Entanglement in SMEFT : Top pair

Rafael Aoude
UCLouvain

UCLouvain

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Based on

Quantum SMEFT tomography: top quark pair production at the LHC

RA, Eric Madge, Fabio Maltoni and Luca Mantani

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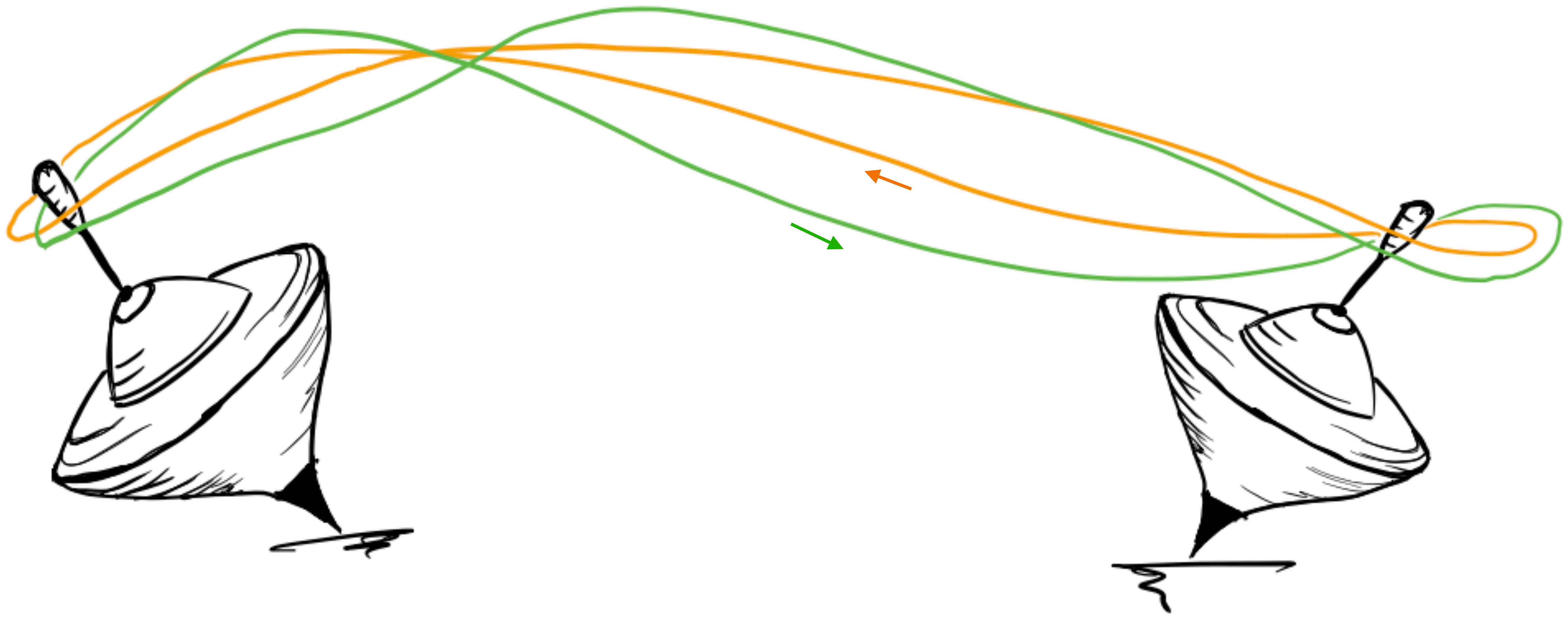
Motivation

- In general, top pair produced entangled at LHC.

- In the SM, there are two point of maximal entanglement and regions of vanishing of entanglement

- What is the picture when SMEFT is considered?



[Afik and de Nova, 21']

[Fabbrichesi, Floreanini, Panizzo, 21']

[Severi, Degli, Maltoni, Sioli, 21']

[Aoude, Madge, Maltoni, Mantani, 22']

[Afik and de Nova, 22']

[Aguilar-Saavedra, Casas, 22']

Spin production density matrix

[Afik and de Nova, 21']

The state-density matrix is obtained from the R-matrix

$$R^I_{\alpha_1\alpha_2,\beta_1\beta_2} \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors} \\ \text{a,b spins}}} \mathcal{M}^*_{\alpha_2\beta_2} \mathcal{M}_{\alpha_1\beta_1}$$

where $\mathcal{M}_{\alpha\beta} \equiv \langle t(k_1,\alpha)\bar{t}(k_2,\beta)|\mathcal{T}|a(p_1)b(p_2)\rangle$

I=gg or qq

Mixed state of qq and gg initiated channels, weighted by the luminosity functions

$$R(\hat{s},\boldsymbol{k}) = \sum_I L^I(\hat{s}) R^I(\hat{s},\boldsymbol{k})$$

Spin production density matrix

[Afik and de Nova, 21']

4x4 matrix in spin-space of the top pair.

Fano decomposition: (spanned by tensor prod. of Pauli and Identity)

$$R = \tilde{A}\,\mathbb{1}_2 \otimes \mathbb{1}_2 + \tilde{B}_i^{+}\,\boldsymbol{\sigma}^i \otimes \mathbb{1}_2 + \tilde{B}_i^{-}\,\mathbb{1}_2 \otimes \boldsymbol{\sigma}^i + \tilde{C}_{ij}\,\boldsymbol{\sigma}^i \otimes \boldsymbol{\sigma}^j\,.$$

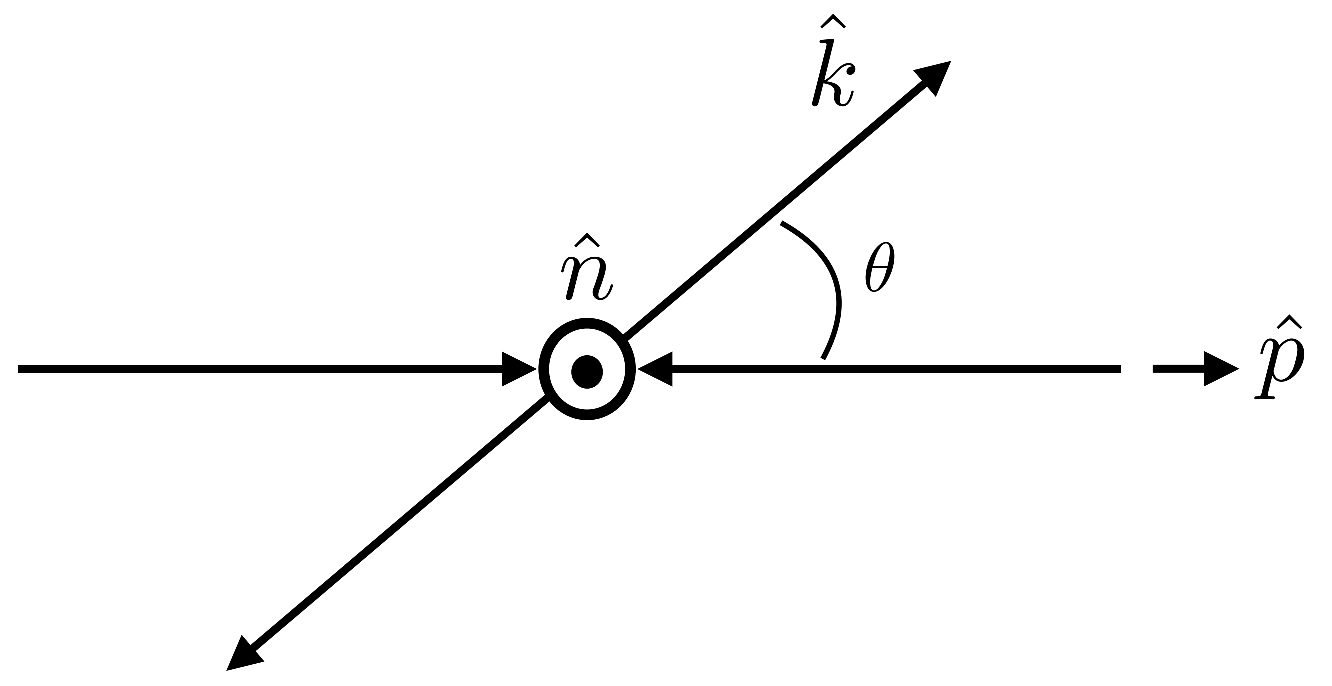
16-coefficients where the norm $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega\mathrm{d}\hat{s}} = \frac{\alpha_s^2 \beta}{\hat{s}^2} \tilde{A}\left(\hat{s}, \boldsymbol{k}\right)$

