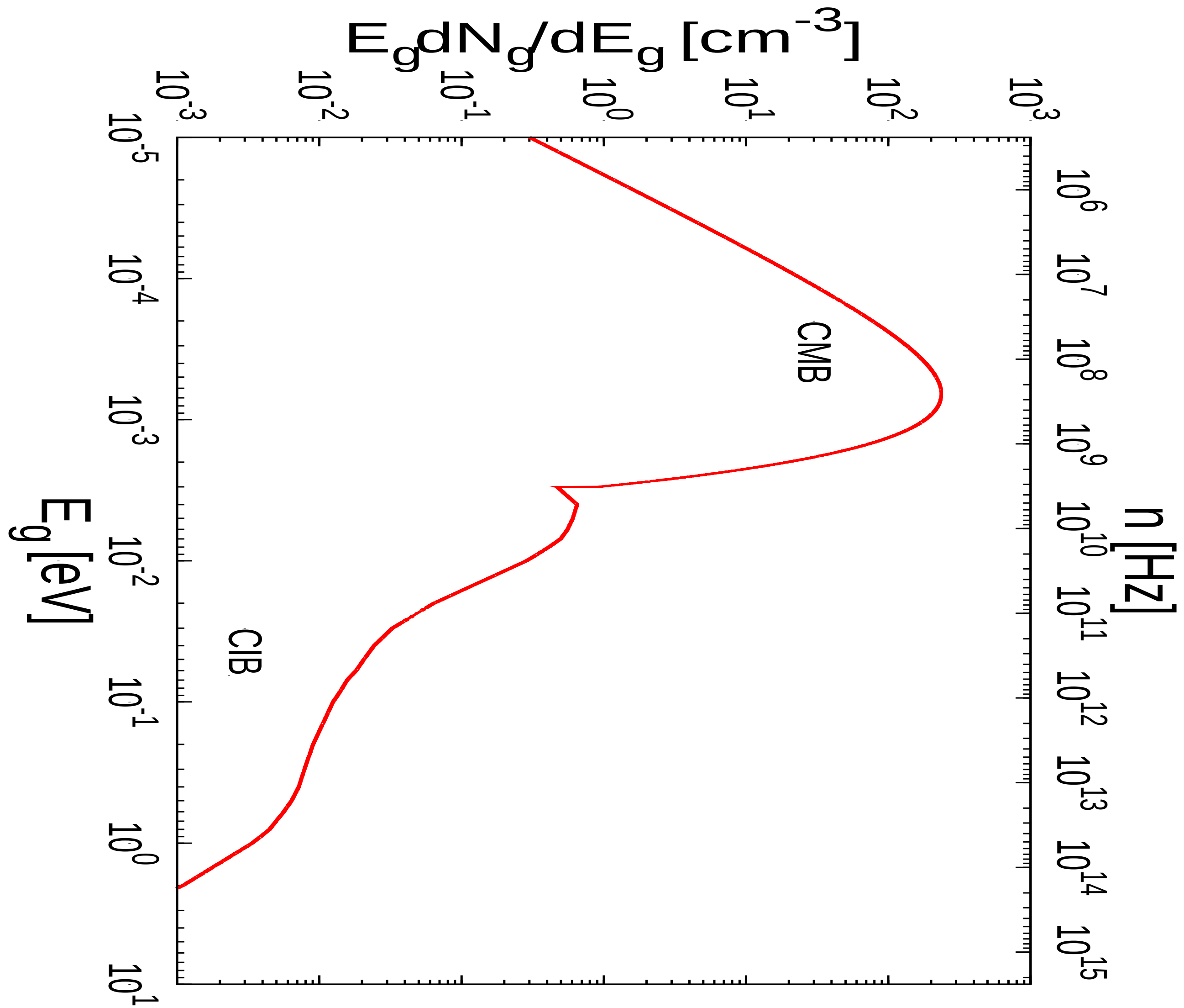
Cosmic Radiation Fields- Number Density

$$\mathbf{n}_\gamma = \int_{\mathbf{0}}^{\infty} \frac{\mathbf{dn}}{\mathbf{dE}_\gamma} \mathbf{dE}_\gamma$$

$$= \int_{\mathbf{0}}^{\infty} \mathbf{E}_\gamma \frac{\mathbf{dn}}{\mathbf{dE}_\gamma} \mathbf{dlnE}_\gamma$$

Cosmic Radiation Fields- Number Density



Cosmic Ray Proton Energy Losses

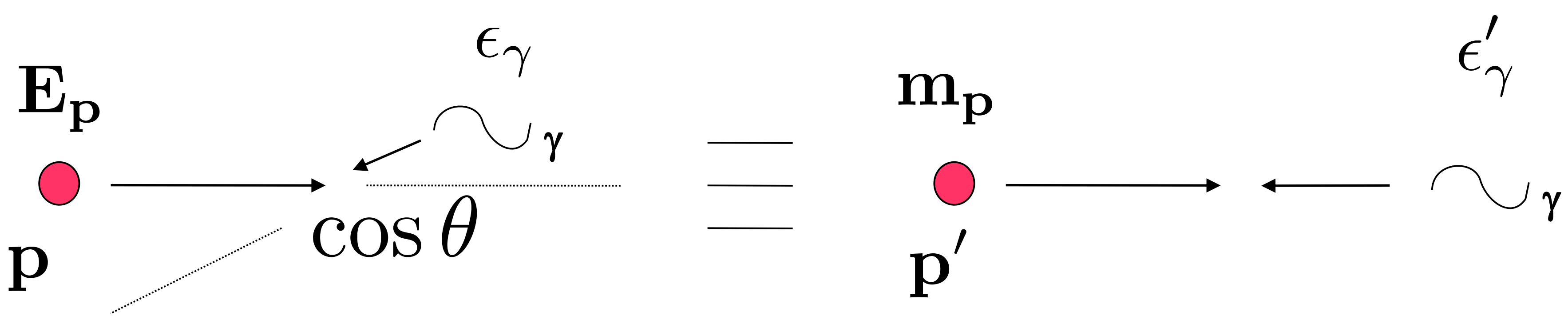
The Interaction Rate

radiation field

cross-section

$$\mathbf{R} = \int_{\mathbf{0}}^{\infty} \mathbf{d}\epsilon_\gamma \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_{-\mathbf{1}}^{\mathbf{1}} \frac{\mathbf{1}}{\mathbf{2}} \mathbf{d}(\cos\theta)\sigma(\cos\theta)\mathbf{1} + \beta\cos\theta)$$

All values above in lab frame



The Interaction Rate

$$\mathbf{R} = \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_{-1}^{1} \frac{1}{2} \mathbf{d}(\cos\theta)\sigma(\cos\theta)(\mathbf{1} + \beta\cos\theta)$$

Since, $\epsilon'_\gamma \mathbf{m_p} = \epsilon_\gamma \mathbf{E_p}(\mathbf{1} + \beta\cos\theta)$

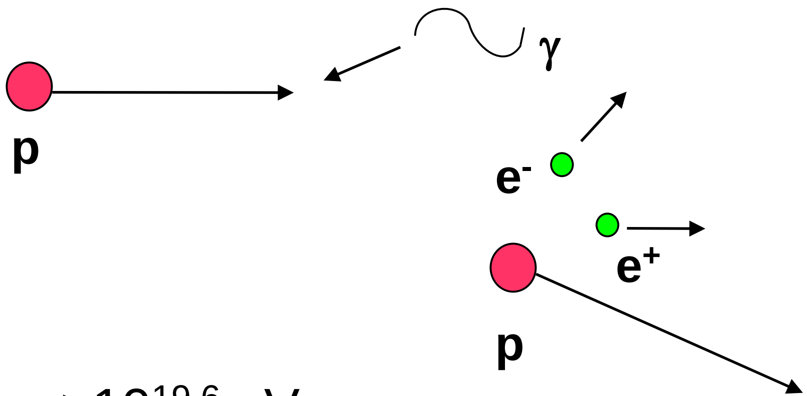
$$(\mathbf{1} + \beta\cos\theta)\mathbf{d}\cos\theta = \frac{\epsilon'_\gamma \mathbf{m_p}}{\epsilon_\gamma \mathbf{E_p}} \frac{\mathbf{d}(\epsilon'_\gamma \mathbf{m_p})}{\epsilon_\gamma \mathbf{E_p}}$$

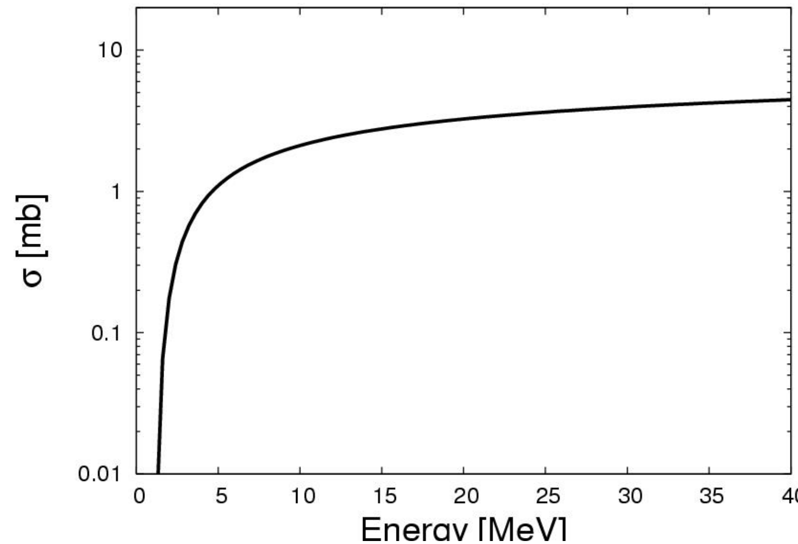
$$\mathbf{R} = \frac{1}{2} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{2\epsilon_\gamma \mathbf{E_p}} \mathbf{d}(\epsilon'_\gamma \mathbf{m_p}) \frac{\epsilon'_\gamma \mathbf{m_p}}{\epsilon_\gamma^2 \mathbf{E_p^2}} \sigma(\epsilon')$$

$$= \frac{\mathbf{m_p^2}}{2\mathbf{E_p^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{1}{\epsilon_\gamma^2} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{2\epsilon_\gamma \frac{\mathbf{E_p}}{\mathbf{m_p}}} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma(\epsilon'_\gamma)$$

Cosmic Ray Proton Interactions

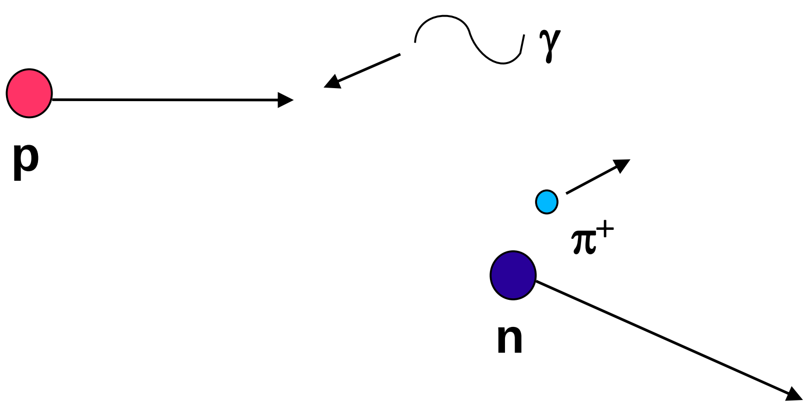
For $E_{proton} < 10^{19.6}$ eV

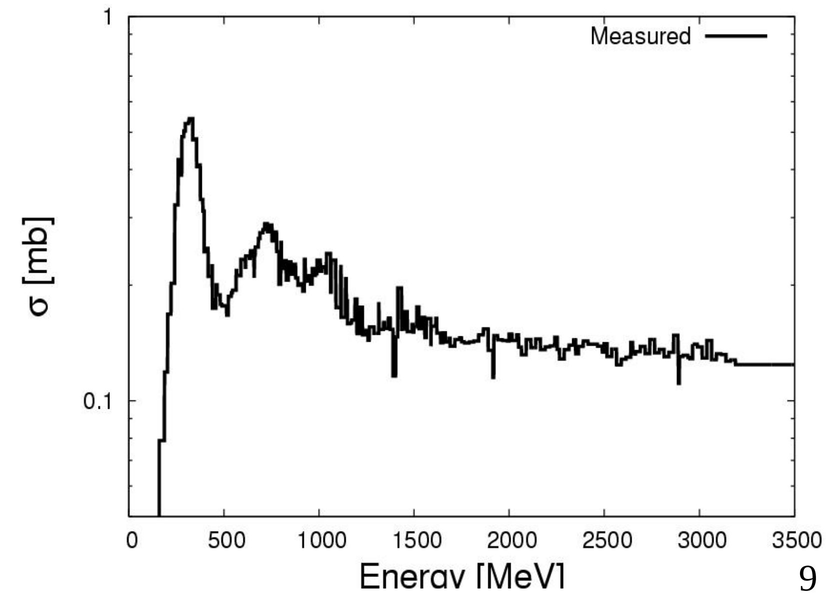




$E_{\gamma}^{th} \sim 1 \text{MeV}$

For $E_{proton} > 10^{19.6}$ eV





$E_{\gamma}^{th} \sim 140 \text{ MeV}$

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Threshold Energy- Proton Pair Production

$$(\mathbf{E_p} + \mathbf{E}_\gamma)^2 - (\mathbf{p_p} - \mathbf{E}_\gamma)^2 = (\mathbf{m_p} + 2\mathbf{m_e})^2$$

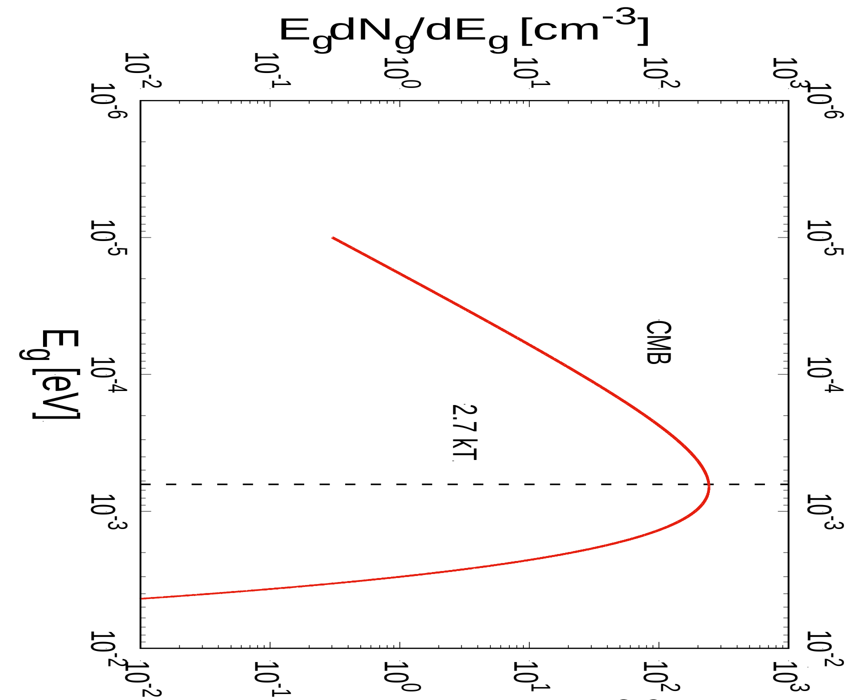
$$\mathbf{m_p^2} + 2\mathbf{E_p}\mathbf{E}_\gamma + 2\mathbf{p_p}\mathbf{E}_\gamma \approx \mathbf{m_p^2} + 4\mathbf{m_p}\mathbf{m_e}$$

$$\boxed{\mathbf{E_p} \approx \frac{\mathbf{m_e}}{\mathbf{E}_\gamma}\mathbf{m_p}} \approx \left(\frac{0.5 \times 10^6}{6 \times 10^{-4}}\right) 0.9 \times 10^9 = 8 \times 10^{17}\ \mathrm{eV}$$

Repeat this calculation for pion production

CMB- Total Number Density

$$\frac{dn}{d\epsilon_\gamma} = \frac{8\pi}{(hc)^3} \frac{\epsilon_\gamma^2}{e^{\epsilon_\gamma/kT} - 1}$$



$$n_\gamma^{BB} = \frac{8\pi(kT)^3}{(hc)^3} \int_0^\infty \frac{x^2}{e^x - 1} dx$$

$$\frac{8\pi(kT_{CMB})^3}{(hc)^3} \approx 170 \text{ cm}^{-3}$$

$$\zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}$$

$$n_\gamma^{CMB} = 8\pi \frac{(kT_{CMB})^3}{(hc)^3} \gamma(3)\zeta(3) \approx 400 \text{ cm}^{-3}$$

CMB- Total Number Density

For a blackbody radiation field distribution, with temperature T,

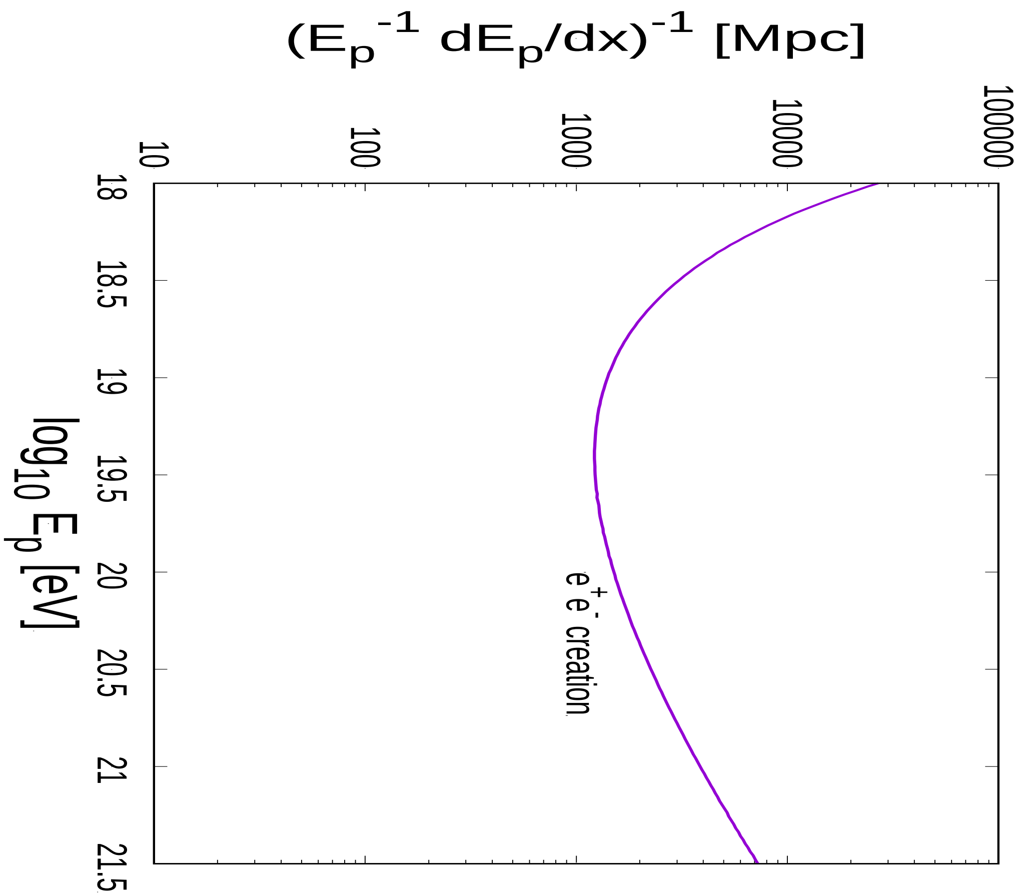
$$\mathbf{n_\gamma = 8\pi\frac{(kT)^3}{(hc)^3}\gamma(3)\zeta(3) \approx 400\ cm^{-3}}$$

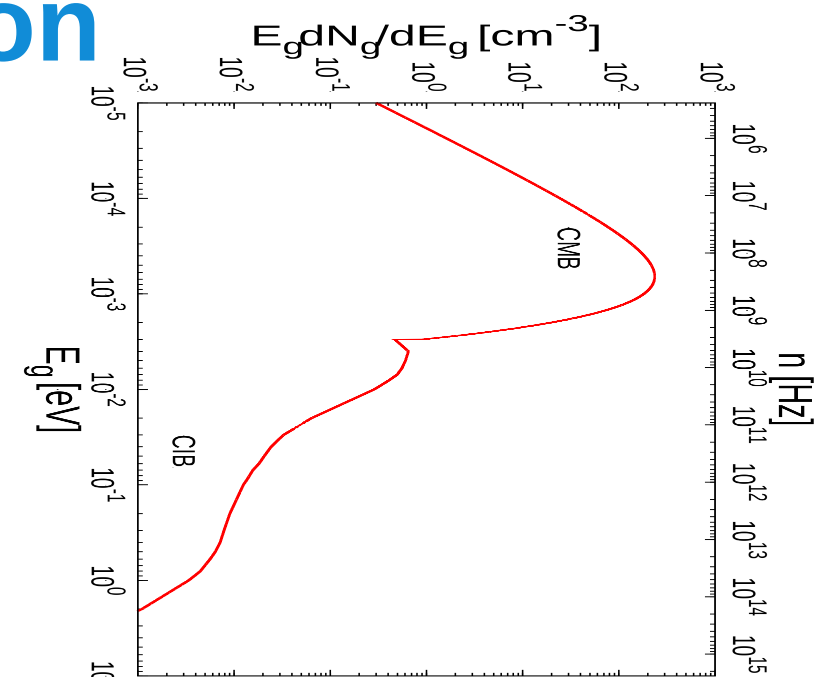
$$\mathbf{U_\gamma = 8\pi\frac{(kT)^4}{(hc)^3}\gamma(4)\zeta(4) = 0.25\ eV\ cm^{-3}}$$

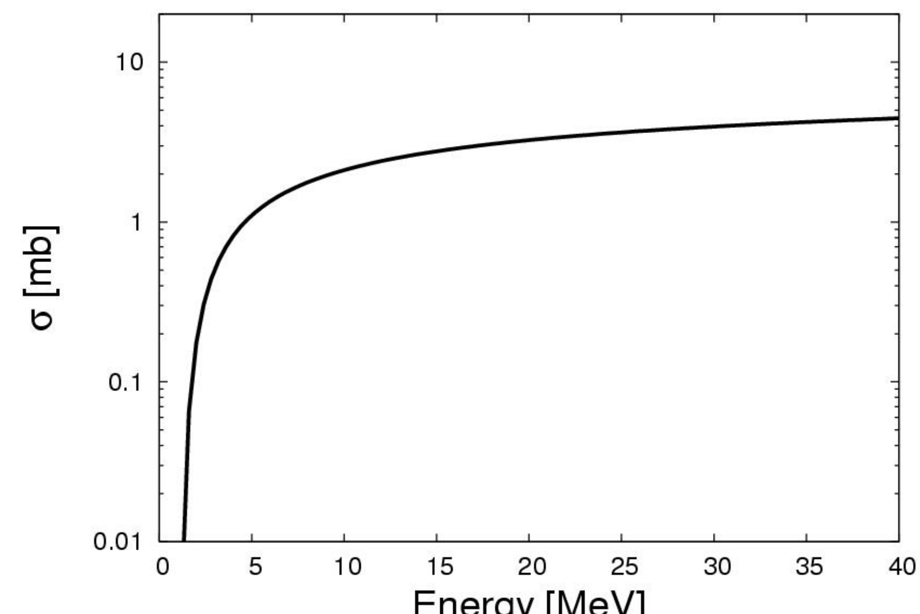
Have a go at demonstrating this to yourself!

