Zero-jettiness resummation for top-quark pair production at the LHC

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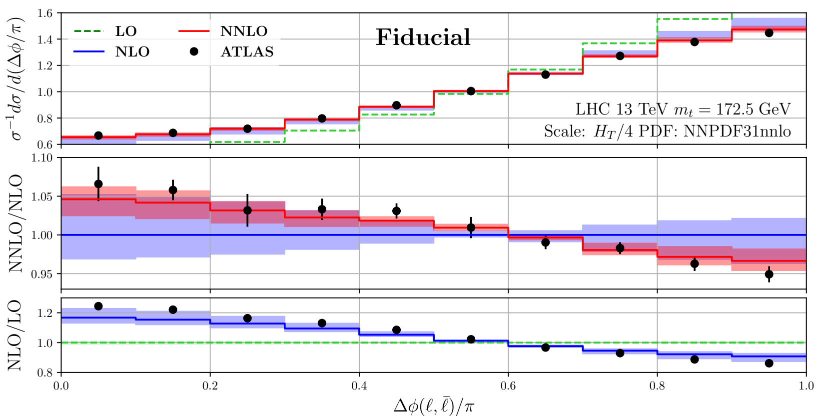
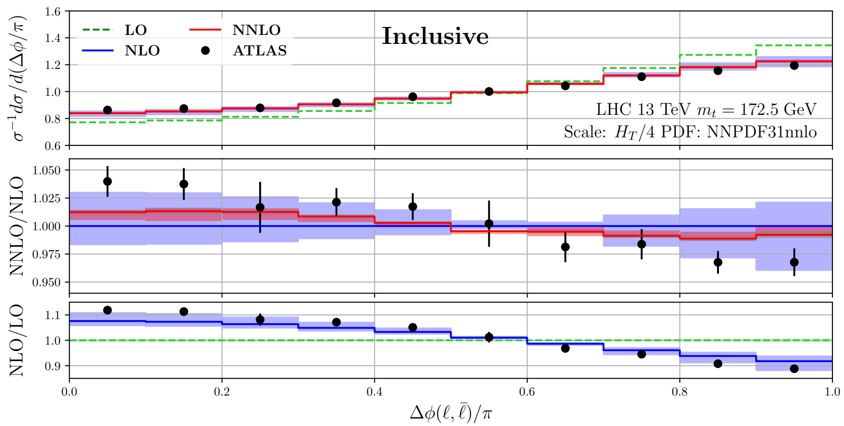
Motivation

- Top-quark is heaviest SM particle: its mass is connected to vacuum stability, it has a large coupling to Higgs sector

- Pair production is already known at NNLO at the fully differential level: why improve resummation and event generators ?

- Experimental measurements are necessarily performed in fiducial phase space regions: extrapolation with lower-order tools (NLO+PS) can create problems





1901.05407 A. Behring, M. Czakon, A. Mitov, A. Papanastasiou, R. Poncelet

Why resumming N-jettiness ?

- N-jettiness is a resolution variable: given an M-particle phase space point with $M \geq N$

$$\mathcal{T}_N(\Phi_M) = \sum_k \min\{\hat{q}_a \cdot p_k, \hat{q}_b \cdot p_k, \hat{q}_1 \cdot p_k, \ldots, \hat{q}_N \cdot p_k\}$$

- The limit $\mathcal{T}_N \to 0$ describes a N-jet event where the unresolved emissions are collinear to the final state jets/initial state beams or soft

- For color-singlet final states, the relevant variable is 0-jettiness

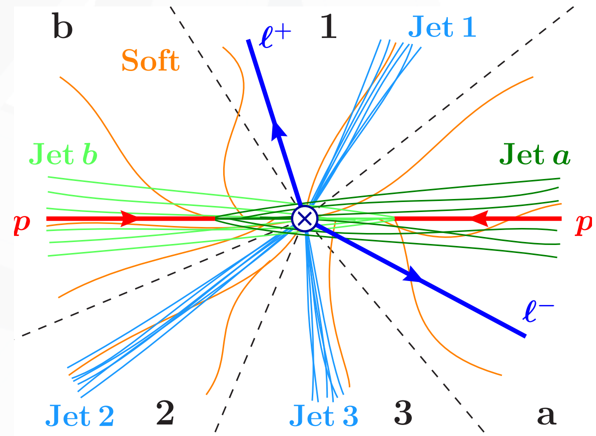
$$\mathcal{T}_0 = \sum_k |\vec{p}_{kT}|\, e^{-|\eta_k - Y|}$$

- Cross section factorizes in $\mathcal{T}_0 \to 0$ limit

[Stewart, Tackmann,Waalewijn `09,`10]

- Three different scales arise in this limit

$$\mu_H = Q, \quad \mu_B = \sqrt{Q\mathcal{T}_0}, \quad \mu_S = \mathcal{T}_0$$



$$\frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0 d\mathcal{T}_0} = \sum_{ij} \underbrace{H_{ij}\,(Q^2, t, \mu_H)}_{\text{NNLO}} U_H(\mu_H, \mu) \left\{ \left[\underbrace{B_i(t_a, x_a, \mu_B)}_{\text{NNLO}} \otimes U_B(\mu_B, \mu)\right] \times \left[\underbrace{B_j(t_b, x_b, \mu_B)}_{} \otimes U_B(\mu_B, \mu)\right] \right\} \otimes \left[\underbrace{S(\mu_s)}_{\text{NNLO}} \otimes U_S(\mu_S, \mu)\right],$$

