Exploring 4D Conformal and Superconformal Field Theories

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May 16, 2011

(w/ David Simmons-Duffin)

SUSY Breaking '11, CERN

Motivation

- Conformal dynamics in 4D could play a role in BSM physics!
  - Walking/Conformal Technicolor [Holdom '81; ...]
  - Warped Extra Dimensions [Randall, Sundrum '99; ...]
  - Flavor Hierarchies [Georgi, Nelson, Manohar '83; Nelson, Strassler '00; DP, DSD '09; ...]
  - Conformal Sequestering [Luty, Sundrum '01; Schmaltz, Sundrum '06]
  - Solution to $\mu/B\mu$ problem [Roy, Schmaltz '07; Murayama, Nomura, DP '07]
  - ...
- Ideas often depend crucially on spectrum of operator dim's!
  - E.g., Conf. Technicolor: Want $\dim H^\dagger H \gtrsim 4$ but $\dim H \sim 1$
- Non-SUSY CFTs: Hard to calculate *anything*
- $\mathcal{N}=1$ SCFTs: Chiral operators have $\Delta = \frac{3}{2}R$

  Non-chiral operator dim's harder

Outline

CFT Review: Primary Operators

- In addition to Poincaré generators P^a and M^{ab}, CFTs have dilatations D and special conformal generators K^a

$$[K^a, P^b] = 2\eta^{ab} D - 2M^{ab}$$

- *Primary* operators $\mathcal{O}^I(0)$ are defined by

$$[K^a, \mathcal{O}^I(0)] = 0$$

(descendants obtained by acting with P^a)

CFT Review: Primary Operators

- Primary 2-pt and 3-pt functions fixed by conformal symmetry in terms of dimensions and spins, up to overall coefficients $\lambda_{\mathcal{O}}$

$$\langle \mathcal{O}^{a_1\ldots a_l}(x_1)\mathcal{O}^{b_1\ldots b_l}(x_2)\rangle = \frac{I^{a_1 b_1}\ldots I^{a_l b_l}}{x_{12}^{2\Delta}}$$

$$\langle \phi(x_1)\phi(x_2)\mathcal{O}^{a_1\ldots a_l}(x_3)\rangle = \frac{\lambda_{\mathcal{O}}}{x_{12}^{2d-\Delta+l}x_{23}^{\Delta-l}x_{13}^{\Delta-l}} Z^{a_1}\ldots Z^{a_l}$$

$$\left(I^{ab} = \eta^{ab} - 2\frac{x_{12}^a x_{12}^b}{x_{12}^2} \quad , \quad Z^a = \frac{x_{31}^a}{x_{31}^2} - \frac{x_{32}^a}{x_{32}^2} \right)$$

- Higher n-pt functions *not* fixed by conformal symmetry alone, but are determined once spectrum and $\lambda_{\mathcal{O}}$'s are known...

CFT Review: Operator Product Expansion

Let ϕ be a scalar primary of dimension d in a 4D CFT:

$$\phi(x)\phi(0) \;=\; \sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}} C_I(x,P)\,\mathcal{O}^I(0) \qquad \text{(OPE)}$$

- Sum runs over *primary* $\mathcal{O}$'s
- $C_I(x,P)$ fixed by conformal symmetry [Dolan, Osborn '00]
- $\mathcal{O}^I = \mathcal{O}^{a_1\ldots a_l}$ can be any spin-l Lorentz representation (traceless symmetric tensor) with $l = 0, 2, \ldots$
- Unitarity tells us that $\Delta_{\mathcal{O}} \geq l + 2 - \delta_{l,0}$ and that $\lambda_{\mathcal{O}}$ is real

