OAK RIDGE National Laboratory

Effect of Three-Dimensional Quadrupole Magnet Model On Beam Dynamics

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HB Workshop

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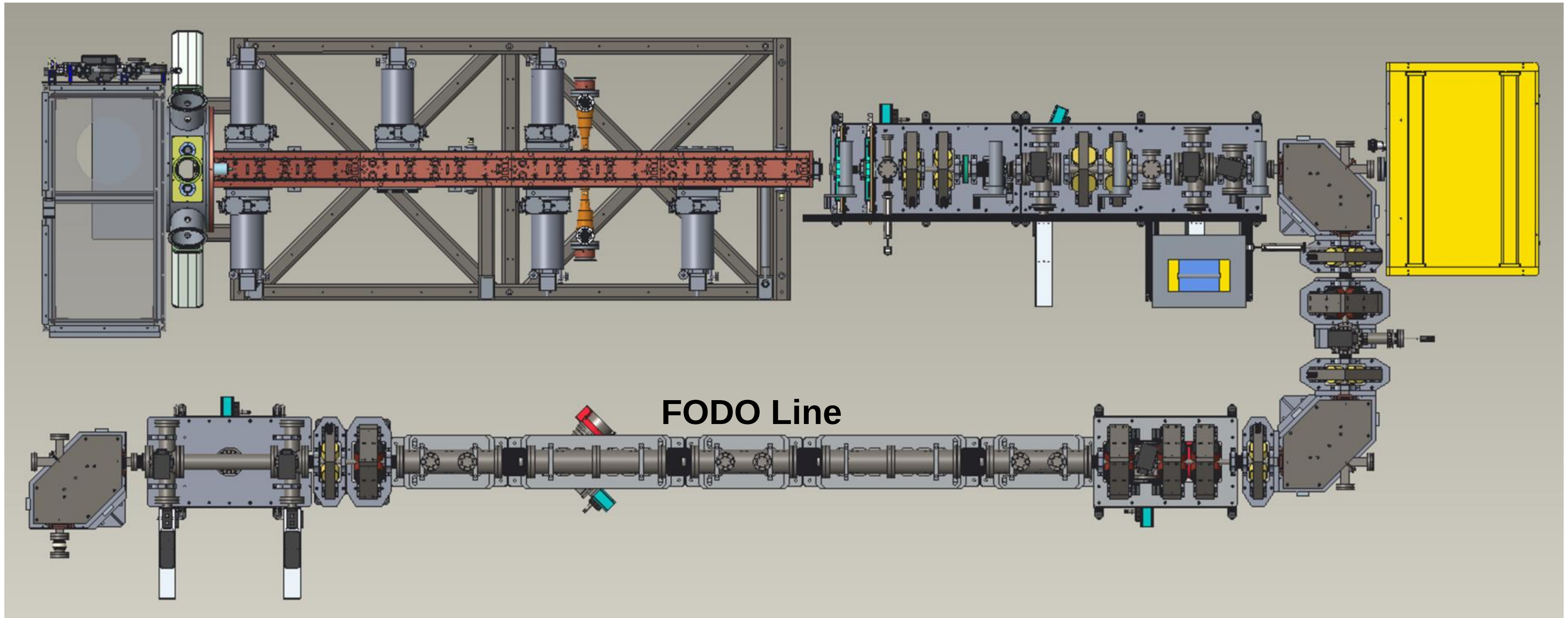
U.S. DEPARTMENT OF ENERGY