Energy Loss Rate- Pair Production

$$R = \frac{m_p^2 c^4}{2E^2} \int_0^\infty d\epsilon_\gamma \frac{1}{\epsilon_\gamma^2} \frac{dn}{d\epsilon_\gamma} \int_0^{2E\epsilon_\gamma/(m_p c^2)} d\epsilon'_\gamma \epsilon'_\gamma \sigma_{p\gamma}(\epsilon'_\gamma) K_p$$







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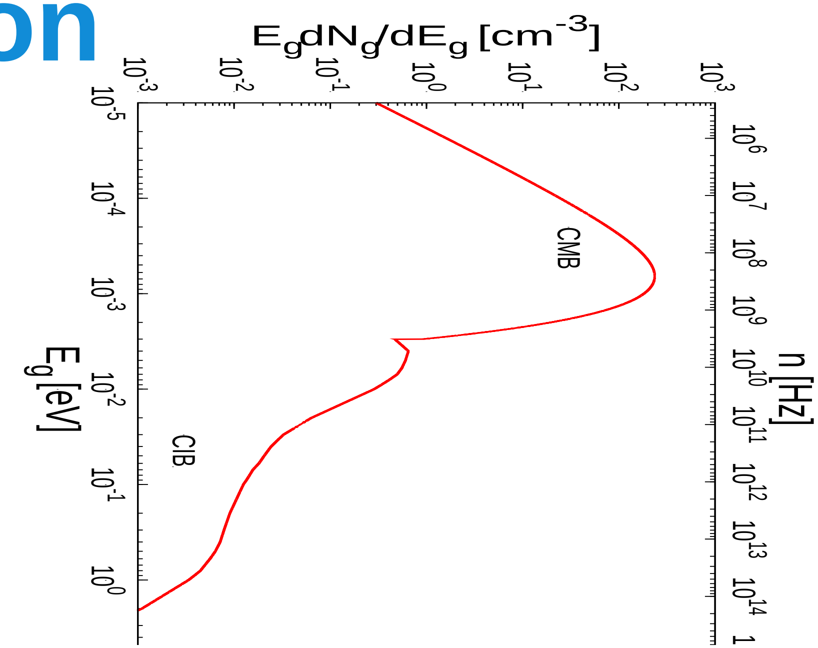
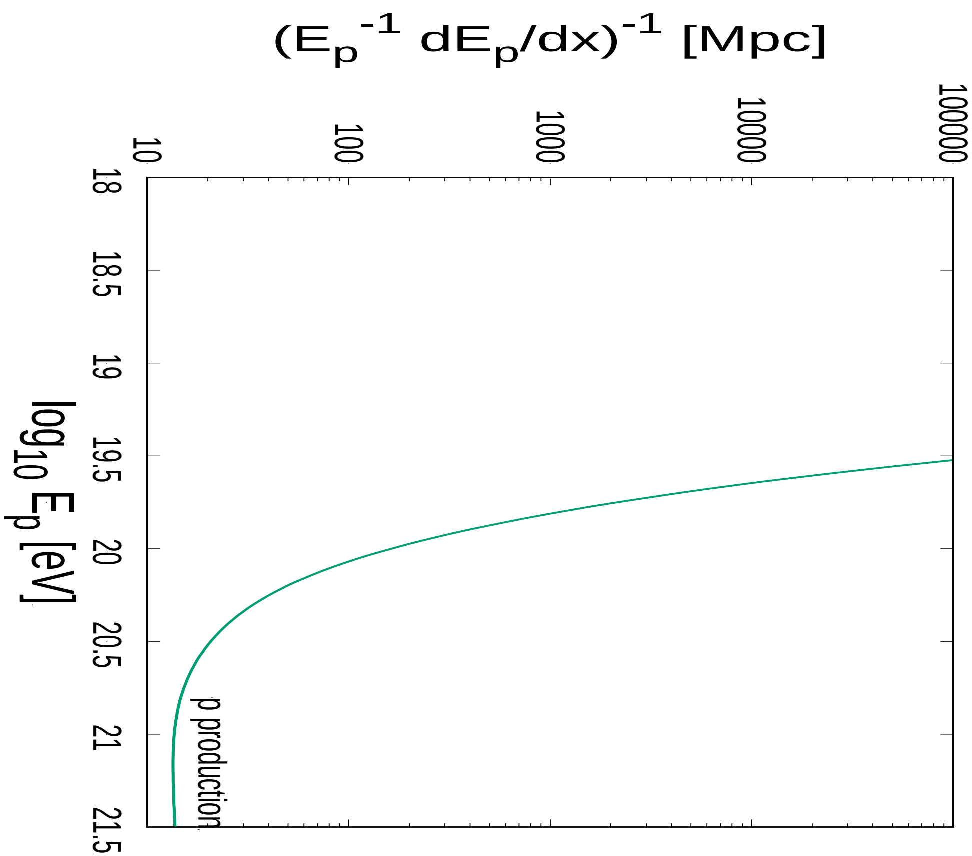
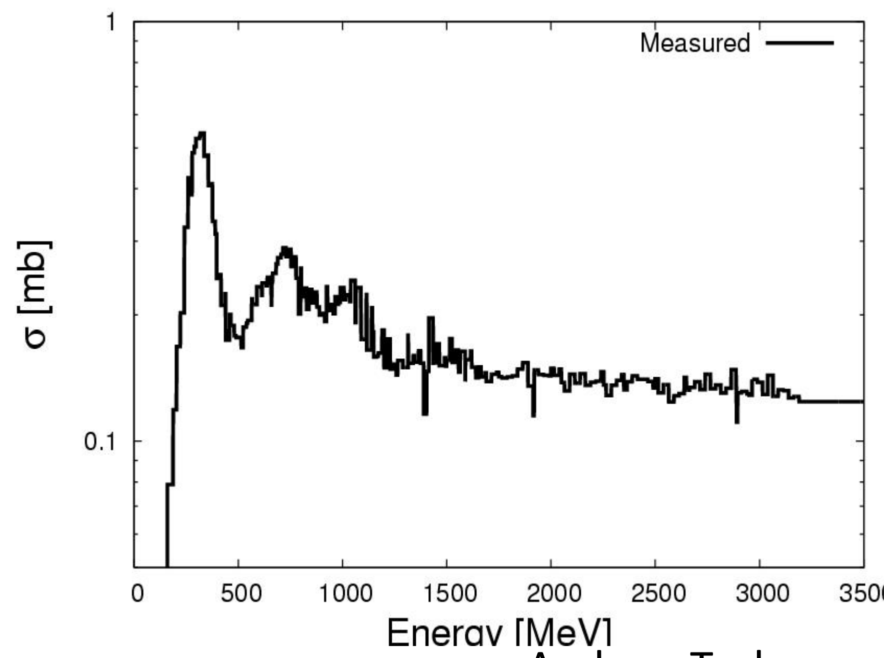
Energy Loss Rate- Pion Production

$$R = \frac{m_p^2 c^4}{2E^2}\int_0^\infty d\epsilon_\gamma \frac{1}{\epsilon_\gamma^2}\frac{dn}{d\epsilon_\gamma}\int_0^{2E\epsilon_\gamma/(m_p c^2)} d\epsilon'_\gamma\, \epsilon'_\gamma\, \sigma_{p\gamma}(\epsilon'_\gamma) K_p$$







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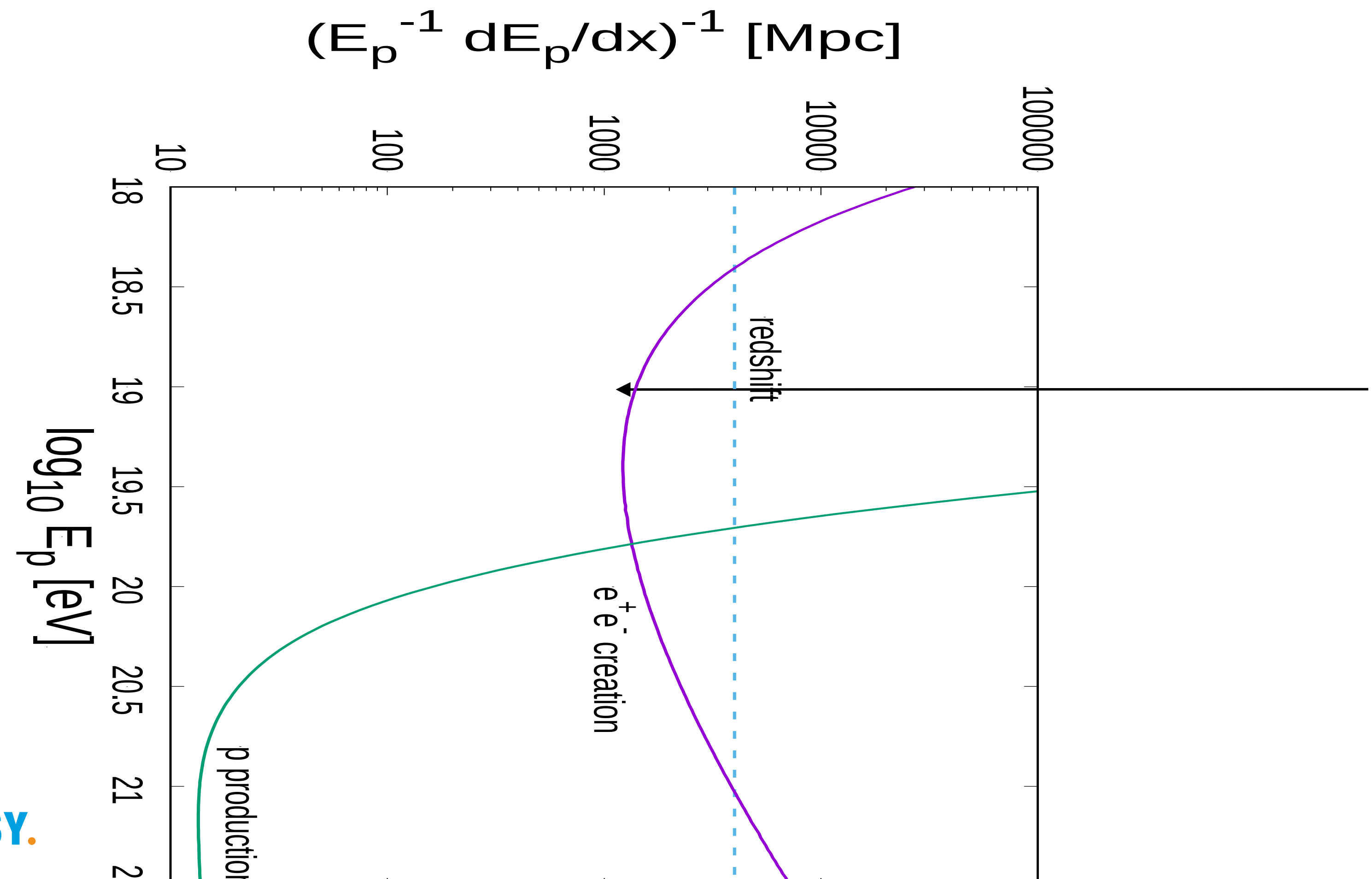
Energy Loss Rates due to Proton Interactions

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^2} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

where R is the energy loss rate

where K_p is the inelasticity



$$\approx \frac{\mathbf{m_p}}{\mathbf{m_e}} \frac{\mathbf{1}}{\mathbf{n_{CMB}}\sigma_{\mathbf{p}\gamma}}$$

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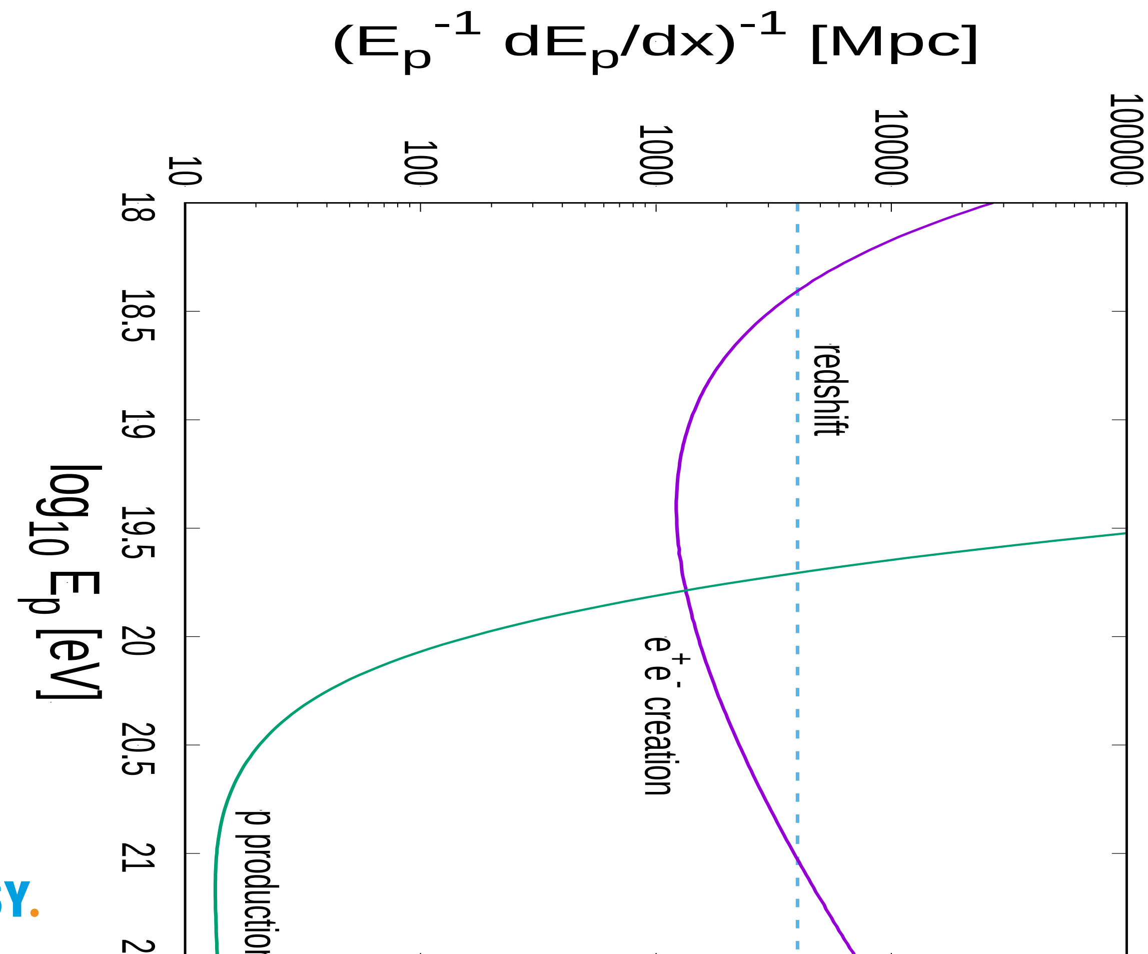
Energy Loss Rates due to Proton Interactions

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^2} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

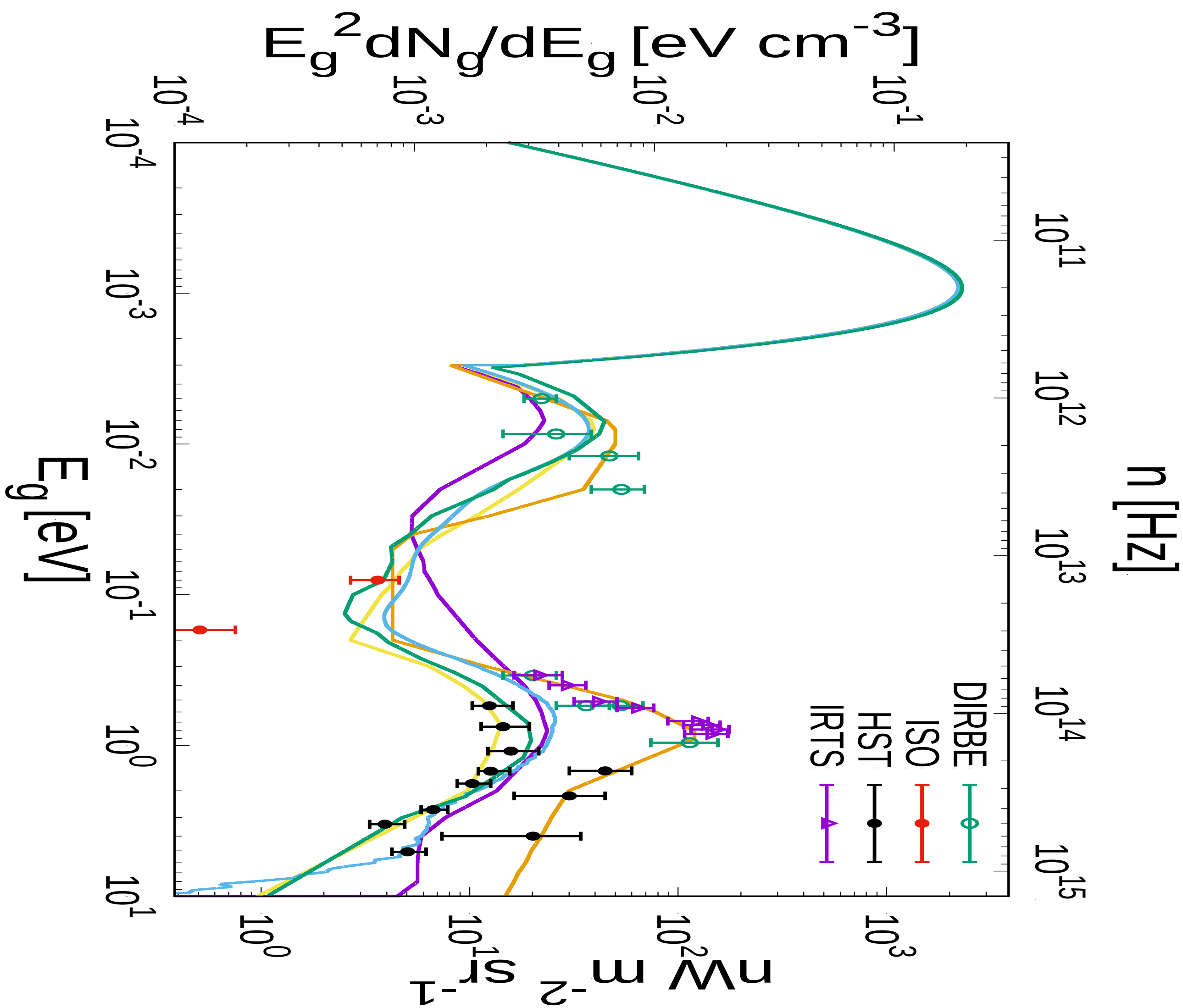
where R is the energy loss rate

where K_p is the inelasticity

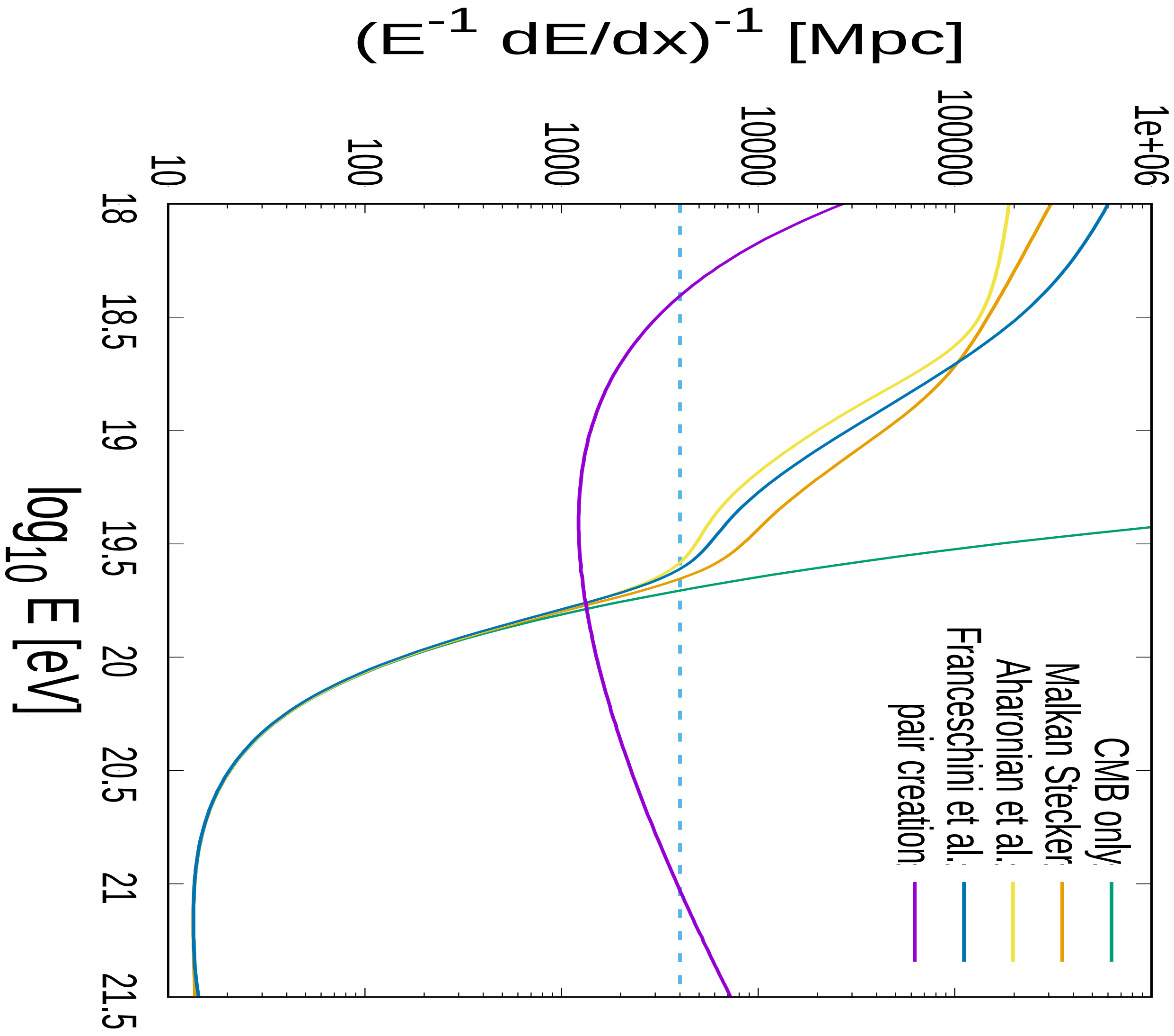


$$\longleftarrow \approx \frac{\mathbf{m_p}}{\mathbf{m_\pi}} \frac{\mathbf{1}}{\mathbf{n_{CMB}}\sigma_{\mathbf{p}\gamma}}$$