Normalize the state as $\rho = R/\mathrm{tr}(R)$

Density matrix and helicity-basis

Helicity basis:

$$\{\boldsymbol{k}, \boldsymbol{n}, \boldsymbol{r}\} : \ \boldsymbol{r} = \frac{(\boldsymbol{p} - z\boldsymbol{k})}{\sqrt{1-z^2}}, \quad \boldsymbol{n} = \boldsymbol{k} \times \boldsymbol{r},$$



To expand in this basis, e.g.

$$C_{nn} = \mathrm{tr}[C_{ij}\, \boldsymbol{n} \otimes \boldsymbol{n}\,]$$

Phase-space parametrized by: $\beta^2 = (1 - 4m_t^2/\hat{s})$ and $\cos\theta$

QCD density-matrices

LO-QCD
and
CP-invariance

(1) C_{ij} symmetric

(2) $B_i^+ = B_i^-$

(3) $C_{kn}, C_{rn}, B_n^\pm$ only at one-loop

(4) $B_k^\pm, B_r^\pm$ vanish

CP-even SMEFT: (1) and (2) still holds

Entanglement in bipartite systems

Given a bipartite system $\mathcal{H}_{\mathrm{ab}} = \mathcal{H}_{\mathrm{a}} \otimes \mathcal{H}_{\mathrm{b}}$

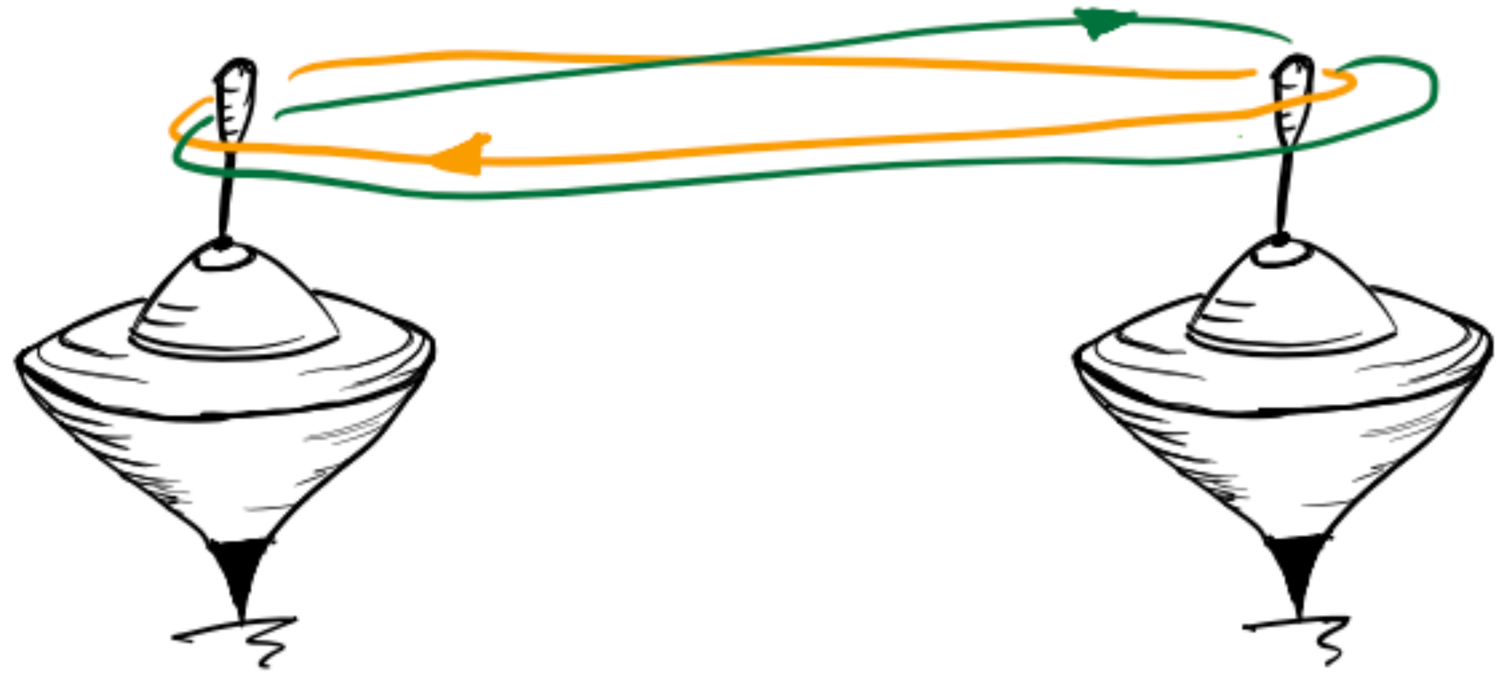
Can you write $|\Psi_{\mathrm{ab}}\rangle = |\Psi_{\mathrm{a}}\rangle \otimes |\Psi_{\mathrm{b}}\rangle$?

No? Then it is entangled.

Or more generally as (mixed states): $\rho_{\mathrm{ab}} = \sum_k p_k \, \rho_{\mathrm{a}}^k \otimes \rho_{\mathrm{b}}^k$

Maximally entangled states (e.g Bell states):

$$|\Phi^\pm\rangle = \frac{|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle}{\sqrt{2}} \quad \text{or} \quad |\Psi^\pm\rangle = \frac{|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle}{\sqrt{2}}$$



Entanglement in bipartite systems

An entanglement measure is more useful than the previous definition:

