

# Naturalness and dark matter predictions for the Higgs mass in MSSM and Beyond

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NPB 835 (2010) 110, arXiv:1103.4793, with G.G. Ross, S. Cassel.

NPB 848 (2011) 1, NPB 831 (2010) 133, with I. Antoniadis, E. Dudas, P. Tziveloglou.

- **Standard Model (SM):** best model so far.

- Agrees with experiment. Open questions: hierarchy, EWSB, unification, DM, gravity,....

- $\delta m_h^2 \sim \mathcal{F}(\alpha_i^0, m_j^0) \Lambda_{UV}^2$ ,  $\Lambda_{UV} \sim M_{Planck}$ :  $\Rightarrow$  **Hierarchy problem**  $\Leftrightarrow$  **fine tuning**.  
- two faces of the same problem.

- **MSSM.** SUSY searches.

- $\delta m_h^2 \sim m_{susy}^2 \log \Lambda_{UV}^2 / m_{susy}^2$   $m_{susy} \sim \text{TeV}$ ;  $m_{susy} \sim \Lambda_{UV} \Rightarrow$  **SM fine-tuning**

$$\mathcal{L} = \int d^4\theta \sum_{\Phi} \mathcal{Z}_{\Phi} \Phi^\dagger e^V \Phi + \left\{ \sum_{i=1,2,3} \int d^2\theta \frac{1}{g_i^2} (1 + m_{1/2}\theta\theta) \text{Tr} W^\alpha W_\alpha|_i + h.c. \right\}$$

$$+ \int d^2\theta \left[ H_2 Q \lambda_U U^c + Q \lambda_D D^c H_1 + L \lambda_E E^c H_1 + \mu H_1 H_2 \right] + h.c., \quad \Phi : H_{1,2}, Q, U^c, D^c, E^c, L$$

$$\mathcal{Z}_{\Phi}(S, S^\dagger) = 1 - S S^\dagger, \quad S = \theta\theta m_0; \quad m_0 = \frac{\langle F_{hid} \rangle}{M_P}, \quad \lambda_F(S) = \lambda_F(0)(1 + A_0 S), \quad \mu(S) = \mu_0 (1 + B_0 S)$$

$$A_0, B_0, \mu_0, m_0, m_{1/2}, \tan \beta = v_2/v_1$$

- MSSM scalar potential:

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - (B_0 \mu_0 H_1 \cdot H_2 + h.c.) + \lambda_1/2 |H_1|^4 + \lambda_2/2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1 \cdot H_2|^2 + \left[ \lambda_5/2 (H_1 \cdot H_2)^2 + \lambda_6 |H_1|^2 (H_1 \cdot H_2) + \lambda_7 |H_2|^2 (H_1 \cdot H_2) + h.c. \right]$$

$$\text{Tree-level: } \lambda_{1,2} = (g_1^2 + g_2^2)/4, \quad \lambda_3 = (g_2^2 - g_1^2)/4, \quad \lambda_4 = -g_2^2/2, \quad \lambda_{5,6,7} = 0$$

$$m^2 \equiv m_1^2 \cos^2 \beta + m_2^2 \sin^2 \beta - B_0 \mu_0 \sin 2\beta, \quad \text{UV : } m_{1,2}^2 = m_0^2 + \mu_0^2$$

$$\lambda \equiv \frac{\lambda_1}{2} \cos^4 \beta + \frac{\lambda_2}{2} \sin^4 \beta + \frac{\lambda_{345}}{4} \sin^2 2\beta + \sin 2\beta (\lambda_6 \cos^2 \beta + \lambda_7 \sin^2 \beta)$$

- The Problem:

$$v^2 = -m^2/\lambda, \quad v = \mathcal{O}(100 \text{ GeV}), \quad \lambda < 1, \quad \text{but } m, m_{1,2}, B_0 \sim \mathcal{O}(1 \text{ TeV}).$$

- “residual” fine-tuning (little hierarchy). Tree level:  $m_h < m_Z$ , LEP2:  $> 114.4 \text{ GeV}$
- Need: large quantum corrections (QC)  $\Rightarrow$  large  $m_{susy}$ . But QC can also increase  $\lambda$  and  $m_h^2 \propto \lambda v^2$
- This is not only a problem of scales but of couplings ( $\lambda_{MSSM}$  small).  $\delta\lambda > 0$  vs.  $m_{susy}^2 \sim m^2$

- EW fine-tuning measures as a test of SUSY

[Ellis et al 1986, Barbieri, Giudice 1988]

$$\Delta \equiv \max \left| \Delta_p \right|_{p=\{\mu_0^2, m_0^2, m_{1/2}^2, A_0^2, B_0^2\}}, \quad \text{or} \quad \Delta' \equiv \left( \sum_p \Delta_p^2 \right)^{1/2}, \quad \Delta_p \equiv \frac{\partial \ln v^2}{\partial \ln p}$$

$$v^2 = -m^2/\lambda, \quad 2\lambda \frac{\partial m^2}{\partial \beta} = m^2 \frac{\partial \lambda}{\partial \beta}, \quad \Rightarrow \quad m^2, \lambda = F(p, \beta(p)),$$

- Exact formula:

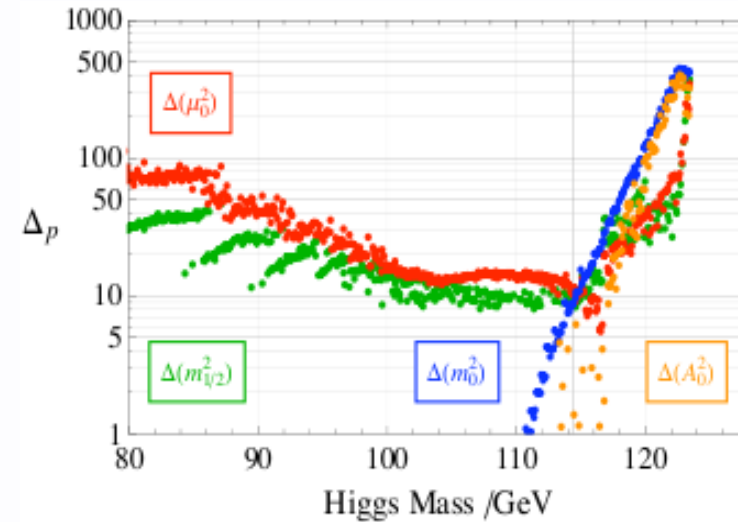
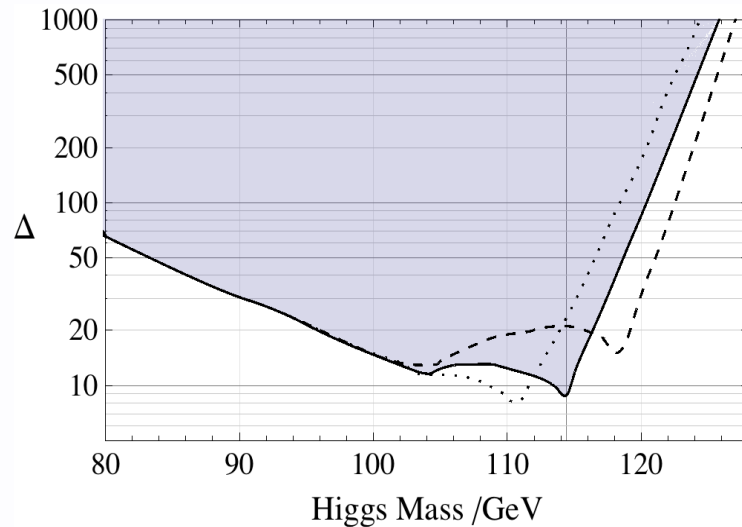
$$\Delta_p = -\frac{p}{z} \left[ \left( 2 \frac{\partial^2 m^2}{\partial \beta^2} + v^2 \frac{\partial^2 \lambda}{\partial \beta^2} \right) \left( \frac{\partial \lambda}{\partial p} + \frac{1}{v^2} \frac{\partial m^2}{\partial p} \right) + \frac{\partial m^2}{\partial \beta} \frac{\partial^2 \lambda}{\partial \beta \partial p} - \frac{\partial \lambda}{\partial \beta} \frac{\partial^2 m^2}{\partial \beta \partial p} \right]$$

$$z \equiv \lambda \left( 2 \frac{\partial^2 m^2}{\partial \beta^2} + v^2 \frac{\partial^2 \lambda}{\partial \beta^2} \right) - \frac{v^2}{2} \left( \frac{\partial \lambda}{\partial \beta} \right)^2, \quad \Delta_p = \frac{p}{m_Z^2(1 + \delta)} + \mathcal{O} \left[ \frac{1}{\tan \beta} \right], \quad \delta : \text{QC top Y.}$$

