Gravitational waves: basics and current challenges

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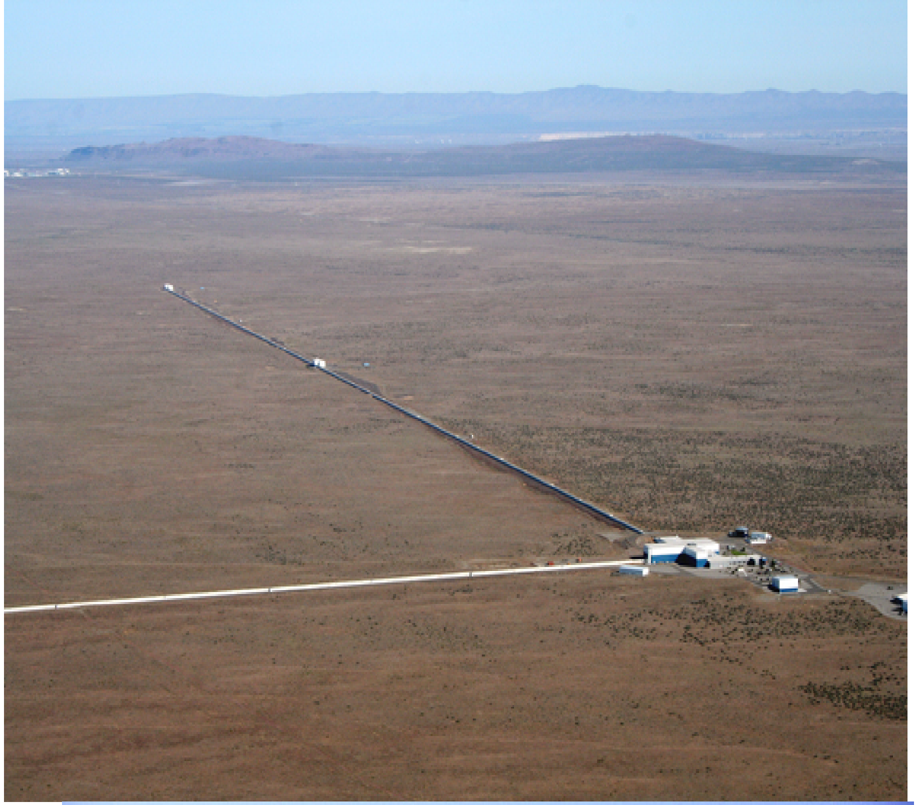
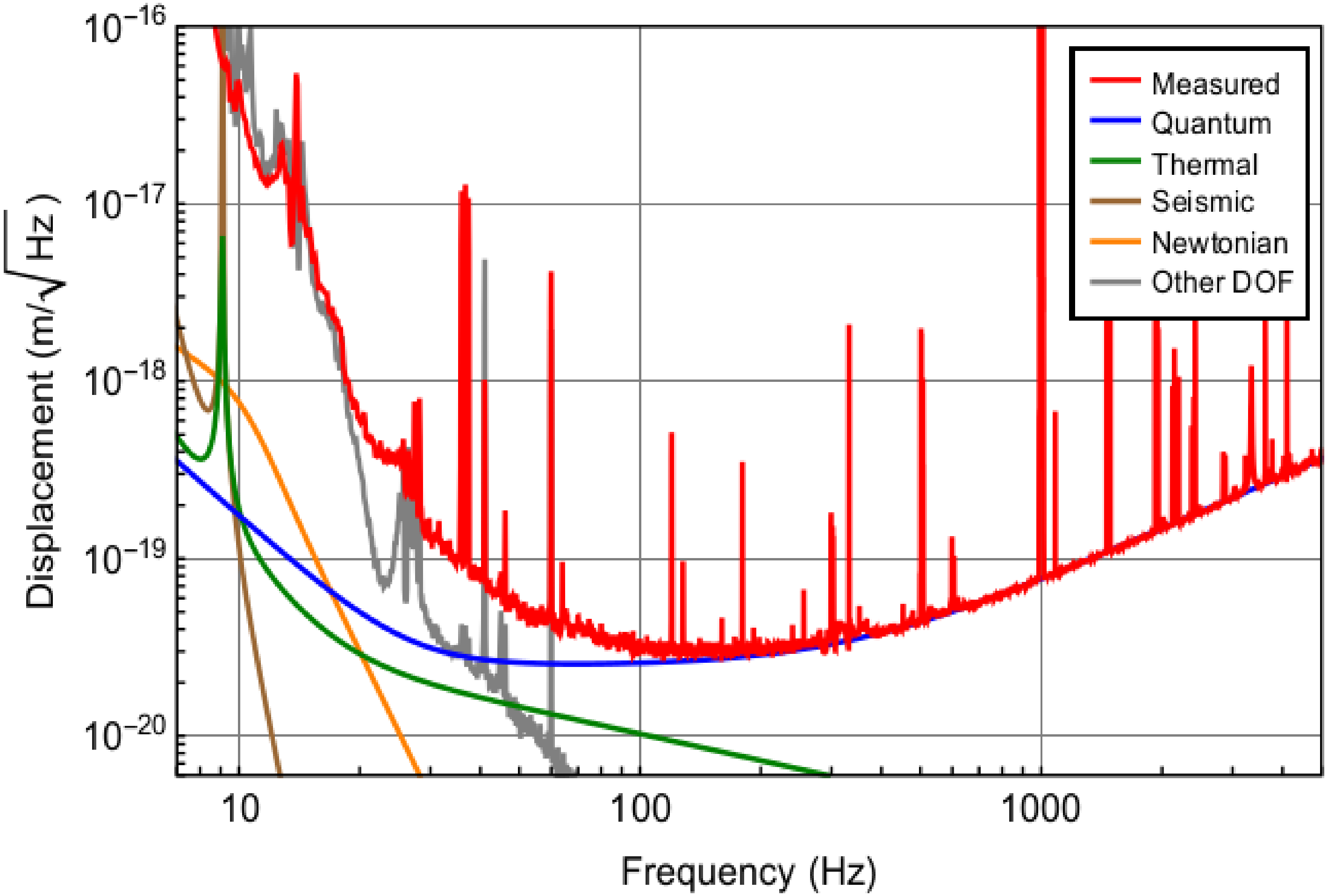
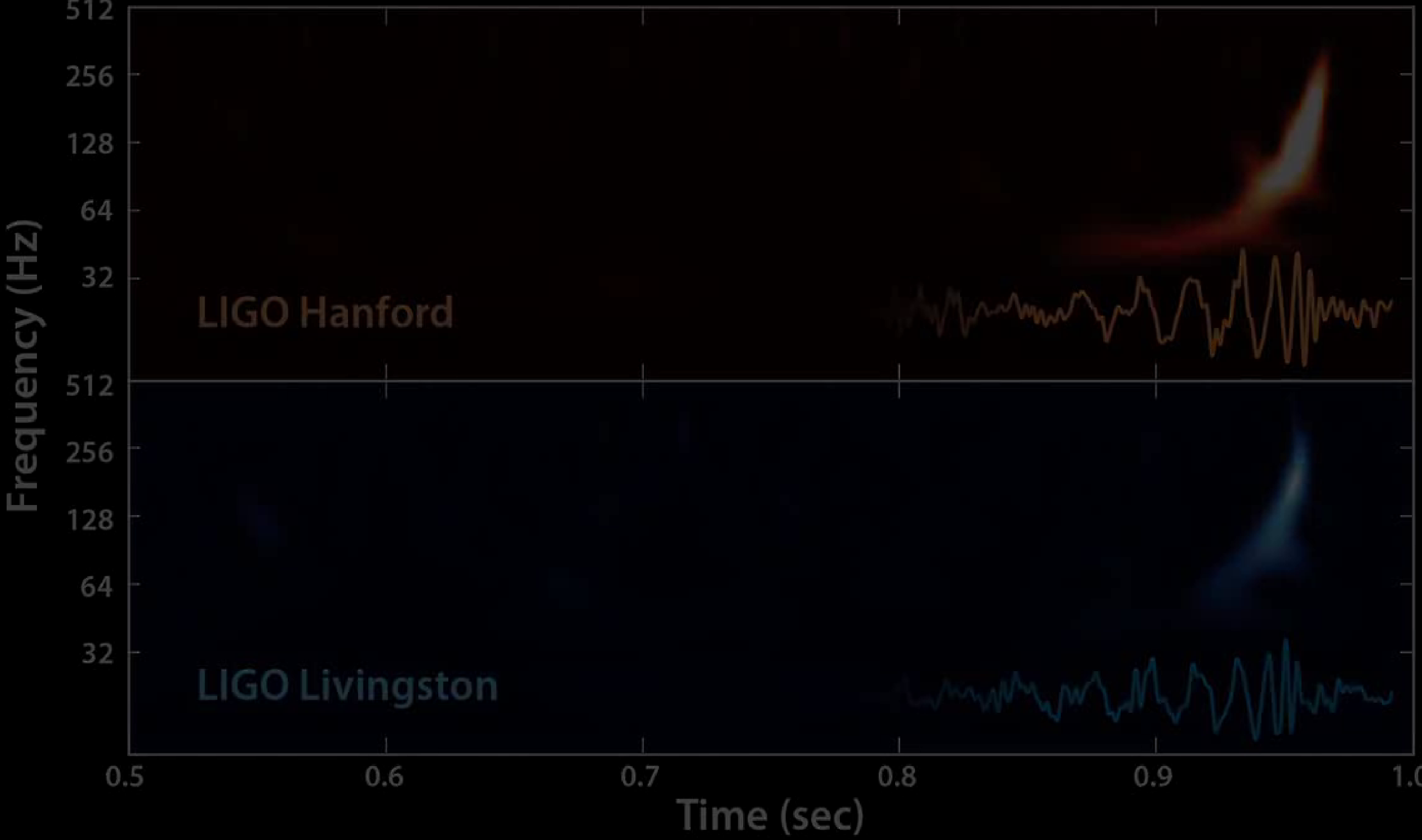