CFT Review: Conformal Block Decomposition

Use OPE to evaluate 4-point function

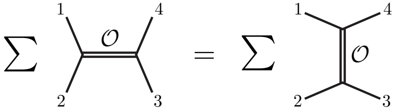
$$\begin{aligned}
&\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle \\
&\qquad = \sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}}^2 C_I(x_{12},\partial_2) C_J(x_{34},\partial_4)\langle \mathcal{O}^I(x_2)\mathcal{O}^J(x_4)\rangle \\
&\qquad \equiv \frac{1}{x_{12}^{2d}x_{34}^{2d}} \sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}}^2\, g_{\Delta,l}(u,v)
\end{aligned}$$

- $u=\frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2}$, $v=\frac{x_{14}^2x_{23}^2}{x_{13}^2x_{24}^2}$ conformally-invariant cross ratios.
- $g_{\Delta,l}(u,v)$ conformal block ($\Delta=\dim\mathcal{O}$ and $l=$ spin of $\mathcal{O}$)
  - Known explicitly in terms of hypergeometric functions [Dolan, Osborn '00; Dolan, Osborn '03]

CFT Review: Crossing Relations

- Four-point function $\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle$ is clearly symmetric under permutations of x_i
- After OPE, symmetry is non-manifest!
- Switching $x_1 \leftrightarrow x_3$ gives the "crossing relation":

$$\sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}}^2 g_{\Delta,l}(u,v) \quad = \quad \left(\frac{u}{v}\right)^d \sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}}^2 g_{\Delta,l}(v,u)$$



- Other permutations give no new information

Method of [Rattazzi et. al. '08]

[Rattazzi, Rychkov, Tonni, Vichi '08]:
crossing + unitarity leads to bounds on CFT quantitites!

- Let's study the OPE coefficient of a particular $\mathcal{O}_0 \in \phi \times \phi$
- We can rewrite crossing relation as

$$\underbrace{\lambda^2_{\mathcal{O}_0} F_{\Delta_0,l_0}(u,v)}_{\mathcal{O}_0} \;=\; \underbrace{1}_{\text{unit op.}} \;-\; \underbrace{\sum_{\mathcal{O}\neq\mathcal{O}_0} \lambda^2_{\mathcal{O}} F_{\Delta,l}(u,v)}_{\text{everything else}},$$

where

$$F_{\Delta,l}(u,v) \equiv \frac{v^d g_{\Delta,l}(u,v) - u^d g_{\Delta,l}(v,u)}{u^d - v^d}.$$

Method of [Rattazzi et. al. '08]

Idea: Find a linear functional α such that

$$\begin{aligned} \alpha(F_{\Delta_0,l_0}) &= 1, \quad \text{and} \\ \alpha(F_{\Delta,l}) &\geq 0, \quad \text{for all other } \mathcal{O} \in \phi \times \phi. \end{aligned}$$

Applying to both sides:

$$\begin{aligned} \alpha\left(\lambda^2_{\mathcal{O}_0} F_{\Delta_0,l_0}\right) &= \alpha(1 - \sum_{\mathcal{O}\neq\mathcal{O}_0} \lambda^2_{\mathcal{O}} F_{\Delta,l}) \\ \lambda^2_{\mathcal{O}_0} &= \alpha(1) - \sum_{\mathcal{O}\neq\mathcal{O}_0} \lambda^2_{\mathcal{O}} \alpha(F_{\Delta,l}) \leq \alpha(1) \end{aligned}$$

since $\lambda^2_{\mathcal{O}} \geq 0$ by unitarity.

Method of [Rattazzi et. al. '08]

- To make the bound $\lambda^2_{\mathcal{O}_0} \leq \alpha(1)$ as strong as possible: *Minimize* $\alpha(1)$ subject to $\alpha(F_{\Delta_0, l_0}) = 1$ and $\alpha(F_{\Delta,l}) \geq 0$
- Take α to be linear combinations of derivatives at some point

$$\alpha : F_{\Delta,l}(z,\overline{z}) \mapsto \sum_{m+n \leq N} a_{mn} \partial_z^m \partial_{\overline{z}}^n F_{\Delta,l}(1/2, 1/2)$$