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From N-jettiness resummation to event generation

- GENEVA [SA,Bauer,Berggren,Tackmann, Walsh `15], [SA,Bauer,Tackmann,Guns `16], [SA,Broggio,Lim, Kallweit,Rottoli `19],[SA,Broggio,Gavardi,Lim,Nagar,Napoletano,Kallweit,Rottoli `20-`21] combines the 3 theoretical tools for QCD predictions (resummation, fixed-order, parton-shower) into a single framework:

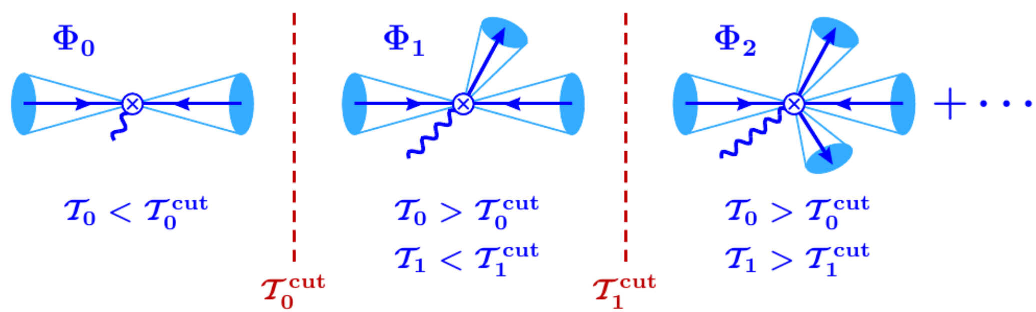
1. fully differential fixed-order calculations, up to NNLO via 0-jettiness or q_T subtraction
2. up to NNLL` resummation four 0-jettiness in SCET or N^{3}LL for q_T via RadISH for colour singlet processes
3. shower and hadronize events (PYTHIA8)

- IR-finite definition of events based on N-jettiness resolution parameters

$$\Phi_0 \text{ events:} \quad \frac{d\sigma_0^{\rm MC}}{d\Phi_0}(\mathcal{T}_0^{\rm cut}),$$

$$\Phi_1 \text{ events:} \quad \frac{d\sigma_1^{\rm MC}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\rm cut}; \mathcal{T}_1^{\rm cut}),$$

$$\Phi_2 \text{ events:} \quad \frac{d\sigma_{\geq 2}^{\rm MC}}{d\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\rm cut}, \mathcal{T}_1 > \mathcal{T}_1^{\rm cut})$$



- When we take $\mathcal{T}_N^{\rm cut} \to 0$, large logarithms of $\mathcal{T}_N^{\rm cut}$,appear that need to be resummed. Matching higher-log resummation to fixed-order improves the accuracy of the predictions across the whole spectrum.

Top-quark pair production: state-of-the-art

- Recently, NNLO+PS for $t\bar{t}$ production available via MiNNLOPS formalism [Mazzitelli, Monni, Nason, Re, Wiesemann, Zanderighi `20,`21]

- Higher-order resummation can improve the description of certain observables (this is the case of the GENEVA generator)

- N-jettiness definition for top-quarks could be extended in two ways:

1. Include top-quark velocities as q_i reference directions. This also resums logs from radiation collinear to the heavy quarks and is appropriate in the boosted regime when $M_{t\bar{t}} \gg m_t$ [Fleming, Hoang, Mantry, Stewart `07, Bachi, Hoang, Mateu, Pathak, Stewart `21]
2. Top quarks are excluded from the sum, collinear radiation is never clustered with heavy quarks. Soft emissions are still correctly accounted for.

We work in this second approach assuming stable top quarks.

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Zero-jettiness factorization for top-quark pairs

Factorization formula (see 2111.03632 Appendix A) derived using SCET+HQET in the region where $M_{t\bar{t}} \sim m_t \sim \sqrt{\hat{s}}$ are all hard scales.

In case of boosted regime $M_{t\bar{t}} \gg m_t$ one would instead need a modified two-jettiness [Fleming, Hoang,Mantry,Stewart `07][Bachu,Hoang,Mateu,Pathak,Stewart `21]

$$\frac{d\sigma}{d\Phi_0 d\tau_B} = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \int dt_a\, dt_b\, B_i(t_a, z_a, \mu)\, B_j(t_b, z_b, \mu)\, \mathrm{Tr}\left[\mathbf{H}_{ij}(\Phi_0, \mu)\, \mathbf{S}_{ij}\left(M\tau_B - \frac{t_a + t_b}{M}, \Phi_0, \mu\right)\right]$$

Beam functions [Stewart, Tackmann, Waalewijn, [1002.2213], known up to N^{3}LO

Hard functions (color matrices)

Soft functions (color matrices)

