Aspects of Supersymmetric Duality

SUSY Breaking 2011

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Overview

- Mapping operators in SUSY theories: chiral ring and conserved currents
- The long U multiplet and the supercurrent multiplet
- SQCD in the free magnetic phase
- Deformed moduli space
- Kutasov theory and the UV superpotential
- Adjoint SQCD
- Soft deformations, $\delta\mathcal{L} \supset m^2 U|$.
- Sketch of a model of EWSB

Mapping Operators in SUSY theories

• UV complete QFT: RG flow between a UV and IR fixed point.

• We will study only asymptotically free theories like SQCD and some simple generalizations. Simple to generalize further.

• Question: given $\mathcal{O}^{UV}$, what is $\mathcal{O}^{IR}$?

• Easy operators to map: short multiplets, like members of the chiral ring, conserved currents.

• Harder operators to map: long multiplets.

• Sometimes can embed these long multiplets inside short multiplets of higher spin and use these larger multiplets to gain traction.

Mapping Operators in SUSY theories (cont...)

• Quantities of interest, real UV bilinears:

$$c^i_j \Phi^\dagger_i \Phi^j + \tilde{c}^i_j \tilde{\Phi}^\dagger_i \tilde{\Phi}^j \ , \tag{1}$$

Appropriate factors of e^V, etc.

• For generic $c, \tilde{c}$ this defines a long multiplet, i.e.,

$$\bar{D}^2 \left(c^i_j \Phi^\dagger_i \Phi^j + \tilde{c}^i_j \tilde{\Phi}^\dagger_i \tilde{\Phi}^j \right) = c \mathrm{Tr} W^2_\alpha + ... \tag{2}$$

• Can we map such an operator to the IR?

• To do that, we need a short multiplet in which to embed it. Natural candidates: symmetry currents of various kinds. R-symmetry current a good option (if present).

• We will study theories with an R-symmetry.

The Role of the R-symmetry Current

• Since $[R,Q] \sim Q$, $\{Q,\bar{Q}\} \sim P$, the R-current transforms in a multiplet with $S_{\mu\alpha}$ and $T_{\mu\nu}$.

$$\bar{D}^{\dot{\alpha}}\mathcal{R}_{\dot{\alpha}\alpha} = \chi_\alpha \ . \tag{3}$$

When $\chi_\alpha = 0$, this is the superconformal R-symmetry.

• There is an ambiguity in the above equation under $\mathcal{R}_{\alpha\dot{\alpha}} \to \mathcal{R}_{\alpha\dot{\alpha}} + [D_\alpha, \bar{D}_{\dot{\alpha}}]\, J$ and $\chi_\alpha \to \chi_\alpha + \frac{3}{2}\bar{D}^2 D_\alpha J$ for conserved J, i.e., $\bar{D}^2 J = 0$. This affects the supercurrent and stress tensor through improvements.

• For the theories we will consider, can write

$$\chi_\alpha = \bar{D}^2 D_\alpha U \ , \tag{4}$$

for a well-defined U.

• Solving the above equations in the UV, we find

$$\begin{aligned} \mathcal{R}^{UV}_{\alpha\dot{\alpha}} &= \sum_i \left(2D_\alpha \Phi_i \bar{D}_{\dot{\alpha}} \bar{\Phi}^i - r_i [D_\alpha, \bar{D}_{\dot{\alpha}}] \Phi_i \bar{\Phi}^i\right), \\ U^{UV} &= \sum_i \left(1 - \frac{3r_i}{2}\right) \bar{\Phi}^i \Phi_i \ . \end{aligned} \tag{5}$$

• U fixed up to $U \to U + Y + \bar{Y}$. Will see later that such terms may appear in the IR.

The R-symmetry Current and the RG Flow

• **Idea:** Use the $\mathcal{R}$-multiplet to follow U along the flow.

• At the IR fixed point, we know what should happen to $\mathcal{R}_{\alpha\dot{\alpha}}$. Indeed, either this multiplet flows to the superconformal R-multiplet or to an object that can be improved to the superconformal R-multiplet:

$$\mathcal{R}^{CFT}_{\alpha\dot{\alpha}} = \mathcal{R}^{IR}_{\alpha\dot{\alpha}} - [D_\alpha, \bar{D}_{\dot{\alpha}}]J \ , \quad U^{CFT} = U^{IR} - \frac{3}{2}J = 0 \ . \qquad (6)$$

Determine $\mathcal{R}^{CFT}_{\alpha\dot{\alpha}}$ from duality or a-maximization.

