Super OPEs and Susy breaking mediation

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to appear.

Outline

- Intro: why study (s)CFTs. QCD Analogy.
- Current OPEs in SCFTs.
- Applications to GGM.
- Example: apply to OGM.

Many 4d (s)CFTs

- SQCD in Seiberg conformal window.
- N=4.
- non-Lagrangian possibilities.
- Observables = spectrum of operators, their dimensions, OPE coefficients (+ non-local, Wilson loops.)

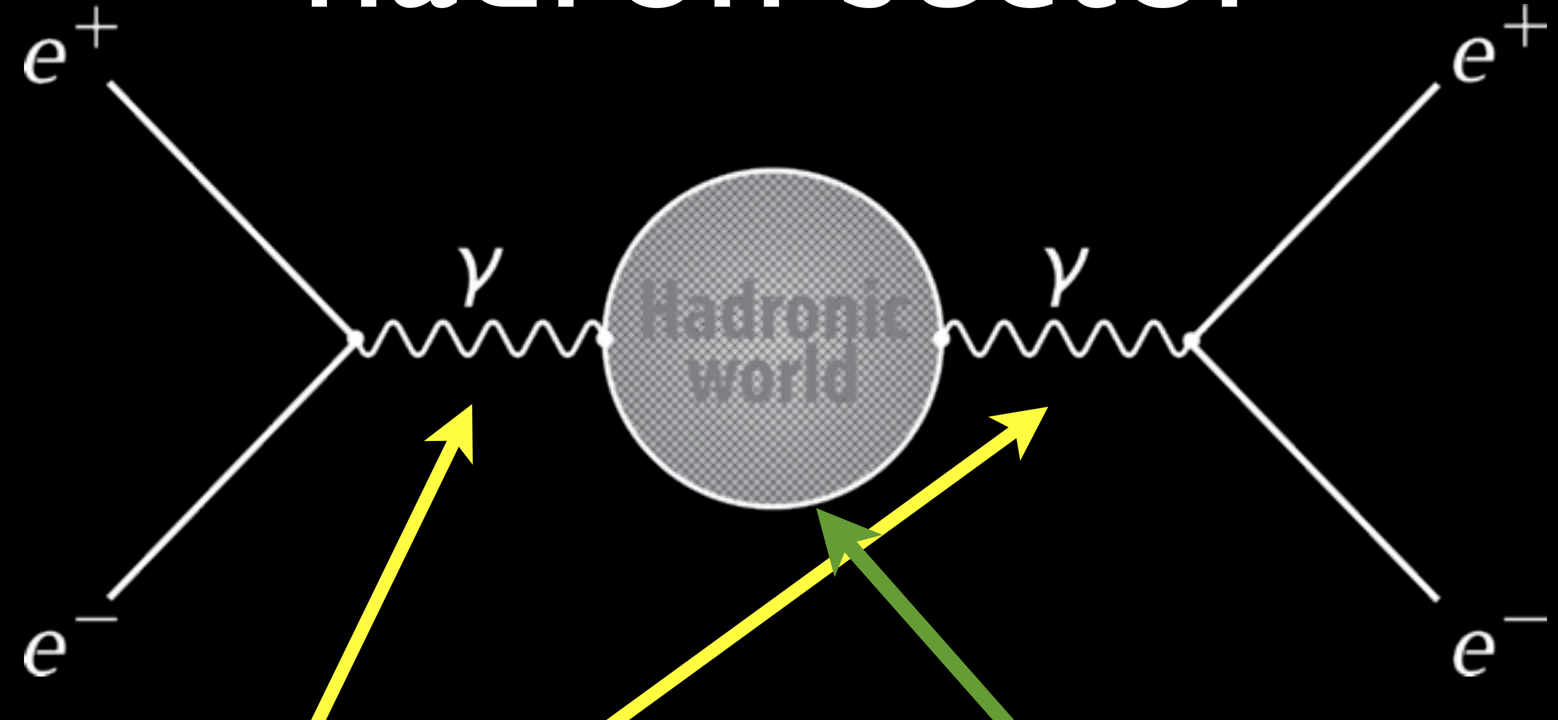
Applications of (s)CFTs (?)

- Help with model building challenges?
- E.g. O(1) anomalous dimensions could help suppress or enhance otherwise finely tuned quantities. Examples: sequestering, flavor hierarchy from anarchy, mu / Bmu in GM, conformal technicolor, etc.
- Or flowing near CFTs. E.g. walking technicolor, unparticles with mass gaps.

Another kind of application of (s)CFTs

- Softly broken symmetries can be regarded as spontaneously broken, get selection rules. IR broken symmetry restored in UV.
- Example: QCD. Not conformal, RG flow. UV: asymptotically free CFT. IR: chiral symmetry breaking, confinement. Would like to relate IR to simpler to UV physics.

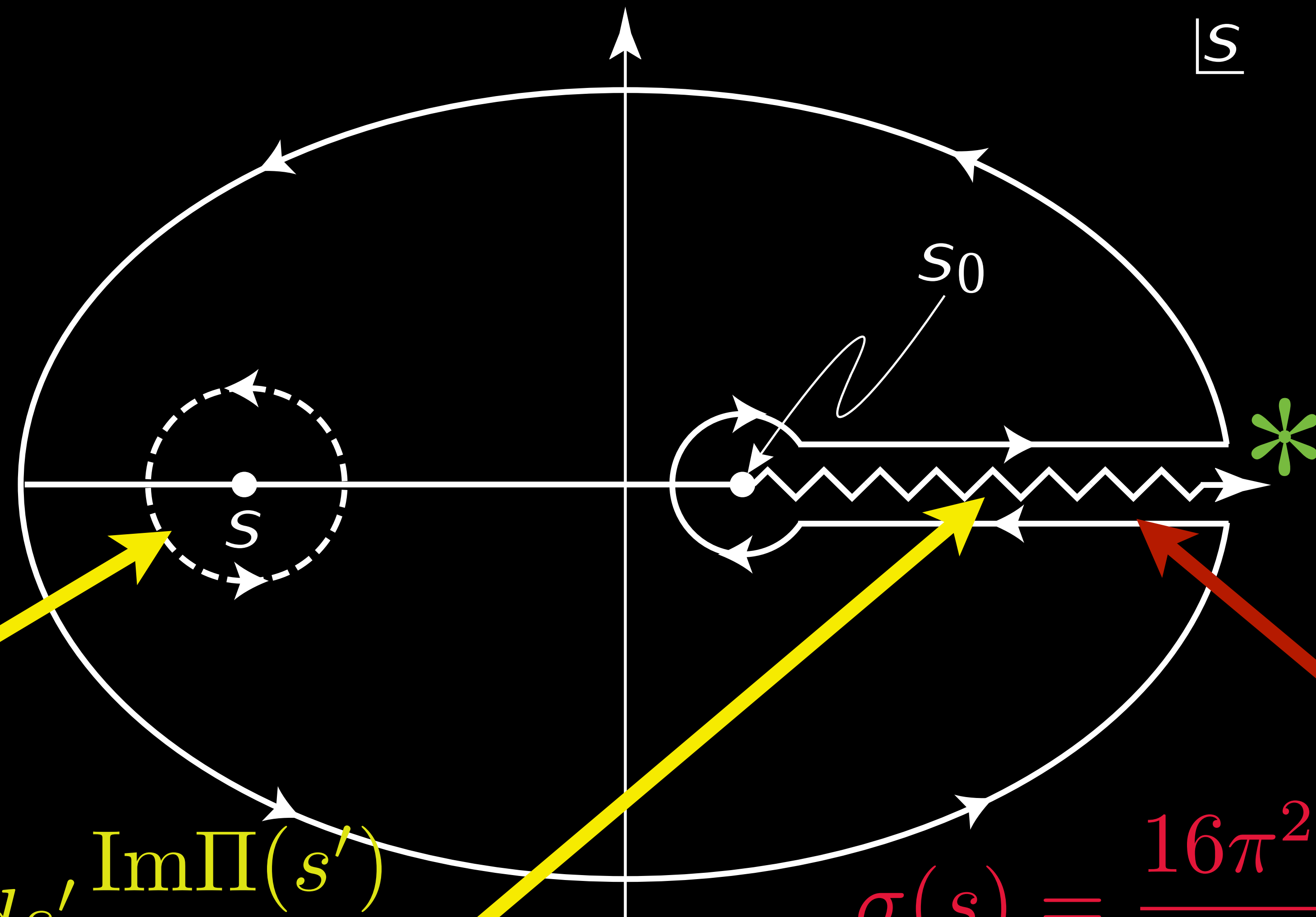
Example: leptons probing the hadron sector