EBL Radiation Field Models



....with Different IR Backgrounds



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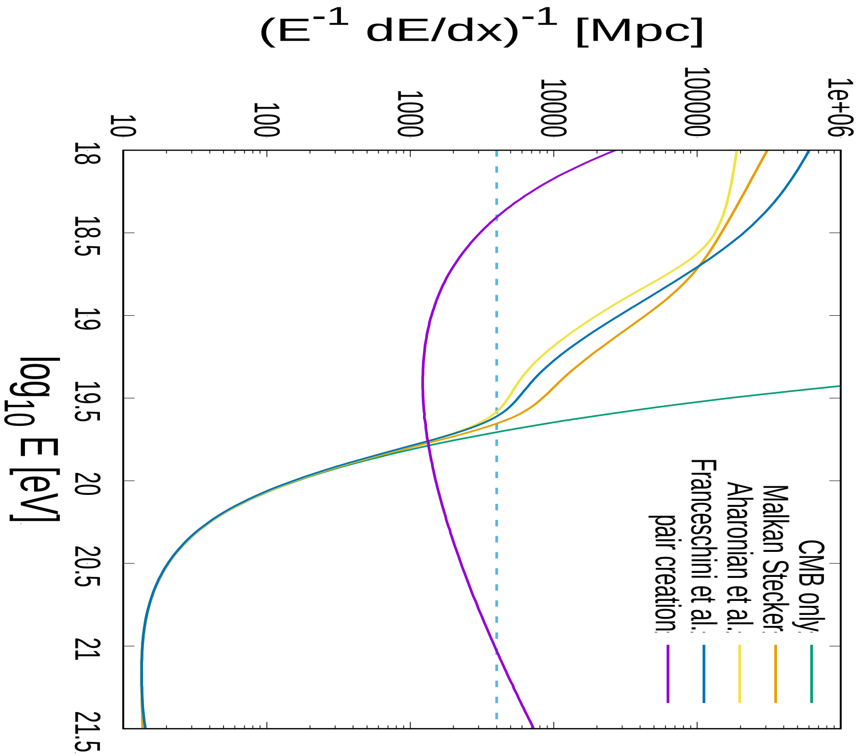
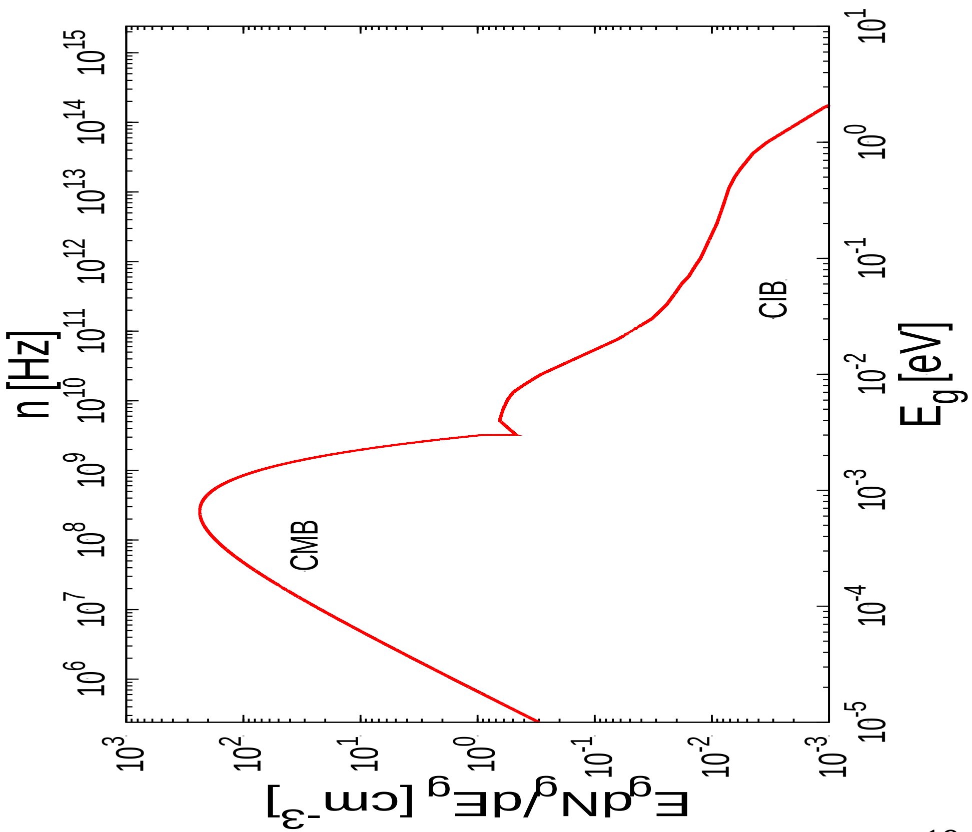
....with Different IR Backgrounds





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Cosmic Ray Nuclei Energy Losses

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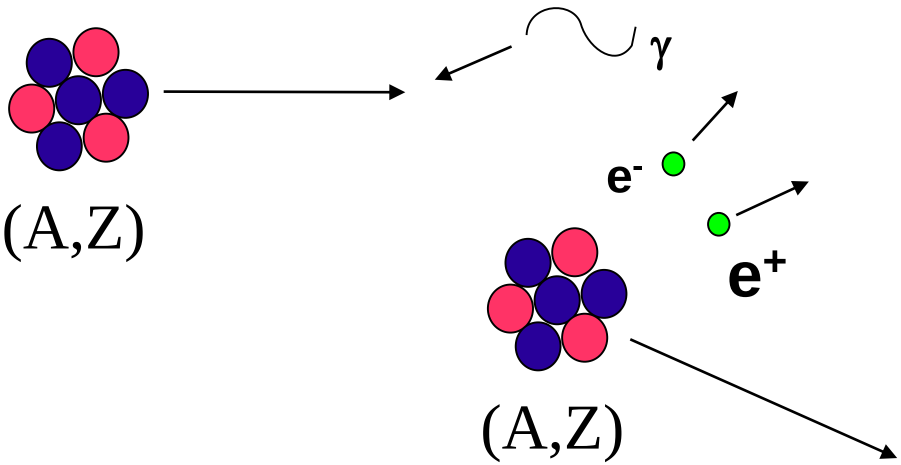
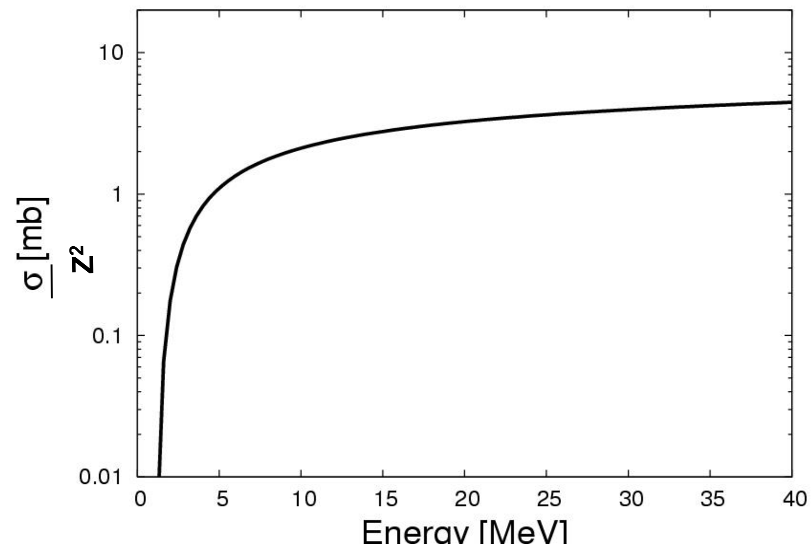
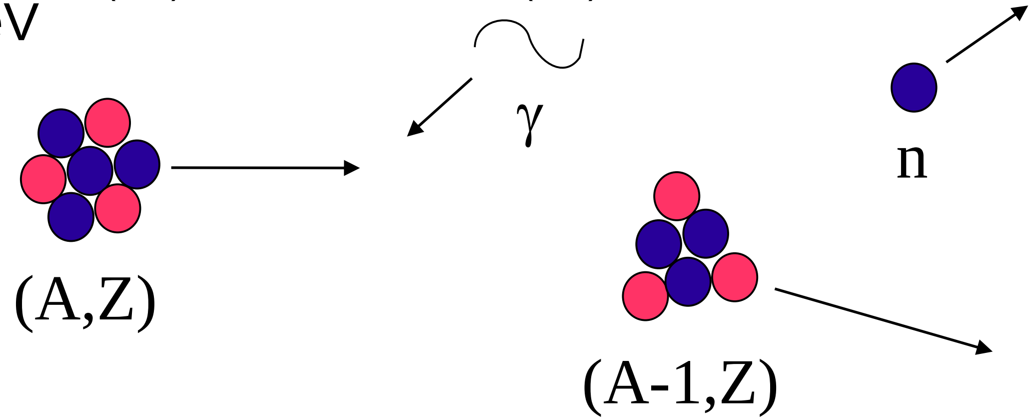
Cosmic Ray Nuclei Interactions

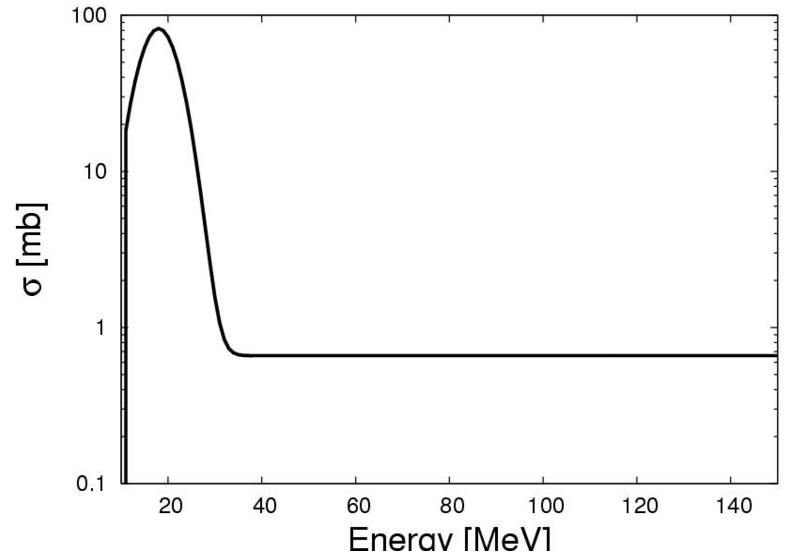
For $10^{19.7} < E_{(A,Z)} < 10^{20.2}$ eV





For $E_{(A,Z)} < 10^{19.7}$ and $E_{(A,Z)} < 10^{20.2}$ eV





Cosmic Ray Nuclei Interactions

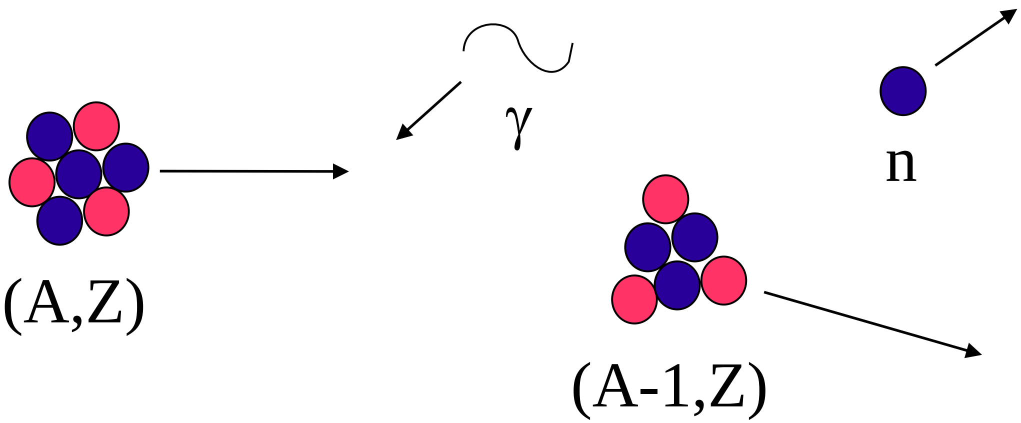


Photo-disintegration-

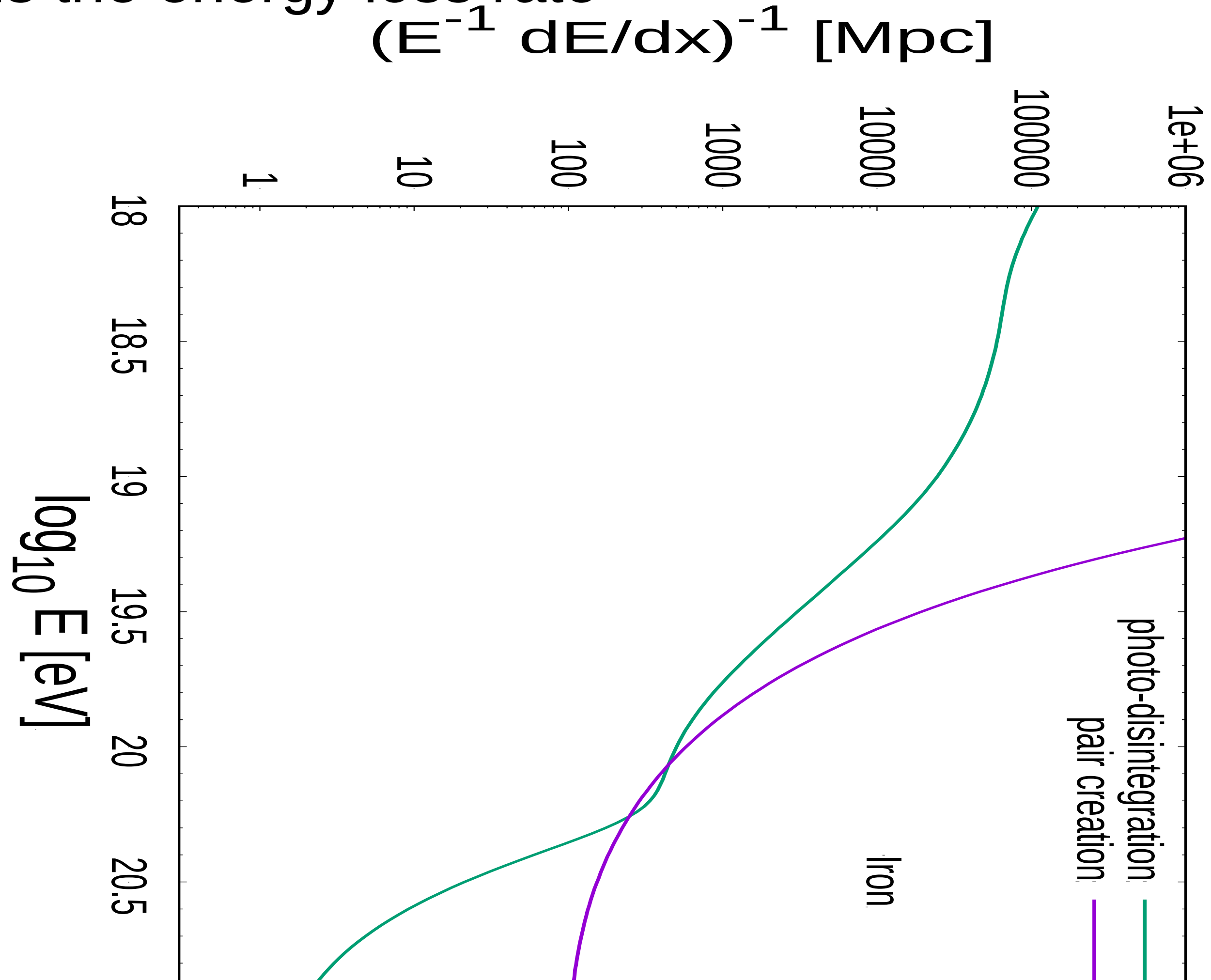
$$N_{(A,Z)} + \gamma \longrightarrow N'_{(A',Z')} + (Z-Z')p + (A-A'+Z'-Z)n, \quad E_\gamma \sim 30\mathrm{MeV}$$

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

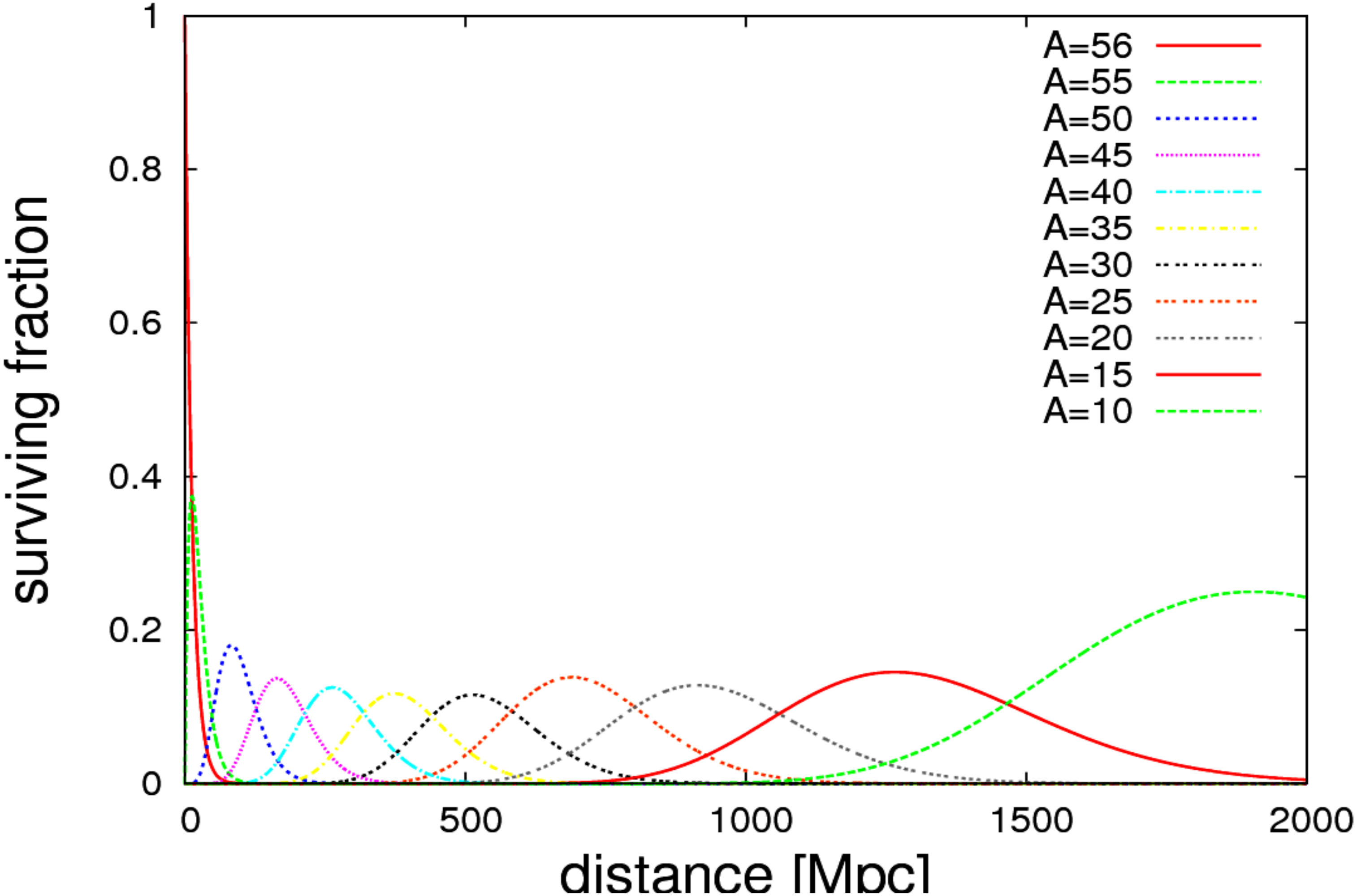
Energy Loss Rates due to Nuclei Interactions

$$\mathbf{R} = \frac{\mathbf{A^2 m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^{\mathbf{2}}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{Am_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{N}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

where R is the energy loss rate



Cosmic Ray Disintegration During Propagation



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High Energy Gamma-Ray and Electron Propagation

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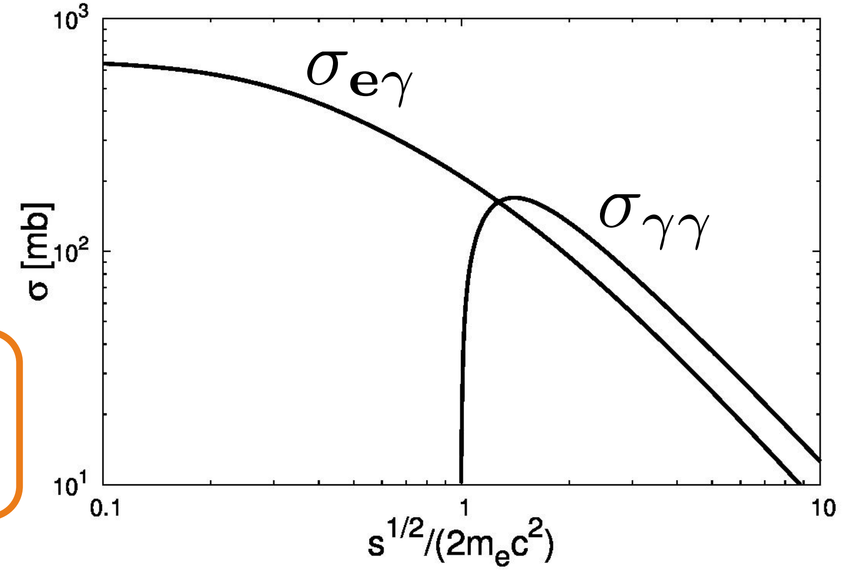
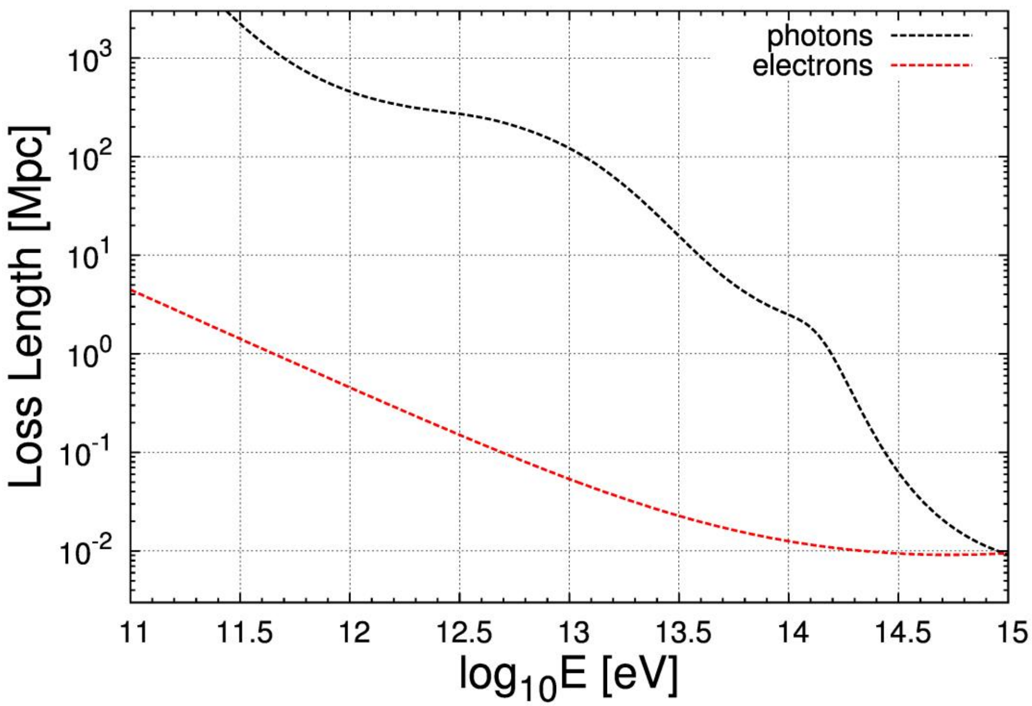
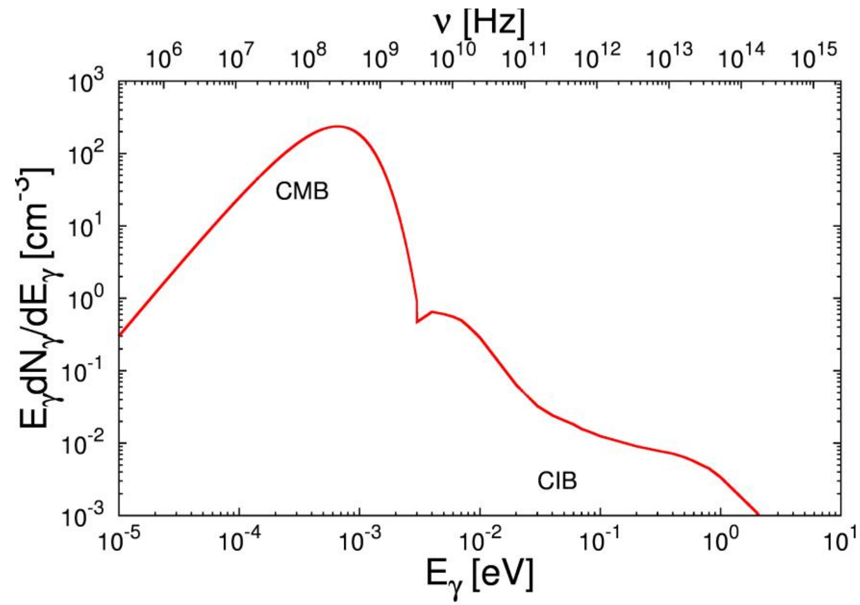
Energy Loss Rates of Electrons and Photons

$$\mathbf{R} = \frac{\mathbf{m_e^2 c^4}}{\mathbf{2E_e^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^{\mathbf{2}}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \mathbf{K_e} \int_0^{\mathbf{2E_e}\epsilon_\gamma/(\mathbf{m_e c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma(\epsilon'_\gamma)$$