- $p_i \sim 1/\Delta$ : "effective prior" emerges automatically in Bayesian fits [Casas et al 2008]
- if  $\Delta$  too large: SUSY fails... Wanted:  $\Delta$  minimal, for low UV sensitivity.
- Many studies:  $\Delta$ : 1-loop.  $\Delta \sim 100$  but  $\Delta \sim m_0^2 \sim \exp(m_h^2/v^2)$ !. [Pokorski, Ellis et al]
- 2-loop study (SoftSusy, micrOMEGAs: 6years  $30 \times 3$  GHz). Impose usual constraints: TH + EXP  
but **NO** LEP bound on  $m_h$ !

- 2-loop results in CMSSM:  $\min \Delta \Rightarrow m_h = ?$

[S. Cassel, DG, G. Ross]



$$(\alpha_3, m_t) = (0.1176, 173.1).$$

$$\Delta \equiv \max |\Delta_p|, \quad p = \mu_0^2, m_0^2, A_0^2, B_0^2, m_{1/2}^2,$$

$$\Delta_{\mu_0^2}, \Delta_{m_0^2} \text{ dominant (EW vs. QCD)}$$

$\Rightarrow$  minimal  $\Delta \propto \exp(m_h^2/v^2) \Rightarrow$  2-loop effects important!

$\Rightarrow$  without LEP2 bound on  $m_h$ ,  $\min \Delta$ :  $\Delta \approx 9 \Rightarrow m_h = 114 \pm 2$  GeV, (just above LEP2 bound!)

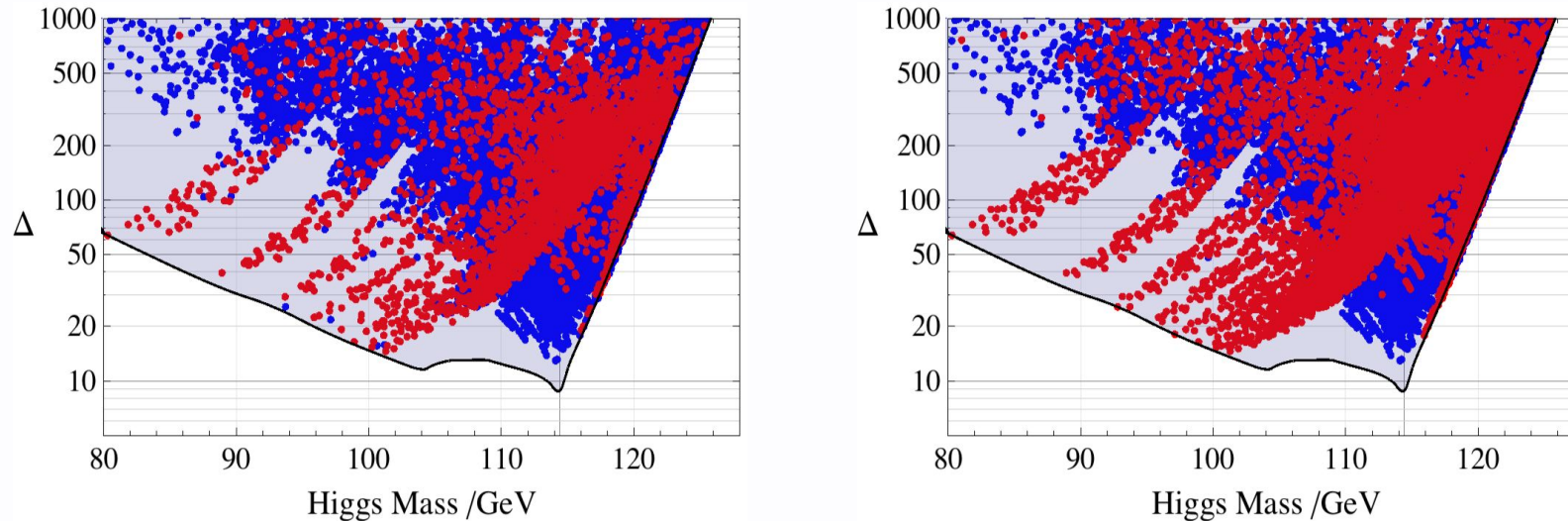
$\Rightarrow$  CMSSM:  $\Delta$  less than usually thought, when using the correct, 2-loop  $\Delta$ .

$\Rightarrow$  Same result ( $m_h$ ) and behaviour if using  $\min$  of  $\Delta' = \sqrt{\sum_p \Delta_p^2}$ , (1 to 2 GeV variations).

$\Rightarrow$  dotted line: increase  $\alpha_3$  ( $1\sigma$ ), reduce  $m_t$  ( $1\sigma$ )  $\Rightarrow$  QCD does not like large  $m_h$ ! (fine-tuning cost)

- 2-loop results in CMSSM:  $\min \Delta + \text{dark matter} \Rightarrow m_h = ?$

[S. Cassel, DG, G. Ross]



LSP: good dark matter candidate  $\Rightarrow$  dark matter constraint:

WMAP:  $\Omega h^2 = 0.1099 \pm 0.0062$ ; blue:  $\Omega h^2$  not-saturated; red:  $\Omega$  saturated:  $1\sigma$  (left);  $3\sigma$  (right).

$\Rightarrow$  Prediction from: Min  $\Delta +$  “right” dark matter abundance, (no LEP2 bound):

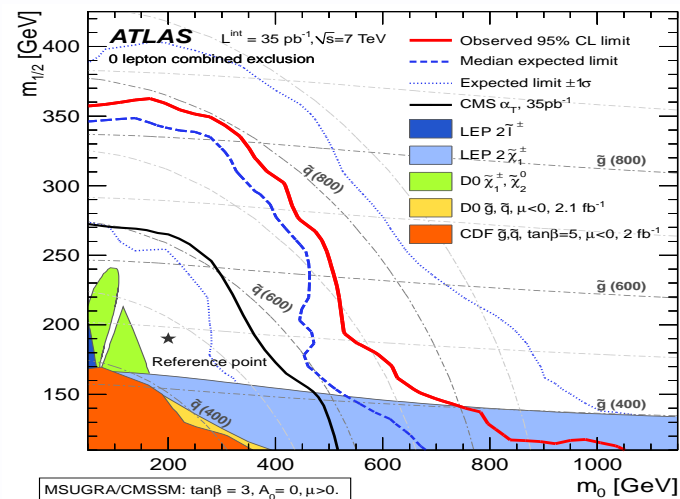
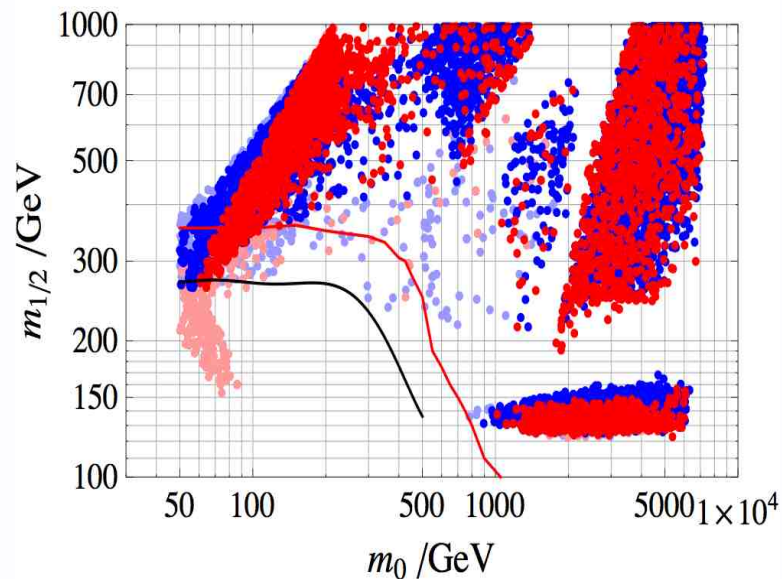
$$m_h = 114.7 \pm 2 \text{ GeV}, \quad \Delta = 15.0, \quad (\text{consistent with WMAP bound}).$$

$$m_h = 115.9 \pm 2 \text{ GeV}, \quad \Delta = 17.8, \quad (\text{saturating WMAP within } 3\sigma).$$