$$i\Pi_h^{\mu\nu} = e^2 \int d^4x\, e^{-ip\cdot x} \langle J^\mu(x) J^\nu(0) \rangle$$

Tool: use OPE to separate UV and IR physics.

Analyticity, optical theorem, OPE



$$\Pi(s) = \frac{1}{\pi}\int_{s_0}^{\infty} ds' \frac{\mathrm{Im}\Pi(s')}{s'-s}$$

$$\sigma(s) = \frac{16\pi^2\alpha^2}{s}\mathrm{Disc}\Pi(s)$$

* Use OPE @ large s. Use approx'ly for all s.

QCD Sum rules, etc. (Shifman, Vainshtein, Zakharov, Valentin.

Change the subject to GGM

Visible sector soft susy breaking masses in GGM from hidden sector current current correlators:

$$M_{\text{gaugino}} \sim \alpha \int d^4x \langle Q^2 (J(x)J(0))\rangle$$

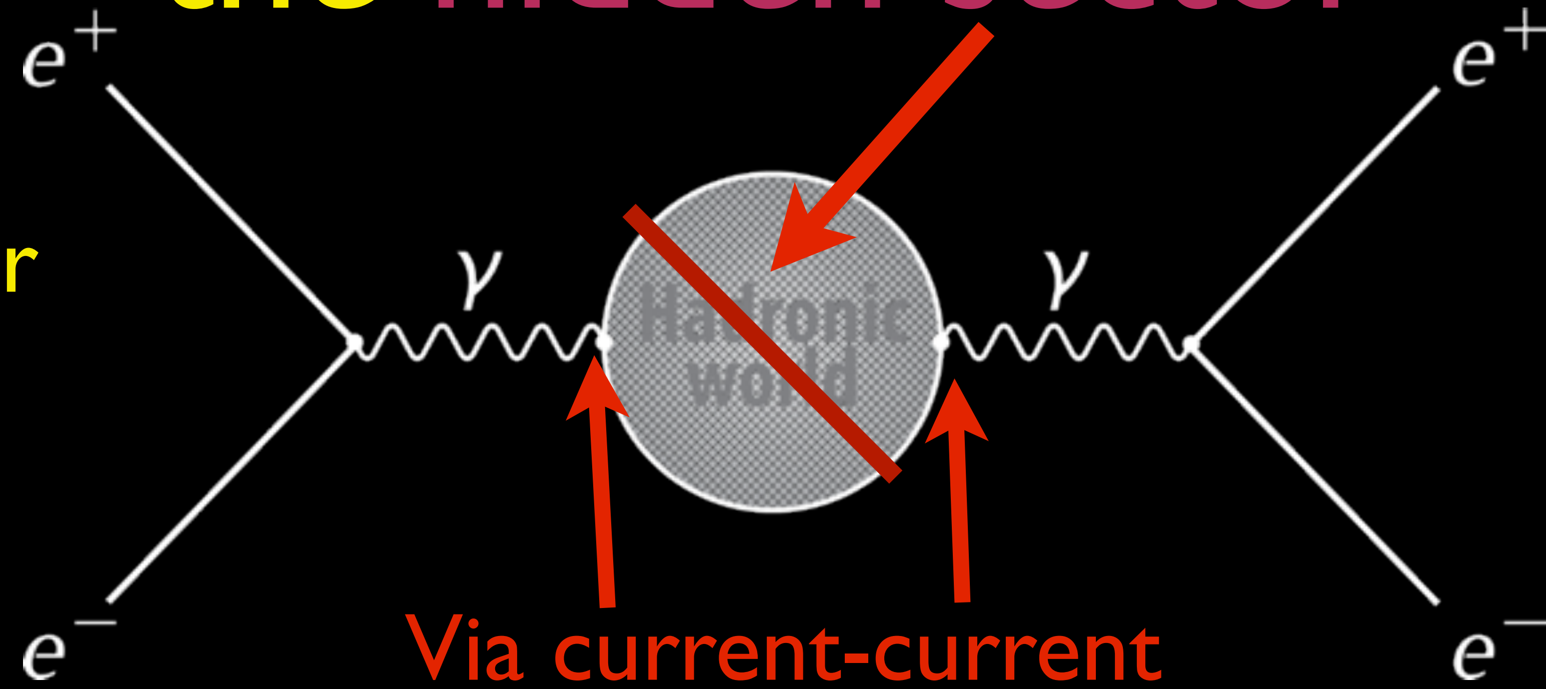
$$m^2_{\text{sfermion}} \sim \alpha^2 \int d^4x \ln(x^2M^2)\langle Q^4 (J(x)J(0))\rangle$$

(Buican, Meade, Seiberg, Shih)

Bottom up: LHC will, we hope, someday measure these few parameters, giving a tiny, indirect peek at the hidden sector. Indulge in some fantasy. Maybe someday humans will learn more about the hidden sector. Hidden sector production? Strongly coupled hidden sector? Dark matter = hidden sector baryons? Top down constraints?