• **Upshot:** Therefore, $U \to \frac{3}{2}J$.

• J may be a conserved current of the full theory or an accidental symmetry of the IR. We will see an extreme version of this for SQCD in the free magnetic range.

• In the case that $U^{IR} = 0$, we can say a bit more using conformal perturbation theory. If approach is via a marginally irrelevant operator, we have $U \sim \gamma J$. Otherwise, we have $U \sim \Lambda^{2-d}\mathcal{O}$ for $d > 2$ (using unitarity).

• In the case of a free magnetic phase, we have

$$U^{IR} = \sum_i \left(1 - \frac{3r_i}{2}\right) \bar{\phi}^i \phi_i \ , \tag{7}$$

for the "emergent" d.o.f's.

Example I: SQCD in the Free Magnetic Range

• Consider $SU(N_c)$ with $N_c + 1 < N_f \leq 3N_c/2$: this is a flow between Gaussian fixed points

• The UV (electric) theory:

$$
\begin{array}{cccccc}
 & SU(N_c) & SU(N_f)\times SU(N_f) & U(1)_R & U(1)_B \\
 & & & & \\
Q & \mathbf{N_c} & \mathbf{N_f}\times\mathbf{1} & 1-\frac{N_c}{N_f} & 1 \\
\tilde{Q} & \mathbf{\bar{N}_c} & \mathbf{1}\times\mathbf{\bar{N}_f} & 1-\frac{N_c}{N_f} & -1
\end{array}
\tag{8}
$$

• Some bilinears that we can write are $c_i^j Q^i Q_j^\dagger + \tilde{c}_i^j \tilde{Q}^i \tilde{Q}_j^\dagger$. What are they in the IR?

- We have the following IR (magnetic) theory

$$
\begin{array}{ccccc}
 & SU(N_f-N_c) & SU(N_f)\times SU(N_f) & U(1)_R & U(1)_B \\
q & \mathbf{N_f-N_c} & \mathbf{\bar{N}_f\times 1} & \frac{N_c}{N_f} & \frac{N_c}{N_f-N_c} \\
\tilde{q} & \mathbf{\bar{N}_f-\bar{N}_c} & \mathbf{1\times\bar{N}_f} & \frac{N_c}{N_f} & -\frac{N_c}{N_f-N_c} \\
M & \mathbf{1} & \mathbf{N_f\times N_f} & 2-2\frac{N_c}{N_f} & 0
\end{array}
\tag{9}
$$

- Some objects are trivial to map, e.g. $QQ^\dagger - \tilde{Q}\tilde{Q}^\dagger \longrightarrow \frac{N_c}{N_f-N_c}\Big(|q|^2-|\tilde{q}|^2\Big)$.

Example I: SQCD in the Free Magnetic Range (cont...)

• But what about $J_A = QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger$? It is not conserved:

$$\bar{D}^2 J_A = \text{Tr} W_\alpha^2 \ . \tag{10}$$

• **Claim:** We can follow this operator using the $\mathcal{R}$ multiplet. Indeed, using the R-charge assignments in the electric table, we find

$$U^{UV} = \left(-\frac{1}{2} + \frac{3N_c}{2N_f}\right)\left(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger\right) \tag{11}$$

• Using the R-charge assignments in the IR, we find

$$U^{IR} = \left(1 - \frac{3N_c}{2N_f}\right)\left(qq^\dagger + \tilde{q}\tilde{q}^\dagger\right) - \left(2 - \frac{3N_c}{N_f}\right) MM^\dagger \tag{12}$$

- Therefore, we find

$$QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger \longrightarrow \frac{2N_f - 3N_c}{3N_c - N_f}\left(qq^\dagger + \tilde{q}\tilde{q}^\dagger - 2MM^\dagger\right) \tag{13}$$

- Acting with $\bar{D}^2$ on both sides of the above equation, we find

$$W^2_{\alpha,\mathrm{el}} \longrightarrow \frac{2N_f - 3N_c}{3N_c - N_f} W^2_{\alpha,\mathrm{mag}} \tag{14}$$

- This gives the physical relation between the two field strengths.