- an upper bound on  $\Delta \Rightarrow$  bounds on SUSY spectrum.  $\Delta < 1000 \Rightarrow m_h < 126 \text{ GeV}$  ( $\pm 2$  to  $3 \text{ GeV}$ )  
 $\Delta < 100 \Rightarrow m_h < 121 \text{ GeV}$  ( $\pm 2$  to  $3 \text{ GeV}$ )

- 2-loop results in CMSSM: min  $\Delta$  & the LHC @ 7 TeV reach.

[S. Cassel, DG, G. Ross]



⇒ Left: points  $\Delta < 1000$ : blue: consistent with WMAP ( $3\sigma$  deviation); red: saturate it within  $3\sigma$ .

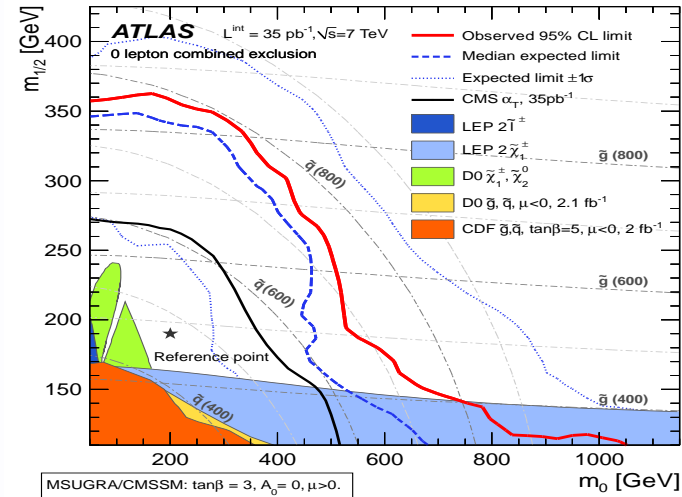
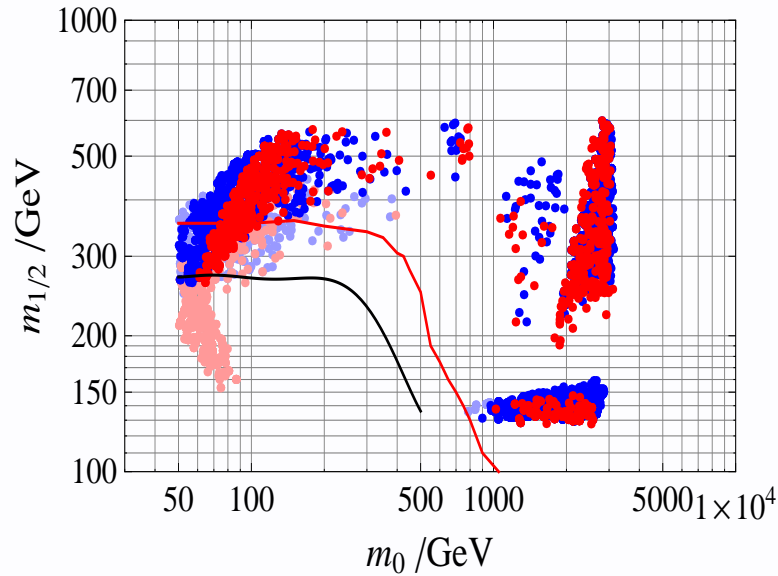
Points in lighter (darker) red/blue have  $m_h$  below (above) LEP2 bound for  $m_h$ .

⇒ Right: Atlas exclusion limits (red curve, observed) for  $\tan\beta = 3$ ,  $A_0 = 0$ ,  $\mu > 0$ . [arXiv:1102.5290](https://arxiv.org/abs/1102.5290).

Atlas is already testing CMSSM points with  $\Delta < 100$  and  $m_h > 114$  GeV. ( $\pm 2$  to  $3$  GeV)

- 2-loop results in CMSSM: min  $\Delta$  & the LHC @ 7 TeV reach.

[S. Cassel, DG, G. Ross]



⇒ Left: points  $\Delta < 100$ : blue: consistent with WMAP ( $3\sigma$  deviation); red: saturate it within  $3\sigma$ .

Points in lighter (darker) red/blue have  $m_h$  below (above) LEP2 bound for  $m_h$ .

⇒ Right: Atlas exclusion limits (red curve, observed) for  $\tan\beta = 3$ ,  $A_0 = 0$ ,  $\mu > 0$ . [arXiv:1102.5290](https://arxiv.org/abs/1102.5290).

Atlas is already testing CMSSM points with  $\Delta < 100$  and  $m_h > 114$  GeV.

⇒  $m_h < 121$  GeV,  $m_0 < 3200$ ,  $120 < m_{1/2} < 720$ ,  $\mu < 680$ ,  $-2000 < A_0 < 2500$ , GeV,  $\tan\beta > 5.5$

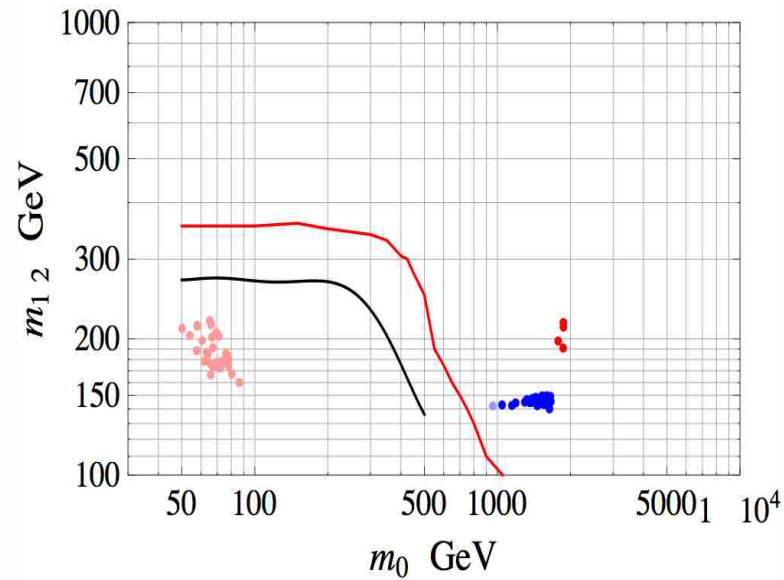
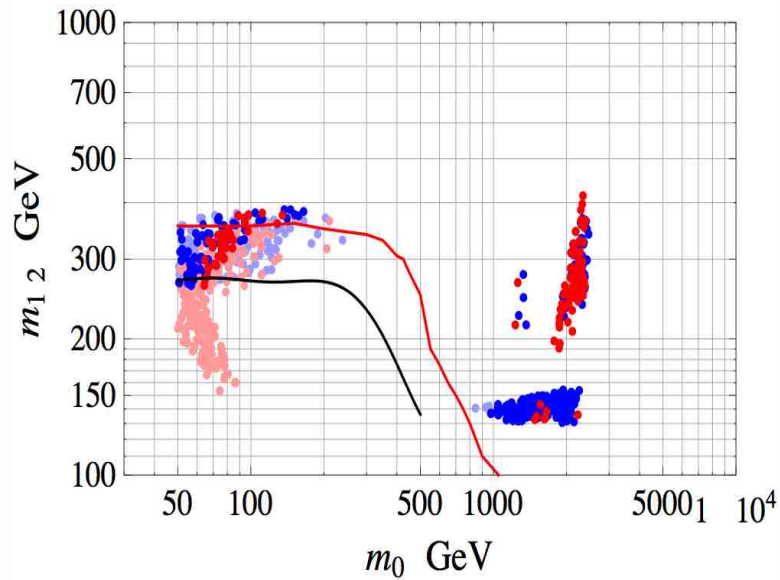
$\chi_1^0 < 305$ ,  $\chi_2^0 < 550$ ,  $\chi_3^0 < 660$ ,  $\chi_4^0 < 665$ ,  $\chi_1^\pm < 550$ ,  $\chi_2^\pm < 670$ ,  $\tilde{g} < 1700$ ,  $\tilde{t}_1 < 2080$ ,  $\tilde{t}_2 < 2660$