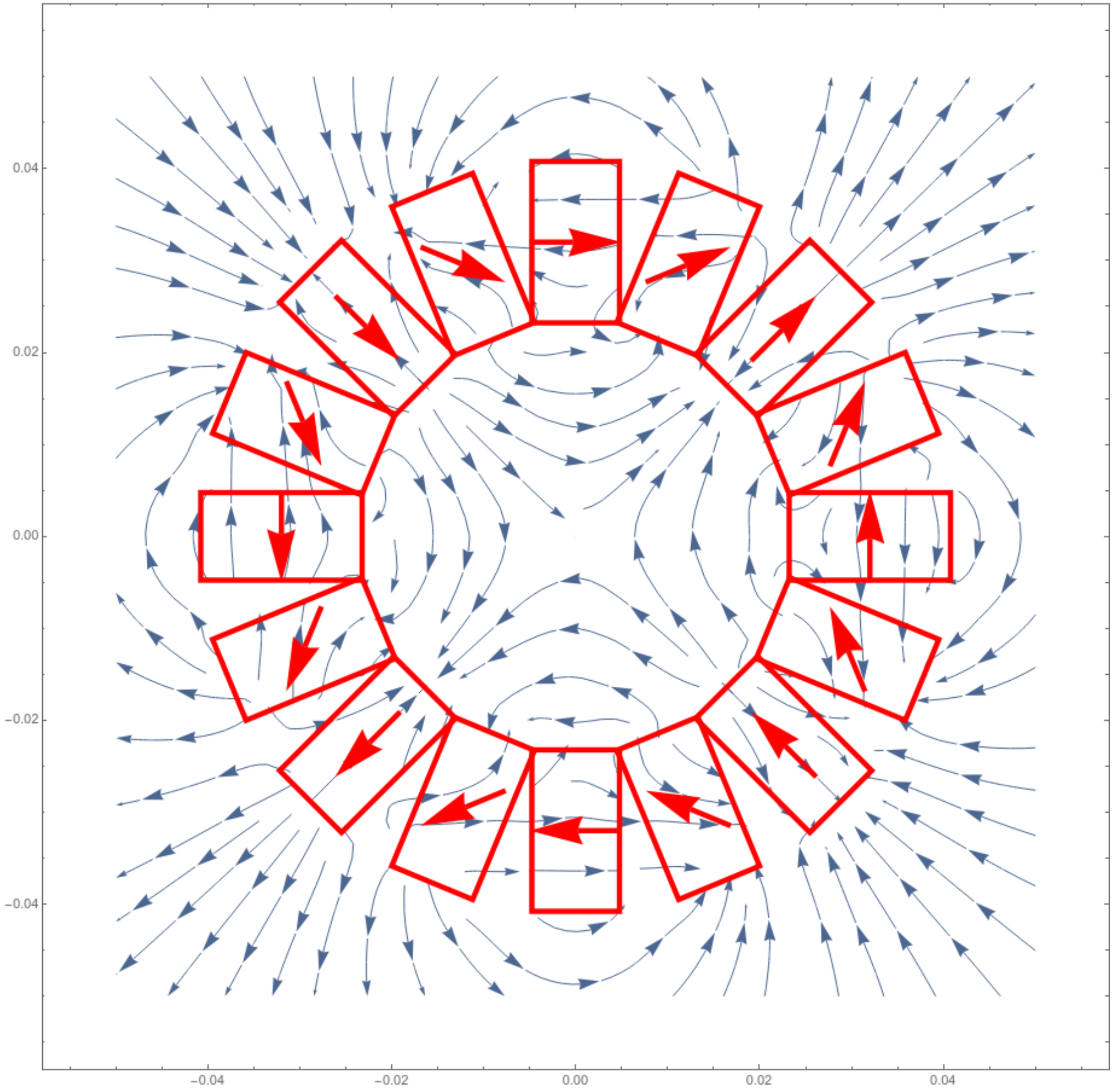
The SNS-BTF at ORNL

- Functional Duplicate of SNS Front End.
- Research into beam distribution, including full 6D measurements, and halo growth.



Permanent Magnet Configuration

- Halbach array quadrupoles in BTF FODO line.
- Allows full magnetic field model to be created.
- Current models use a perfect quadrupole.

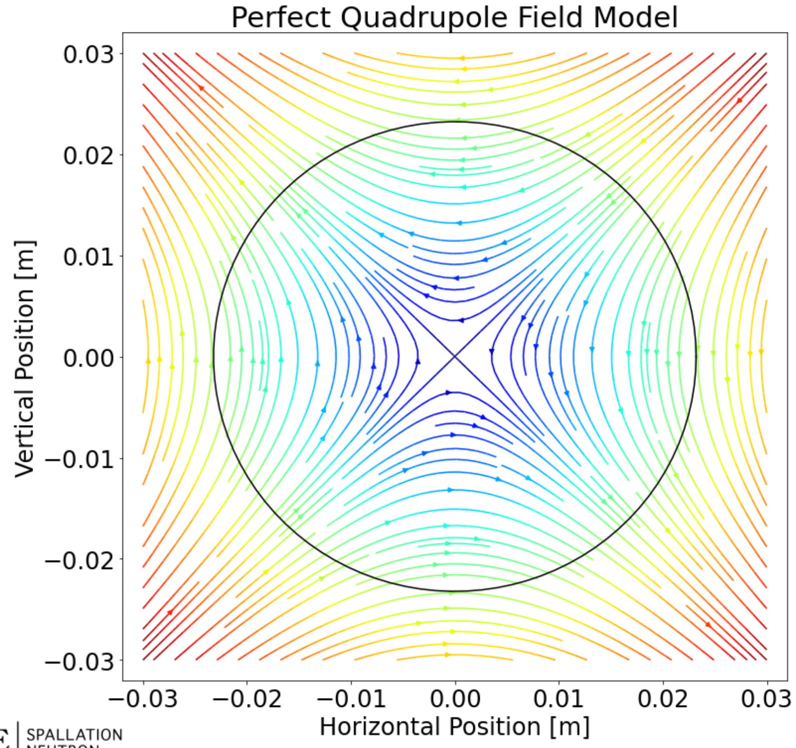


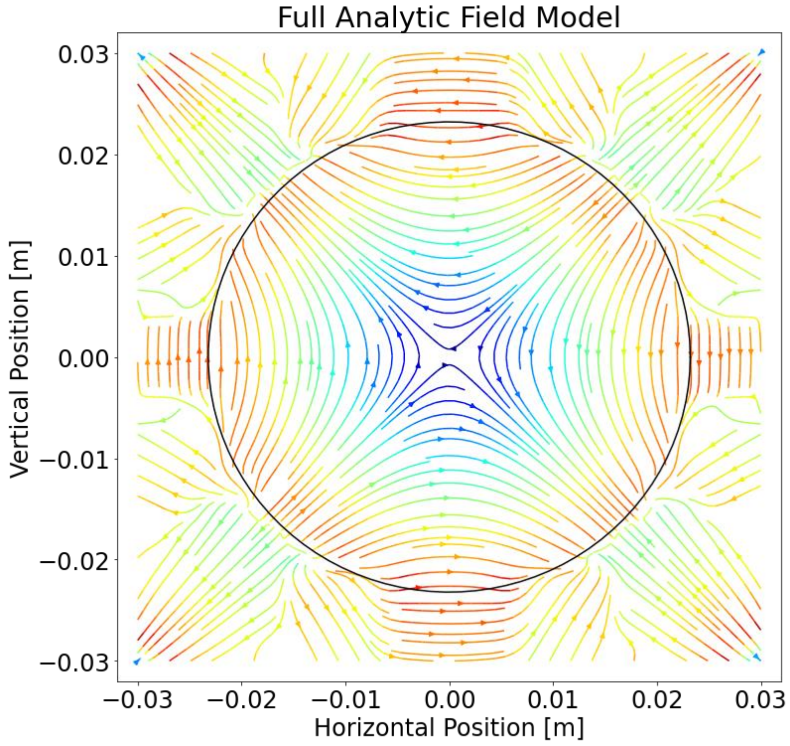
Purpose of Research into the Magnetic Models

- How accurate is the simplified model, specifically in the near aperture region?
- Is there a benefit to using the full model?