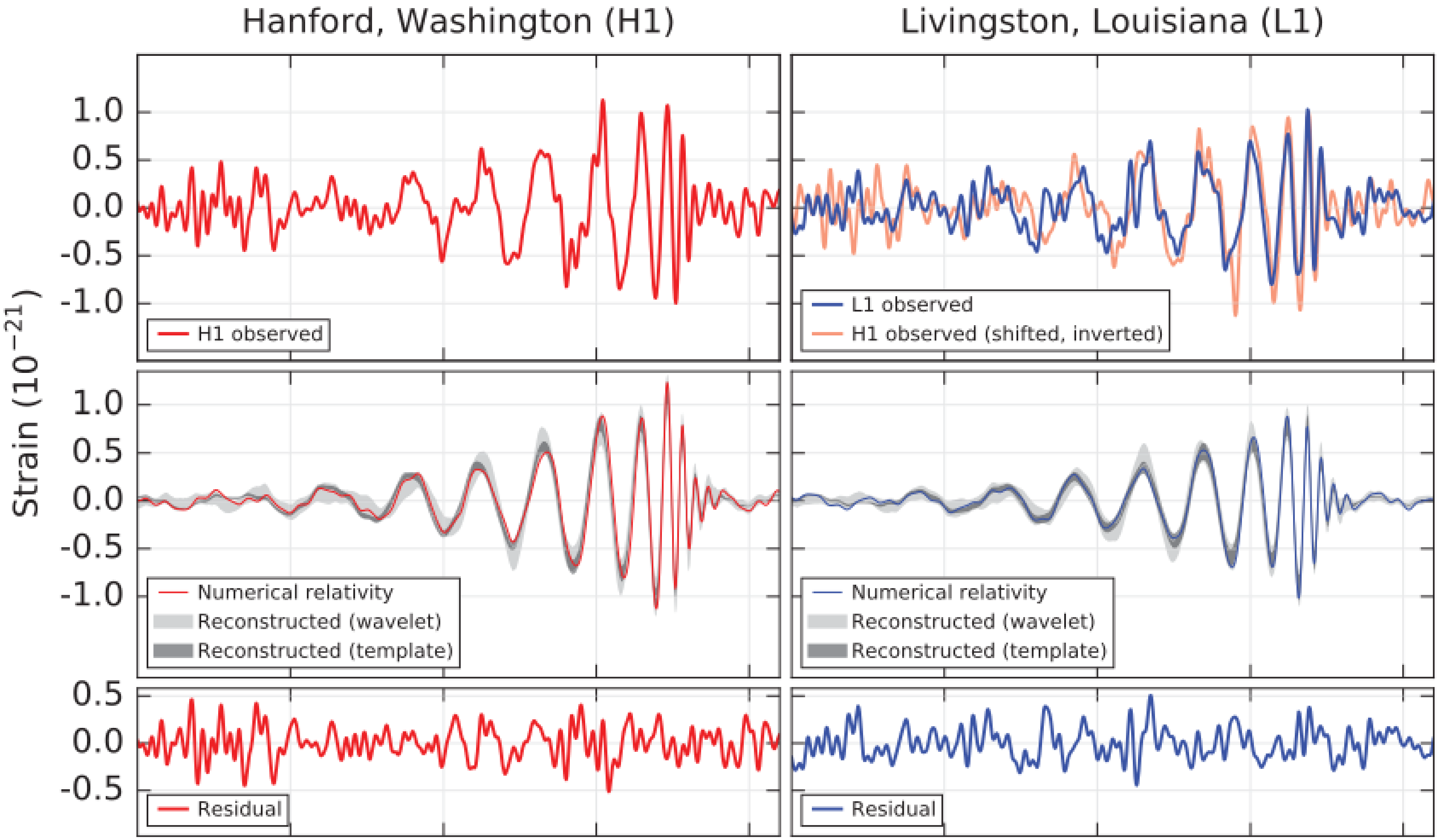
LIGO Sensitivity in 2015



$$h_{min}(100\text{Hz}) \approx 10^{-22}$$


512
256
128
64
32
LIGO Hanford
512
256
128
64
32
LIGO Livingston
Frequency (Hz)
0.5
0.6
0.7
0.8
0.9
1.0
Time (sec)

September 14, 2015 event

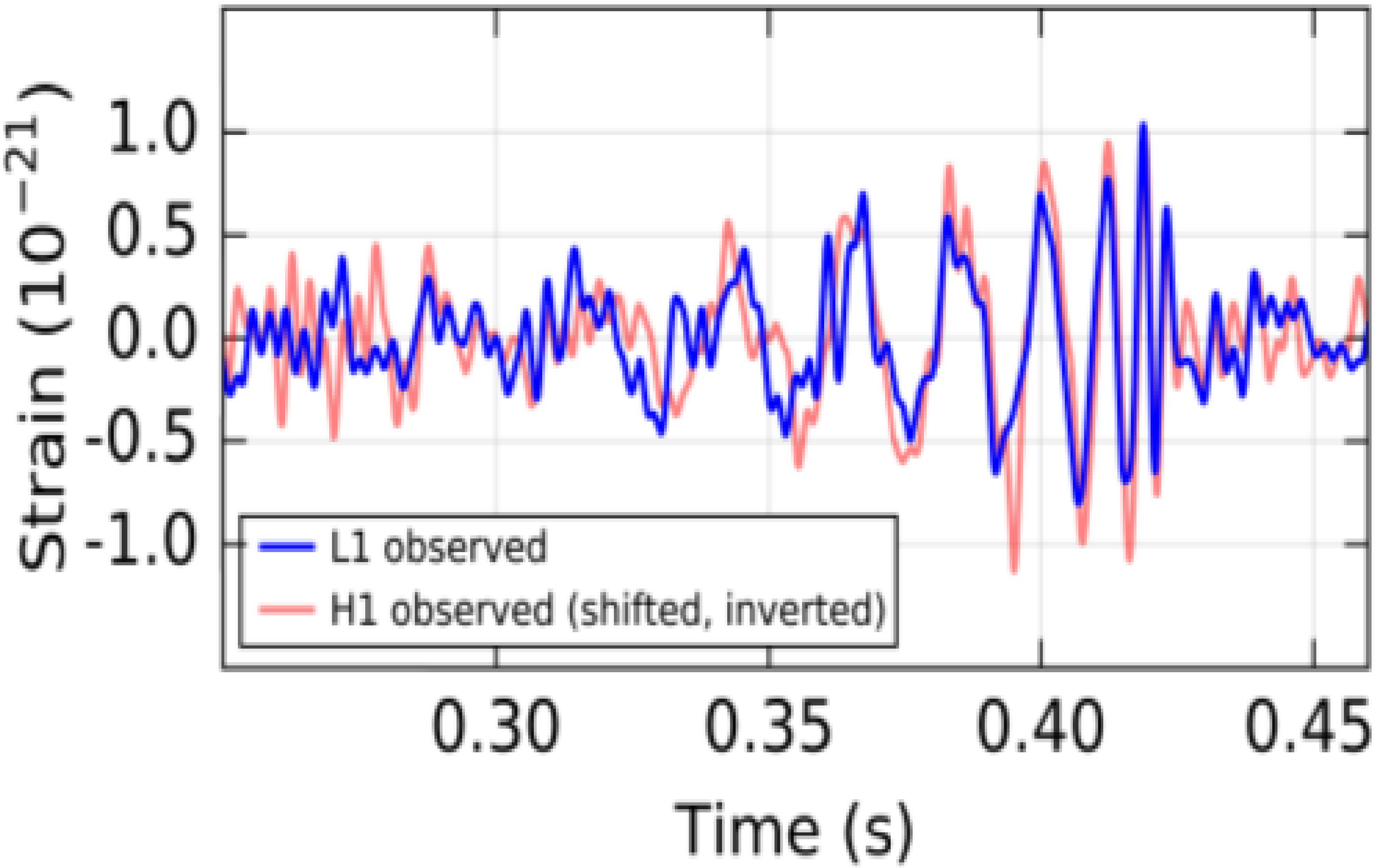


GW150914

- Significant signal in both detectors
- Shape of the signal consistent in both detectors
- 6.9 ms delay with 3000km (10ms) distance

- This is a real astrophysical signal!

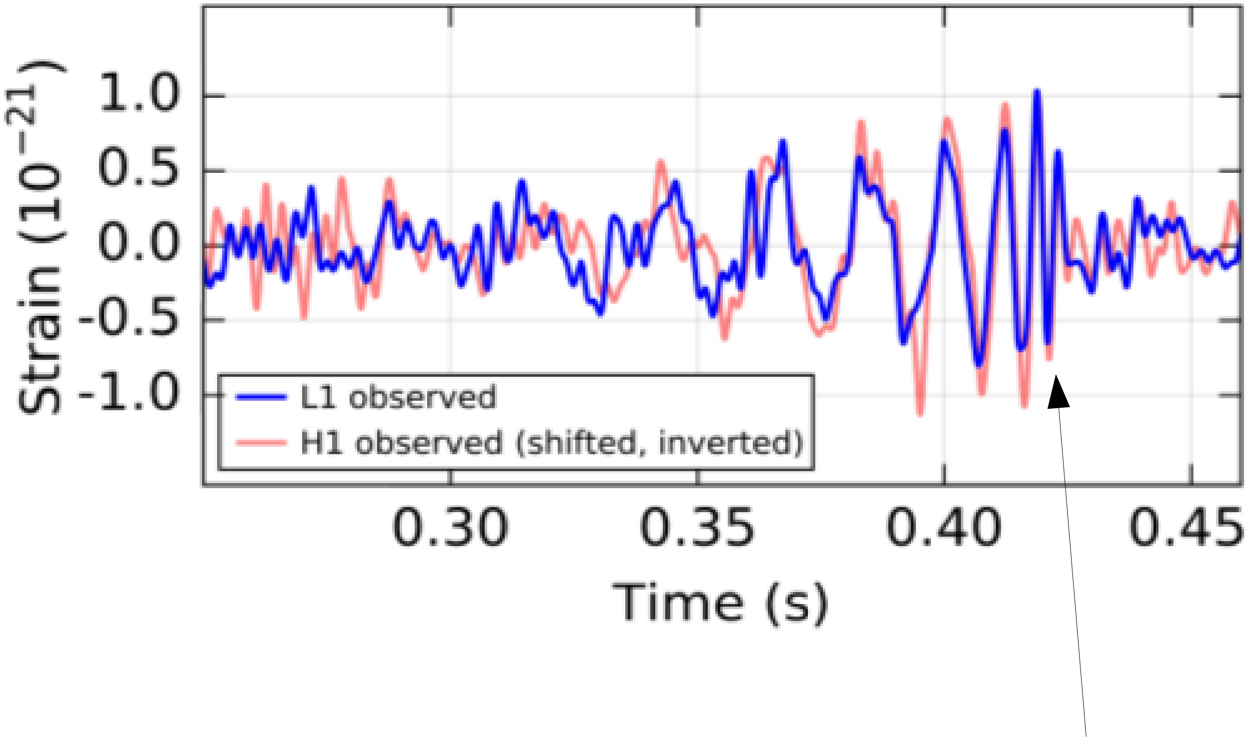
What is it?


Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
0.30
0.35
0.40
0.45
Time (s)
L1 observed
H1 observed (shifted, inverted)

What can you see with naked eye?

- Oscillations
- Increasing frequency
- Increasing amplitude
- Maximum and fast decay with a constant frequency
- Hipothesis:
  - Oscillations of a physical system ?
  - Orbital motion

Maximum frequency



$$P_{min} \approx 7\text{ms} \qquad f_{GW,max} = 150\text{Hz}$$

$$\omega_{Kep,max} = 2\pi \frac{f_{GW,max}}{2} = 2\pi \times 75\text{Hz} \qquad \omega = \sqrt{\frac{GM}{A^3}}$$