It is convenient to transform the soft and beam functions in Laplace space to solve the RG equations, the factorization formula is turn into a product of (matrix) functions

$$\mathscr{L}\left[\frac{d\sigma}{d\Phi_0 d\tau_B}\right] = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \tilde{B}_i\left(\ln\frac{M\kappa}{\mu^2}, z_a\right) \tilde{B}_j\left(\ln\frac{M\kappa}{\mu^2}, z_b\right) \mathrm{Tr}\left[\mathbf{H}_{ij}\left(\ln\frac{M^2}{\mu^2}, \Phi_0\right) \tilde{\mathbf{S}}_{ij}\left(\ln\frac{\mu^2}{\kappa^2}, \Phi_0\right)\right]$$

Beam Functions

The beam functions are given by convolutions of perturbative kernels with the standard PDFs $f_i(x,\mu)$

$$B_i(t,z,\mu) = \sum_j \int_z^1 \frac{d\xi}{\xi} I_{ij}(t, z/\xi, \mu) f_j(\xi,\mu)$$

I_{ij} kernels are known up to N^{3}LO, process independent

RG equation in Laplace space is given by

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{B}_i(L_c,z,\mu) = \Big[-2\,\Gamma_{\rm cusp}(\alpha_s)\,L_c + \gamma_i^B(\alpha_s)\Big]\tilde{B}_i(L_c,z,\mu)$$

with solution in momentum space

$$B(t,z,\mu) = \exp\left[-4S(\mu_B,\mu) - a_{\gamma^B}(\mu_B,\mu)\right]\tilde{B}(\partial_{\eta_B},z,\mu_B)\frac{1}{t}\left(\frac{t}{\mu_B^2}\right)^{\eta_B}\frac{e^{-\gamma_E\eta_B}}{\Gamma(\eta_B)}$$

where $\eta_B \equiv 2a_\Gamma(\mu_B,\mu)$ and the collinear log is given by $L_c = \ln\left(M\kappa/\mu^2\right)$

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Hard Function

The hard functions arise from matching the full theory onto the EFT, they can be extracted from colour decomposed loop amplitudes. At NLO it was first computed in [Ahrens, Ferroglia, Neubert, Pecjak, Yang, 1003.5827]. They satisfy the RG equations

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{H}(M,\beta_t,\theta,\mu) = \mathbf{\Gamma}_H(M,\beta_t,\theta,\mu)\,\mathbf{H}(M,\beta_t,\theta,\mu) + \mathbf{H}(M,\beta_t,\theta,\mu)\,\mathbf{\Gamma}_H^\dagger(M,\beta_t,\theta,\mu)$$

whose solution is

$$\mathbf{H}(M,\beta_t,\theta,\mu) = \mathbf{U}(M,\beta_t,\theta,\mu_h,\mu)\mathbf{H}(M,\beta_t,\theta,\mu_h)\mathbf{U}^\dagger(M,\beta_t,\theta,\mu_h,\mu)$$

We can split the evolution and the anomalous dimension into a cusp (diagonal in colour space) and non-cusp (not diagonal) part

$$\mathbf{U}(M,\beta_t,\theta,\mu_h,\mu) = \exp\left[2S(\mu_h,\mu) - a_\Gamma(\mu_h,\mu)\left(\ln\frac{M^2}{\mu_h^2} - i\pi\right)\right]\mathbf{u}(M,\beta_t,\theta,\mu_h,\mu)$$

$$\mathbf{\Gamma}_H(M,\beta_t,\theta,\mu) = \Gamma_{\rm cusp}(\alpha_s)\left(\ln\frac{M^2}{\mu^2} - i\pi\right) + \boldsymbol{\gamma}^h(M,\beta_t,\theta,\alpha_s)$$

$$\mathbf{u}(M,\beta_t,\theta,\mu_h,\mu) = \mathcal{P}\exp\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu)}\frac{\mathrm{d}\alpha}{\beta(\alpha)}\boldsymbol{\gamma}^h(M,\beta_t,\theta,\alpha)$$

We evaluate the matrix exponential **u** as a series expansion in α_s [Buchalla,Buras,Lautenbacher `96]

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Soft Function

We computed the soft functions matrices at NLO which were unknown for this observable

$$\mathbf{S}^{(1)}_{\text{bare},\,ij}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu) = \sum_{\alpha,\,\beta} \boldsymbol{w}_{ij}^{\alpha\beta}\, \hat{\mathcal{I}}_{\alpha\beta}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu)$$

$$\hat{\mathcal{I}}_{\alpha\beta}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu) = -\frac{2(\mu^2 e^{\gamma_E})^\epsilon}{\pi^{1-\epsilon}} \int \mathrm{d}^d k \frac{v_\alpha \cdot v_\beta}{v_\alpha \cdot k\, v_\beta \cdot k}\, \delta(k^2)\Theta(k^0)$$
$$\times \left[\delta(k_a^+ - k\cdot n_a)\Theta(k\cdot n_b - k\cdot n_a)\,\delta(k_b^+) + \delta(k_b^+ - k\cdot n_b)\Theta(k\cdot n_a - k\cdot n_b)\,\delta(k_a^+)\right]$$

Averaging over the two hemisphere momenta, the soft functions satisfy the RG equation in Laplace space

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) = \left[\Gamma_{\text{cusp}} L \; - \boldsymbol{\gamma}^{s\dagger}\right] \tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) + \tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) \left[\Gamma_{\text{cusp}} L \; - \boldsymbol{\gamma}^{s}\right]$$