Magnet Model Field Diagram

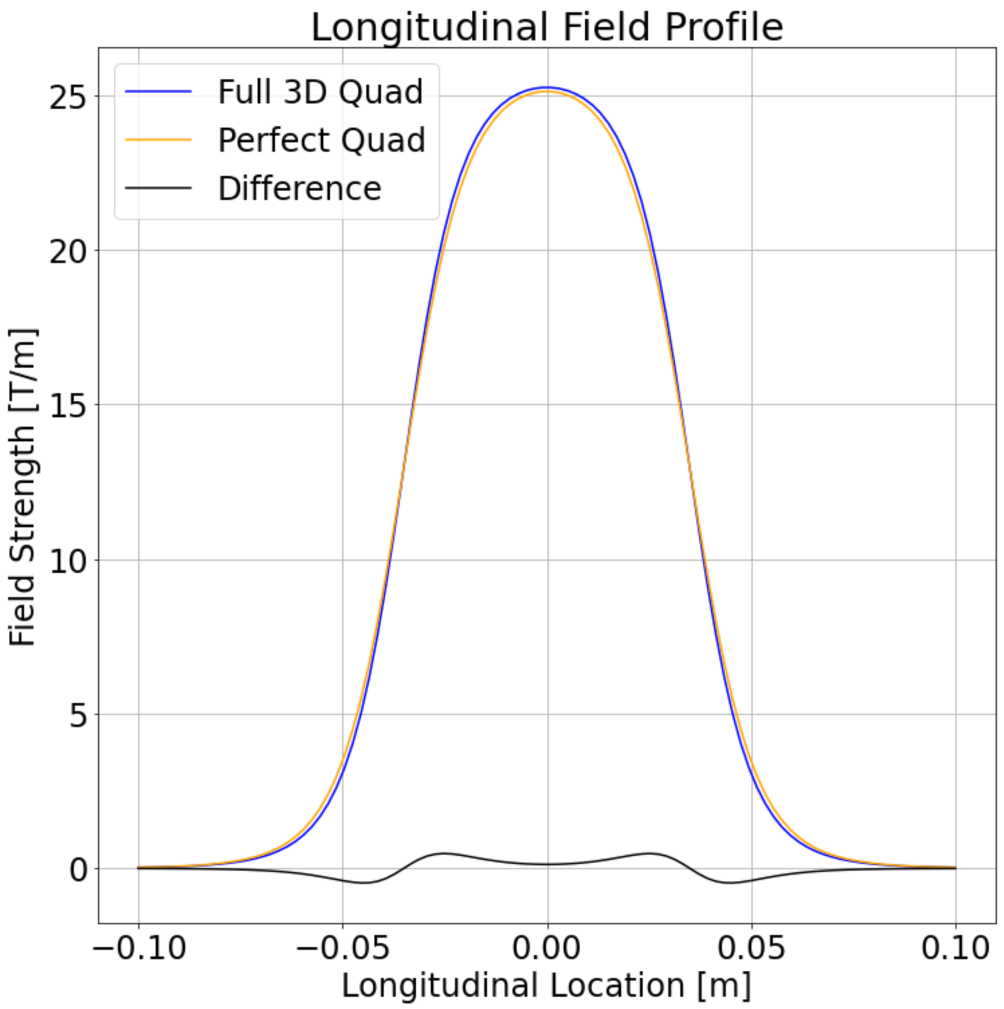
- Using the two magnet models the magnet field is determined at every position.
- The perfect quadrupole scales linearly in the transverse plain.