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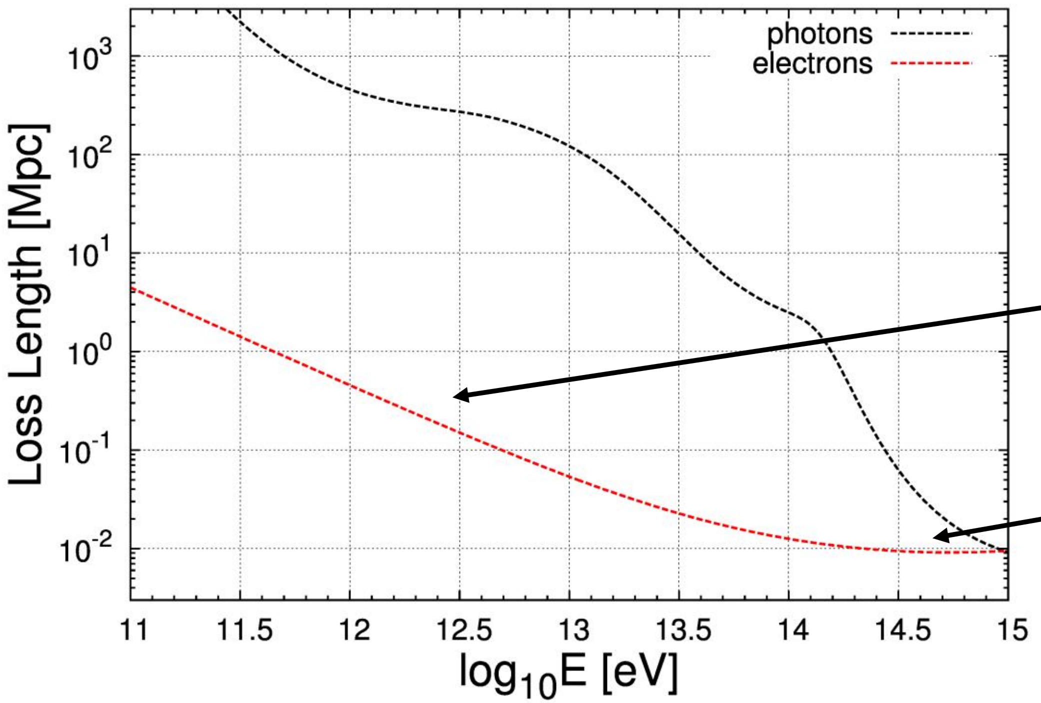
Energy Loss Rates of Electrons and Photons



$$\tau_e \propto E_e^{-1}$$

onset of Klein Nishina effect

Thomson regime electron cooling:

$$K_e = \frac{E_\gamma}{E_e} = b \quad \Longrightarrow \quad b = \frac{E_e E_\gamma^{bg}}{(m_e c^2)^2}$$

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Cosmic Ray Spectra

Assumptions on Source Population

Spatial Distribution

$$\frac{dN}{dV_C} \propto (1+z)^n$$

$$z < z_{max}$$

$$n = -6,\ -3,\ 0,\ 3$$

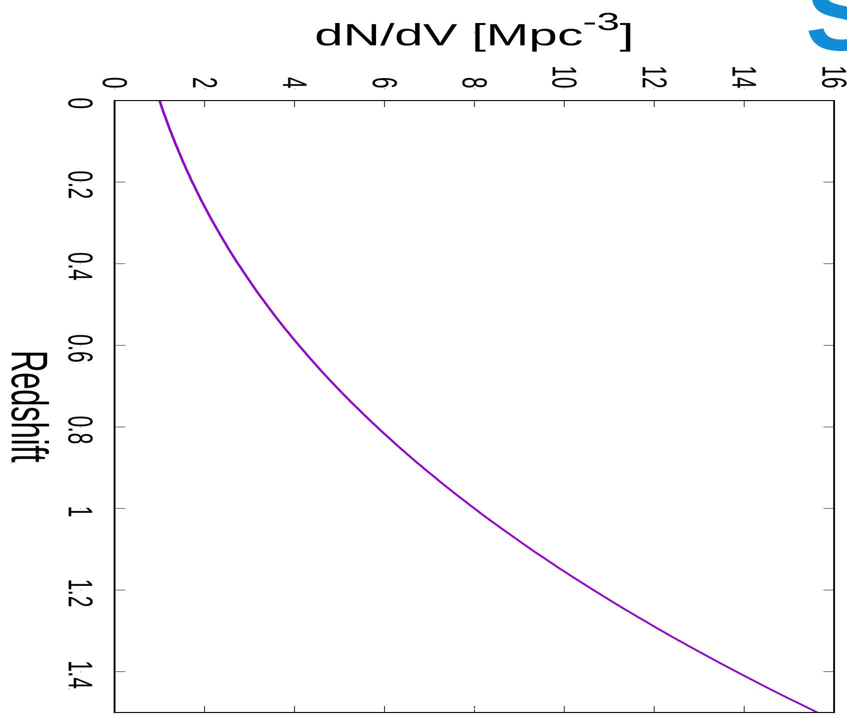
Energy Distribution

$$\frac{dN}{dE} \propto E^{-\alpha} \exp[-E/E_{Z,max}]$$

$$E_{Z,max} = (Z/26) \times E_{Fe,max}$$

Note- magnetic field horizon effects are neglected in the following.
This amounts to assuming: $d_s < (ct_H \lambda_{scat})^{1/2}$
ie. the source distribution may be approximated to be spatially continuous (also note, presence of t_H term comes from temporally continuous assumption)

A Cosmological Distribution of Sources

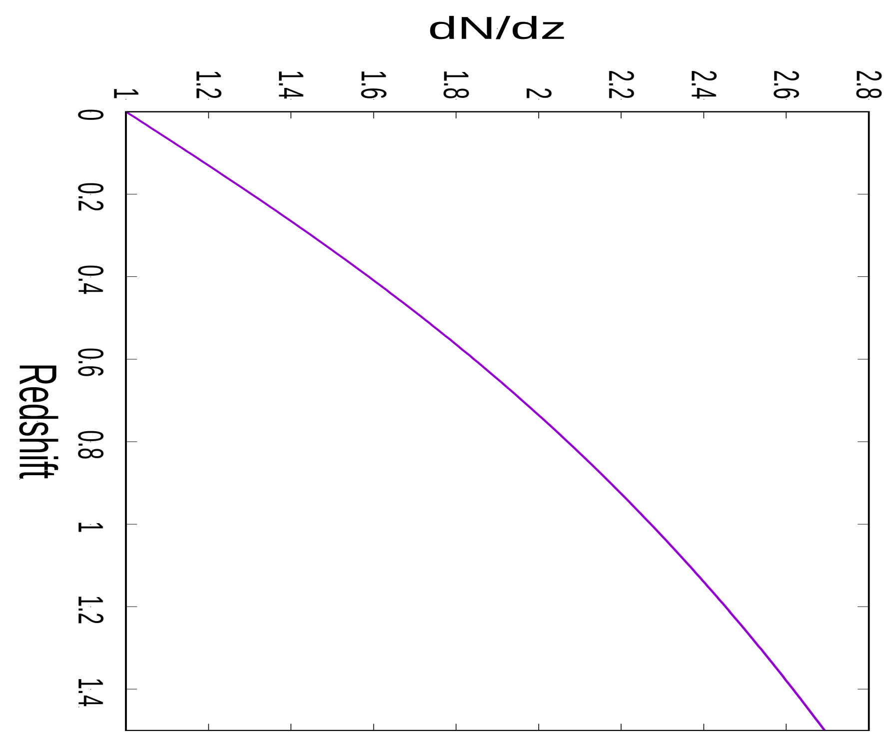


Distribution of sources in a comoving volume

$$\mathbf{dV_c = 4\pi\chi^2 d\chi}$$

$$\mathbf{d\chi = \frac{dz}{H}}$$

$$\mathbf{\approx \frac{dz}{H_0(\Omega_M(1+z)^3 + \Omega_\Lambda)^{1/2}}}$$



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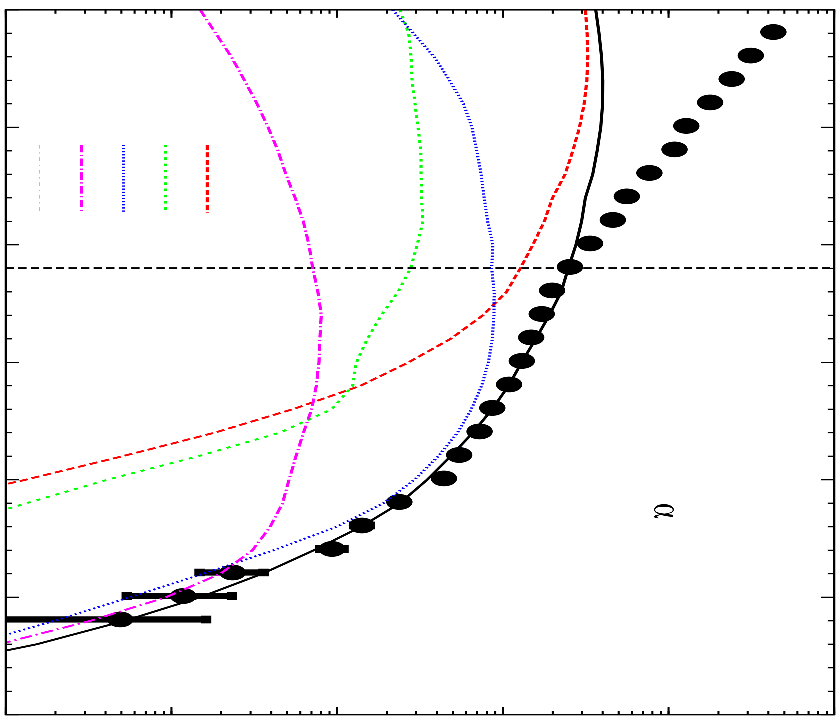
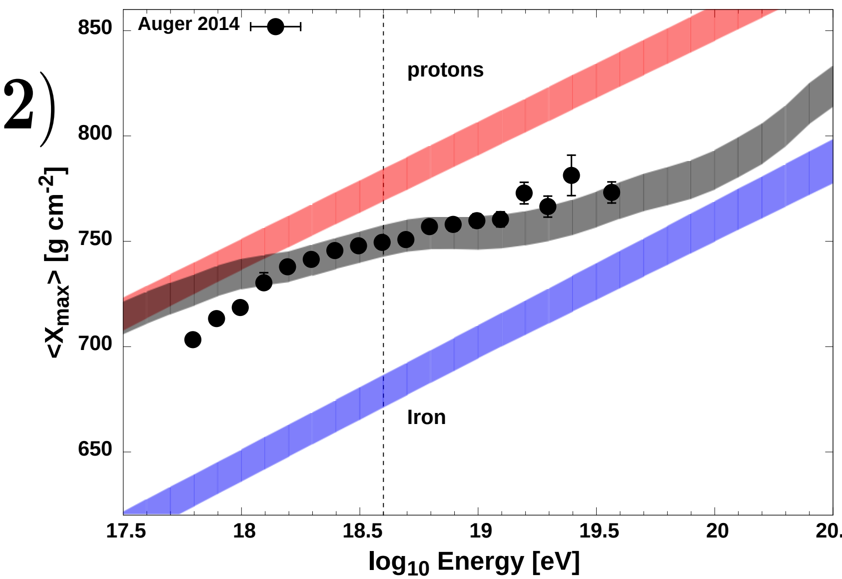
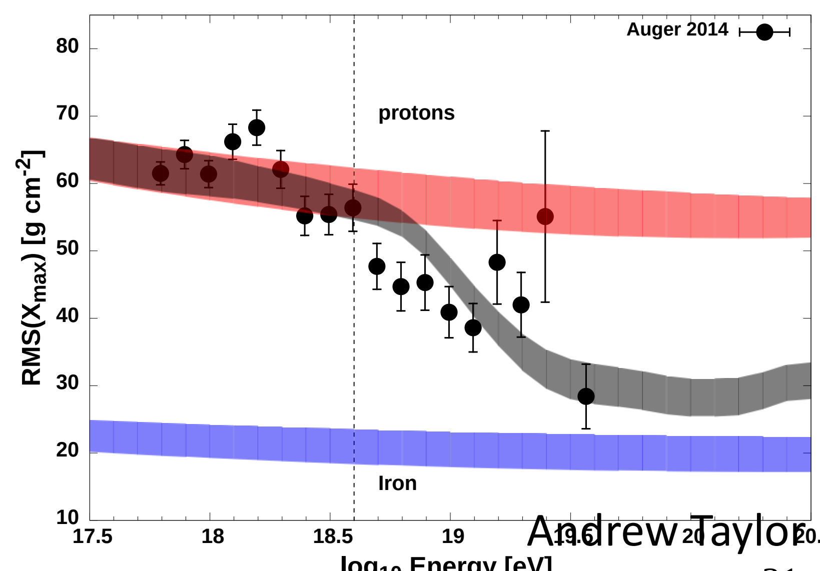
MCMC Likelihood Scan: Spectral + Composition Fits

$$\mathbf{L}(\mathbf{f_p}, \mathbf{f_{He}}, \mathbf{f_N}, \mathbf{f_{Si}}, \mathbf{E_{max}}, \alpha) \propto \exp(-\chi^2/2)$$

n=3 evolution result







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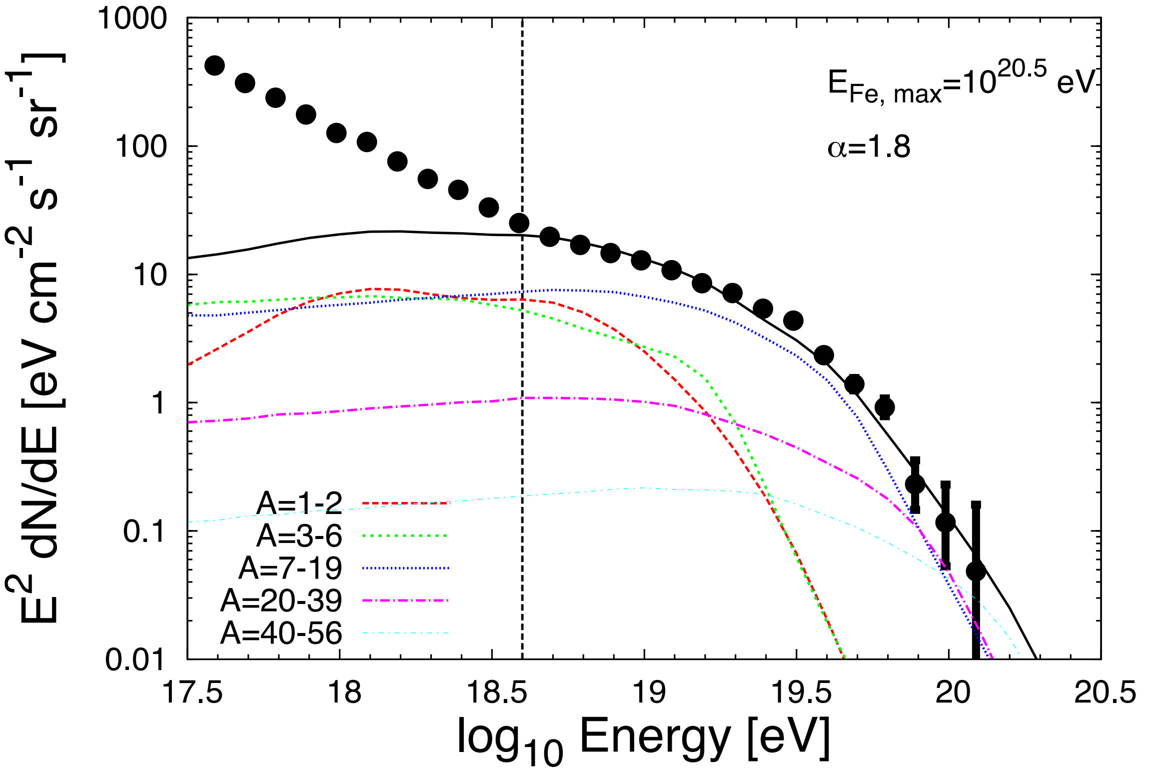
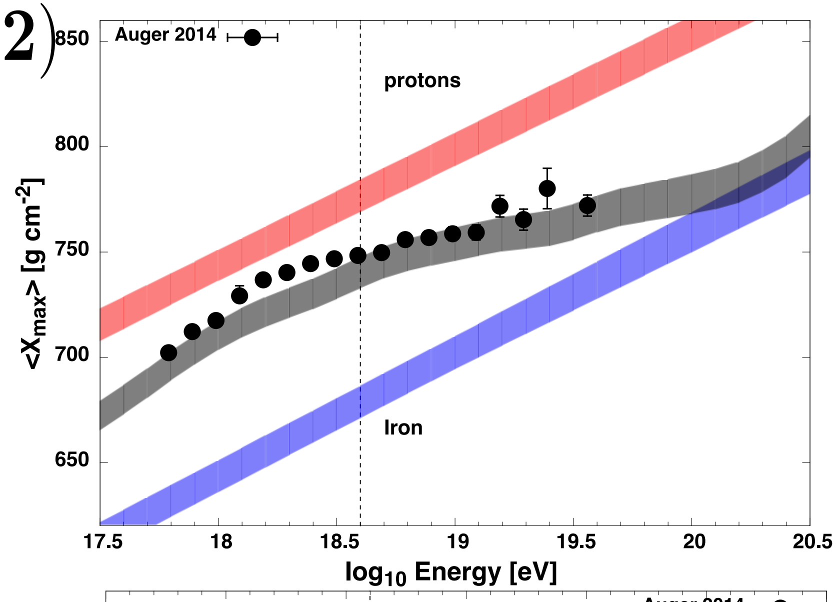
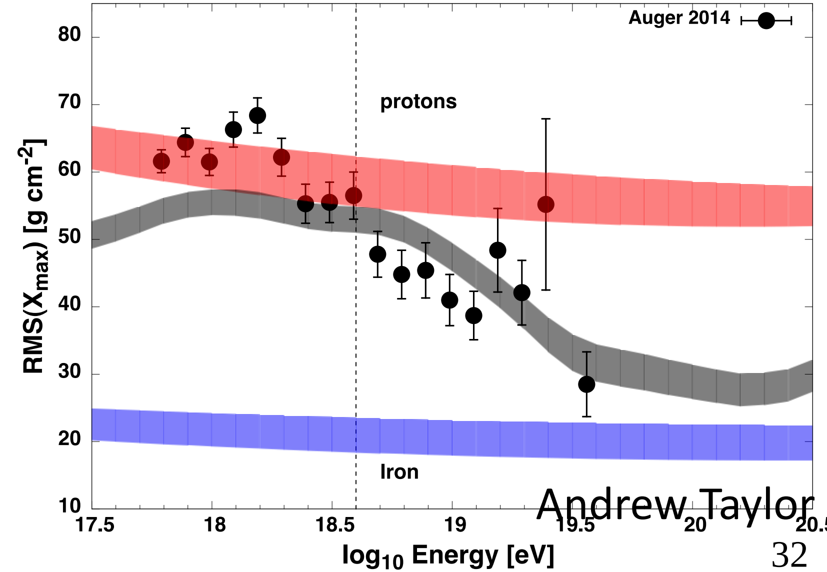
MCMC Likelihood Scan: “Soft” Spectra Solutions

$$\mathbf{L}(\mathbf{f_p}, \mathbf{f_{He}}, \mathbf{f_N}, \mathbf{f_{Si}}, \mathbf{E_{max}}, \alpha) \propto \exp(-\chi^2/2)$$

n=-6 evolution result







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MCMC Results Table

From astro-ph/1505.06090 (Taylor et al. 2015)

Similar conclusion arrived at by others too: astro-ph/1612.07155 (Aab et al. 2017)