- Peres-Horodecki Criterion: $\Delta \equiv -C_{nn} + |C_{kk} + C_{rr}| - 1 > 0$

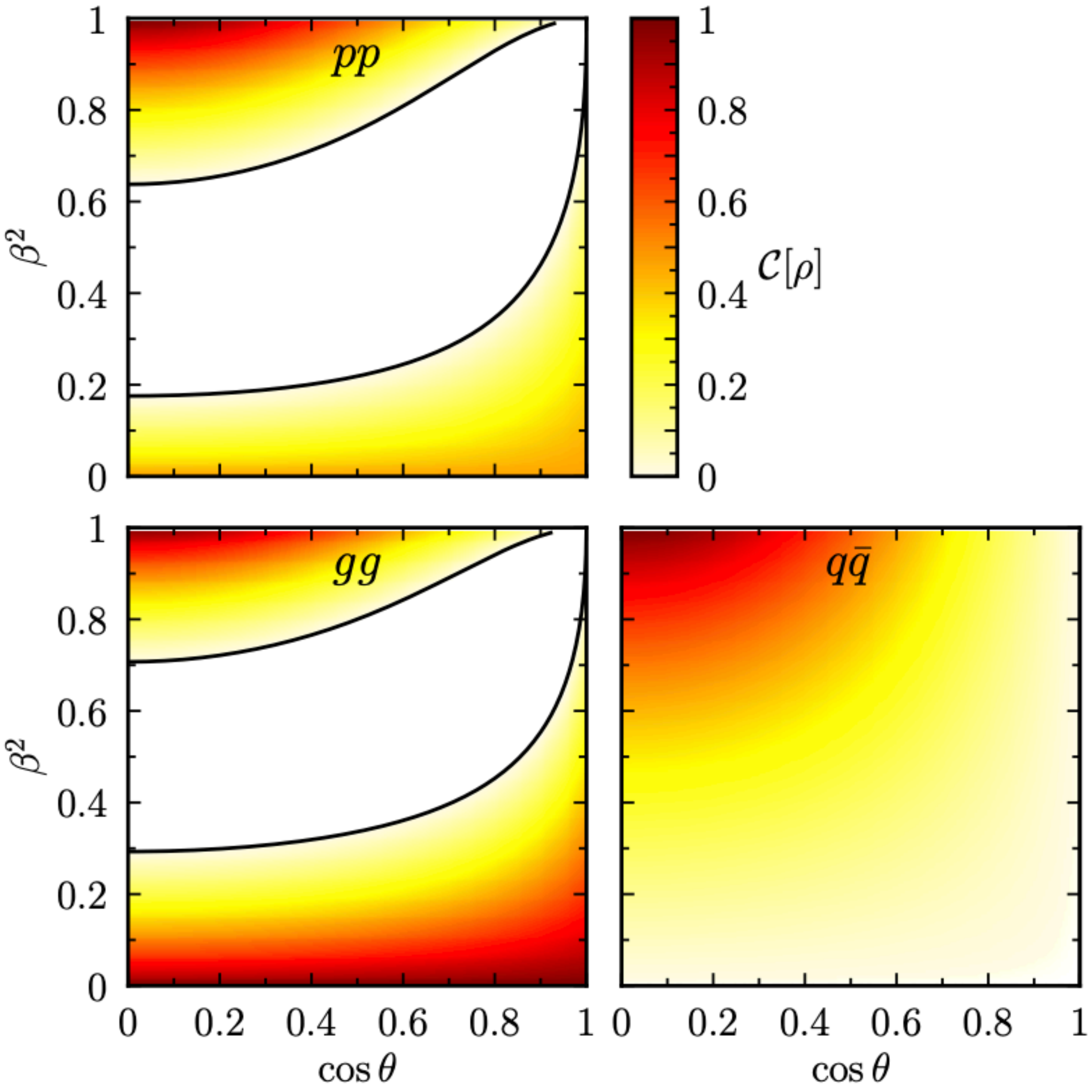
(in the helicity-basis)

- Concurrence: $C[\rho] = \max(\Delta/2, 0)$

$C[\rho] = 1$ (maximally entangled)

What's the story for the SM?

[Afik and de Nova, 21']



White regions: zero-entanglement

Maximal entanglement points/regions

- At threshold: $\beta^2 = 0, \forall\theta$
- high-E: $\beta^2 \to 1, \cos\theta = 0$

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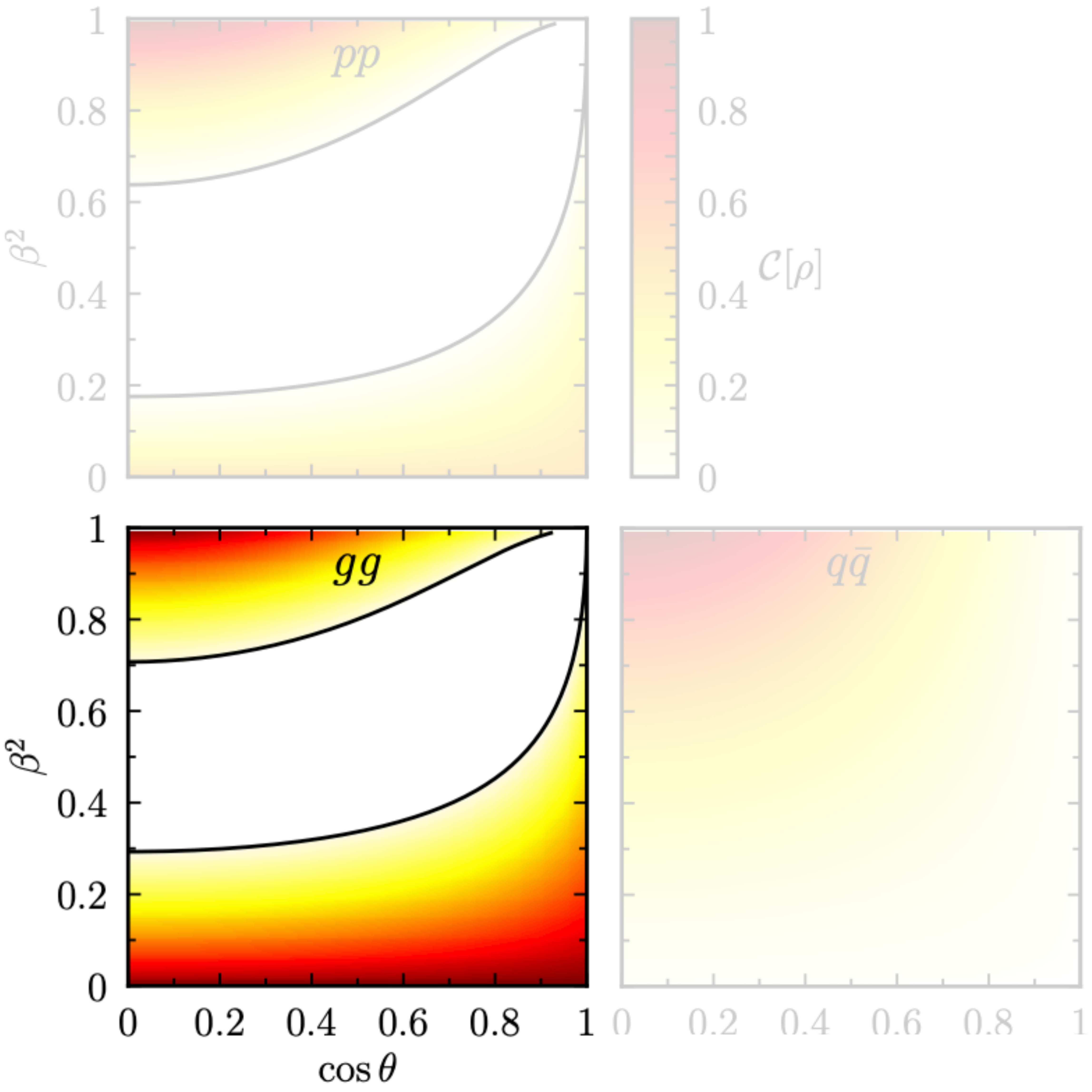