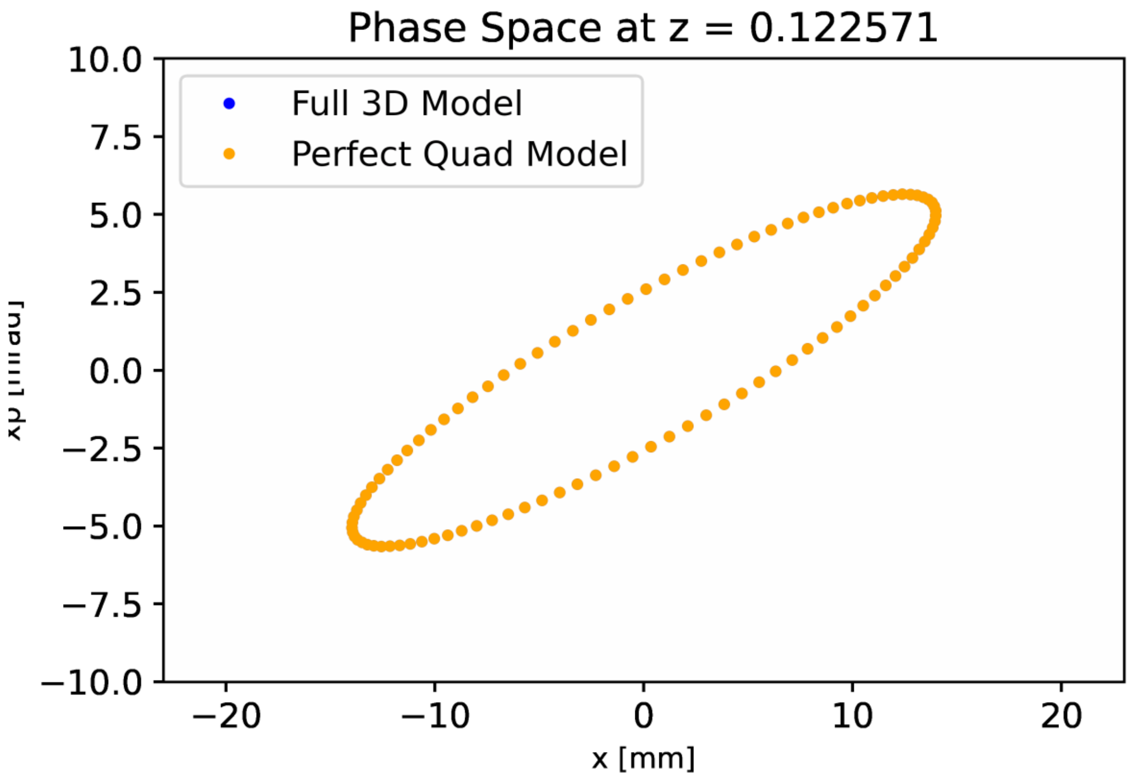
Integrated Field Strengths

- Scaling of Magnetic Field Strengths by scalar factor.
- Matched to BTF magnets integrated field value of 1.817 T.
- Virtual BTF FODO line created.



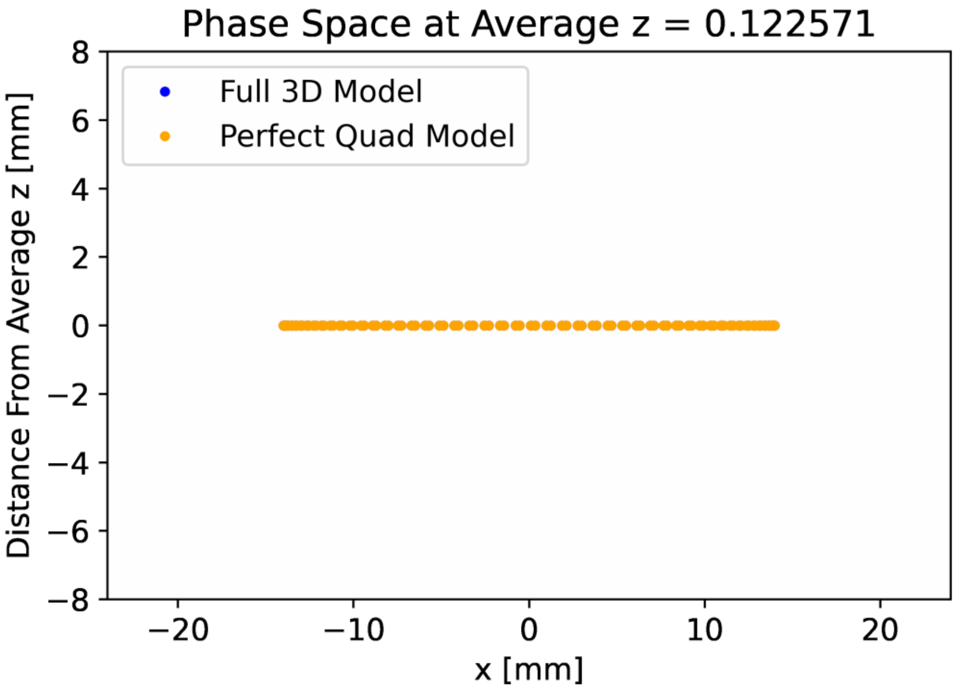
Initial Tracking of Particles

- Runge-Kutta r and p tracking of particles in x,x’ phase space.
- Phase spaces from transforming normalized circle using matched condition twiss parameters.
- Models hold similar phase spaces, though distortions in phase spaces appear.



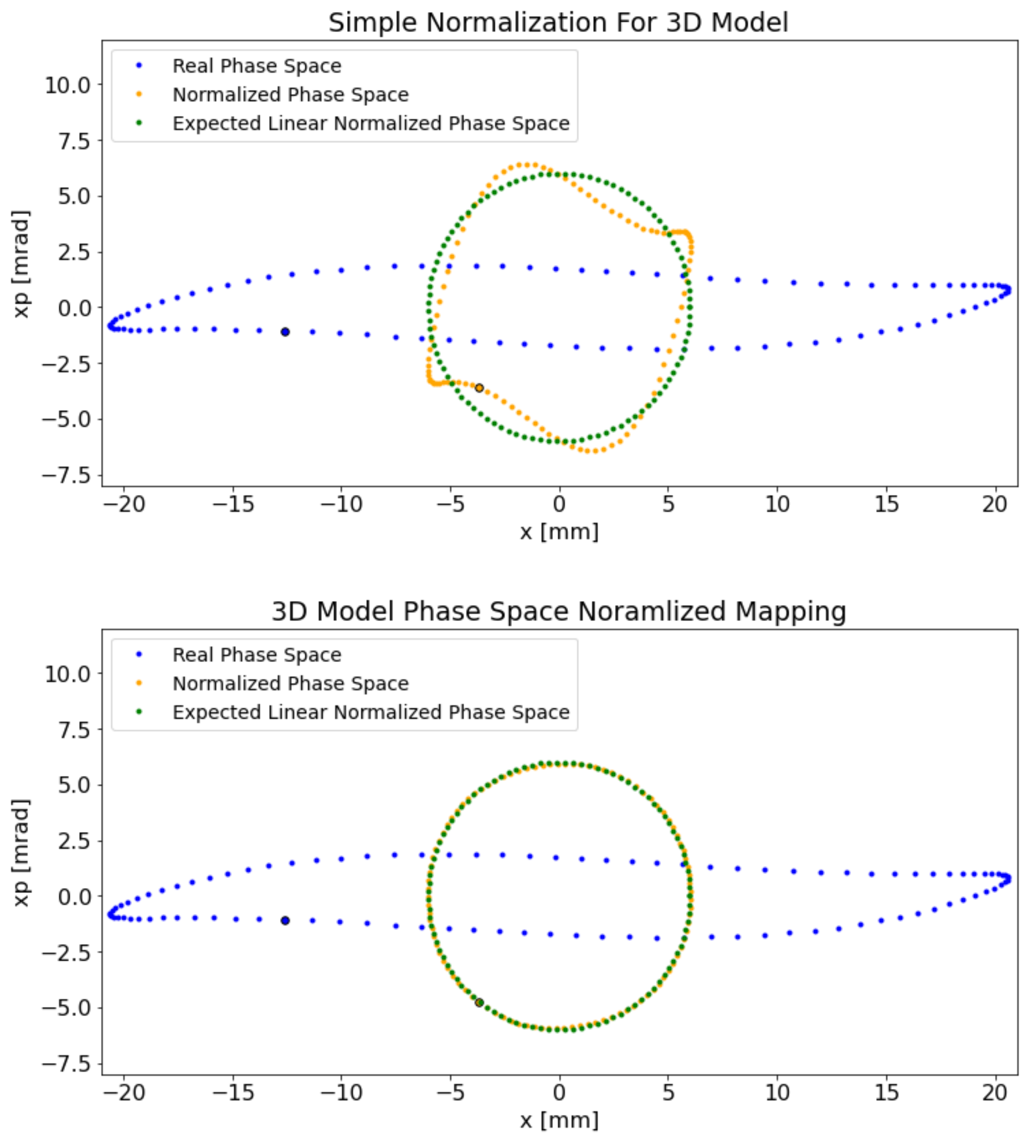
Why Are There Distortions in Phase Space?