QCD Analogy: visibles probing the hidden sector

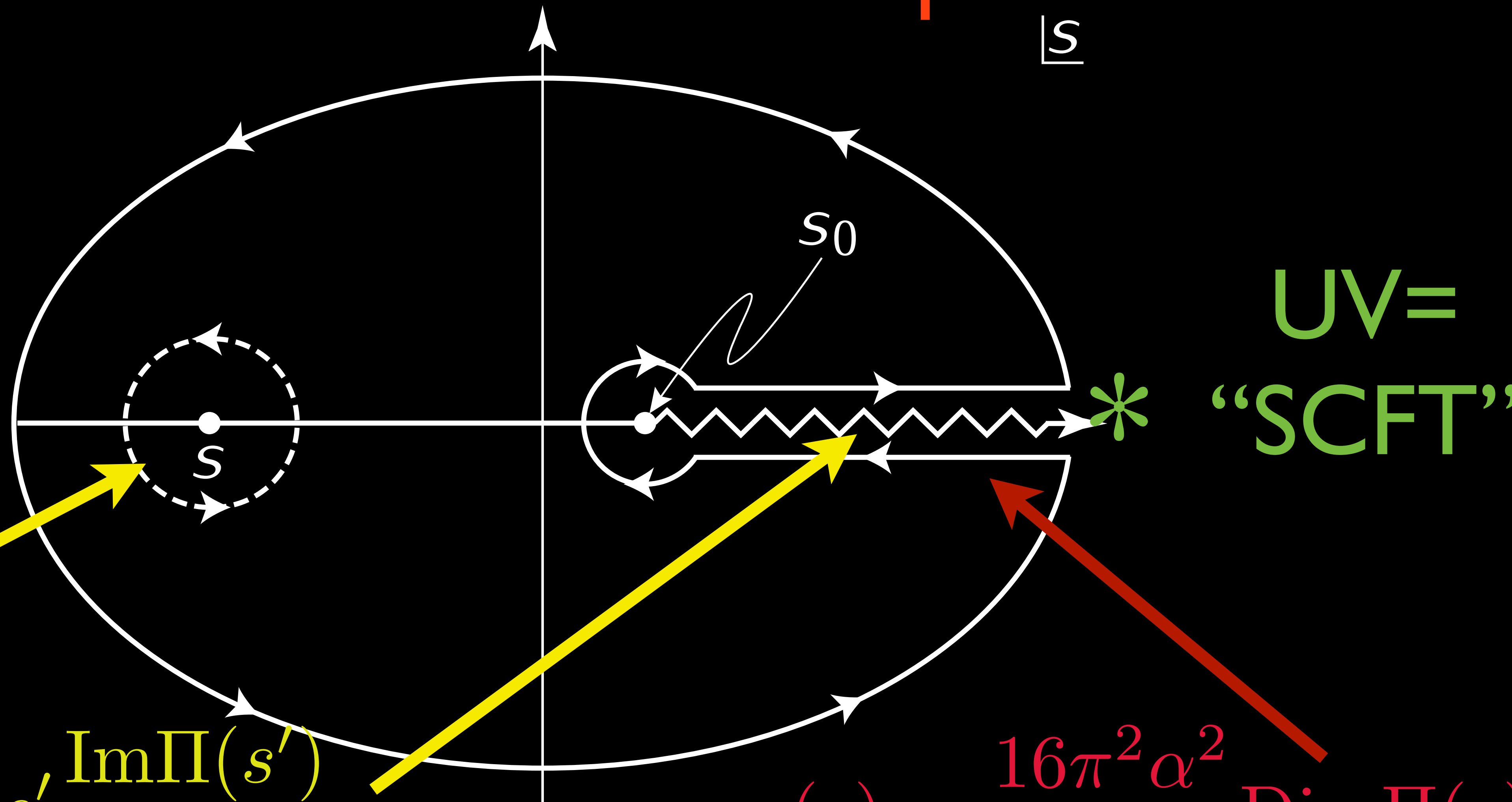
Visible sector matter



Via current-current 2-point function.

Visible sector soft masses in GGM is an example of this?
Other possible visible signatures of the hidden sector?
General constraints on current-current 2-point functions?

We use optical thm. etc. for hidden sector correlators + visible consequences



$$\Pi(s) = \frac{1}{\pi}\int_{s_0}^{\infty} ds' \frac{\mathrm{Im}\Pi(s')}{s'-s}$$

gaugino or sfermion masses; hidden sec. bound states, a-la QCD sum rules?

$$\sigma(s) = \frac{16\pi^2\alpha^2}{s}\mathrm{Disc}\Pi(s)$$

Visibles to hiddens production x-section. Constrains Disc.’s

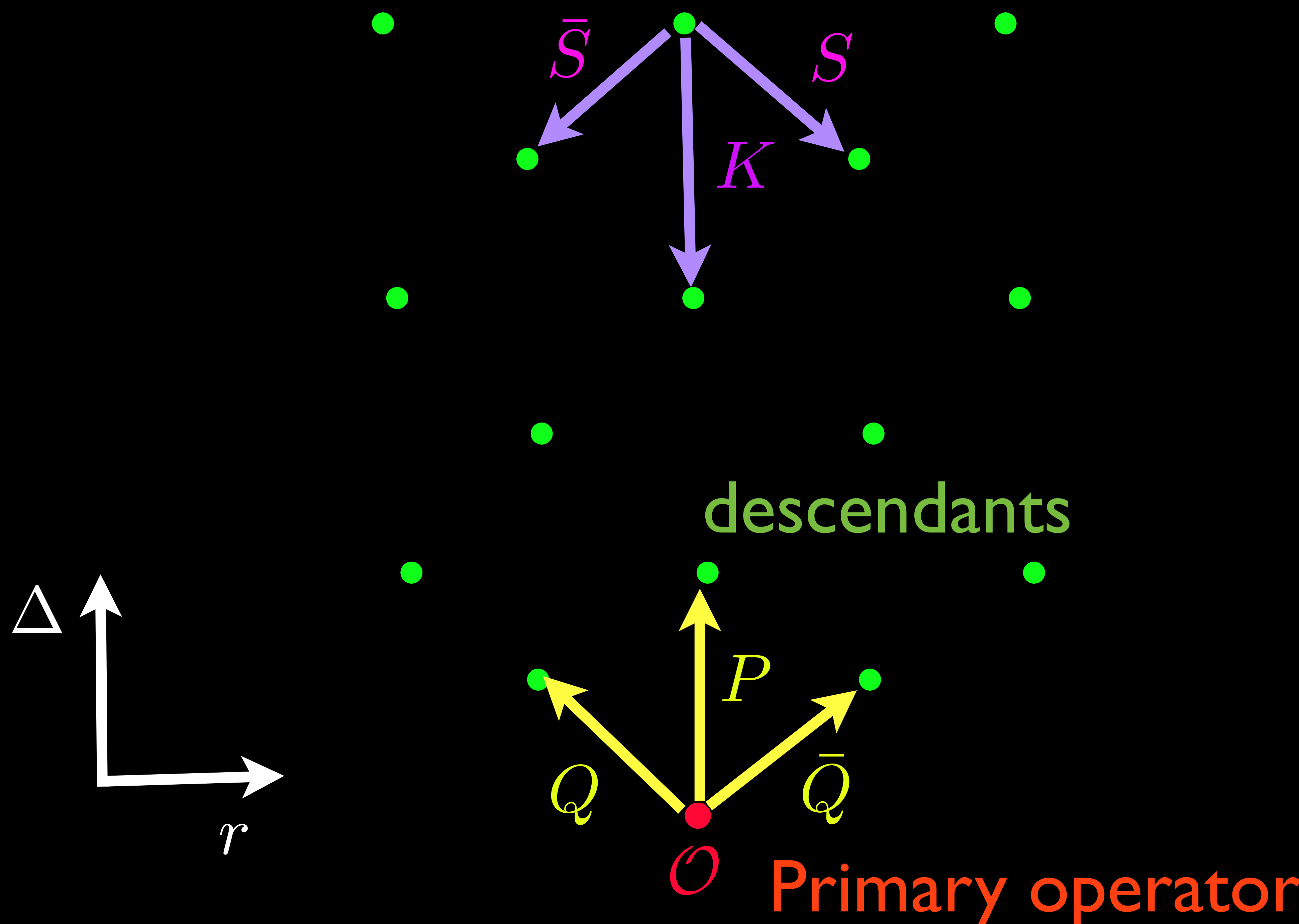
UV= "SCFT"