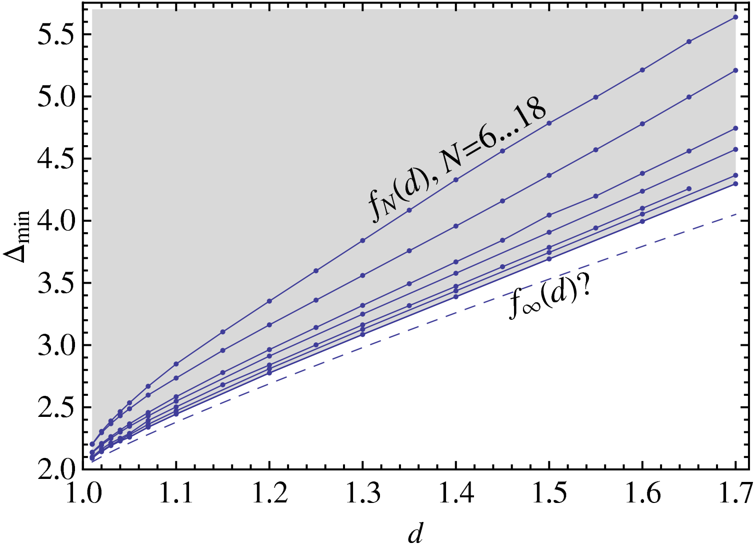
- Discretize constraints to $\alpha(F_{\Delta_i, l_i}) \geq 0$ for $D = \{(\Delta_i, l_i)\}$ (to make the problem numerically tractable)
- Take $N \to \infty$ to recover "optimal" bound

Method of [Rattazzi et. al. '08]

- Can make any assumptions about the spectrum that we want!
- Let's assume that all scalars appearing in the OPE $\phi \times \phi$ have dimension larger than some $\Delta_{\min} = \dim \mathcal{O}_0$
- If $\lambda^2_{\mathcal{O}_0} \leq \alpha(1) < 0$, there is a contradiction with unitarity and the assumed spectrum can be ruled out

By scanning over different $\Delta_{\min}$, one can obtain bounds on $\dim \phi^2$ as a function of $d = \dim \phi$

Bounds on $\dim \phi^2$ (taken from [Rychkov, Vichi '09])



Extensions

Method has now been extended to obtain:

- Bounds in $\mathcal{N} = 1$ SCFTs
  [DP, Simmons-Duffin '10]
  - Uses *superconformal block* expansion of $\langle \Phi\Phi^\dagger\Phi\Phi^\dagger \rangle$, Φ chiral
- Bounds in the presence of global symmetries
  [Rattazzi, Rychkov, Vichi '10]
  - Considering $\langle \phi_i\phi_j\phi_k\phi_l \rangle$ leads to *system* of crossing relations
- Bounds on various OPE coefficients
  - Scalar 3-pt functions [Caracciolo, Rychkov '09]
  - Flavor Symmetry Currents [DP, Simmons-Duffin '10]
  - Stress Tensor [DP, Simmons-Duffin; Rattazzi, Rychkov, Vichi '10]

$\mathcal{N}=1$ Superconformal Algebra

$$
\begin{array}{ccccc}
\text{dim} & & & & \\
+1 & & & P_a & & \\
+1/2 & & Q_\alpha & & \overline{Q}_{\dot\alpha} & \\
0 & M_{\alpha\beta} & & D, R & & M_{\dot\alpha\dot\beta} \\
-1/2 & & S_\alpha & & \overline{S}_{\dot\alpha} & \\
-1 & & & K_a, & &
\end{array}
$$

$$\{Q, \overline{Q}\} = P \qquad\qquad \{S, \overline{S}\} = K$$

- Superconformal primary means $[S, \mathcal{O}(0)] = [\overline{S}, \mathcal{O}(0)] = 0$
- Descendants obtained by acting with $P, Q, \overline{Q}$
- Chiral means $[\overline{Q}, \phi(0)] = 0$