Solution in momentum space, where we used the consistency relation among anomalous dimensions $\boldsymbol{\gamma}^s = \boldsymbol{\gamma}^h + \gamma^B\,\mathbf{1}$

$$\eta_s \equiv -2a_\Gamma(\mu_s, \mu)$$

$$\mathbf{S}_B(l^+, \beta_t, \theta, \mu) = \exp\left[4S(\mu_s, \mu) + 2a_{\gamma^B}(\mu_s, \mu)\right]$$
$$\times\; \mathbf{u}^\dagger(\beta_t, \theta, \mu, \mu_s)\, \tilde{\mathbf{S}}_B(\partial_{\eta_s}, \beta_t, \theta, \mu_s)\, \mathbf{u}(\beta_t, \theta, \mu, \mu_s)\, \frac{1}{l^+}\left(\frac{l^+}{\mu_s}\right)^{2\eta_s} \frac{e^{-2\gamma_E \eta_s}}{\Gamma(2\eta_s)}$$

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Final resummed result

We can combine the solutions for the hard, soft and beam functions to obtain a resummed formula valid at any logarithmic order

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_0 \mathrm{d}\tau_B} = U(\mu_h, \mu_B, \mu_s, L_h, L_s) \times \mathrm{Tr}\Big\{ \mathbf{u}(\beta_t, \theta, \mu_h, \mu_s)\, \mathbf{H}(M, \beta_t, \theta, \mu_h)\, \mathbf{u}^\dagger(\beta_t, \theta, \mu_h, \mu_s)\, \tilde{\mathbf{S}}_B(\partial_{\eta_s} + L_s, \beta_t, \theta, \mu_s) \Big\} \times \tilde{B}_a(\partial_{\eta_B} + L_B, z_a, \mu_B) \tilde{B}_b(\partial_{\eta'_B} + L_B, z_b, \mu_B) \frac{1}{\tau_B^{1-\eta_{\mathrm{tot}}}} \frac{e^{-\gamma_E \eta_{\mathrm{tot}}}}{\Gamma(\eta_{\mathrm{tot}})} .$$

where

$$U(\mu_h, \mu_B, \mu_s, L_h, L_s) = \exp\Big[4S(\mu_h, \mu_B) + 4S(\mu_s, \mu_B) + 2a_{\gamma^B}(\mu_s, \mu_B) - 2a_\Gamma(\mu_h, \mu_B)\, L_h - 2a_\Gamma(\mu_s, \mu_B)\, L_s \Big]$$

and $L_s = \ln(M^2/\mu_s^2)$, $L_h = \ln(M^2/\mu_h^2)$, $L_B = \ln(M^2/\mu_B^2)$ and $\eta_{\mathrm{tot}} = 2\eta_S + \eta_B + \eta_{B'}$

The final accuracy depends on the availability of the perturbative ingredients

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NNLL'$_a$ resummation

- We currently have:
  - hard functions at NLO
  - soft functions at NLO, by knowing the two-loop soft anomalous dimensions we can solve the RG equations order by order and obtain all the NNLO logarithmic contributions, we only miss $\delta(\mathcal{T}_0)$ terms at NNLO
  - beam functions at NNLO (for initial states with quarks and gluons)
  - two-loop anomalous dimensions
- We have everything for NNLL but we are missing $\delta(\mathcal{T}_0)$ terms (NNLO hard functions and NNLO soft) for NNLL'. If we include everything we know we obtain what we call an approximate $\mathrm{NNLL}'_{\mathrm{a}}$ result
- We can however already construct an approximate (N)NLO formula which reproduces the fixed-order behaviour of the spectrum (for $\mathcal{T}_0 > 0$) and match it to $t\bar{t} + jet$ at NLO.