- Hidden sector: susy and conformal symm. is broken. But imagine they're restored in UV.
- Bigger UV symmetry implies UV relations. Some vestige can survive to IR.
- Broken symmetry in IR by operator or spurion vevs. $\langle \mathcal{O}_i \rangle \neq \delta_{i0}$ ➔ SC broken
- Apply the sOPE in the "sCFT", with $\langle \mathcal{O}_i \rangle \neq \delta_{i0}$ on the RHS.

sOPE in SCFTs

- Relate to 2-point and 3-point functions.
- Superconformal symmetry constrains their form (Osborn).
- Apply to our case of interest, correlators for conserved current OPEs.
- Relations among superconformal primaries and descendant sOPE coefficients.

Superconformal reps



Conserved current supermultiplets

$\mathcal{T}_\mu = j_R^\mu + \theta\sigma^\mu\bar\theta \;\; T_{\mu\nu} + \ldots$ Ferrara -Zumino multiplet

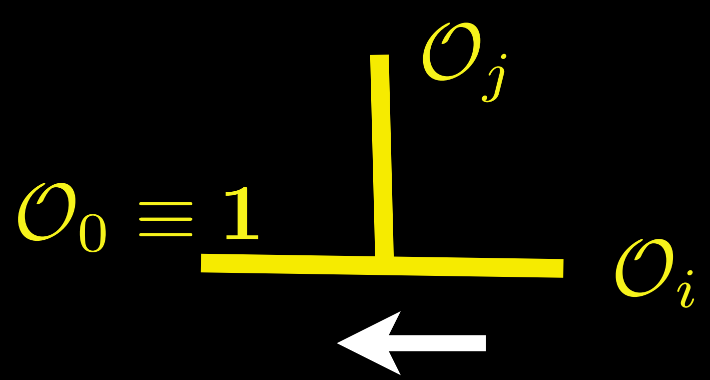
(Primary op. compt.)

$\mathcal{J} = J + i\theta j - i\bar\theta\bar j - \theta\sigma^\mu\bar\theta \;\; j^\mu + \ldots$ Conserved current multiplet

$\bar D^{\dot\alpha}\mathcal{T}_{\alpha\dot\alpha} = 0$ "SCFT," $D^2\mathcal{J} = \bar D^2\mathcal{J} = 0$ Conserved

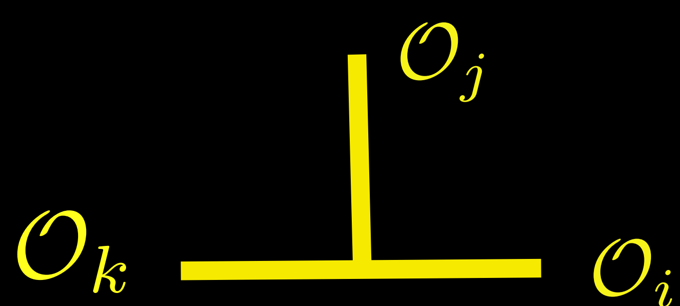
➔ Ward identities for their correlation fns.

2 and 3 point funs, OPE



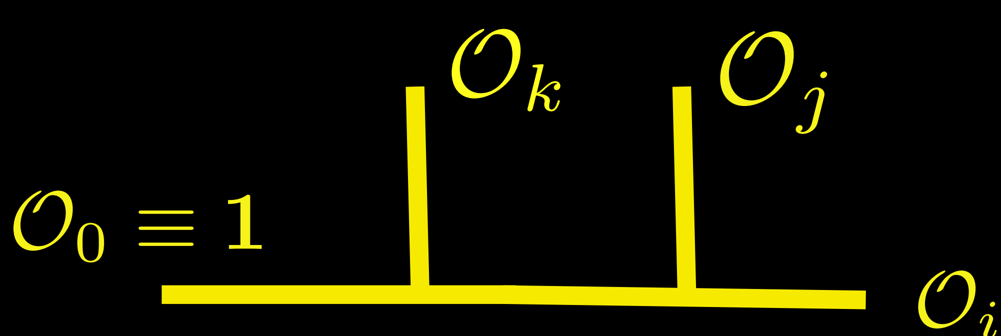
$\sim \quad c_{ij}^{0} = g_{ij}$

2-point fn coefficients
~ metric (Zamolodchikov)



