New understanding of longitudinal (bunched) beam instabilities and comparison with measurements

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Acknowledgments:

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68th ICFA ABDW on High-intensity and high-brightness hadron beams, 9-13 October 2023

Outline

1. Loss of Landau Damping

I. Karpov, T. Argyropoulos, E. Shaposhnikova, Thresholds for loss of Landau damping in longitudinal plane, 2021

2. Single-bunch instabilities

I. Karpov, Longitudinal mode-coupling instabilities of proton bunches in the CERN Super Proton Synchrotron, 2023

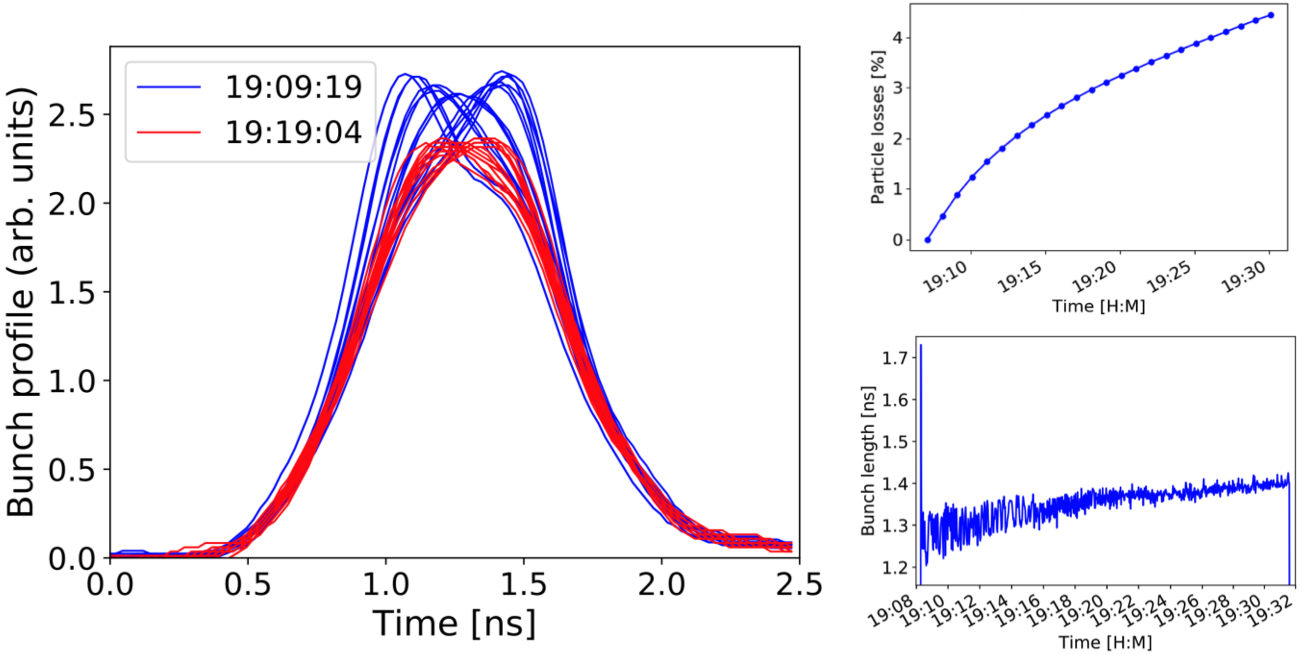
3. Multi-bunch instabilities

I. Karpov, E. Shaposhnikova, Generalized threshold of longitudinal multi-bunch instability in synchrotrons, 2023

Loss of Landau damping

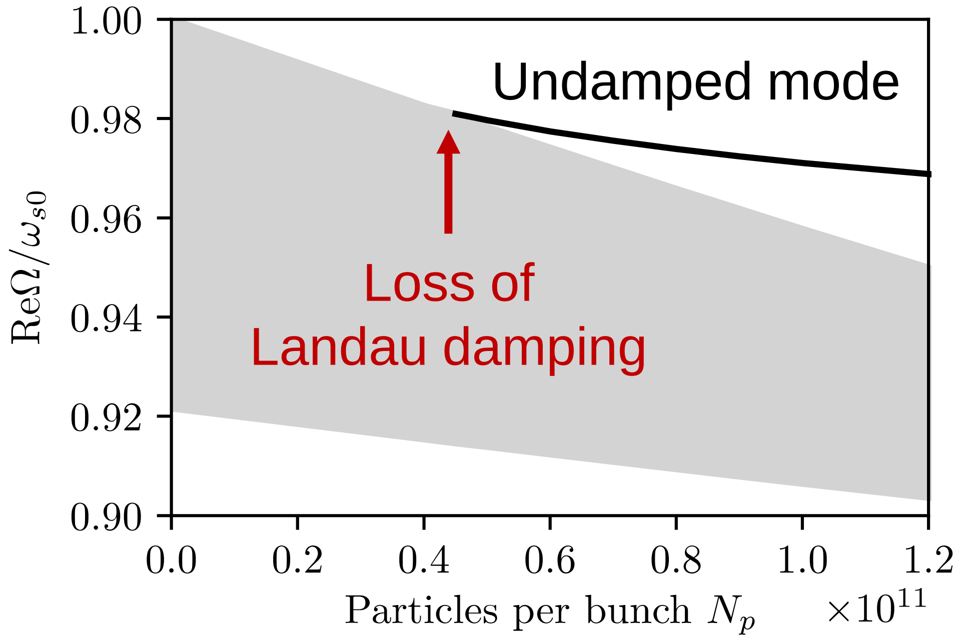
Loss of Landau damping (LLD)

Long-lasting oscillations were observed in SPS, RHIC, Tevatron, LHC, …



H. Timko et al, Beam instabilities after injection to the LHC, 2018

Longitudinal particle oscillations can be described as van Kampen modes*



Dominant inductive impedance above transition

**Y. H. Chin, K. Satoh, and K. Yokoya, Instability of a bunched beam with synchrotron frequency spread, 1983, and A. Burov, Van Kampen modes for bunch longitudinal motion, 2010*

Lebedev equation*

A system of equations for line-density harmonics $\tilde{\lambda}_k$ for coherent mode Ω

Beam and RF parameters

Impedance at $k\omega_0 + \Omega$

$$\sum_{k=-\infty}^{\infty} \left[\delta_{k'k} + \frac{qN_p\omega_{\mathrm{rf}}}{V_0} G_{k'k}(\Omega) \frac{Z_k(\Omega)}{k} \right] \tilde{\lambda}_k(\Omega) \equiv \sum_{k=-\infty}^{\infty} \mathcal{M}_{k'k}(\Omega)\tilde{\lambda}_k(\Omega) = 0$$

Beam transfer matrix

$k = \omega/\omega_0$
N_p – number of particles
q – charge
ω_0 – revolution frequency
ω_{rf} – rf frequency
V_0 – rf voltage

→ The mode Ω is a solution if the determinant is zero, $\det \mathcal{M} = 0$

**A. N. Lebedev, Coherent synchrotron oscillations in the presence of a space charge, 1968*

Approximate analytic solution

$$\det\mathcal{M} = \det[I + \varepsilon X(\varepsilon)] = \det(\exp\{\ln[I + \varepsilon X(\varepsilon)]\}) = \exp(\mathrm{tr}\{\ln[I + \varepsilon X(\varepsilon)]\})$$

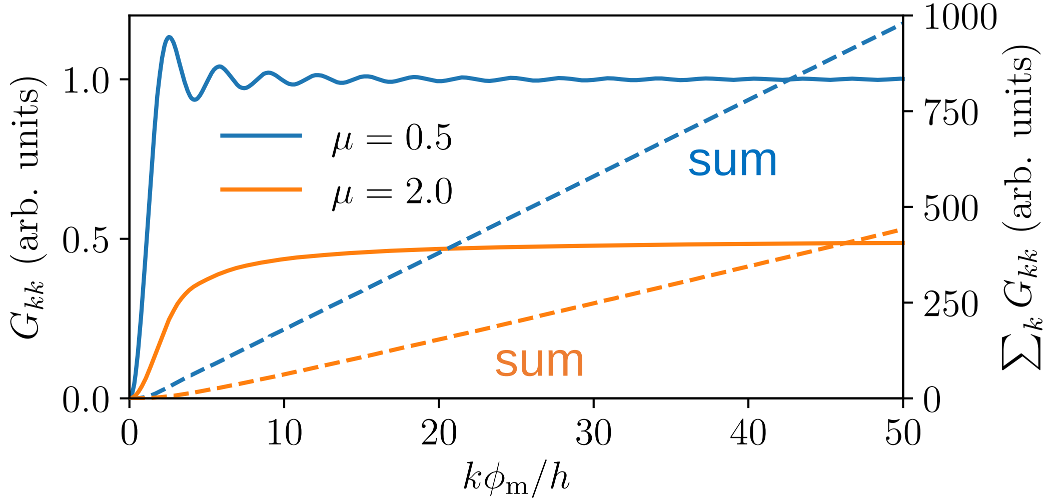
$$= 1 + \varepsilon\,\mathrm{tr}[X(0)] + \mathcal{O}(\varepsilon^2)$$

$\det(\exp A) = \exp(\mathrm{tr}A)$

The LLD threshold for dipole mode: $N_{\mathrm{LLD}} \approx \frac{V_0}{q\omega_{\mathrm{rf}}}\left[\sum_{k=-\infty}^{\infty} G_{kk}(\Omega)\frac{Z_k(\Omega)}{k}\right]^{-1}$

Assuming:

- Reactive impedance $Z_k/k = i\mathrm{Im}Z/k =$ const.
- Beam above transition in single rf: $\Omega = \omega_s(0)$
- Short bunch approximation $\phi_{\mathrm{m}} = \omega_{\mathrm{rf}}\,\tau/2 \ll \pi$
- Binomial distribution $\lambda(\phi) \propto [1 - \phi^2/\phi_{\mathrm{m}}^2]^{\mu+1/2}$



→ Elements G_{kk} saturate for $k \to \infty$

→ LLD threshold is zero for commonly used inductive impedance $\mathrm{Im}Z/k =$ const

LLD threshold

One needs to introduce a cutoff frequency $f_c = k_c f_0$ and then

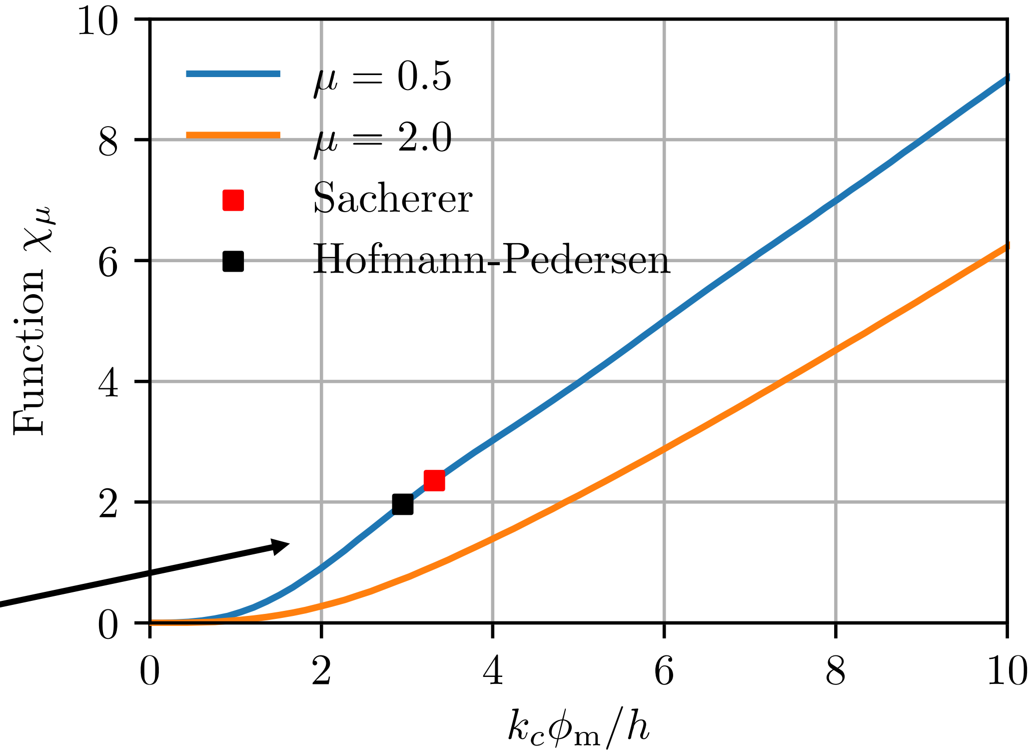
$$N_{\mathrm{LLD}} \approx \frac{\pi}{32qh\omega_{\mathrm{rf}}\mu(\mu+1)} \frac{V_0\phi_{\mathrm{m}}^5}{\chi_\mu(k_c\phi_{\mathrm{m}}/h)\mathrm{Im}Z/k}$$