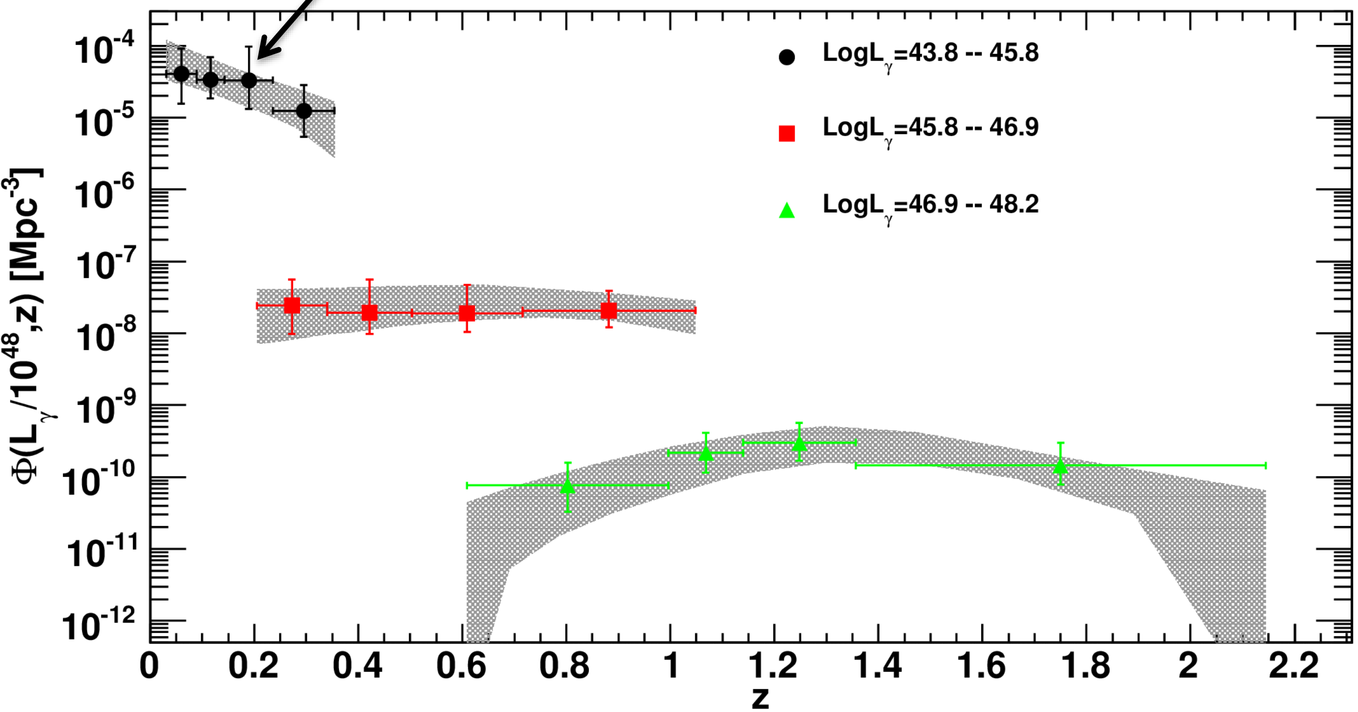
Parameter	$n=-6$		$n=-3$		$n=0$		$n=3$	
	Best-fit Value	Posterior Mean & Standard Deviation	Best-fit Value	Posterior Mean & Standard Deviation	Best-fit Value	Posterior Mean & Standard Deviation	Best-fit Value	Posterior Mean & Standard Deviation
f_p	0.03	0.14 ± 0.12	0.08	0.15 ± 0.13	0.17	0.17 ± 0.16	0.19	0.20 ± 0.16
f_{He}	0.50	0.21 ± 0.17	0.42	0.17 ± 0.16	0.53	0.20 ± 0.17	0.32	0.23 ± 0.20
f_{N}	0.40	0.50 ± 0.18	0.42	0.51 ± 0.19	0.29	0.47 ± 0.19	0.43	0.45 ± 0.21
f_{Si}	0.06	0.11 ± 0.12	0.08	0.12 ± 0.13	0.0	0.11 ± 0.12	0.06	0.078 ± 0.086
f_{Fe}	0.01	0.052 ± 0.039	0.0	0.053 ± 0.042	0.01	0.050 ± 0.038	0.0	0.044 ± 0.034
α	1.8	1.83 ± 0.31	1.6	1.67 ± 0.36	1.1	1.33 ± 0.41	0.6	0.64 ± 0.44
$\log_{10}\left(\frac{E_{\mathrm{Fe,max}}}{\mathrm{eV}}\right)$	20.5	20.55 ± 0.26	20.5	20.52 ± 0.27	20.2	20.38 ± 0.25	20.2	20.16 ± 0.18

Flatter spectra preferred for negative source evolution

Hard spectra preferred for source evolution following that of the SFR

High Spectral Peaked Blazar Evolution

n=-6 evolution result

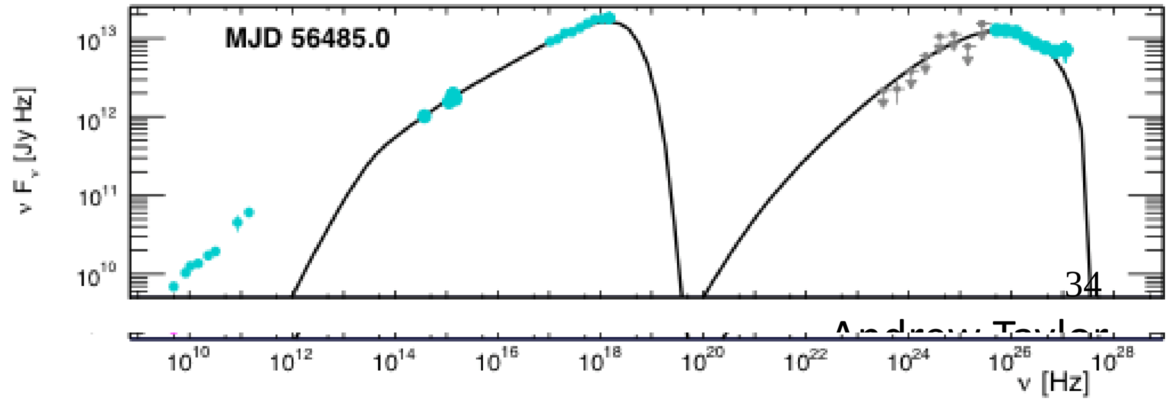


From astro-ph/1310.0006 (Ajello et al. 2014)

- Reminder:
  Blazar -> BL Lac (FR1) -> HSP

- Supports idea that FSRQ (gas accreting) AGN evolve into BL Lac (gas starved) AGN

Archetypal HSP example Mrk 501



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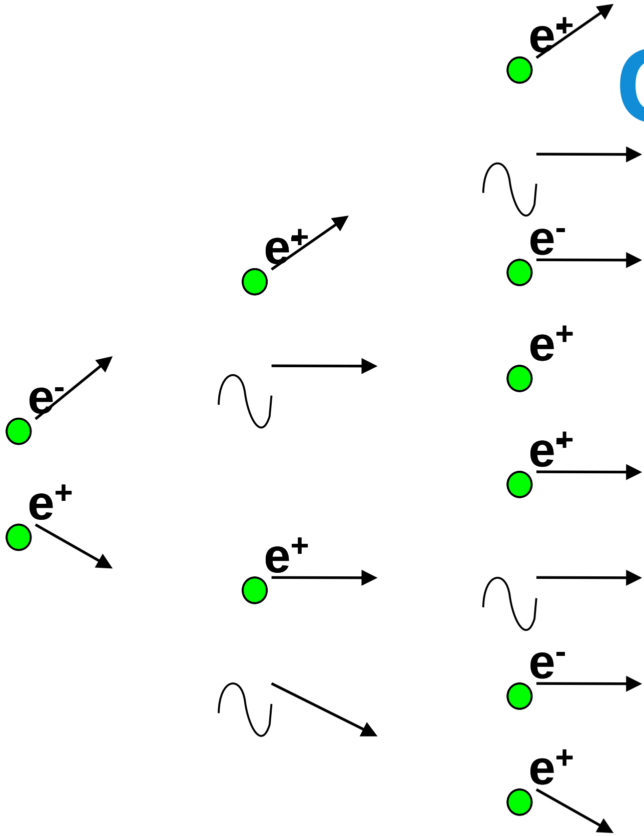
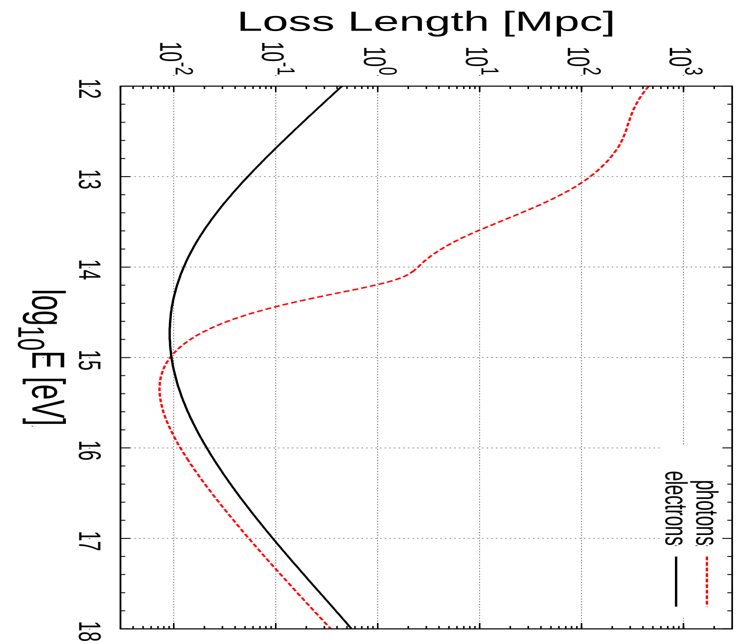
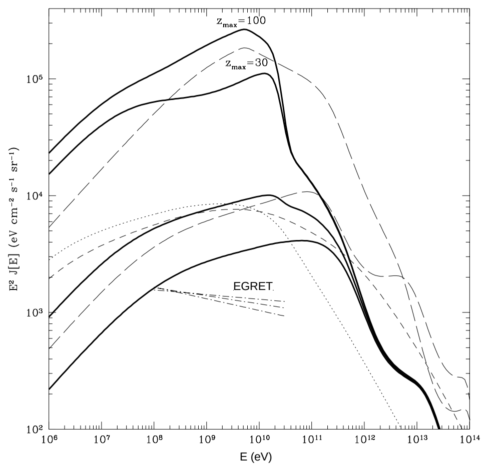
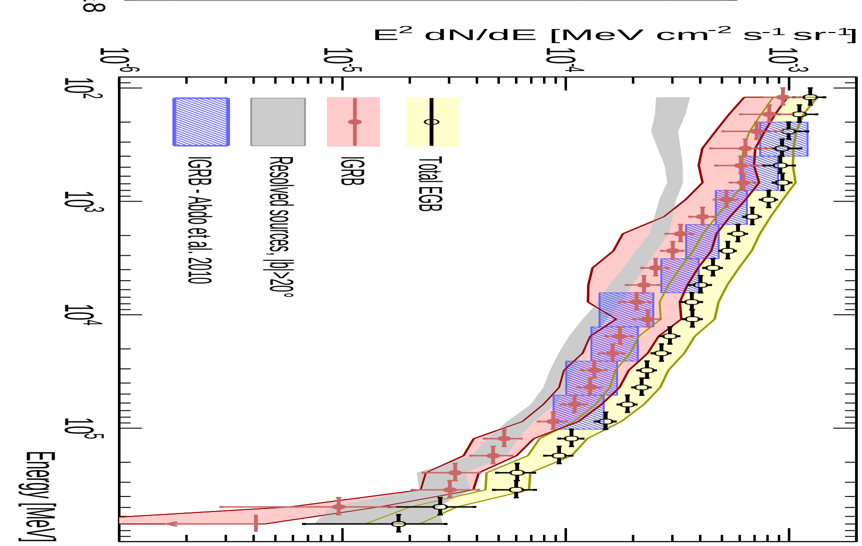
Cascade Spectra + the IGRB





From Coppi et al. astro-ph/9610176

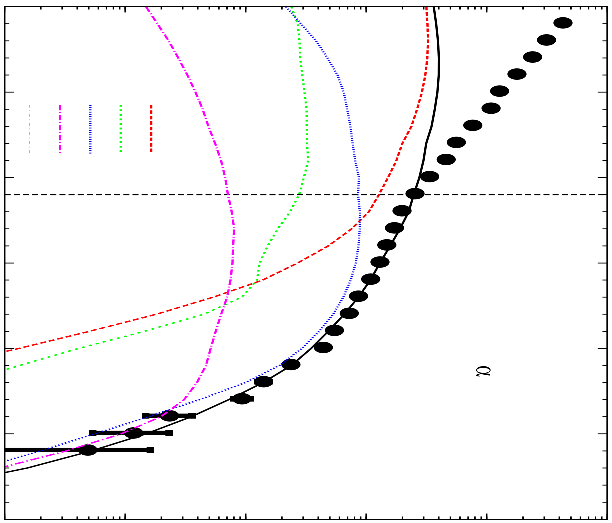




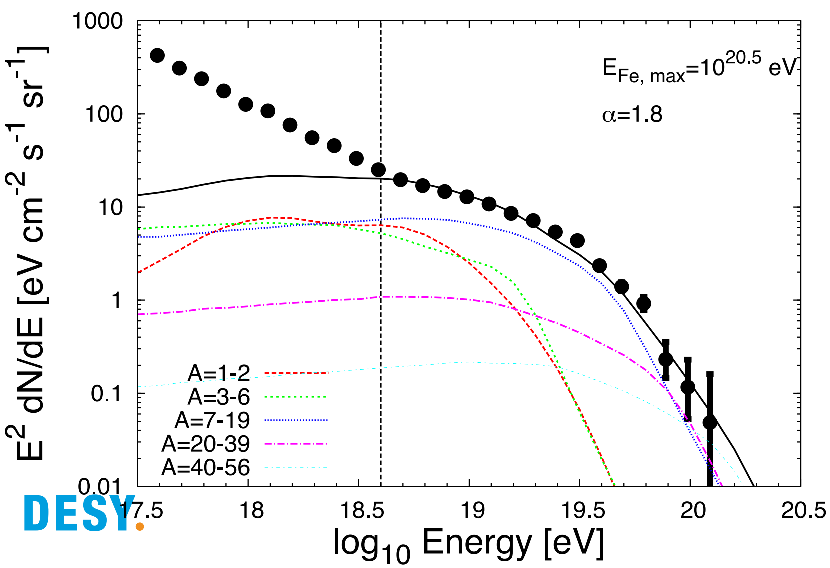
Regardless of where the energy is injected (ie independent of source z), the arriving flux possesses a ~universal shape

Secondary (Guaranteed) Gamma-Ray Fluxes From $>10^{18.6}$eV UHECR

n=3 evolution result

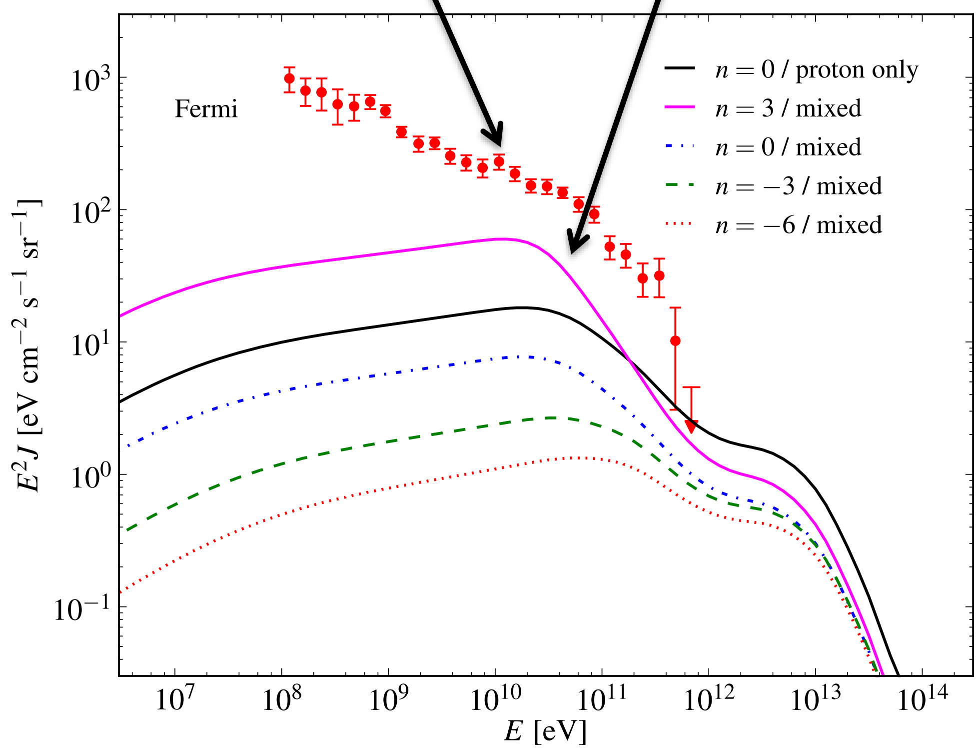


n=-6 evolution result



IGRB (EGB with resolved points sources removed)

n=3 to -6 evolution scenarios give rise to between **40%** and **12%** of Fermi limit



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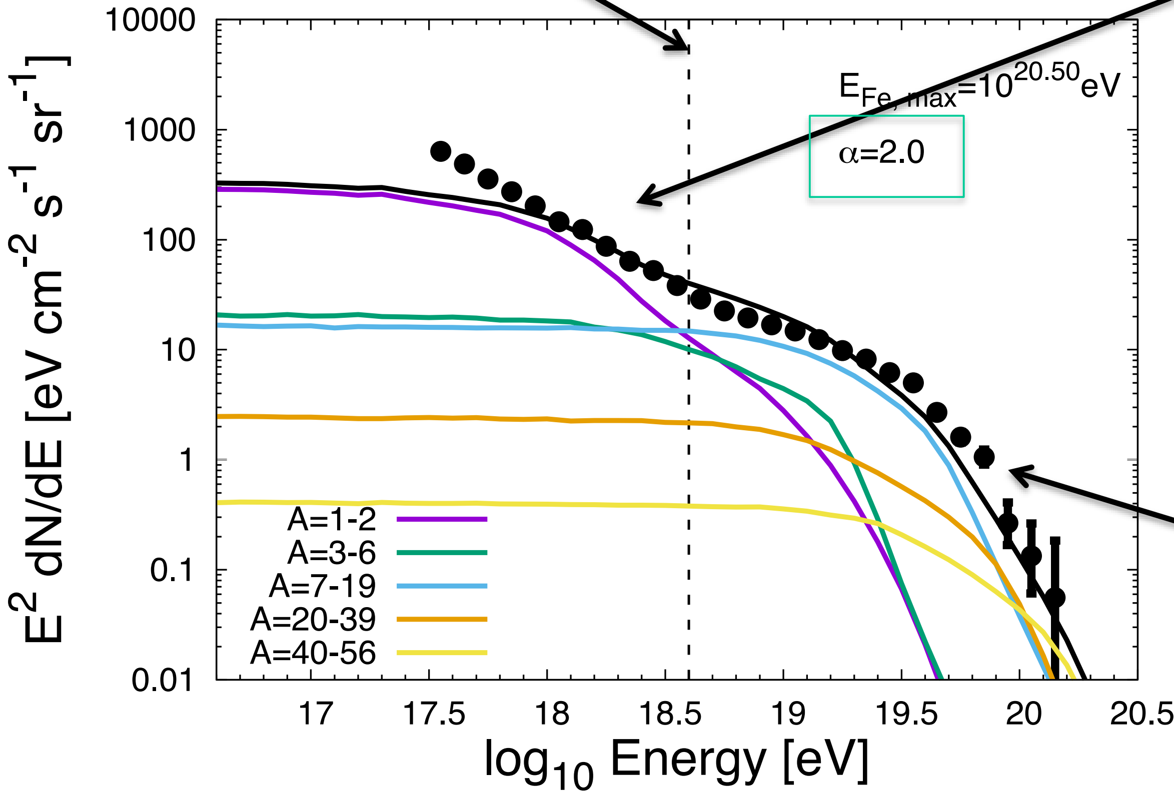
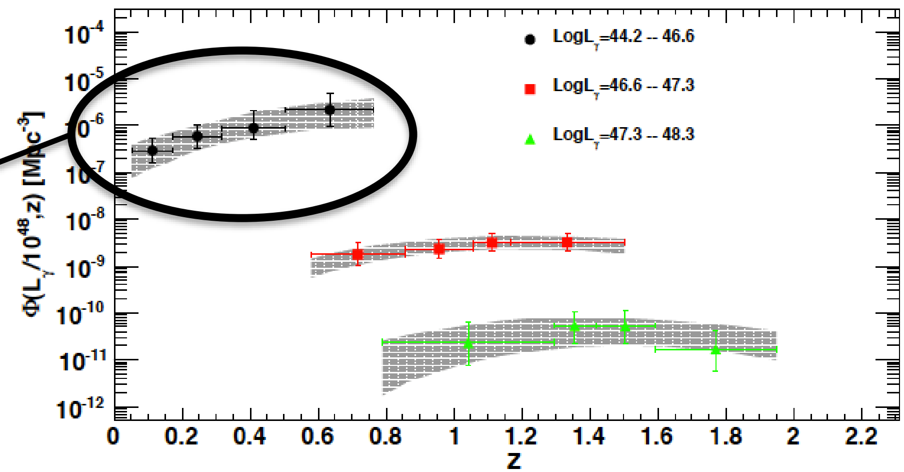
Does a Separate Class of Extragalactic Source Dominate at Sub-Ankle Energies?