Short calculation of energy losses

$$L_{GW} = -\frac{dE}{dt} = \frac{32}{5}\frac{G^4}{c^5}\frac{M^3\mu^2}{A^5}$$

$$E = -\frac{1}{2}\frac{GM\mu}{A}$$

$$\Omega^2 = \frac{GM}{A^3}$$

$$\frac{d\Omega}{dt} = \frac{96}{5}\frac{G^{5/3}M^{2/3}\mu}{c^5}\Omega^{11/3}$$

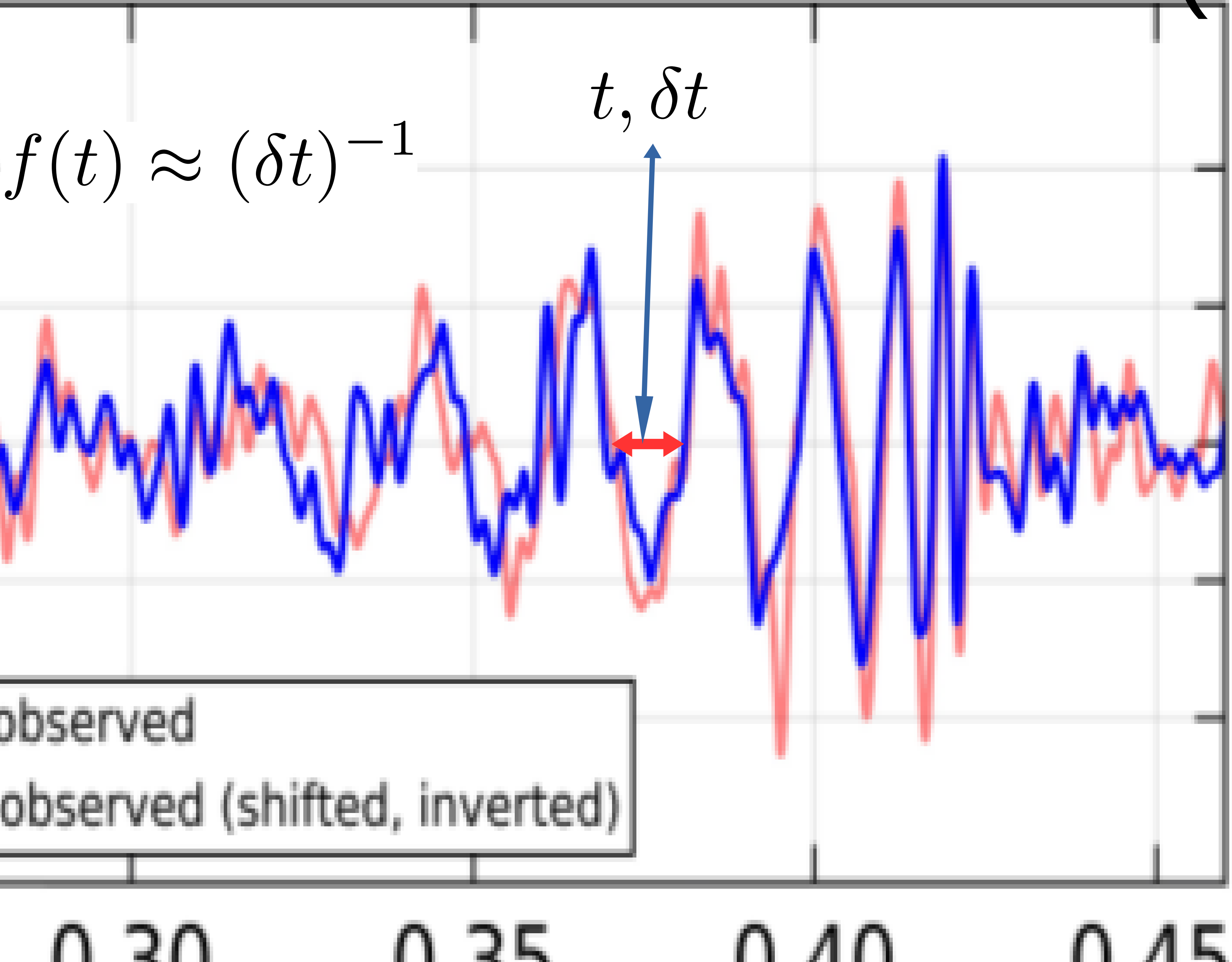
Frequency increase rate

$$\mathcal{M}^5 = \left(\frac{c^3}{G}\right)^5 \left(\frac{5}{96}\right)^3 (2\pi)^{-8} f_{GW}^{-11} \dot{f}_{GW}^3$$

Masa chirp: $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$

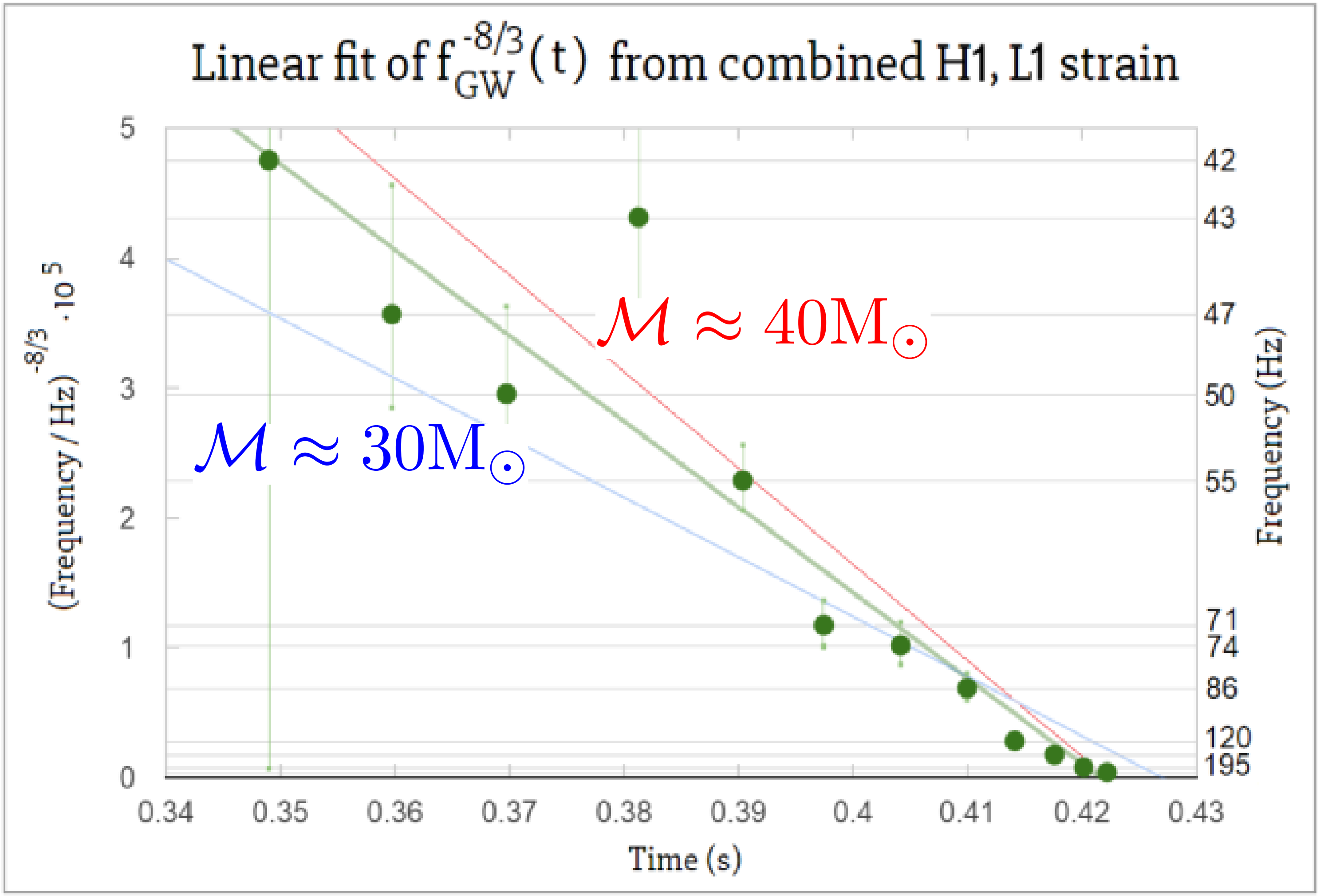
$$f_{GW}^{-8/3} = \frac{3}{8} \left(\frac{96}{5}\right)^{1/3} (2\pi)^{8/3} \left(\frac{G\mathcal{M}}{c^3}\right)^{5/3} (t_c - t)$$

How to determine f(t) ?



$$f(t) \approx (\delta t)^{-1}$$

$t, \delta t$



Chirp mass: $\mathcal{M} \approx 37\mathrm{M}_\odot$

IF $m_1 = m_2$ then $m_1 = m_2 = 2^{1/5}\mathcal{M} = 42.5\mathrm{M}_\odot$

Size of the system

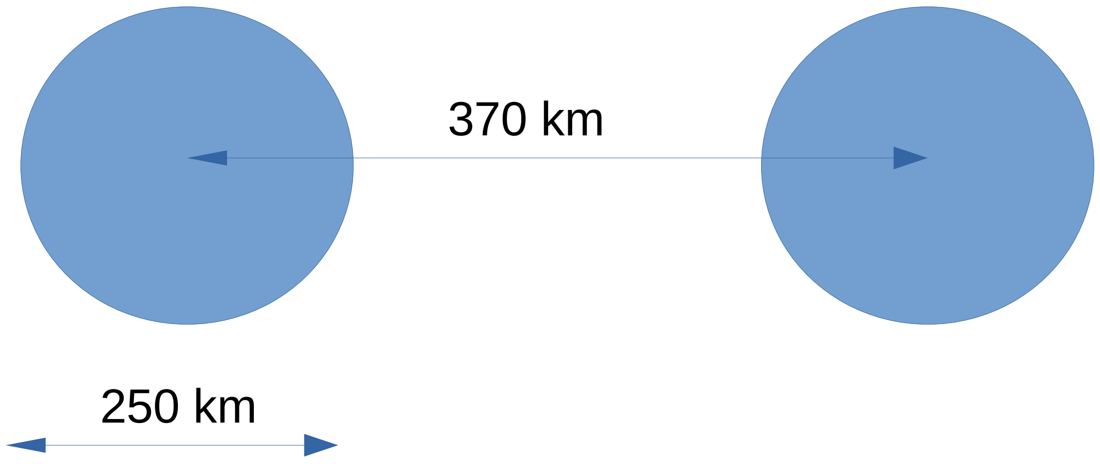
$$R_{Sch} = \frac{2GM}{c^2} = 2.95 \frac{M}{M_\odot}\,\mathrm{km}$$

$$R_{sch}(42.5 M_\odot) = 125\mathrm{km}$$

Orbit size:

$$A = \left(\frac{GM}{\omega^2}\right)^{1/3} = 370\,\mathrm{km}\left(\frac{M}{85 M_\odot}\right)^{1/3}\left(\frac{f}{150\mathrm{Hz}}\right)^{-2/3}$$

Compactness

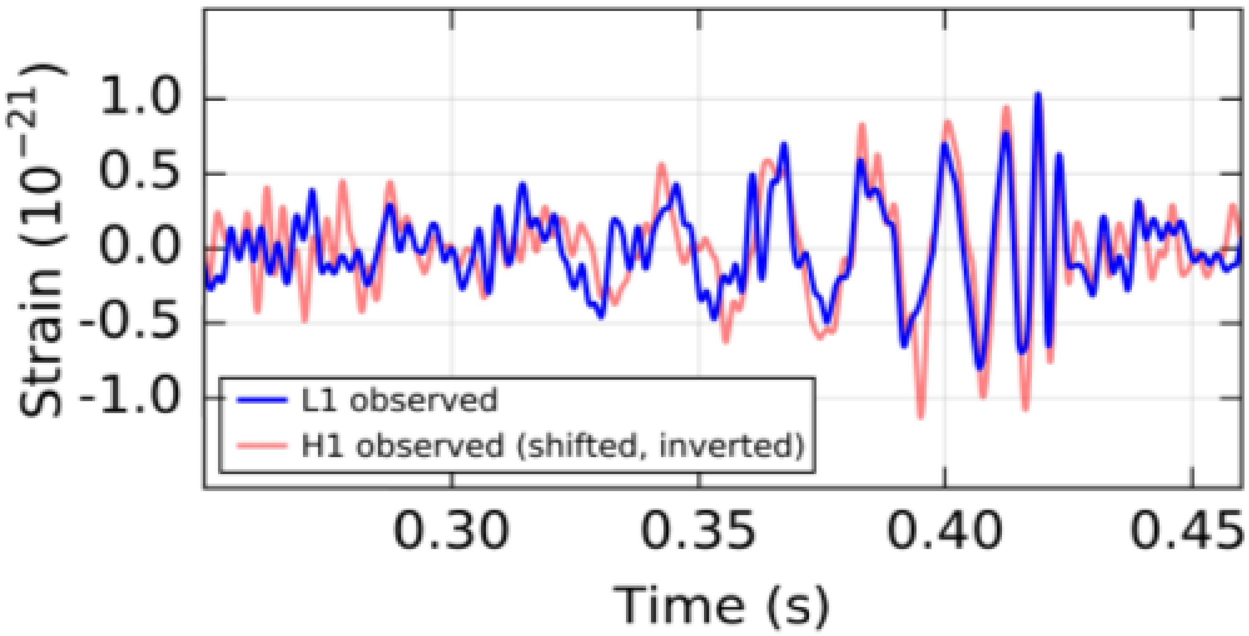


$$\mathcal{R} = \frac{A}{R_{Sch}} \approx 3.4$$

From Kepler law:

$$v = \omega r \approx \sqrt{\frac{1}{2\mathcal{R}}} \approx 0.5c$$

Luminosity, flux → distance



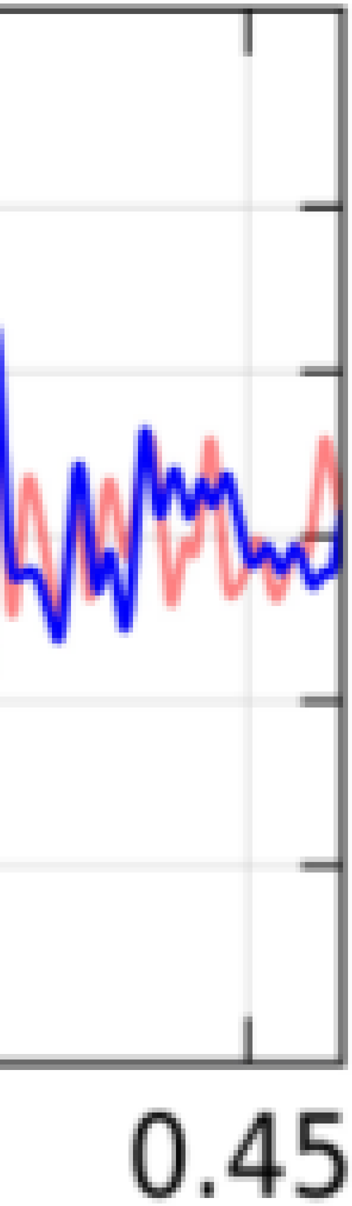
Amplitude:

$$h_{max} \approx 10^{-21}$$

$$L_{GW} = \frac{c^3}{16\pi G} \int\int |\dot{h}|^2 dS \approx \frac{c^5}{G} \frac{d_L^2 h^2 \omega_{GW}^2}{4c^2}$$