For $f_c \to \infty$ $N_{\mathrm{LLD}} \approx \frac{\pi}{32q\omega_{\mathrm{rf}}\mu(\mu+1)} \frac{V_0\phi_{\mathrm{m}}^4}{k_c \mathrm{Im}Z/k}$

so that $N_{\mathrm{LLD}} \propto 1/f_c$ and $\phi_{\mathrm{m}}^5 \to \phi_{\mathrm{m}}^4$

→ N_{LLD} based on Sacherer* and Hofmann-Pedersen** formalisms ($\mu = 0.5$) is reproduced for $f_c \approx 1/\tau$ ($k_c\phi_{\mathrm{m}} \approx \pi$)

Function $\chi_\mu(y) = y\left[1 - {}_2F_3\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},2,\mu;-y^2\right)\right]$



*F.J. Sacherer, Methods for computing bunched-beam instabilities, 1972

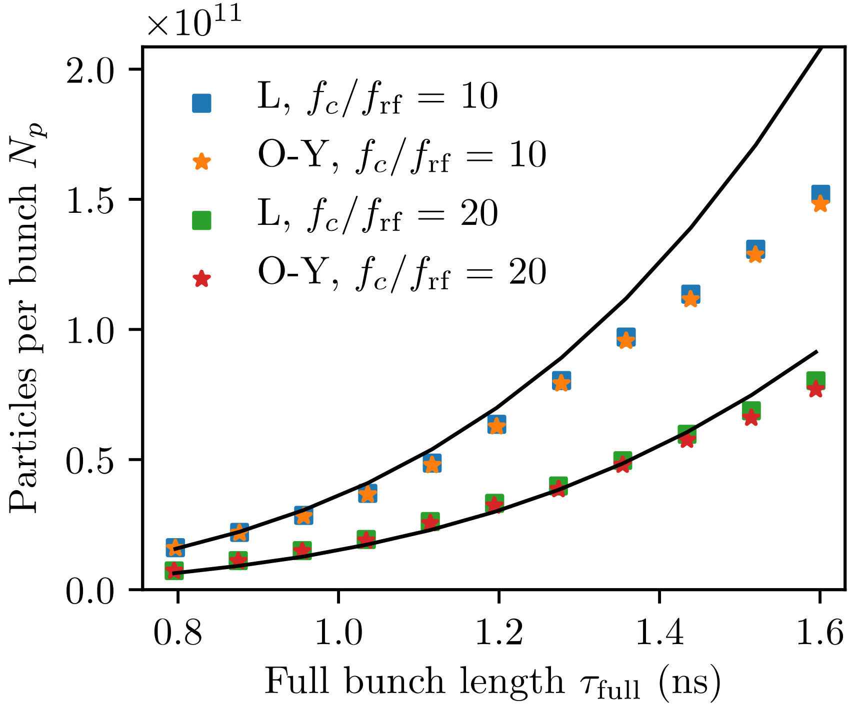
**A. Hofmann and F. Pedersen, Bunches with local elliptic energy distributions, 1979

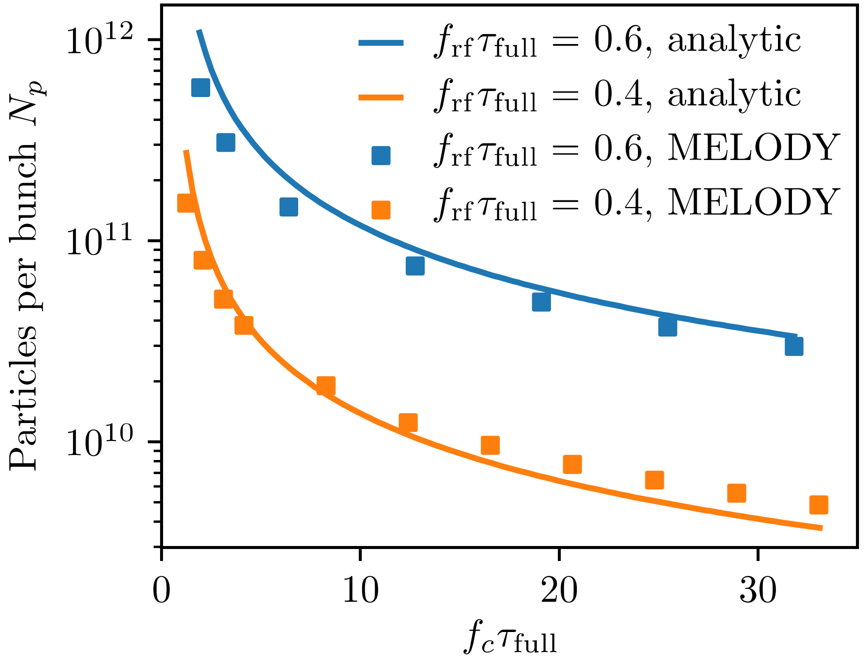
LLD threshold: numerical approaches

(1) The Oide-Yokoya discretization method (O-Y)*: Originally applied for analysis of single-bunch instabilities and later for LLD studies**

(2) Direct solution of the Lebedev equation (L): recently implemented in code MELODY***

Example for LHC: 450 GeV, $\mu = 2$, truncated inductive impedance with $\mathrm{Im}Z/k = 0.07$ Ohm





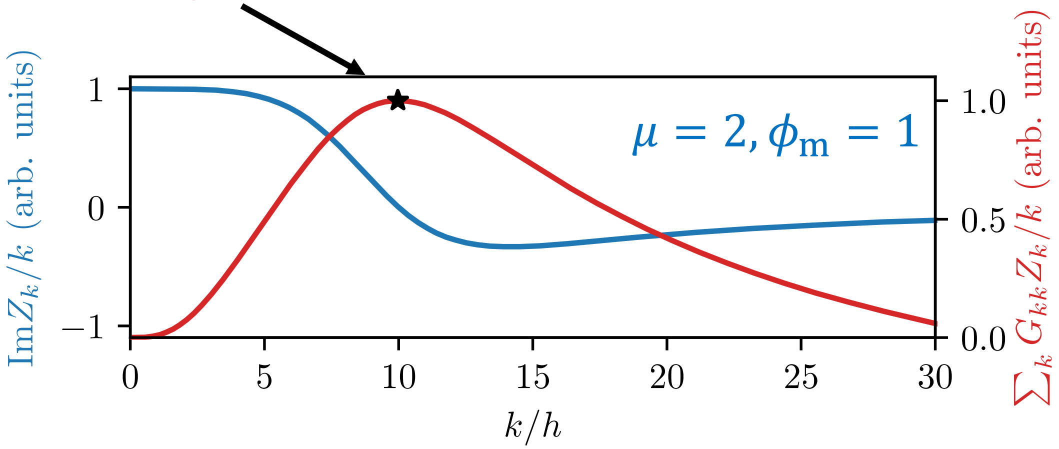
**K. Oide and K. Yokoya, Longitudinal single bunch instability in electron storage rings, 1990*
***A. Burov, Van Kampen modes for bunch longitudinal motion, 2010*
****IK, Matrix Equations for LOngitudinal beam DYnamics*

LLD for effective impedance

Since $N_{\mathrm{LLD}} \approx \frac{V_0}{qh\omega_0}\left[\sum_{k=-\infty}^{\infty} G_{kk}(\Omega)\frac{Z_k(\Omega)}{k}\right]^{-1}$

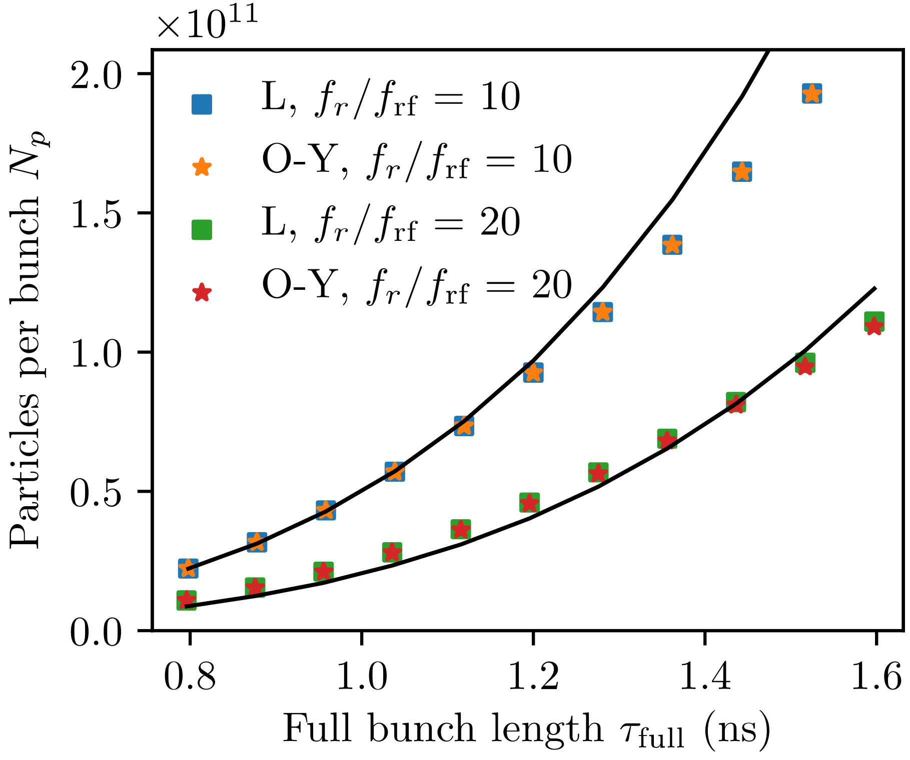
naturally $(\mathrm{Im}Z/k)_{\mathrm{eff}} = \frac{\sum_{k=1}^{k_{\mathrm{eff}}} G_{kk}\mathrm{Im}(Z_k/k)}{\sum_{k=1}^{k_{\mathrm{eff}}} G_{kk}}$

where k_{eff} maximizes the nominator*



Broadband resonator with $Q = 1,\ f_r = 10f_{\mathrm{rf}}$

All work with $k_c \rightarrow k_{\mathrm{eff}}$ & $\mathrm{Im}Z/k \rightarrow (\mathrm{Im}Z/k)_{\mathrm{eff}}$



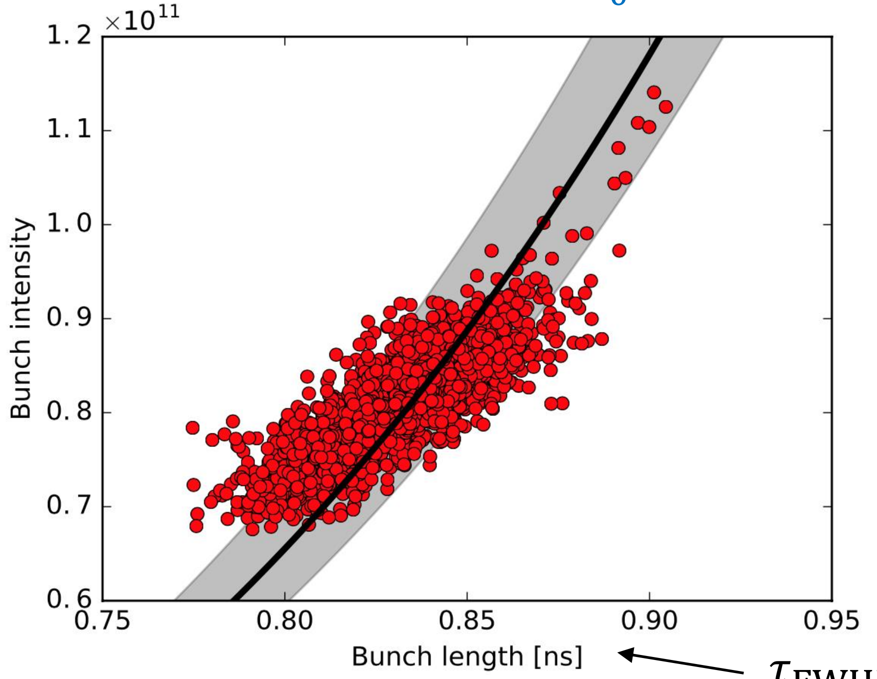
LHC, 450 GeV, $\mu = 2$, broadband impedance with $R = 0.07 f_r/f_0$ Ohm and $Q = 1$

*S. Nese, Effective impedance for the threshold of loss of Landau damping, 2021

Beam measurements of LLD

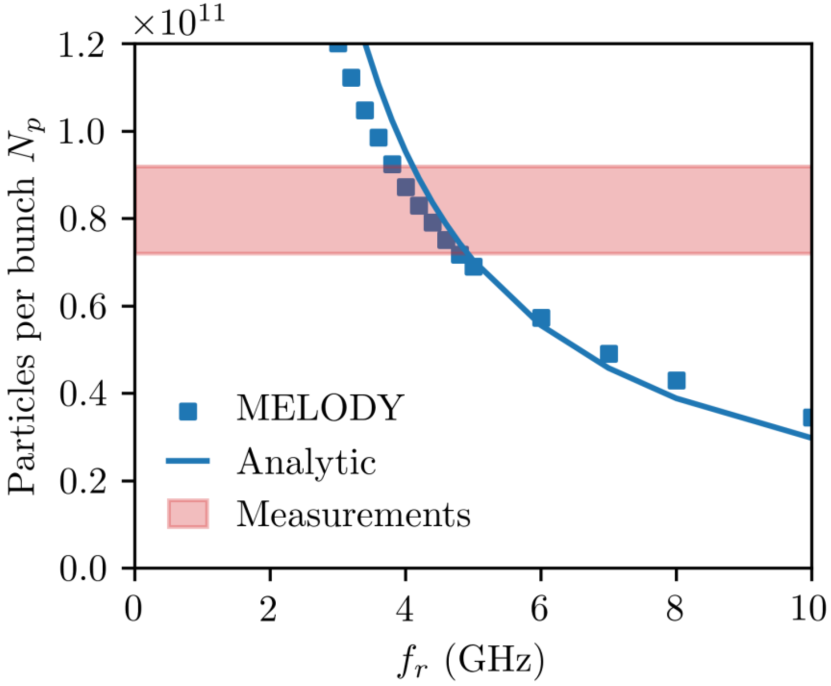
LLD was the first and only intensity effect observed in the LHC in the longitudinal plane*