- All particles have same longitudinal velocity.
- Each particles starting position causes there to be different path lengths.
- With r and p tracking particles are at different z positions at one time.



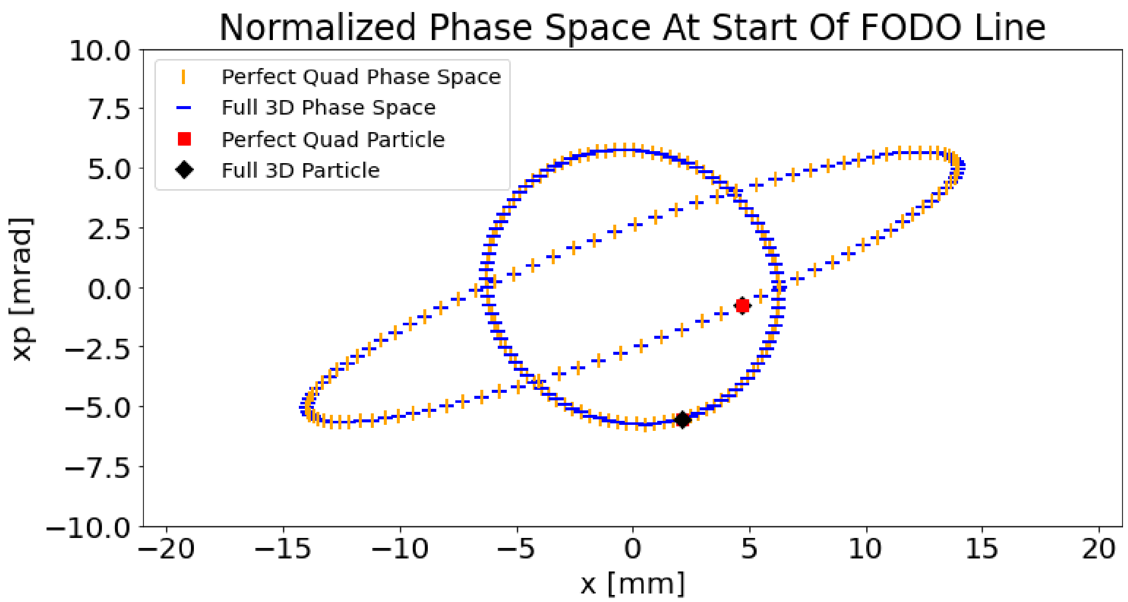
Compensating for Z lag in Normalized Space

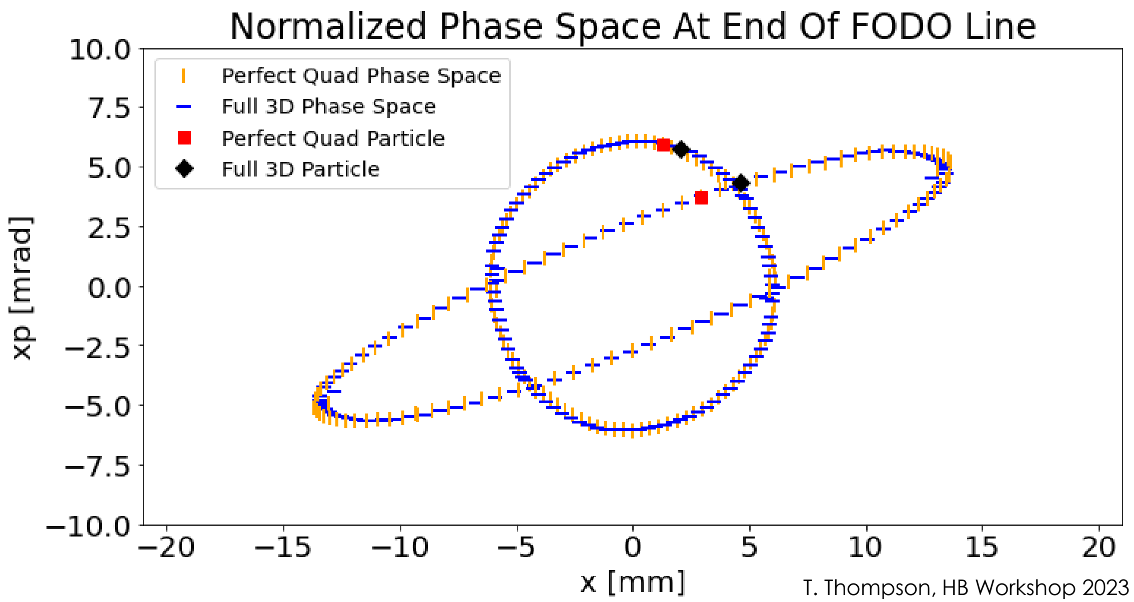
- Phase spaces can be normalized using twiss parameters.
- These twiss parameters depend on the z position of particles.
- Adjustment of twiss parameters for each particles allows correct normalization.