Maximal entanglement points/regions

- At threshold: $\beta^2 = 0, \forall\theta$

(singlet)

$$\rho_{gg}^{\rm SM}(0,z) = |\Psi^+\rangle_{\boldsymbol{n}}\langle\Psi^+|_{\boldsymbol{n}},$$

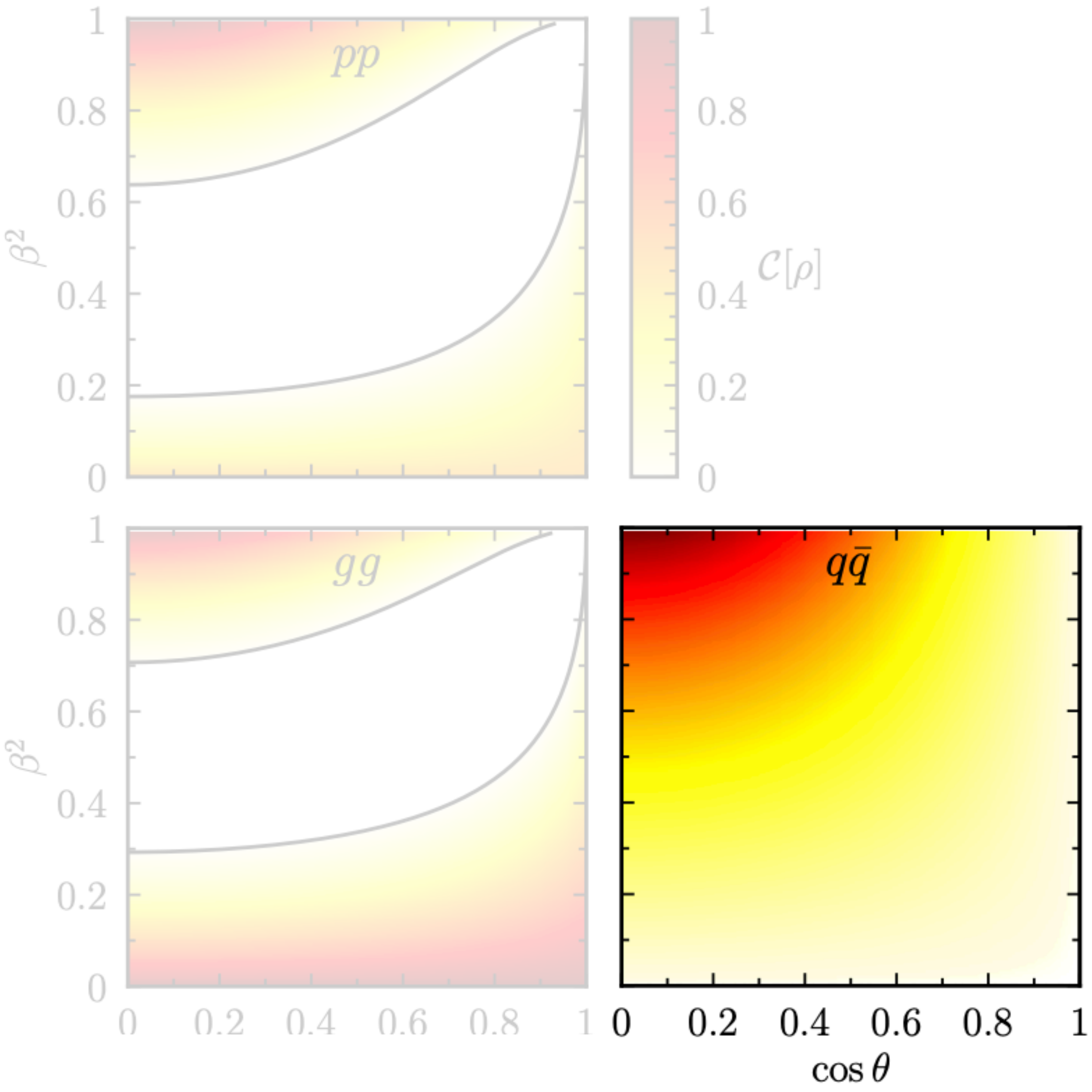
- high-E: $\beta^2 \to 1, \cos\theta = 0$

(triplet)

$$\rho_{gg}^{\rm SM}(1,0) = |\Psi^-\rangle_{\boldsymbol{n}}\langle\Psi^-|_{\boldsymbol{n}}$$

What’s the story for the SM?

[Afik and de Nova, 21’]



Maximal entanglement points/regions

- At threshold: $\beta^2 = 0, \forall\theta$

  mixed but separable

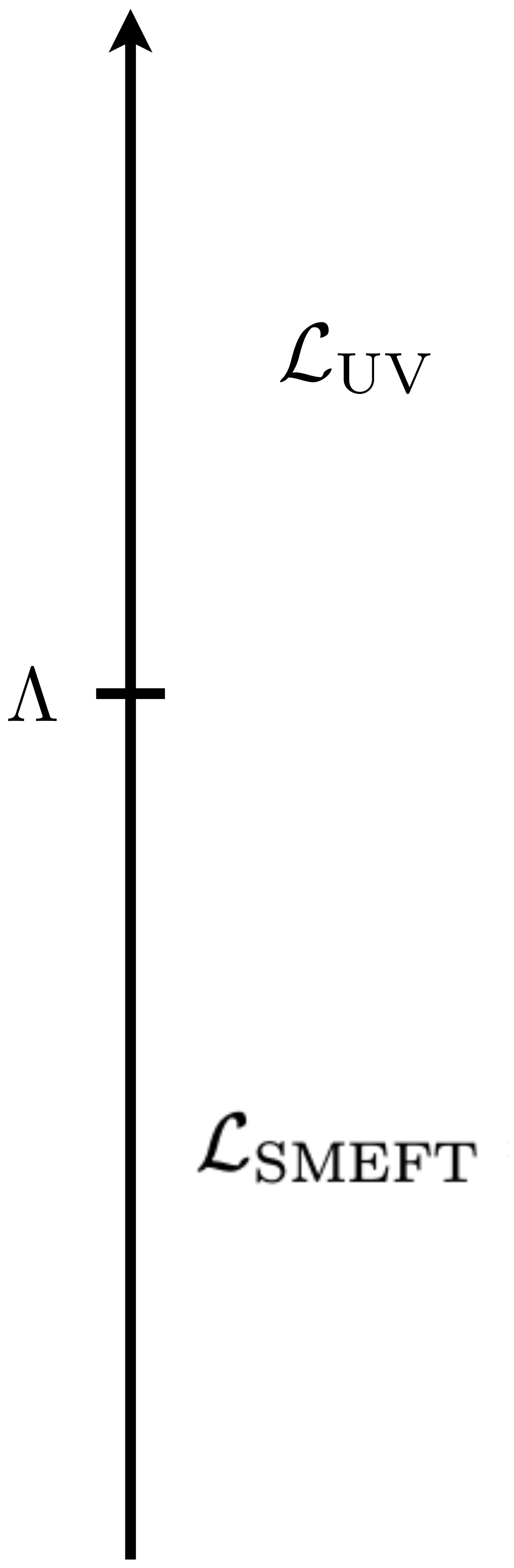
- high-E: $\beta^2 \to 1, \cos\theta = 0$

  (triplet: same as gg)

$$\rho_{q\bar{q}}^{\rm SM}(1,0) = |\Psi^-\rangle_{\boldsymbol{n}}\langle\Psi^-|_{\boldsymbol{n}}$$