Measured parameters of bunches with LLD in LHC at 6.5 TeV with $V_0 = 10$ MV*



$\tau_{\mathrm{FWHM}}\sqrt{2/\ln 2}$

LLD threshold for LHC at 6.5 TeV with $V_0 = 10$ MV, $\mu = 2$, $\mathrm{Im}Z/k = 0.076$ Ohm



→ Calculations are consistent with observations for $f_r \approx 5$ GHz (cutoff of LHC beam pipe)

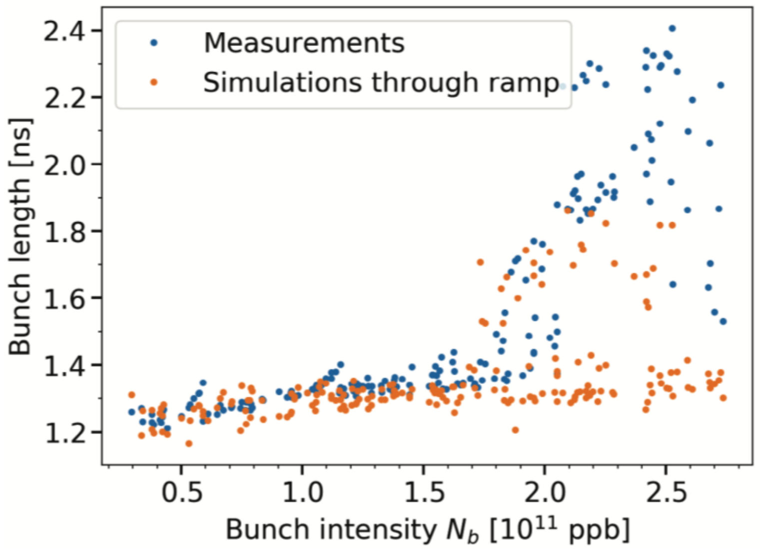
*E. Shaposhnikova et al, Loss of Landau damping in the LHC, 2011
J.F. Esteban Müller, Longitudinal intensity effects in the CERN Large Hadron Collider, PhD, 2016

Single-bunch instabilities

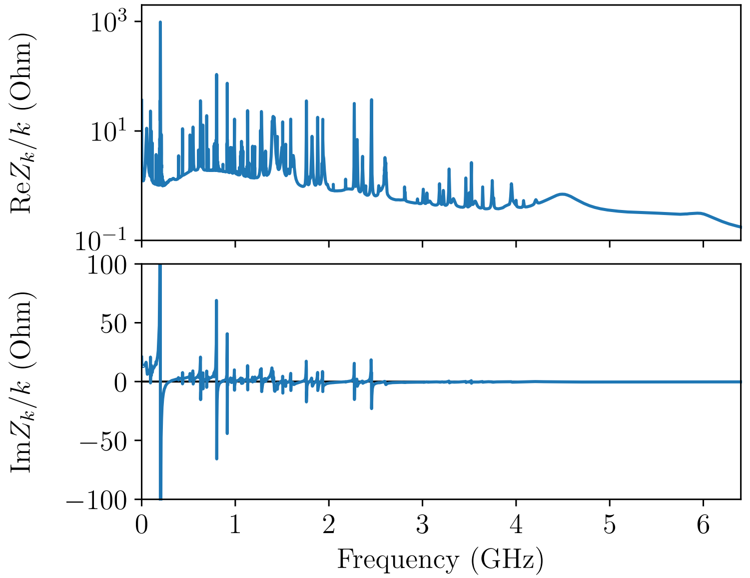
Instability of proton bunch in SPS

Uncontrolled emittance blowup during the acceleration of single bunches was observed

Bunch parameters after acceleration from 26 to 450 GeV*



SPS impedance model (2018)***

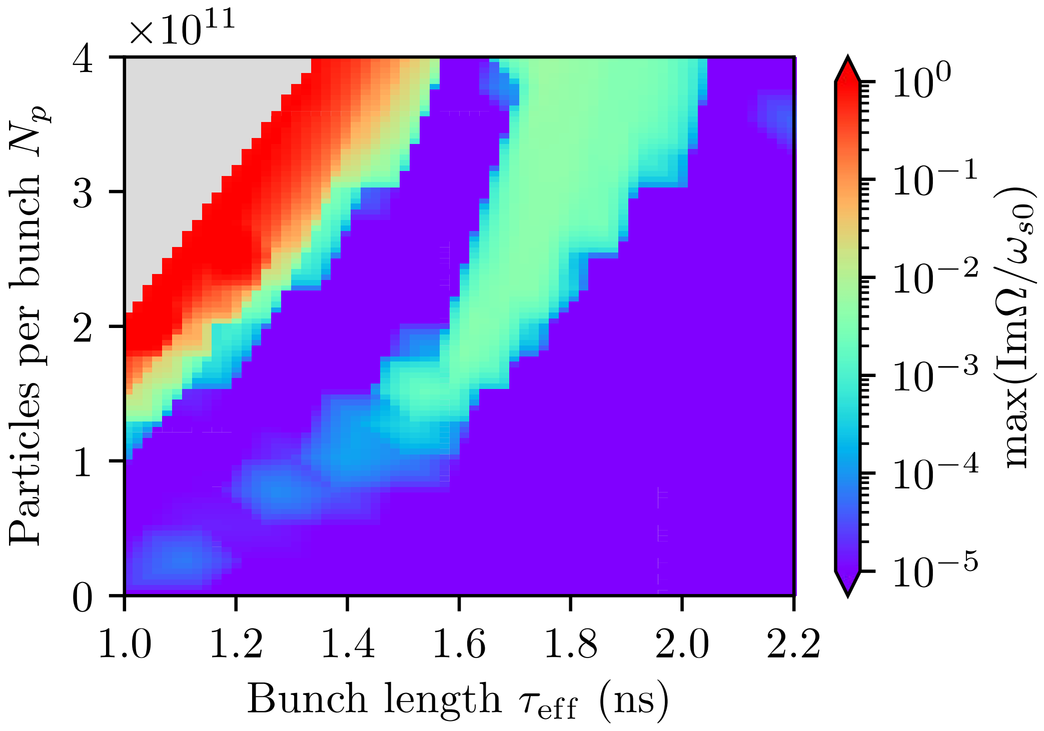


The simulation results (with code BLonD**) for the complicated impedance model were consistent with the measured instability threshold*, however, the instability mechanism was not known

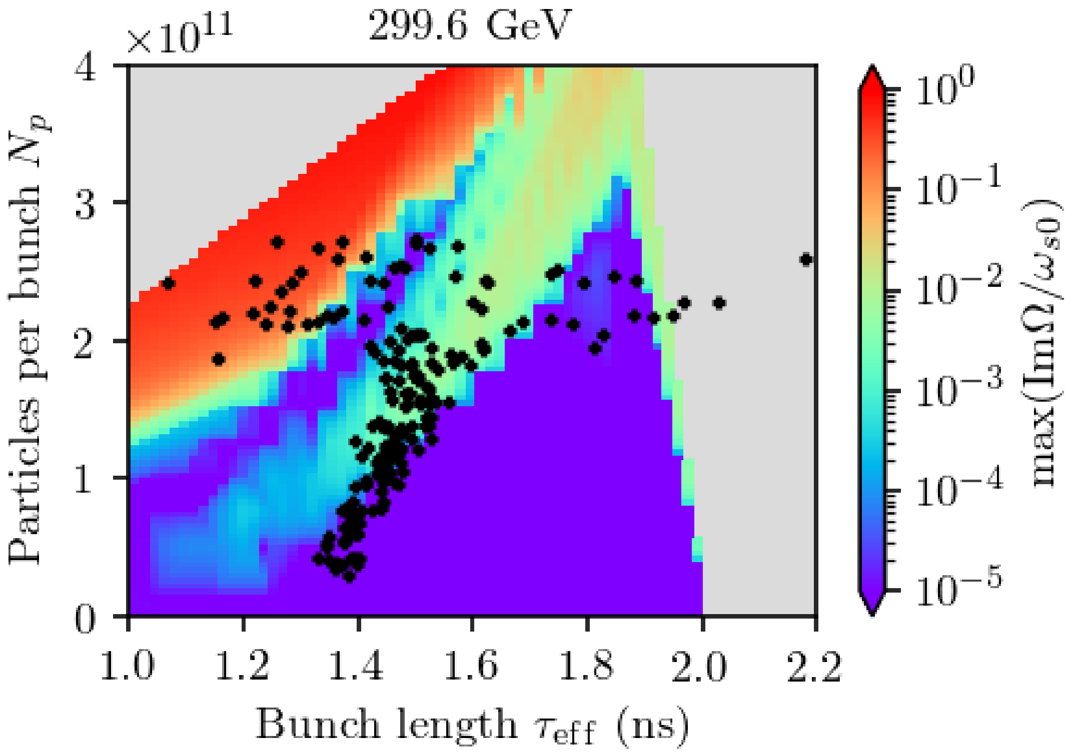
**A. Lasheen, Beam measurements of the longitudinal impedance of the CERN Super Proton Synchrotron, PhD, 2017*
J. Repond, Possible mitigations of longitudinal intensity limitations for HL-LHC beam in the CERN SPS, PhD, 2019
***H. Timko et al, Beam Longitudinal Dynamics Simulation Suite BLonD, 2022*
****CERN SPS Longitudinal Impedance Model, https:// gitlab.cern.ch/longitudinal-impedance/SPS*

Stability maps during acceleration

Calculations at flattop



Calculations during ramp (✚ - measurements)

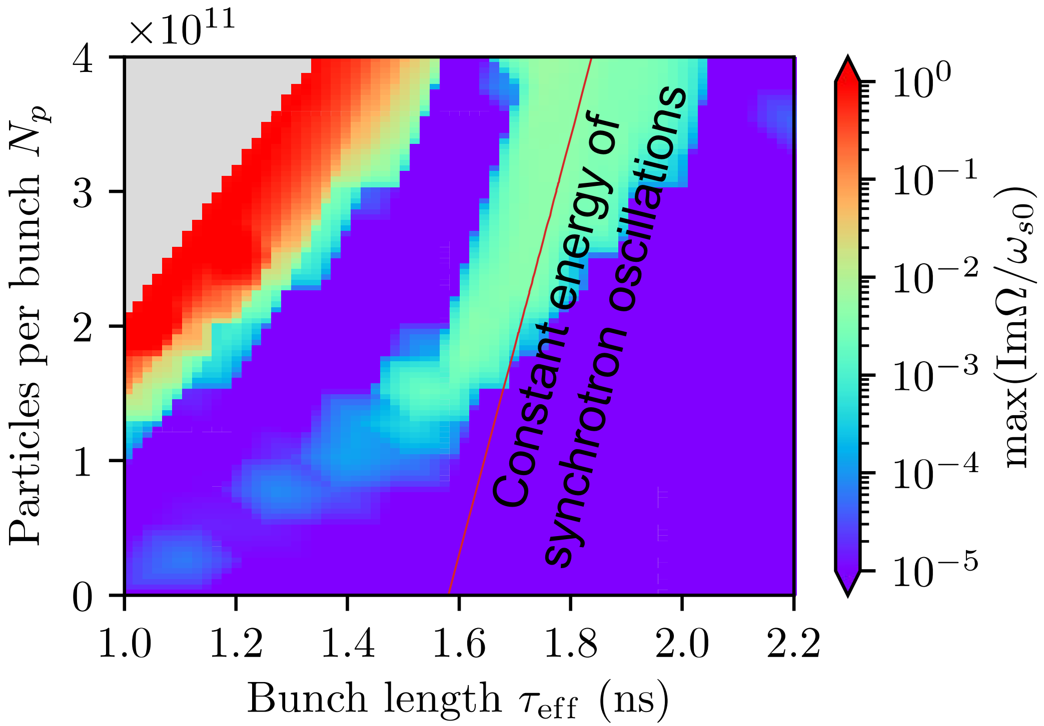


The island found in simulations at 450 GeV* is also present earlier in the acceleration cycle**
→ Measured parameters of unstable bunches (✚) are crossing the island