Example II: The Deformed Moduli Space

- Consider $SQCD$ with $N_f = N_c > 2$

- The IR is described by M and B satisfying

$$\det M - B\tilde{B} = \Lambda^{2N_c} \ . \tag{15}$$

Therefore some of the short distance symmetries are spontaneously broken.

- Will find some ambiguities in following U. In some vacua we will have enough (broken) symmetry to fix U. In others we won't, but we won't discuss these cases here.

Example II: The Deformed Moduli Space (cont...)

- Consider first the following vacuum

$$M = 0 \; , \qquad B = \tilde{B} = \Lambda^{N_c} \; . \tag{16}$$

- In this vacuum the symmetry is broken as follows

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R \hookrightarrow SU(N_f)_L \times SU(N_f)_R \times U(1)_R \tag{17}$$

- We can use our previous techniques to fix U as follows:

$$U = \delta M \delta M^\dagger + \delta b \delta b^\dagger \; , \tag{18}$$

where δb is the Goldstone superfield for the $U(1)_B$ breaking.

- Demanding invariance under the (non-linearly realized) $U(1)_B$ symmetry requires

$$QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger \longrightarrow \mathrm{Tr}\left(\delta M \delta M^\dagger\right) + \frac{1}{2}(\delta b + \delta b^\dagger)^2 \; . \tag{19}$$

• Note that this fixes the holomorphic + anti-holomorphic ambiguity.

Example III: The Kutasov Theory

• We consider the following electric theory with $\frac{N_c}{k} < N_f < \frac{2N_c}{2k-1}$

$$
\begin{array}{cccccc}
 & SU(N_c) & SU(N_f)\times SU(N_f) & U(1)_R & U(1)_B \\
 & & & & \\
Q & \mathbf{N_c} & \mathbf{N_f}\times\mathbf{1} & 1-\frac{2}{k+1}\frac{N_c}{N_f} & 1 \\
\tilde{Q} & \bar{\mathbf{N}}_\mathbf{c} & \mathbf{1}\times\bar{\mathbf{N}}_\mathbf{f} & 1-\frac{2}{k+1}\frac{N_c}{N_f} & -1 \\
X & \mathbf{N_c^2}-\mathbf{1} & \mathbf{1}\times\mathbf{1} & \frac{2}{k+1} & 0
\end{array}
\tag{20}
$$

and the following superpotential

$$W = s_0 \mathrm{Tr}(X^{k+1}) \ . \tag{21}$$

- And the following magnetic theory

	$SU(kN_f - N_c)$	$SU(N_f) \times SU(N_f)$	$U(1)_R$
q	$\mathbf{kN_f - N_c}$	$\mathbf{\bar{N}_f \times 1}$	$1 - \frac{2}{k+1}\frac{kN_f - N_c}{N_f}$
$\tilde{q}$	$\overline{kN_f - N_c}$	$\mathbf{1 \times N_f}$	$1 - \frac{2}{k+1}\frac{kN_f - N_c}{N_f}$
Y	$\mathbf{(kN_f - N_c)^2 - 1}$	$\mathbf{1 \times 1}$	$\frac{2}{k+1}$
M_j	$\mathbf{1}$	$\mathbf{N_f \times \bar{N}_f}$	$2 - \frac{4}{k+1}\frac{N_c}{N_f} + \frac{2}{k+1}(j-1)$

(22)

and the following superpotential

$$W_{\text{mag}} = -\frac{s_0}{k+1}\text{Tr } Y^{k+1} + \frac{s_0}{\mu^2}\sum_{j=1}^{k} M_j \tilde{q} Y^{k-j} q \ . \qquad (23)$$

Example III: The Kutasov Theory (cont...)

• The UV superpotential breaks the symmetry associated with the current

$$J_X = \frac{N_c}{N_f}\left(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger\right) - XX^\dagger \ . \tag{24}$$

• Using baryon matching we can fix the coefficient of $YY^\dagger$ in the IR.