SMEFT

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2}\sum_i c_i \mathcal{O}_i$$



LO-QCD in ttbar prod. (SMEFTatNLO) [Degrande et. al, 08’]

$$\mathcal{O}_G = g_s f^{ABC} G^{A,\mu}_{\nu} G^{B,\nu}_{\rho} G^{C,\rho}_{\mu}$$

$$\mathcal{O}_{\varphi G} = \left(\varphi^\dagger\varphi - \frac{v^2}{2}\right) G^{\mu\nu}_A G^A_{\mu\nu}$$

$$\mathcal{O}_{tG} = g_s(\bar{Q}\sigma^{\mu\nu}T^A\, t)\tilde{\varphi}G^A_{\mu\nu} + \text{h.c.}$$

+4F operators

$$\mathcal{O}^{(8,1)}_{Qq}, \mathcal{O}^{(8,3)}_{Qq}, \mathcal{O}^{(8)}_{tu}, \mathcal{O}^{(8)}_{td}, \mathcal{O}^{(8)}_{Qu}, \mathcal{O}^{(8)}_{Qd}, \mathcal{O}^{(8)}_{tq}$$

SMEFT

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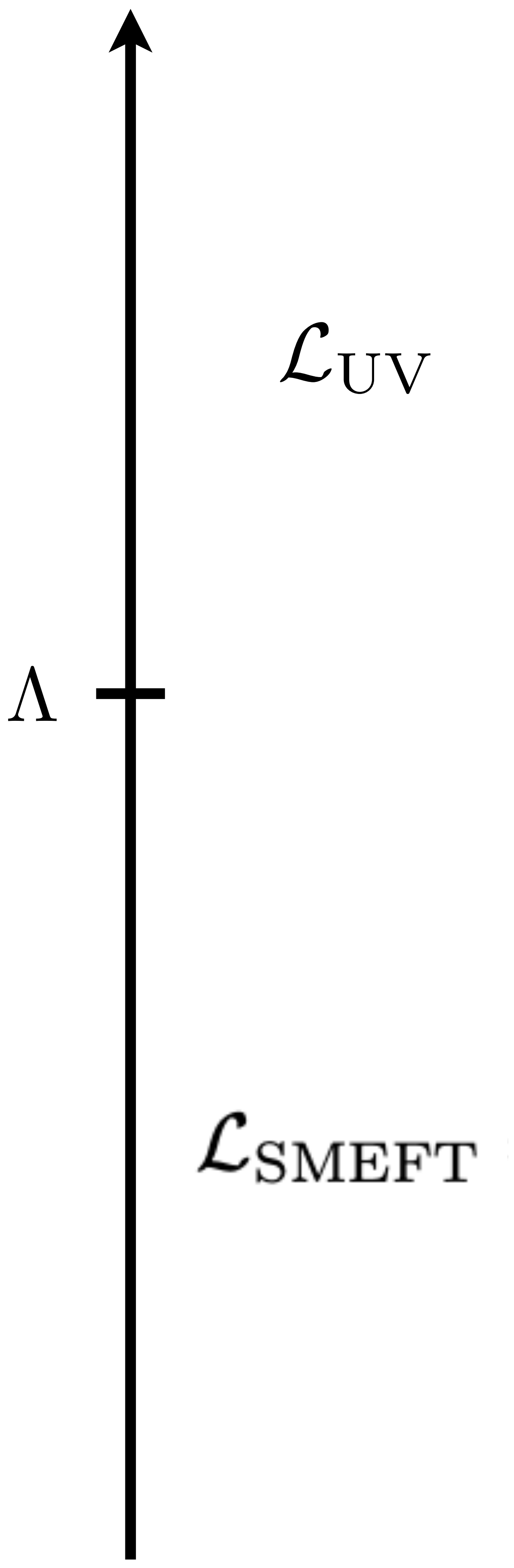

LO-QCD in ttbar prod. (SMEFTatNLO) [Degrande et. al, 08']

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+4F operators

$$\mathcal{O}_{Qq}^{(8,1)}, \mathcal{O}_{Qq}^{(8,3)}, \mathcal{O}_{tu}^{(8)}, \mathcal{O}_{td}^{(8)}, \mathcal{O}_{Qu}^{(8)}, \mathcal{O}_{Qd}^{(8)}, \mathcal{O}_{tq}^{(8)}$$

Maximal points are affected by SMEFT?

Can SMEFT induce new regions?

SMEFT

Back to the R-matrix… $R^I_{\alpha_1\alpha_2,\beta_1\beta_2} \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors} \\ \text{a,b spins}}} \mathcal{M}^*_{\alpha_2\beta_2} \mathcal{M}_{\alpha_1\beta_1}$

With dim-six contributions:

$$\mathcal{M}_{\alpha\beta} = \mathcal{M}^{\text{SM}}_{\alpha\beta} + \frac{1}{\Lambda^2}\mathcal{M}^{(\text{d6})}_{\alpha\beta} \longrightarrow \rho = \frac{R^{\text{SM}} + R^{\text{EFT}}}{\text{tr}(R^{\text{SM}}) + \text{tr}(R^{\text{EFT}})}$$

The Fano coefficients $X = X^{(0)} + \frac{1}{\Lambda^2}X^{(1)} + \frac{1}{\Lambda^4}X^{(2)}$ where

$X = \tilde{A},\ \tilde{C}_{ij}$ and $\tilde{B}^{\pm}_i$

linear $\mathcal{O}(\Lambda^{-2})$ quadratics* $\mathcal{O}(\Lambda^{-4})$

* from dim-six

SMEFT

Back to the R-matrix…

$$R^I_{\alpha_1\alpha_2,\beta_1\beta_2} \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors}\\ \text{a,b spins}}} \mathcal{M}^*_{\alpha_2\beta_2}\, \mathcal{M}_{\alpha_1\beta_1}$$

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At $\mathcal{O}(\Lambda^{-2})$

$$\tilde{C}^{gg,(1)}_{nn} = \frac{g_s^2}{\Lambda^2}\frac{1}{1-\beta^2 z^2}\left[\frac{-7g_s^2 v m_t}{12\sqrt{2}}c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2 - (1-\beta^2)m_h^2}c_{\varphi G} + \frac{9g_s^2\beta^2 m_t^2 z^2}{8}c_G\right]$$

SMEFT entanglement: gg-initiated

[Aoude, Madge, Maltoni, Mantani, 22']

only $\mathcal{O}_{tG}, \mathcal{O}_G, \mathcal{O}_{\varphi G}$ contributes

$$\rho = \frac{R^{\mathrm{SM}} + R^{\mathrm{EFT}}}{\mathrm{tr}(R^{\mathrm{SM}}) + \mathrm{tr}(R^{\mathrm{EFT}})}$$

gg-initiated at threshold $\beta^2 = 0$

- linear interference exactly cancel, maximally entangled state unchanged
- quadratics vanish for $\mathcal{O}_{\varphi G}$ and decreases for $\mathcal{O}_{tG}, \mathcal{O}_G$

gg-initiated at high-E: $\beta^2 \to 1$: EFT not valid but $m_t^2 \ll \hat{s} \ll \Lambda^2$