$\tilde{b}_1 < 2660$ ,  $\tilde{b}_2 < 3140$  GeV



- 2-loop results in CMSSM: min  $\Delta$  & the LHC @ 7 TeV reach.

[S. Cassel, DG, G. Ross]



⇒ Left: points  $\Delta < 50$ : blue: consistent with WMAP ( $3\sigma$  deviation); red: saturate it within  $3\sigma$ .

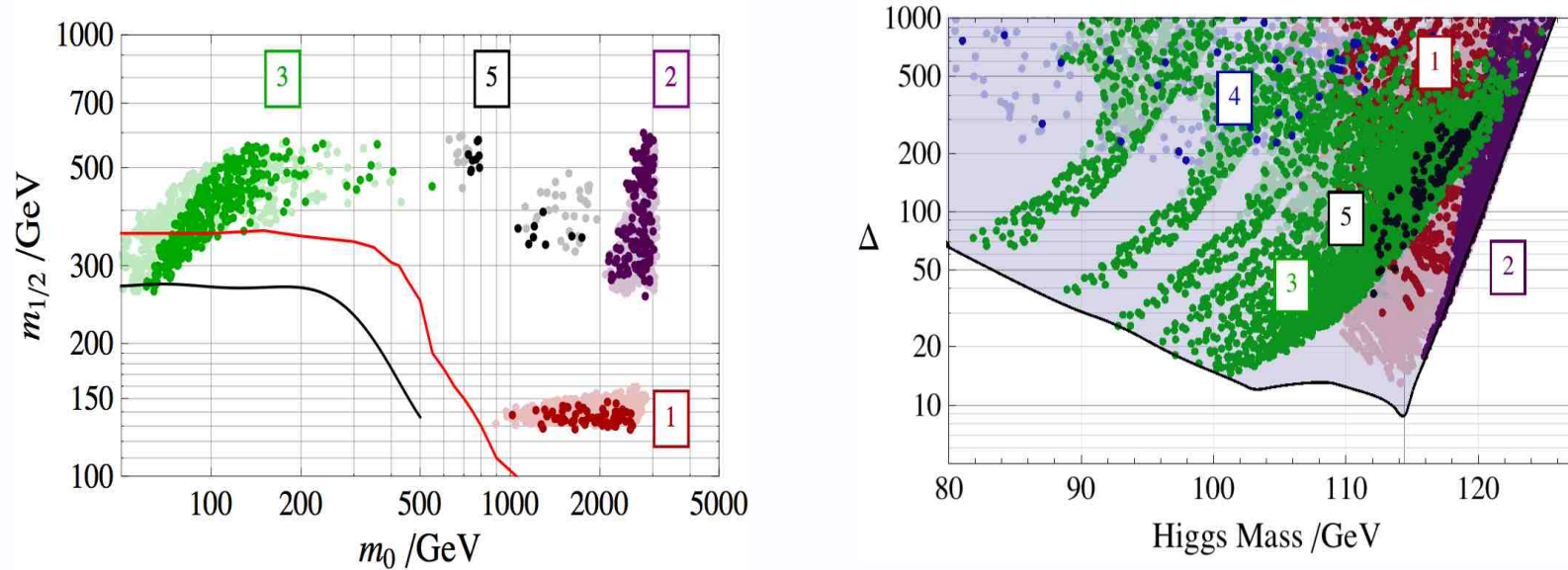
Points in lighter (darker) red/blue have  $m_h$  below (above) LEP2 bound for  $m_h$ .

⇒ Right: points  $\Delta < 20$ .

⇒ small(er) area in moduli space  $(m_0, m_{1/2})$ : not an indication/measure of (large) fine-tuning!

"moduli fixing"  $\neq$  fine tuning

- Two-loop results in CMSSM: min  $\Delta$  and LHC @ 7 TeV reach;  $m_h > 114$  GeV.



$\Delta < 100$ ; dark colours:  $\Omega h^2$  within  $3\sigma$ ; light colours: below this bound.

LHC 7 TeV:  $\mathcal{O}(1) \text{ fb}^{-1}$ ;  $m_{\tilde{g}} \sim 1.1 \text{ TeV}$  if  $m_{\tilde{q}} \sim m_{\tilde{g}}$ ;  $m_{\tilde{g}} \sim 620 \text{ GeV}$ ,  $m_{\tilde{q}} \gg m_{\tilde{g}}$  [Baer, Barger, Tata, Lessa]

- purple: LSP: higgsino 10%, heavy squarks

- red: LSP: bino-like, heavy squarks (TeV), focus point region.

- green: LSP: bino-like. light squarks.

[S. Cassel, DG, S. Kraml, G. Ross, A. Lessa]

- How do these results  $m_h$  change under “new physics” ?

- large(r)  $m_h$  at low  $\Delta$  ?

(a). to relax CMSSM constraints (gaugino univ); symmetries... [Kane, King, Ross, Horton, 09]

(b). “new physics” beyond MSSM, that increases  $m_h$  and reduces  $\Delta$ :  $U(1)'$ , extra S, D of  $SU(2)$ ...

Effective approach: MSSM Higgs+ effective operators ( $X_{n,i}$ )

$$\mathcal{L} = \mathcal{L}_{MSSM}^{Higgs} + \sum_{n \geq 1, i} \frac{\rho_{n,i}}{M_*^n} X_{n,i}, \quad \rho_{i,n} \sim \mathcal{O}(1); \quad M_* : \text{scale of "new physics"}$$

$X_{n,i}$  effective operators, parametrize new physics beyond MSSM Higgs sector.

- generated in renormalisable theories, after integrating some massive states.

- We consider: (d=5), (d=6) operators; corrections to  $m_h$ ?

$$\mathcal{L}_{MSSM}^{Higgs} + \mathcal{L}_i(\propto 1/M_*) + \mathcal{O}_i(\propto 1/M_*^2)$$

• d=5 operators beyond MSSM Higgs:

[DG, Antoniadis, Dudas, Tziveloglou]

$$\mathcal{L}_1 = \frac{1}{M_*} \int d^2\theta \zeta(S) (H_2 \cdot H_1)^2 + h.c., \quad [S, T]$$

$$\mathcal{L}_2 = \frac{1}{M_*} \int d^4\theta \left\{ a(S, S^\dagger) D^\alpha \left[ b(S, S^\dagger) H_2 e^{-V_1} \right] D_\alpha \left[ c(S, S^\dagger) e^{V_1} H_1 \right] + h.c. \right\}, \quad [D]$$