• This operator cannot be followed using the $\mathcal{R}$-multiplet

• But, using our previous tricks, there is another interesting long multiplet that we can follow

$$U^{UV} = \left(-\frac{1}{2} + \frac{3}{k+1}\frac{N_c}{N_f}\right)\left(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger\right) + \left(1 - \frac{3}{k+1}\right)XX^\dagger. \tag{25}$$

- Using the $\mathcal{R}$ multiplet we find that in the IR

$$
\begin{aligned}
U^{IR} \;=\; & \left(-\frac{1}{2}+\frac{3}{k+1}\frac{kN_f-N_c}{N_f}\right)\left(qq^{\dagger}+\tilde{q}\tilde{q}^{\dagger}\right)+\left(1-\frac{3}{k+1}\right)YY^{\dagger} \\
+ \; & \sum_j\left(-2+\frac{6}{k+1}\frac{N_c}{N_f}-\frac{3(j-1)}{k+1}\right)M_jM_j^{\dagger}\ . \qquad (26)
\end{aligned}
$$

- Therefore:

$$
\begin{aligned}
& \left(-\frac{1}{2}+\frac{3}{k+1}\frac{N_c}{N_f}\right)\left(QQ^{\dagger}+\tilde{Q}\tilde{Q}^{\dagger}\right)+\left(1-\frac{3}{k+1}\right)XX^{\dagger}\rightarrow \\
& \left(-\frac{1}{2}+\frac{3}{k+1}\frac{kN_f-N_c}{N_f}\right)\left(qq^{\dagger}+\tilde{q}\tilde{q}^{\dagger}\right)+\left(1-\frac{3}{k+1}\right)YY^{\dagger} \\
+ \; & \sum_j\left(-2+\frac{6}{k+1}\frac{N_c}{N_f}-\frac{3(j-1)}{k+1}\right)M_jM_j^{\dagger} \qquad (27)
\end{aligned}
$$

Example IV: Adjoint SQCD

• We have focused mostly on theories with a free IR description. Here we will discuss adjoint SQCD. It is believed to flow to an interacting IR SCFT.

	$SU(N_c)$	$SU(N_f)\times SU(N_f)$	$U(1)_R$	$U(1)'$	$U(1)_B$
Q	$\mathbf{N_c}$	$\mathbf{N_f}\times\mathbf{1}$	$1-\frac{2N_c}{3N_f}$	1	1
$\tilde{Q}$	$\mathbf{\bar{N}_c}$	$\mathbf{1}\times\mathbf{\bar{N}_f}$	$1-\frac{2N_c}{3N_f}$	1	−1
X	$\mathbf{N_c^2}-\mathbf{1}$	$\mathbf{1}\times\mathbf{1}$	2/3	−1	0

(28)

• Don't know much about the IR, but we can infer when some fields $M^i = QX^i\tilde{Q}$ become free—e.g. for $N_f/N_c < (3+\sqrt{7})^{-1}$, $M_0 = Q\tilde{Q}$ becomes free.

- Using out techniques, we can map

$$\left(-\frac{1}{2}+\frac{N_c}{N_f}\right)\left(QQ^\dagger+\tilde{Q}\tilde{Q}^\dagger\right) \longrightarrow \sum_{j=0}^{P(N_f/N_c)}\left(1-\frac{3R(M_j)}{2}\right) M_j M_j^\dagger + \ldots$$

$$= -\sum_{i=0}^{P(N_f/N_c)}\left(j+2-2\frac{N_c}{N_f}\right) M_j M_j^\dagger + \ldots \quad (29)$$

Applications: Soft SUSY Breaking Terms

• At weak coupling, soft breaking terms can be thought of as bottom components of current multiplets.

• **Idea:** Deform the UV by adding a probe soft term and see what happens in the IR. $\delta\mathcal{L} = -m^2 J|$ straightforward to follow for conserved J. We can use our results to follow what happens when we deform by a (non)-conserved current, $\delta\mathcal{L} = -m^2 U|$, and make contact with the literature (e.g., [Arkani-Hamed, Rattazzi], [Luty, Rattazzi], [Cheng, Shadmi], [Fukushima, Kitano, Yamaguchi],...