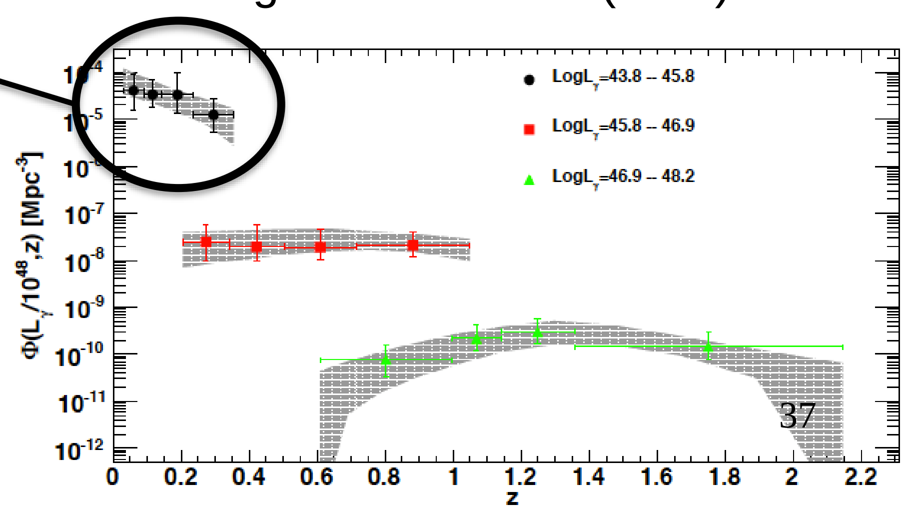
Ankle Energy



Positive evolution (ISP + LSP)

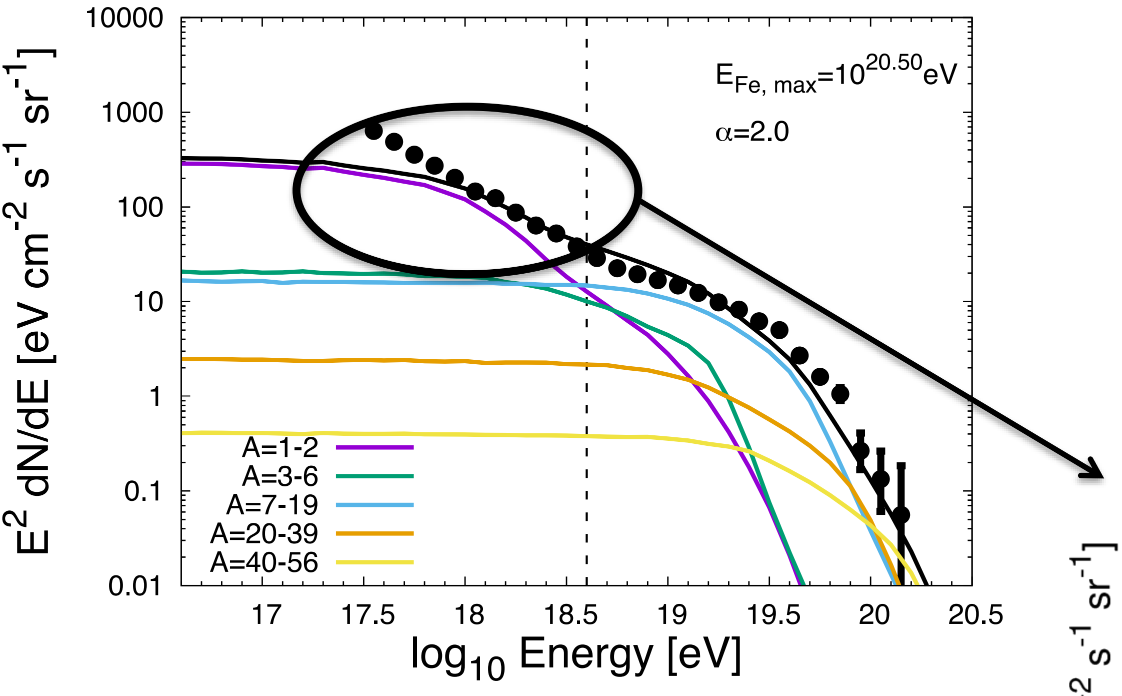


Negative evolution (HSP)

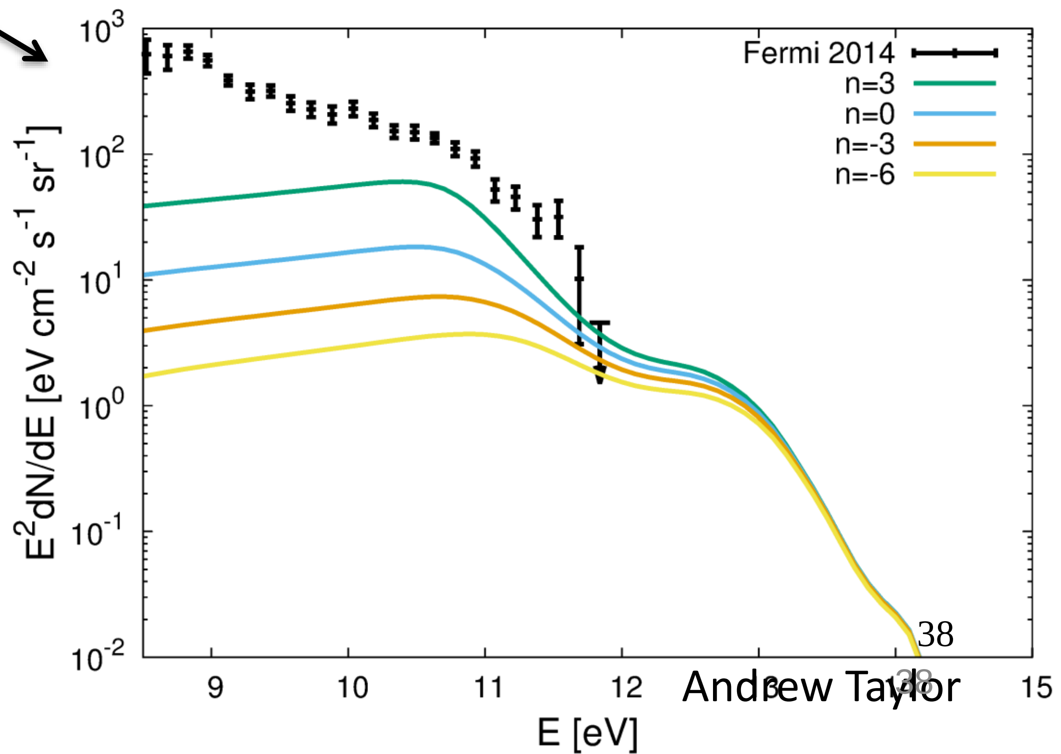


DESY.

Cascade Contribution from Second Source Population



n=3 to -6 evolution scenarios give rise to between **100%** and **40%** of Fermi limit



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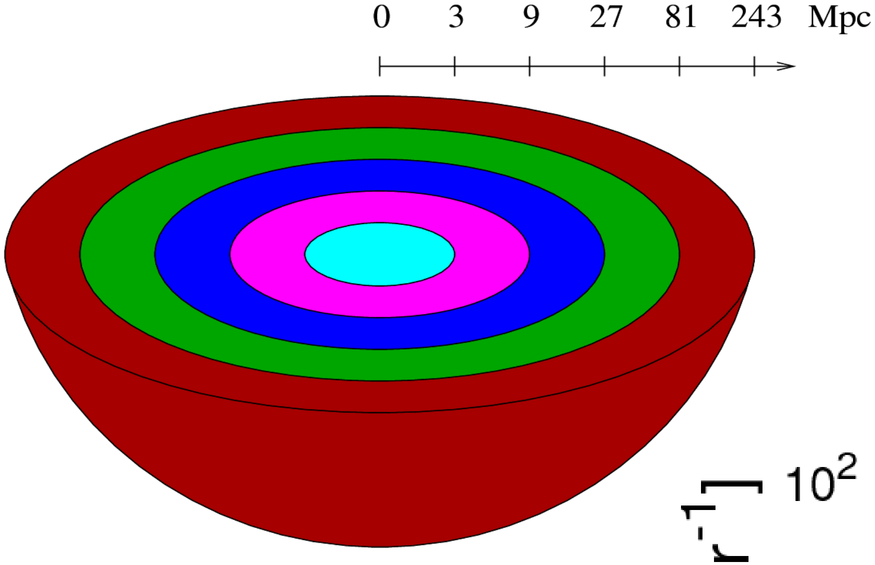
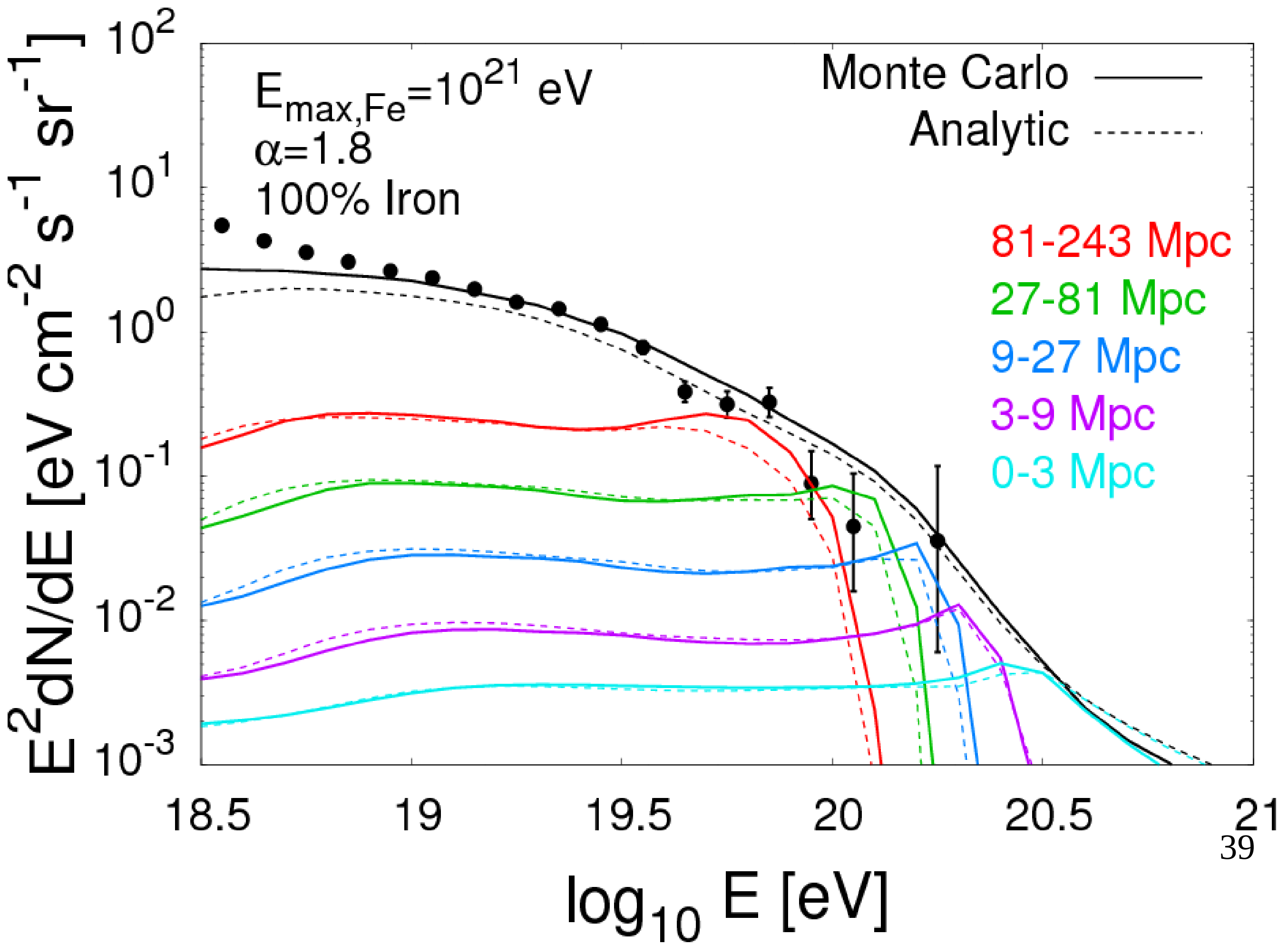
Local Scales Effect Highest Energies



(logarithmic scale)



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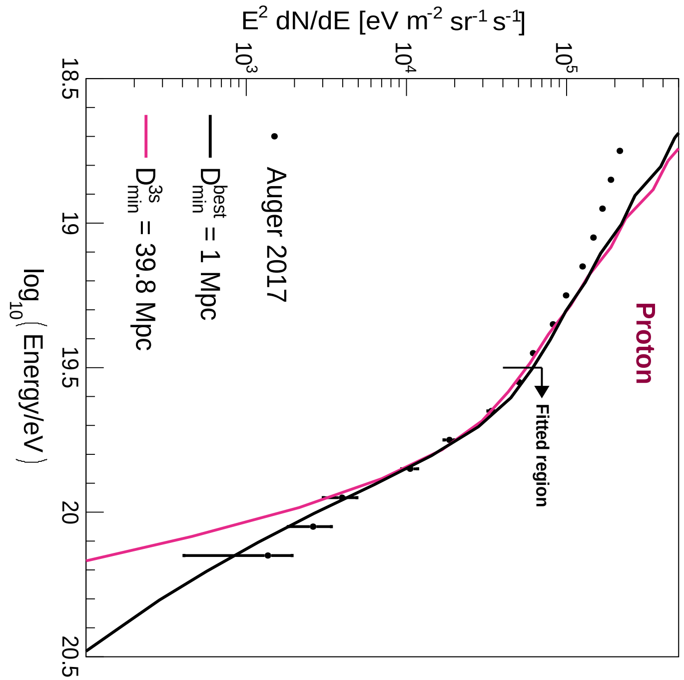
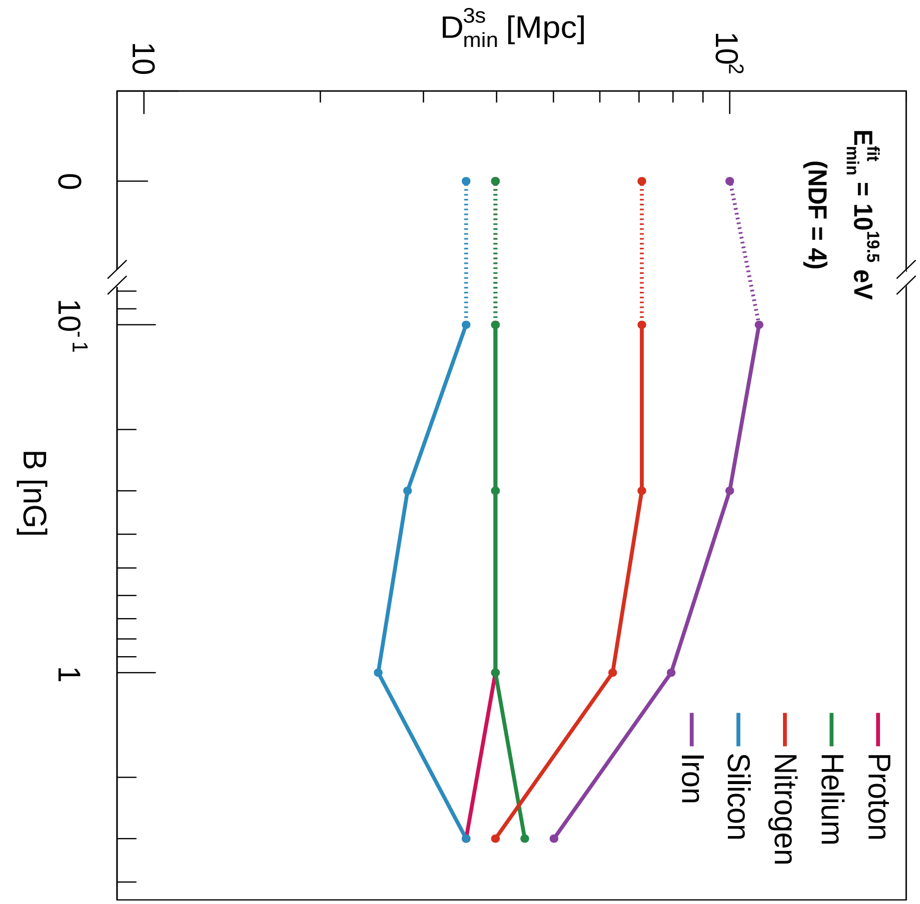
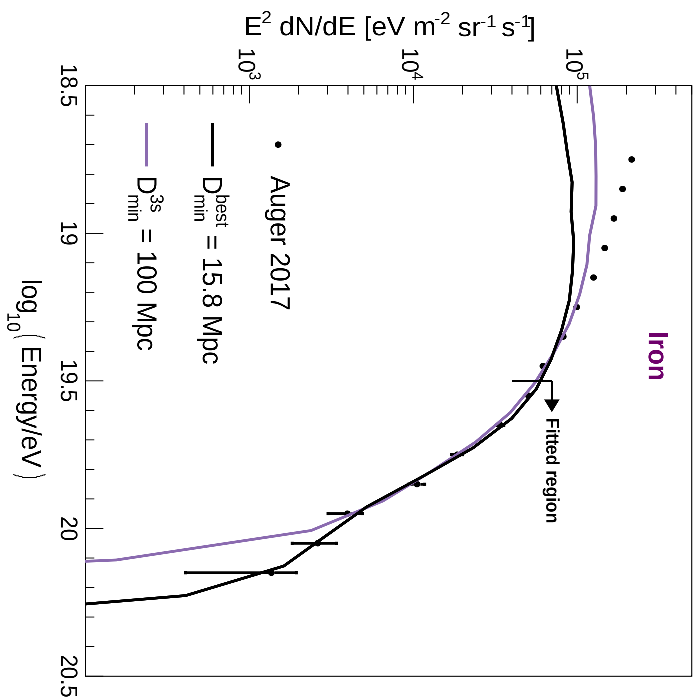
Proximity of Local Sources?



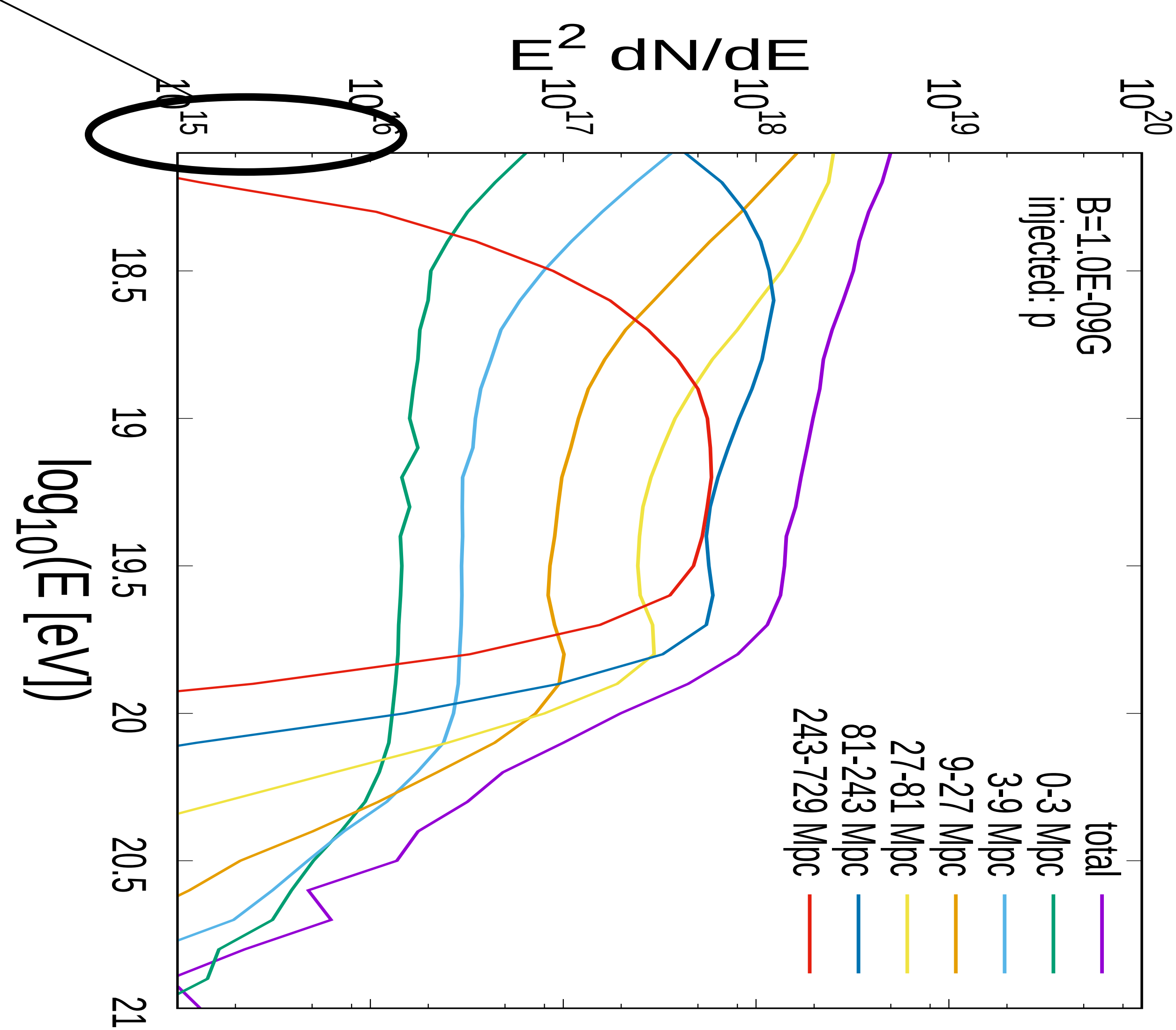
From astro-ph/2005.14275 (Lang et al. 2020)





Magnetic Horizon Effect

Note strong B-field strength considered



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Conclusions

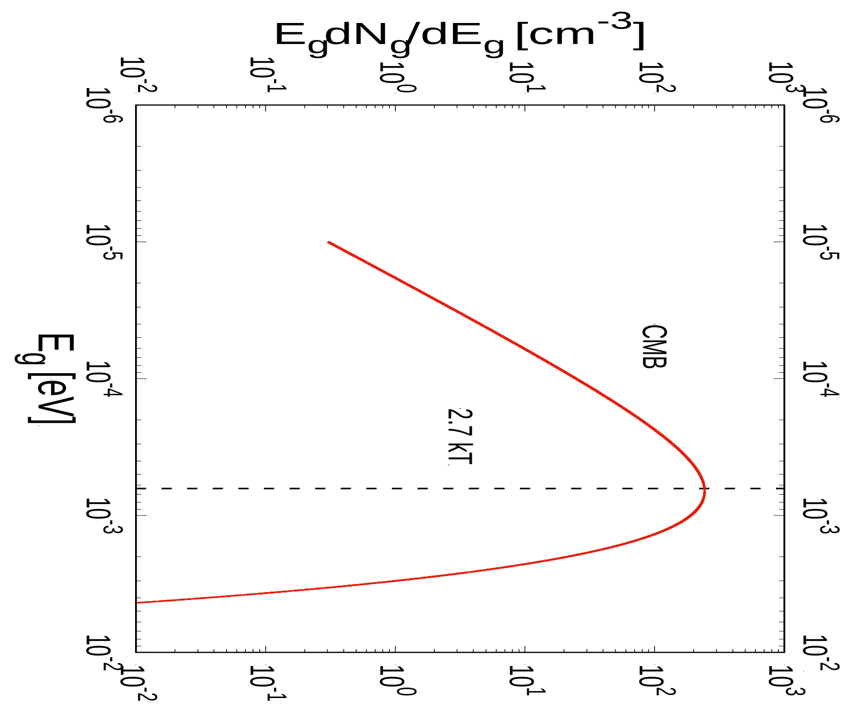
- The attenuation of cosmic ray protons/nuclei/photons/electrons due to the presence of background radiation fields is well understood
- The largest limitation presently is the EBL (dust and stellar emission components)
- Despite these limitations, calculations for the propagation ultra high energy cosmic rays in these background radiation fields are predictive
- A negative evolution of sources allows for softer source injection spectra (more consistent with the Fermi acceleration model)
- A negative evolution of sources gives rise to a reduced level of diffuse gamma-ray background contribution
- The current cosmic ray data at the highest energies is suggestive that the sources should be no further than a few 10s of Mpc

End of Lecture

Blackbody- Total Number Density

$$\mathbf{n_\gamma^{BB}} = 8\pi\frac{(\mathbf{kT})^3}{(\mathbf{hc})^3}\gamma(3)\zeta(3)$$

$$\mathbf{n_\gamma^{BB}} = \frac{8\pi(\mathbf{kT})^3}{(\mathbf{hc})^3}\int_0^\infty \frac{\mathbf{x}^2}{\mathbf{e}^\mathbf{x}-1}\mathbf{dx}$$



$$\int_0^\infty \mathbf{x}^2\mathbf{e}^{-\mathbf{x}}\mathbf{dx} = \gamma(3)$$

$$\frac{\mathbf{x}^\mathbf{n}}{\mathbf{e}^\mathbf{x}-1} = \frac{\mathbf{e}^{-\mathbf{x}}\mathbf{x}^\mathbf{n}}{1-\mathbf{e}^{-\mathbf{x}}}$$