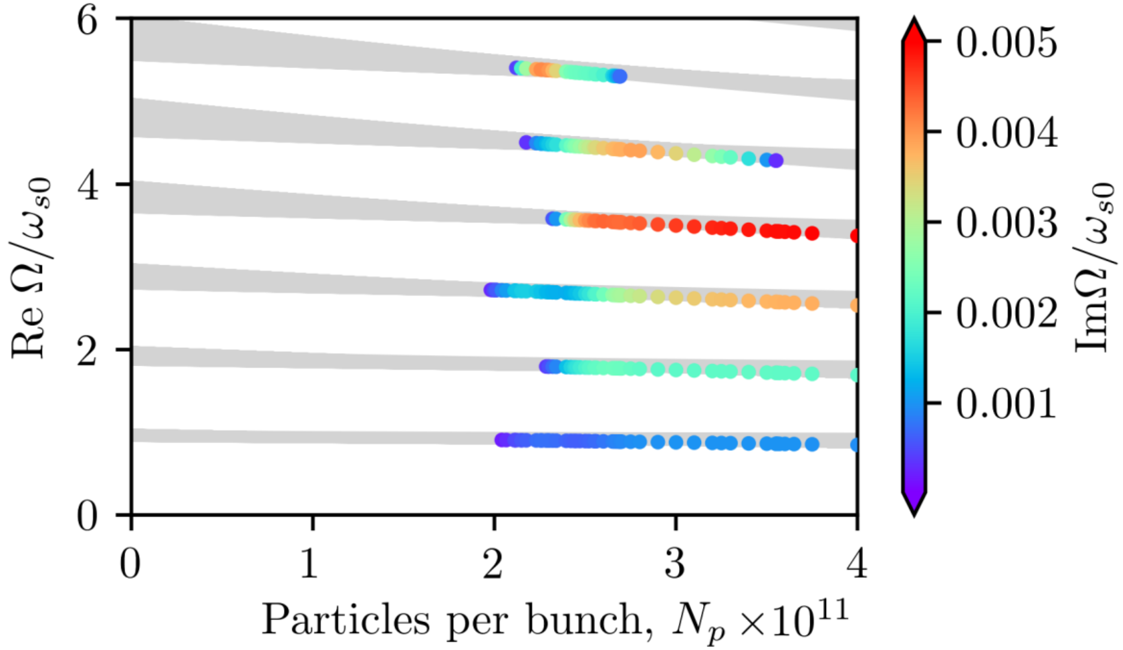
***M.Gadioux, Evaluation of longitudinal single-bunch stability in the SPS and bunch optimization for AWAKE, 2020*

Unstable island

Calculations at flattop



van Kampen modes



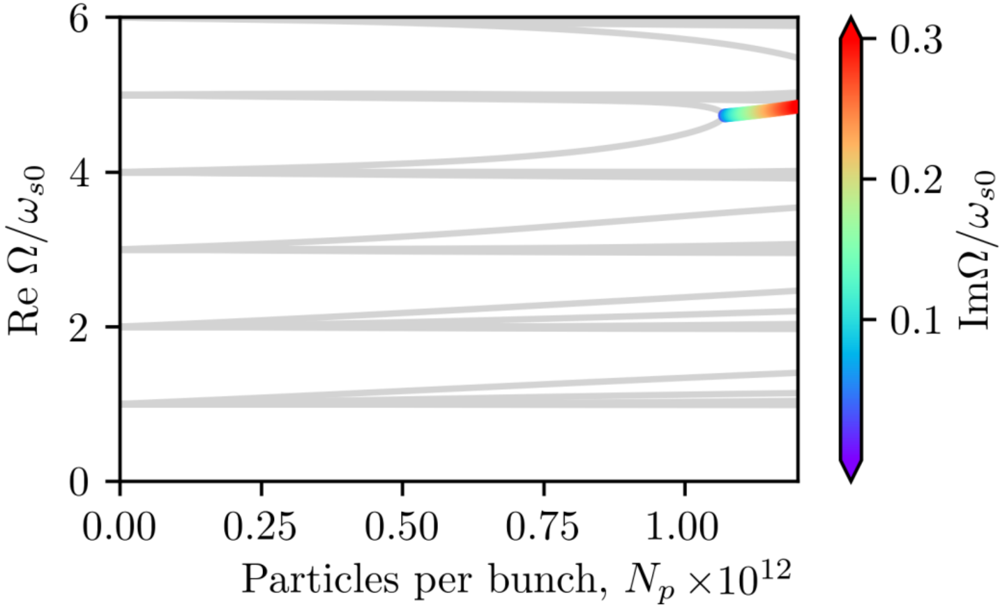
→ Radial mode-coupling instability** since there is no overlap of modes from different azimuthal bands

→ Coupling is present in many azimuthal modes simultaneously (microwave regime)

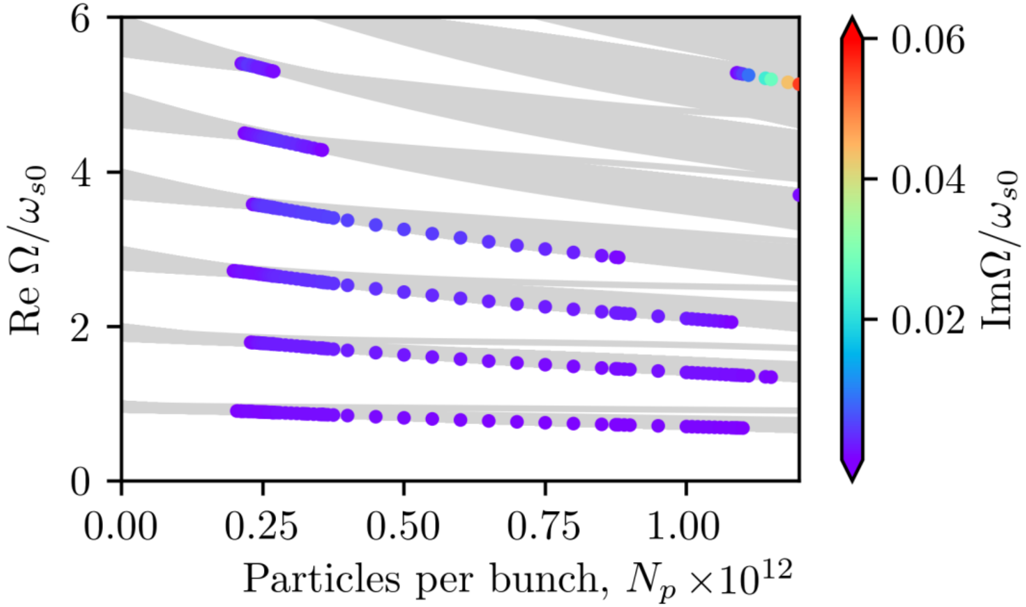
**K. Oide and K. Yokoya, Longitudinal single bunch instability in electron storage rings, 1990*

Role of rf nonlinearity

van Kampen mode for linear rf without potential-well distortion (PWD)



van Kampen modes



If PWD and rf nonlinearity are neglected, the instability threshold is 5 times higher (azimuthal mode-coupling instability*) than for radial mode-coupling instability

In a self-consistent approach, a strong radial mode-coupling instability emerges at this intensity
→ rf nonlinearity can significantly reduce the threshold

**F. J. Sacherer, Bunch lengthening and microwave instability, 1977*

Multi-bunch instabilities

Instability due to narrowband impedance

Coupled-bunch mode l of M equidistant bunches can be driven by impedance with $k_{\mathrm{nb}} = \lfloor f_{r,\mathrm{nb}}/f_0 \rfloor = pM + l$

The threshold can be obtained from the Lebedev equation. If the resonator bandwidth $\Delta\omega \ll M\omega_0$ and k_{nb} is far from $M/2$ harmonics*

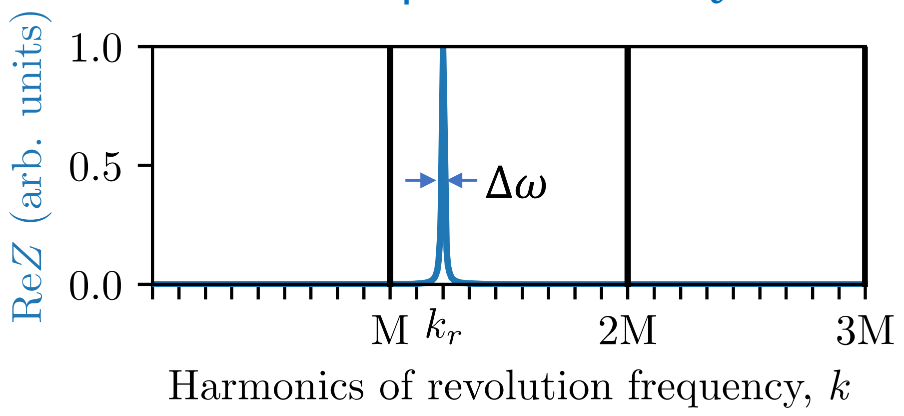
The coupled-bunch instability (CBI) threshold for the binomial distribution is the lowest for $m = 1$**

$$N_{\mathrm{CBI}} \approx \frac{V_0 \phi_{\max}^4 k_{\mathrm{nb}}}{16 q h \omega_0 M R_{\mathrm{nb}}} \min_{y\in[0,1]} \left[\frac{(1-y^2)^{1-\mu}}{\mu(\mu+1)} J_1^{-2}\left(\frac{y k_{\mathrm{nb}} \phi_{\max}}{h} \right) \right]$$

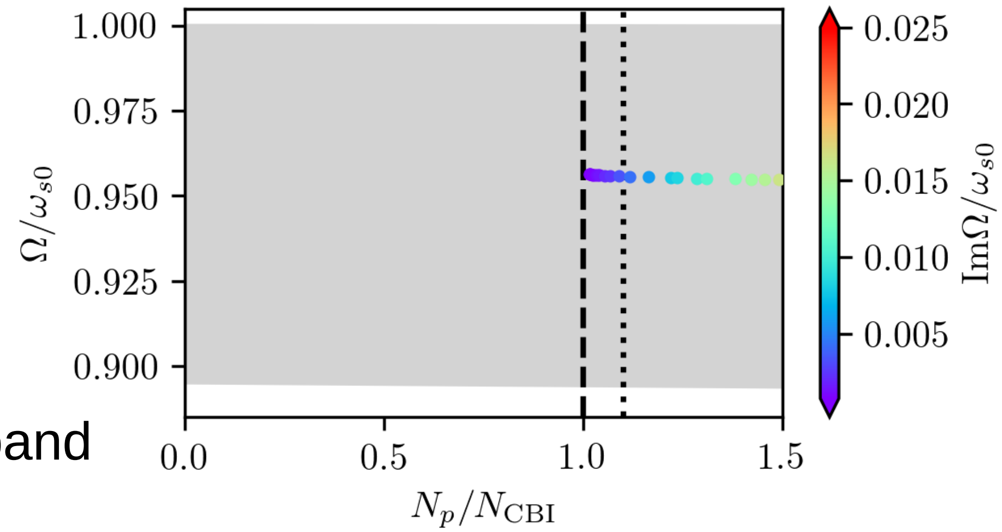
Bessel function

→ Unstable mode Ω_{CBI} is inside the incoherent frequency band

Resonator impedance with $Q = 100$



Example of unstable dipole mode



*V. I. Balbekov and S. V. Ivanov, Longitudinal beam instability threshold beam in proton synchrotrons, 1986

**IK and E. Shaposhnikova, "Longitudinal coupled-bunch instability evaluation for FCC-hh, 2019

Generalized threshold

Typically, broadband (bb) and narrowband (nb) impedance sources are treated separately, except in a few examples of CBI growth rate calculations*

Including them in the Lebedev equation simultaneously

$$N_g(\Omega_g) \approx \frac{V_0}{q\omega_{\mathrm{rf}}}\left[\sum_{k=-\infty}^{\infty} G_{kk}(\Omega_g)\frac{Z_k^{\mathrm{bb}}(\Omega_g)+Z_k^{\mathrm{nb}}(\Omega_g)}{k}\right]^{-1}$$