• More concretely, in SQCD in the free magnetic range suppose we add in the UV

$$\delta\mathcal{L}_{\text{el}} = -m^2 J_A| - m_\lambda (W^2_{\alpha,\text{el}} + \text{hc})| = -m^2 \left(Q Q^\dagger + \tilde{Q}\tilde{Q}^\dagger \right) + m_\lambda (\lambda^2_{\text{el}} + c.c.) \,, \tag{30}$$

• Then, we find that in the IR

$$\delta\mathcal{L}_{\text{mag}} = -m^2 \cdot \frac{2N_f - 3N_c}{3N_c - N_f}\left(qq^\dagger + \tilde{q}\tilde{q}^\dagger - 2MM^\dagger\right) + m_\lambda \cdot \frac{2N_f - 3N_c}{3N_c - N_f}(\lambda^2_{\text{mag}} + c.c.) \ , \tag{31}$$

• In particular, we find the well-known tachyonic squarks in the IR.

• All is not lost, however—even though the magnetic D-terms and superpotential don't help. Indeed, we can weakly gauge baryon number and find a minimum near the origin!

$$q \sim \frac{m}{g_B}\mathbf{1}, \tilde{q} = 0, \quad M = 0 \tag{32}$$

Here $\mathbf{1}$ means the matrix with the upper left $(N_f - N_c) \times (N_f - N_c)$ block set to be proportional to the unit matrix and the rest of the entries are set to zero.

• Calculability just requires $g, h > g_B >> m/\Lambda$ and much smaller than all the other couplings in the theory. This vacuum breaks the magnetic gauge symmetry and Higgses baryon number too. We find: $SU(N_f - N_c) \times SU(N_c) \times SU(N_f)$ left over. Note color-flavor locking.

• On the other hand, if $g_B >\sim g, h$, then we find a vacuum with

$$q \sim \tilde{q} \sim m, \quad M = 0 \ . \tag{33}$$

• Can add more general soft terms using baryonic current and $SU(N_f) \times SU(N_f)$ currents.

Applications: Soft SUSY Breaking Terms (cont...)

• Can find a stable vacuum easily in deformed moduli space example on baryonic branch [Luty, Rattazzi]

$$\delta\mathcal{L} = -m^2\left(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger\right) \to -m^2\left(\mathrm{Tr}\left(\delta M \delta M^\dagger\right) + \frac{1}{2}(\delta b + \delta b^\dagger)^2\right) \tag{34}$$

• Surprisingly, acting with $\bar{D}^2$ on both sides of the above equation, we find that the gaugino mass gets mapped into an enhanced mass for the fermionic partner of the Goldstone boson

$$\frac{1}{8\pi^2}W_\alpha^2 \longrightarrow \bar{D}_{\dot{\alpha}} b^\dagger \bar{D}^{\dot{\alpha}} b^\dagger \tag{35}$$

Applications: EWSB

• These ideas also give rise to a potentially simple model of EWSB

$$\begin{array}{cccc} & SU(N_c) & SU(2)_L & U(1)_Y \\ & & & \\ H & \mathbf{N_c} & \mathbf{2} & \frac{1}{2} \\ \tilde{H} & \bar{\mathbf{N}}_\mathbf{c} & \mathbf{2} & -\frac{1}{2} \\ \Phi_{i=1 \ldots N_c-1} & \mathbf{N_c} & \mathbf{1} & \frac{1}{2} \\ \tilde{\Phi}_{i=1 \ldots N_c-1} & \bar{\mathbf{N}}_\mathbf{c} & \mathbf{1} & -\frac{1}{2} \end{array} \tag{36}$$

Here $N_f = N_c + 1 \geq 4$.

• Baryon number is identified with hypercharge.

• The IR is confining with baryons and mesons

• We have the usual

$$W^{(conf)} = \tilde{B} M B - \Lambda^{3-N_f} \det M \qquad (37)$$

• Since baryon number is identified with the hypercharge gauge symmetry we have the previous vacuum with $B \sim \tilde{B} \sim m, \quad M = 0$.

• Can naturally break EW symmetry (vevs of order m_{soft}) and keep triplets (mesons) from getting a vev.

• May get naturally large Higgs self-couplings from NMSSM-like effect. The NMSSM term is emergent.

Conclusions and Open Questions

• Simple mapping of operators from UV to IR. Gained some mileage and mapped some long multiplets using short higher spin multiplets (the R-current).

• Serious study of the pheno.

• Would like to say more about interacting SCFTs and SUSY breaking in such theories.

• Move beyond probe approximation.