• d=6 operators:

$$\mathcal{O}_i = \frac{1}{M_*^2} \int d^4\theta \mathcal{Z}_i(S, S^\dagger) (H_i^\dagger e^{V_i} H_i)^2, \quad i = 1, 2. \quad (T, U(1))$$

$$\mathcal{O}_3 = \frac{1}{M_*^2} \int d^4\theta \mathcal{Z}_3(S, S^\dagger) (H_1^\dagger e^{V_1} H_1) (H_2^\dagger e^{V_2} H_2), \quad (T, U(1))$$

$$\mathcal{O}_4 = \frac{1}{M_*^2} \int d^4\theta \mathcal{Z}_4(S, S^\dagger) (H_2 H_1) (H_2 H_1)^\dagger, \quad (S)$$

$$\mathcal{O}_5 = \frac{1}{M_*^2} \int d^4\theta \mathcal{Z}_5(S, S^\dagger) (H_1^\dagger e^{V_1} H_1) (H_2 H_1 + h.c.), \quad (2D, S)$$

$$\mathcal{O}_6 = \frac{1}{M_*^2} \int d^4\theta \mathcal{Z}_6(S, S^\dagger) (H_2^\dagger e^{V_2} H_2) (H_2 H_1 + h.c.), \quad (2D, S)$$

$$\mathcal{O}_7 = \frac{1}{M_*^2} \int d^2\theta \frac{1}{16\kappa g_i^2} \mathcal{Z}_7(S, 0) \text{Tr} W_i^\alpha W_{i,\alpha} (H_2 H_1) + h.c.,$$

$$\mathcal{O}_8 = \frac{1}{M_*^2} \int d^4\theta \left[ \mathcal{Z}_8(0, S^\dagger) (H_2 H_1)^2 + h.c. \right]$$

• Other operators:

$$\begin{aligned}
\mathcal{O}_9 &= \frac{1}{M_*^2} \int d^4\theta \mathcal{Z}_9(S, S^\dagger) H_1^\dagger \bar{\nabla}^2 e^{V_1} \nabla^2 H_1 \\
\mathcal{O}_{10} &= \frac{1}{M_*^2} \int d^4\theta \mathcal{Z}_{10}(S, S^\dagger) H_2^\dagger \bar{\nabla}^2 e^{V_2} \nabla^2 H_2 \\
\mathcal{O}_{11} &= \frac{1}{M_*^2} \int d^4\theta \mathcal{Z}_{11}(S, S^\dagger) H_1^\dagger e^{V_1} \nabla^\alpha W_\alpha^{(1)} H_1 \\
\mathcal{O}_{12} &= \frac{1}{M_*^2} \int d^4\theta \mathcal{Z}_{12}(S, S^\dagger) H_2^\dagger e^{V_2} \nabla^\alpha W_\alpha^{(2)} H_2 \\
\mathcal{O}_{13} &= \frac{1}{M_*^2} \int d^4\theta \mathcal{Z}_{13}(S, S^\dagger) H_1^\dagger e^{V_1} W_\alpha^{(1)} \nabla^\alpha H_1 \\
\mathcal{O}_{14} &= \frac{1}{M_*^2} \int d^4\theta \mathcal{Z}_{14}(S, S^\dagger) H_2^\dagger e^{V_2} W_\alpha^{(2)} \nabla^\alpha H_2 \\
\mathcal{O}_{15} &= \frac{1}{M_*^2} \int d^4\theta \mathcal{Z}_{15}(S, S^\dagger) \text{Tr} e^V W^\alpha e^{-V} D^2 e^V W_\alpha e^{-V}
\end{aligned}$$

where  $\nabla_\alpha H_i \equiv e^{-V_i} D_\alpha e^{V_i} H_i$ ,  $V_i = V_W^a \sigma^i / 2 + (\mp 1/2) V_Y$ ;  $i = 1, 2$  .

$$\frac{1}{M_*^2} \mathcal{Z}_j(S, S^\dagger) = \alpha_{j0} + \alpha_{j1} m_0 \theta\theta + \alpha_{j1}^* m_0 \bar{\theta}\bar{\theta} + \alpha_{j2} m_0^2 \theta\theta\bar{\theta}\bar{\theta}, \quad \alpha_{jk} \sim 1/M_*^2, \quad S = m_0 \theta\theta$$

$\Rightarrow \mathcal{L}_2, \mathcal{O}_{9,\dots,15}$  removed by field redefinitions up to wavefunction ren, redefinition of soft terms,  $\mu!$

$$\Rightarrow \mathcal{L}_{total} = \mathcal{L}_{MSSM}^{Higgs} + \mathcal{L}_1 + \sum_{i=1}^8 \mathcal{O}_i$$

• **Comment: removing redundant operators:**

[DG, Antoniadis, Dudas]

$$\mathcal{O}_9 = \frac{1}{M_*^2} \int d^4\theta \mathcal{Z}_9(S, S^\dagger) H_1^\dagger \bar{\nabla}^2 e^{V_1} \nabla^2 H_1 \sim \frac{1}{M_*^2} \int d^4\theta H_1^\dagger \square H_1 \supset -(1/M_*^2) h_1^* \square^2 h_1 - h_1^* \square h_1 \cdots \dots$$

$$\bullet \mathcal{L} = \int d^4\theta \Phi^\dagger (1 + \square/M_*^2) \Phi + \left[ \int d^2\theta W(\Phi) + h.c. \right], \quad \Phi^\dagger \square \Phi \rightarrow (-1/16) \bar{D}^2 \Phi^\dagger D^2 \Phi$$

$$\Phi = a_1 \Phi_1 + a_2 \Phi_2$$

$$(1/m) \bar{D}^2 \Phi^\dagger = b_1 \Phi_1 + b_2 \Phi_2 \Rightarrow \delta\mathcal{L} = \int d^2\theta \left[ (1/m) \bar{D}^2 (a_1 \Phi_1 + a_2 \Phi_2)^\dagger - (b_1 \Phi_1 + b_2 \Phi_2) \right] \Phi_3 \frac{m^2}{4 M_*}$$

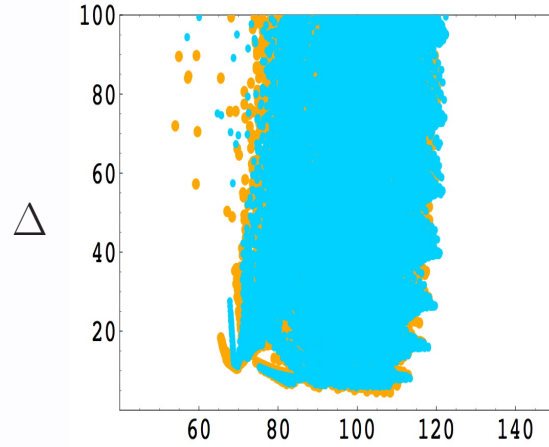
$$\Rightarrow \mathcal{L} + \delta\mathcal{L} = \int d^4\theta \left[ \tilde{\Phi}_1^\dagger \tilde{\Phi}_1 - \tilde{\Phi}_2^\dagger \tilde{\Phi}_2 - \tilde{\Phi}_3^\dagger \tilde{\Phi}_3 \right] + \int d^2\theta \left[ (-M_*) \tilde{\Phi}_2 \tilde{\Phi}_3 + W(\Phi(\tilde{\Phi}_{1,2})) \right] + h.c. + \mathcal{O}(1/M_*^3)$$

where  $\Phi = \tilde{\Phi}_2 - \tilde{\Phi}_1$ ; then integrate  $\tilde{\Phi}_{1,2}$  (massive):

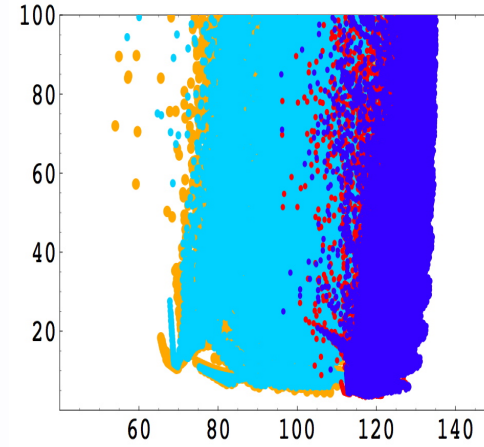
$$\mathcal{L} + \delta\mathcal{L} = \int d^4\theta \left[ \tilde{\Phi}_1^\dagger \tilde{\Phi}_1 - \frac{1}{M_*^2} W'^\dagger(\tilde{\Phi}_1) W'(\tilde{\Phi}_1) \right] + \int d^2\theta W(\tilde{\Phi}_1) + h.c.$$