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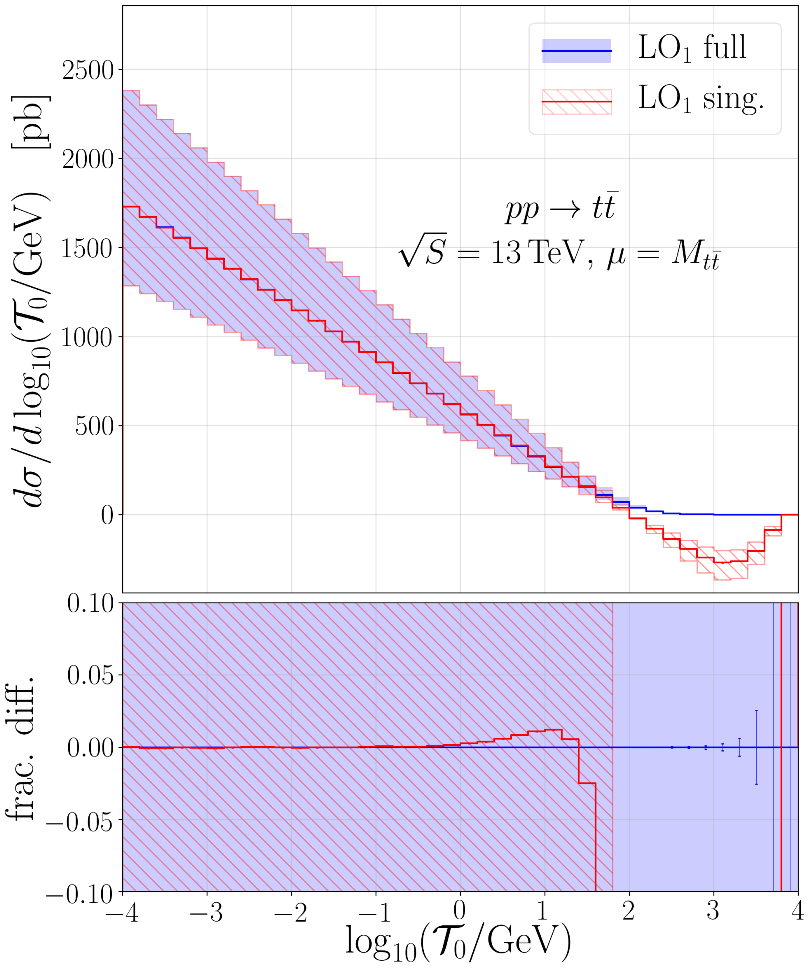
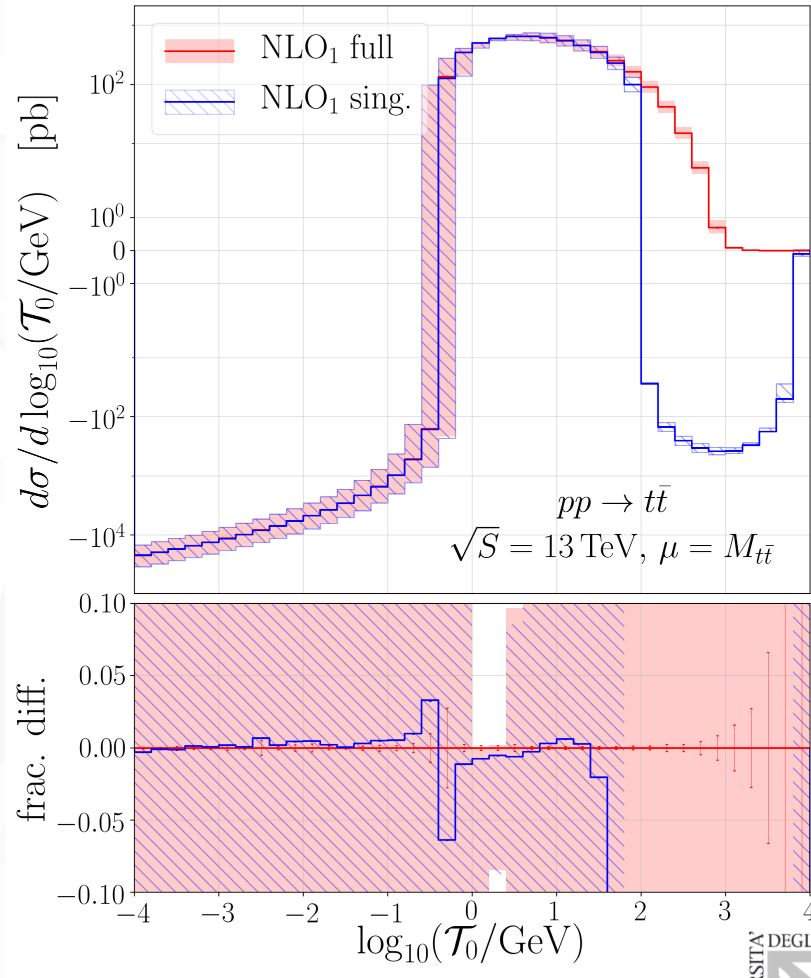
Singular cross section