Superconformal Block Decomposition

ϕ: scalar chiral superconformal primary of dimension d in an SCFT (lowest component of chiral superfield Φ)

$$\langle \phi(x_1)\phi^\dagger(x_2)\phi(x_3)\phi^\dagger(x_4)\rangle = \frac{1}{x_{12}^{2d}x_{34}^{2d}} \sum_{\mathcal{O}\in\Phi\times\Phi^\dagger} |\lambda_{\mathcal{O}}|^2(-1)^l \mathcal{G}_{\Delta,l}(u,v)$$

- Sum over superconformal primaries $\mathcal{O}^I$ with zero R-charge
- $\lambda_{\mathcal{O}}$ real for even spin $\mathcal{O}^I$, imaginary for odd spin $\mathcal{O}^I$
- $x_1 \leftrightarrow x_3$ gives crossing relation only involving $\mathcal{O}^I \in \Phi\times\Phi^\dagger$

- $\mathcal{G}_{\Delta,l}(u,v)$ finite sum of conformal blocks, since $\mathcal{O}$ has finite number of descendants that are conformal primaries...

Superconformal Block Derivation

Multiplet built from $\mathcal{O}$ (generically) contains four conformal primaries with vanishing R-charge and definite spin:

name	operator	dim	spin
$\mathcal{O}$	$\mathcal{O}$	Δ	l
J, N	$Q\overline{Q}\mathcal{O} + \#P\mathcal{O}$	$\Delta + 1$	$l+1, l-1$
D	$Q^2\overline{Q}^2\mathcal{O} + \#PQ\overline{Q}\mathcal{O} + \#PP\mathcal{O}$	$\Delta + 2$	l

- Superconformal symmetry fixes coefficients of $\langle \phi\phi^\dagger J\rangle, \langle \phi\phi^\dagger N\rangle, \langle \phi\phi^\dagger D\rangle$ in terms of $\langle \phi\phi^\dagger \mathcal{O}\rangle$
- Must also normalize J, N, D to have canonical 2-pt functions
- Superconformal block is then a sum of $g_{\Delta,l}$'s for $\mathcal{O}, J, N, D$

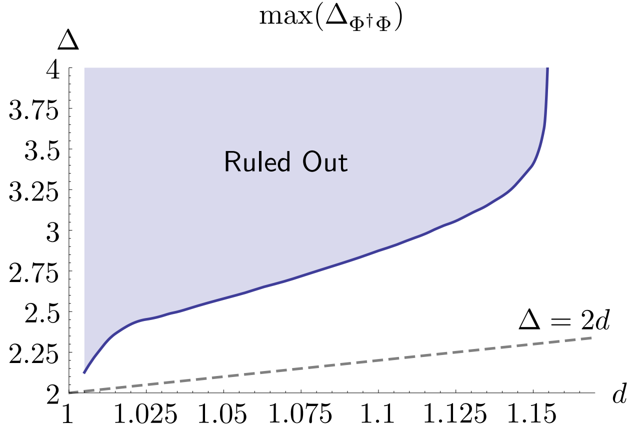
Superconformal Blocks

We find,

$$\mathcal{G}_{\Delta,l} = g_{\Delta,l} - \frac{(\Delta+l)}{2(\Delta+l+1)} g_{\Delta+1,l+1} - \frac{(\Delta-l-2)}{8(\Delta-l-1)} g_{\Delta+1,l-1} + \frac{(\Delta+l)(\Delta-l-2)}{16(\Delta+l+1)(\Delta-l-1)} g_{\Delta+2,l}$$

- When unitarity bound $\Delta \geq l+2$ is saturated, multiplet is shortened
- $\mathcal{G}_{\Delta,l}$ can also be determined from consistency with $\mathcal{N}=2$ superconformal blocks computed by [Dolan, Osborn '01]

Upper Bound on Dimension of $\Phi^\dagger\Phi$



- Weaker than before due to odd-l operators in $\Phi \times \Phi^\dagger$ OPE
- Bounds improve further by using relation to $\Phi \times \Phi$ OPE, but trickier numerically [Vichi, in progress]