**MSSM +  $\mathcal{O}_9$ :  $\int d^4\theta W'^\dagger W' \rightarrow \int d^4\theta (\mu^2/M_*^2) H_2^\dagger H_2$ : wavefunc ren (Susy broken: soft terms ren)**

- Fine-tuning in MSSM + (d=5) operator:  $\mathcal{L}_1 = \int d^2\theta (\zeta_0 + \zeta_1 S)(H_1 \cdot H_2)^2$  [Cassel, DG, Ross]



MSSM

MSSM +  $\mathcal{L}_1$ ,  $\mu_0 \zeta_0 = 0.035$ ,  $\zeta_1 = 0$ 

$$m_{h,H}^2 = \left(m_h^2\right)_{MSSM}^{1-loop} + 2\zeta_0 \mu_0 v^2 \sin 2\beta \left(1 \pm \frac{m_A^2 + m_Z^2}{\sqrt{w}}\right) + \zeta_1 m_0 v^2 \left(1 \mp \frac{(m_A^2 - m_Z^2) \cos^2 2\beta}{\sqrt{w}}\right) + \delta m_h^2$$

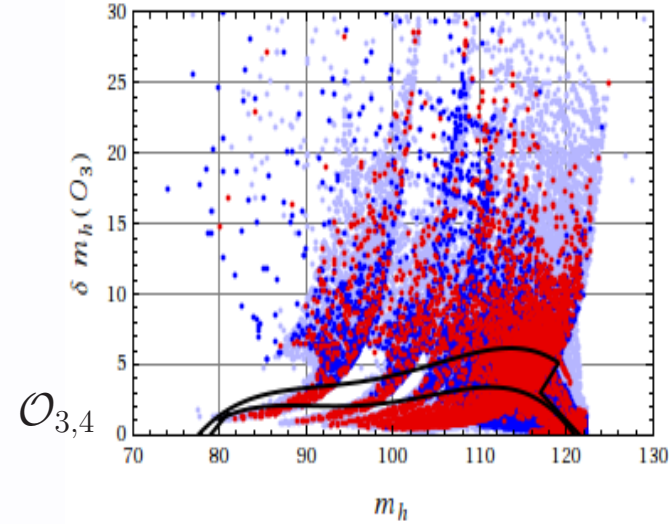
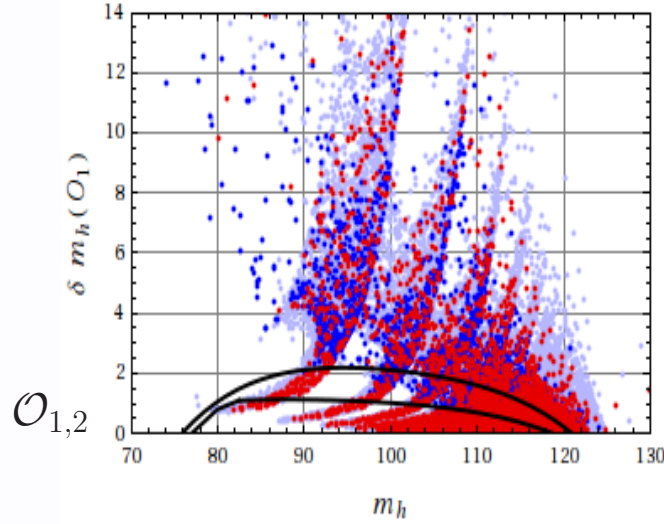
$$\text{where } \zeta_0, \zeta_1 \sim \mathcal{O}(1/M_*), \quad \delta m_h^2 = \mathcal{O}(1/M_*^2), \quad w \equiv [m_A^2 + m_Z^2]^2 - 4m_A^2 m_Z^2 \cos^2 2\beta$$

$$\Rightarrow \Delta < 10, \quad 114.4 \leq m_h \leq 130 \text{ GeV}, \quad M_* \approx 1/\zeta_0 \approx 65 \times \mu_0 = 5 \text{ to } 10 \text{ TeV}, \quad \tan \beta < 6.$$

$$\Rightarrow \text{(d=5) op: massive singlet: } S H_1 H_2 + M_* S^2. \text{ Re-do analysis in NMSSM with } M_* S^2 \text{ F-term.}$$

$$\Rightarrow \text{At large } \tan \beta: \text{ d=6 operators relevant: } \lambda \propto (2\mu_0 \zeta_0)^2 \sim (2\zeta_0 \mu_0) / \tan \beta \Rightarrow (2\mu_0 \zeta_0) < 1 / \tan \beta$$

- d=6 effective operators: corrections to  $m_h$  ( $\alpha_{jk} \sim 1/M_*^2$ )



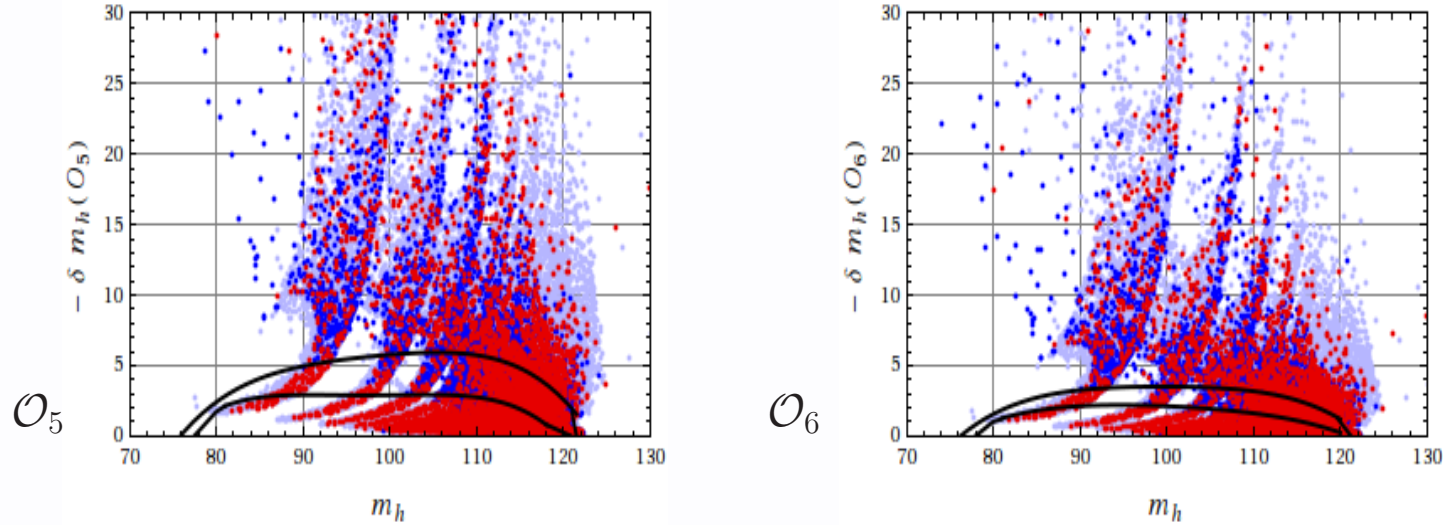
$$\begin{aligned}
\delta m_h^2 &= -2v^2 \left[ \alpha_{22} m_0^2 + (\alpha_{30} + \alpha_{40}) \mu_0^2 + 2\alpha_{61} m_0 \mu_0 - \alpha_{20} m_Z^2 \right] - \frac{(2\zeta_0 \mu_0)^2 v^4}{m_A^2 - m_Z^2} \\
&+ \frac{v^2}{\tan \beta} \left[ \frac{1}{(m_A^2 - m_Z^2)} \left( 4m_A^2 \left( (2\alpha_{21} + \alpha_{31} + \alpha_{41} + 2\alpha_{81}) m_0 \mu_0 + (2\alpha_{50} + \alpha_{60}) \mu_0^2 + \alpha_{62} m_0^2 \right) \right. \right. \\
&\left. \left. - (2\alpha_{60} - 3\alpha_{70}) m_A^2 m_Z^2 - (2\alpha_{60} + \alpha_{70}) m_Z^4 \right) + \frac{8(m_A^2 + m_Z^2) (\mu_0 m_0 \zeta_0 \zeta_1) v^2}{(m_A^2 - m_Z^2)^2} \right] + \mathcal{O} \left[ \frac{\tilde{m}^2}{M_*^2 \tan^2 \beta} \right] \\
\Rightarrow \delta m_h &= (m_h^2 + \delta m_h^2)^{1/2} - m_h = (1/2) (\delta m_h^2 / m_h) + \mathcal{O}(1/M_*^4) \quad m_h : \text{MSSM 2-loop LL value}
\end{aligned}$$