- linear interference: sign dependent
- quadratics always decreases

SMEFT entanglement: qq-initiated

[Aoude, Madge, Maltoni, Mantani, 22']

only $\mathcal{O}_{tG}$ and 4F contributes

$$\rho = \frac{R^{\rm SM} + R^{\rm EFT}}{\mathrm{tr}(R^{\rm SM}) + \mathrm{tr}(R^{\rm EFT})}$$

qq-initiated at threshold $\beta^2 = 0$

- no contributions for linear and quad

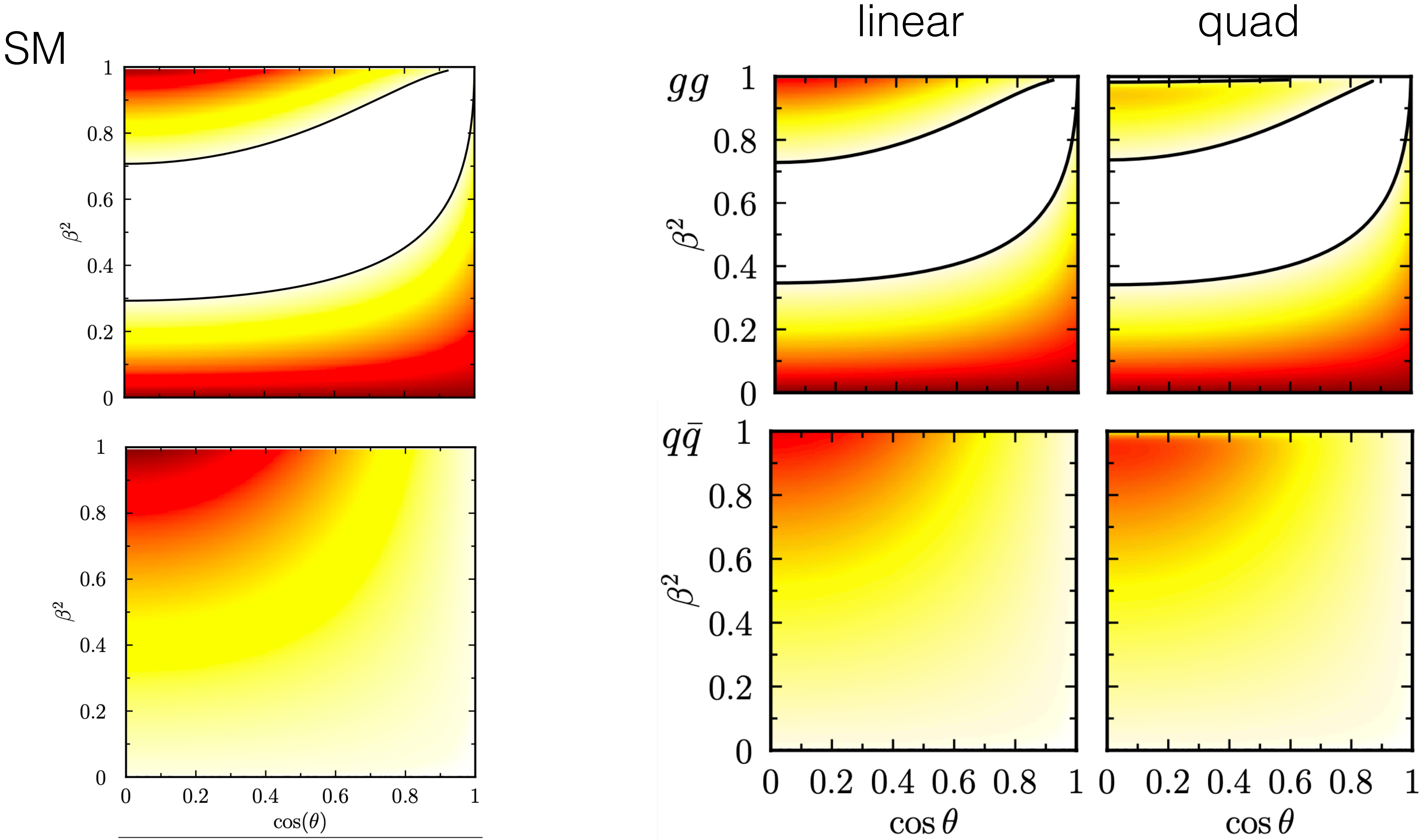
qq-initiated at high-E: $m_t^2 \ll \hat{s} \ll \Lambda^2$

- sign dependent for linear and quadratics always decreases

everything gets more involved for pp

SMEFT entanglement

$$\mathcal{O}_{tG} = g_s(\bar{Q}\sigma^{\mu\nu}T^A\, t)\tilde{\varphi}G^A_{\mu\nu} + \text{h.c.}$$



SMEFT entanglement

$$\rho = \frac{R^{\mathrm{SM}} + R^{\mathrm{EFT}}}{\mathrm{tr}(R^{\mathrm{SM}}) + \mathrm{tr}(R^{\mathrm{EFT}})}$$

Δ_0 calculated with SM R's

$$\Delta_1 \equiv \Delta - \Delta_0$$

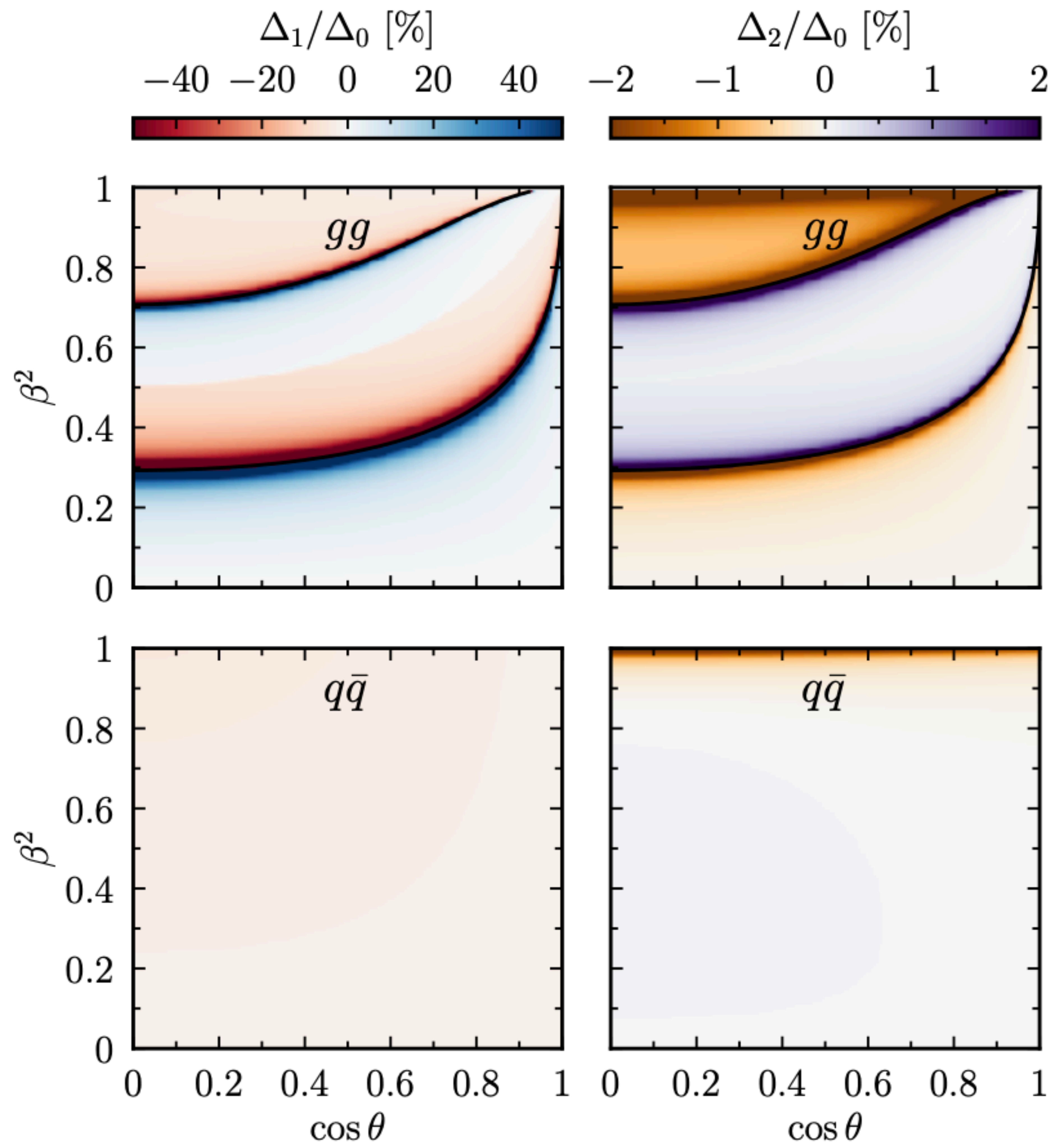
→ calculated with SMEFT R's up to $\mathcal{O}(\Lambda^{-2})$