→ $d_L \approx 300\mathrm{Mpc}$

$$F_{GW} \approx \frac{c^5}{G} \frac{h^2 \omega_{GW}^2}{16\pi c^2} \approx 4 \frac{\mathrm{erg}}{\mathrm{cm^2 s}}$$



Ringdown

Reaching the Kerr form of the BH

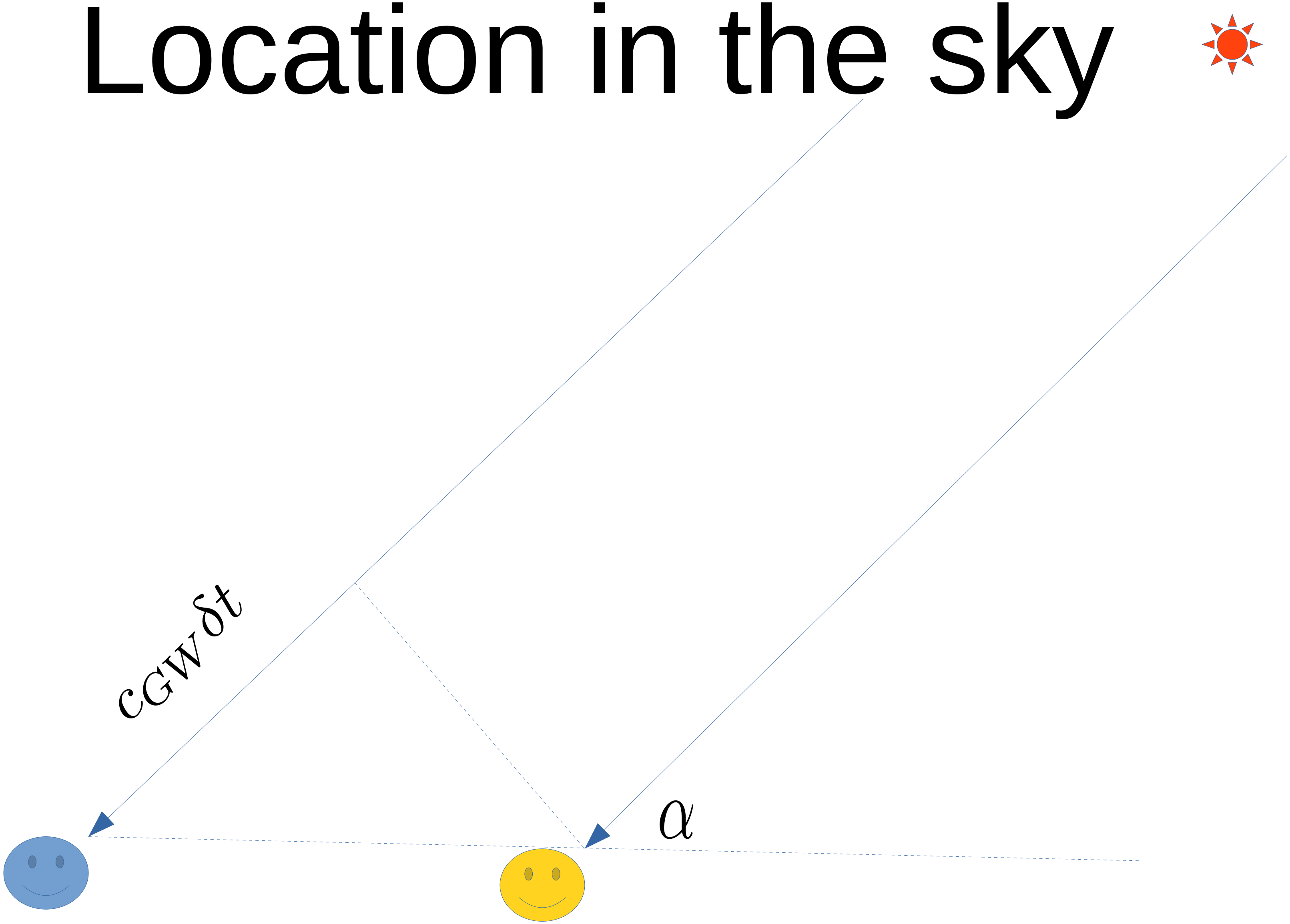
$$\frac{GM}{c^3}\omega_R = x + iy = 0.53 + 0.081i$$

Mod l=2 m=2 n=0, for a BH with spin 0.7

$$f_{GW} = 210Hz\left(\frac{80M_\odot}{M}\right)$$

$$\tau_{damp} = 5ms\left(\frac{M}{80M_\odot}\right)$$

Location in the sky



We assume that the gravitational wave speed is c.

A list of breakthroughs

- Detection of gravitational waves
- Detection of a black hole
- Detection of black hole binary
- Evidence for BHs with masses of 30 and and up to 60 solar masses
- Possibility to test General Relativity
- Possibility to test Quantum Gravity(?)
- The brightest source ever seen in the sky:

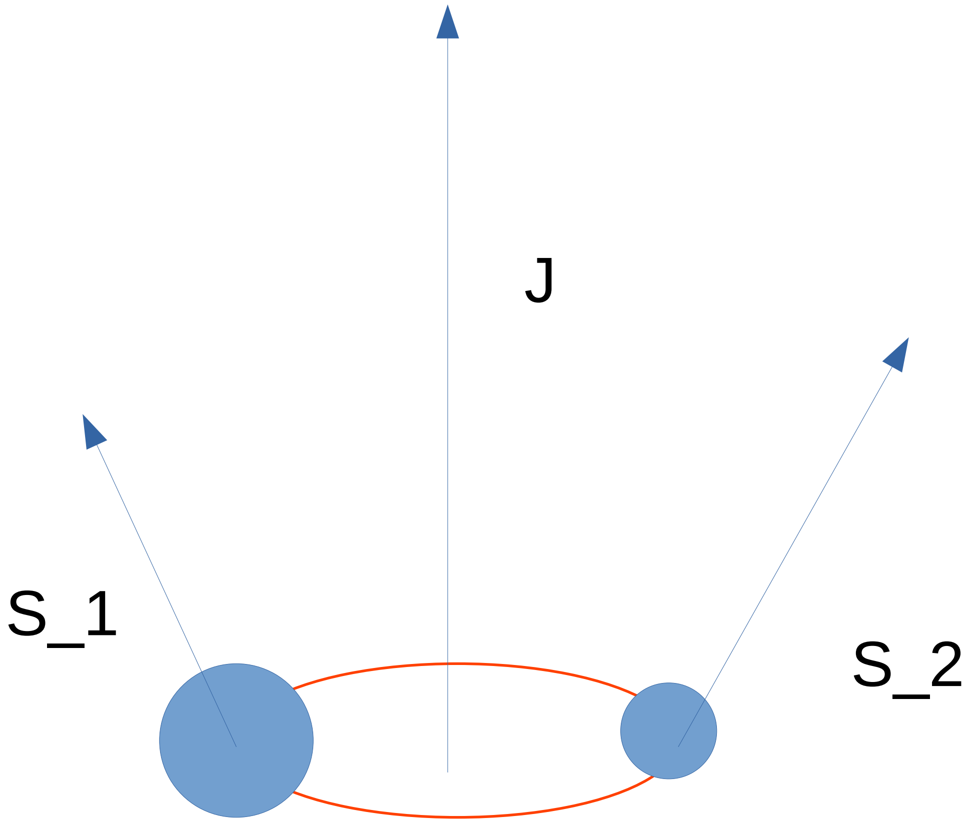
$$L_{GW} = 200^{+30}_{-20} \mathrm{M}_{\odot} s^{-1} = 3.6^{+0.5}_{-0.4} \times 10^{56} \mathrm{erg\ s}^{-1}$$

Current status of detections

- What can be measured:
  - Chirp mass
  - Mass and mass ratio
  - Effective spin
  - Effective precession
  - Statistical proporties

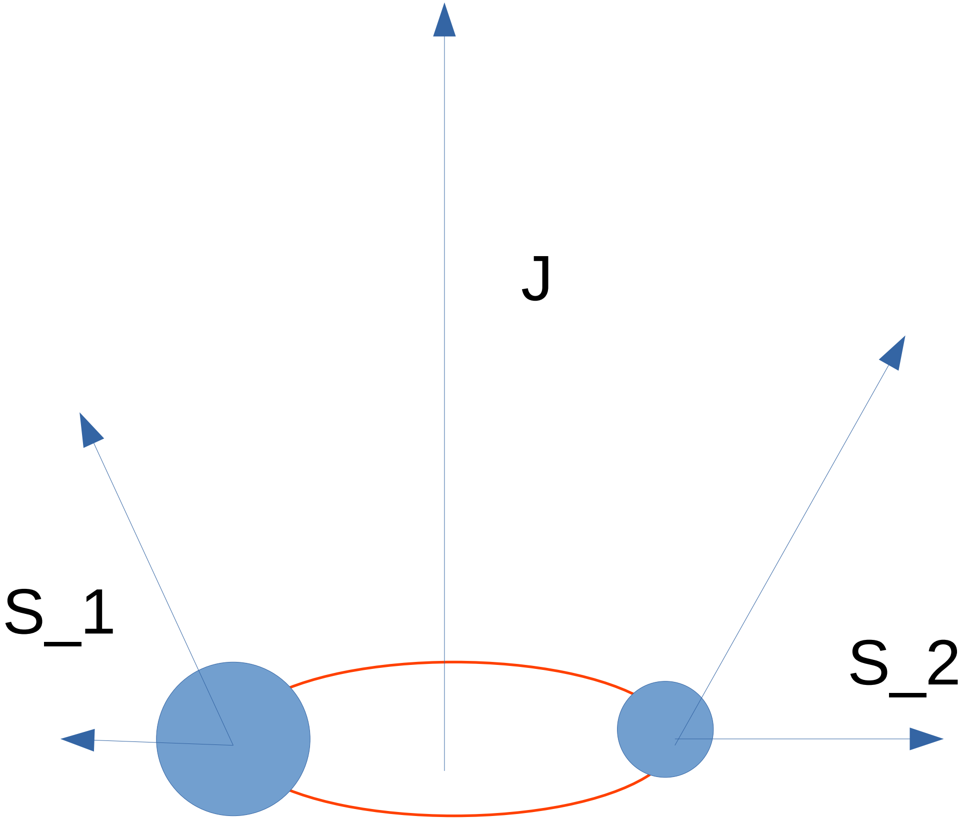
$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}.$$