Fixed-order comparisons, approximate NLO and approximate NNLO vs LO_1 and NLO_1





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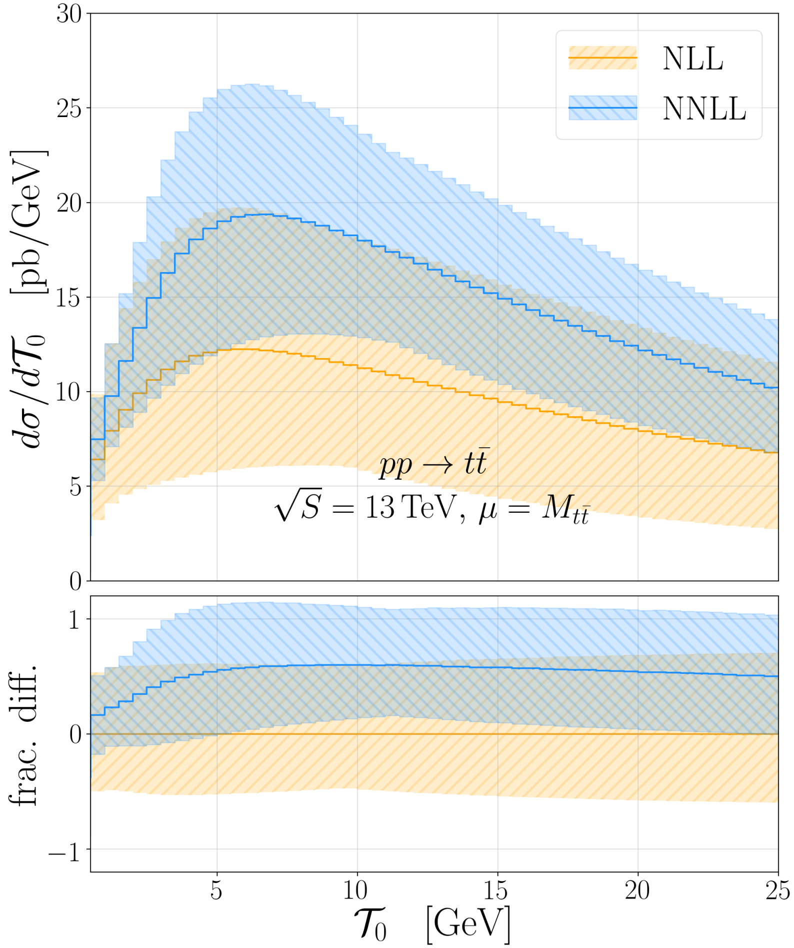
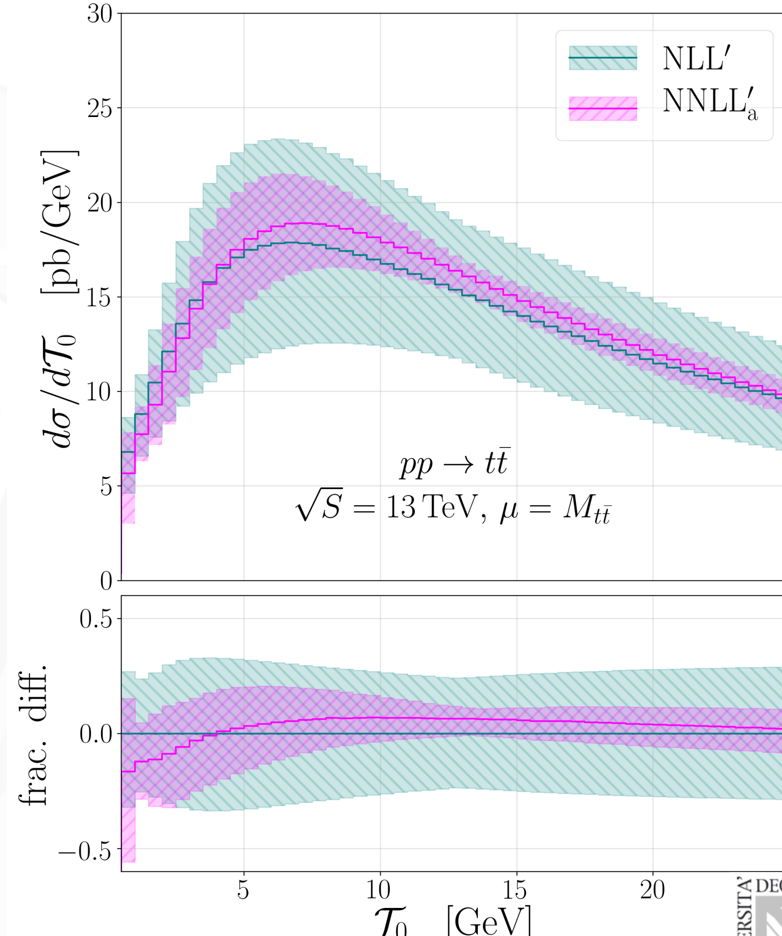
Resummed results

$NNLL'_a$ is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales





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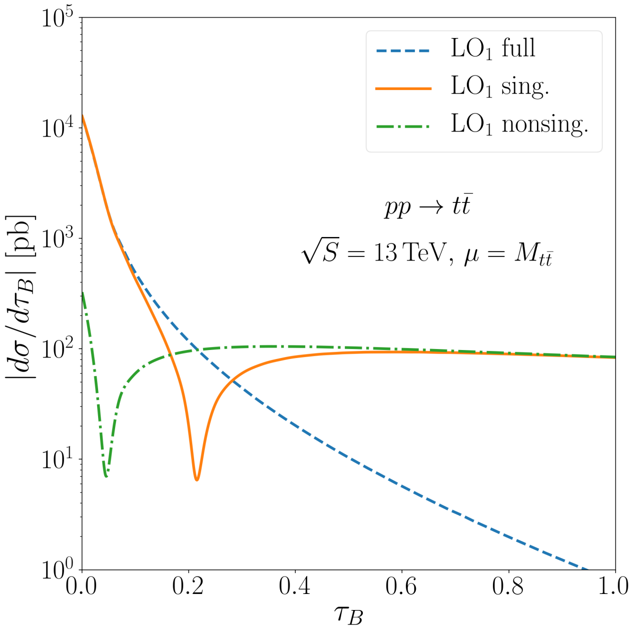
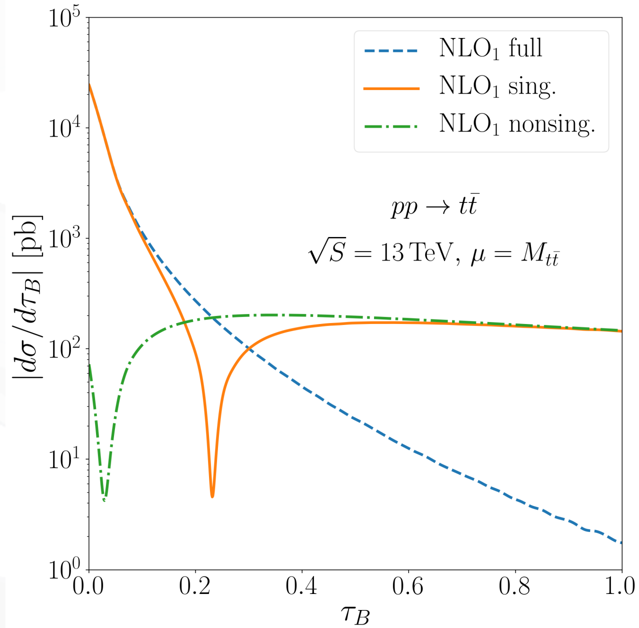
Profile scales