The Stress Tensor

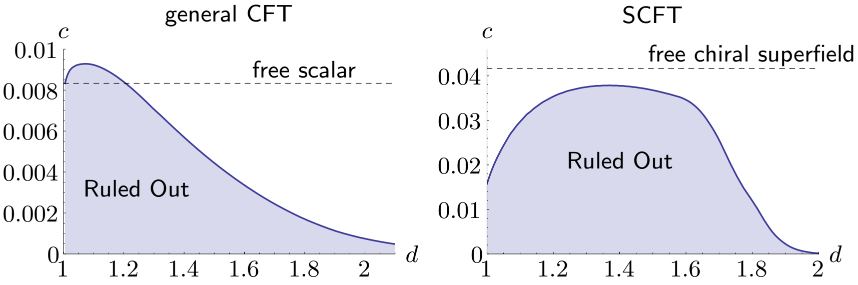
- Ward identity fixes $\langle \phi\phi T \rangle \propto d$
- Only unknown: $\langle TT \rangle \propto c$, the central charge (trace anomaly $16\pi^2 \langle T^a_a \rangle = c(\mathrm{Weyl})^2 - a(\mathrm{Euler})$)
- In an SCFT, T lives in the $U(1)_R$ current multiplet $\mathcal{J}^a = J^a_R + \theta\sigma_b\overline{\theta}T^{ab} + \ldots$, and c determined by $U(1)_R$

- Conformal block contributions are

$$\langle \phi\phi\phi\phi \rangle \sim \frac{d^2}{90c}\, g_{4,2} \qquad \text{(general CFTs)}$$

$$\langle \phi\phi^\dagger\phi\phi^\dagger \rangle \sim -\frac{d^2}{36c}\, \mathcal{G}_{3,1} \qquad \text{(SCFTs)}$$

Lower Bound on c



(See also [Rattazzi, Rychkov, Vichi '10])

- In dual AdS_5, $c \sim \pi^2 L^3 M_P^3$. Gravity can't be arbitrarily strong in presence of light bulk scalar!
- Similar bounds can be placed on coefficients of flavor current 2-point functions $\langle J^I J^J \rangle \propto \tau^{IJ}$

Future Directions

Some future directions for this program:

- Obtain stronger bounds to make contact with BSM scenarios
- Better algorithms (esp. to deal with global symmetries)
- Bounds in other dimensions
- 4-point functions of operators with spin [in progress...]
- AdS dual interpretation?

Another Strategy...

- Another route to progress: look for theories that have extra hidden structure that makes them more solvable?
- Success story in (planar) $\mathcal{N} = 4$ SYM: *Integrability* lets one calculate operators dimensions at $\lambda \sim O(1)$
  [Beisert, Staudacher, Gromov, Kazakov, Vieira... + many others]
- First evidence: [Minahan, Zarembo '02] found that the 1-loop dilatation operator acting on scalars $\mathrm{Tr}(X_i \dots X_j)$ is equivalent to an exactly solvable Heisenberg spin chain
- As an exploration, let's repeat this study for $\mathcal{N} = 1$ SQCD...
  [DP, Simmons-Duffin '11]

$\mathcal{N} = 1$ SQCD

- Matter content:

	$\mathrm{SU}(N_c)$	$\mathrm{SU}(N_f)_L$	$\mathrm{SU}(N_f)_R$	$U(1)_B$	$U(1)_R$
Q_{ai}	$\square$	$\square$	1	1	$1 - \frac{N_c}{N_f}$
$\widetilde{Q}^{\tilde{\imath}a}$	$\overline{\square}$	1	$\overline{\square}$	-1	$1 - \frac{N_c}{N_f}$

$$\begin{aligned} i, \tilde{\imath} &= 1, \ldots, N_f \\ a &= 1, \ldots, N_c \end{aligned}$$