Effective spin



$$\chi_{eff} = \frac{m_1\vec{s}_1 + m_2\vec{s}_2}{m_1 + m_2}\,\frac{\vec{J}}{|J|}$$

Effective precession spin

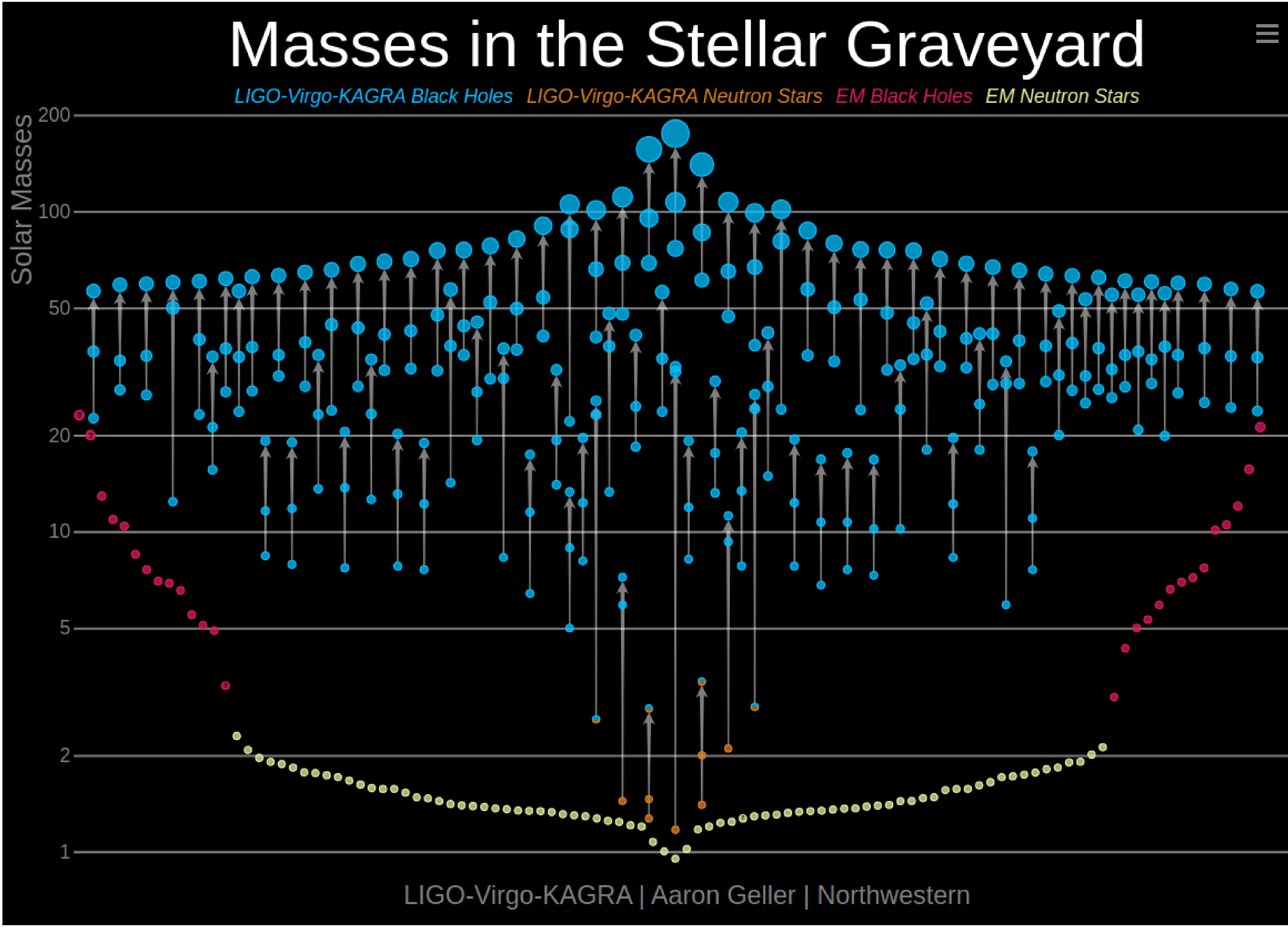


J

S_1

S_2

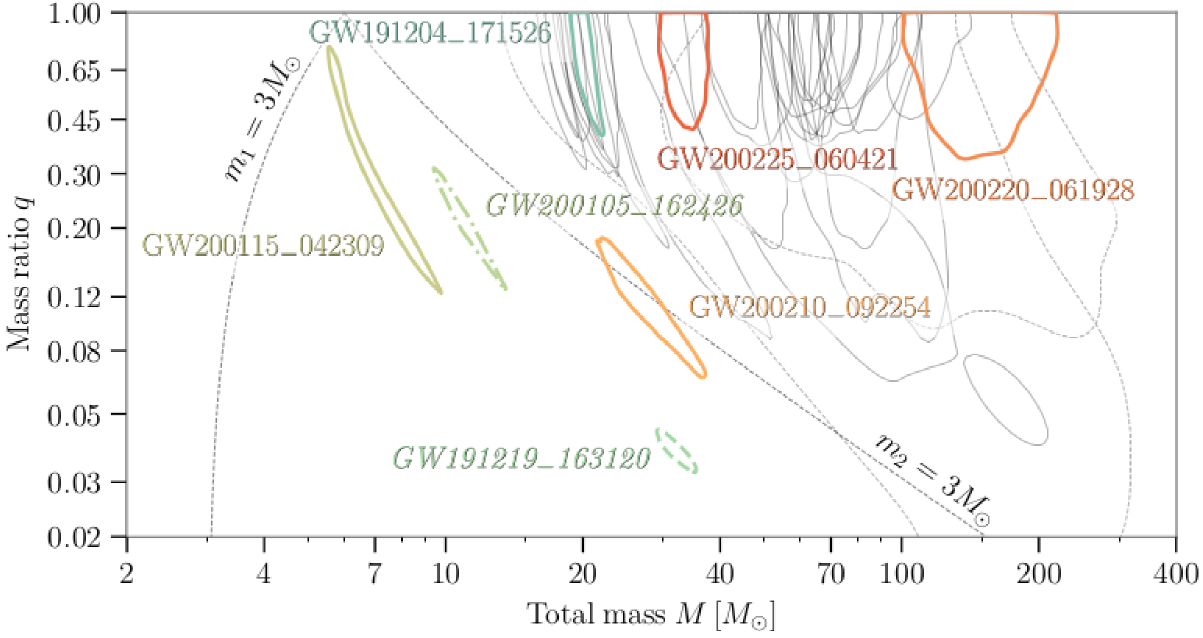
$$\chi_p = \max\left[|s_1|\sin\theta_1, \left(\frac{4q+3}{4+3q}\right) q|s_2|\sin\theta_2\right]$$

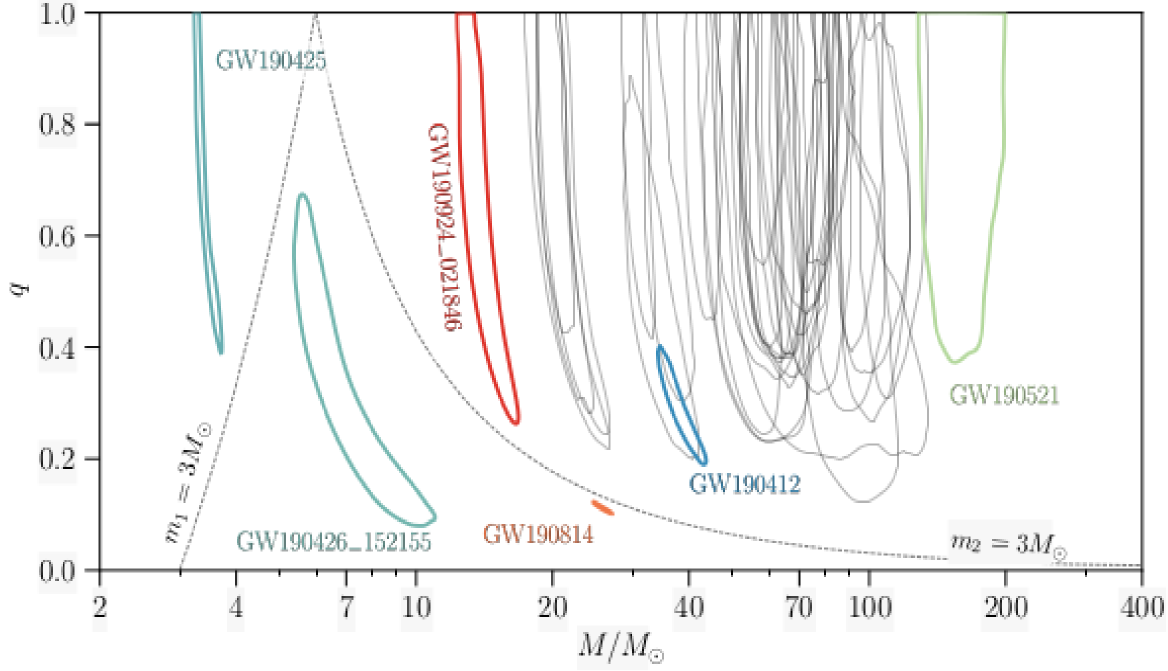


Masses in the Stellar Graveyard

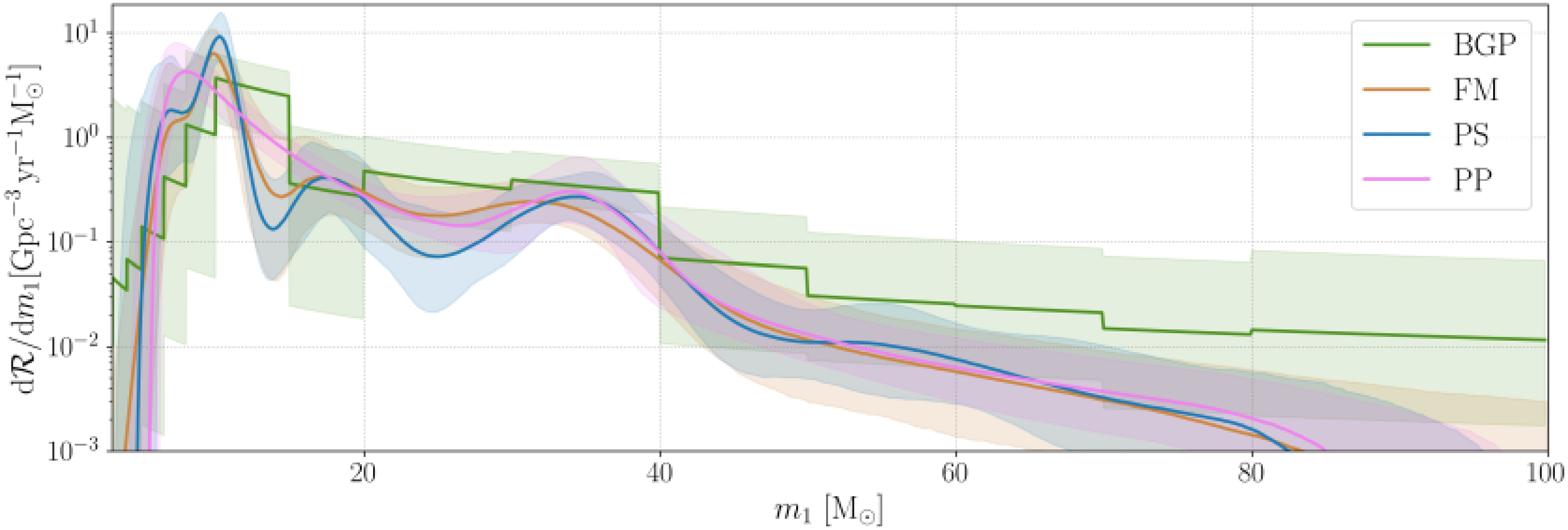
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