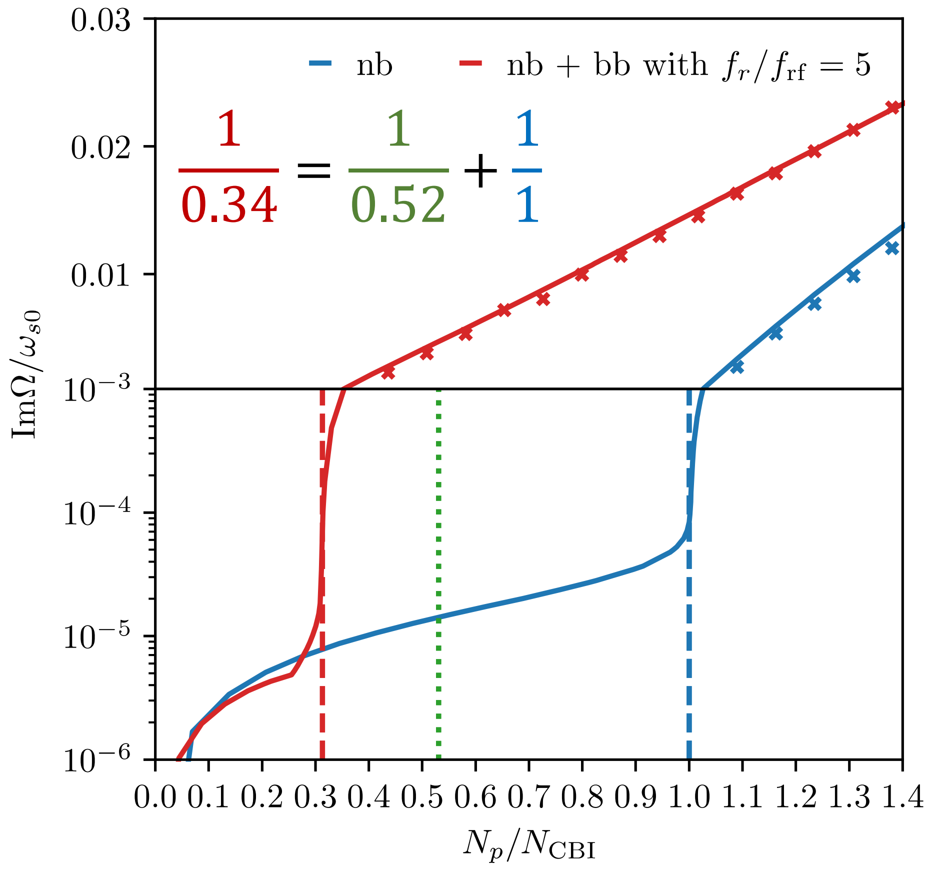
$$\Omega_g \neq \Omega_{\mathrm{LLD}} \text{ and } \Omega_g \neq \Omega_{\mathrm{CBI}}$$

→ Approximate threshold (first estimate)

$$\frac{1}{N_g} \approx \frac{1}{N_{\mathrm{LLD}}} + \frac{1}{N_{\mathrm{CBI}}}$$

→ Instability develops below the LLD threshold

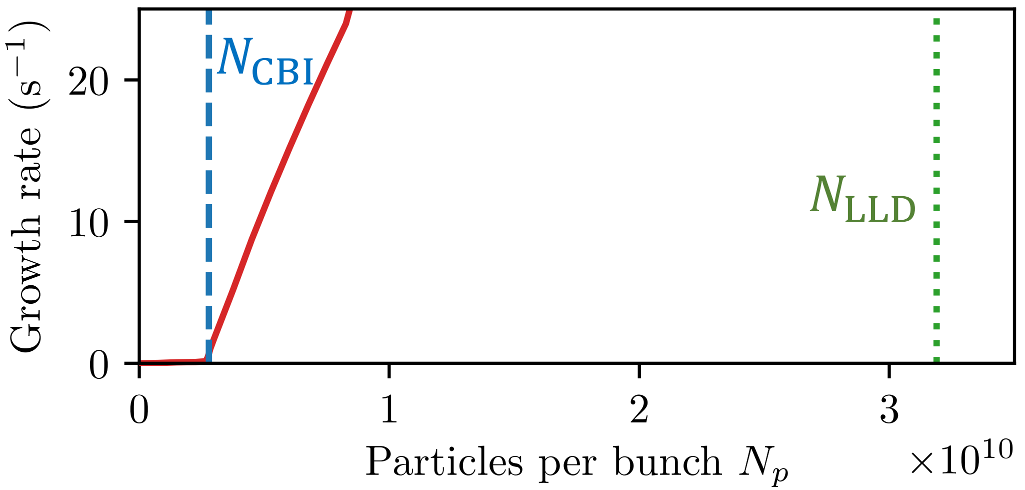
Growth rates of most unstable modes for 9 bunches (MELODY - lines, BLonD - crosses)



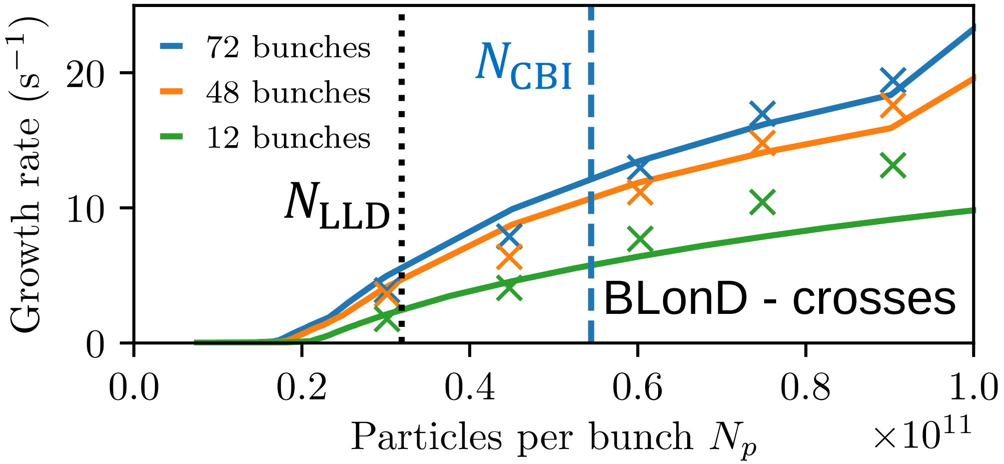
*M. Blaskiewicz, Longitudinal stability calculations, 2009, and recently in A. Burov, Longitudinal modes of bunched beams with weak space charge, 2021

Multi-bunch instabilities in the SPS

Growth rates of most unstable modes for full ring (5 ns bunch spacing)



Growth rates of most unstable modes for LHC-type trains (25 ns bunch spacing)



Instability of fixed-target beams (5 ns spacing) is driven by Higher Order Mode (HOM) of 200 MHz rf system at 914 MHz*

→ LLD has no impact since N_{CBI} is very low

→ Instability of bunch trains is enhanced by LLD (weak dependence on number of bunches)
→ Stability is improved with an additional 800 MHz rf system and controlled emittance blowup (LLD threshold is increased)**

*E. Shaposhnikova, Analysis of coupled bunch instability spectra, 1999
**LHC Injectors Upgrade, Technical Design Report, Vol. I: Protons, 2014

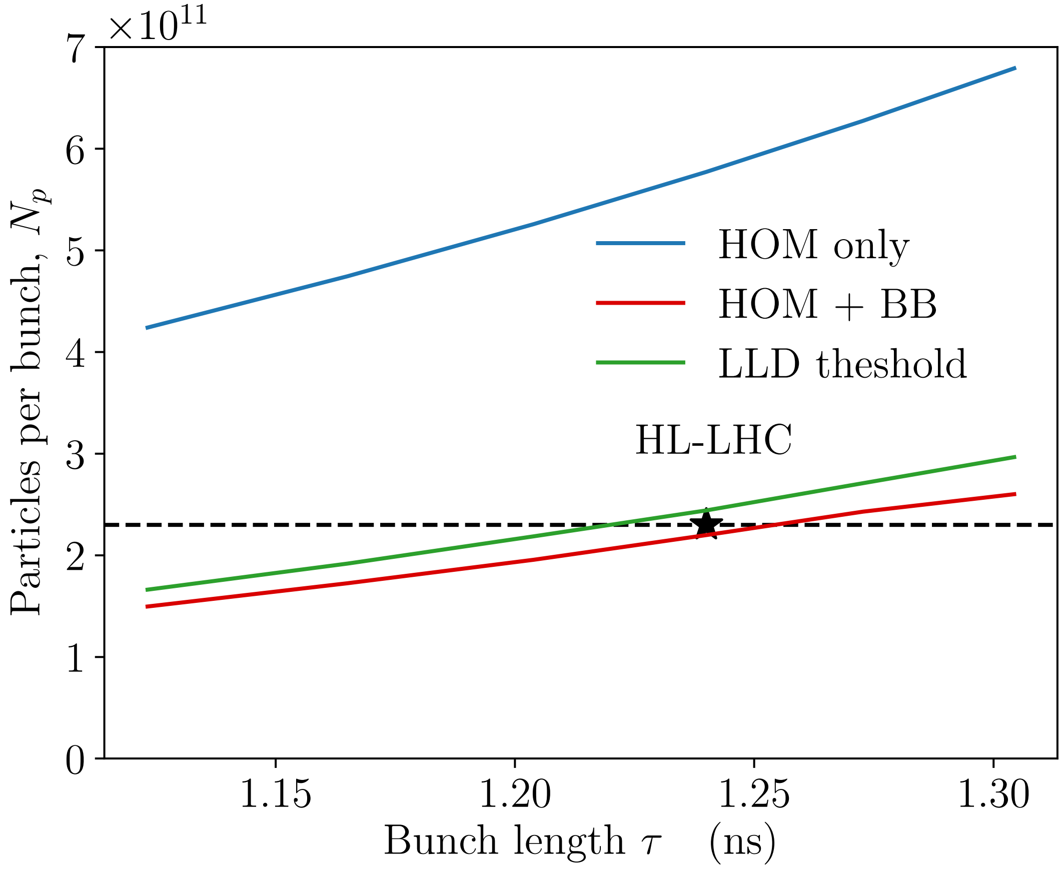
Expectations for HL-LHC

Coupled-bunch instabilities (CBI) driven by higher-order modes (HOM) have not been observed in the LHC so far

Bunch intensity for HL-LHC is doubled compared to LHC, and crab cavities with strongly damped HOMs will be installed

→ In the presence of BB impedance, the instability threshold is reduced below the LLD threshold
→ Precise BB impedance model (f_c) is necessary to predict stability margins

Instability thresholds at $E = 450$ GeV for V_0 = 8 MV:
nb - $R_{\mathrm{nb}} = 4 \times 71$ kOhm, f_r = 582 MHz
bb - $(\mathrm{Im}Z/k)_{\mathrm{eff}} \approx 0.075$ Ohm, f_r = 5 GHz



Summary

Threshold for loss of Landau Damping (LLD) for binomial distribution:

- is inversely proportional to cutoff frequency (vanishes for ImZ/k = const)
- has weaker dependence on the bunch length (4th instead of 5th power)
- can be evaluated for arbitrary impedance using effective-impedance parameters

Single bunch instability threshold:

- is mainly determined by the radial mode-coupling mechanism
- can be reduced by rf nonlinearity

Multi-bunch instability threshold:

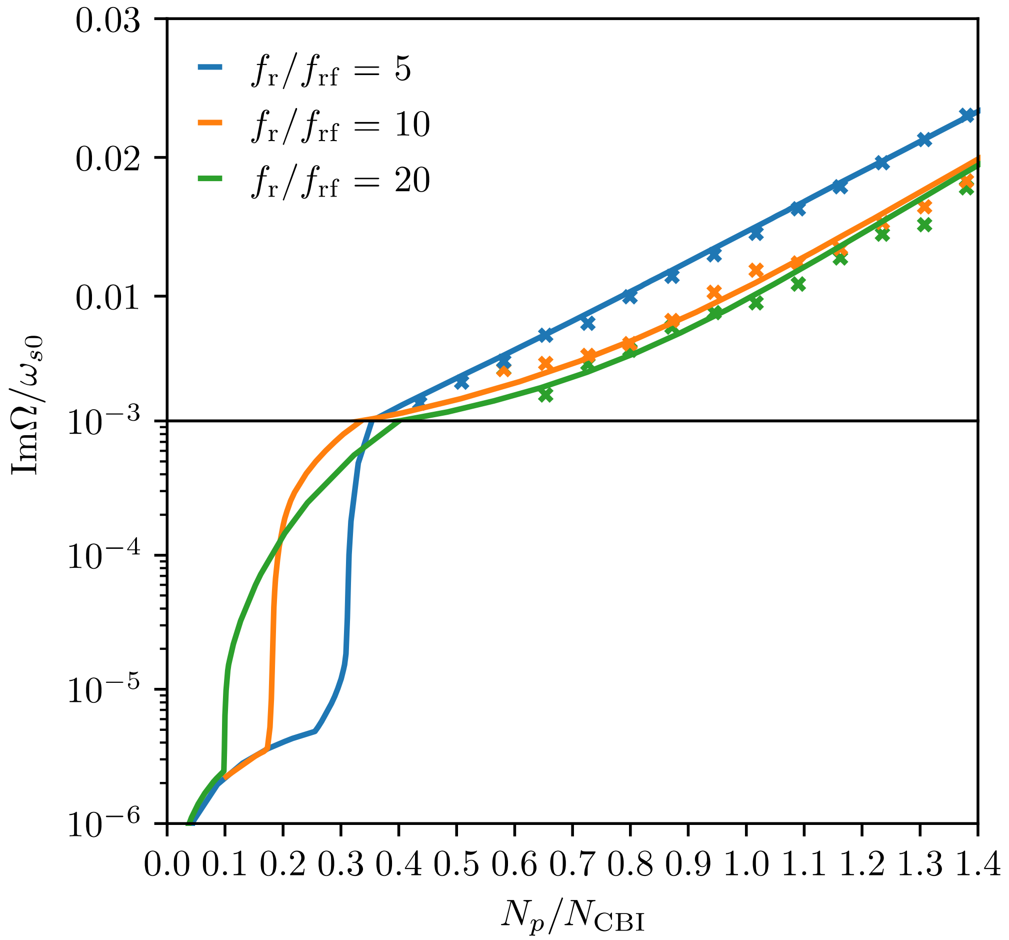
- is defined by both broadband and narrowband impedance contributions
- can be below the LLD threshold

These findings are supported by numerical calculations and beam measurements

Thank you for your attention!