- Conformal window: $\frac{3}{2} < \frac{N_f}{N_c} < 3$ [Seiberg '94]
- Veneziano limit: $N_f, N_c \to \infty$, with $\frac{N_f}{N_c}$ fixed
  Weak-coupling [Banks, Zaks '82] regime: $\epsilon = \frac{3N_c}{N_f} - 1 \ll 1$

$$\lambda \equiv \frac{g^2 N_c}{8\pi^2} = \epsilon + \frac{\epsilon^2}{2} + \frac{9}{4}(1 + 2\zeta_3)\epsilon^3 + \ldots$$

Generalized Single Trace Operators

Basic objects when $N_c, N_f \to \infty$ are "generalized single-trace" operators. Using only scalars, the building blocks are:

$$X \equiv (QQ^\dagger)^a_b, \quad Y \equiv (\widetilde{Q}^\dagger \widetilde{Q})^a_b$$

We then have flavor singlets

$$Q_{ai}\, Q^{\dagger ib}\, \widetilde{Q}^\dagger_{b\tilde{\jmath}} \widetilde{Q}^{\tilde{\jmath}c} \ldots Q_{dk}\, Q^{\dagger ka} \quad = \quad \mathrm{Tr}(XY\ldots X) \quad \text{“closed”}$$

and flavor adjoint+bifundamentals

$$\left.\begin{array}{c} Q^\dagger XY\ldots XQ \\ \widetilde{Q}XY\ldots XQ \\ Q^\dagger XY\ldots X\widetilde{Q}^\dagger \\ \widetilde{Q}XY\ldots X\widetilde{Q}^\dagger \end{array}\right\} \quad \text{“open”}$$

1-Loop Dilatation Operator as a Spin Chain

- Useful notation: $XYX\ldots \longrightarrow |\uparrow\downarrow\uparrow\ldots\rangle$ (spin chain state)
- Represent operators acting on spins in terms of Pauli Matrices
- Evaluating 1-loop diagrams, we obtain the Dilatation operator:

$$D_{closed} = 2L + \lambda\frac{N_f - N_c}{2N_c}L + \frac{\lambda}{2}\underbrace{\sum_{n=0}^{L-1}(\sigma_n^z\sigma_{n+1}^z - \frac{N_f}{N_c}\sigma_n^x)}_{H} + \ldots$$

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- H is the Hamiltonian for the 1D Ising Model in a transverse magnetic field: exactly solvable [Pfeuty '70]!

Solution

Map to fermionic oscillators [Jordan, Wigner '28]:

$$c_n^\dagger = \left(\prod_{m=0}^{n-1} \sigma_m^x\right)\sigma_n^+ \qquad c_n = \left(\prod_{m=0}^{n-1} \sigma_m^x\right)\sigma_n^-$$

$$\{c_n^\dagger, c_m\} = \delta_{nm}, \qquad \{c_n, c_m\} = \{c_n^\dagger, c_m^\dagger\} = 0$$

($\sigma_n^\pm \equiv \frac{1}{2}(\sigma_n^y \pm i\sigma_n^z)$, $\prod_{m=0}^{n-1}\sigma_m^x$ counts parity ($Q \leftrightarrow \widetilde{Q}^\dagger$) to the left)

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Key fact: Hamiltonian quadratic in $c, c^\dagger$

$$\begin{aligned} H &= \sum_{n=0}^{L-1} \left(\sigma_n^z \sigma_{n+1}^z - h\sigma_n^x\right) \\ &= \sum_{n=0}^{L-1} (c_n^\dagger + c_n)(c_{n+1}^\dagger - c_{n+1}) - h(2c_n^\dagger c_n - 1). \end{aligned}$$

Diagonalizing the Fermions

Now go to momentum space and do Bogoliubov transformation:

$$\begin{aligned} H &= \sum_{n=0}^{L-1} (c_n^\dagger + c_n)(c_{n+1}^\dagger - c_{n+1}) - h(2c_n^\dagger c_n - 1) \\ &= \sum_k \left[-(2\cos k + 2h) c_k^\dagger c_k - i \sin k (c_{-k}^\dagger c_k^\dagger + c_{-k} c_k) \right] + Lh \\ &= \sum_k \epsilon(k) \left(b_k^\dagger b_k - \frac{1}{2} \right) \qquad \text{(Bogoliubov)} \end{aligned}$$