$\sim \quad c_{ij}^{k}$

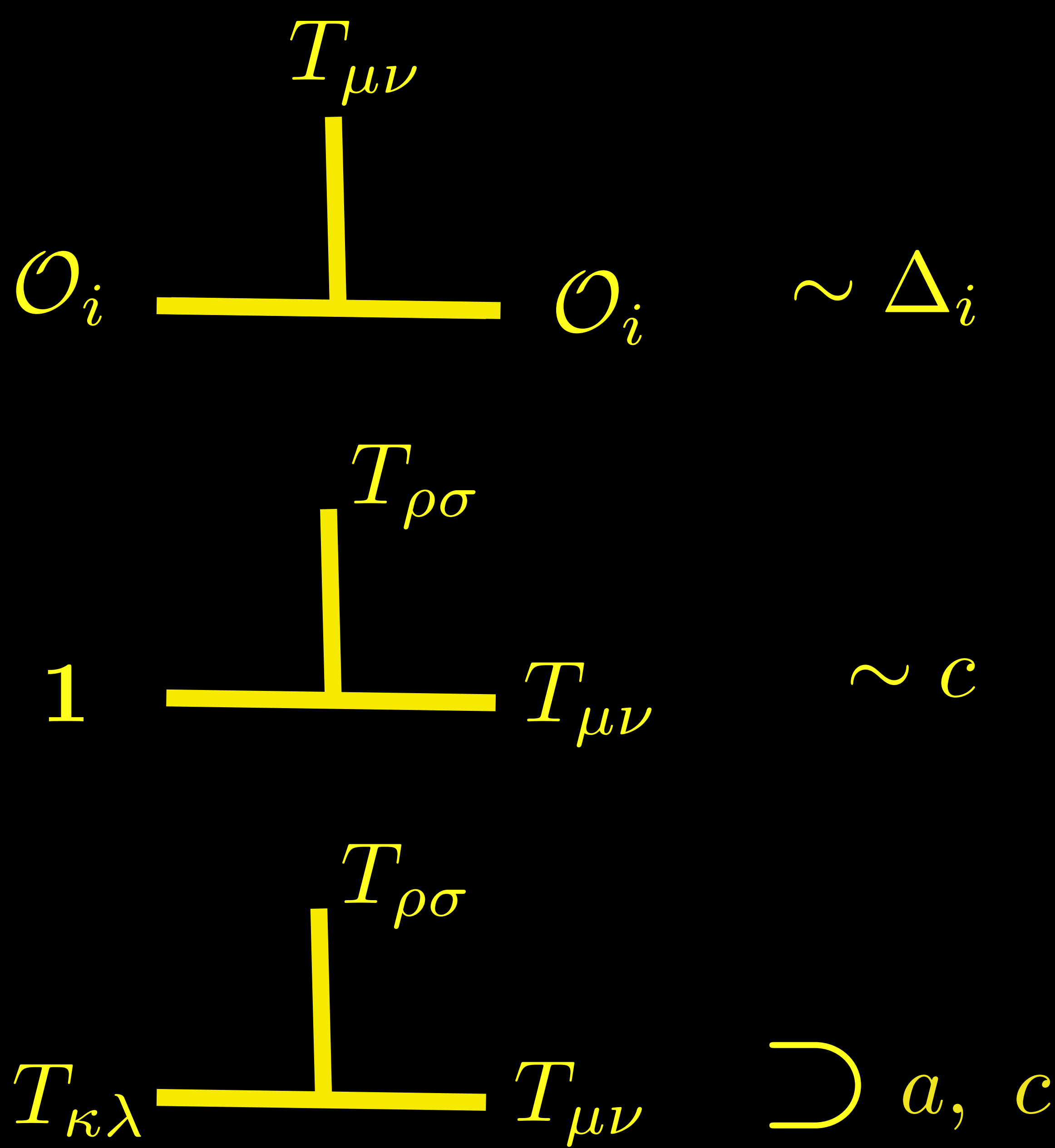
OPE coefficients



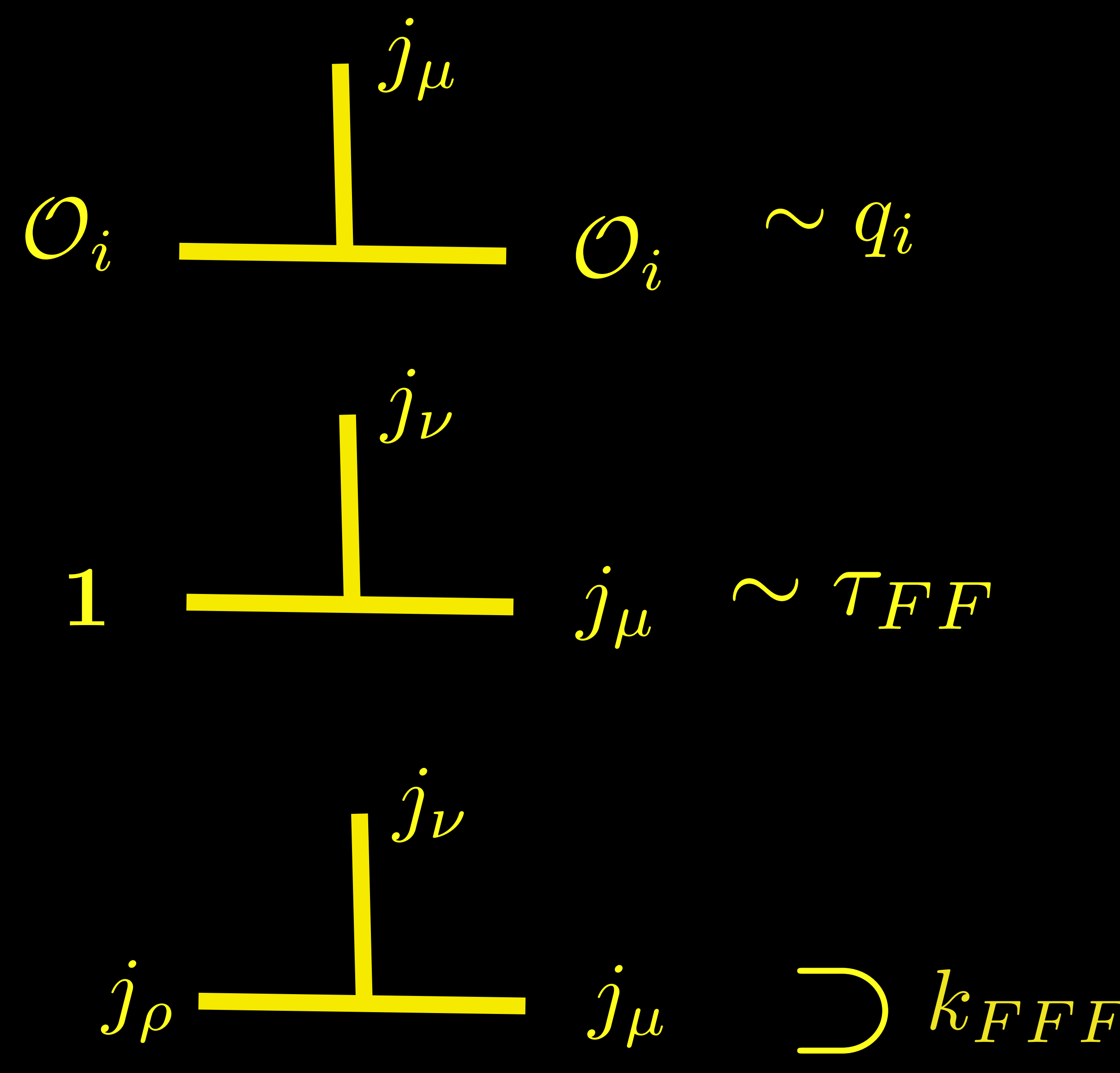
$\sim \quad c_{ij}^{\ell} g_{\ell k} = c_{ijk}$

3-point fn
coefficients

Examples (4d N=0, Osborn Petkou)