Resummation is switched off when nonsingular contributions become important in the zero-jettiness spectrum





$$\mu_H = \mu_{\rm NS}\,,$$
$$\mu_S(\mathcal{T}_0) = \mu_{\rm NS}\; f_{\rm run}(\mathcal{T}_0/M)\,,$$
$$\mu_B(\mathcal{T}_0) = \mu_{\rm NS}\; \sqrt{f_{\rm run}(\mathcal{T}_0/M)}$$

$$f_{\rm run}(y) = \begin{cases} y_0\left[1+(y/y_0)^2/4\right] & y \le 2y_0\,, \\ y & 2y_0 \le y \le y_1\,, \\ y + \frac{(2-y_2-y_3)(y-y_1)^2}{2(y_2-y_1)(y_3-y_1)} & y_1 \le y \le y_2\,, \\ 1 - \frac{(2-y_1-y_2)(y-y_3)^2}{2(y_3-y_1)(y_3-y_2)} & y_2 \le y \le y_3\,, \\ 1 & y_3 \le y\,. \end{cases}$$

Theory uncertainties are evaluated via symmetric envelopes of profile scale variations (later added in quadrature with FO variations)

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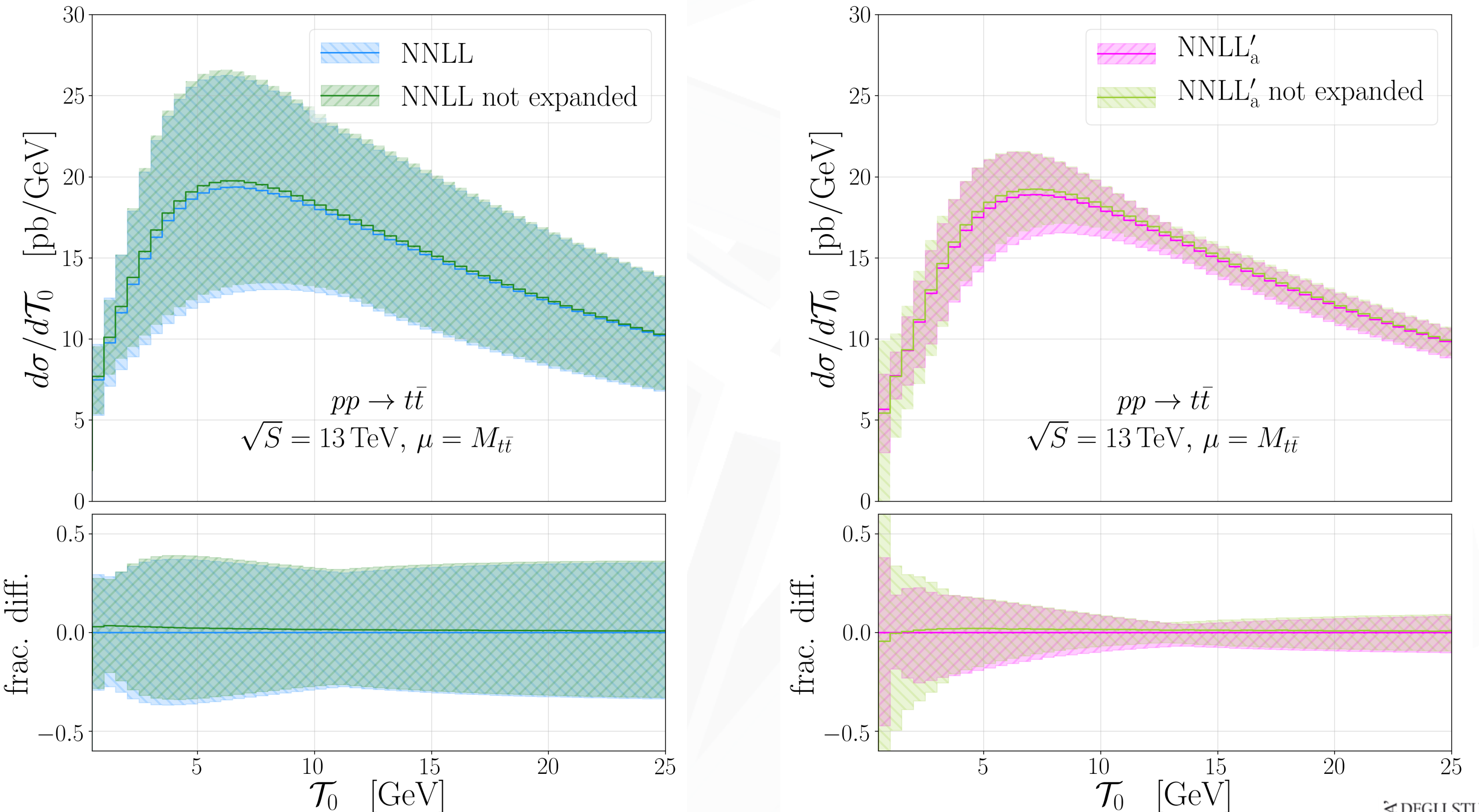
Resummed results - higher order effects

The off-diagonal colors evolution matrix **u** can only be evaluated as an α_s expansion.