Masses and mass ratios





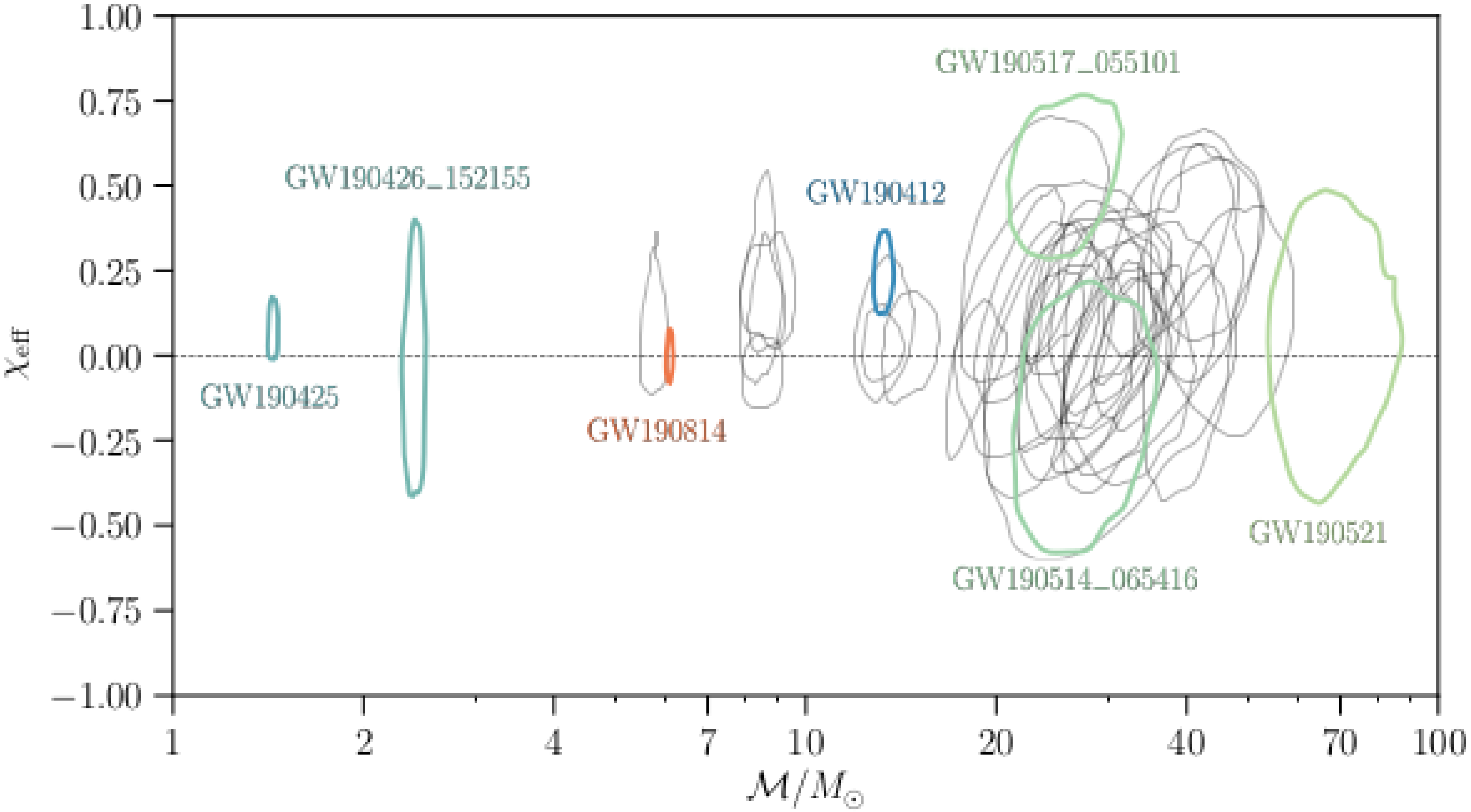
Primary mass



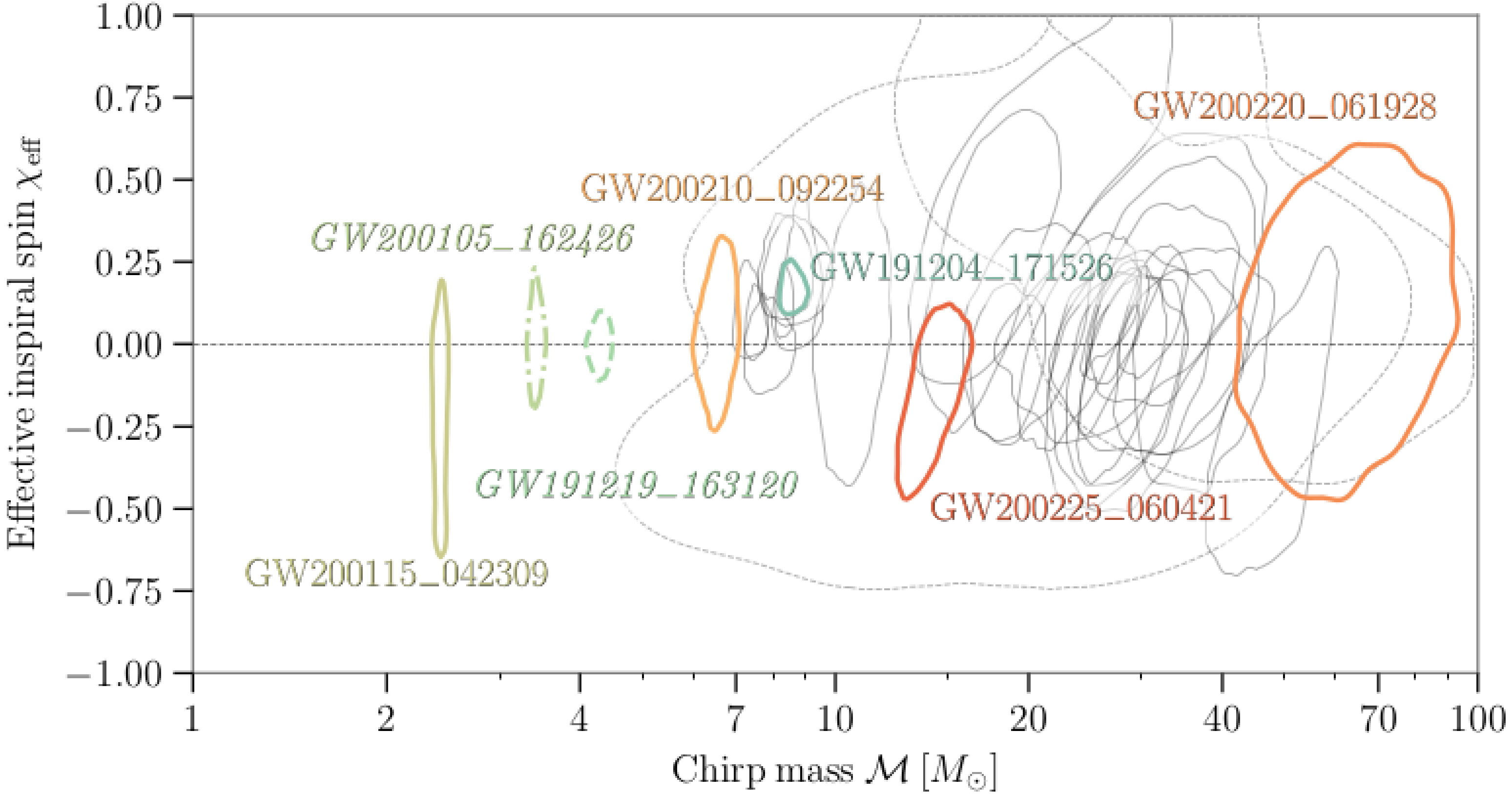
Peaks in the stellar mass region

Long tail to high masses

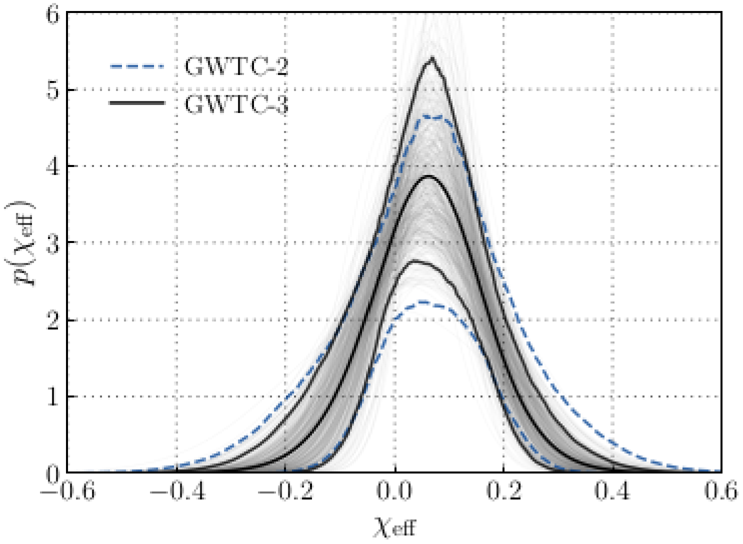
Spins and masses

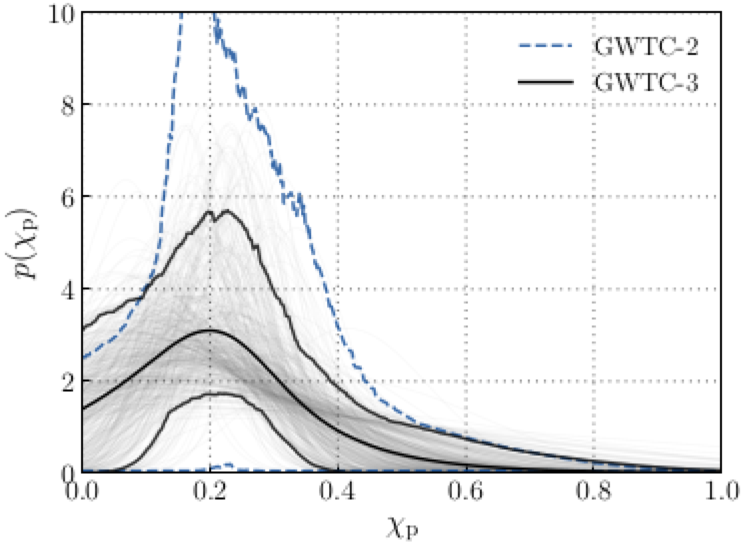


Spins and masses



Spin distribution

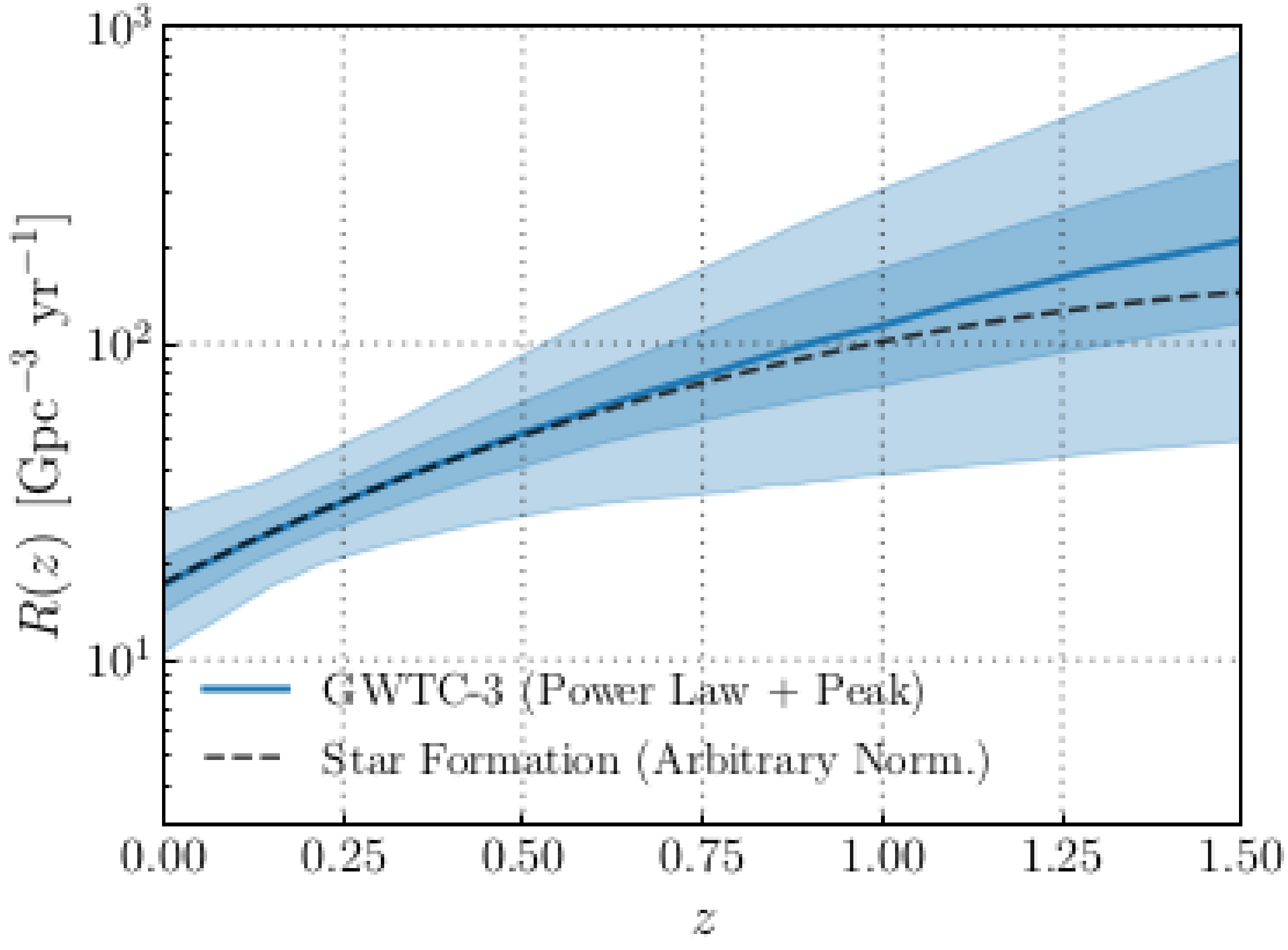




Slight tendency toward positive values

Spins are small

Rates vs redshift



Challenges in formation

- Black hole masses and spins
  - Not a real problem...
- Orbital separation
  - Need to work a little…
- Rate
  - There is quite a lot of them...

What is their origin?

- Stellar models
  - Binary evolution (filed, chemically homogenous, etc.)
  - Cluster evolution (including nuclear cluster
- Primordial BHs

Isolated binary evolution

- Masses
  - must come from stellar evolution
  - PPS mass maximum ~ 60-70 Msun
- Effective spins
  - should be aligned at least partially
  - Small or large?
- Rates
  - Should follow SFR

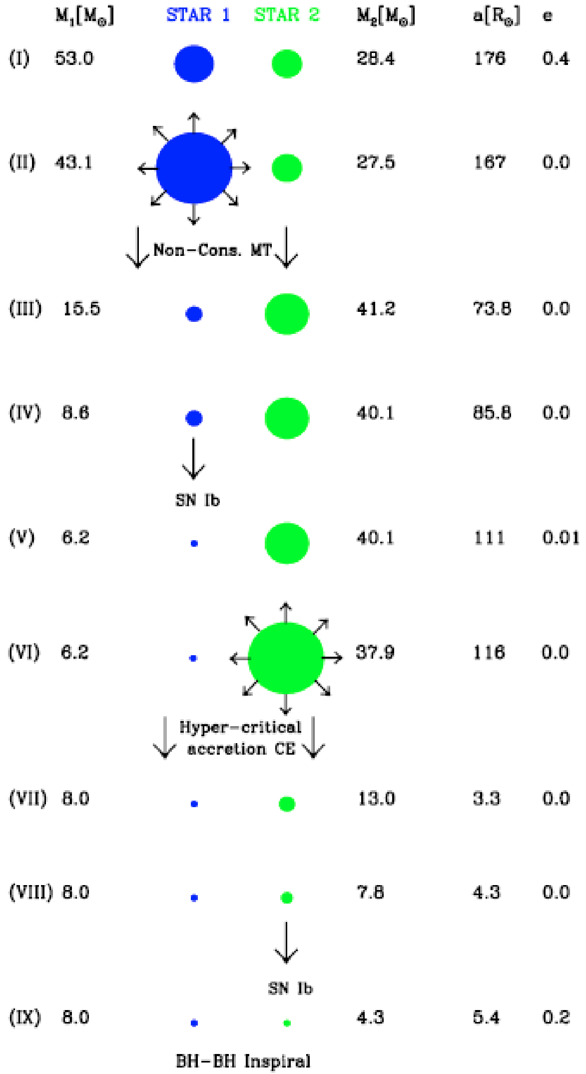
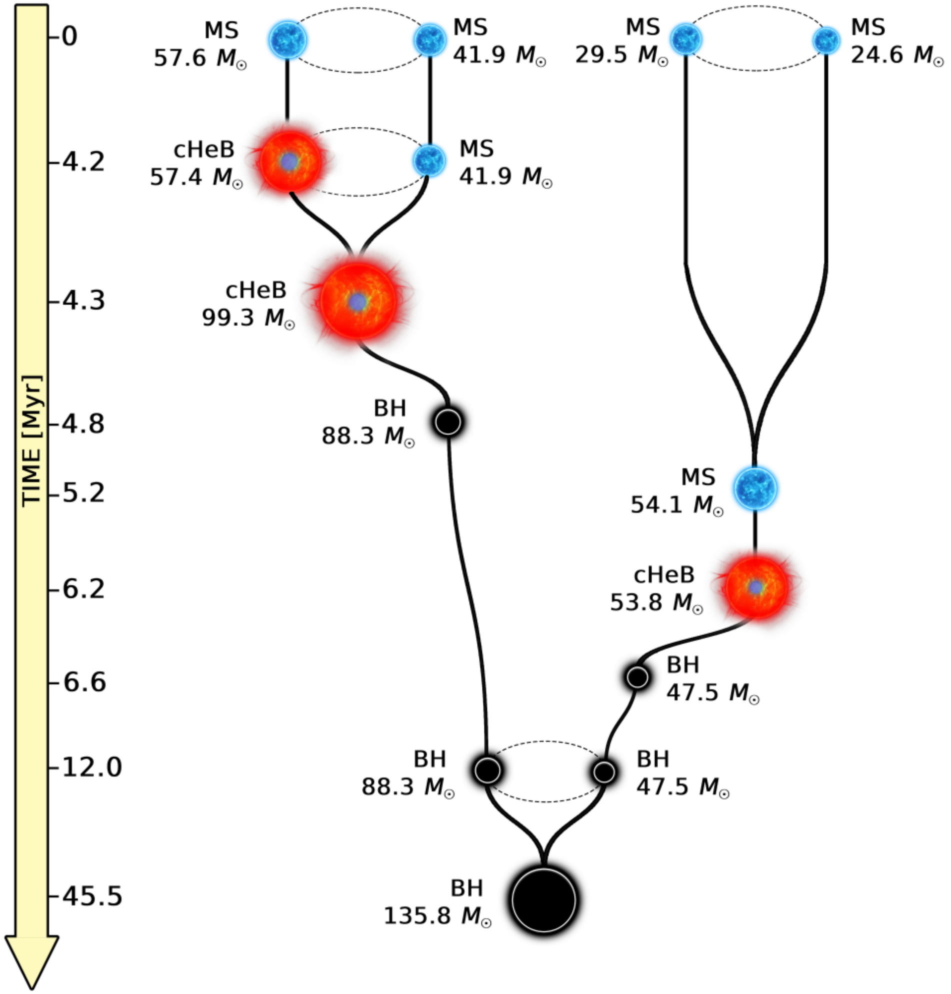