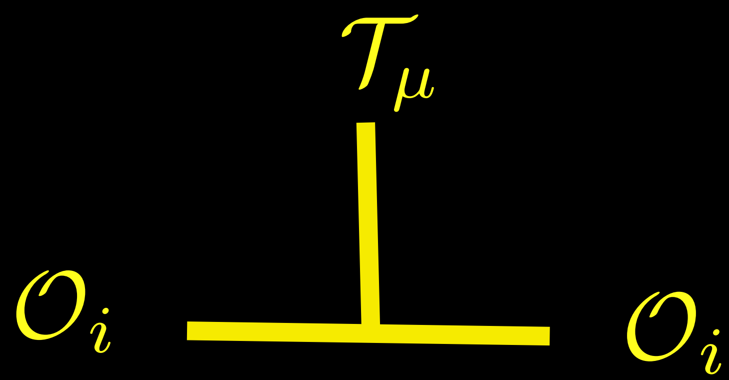


conf'l anomalies

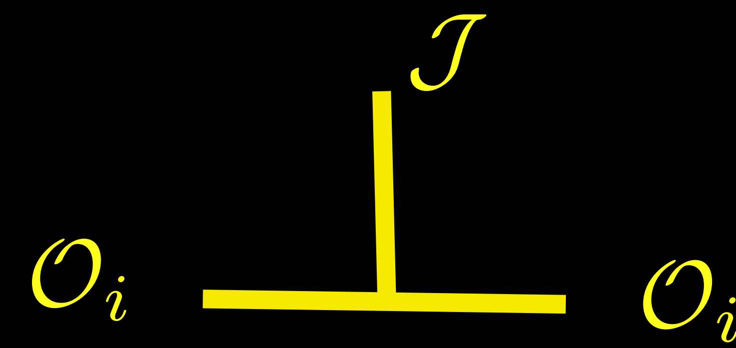


't Hooft anomalies

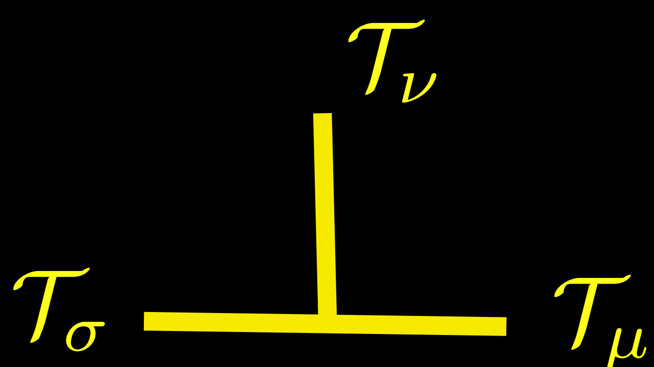
4d N=1 SCFT, e.g.



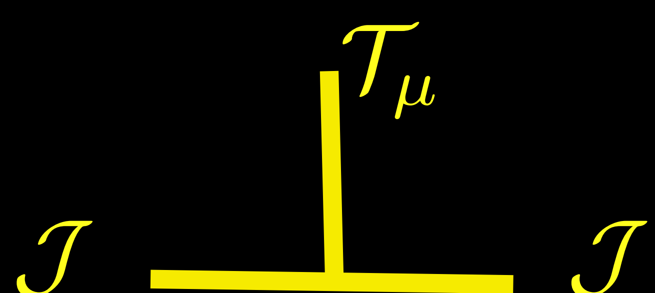
$\sim \Delta_i,\ r_i$



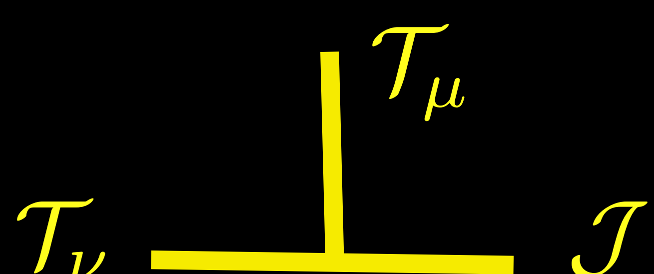
$\sim q_i$



Relates a, c to Tr R^3 and Tr R 't Hooft anomalies, Aneslmi et. al.; Osborn



Relates τ_{FF} to Tr RFF 't Hooft anomalies, Aneslmi et. al.; Osborn



Relates Tr RRF to Tr F anomalies: our proof of a-maximixation, KI+ Wecht.

Determining descendant OPEs from the primaries

Without susy, this was fully worked out in the '70s by Sergio Ferrara and collaborators. E.g. fixed coeffs

$$\mathcal{O}_i(x_1)\mathcal{O}_j(x_2) = \sum_{[k]} \frac{c_{ij}^k}{r_{12}^{\Delta_i+\Delta_j-\Delta_k}} \left(1 - \frac{i}{2}\left(\frac{\Delta_k+\Delta_i-\Delta_j}{\Delta_k}\right) x\cdot P + \dots\right)\mathcal{O}_k(x_2)$$

You'd expect: (i) susy case is similar (ii) worked out long ago.

But both these are wrong. In fact, superconformal descendants are generally NOT fully determined from those of the primaries. In some special cases, including ours, it happens that the descendants are indeed fully determined by the primaries, but in generic OPEs they're not. Surprising.

Superconformal 3-point functions and invariants

Implement the superconformal symmetry constraints using Osborn's superspace formalism and results. 3-point functions depend on $z_i = (x_i^\mu, \theta_i^\alpha, \bar{\theta}_i^{\dot{\alpha}}), \qquad i = 1, 2, 3.$

Make e.g. conformally nice coordinate $Z_3^\mu = \frac{x_{31}^\mu}{x_{31}^2} - \frac{x_{32}^\mu}{x_{32}^2}$

Complete it with corresponding theta terms to make ~ chiral or anti-chiral versions, called $X_3, \ \bar{X}_3$ (don't confuse with the chiral superfield X). These are coordinates, not fields.

There are corresponding nice combinations of coordinates that start with theta's, so they're nilpotent $\Theta_3, \ \bar{\Theta}_3$

General 3-point functions and the Theta invariant

Super operator 3-point functions

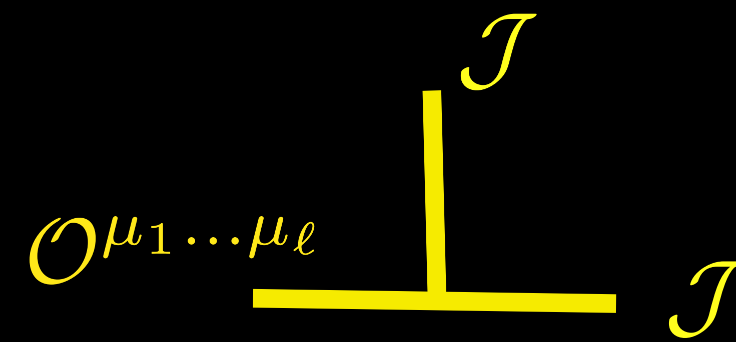
Determined dep on super-coords

$$\langle \mathcal{O}_i(z_1)\mathcal{O}_j(z_2)\mathcal{O}_k(z_3)\rangle = c_{ijk}\,(\dots)\,f(X_3,\Theta_3,\bar{\Theta}_3)$$

Generally underdetermined function of Theta, means that superdescendant 3-point functions are not fully determined by the coefficient c_{ijk} of the three superconformal primaries.