$\Longrightarrow$ A system of free fermions, with dispersion relation

$$\epsilon(k) = 2\sqrt{h^2 + 1 + 2h\cos k} \qquad \left(h = \frac{N_f}{N_c} = 3 + O(\lambda) \right)$$

Quasimomenta Quantization

- Allowed values of k depends on overall parity, which determines the boundary conditions for the c's

$$P \equiv \prod_{n=0}^{L-1} \sigma_n^x = \begin{cases} -1 : & k = \frac{2\pi m}{L} & \text{(periodic)} \\ +1 : & k = \frac{(2m+1)\pi}{L} & \text{(anti-periodic)} \end{cases}$$

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- Similarly, can think of operators with open flavor indices as states in an open chain with "fixed" boundary conditions,

$$Q^\dagger XY \ldots YQ \qquad \Longrightarrow \qquad |\uparrow \uparrow\downarrow \ldots \downarrow \uparrow\rangle$$

- System is still solvable [Douçot, Feigel'man, Ioffe, Ioselevich '04]! Same H, but with an interesting quantization condition:

$$\frac{\sin(k(L+2))}{\sin(k(L+1))} = -h \qquad \text{(open chains)}$$

Example: Closed 4-field Operators

4-field flavor singlet operators: $\mathrm{Tr}(X^2)$, $\mathrm{Tr}(Y^2)$, $\mathrm{Tr}(XY)$.

- Odd parity

$$b^\dagger_\pi|0\rangle \qquad \underbrace{b^\dagger_0|0\rangle}_{\sum k=0} \longleftrightarrow \mathrm{Tr}(X^2-Y^2)$$

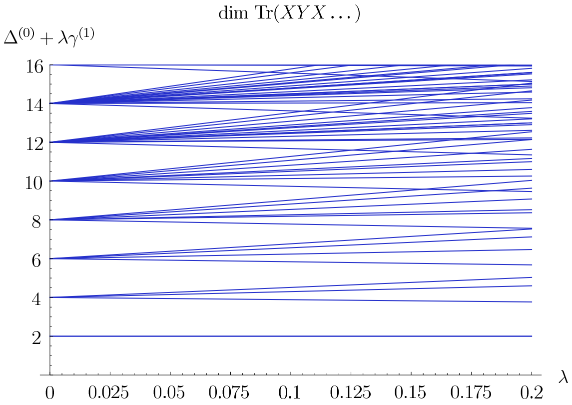
$$\gamma = \left(2+\frac{1}{2}\sqrt{10+6\cos 0}-\frac{1}{2}\sqrt{10+6\cos\pi}\right)\lambda = 3\lambda$$

- Even parity

$$\underbrace{|0\rangle, \quad b^\dagger_{\frac{\pi}{2}}b^\dagger_{\frac{3\pi}{2}}|0\rangle}_{\sum k=0} \qquad \longleftrightarrow \qquad \mathrm{Tr}(X^2+Y^2), \quad \mathrm{Tr}(XY)$$

$$\gamma = \left(2\pm\frac{1}{2}\sqrt{10+6\cos\frac{\pi}{2}}\pm\frac{1}{2}\sqrt{10+6\cos\frac{3\pi}{2}}\right)\lambda = (2\pm\sqrt{10})\lambda$$

Asymptotics



$$\gamma_\pm = \lambda L \pm \frac{\lambda L}{4\pi}\int_0^{2\pi} dk\sqrt{10 + 6\cos k} \qquad \text{(Large } L\text{)}$$

$$\approx (1 \pm 1.54)\lambda L$$

Outlook

Possible directions

- At 2-loops, mixing with fermions+gauge fields, but still nearest neighbor in X, Y. Nontrivial S-matrix for $b^\dagger, b$?
- Magnetic dual is weakly-coupled when $N_f/N_c \sim 3/2$
  - Building blocks of generalized single traces: $q(MM^\dagger)^k q^\dagger$, $\widetilde{q}^\dagger(M^\dagger M)^k \widetilde{q}$, $q(MM^\dagger)^k M\widetilde{q}$, $\widetilde{q}^\dagger(M^\dagger M)^k M^\dagger q^\dagger$
  - Noncompact spin chain?
- Speculation: TFIM has quantum phase transition at $h=1$ (naively $N_f/N_c=1$). Could this be relevant in SQCD?