$$\Delta_2 \equiv \Delta - \Delta_1 - \Delta_0$$

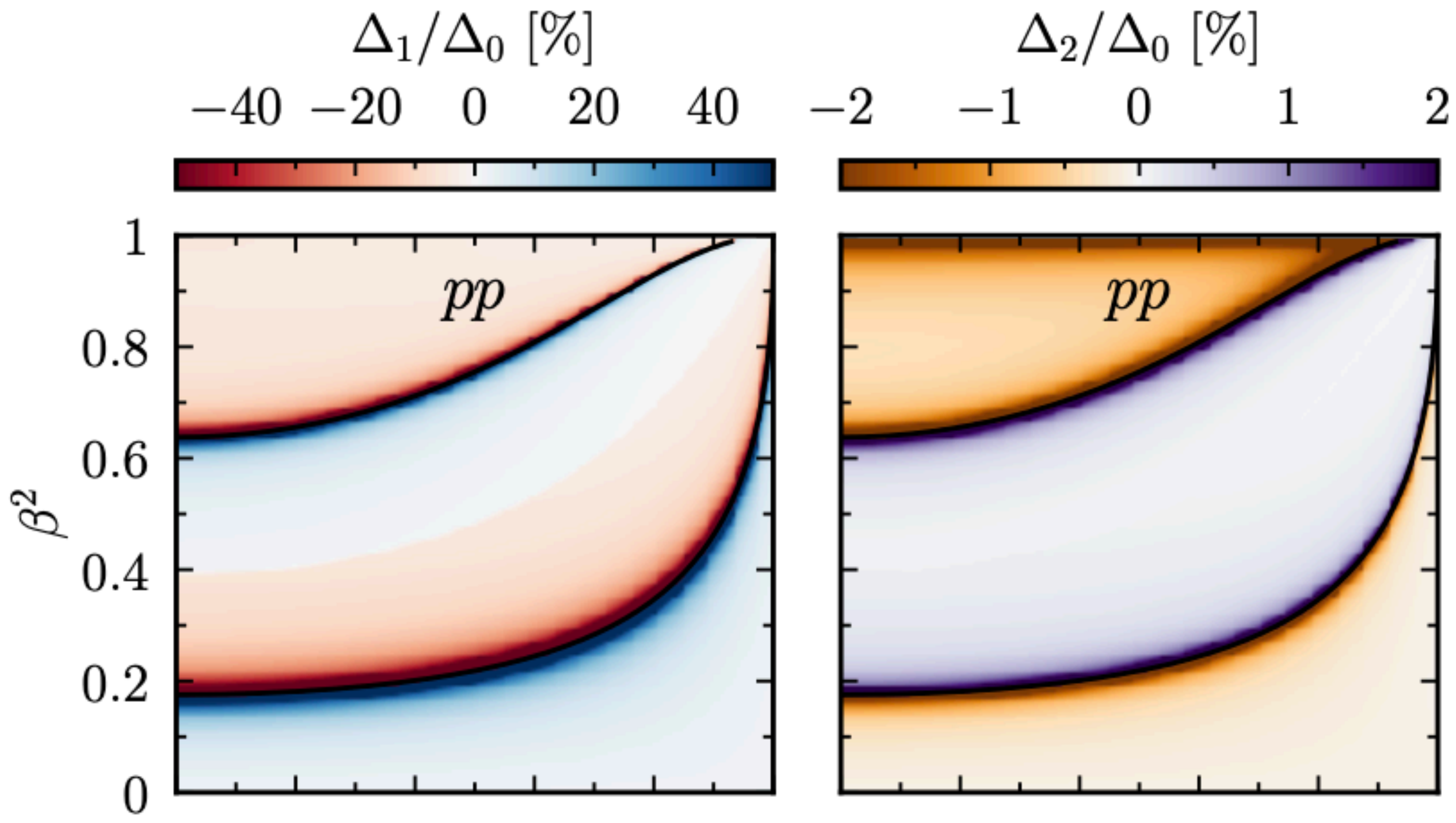
→ calculated with SMEFT R's up to $\mathcal{O}(\Lambda^{-4})$