Now consider J(x) J(0) OPE (KI, **Fortin, Stergiou**)

Real, R=0, spin *l* primaries + descendants on RHS, only



$$\langle \mathcal{J}(z_1)\mathcal{J}(z_2)\mathcal{O}_i^{(\ell)(z_3)} \rangle = c_{JJ\mathcal{O}} f_{\Delta_\mathcal{O},\ell}(z_1, z_2, z_3)$$

$$f = \frac{t^{\mu_1 \ldots \mu_\ell}(X_3, \Theta_3, \bar{\Theta}_3)}{x_{\bar{1}3}^2 x_{\bar{3}1}^2 x_{\bar{2}3}^2 x_{\bar{3}2}^2}$$

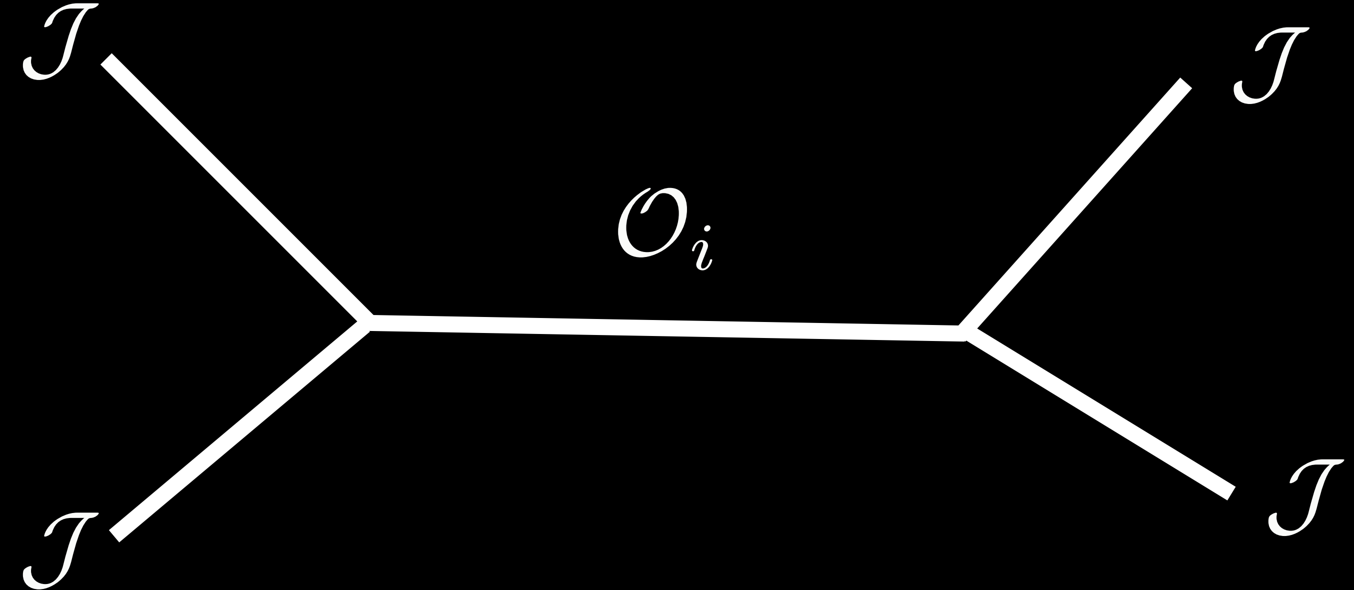
determined fn. of supercoordinates, via J conservation, e.g. for spin 0,

$$t = (X \cdot \bar{X})^{\frac{1}{2}\Delta_\mathcal{O} - 2} \left(1 - (\frac{\Delta_\mathcal{O}}{2} - 2)(\frac{\Delta_\mathcal{O}}{2} - 3)\frac{\Theta^2 \bar{\Theta}^2}{X \cdot \bar{X}}\right)$$

$X, \Theta =$ combinations super coords (Osborn)

Implies relations in supermultipet of JJ OPEs, from superconformal symmetry + current conservation.

(Correspondingly current 4-pt conformal blocks determined)



Can write in terms of N=0 blocks of Dolan & Osborn, like in talk / paper of Davids Poland and Simmons-Duffin.

Relations, seen from algebra

$$j^\alpha(x) j_\alpha(0) = Q^2 (J(x)J(0)) = \frac{1}{x^2} Q(x \cdot \bar{S})(J(x)J(0))$$ etc

→ Relations among different OPEs on LHS

$$S^2(J(x)J(0)) = \bar{S}^2(J(x)J(0)) = 0$$ etc

→ Relations among different terms on RHS of OPE.

Simple Example: MGM as "SCFT"

Charged messenger pair + hidden goldstino field or spurion X

$W = X\Phi\widetilde{\Phi}$ $\quad\quad$ $\mathcal{J} = \Phi^\dagger\Phi - \widetilde{\Phi}^\dagger\widetilde{\Phi}$

X $\longrightarrow$ mess' $\rightarrow$ $\mathcal{J}(z_1)\mathcal{J}(z_2) = \text{OPE}$

visible

IR: $\langle X \rangle = M + \theta^2 F$

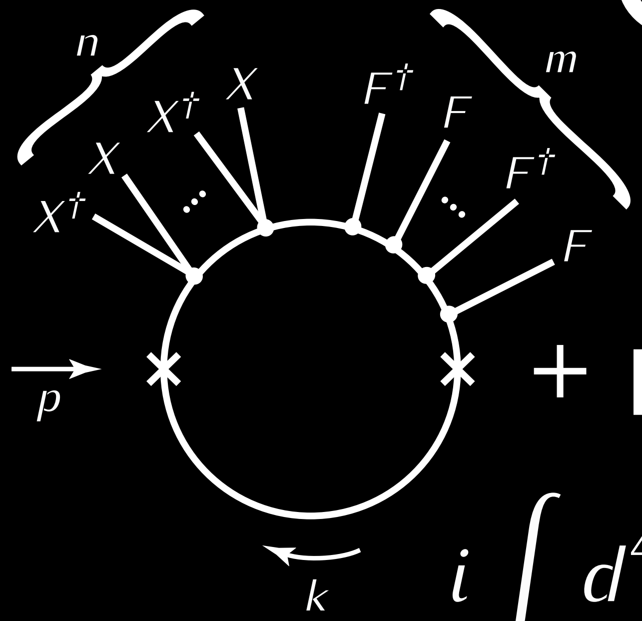
$m_\pm^2 = M^2 \pm F$

$m_f = M$

spont conf'l + susy breaking

Compute OPE coeffs in UV "SCFT", including X, F ops on RHS. Coeffs = UV data, unaffected by IR op vevs.