$\Rightarrow$  lower curve:  $\Delta < 100$ :  $m_h < 121$  GeV,  $\delta m_h < 4$  GeV.  $M_* = 8$  TeV. note:  $\alpha_{j0} \tilde{m} \leq 1/4$

$\Rightarrow$  top curve:  $\Delta < 200$ :  $m_h < 122$  GeV,  $\delta m_h < 6$  GeV.  $\pm 1$  GeV ( $\delta m_h$ )  $\leftrightarrow$   $\mp 1$  TeV ( $\delta M_*$ ).



- d=6 effective operators: corrections to  $m_h$ , ( $\alpha_{jk} \sim 1/M_*^2$ )



$$\rho - 1 = -(v^2/M_*^2) [\alpha_{10} \cos^4 \beta + \alpha_{20} \sin^4 \beta - \alpha_{30} \sin^2 \beta \cos^2 \beta] + \mathcal{O}(v^4/M_*^4), \quad M_* \sim 8 \text{ TeV} \quad [\text{Blum et al}]$$

$\Rightarrow$  large  $\tan \beta$ : larger  $\alpha_{30}, \alpha_{10}$  allowed  $\Rightarrow \alpha_{30}, \alpha_{40}$  largest SUSY correction to  $m_h$

$$\mathcal{O}_3 \sim \alpha_{30} \int d^4\theta (H_1^\dagger e^{V_1} H_1) (H_2^\dagger e^{V_2} H_2), \quad [T, U(1)] \quad \mathcal{O}_4 \sim \alpha_{40} \int d^4\theta (H_2 H_1) (H_2 H_1)^\dagger, \quad [S]$$

$\Rightarrow$  difficult to generate  $\alpha_{30}, \alpha_{40}$  with the “right” sign, by integrating massive  $T, U(1), S$  in ren model

$\Rightarrow$  neutralino mass corrections very small (LSP):  $\sim \mu/M_* \sim \text{few } (\leq 1 \text{ to } 2 \text{ GeV})!$

$$\Rightarrow \Delta = 18 (3\sigma \text{ WMAP}), \quad m_h = 115.9 \pm 2 \text{ GeV} \Rightarrow m_h + \delta m_h = 119.9 \pm 2 \text{ GeV}, \quad \Delta' = \Delta \frac{m_h^2}{(m_h + \delta m_h)^2}$$

- **Conclusions:**

- **Hierarchy problem  $\leftrightarrow$  fine tuning. Test of SUSY.**

$\Rightarrow$  min  $\Delta$  + DM consistency, in **constrained** MSSM, but no LEP2 bound on  $m_h$ :

$$m_h = 114 \pm 2 \text{ GeV}, \quad \Delta \approx 9, \quad (\text{no DM constraint}).$$

$$m_h = 115.9 \pm 2 \text{ GeV}, \quad \Delta = 17.8, \quad (\text{WMAP within } 3\sigma).$$

$\Rightarrow$   $\Delta$  minimal at LEP2 bound! similar result if using  $\Delta'$  (quadrature).

$\Rightarrow$  QCD does not like large  $m_h$  without fine-tuning cost:  $\Delta < 100$  (1000),  $m_h < 121$  (126) GeV.

- **Beyond MSSM Higgs with all effective operators of  $d=5$ ,  $d=6$  (new U(1)'s, S, D...):**

$\Rightarrow$   **$d=5$**  operators: small  $\Delta < 10$  allowed for  $114.4 \leq m_h \leq 130$  GeV. **Massive S? NMSSM with  $M_* S^2$**

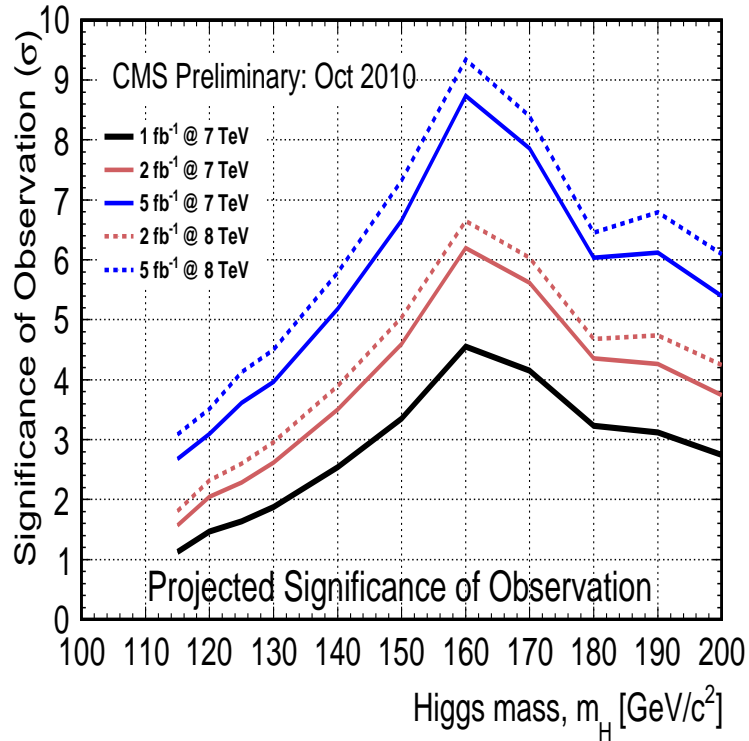
$\Rightarrow$   **$d=6$**  operators:

Points of  $\Delta < 100$  (200)  $\Rightarrow$  SUSY  $\delta m_h \leq 4$  GeV (6 GeV).  $M_* = 8$  TeV; ( $\pm 1$  GeV for  $\mp 1$  TeV)

**Extra U(1) or S ?**

- CMS significance of observation

- Our CMSSM spectrum and parameter space.



	SUG0	SUG1	SUG2	SUG3	SUG5
$m_0$	1455	1508	2270	113	725
$m_{1/2}$	160	135	329	383	535
$A_0$	238	1492	30	-220	1138
$\tan \beta$	22.5	22.5	35	15	50
$\mu$	191	433	187	529	581
$m_{\tilde{g}}$	482	414	900	898	1252
$m_{\tilde{u}_L}$	1469	1509	2331	826	1315
$m_{\tilde{t}_1}$	876	831	1423	602	1000
$m_{\tilde{\chi}_1^+}$	106	104	168	293	416
$m_{\tilde{\chi}_2^0}$	108	104	181	293	416
$m_{\tilde{\chi}_1^0}$	60	53	123	155	222
$\Delta$	9	50	45	68	84
$\Omega_{\tilde{\chi}_1^0} h^2$	0.41	0.13	0.10	0.13	0.10
$10^4 \text{BR}(b \rightarrow s\gamma)$	3.4	3.7	3.4	3.2	3.2
$10^9 \text{BR}(B_s \rightarrow \mu\mu)$	3.0	2.9	2.9	3.4	1.7
$\delta a_\mu \times 10^{10}$	4.5	3.2	3.2	22.5	16.6

CMSSM parameters, sparticle masses (GeV) for regions 1,...5

- The favoured CMSSM spectrum of minimal  $\Delta = 15$  consistent with  $\Omega h^2$ .