...lots of work to be done!

Backup Slides

CFT Review: Conformal Blocks

Explicit formula [Dolan, Osborn '00]

$$
\begin{aligned}
g_{\Delta,l}(u,v) &= \frac{(-1)^l}{2^l}\frac{z\bar{z}}{z-\bar{z}}[k_{\Delta+l}(z)k_{\Delta-l-2}(\bar{z}) - z \leftrightarrow \bar{z}] \\
k_\beta(x) &= x^{\beta/2}{}_2F_1(\beta/2,\beta/2,\beta;x),
\end{aligned}
$$

where $u = z\bar{z}$ and $v = (1-z)(1-\bar{z})$.

- Similar expressions in other even dimensions, recursion relations known in odd dimensions
- Alternatively can be viewed as eigenfunctions of the quadratic casimir of the conformal group [Dolan, Osborn '03]

Flavor Currents

- If ϕ transforms under flavor symmetry with charges T^I, conserved currents J^I appear in the $\phi \times \phi^\dagger$ OPE:

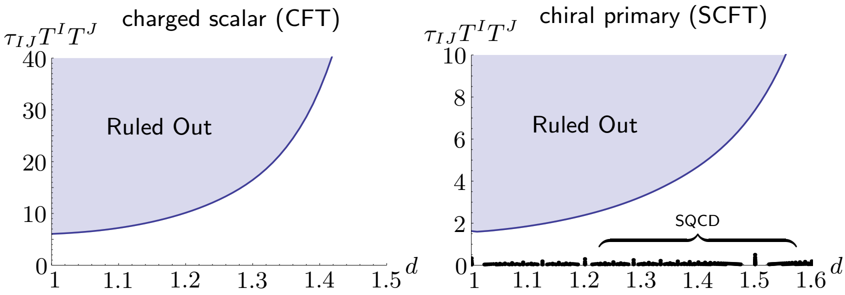
$$\langle \phi\phi^\dagger J^I \rangle \quad \sim \quad -\frac{i}{2\pi^2} T^I \qquad \text{(Ward id.)}$$

- Flavor current conformal blocks are then determined by current 2-pt functions

$$\begin{aligned}
\langle J^I J^J \rangle &\sim \frac{3}{4\pi^4} \tau^{IJ} \\
\langle \phi\phi^\dagger \phi\phi^\dagger \rangle &\sim -\frac{1}{3} \tau_{IJ} T^I T^J \, g_{3,1} \qquad \text{(general CFTs)}, \\
\langle \phi\phi^\dagger \phi\phi^\dagger \rangle &\sim \quad \tau_{IJ} T^I T^J \, \mathcal{G}_{2,0} \qquad \text{(SCFTs)},
\end{aligned}$$

where $\tau_{IJ} = (\tau^{IJ})^{-1}$ (in SCFTs, $\tau^{IJ} = -3\mathrm{Tr}(R T^I T^J)$).

Upper Bounds on $\tau_{IJ}T^I T^J$



- Example: SUSY QCD with $\frac{3}{2}N_c < N_f < 3N_c$, $M = Q\tilde{Q}$
  $\langle MM^\dagger MM^\dagger \rangle$: $d = 3 - \frac{3N_c}{N_f}$ and $\tau_{IJ}T^I T^J = \frac{2}{3}\frac{N_f - 1}{N_c^2}$
- In dual AdS$_5$, $(8\pi^2 L)\tau_{IJ} = g_{IJ}^2$. Gauge coupling can't be too strong in presence of charged scalar.