OPE Wilson Coeffs, Relns:



+ permutations

$$i\int d^4x\, e^{-ip\cdot x} J(x)J(0) \to \sum_{m,n=0}^{\infty} \tilde{c}_0(m,n;s,\mu)(F^\dagger F)^m (X^\dagger X)^n(0)$$

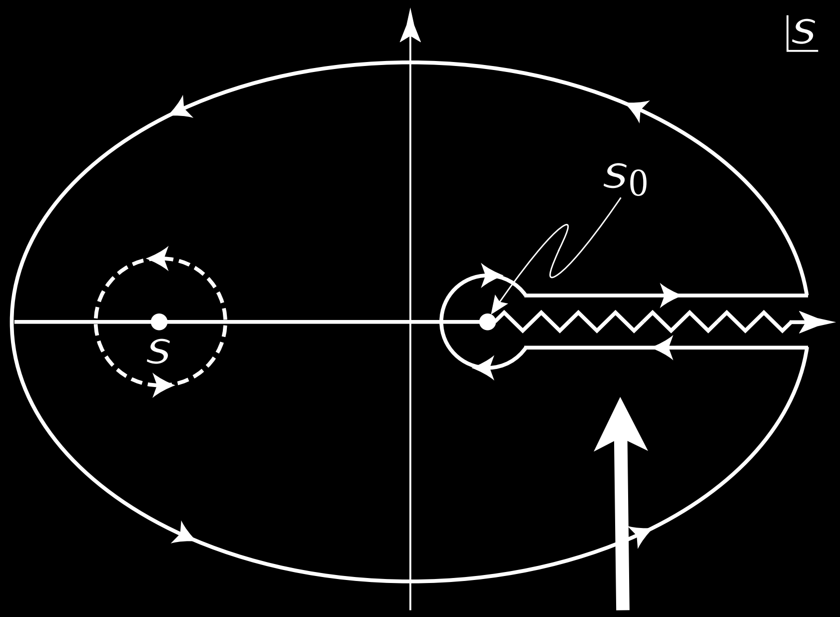
$$+ \sum_{m,n=0}^{\infty} \tilde{d}_0(m,n;s,\mu)(F^\dagger F)^m (X^\dagger X)^n X^\dagger F^\dagger \chi^2(0) + \cdots$$

Use / check: $j^\alpha(x)j_\alpha(0) = Q^2(J(x)J(0)) = \frac{1}{x^2} Q(x\cdot\bar{S})(J(x)J(0))$

$$i\int d^4x\, e^{-ip\cdot x} j_\alpha(x)j_\beta(0) \to \epsilon_{\alpha\beta} F X^\dagger \sum_{m,n=0}^{\infty} \tilde{c}_{1/2}(m,n;s,\mu)(F^\dagger F)^m (X^\dagger X)^n$$

With: $\tilde{c}_{1/2}(m,n) = (n+1)\tilde{c}_0(m,n+1) + 2\tilde{d}_0(m-1,n)$ It works.

Only need JJ OPE



Cross sections

For GGM fns

$$\widetilde{C}_0(s) = \mathrm{F.T.}\langle J(x)J(0)\rangle$$
$$\widetilde{C}_{1/2}(s),\ \widetilde{C}_1(s)$$
$$\widetilde{B}_{1/2}(s) = \mathrm{F.T.}\langle j_\alpha(x)j^\alpha(0)\rangle$$

Cut: make on-shell hidden / messengers, disc = total cross sect.

$$\sigma_a(\mathrm{vis}\to\mathrm{hid}) = \frac{8\pi^2\alpha^2}{s}\mathrm{Disc}\widetilde{C}_a(s)$$ Also, e.g. $$\sigma_0(s) = \frac{\lambda^{1/2}(s,m_1,m_2)}{8\pi s^2}|\mathcal{M}|^2$$

$$\lambda = 4s|\vec{p}_{os}|^2 = (s-(m_1+m_2)^2)(s-(m_1-m_2)^2)$$ Textbook kinematic factor

Our example: $$\sigma_0(\mathrm{vis}\to\mathrm{hid}) = \frac{2\pi\alpha^2}{s^2}\lambda^{1/2}(s,m_+,m_-)$$

$$\sigma_0(s) = \frac{1}{s}\,\mathrm{Im}\sum_{m,n=0}^{\infty}\tilde{c}_0(m,n;s,\Lambda)(F^\dagger F)^m(X^\dagger X)^n$$

Compare with the Disc of our Wilson coefficients. It works. Use symms to find all other cross sects.

GGM soft masses from OPE

GGM:

$$M_r = g_r^2 M \widetilde{B}^{(r)}_{1/2}(0) \qquad m_{\tilde{f}}^2 = \sum_{r=1}^{3} g_r^4 c_2(f;r) A_r$$

$$A_r = -\frac{M^2}{16\pi^2}\int dy\, \left[\widetilde{C}_0^{(r)}(y) - 4\widetilde{C}_{1/2}^{(r)}(y) + 3\widetilde{C}_1^{(r)}(y)\right]$$

original MSS expressions

Use the OPE + dispersion relations, e.g. $\widetilde{B}(s=0) = \int_{s_0}^{\infty} \frac{ds}{\pi} \frac{\mathrm{Im}(\widetilde{B}(s))}{s}$

IR :(for OPE

UV :) for OPE.

$$M_{\text{gaugino}} = \sum_k \frac{\alpha\, \mathrm{Im}[s^{d_k/2}\, \tilde{c}^k_{JJ}(s)]}{2^{d_k-1} d_k M^{d_k}} \langle Q^2(\mathcal{O}_k(0))\rangle$$

$$m^2_{\text{sfermion}} = -\sum_k \frac{\alpha^2 c_2\, \mathrm{Im}[s^{d_k/2}\, \tilde{c}^k_{JJ}(s)]}{2^{d_k+1}\pi d_k^2 M^{d_k}} \langle \bar{Q}^2 Q^2(\mathcal{O}_k(0))\rangle$$

In our example, these reproduce the usual GM f(x), g(x), x=F/M²

Get good approx. by keeping only few leading terms in OPE.

Approx. soft masses from OPE

$$J_a(x)J_b(0) = \tau\frac{\delta_{ab}\mathbb{1}}{16\pi^4x^4} + \tau^{-1}kd^c_{ab}\frac{J_c(0)}{4\pi^2x^2} + w\frac{\delta_{ab}K(0)}{4\pi^2x^{2-\gamma_K}} + c^i_{ab}\frac{\mathcal{O}_i(0)}{x^{4-\Delta_i}} + \cdots$$

QQ(*)=0

"Konishi," lowest dim op with QQ(K) $\neq 0$

higher dim ops + descendants

We find:

$$M_{\text{gaugino}} \approx -\frac{\alpha\pi w\gamma_{Ki}}{8M^2}\langle Q^2(\mathcal{O}_i(0))\rangle$$

$$m^2_{\text{sfermion}} \approx \frac{\alpha^2 c_2 w\gamma_{Ki}}{64M^2}\langle \bar{Q}^2Q^2(\mathcal{O}_i(0))\rangle$$

Leading term from Konishi operator, and its mixing operators.

E.g. in our OGM example, standard soft mass functions f(x), g(x) ~ 1. The OPE approximation from including just a single term already accounts for ~ 1/2 of these functions. The higher dimension operators and descendants give just small corrections. Power of the OPE.

Summary

- Constrain current-current OPE in SCFTs, showed superconformal descendant coeffs are fully determined from primaries.
- Can apply to "SCFTs" with broken susy and conf'l symmetry. OPE coeffs =UV, so don't notice IR breaking by vevs.
- Can apply to GGM. Use OPE to find cross sections and approximations to soft masses.

Thank you.