CMB- Total Number Density

$$\mathbf{n_\gamma^{BB}} = \mathbf{8}\pi \frac{\mathbf{(kT)^3}}{\mathbf{(hc)^3}} \gamma(\mathbf{3})\zeta(\mathbf{3})$$

$$\frac{\mathbf{x^n}}{\mathbf{e^x - 1}} = \frac{\mathbf{e^{-x}x^n}}{\mathbf{1 - e^{-x}}}$$

$$= \sum_{\mathbf{m=0}}^{\infty} \mathbf{e^{-mx}e^{-x}x^n}$$

$$= \sum_{\mathbf{m=1}}^{\infty} \mathbf{e^{-mx}x^n}$$

CMB- Total Number Density

$$n_\gamma^{BB} = 8\pi \frac{(kT)^3}{(hc)^3}\gamma(3)\zeta(3)$$

$$\int \frac{x^n}{e^x - 1}dx = \sum_{m=1}^{\infty} \int e^{-mx}x^n dx$$

Let $y = mx$

$$\int \frac{x^n}{e^x - 1}dx = \sum_{m=1}^{\infty} \int e^{-y}\left(\frac{y}{m}\right)^n d\left(\frac{y}{m}\right)$$

$$\int \frac{x^n}{e^x - 1}dx = \sum_{m=1}^{\infty} \frac{1}{m^{n+1}} \int y^n e^{-y} dy = \gamma(n+1)\zeta(n+1)$$

Threshold Energy- Proton Pion Production

$$(\mathbf{E_p} + \mathbf{E}_\gamma)^2 - (\mathbf{p_p} - \mathbf{E}_\gamma)^2 = (\mathbf{m_p} + \mathbf{m}_\pi)^2$$

$$\mathbf{m_p^2} + \mathbf{2E_pE}_\gamma + \mathbf{2p_pE}_\gamma \approx \mathbf{m_p^2} + \mathbf{2m_pm}_\pi$$

$$\mathbf{E_p} \approx \frac{\mathbf{m}_\pi}{\mathbf{2E}_\gamma}\mathbf{m_p} \approx \left(\frac{\mathbf{135 \times 10^6}}{\mathbf{2 \times 6 \times 10^{-4}}}\right)\mathbf{0.9 \times 10^9} = \mathbf{10^{20}\ eV}$$

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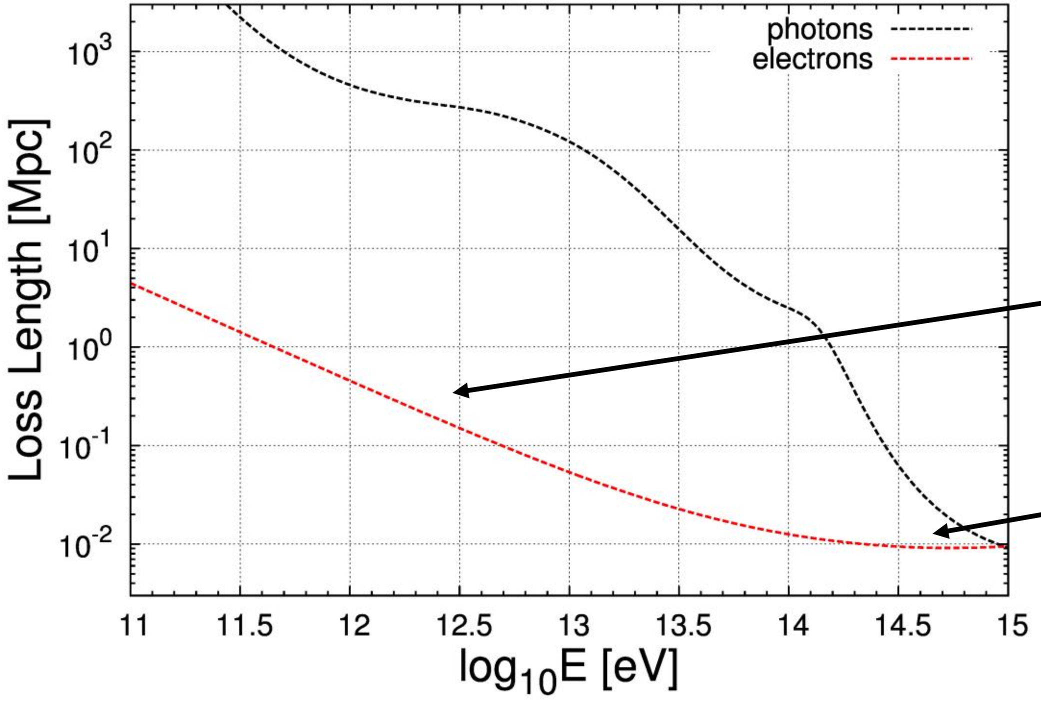
Energy Loss Rates of Electrons and Photons



$\tau_e \propto E_e^{-1}$

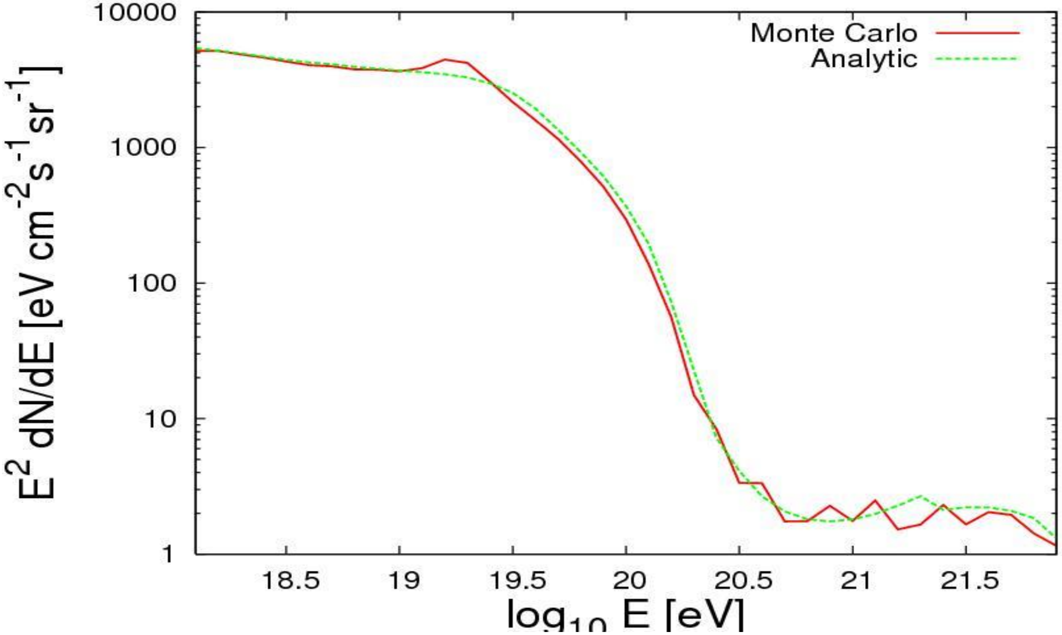
onset of Klein Nishina effect

Thomson regime electron cooling:

$$\tau_e^{-1} = \frac{m_e^2 c^4}{2E_e^2} \int_0^\infty d\epsilon_\gamma \frac{1}{\epsilon_\gamma^2} \frac{dn}{d\epsilon_\gamma} K_e \int_0^{2E_e\epsilon_\gamma/(m_e c^2)} d\epsilon'_\gamma \epsilon'_\gamma \sigma(\epsilon'_\gamma)$$

$$\approx \sigma_T \int_0^\infty b \frac{dn}{d\epsilon_\gamma} d\epsilon_\gamma$$

Comparison of Analytic and Monte Carlo Results



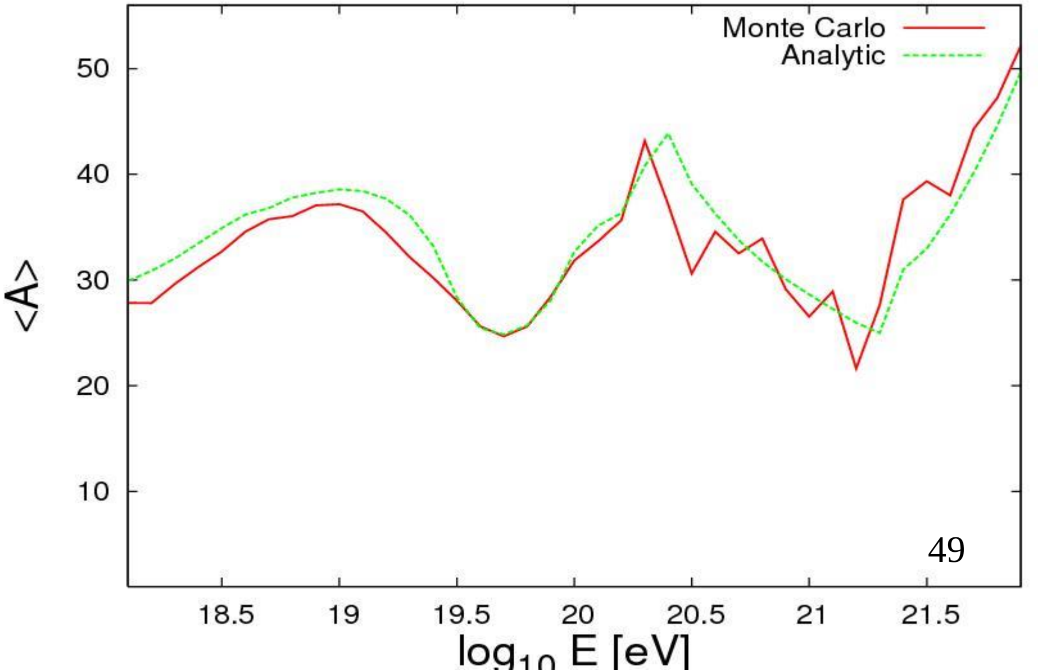


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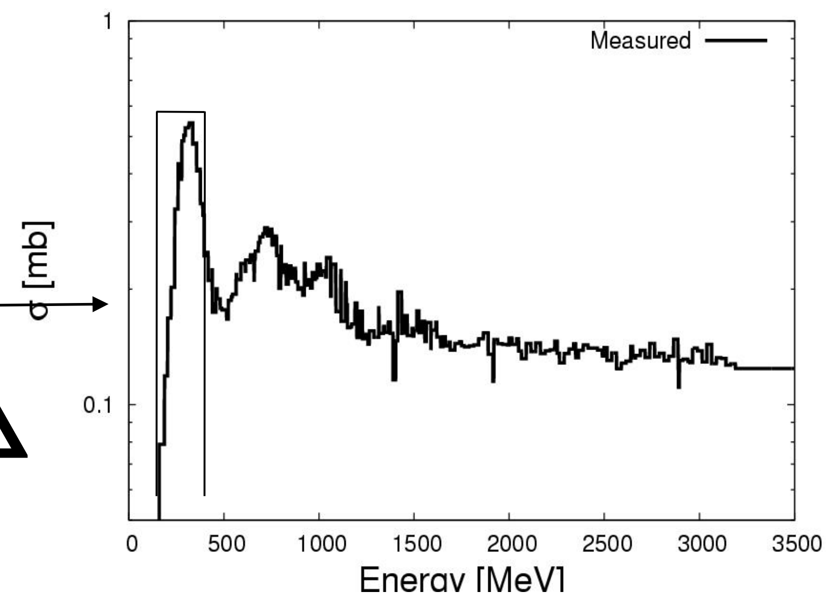
Photo-Pion Production Rate

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_{\mathbf{0}}^{\infty} \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^{\mathbf{2}}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_{\mathbf{0}}^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma)\mathbf{K_p}$$

Assuming the cross-section is approximately:

$$\sigma_{\mathbf{p}\gamma}(\epsilon_\gamma) = \mathbf{0} \qquad \begin{matrix} \epsilon_\gamma < \mathbf{E} - \mathbf{\Delta} \\ \epsilon_\gamma > \mathbf{E} + \mathbf{\Delta} \end{matrix}$$

$$\sigma_{\mathbf{p}\gamma}(\epsilon_\gamma) = \sigma_{\mathbf{p}\gamma} \quad \mathbf{E} - \mathbf{\Delta} < \epsilon_\gamma < \mathbf{E} + \mathbf{\Delta}$$

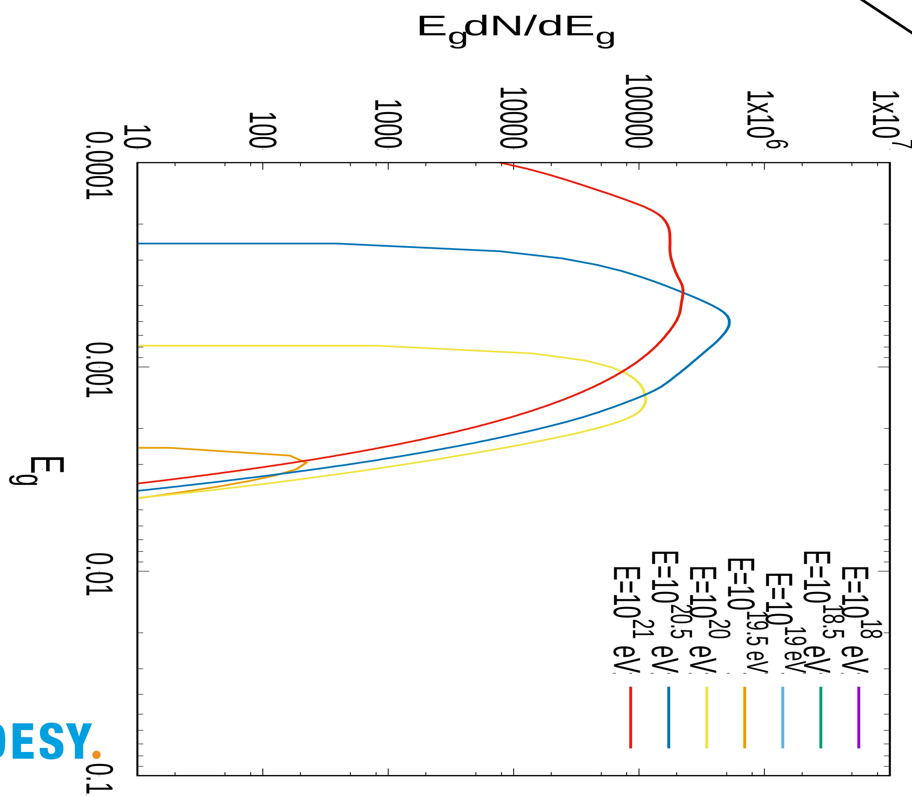


Where $\sigma_{\mathbf{p}\gamma} = \mathbf{0.5\ mb}$, $\mathbf{E} = \mathbf{300\ MeV}$, $\mathbf{\Delta} = \mathbf{100\ MeV}$

Photo-Pion Production Rate

$$\mathbf{R}(\boldsymbol{\Gamma}) \approx \sigma_0 \int_{(\mathbf{E_0}-\boldsymbol{\Delta_0})/2\boldsymbol{\Gamma}}^{(\mathbf{E_0}+\boldsymbol{\Delta_0})/2\boldsymbol{\Gamma}} \left(\frac{\epsilon^2 - [(\mathbf{E_0}-\boldsymbol{\Delta_0})/2\boldsymbol{\Gamma}]^2}{\epsilon^2}\right)\frac{\mathbf{dn}}{\mathbf{d}\epsilon}\mathbf{d}\epsilon +$$

$$\sigma_0 \int_{(\mathbf{E_0}+\boldsymbol{\Delta_0})/2\boldsymbol{\Gamma}}^{\infty} \left(\frac{[(\mathbf{E_0}+\boldsymbol{\Delta_0})/2\boldsymbol{\Gamma}]^2 - [(\mathbf{E_0}-\boldsymbol{\Delta_0})/2\boldsymbol{\Gamma}]^2}{\epsilon^2}\right)\frac{\mathbf{dn}}{\mathbf{d}\epsilon}\mathbf{d}\epsilon$$



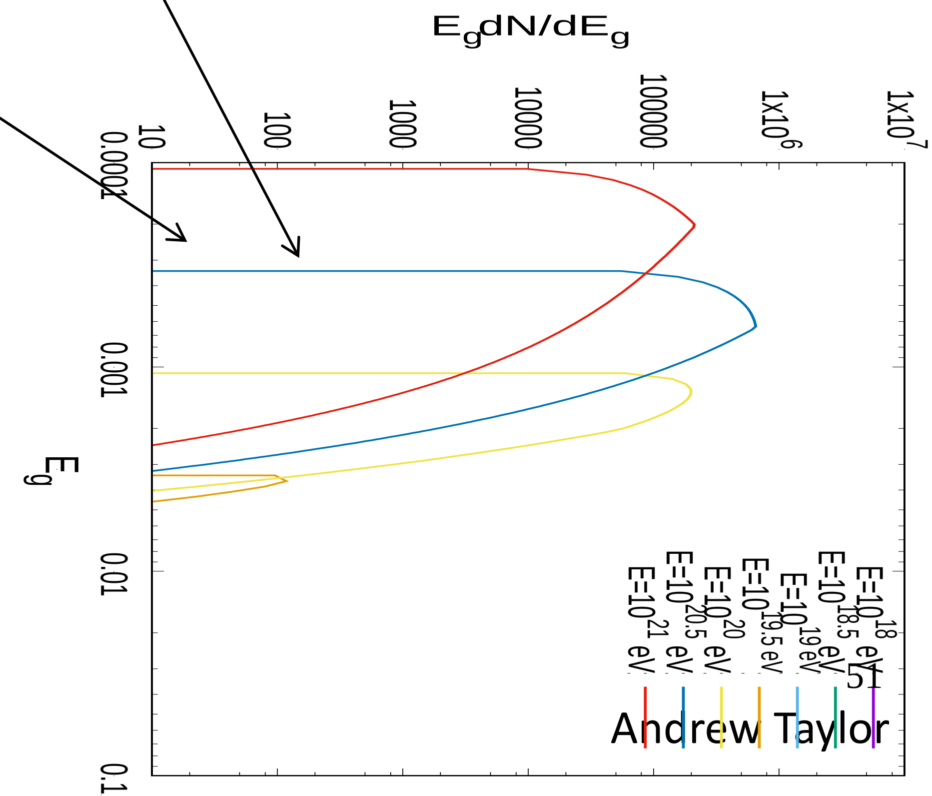


Photo-Pion Production Rate

$$\mathbf{R(\Gamma) \approx n_0 \sigma_0 \int_{x_1(\Gamma)}^{x_2(\Gamma)} \frac{\left(x^2 - x_1(\Gamma)^2\right)}{e^x - 1} dx +}$$

$$\mathbf{n_0 \sigma_0 \int_{x_2(\Gamma)}^{\infty} \frac{\left(x_2^2(\Gamma) - x_1^2(\Gamma)\right)}{e^x - 1}}$$

$$\mathbf{R(\Gamma) \approx \frac{1}{l_0} \left[\, (\gamma_i(3, x_2(\Gamma)) - \gamma_i(3, x_1(\Gamma))) - x_1(\Gamma)^2 (\gamma_i(1, x_2(\Gamma)) - \gamma_i(1, x_1(\Gamma))) + \right.}$$
$$\mathbf{\left. x_2(\Gamma)^2 (1 - \gamma_i(1, x_2(\Gamma))) - x_1(\Gamma)^2 (1 - \gamma_i(1, x_2(\Gamma))) \, \right]}$$

$$\mathbf{\gamma_i(3, x) = 2 - (2 + 2x + x^2) \exp(-x) \qquad \gamma_i(1, x) = 1 - \exp(-x)}$$

$$\mathbf{R(\Gamma) \approx \frac{2}{l_0} \left[e^{-x_1} \left(1 - e^{-x_1} + x_1 (1 - 2e^{-x_1})\right)\right]}$$

Photo-Pion Production Rate: Blackbody Interactions

$$\mathbf{R(\Gamma) \approx n_0\sigma_0 \int_{x_1(\Gamma)}^{x_2(\Gamma)} \frac{\left(x^2 - x_1(\Gamma)^2\right)}{e^x - 1} dx +}$$

$$\mathbf{n_0\sigma_0 \int_{x_2(\Gamma)}^{\infty} \frac{\left(x_2^2(\Gamma) - x_1^2(\Gamma)\right)}{e^x - 1}}$$

$$\mathbf{R(\Gamma) \approx \frac{2}{l_0}\left[e^{-x_1}(1 - e^{-x_1} + x_1(1 - 2e^{-x_1}))\right]}$$

Where, $\mathbf{l_0 = 10\ Mpc}$ $\quad$ $\mathbf{x_1 = \frac{(E - \Delta)m_p}{2kT_{CMB}E_p} = \frac{10^{20.5\ eV}}{E_p}}$

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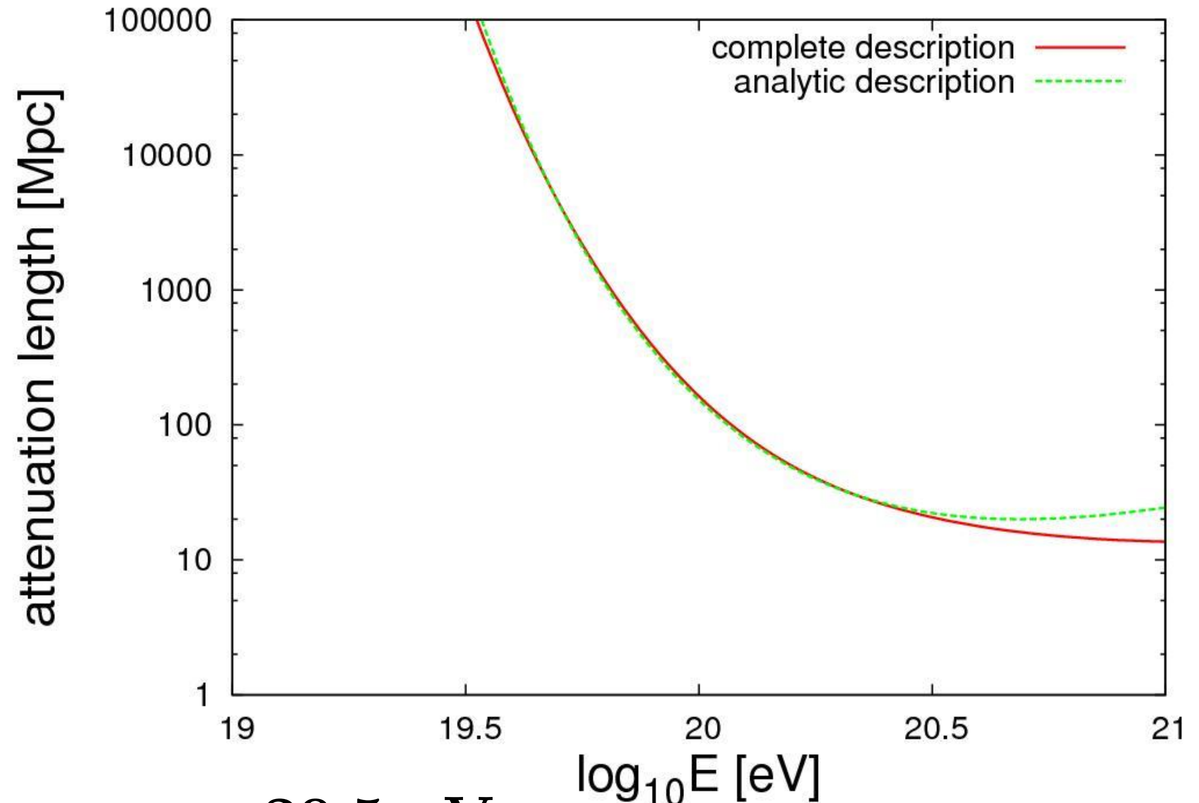
Photo-Pion Production Rate: Blackbody Interactions

With, $\mathbf{kT_{CMB} \approx 2 \times 10^{-4}\ eV}$

$$\mathbf{R \approx 0.2\sigma_{p\gamma} \int_{\frac{E-\Delta}{2\Gamma}}^{\frac{E+\Delta}{2\Gamma}} d\epsilon_\gamma \frac{dn}{d\epsilon_\gamma}}$$

$$\mathbf{\approx \left(\frac{l_0}{e^{-x_1}(1 - e^{-x_1})} \right)^{-1}}$$



Where $\mathbf{l_0}$ is 5 Mpc and $\mathbf{x_1 = \frac{10^{20.5}\ eV}{E_p}}$

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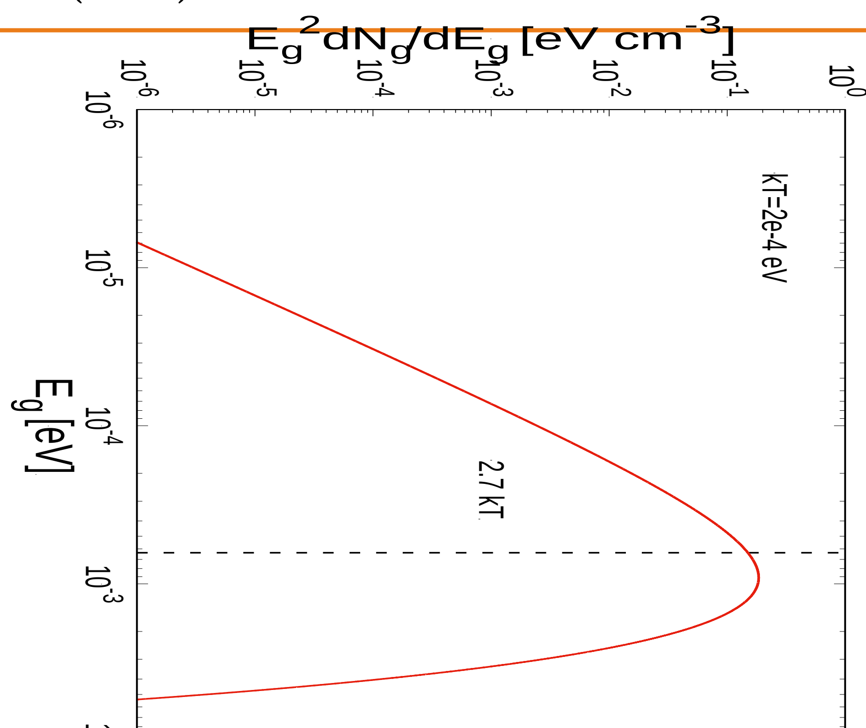
DESY.