1D and 3D Models in Normalized Space

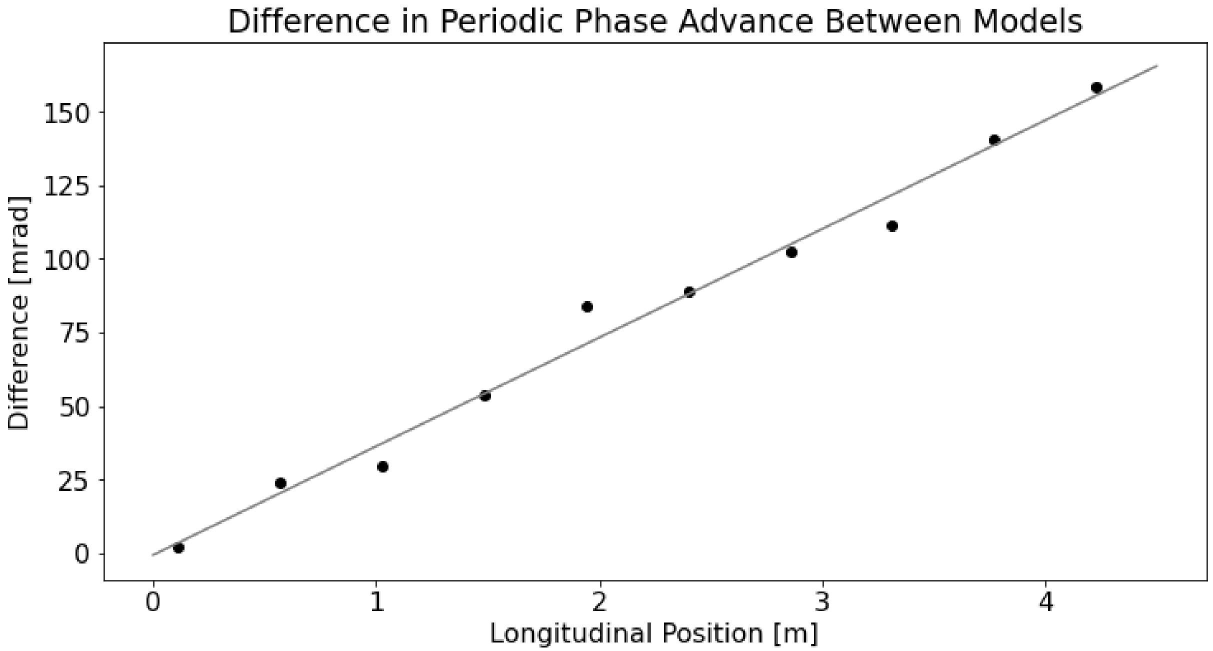
- They both normalize correctly along the length of the FODO line.
- Tracking a single particle reveals a phase advance difference.





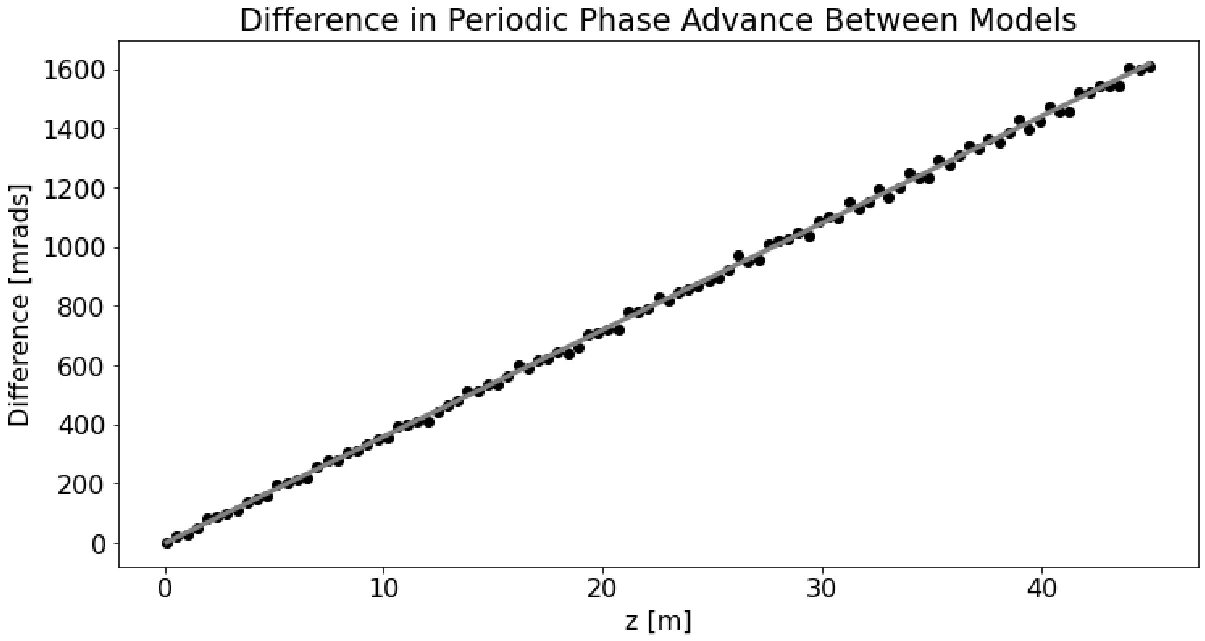
Quantifying The Phase Advance Difference