$h^0$	114.7	$\tilde{\chi}_1^0$	79	$\tilde{b}_1$	1147	$\tilde{u}_L$	1444
$H^0$	1264	$\tilde{\chi}_2^0$	142	$\tilde{b}_2$	1369	$\tilde{u}_R$	1446
$H^\pm$	1267	$\tilde{\chi}_3^0$	255	$\tilde{\tau}_1$	1328	$\tilde{d}_L$	1448
$A^0$	1264	$\tilde{\chi}_4^0$	280	$\tilde{\tau}_2$	1368	$\tilde{d}_R$	1446
$\tilde{g}$	549	$\tilde{\chi}_1^\pm$	142	$\tilde{\mu}_L$	1406	$\tilde{s}_L$	1448
$\tilde{\nu}_\tau$	1366	$\tilde{\chi}_2^\pm$	280	$\tilde{\mu}_R$	1406	$\tilde{s}_R$	1446
$\tilde{\nu}_\mu$	1404	$\tilde{t}_1$	873	$\tilde{e}_L$	1406	$\tilde{c}_L$	1444
$\tilde{\nu}_e$	1404	$\tilde{t}_2$	1158	$\tilde{e}_R$	1406	$\tilde{c}_R$	1446

(focus point region).

- Generating d=5 operators beyond the MSSM the Higgs sector:

- $\mathcal{L}_1$ ?: massive gauge singlet  $\Sigma$  or SU(2) triplet ( $M_* \gg \mu$ ):

$$\mathcal{L} \supset \int d^4\theta \Sigma^\dagger \Sigma + \int d^2\theta \left[ \mu H_1 \cdot H_2 - M_* \Sigma^2 + \lambda \Sigma H_1 H_2 \right] + h.c. \Rightarrow \mathcal{L}_1 = \frac{1}{4M_*} \int d^2\theta \lambda^2 (H_1 \cdot H_2)^2 + h.c.$$

- $\mathcal{L}_2$ ?: massive Higgs doublets  $H_{3,4}$  beyond MSSM  $H_{1,2}$

$$\begin{aligned} \mathcal{L} = & \int d^4\theta \left[ \sum_{j=1,3} (H_j^\dagger e^{V_1} H_j + H_{j+1}^\dagger e^{V_2} H_{j+1}) + (\nu_1 H_1^\dagger e^{V_1} H_3 + \nu_2 H_2^\dagger e^{V_2} H_4 + h.c.) \right]; \\ & + \int d^2\theta \left[ \mu H_1 H_2 + M_* H_3 H_4 \right] + h.c.; \quad -\frac{1}{4} \overline{D}^2 [H_3^\dagger e^{V_1}] - \frac{1}{4} \overline{D}^2 [\nu_1 H_1^\dagger e^{V_1}] + M_* H_4 = 0 \quad (H_3) \end{aligned}$$

$$\Rightarrow \mathcal{L}_2 = \int d^4\theta \left[ \sum_{j=1,2} H_j^\dagger e^{V_j} H_j + \left( \frac{\nu_1 \nu_2}{4M_*} H_2 e^{-V_1} D^2 e^{V_1} H_1 + h.c. \right) \right] + \left[ \int d^2\theta \mu H_1 \cdot H_2 + h.c. \right]$$

$$H_2 e^{-V_1} D^2 e^{V_1} H_1 \sim D^\alpha [H_2 e^{-V_1}] D_\alpha e^{V_1} H_1.$$

- ? “onshell”:  $D^2 [e^{V_1} H_1] = 4\mu H_2^\dagger \Rightarrow$  wavefunc ren only. [Politzer, Georgi, Dixon, Taylor]

- Removing redundant operators by field redefinitions: [Antoniadis, Dudas, DG, Tziveloglou]

$$\mathcal{L}_1 = \frac{1}{M_*} \int d^2\theta \zeta(S) (H_2 \cdot H_1)^2 + h.c., \quad [S, T]$$

$$\mathcal{L}_2 = \frac{1}{M_*} \int d^4\theta \left\{ a(S, S^\dagger) D^\alpha \left[ b(S, S^\dagger) H_2 e^{-V_1} \right] D_\alpha \left[ c(S, S^\dagger) e^{V_1} H_1 \right] + h.c. \right\}, \quad [D]$$

$$\frac{1}{M_*} \zeta(S) = \zeta_0 + \zeta_1 m_0 \theta\theta, \quad \zeta_0, \zeta_1 \sim 1/M_*,$$

$$a(S, S^\dagger) = a_0 + a_1 S + a_1^* S^\dagger + a_2 S S^\dagger, \quad S = \theta\theta m_0, \text{ spurion}$$

$\Rightarrow \mathcal{L}_2$  removed by general non-linear, field redefinitions in  $\mathcal{L}_{MSSM}$

$$H_1 \rightarrow H_1 - \frac{1}{M_*} \bar{D}^2 \left[ \delta_1(S, S^\dagger) H_2^\dagger e^{V_2} (i\sigma_2) \right]^T$$

$$H_2 \rightarrow H_2 + \frac{1}{M_*} \bar{D}^2 \left[ \delta_2(S, S^\dagger) H_1^\dagger e^{V_1} (i\sigma_2) \right]^T$$

$$\delta_1 = s_0 + s_1 S + s_2 S^\dagger + s_3 S S^\dagger, \quad \delta_2 = s'_0 + s'_1 S + s'_2 S^\dagger + s'_3 S S^\dagger, \quad F: U^c, D^c, E^c$$

$\mathcal{L}_2$  removed by suitably chosen  $s_i, s'_i$ .  $\Rightarrow$  soft terms &  $\mu$ -term redefinition:  $\Rightarrow$  only  $\mathcal{L}_1$  left (d=5)

• Other physical consequences:

$$\mathcal{L}_{MSSM} \supset \int d^2\theta \left[ H_2 Q \lambda_U(S) U^c + Q \lambda_D(S) D^c H_1 + L \lambda_E(S) E^c H_1 + \mu(S) H_1 H_2 \right] + h.c.$$

$$H_1 \rightarrow H_1 - \frac{1}{M_*} \bar{D}^2 \left[ \delta_1(S, S^\dagger) H_2^\dagger e^{V_2} (i\sigma_2) \right]^T$$

$$H_2 \rightarrow H_2 + \frac{1}{M_*} \bar{D}^2 \left[ \delta_2(S, S^\dagger) H_1^\dagger e^{V_1} (i\sigma_2) \right]^T$$

$$\Rightarrow \mathcal{L}_{eff} \supset -\frac{1}{M_*} \int d^4\theta \left[ H_1^\dagger e^{V_1} Q \lambda_U(S) U^c + H_2^\dagger e^{V_2} (Q \lambda_D(S) D^c) + H_2^\dagger e^{V_2} (L \lambda_E(S) E^c) + h.c. \right]$$

⇒ “wrong”-Higgs couplings:

$$\frac{m_0}{M_*} (\lambda_U(0))_{ij} (h_1^\dagger q_{Li}) u_{Rj}^c + \frac{m_0}{M_*} (\lambda_D(0))_{ij} (h_2^\dagger q_{Li}) d_{Rj}^c + \frac{m_0}{M_*} (\lambda_E(0))_{ij} (h_2^\dagger l_{Li}) e_{Rj}^c + h.c.,$$

$$\Rightarrow m_b = \frac{v \cos \beta}{\sqrt{2}} \lambda_b \left( 1 + \frac{\delta \lambda_b}{\lambda_b} + \frac{\Delta \lambda_b \tan \beta}{\lambda_b} \right)$$

“Wrong”-Higgs couplings also in MSSM at 1-loop:

[Haber, Mason; Pokorski, Hall, Pierce, Katz].