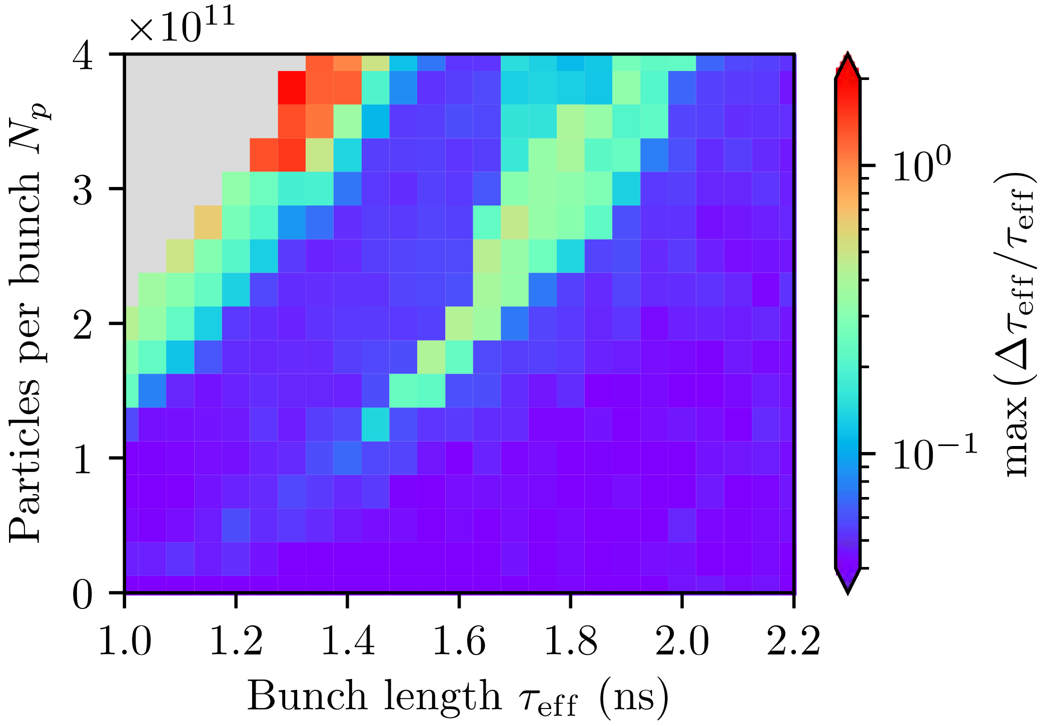
Spare slides

Growth rate vs cutoff

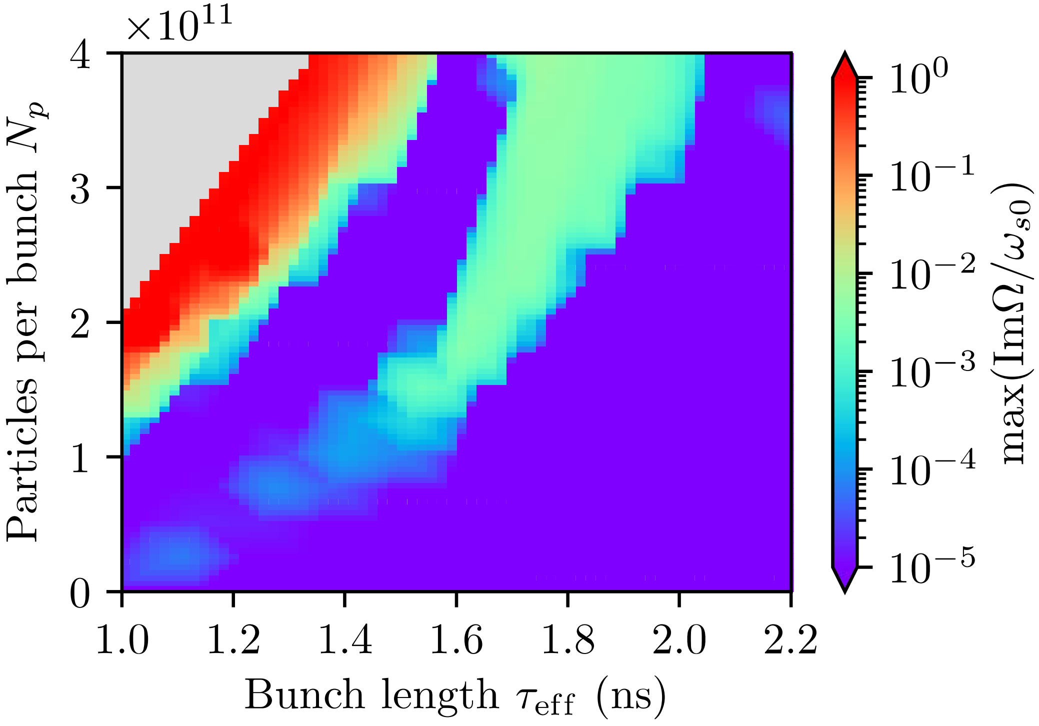


Beam stability at SPS flattop

Stability map based on simulations with code BLonD*



Stability map based on calculations with code MELODY



An unstable island was observed in simulations at 450 GeV** and reproduced with MELODY

**H. Timko et al, Beam Longitudinal Dynamics Simulation Suite BLonD, 2022*
***E. Radvilas, Simulations of single-bunch instability on flat top, 2015*

Generalized threshold

Typically, broadband (bb) and narrowband (nb) impedance sources are treated separately, except in a few analyses of CBI growth rates*

Including both bb and nb sources in the Lebedev equation simultaneously

$$N_g \approx \frac{V_0}{qh\omega_0}\left[\sum_{k=-\infty}^{\infty} G_{kk}(\Omega_{\mathrm{LLD}})\frac{Z_k^{\mathrm{bb}}(\Omega_{\mathrm{LLD}})}{k} + G_{k_{\mathrm{nb}}k_{\mathrm{nb}}}(\Omega_{\mathrm{CBI}})\frac{Z_k^{\mathrm{nb}}(\Omega_{\mathrm{CBI}})}{k_{\mathrm{nb}}}\right]^{-1}$$

$\Omega_g \neq \Omega_{\mathrm{LLD}}$ and $\Omega_g \neq \Omega_{\mathrm{CBI}}$

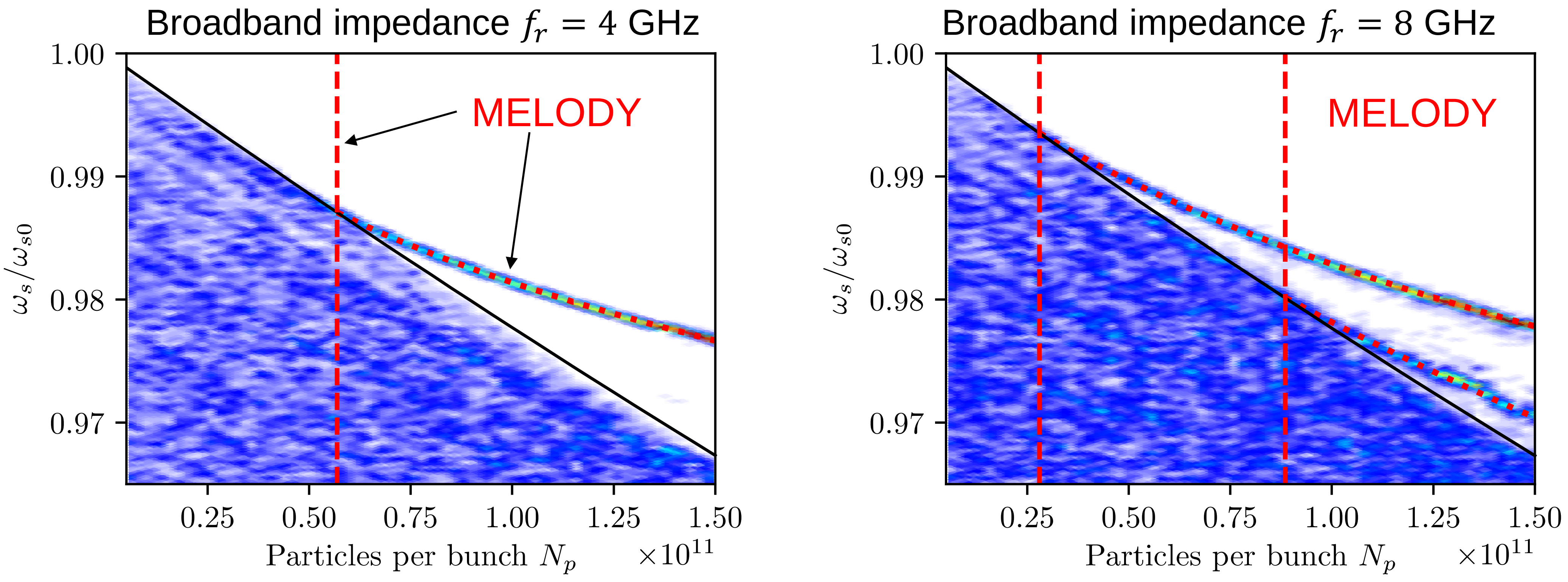
$\neq 0$ only for $k = k_{\mathrm{nb}}$

→ Proposed approximate threshold $$\frac{1}{N_g} \approx \frac{1}{N_{\mathrm{LLD}}} + \frac{1}{N_{\mathrm{CBI}}}$$

**M. Blaskiewicz, Longitudinal stability calculations, 2009, and recently in*
A. Burov, Longitudinal modes of bunched beams with weak space charge, 2021

LLD in macroparticle simulations

The matched bunch is tracked using code BLonD* for ~5000 synchrotron periods
FFT of mean position is computed for various bunch intensities



→ Numerical predictions are supported by macroparticle simulations

**H. Timko et al, Beam Longitudinal Dynamics Simulation Suite BLonD, 2022*

Impact on beam

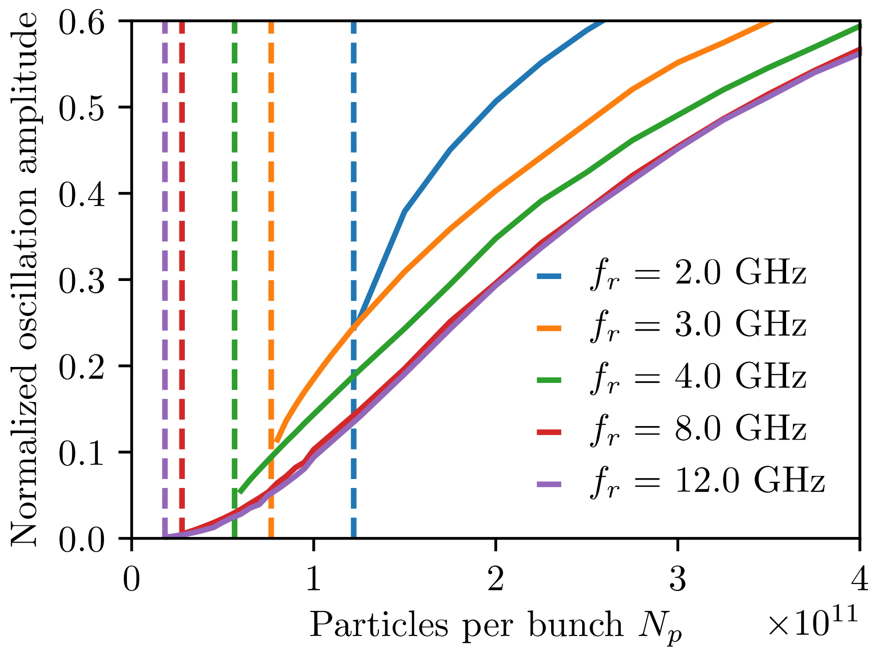
Rigid bunch perturbation is common for accelerators (phase error or noise)

→ For $N_p > N_{\mathrm{LLD}}$, the residual oscillation amplitude, A_{res}, depends on intensity

→ For $N_p = N_{\mathrm{LLD}}$, A_{res} is smaller for higher cutoff frequency

→ Obtaining N_{LLD} and A_{res} in measurements, $(\mathrm{Im}Z/k)_{\mathrm{eff}}$ and k_{eff} can be probed (recently applied in PS* and SPS**)

Residual amplitude evaluated with MELODY: LHC, 450 GeV, $\mu = 2$, broadband impedance with $R = 0.07 f_r / f_0$ Ohm, and $Q = 1$



*L.Intelisano, H.Damerau, and IK, Measurements of longitudinal loss of Landau damping in the CERN Proton Synchrotron, 2023

** L.Intelisano, H.Damerau, and IK, Longitudinal loss of Landau damping in the CERN Super Proton Synchrotron at 200 GeV, 2023