- Take the angle between a similar point in the normalized phase space.
- Repeat this along the FODO line to visualize the trend.
- Phase Advance Difference accrues by ~36 mrad/m (2.06 degrees/m)



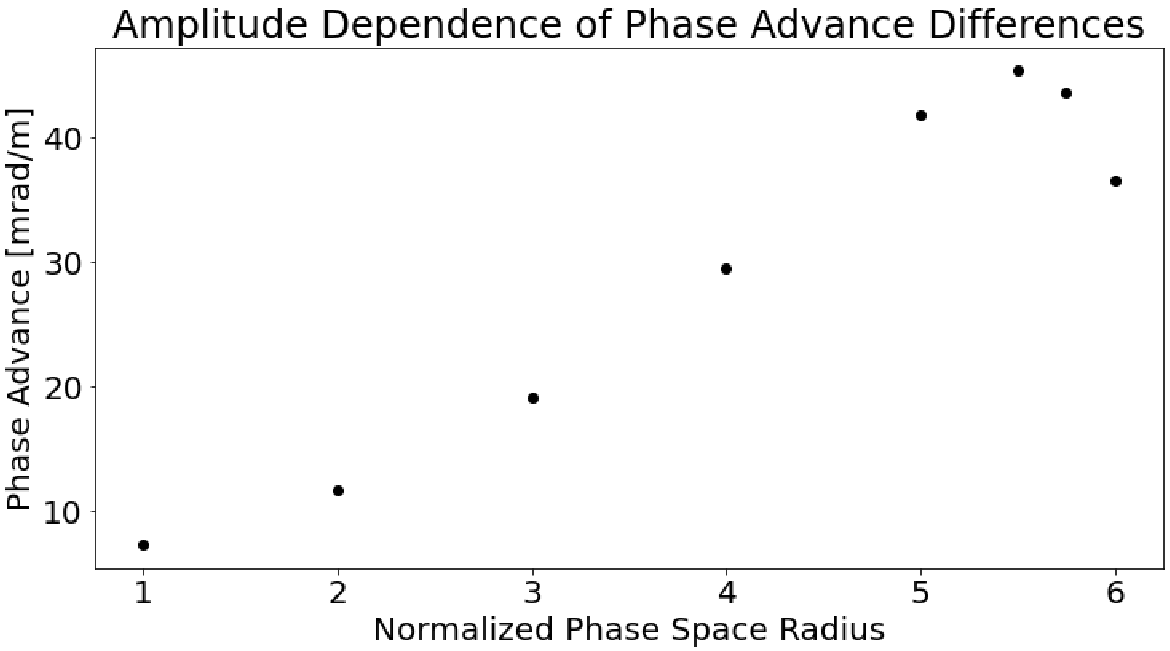
Phase Advance Difference Extended

- Extending the FODO line to ~10x the length.
- Repeat the same analysis process.
- The Phase Advance Difference Trend is consistently ~36 mrad/m.



Phase Advance Difference at Amplitudes

- Phase advance difference of ~36 mrad/m at aperture.
- In general, this decreases as maximum particle amplitude decreases.