Fig. 1. An example evolutionary scenario leading to formation of a double black hole binary. For details see the text.

Cluster evolution

- Masses
  - Can be much larger (hierarchical mergers)
- Spins
  - Random – not aligned
  - Small, large (2nd generation)
- Rates
  - Should peak at higher redshift (peak of GC formation)



Mapelli, 21

Primordial binaries

- Masses
  - Correspond to phase transitions in the Early universe (can be below 3Msun)
- Spins
  - Random, small
- Rates
  - Do not have to follow SFR

Comparison with observations

The merger rate densities

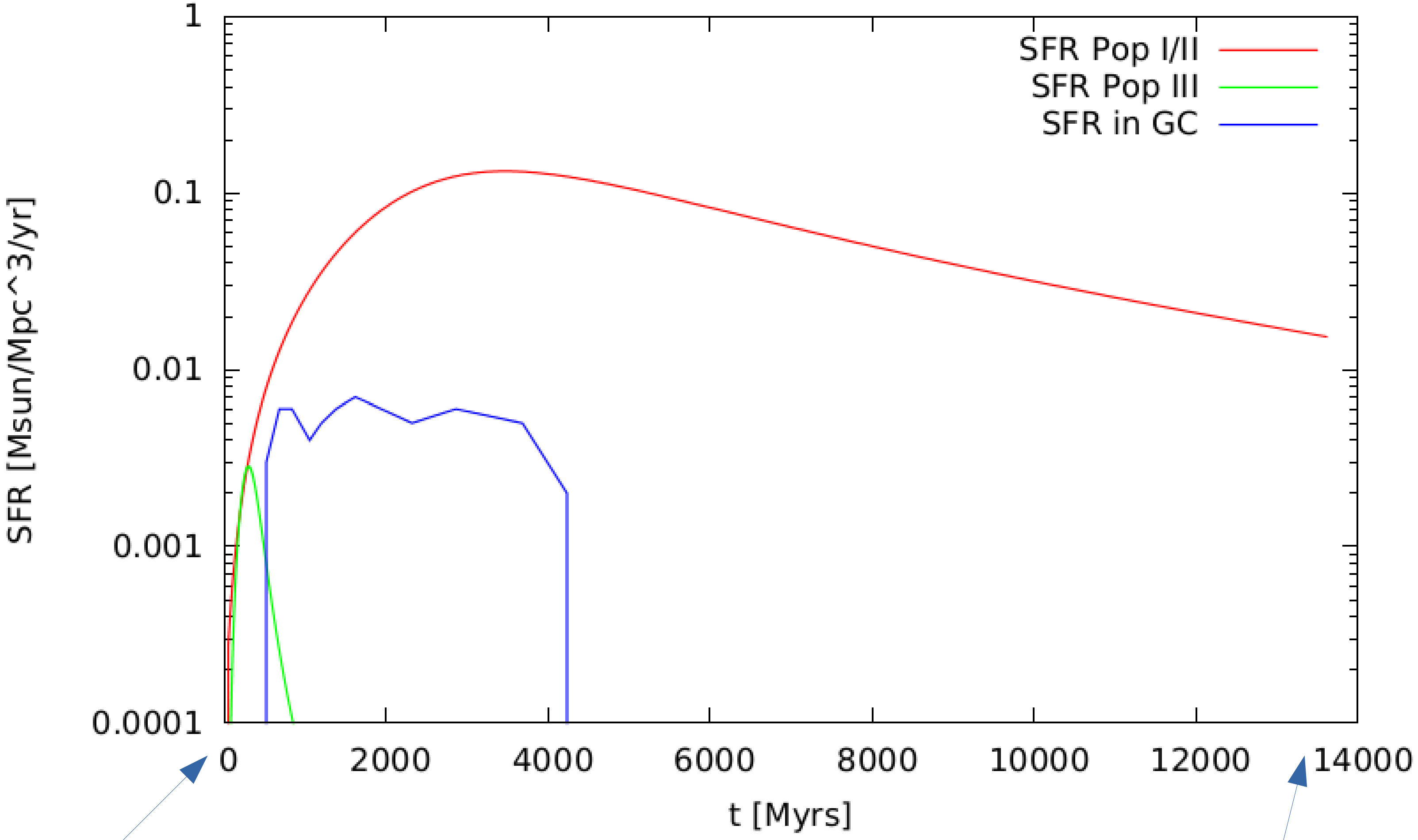
- BBH estimate $R = 17 - 45 \mathrm{Gpc}^{-3} \mathrm{yr}^{-1}$
- BNS estimate $R = 13 - 1900 \mathrm{Gpc}^{-3} \mathrm{yr}^{-1}$
- BHNS estimate $R = 7.4 - 320 \mathrm{Gpc}^{-3} \mathrm{yr}^{-1}$
- The local supernova rate ~ $10^5 \mathrm{Gpc}^{-3} \mathrm{yr}^{-1}$
- The BH formation rate is ~ $10^4 \mathrm{Gpc}^{-3} \mathrm{yr}^{-1}$
- About 1 black hole in a 100-1000 ends up in a merging binary
- Similarly NS: 1 in 100-1000 is in a merging binary!