We can similarly choose to expand or not expand also U, the difference is quite small



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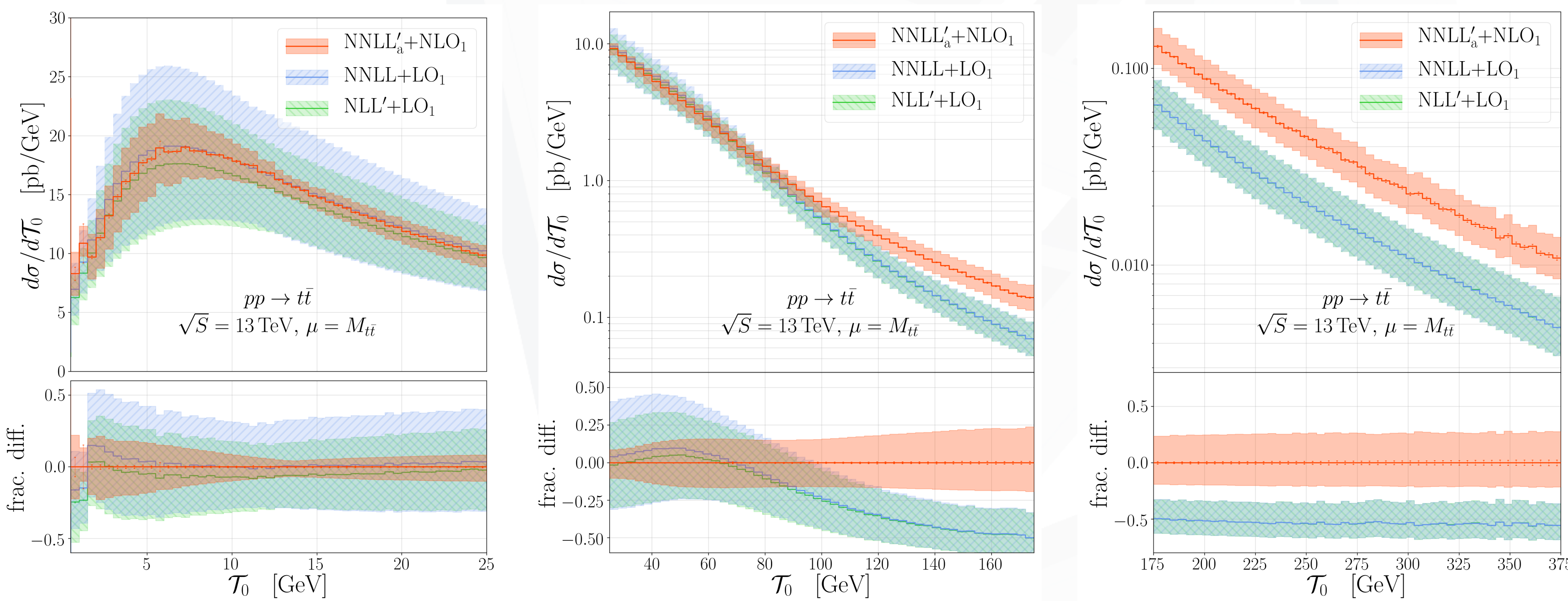
Matched results

Matching to $t\bar{t}+j$ @NLO improves the perturbative accuracy across the whole spectrum

$$\frac{\mathrm{d}\sigma^{\mathrm{match}}}{\mathrm{d}\mathcal{T}_0} = \frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0} + \frac{\mathrm{d}\sigma^{\mathrm{FO}}}{\mathrm{d}\mathcal{T}_0} - \left[\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0}\right]_{\mathrm{FO}}$$



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Conclusion and outlook

- First implementation of approximate NNLL' zero-jettiness resummation for heavy quarks.
- Calculate and extract all the missing boundary conditions to reach full NNLL$'$ accuracy for the top-quark pair production process (two-loop hard and soft functions).
- Implement it in GENEVA event generator to provide full NNLO+PS
- Extended the study to the boosted top regime $m_t \ll M_{t\bar{t}}$ at the LHC
- Include top-quark decay product
- Study associated production of a top-pair and a heavy boson $t\bar{t}V$ $(V = H, W^{\pm}, Z)$ [Broggio,Ferroglia,Pecjak,Signer, Yang `15], [Broggio,Ferroglia,Pecjak,Ossola `16], [Broggio,Ferroglia,Pecjak,Yang `16],[Broggio,Ferroglia,Pecjak,Ossola,Sameshima `17]