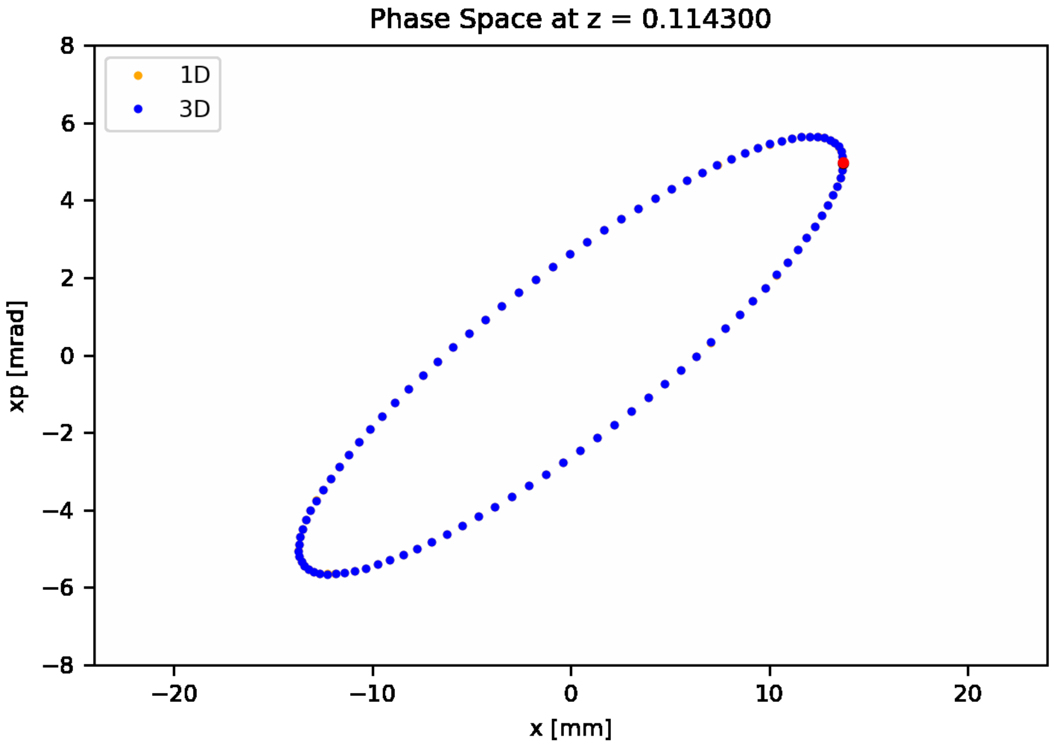


Conclusion

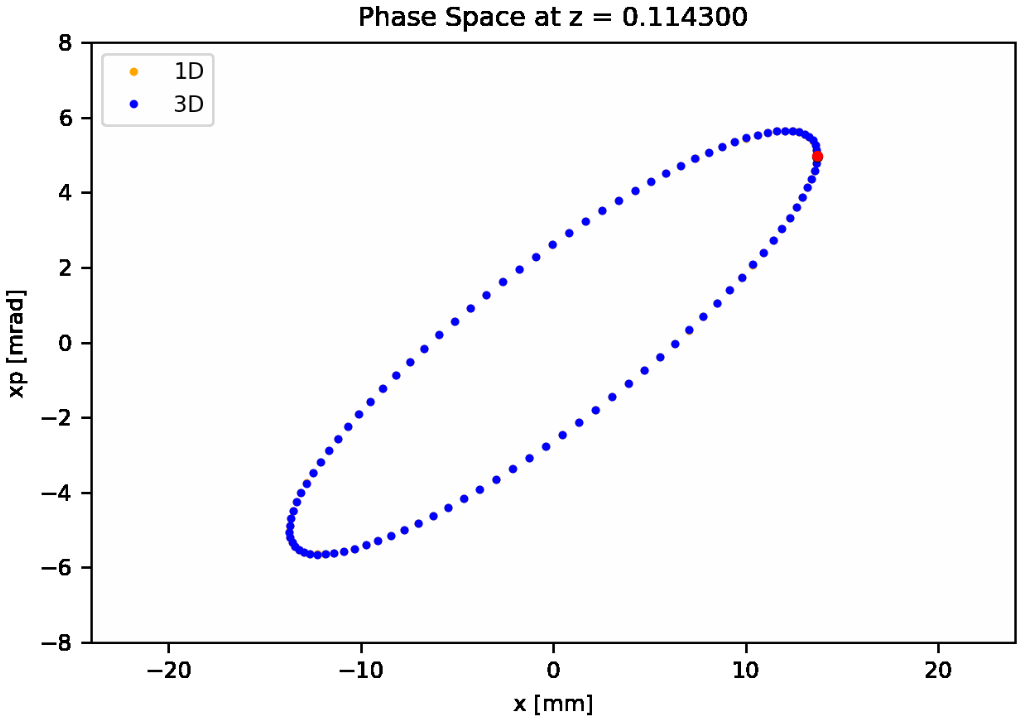
- The two models simulate near identical phase space ellipses.
- There is a difference in phase advance of ~36 mrad/m at aperture, that peaks at ~45 mrad/m slightly within aperture and decrease as particle amplitude decrease.

Extra Slides

Extended Beamline Simulation



Above: All particles at one time step but different z positions
Right: All particles near one z position but different time steps



3D Model Analytic Formula

```
Bp[x_, y_, z_, at_, bt_] := ({k, p, m, n} = {at + 2 x, at - 2 x, bt + 2 z, bt - 2 z};
  {s, t, g, q} = {Sqrt[p^2 + 4 y^2 + n^2], Sqrt[k^2 + 4 y^2 + m^2], Sqrt[k^2 + 4 y^2 + n^2], Sqrt[p^2 + 4 y^2 + m^2]};
  {Log[((q+m) (g-n))/((s-n) (t+m))],
   Sign[y] (ArcTan[2 s Abs[y], p n] + ArcTan[2 g Abs[y], k n] + ArcTan[2 q Abs[y], p m] + ArcTan[2 t Abs[y], k m]),
   Log[((q-p) (g+k))/((s-p) (t+k))]});

B[x_, y_, z_, M_] := M[[2]] (Bp[x, y, z, a, b] - Bp[x, h + y, z, a, b]) + M[[1]] {{0, -1, 0}, {1, 0, 0}, {0, 0, 1}}.(Bp[h/2 + y, a/2 - x, z, h, b] - Bp[h/2 + y, -a/2 - x, z, h, b]);

Br[x_, y_, z_, M_, dx_, dy_, ϕ_] := {{Cos[ϕ], -Sin[ϕ], 0}, {Sin[ϕ], Cos[ϕ], 0}, {0, 0, 1}}.B[x Cos[ϕ] + y Sin[ϕ] - dx, y Cos[ϕ] - x Sin[ϕ] - dy, z, M];

Bs[x_, y_, z_] := Sum[Br[x, y, z, {-Cos[i π/4], Sin[i π/4]}, 0, -r, i π/8], {i, 0, 15}];
```

```
itm = 0.0254;

r = 0.914 itm;

a = 0.375 itm;
b = 1.378 itm;
h = 0.691 itm;
```

For BTF accuracy, b (quad z length) is doubled representing BTF quads being a pancake of two quads

1D Model Analytic Formula

$$\mathrm{npancakes} = 2$$

$$\mathrm{inch2meter} = 0.0254$$

$$\mathrm{r_i} = 0.914 * \mathrm{inch2meter}$$

$$\mathrm{r_0} = 1.605 * \mathrm{inch2meter}$$

$$\mathrm{lq} = \mathrm{npancakes} * 1.378 * \mathrm{inch2meter}$$

$$\mathrm{v_1} = \frac{1}{\sqrt{1 + (\frac{z}{r_i})^2}}$$

$$\mathrm{v_2} = \frac{1}{\sqrt{1 + (\frac{z}{r_o})^2}}$$

$$\mathrm{func(z)} = \frac{1}{2}\left(1 - \frac{z}{8}\left(\frac{1}{r_i} + \frac{1}{r_o}\right) * v_1^2 * v_2^2 * \frac{v_1^2 + v_1 * v_2 + v_2^2 + 4 + \frac{8}{v_1/v_2}}{v_1 + v_2}\right)$$

$$\mathrm{PMQ\ Function} = \mathrm{func}(z - \frac{\mathrm{lq}}{2}) - \mathrm{func}(z + \frac{\mathrm{lq}}{2})$$

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