SMEFT entanglement marker

separate channels



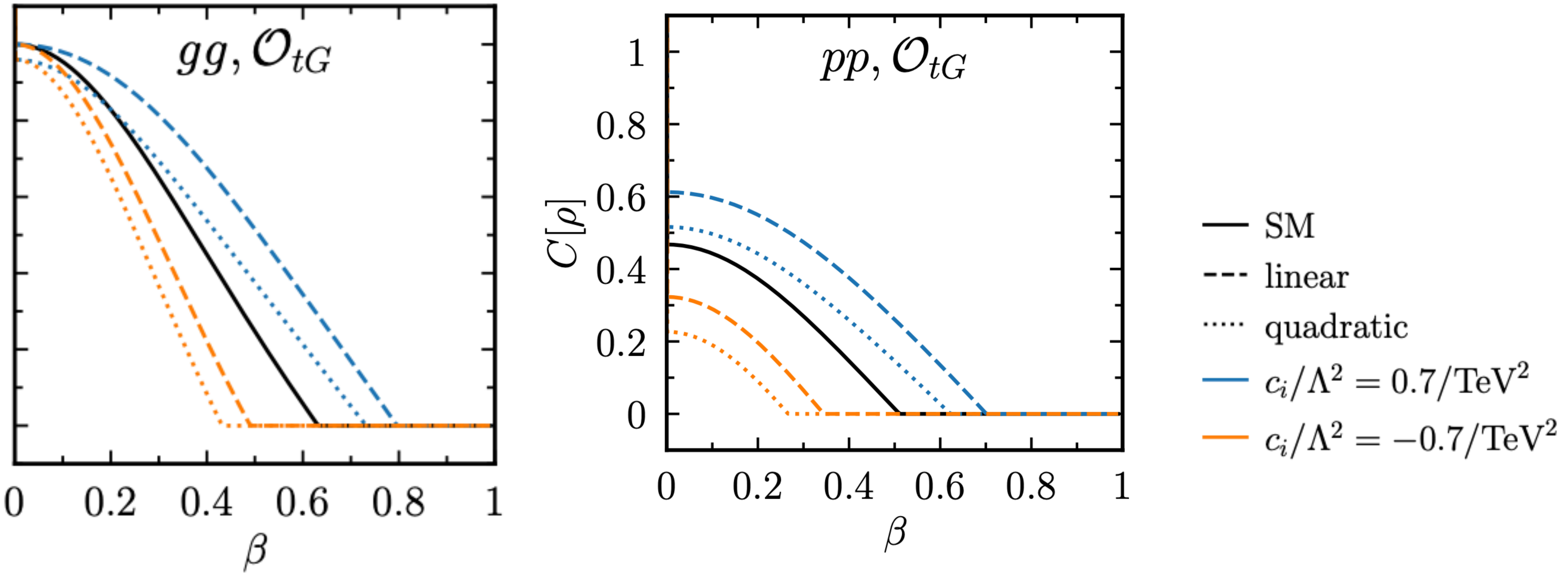
mixed state



SMEFT averaged concurrence

Average over the solid angle

$$\bar{R} = (4\pi)^{-1} \int \mathrm{d}\Omega\, R(\hat{s}, \boldsymbol{k}) \,,$$



SMEFT quantum state

At threshold

$$\rho_{gg}^{\text{EFT}}(0,z) = p_{gg}|\Psi^-\rangle_{\boldsymbol{p}}\langle\Psi^-|_{\boldsymbol{p}} + (1-p_{gg})|\Psi^+\rangle_{\boldsymbol{p}}\langle\Psi^+|_{\boldsymbol{p}}\,.$$

(Induces a triplet)

$$\rho_{q\bar{q}}^{\text{EFT}}(0,z) = p_{q\bar{q}}\,|\uparrow\uparrow\rangle_{\boldsymbol{p}}\,\langle\uparrow\uparrow|_{\boldsymbol{p}} + (1-p_{q\bar{q}})\,|\downarrow\downarrow\rangle_{\boldsymbol{p}}\,\langle\downarrow\downarrow|_{\boldsymbol{p}}\ ,$$

(changes the mixed state)

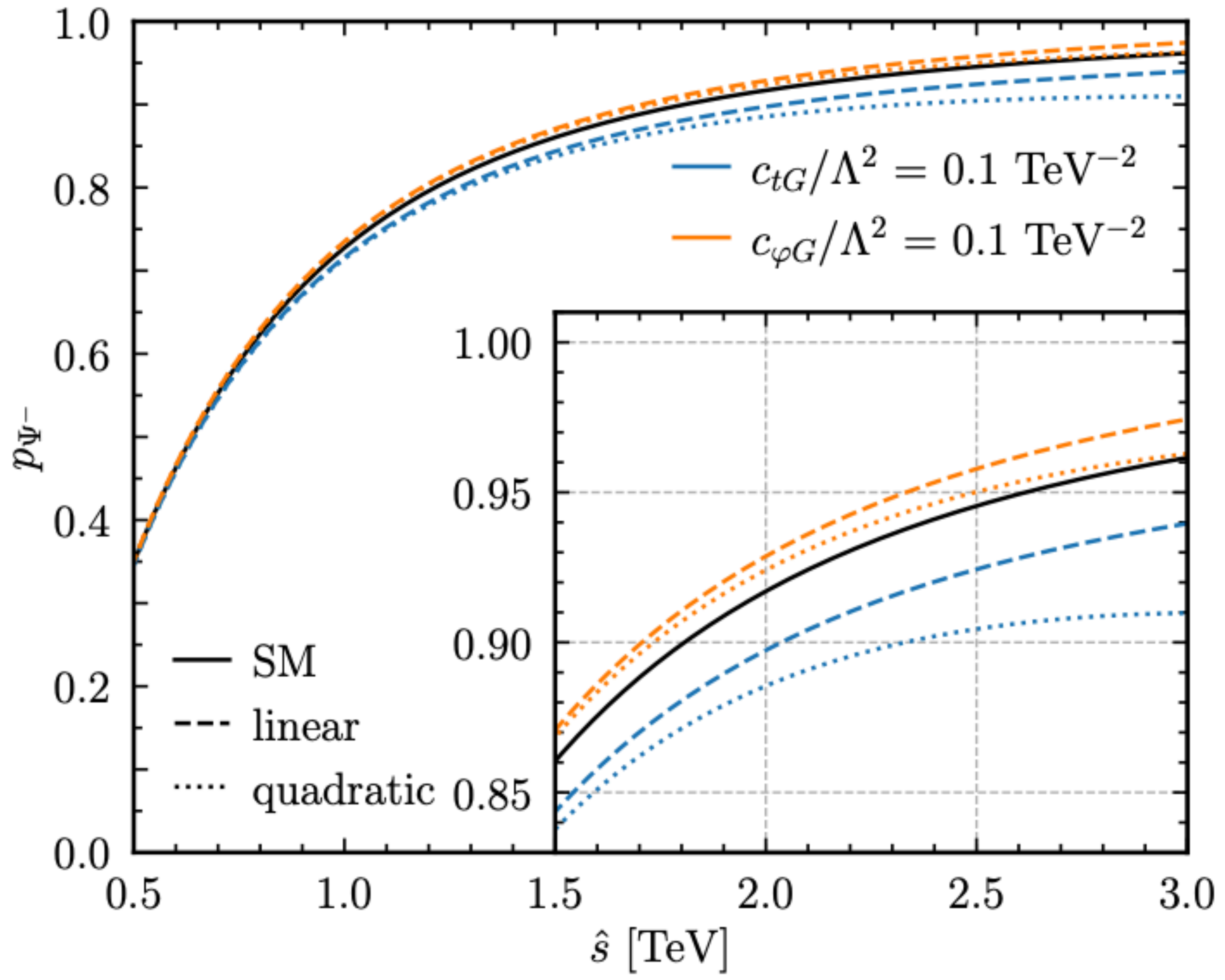
where

$$p_{gg} = \frac{72}{7\Lambda^4} m_t^2 (3\sqrt{2} m_t\, c_G + v\, c_{tG})^2\,,$$

$$p_{q\bar{q}} = \frac{1}{2} - 4\frac{c_{VA}^{(8),u}}{\Lambda^2} + \frac{8m_t^4}{\Lambda^4}\Big(\frac{v\sqrt{2}}{m_t} c_{VA}^{(8),u} c_{tG} - 9 c_{VA}^{(1),u} c_{VV}^{(1),u} + 2 c_{VA}^{(8),u} c_{VV}^{(8),u}\Big),$$

At high-pT

(triplet prob.) $p_{\Psi^-} = \langle\Psi^-|_{\boldsymbol{n}}\,\rho\,|\Psi^-\rangle_{\boldsymbol{n}}$



Conclusions

SM induces maximal entanglement points/regions in ttbar

Purely linear interference SMEFT effects vanish in these regions!

Quadratic interference decrease the entanglement at these points

Missing dim-8 linear interference and double-insertions at $\mathcal{O}(\Lambda^{-4})$

Questions?

QI observables can help contraint SMEFT ops?

Other processes?

All this effects due to approxs? Tree-level, only dim-six, no double insertions

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