CMB- Total Energy Density

$$\rho_{\gamma}^{\mathbf{BB}} = \frac{8\pi(\mathbf{kT})^4}{(\mathbf{hc})^3}\int_0^{\infty}\frac{\mathbf{x}^3}{\mathbf{e}^{\mathbf{x}}-1}\mathbf{dx}$$

$$\rho_{\gamma}^{\mathbf{CMB}} = 8\pi\frac{(\mathbf{kT_{CMB}})^4}{(\mathbf{hc})^3}\gamma(4)\zeta(4) \approx \mathbf{0.25\ eV\ cm^{-3}}$$



An Analytic Description of these Results

Differential Equation Describing System State

$$\frac{\mathbf{d}}{\mathbf{dt}}\begin{pmatrix} f_{56} \\ f_{55} \\ f_{54} \end{pmatrix} = \mathbf{\Lambda}\begin{pmatrix} f_{56} \\ f_{55} \\ f_{54} \end{pmatrix}$$

$$\mathbf{\Lambda} = \begin{pmatrix} -\left(\frac{1}{\tau_{56\to55}}+\frac{1}{\tau_{56\to54}}+\dots\right) & 0 & 0 \\ \frac{1}{\tau_{56\to55}} & -\left(\frac{1}{\tau_{55\to54}}+\frac{1}{\tau_{55\to53}}+\dots\right) & 0 \\ \frac{1}{\tau_{56\to54}} & \frac{1}{\tau_{55\to54}} & -\left(\frac{1}{\tau_{54\to53}}+\frac{1}{\tau_{54\to52}}+\dots\right) \end{pmatrix}$$

by $$\mathbf{f_q(t) = \sum_{n=q}^{56} A_n f_n(t)}$$

then $$\mathbf{f_q(t) = \sum_{n=q}^{56} A_n e^{-\lambda_n t} f_n(0)}$$

(where A_n values are set by the initial conditions)

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Only Considering Single Nucleon Losses

$$\mathbf{\Lambda} = \begin{pmatrix} -\frac{1}{\tau_{56\to55}} & 0 & 0 \\ \frac{1}{\tau_{56\to55}} & -\frac{1}{\tau_{55\to54}} & 0 \\ 0 & \frac{1}{\tau_{55\to54}} & -\frac{1}{\tau_{54\to53}} \end{pmatrix}$$

and

$$\mathbf{f_q(t) = \sum_{n=q}^{56} f_{56}(0) \frac{\tau_q \tau_n^{56-q-1}}{\prod_{p=q}^{56}(\tau_n - \tau_p)} e^{-\frac{t}{\tau_n}}}$$

Nuclear Cascade Description

Consider

$$\frac{\mathbf{df_q}}{\mathbf{dt}} + \frac{\mathbf{f_q}}{\tau_\mathbf{q}} = \frac{\mathbf{f_{q+1}}}{\tau_\mathbf{q+1}}$$

$$\mathbf{e}^{\left(\frac{-\mathbf{t}}{\tau_\mathbf{q}}\right)} \frac{\mathbf{d}}{\mathbf{dt}} \left[\mathbf{e}^{\left(\frac{\mathbf{t}}{\tau_\mathbf{q}}\right)} \mathbf{f_q}\right] = \frac{\mathbf{f_{q+1}}}{\tau_\mathbf{q+1}}$$

$$\mathbf{f_q} = \mathbf{e}^{\left(\frac{-\mathbf{t}}{\tau_\mathbf{q}}\right)} \int \mathbf{e}^{\left(\frac{\mathbf{t}}{\tau_\mathbf{q}}\right)} \frac{\mathbf{f_{q+1}}}{\tau_\mathbf{q+1}} \mathbf{dt}$$

Assume solution is true for q, apply to q+1

$$\frac{\mathbf{f_{q+1}(t)}}{\mathbf{f_{56}(0)}} = \sum_{\mathbf{n=q+1}}^{\mathbf{56}} \frac{\tau_\mathbf{q+1} \tau_\mathbf{n}^{\mathbf{56-q-2}}}{\prod_{\mathbf{p=q+1}}^{\mathbf{56}} (\tau_\mathbf{n} - \tau_\mathbf{p})} \mathbf{e}^{-\frac{\mathbf{t}}{\tau_\mathbf{n}}}$$

Nuclear Cascade Description

Assume solution is true

$$\frac{\mathbf{f_{q+1}(t)}}{\mathbf{f_{56}(0)}} = \sum_{\mathbf{n=q+1}}^{\mathbf{56}} \frac{\tau_{\mathbf{q+1}} \tau_{\mathbf{n}}^{\mathbf{56-q-2}}}{\prod_{\mathbf{p=q+1}}^{\mathbf{56}} (\tau_{\mathbf{n}} - \tau_{\mathbf{p}})} \mathbf{e}^{-\frac{\mathbf{t}}{\tau_{\mathbf{n}}}}$$

$$\mathbf{f_q} = \mathbf{e}^{\left(\frac{-\mathbf{t}}{\tau_{\mathbf{q}}}\right)} \int \mathbf{e}^{\left(\frac{\mathbf{t}}{\tau_{\mathbf{q}}}\right)} \frac{\mathbf{f_{q+1}}}{\tau_{\mathbf{q+1}}} \mathbf{dt}$$

$$\frac{\mathbf{f_q(t)}}{\mathbf{f_{56}(0)}} = \sum_{\mathbf{n=q+1}}^{\mathbf{56}} \frac{\tau_{\mathbf{n}}^{\mathbf{56-q-2}}}{\prod_{\mathbf{p=q+1}}^{\mathbf{56}} (\tau_{\mathbf{n}} - \tau_{\mathbf{p}})} \left[\left(\frac{\mathbf{1}}{\tau_{\mathbf{q}}} - \frac{\mathbf{1}}{\tau_{\mathbf{n}}} \right)^{-\mathbf{1}} \mathbf{e}^{\frac{-\mathbf{t}}{\tau_{\mathbf{n}}}} \right] - \mathbf{c}\mathbf{e}^{\frac{-\mathbf{t}}{\tau_{\mathbf{q}}}}$$

Since $\mathbf{f_q(0)} = \mathbf{0}$

$$\mathbf{c} = \sum_{\mathbf{n=q+1}}^{\mathbf{56}} \frac{\tau_{\mathbf{q}} \tau_{\mathbf{n}}^{\mathbf{56-q-1}}}{\prod_{\mathbf{p=q}}^{\mathbf{56}} (\tau_{\mathbf{n}} - \tau_{\mathbf{p}})}$$

Nuclear Cascade Description

$$\frac{\mathbf{f_q(t)}}{\mathbf{f_{56}(0)}} = \sum_{\mathbf{n=q+1}}^{\mathbf{56}} \frac{\tau_{\mathbf{n}}^{\mathbf{56-q-2}}}{\prod_{\mathbf{p=q}}^{\mathbf{56}}(\tau_{\mathbf{n}} - \tau_{\mathbf{p}})} \mathbf{e}^{\frac{-\mathbf{t}}{\tau_{\mathbf{n}}}} - \sum_{\mathbf{n=q+1}}^{\mathbf{56}} \frac{\tau_{\mathbf{q}}\tau_{\mathbf{n}}^{\mathbf{56-q-1}}}{\prod_{\mathbf{p=q}}^{\mathbf{56}}(\tau_{\mathbf{n}} - \tau_{\mathbf{p}})} \mathbf{e}^{\frac{-\mathbf{t}}{\tau_{\mathbf{q}}}}$$

$$\frac{\mathbf{f_q(t)}}{\mathbf{f_{56}(0)}} = \sum_{\mathbf{n=q}}^{\mathbf{56}} \frac{\tau_{\mathbf{q}}\tau_{\mathbf{n}}^{\mathbf{56-q-1}}}{\prod_{\mathbf{p=q}}^{\mathbf{56}}(\tau_{\mathbf{n}} - \tau_{\mathbf{p}})} \mathbf{e}^{-\frac{\mathbf{t}}{\tau_{\mathbf{n}}}}$$

These are equivalent if:

$$\sum_{\mathbf{n=q+1}}^{\mathbf{56}} \frac{\tau_{\mathbf{q}}\tau_{\mathbf{n}}^{\mathbf{56-q-1}}}{\prod_{\mathbf{p=q}}^{\mathbf{56}}(\tau_{\mathbf{n}} - \tau_{\mathbf{p}})} = \frac{\tau_{\mathbf{q}}\tau_{\mathbf{q}}^{\mathbf{56-q-1}}}{\prod_{\mathbf{p=q}}^{\mathbf{56}}(\tau_{\mathbf{q}} - \tau_{\mathbf{p}})}$$

Consider:

$$\frac{\mathbf{w^2}}{(\mathbf{w-x})(\mathbf{w-y})(\mathbf{w-z})} + \frac{\mathbf{x^2}}{(\mathbf{x-w})(\mathbf{x-y})(\mathbf{x-z})} + \frac{\mathbf{y^2}}{(\mathbf{y-w})(\mathbf{y-x})(\mathbf{y-z})} = -\frac{\mathbf{z^2}}{(\mathbf{z-w})(\mathbf{z-x})(\mathbf{z-y})}$$

Nuclear Cascade Description

$$\sum_{\mathbf{n=q+1}}^{\mathbf{56}} \frac{\tau_{\mathbf{q}}\tau_{\mathbf{n}}^{\mathbf{56-q-1}}}{\prod_{\mathbf{p=q}}^{\mathbf{56}}(\tau_{\mathbf{n}}-\tau_{\mathbf{p}})} = \frac{\tau_{\mathbf{q}}\tau_{\mathbf{q}}^{\mathbf{56-q-1}}}{\prod_{\mathbf{p=q}}^{\mathbf{56}}(\tau_{\mathbf{q}}-\tau_{\mathbf{p}})}$$

Consider the case

$$\frac{\mathbf{w^2}}{(\mathbf{w}-\mathbf{x})(\mathbf{w}-\mathbf{y})(\mathbf{w}-\mathbf{z})} + \frac{\mathbf{x^2}}{(\mathbf{x}-\mathbf{w})(\mathbf{x}-\mathbf{y})(\mathbf{x}-\mathbf{z})} + \frac{\mathbf{y^2}}{(\mathbf{y}-\mathbf{w})(\mathbf{y}-\mathbf{x})(\mathbf{y}-\mathbf{z})} = -\frac{\mathbf{z^2}}{(\mathbf{z}-\mathbf{w})(\mathbf{z}-\mathbf{x})(\mathbf{z}-\mathbf{y})}$$

$$\begin{vmatrix} 1 & w & w^2 & w^2 \\ 1 & x & x^2 & x^2 \\ 1 & y & y^2 & y^2 \\ 1 & z & z^2 & z^2 \end{vmatrix} = \mathbf{0}$$

Cascade of Nuclei Through Species- single nucleon loss

Since nuclei Lorentz factor remains ~conserved, and cross-section varies mildly with A (nuclear mass)

$$\tau_{\mathbf{56 \rightarrow 55}} \approx \tau_{\mathbf{55 \rightarrow 54 \cdots}}$$

For the case $\tau_{\mathbf{56 \rightarrow 55}} = \tau_{\mathbf{55 \rightarrow 54 \cdots}}$

$$f_q = \frac{t^{(q_{max}-q)}}{\tau_q (q_{max}-q)!} e^{-t/\tau_q}$$

ie. Gaisser-Hillas type function!
(used to describe air showers)

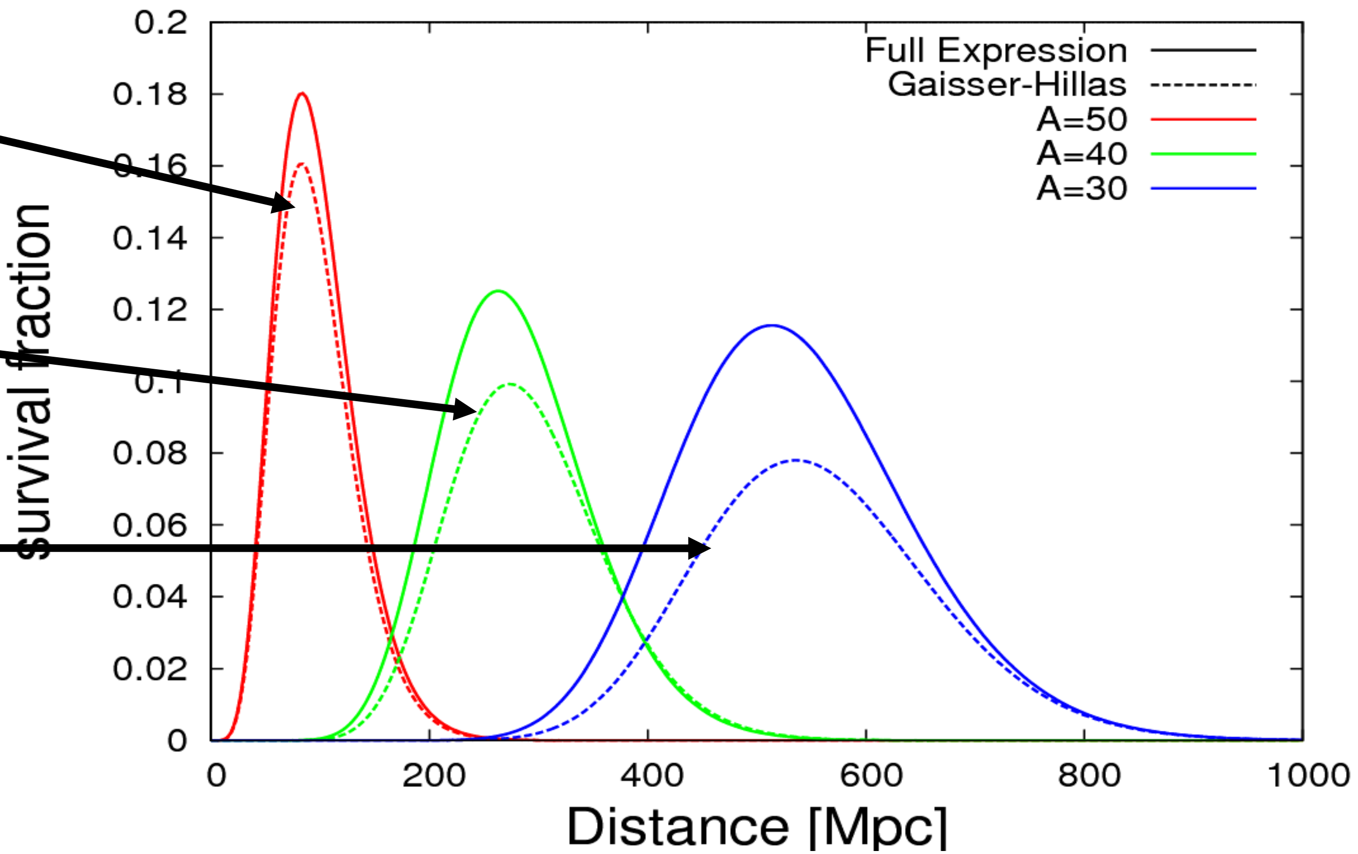
Cascade of Nuclei Through Species-Comparison of Approximation

Starting with Fe, $q_{\mathbf{max}} = \mathbf{56}$

$$f_{50} = \frac{t^6}{6!} e^{-\frac{t}{\tau_{50}}}$$

$$f_{40} = \frac{t^{16}}{16!} e^{-\frac{t}{\tau_{40}}}$$

$$f_{30} = \frac{t^{26}}{26!} e^{-\frac{t}{\tau_{30}}}$$



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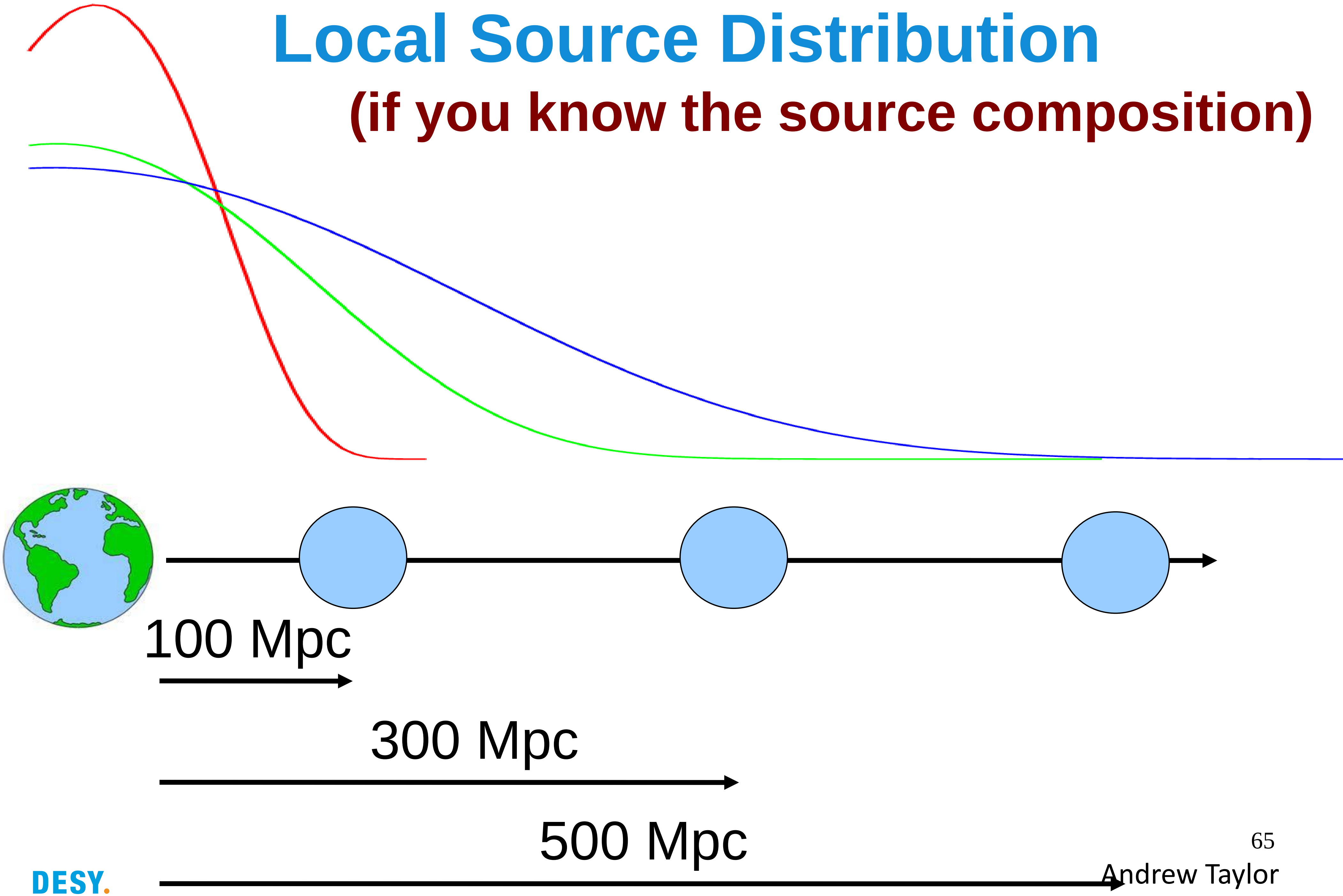
Composition – an Excellent Probe of the Local Source Distribution

(if you know the source composition)



Assumptions on Source Population

Spatial Distribution

motivated by star formation rate evolution

$$\frac{dN}{dV_C} \propto (1+z)^3 \qquad z < 1.9$$

$$\frac{dN}{dV_C} \propto (1+1.9)^3 \qquad 1.9 < z < 2.7$$

$$\frac{dN}{dV_C} \propto (1+1.9)^3 e^{-z/1.7} \qquad z > 2.7$$

Energy Distribution

motivated by Fermi acceleration theory

$$\frac{dN}{dE} \propto E^{-\alpha} \exp[-E/E_{Z,max}]$$

$$E_{Z,max} = (Z/26) \times E_{Fe,max}$$

Note- magnetic field horizon effects are neglected in the following.
This amounts to assuming: $d_s < (ct_H \lambda_{scat})^{1/2}$
ie. the source distribution may be approximated to be spatially continuous (also note, presence of t_H term comes from temporally continuous assumption)