Comparisons with LHC measurements

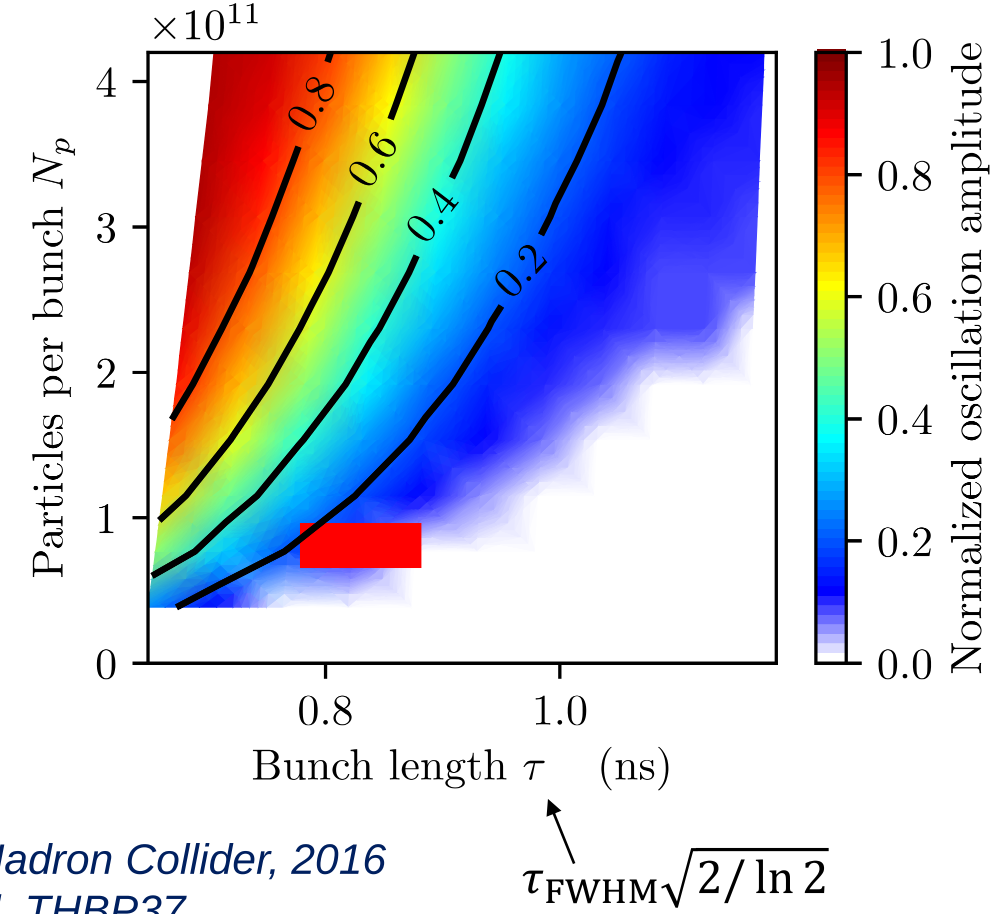
Different measurements have been performed since 2010*

The threshold was determined as an onset of slowly growing oscillations

→ Calculations for $f_r = 5$ GHz are consistent with observations

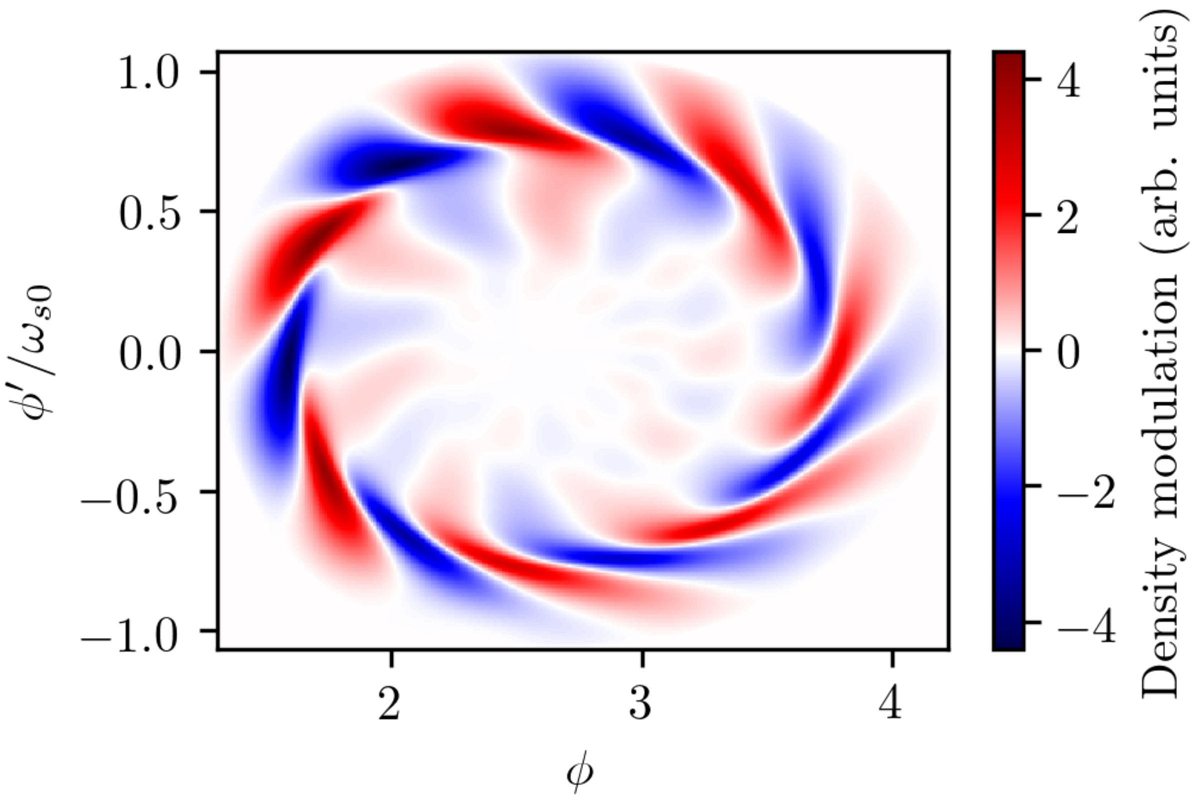
→ Revision of the LHC impedance model at high frequencies is ongoing***

Residual oscillation amplitude computed with MELODY LHC at 6.5 TeV with $V_0 = 10$ MV, $\mu = 2$



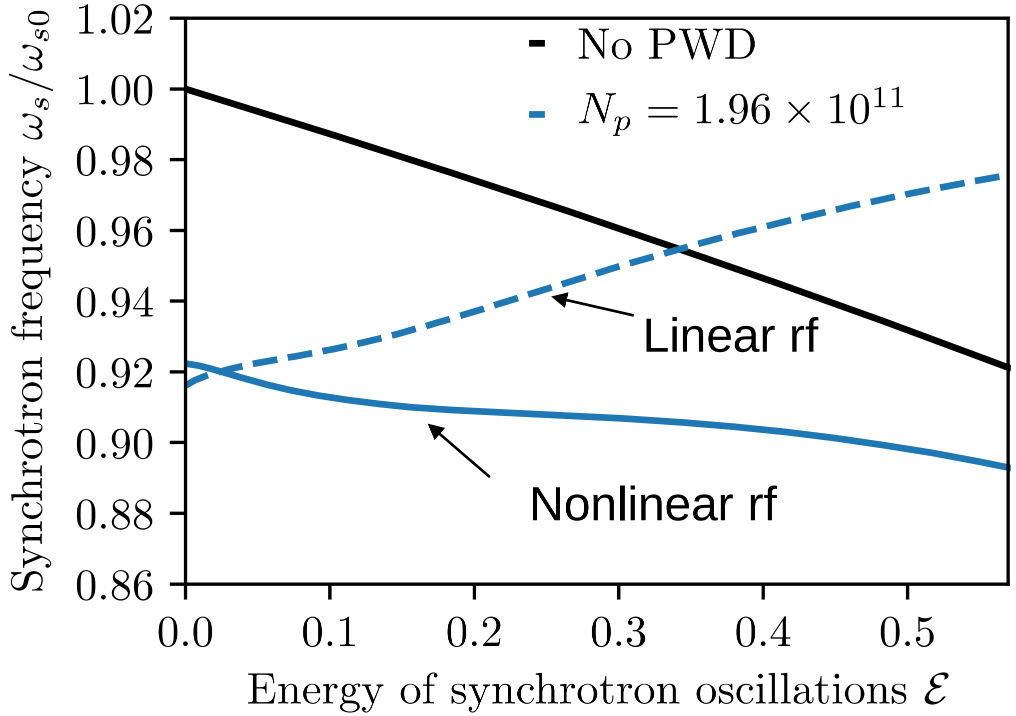
*E. Shaposhnikova et al, Loss of Landau damping in the LHC, 2011
**J.F. Esteban Müller, Longitudinal intensity effects in the CERN Large Hadron Collider, 2016
***M. Zampetakis et al, Refining the LHC Longitudinal Impedance Model, THBP37

Strong radial mode-coupling



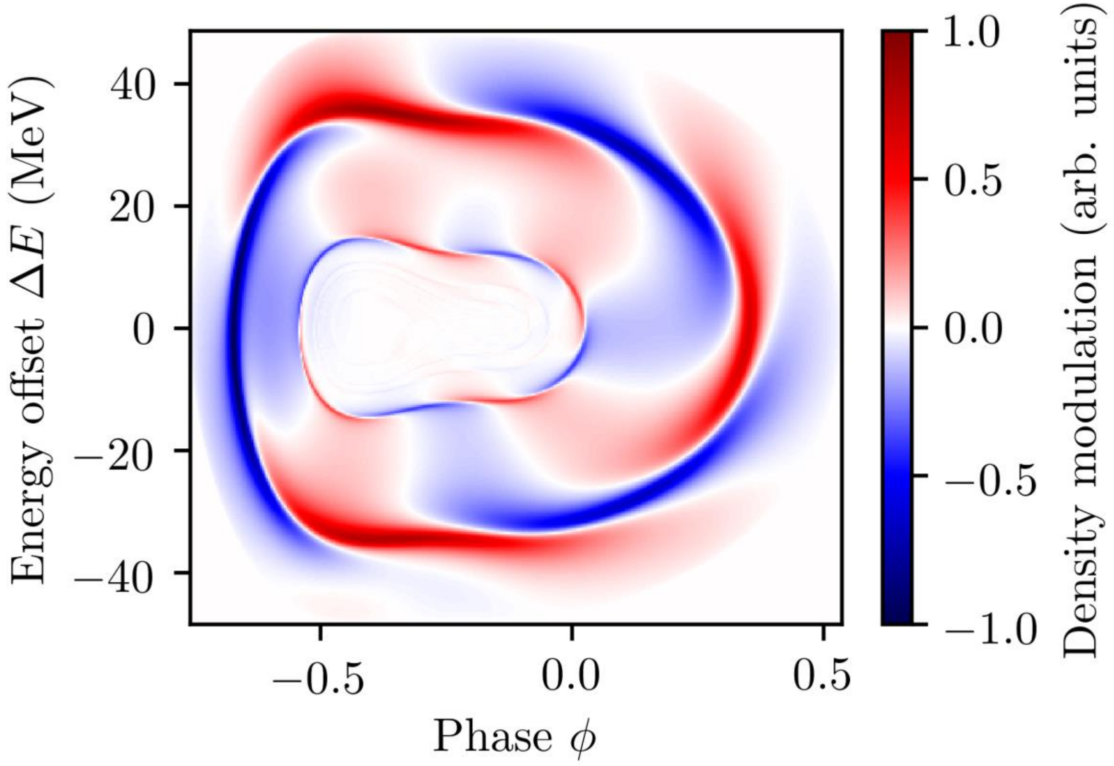
Nonmonotonicity

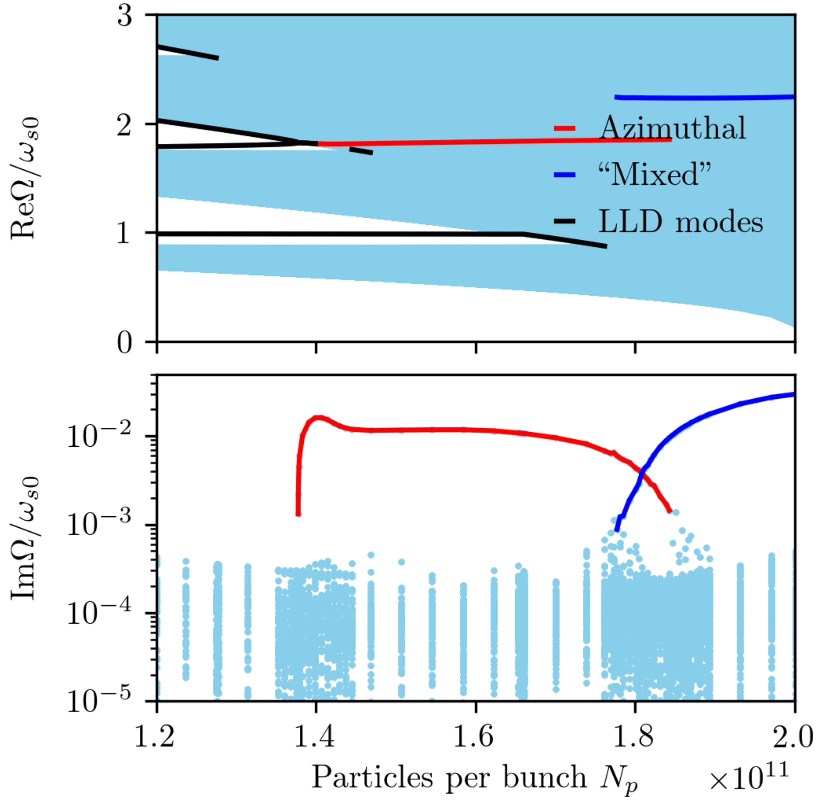
Synchrotron frequency distribution at instability threshold