Rates

- BHBH production efficiency:
  - Number of merging BBH per unit mass
- Delay times
- Mass distribution
  - Intrinsic vs observed: range and redshift effect

- Rate density: local and as a function of redshift

SFR



Big Bang

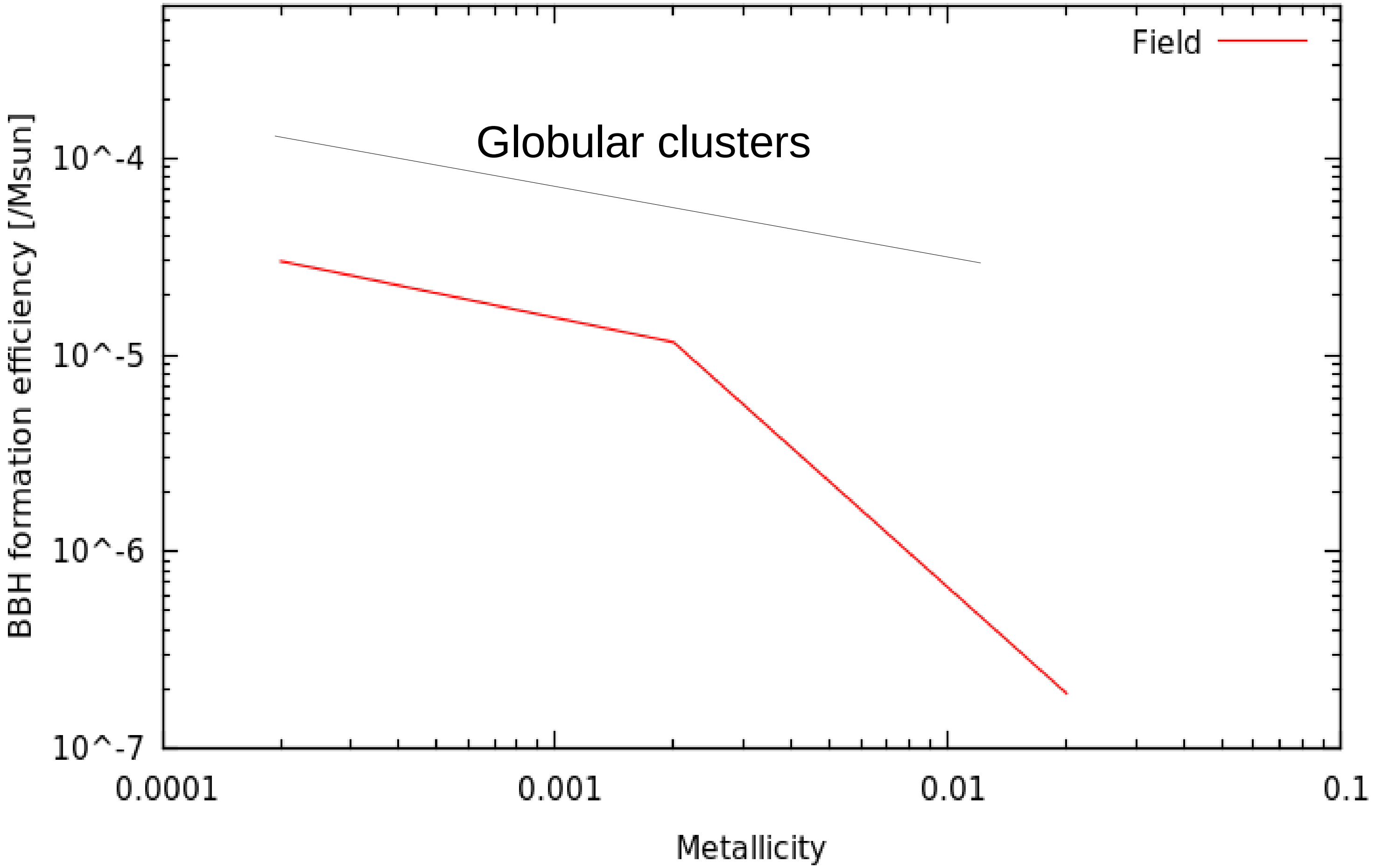
Today

BHBH formation efficiency

$$X_{BHBH} = \frac{N_{BHBH}}{M_*}$$

If all BHs end up in merging binaries
and with Salpeter IMF

$$X_{BHBH}^{max} = 1.8 \times 10^{-3} M_\odot^{-1}$$



Basic rate arguments

- Formation scenario must be generic
- Exceptional environments must produce BBH and BNS with very high efficiency
- Dense regions are not favored, but do contribute
- I am skeptical about exotic models

Binary evolution

- Masses –we see too heavy BHs
- Spins
  - slightly positive
  - are small spins a problem?
- Rates increase with z

Small spins

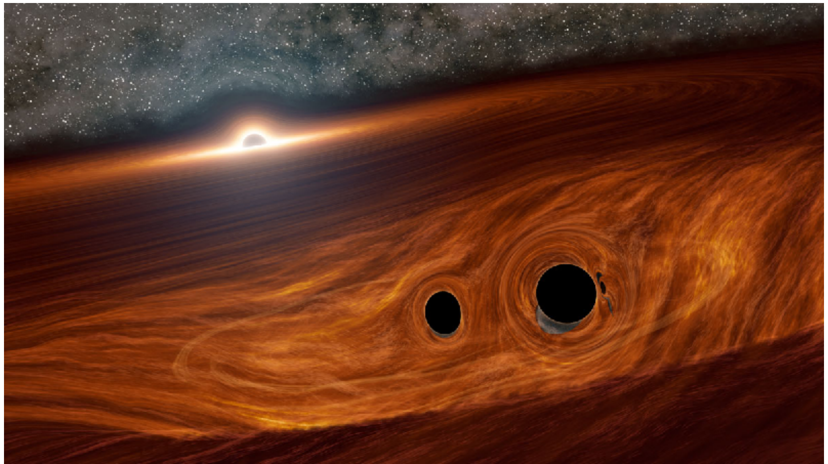
- BH spins measured in accreting binaries are large
- But:
  - Spins of young pulsars
  - Supernova vs GRB rate → spins

Cluster evolution

- Masses – extend above PPSN gap
- Spins
  - why positive?, consistent with an isotropic subpopulation
  - In hierachical merges should be ~0.7
- Rates
  - increase but follow SFR
  - Is there a peak at z=2-3?

AGN model of formation

- GW190521 – quasar flare after 35 days.
- Possibility of forming eccentric binaries
- Rates – very low… (in my opinion)



Primordial

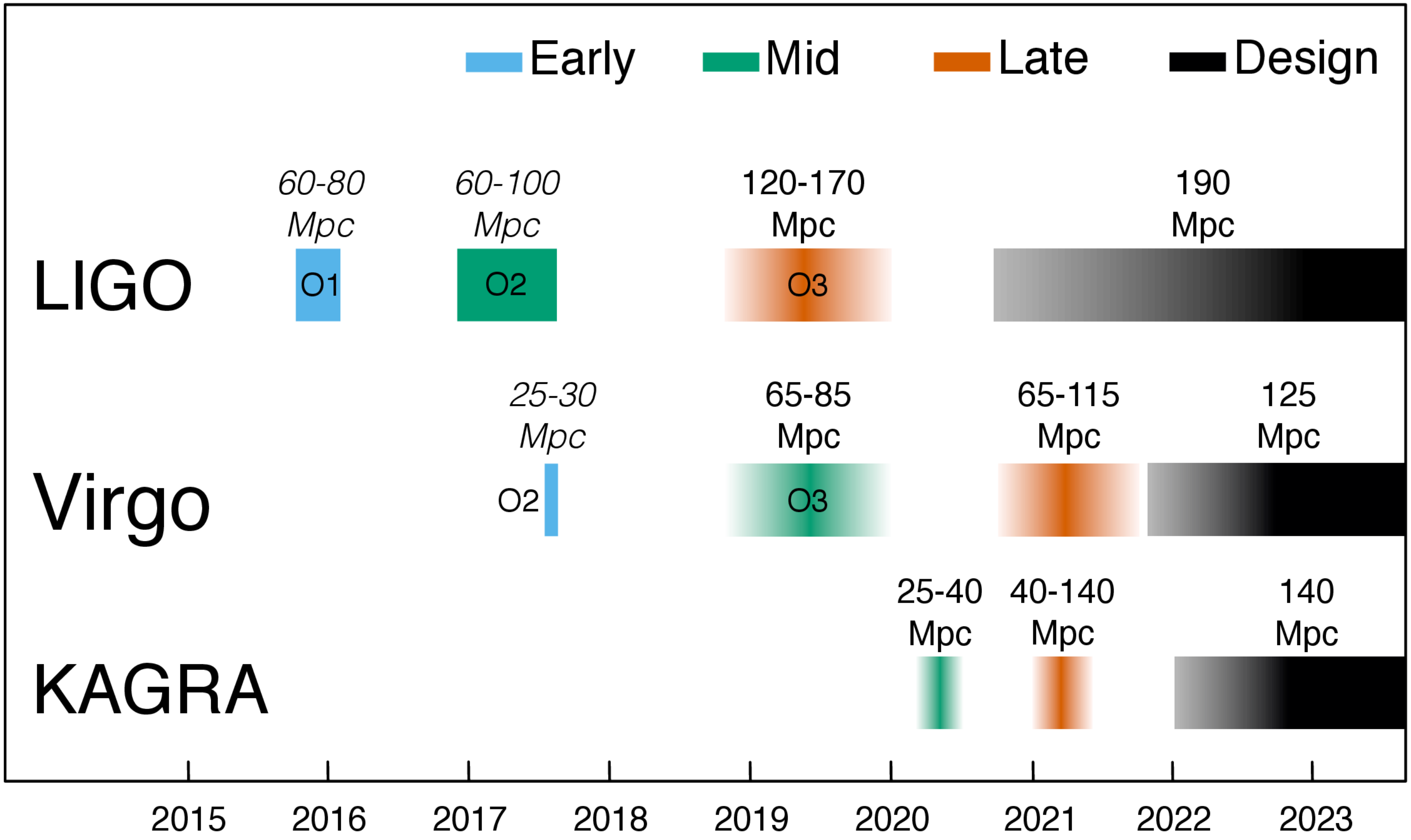
- Distribution of masses, lack of BHs below the stellar limit.
- Spins positive
  - But a sub-population possible
- Why do the rates follow SFR?
  - Rate conspiracy?

How does it look

Model	Masses	Spins	Rates
Binary	🟨	🟩	🟩
Cluster	🟩	🟨	🟩
Primordial	🟨	🟨	🟨

My conclusion is that we may need more than one scenario to explain observations.

What next



ET and Cosmic Explorer needed!

Open questions

- Origin of sources – BBH, BNS, BHNS
- Mass spectrum – IMBHs
- Formation redshifts
- Cosmology with GW
- Fundamental science with GW – validity of GR
- Promordial GW background – echoes from inflation
- Other sources: pulsars, supernovae
- Other wavelengths: LISA, PTA, Moon missions
- …..