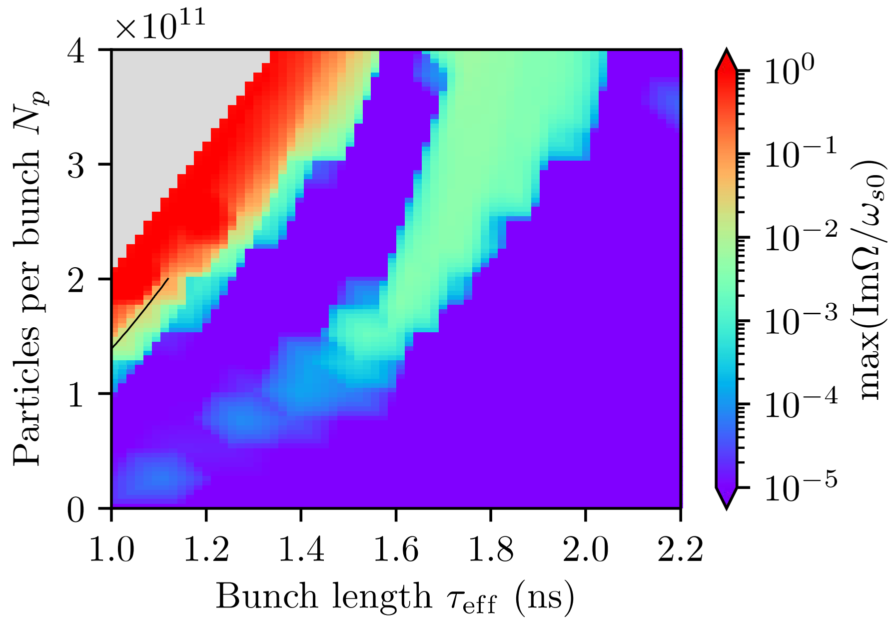
Mixed mode-coupling instability

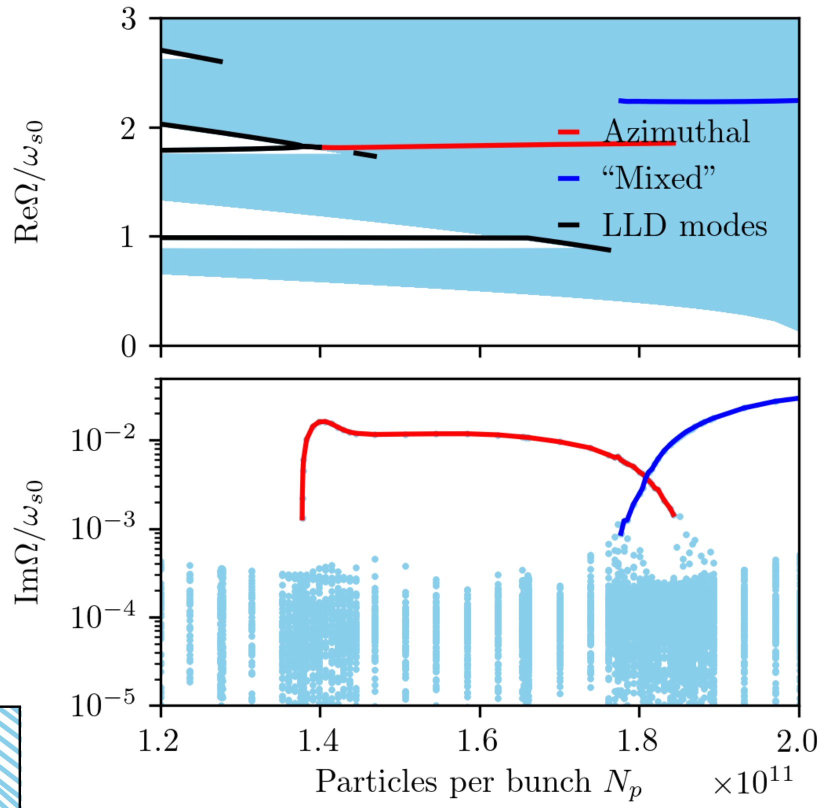
Once synchrotron frequency bands fully overlap, ‘mixed’ mode coupling instability can emerge



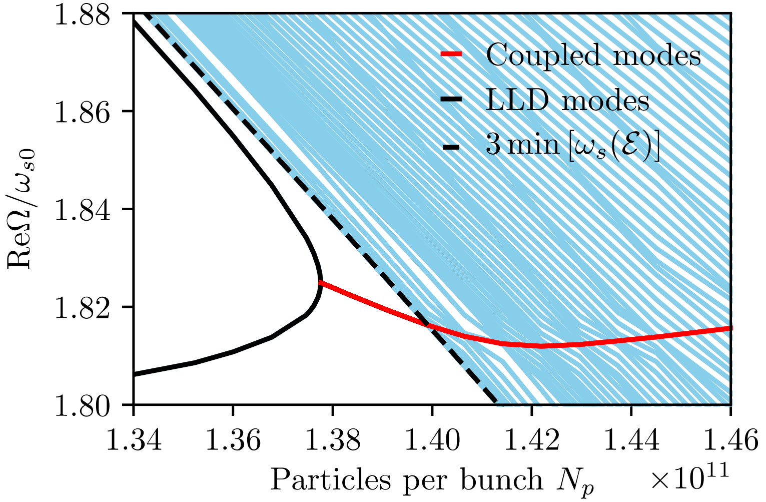


Azimuthal mode-coupling



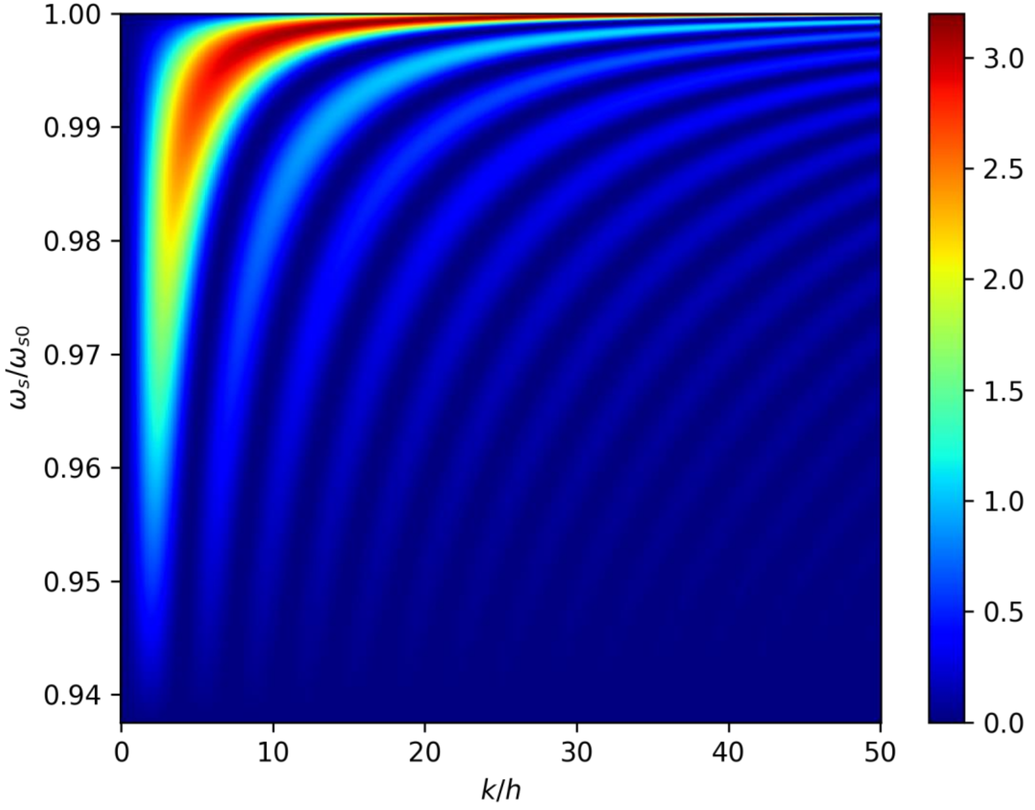


Azimuthal mode-coupling is possible in SPS for very short bunches (~1 ns). It is a coupling of LLD modes and can be suppressed by an increase in bunch intensity or a change in distribution



van Kampen mode spectra

Binomial mu = 2

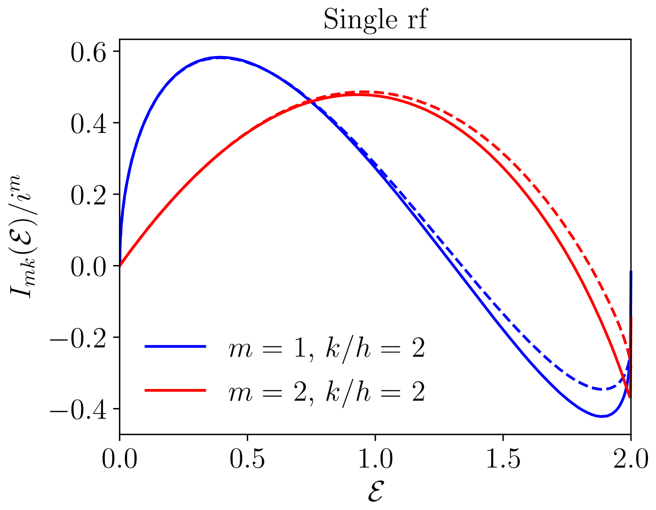


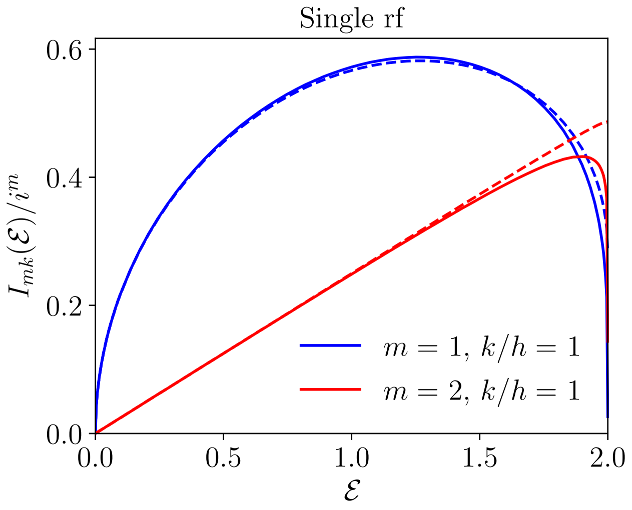
$$C_m(\mathcal{E},\Omega) = \sqrt{-\omega_s(\mathcal{E})\frac{d\mathcal{F}(\mathcal{E})}{d\mathcal{E}}}\, m\,\frac{\omega_s(\mathcal{E})}{\omega_{s0}^2}\tilde{C}_m(\mathcal{E},\Omega),$$

$$\tilde{C}_m(\mathcal{E},\Omega) = -\left\{\mathrm{P}\frac{1}{\Omega^2 - m^2\omega_s^2(\mathcal{E})} + \alpha(\mathcal{E},\Omega)\delta[\Omega^2 - m^2\omega_s^2(\mathcal{E})]\right\}$$

$$\times 2i\zeta\omega_{s0}^2 \sum_{m'=1}^{\infty}\sum_{n=0}^{\infty}\sum_{n'=0}^{\infty} K_{mm'}^{nn'}(\Omega)\, a_{m'}^{n'}(\Omega)\, s_n^{(m)}(\mathcal{E}),$$

$$\tilde{\lambda}_k(\Omega) = \frac{\omega_{s0}^2}{h}\sum_{m=1}^{\infty}\int_0^{\mathcal{E}_{\max}} \frac{C_m(\mathcal{E},\Omega) I_{mk}^*(\mathcal{E})}{\omega_s(\mathcal{E})}\, d\mathcal{E}.$$





$$G_{k'k} = -i\omega_{s0}^2 \sum_{m=-\infty}^{\infty} m \int_0^{\mathcal{E}_{\max}} \frac{d\mathcal{F}(\mathcal{E})}{d\mathcal{E}} \frac{I_{mk}^*(\mathcal{E}) I_{mk'}(\mathcal{E})}{\Omega - m\omega_s(\mathcal{E})} d\mathcal{E},$$

$$I_{mk}(\mathcal{E}) = \frac{1}{\pi} \int_0^{\pi} e^{i\frac{k}{h}\phi(\mathcal{E},\psi)} \cos m\psi \, d\psi.$$